Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/7.3.6-Exponentials-of-inverse-hyperbolic-tangent-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1378]. This is test number [196].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (1378)	% 0. (0)
Mathematica	% 98.19 (1353)	% 1.81 (25)
Maple	% 79.83 (1100)	% 20.17 (278)
Maxima	% 37.37 (515)	% 62.63 (863)
Fricas	% 80.55 (1110)	% 19.45 (268)
Sympy	% 32.29 (445)	% 67.71 (933)
Giac	% 52.83 (728)	% 47.17 (650)

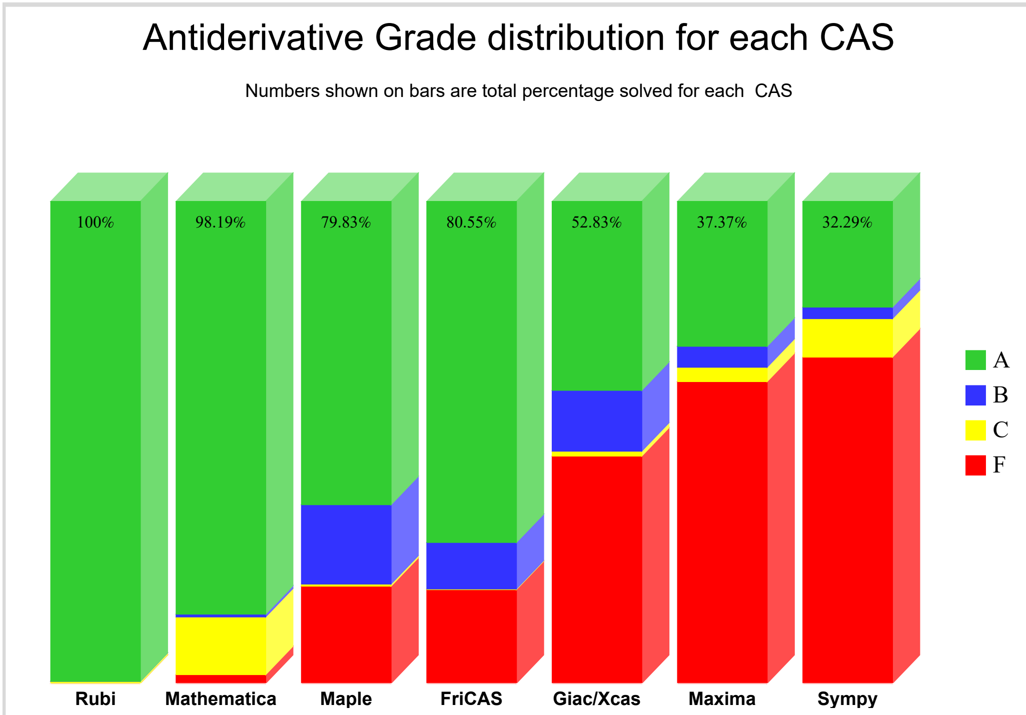
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

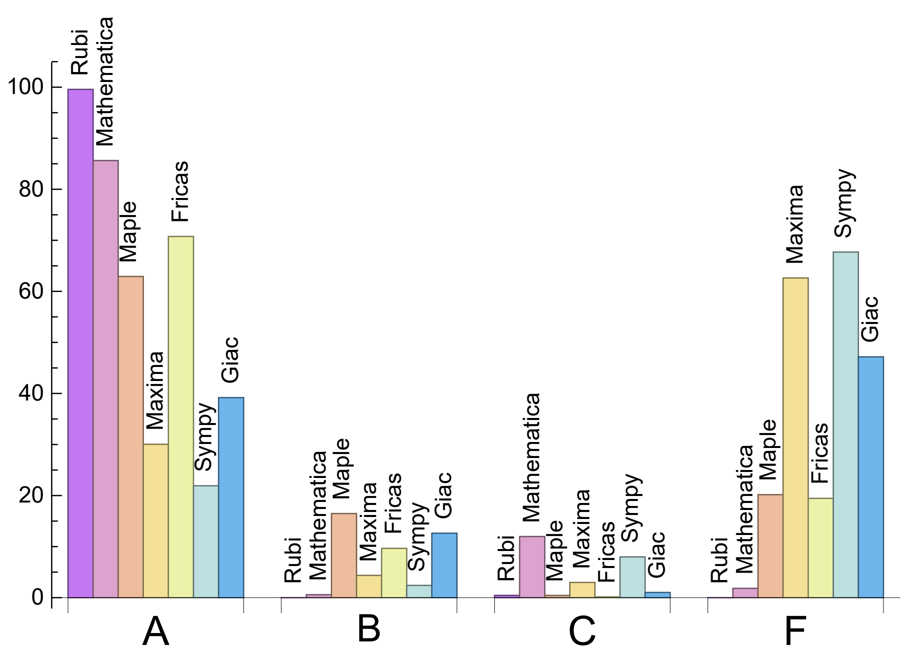
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.56	0.	0.44	0.
Mathematica	85.63	0.58	11.97	1.81
Maple	62.92	16.47	0.44	20.17
Maxima	30.04	4.35	2.98	62.63
Fricas	70.75	9.65	0.15	19.45
Sympy	21.92	2.39	7.98	67.71
Giac	39.19	12.63	1.02	47.17

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	118.01	1.	99.	1.
Mathematica	0.14	75.45	0.75	70.	0.74
Maple	0.07	151.34	1.43	90.5	1.03
Maxima	1.14	145.6	1.95	89.	1.4
Fricas	2.11	411.44	3.56	273.	3.07
Sympy	8.64	199.05	2.27	82.	1.19
Giac	1.67	173.51	1.92	109.	1.48

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {373, 620, 621, 791, 792, 795, 796, 1313, 1316, 1317, 1323, 1324, 1331, 1332, 1345, 1346, 1347, 1352, 1353, 1354}

Mathematica {6, 7, 8, 9, 10, 22, 23, 24, 25, 38, 39, 40, 41, 42, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 142, 144, 145, 147, 149, 152, 158, 159, 160, 162, 163, 164, 165, 166, 178, 180, 181, 182, 183, 184, 185, 186, 198, 199, 200, 202, 203, 204, 205, 206, 217, 218, 219, 220, 221, 226, 227, 228, 229, 231, 232, 233, 234, 243, 244, 245, 247, 248, 249, 250, 251, 258, 259, 260, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 291, 295, 296, 297, 298, 299, 300, 301, 306, 307, 308, 309, 310, 311, 312, 313, 316, 317, 318, 319, 320, 321, 322, 323, 324, 327, 328, 329, 330, 331, 336, 337, 338, 339, 340, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 363, 364, 365, 366, 387, 388, 389, 391, 392, 405, 411, 415, 416, 417, 418, 431, 432, 434, 435, 436, 448, 449, 450, 451, 452, 457, 458, 459, 466, 467, 468, 469, 470, 475, 477, 478, 485, 486, 487, 488, 494, 495, 496, 499, 502, 503, 504, 505, 510, 513, 514, 523, 524, 525, 526, 527, 528, 529, 530, 535, 539, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 573, 574, 575, 591, 592, 593, 594, 595, 596, 597, 598, 608, 609, 610, 611, 612, 620, 621, 623, 628, 629, 630, 645, 646, 662, 663, 664, 678, 679, 689, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 707, 717, 723, 724, 725, 726, 727, 728, 729, 730, 731, 734, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 791, 792, 796, 799, 801, 802, 803, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 822, 823, 824, 825, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 847, 848, 849, 850, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 873, 874, 876, 879, 883, 884, 885, 886, 891, 918, 919, 954, 1080, 1081, 1082, 1083, 1084, 1085, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1111, 1112, 1113, 1114, 1115, 1119, 1120, 1121, 1122, 1131, 1132, 1133, 1134, 1135, 1140, 1145, 1146, 1147, 1148, 1149, 1152, 1153, 1165, 1174, 1175, 1176, 1177, 1179, 1180, 1181, 1192, 1193, 1194, 1195, 1196, 1199, 1200, 1207, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1274, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1300, 1301, 1302, 1304, 1311, 1313, 1315, 1316, 1317, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1331, 1332, 1342, 1343, 1344, 1345, 1346, 1347, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1358, 1367}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-i

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

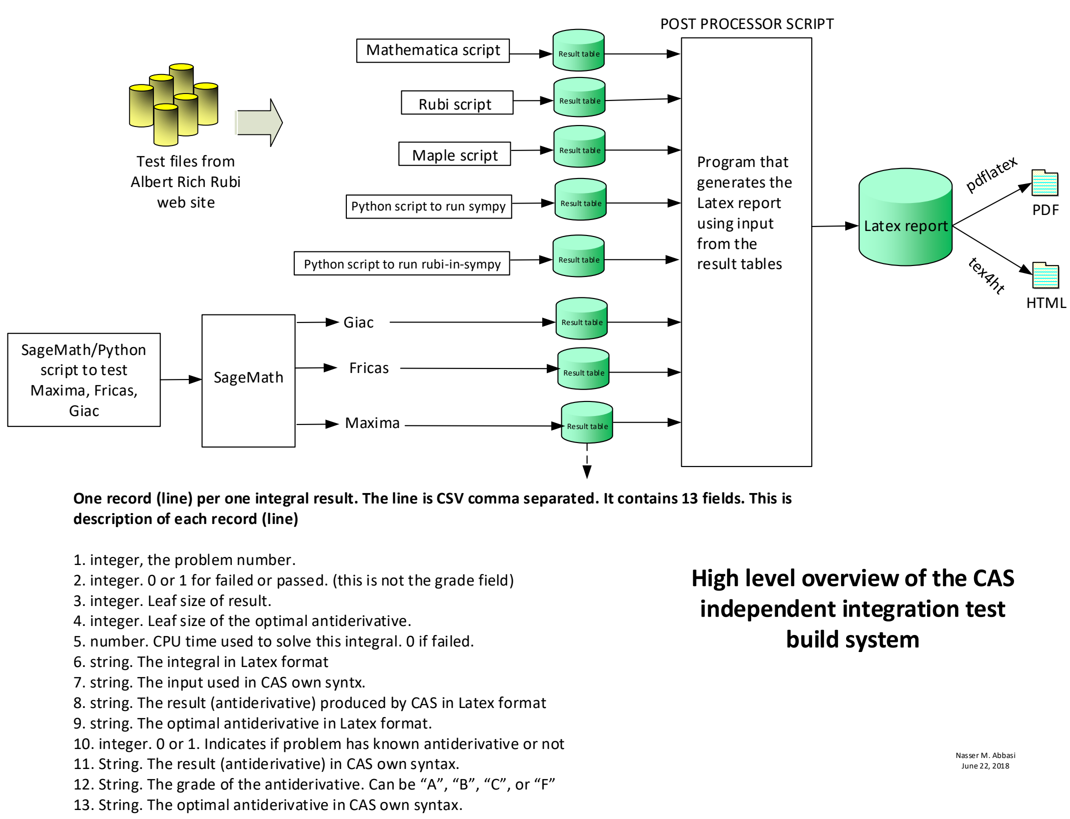
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 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1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378 }

B grade: { }

C grade: { 172, 620, 791, 792, 795, 1332 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 63, 84, 104, 130, 141, 143, 146, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 252, 253, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 270, 271, 272, 273, 274, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 300, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 341, 342, 343, 344, 346, 348, 349, 351, 352, 353, 354, 356, 357, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 433, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 556, 557, 558, 559, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 619, 621, 622, 623, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 802, 804, 805, 807, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 869, 870, 871, 873, 874, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1285, 1286, 1287, 1288, 1293, 1294, 1295, 1296, 1299, 1303, 1304, 1305, 1306, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378 }

B grade: { 291, 299, 301, 499, 620, 868, 1185, 1358 }

C grade: { 61, 62, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 128, 129, 131, 132, 133, 135, 136, 137, 138, 139, 140, 142, 144, 145, 147, 172, 182, 217, 218, 219, 233, 234, 248, 250, 251, 260, 267, 268, 269, 275, 276, 277, 339, 345, 347, 350, 355, 360, 418, 432, 434, 435, 436, 470, 510, 523, 524, 525, 526, 527, 528, 529, 530, 551, 552, 553, 554, 555, 560, 561, 562, 573, 574, 628, 629, 630, 645, 646, 662, 663, 664, 678, 679, 801, 803, 806, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 872, 1053, 1133, 1134, 1135, 1145, 1146, 1232, 1252, 1281, 1282, 1283, 1284, 1289, 1290, 1291, 1292, 1297, 1298, 1300, 1301, 1302 }

F grade: { 60, 70, 80, 90, 100, 110, 120, 127, 134, 148, 447, 484, 617, 618, 624, 625, 626, 627, 800, 875, 1307, 1356, 1357, 1359, 1360}

2.1.3 Maple

A grade: { 1, 2, 3, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 43, 44, 45, 46, 47, 48, 49, 50, 142, 144, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 179, 181, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 202, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 217, 218, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 352, 354, 355, 356, 357, 358, 359, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 379, 381, 383, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 438, 444, 445, 446, 448, 449, 450, 451, 452, 453, 454, 455, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 469, 470, 471, 472, 473, 475, 476, 477, 478, 479, 480, 481, 482, 483, 485, 489, 494, 495, 496, 497, 498, 499, 500, 501, 507, 510, 511, 512, 513, 514, 515, 516, 518, 519, 520, 521, 527, 528, 529, 530, 531, 532, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 549, 550, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 598, 599, 600, 608, 609, 610, 611, 612, 613, 614, 615, 616, 628, 629, 630, 631, 632, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 686, 687, 688, 689, 690, 691, 692, 693, 694, 699, 700, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 727, 728, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 780, 781, 782, 783, 784, 785, 786, 787, 788, 821, 827, 828, 829, 830, 831, 832, 833, 834, 853, 854, 855, 856, 857, 858, 859, 872, 885, 886, 887, 888, 889, 891, 892, 893, 894, 895, 896, 900, 901, 902, 904, 905, 906, 910, 911, 912, 913, 914, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 994, 998, 999, 1000, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1080, 1081, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1118, 1119, 1121, 1122, 1123, 1124, 1127, 1137, 1138, 1139, 1140, 1143, 1147, 1148, 1149, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1182, 1183, 1184, 1185, 1186, 1188, 1189, 1190, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1239, 1240, 1245, 1248, 1249, 1250, 1251, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1285, 1286, 1287, 1288, 1293, 1294, 1295, 1296, 1304, 1315, 1320, 1321, 1322, 1325, 1326, 1343, 1344, 1348, 1349, 1350, 1351, 1355, 1363, 1364, 1365, 1366, 1367, 1370, 1371, 1373, 1374, 1375, 1376, 1377, 1378 }

B grade: { 4, 6, 34, 35, 36, 37, 38, 39, 40, 41, 42, 51, 52, 53, 54, 55, 56, 57, 58, 59, 174, 178, 180, 182, 200, 201, 212, 219, 220, 341, 350, 351, 353, 360, 361, 362, 378, 380, 382, 384, 385, 386, 468, 486, 487, 488, 490, 491, 492, 502, 503, 504, 505, 506, 508, 509, 517, 522, 523, 524, 525, 526, 533, 534, 545, 546, 547, 548, 551, 552, 553, 575, 576, 577, 601, 602, 603, 604, 605, 606, 607, 633, 634, 635, 652, 666, 667, 668, 669, 680, 681, 682, 683, 684, 685, 695, 696, 697, 698, 701, 702, 703, 704, 723, 724, 725, 726, 729, 730, 731, 751, 752, 753, 754, 755, 775, 776, 777, 778, 779, 818, 819, 820, 822, 823, 824, 825, 826, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 873, 874, 883, 884, 890, 897, 898, 899, 903, 907, 908, 909, 915, 916, 917, 987, 988, 992, 993, 1078, 1079, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1117, 1120, 1125, 1126, 1141, 1142, 1144, 1145, 1146, 1150, 1151, 1187, 1238, 1241, 1242, 1243, 1244, 1246, 1247, 1368, 1369, 1372 }

C grade: { 141, 143, 146, 995, 996, 997 }

F grade: { 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 278, 279, 280, 281, 282, 283, 284, 285, 387, 411, 437, 439, 440, 441, 442, 443, 447, 456, 474, 484, 493, 564, 591, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 875, 876, 877, 878, 879, 880, 881, 882, 989, 990, 991, 1001, 1002, 1003, 1004, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1191, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1252, 1253, 1279, 1280, 1281, 1282, 1283, 1284, 1289, 1290, 1291, 1292, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1316, 1317, 1318, 1319, 1323, 1324, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1345, 1346, 1347, 1352, 1353, 1354, 1356, 1357, 1358, 1359, 1360, 1361, 1362 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 53, 54, 158, 159, 160, 161, 167, 168, 169, 170, 171, 172, 173, 175, 176, 179, 181, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 200, 207, 208, 209, 210, 211, 213, 214, 215, 219, 226, 227, 228, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 252, 253, 254, 255, 256, 257, 270, 271, 272, 273, 274, 286, 287, 288, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 375, 376, 379, 380, 388, 389, 390, 393, 394, 395, 396, 412, 413, 414, 428, 429, 430, 431, 438, 448, 449, 450, 451, 457, 458, 459, 460, 461, 462, 463, 464, 465, 475, 476, 477, 478, 479, 480, 481, 482, 483, 489, 494, 495, 496, 497, 498, 499, 500, 501, 507, 631, 636, 637, 638, 639, 640, 641, 642, 643, 644, 653, 654, 655, 656, 657, 658, 659, 660, 661, 670, 671, 672, 673, 674, 675, 676, 677, 826, 827, 828, 829, 830, 831, 832, 833, 834, 844, 845, 846, 851, 852, 853, 854, 855, 856, 857, 858, 859, 863, 884, 885, 886, 893, 895, 904, 906, 916, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 954, 955, 956, 998, 999, 1000, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1126, 1127, 1137, 1138, 1139, 1140, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1182, 1183, 1184, 1186, 1187, 1189, 1190, 1192, 1193, 1194, 1195, 1201, 1202, 1203, 1204, 1211, 1212, 1213, 1214, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1254, 1255, 1274, 1275, 1276, 1277, 1278, 1304, 1315, 1363, 1364, 1365, 1366, 1367, 1370, 1371, 1376, 1377

B grade: { 6, 174, 178, 180, 183, 184, 185, 186, 194, 201, 212, 243, 244, 245, 246, 289, 294, 373, 374, 377, 378, 466, 467, 468, 469, 628, 629, 630, 645, 646, 647, 648, 843, 862, 869, 870, 871, 872, 883, 887, 888, 889, 890, 891, 1035, 1125, 1141, 1142, 1150, 1151, 1152, 1166, 1167, 1168, 1185, 1188, 1368, 1369, 1372, 1378 }

C grade: { 51, 52, 217, 218, 690, 691, 709, 719, 737, 741, 742, 743, 744, 745, 746, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 860, 861, 1256, 1257 }

F grade: { 39, 40, 41, 42, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 162, 163, 164, 165, 166, 177, 182, 197, 202, 203, 204, 205, 206, 216, 220, 221, 222, 223, 224, 225, 231, 232, 233, 234, 247, 248, 249, 250, 251, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 381, 382, 383, 384, 385, 386, 387, 391, 392, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 452, 453, 454, 455, 456, 470, 471, 472, 473, 474, 484, 485, 486, 487, 488, 490, 491, 492, 493, 502, 503, 504, 505, 506, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 632, 633, 634, 635, 649, 650, 651, 652, 662, 663, 664, 665, 666, 667, 668, 669, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 710, 711, 712, 713, 714, 715, 716, 717, 718, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 738, 739, 740, 747, 748, 749, 750, 751, 752, 753, 754, 755, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 818, 819, 820, 821, 822, 823, 824, 825, 835, 836, 837, 838, 839, 840, 841, 842, 847, 848, 849, 850, 864, 865, 866, 867, 868, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 892, 894, 896, 897, 898, 899, 900, 901, 902, 903, 905, 907, 908, 909, 910, 911, 912, 913, 914, 915, 917, 918, 919, 949, 950, 951, 952, 953, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1191, 1196, 1197, 1198, 1199, 1200, 1205, 1206, 1207, 1208, 1209, 1210, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1373, 1374, 1375 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 66, 67, 68, 69, 77, 78, 79, 86, 87, 88, 89, 97, 98, 99, 107, 108, 109, 117, 118, 119, 121, 122, 123, 124, 125, 126, 128, 129, 131, 133, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 180, 181, 182, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 200, 202, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 370, 371, 372, 373, 375, 376, 379, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 438, 444, 445, 446, 448, 449, 450, 451, 452, 453, 454, 455, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 475, 476, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 535, 536, 537, 538, 539, 540, 541, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 563, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 583, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 710, 711, 714, 715, 716, 717, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 737, 738, 741, 742, 743, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 761, 762, 763, 764, 765, 766, 767, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 785, 786, 787, 788, 818, 819, 820, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 840, 841, 842, 843, 844, 845, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 865, 866, 867, 869, 870, 872, 873, 874, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 900, 901, 903, 904, 905, 906, 907, 908, 912, 916, 917, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 949, 950, 951, 954, 955, 956, 957, 958, 959, 962, 963, 964, 965, 970, 971, 979, 980, 981, 982, 986, 994, 998, 999, 1000, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1072, 1073, 1074, 1075, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1127, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1170, 1171, 1172, 1182, 1183, 1184, 1186, 1187, 1189, 1190, 1192, 1193, 1194, 1195, 1196, 1197, 1201, 1202, 1203, 1204, 1207, 1208, 1209, 1210, 1212, 1213, 1214, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1254, 1255, 1256, 1257, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1276, 1277, 1278, 1285, 1286, 1287, 1288, 1315, 1320, 1321, 1322, 1325, 1326, 1343, 1344, 1348, 1349, 1350, 1351, 1355, 1363, 1364, 1365, 1366, 1367, 1370, 1371 }

B grade: { 6, 55, 61, 62, 63, 64, 65, 71, 72, 73, 74, 75, 76, 81, 82, 83, 84, 85, 91, 92, 93, 94, 95, 96, 101, 102, 103, 104, 105, 106, 111, 112, 113, 114, 115, 116, 130, 132, 135, 136, 137, 138, 139, 140, 163, 174, 183, 184, 194, 201, 212, 242, 340, 368, 369, 374, 377, 378, 381, 382, 383, 384, 385, 386, 690, 691, 718, 719, 744, 745, 768, 769, 821, 822, 839, 846, 847, 864, 868, 871, 899, 902, 909, 910, 911, 913, 914, 915, 918, 919, 941, 942, 943, 944, 945, 946, 947, 948, 952, 953, 960, 961, 992, 993, 1035, 1071, 1076, 1125, 1126, 1152, 1169, 1185, 1188, 1198, 1199, 1200, 1205, 1206, 1211, 1235, 1258, 1275, 1281, 1282, 1283, 1284, 1368, 1369, 1372, 1373, 1374, 1377, 1378 }

C grade: { 367, 380 }

F grade: { 60, 70, 80, 90, 100, 110, 120, 127, 134, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 278, 279, 280, 281, 282, 283, 284, 285, 387, 411, 437, 439, 440, 441, 442, 443, 447, 456, 474, 484, 493, 515, 533, 534, 542, 543, 562, 564, 582, 584, 591, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 692, 693, 694, 709, 712, 713, 720, 721, 722, 736, 739, 740, 759, 760, 783, 784, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 875, 876, 877, 878, 879, 880, 881, 882, 966, 967, 968, 969, 972, 973, 974, 975, 976, 977, 978, 983, 984, 985, 987, 988, 989, 990, 991, 995, 996, 997, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1160, 1161, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1191, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1252, 1253, 1266, 1267, 1279, 1280, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1316, 1317, 1318, 1319, 1323, 1324, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1345, 1346, 1347, 1352, 1353, 1354, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1375, 1376 }

2.1.6 Sympy

A grade: { 1, 3, 5, 11, 12, 13, 14, 15, 16, 17, 18, 26, 27, 28, 29, 30, 31, 32, 33, 43, 44, 45, 46, 47, 48, 49, 50, 158, 159, 160, 161, 167, 169, 170, 171, 172, 173, 175, 176, 178, 179, 180, 181, 187, 188, 189, 190, 191, 192, 193, 195, 196, 201, 207, 208, 209, 210, 211, 213, 214, 215, 235, 236, 237, 238, 239, 240, 241, 242, 262, 263, 264, 265, 266, 267, 268, 287, 289, 290, 291, 295, 297, 298, 306, 308, 309, 316, 318, 319, 363, 364, 365, 366, 367, 368, 393, 394, 395, 396, 397, 419, 420, 421, 422, 423, 448, 449, 450, 451, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 480, 481, 482, 483, 489, 494, 495, 496, 497, 498, 500, 501, 507, 577, 603, 628, 629, 630, 631, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 653, 654, 655, 656, 657, 658, 659, 660, 661, 670, 671, 672, 673, 674, 675, 676, 677, 826, 827, 828, 829, 830, 851, 852, 853, 854, 855, 883, 884, 885, 886, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 992, 993, 994, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1086, 1088, 1098, 1100, 1125, 1126, 1127, 1137, 1139, 1140, 1186, 1189, 1190, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1370, 1371 }

B grade: { 6, 143, 168, 174, 194, 212, 293, 398, 399, 400, 401, 424, 425, 426, 499, 831, 832, 833, 834, 856, 857, 858, 859, 995, 1035, 1182, 1183, 1184, 1185, 1187, 1188, 1369, 1372 }

C grade: { 2, 4, 7, 8, 9, 10, 144, 146, 286, 288, 292, 294, 296, 299, 300, 301, 302, 303, 304, 305, 307, 310, 311, 312, 313, 314, 315, 317, 320, 321, 322, 323, 324, 325, 326, 456, 518, 519, 520, 521, 662, 663, 664, 665, 679, 680, 724, 725, 726, 803, 804, 806, 987, 988, 996, 997, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1087, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1099, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1131, 1132, 1136, 1138, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1192, 1193, 1194, 1195, 1245, 1246, 1253, 1255, 1256 }

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2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 48, 49, 50, 51, 52, 53, 54, 158, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 180, 181, 182, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 200, 201, 203, 204, 207, 208, 209, 211, 213, 214, 215, 217, 218, 219, 220, 222, 226, 227, 228, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 286, 287, 288, 289, 290, 291, 293, 295, 296, 297, 298, 299, 306, 307, 308, 309, 310, 316, 317, 318, 319, 320, 321, 327, 328, 329, 330, 331, 332, 345, 346, 347, 354, 355, 356, 357, 358, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 385, 386, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 444, 451, 452, 453, 454, 455, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 469, 470, 471, 472, 473, 475, 476, 477, 478, 479, 480, 481, 482, 483, 487, 488, 489, 490, 491, 492, 494, 495, 496, 499, 500, 501, 503, 504, 505, 506, 507, 524, 525, 526, 573, 574, 575, 632, 636, 637, 638, 639, 640, 641, 642, 643, 644, 648, 649, 650, 653, 654, 655, 656, 657, 658, 659, 660, 661, 666, 670, 671, 674, 675, 676, 677, 681, 682, 683, 695, 696, 697, 698, 699, 723, 724, 725, 726, 727, 747, 748, 749, 750, 751, 753, 771, 772, 773, 774, 775, 777, 818, 819, 820, 821, 822, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 843, 844, 845, 846, 847, 860, 861, 862, 863, 864, 868, 869, 870, 871, 872, 873, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 994, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1086, 1087, 1088, 1089, 1090, 1091, 1098, 1099, 1100, 1101, 1102, 1103, 1107, 1116, 1117, 1121, 1122, 1137, 1138, 1139, 1140, 1141, 1147, 1148, 1149, 1150, 1151, 1153, 1182, 1183, 1184, 1186, 1187, 1188, 1189, 1190, 1192, 1193, 1194, 1195, 1196, 1228, 1229, 1230, 1232, 1233, 1234, 1235, 1239, 1240, 1254, 1255, 1256, 1257, 1259, 1370 }

B grade: { 6, 7, 8, 9, 10, 22, 23, 24, 25, 38, 39, 40, 41, 42, 45, 46, 47, 55, 56, 57, 58, 59, 164, 165, 174, 184, 185, 194, 210, 212, 235, 236, 292, 294, 300, 301, 302, 303, 304, 305, 311, 312, 313, 314, 315, 322, 323, 324, 325, 326, 333, 334, 335, 348, 349, 350, 351, 352, 353, 359, 360, 361, 362, 384, 448, 449, 450, 466, 468, 485, 486, 497, 498, 502, 522, 523, 549, 550, 551, 552, 553, 563, 576, 605, 606, 607, 628, 629, 630, 631, 645, 646, 647, 662, 663, 664, 665, 672, 673, 678, 679, 680, 752, 754, 755, 776, 778, 779, 823, 824, 825, 840, 841, 842, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 865, 866, 867, 874, 893, 894, 895, 992, 993, 1035, 1083, 1084, 1085, 1092, 1093, 1094, 1095, 1096, 1097, 1104, 1105, 1106, 1118, 1119, 1142, 1143, 1144, 1145, 1146, 1152, 1185, 1231, 1236, 1237, 1238, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1258, 1368, 1369, 1371, 1372, 1378 }

C grade: { 202, 205, 221, 224, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817 }

F grade: { 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 163, 166, 167, 177, 183, 186, 187, 197, 206, 216, 223, 225, 278, 279, 280, 281, 282, 283, 284, 285, 336, 337, 338, 339, 340, 341, 342, 343, 344, 387, 411, 437, 438, 439, 440, 441, 442, 443, 445, 446, 447, 456, 474, 484, 493, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 554, 555, 556, 557, 558, 559, 560, 561, 562, 564, 565, 566, 567, 568, 569, 570, 571, 572, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 633, 634, 635, 651, 652, 667, 668, 669, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 875, 876, 877, 878, 879, 880, 881, 882, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1120, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1191, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1248, 1249, 1250, 1251, 1252, 1253, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1373, 1374, 1375, 1376, 1377 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	60	127	158	176	221	92
normalized size	1	1.	0.54	1.14	1.42	1.59	1.99	0.83
time (sec)	N/A	0.105	0.052	0.039	1.457	2.365	5.941	1.26

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	52	107	131	153	199	80
normalized size	1	1.	0.6	1.23	1.51	1.76	2.29	0.92
time (sec)	N/A	0.075	0.037	0.037	1.425	2.122	5.776	1.227

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	44	87	104	132	133	68
normalized size	1	1.	0.59	1.18	1.41	1.78	1.8	0.92
time (sec)	N/A	0.054	0.032	0.036	1.438	2.134	4.09	1.212

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	67	77	113	110	55
normalized size	1	1.	0.87	1.76	2.03	2.97	2.89	1.45
time (sec)	N/A	0.022	0.025	0.036	1.432	2.089	4.006	1.215

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	25	45	47	92	19	39
normalized size	1	1.	0.89	1.61	1.68	3.29	0.68	1.39
time (sec)	N/A	0.011	0.01	0.032	1.418	2.097	1.63	1.176

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	22	22	26	44	63	104	70	69
normalized size	1	1.	1.18	2.	2.86	4.73	3.18	3.14
time (sec)	N/A	0.042	0.014	0.033	1.443	2.029	9.425	1.208

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	38	38	44	35	63	84	65	130
normalized size	1	1.	1.16	0.92	1.66	2.21	1.71	3.42
time (sec)	N/A	0.044	0.029	0.035	1.429	1.951	6.429	1.179

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	58	55	90	113	136	213
normalized size	1	1.	0.91	0.86	1.41	1.77	2.12	3.33
time (sec)	N/A	0.063	0.04	0.037	1.434	1.733	4.391	1.208

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	67	77	117	132	185	284
normalized size	1	1.	0.74	0.86	1.3	1.47	2.06	3.16
time (sec)	N/A	0.085	0.052	0.037	1.425	1.602	6.055	1.223

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	75	100	144	151	258	369
normalized size	1	1.	0.66	0.88	1.26	1.32	2.26	3.24
time (sec)	N/A	0.107	0.06	0.038	1.425	1.725	9.577	1.199

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	40	58	101	39	63
normalized size	1	1.	1.	0.91	1.32	2.3	0.89	1.43
time (sec)	N/A	0.042	0.018	0.027	0.939	1.638	0.429	1.156

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	32	46	77	29	51
normalized size	1	1.	1.	0.94	1.35	2.26	0.85	1.5
time (sec)	N/A	0.034	0.013	0.027	0.96	1.69	0.385	1.18

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	35	61	22	41
normalized size	1	1.	1.	0.92	1.35	2.35	0.85	1.58
time (sec)	N/A	0.029	0.011	0.032	0.963	1.618	0.387	1.143

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	20	36	12	22
normalized size	1	1.	1.	1.	1.25	2.25	0.75	1.38
time (sec)	N/A	0.012	0.009	0.027	0.953	1.744	0.187	1.172

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	15	35	10	18
normalized size	1	1.	1.	1.	1.25	2.92	0.83	1.5
time (sec)	N/A	0.025	0.007	0.033	0.952	1.768	0.253	1.203

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	21	27	59	17	30
normalized size	1	1.	1.	1.	1.29	2.81	0.81	1.43
time (sec)	N/A	0.028	0.01	0.052	0.95	1.735	0.554	1.167

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	41	89	27	43
normalized size	1	1.	1.	0.94	1.24	2.7	0.82	1.3
time (sec)	N/A	0.032	0.011	0.033	0.947	1.675	0.49	1.15

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	39	51	105	36	54
normalized size	1	1.	1.	0.95	1.24	2.56	0.88	1.32
time (sec)	N/A	0.036	0.013	0.037	0.952	1.694	0.486	1.148

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	58	122	151	197	0	117
normalized size	1	1.	0.63	1.33	1.64	2.14	0.	1.27
time (sec)	N/A	0.649	0.055	0.047	1.446	1.799	0.	1.182

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	53	102	124	177	0	105
normalized size	1	1.	0.6	1.16	1.41	2.01	0.	1.19
time (sec)	N/A	0.383	0.036	0.04	1.434	1.694	0.	1.207

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	39	79	93	147	0	85
normalized size	1	1.	0.71	1.44	1.69	2.67	0.	1.55
time (sec)	N/A	0.049	0.029	0.037	1.437	1.712	0.	1.216

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	48	48	51	75	105	193	0	117
normalized size	1	1.	1.06	1.56	2.19	4.02	0.	2.44
time (sec)	N/A	0.787	0.04	0.037	1.44	1.852	0.	1.192

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	57	82	108	159	0	203
normalized size	1	1.	0.9	1.3	1.71	2.52	0.	3.22
time (sec)	N/A	0.697	0.053	0.041	0.954	1.856	0.	1.187

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	75	108	138	196	0	288
normalized size	1	1.	0.82	1.19	1.52	2.15	0.	3.16
time (sec)	N/A	0.741	0.07	0.043	0.953	2.019	0.	1.215

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	81	146	165	217	0	358
normalized size	1	1.	0.69	1.25	1.41	1.85	0.	3.06
time (sec)	N/A	0.738	0.08	0.045	0.956	1.889	0.	1.238

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	78	155	49	82
normalized size	1	1.	1.	0.91	1.37	2.72	0.86	1.44
time (sec)	N/A	0.05	0.045	0.039	0.95	1.879	0.442	1.179

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	66	130	39	70
normalized size	1	1.	1.	0.94	1.4	2.77	0.83	1.49
time (sec)	N/A	0.044	0.037	0.037	0.947	1.812	0.517	1.163

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	55	109	31	59
normalized size	1	1.	1.	0.92	1.41	2.79	0.79	1.51
time (sec)	N/A	0.029	0.029	0.036	0.967	1.836	0.39	1.177

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	35	81	19	35
normalized size	1	1.	0.96	0.96	1.3	3.	0.7	1.3
time (sec)	N/A	0.014	0.017	0.032	0.942	1.806	0.492	1.175

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	16	46	8	18
normalized size	1	1.	1.	1.	1.23	3.54	0.62	1.38
time (sec)	N/A	0.025	0.009	0.033	0.984	1.983	0.475	1.247

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	46	116	26	49
normalized size	1	1.	1.	0.97	1.44	3.62	0.81	1.53
time (sec)	N/A	0.033	0.022	0.038	0.948	2.037	0.61	1.176

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	43	65	155	41	63
normalized size	1	1.	1.	0.93	1.41	3.37	0.89	1.37
time (sec)	N/A	0.037	0.028	0.037	0.958	1.855	0.679	1.184

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	76	173	49	74
normalized size	1	1.	1.	0.94	1.41	3.2	0.91	1.37
time (sec)	N/A	0.045	0.043	0.037	0.947	1.918	0.689	1.221

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	51	154	108	151	0	80
normalized size	1	1.	0.59	1.77	1.24	1.74	0.	0.92
time (sec)	N/A	0.077	0.041	0.043	1.439	1.956	0.	1.192

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	43	134	82	131	0	68
normalized size	1	1.	0.59	1.84	1.12	1.79	0.	0.93
time (sec)	N/A	0.055	0.032	0.038	1.433	1.859	0.	1.171

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	34	119	61	112	0	55
normalized size	1	1.	0.87	3.05	1.56	2.87	0.	1.41
time (sec)	N/A	0.023	0.025	0.04	1.422	1.976	0.	1.162

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	66	34	90	0	38
normalized size	1	1.	0.85	2.44	1.26	3.33	0.	1.41
time (sec)	N/A	0.011	0.012	0.031	1.441	1.94	0.	1.174

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	24	24	28	93	57	103	0	70
normalized size	1	1.	1.17	3.88	2.38	4.29	0.	2.92
time (sec)	N/A	0.042	0.015	0.041	1.441	1.997	0.	1.218

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	37	37	44	162	0	85	0	128
normalized size	1	1.	1.19	4.38	0.	2.3	0.	3.46
time (sec)	N/A	0.042	0.026	0.059	0.	1.898	0.	1.193

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	57	186	0	113	0	215
normalized size	1	1.	0.9	2.95	0.	1.79	0.	3.41
time (sec)	N/A	0.06	0.042	0.043	0.	1.935	0.	1.219

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	66	207	0	134	0	284
normalized size	1	1.	0.73	2.3	0.	1.49	0.	3.16
time (sec)	N/A	0.083	0.053	0.055	0.	1.919	0.	1.232

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	74	226	0	151	0	369
normalized size	1	1.	0.65	1.98	0.	1.32	0.	3.24
time (sec)	N/A	0.103	0.057	0.054	0.	1.986	0.	1.205

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	58	101	37	89
normalized size	1	1.	1.	0.93	1.35	2.35	0.86	2.07
time (sec)	N/A	0.039	0.019	0.033	0.945	1.762	0.339	1.232

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	46	77	27	77
normalized size	1	1.	1.	0.97	1.44	2.41	0.84	2.41
time (sec)	N/A	0.033	0.014	0.03	0.974	1.911	0.356	1.173

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	35	61	20	70
normalized size	1	1.	1.	0.96	1.4	2.44	0.8	2.8
time (sec)	N/A	0.024	0.011	0.032	0.947	1.842	0.287	1.169

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	36	10	86
normalized size	1	1.	1.	1.07	1.33	2.4	0.67	5.73
time (sec)	N/A	0.01	0.01	0.026	0.958	1.871	0.136	1.174

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	35	10	58
normalized size	1	1.	1.	1.09	1.36	3.18	0.91	5.27
time (sec)	N/A	0.024	0.006	0.031	0.957	1.756	0.154	1.143

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	27	58	17	41
normalized size	1	1.	1.	1.05	1.35	2.9	0.85	2.05
time (sec)	N/A	0.027	0.009	0.034	0.946	1.969	1.073	1.195

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	41	89	26	68
normalized size	1	1.	1.	0.97	1.28	2.78	0.81	2.12
time (sec)	N/A	0.03	0.01	0.035	0.992	1.82	0.669	1.151

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	38	51	104	36	84
normalized size	1	1.	1.	0.97	1.31	2.67	0.92	2.15
time (sec)	N/A	0.032	0.012	0.038	0.949	1.828	0.603	1.194

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	70	235	290	216	0	128
normalized size	1	1.	0.53	1.79	2.21	1.65	0.	0.98
time (sec)	N/A	0.689	0.055	0.054	1.486	2.054	0.	1.226

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	58	170	239	198	0	117
normalized size	1	1.	0.61	1.79	2.52	2.08	0.	1.23
time (sec)	N/A	0.638	0.069	0.048	1.492	1.876	0.	1.243

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	44	169	149	177	0	105
normalized size	1	1.	0.51	1.97	1.73	2.06	0.	1.22
time (sec)	N/A	0.364	0.053	0.043	1.462	1.93	0.	1.177

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	39	164	85	149	0	86
normalized size	1	1.	0.7	2.93	1.52	2.66	0.	1.54
time (sec)	N/A	0.049	0.03	0.041	1.43	2.019	0.	1.213

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	49	200	0	193	0	116
normalized size	1	1.	1.09	4.44	0.	4.29	0.	2.58
time (sec)	N/A	0.708	0.037	0.049	0.	2.062	0.	1.164

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	57	261	0	161	0	203
normalized size	1	1.	0.92	4.21	0.	2.6	0.	3.27
time (sec)	N/A	0.69	0.052	0.052	0.	1.943	0.	1.166

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	75	319	0	196	0	289
normalized size	1	1.	0.83	3.54	0.	2.18	0.	3.21
time (sec)	N/A	0.735	0.061	0.066	0.	1.959	0.	1.21

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	82	338	0	219	0	358
normalized size	1	1.	0.71	2.91	0.	1.89	0.	3.09
time (sec)	N/A	0.748	0.075	0.057	0.	1.863	0.	1.215

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	89	359	0	235	0	440
normalized size	1	1.	0.66	2.66	0.	1.74	0.	3.26
time (sec)	N/A	0.822	0.079	0.066	0.	1.783	0.	1.201

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.299	0.126	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	282	282	69	0	0	1350	0	0
normalized size	1	1.	0.24	0.	0.	4.79	0.	0.
time (sec)	N/A	0.214	0.036	0.118	0.	1.88	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	255	255	56	0	0	1303	0	0
normalized size	1	1.	0.22	0.	0.	5.11	0.	0.
time (sec)	N/A	0.173	0.02	0.088	0.	1.982	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	222	222	149	0	0	1261	0	0
normalized size	1	1.	0.67	0.	0.	5.68	0.	0.
time (sec)	N/A	0.146	0.202	0.087	0.	1.771	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	83	0	0	999	0	0
normalized size	1	1.	0.37	0.	0.	4.4	0.	0.
time (sec)	N/A	0.163	0.03	0.09	0.	1.767	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	58	0	0	286	0	0
normalized size	1	1.	0.79	0.	0.	3.92	0.	0.
time (sec)	N/A	0.034	0.014	0.089	0.	1.803	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	70	0	0	321	0	0
normalized size	1	1.	0.64	0.	0.	2.92	0.	0.
time (sec)	N/A	0.044	0.017	0.088	0.	1.842	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	78	0	0	347	0	0
normalized size	1	1.	0.56	0.	0.	2.5	0.	0.
time (sec)	N/A	0.062	0.022	0.09	0.	1.729	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	168	168	86	0	0	373	0	0
normalized size	1	1.	0.51	0.	0.	2.22	0.	0.
time (sec)	N/A	0.079	0.026	0.098	0.	1.781	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	94	0	0	400	0	0
normalized size	1	1.	0.48	0.	0.	2.03	0.	0.
time (sec)	N/A	0.096	0.03	0.097	0.	1.77	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.303	0.118	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	290	290	131	0	0	1403	0	0
normalized size	1	1.	0.45	0.	0.	4.84	0.	0.
time (sec)	N/A	0.204	0.103	0.098	0.	1.916	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	282	282	69	0	0	1374	0	0
normalized size	1	1.	0.24	0.	0.	4.87	0.	0.
time (sec)	N/A	0.2	0.032	0.092	0.	1.864	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	255	255	54	0	0	1323	0	0
normalized size	1	1.	0.21	0.	0.	5.19	0.	0.
time (sec)	N/A	0.169	0.017	0.096	0.	1.891	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	223	223	48	0	0	1281	0	0
normalized size	1	1.	0.22	0.	0.	5.74	0.	0.
time (sec)	N/A	0.134	0.059	0.092	0.	1.879	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	83	0	0	999	0	0
normalized size	1	1.	0.37	0.	0.	4.4	0.	0.
time (sec)	N/A	0.158	0.029	0.093	0.	1.838	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	55	0	0	302	0	0
normalized size	1	1.	0.75	0.	0.	4.14	0.	0.
time (sec)	N/A	0.033	0.013	0.092	0.	1.754	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	70	0	0	339	0	0
normalized size	1	1.	0.64	0.	0.	3.08	0.	0.
time (sec)	N/A	0.044	0.016	0.097	0.	1.69	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	78	0	0	363	0	0
normalized size	1	1.	0.56	0.	0.	2.61	0.	0.
time (sec)	N/A	0.061	0.024	0.106	0.	1.74	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	168	168	86	0	0	386	0	0
normalized size	1	1.	0.51	0.	0.	2.3	0.	0.
time (sec)	N/A	0.078	0.027	0.095	0.	1.737	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.332	0.115	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	317	317	74	0	0	1400	0	0
normalized size	1	1.	0.23	0.	0.	4.42	0.	0.
time (sec)	N/A	0.241	0.046	0.102	0.	1.84	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	305	305	66	0	0	1364	0	0
normalized size	1	1.	0.22	0.	0.	4.47	0.	0.
time (sec)	N/A	0.226	0.031	0.102	0.	1.874	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	61	0	0	1319	0	0
normalized size	1	1.	0.22	0.	0.	4.73	0.	0.
time (sec)	N/A	0.194	0.025	0.102	0.	1.88	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	247	247	174	0	0	1268	0	0
normalized size	1	1.	0.7	0.	0.	5.13	0.	0.
time (sec)	N/A	0.165	0.22	0.112	0.	1.789	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	248	248	93	0	0	1052	0	0
normalized size	1	1.	0.38	0.	0.	4.24	0.	0.
time (sec)	N/A	0.205	0.036	0.103	0.	1.82	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	74	0	0	296	0	0
normalized size	1	1.	0.78	0.	0.	3.12	0.	0.
time (sec)	N/A	0.039	0.019	0.106	0.	1.7	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	86	0	0	335	0	0
normalized size	1	1.	0.63	0.	0.	2.46	0.	0.
time (sec)	N/A	0.049	0.024	0.109	0.	2.124	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	91	0	0	360	0	0
normalized size	1	1.	0.55	0.	0.	2.18	0.	0.
time (sec)	N/A	0.079	0.028	0.117	0.	2.005	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	99	0	0	390	0	0
normalized size	1	1.	0.51	0.	0.	2.01	0.	0.
time (sec)	N/A	0.094	0.032	0.112	0.	2.008	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.298	0.092	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	290	290	116	0	0	1399	0	0
normalized size	1	1.	0.4	0.	0.	4.82	0.	0.
time (sec)	N/A	0.213	0.104	0.11	0.	2.284	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	282	282	62	0	0	1369	0	0
normalized size	1	1.	0.22	0.	0.	4.85	0.	0.
time (sec)	N/A	0.199	0.026	0.111	0.	2.181	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	255	255	55	0	0	1314	0	0
normalized size	1	1.	0.22	0.	0.	5.15	0.	0.
time (sec)	N/A	0.168	0.019	0.099	0.	2.175	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	221	221	35	0	0	1273	0	0
normalized size	1	1.	0.16	0.	0.	5.76	0.	0.
time (sec)	N/A	0.138	0.035	0.092	0.	1.826	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	83	0	0	998	0	0
normalized size	1	1.	0.37	0.	0.	4.4	0.	0.
time (sec)	N/A	0.161	0.026	0.099	0.	1.81	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	55	0	0	298	0	0
normalized size	1	1.	0.76	0.	0.	4.14	0.	0.
time (sec)	N/A	0.036	0.014	0.092	0.	1.743	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	69	0	0	332	0	0
normalized size	1	1.	0.63	0.	0.	3.02	0.	0.
time (sec)	N/A	0.046	0.021	0.107	0.	1.749	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	78	0	0	360	0	0
normalized size	1	1.	0.56	0.	0.	2.59	0.	0.
time (sec)	N/A	0.062	0.022	0.092	0.	1.821	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	168	168	86	0	0	382	0	0
normalized size	1	1.	0.51	0.	0.	2.27	0.	0.
time (sec)	N/A	0.079	0.028	0.093	0.	1.742	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.317	0.086	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	290	290	116	0	0	1388	0	0
normalized size	1	1.	0.4	0.	0.	4.79	0.	0.
time (sec)	N/A	0.208	0.104	0.107	0.	1.954	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	282	282	62	0	0	1361	0	0
normalized size	1	1.	0.22	0.	0.	4.83	0.	0.
time (sec)	N/A	0.207	0.028	0.108	0.	1.748	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	255	255	56	0	0	1312	0	0
normalized size	1	1.	0.22	0.	0.	5.15	0.	0.
time (sec)	N/A	0.17	0.02	0.109	0.	1.844	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	222	222	150	0	0	1269	0	0
normalized size	1	1.	0.68	0.	0.	5.72	0.	0.
time (sec)	N/A	0.14	0.227	0.096	0.	1.788	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	83	0	0	998	0	0
normalized size	1	1.	0.37	0.	0.	4.4	0.	0.
time (sec)	N/A	0.163	0.028	0.104	0.	1.877	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	55	0	0	290	0	0
normalized size	1	1.	0.75	0.	0.	3.97	0.	0.
time (sec)	N/A	0.033	0.014	0.115	0.	1.697	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	70	0	0	331	0	0
normalized size	1	1.	0.64	0.	0.	3.01	0.	0.
time (sec)	N/A	0.042	0.021	0.105	0.	1.675	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	78	0	0	355	0	0
normalized size	1	1.	0.56	0.	0.	2.55	0.	0.
time (sec)	N/A	0.061	0.022	0.107	0.	1.775	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	168	168	86	0	0	379	0	0
normalized size	1	1.	0.51	0.	0.	2.26	0.	0.
time (sec)	N/A	0.079	0.029	0.114	0.	2.044	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.532	0.14	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	317	317	79	0	0	1500	0	0
normalized size	1	1.	0.25	0.	0.	4.73	0.	0.
time (sec)	N/A	0.238	0.036	0.147	0.	2.296	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	305	305	70	0	0	1462	0	0
normalized size	1	1.	0.23	0.	0.	4.79	0.	0.
time (sec)	N/A	0.225	0.035	0.129	0.	2.097	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	64	0	0	1413	0	0
normalized size	1	1.	0.23	0.	0.	5.06	0.	0.
time (sec)	N/A	0.193	0.024	0.121	0.	1.835	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	247	247	33	0	0	1361	0	0
normalized size	1	1.	0.13	0.	0.	5.51	0.	0.
time (sec)	N/A	0.158	0.054	0.092	0.	1.976	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	248	248	90	0	0	1191	0	0
normalized size	1	1.	0.36	0.	0.	4.8	0.	0.
time (sec)	N/A	0.204	0.063	0.118	0.	1.972	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	55	0	0	383	0	0
normalized size	1	1.	0.58	0.	0.	4.03	0.	0.
time (sec)	N/A	0.039	0.018	0.117	0.	1.737	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	70	0	0	421	0	0
normalized size	1	1.	0.51	0.	0.	3.1	0.	0.
time (sec)	N/A	0.054	0.023	0.117	0.	1.761	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	78	0	0	448	0	0
normalized size	1	1.	0.47	0.	0.	2.72	0.	0.
time (sec)	N/A	0.075	0.023	0.127	0.	1.798	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	86	0	0	477	0	0
normalized size	1	1.	0.44	0.	0.	2.46	0.	0.
time (sec)	N/A	0.098	0.032	0.118	0.	1.798	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.231	0.038	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	245	245	59	0	0	882	0	0
normalized size	1	1.	0.24	0.	0.	3.6	0.	0.
time (sec)	N/A	0.385	0.027	0.033	0.	1.933	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	224	224	49	0	0	810	0	0
normalized size	1	1.	0.22	0.	0.	3.62	0.	0.
time (sec)	N/A	0.34	0.013	0.033	0.	1.738	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	202	202	39	0	0	791	0	0
normalized size	1	1.	0.19	0.	0.	3.92	0.	0.
time (sec)	N/A	0.329	0.043	0.035	0.	1.795	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	346	346	74	0	0	1268	0	0
normalized size	1	1.	0.21	0.	0.	3.66	0.	0.
time (sec)	N/A	0.455	0.026	0.031	0.	1.923	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	50	0	0	626	0	0
normalized size	1	1.	0.26	0.	0.	3.23	0.	0.
time (sec)	N/A	0.157	0.011	0.037	0.	1.738	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	224	224	60	0	0	657	0	0
normalized size	1	1.	0.27	0.	0.	2.93	0.	0.
time (sec)	N/A	0.175	0.015	0.036	0.	1.734	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.236	0.035	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	59	0	0	428	0	0
normalized size	1	1.	0.44	0.	0.	3.22	0.	0.
time (sec)	N/A	0.05	0.027	0.034	0.	1.68	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	46	0	0	406	0	0
normalized size	1	1.	0.41	0.	0.	3.62	0.	0.
time (sec)	N/A	0.032	0.013	0.035	0.	1.639	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	87	0	0	387	0	0
normalized size	1	1.	1.04	0.	0.	4.61	0.	0.
time (sec)	N/A	0.017	0.135	0.035	0.	1.674	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	74	0	0	405	0	0
normalized size	1	1.	0.55	0.	0.	3.	0.	0.
time (sec)	N/A	0.032	0.023	0.033	0.	1.671	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	45	0	0	398	0	0
normalized size	1	1.	0.53	0.	0.	4.68	0.	0.
time (sec)	N/A	0.031	0.011	0.033	0.	1.686	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	57	0	0	427	0	0
normalized size	1	1.	0.49	0.	0.	3.68	0.	0.
time (sec)	N/A	0.038	0.013	0.033	0.	1.729	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.302	0.036	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	646	646	70	0	0	6882	0	0
normalized size	1	1.	0.11	0.	0.	10.65	0.	0.
time (sec)	N/A	0.702	0.035	0.036	0.	2.398	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	619	619	56	0	0	6819	0	0
normalized size	1	1.	0.09	0.	0.	11.02	0.	0.
time (sec)	N/A	0.482	0.021	0.035	0.	2.286	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	591	591	48	0	0	6561	0	0
normalized size	1	1.	0.08	0.	0.	11.1	0.	0.
time (sec)	N/A	0.437	0.053	0.034	0.	2.249	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	759	759	83	0	0	5937	0	0
normalized size	1	1.	0.11	0.	0.	7.82	0.	0.
time (sec)	N/A	0.527	0.03	0.039	0.	2.247	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	58	0	0	1359	0	0
normalized size	1	1.	0.21	0.	0.	5.01	0.	0.
time (sec)	N/A	0.129	0.017	0.035	0.	1.929	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	312	312	73	0	0	1423	0	0
normalized size	1	1.	0.23	0.	0.	4.56	0.	0.
time (sec)	N/A	0.153	0.021	0.036	0.	2.376	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	47	461	0	0	0	0
normalized size	1	1.	1.04	10.24	0.	0.	0.	0.
time (sec)	N/A	0.043	0.023	0.444	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	92	139	0	0	0	0
normalized size	1	1.	0.61	0.92	0.	0.	0.	0.
time (sec)	N/A	0.881	0.079	0.343	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	26	184	0	0	99	0
normalized size	1	1.	0.72	5.11	0.	0.	2.75	0.
time (sec)	N/A	0.025	0.007	0.359	0.	0.	3.268	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	70	67	0	0	97	0
normalized size	1	1.	0.95	0.91	0.	0.	1.31	0.
time (sec)	N/A	0.042	0.03	0.234	0.	0.	3.796	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	31	0	0	0	0	0
normalized size	1	1.	0.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.027	0.381	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	27	126	0	0	119	0
normalized size	1	1.	0.73	3.41	0.	0.	3.22	0.
time (sec)	N/A	0.025	0.007	0.36	0.	0.	3.691	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	55	0	0	0	0	0
normalized size	1	1.	0.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.814	0.052	0.332	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.225	0.069	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	182	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	0.172	0.059	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	96	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.051	0.059	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	86	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.024	0.05	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	46	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.03	0.04	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	95	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.02	0.043	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	65	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.016	0.059	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	91	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.029	0.058	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	101	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.044	0.092	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	53	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.031	0.378	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	75	137	171	209	226	105
normalized size	1	1.	0.61	1.11	1.39	1.7	1.84	0.85
time (sec)	N/A	0.08	0.106	0.045	1.437	1.573	8.069	1.239

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	67	114	140	178	136	89
normalized size	1	1.	0.74	1.25	1.54	1.96	1.49	0.98
time (sec)	N/A	0.056	0.088	0.039	1.432	1.646	5.608	1.177

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	59	91	109	151	102	73
normalized size	1	1.	0.97	1.49	1.79	2.48	1.67	1.2
time (sec)	N/A	0.037	0.08	0.036	1.477	1.641	4.666	1.202

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	30	46	49	107	46	41
normalized size	1	1.	0.91	1.39	1.48	3.24	1.39	1.24
time (sec)	N/A	0.019	0.012	0.032	1.434	1.623	2.661	1.192

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	43	43	46	76	0	136	0	72
normalized size	1	1.	1.07	1.77	0.	3.16	0.	1.67
time (sec)	N/A	0.041	0.024	0.036	0.	1.666	0.	1.239

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	32	32	29	35	0	127	0	0
normalized size	1	1.	0.91	1.09	0.	3.97	0.	0.
time (sec)	N/A	0.033	0.011	0.032	0.	1.535	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	35	40	0	188	0	196
normalized size	1	1.	0.54	0.62	0.	2.89	0.	3.02
time (sec)	N/A	0.05	0.017	0.033	0.	1.629	0.	1.206

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	43	49	0	254	0	269
normalized size	1	1.	0.44	0.51	0.	2.62	0.	2.77
time (sec)	N/A	0.069	0.021	0.03	0.	1.668	0.	1.229

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	51	57	0	319	0	0
normalized size	1	1.	0.4	0.44	0.	2.47	0.	0.
time (sec)	N/A	0.091	0.025	0.035	0.	1.706	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	29	30	47	63	126	0
normalized size	1	1.	0.71	0.73	1.15	1.54	3.07	0.
time (sec)	N/A	0.044	0.021	0.033	1.136	1.682	1.131	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	23	45	80	128	66	80
normalized size	1	1.	0.62	1.22	2.16	3.46	1.78	2.16
time (sec)	N/A	0.034	0.019	0.028	0.947	1.491	0.105	1.249

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	32	29	50	76	36	50
normalized size	1	1.	0.86	0.78	1.35	2.05	0.97	1.35
time (sec)	N/A	0.032	0.016	0.025	0.947	1.545	0.092	1.208

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	29	50	80	37	50
normalized size	1	1.	0.81	0.78	1.35	2.16	1.	1.35
time (sec)	N/A	0.035	0.014	0.032	0.957	1.561	0.089	1.184

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	15	23	35	15	23
normalized size	1	1.	0.84	0.79	1.21	1.84	0.79	1.21
time (sec)	N/A	0.026	0.008	0.026	0.96	1.541	0.085	1.231

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	26	26	11	15	26	10	15
normalized size	1	2.	2.	0.85	1.15	2.	0.77	1.15
time (sec)	N/A	0.012	0.008	0.027	0.944	1.513	0.079	1.174

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	25	30	39	62	20	41
normalized size	1	1.	0.81	0.97	1.26	2.	0.65	1.32
time (sec)	N/A	0.038	0.016	0.036	0.935	1.68	0.334	1.203

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	28	34	47	22	43
normalized size	1	1.	1.92	2.15	2.62	3.62	1.69	3.31
time (sec)	N/A	0.026	0.007	0.039	0.939	1.682	0.378	1.23

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	23	30	63	95	48	28
normalized size	1	1.	0.62	0.81	1.7	2.57	1.3	0.76
time (sec)	N/A	0.039	0.014	0.035	0.948	1.925	0.49	1.206

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	23	30	77	115	58	28
normalized size	1	1.	0.62	0.81	2.08	3.11	1.57	0.76
time (sec)	N/A	0.039	0.014	0.033	0.969	1.781	0.525	1.171

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	58	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.024	0.47	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	75	183	234	201	459	105
normalized size	1	1.	0.9	2.2	2.82	2.42	5.53	1.27
time (sec)	N/A	0.049	0.093	0.069	1.454	1.933	20.223	1.285

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	44	96	116	140	301	65
normalized size	1	1.	0.75	1.63	1.97	2.37	5.1	1.1
time (sec)	N/A	0.036	0.036	0.044	1.476	1.892	8.584	1.203

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	59	137	171	151	221	73
normalized size	1	1.	0.97	2.25	2.8	2.48	3.62	1.2
time (sec)	N/A	0.046	0.074	0.047	1.443	1.823	12.462	1.226

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	48	104	127	119	165	51
normalized size	1	1.	0.74	1.6	1.95	1.83	2.54	0.78
time (sec)	N/A	0.048	0.033	0.039	1.442	1.963	8.497	1.206

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	45	146	0	216	0	107
normalized size	1	1.	0.61	1.97	0.	2.92	0.	1.45
time (sec)	N/A	0.059	0.012	0.039	0.	2.003	0.	1.236

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	32	32	29	35	197	181	0	0
normalized size	1	1.	0.91	1.09	6.16	5.66	0.	0.
time (sec)	N/A	0.037	0.01	0.031	1.022	1.926	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	34	40	292	244	0	269
normalized size	1	1.	0.52	0.62	4.49	3.75	0.	4.14
time (sec)	N/A	0.051	0.018	0.033	0.994	1.81	0.	1.277

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	43	49	441	319	0	342
normalized size	1	1.	0.44	0.51	4.55	3.29	0.	3.53
time (sec)	N/A	0.069	0.024	0.033	1.006	1.763	0.	1.204

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	51	57	624	389	0	0
normalized size	1	1.	0.4	0.44	4.84	3.02	0.	0.
time (sec)	N/A	0.092	0.027	0.036	1.029	1.789	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	74	104	161	570	0
normalized size	1	1.	0.76	1.12	1.58	2.44	8.64	0.
time (sec)	N/A	0.058	0.075	0.035	1.176	1.732	3.001	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	31	45	80	130	63	80
normalized size	1	1.	0.58	0.85	1.51	2.45	1.19	1.51
time (sec)	N/A	0.044	0.02	0.029	1.029	1.515	0.113	1.237

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	26	23	38	58	29	38
normalized size	1	1.	0.81	0.72	1.19	1.81	0.91	1.19
time (sec)	N/A	0.033	0.014	0.026	0.959	1.425	0.096	1.22

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	29	50	81	37	50
normalized size	1	1.	0.86	0.83	1.43	2.31	1.06	1.43
time (sec)	N/A	0.038	0.014	0.032	0.947	1.463	0.096	1.221

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	21	16	34	50	24	34
normalized size	1	1.	1.24	0.94	2.	2.94	1.41	2.
time (sec)	N/A	0.025	0.012	0.027	0.95	1.482	0.1	1.146

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	25	32	66	26	47
normalized size	1	1.	0.96	0.93	1.19	2.44	0.96	1.74
time (sec)	N/A	0.023	0.01	0.031	0.949	1.545	0.3	1.196

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	36	46	59	108	36	50
normalized size	1	1.	0.75	0.96	1.23	2.25	0.75	1.04
time (sec)	N/A	0.045	0.018	0.035	0.953	1.649	0.435	1.185

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	42	69	100	51	68
normalized size	1	1.	1.	1.68	2.76	4.	2.04	2.72
time (sec)	N/A	0.028	0.008	0.036	0.952	1.691	0.476	1.178

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	31	41	88	131	66	39
normalized size	1	1.	0.6	0.79	1.69	2.52	1.27	0.75
time (sec)	N/A	0.043	0.017	0.032	0.951	1.555	0.536	1.171

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	31	42	104	158	80	39
normalized size	1	1.	0.58	0.79	1.96	2.98	1.51	0.74
time (sec)	N/A	0.042	0.018	0.033	0.965	1.528	0.652	1.179

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	59	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.023	0.451	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	80	160	120	182	0	90
normalized size	1	1.	0.6	1.2	0.9	1.37	0.	0.68
time (sec)	N/A	0.096	0.069	0.039	1.472	1.629	0.	1.189

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	72	142	96	154	0	73
normalized size	1	1.	0.71	1.41	0.95	1.52	0.	0.72
time (sec)	N/A	0.07	0.082	0.041	1.448	1.844	0.	1.205

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	61	114	61	119	0	51
normalized size	1	1.	0.97	1.81	0.97	1.89	0.	0.81
time (sec)	N/A	0.043	0.049	0.036	1.437	1.658	0.	1.205

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	154	42	66	44	19
normalized size	1	1.	1.	14.	3.82	6.	4.	1.73
time (sec)	N/A	0.028	0.007	0.041	1.471	1.589	4.229	1.196

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	29	29	26	28	0	70	0	95
normalized size	1	1.	0.9	0.97	0.	2.41	0.	3.28
time (sec)	N/A	0.036	0.009	0.031	0.	1.669	0.	1.198

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	34	33	0	130	0	123
normalized size	1	1.	0.52	0.51	0.	2.	0.	1.89
time (sec)	N/A	0.052	0.017	0.032	0.	1.613	0.	1.235

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	43	42	0	190	0	196
normalized size	1	1.	0.44	0.43	0.	1.96	0.	2.02
time (sec)	N/A	0.068	0.02	0.03	0.	1.704	0.	1.211

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	51	50	0	251	0	221
normalized size	1	1.	0.4	0.39	0.	1.95	0.	1.71
time (sec)	N/A	0.09	0.024	0.03	0.	1.592	0.	1.252

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.015	0.359	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	56	64	85	155	68	127
normalized size	1	1.	0.62	0.7	0.93	1.7	0.75	1.4
time (sec)	N/A	0.049	0.019	0.028	0.941	1.547	0.346	1.195

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	48	53	70	128	56	111
normalized size	1	1.	0.66	0.73	0.96	1.75	0.77	1.52
time (sec)	N/A	0.043	0.017	0.029	0.945	1.664	0.335	1.185

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	39	42	55	99	41	92
normalized size	1	1.	0.72	0.78	1.02	1.83	0.76	1.7
time (sec)	N/A	0.036	0.013	0.029	0.944	1.668	0.312	1.173

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	25	32	65	24	68
normalized size	1	1.	0.96	0.96	1.23	2.5	0.92	2.62
time (sec)	N/A	0.023	0.013	0.036	0.944	1.606	0.287	1.235

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	27	10	36
normalized size	1	1.	1.	1.08	1.38	2.08	0.77	2.77
time (sec)	N/A	0.026	0.006	0.03	0.944	1.571	0.085	1.235

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	30	39	58	22	34
normalized size	1	1.	1.	2.73	3.55	5.27	2.	3.09
time (sec)	N/A	0.027	0.007	0.032	0.945	1.573	0.158	1.275

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	31	45	65	107	39	58
normalized size	1	1.	0.94	1.36	1.97	3.24	1.18	1.76
time (sec)	N/A	0.04	0.018	0.036	0.94	1.582	0.423	1.2

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	35	60	85	173	56	78
normalized size	1	1.	0.69	1.18	1.67	3.39	1.1	1.53
time (sec)	N/A	0.047	0.022	0.035	0.956	1.608	0.5	1.203

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	44	75	113	252	76	120
normalized size	1	1.	0.64	1.09	1.64	3.65	1.1	1.74
time (sec)	N/A	0.053	0.029	0.039	0.973	1.673	0.614	1.226

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	60	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.028	0.474	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	45	245	315	267	0	139
normalized size	1	1.	0.28	1.5	1.93	1.64	0.	0.85
time (sec)	N/A	0.125	0.02	0.049	1.496	1.678	0.	1.289

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	45	179	265	243	0	123
normalized size	1	1.	0.34	1.37	2.02	1.85	0.	0.94
time (sec)	N/A	0.094	0.017	0.045	1.462	1.68	0.	1.291

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	43	169	147	192	0	99
normalized size	1	1.	0.47	1.86	1.62	2.11	0.	1.09
time (sec)	N/A	0.064	0.015	0.043	1.447	1.649	0.	1.246

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	41	41	59	292	0	138	0	72
normalized size	1	1.	1.44	7.12	0.	3.37	0.	1.76
time (sec)	N/A	0.041	0.052	0.052	0.	1.673	0.	1.232

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	28	28	27	34	0	72	0	93
normalized size	1	1.	0.96	1.21	0.	2.57	0.	3.32
time (sec)	N/A	0.033	0.01	0.03	0.	1.655	0.	1.224

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	32	0	58	0	39
normalized size	1	1.	1.	1.68	0.	3.05	0.	2.05
time (sec)	N/A	0.028	0.011	0.03	0.	1.659	0.	1.214

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	45	49	0	173	0	0
normalized size	1	1.	0.82	0.89	0.	3.15	0.	0.
time (sec)	N/A	0.045	0.017	0.031	0.	1.628	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	52	56	0	200	0	235
normalized size	1	1.	0.6	0.64	0.	2.3	0.	2.7
time (sec)	N/A	0.062	0.019	0.033	0.	1.615	0.	1.386

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	61	65	0	308	0	0
normalized size	1	1.	0.51	0.55	0.	2.59	0.	0.
time (sec)	N/A	0.076	0.023	0.031	0.	1.66	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	70	71	203	212	0	117
normalized size	1	1.	0.4	0.4	1.15	1.2	0.	0.66
time (sec)	N/A	0.129	0.05	0.038	1.03	1.702	0.	1.482

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	62	63	173	176	0	99
normalized size	1	1.	0.44	0.45	1.23	1.25	0.	0.7
time (sec)	N/A	0.104	0.041	0.033	1.026	1.589	0.	1.312

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	54	55	143	151	0	82
normalized size	1	1.	0.51	0.52	1.35	1.42	0.	0.77
time (sec)	N/A	0.082	0.034	0.031	1.022	1.681	0.	1.295

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	44	47	111	113	0	63
normalized size	1	1.	0.62	0.66	1.56	1.59	0.	0.89
time (sec)	N/A	0.061	0.025	0.03	1.031	1.782	0.	1.22

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	37	34	78	86	0	43
normalized size	1	1.	1.06	0.97	2.23	2.46	0.	1.23
time (sec)	N/A	0.041	0.013	0.027	1.033	1.865	0.	1.189

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	62	84	0	491	0	123
normalized size	1	1.	0.75	1.01	0.	5.92	0.	1.48
time (sec)	N/A	0.082	0.025	0.11	0.	1.941	0.	1.291

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	80	111	0	590	0	78
normalized size	1	1.	0.98	1.35	0.	7.2	0.	0.95
time (sec)	N/A	0.08	0.053	0.102	0.	1.972	0.	1.274

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	57	156	0	695	0	103
normalized size	1	1.	0.47	1.28	0.	5.7	0.	0.84
time (sec)	N/A	0.106	0.018	0.102	0.	2.027	0.	1.303

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	57	208	0	822	0	124
normalized size	1	1.	0.36	1.32	0.	5.24	0.	0.79
time (sec)	N/A	0.13	0.02	0.105	0.	2.14	0.	1.317

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	43	131	172	234
normalized size	1	1.	0.85	0.52	1.08	3.28	4.3	5.85
time (sec)	N/A	0.048	0.04	0.033	0.957	2.262	29.91	1.189

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	43	104	76	108
normalized size	1	1.	0.85	0.52	1.08	2.6	1.9	2.7
time (sec)	N/A	0.047	0.037	0.028	0.948	2.221	12.587	1.195

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	30	21	43	74	58	76
normalized size	1	1.	0.75	0.52	1.08	1.85	1.45	1.9
time (sec)	N/A	0.049	0.036	0.035	0.948	2.205	12.322	1.19

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	23	20	41	47	31	41
normalized size	1	1.	0.61	0.53	1.08	1.24	0.82	1.08
time (sec)	N/A	0.043	0.026	0.038	0.953	2.169	3.805	1.191

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	21	20	41	62	48	43
normalized size	1	1.	0.58	0.56	1.14	1.72	1.33	1.19
time (sec)	N/A	0.047	0.023	0.033	0.95	2.168	29.837	1.175

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	34	21	35	95	29	49
normalized size	1	1.	0.89	0.55	0.92	2.5	0.76	1.29
time (sec)	N/A	0.048	0.043	0.037	0.942	2.208	32.431	1.21

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	32	119	31	46
normalized size	1	1.	0.85	0.52	0.8	2.98	0.78	1.15
time (sec)	N/A	0.049	0.054	0.033	0.946	2.082	73.439	1.208

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	35	139	31	49
normalized size	1	1.	0.85	0.52	0.88	3.48	0.78	1.22
time (sec)	N/A	0.052	0.052	0.031	0.954	2.156	54.737	1.164

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	62	63	343	204	0	99
normalized size	1	1.	0.44	0.45	2.43	1.45	0.	0.7
time (sec)	N/A	0.109	0.045	0.039	1.155	2.22	0.	1.302

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	54	55	284	177	0	82
normalized size	1	1.	0.51	0.52	2.68	1.67	0.	0.77
time (sec)	N/A	0.088	0.036	0.032	1.132	2.222	0.	1.276

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	46	47	221	142	0	63
normalized size	1	1.	0.65	0.66	3.11	2.	0.	0.89
time (sec)	N/A	0.067	0.03	0.028	1.143	2.215	0.	1.239

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	37	34	163	108	0	46
normalized size	1	1.	1.06	0.97	4.66	3.09	0.	1.31
time (sec)	N/A	0.05	0.022	0.034	1.111	2.25	0.	1.172

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	67	95	0	518	0	142
normalized size	1	1.	0.56	0.8	0.	4.35	0.	1.19
time (sec)	N/A	0.104	0.033	0.096	0.	2.234	0.	1.262

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	57	127	0	608	0	95
normalized size	1	1.	0.5	1.1	0.	5.29	0.	0.83
time (sec)	N/A	0.101	0.022	0.105	0.	2.27	0.	1.265

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	91	158	0	703	0	99
normalized size	1	1.	0.75	1.3	0.	5.76	0.	0.81
time (sec)	N/A	0.105	0.075	0.109	0.	2.342	0.	1.374

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	57	208	0	817	0	124
normalized size	1	1.	0.36	1.32	0.	5.2	0.	0.79
time (sec)	N/A	0.134	0.024	0.112	0.	2.274	0.	1.387

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	192	192	57	258	0	944	0	142
normalized size	1	1.	0.3	1.34	0.	4.92	0.	0.74
time (sec)	N/A	0.159	0.024	0.109	0.	2.33	0.	1.462

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	73	72	111	209	0	132
normalized size	1	1.	0.35	0.35	0.54	1.01	0.	0.64
time (sec)	N/A	0.162	0.055	0.031	1.02	2.198	0.	1.328

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	65	64	96	182	0	115
normalized size	1	1.	0.38	0.37	0.56	1.06	0.	0.67
time (sec)	N/A	0.134	0.04	0.034	1.011	2.251	0.	1.306

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	57	56	81	149	0	97
normalized size	1	1.	0.42	0.41	0.6	1.1	0.	0.71
time (sec)	N/A	0.106	0.035	0.03	1.013	2.119	0.	1.263

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	49	48	66	117	0	80
normalized size	1	1.	0.49	0.48	0.65	1.16	0.	0.79
time (sec)	N/A	0.087	0.028	0.03	0.998	2.235	0.	1.222

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	38	39	50	85	0	58
normalized size	1	1.	0.58	0.59	0.76	1.29	0.	0.88
time (sec)	N/A	0.062	0.02	0.029	1.	2.133	0.	1.174

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	20	76	0	42
normalized size	1	1.	1.	0.9	0.67	2.53	0.	1.4
time (sec)	N/A	0.047	0.016	0.028	0.981	2.129	0.	1.238

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	51	51	44	68	0	321	0	84
normalized size	1	1.	0.86	1.33	0.	6.29	0.	1.65
time (sec)	N/A	0.062	0.024	0.096	0.	2.164	0.	1.222

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	70	111	0	589	0	92
normalized size	1	1.	0.78	1.23	0.	6.54	0.	1.02
time (sec)	N/A	0.082	0.058	0.103	0.	2.245	0.	1.51

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	52	158	0	705	0	112
normalized size	1	1.	0.42	1.26	0.	5.64	0.	0.9
time (sec)	N/A	0.105	0.024	0.106	0.	2.205	0.	1.461

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	88	101	0	517	0	217
normalized size	1	1.	0.64	0.74	0.	3.77	0.	1.58
time (sec)	N/A	0.111	0.074	0.035	0.	2.259	0.	1.256

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	80	87	0	460	109	181
normalized size	1	1.	0.69	0.75	0.	3.97	0.94	1.56
time (sec)	N/A	0.098	0.056	0.034	0.	2.341	55.409	1.214

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	71	73	0	379	90	144
normalized size	1	1.	0.75	0.77	0.	3.99	0.95	1.52
time (sec)	N/A	0.079	0.042	0.033	0.	2.374	42.435	1.253

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	61	59	0	312	75	104
normalized size	1	1.	0.8	0.78	0.	4.11	0.99	1.37
time (sec)	N/A	0.066	0.04	0.032	0.	2.59	6.77	1.273

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	45	0	294	58	69
normalized size	1	1.	1.	0.78	0.	5.07	1.	1.19
time (sec)	N/A	0.058	0.025	0.033	0.	2.572	23.623	1.221

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	30	0	232	39	47
normalized size	1	1.	1.	0.79	0.	6.11	1.03	1.24
time (sec)	N/A	0.051	0.017	0.032	0.	2.498	32.398	1.197

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	36	50	0	360	60	72
normalized size	1	1.	0.63	0.88	0.	6.32	1.05	1.26
time (sec)	N/A	0.06	0.022	0.035	0.	2.544	21.771	1.257

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	39	64	0	477	80	99
normalized size	1	1.	0.47	0.77	0.	5.75	0.96	1.19
time (sec)	N/A	0.07	0.025	0.043	0.	2.325	58.366	1.193

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	39	78	0	598	0	126
normalized size	1	1.	0.38	0.75	0.	5.75	0.	1.21
time (sec)	N/A	0.082	0.03	0.044	0.	2.322	0.	1.156

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	70	71	99	180	0	115
normalized size	1	1.	0.41	0.42	0.58	1.05	0.	0.67
time (sec)	N/A	0.137	0.039	0.032	1.031	1.986	0.	1.328

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	61	62	82	134	0	95
normalized size	1	1.	0.45	0.46	0.6	0.99	0.	0.7
time (sec)	N/A	0.113	0.034	0.033	1.027	1.99	0.	1.281

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	51	54	68	108	0	77
normalized size	1	1.	0.5	0.52	0.66	1.05	0.	0.75
time (sec)	N/A	0.084	0.028	0.031	1.043	1.598	0.	1.252

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	40	46	41	90	0	59
normalized size	1	1.	0.6	0.69	0.61	1.34	0.	0.88
time (sec)	N/A	0.064	0.024	0.029	0.988	1.575	0.	1.215

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	33	33	35	34	38	82	0	41
normalized size	1	1.	1.06	1.03	1.15	2.48	0.	1.24
time (sec)	N/A	0.05	0.024	0.031	0.985	1.57	0.	1.194

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	55	82	0	529	0	127
normalized size	1	1.	0.65	0.96	0.	6.22	0.	1.49
time (sec)	N/A	0.081	0.024	0.102	0.	1.634	0.	1.199

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	57	124	0	682	0	111
normalized size	1	1.	0.46	0.99	0.	5.46	0.	0.89
time (sec)	N/A	0.102	0.029	0.118	0.	1.709	0.	1.315

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	57	173	0	747	0	134
normalized size	1	1.	0.36	1.08	0.	4.67	0.	0.84
time (sec)	N/A	0.13	0.03	0.119	0.	1.636	0.	1.301

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	80	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.054	0.246	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	80	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.043	0.233	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.035	0.23	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.031	0.224	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.026	0.231	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.024	0.212	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	80	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.032	0.219	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	80	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.042	0.228	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	50	91	109	155	357	77
normalized size	1	1.	0.6	1.1	1.31	1.87	4.3	0.93
time (sec)	N/A	0.071	0.057	0.041	1.435	1.727	11.942	1.2

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	32	43	78	82	66	63
normalized size	1	1.	0.71	0.96	1.73	1.82	1.47	1.4
time (sec)	N/A	0.065	0.024	0.033	1.425	1.582	1.002	1.209

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	40	70	81	132	245	61
normalized size	1	1.	0.69	1.21	1.4	2.28	4.22	1.05
time (sec)	N/A	0.057	0.041	0.039	1.435	1.605	7.477	1.221

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	33	50	59	42	24
normalized size	1	1.	1.	1.5	2.27	2.68	1.91	1.09
time (sec)	N/A	0.028	0.013	0.029	1.417	1.512	0.58	1.226

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	30	46	49	107	46	41
normalized size	1	1.	0.91	1.39	1.48	3.24	1.39	1.24
time (sec)	N/A	0.02	0.009	0.043	1.41	1.754	3.391	1.2

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	35	35	79	32	59	78	66	70
normalized size	1	1.	2.26	0.91	1.69	2.23	1.89	2.
time (sec)	N/A	0.053	0.078	0.035	1.413	1.723	20.426	1.144

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	28	51	55	101	88	100
normalized size	1	1.	0.97	1.76	1.9	3.48	3.03	3.45
time (sec)	N/A	0.043	0.029	0.039	1.444	1.688	2.963	1.151

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	67	40	69	104	78	82
normalized size	1	1.	1.46	0.87	1.5	2.26	1.7	1.78
time (sec)	N/A	0.058	0.023	0.039	1.426	1.682	21.631	1.281

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	33	54	59	133	167
normalized size	1	1.	1.	1.5	2.45	2.68	6.05	7.59
time (sec)	N/A	0.039	0.014	0.03	1.437	1.625	3.355	1.291

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	89	163	207	230	486	124
normalized size	1	1.	0.72	1.31	1.67	1.85	3.92	1.
time (sec)	N/A	0.136	0.192	0.05	1.432	1.612	14.691	1.368

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	75	140	176	207	374	109
normalized size	1	1.	0.66	1.24	1.56	1.83	3.31	0.96
time (sec)	N/A	0.121	0.079	0.044	1.429	1.646	10.218	1.398

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	67	117	144	176	330	93
normalized size	1	1.	0.96	1.67	2.06	2.51	4.71	1.33
time (sec)	N/A	0.062	0.096	0.041	1.432	1.6	9.61	1.355

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	59	91	109	151	102	73
normalized size	1	1.	0.97	1.49	1.79	2.48	1.67	1.2
time (sec)	N/A	0.037	0.074	0.039	1.426	1.626	5.305	1.31

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	125	86	120	169	201	113
normalized size	1	1.	2.12	1.46	2.03	2.86	3.41	1.92
time (sec)	N/A	0.101	0.078	0.037	1.424	1.71	12.066	1.44

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	84	91	127	193	153	189
normalized size	1	1.	1.45	1.57	2.19	3.33	2.64	3.26
time (sec)	N/A	0.103	0.175	0.039	1.435	1.6	4.607	1.255

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	147	95	132	203	226	259
normalized size	1	1.	2.19	1.42	1.97	3.03	3.37	3.87
time (sec)	N/A	0.105	0.101	0.048	1.425	1.646	7.363	1.436

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	91	100	134	154	270	315
normalized size	1	1.	1.21	1.33	1.79	2.05	3.6	4.2
time (sec)	N/A	0.093	0.029	0.039	1.444	1.519	7.482	1.388

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	99	125	165	178	415	324
normalized size	1	1.	0.97	1.23	1.62	1.75	4.07	3.18
time (sec)	N/A	0.119	0.03	0.041	1.461	1.557	10.617	1.408

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	107	168	196	207	522	401
normalized size	1	1.	0.83	1.3	1.52	1.6	4.05	3.11
time (sec)	N/A	0.14	0.035	0.043	1.43	1.695	11.008	1.365

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	115	193	227	232	644	572
normalized size	1	1.	0.74	1.24	1.46	1.49	4.13	3.67
time (sec)	N/A	0.173	0.039	0.049	1.425	1.703	15.382	1.253

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	91	186	238	261	512	140
normalized size	1	1.	0.61	1.26	1.61	1.76	3.46	0.95
time (sec)	N/A	0.238	0.123	0.055	1.441	1.674	14.493	1.297

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	83	163	207	231	423	124
normalized size	1	1.	0.69	1.35	1.71	1.91	3.5	1.02
time (sec)	N/A	0.201	0.086	0.046	1.431	1.649	12.148	1.195

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	75	140	176	205	355	109
normalized size	1	1.	0.8	1.49	1.87	2.18	3.78	1.16
time (sec)	N/A	0.128	0.104	0.046	1.438	1.624	11.21	1.204

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	67	114	140	178	136	89
normalized size	1	1.	0.74	1.25	1.54	1.96	1.49	0.98
time (sec)	N/A	0.057	0.083	0.036	1.446	1.632	6.464	1.392

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	135	110	153	193	226	128
normalized size	1	1.	1.8	1.47	2.04	2.57	3.01	1.71
time (sec)	N/A	0.173	0.104	0.036	1.435	1.628	13.208	1.258

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	143	113	157	228	199	205
normalized size	1	1.	1.72	1.36	1.89	2.75	2.4	2.47
time (sec)	N/A	0.174	0.102	0.042	1.44	1.542	7.113	1.277

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	155	116	161	246	228	286
normalized size	1	1.	1.68	1.26	1.75	2.67	2.48	3.11
time (sec)	N/A	0.179	0.124	0.04	1.432	1.619	5.463	1.308

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	156	119	165	225	279	338
normalized size	1	1.	1.77	1.35	1.88	2.56	3.17	3.84
time (sec)	N/A	0.179	0.104	0.042	1.441	1.652	6.576	1.238

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	99	144	165	182	347	405
normalized size	1	1.	0.97	1.41	1.62	1.78	3.4	3.97
time (sec)	N/A	0.163	0.032	0.046	1.437	1.566	8.72	1.164

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	107	190	196	207	476	401
normalized size	1	1.	0.83	1.47	1.52	1.6	3.69	3.11
time (sec)	N/A	0.19	0.033	0.043	1.43	1.627	10.122	1.187

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	99	209	269	302	842	158
normalized size	1	1.	0.57	1.21	1.55	1.75	4.87	0.91
time (sec)	N/A	0.333	0.222	0.071	1.45	1.583	23.78	1.197

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	91	186	238	261	683	140
normalized size	1	1.	0.62	1.27	1.63	1.79	4.68	0.96
time (sec)	N/A	0.304	0.103	0.062	1.452	1.619	16.703	1.207

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	158	158	83	163	207	236	617	127
normalized size	1	1.	0.53	1.03	1.31	1.49	3.91	0.8
time (sec)	N/A	0.141	0.124	0.052	1.433	1.62	15.639	1.221

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	75	137	171	209	226	105
normalized size	1	1.	0.61	1.11	1.39	1.7	1.84	0.85
time (sec)	N/A	0.081	0.095	0.043	1.445	1.581	8.737	1.252

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	142	115	159	212	420	135
normalized size	1	1.	1.41	1.14	1.57	2.1	4.16	1.34
time (sec)	N/A	0.246	0.096	0.04	1.441	1.664	20.69	1.304

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	152	136	188	258	306	221
normalized size	1	1.	1.43	1.28	1.77	2.43	2.89	2.08
time (sec)	N/A	0.242	0.127	0.046	1.446	1.585	7.413	1.285

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	106	138	190	269	357	302
normalized size	1	1.	0.91	1.19	1.64	2.32	3.08	2.6
time (sec)	N/A	0.247	0.215	0.046	1.434	1.697	8.898	1.243

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	164	140	193	274	359	365
normalized size	1	1.	1.37	1.17	1.61	2.28	2.99	3.04
time (sec)	N/A	0.249	0.135	0.042	1.439	1.65	7.753	1.243

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	116	125	118	163	234	505	427
normalized size	1	1.05	1.14	1.07	1.48	2.13	4.59	3.88
time (sec)	N/A	0.248	0.21	0.046	1.428	1.548	11.285	1.179

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	107	207	196	212	607	478
normalized size	1	1.	0.83	1.6	1.52	1.64	4.71	3.71
time (sec)	N/A	0.246	0.036	0.053	1.43	1.583	12.392	1.187

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	115	255	227	239	801	572
normalized size	1	1.	0.74	1.63	1.46	1.53	5.13	3.67
time (sec)	N/A	0.264	0.037	0.051	1.436	1.605	17.7	1.18

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	81	166	0	228	0	154
normalized size	1	1.	0.55	1.14	0.	1.56	0.	1.05
time (sec)	N/A	0.343	0.065	0.046	0.	1.566	0.	1.214

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	72	142	0	198	0	136
normalized size	1	1.	0.63	1.25	0.	1.74	0.	1.19
time (sec)	N/A	0.286	0.051	0.042	0.	1.55	0.	1.322

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	64	120	0	178	0	122
normalized size	1	1.	0.89	1.67	0.	2.47	0.	1.69
time (sec)	N/A	0.201	0.044	0.044	0.	1.579	0.	1.189

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	53	98	0	155	0	105
normalized size	1	1.	0.83	1.53	0.	2.42	0.	1.64
time (sec)	N/A	0.078	0.035	0.037	0.	1.505	0.	1.237

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	43	43	46	76	0	136	0	72
normalized size	1	1.	1.07	1.77	0.	3.16	0.	1.67
time (sec)	N/A	0.041	0.023	0.04	0.	1.549	0.	1.216

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	55	61	0	124	0	108
normalized size	1	1.	1.22	1.36	0.	2.76	0.	2.4
time (sec)	N/A	0.168	0.025	0.04	0.	1.531	0.	1.274

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	68	77	0	165	0	215
normalized size	1	1.	0.99	1.12	0.	2.39	0.	3.12
time (sec)	N/A	0.193	0.025	0.042	0.	1.611	0.	1.252

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	83	99	0	200	0	302
normalized size	1	1.	0.83	0.99	0.	2.	0.	3.02
time (sec)	N/A	0.258	0.04	0.05	0.	1.527	0.	1.269

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	91	142	0	217	0	382
normalized size	1	1.	0.73	1.14	0.	1.74	0.	3.06
time (sec)	N/A	0.329	0.041	0.046	0.	1.58	0.	1.265

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	80	187	0	286	0	0
normalized size	1	1.	0.5	1.18	0.	1.8	0.	0.
time (sec)	N/A	0.517	0.094	0.049	0.	1.633	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	72	164	0	267	0	0
normalized size	1	1.	0.69	1.58	0.	2.57	0.	0.
time (sec)	N/A	0.304	0.083	0.044	0.	1.61	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	64	143	0	248	0	0
normalized size	1	1.	0.62	1.38	0.	2.38	0.	0.
time (sec)	N/A	0.188	0.071	0.046	0.	1.617	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	57	122	0	225	0	0
normalized size	1	1.	0.77	1.65	0.	3.04	0.	0.
time (sec)	N/A	0.08	0.066	0.042	0.	1.58	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	32	32	29	35	0	127	0	0
normalized size	1	1.	0.91	1.09	0.	3.97	0.	0.
time (sec)	N/A	0.034	0.009	0.03	0.	1.607	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	78	147	0	207	0	0
normalized size	1	1.	1.05	1.99	0.	2.8	0.	0.
time (sec)	N/A	0.2	0.034	0.04	0.	1.572	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	91	118	0	252	0	0
normalized size	1	1.	0.92	1.19	0.	2.55	0.	0.
time (sec)	N/A	0.267	0.042	0.045	0.	1.603	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	103	181	0	285	0	0
normalized size	1	1.	0.78	1.37	0.	2.16	0.	0.
time (sec)	N/A	0.334	0.046	0.055	0.	1.584	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	111	226	0	304	0	0
normalized size	1	1.	0.69	1.4	0.	1.89	0.	0.
time (sec)	N/A	0.423	0.053	0.048	0.	1.629	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	122	208	0	355	0	267
normalized size	1	1.	0.9	1.54	0.	2.63	0.	1.98
time (sec)	N/A	0.415	0.136	0.052	0.	1.691	0.	1.242

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	72	186	0	329	0	251
normalized size	1	1.	0.53	1.36	0.	2.4	0.	1.83
time (sec)	N/A	0.333	0.072	0.049	0.	1.684	0.	1.204

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	77	167	0	300	0	225
normalized size	1	1.	0.72	1.56	0.	2.8	0.	2.1
time (sec)	N/A	0.22	0.058	0.058	0.	1.567	0.	1.186

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	35	41	0	189	0	163
normalized size	1	1.	0.54	0.63	0.	2.91	0.	2.51
time (sec)	N/A	0.078	0.018	0.034	0.	1.566	0.	1.219

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	35	40	0	188	0	196
normalized size	1	1.	0.54	0.62	0.	2.89	0.	3.02
time (sec)	N/A	0.052	0.016	0.033	0.	1.575	0.	1.162

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	71	275	0	284	0	255
normalized size	1	1.	0.73	2.84	0.	2.93	0.	2.63
time (sec)	N/A	0.274	0.086	0.048	0.	1.65	0.	1.22

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	101	248	0	338	0	363
normalized size	1	1.	0.8	1.95	0.	2.66	0.	2.86
time (sec)	N/A	0.351	0.055	0.048	0.	1.488	0.	1.23

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	113	223	0	375	0	456
normalized size	1	1.	0.7	1.38	0.	2.31	0.	2.81
time (sec)	N/A	0.43	0.059	0.055	0.	1.631	0.	1.26

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	121	355	0	389	0	531
normalized size	1	1.	0.65	1.9	0.	2.08	0.	2.84
time (sec)	N/A	0.525	0.062	0.054	0.	1.667	0.	1.344

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	126	252	0	454	0	340
normalized size	1	1.	0.76	1.52	0.	2.73	0.	2.05
time (sec)	N/A	0.533	0.324	0.057	0.	1.734	0.	1.277

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	168	168	95	231	0	423	0	325
normalized size	1	1.	0.57	1.38	0.	2.52	0.	1.93
time (sec)	N/A	0.403	0.087	0.056	0.	1.591	0.	1.199

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	94	210	0	394	0	297
normalized size	1	1.	0.68	1.52	0.	2.86	0.	2.15
time (sec)	N/A	0.272	0.233	0.065	0.	1.643	0.	1.22

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	43	49	0	250	0	200
normalized size	1	1.	0.44	0.51	0.	2.58	0.	2.06
time (sec)	N/A	0.208	0.026	0.034	0.	1.794	0.	1.295

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	42	48	0	240	0	200
normalized size	1	1.	0.43	0.49	0.	2.47	0.	2.06
time (sec)	N/A	0.101	0.019	0.038	0.	1.883	0.	1.2

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	43	49	0	254	0	269
normalized size	1	1.	0.44	0.51	0.	2.62	0.	2.77
time (sec)	N/A	0.07	0.02	0.035	0.	1.886	0.	1.295

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	79	451	0	379	0	328
normalized size	1	1.	0.62	3.52	0.	2.96	0.	2.56
time (sec)	N/A	0.311	0.172	0.047	0.	1.78	0.	1.348

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	109	423	0	429	0	436
normalized size	1	1.	0.7	2.73	0.	2.77	0.	2.81
time (sec)	N/A	0.449	0.061	0.053	0.	1.971	0.	1.266

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	121	397	0	471	0	529
normalized size	1	1.	0.63	2.07	0.	2.45	0.	2.76
time (sec)	N/A	0.524	0.065	0.052	0.	1.979	0.	1.229

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	42	41	54	97	37	28
normalized size	1	1.	0.69	0.67	0.89	1.59	0.61	0.46
time (sec)	N/A	0.032	0.03	0.036	1.426	1.755	0.421	1.158

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	47	47	37	29	38	86	27	26
normalized size	1	1.	0.79	0.62	0.81	1.83	0.57	0.55
time (sec)	N/A	0.017	0.012	0.034	1.433	1.799	0.236	1.124

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	51	57	76	117	54	38
normalized size	1	1.	0.59	0.66	0.87	1.34	0.62	0.44
time (sec)	N/A	0.048	0.031	0.035	1.43	1.865	0.865	1.286

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	44	43	57	101	44	34
normalized size	1	1.	0.66	0.64	0.85	1.51	0.66	0.51
time (sec)	N/A	0.029	0.022	0.045	1.448	1.737	0.419	1.176

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	13	17	15	23	8	15
normalized size	1	1.	0.72	0.94	0.83	1.28	0.44	0.83
time (sec)	N/A	0.034	0.005	0.027	0.938	1.752	0.14	1.143

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	47	2	3
normalized size	1	1.	1.	1.5	1.5	23.5	1.	1.5
time (sec)	N/A	0.019	0.003	0.033	1.425	1.681	0.136	1.131

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	24	24	105	0	32
normalized size	1	1.	1.	1.2	1.2	5.25	0.	1.6
time (sec)	N/A	0.034	0.034	0.032	1.423	1.875	0.	1.239

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	14	22	47	0	28
normalized size	1	1.	1.	0.78	1.22	2.61	0.	1.56
time (sec)	N/A	0.018	0.004	0.029	1.419	1.731	0.	1.275

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	28	35	39	85	0	70
normalized size	1	1.	0.57	0.71	0.8	1.73	0.	1.43
time (sec)	N/A	0.048	0.011	0.03	0.95	1.802	0.	1.29

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	23	30	32	73	0	51
normalized size	1	1.	0.61	0.79	0.84	1.92	0.	1.34
time (sec)	N/A	0.029	0.009	0.03	0.953	1.672	0.	1.195

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	34	47	21	34	65	99	0	36
normalized size	1	1.38	0.62	1.	1.91	2.91	0.	1.06
time (sec)	N/A	0.062	0.01	0.028	0.972	1.729	0.	1.246

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	29	49	81	0	27
normalized size	1	1.	0.7	1.26	2.13	3.52	0.	1.17
time (sec)	N/A	0.034	0.007	0.03	0.973	1.807	0.	1.204

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	21	28	30	66	0	51
normalized size	1	1.	0.58	0.78	0.83	1.83	0.	1.42
time (sec)	N/A	0.042	0.008	0.036	0.943	1.777	0.	1.289

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	16	23	23	55	0	32
normalized size	1	1.	0.64	0.92	0.92	2.2	0.	1.28
time (sec)	N/A	0.023	0.006	0.029	0.947	1.708	0.	1.234

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	29	51	80	0	27
normalized size	1	1.	0.7	1.26	2.22	3.48	0.	1.17
time (sec)	N/A	0.039	0.007	0.031	0.984	1.861	0.	1.181

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	24	31	68	0	18
normalized size	1	1.	1.	2.18	2.82	6.18	0.	1.64
time (sec)	N/A	0.02	0.004	0.029	0.98	1.877	0.	1.194

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	16	23	20	55	0	32
normalized size	1	1.	0.64	0.92	0.8	2.2	0.	1.28
time (sec)	N/A	0.039	0.007	0.032	0.962	2.053	0.	1.154

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	20	16	42	0	20
normalized size	1	1.	1.	1.82	1.45	3.82	0.	1.82
time (sec)	N/A	0.019	0.004	0.029	0.97	1.85	0.	1.151

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	36	61	0	204	0	59
normalized size	1	1.	0.86	1.45	0.	4.86	0.	1.4
time (sec)	N/A	0.046	0.02	0.042	0.	1.818	0.	1.231

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	52	0	185	0	50
normalized size	1	1.	1.	1.68	0.	5.97	0.	1.61
time (sec)	N/A	0.034	0.005	0.037	0.	1.823	0.	1.209

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	50	0	188	0	58
normalized size	1	1.	1.	1.47	0.	5.53	0.	1.71
time (sec)	N/A	0.045	0.007	0.065	0.	1.735	0.	1.25

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	40	0	122	0	50
normalized size	1	1.	1.	1.74	0.	5.3	0.	2.17
time (sec)	N/A	0.028	0.004	0.062	0.	1.786	0.	1.203

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	40	78	0	228	0	66
normalized size	1	1.	0.78	1.53	0.	4.47	0.	1.29
time (sec)	N/A	0.055	0.02	0.045	0.	1.774	0.	1.26

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	36	69	0	212	0	57
normalized size	1	1.	0.97	1.86	0.	5.73	0.	1.54
time (sec)	N/A	0.037	0.023	0.043	0.	1.921	0.	1.185

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	46	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.018	0.368	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	51	48	143	128	0	88
normalized size	1	1.	0.48	0.45	1.34	1.2	0.	0.82
time (sec)	N/A	0.158	0.028	0.039	1.034	1.807	0.	1.376

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	43	40	112	107	0	70
normalized size	1	1.	0.62	0.58	1.62	1.55	0.	1.01
time (sec)	N/A	0.09	0.023	0.033	1.03	1.833	0.	1.365

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	37	34	78	86	0	43
normalized size	1	1.	1.06	0.97	2.23	2.46	0.	1.23
time (sec)	N/A	0.045	0.013	0.03	1.017	1.78	0.	1.358

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	68	68	46	71	0	417	0	108
normalized size	1	1.	0.68	1.04	0.	6.13	0.	1.59
time (sec)	N/A	0.152	0.02	0.097	0.	1.896	0.	1.263

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	57	78	0	441	0	136
normalized size	1	1.	0.79	1.08	0.	6.12	0.	1.89
time (sec)	N/A	0.157	0.024	0.1	0.	1.94	0.	1.288

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	48	45	100	115	83	140
normalized size	1	1.	0.48	0.45	0.99	1.14	0.82	1.39
time (sec)	N/A	0.132	0.066	0.035	0.965	1.763	10.948	1.231

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	40	37	81	95	68	108
normalized size	1	1.	0.5	0.46	1.01	1.19	0.85	1.35
time (sec)	N/A	0.163	0.052	0.031	0.963	1.803	8.013	1.224

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	31	28	59	66	48	74
normalized size	1	1.	0.54	0.49	1.04	1.16	0.84	1.3
time (sec)	N/A	0.086	0.04	0.031	0.948	1.909	6.45	1.226

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	23	20	41	47	31	41
normalized size	1	1.	0.61	0.53	1.08	1.24	0.82	1.08
time (sec)	N/A	0.048	0.023	0.033	0.952	1.818	4.187	1.185

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	0	207	39	49
normalized size	1	1.	1.	0.82	0.	5.31	1.	1.26
time (sec)	N/A	0.105	0.03	0.035	0.	1.861	6.206	1.279

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	45	0	234	119	54
normalized size	1	1.	1.	1.05	0.	5.44	2.77	1.26
time (sec)	N/A	0.107	0.028	0.04	0.	1.989	12.968	1.234

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	55	65	0	288	270	97
normalized size	1	1.	0.81	0.96	0.	4.24	3.97	1.43
time (sec)	N/A	0.116	0.045	0.043	0.	1.945	23.728	1.172

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	63	80	0	331	439	134
normalized size	1	1.	0.71	0.9	0.	3.72	4.93	1.51
time (sec)	N/A	0.128	0.056	0.043	0.	1.849	42.367	1.215

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	71	93	0	370	639	170
normalized size	1	1.	0.65	0.85	0.	3.36	5.81	1.55
time (sec)	N/A	0.145	0.067	0.043	0.	1.947	102.727	1.222

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	92	146	0	659	0	201
normalized size	1	1.	0.37	0.59	0.	2.65	0.	0.81
time (sec)	N/A	0.18	0.08	0.099	0.	1.95	0.	1.362

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	84	129	0	605	0	169
normalized size	1	1.	0.5	0.76	0.	3.58	0.	1.
time (sec)	N/A	0.16	0.064	0.13	0.	1.844	0.	1.365

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	76	112	0	572	0	166
normalized size	1	1.	0.45	0.66	0.	3.38	0.	0.98
time (sec)	N/A	0.114	0.042	0.118	0.	1.893	0.	1.278

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	67	95	0	518	0	142
normalized size	1	1.	0.56	0.8	0.	4.35	0.	1.19
time (sec)	N/A	0.114	0.032	0.108	0.	1.785	0.	1.181

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	66	98	0	753	0	188
normalized size	1	1.	0.55	0.82	0.	6.33	0.	1.58
time (sec)	N/A	0.136	0.032	0.104	0.	1.945	0.	1.231

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	75	105	0	799	0	211
normalized size	1	1.	0.6	0.85	0.	6.44	0.	1.7
time (sec)	N/A	0.135	0.036	0.124	0.	1.989	0.	1.286

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	92	131	0	869	0	244
normalized size	1	1.	0.53	0.76	0.	5.02	0.	1.41
time (sec)	N/A	0.149	0.054	0.11	0.	1.999	0.	1.323

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	100	151	0	913	0	262
normalized size	1	1.	0.46	0.7	0.	4.23	0.	1.21
time (sec)	N/A	0.166	0.064	0.112	0.	2.092	0.	1.377

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	108	171	0	967	0	279
normalized size	1	1.	0.42	0.66	0.	3.73	0.	1.08
time (sec)	N/A	0.192	0.075	0.109	0.	2.139	0.	1.414

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	77	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.185	0.041	0.497	0.	0.	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	55	56	84	132	0	100
normalized size	1	1.	0.41	0.41	0.62	0.98	0.	0.74
time (sec)	N/A	0.213	0.036	0.032	1.03	1.741	0.	1.19

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	46	47	68	104	0	81
normalized size	1	1.	0.46	0.47	0.67	1.03	0.	0.8
time (sec)	N/A	0.121	0.029	0.032	1.02	1.847	0.	1.191

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	38	39	50	85	0	58
normalized size	1	1.	0.58	0.59	0.76	1.29	0.	0.88
time (sec)	N/A	0.065	0.019	0.03	0.999	1.796	0.	1.197

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	68	68	44	69	0	417	0	109
normalized size	1	1.	0.65	1.01	0.	6.13	0.	1.6
time (sec)	N/A	0.158	0.02	0.097	0.	1.901	0.	1.215

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	52	79	0	446	0	131
normalized size	1	1.	0.72	1.1	0.	6.19	0.	1.82
time (sec)	N/A	0.161	0.03	0.1	0.	1.864	0.	1.206

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	64	101	0	501	0	159
normalized size	1	1.	0.57	0.9	0.	4.47	0.	1.42
time (sec)	N/A	0.196	0.033	0.102	0.	1.929	0.	1.312

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	56	121	0	543	0	177
normalized size	1	1.	0.38	0.82	0.	3.67	0.	1.2
time (sec)	N/A	0.237	0.022	0.105	0.	1.961	0.	1.318

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	87	101	0	454	126	215
normalized size	1	1.	0.63	0.73	0.	3.27	0.91	1.55
time (sec)	N/A	0.173	0.121	0.036	0.	1.976	13.19	1.292

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	78	75	0	398	97	142
normalized size	1	1.	0.8	0.77	0.	4.1	1.	1.46
time (sec)	N/A	0.158	0.084	0.042	0.	1.904	10.483	1.199

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	70	73	0	367	94	142
normalized size	1	1.	0.72	0.75	0.	3.78	0.97	1.46
time (sec)	N/A	0.107	0.076	0.035	0.	1.721	8.682	1.291

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	61	59	0	312	75	104
normalized size	1	1.	0.8	0.78	0.	4.11	0.99	1.37
time (sec)	N/A	0.066	0.039	0.034	0.	1.608	6.072	1.297

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	58	0	417	80	90
normalized size	1	1.	1.	0.78	0.	5.64	1.08	1.22
time (sec)	N/A	0.138	0.026	0.041	0.	1.591	7.312	1.233

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	71	0	454	162	97
normalized size	1	1.	1.	0.9	0.	5.75	2.05	1.23
time (sec)	N/A	0.14	0.042	0.043	0.	1.666	11.323	1.209

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	93	95	0	525	352	143
normalized size	1	1.	0.88	0.9	0.	4.95	3.32	1.35
time (sec)	N/A	0.159	0.07	0.043	0.	1.684	28.206	1.205

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	101	110	0	567	614	180
normalized size	1	1.	0.8	0.87	0.	4.46	4.83	1.42
time (sec)	N/A	0.182	0.092	0.045	0.	1.706	131.152	1.161

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	109	123	0	622	0	216
normalized size	1	1.	0.74	0.83	0.	4.2	0.	1.46
time (sec)	N/A	0.209	0.105	0.047	0.	1.685	0.	1.167

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	76	79	116	170	0	146
normalized size	1	1.	0.32	0.34	0.49	0.72	0.	0.62
time (sec)	N/A	0.154	0.049	0.033	1.042	1.536	0.	1.29

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	68	71	101	158	0	126
normalized size	1	1.	0.35	0.36	0.51	0.8	0.	0.64
time (sec)	N/A	0.15	0.043	0.032	1.037	1.535	0.	1.329

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	60	63	86	134	0	105
normalized size	1	1.	0.38	0.4	0.55	0.85	0.	0.67
time (sec)	N/A	0.105	0.033	0.033	1.028	1.604	0.	1.261

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	51	54	68	108	0	77
normalized size	1	1.	0.5	0.52	0.66	1.05	0.	0.75
time (sec)	N/A	0.096	0.026	0.033	1.022	1.515	0.	1.258

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	51	78	0	466	0	127
normalized size	1	1.	0.48	0.73	0.	4.36	0.	1.19
time (sec)	N/A	0.129	0.035	0.108	0.	1.6	0.	1.221

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	64	82	0	483	0	144
normalized size	1	1.	0.57	0.73	0.	4.31	0.	1.29
time (sec)	N/A	0.119	0.029	0.128	0.	1.681	0.	1.239

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	65	100	0	547	0	178
normalized size	1	1.	0.4	0.61	0.	3.36	0.	1.09
time (sec)	N/A	0.13	0.027	0.113	0.	1.842	0.	1.231

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	206	209	65	111	0	591	0	198
normalized size	1	1.01	0.32	0.54	0.	2.87	0.	0.96
time (sec)	N/A	0.138	0.028	0.112	0.	1.943	0.	1.25

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	252	65	122	0	647	0	216
normalized size	1	1.01	0.26	0.49	0.	2.6	0.	0.87
time (sec)	N/A	0.155	0.029	0.125	0.	1.927	0.	1.325

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.026	0.343	0.	0.	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	32	32	41	81	0	0
normalized size	1	1.	0.86	0.86	1.11	2.19	0.	0.
time (sec)	N/A	0.035	0.024	0.027	0.988	1.901	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	77	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.024	0.261	0.	0.	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	65	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.02	0.148	0.	0.	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	65	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.017	0.159	0.	0.	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	63	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.017	0.06	0.	0.	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.01	0.204	0.	0.	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	0	120	0	43
normalized size	1	1.	1.	0.85	0.	3.08	0.	1.1
time (sec)	N/A	0.039	0.019	0.033	0.	1.928	0.	1.119

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	51	46	0	259	0	0
normalized size	1	1.	0.61	0.55	0.	3.08	0.	0.
time (sec)	N/A	0.056	0.031	0.032	0.	2.255	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	74	68	0	483	0	0
normalized size	1	1.	0.55	0.5	0.	3.58	0.	0.
time (sec)	N/A	0.081	0.044	0.04	0.	2.264	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.803	0.369	0.	0.	0.	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	86	142	269	282	357	355
normalized size	1	1.	0.69	1.14	2.15	2.26	2.86	2.84
time (sec)	N/A	0.256	0.242	0.044	1.467	2.173	11.207	1.179

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	75	119	162	252	228	279
normalized size	1	1.	0.77	1.23	1.67	2.6	2.35	2.88
time (sec)	N/A	0.191	0.186	0.04	1.469	2.256	6.933	1.207

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	78	95	132	201	151	188
normalized size	1	1.	1.2	1.46	2.03	3.09	2.32	2.89
time (sec)	N/A	0.123	0.1	0.039	1.443	2.214	6.37	1.194

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	41	41	42	34	68	85	61	74
normalized size	1	1.	1.02	0.83	1.66	2.07	1.49	1.8
time (sec)	N/A	0.068	0.04	0.036	0.947	2.101	16.261	1.123

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	52	96	0	154	0	99
normalized size	1	1.	0.8	1.48	0.	2.37	0.	1.52
time (sec)	N/A	0.096	0.038	0.043	0.	2.093	0.	1.177

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	53	140	0	246	0	171
normalized size	1	1.	0.51	1.35	0.	2.37	0.	1.64
time (sec)	N/A	0.185	0.133	0.047	0.	2.117	0.	1.19

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	61	184	0	328	0	244
normalized size	1	1.	0.45	1.35	0.	2.41	0.	1.79
time (sec)	N/A	0.282	0.147	0.046	0.	2.266	0.	1.173

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	69	228	0	420	0	317
normalized size	1	1.	0.41	1.36	0.	2.5	0.	1.89
time (sec)	N/A	0.352	0.179	0.05	0.	2.255	0.	1.163

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	46	0	0	0	274	0
normalized size	1	1.	0.78	0.	0.	0.	4.64	0.
time (sec)	N/A	0.08	0.028	0.335	0.	0.	7.343	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	64	61	78	143	63	80
normalized size	1	1.	1.03	0.98	1.26	2.31	1.02	1.29
time (sec)	N/A	0.113	0.217	0.036	0.948	2.07	0.461	1.19

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	40	40	42	39	51	99	39	53
normalized size	1	1.	1.05	0.98	1.27	2.48	0.98	1.32
time (sec)	N/A	0.102	0.148	0.034	0.949	2.061	0.363	1.201

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	40	40	42	39	50	99	37	51
normalized size	1	1.	1.05	0.98	1.25	2.48	0.92	1.27
time (sec)	N/A	0.098	0.121	0.037	0.949	2.018	0.337	1.138

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	20	24	41	17	24
normalized size	1	1.	1.	1.11	1.33	2.28	0.94	1.33
time (sec)	N/A	0.09	0.087	0.033	0.956	1.931	0.262	1.129

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	31	12	19
normalized size	1	1.	1.	1.08	1.38	2.38	0.92	1.46
time (sec)	N/A	0.054	0.034	0.029	0.937	2.009	0.096	1.13

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	31	37	49	88	26	50
normalized size	1	1.	0.82	0.97	1.29	2.32	0.68	1.32
time (sec)	N/A	0.088	0.029	0.038	0.949	2.031	0.338	1.17

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	51	74	150	49	57
normalized size	1	1.	0.96	0.96	1.4	2.83	0.92	1.08
time (sec)	N/A	0.114	0.09	0.037	0.945	2.176	0.427	1.155

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	63	67	103	219	73	69
normalized size	1	1.	0.85	0.91	1.39	2.96	0.99	0.93
time (sec)	N/A	0.133	0.122	0.035	0.972	2.18	0.551	1.167

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	71	82	127	273	94	80
normalized size	1	1.	0.81	0.93	1.44	3.1	1.07	0.91
time (sec)	N/A	0.139	0.147	0.035	0.957	2.189	0.652	1.138

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	85	180	487	282	354	354
normalized size	1	1.	0.83	1.75	4.73	2.74	3.44	3.44
time (sec)	N/A	0.168	0.244	0.052	1.475	2.152	13.383	1.19

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	77	118	254	165	104	107
normalized size	1	1.	1.	1.53	3.3	2.14	1.35	1.39
time (sec)	N/A	0.121	0.13	0.043	1.445	2.088	21.552	1.161

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	82	133	298	201	151	188
normalized size	1	1.	1.22	1.99	4.45	3.	2.25	2.81
time (sec)	N/A	0.158	0.111	0.044	1.456	2.117	9.193	1.194

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	50	50	47	84	205	144	104	92
normalized size	1	1.	0.94	1.68	4.1	2.88	2.08	1.84
time (sec)	N/A	0.164	0.053	0.041	1.481	2.119	7.769	1.192

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	99	99	62	168	0	236	0	170
normalized size	1	1.	0.63	1.7	0.	2.38	0.	1.72
time (sec)	N/A	0.12	0.034	0.046	0.	2.177	0.	1.163

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	61	212	0	331	0	243
normalized size	1	1.	0.48	1.66	0.	2.59	0.	1.9
time (sec)	N/A	0.257	0.123	0.051	0.	2.208	0.	1.227

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	69	256	0	398	0	316
normalized size	1	1.	0.45	1.65	0.	2.57	0.	2.04
time (sec)	N/A	0.334	0.148	0.052	0.	2.209	0.	1.229

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	77	300	0	516	0	389
normalized size	1	1.	0.41	1.58	0.	2.72	0.	2.05
time (sec)	N/A	0.443	0.182	0.059	0.	2.262	0.	1.315

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	81	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.044	0.353	0.	0.	0.	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	66	61	80	150	63	81
normalized size	1	1.	1.03	0.95	1.25	2.34	0.98	1.27
time (sec)	N/A	0.111	0.277	0.036	0.956	1.971	0.512	1.124

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	42	72	31	42
normalized size	1	1.	1.	0.9	1.4	2.4	1.03	1.4
time (sec)	N/A	0.098	0.182	0.033	0.943	1.908	0.371	1.132

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	38	38	40	37	46	97	37	47
normalized size	1	1.	1.05	0.97	1.21	2.55	0.97	1.24
time (sec)	N/A	0.101	0.134	0.031	0.939	1.964	0.387	1.17

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	27	27	29	28	36	65	26	38
normalized size	1	1.	1.07	1.04	1.33	2.41	0.96	1.41
time (sec)	N/A	0.096	0.094	0.034	0.951	2.024	0.329	1.174

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	32	55	17	35
normalized size	1	1.	1.	1.	1.28	2.2	0.68	1.4
time (sec)	N/A	0.065	0.038	0.035	0.959	2.055	0.42	1.118

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	51	66	140	41	57
normalized size	1	1.	0.96	0.96	1.25	2.64	0.77	1.08
time (sec)	N/A	0.102	0.029	0.036	0.953	1.964	0.454	1.21

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	63	66	101	216	73	68
normalized size	1	1.	0.89	0.93	1.42	3.04	1.03	0.96
time (sec)	N/A	0.127	0.105	0.034	0.96	1.958	0.575	1.149

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	71	81	126	275	94	78
normalized size	1	1.	0.8	0.91	1.42	3.09	1.06	0.88
time (sec)	N/A	0.145	0.137	0.037	0.975	1.99	0.704	1.169

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	79	96	153	347	114	89
normalized size	1	1.	0.75	0.91	1.46	3.3	1.09	0.85
time (sec)	N/A	0.149	0.16	0.052	0.973	2.013	0.891	1.168

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	1.043	0.332	0.	0.	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	85	232	0	286	0	354
normalized size	1	1.	0.61	1.66	0.	2.04	0.	2.53
time (sec)	N/A	0.42	0.239	0.052	0.	2.135	0.	1.187

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	77	209	0	259	0	278
normalized size	1	1.	0.69	1.88	0.	2.33	0.	2.5
time (sec)	N/A	0.312	0.181	0.053	0.	2.124	0.	1.2

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	83	186	0	205	0	188
normalized size	1	1.	1.01	2.27	0.	2.5	0.	2.29
time (sec)	N/A	0.238	0.116	0.044	0.	2.183	0.	1.14

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	50	50	47	106	0	146	0	92
normalized size	1	1.	0.94	2.12	0.	2.92	0.	1.84
time (sec)	N/A	0.151	0.053	0.045	0.	2.129	0.	1.136

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	30	35	53	26
normalized size	1	1.	1.	0.95	1.43	1.67	2.52	1.24
time (sec)	N/A	0.074	0.013	0.032	0.975	1.975	8.538	1.135

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	47	198	0	158	0	97
normalized size	1	1.	0.75	3.14	0.	2.51	0.	1.54
time (sec)	N/A	0.179	0.095	0.046	0.	2.109	0.	1.145

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	53	242	0	244	0	170
normalized size	1	1.	0.56	2.57	0.	2.6	0.	1.81
time (sec)	N/A	0.26	0.126	0.049	0.	2.183	0.	1.212

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	61	286	0	317	0	243
normalized size	1	1.	0.49	2.29	0.	2.54	0.	1.94
time (sec)	N/A	0.354	0.158	0.053	0.	2.106	0.	1.188

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	87	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	0.048	0.233	0.	0.	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	68	65	82	163	58	151
normalized size	1	1.	1.03	0.98	1.24	2.47	0.88	2.29
time (sec)	N/A	0.118	0.165	0.039	0.958	2.177	0.726	1.14

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	57	54	70	142	44	135
normalized size	1	1.	1.04	0.98	1.27	2.58	0.8	2.45
time (sec)	N/A	0.116	0.124	0.037	0.966	2.222	0.63	1.162

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	42	42	44	43	57	100	32	97
normalized size	1	1.	1.05	1.02	1.36	2.38	0.76	2.31
time (sec)	N/A	0.111	0.097	0.036	0.959	2.242	0.522	1.203

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	34	57	19	76
normalized size	1	1.	1.	1.04	1.36	2.28	0.76	3.04
time (sec)	N/A	0.066	0.036	0.034	0.961	2.291	0.427	1.187

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	21	27	39	19	55
normalized size	1	1.	0.9	1.05	1.35	1.95	0.95	2.75
time (sec)	N/A	0.088	0.015	0.028	0.941	2.21	0.307	1.258

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	18	18	40	36	47	70	36	47
normalized size	1	1.	2.22	2.	2.61	3.89	2.	2.61
time (sec)	N/A	0.11	0.073	0.036	0.965	2.202	0.353	1.219

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	51	73	138	58	115
normalized size	1	1.	0.98	0.88	1.26	2.38	1.	1.98
time (sec)	N/A	0.125	0.107	0.038	0.957	2.219	0.557	1.232

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	69	66	95	212	75	128
normalized size	1	1.	0.91	0.87	1.25	2.79	0.99	1.68
time (sec)	N/A	0.133	0.138	0.037	0.948	2.28	0.675	1.167

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	93	352	0	365	0	358
normalized size	1	1.	0.66	2.51	0.	2.61	0.	2.56
time (sec)	N/A	0.368	0.282	0.063	0.	2.514	0.	1.173

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	81	329	0	313	0	266
normalized size	1	1.	0.73	2.96	0.	2.82	0.	2.4
time (sec)	N/A	0.295	0.217	0.052	0.	2.476	0.	1.202

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	61	223	0	224	0	140
normalized size	1	1.	0.79	2.9	0.	2.91	0.	1.82
time (sec)	N/A	0.206	0.137	0.052	0.	2.503	0.	1.337

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	67	292	0	154	0	99
normalized size	1	1.	1.03	4.49	0.	2.37	0.	1.52
time (sec)	N/A	0.104	0.033	0.052	0.	2.216	0.	1.246

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	46	336	0	159	0	99
normalized size	1	1.	0.73	5.33	0.	2.52	0.	1.57
time (sec)	N/A	0.125	0.134	0.053	0.	2.333	0.	1.175

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	30	43	61	101	34	45
normalized size	1	1.	0.67	0.96	1.36	2.24	0.76	1.
time (sec)	N/A	0.119	0.131	0.034	0.982	2.195	23.522	1.172

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	68	424	0	297	0	0
normalized size	1	1.	0.71	4.42	0.	3.09	0.	0.
time (sec)	N/A	0.165	0.174	0.064	0.	2.473	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	76	468	0	342	0	0
normalized size	1	1.	0.61	3.74	0.	2.74	0.	0.
time (sec)	N/A	0.318	0.216	0.086	0.	2.444	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	225	225	101	172	0	810	0	0
normalized size	1	1.	0.45	0.76	0.	3.6	0.	0.
time (sec)	N/A	0.209	0.079	0.18	0.	3.031	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	95	154	0	745	0	0
normalized size	1	1.	0.52	0.85	0.	4.12	0.	0.
time (sec)	N/A	0.165	0.074	0.145	0.	2.925	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	87	136	0	676	0	0
normalized size	1	1.	0.51	0.8	0.	3.95	0.	0.
time (sec)	N/A	0.158	0.064	0.144	0.	2.771	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	70	108	0	559	0	0
normalized size	1	1.	0.55	0.84	0.	4.37	0.	0.
time (sec)	N/A	0.192	0.05	0.138	0.	2.673	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	85	69	89	0	529	0	0
normalized size	1	1.01	0.82	1.06	0.	6.3	0.	0.
time (sec)	N/A	0.144	0.038	0.128	0.	2.666	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	105	168	0	0	0	0
normalized size	1	1.	0.67	1.07	0.	0.	0.	0.
time (sec)	N/A	0.158	0.064	0.157	0.	0.	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	126	276	0	1146	0	0
normalized size	1	1.	0.64	1.39	0.	5.79	0.	0.
time (sec)	N/A	0.187	0.149	0.152	0.	3.133	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	139	390	0	1305	0	0
normalized size	1	1.	0.56	1.57	0.	5.24	0.	0.
time (sec)	N/A	0.203	0.171	0.148	0.	3.004	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	91	163	0	517	2210	0
normalized size	1	1.	0.63	1.12	0.	3.57	15.24	0.
time (sec)	N/A	0.185	0.14	0.131	0.	2.114	30.984	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	83	144	0	462	729	0
normalized size	1	1.	0.69	1.2	0.	3.85	6.08	0.
time (sec)	N/A	0.161	0.086	0.138	0.	2.17	23.473	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	75	108	0	398	143	0
normalized size	1	1.	0.78	1.12	0.	4.15	1.49	0.
time (sec)	N/A	0.143	0.062	0.127	0.	2.134	7.318	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	57	103	0	297	163	0
normalized size	1	1.	0.8	1.45	0.	4.18	2.3	0.
time (sec)	N/A	0.121	0.047	0.121	0.	2.151	21.405	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	120	0	275	0	132
normalized size	1	1.	1.	2.35	0.	5.39	0.	2.59
time (sec)	N/A	0.101	0.03	0.131	0.	2.194	0.	1.209

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	44	194	0	375	0	166
normalized size	1	1.	0.62	2.73	0.	5.28	0.	2.34
time (sec)	N/A	0.162	0.026	0.144	0.	2.106	0.	1.252

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	55	260	0	513	0	193
normalized size	1	1.	0.57	2.71	0.	5.34	0.	2.01
time (sec)	N/A	0.15	0.028	0.128	0.	1.893	0.	1.283

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	59	328	0	628	0	220
normalized size	1	1.	0.5	2.76	0.	5.28	0.	1.85
time (sec)	N/A	0.159	0.03	0.129	0.	1.837	0.	1.288

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	46	396	0	772	0	250
normalized size	1	1.	0.32	2.71	0.	5.29	0.	1.71
time (sec)	N/A	0.19	0.04	0.139	0.	1.773	0.	1.313

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	223	223	85	172	0	802	0	0
normalized size	1	1.	0.38	0.77	0.	3.6	0.	0.
time (sec)	N/A	0.184	0.064	0.137	0.	2.154	0.	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	77	154	0	744	0	0
normalized size	1	1.	0.35	0.71	0.	3.43	0.	0.
time (sec)	N/A	0.169	0.053	0.141	0.	2.076	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	66	136	0	674	0	0
normalized size	1	1.	0.38	0.77	0.	3.83	0.	0.
time (sec)	N/A	0.206	0.04	0.142	0.	2.153	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	42	109	0	564	0	0
normalized size	1	1.	0.33	0.86	0.	4.44	0.	0.
time (sec)	N/A	0.173	0.025	0.135	0.	2.127	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	105	165	0	975	0	0
normalized size	1	1.	0.68	1.06	0.	6.29	0.	0.
time (sec)	N/A	0.154	0.074	0.137	0.	2.576	0.	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	120	276	0	1131	0	0
normalized size	1	1.	0.62	1.42	0.	5.8	0.	0.
time (sec)	N/A	0.17	0.099	0.146	0.	2.633	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	139	390	0	0	0	0
normalized size	1	1.	0.56	1.57	0.	0.	0.	0.
time (sec)	N/A	0.205	0.14	0.148	0.	0.	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	147	504	0	0	0	0
normalized size	1	1.	0.5	1.72	0.	0.	0.	0.
time (sec)	N/A	0.231	0.16	0.177	0.	0.	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	225	225	108	172	0	819	0	0
normalized size	1	1.	0.48	0.76	0.	3.64	0.	0.
time (sec)	N/A	0.188	2.63	0.145	0.	2.355	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	95	154	0	744	0	0
normalized size	1	1.	0.53	0.86	0.	4.16	0.	0.
time (sec)	N/A	0.167	0.089	0.164	0.	2.52	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	87	136	0	680	0	0
normalized size	1	1.	0.64	0.99	0.	4.96	0.	0.
time (sec)	N/A	0.154	0.061	0.159	0.	2.469	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	71	109	0	566	0	0
normalized size	1	1.	0.56	0.87	0.	4.49	0.	0.
time (sec)	N/A	0.151	0.046	0.14	0.	2.145	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	67	91	0	532	0	0
normalized size	1	1.	0.74	1.01	0.	5.91	0.	0.
time (sec)	N/A	0.164	0.048	0.142	0.	2.136	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	71	92	0	537	0	0
normalized size	1	1.	0.81	1.05	0.	6.1	0.	0.
time (sec)	N/A	0.146	0.037	0.125	0.	2.085	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	105	168	0	986	0	0
normalized size	1	1.	0.66	1.06	0.	6.2	0.	0.
time (sec)	N/A	0.18	0.058	0.145	0.	2.453	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	127	276	0	0	0	0
normalized size	1	1.	0.61	1.33	0.	0.	0.	0.
time (sec)	N/A	0.189	0.167	0.152	0.	0.	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	139	390	0	0	0	0
normalized size	1	1.	0.55	1.55	0.	0.	0.	0.
time (sec)	N/A	0.205	0.216	0.151	0.	0.	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	133	305	0	811	0	0
normalized size	1	1.	0.7	1.61	0.	4.29	0.	0.
time (sec)	N/A	0.265	0.323	0.155	0.	1.736	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	125	281	0	741	0	0
normalized size	1	1.	0.76	1.71	0.	4.52	0.	0.
time (sec)	N/A	0.226	0.171	0.128	0.	1.974	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	116	257	0	655	0	0
normalized size	1	1.	0.83	1.85	0.	4.71	0.	0.
time (sec)	N/A	0.198	0.101	0.136	0.	1.905	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	96	229	0	539	0	0
normalized size	1	1.	0.85	2.03	0.	4.77	0.	0.
time (sec)	N/A	0.213	0.081	0.124	0.	1.969	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	190	0	509	0	0
normalized size	1	1.	1.	2.04	0.	5.47	0.	0.
time (sec)	N/A	0.163	0.049	0.124	0.	1.969	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	136	0	535	0	227
normalized size	1	1.	1.	1.42	0.	5.57	0.	2.36
time (sec)	N/A	0.161	0.047	0.126	0.	1.781	0.	1.328

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	134	0	529	0	225
normalized size	1	1.	1.	1.41	0.	5.57	0.	2.37
time (sec)	N/A	0.163	0.051	0.148	0.	1.902	0.	1.437

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	67	370	0	649	0	301
normalized size	1	1.	0.56	3.11	0.	5.45	0.	2.53
time (sec)	N/A	0.19	0.052	0.128	0.	1.948	0.	1.774

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	77	497	0	811	0	346
normalized size	1	1.	0.52	3.36	0.	5.48	0.	2.34
time (sec)	N/A	0.228	0.056	0.133	0.	2.	0.	2.302

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	82	626	0	973	0	389
normalized size	1	1.	0.47	3.62	0.	5.62	0.	2.25
time (sec)	N/A	0.257	0.058	0.161	0.	1.984	0.	3.871

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	234	227	0	848	0	0
normalized size	1	1.	0.88	0.85	0.	3.18	0.	0.
time (sec)	N/A	0.243	10.058	0.158	0.	2.199	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	225	225	200	209	0	802	0	0
normalized size	1	1.	0.89	0.93	0.	3.56	0.	0.
time (sec)	N/A	0.207	3.544	0.153	0.	2.216	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	97	191	0	728	0	0
normalized size	1	1.	0.54	1.06	0.	4.02	0.	0.
time (sec)	N/A	0.19	0.078	0.157	0.	2.177	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	80	164	0	621	0	0
normalized size	1	1.	0.6	1.23	0.	4.67	0.	0.
time (sec)	N/A	0.175	0.051	0.157	0.	2.127	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	80	140	0	586	0	0
normalized size	1	1.	0.65	1.14	0.	4.76	0.	0.
time (sec)	N/A	0.159	0.051	0.139	0.	2.25	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	86	143	0	598	0	0
normalized size	1	1.	0.68	1.13	0.	4.71	0.	0.
time (sec)	N/A	0.204	0.06	0.142	0.	2.3	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	44	143	0	609	0	0
normalized size	1	1.	0.34	1.09	0.	4.65	0.	0.
time (sec)	N/A	0.194	0.061	0.137	0.	2.138	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	199	199	162	279	0	1057	0	0
normalized size	1	1.	0.81	1.4	0.	5.31	0.	0.
time (sec)	N/A	0.205	0.154	0.147	0.	2.654	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	251	234	315	0	0	0	0
normalized size	1	1.	0.93	1.25	0.	0.	0.	0.
time (sec)	N/A	0.221	0.599	0.157	0.	0.	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	92	165	0	258	0	293
normalized size	1	1.	0.88	1.57	0.	2.46	0.	2.79
time (sec)	N/A	0.322	0.044	0.049	0.	1.896	0.	1.236

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	56	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.23	0.027	0.296	0.	0.	0.	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	88	125	0	617	0	0
normalized size	1	1.	0.49	0.7	0.	3.45	0.	0.
time (sec)	N/A	0.251	0.063	0.133	0.	2.024	0.	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	80	105	0	571	0	0
normalized size	1	1.	0.59	0.78	0.	4.23	0.	0.
time (sec)	N/A	0.189	0.051	0.128	0.	2.168	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	69	89	0	529	0	0
normalized size	1	1.	0.81	1.05	0.	6.22	0.	0.
time (sec)	N/A	0.142	0.04	0.13	0.	2.187	0.	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	61	90	0	506	0	0
normalized size	1	1.	0.71	1.05	0.	5.88	0.	0.
time (sec)	N/A	0.249	0.037	0.134	0.	2.27	0.	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	40	0	95	0	0
normalized size	1	1.	1.	0.98	0.	2.32	0.	0.
time (sec)	N/A	0.227	0.021	0.083	0.	1.904	0.	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	47	46	0	116	0	0
normalized size	1	1.	0.56	0.55	0.	1.38	0.	0.
time (sec)	N/A	0.216	0.028	0.08	0.	1.841	0.	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	55	54	0	136	0	0
normalized size	1	1.	0.43	0.42	0.	1.06	0.	0.
time (sec)	N/A	0.231	0.028	0.094	0.	1.892	0.	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	63	62	0	155	0	0
normalized size	1	1.	0.37	0.36	0.	0.9	0.	0.
time (sec)	N/A	0.233	0.032	0.089	0.	2.01	0.	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	50	172	0	412	0	193
normalized size	1	1.	0.38	1.32	0.	3.17	0.	1.48
time (sec)	N/A	0.245	0.033	0.131	0.	1.864	0.	1.333

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	50	155	0	373	0	173
normalized size	1	1.	0.48	1.48	0.	3.55	0.	1.65
time (sec)	N/A	0.217	0.035	0.128	0.	1.883	0.	1.288

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	77	139	0	329	0	153
normalized size	1	1.	0.96	1.74	0.	4.11	0.	1.91
time (sec)	N/A	0.143	0.071	0.118	0.	1.925	0.	1.297

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	118	0	275	0	132
normalized size	1	1.	1.	2.31	0.	5.39	0.	2.59
time (sec)	N/A	0.096	0.039	0.152	0.	1.779	0.	1.25

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	98	0	246	39	0
normalized size	1	1.	1.	2.09	0.	5.23	0.83	0.
time (sec)	N/A	0.191	0.032	0.135	0.	1.786	13.24	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	28	27	0	59	0	0
normalized size	1	1.	0.67	0.64	0.	1.4	0.	0.
time (sec)	N/A	0.216	0.034	0.082	0.	1.756	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	36	35	0	78	0	0
normalized size	1	1.	0.52	0.51	0.	1.13	0.	0.
time (sec)	N/A	0.222	0.041	0.086	0.	1.825	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	44	43	0	104	0	0
normalized size	1	1.	0.46	0.45	0.	1.08	0.	0.
time (sec)	N/A	0.224	0.049	0.087	0.	1.793	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	52	51	0	124	0	0
normalized size	1	1.	0.43	0.42	0.	1.02	0.	0.
time (sec)	N/A	0.238	0.059	0.089	0.	1.798	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	130	247	0	0	0	0
normalized size	1	1.	0.45	0.85	0.	0.	0.	0.
time (sec)	N/A	0.313	0.099	0.149	0.	0.	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	122	219	0	1073	0	0
normalized size	1	1.	0.49	0.88	0.	4.33	0.	0.
time (sec)	N/A	0.292	0.083	0.138	0.	2.696	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	114	191	0	0	0	0
normalized size	1	1.	0.56	0.94	0.	0.	0.	0.
time (sec)	N/A	0.228	0.072	0.138	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	105	165	0	975	0	0
normalized size	1	1.	0.68	1.06	0.	6.29	0.	0.
time (sec)	N/A	0.163	0.065	0.142	0.	2.999	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	103	166	0	949	0	0
normalized size	1	1.	0.67	1.08	0.	6.16	0.	0.
time (sec)	N/A	0.251	0.047	0.15	0.	2.955	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	93	150	0	668	0	0
normalized size	1	1.	0.63	1.02	0.	4.54	0.	0.
time (sec)	N/A	0.246	0.054	0.154	0.	2.597	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	101	181	0	728	0	0
normalized size	1	1.	0.53	0.95	0.	3.81	0.	0.
time (sec)	N/A	0.279	0.06	0.145	0.	2.572	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	109	209	0	760	0	0
normalized size	1	1.	0.46	0.88	0.	3.21	0.	0.
time (sec)	N/A	0.277	0.071	0.145	0.	2.518	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	117	237	0	814	0	0
normalized size	1	1.	0.42	0.84	0.	2.9	0.	0.
time (sec)	N/A	0.307	0.078	0.168	0.	2.66	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	87	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.293	0.056	0.381	0.	0.	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	182	182	88	125	0	625	0	0
normalized size	1	1.	0.48	0.69	0.	3.43	0.	0.
time (sec)	N/A	0.308	0.084	0.132	0.	3.238	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	80	107	0	582	0	0
normalized size	1	1.	0.58	0.78	0.	4.22	0.	0.
time (sec)	N/A	0.23	0.051	0.123	0.	3.003	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	67	91	0	532	0	0
normalized size	1	1.	0.74	1.01	0.	5.91	0.	0.
time (sec)	N/A	0.176	0.049	0.135	0.	2.906	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	60	91	0	505	0	0
normalized size	1	1.	0.67	1.02	0.	5.67	0.	0.
time (sec)	N/A	0.266	0.052	0.137	0.	2.721	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	47	46	0	99	0	0
normalized size	1	1.	0.56	0.55	0.	1.18	0.	0.
time (sec)	N/A	0.26	0.031	0.093	0.	2.338	0.	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	55	54	0	116	0	0
normalized size	1	1.	0.43	0.42	0.	0.91	0.	0.
time (sec)	N/A	0.267	0.03	0.089	0.	2.439	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	63	62	0	143	0	0
normalized size	1	1.	0.37	0.36	0.	0.83	0.	0.
time (sec)	N/A	0.263	0.033	0.084	0.	2.51	0.	0.

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	116	259	0	666	0	0
normalized size	1	1.	0.67	1.51	0.	3.87	0.	0.
time (sec)	N/A	0.352	0.136	0.117	0.	2.546	0.	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	108	237	0	609	0	0
normalized size	1	1.	0.73	1.61	0.	4.14	0.	0.
time (sec)	N/A	0.314	0.099	0.118	0.	2.683	0.	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	100	215	0	568	0	0
normalized size	1	1.	0.82	1.76	0.	4.66	0.	0.
time (sec)	N/A	0.24	0.079	0.115	0.	2.523	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	190	0	509	0	0
normalized size	1	1.	1.	2.04	0.	5.47	0.	0.
time (sec)	N/A	0.158	0.045	0.122	0.	2.564	0.	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	227	0	482	80	0
normalized size	1	1.	1.	2.64	0.	5.6	0.93	0.
time (sec)	N/A	0.237	0.034	0.126	0.	2.657	21.554	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	69	254	0	371	0	0
normalized size	1	1.	0.84	3.1	0.	4.52	0.	0.
time (sec)	N/A	0.236	0.062	0.125	0.	2.457	0.	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	79	278	0	433	0	375
normalized size	1	1.	0.7	2.46	0.	3.83	0.	3.32
time (sec)	N/A	0.253	0.075	0.128	0.	2.366	0.	2.176

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	87	302	0	463	0	481
normalized size	1	1.	0.77	2.67	0.	4.1	0.	4.26
time (sec)	N/A	0.281	0.125	0.127	0.	2.141	0.	2.439

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	95	326	0	520	0	586
normalized size	1	1.	0.58	2.	0.	3.19	0.	3.6
time (sec)	N/A	0.303	0.128	0.131	0.	2.301	0.	2.888

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	262	262	108	194	0	738	0	0
normalized size	1	1.	0.41	0.74	0.	2.82	0.	0.
time (sec)	N/A	0.267	0.062	0.149	0.	2.526	0.	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	218	218	100	176	0	683	0	0
normalized size	1	1.	0.46	0.81	0.	3.13	0.	0.
time (sec)	N/A	0.25	0.059	0.138	0.	2.522	0.	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	92	158	0	640	0	0
normalized size	1	1.	0.53	0.91	0.	3.68	0.	0.
time (sec)	N/A	0.199	0.056	0.151	0.	2.541	0.	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	80	140	0	586	0	0
normalized size	1	1.	0.65	1.14	0.	4.76	0.	0.
time (sec)	N/A	0.145	0.049	0.145	0.	2.431	0.	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	70	142	0	560	0	0
normalized size	1	1.	0.56	1.15	0.	4.52	0.	0.
time (sec)	N/A	0.237	0.051	0.152	0.	2.457	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	50	61	0	120	0	0
normalized size	1	1.	0.41	0.5	0.	0.98	0.	0.
time (sec)	N/A	0.223	0.03	0.087	0.	2.122	0.	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	58	69	0	142	0	0
normalized size	1	1.	0.35	0.42	0.	0.86	0.	0.
time (sec)	N/A	0.234	0.031	0.086	0.	2.124	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	66	77	0	167	0	0
normalized size	1	1.	0.31	0.36	0.	0.78	0.	0.
time (sec)	N/A	0.244	0.033	0.082	0.	2.108	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	74	85	0	178	0	0
normalized size	1	1.	0.29	0.33	0.	0.69	0.	0.
time (sec)	N/A	0.249	0.036	0.088	0.	2.142	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.619	0.207	0.	0.	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.425	0.147	0.	0.	0.	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.02	0.161	0.	0.	0.	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	130	71	262	0	0	0	0	0
normalized size	1	0.55	2.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.479	0.054	0.	0.	0.	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	187	184	180	0	0	0	0	0
normalized size	1	0.98	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.238	0.049	0.	0.	0.	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	95	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	0.04	0.049	0.	0.	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	194	0	0	0	0	0
normalized size	1	1.	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.172	0.443	0.046	0.	0.	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.169	180.004	0.118	0.	0.	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.15	180.007	0.122	0.	0.	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	180.006	0.121	0.	0.	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.17	180.005	0.122	0.	0.	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	70	233	707	413	1119	682
normalized size	1	1.	0.41	1.38	4.18	2.44	6.62	4.04
time (sec)	N/A	0.219	0.032	0.061	1.506	2.112	133.666	1.209

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	70	187	460	347	687	520
normalized size	1	1.	0.51	1.38	3.38	2.55	5.05	3.82
time (sec)	N/A	0.187	0.026	0.053	1.482	2.174	54.05	1.224

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	70	141	267	282	354	355
normalized size	1	1.	0.68	1.37	2.59	2.74	3.44	3.45
time (sec)	N/A	0.158	0.032	0.048	1.466	2.186	16.869	1.224

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	85	119	189	144	173
normalized size	1	1.	0.95	1.47	2.05	3.26	2.48	2.98
time (sec)	N/A	0.101	0.045	0.044	1.472	2.12	9.036	1.2

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	53	94	0	154	0	97
normalized size	1	1.	0.87	1.54	0.	2.52	0.	1.59
time (sec)	N/A	0.129	0.028	0.039	0.	2.072	0.	1.177

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	78	177	0	297	0	0
normalized size	1	1.	0.81	1.84	0.	3.09	0.	0.
time (sec)	N/A	0.15	0.05	0.048	0.	2.118	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	108	259	0	456	0	0
normalized size	1	1.	0.84	2.01	0.	3.53	0.	0.
time (sec)	N/A	0.197	0.076	0.053	0.	2.22	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	126	341	0	633	0	0
normalized size	1	1.	0.78	2.1	0.	3.91	0.	0.
time (sec)	N/A	0.223	0.097	0.059	0.	2.576	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	128	117	155	298	126	157
normalized size	1	1.	1.	0.91	1.21	2.33	0.98	1.23
time (sec)	N/A	0.15	0.042	0.038	0.965	2.073	2.038	1.166

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	84	111	208	90	112
normalized size	1	1.	1.	0.92	1.22	2.29	0.99	1.23
time (sec)	N/A	0.132	0.027	0.038	0.972	1.985	3.202	1.164

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	73	96	176	78	97
normalized size	1	1.	1.	0.94	1.23	2.26	1.	1.24
time (sec)	N/A	0.124	0.022	0.036	0.959	1.959	2.609	1.206

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	40	49	99	41	50
normalized size	1	1.	1.	0.98	1.2	2.41	1.	1.22
time (sec)	N/A	0.112	0.016	0.033	0.988	1.968	0.73	1.125

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	28	58	20	30
normalized size	1	1.	1.	1.05	1.33	2.76	0.95	1.43
time (sec)	N/A	0.068	0.012	0.033	0.966	1.998	0.449	1.17

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	36	46	88	37	49
normalized size	1	1.	1.	0.95	1.21	2.32	0.97	1.29
time (sec)	N/A	0.124	0.02	0.033	0.962	2.016	0.875	1.177

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	69	66	95	212	75	78
normalized size	1	1.	0.91	0.87	1.25	2.79	0.99	1.03
time (sec)	N/A	0.146	0.041	0.036	0.973	2.035	3.914	1.184

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	103	96	132	308	104	109
normalized size	1	1.	0.93	0.86	1.19	2.77	0.94	0.98
time (sec)	N/A	0.169	0.087	0.046	0.973	2.002	3.648	1.152

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	123	126	197	510	158	131
normalized size	1	1.	0.84	0.86	1.35	3.49	1.08	0.9
time (sec)	N/A	0.199	0.088	0.046	0.986	2.078	3.234	1.239

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	191	273	1022	398	935	682
normalized size	1	1.	1.	1.43	5.35	2.08	4.9	3.57
time (sec)	N/A	0.366	0.209	0.089	1.531	2.22	40.36	1.238

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	186	227	614	338	687	520
normalized size	1	1.	1.18	1.45	3.91	2.15	4.38	3.31
time (sec)	N/A	0.315	0.1	0.072	1.467	2.274	25.356	1.268

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	128	181	485	282	357	354
normalized size	1	1.	1.03	1.46	3.91	2.27	2.88	2.85
time (sec)	N/A	0.271	0.065	0.053	1.576	2.1	23.643	1.184

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	56	121	286	190	150	174
normalized size	1	1.	0.77	1.66	3.92	2.6	2.05	2.38
time (sec)	N/A	0.209	0.044	0.043	1.467	2.123	13.141	1.236

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	78	168	0	236	0	170
normalized size	1	1.	0.82	1.77	0.	2.48	0.	1.79
time (sec)	N/A	0.226	0.076	0.043	0.	2.12	0.	1.232

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	88	212	0	317	0	243
normalized size	1	1.	0.7	1.7	0.	2.54	0.	1.94
time (sec)	N/A	0.335	0.099	0.05	0.	2.153	0.	1.189

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	96	256	0	482	0	0
normalized size	1	1.	0.62	1.65	0.	3.11	0.	0.
time (sec)	N/A	0.44	0.119	0.058	0.	2.235	0.	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	124	367	0	645	0	0
normalized size	1	1.	0.67	1.98	0.	3.49	0.	0.
time (sec)	N/A	0.564	0.146	0.061	0.	2.58	0.	0.

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	116	105	139	269	112	140
normalized size	1	1.	1.	0.91	1.2	2.32	0.97	1.21
time (sec)	N/A	0.143	0.036	0.036	0.981	2.085	3.897	1.145

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	100	93	124	231	100	126
normalized size	1	1.	1.	0.93	1.24	2.31	1.	1.26
time (sec)	N/A	0.137	0.028	0.037	1.007	2.273	3.268	1.217

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	60	80	151	65	81
normalized size	1	1.	1.	0.95	1.27	2.4	1.03	1.29
time (sec)	N/A	0.123	0.021	0.036	0.978	2.309	0.741	1.159

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	50	62	122	51	63
normalized size	1	1.	1.	0.98	1.22	2.39	1.	1.24
time (sec)	N/A	0.119	0.017	0.036	0.985	2.244	0.622	1.214

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	33	43	88	26	46
normalized size	1	1.	1.	1.	1.3	2.67	0.79	1.39
time (sec)	N/A	0.077	0.018	0.043	0.965	1.948	0.828	1.152

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	51	66	140	41	57
normalized size	1	1.	1.	0.96	1.25	2.64	0.77	1.08
time (sec)	N/A	0.134	0.03	0.039	0.959	1.792	1.23	1.185

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	63	66	101	215	83	68
normalized size	1	1.	0.89	0.93	1.42	3.03	1.17	0.96
time (sec)	N/A	0.136	0.035	0.034	0.985	1.879	2.376	1.172

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	89	95	144	370	114	99
normalized size	1	1.	0.8	0.86	1.3	3.33	1.03	0.89
time (sec)	N/A	0.166	0.065	0.039	1.012	1.865	2.86	1.175

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	98	125	182	479	144	130
normalized size	1	1.	0.67	0.86	1.25	3.28	0.99	0.89
time (sec)	N/A	0.192	0.097	0.043	1.006	1.921	2.248	1.315

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	70	249	0	413	1110	680
normalized size	1	1.	0.41	1.47	0.	2.44	6.57	4.02
time (sec)	N/A	0.232	0.028	0.085	0.	1.986	25.636	1.282

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	70	203	0	347	692	518
normalized size	1	1.	0.51	1.49	0.	2.55	5.09	3.81
time (sec)	N/A	0.223	0.025	0.056	0.	2.054	36.58	1.219

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	70	157	0	282	381	354
normalized size	1	1.	0.68	1.52	0.	2.74	3.7	3.44
time (sec)	N/A	0.17	0.025	0.046	0.	1.826	15.089	1.225

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	100	0	189	177	173
normalized size	1	1.	0.95	1.72	0.	3.26	3.05	2.98
time (sec)	N/A	0.112	0.038	0.039	0.	2.242	7.114	1.196

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	192	0	153	0	96
normalized size	1	1.	0.9	3.2	0.	2.55	0.	1.6
time (sec)	N/A	0.141	0.029	0.047	0.	2.002	0.	1.261

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	78	274	0	296	0	0
normalized size	1	1.	0.8	2.82	0.	3.05	0.	0.
time (sec)	N/A	0.163	0.054	0.053	0.	1.839	0.	0.

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	108	356	0	455	0	0
normalized size	1	1.	0.83	2.74	0.	3.5	0.	0.
time (sec)	N/A	0.199	0.076	0.062	0.	1.909	0.	0.

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	126	438	0	632	0	0
normalized size	1	1.	0.77	2.69	0.	3.88	0.	0.
time (sec)	N/A	0.241	0.096	0.069	0.	2.387	0.	0.

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	84	111	208	88	216
normalized size	1	1.	1.	0.92	1.22	2.29	0.97	2.37
time (sec)	N/A	0.139	0.026	0.042	0.962	1.926	3.565	1.171

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	73	96	176	76	184
normalized size	1	1.	1.	0.94	1.23	2.26	0.97	2.36
time (sec)	N/A	0.128	0.022	0.037	0.967	1.859	2.22	1.234

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	51	99	39	151
normalized size	1	1.	1.	0.98	1.27	2.48	0.98	3.78
time (sec)	N/A	0.116	0.016	0.035	0.96	1.736	1.935	1.347

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	28	58	20	119
normalized size	1	1.	1.	1.05	1.33	2.76	0.95	5.67
time (sec)	N/A	0.07	0.012	0.034	0.967	1.919	0.565	1.233

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	36	45	88	37	74
normalized size	1	1.	0.8	1.03	1.29	2.51	1.06	2.11
time (sec)	N/A	0.132	0.027	0.035	0.963	1.795	0.733	1.318

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	68	66	95	212	75	136
normalized size	1	1.	0.92	0.89	1.28	2.86	1.01	1.84
time (sec)	N/A	0.145	0.044	0.039	0.971	1.875	2.008	1.218

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	104	96	132	308	104	189
normalized size	1	1.	0.95	0.88	1.21	2.83	0.95	1.73
time (sec)	N/A	0.166	0.062	0.041	0.972	1.863	3.706	1.198

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	121	126	197	510	158	221
normalized size	1	1.	0.84	0.88	1.37	3.54	1.1	1.53
time (sec)	N/A	0.201	0.09	0.047	0.992	1.773	5.427	1.215

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	191	289	0	398	0	683
normalized size	1	1.	1.	1.51	0.	2.08	0.	3.58
time (sec)	N/A	0.362	0.183	0.104	0.	2.215	0.	1.258

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	186	243	0	338	695	521
normalized size	1	1.	1.18	1.55	0.	2.15	4.43	3.32
time (sec)	N/A	0.321	0.099	0.066	0.	2.184	45.898	1.272

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	128	299	0	282	384	355
normalized size	1	1.	1.02	2.39	0.	2.26	3.07	2.84
time (sec)	N/A	0.276	0.081	0.055	0.	2.296	33.257	1.223

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	57	266	0	190	0	176
normalized size	1	1.	0.77	3.59	0.	2.57	0.	2.38
time (sec)	N/A	0.208	0.063	0.057	0.	2.185	0.	1.257

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	78	330	0	238	0	171
normalized size	1	1.	0.8	3.4	0.	2.45	0.	1.76
time (sec)	N/A	0.234	0.079	0.062	0.	2.156	0.	1.249

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	86	412	0	319	0	244
normalized size	1	1.	0.67	3.19	0.	2.47	0.	1.89
time (sec)	N/A	0.363	0.101	0.066	0.	2.366	0.	1.217

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	94	494	0	483	0	0
normalized size	1	1.	0.59	3.11	0.	3.04	0.	0.
time (sec)	N/A	0.451	0.126	0.076	0.	2.631	0.	0.

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	122	576	0	647	0	0
normalized size	1	1.	0.65	3.05	0.	3.42	0.	0.
time (sec)	N/A	0.636	0.148	0.095	0.	2.633	0.	0.

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	114	118	0	1355	0	0
normalized size	1	1.	0.3	0.32	0.	3.62	0.	0.
time (sec)	N/A	0.204	0.102	0.213	0.	2.599	0.	0.

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	98	102	0	1142	0	0
normalized size	1	1.	0.33	0.34	0.	3.82	0.	0.
time (sec)	N/A	0.19	0.059	0.164	0.	2.562	0.	0.

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	82	86	0	984	0	0
normalized size	1	1.	0.37	0.39	0.	4.49	0.	0.
time (sec)	N/A	0.179	0.046	0.153	0.	2.634	0.	0.

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	72	70	0	778	0	0
normalized size	1	1.	0.49	0.48	0.	5.33	0.	0.
time (sec)	N/A	0.167	0.035	0.157	0.	2.551	0.	0.

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	37	52	23	648	0	0
normalized size	1	1.	0.54	0.76	0.34	9.53	0.	0.
time (sec)	N/A	0.144	0.021	0.15	1.181	2.43	0.	0.

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	48	50	28	765	0	0
normalized size	1	1.	0.61	0.63	0.35	9.68	0.	0.
time (sec)	N/A	0.123	0.03	0.151	1.347	2.55	0.	0.

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	91	94	0	0	0	0
normalized size	1	1.	0.52	0.54	0.	0.	0.	0.
time (sec)	N/A	0.184	0.052	0.174	0.	0.	0.	0.

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	87	167	0	0	0	0
normalized size	1	1.	0.33	0.63	0.	0.	0.	0.
time (sec)	N/A	0.205	0.133	0.178	0.	0.	0.	0.

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	147	239	0	0	0	0
normalized size	1	1.	0.41	0.66	0.	0.	0.	0.
time (sec)	N/A	0.298	0.117	0.174	0.	0.	0.	0.

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	450	450	166	965	0	1129	0	954
normalized size	1	1.	0.37	2.14	0.	2.51	0.	2.12
time (sec)	N/A	0.557	0.186	0.274	0.	2.438	0.	57.171

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	372	372	150	795	0	972	0	757
normalized size	1	1.	0.4	2.14	0.	2.61	0.	2.03
time (sec)	N/A	0.497	0.153	0.163	0.	2.175	0.	84.537

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	294	294	134	625	0	856	0	562
normalized size	1	1.	0.46	2.13	0.	2.91	0.	1.91
time (sec)	N/A	0.454	0.125	0.15	0.	2.161	0.	3.585

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	115	455	0	694	0	358
normalized size	1	1.	0.54	2.13	0.	3.24	0.	1.67
time (sec)	N/A	0.395	0.108	0.142	0.	1.969	0.	1.609

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	81	198	0	579	0	208
normalized size	1	1.	0.69	1.68	0.	4.91	0.	1.76
time (sec)	N/A	0.305	0.079	0.144	0.	1.952	0.	1.24

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	69	178	0	447	0	0
normalized size	1	1.	0.63	1.62	0.	4.06	0.	0.
time (sec)	N/A	0.244	0.068	0.145	0.	2.02	0.	0.

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	95	326	0	590	0	0
normalized size	1	1.	0.77	2.65	0.	4.8	0.	0.
time (sec)	N/A	0.373	0.083	0.134	0.	1.945	0.	0.

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	203	203	105	462	0	744	0	0
normalized size	1	1.	0.52	2.28	0.	3.67	0.	0.
time (sec)	N/A	0.405	0.098	0.138	0.	2.235	0.	0.

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	283	283	133	572	0	1044	0	0
normalized size	1	1.	0.47	2.02	0.	3.69	0.	0.
time (sec)	N/A	0.427	0.115	0.151	0.	3.078	0.	0.

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	363	363	151	682	0	1235	0	0
normalized size	1	1.	0.42	1.88	0.	3.4	0.	0.
time (sec)	N/A	0.475	0.158	0.195	0.	4.278	0.	0.

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	98	102	0	1181	0	0
normalized size	1	1.	0.33	0.34	0.	3.94	0.	0.
time (sec)	N/A	0.194	0.065	0.16	0.	2.371	0.	0.

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	98	102	0	1148	0	0
normalized size	1	1.	0.33	0.34	0.	3.81	0.	0.
time (sec)	N/A	0.197	0.063	0.16	0.	2.312	0.	0.

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	220	220	90	86	0	959	0	0
normalized size	1	1.	0.41	0.39	0.	4.36	0.	0.
time (sec)	N/A	0.185	0.046	0.16	0.	2.35	0.	0.

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	64	70	0	779	0	0
normalized size	1	1.	0.44	0.48	0.	5.37	0.	0.
time (sec)	N/A	0.17	0.038	0.156	0.	2.234	0.	0.

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	47	61	200	0	0	0
normalized size	1	1.	0.44	0.56	1.85	0.	0.	0.
time (sec)	N/A	0.152	0.03	0.158	1.353	0.	0.	0.

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	59	77	0	910	0	0
normalized size	1	1.	0.48	0.63	0.	7.4	0.	0.
time (sec)	N/A	0.14	0.045	0.151	0.	2.051	0.	0.

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	87	106	0	994	0	0
normalized size	1	1.	0.5	0.61	0.	5.71	0.	0.
time (sec)	N/A	0.192	0.056	0.182	0.	2.18	0.	0.

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	113	176	0	0	0	0
normalized size	1	1.	0.42	0.65	0.	0.	0.	0.
time (sec)	N/A	0.219	0.082	0.161	0.	0.	0.	0.

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	146	248	0	0	0	0
normalized size	1	1.	0.4	0.68	0.	0.	0.	0.
time (sec)	N/A	0.247	0.115	0.161	0.	0.	0.	0.

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	115	118	0	1355	0	0
normalized size	1	1.	0.31	0.31	0.	3.61	0.	0.
time (sec)	N/A	0.207	0.106	0.161	0.	2.265	0.	0.

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	98	102	0	1142	0	0
normalized size	1	1.	0.33	0.34	0.	3.82	0.	0.
time (sec)	N/A	0.192	0.057	0.155	0.	2.277	0.	0.

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	82	86	0	984	0	0
normalized size	1	1.	0.37	0.39	0.	4.47	0.	0.
time (sec)	N/A	0.179	0.048	0.158	0.	2.126	0.	0.

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	72	70	0	778	0	0
normalized size	1	1.	0.5	0.49	0.	5.4	0.	0.
time (sec)	N/A	0.167	0.035	0.161	0.	2.191	0.	0.

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	38	53	0	648	0	0
normalized size	1	1.	0.55	0.77	0.	9.39	0.	0.
time (sec)	N/A	0.143	0.021	0.143	0.	2.171	0.	0.

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	50	51	28	763	0	0
normalized size	1	1.	0.65	0.66	0.36	9.91	0.	0.
time (sec)	N/A	0.128	0.029	0.148	1.113	2.248	0.	0.

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	91	95	0	0	0	0
normalized size	1	1.	0.52	0.54	0.	0.	0.	0.
time (sec)	N/A	0.186	0.053	0.168	0.	0.	0.	0.

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	88	167	0	0	0	0
normalized size	1	1.	0.33	0.63	0.	0.	0.	0.
time (sec)	N/A	0.211	0.124	0.161	0.	0.	0.	0.

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	145	239	0	0	0	0
normalized size	1	1.	0.4	0.66	0.	0.	0.	0.
time (sec)	N/A	0.24	0.118	0.168	0.	0.	0.	0.

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	455	455	166	965	0	1130	0	954
normalized size	1	1.	0.36	2.12	0.	2.48	0.	2.1
time (sec)	N/A	0.537	0.199	0.26	0.	2.48	0.	57.762

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	375	375	150	795	0	973	1059	757
normalized size	1	1.	0.4	2.12	0.	2.59	2.82	2.02
time (sec)	N/A	0.467	0.158	0.161	0.	2.174	39.181	86.447

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	293	293	134	625	0	857	500	562
normalized size	1	1.	0.46	2.13	0.	2.92	1.71	1.92
time (sec)	N/A	0.427	0.135	0.147	0.	2.089	25.508	3.391

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	212	212	115	454	0	695	376	358
normalized size	1	1.	0.54	2.14	0.	3.28	1.77	1.69
time (sec)	N/A	0.384	0.126	0.135	0.	1.933	14.8	1.684

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	80	198	0	579	0	207
normalized size	1	1.	0.68	1.68	0.	4.91	0.	1.75
time (sec)	N/A	0.296	0.089	0.138	0.	2.022	0.	1.271

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	69	177	0	448	0	0
normalized size	1	1.	0.62	1.59	0.	4.04	0.	0.
time (sec)	N/A	0.241	0.069	0.145	0.	1.979	0.	0.

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	95	326	0	591	0	0
normalized size	1	1.	0.77	2.63	0.	4.77	0.	0.
time (sec)	N/A	0.366	0.086	0.134	0.	2.125	0.	0.

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	105	462	0	745	0	0
normalized size	1	1.	0.54	2.38	0.	3.84	0.	0.
time (sec)	N/A	0.398	0.101	0.144	0.	2.281	0.	0.

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	131	572	0	1045	0	0
normalized size	1	1.	0.49	2.12	0.	3.87	0.	0.
time (sec)	N/A	0.427	0.121	0.15	0.	2.971	0.	0.

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	97	102	0	1181	0	0
normalized size	1	1.	0.32	0.34	0.	3.95	0.	0.
time (sec)	N/A	0.201	0.093	0.16	0.	2.157	0.	0.

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	98	102	0	1148	0	0
normalized size	1	1.	0.33	0.34	0.	3.81	0.	0.
time (sec)	N/A	0.19	0.06	0.166	0.	2.303	0.	0.

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	218	218	90	86	0	959	0	0
normalized size	1	1.	0.41	0.39	0.	4.4	0.	0.
time (sec)	N/A	0.185	0.053	0.152	0.	2.091	0.	0.

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	64	70	0	779	0	0
normalized size	1	1.	0.44	0.48	0.	5.34	0.	0.
time (sec)	N/A	0.167	0.037	0.162	0.	2.082	0.	0.

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	45	60	0	0	0	0
normalized size	1	1.	0.42	0.57	0.	0.	0.	0.
time (sec)	N/A	0.147	0.026	0.154	0.	0.	0.	0.

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	58	78	72	910	0	0
normalized size	1	1.	0.48	0.64	0.59	7.46	0.	0.
time (sec)	N/A	0.137	0.048	0.158	1.057	2.209	0.	0.

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	86	106	0	994	0	0
normalized size	1	1.	0.51	0.62	0.	5.85	0.	0.
time (sec)	N/A	0.197	0.056	0.177	0.	2.198	0.	0.

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	110	176	0	0	0	0
normalized size	1	1.	0.41	0.66	0.	0.	0.	0.
time (sec)	N/A	0.22	0.091	0.187	0.	0.	0.	0.

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	146	248	0	0	0	0
normalized size	1	1.	0.41	0.69	0.	0.	0.	0.
time (sec)	N/A	0.251	0.122	0.173	0.	0.	0.	0.

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	51	53	41	153	0	0
normalized size	1	1.	0.64	0.66	0.51	1.91	0.	0.
time (sec)	N/A	0.22	0.031	0.087	1.16	1.877	0.	0.

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	42	43	27	120	0	0
normalized size	1	1.	0.57	0.58	0.36	1.62	0.	0.
time (sec)	N/A	0.212	0.02	0.105	1.147	1.79	0.	0.

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	41	42	24	117	0	0
normalized size	1	1.	0.58	0.59	0.34	1.65	0.	0.
time (sec)	N/A	0.128	0.02	0.092	1.138	1.871	0.	0.

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	37	52	23	648	0	0
normalized size	1	1.	0.54	0.76	0.34	9.53	0.	0.
time (sec)	N/A	0.13	0.018	0.166	1.155	2.064	0.	0.

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	37	52	26	621	0	0
normalized size	1	1.	0.56	0.79	0.39	9.41	0.	0.
time (sec)	N/A	0.223	0.021	0.182	1.146	2.04	0.	0.

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	43	43	42	43	27	134	0	0
normalized size	1	1.	0.98	1.	0.63	3.12	0.	0.
time (sec)	N/A	0.212	0.019	0.091	1.159	1.859	0.	0.

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	93	196	0	490	0	173
normalized size	1	1.	0.58	1.22	0.	3.06	0.	1.08
time (sec)	N/A	0.408	0.09	0.148	0.	2.023	0.	1.226

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	84	174	0	439	0	157
normalized size	1	1.	0.68	1.41	0.	3.57	0.	1.28
time (sec)	N/A	0.331	0.063	0.124	0.	1.905	0.	1.262

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	77	147	0	406	0	143
normalized size	1	1.	0.79	1.5	0.	4.14	0.	1.46
time (sec)	N/A	0.21	0.062	0.112	0.	1.944	0.	1.214

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	81	198	0	579	0	208
normalized size	1	1.	0.69	1.68	0.	4.91	0.	1.76
time (sec)	N/A	0.288	0.085	0.118	0.	1.973	0.	1.219

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	85	305	0	552	0	173
normalized size	1	1.	0.72	2.58	0.	4.68	0.	1.47
time (sec)	N/A	0.383	0.07	0.122	0.	1.868	0.	1.411

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	79	347	0	392	0	263
normalized size	1	1.	0.71	3.13	0.	3.53	0.	2.37
time (sec)	N/A	0.382	0.068	0.119	0.	1.96	0.	1.723

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	71	378	0	440	0	312
normalized size	1	1.	0.51	2.72	0.	3.17	0.	2.24
time (sec)	N/A	0.388	0.083	0.125	0.	1.946	0.	2.171

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	95	410	0	486	0	427
normalized size	1	1.	0.61	2.63	0.	3.12	0.	2.74
time (sec)	N/A	0.411	0.086	0.13	0.	2.29	0.	3.744

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	103	447	0	521	0	489
normalized size	1	1.	0.57	2.47	0.	2.88	0.	2.7
time (sec)	N/A	0.43	0.087	0.13	0.	2.606	0.	3.736

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	71	85	265	872	0	0
normalized size	1	1.	0.38	0.45	1.41	4.64	0.	0.
time (sec)	N/A	0.265	0.048	0.144	1.324	2.631	0.	0.

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	65	78	232	846	0	0
normalized size	1	1.	0.42	0.51	1.52	5.53	0.	0.
time (sec)	N/A	0.232	0.042	0.141	1.329	2.52	0.	0.

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	55	69	201	803	0	0
normalized size	1	1.	0.48	0.61	1.76	7.04	0.	0.
time (sec)	N/A	0.148	0.026	0.143	1.305	2.577	0.	0.

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	47	61	200	0	0	0
normalized size	1	1.	0.44	0.56	1.85	0.	0.	0.
time (sec)	N/A	0.147	0.029	0.142	1.311	0.	0.	0.

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	49	63	194	0	0	0
normalized size	1	1.	0.46	0.59	1.81	0.	0.	0.
time (sec)	N/A	0.245	0.033	0.148	1.324	0.	0.	0.

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	64	78	215	1023	0	0
normalized size	1	1.	0.43	0.53	1.45	6.91	0.	0.
time (sec)	N/A	0.25	0.045	0.148	1.325	2.677	0.	0.

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	74	86	248	1102	0	0
normalized size	1	1.	0.4	0.46	1.33	5.89	0.	0.
time (sec)	N/A	0.257	0.05	0.146	1.328	2.666	0.	0.

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	80	94	282	1154	0	0
normalized size	1	1.	0.36	0.42	1.27	5.2	0.	0.
time (sec)	N/A	0.267	0.055	0.154	1.367	2.688	0.	0.

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	92	102	324	1247	0	0
normalized size	1	1.	0.35	0.39	1.23	4.74	0.	0.
time (sec)	N/A	0.274	0.067	0.151	1.335	2.73	0.	0.

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	52	69	74	151	0	0
normalized size	1	1.	0.64	0.85	0.91	1.86	0.	0.
time (sec)	N/A	0.234	0.039	0.08	1.15	2.206	0.	0.

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	42	57	58	119	0	0
normalized size	1	1.	0.57	0.77	0.78	1.61	0.	0.
time (sec)	N/A	0.209	0.022	0.079	1.137	2.159	0.	0.

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	41	56	69	116	0	0
normalized size	1	1.	0.58	0.79	0.97	1.63	0.	0.
time (sec)	N/A	0.131	0.019	0.082	1.125	2.098	0.	0.

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	38	53	0	648	0	0
normalized size	1	1.	0.55	0.77	0.	9.39	0.	0.
time (sec)	N/A	0.136	0.02	0.135	0.	2.445	0.	0.

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	38	51	0	620	0	0
normalized size	1	1.	0.57	0.76	0.	9.25	0.	0.
time (sec)	N/A	0.231	0.021	0.141	0.	2.377	0.	0.

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	44	57	0	132	0	0
normalized size	1	1.	1.	1.3	0.	3.	0.	0.
time (sec)	N/A	0.222	0.02	0.078	0.	2.117	0.	0.

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	93	196	0	490	0	173
normalized size	1	1.	0.57	1.2	0.	3.01	0.	1.06
time (sec)	N/A	0.403	0.082	0.141	0.	2.235	0.	1.252

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	84	173	0	439	0	158
normalized size	1	1.	0.67	1.37	0.	3.48	0.	1.25
time (sec)	N/A	0.327	0.067	0.112	0.	2.198	0.	1.332

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	99	99	100	147	0	406	0	143
normalized size	1	1.	1.01	1.48	0.	4.1	0.	1.44
time (sec)	N/A	0.213	0.056	0.109	0.	2.28	0.	1.223

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	80	198	0	579	0	207
normalized size	1	1.	0.68	1.68	0.	4.91	0.	1.75
time (sec)	N/A	0.29	0.08	0.116	0.	2.299	0.	1.244

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	83	307	0	554	0	170
normalized size	1	1.	0.7	2.6	0.	4.69	0.	1.44
time (sec)	N/A	0.418	0.066	0.115	0.	2.242	0.	1.413

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	78	348	0	392	0	263
normalized size	1	1.	0.7	3.11	0.	3.5	0.	2.35
time (sec)	N/A	0.391	0.065	0.124	0.	2.22	0.	1.604

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	86	378	0	441	0	312
normalized size	1	1.	0.61	2.7	0.	3.15	0.	2.23
time (sec)	N/A	0.393	0.073	0.129	0.	2.352	0.	1.988

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	94	410	0	486	0	427
normalized size	1	1.	0.6	2.63	0.	3.12	0.	2.74
time (sec)	N/A	0.415	0.079	0.126	0.	2.315	0.	3.49

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	102	447	0	522	0	489
normalized size	1	1.	0.56	2.47	0.	2.88	0.	2.7
time (sec)	N/A	0.44	0.084	0.126	0.	2.295	0.	3.699

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	70	85	0	873	0	0
normalized size	1	1.	0.37	0.45	0.	4.67	0.	0.
time (sec)	N/A	0.266	0.049	0.138	0.	2.517	0.	0.

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	64	78	0	846	0	0
normalized size	1	1.	0.42	0.51	0.	5.57	0.	0.
time (sec)	N/A	0.24	0.046	0.139	0.	2.456	0.	0.

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	54	69	0	805	0	0
normalized size	1	1.	0.48	0.61	0.	7.12	0.	0.
time (sec)	N/A	0.176	0.033	0.148	0.	2.572	0.	0.

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	45	60	0	0	0	0
normalized size	1	1.	0.42	0.57	0.	0.	0.	0.
time (sec)	N/A	0.169	0.025	0.141	0.	0.	0.	0.

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	48	62	0	0	0	0
normalized size	1	1.	0.45	0.58	0.	0.	0.	0.
time (sec)	N/A	0.261	0.033	0.145	0.	0.	0.	0.

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	63	78	0	1025	0	0
normalized size	1	1.	0.43	0.53	0.	6.97	0.	0.
time (sec)	N/A	0.269	0.041	0.151	0.	2.701	0.	0.

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	73	86	0	1102	0	0
normalized size	1	1.	0.39	0.46	0.	5.92	0.	0.
time (sec)	N/A	0.281	0.044	0.15	0.	2.688	0.	0.

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	78	94	0	1156	0	0
normalized size	1	1.	0.35	0.43	0.	5.25	0.	0.
time (sec)	N/A	0.259	0.055	0.146	0.	2.716	0.	0.

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	87	102	0	1247	0	0
normalized size	1	1.	0.33	0.39	0.	4.76	0.	0.
time (sec)	N/A	0.262	0.064	0.148	0.	2.701	0.	0.

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	0.019	0.227	0.	0.	0.	0.

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	0.014	0.144	0.	0.	0.	0.

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	331	71	229	0	0	0	0	0
normalized size	1	0.21	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.801	0.059	0.	0.	0.	0.

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	137	70	126	0	0	0	0	0
normalized size	1	0.51	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.23	0.052	0.	0.	0.	0.

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	82	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.171	0.105	0.044	0.	0.	0.	0.

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	178	0	0	0	0	0
normalized size	1	1.	0.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.423	0.157	0.054	0.	0.	0.	0.

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	F	F	F(-1)	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	430	103	190	0	0	0	0	0
normalized size	1	0.24	0.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.206	1.687	0.133	0.	0.	0.	0.

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	272	302	208	0	0	0	0	0
normalized size	1	1.11	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.26	0.172	0.116	0.	0.	0.	0.

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	130	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.19	0.098	0.12	0.	0.	0.	0.

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	186	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.291	0.245	0.118	0.	0.	0.	0.

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1039	1039	227	0	0	0	0	0
normalized size	1	1.	0.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.77	6.317	0.116	0.	0.	0.	0.

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	0.355	0.163	0.	0.	0.	0.

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	339	339	217	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.343	0.199	0.332	0.	0.	0.	0.

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	175	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.33	0.158	0.298	0.	0.	0.	0.

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	142	0	0	0	697	0
normalized size	1	1.	0.65	0.	0.	0.	3.21	0.
time (sec)	N/A	0.261	0.098	0.317	0.	0.	11.433	0.

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	112	0	0	0	178	0
normalized size	1	1.	0.82	0.	0.	0.	1.3	0.
time (sec)	N/A	0.133	0.042	0.211	0.	0.	16.929	0.

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	112	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	0.029	0.324	0.	0.	0.	0.

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	218	218	142	0	0	0	695	0
normalized size	1	1.	0.65	0.	0.	0.	3.19	0.
time (sec)	N/A	0.264	0.102	0.219	0.	0.	12.538	0.

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	216	216	173	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.322	0.138	0.336	0.	0.	0.	0.

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	C	F	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	240	240	184	0	863	0	0	167
normalized size	1	1.	0.77	0.	3.6	0.	0.	0.7
time (sec)	N/A	0.397	8.951	0.276	1.311	0.	0.	1.237

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	C	F	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	134	0	471	0	0	100
normalized size	1	1.	0.95	0.	3.34	0.	0.	0.71
time (sec)	N/A	0.17	7.987	0.277	1.215	0.	0.	1.224

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	C	F	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	168	0	1227	0	0	146
normalized size	1	1.	1.03	0.	7.53	0.	0.	0.9
time (sec)	N/A	0.298	8.523	0.305	1.465	0.	0.	1.246

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	C	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	77	0	672	0	0	89
normalized size	1	1.	1.07	0.	9.33	0.	0.	1.24
time (sec)	N/A	0.122	0.024	0.302	1.297	0.	0.	1.181

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	C	F	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	335	335	200	0	1364	0	0	273
normalized size	1	1.	0.6	0.	4.07	0.	0.	0.81
time (sec)	N/A	0.476	9.477	0.28	1.403	0.	0.	1.289

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	C	F	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	236	236	178	0	863	0	0	165
normalized size	1	1.	0.75	0.	3.66	0.	0.	0.7
time (sec)	N/A	0.263	8.356	0.27	1.323	0.	0.	1.269

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	C	F	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	193	193	193	0	2009	0	0	165
normalized size	1	1.	1.	0.	10.41	0.	0.	0.85
time (sec)	N/A	0.401	8.273	0.295	1.69	0.	0.	1.271

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	C	F	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	176	0	1231	0	0	116
normalized size	1	1.	1.12	0.	7.84	0.	0.	0.74
time (sec)	N/A	0.233	7.014	0.284	1.482	0.	0.	1.208

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	C	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	162	0	468	0	0	100
normalized size	1	1.	1.16	0.	3.34	0.	0.	0.71
time (sec)	N/A	0.188	7.881	0.27	1.217	0.	0.	1.212

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	C	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	70	0	227	0	0	65
normalized size	1	1.	1.13	0.	3.66	0.	0.	1.05
time (sec)	N/A	0.11	0.027	0.281	1.137	0.	0.	1.207

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	149	487	0	338	0	188
normalized size	1	1.	0.96	3.12	0.	2.17	0.	1.21
time (sec)	N/A	0.16	0.393	0.067	0.	2.147	0.	1.216

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	159	315	0	267	0	131
normalized size	1	1.	1.22	2.42	0.	2.05	0.	1.01
time (sec)	N/A	0.167	0.175	0.062	0.	2.165	0.	1.172

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	130	178	0	209	0	80
normalized size	1	1.	1.55	2.12	0.	2.49	0.	0.95
time (sec)	N/A	0.064	0.091	0.058	0.	2.15	0.	1.207

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	28	71	0	170	0	49
normalized size	1	1.	0.72	1.82	0.	4.36	0.	1.26
time (sec)	N/A	0.026	0.016	0.059	0.	2.269	0.	1.203

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	100	168	0	717	0	107
normalized size	1	1.	1.56	2.62	0.	11.2	0.	1.67
time (sec)	N/A	0.075	0.103	0.057	0.	2.639	0.	1.21

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	83	265	0	633	0	257
normalized size	1	1.	0.85	2.7	0.	6.46	0.	2.62
time (sec)	N/A	0.056	0.05	0.063	0.	2.663	0.	1.27

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	117	453	0	821	0	840
normalized size	1	1.	0.72	2.8	0.	5.07	0.	5.19
time (sec)	N/A	0.108	0.112	0.072	0.	2.631	0.	1.308

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	186	683	0	1094	0	1858
normalized size	1	1.	0.87	3.21	0.	5.14	0.	8.72
time (sec)	N/A	0.183	0.271	0.067	0.	2.285	0.	1.337

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	75	161	127	234	78	165
normalized size	1	1.	0.9	1.94	1.53	2.82	0.94	1.99
time (sec)	N/A	0.083	0.064	0.03	0.961	1.889	0.444	1.153

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	108	92	171	56	111
normalized size	1	1.	1.	1.64	1.39	2.59	0.85	1.68
time (sec)	N/A	0.06	0.043	0.027	0.966	1.831	0.404	1.214

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	68	62	115	37	70
normalized size	1	1.	0.9	1.39	1.27	2.35	0.76	1.43
time (sec)	N/A	0.051	0.031	0.028	0.94	1.738	0.361	1.155

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	38	41	77	26	46
normalized size	1	1.	1.	1.12	1.21	2.26	0.76	1.35
time (sec)	N/A	0.031	0.017	0.03	0.951	1.718	0.319	1.185

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	22	42	14	23
normalized size	1	1.	1.	0.89	1.16	2.21	0.74	1.21
time (sec)	N/A	0.012	0.01	0.03	0.952	1.87	0.123	1.165

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	26	35	36	65	88	46
normalized size	1	1.	0.79	1.06	1.09	1.97	2.67	1.39
time (sec)	N/A	0.038	0.017	0.033	0.954	1.881	0.533	1.201

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	34	46	65	97	143	77
normalized size	1	1.	0.71	0.96	1.35	2.02	2.98	1.6
time (sec)	N/A	0.041	0.022	0.05	0.961	1.834	0.539	1.166

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	63	100	161	209	123
normalized size	1	1.	0.81	0.94	1.49	2.4	3.12	1.84
time (sec)	N/A	0.051	0.033	0.037	0.958	1.889	0.669	1.144

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	75	76	146	216	260	162
normalized size	1	1.	0.91	0.93	1.78	2.63	3.17	1.98
time (sec)	N/A	0.062	0.047	0.038	0.956	1.875	0.795	1.187

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	203	756	0	456	0	261
normalized size	1	1.	1.09	4.04	0.	2.44	0.	1.4
time (sec)	N/A	0.182	0.285	0.088	0.	2.594	0.	1.261

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	168	168	170	552	0	375	0	200
normalized size	1	1.	1.01	3.29	0.	2.23	0.	1.19
time (sec)	N/A	0.191	0.217	0.051	0.	2.28	0.	1.187

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	142	381	0	300	0	147
normalized size	1	1.	1.17	3.15	0.	2.48	0.	1.21
time (sec)	N/A	0.096	0.156	0.042	0.	2.186	0.	1.232

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	68	68	43	388	0	234	0	101
normalized size	1	1.	0.63	5.71	0.	3.44	0.	1.49
time (sec)	N/A	0.034	0.039	0.037	0.	1.995	0.	1.247

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	160	1019	0	1058	0	188
normalized size	1	1.	1.5	9.52	0.	9.89	0.	1.76
time (sec)	N/A	0.091	0.487	0.042	0.	2.144	0.	1.252

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	101	1520	0	871	0	672
normalized size	1	1.	0.75	11.34	0.	6.5	0.	5.01
time (sec)	N/A	0.075	0.146	0.042	0.	2.117	0.	1.351

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	202	202	136	2194	0	1184	0	933
normalized size	1	1.	0.67	10.86	0.	5.86	0.	4.62
time (sec)	N/A	0.122	0.134	0.045	0.	2.515	0.	1.284

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	260	260	194	2947	0	1611	0	2053
normalized size	1	1.	0.75	11.33	0.	6.2	0.	7.9
time (sec)	N/A	0.21	0.337	0.047	0.	2.769	0.	1.441

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	160	809	456	338	0	200
normalized size	1	1.	1.03	5.19	2.92	2.17	0.	1.28
time (sec)	N/A	0.166	0.438	0.043	1.478	2.037	0.	1.221

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	126	535	235	267	0	143
normalized size	1	1.	0.97	4.12	1.81	2.05	0.	1.1
time (sec)	N/A	0.146	0.157	0.04	1.461	1.895	0.	1.2

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	99	302	144	209	0	92
normalized size	1	1.	1.18	3.6	1.71	2.49	0.	1.1
time (sec)	N/A	0.063	0.109	0.036	1.443	1.897	0.	1.192

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	26	95	50	169	0	59
normalized size	1	1.	0.68	2.5	1.32	4.45	0.	1.55
time (sec)	N/A	0.028	0.018	0.03	1.435	1.847	0.	1.293

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	68	68	100	249	0	718	0	120
normalized size	1	1.	1.47	3.66	0.	10.56	0.	1.76
time (sec)	N/A	0.06	0.067	0.042	0.	1.911	0.	1.306

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	89	565	0	633	0	301
normalized size	1	1.	0.95	6.01	0.	6.73	0.	3.2
time (sec)	N/A	0.057	0.062	0.083	0.	1.703	0.	1.278

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	117	1116	0	821	0	1013
normalized size	1	1.	0.72	6.89	0.	5.07	0.	6.25
time (sec)	N/A	0.098	0.124	0.049	0.	1.776	0.	1.273

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	210	210	187	1711	0	1094	0	2240
normalized size	1	1.	0.89	8.15	0.	5.21	0.	10.67
time (sec)	N/A	0.168	0.273	0.05	0.	1.861	0.	1.364

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	159	127	234	78	273
normalized size	1	1.	1.	2.24	1.79	3.3	1.1	3.85
time (sec)	N/A	0.079	0.057	0.029	0.956	1.411	0.448	1.209

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	109	92	171	56	200
normalized size	1	1.	1.	1.91	1.61	3.	0.98	3.51
time (sec)	N/A	0.056	0.041	0.026	0.968	1.416	0.384	1.138

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	67	62	115	37	138
normalized size	1	1.	1.	1.63	1.51	2.8	0.9	3.37
time (sec)	N/A	0.046	0.03	0.028	0.967	1.445	0.354	1.193

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	38	41	77	26	93
normalized size	1	1.	1.	1.31	1.41	2.66	0.9	3.21
time (sec)	N/A	0.032	0.017	0.028	0.959	1.485	0.316	1.134

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	22	42	12	51
normalized size	1	1.	1.	1.06	1.38	2.62	0.75	3.19
time (sec)	N/A	0.012	0.011	0.029	0.959	1.448	0.123	1.177

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	23	34	36	65	90	89
normalized size	1	1.	0.82	1.21	1.29	2.32	3.21	3.18
time (sec)	N/A	0.035	0.016	0.033	0.954	1.526	0.524	1.195

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	31	47	65	96	143	108
normalized size	1	1.	0.76	1.15	1.59	2.34	3.49	2.63
time (sec)	N/A	0.043	0.021	0.035	0.954	1.553	0.518	1.164

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	51	63	100	162	209	163
normalized size	1	1.	0.88	1.09	1.72	2.79	3.6	2.81
time (sec)	N/A	0.05	0.033	0.034	0.951	1.568	0.652	1.196

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	75	146	215	262	215
normalized size	1	1.	1.	1.07	2.09	3.07	3.74	3.07
time (sec)	N/A	0.058	0.046	0.036	0.956	1.547	0.768	1.172

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	231	1271	1330	456	0	285
normalized size	1	1.	1.24	6.8	7.11	2.44	0.	1.52
time (sec)	N/A	0.188	0.206	0.051	1.517	2.097	0.	1.235

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	167	167	190	830	840	375	0	224
normalized size	1	1.	1.14	4.97	5.03	2.25	0.	1.34
time (sec)	N/A	0.201	0.169	0.048	1.491	2.015	0.	1.197

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	157	543	404	300	0	171
normalized size	1	1.	1.32	4.56	3.39	2.52	0.	1.44
time (sec)	N/A	0.092	0.135	0.043	1.454	1.833	0.	1.225

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	68	68	43	264	140	234	0	127
normalized size	1	1.	0.63	3.88	2.06	3.44	0.	1.87
time (sec)	N/A	0.034	0.038	0.035	1.454	1.638	0.	1.176

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	131	1062	0	1057	0	208
normalized size	1	1.	1.27	10.31	0.	10.26	0.	2.02
time (sec)	N/A	0.085	0.442	0.066	0.	1.835	0.	1.213

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	101	1710	0	871	0	815
normalized size	1	1.	0.78	13.15	0.	6.7	0.	6.27
time (sec)	N/A	0.077	0.098	0.08	0.	1.824	0.	1.224

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	200	200	134	2848	0	1184	0	1110
normalized size	1	1.	0.67	14.24	0.	5.92	0.	5.55
time (sec)	N/A	0.129	0.154	0.057	0.	2.075	0.	1.218

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	257	257	200	4212	0	1611	0	2483
normalized size	1	1.	0.78	16.39	0.	6.27	0.	9.66
time (sec)	N/A	0.216	0.343	0.067	0.	2.386	0.	1.325

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	10	10	37	34	0	58	0	20
normalized size	1	1.	3.7	3.4	0.	5.8	0.	2.
time (sec)	N/A	0.035	0.012	0.032	0.	1.48	0.	1.203

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	90	499	1358	344	0	169
normalized size	1	1.	0.83	4.58	12.46	3.16	0.	1.55
time (sec)	N/A	0.169	0.339	0.043	2.104	1.845	0.	1.268

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	64	325	954	273	0	127
normalized size	1	1.	0.82	4.17	12.23	3.5	0.	1.63
time (sec)	N/A	0.137	0.215	0.04	1.812	1.726	0.	1.223

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	49	160	594	225	0	89
normalized size	1	1.	1.11	3.64	13.5	5.11	0.	2.02
time (sec)	N/A	0.082	0.145	0.037	1.615	1.637	0.	1.197

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	12	42	286	77	0	54
normalized size	1	1.	0.44	1.56	10.59	2.85	0.	2.
time (sec)	N/A	0.037	0.096	0.032	1.512	1.665	0.	1.207

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	118	391	0	730	0	151
normalized size	1	1.	1.27	4.2	0.	7.85	0.	1.62
time (sec)	N/A	0.132	0.242	0.037	0.	1.76	0.	1.213

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	149	671	0	1021	0	701
normalized size	1	1.	0.99	4.47	0.	6.81	0.	4.67
time (sec)	N/A	0.159	0.299	0.062	0.	1.83	0.	1.224

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.791	0.068	0.	0.	0.	0.

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	206	206	220	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	0.251	0.053	0.	0.	0.	0.

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	127	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	0.113	0.053	0.	0.	0.	0.

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	96	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.04	0.047	0.	0.	0.	0.

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	50	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.041	0.043	0.	0.	0.	0.

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	111	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.035	0.043	0.	0.	0.	0.

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	83	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.022	0.048	0.	0.	0.	0.

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	123	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.055	0.049	0.	0.	0.	0.

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	107	229	296	312	454	171
normalized size	1	1.	0.84	1.8	2.33	2.46	3.57	1.35
time (sec)	N/A	0.064	0.145	0.088	1.472	1.587	20.968	1.199

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	91	183	234	258	267	139
normalized size	1	1.	0.87	1.74	2.23	2.46	2.54	1.32
time (sec)	N/A	0.056	0.12	0.053	1.452	1.596	11.375	1.174

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	75	137	171	201	144	105
normalized size	1	1.	0.9	1.65	2.06	2.42	1.73	1.27
time (sec)	N/A	0.046	0.093	0.04	1.449	1.553	4.707	1.174

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	57	83	99	140	53	62
normalized size	1	1.	1.04	1.51	1.8	2.55	0.96	1.13
time (sec)	N/A	0.029	0.069	0.036	1.436	1.524	2.549	1.235

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	74	143	502	198	0	138
normalized size	1	1.	0.73	1.42	4.97	1.96	0.	1.37
time (sec)	N/A	0.132	0.047	0.04	1.846	1.592	0.	1.2

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	65	119	414	174	0	122
normalized size	1	1.	0.88	1.61	5.59	2.35	0.	1.65
time (sec)	N/A	0.108	0.039	0.039	1.712	1.595	0.	1.208

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	98	320	155	0	105
normalized size	1	1.	0.9	1.63	5.33	2.58	0.	1.75
time (sec)	N/A	0.107	0.031	0.036	1.601	1.572	0.	1.242

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	45	79	281	139	0	80
normalized size	1	1.	1.15	2.03	7.21	3.56	0.	2.05
time (sec)	N/A	0.064	0.029	0.036	1.542	1.546	0.	1.207

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	13	13	26	25	220	65	0	50
normalized size	1	1.	2.	1.92	16.92	5.	0.	3.85
time (sec)	N/A	0.029	0.008	0.03	1.5	1.538	0.	1.182

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	55	60	0	119	0	108
normalized size	1	1.	1.25	1.36	0.	2.7	0.	2.45
time (sec)	N/A	0.099	0.021	0.039	0.	1.569	0.	1.22

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	67	75	140	157	0	215
normalized size	1	1.	0.96	1.07	2.	2.24	0.	3.07
time (sec)	N/A	0.117	0.026	0.042	1.009	1.582	0.	1.213

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	83	97	0	197	0	302
normalized size	1	1.	0.84	0.98	0.	1.99	0.	3.05
time (sec)	N/A	0.139	0.036	0.041	0.	1.544	0.	1.21

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	91	140	200	215	0	382
normalized size	1	1.	0.71	1.09	1.56	1.68	0.	2.98
time (sec)	N/A	0.166	0.037	0.043	1.018	1.541	0.	1.187

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	93	225	0	343	0	0
normalized size	1	1.	0.68	1.64	0.	2.5	0.	0.
time (sec)	N/A	0.154	0.066	0.047	0.	1.644	0.	0.

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	86	202	0	323	0	0
normalized size	1	1.	0.78	1.84	0.	2.94	0.	0.
time (sec)	N/A	0.136	0.05	0.045	0.	1.569	0.	0.

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	78	180	0	300	0	0
normalized size	1	1.	0.79	1.82	0.	3.03	0.	0.
time (sec)	N/A	0.122	0.05	0.046	0.	1.589	0.	0.

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	69	160	0	281	0	0
normalized size	1	1.	0.93	2.16	0.	3.8	0.	0.
time (sec)	N/A	0.104	0.04	0.042	0.	1.595	0.	0.

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	45	41	0	182	0	0
normalized size	1	1.	0.79	0.72	0.	3.19	0.	0.
time (sec)	N/A	0.094	0.02	0.029	0.	1.624	0.	0.

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	44	41	0	170	0	0
normalized size	1	1.	0.8	0.75	0.	3.09	0.	0.
time (sec)	N/A	0.064	0.019	0.03	0.	1.527	0.	0.

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	45	42	0	173	0	0
normalized size	1	1.	0.87	0.81	0.	3.33	0.	0.
time (sec)	N/A	0.04	0.013	0.03	0.	1.524	0.	0.

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	77	182	0	262	0	0
normalized size	1	1.	1.04	2.46	0.	3.54	0.	0.
time (sec)	N/A	0.116	0.032	0.042	0.	1.596	0.	0.

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	91	150	194	305	0	0
normalized size	1	1.	0.87	1.43	1.85	2.9	0.	0.
time (sec)	N/A	0.142	0.042	0.046	1.039	1.591	0.	0.

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	103	214	0	342	0	0
normalized size	1	1.	0.77	1.6	0.	2.55	0.	0.
time (sec)	N/A	0.159	0.046	0.046	0.	1.545	0.	0.

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	110	260	258	360	0	0
normalized size	1	1.	0.68	1.61	1.6	2.24	0.	0.
time (sec)	N/A	0.189	0.051	0.046	1.033	1.536	0.	0.

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	116	283	0	485	0	0
normalized size	1	1.	0.81	1.98	0.	3.39	0.	0.
time (sec)	N/A	0.181	0.077	0.053	0.	1.692	0.	0.

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	108	262	0	462	0	0
normalized size	1	1.	0.81	1.97	0.	3.47	0.	0.
time (sec)	N/A	0.154	0.073	0.05	0.	1.584	0.	0.

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	100	243	0	433	0	0
normalized size	1	1.	0.93	2.25	0.	4.01	0.	0.
time (sec)	N/A	0.129	0.056	0.049	0.	1.67	0.	0.

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	68	58	0	300	0	0
normalized size	1	1.	0.84	0.72	0.	3.7	0.	0.
time (sec)	N/A	0.116	0.026	0.03	0.	1.514	0.	0.

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	88	68	58	0	294	0	0
normalized size	1	1.19	0.92	0.78	0.	3.97	0.	0.
time (sec)	N/A	0.106	0.024	0.03	0.	1.568	0.	0.

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	68	58	0	294	0	0
normalized size	1	1.	0.77	0.66	0.	3.34	0.	0.
time (sec)	N/A	0.108	0.026	0.033	0.	1.579	0.	0.

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	68	58	0	293	0	0
normalized size	1	1.	0.85	0.72	0.	3.66	0.	0.
time (sec)	N/A	0.069	0.031	0.032	0.	1.586	0.	0.

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	59	58	0	293	0	0
normalized size	1	1.	0.8	0.78	0.	3.96	0.	0.
time (sec)	N/A	0.045	0.022	0.033	0.	1.522	0.	0.

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	108	384	0	419	0	0
normalized size	1	1.	1.07	3.8	0.	4.15	0.	0.
time (sec)	N/A	0.14	0.05	0.048	0.	1.678	0.	0.

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	121	314	240	464	0	0
normalized size	1	1.	0.9	2.33	1.78	3.44	0.	0.
time (sec)	N/A	0.171	0.059	0.052	1.055	1.701	0.	0.

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	133	326	0	505	0	0
normalized size	1	1.	0.81	1.99	0.	3.08	0.	0.
time (sec)	N/A	0.194	0.067	0.052	0.	1.623	0.	0.

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	75	74	0	412	0	0
normalized size	1	1.	0.78	0.77	0.	4.29	0.	0.
time (sec)	N/A	0.053	0.033	0.032	0.	1.787	0.	0.

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	91	90	0	554	0	0
normalized size	1	1.	0.77	0.76	0.	4.69	0.	0.
time (sec)	N/A	0.064	0.031	0.032	0.	2.024	0.	0.

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	43	58	100	39	59
normalized size	1	1.	1.	0.88	1.18	2.04	0.8	1.2
time (sec)	N/A	0.1	0.025	0.029	0.951	1.453	0.263	1.216

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	35	47	80	31	49
normalized size	1	1.	1.	0.9	1.21	2.05	0.79	1.26
time (sec)	N/A	0.096	0.02	0.026	0.952	1.482	0.257	1.131

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	27	35	61	22	36
normalized size	1	1.	1.	0.93	1.21	2.1	0.76	1.24
time (sec)	N/A	0.091	0.016	0.026	0.943	1.498	0.252	1.153

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	24	36	14	26
normalized size	1	1.	1.	1.	1.26	1.89	0.74	1.37
time (sec)	N/A	0.062	0.013	0.026	0.945	1.712	0.248	1.179

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	15	23	8	16
normalized size	1	1.	1.	1.	1.25	1.92	0.67	1.33
time (sec)	N/A	0.034	0.005	0.026	0.963	1.681	0.063	1.152

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	15	32	8	18
normalized size	1	1.	1.	1.	1.25	2.67	0.67	1.5
time (sec)	N/A	0.08	0.006	0.032	0.95	1.763	0.111	1.15

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	20	26	54	15	28
normalized size	1	1.	1.	1.	1.3	2.7	0.75	1.4
time (sec)	N/A	0.087	0.013	0.033	0.943	1.738	0.138	1.153

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	30	39	89	26	42
normalized size	1	1.	1.	0.94	1.22	2.78	0.81	1.31
time (sec)	N/A	0.09	0.014	0.031	0.955	1.673	0.311	1.166

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	50	105	34	53
normalized size	1	1.	1.	0.9	1.19	2.5	0.81	1.26
time (sec)	N/A	0.091	0.015	0.031	0.957	1.854	0.34	1.134

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	45	49	70	144	48	76
normalized size	1	1.	0.78	0.84	1.21	2.48	0.83	1.31
time (sec)	N/A	0.115	0.061	0.035	0.943	1.779	0.427	1.16

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	39	41	58	128	39	57
normalized size	1	1.	0.81	0.85	1.21	2.67	0.81	1.19
time (sec)	N/A	0.113	0.031	0.033	0.949	1.773	0.413	1.158

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	36	51	101	34	50
normalized size	1	1.	0.77	0.84	1.19	2.35	0.79	1.16
time (sec)	N/A	0.113	0.024	0.033	0.969	1.747	0.385	1.196

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	36	51	100	32	50
normalized size	1	1.	0.81	1.33	1.89	3.7	1.19	1.85
time (sec)	N/A	0.078	0.019	0.035	0.946	1.705	0.336	1.212

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	36	49	96	29	50
normalized size	1	1.	0.74	1.33	1.81	3.56	1.07	1.85
time (sec)	N/A	0.048	0.016	0.033	0.943	1.697	0.353	1.167

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	32	29	38	126	29	42
normalized size	1	1.	0.89	0.81	1.06	3.5	0.81	1.17
time (sec)	N/A	0.109	0.031	0.036	0.945	1.835	0.419	1.137

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	57	163	42	59
normalized size	1	1.	1.	0.85	1.24	3.54	0.91	1.28
time (sec)	N/A	0.11	0.035	0.037	0.962	1.732	0.525	1.14

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	54	80	198	58	80
normalized size	1	1.	0.94	0.86	1.27	3.14	0.92	1.27
time (sec)	N/A	0.118	0.048	0.039	0.954	1.694	0.584	1.177

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	67	62	90	224	66	90
normalized size	1	1.	0.92	0.85	1.23	3.07	0.9	1.23
time (sec)	N/A	0.122	0.051	0.036	0.964	1.719	0.632	1.147

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	65	74	108	271	80	104
normalized size	1	1.	0.74	0.84	1.23	3.08	0.91	1.18
time (sec)	N/A	0.14	0.09	0.035	0.955	1.776	0.626	1.174

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	55	65	97	255	70	86
normalized size	1	1.	0.72	0.86	1.28	3.36	0.92	1.13
time (sec)	N/A	0.127	0.072	0.036	0.953	1.784	0.615	1.216

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	55	60	89	217	66	78
normalized size	1	1.	0.76	0.83	1.24	3.01	0.92	1.08
time (sec)	N/A	0.125	0.057	0.036	0.95	2.063	0.548	1.207

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	53	60	89	215	65	78
normalized size	1	1.	0.95	1.07	1.59	3.84	1.16	1.39
time (sec)	N/A	0.124	0.032	0.035	0.95	2.37	0.489	1.228

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	52	60	88	208	61	77
normalized size	1	1.	0.93	1.07	1.57	3.71	1.09	1.38
time (sec)	N/A	0.123	0.03	0.035	0.945	2.33	0.476	1.181

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	28	48	88	208	60	77
normalized size	1	1.	0.68	1.17	2.15	5.07	1.46	1.88
time (sec)	N/A	0.084	0.028	0.036	0.967	2.138	0.479	1.208

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	53	60	86	212	65	78
normalized size	1	1.	0.95	1.07	1.54	3.79	1.16	1.39
time (sec)	N/A	0.057	0.025	0.036	0.968	2.021	0.49	1.169

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	47	77	269	60	69
normalized size	1	1.	0.92	0.8	1.31	4.56	1.02	1.17
time (sec)	N/A	0.118	0.045	0.038	0.959	2.136	0.591	1.165

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	65	59	97	316	78	90
normalized size	1	1.	0.92	0.83	1.37	4.45	1.1	1.27
time (sec)	N/A	0.127	0.06	0.038	0.952	2.034	0.772	1.207

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	83	78	120	348	95	111
normalized size	1	1.	0.93	0.88	1.35	3.91	1.07	1.25
time (sec)	N/A	0.135	0.072	0.041	0.961	2.14	0.917	1.237

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	91	86	131	378	104	122
normalized size	1	1.	0.89	0.84	1.28	3.71	1.02	1.2
time (sec)	N/A	0.145	0.083	0.042	0.964	2.039	1.17	1.177

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	42	37	0	107	0	0
normalized size	1	1.	0.57	0.5	0.	1.45	0.	0.
time (sec)	N/A	0.184	0.023	0.027	0.	2.029	0.	0.

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	42	37	0	105	0	0
normalized size	1	1.	0.57	0.5	0.	1.42	0.	0.
time (sec)	N/A	0.128	0.019	0.028	0.	2.222	0.	0.

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	40	34	0	100	0	0
normalized size	1	1.	0.58	0.49	0.	1.45	0.	0.
time (sec)	N/A	0.072	0.015	0.025	0.	2.329	0.	0.

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	36	48	0	555	0	0
normalized size	1	1.	0.55	0.74	0.	8.54	0.	0.
time (sec)	N/A	0.176	0.019	0.093	0.	2.496	0.	0.

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	40	50	0	555	0	0
normalized size	1	1.	0.59	0.74	0.	8.16	0.	0.
time (sec)	N/A	0.178	0.019	0.089	0.	2.377	0.	0.

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	57	65	178	147	0	0
normalized size	1	1.	0.64	0.73	2.	1.65	0.	0.
time (sec)	N/A	0.087	0.027	0.032	1.033	2.089	0.	0.

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	60	81	240	207	0	0
normalized size	1	1.	0.44	0.6	1.76	1.52	0.	0.
time (sec)	N/A	0.095	0.038	0.03	1.019	2.129	0.	0.

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	68	97	297	263	0	0
normalized size	1	1.	0.37	0.53	1.62	1.44	0.	0.
time (sec)	N/A	0.105	0.046	0.03	1.028	2.188	0.	0.

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	71	83	0	828	0	0
normalized size	1	1.	0.37	0.43	0.	4.27	0.	0.
time (sec)	N/A	0.213	0.045	0.085	0.	2.465	0.	0.

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	63	75	0	788	0	0
normalized size	1	1.	0.41	0.48	0.	5.08	0.	0.
time (sec)	N/A	0.21	0.037	0.085	0.	2.447	0.	0.

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	54	66	0	748	0	0
normalized size	1	1.	0.47	0.57	0.	6.45	0.	0.
time (sec)	N/A	0.201	0.031	0.085	0.	2.383	0.	0.

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	45	55	0	703	0	0
normalized size	1	1.	0.58	0.71	0.	9.13	0.	0.
time (sec)	N/A	0.134	0.027	0.082	0.	2.516	0.	0.

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	51	0	494	0	0
normalized size	1	1.	1.	1.24	0.	12.05	0.	0.
time (sec)	N/A	0.071	0.012	0.082	0.	2.48	0.	0.

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	42	53	0	662	0	0
normalized size	1	1.	0.59	0.75	0.	9.32	0.	0.
time (sec)	N/A	0.182	0.017	0.087	0.	2.629	0.	0.

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	50	62	0	894	0	0
normalized size	1	1.	0.47	0.58	0.	8.36	0.	0.
time (sec)	N/A	0.19	0.03	0.089	0.	2.645	0.	0.

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	62	76	0	969	0	0
normalized size	1	1.	0.42	0.51	0.	6.55	0.	0.
time (sec)	N/A	0.197	0.041	0.099	0.	2.708	0.	0.

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	72	84	0	1026	0	0
normalized size	1	1.	0.39	0.45	0.	5.49	0.	0.
time (sec)	N/A	0.199	0.04	0.092	0.	2.569	0.	0.

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	87	119	0	0	0	0
normalized size	1	1.	0.33	0.46	0.	0.	0.	0.
time (sec)	N/A	0.246	0.073	0.098	0.	0.	0.	0.

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	77	110	0	0	0	0
normalized size	1	1.	0.35	0.5	0.	0.	0.	0.
time (sec)	N/A	0.236	0.06	0.101	0.	0.	0.	0.

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	71	102	0	0	0	0
normalized size	1	1.	0.4	0.58	0.	0.	0.	0.
time (sec)	N/A	0.232	0.052	0.092	0.	0.	0.	0.

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	76	90	0	0	0	0
normalized size	1	1.	0.55	0.65	0.	0.	0.	0.
time (sec)	N/A	0.222	0.041	0.095	0.	0.	0.	0.

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	60	88	0	710	0	0
normalized size	1	1.	0.66	0.97	0.	7.8	0.	0.
time (sec)	N/A	0.151	0.035	0.092	0.	2.104	0.	0.

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	60	88	0	705	0	0
normalized size	1	1.	0.66	0.97	0.	7.75	0.	0.
time (sec)	N/A	0.093	0.03	0.09	0.	2.01	0.	0.

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	65	96	0	0	0	0
normalized size	1	1.	0.39	0.58	0.	0.	0.	0.
time (sec)	N/A	0.219	0.051	0.095	0.	0.	0.	0.

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	76	122	0	0	0	0
normalized size	1	1.	0.37	0.59	0.	0.	0.	0.
time (sec)	N/A	0.226	0.058	0.096	0.	0.	0.	0.

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	91	142	0	0	0	0
normalized size	1	1.	0.36	0.56	0.	0.	0.	0.
time (sec)	N/A	0.238	0.061	0.096	0.	0.	0.	0.

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	99	151	0	0	0	0
normalized size	1	1.	0.33	0.51	0.	0.	0.	0.
time (sec)	N/A	0.241	0.073	0.097	0.	0.	0.	0.

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	97	190	0	0	0	0
normalized size	1	1.	0.31	0.61	0.	0.	0.	0.
time (sec)	N/A	0.255	0.117	0.096	0.	0.	0.	0.

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	87	182	0	0	0	0
normalized size	1	1.	0.32	0.68	0.	0.	0.	0.
time (sec)	N/A	0.246	0.098	0.095	0.	0.	0.	0.

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	87	166	0	0	0	0
normalized size	1	1.	0.38	0.72	0.	0.	0.	0.
time (sec)	N/A	0.238	0.091	0.093	0.	0.	0.	0.

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	85	166	0	945	0	0
normalized size	1	1.	0.46	0.9	0.	5.14	0.	0.
time (sec)	N/A	0.235	0.058	0.099	0.	2.057	0.	0.

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	84	161	0	930	0	0
normalized size	1	1.	0.46	0.88	0.	5.05	0.	0.
time (sec)	N/A	0.237	0.057	0.095	0.	2.069	0.	0.

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	60	161	0	936	0	0
normalized size	1	1.	0.44	1.18	0.	6.83	0.	0.
time (sec)	N/A	0.165	0.065	0.095	0.	2.085	0.	0.

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	85	166	0	934	0	0
normalized size	1	1.	0.46	0.9	0.	5.08	0.	0.
time (sec)	N/A	0.112	0.05	0.097	0.	2.126	0.	0.

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	86	193	0	0	0	0
normalized size	1	1.	0.34	0.77	0.	0.	0.	0.
time (sec)	N/A	0.244	0.073	0.093	0.	0.	0.	0.

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	97	222	0	0	0	0
normalized size	1	1.	0.33	0.75	0.	0.	0.	0.
time (sec)	N/A	0.245	0.089	0.1	0.	0.	0.	0.

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	115	242	0	0	0	0
normalized size	1	1.	0.33	0.7	0.	0.	0.	0.
time (sec)	N/A	0.257	0.125	0.105	0.	0.	0.	0.

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	103	238	0	1165	0	0
normalized size	1	1.	0.37	0.86	0.	4.21	0.	0.
time (sec)	N/A	0.132	0.079	0.103	0.	2.137	0.	0.

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	82	227	0	0	223	0
normalized size	1	1.	1.02	2.84	0.	0.	2.79	0.
time (sec)	N/A	0.096	0.036	0.323	0.	0.	11.68	0.

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	72	143	0	0	104	0
normalized size	1	1.	0.95	1.88	0.	0.	1.37	0.
time (sec)	N/A	0.079	0.042	0.185	0.	0.	4.915	0.

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	82	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.03	0.293	0.	0.	0.	0.

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	82	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.031	0.291	0.	0.	0.	0.

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	82	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.03	0.29	0.	0.	0.	0.

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	70	338	0	682	1760	621
normalized size	1	1.	0.85	4.12	0.	8.32	21.46	7.57
time (sec)	N/A	0.118	0.046	0.032	0.	1.957	1.545	1.2

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	142	0	278	585	266
normalized size	1	1.	1.	2.63	0.	5.15	10.83	4.93
time (sec)	N/A	0.099	0.094	0.031	0.	1.99	0.808	1.171

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	20	27	0	66	82	55
normalized size	1	1.	0.83	1.12	0.	2.75	3.42	2.29
time (sec)	N/A	0.08	0.013	0.027	0.	2.135	0.278	1.136

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	100	0	0	44	0
normalized size	1	1.	1.	4.55	0.	0.	2.	0.
time (sec)	N/A	0.082	0.011	0.193	0.	0.	2.048	0.

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	67	177	0	0	673	0
normalized size	1	1.	0.96	2.53	0.	0.	9.61	0.
time (sec)	N/A	0.104	0.035	0.207	0.	0.	4.737	0.

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	67	224	0	0	2152	0
normalized size	1	1.	0.96	3.2	0.	0.	30.74	0.
time (sec)	N/A	0.116	0.033	0.277	0.	0.	7.95	0.

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	102	377	194	1058	0	0
normalized size	1	1.	0.37	1.38	0.71	3.86	0.	0.
time (sec)	N/A	0.23	0.093	0.033	1.012	2.268	0.	0.

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	84	180	108	459	0	0
normalized size	1	1.	0.48	1.03	0.62	2.64	0.	0.
time (sec)	N/A	0.203	0.09	0.03	1.001	2.371	0.	0.

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	49	52	41	167	0	0
normalized size	1	1.	0.6	0.63	0.5	2.04	0.	0.
time (sec)	N/A	0.174	0.033	0.027	1.037	2.18	0.	0.

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	0.025	0.333	0.	0.	0.	0.

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	107	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.216	0.042	0.328	0.	0.	0.	0.

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	107	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.212	0.04	0.344	0.	0.	0.	0.

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	114	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	0.042	0.329	0.	0.	0.	0.

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	77	47	0	0	258	0
normalized size	1	1.	0.91	0.55	0.	0.	3.04	0.
time (sec)	N/A	0.114	0.07	0.357	0.	0.	29.652	0.

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	76	47	0	0	255	0
normalized size	1	1.	0.9	0.56	0.	0.	3.04	0.
time (sec)	N/A	0.115	0.048	0.322	0.	0.	19.257	0.

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	60	47	0	0	301	0
normalized size	1	1.	1.03	0.81	0.	0.	5.19	0.
time (sec)	N/A	0.061	0.024	0.342	0.	0.	13.386	0.

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	44	0	0	292	0
normalized size	1	1.	1.	0.75	0.	0.	4.95	0.
time (sec)	N/A	0.039	0.014	0.321	0.	0.	10.842	0.

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	74	90	0	0	286	0
normalized size	1	1.	1.03	1.25	0.	0.	3.97	0.
time (sec)	N/A	0.093	0.025	0.343	0.	0.	14.515	0.

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	77	93	0	0	280	0
normalized size	1	1.	1.03	1.24	0.	0.	3.73	0.
time (sec)	N/A	0.1	0.026	0.331	0.	0.	15.531	0.

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	80	112	0	0	287	0
normalized size	1	1.	1.03	1.44	0.	0.	3.68	0.
time (sec)	N/A	0.099	0.022	0.339	0.	0.	23.739	0.

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	105	0	0	0	272	0
normalized size	1	1.	0.78	0.	0.	0.	2.03	0.
time (sec)	N/A	0.181	0.087	0.322	0.	0.	30.024	0.

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	102	0	0	0	269	0
normalized size	1	1.	0.77	0.	0.	0.	2.02	0.
time (sec)	N/A	0.181	0.084	0.318	0.	0.	20.058	0.

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	88	0	0	0	314	0
normalized size	1	1.	0.92	0.	0.	0.	3.27	0.
time (sec)	N/A	0.109	0.034	0.313	0.	0.	13.535	0.

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	0	0	0	306	0
normalized size	1	1.	1.	0.	0.	0.	3.56	0.
time (sec)	N/A	0.064	0.017	0.322	0.	0.	10.821	0.

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	102	0	0	0	299	0
normalized size	1	1.	0.93	0.	0.	0.	2.72	0.
time (sec)	N/A	0.16	0.029	0.315	0.	0.	15.687	0.

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	105	0	0	0	294	0
normalized size	1	1.	0.93	0.	0.	0.	2.6	0.
time (sec)	N/A	0.167	0.036	0.328	0.	0.	16.23	0.

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	108	0	0	0	301	0
normalized size	1	1.	0.93	0.	0.	0.	2.59	0.
time (sec)	N/A	0.161	0.022	0.319	0.	0.	24.624	0.

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	22	23	31	55	24	31
normalized size	1	1.	0.76	0.79	1.07	1.9	0.83	1.07
time (sec)	N/A	0.055	0.017	0.026	0.947	1.861	0.078	1.181

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	22	23	31	55	26	31
normalized size	1	1.	0.76	0.79	1.07	1.9	0.9	1.07
time (sec)	N/A	0.055	0.017	0.026	0.935	1.921	0.078	1.163

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	22	23	31	55	24	31
normalized size	1	1.	0.76	0.79	1.07	1.9	0.83	1.07
time (sec)	N/A	0.053	0.015	0.024	0.939	1.941	0.077	1.127

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	22	23	31	55	26	31
normalized size	1	1.	0.76	0.79	1.07	1.9	0.9	1.07
time (sec)	N/A	0.04	0.014	0.025	0.955	1.917	0.076	1.138

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	19	14	26	42	19	26
normalized size	1	1.	1.27	0.93	1.73	2.8	1.27	1.73
time (sec)	N/A	0.019	0.012	0.025	0.942	1.839	0.076	1.141

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	20	26	49	20	27
normalized size	1	1.	0.9	0.95	1.24	2.33	0.95	1.29
time (sec)	N/A	0.049	0.015	0.026	0.945	2.015	0.106	1.161

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	20	26	49	17	27
normalized size	1	1.	0.95	1.05	1.37	2.58	0.89	1.42
time (sec)	N/A	0.049	0.013	0.032	0.941	1.715	0.277	1.156

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	27	59	20	28
normalized size	1	1.	0.96	0.96	1.17	2.57	0.87	1.22
time (sec)	N/A	0.052	0.017	0.032	0.952	1.711	0.322	1.138

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	23	28	51	24	28
normalized size	1	1.	1.	1.53	1.87	3.4	1.6	1.87
time (sec)	N/A	0.045	0.009	0.03	0.941	1.664	0.331	1.138

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	32	33	54	89	41	54
normalized size	1	1.	0.67	0.69	1.12	1.85	0.85	1.12
time (sec)	N/A	0.088	0.024	0.024	0.941	1.668	0.088	1.145

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	40	33	54	89	44	54
normalized size	1	1.	0.83	0.69	1.12	1.85	0.92	1.12
time (sec)	N/A	0.089	0.022	0.023	0.944	1.632	0.089	1.111

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	32	33	54	89	41	54
normalized size	1	1.	0.67	0.69	1.12	1.85	0.85	1.12
time (sec)	N/A	0.09	0.021	0.025	0.944	1.688	0.087	1.124

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	32	33	54	89	44	54
normalized size	1	1.	0.67	0.69	1.12	1.85	0.92	1.12
time (sec)	N/A	0.067	0.018	0.024	0.949	1.695	0.087	1.141

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	23	28	49	76	36	49
normalized size	1	1.	0.66	0.8	1.4	2.17	1.03	1.4
time (sec)	N/A	0.037	0.013	0.023	0.951	1.695	0.086	1.157

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	33	37	49	82	39	50
normalized size	1	1.	0.82	0.92	1.22	2.05	0.98	1.25
time (sec)	N/A	0.075	0.015	0.026	0.945	1.777	0.292	1.126

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	40	53	88	36	54
normalized size	1	1.	1.	0.98	1.29	2.15	0.88	1.32
time (sec)	N/A	0.082	0.013	0.033	0.949	1.671	0.307	1.133

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	31	50	78	39	50
normalized size	1	1.	1.	1.82	2.94	4.59	2.29	2.94
time (sec)	N/A	0.069	0.015	0.031	0.949	1.644	0.315	1.118

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	38	49	90	37	50
normalized size	1	1.	1.	0.97	1.26	2.31	0.95	1.28
time (sec)	N/A	0.08	0.015	0.031	0.949	1.671	0.356	1.133

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	35	40	54	97	39	55
normalized size	1	1.	0.81	0.93	1.26	2.26	0.91	1.28
time (sec)	N/A	0.079	0.017	0.03	0.955	1.67	0.397	1.12

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	23	31	54	86	41	54
normalized size	1	1.	0.55	0.74	1.29	2.05	0.98	1.29
time (sec)	N/A	0.081	0.01	0.032	0.954	1.652	0.412	1.114

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	70	57	99	165	76	99
normalized size	1	1.	0.8	0.66	1.14	1.9	0.87	1.14
time (sec)	N/A	0.102	0.038	0.026	0.956	1.636	0.099	1.145

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	70	57	99	163	82	99
normalized size	1	1.	0.8	0.66	1.14	1.87	0.94	1.14
time (sec)	N/A	0.099	0.033	0.026	0.942	1.637	0.097	1.129

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	70	57	99	161	78	99
normalized size	1	1.	0.83	0.68	1.18	1.92	0.93	1.18
time (sec)	N/A	0.103	0.032	0.026	0.946	1.617	0.098	1.138

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	70	57	99	161	82	99
normalized size	1	1.	1.01	0.83	1.43	2.33	1.19	1.43
time (sec)	N/A	0.08	0.03	0.024	0.961	1.716	0.096	1.139

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	31	52	93	142	70	93
normalized size	1	1.	0.6	1.	1.79	2.73	1.35	1.79
time (sec)	N/A	0.05	0.021	0.025	0.963	1.689	0.108	1.129

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	51	70	93	154	76	95
normalized size	1	1.	0.65	0.89	1.18	1.95	0.96	1.2
time (sec)	N/A	0.086	0.029	0.026	0.961	1.962	0.32	1.18

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	59	71	95	163	70	96
normalized size	1	1.	0.78	0.93	1.25	2.14	0.92	1.26
time (sec)	N/A	0.091	0.026	0.031	0.957	1.932	0.343	1.15

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	71	93	162	73	95
normalized size	1	1.	0.74	0.91	1.19	2.08	0.94	1.22
time (sec)	N/A	0.095	0.024	0.033	0.966	2.012	0.371	1.167

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	55	69	95	153	70	96
normalized size	1	1.	0.76	0.96	1.32	2.12	0.97	1.33
time (sec)	N/A	0.093	0.029	0.031	0.943	2.035	0.409	1.15

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	39	59	107	167	87	107
normalized size	1	1.	0.57	0.86	1.55	2.42	1.26	1.55
time (sec)	N/A	0.056	0.025	0.024	0.944	1.925	0.111	1.189

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	49	61	77	131	53	95
normalized size	1	1.	0.78	0.97	1.22	2.08	0.84	1.51
time (sec)	N/A	0.112	0.037	0.034	0.96	1.948	0.357	1.189

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	41	51	66	113	44	72
normalized size	1	1.	0.77	0.96	1.25	2.13	0.83	1.36
time (sec)	N/A	0.104	0.03	0.034	0.965	1.901	0.354	1.164

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	28	39	54	89	32	53
normalized size	1	1.	0.72	1.	1.38	2.28	0.82	1.36
time (sec)	N/A	0.096	0.025	0.035	0.953	2.021	0.33	1.134

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	23	30	42	65	24	41
normalized size	1	1.	0.77	1.	1.4	2.17	0.8	1.37
time (sec)	N/A	0.065	0.019	0.03	0.954	1.93	0.303	1.136

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	18	16	20	27	12	20
normalized size	1	1.	1.2	1.07	1.33	1.8	0.8	1.33
time (sec)	N/A	0.033	0.012	0.026	0.966	1.978	0.298	1.148

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	24	31	41	84	19	43
normalized size	1	1.	0.77	1.	1.32	2.71	0.61	1.39
time (sec)	N/A	0.087	0.02	0.031	0.958	2.032	0.364	1.206

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	35	43	57	122	31	61
normalized size	1	1.	0.81	1.	1.33	2.84	0.72	1.42
time (sec)	N/A	0.094	0.03	0.033	0.967	2.008	0.411	1.14

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	58	76	157	46	76
normalized size	1	1.	0.87	0.97	1.27	2.62	0.77	1.27
time (sec)	N/A	0.101	0.048	0.035	0.956	2.033	0.445	1.123

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	69	86	177	54	86
normalized size	1	1.	1.	0.97	1.21	2.49	0.76	1.21
time (sec)	N/A	0.11	0.039	0.034	0.945	2.048	0.493	1.151

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	68	69	101	215	70	82
normalized size	1	1.	0.86	0.87	1.28	2.72	0.89	1.04
time (sec)	N/A	0.119	0.045	0.036	0.956	2.041	0.595	1.164

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	53	60	89	178	61	70
normalized size	1	1.	0.76	0.86	1.27	2.54	0.87	1.
time (sec)	N/A	0.107	0.036	0.033	0.959	2.222	0.537	1.141

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	35	60	89	174	60	70
normalized size	1	1.	0.69	1.18	1.75	3.41	1.18	1.37
time (sec)	N/A	0.105	0.027	0.032	0.947	2.324	0.468	1.155

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	34	60	80	169	53	63
normalized size	1	1.	0.67	1.18	1.57	3.31	1.04	1.24
time (sec)	N/A	0.08	0.025	0.033	0.962	2.322	0.449	1.141

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	35	60	85	173	56	69
normalized size	1	1.	0.69	1.18	1.67	3.39	1.1	1.35
time (sec)	N/A	0.053	0.019	0.032	0.972	2.282	0.475	1.145

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	48	54	81	215	58	68
normalized size	1	1.	0.75	0.84	1.27	3.36	0.91	1.06
time (sec)	N/A	0.099	0.038	0.038	0.956	2.402	0.669	1.136

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	57	68	103	266	76	88
normalized size	1	1.	0.73	0.87	1.32	3.41	0.97	1.13
time (sec)	N/A	0.112	0.051	0.039	0.955	2.304	0.796	1.14

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	72	87	124	301	92	107
normalized size	1	1.	0.73	0.88	1.25	3.04	0.93	1.08
time (sec)	N/A	0.122	0.096	0.039	0.957	2.346	0.884	1.136

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	87	98	135	328	100	117
normalized size	1	1.	0.79	0.89	1.23	2.98	0.91	1.06
time (sec)	N/A	0.134	0.079	0.038	0.967	2.358	1.005	1.128

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	87	90	127	277	92	101
normalized size	1	1.	0.83	0.86	1.21	2.64	0.88	0.96
time (sec)	N/A	0.133	0.054	0.039	0.959	2.373	0.787	1.15

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	64	90	127	273	88	101
normalized size	1	1.	0.74	1.05	1.48	3.17	1.02	1.17
time (sec)	N/A	0.138	0.041	0.036	0.97	2.382	0.685	1.138

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	64	90	127	271	88	101
normalized size	1	1.	0.74	1.05	1.48	3.15	1.02	1.17
time (sec)	N/A	0.123	0.042	0.036	0.966	2.255	0.669	1.167

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	54	66	97	49	61
normalized size	1	1.	0.97	1.74	2.13	3.13	1.58	1.97
time (sec)	N/A	0.089	0.029	0.034	0.969	2.217	0.581	1.139

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	60	75	126	269	87	100
normalized size	1	1.	0.88	1.1	1.85	3.96	1.28	1.47
time (sec)	N/A	0.088	0.034	0.036	0.968	2.312	0.64	1.123

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	63	90	123	267	83	100
normalized size	1	1.	0.73	1.05	1.43	3.1	0.97	1.16
time (sec)	N/A	0.068	0.031	0.036	0.963	2.119	0.657	1.134

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	66	78	120	329	87	99
normalized size	1	1.	0.71	0.84	1.29	3.54	0.94	1.06
time (sec)	N/A	0.119	0.067	0.039	0.98	2.383	0.925	1.135

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	78	94	140	377	104	119
normalized size	1	1.	0.72	0.86	1.28	3.46	0.95	1.09
time (sec)	N/A	0.135	0.083	0.04	0.992	2.662	1.12	1.153

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	98	117	162	423	121	138
normalized size	1	1.	0.73	0.87	1.21	3.16	0.9	1.03
time (sec)	N/A	0.151	0.109	0.042	0.968	3.109	1.432	1.144

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	82	120	189	458	143	123
normalized size	1	1.	0.68	0.99	1.56	3.79	1.18	1.02
time (sec)	N/A	0.091	0.06	0.036	0.983	2.774	0.98	1.161

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	96	210	0	433	0	124
normalized size	1	1.	0.7	1.53	0.	3.16	0.	0.91
time (sec)	N/A	0.329	0.143	0.043	0.	2.889	0.	1.182

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	88	186	0	398	0	113
normalized size	1	1.	0.79	1.66	0.	3.55	0.	1.01
time (sec)	N/A	0.287	0.107	0.041	0.	3.232	0.	1.175

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	79	164	0	347	0	99
normalized size	1	1.	0.94	1.95	0.	4.13	0.	1.18
time (sec)	N/A	0.182	0.09	0.038	0.	2.862	0.	1.179

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	76	134	0	309	0	84
normalized size	1	1.	0.88	1.56	0.	3.59	0.	0.98
time (sec)	N/A	0.078	0.051	0.035	0.	2.743	0.	1.151

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	99	128	0	444	0	131
normalized size	1	1.	1.27	1.64	0.	5.69	0.	1.68
time (sec)	N/A	0.245	0.078	0.04	0.	2.719	0.	1.173

Problem 1082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	106	211	0	479	0	180
normalized size	1	1.	1.29	2.57	0.	5.84	0.	2.2
time (sec)	N/A	0.247	0.087	0.042	0.	2.726	0.	1.178

Problem 1083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	79	239	0	339	0	271
normalized size	1	1.	1.01	3.06	0.	4.35	0.	3.47
time (sec)	N/A	0.246	0.102	0.045	0.	2.585	0.	1.149

Problem 1084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	82	262	0	371	0	338
normalized size	1	1.	0.81	2.59	0.	3.67	0.	3.35
time (sec)	N/A	0.272	0.114	0.049	0.	2.658	0.	1.168

Problem 1085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	95	287	0	417	0	437
normalized size	1	1.	0.73	2.21	0.	3.21	0.	3.36
time (sec)	N/A	0.309	0.137	0.05	0.	2.625	0.	1.184

Problem 1086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	113	268	0	566	420	158
normalized size	1	1.	0.7	1.66	0.	3.52	2.61	0.98
time (sec)	N/A	0.336	0.137	0.046	0.	2.751	11.915	1.18

Problem 1087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	105	244	0	508	515	144
normalized size	1	1.	0.77	1.79	0.	3.74	3.79	1.06
time (sec)	N/A	0.308	0.119	0.04	0.	2.662	13.317	1.169

Problem 1088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	97	222	0	466	306	132
normalized size	1	1.	0.87	2.	0.	4.2	2.76	1.19
time (sec)	N/A	0.195	0.099	0.037	0.	2.853	8.753	1.164

Problem 1089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	117	186	0	413	340	115
normalized size	1	1.	1.09	1.74	0.	3.86	3.18	1.07
time (sec)	N/A	0.086	0.09	0.036	0.	2.689	9.04	1.139

Problem 1090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	115	179	0	539	267	157
normalized size	1	1.	1.14	1.77	0.	5.34	2.64	1.55
time (sec)	N/A	0.286	0.108	0.041	0.	2.78	8.74	1.174

Problem 1091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	124	286	0	567	350	223
normalized size	1	1.	1.11	2.55	0.	5.06	3.12	1.99
time (sec)	N/A	0.285	0.151	0.042	0.	2.769	8.593	1.197

Problem 1092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	129	316	0	594	366	370
normalized size	1	1.	1.07	2.61	0.	4.91	3.02	3.06
time (sec)	N/A	0.289	0.209	0.046	0.	2.684	7.438	1.184

Problem 1093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	127	339	0	601	359	350
normalized size	1	1.	1.1	2.95	0.	5.23	3.12	3.04
time (sec)	N/A	0.282	0.156	0.049	0.	2.839	10.081	1.19

Problem 1094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	96	364	0	437	447	501
normalized size	1	1.	0.91	3.43	0.	4.12	4.22	4.73
time (sec)	N/A	0.257	0.149	0.054	0.	2.706	8.464	1.178

Problem 1095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	104	388	0	485	484	559
normalized size	1	1.	0.79	2.96	0.	3.7	3.69	4.27
time (sec)	N/A	0.293	0.15	0.063	0.	2.773	14.569	1.163

Problem 1096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	109	412	0	527	636	598
normalized size	1	1.	0.7	2.64	0.	3.38	4.08	3.83
time (sec)	N/A	0.33	0.231	0.074	0.	2.848	12.535	1.188

Problem 1097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	120	436	0	585	660	714
normalized size	1	1.	0.66	2.41	0.	3.23	3.65	3.94
time (sec)	N/A	0.371	0.188	0.093	0.	2.78	20.062	1.193

Problem 1098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	131	330	0	726	763	209
normalized size	1	1.	0.7	1.76	0.	3.88	4.08	1.12
time (sec)	N/A	0.37	0.214	0.046	0.	2.88	25.471	1.182

Problem 1099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	123	306	0	683	687	193
normalized size	1	1.	0.76	1.89	0.	4.22	4.24	1.19
time (sec)	N/A	0.325	0.174	0.04	0.	2.853	20.351	1.203

Problem 1100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	115	284	0	609	586	177
normalized size	1	1.	0.84	2.07	0.	4.45	4.28	1.29
time (sec)	N/A	0.216	0.151	0.039	0.	2.854	18.47	1.183

Problem 1101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	135	242	0	545	478	157
normalized size	1	1.	1.04	1.86	0.	4.19	3.68	1.21
time (sec)	N/A	0.098	0.136	0.036	0.	2.817	14.351	1.148

Problem 1102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	136	235	0	671	508	203
normalized size	1	1.	1.	1.73	0.	4.93	3.74	1.49
time (sec)	N/A	0.334	0.148	0.042	0.	3.317	21.59	1.158

Problem 1103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	143	367	0	702	483	263
normalized size	1	1.	1.01	2.6	0.	4.98	3.43	1.87
time (sec)	N/A	0.335	0.232	0.045	0.	2.797	13.313	1.203

Problem 1104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	151	399	0	717	401	408
normalized size	1	1.	1.	2.64	0.	4.75	2.66	2.7
time (sec)	N/A	0.332	0.26	0.048	0.	2.969	10.295	1.184

Problem 1105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	149	423	0	717	478	394
normalized size	1	1.	0.96	2.73	0.	4.63	3.08	2.54
time (sec)	N/A	0.342	0.263	0.05	0.	2.794	14.124	1.204

Problem 1106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	151	447	0	729	575	594
normalized size	1	1.	0.97	2.88	0.	4.7	3.71	3.83
time (sec)	N/A	0.334	0.262	0.061	0.	2.844	11.295	1.206

Problem 1107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	151	296	0	655	1091	190
normalized size	1	1.	0.99	1.93	0.	4.28	7.13	1.24
time (sec)	N/A	0.11	0.133	0.037	0.	2.929	28.103	1.203

Problem 1108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	97	149	0	446	0	0
normalized size	1	1.	0.71	1.09	0.	3.26	0.	0.
time (sec)	N/A	0.356	0.171	0.041	0.	2.952	0.	0.

Problem 1109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	94	126	0	410	0	0
normalized size	1	1.	1.01	1.35	0.	4.41	0.	0.
time (sec)	N/A	0.249	0.103	0.039	0.	2.83	0.	0.

Problem 1110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	78	103	0	362	0	0
normalized size	1	1.	0.93	1.23	0.	4.31	0.	0.
time (sec)	N/A	0.119	0.108	0.036	0.	2.829	0.	0.

Problem 1111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	82	80	0	338	0	0
normalized size	1	1.	1.37	1.33	0.	5.63	0.	0.
time (sec)	N/A	0.066	0.033	0.034	0.	2.796	0.	0.

Problem 1112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	52	52	66	80	0	325	0	0
normalized size	1	1.	1.27	1.54	0.	6.25	0.	0.
time (sec)	N/A	0.228	0.111	0.037	0.	2.778	0.	0.

Problem 1113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	78	99	0	377	0	0
normalized size	1	1.	1.01	1.29	0.	4.9	0.	0.
time (sec)	N/A	0.25	0.133	0.04	0.	2.752	0.	0.

Problem 1114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	94	124	0	441	0	0
normalized size	1	1.	0.86	1.14	0.	4.05	0.	0.
time (sec)	N/A	0.323	0.161	0.042	0.	2.699	0.	0.

Problem 1115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	101	150	0	477	0	0
normalized size	1	1.	0.75	1.11	0.	3.53	0.	0.
time (sec)	N/A	0.407	0.166	0.045	0.	2.863	0.	0.

Problem 1116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	90	190	0	506	0	89
normalized size	1	1.	0.77	1.62	0.	4.32	0.	0.76
time (sec)	N/A	0.321	0.166	0.04	0.	2.799	0.	1.215

Problem 1117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	82	166	0	471	0	58
normalized size	1	1.	0.88	1.78	0.	5.06	0.	0.62
time (sec)	N/A	0.263	0.116	0.039	0.	2.75	0.	1.211

Problem 1118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	38	32	0	103	0	158
normalized size	1	1.	0.63	0.53	0.	1.72	0.	2.63
time (sec)	N/A	0.117	0.056	0.032	0.	2.655	0.	1.207

Problem 1119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	51	51	63	31	0	99	0	200
normalized size	1	1.	1.24	0.61	0.	1.94	0.	3.92
time (sec)	N/A	0.064	0.03	0.031	0.	3.035	0.	1.188

Problem 1120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	75	152	0	452	0	0
normalized size	1	1.	0.91	1.85	0.	5.51	0.	0.
time (sec)	N/A	0.268	0.111	0.041	0.	2.953	0.	0.

Problem 1121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	89	178	0	514	0	135
normalized size	1	1.	0.82	1.65	0.	4.76	0.	1.25
time (sec)	N/A	0.337	0.131	0.042	0.	3.196	0.	1.226

Problem 1122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	105	205	0	570	0	275
normalized size	1	1.	0.74	1.44	0.	4.01	0.	1.94
time (sec)	N/A	0.42	0.145	0.043	0.	3.846	0.	1.244

Problem 1123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	53	47	0	153	0	0
normalized size	1	1.	0.72	0.64	0.	2.07	0.	0.
time (sec)	N/A	0.072	0.036	0.03	0.	3.682	0.	0.

Problem 1124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	96	64	0	251	0	0
normalized size	1	1.	0.99	0.66	0.	2.59	0.	0.
time (sec)	N/A	0.08	0.05	0.031	0.	5.552	0.	0.

Problem 1125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	88	476	410	1233	3009	0
normalized size	1	1.	0.73	3.97	3.42	10.28	25.08	0.
time (sec)	N/A	0.113	0.064	0.031	1.144	3.262	4.081	0.

Problem 1126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	69	146	154	374	706	0
normalized size	1	1.	1.03	2.18	2.3	5.58	10.54	0.
time (sec)	N/A	0.087	0.105	0.029	1.133	3.05	2.195	0.

Problem 1127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	34	74	84	178	299	0
normalized size	1	1.	0.81	1.76	2.	4.24	7.12	0.
time (sec)	N/A	0.063	0.039	0.028	1.119	2.829	1.294	0.

Problem 1128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.013	0.372	0.	0.	0.	0.

Problem 1129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	92	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.178	0.068	0.418	0.	0.	0.	0.

Problem 1130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	194	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.349	0.15	0.403	0.	0.	0.	0.

Problem 1131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	180	0	0	0	226	0
normalized size	1	1.	1.02	0.	0.	0.	1.28	0.
time (sec)	N/A	0.314	0.221	0.402	0.	0.	88.096	0.

Problem 1132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	158	0	0	0	172	0
normalized size	1	1.	0.92	0.	0.	0.	1.	0.
time (sec)	N/A	0.3	0.153	0.398	0.	0.	17.943	0.

Problem 1133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	130	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.291	0.222	0.395	0.	0.	0.	0.

Problem 1134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	66	0	0	0	0	0
normalized size	1	1.	0.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.293	0.114	0.397	0.	0.	0.	0.

Problem 1135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	173	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.32	0.242	0.401	0.	0.	0.	0.

Problem 1136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	68	0	0	0	653	0
normalized size	1	1.	1.24	0.	0.	0.	11.87	0.
time (sec)	N/A	0.063	0.018	0.423	0.	0.	10.183	0.

Problem 1137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	70	191	244	220	483	105
normalized size	1	1.	0.51	1.4	1.79	1.62	3.55	0.77
time (sec)	N/A	0.276	0.107	0.083	1.451	2.701	20.157	1.167

Problem 1138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	62	170	216	197	371	93
normalized size	1	1.	0.56	1.53	1.95	1.77	3.34	0.84
time (sec)	N/A	0.247	0.086	0.065	1.456	2.673	16.775	1.177

Problem 1139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	54	148	186	166	326	78
normalized size	1	1.	0.45	1.23	1.55	1.38	2.72	0.65
time (sec)	N/A	0.096	0.073	0.052	1.439	2.589	14.054	1.17

Problem 1140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	57	125	155	143	218	62
normalized size	1	1.	0.63	1.37	1.7	1.57	2.4	0.68
time (sec)	N/A	0.057	0.048	0.049	1.466	2.638	10.986	1.229

Problem 1141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	49	121	167	162	197	103
normalized size	1	1.	0.74	1.83	2.53	2.45	2.98	1.56
time (sec)	N/A	0.195	0.045	0.043	1.48	2.646	10.294	1.187

Problem 1142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	52	122	169	184	150	177
normalized size	1	1.	0.79	1.85	2.56	2.79	2.27	2.68
time (sec)	N/A	0.191	0.063	0.043	1.457	2.614	9.772	1.16

Problem 1143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	60	125	173	194	223	242
normalized size	1	1.	0.79	1.64	2.28	2.55	2.93	3.18
time (sec)	N/A	0.194	0.074	0.045	1.465	2.683	7.56	1.183

Problem 1144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	60	184	173	146	267	294
normalized size	1	1.	0.64	1.96	1.84	1.55	2.84	3.13
time (sec)	N/A	0.2	0.062	0.042	0.975	2.625	17.375	1.165

Problem 1145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	97	231	201	166	411	378
normalized size	1	1.	0.84	2.01	1.75	1.44	3.57	3.29
time (sec)	N/A	0.223	0.11	0.044	0.963	2.583	10.241	1.165

Problem 1146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	110	292	230	197	518	448
normalized size	1	1.	0.76	2.03	1.6	1.37	3.6	3.11
time (sec)	N/A	0.25	0.121	0.049	0.98	2.619	31.363	1.162

Problem 1147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	75	183	234	209	340	105
normalized size	1	1.	0.62	1.51	1.93	1.73	2.81	0.87
time (sec)	N/A	0.072	0.099	0.066	1.467	2.623	16.927	1.144

Problem 1148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	91	229	296	261	632	138
normalized size	1	1.	0.64	1.6	2.07	1.83	4.42	0.97
time (sec)	N/A	0.082	0.119	0.104	1.463	2.651	22.772	1.175

Problem 1149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	107	275	358	325	996	169
normalized size	1	1.	0.65	1.67	2.17	1.97	6.04	1.02
time (sec)	N/A	0.09	0.147	0.24	1.479	2.729	39.686	1.186

Problem 1150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	78	178	782	240	0	180
normalized size	1	1.	0.82	1.87	8.23	2.53	0.	1.89
time (sec)	N/A	0.187	0.085	0.04	1.921	2.589	0.	1.18

Problem 1151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	155	572	217	0	151
normalized size	1	1.	1.	2.21	8.17	3.1	0.	2.16
time (sec)	N/A	0.086	0.062	0.038	1.752	2.62	0.	1.19

Problem 1152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	18	18	29	28	680	119	0	89
normalized size	1	1.	1.61	1.56	37.78	6.61	0.	4.94
time (sec)	N/A	0.031	0.011	0.029	1.674	2.526	0.	1.161

Problem 1153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	43	49	0	190	0	196
normalized size	1	1.	0.44	0.51	0.	1.96	0.	2.02
time (sec)	N/A	0.074	0.018	0.03	0.	2.593	0.	1.191

Problem 1154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	61	63	0	308	0	0
normalized size	1	1.	0.51	0.53	0.	2.59	0.	0.
time (sec)	N/A	0.089	0.021	0.033	0.	2.679	0.	0.

Problem 1155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	75	74	0	417	0	0
normalized size	1	1.	0.53	0.52	0.	2.96	0.	0.
time (sec)	N/A	0.091	0.028	0.033	0.	2.984	0.	0.

Problem 1156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	79	88	0	869	0	0
normalized size	1	1.	0.35	0.39	0.	3.85	0.	0.
time (sec)	N/A	0.221	0.052	0.089	0.	3.055	0.	0.

Problem 1157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	70	79	0	809	0	0
normalized size	1	1.	0.38	0.43	0.	4.37	0.	0.
time (sec)	N/A	0.211	0.042	0.087	0.	2.95	0.	0.

Problem 1158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	64	72	0	783	0	0
normalized size	1	1.	0.43	0.48	0.	5.22	0.	0.
time (sec)	N/A	0.152	0.038	0.085	0.	2.94	0.	0.

Problem 1159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	54	63	0	734	0	0
normalized size	1	1.	0.49	0.57	0.	6.61	0.	0.
time (sec)	N/A	0.085	0.026	0.085	0.	2.915	0.	0.

Problem 1160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	46	55	0	0	0	0
normalized size	1	1.	0.44	0.53	0.	0.	0.	0.
time (sec)	N/A	0.194	0.03	0.096	0.	0.	0.	0.

Problem 1161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	51	60	0	0	0	0
normalized size	1	1.	0.47	0.56	0.	0.	0.	0.
time (sec)	N/A	0.2	0.034	0.089	0.	0.	0.	0.

Problem 1162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	63	73	0	961	0	0
normalized size	1	1.	0.42	0.49	0.	6.45	0.	0.
time (sec)	N/A	0.206	0.042	0.089	0.	3.207	0.	0.

Problem 1163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	73	81	0	1023	0	0
normalized size	1	1.	0.39	0.43	0.	5.44	0.	0.
time (sec)	N/A	0.21	0.046	0.091	0.	3.147	0.	0.

Problem 1164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	79	89	0	1076	0	0
normalized size	1	1.	0.35	0.4	0.	4.83	0.	0.
time (sec)	N/A	0.22	0.053	0.093	0.	3.3	0.	0.

Problem 1165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	58	50	0	143	0	0
normalized size	1	1.	1.32	1.14	0.	3.25	0.	0.
time (sec)	N/A	0.082	0.028	0.027	0.	2.597	0.	0.

Problem 1166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	52	81	320	207	0	0
normalized size	1	1.	0.56	0.87	3.44	2.23	0.	0.
time (sec)	N/A	0.091	0.038	0.028	1.105	2.516	0.	0.

Problem 1167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	60	97	436	262	0	0
normalized size	1	1.	0.43	0.7	3.14	1.88	0.	0.
time (sec)	N/A	0.104	0.055	0.031	1.116	2.534	0.	0.

Problem 1168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	68	97	552	265	0	0
normalized size	1	1.	0.37	0.52	2.98	1.43	0.	0.
time (sec)	N/A	0.115	0.053	0.03	1.148	2.597	0.	0.

Problem 1169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	51	70	0	803	0	0
normalized size	1	1.	0.61	0.84	0.	9.67	0.	0.
time (sec)	N/A	0.089	0.033	0.087	0.	2.955	0.	0.

Problem 1170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	53	43	0	144	0	0
normalized size	1	1.	1.13	0.91	0.	3.06	0.	0.
time (sec)	N/A	0.084	0.04	0.027	0.	2.562	0.	0.

Problem 1171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	73	159	0	969	0	0
normalized size	1	1.	0.39	0.86	0.	5.24	0.	0.
time (sec)	N/A	0.116	0.052	0.095	0.	3.048	0.	0.

Problem 1172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	101	238	0	1175	0	0
normalized size	1	1.	0.36	0.86	0.	4.23	0.	0.
time (sec)	N/A	0.139	0.077	0.099	0.	2.93	0.	0.

Problem 1173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	74	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.207	0.05	0.421	0.	0.	0.	0.

Problem 1174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	251	186	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.416	0.167	0.406	0.	0.	0.	0.

Problem 1175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	224	224	176	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.355	0.202	0.411	0.	0.	0.	0.

Problem 1176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	222	222	179	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.339	0.23	0.404	0.	0.	0.	0.

Problem 1177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	134	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.169	0.363	0.401	0.	0.	0.	0.

Problem 1178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	83	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.029	0.395	0.	0.	0.	0.

Problem 1179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	193	193	159	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.292	0.223	0.396	0.	0.	0.	0.

Problem 1180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	133	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.331	0.25	0.396	0.	0.	0.	0.

Problem 1181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	154	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.367	0.289	0.395	0.	0.	0.	0.

Problem 1182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	39	75	136	215	109	136
normalized size	1	1.	0.59	1.14	2.06	3.26	1.65	2.06
time (sec)	N/A	0.064	0.032	0.026	0.97	2.261	0.122	1.146

Problem 1183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	31	69	124	190	100	124
normalized size	1	1.	0.6	1.33	2.38	3.65	1.92	2.38
time (sec)	N/A	0.057	0.025	0.026	0.955	2.254	0.117	1.157

Problem 1184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	23	45	80	122	63	80
normalized size	1	1.	0.66	1.29	2.29	3.49	1.8	2.29
time (sec)	N/A	0.039	0.019	0.024	0.978	2.18	0.105	1.136

Problem 1185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	37	16	63	93	48	63
normalized size	1	1.	2.18	0.94	3.71	5.47	2.82	3.71
time (sec)	N/A	0.03	0.018	0.026	0.973	2.294	0.1	1.145

Problem 1186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	46	36	34	45	88	36	59
normalized size	1	1.28	1.	0.94	1.25	2.44	1.	1.64
time (sec)	N/A	0.032	0.012	0.026	0.965	2.299	0.303	1.122

Problem 1187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	28	26	39	17	16
normalized size	1	1.	1.92	2.15	2.	3.	1.31	1.23
time (sec)	N/A	0.034	0.008	0.03	0.967	2.221	0.352	1.161

Problem 1188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	17	16	55	78	42	20
normalized size	1	1.	0.94	0.89	3.06	4.33	2.33	1.11
time (sec)	N/A	0.035	0.016	0.026	0.955	2.177	0.402	1.173

Problem 1189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	52	90	138	324	99	92
normalized size	1	1.	0.6	1.03	1.59	3.72	1.14	1.06
time (sec)	N/A	0.068	0.034	0.033	0.973	2.2	0.688	1.17

Problem 1190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	80	120	176	421	129	123
normalized size	1	1.	0.66	0.98	1.44	3.45	1.06	1.01
time (sec)	N/A	0.089	0.055	0.037	0.998	2.414	0.974	1.146

Problem 1191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	72	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.02	0.434	0.	0.	0.	0.

Problem 1192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	107	201	246	312	996	167
normalized size	1	1.	0.84	1.58	1.94	2.46	7.84	1.31
time (sec)	N/A	0.068	0.144	0.07	1.484	2.223	20.046	1.195

Problem 1193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	91	155	184	258	629	136
normalized size	1	1.	0.87	1.48	1.75	2.46	5.99	1.3
time (sec)	N/A	0.059	0.122	0.046	1.501	2.427	11.581	1.225

Problem 1194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	75	109	122	201	337	105
normalized size	1	1.	0.9	1.31	1.47	2.42	4.06	1.27
time (sec)	N/A	0.051	0.095	0.037	1.458	2.401	6.203	1.21

Problem 1195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	57	64	61	140	109	62
normalized size	1	1.	1.04	1.16	1.11	2.55	1.98	1.13
time (sec)	N/A	0.03	0.081	0.03	1.451	2.166	3.66	1.269

Problem 1196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	16	16	27	28	0	66	0	50
normalized size	1	1.	1.69	1.75	0.	4.12	0.	3.12
time (sec)	N/A	0.031	0.01	0.03	0.	2.077	0.	1.195

Problem 1197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	42	0	174	0	0
normalized size	1	1.	0.81	0.79	0.	3.28	0.	0.
time (sec)	N/A	0.043	0.017	0.03	0.	2.335	0.	0.

Problem 1198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	59	58	0	294	0	0
normalized size	1	1.	0.79	0.77	0.	3.92	0.	0.
time (sec)	N/A	0.05	0.028	0.03	0.	2.481	0.	0.

Problem 1199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	75	74	0	413	0	0
normalized size	1	1.	0.77	0.76	0.	4.26	0.	0.
time (sec)	N/A	0.057	0.034	0.03	0.	2.724	0.	0.

Problem 1200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	91	90	0	555	0	0
normalized size	1	1.	0.76	0.76	0.	4.66	0.	0.
time (sec)	N/A	0.066	0.04	0.032	0.	3.505	0.	0.

Problem 1201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	50	68	85	166	0	0
normalized size	1	1.	0.6	0.82	1.02	2.	0.	0.
time (sec)	N/A	0.178	0.045	0.03	1.041	2.654	0.	0.

Problem 1202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	42	51	55	105	0	0
normalized size	1	1.	0.57	0.69	0.74	1.42	0.	0.
time (sec)	N/A	0.189	0.024	0.027	1.003	2.296	0.	0.

Problem 1203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	42	51	55	104	0	0
normalized size	1	1.	0.57	0.69	0.74	1.41	0.	0.
time (sec)	N/A	0.13	0.022	0.028	1.009	2.396	0.	0.

Problem 1204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	40	48	65	99	0	0
normalized size	1	1.	0.58	0.7	0.94	1.43	0.	0.
time (sec)	N/A	0.071	0.016	0.027	1.008	2.428	0.	0.

Problem 1205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	37	47	0	555	0	0
normalized size	1	1.	0.56	0.71	0.	8.41	0.	0.
time (sec)	N/A	0.177	0.019	0.085	0.	2.583	0.	0.

Problem 1206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	41	49	0	556	0	0
normalized size	1	1.	0.59	0.71	0.	8.06	0.	0.
time (sec)	N/A	0.179	0.023	0.088	0.	2.469	0.	0.

Problem 1207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	57	65	0	149	0	0
normalized size	1	1.	0.63	0.71	0.	1.64	0.	0.
time (sec)	N/A	0.088	0.031	0.029	0.	2.205	0.	0.

Problem 1208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	60	81	0	205	0	0
normalized size	1	1.	0.43	0.58	0.	1.46	0.	0.
time (sec)	N/A	0.097	0.042	0.029	0.	2.345	0.	0.

Problem 1209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	68	97	0	265	0	0
normalized size	1	1.	0.36	0.52	0.	1.42	0.	0.
time (sec)	N/A	0.116	0.048	0.03	0.	2.276	0.	0.

Problem 1210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	76	113	0	313	0	0
normalized size	1	1.	0.32	0.48	0.	1.34	0.	0.
time (sec)	N/A	0.119	0.06	0.03	0.	2.447	0.	0.

Problem 1211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	40	18	493	0	0
normalized size	1	1.	1.	1.03	0.46	12.64	0.	0.
time (sec)	N/A	0.075	0.02	0.081	0.988	2.606	0.	0.

Problem 1212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	59	88	68	707	0	0
normalized size	1	1.	0.66	0.98	0.76	7.86	0.	0.
time (sec)	N/A	0.099	0.04	0.093	1.014	2.751	0.	0.

Problem 1213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	83	166	112	934	0	0
normalized size	1	1.	0.45	0.91	0.61	5.1	0.	0.
time (sec)	N/A	0.115	0.065	0.094	1.013	2.632	0.	0.

Problem 1214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	101	238	182	1165	0	0
normalized size	1	1.	0.37	0.86	0.66	4.22	0.	0.
time (sec)	N/A	0.138	0.087	0.095	1.028	2.649	0.	0.

Problem 1215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	115	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.164	0.047	0.44	0.	0.	0.	0.

Problem 1216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.072	0.431	0.	0.	0.	0.

Problem 1217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	75	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	0.056	0.424	0.	0.	0.	0.

Problem 1218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.025	0.429	0.	0.	0.	0.

Problem 1219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	56	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.023	0.422	0.	0.	0.	0.

Problem 1220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.027	0.431	0.	0.	0.	0.

Problem 1221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.026	0.424	0.	0.	0.	0.

Problem 1222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	119	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.188	0.06	0.421	0.	0.	0.	0.

Problem 1223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	102	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.187	0.08	0.418	0.	0.	0.	0.

Problem 1224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	88	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.03	0.43	0.	0.	0.	0.

Problem 1225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	83	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.031	0.422	0.	0.	0.	0.

Problem 1226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	103	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	0.032	0.42	0.	0.	0.	0.

Problem 1227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	102	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.174	0.035	0.419	0.	0.	0.	0.

Problem 1228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	39	61	109	167	87	105
normalized size	1	1.	0.53	0.84	1.49	2.29	1.19	1.44
time (sec)	N/A	0.055	0.027	0.024	0.963	2.072	0.113	1.136

Problem 1229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	31	52	93	142	70	89
normalized size	1	1.	0.56	0.95	1.69	2.58	1.27	1.62
time (sec)	N/A	0.047	0.023	0.026	0.976	2.201	0.106	1.151

Problem 1230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	32	29	50	76	36	73
normalized size	1	1.	0.86	0.78	1.35	2.05	0.97	1.97
time (sec)	N/A	0.037	0.016	0.027	0.968	2.192	0.097	1.126

Problem 1231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	20	14	27	42	19	46
normalized size	1	1.	1.25	0.88	1.69	2.62	1.19	2.88
time (sec)	N/A	0.019	0.01	0.026	0.951	2.134	0.15	1.136

Problem 1232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	18	16	19	27	12	20
normalized size	1	1.	1.2	1.07	1.27	1.8	0.8	1.33
time (sec)	N/A	0.033	0.016	0.024	0.959	2.066	0.341	1.149

Problem 1233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	33	60	85	173	56	74
normalized size	1	1.	0.67	1.22	1.73	3.53	1.14	1.51
time (sec)	N/A	0.05	0.025	0.034	0.961	2.075	0.522	1.166

Problem 1234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	61	90	123	267	83	131
normalized size	1	1.	0.73	1.07	1.46	3.18	0.99	1.56
time (sec)	N/A	0.068	0.042	0.036	0.968	2.343	0.689	1.144

Problem 1235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	80	120	189	458	143	165
normalized size	1	1.	0.67	1.01	1.59	3.85	1.2	1.39
time (sec)	N/A	0.087	0.062	0.036	1.005	2.35	1.046	1.164

Problem 1236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	96	202	0	433	0	371
normalized size	1	1.	0.7	1.47	0.	3.16	0.	2.71
time (sec)	N/A	0.327	0.127	0.042	0.	2.488	0.	1.311

Problem 1237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	88	178	0	398	0	305
normalized size	1	1.	0.79	1.59	0.	3.55	0.	2.72
time (sec)	N/A	0.284	0.095	0.039	0.	2.364	0.	1.264

Problem 1238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	154	0	347	0	242
normalized size	1	1.	0.94	1.81	0.	4.08	0.	2.85
time (sec)	N/A	0.183	0.081	0.036	0.	2.419	0.	1.301

Problem 1239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	99	126	0	309	0	177
normalized size	1	1.	1.14	1.45	0.	3.55	0.	2.03
time (sec)	N/A	0.075	0.051	0.034	0.	2.433	0.	1.209

Problem 1240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	99	120	0	443	0	167
normalized size	1	1.	1.27	1.54	0.	5.68	0.	2.14
time (sec)	N/A	0.245	0.08	0.04	0.	2.402	0.	1.237

Problem 1241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	106	203	0	478	0	227
normalized size	1	1.	1.29	2.48	0.	5.83	0.	2.77
time (sec)	N/A	0.248	0.089	0.04	0.	2.542	0.	1.199

Problem 1242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	76	231	0	339	0	205
normalized size	1	1.	0.97	2.96	0.	4.35	0.	2.63
time (sec)	N/A	0.237	0.126	0.042	0.	2.357	0.	1.171

Problem 1243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	99	99	82	253	0	370	0	284
normalized size	1	1.	0.83	2.56	0.	3.74	0.	2.87
time (sec)	N/A	0.275	0.128	0.047	0.	2.46	0.	1.212

Problem 1244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	95	279	0	417	0	348
normalized size	1	1.	0.73	2.15	0.	3.21	0.	2.68
time (sec)	N/A	0.308	0.134	0.052	0.	2.479	0.	1.17

Problem 1245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	117	174	0	413	340	302
normalized size	1	1.	1.08	1.61	0.	3.82	3.15	2.8
time (sec)	N/A	0.087	0.097	0.035	0.	2.453	8.391	1.254

Problem 1246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	135	226	0	545	478	432
normalized size	1	1.	1.03	1.73	0.	4.16	3.65	3.3
time (sec)	N/A	0.097	0.139	0.036	0.	2.679	13.758	1.388

Problem 1247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	151	276	0	655	0	562
normalized size	1	1.	0.98	1.79	0.	4.25	0.	3.65
time (sec)	N/A	0.116	0.129	0.037	0.	2.51	0.	1.422

Problem 1248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	100	74	0	338	0	0
normalized size	1	1.	1.64	1.21	0.	5.54	0.	0.
time (sec)	N/A	0.067	0.064	0.033	0.	2.51	0.	0.

Problem 1249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	52	52	63	31	0	99	0	0
normalized size	1	1.	1.21	0.6	0.	1.9	0.	0.
time (sec)	N/A	0.064	0.036	0.029	0.	2.469	0.	0.

Problem 1250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	79	47	0	153	0	0
normalized size	1	1.	1.05	0.63	0.	2.04	0.	0.
time (sec)	N/A	0.071	0.046	0.03	0.	2.919	0.	0.

Problem 1251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	96	64	0	251	0	0
normalized size	1	1.	0.98	0.65	0.	2.56	0.	0.
time (sec)	N/A	0.081	0.054	0.03	0.	4.602	0.	0.

Problem 1252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	111	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.276	0.167	0.312	0.	0.	0.	0.

Problem 1253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	74	0	0	0	651	0
normalized size	1	1.	1.37	0.	0.	0.	12.06	0.
time (sec)	N/A	0.059	0.024	0.327	0.	0.	11.127	0.

Problem 1254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	167	167	107	173	208	325	0	170
normalized size	1	1.	0.64	1.04	1.25	1.95	0.	1.02
time (sec)	N/A	0.094	0.147	0.05	1.467	2.654	0.	1.173

Problem 1255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	91	127	146	261	632	138
normalized size	1	1.	0.63	0.88	1.01	1.8	4.36	0.95
time (sec)	N/A	0.087	0.119	0.038	1.459	2.615	34.405	1.16

Problem 1256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	75	189	198	209	340	105
normalized size	1	1.	0.61	1.54	1.61	1.7	2.76	0.85
time (sec)	N/A	0.075	0.1	0.034	1.455	2.573	12.523	1.177

Problem 1257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	70	133	165	143	0	62
normalized size	1	1.	0.75	1.43	1.77	1.54	0.	0.67
time (sec)	N/A	0.057	0.086	0.04	1.464	2.72	0.	1.167

Problem 1258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	18	18	29	28	0	120	0	89
normalized size	1	1.	1.61	1.56	0.	6.67	0.	4.94
time (sec)	N/A	0.033	0.015	0.029	0.	2.699	0.	1.155

Problem 1259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	43	42	0	192	0	196
normalized size	1	1.	0.46	0.45	0.	2.04	0.	2.09
time (sec)	N/A	0.08	0.019	0.029	0.	3.116	0.	1.176

Problem 1260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	59	58	0	309	0	0
normalized size	1	1.	0.51	0.5	0.	2.66	0.	0.
time (sec)	N/A	0.085	0.023	0.03	0.	2.728	0.	0.

Problem 1261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	75	74	0	419	0	0
normalized size	1	1.	0.54	0.54	0.	3.04	0.	0.
time (sec)	N/A	0.093	0.032	0.03	0.	3.077	0.	0.

Problem 1262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	81	88	0	869	0	0
normalized size	1	1.	0.36	0.39	0.	3.86	0.	0.
time (sec)	N/A	0.225	0.057	0.086	0.	3.561	0.	0.

Problem 1263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	69	79	0	807	0	0
normalized size	1	1.	0.38	0.43	0.	4.39	0.	0.
time (sec)	N/A	0.219	0.047	0.085	0.	2.957	0.	0.

Problem 1264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	63	72	0	783	0	0
normalized size	1	1.	0.42	0.48	0.	5.26	0.	0.
time (sec)	N/A	0.149	0.048	0.085	0.	2.984	0.	0.

Problem 1265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	53	63	0	733	0	0
normalized size	1	1.	0.48	0.57	0.	6.66	0.	0.
time (sec)	N/A	0.083	0.026	0.084	0.	2.896	0.	0.

Problem 1266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	44	56	0	0	0	0
normalized size	1	1.	0.43	0.55	0.	0.	0.	0.
time (sec)	N/A	0.197	0.025	0.088	0.	0.	0.	0.

Problem 1267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	50	60	0	0	0	0
normalized size	1	1.	0.47	0.56	0.	0.	0.	0.
time (sec)	N/A	0.202	0.032	0.095	0.	0.	0.	0.

Problem 1268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	62	73	0	959	0	0
normalized size	1	1.	0.42	0.49	0.	6.48	0.	0.
time (sec)	N/A	0.206	0.042	0.094	0.	3.237	0.	0.

Problem 1269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	72	81	0	1022	0	0
normalized size	1	1.	0.39	0.43	0.	5.47	0.	0.
time (sec)	N/A	0.208	0.043	0.094	0.	3.211	0.	0.

Problem 1270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	77	89	0	1073	0	0
normalized size	1	1.	0.35	0.4	0.	4.86	0.	0.
time (sec)	N/A	0.212	0.049	0.089	0.	3.185	0.	0.

Problem 1271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	68	97	0	266	0	0
normalized size	1	1.	0.36	0.51	0.	1.41	0.	0.
time (sec)	N/A	0.109	0.054	0.03	0.	2.889	0.	0.

Problem 1272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	60	97	0	261	0	0
normalized size	1	1.	0.42	0.68	0.	1.84	0.	0.
time (sec)	N/A	0.104	0.049	0.03	0.	3.166	0.	0.

Problem 1273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	52	81	0	208	0	0
normalized size	1	1.	0.55	0.85	0.	2.19	0.	0.
time (sec)	N/A	0.089	0.039	0.028	0.	3.056	0.	0.

Problem 1274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	58	64	95	142	0	0
normalized size	1	1.	1.29	1.42	2.11	3.16	0.	0.
time (sec)	N/A	0.08	0.031	0.026	1.043	3.089	0.	0.

Problem 1275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	54	69	45	806	0	0
normalized size	1	1.	0.66	0.84	0.55	9.83	0.	0.
time (sec)	N/A	0.088	0.033	0.087	0.993	3.023	0.	0.

Problem 1276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	53	38	39	146	0	0
normalized size	1	1.	1.15	0.83	0.85	3.17	0.	0.
time (sec)	N/A	0.081	0.043	0.027	0.997	2.459	0.	0.

Problem 1277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	73	159	126	969	0	0
normalized size	1	1.	0.4	0.87	0.69	5.32	0.	0.
time (sec)	N/A	0.111	0.061	0.092	1.007	2.921	0.	0.

Problem 1278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	101	238	165	1175	0	0
normalized size	1	1.	0.37	0.87	0.6	4.27	0.	0.
time (sec)	N/A	0.14	0.085	0.098	1.031	2.942	0.	0.

Problem 1279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	74	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.202	0.05	0.435	0.	0.	0.	0.

Problem 1280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.029	0.431	0.	0.	0.	0.

Problem 1281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	359	359	42	0	0	1405	0	0
normalized size	1	1.	0.12	0.	0.	3.91	0.	0.
time (sec)	N/A	0.306	0.019	0.201	0.	2.965	0.	0.

Problem 1282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	307	307	42	0	0	1346	0	0
normalized size	1	1.	0.14	0.	0.	4.38	0.	0.
time (sec)	N/A	0.24	0.014	0.201	0.	2.957	0.	0.

Problem 1283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	255	255	42	0	0	1299	0	0
normalized size	1	1.	0.16	0.	0.	5.09	0.	0.
time (sec)	N/A	0.188	0.011	0.198	0.	2.857	0.	0.

Problem 1284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	193	193	40	0	0	1183	0	0
normalized size	1	1.	0.21	0.	0.	6.13	0.	0.
time (sec)	N/A	0.142	0.011	0.21	0.	2.776	0.	0.

Problem 1285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	37	37	32	54	0	117	0	0
normalized size	1	1.	0.86	1.46	0.	3.16	0.	0.
time (sec)	N/A	0.04	0.013	0.031	0.	2.451	0.	0.

Problem 1286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	48	70	0	170	0	0
normalized size	1	1.	0.64	0.93	0.	2.27	0.	0.
time (sec)	N/A	0.079	0.02	0.033	0.	2.599	0.	0.

Problem 1287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	64	86	0	234	0	0
normalized size	1	1.	0.57	0.77	0.	2.09	0.	0.
time (sec)	N/A	0.117	0.03	0.03	0.	2.567	0.	0.

Problem 1288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	80	102	0	300	0	0
normalized size	1	1.	0.54	0.68	0.	2.01	0.	0.
time (sec)	N/A	0.168	0.03	0.031	0.	2.533	0.	0.

Problem 1289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	679	679	74	0	0	0	0	0
normalized size	1	1.	0.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.392	0.036	0.198	0.	0.	0.	0.

Problem 1290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	547	547	72	0	0	0	0	0
normalized size	1	1.	0.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.315	0.03	0.202	0.	0.	0.	0.

Problem 1291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	429	429	71	0	0	0	0	0
normalized size	1	1.	0.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.257	0.024	0.203	0.	0.	0.	0.

Problem 1292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	309	309	69	0	0	0	0	0
normalized size	1	1.	0.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.205	0.022	0.202	0.	0.	0.	0.

Problem 1293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	41	41	64	55	0	0	0	0
normalized size	1	1.	1.56	1.34	0.	0.	0.	0.
time (sec)	N/A	0.045	0.025	0.028	0.	0.	0.	0.

Problem 1294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	80	71	0	0	0	0
normalized size	1	1.	0.96	0.86	0.	0.	0.	0.
time (sec)	N/A	0.094	0.037	0.03	0.	0.	0.	0.

Problem 1295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	96	87	0	0	0	0
normalized size	1	1.	0.77	0.7	0.	0.	0.	0.
time (sec)	N/A	0.143	0.047	0.03	0.	0.	0.	0.

Problem 1296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	112	103	0	0	0	0
normalized size	1	1.	0.68	0.62	0.	0.	0.	0.
time (sec)	N/A	0.197	0.053	0.031	0.	0.	0.	0.

Problem 1297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	84	0	0	0	0	0
normalized size	1	1.	0.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.299	0.064	0.217	0.	0.	0.	0.

Problem 1298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	70	0	0	0	0	0
normalized size	1	1.	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.26	0.043	0.217	0.	0.	0.	0.

Problem 1299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	74	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.162	0.053	0.214	0.	0.	0.	0.

Problem 1300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	64	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.017	0.213	0.	0.	0.	0.

Problem 1301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	79	0	0	0	0	0
normalized size	1	1.	0.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.235	0.032	0.211	0.	0.	0.	0.

Problem 1302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	86	0	0	0	0	0
normalized size	1	1.	0.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.249	0.035	0.223	0.	0.	0.	0.

Problem 1303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	107	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.254	0.069	0.215	0.	0.	0.	0.

Problem 1304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	41	41	63	54	38	0	0	0
normalized size	1	1.	1.54	1.32	0.93	0.	0.	0.
time (sec)	N/A	0.119	0.033	0.028	2.533	0.	0.	0.

Problem 1305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	93	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.169	0.039	0.22	0.	0.	0.	0.

Problem 1306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	69	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.026	0.227	0.	0.	0.	0.

Problem 1307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.225	0.564	0.217	0.	0.	0.	0.

Problem 1308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	65	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.019	0.049	0.	0.	0.	0.

Problem 1309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	67	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.021	0.144	0.	0.	0.	0.

Problem 1310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	67	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.025	0.148	0.	0.	0.	0.

Problem 1311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	172	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.257	0.203	0.203	0.	0.	0.	0.

Problem 1312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	120	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.205	0.061	0.188	0.	0.	0.	0.

Problem 1313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	86	127	82	0	0	0	0	0
normalized size	1	1.48	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	0.063	0.185	0.	0.	0.	0.

Problem 1314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	74	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.056	0.184	0.	0.	0.	0.

Problem 1315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	18	18	33	18	41	53	0	0
normalized size	1	1.	1.83	1.	2.28	2.94	0.	0.
time (sec)	N/A	0.034	0.007	0.029	0.964	2.193	0.	0.

Problem 1316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	90	100	85	0	0	0	0	0
normalized size	1	1.11	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.035	0.19	0.	0.	0.	0.

Problem 1317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	123	137	103	0	0	0	0	0
normalized size	1	1.11	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.144	0.044	0.185	0.	0.	0.	0.

Problem 1318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	178	0	0	0	0	0
normalized size	1	1.	0.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.388	0.156	0.191	0.	0.	0.	0.

Problem 1319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	311	311	122	0	0	0	0	0
normalized size	1	1.	0.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.252	5.193	0.192	0.	0.	0.	0.

Problem 1320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	65	62	0	178	0	0
normalized size	1	1.	0.82	0.78	0.	2.25	0.	0.
time (sec)	N/A	0.127	0.052	0.028	0.	2.176	0.	0.

Problem 1321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	102	56	47	0	151	0	0
normalized size	1	1.48	0.81	0.68	0.	2.19	0.	0.
time (sec)	N/A	0.149	0.034	0.03	0.	2.232	0.	0.

Problem 1322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	68	55	0	163	0	0
normalized size	1	1.	0.94	0.76	0.	2.26	0.	0.
time (sec)	N/A	0.074	0.029	0.029	0.	2.342	0.	0.

Problem 1323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	190	200	121	0	0	0	0	0
normalized size	1	1.05	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.202	0.07	0.183	0.	0.	0.	0.

Problem 1324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	239	253	163	0	0	0	0	0
normalized size	1	1.06	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.251	0.088	0.187	0.	0.	0.	0.

Problem 1325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	111	101	0	371	0	0
normalized size	1	1.	0.87	0.8	0.	2.92	0.	0.
time (sec)	N/A	0.134	0.076	0.032	0.	2.555	0.	0.

Problem 1326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	170	167	0	682	0	0
normalized size	1	1.	0.86	0.85	0.	3.46	0.	0.
time (sec)	N/A	0.186	0.1	0.033	0.	2.595	0.	0.

Problem 1327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	256	256	236	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.34	0.506	0.203	0.	0.	0.	0.

Problem 1328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	135	0	0	0	0	0
normalized size	1	1.	0.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.313	0.191	0.207	0.	0.	0.	0.

Problem 1329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	125	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.187	0.128	0.2	0.	0.	0.	0.

Problem 1330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	101	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.049	0.205	0.	0.	0.	0.

Problem 1331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	269	299	207	0	0	0	0	0
normalized size	1	1.11	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.293	0.157	0.201	0.	0.	0.	0.

Problem 1332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	268	97	138	0	0	0	0	0
normalized size	1	0.36	0.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.234	0.423	0.207	0.	0.	0.	0.

Problem 1333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	102	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.074	0.21	0.	0.	0.	0.

Problem 1334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	187	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.381	0.262	0.209	0.	0.	0.	0.

Problem 1335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	141	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.329	0.165	0.213	0.	0.	0.	0.

Problem 1336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	124	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.202	0.093	0.211	0.	0.	0.	0.

Problem 1337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	101	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.035	0.203	0.	0.	0.	0.

Problem 1338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	99	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.231	0.048	0.205	0.	0.	0.	0.

Problem 1339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	117	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.249	0.068	0.202	0.	0.	0.	0.

Problem 1340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	134	0	0	0	0	0
normalized size	1	1.	0.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.274	0.084	0.208	0.	0.	0.	0.

Problem 1341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	186	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.363	0.214	0.207	0.	0.	0.	0.

Problem 1342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	155	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.246	0.218	0.203	0.	0.	0.	0.

Problem 1343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	46	46	81	49	0	159	0	0
normalized size	1	1.	1.76	1.07	0.	3.46	0.	0.
time (sec)	N/A	0.093	0.058	0.029	0.	2.254	0.	0.

Problem 1344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	46	46	81	49	0	153	0	0
normalized size	1	1.	1.76	1.07	0.	3.33	0.	0.
time (sec)	N/A	0.053	0.053	0.027	0.	2.227	0.	0.

Problem 1345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	243	247	149	0	0	0	0	0
normalized size	1	1.02	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.324	0.144	0.209	0.	0.	0.	0.

Problem 1346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	321	325	173	0	0	0	0	0
normalized size	1	1.01	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.351	0.167	0.21	0.	0.	0.	0.

Problem 1347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	417	422	219	0	0	0	0	0
normalized size	1	1.01	0.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.421	0.216	0.201	0.	0.	0.	0.

Problem 1348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	112	93	0	352	0	0
normalized size	1	1.	0.28	0.23	0.	0.86	0.	0.
time (sec)	N/A	0.486	0.197	0.028	0.	2.417	0.	0.

Problem 1349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	125	96	0	356	0	0
normalized size	1	1.	1.23	0.94	0.	3.49	0.	0.
time (sec)	N/A	0.199	0.184	0.03	0.	2.244	0.	0.

Problem 1350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	114	86	0	344	0	0
normalized size	1	1.	0.86	0.65	0.	2.59	0.	0.
time (sec)	N/A	0.214	0.165	0.03	0.	2.105	0.	0.

Problem 1351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	123	84	0	335	0	0
normalized size	1	1.	1.21	0.82	0.	3.28	0.	0.
time (sec)	N/A	0.106	0.143	0.03	0.	2.13	0.	0.

Problem 1352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	417	421	222	0	0	0	0	0
normalized size	1	1.01	0.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.458	0.243	0.211	0.	0.	0.	0.

Problem 1353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	507	511	268	0	0	0	0	0
normalized size	1	1.01	0.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.535	0.304	0.211	0.	0.	0.	0.

Problem 1354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	623	628	267	0	0	0	0	0
normalized size	1	1.01	0.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.647	0.659	0.204	0.	0.	0.	0.

Problem 1355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	182	140	0	622	0	0
normalized size	1	1.	1.1	0.84	0.	3.75	0.	0.
time (sec)	N/A	0.186	0.221	0.03	0.	2.229	0.	0.

Problem 1356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.29	0.162	0.	0.	0.	0.

Problem 1357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.262	0.066	0.	0.	0.	0.

Problem 1358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	42	42	106	0	0	0	0	0
normalized size	1	1.	2.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.196	0.203	0.	0.	0.	0.

Problem 1359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.445	0.21	0.	0.	0.	0.

Problem 1360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.161	0.479	0.307	0.	0.	0.	0.

Problem 1361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	136	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.184	0.098	0.27	0.	0.	0.	0.

Problem 1362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	102	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.025	0.24	0.	0.	0.	0.

Problem 1363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	31	59	54	158	0	0
normalized size	1	1.	0.76	1.44	1.32	3.85	0.	0.
time (sec)	N/A	0.048	0.019	0.026	1.003	2.148	0.	0.

Problem 1364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	58	60	55	161	0	0
normalized size	1	1.	0.61	0.63	0.58	1.69	0.	0.
time (sec)	N/A	0.09	0.021	0.029	0.996	2.209	0.	0.

Problem 1365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	36	38	46	89	0	0
normalized size	1	1.	0.73	0.78	0.94	1.82	0.	0.
time (sec)	N/A	0.073	0.025	0.024	0.983	2.148	0.	0.

Problem 1366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	36	40	49	92	0	0
normalized size	1	1.	0.71	0.78	0.96	1.8	0.	0.
time (sec)	N/A	0.071	0.028	0.026	0.991	2.132	0.	0.

Problem 1367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	53	92	58	101	157	0	0
normalized size	1	1.	1.74	1.09	1.91	2.96	0.	0.
time (sec)	N/A	0.109	0.074	0.03	1.028	2.185	0.	0.

Problem 1368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	426	512	987	0	404
normalized size	1	1.	0.97	13.74	16.52	31.84	0.	13.03
time (sec)	N/A	0.091	0.975	0.059	2.004	5.528	0.	1.225

Problem 1369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	186	228	370	178	188
normalized size	1	1.	0.97	6.	7.35	11.94	5.74	6.06
time (sec)	N/A	0.089	0.175	0.04	1.114	2.148	132.855	1.134

Problem 1370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	54	66	97	49	61
normalized size	1	1.	0.97	1.74	2.13	3.13	1.58	1.97
time (sec)	N/A	0.088	0.031	0.034	0.985	1.906	0.568	1.126

Problem 1371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	54	66	97	49	103
normalized size	1	1.	0.97	1.74	2.13	3.13	1.58	3.32
time (sec)	N/A	0.089	0.03	0.033	0.96	1.901	0.58	1.136

Problem 1372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	186	228	370	178	188
normalized size	1	1.	0.97	6.	7.35	11.94	5.74	6.06
time (sec)	N/A	0.087	0.169	0.04	1.103	2.084	100.454	1.141

Problem 1373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	59	49	0	1227	0	0
normalized size	1	1.	0.98	0.82	0.	20.45	0.	0.
time (sec)	N/A	0.231	0.447	0.04	0.	2.908	0.	0.

Problem 1374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	59	49	0	393	0	0
normalized size	1	1.	0.98	0.82	0.	6.55	0.	0.
time (sec)	N/A	0.24	0.094	0.031	0.	2.236	0.	0.

Problem 1375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	76	90	0	0	0	0
normalized size	1	1.	0.55	0.65	0.	0.	0.	0.
time (sec)	N/A	0.24	0.047	0.09	0.	0.	0.	0.

Problem 1376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	72	88	70	0	0	0
normalized size	1	1.	0.52	0.64	0.51	0.	0.	0.
time (sec)	N/A	0.257	0.05	0.094	0.99	0.	0.	0.

Problem 1377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	59	49	126	392	0	0
normalized size	1	1.	0.98	0.82	2.1	6.53	0.	0.
time (sec)	N/A	0.237	0.092	0.032	1.102	2.179	0.	0.

Problem 1378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	59	49	369	1226	0	664
normalized size	1	1.	0.98	0.82	6.15	20.43	0.	11.07
time (sec)	N/A	0.238	0.466	0.037	3.732	3.007	0.	29.057

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [138] had the largest ratio of [1.429]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.	10	0.4
2	A	5	4	1.	10	0.4
3	A	7	5	1.	10	0.5
4	A	3	3	1.	8	0.375
5	A	3	3	1.	6	0.5
6	A	6	6	1.	10	0.6
7	A	5	5	1.	10	0.5
8	A	6	6	1.	10	0.6
9	A	7	6	1.	10	0.6
10	A	8	6	1.	10	0.6
11	A	3	2	1.	12	0.167
12	A	3	2	1.	12	0.167
13	A	3	2	1.	10	0.2
14	A	3	2	1.	8	0.25
15	A	3	2	1.	12	0.167
16	A	3	2	1.	12	0.167
17	A	3	2	1.	12	0.167
18	A	3	2	1.	12	0.167
19	A	10	9	1.	12	0.75
20	A	9	7	1.	10	0.7
21	A	5	5	1.	8	0.625
22	A	8	7	1.	12	0.583
23	A	8	7	1.	12	0.583
24	A	12	8	1.	12	0.667
25	A	14	9	1.	12	0.75
26	A	3	2	1.	12	0.167
27	A	3	2	1.	12	0.167
28	A	3	2	1.	10	0.2
29	A	3	2	1.	8	0.25
30	A	3	2	1.	12	0.167
31	A	3	2	1.	12	0.167
32	A	3	2	1.	12	0.167
33	A	3	2	1.	12	0.167
34	A	5	4	1.	12	0.333
35	A	7	5	1.	12	0.417
36	A	3	3	1.	10	0.3
37	A	3	3	1.	8	0.375
38	A	6	6	1.	12	0.5
39	A	5	5	1.	12	0.417
40	A	6	6	1.	12	0.5
41	A	7	6	1.	12	0.5
42	A	8	6	1.	12	0.5
43	A	3	2	1.	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	3	2	1.	12	0.167
45	A	3	2	1.	10	0.2
46	A	3	2	1.	8	0.25
47	A	3	2	1.	12	0.167
48	A	3	2	1.	12	0.167
49	A	3	2	1.	12	0.167
50	A	3	2	1.	12	0.167
51	A	14	11	1.	12	0.917
52	A	10	9	1.	12	0.75
53	A	9	7	1.	10	0.7
54	A	5	5	1.	8	0.625
55	A	8	7	1.	12	0.583
56	A	8	7	1.	12	0.583
57	A	12	8	1.	12	0.667
58	A	14	9	1.	12	0.75
59	A	19	9	1.	12	0.75
60	A	2	2	1.	14	0.143
61	A	15	12	1.	14	0.857
62	A	14	11	1.	12	0.917
63	A	13	10	1.	10	1.
64	A	17	14	1.	14	1.
65	A	6	6	1.	14	0.429
66	A	7	7	1.	14	0.5
67	A	9	8	1.	14	0.571
68	A	10	8	1.	14	0.571
69	A	11	8	1.	14	0.571
70	A	2	2	1.	14	0.143
71	A	15	12	1.	14	0.857
72	A	15	12	1.	14	0.857
73	A	14	11	1.	12	0.917
74	A	13	10	1.	10	1.
75	A	17	14	1.	14	1.
76	A	6	6	1.	14	0.429
77	A	7	7	1.	14	0.5
78	A	9	8	1.	14	0.571
79	A	10	8	1.	14	0.571
80	A	2	2	1.	14	0.143
81	A	16	13	1.	14	0.929
82	A	16	12	1.	14	0.857
83	A	15	11	1.	12	0.917
84	A	14	11	1.	10	1.1
85	A	19	16	1.	14	1.143
86	A	7	6	1.	14	0.429
87	A	8	7	1.	14	0.5

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
88	A	10	9	1.	14	0.643
89	A	11	9	1.	14	0.643
90	A	2	2	1.	14	0.143
91	A	15	12	1.	14	0.857
92	A	15	12	1.	14	0.857
93	A	14	11	1.	12	0.917
94	A	13	10	1.	10	1.
95	A	17	14	1.	14	1.
96	A	6	6	1.	14	0.429
97	A	7	7	1.	14	0.5
98	A	9	8	1.	14	0.571
99	A	10	8	1.	14	0.571
100	A	2	2	1.	14	0.143
101	A	15	12	1.	14	0.857
102	A	15	12	1.	14	0.857
103	A	14	11	1.	12	0.917
104	A	13	10	1.	10	1.
105	A	17	14	1.	14	1.
106	A	6	6	1.	14	0.429
107	A	7	7	1.	14	0.5
108	A	9	8	1.	14	0.571
109	A	10	8	1.	14	0.571
110	A	2	2	1.	14	0.143
111	A	16	13	1.	14	0.929
112	A	16	12	1.	14	0.857
113	A	15	11	1.	12	0.917
114	A	14	11	1.	10	1.1
115	A	19	16	1.	14	1.143
116	A	7	6	1.	14	0.429
117	A	8	7	1.	14	0.5
118	A	10	9	1.	14	0.643
119	A	11	9	1.	14	0.643
120	A	2	2	1.	12	0.167
121	A	16	12	1.	12	1.
122	A	15	11	1.	10	1.1
123	A	14	10	1.	8	1.25
124	A	25	13	1.	12	1.083
125	A	13	9	1.	12	0.75
126	A	14	10	1.	12	0.833
127	A	2	2	1.	12	0.167
128	A	5	5	1.	12	0.417
129	A	4	4	1.	10	0.4
130	A	3	3	1.	8	0.375
131	A	4	4	1.	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
132	A	3	3	1.	12	0.25
133	A	4	4	1.	12	0.333
134	A	2	2	1.	14	0.143
135	A	27	13	1.	14	0.929
136	A	26	12	1.	12	1.
137	A	25	11	1.	10	1.1
138	A	39	20	1.	14	1.429
139	A	16	13	1.	14	0.929
140	A	17	14	1.	14	1.
141	A	4	4	1.	12	0.333
142	A	9	5	1.	12	0.417
143	A	3	3	1.	12	0.25
144	A	4	3	1.	10	0.3
145	A	4	3	1.	12	0.25
146	A	3	3	1.	12	0.25
147	A	9	5	1.	12	0.417
148	A	2	2	1.	12	0.167
149	A	4	4	1.	12	0.333
150	A	4	4	1.	12	0.333
151	A	3	3	1.	10	0.3
152	A	2	2	1.	8	0.25
153	A	4	4	1.	12	0.333
154	A	2	2	1.	12	0.167
155	A	3	3	1.	12	0.25
156	A	5	5	1.	12	0.417
157	A	3	3	1.	16	0.188
158	A	6	5	1.	16	0.312
159	A	5	5	1.	16	0.312
160	A	4	4	1.	16	0.25
161	A	3	3	1.	14	0.214
162	A	3	3	1.	16	0.188
163	A	2	2	1.	16	0.125
164	A	3	3	1.	16	0.188
165	A	4	3	1.	16	0.188
166	A	5	3	1.	16	0.188
167	A	4	3	1.	18	0.167
168	A	3	2	1.	18	0.111
169	A	3	2	1.	18	0.111
170	A	3	2	1.	18	0.111
171	A	3	2	1.	18	0.111
172	C	1	1	2.	16	0.062
173	A	3	2	1.	18	0.111
174	A	2	2	1.	18	0.111
175	A	3	2	1.	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	3	2	1.	18	0.111
177	A	3	3	1.	18	0.167
178	A	5	4	1.	18	0.222
179	A	4	3	1.	18	0.167
180	A	4	4	1.	18	0.222
181	A	4	3	1.	16	0.188
182	A	4	3	1.	18	0.167
183	A	2	2	1.	18	0.111
184	A	3	3	1.	18	0.167
185	A	4	3	1.	18	0.167
186	A	5	3	1.	18	0.167
187	A	4	3	1.	18	0.167
188	A	3	2	1.	18	0.111
189	A	4	3	1.	18	0.167
190	A	3	2	1.	18	0.111
191	A	2	2	1.	18	0.111
192	A	3	2	1.	16	0.125
193	A	3	2	1.	18	0.111
194	A	2	2	1.	18	0.111
195	A	3	2	1.	18	0.111
196	A	3	2	1.	18	0.111
197	A	3	3	1.	18	0.167
198	A	6	4	1.	18	0.222
199	A	5	4	1.	18	0.222
200	A	4	4	1.	16	0.25
201	A	2	2	1.	18	0.111
202	A	2	2	1.	18	0.111
203	A	3	3	1.	18	0.167
204	A	4	3	1.	18	0.167
205	A	5	3	1.	18	0.167
206	A	3	3	1.	18	0.167
207	A	3	2	1.	18	0.111
208	A	3	2	1.	18	0.111
209	A	3	2	1.	18	0.111
210	A	3	2	1.	16	0.125
211	A	2	2	1.	18	0.111
212	A	3	3	1.	18	0.167
213	A	4	3	1.	18	0.167
214	A	4	3	1.	18	0.167
215	A	4	3	1.	18	0.167
216	A	3	3	1.	18	0.167
217	A	7	5	1.	18	0.278
218	A	6	5	1.	18	0.278
219	A	5	5	1.	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
220	A	3	3	1.	18	0.167
221	A	2	2	1.	18	0.111
222	A	2	2	1.	18	0.111
223	A	3	3	1.	18	0.167
224	A	4	3	1.	18	0.167
225	A	5	3	1.	18	0.167
226	A	6	3	1.	18	0.167
227	A	5	3	1.	18	0.167
228	A	4	3	1.	18	0.167
229	A	3	3	1.	18	0.167
230	A	2	2	1.	18	0.111
231	A	4	4	1.	18	0.222
232	A	4	4	1.	18	0.222
233	A	5	5	1.	18	0.278
234	A	6	5	1.	18	0.278
235	A	4	3	1.	20	0.15
236	A	4	3	1.	20	0.15
237	A	4	3	1.	20	0.15
238	A	4	3	1.	20	0.15
239	A	4	3	1.	20	0.15
240	A	4	3	1.	20	0.15
241	A	4	3	1.	20	0.15
242	A	4	3	1.	20	0.15
243	A	5	3	1.	20	0.15
244	A	4	3	1.	20	0.15
245	A	3	3	1.	20	0.15
246	A	2	2	1.	20	0.1
247	A	5	4	1.	20	0.2
248	A	5	5	1.	20	0.25
249	A	5	4	1.	20	0.2
250	A	6	5	1.	20	0.25
251	A	7	5	1.	20	0.25
252	A	7	3	1.	20	0.15
253	A	6	3	1.	20	0.15
254	A	5	3	1.	20	0.15
255	A	4	3	1.	20	0.15
256	A	3	3	1.	20	0.15
257	A	2	2	1.	20	0.1
258	A	3	3	1.	20	0.15
259	A	4	4	1.	20	0.2
260	A	5	4	1.	20	0.2
261	A	9	5	1.	20	0.25
262	A	8	5	1.	20	0.25
263	A	7	5	1.	20	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
264	A	6	5	1.	20	0.25
265	A	5	5	1.	20	0.25
266	A	4	4	1.	20	0.2
267	A	5	5	1.	20	0.25
268	A	6	5	1.	20	0.25
269	A	7	5	1.	20	0.25
270	A	6	3	1.	20	0.15
271	A	5	3	1.	20	0.15
272	A	4	3	1.	20	0.15
273	A	3	3	1.	20	0.15
274	A	2	2	1.	20	0.1
275	A	4	4	1.	20	0.2
276	A	5	5	1.	20	0.25
277	A	6	5	1.	20	0.25
278	A	3	3	1.	20	0.15
279	A	3	3	1.	20	0.15
280	A	3	3	1.	20	0.15
281	A	3	3	1.	20	0.15
282	A	3	3	1.	20	0.15
283	A	3	3	1.	20	0.15
284	A	3	3	1.	20	0.15
285	A	3	3	1.	20	0.15
286	A	5	4	1.	17	0.235
287	A	4	3	1.	17	0.176
288	A	4	4	1.	17	0.235
289	A	2	2	1.	15	0.133
290	A	3	3	1.	14	0.214
291	A	5	5	1.	17	0.294
292	A	3	3	1.	17	0.176
293	A	5	5	1.	17	0.294
294	A	2	2	1.	17	0.118
295	A	6	5	1.	19	0.263
296	A	9	5	1.	19	0.263
297	A	4	4	1.	17	0.235
298	A	4	4	1.	16	0.25
299	A	7	7	1.	19	0.368
300	A	7	7	1.	19	0.368
301	A	7	7	1.	19	0.368
302	A	6	6	1.	19	0.316
303	A	7	7	1.	19	0.368
304	A	8	7	1.	19	0.368
305	A	9	7	1.	19	0.368
306	A	7	6	1.	19	0.316
307	A	6	6	1.	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
308	A	5	5	1.	17	0.294
309	A	5	5	1.	16	0.312
310	A	8	8	1.	19	0.421
311	A	8	8	1.	19	0.421
312	A	8	8	1.	19	0.421
313	A	8	8	1.	19	0.421
314	A	7	7	1.	19	0.368
315	A	8	8	1.	19	0.421
316	A	8	6	1.	19	0.316
317	A	7	6	1.	19	0.316
318	A	7	6	1.	17	0.353
319	A	6	5	1.	16	0.312
320	A	9	8	1.	19	0.421
321	A	9	9	1.	19	0.474
322	A	9	8	1.	19	0.421
323	A	9	8	1.	19	0.421
324	A	9	8	1.05	19	0.421
325	A	8	7	1.	19	0.368
326	A	9	8	1.	19	0.421
327	A	8	6	1.	19	0.316
328	A	7	6	1.	19	0.316
329	A	5	5	1.	19	0.263
330	A	4	4	1.	17	0.235
331	A	3	3	1.	16	0.188
332	A	7	7	1.	19	0.368
333	A	7	7	1.	19	0.368
334	A	8	8	1.	19	0.421
335	A	9	8	1.	19	0.421
336	A	10	9	1.	19	0.474
337	A	6	5	1.	19	0.263
338	A	5	5	1.	19	0.263
339	A	4	4	1.	17	0.235
340	A	2	2	1.	16	0.125
341	A	8	8	1.	19	0.421
342	A	8	7	1.	19	0.368
343	A	9	8	1.	19	0.421
344	A	10	8	1.	19	0.421
345	A	7	5	1.	19	0.263
346	A	9	7	1.	19	0.368
347	A	8	6	1.	19	0.316
348	A	3	3	1.	17	0.176
349	A	3	3	1.	16	0.188
350	A	9	8	1.	19	0.421
351	A	9	7	1.	19	0.368

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
352	A	10	8	1.	19	0.421
353	A	11	8	1.	19	0.421
354	A	8	5	1.	19	0.263
355	A	12	7	1.	19	0.368
356	A	11	6	1.	19	0.316
357	A	5	5	1.	19	0.263
358	A	4	4	1.	17	0.235
359	A	4	3	1.	16	0.188
360	A	10	8	1.	19	0.421
361	A	10	7	1.	19	0.368
362	A	11	8	1.	19	0.421
363	A	6	5	1.	9	0.556
364	A	5	4	1.	8	0.5
365	A	7	5	1.	11	0.454
366	A	6	4	1.	10	0.4
367	A	2	2	1.	11	0.182
368	A	3	3	1.	10	0.3
369	A	4	4	1.	11	0.364
370	A	2	2	1.	10	0.2
371	A	3	2	1.	13	0.154
372	A	3	2	1.	12	0.167
373	A	5	4	1.38	15	0.267
374	A	4	3	1.	14	0.214
375	A	3	2	1.	13	0.154
376	A	3	2	1.	12	0.167
377	A	4	3	1.	15	0.2
378	A	3	3	1.	14	0.214
379	A	3	2	1.	13	0.154
380	A	2	2	1.	12	0.167
381	A	5	5	1.	15	0.333
382	A	5	5	1.	14	0.357
383	A	4	4	1.	13	0.308
384	A	3	3	1.	12	0.25
385	A	5	5	1.	15	0.333
386	A	5	5	1.	14	0.357
387	A	4	4	1.	21	0.19
388	A	4	4	1.	21	0.19
389	A	3	3	1.	19	0.158
390	A	2	2	1.	18	0.111
391	A	4	4	1.	21	0.19
392	A	4	4	1.	21	0.19
393	A	4	3	1.	23	0.13
394	A	4	3	1.	23	0.13
395	A	4	3	1.	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
396	A	4	3	1.	20	0.15
397	A	5	5	1.	23	0.217
398	A	5	5	1.	23	0.217
399	A	6	6	1.	23	0.261
400	A	7	6	1.	23	0.261
401	A	8	6	1.	23	0.261
402	A	8	6	1.	23	0.261
403	A	8	6	1.	23	0.261
404	A	7	6	1.	21	0.286
405	A	5	4	1.	20	0.2
406	A	8	6	1.	23	0.261
407	A	8	6	1.	23	0.261
408	A	9	7	1.	23	0.304
409	A	10	7	1.	23	0.304
410	A	11	7	1.	23	0.304
411	A	5	5	1.	23	0.217
412	A	5	5	1.	23	0.217
413	A	4	4	1.	21	0.19
414	A	3	3	1.	20	0.15
415	A	4	4	1.	23	0.174
416	A	4	4	1.	23	0.174
417	A	5	5	1.	23	0.217
418	A	6	5	1.	23	0.217
419	A	8	6	1.	23	0.261
420	A	8	6	1.	23	0.261
421	A	7	6	1.	21	0.286
422	A	6	5	1.	20	0.25
423	A	8	7	1.	23	0.304
424	A	8	7	1.	23	0.304
425	A	9	8	1.	23	0.348
426	A	10	8	1.	23	0.348
427	A	11	8	1.	23	0.348
428	A	4	3	1.	23	0.13
429	A	4	3	1.	23	0.13
430	A	4	3	1.	21	0.143
431	A	4	3	1.	20	0.15
432	A	6	5	1.	23	0.217
433	A	6	6	1.	23	0.261
434	A	7	7	1.	23	0.304
435	A	8	7	1.01	23	0.304
436	A	9	7	1.01	23	0.304
437	A	3	3	1.	19	0.158
438	A	3	3	1.	19	0.158
439	A	3	3	1.	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
440	A	2	2	1.	18	0.111
441	A	2	2	1.	18	0.111
442	A	2	2	1.	16	0.125
443	A	2	2	1.	18	0.111
444	A	2	2	1.	18	0.111
445	A	3	3	1.	18	0.167
446	A	4	3	1.	18	0.167
447	A	3	3	1.	20	0.15
448	A	10	9	1.	20	0.45
449	A	9	9	1.	20	0.45
450	A	8	8	1.	20	0.4
451	A	6	6	1.	18	0.333
452	A	5	5	1.	20	0.25
453	A	6	6	1.	20	0.3
454	A	10	8	1.	20	0.4
455	A	13	8	1.	20	0.4
456	A	6	6	1.	22	0.273
457	A	4	3	1.	22	0.136
458	A	4	3	1.	22	0.136
459	A	4	3	1.	22	0.136
460	A	5	4	1.	22	0.182
461	A	4	3	1.	20	0.15
462	A	4	3	1.	22	0.136
463	A	4	3	1.	22	0.136
464	A	4	3	1.	22	0.136
465	A	4	3	1.	22	0.136
466	A	9	9	1.	22	0.409
467	A	7	7	1.	22	0.318
468	A	9	9	1.	22	0.409
469	A	9	9	1.	20	0.45
470	A	6	6	1.	22	0.273
471	A	9	8	1.	22	0.364
472	A	9	8	1.	22	0.364
473	A	10	8	1.	22	0.364
474	A	7	7	1.	22	0.318
475	A	4	3	1.	22	0.136
476	A	5	4	1.	22	0.182
477	A	4	3	1.	22	0.136
478	A	4	3	1.	22	0.136
479	A	4	3	1.	20	0.15
480	A	4	3	1.	22	0.136
481	A	4	3	1.	22	0.136
482	A	4	3	1.	22	0.136
483	A	4	3	1.	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
484	A	3	3	1.	22	0.136
485	A	11	9	1.	22	0.409
486	A	10	9	1.	22	0.409
487	A	9	9	1.	22	0.409
488	A	8	8	1.	20	0.4
489	A	3	3	1.	22	0.136
490	A	6	6	1.	22	0.273
491	A	7	6	1.	22	0.273
492	A	8	6	1.	22	0.273
493	A	8	8	1.	22	0.364
494	A	4	3	1.	22	0.136
495	A	4	3	1.	22	0.136
496	A	4	3	1.	22	0.136
497	A	4	3	1.	20	0.15
498	A	4	3	1.	22	0.136
499	A	5	4	1.	22	0.182
500	A	4	3	1.	22	0.136
501	A	4	3	1.	22	0.136
502	A	11	10	1.	22	0.454
503	A	10	10	1.	22	0.454
504	A	9	9	1.	20	0.45
505	A	5	5	1.	22	0.227
506	A	6	6	1.	22	0.273
507	A	5	4	1.	22	0.182
508	A	7	6	1.	22	0.273
509	A	8	7	1.	22	0.318
510	A	8	7	1.	22	0.318
511	A	7	7	1.	22	0.318
512	A	7	7	1.	22	0.318
513	A	7	7	1.	22	0.318
514	A	6	6	1.01	22	0.273
515	A	8	8	1.	22	0.364
516	A	9	9	1.	22	0.409
517	A	10	10	1.	22	0.454
518	A	11	8	1.	24	0.333
519	A	10	8	1.	24	0.333
520	A	9	8	1.	24	0.333
521	A	8	8	1.	24	0.333
522	A	7	7	1.	24	0.292
523	A	8	8	1.	24	0.333
524	A	9	8	1.	24	0.333
525	A	10	8	1.	24	0.333
526	A	11	8	1.	24	0.333
527	A	8	8	1.	24	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
528	A	8	8	1.	24	0.333
529	A	8	8	1.	24	0.333
530	A	7	7	1.	24	0.292
531	A	8	8	1.	24	0.333
532	A	9	9	1.	24	0.375
533	A	10	10	1.	24	0.417
534	A	11	10	1.	24	0.417
535	A	8	7	1.	24	0.292
536	A	7	7	1.	24	0.292
537	A	6	6	1.	24	0.25
538	A	6	6	1.	24	0.25
539	A	6	6	1.	24	0.25
540	A	6	6	1.	24	0.25
541	A	9	9	1.	24	0.375
542	A	9	9	1.	24	0.375
543	A	10	10	1.	24	0.417
544	A	14	9	1.	24	0.375
545	A	13	9	1.	24	0.375
546	A	12	9	1.	24	0.375
547	A	11	9	1.	24	0.375
548	A	10	8	1.	24	0.333
549	A	10	8	1.	24	0.333
550	A	11	9	1.	24	0.375
551	A	11	9	1.	24	0.375
552	A	12	9	1.	24	0.375
553	A	13	9	1.	24	0.375
554	A	9	7	1.	24	0.292
555	A	8	7	1.	24	0.292
556	A	7	7	1.	24	0.292
557	A	6	6	1.	24	0.25
558	A	6	6	1.	24	0.25
559	A	7	7	1.	24	0.292
560	A	7	7	1.	24	0.292
561	A	9	9	1.	24	0.375
562	A	10	10	1.	24	0.417
563	A	9	8	1.	25	0.32
564	A	4	4	1.	25	0.16
565	A	8	6	1.	25	0.24
566	A	7	6	1.	23	0.261
567	A	6	6	1.	22	0.273
568	A	6	6	1.	25	0.24
569	A	4	4	1.	25	0.16
570	A	5	5	1.	25	0.2
571	A	6	5	1.	25	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
572	A	7	5	1.	25	0.2
573	A	10	8	1.	27	0.296
574	A	9	8	1.	27	0.296
575	A	8	8	1.	25	0.32
576	A	7	7	1.	24	0.292
577	A	7	7	1.	27	0.259
578	A	6	5	1.	27	0.185
579	A	6	5	1.	27	0.185
580	A	6	5	1.	27	0.185
581	A	6	5	1.	27	0.185
582	A	11	9	1.	27	0.333
583	A	10	9	1.	27	0.333
584	A	9	9	1.	25	0.36
585	A	8	8	1.	24	0.333
586	A	8	8	1.	27	0.296
587	A	6	5	1.	27	0.185
588	A	7	6	1.	27	0.222
589	A	9	7	1.	27	0.259
590	A	10	7	1.	27	0.259
591	A	5	5	1.	27	0.185
592	A	8	7	1.	27	0.259
593	A	7	7	1.	25	0.28
594	A	6	6	1.	24	0.25
595	A	6	6	1.	27	0.222
596	A	5	5	1.	27	0.185
597	A	6	6	1.	27	0.222
598	A	7	6	1.	27	0.222
599	A	13	9	1.	27	0.333
600	A	12	9	1.	27	0.333
601	A	11	9	1.	25	0.36
602	A	10	8	1.	24	0.333
603	A	10	8	1.	27	0.296
604	A	8	7	1.	27	0.259
605	A	9	8	1.	27	0.296
606	A	10	8	1.	27	0.296
607	A	10	8	1.	27	0.296
608	A	9	7	1.	27	0.259
609	A	8	7	1.	27	0.259
610	A	7	7	1.	25	0.28
611	A	6	6	1.	24	0.25
612	A	6	6	1.	27	0.222
613	A	5	5	1.	27	0.185
614	A	6	6	1.	27	0.222
615	A	7	6	1.	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
616	A	8	6	1.	27	0.222
617	A	3	3	1.	22	0.136
618	A	3	3	1.	23	0.13
619	A	3	3	1.	23	0.13
620	C	3	3	0.55	22	0.136
621	A	7	5	0.98	20	0.25
622	A	4	4	1.	22	0.182
623	A	5	5	1.	22	0.227
624	A	3	3	1.	24	0.125
625	A	3	3	1.	24	0.125
626	A	3	3	1.	24	0.125
627	A	3	3	1.	24	0.125
628	A	11	9	1.	20	0.45
629	A	10	9	1.	20	0.45
630	A	9	9	1.	20	0.45
631	A	8	8	1.	18	0.444
632	A	6	6	1.	20	0.3
633	A	6	5	1.	20	0.25
634	A	7	5	1.	20	0.25
635	A	8	5	1.	20	0.25
636	A	4	3	1.	22	0.136
637	A	4	3	1.	22	0.136
638	A	4	3	1.	22	0.136
639	A	4	3	1.	22	0.136
640	A	4	3	1.	20	0.15
641	A	4	3	1.	22	0.136
642	A	4	3	1.	22	0.136
643	A	4	3	1.	22	0.136
644	A	4	3	1.	22	0.136
645	A	12	10	1.	22	0.454
646	A	11	10	1.	22	0.454
647	A	10	9	1.	22	0.409
648	A	9	9	1.	20	0.45
649	A	7	7	1.	22	0.318
650	A	7	5	1.	22	0.227
651	A	8	6	1.	22	0.273
652	A	9	6	1.	22	0.273
653	A	4	3	1.	22	0.136
654	A	4	3	1.	22	0.136
655	A	4	3	1.	22	0.136
656	A	4	3	1.	22	0.136
657	A	4	3	1.	20	0.15
658	A	4	3	1.	22	0.136
659	A	4	3	1.	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
660	A	4	3	1.	22	0.136
661	A	4	3	1.	22	0.136
662	A	11	9	1.	22	0.409
663	A	10	9	1.	22	0.409
664	A	9	9	1.	22	0.409
665	A	8	8	1.	20	0.4
666	A	6	6	1.	22	0.273
667	A	6	5	1.	22	0.227
668	A	7	5	1.	22	0.227
669	A	8	5	1.	22	0.227
670	A	4	3	1.	22	0.136
671	A	4	3	1.	22	0.136
672	A	4	3	1.	22	0.136
673	A	4	3	1.	20	0.15
674	A	4	3	1.	22	0.136
675	A	4	3	1.	22	0.136
676	A	4	3	1.	22	0.136
677	A	4	3	1.	22	0.136
678	A	12	10	1.	22	0.454
679	A	11	10	1.	22	0.454
680	A	10	9	1.	22	0.409
681	A	9	9	1.	20	0.45
682	A	7	7	1.	22	0.318
683	A	7	5	1.	22	0.227
684	A	8	6	1.	22	0.273
685	A	9	6	1.	22	0.273
686	A	4	3	1.	22	0.136
687	A	4	3	1.	22	0.136
688	A	4	3	1.	22	0.136
689	A	4	3	1.	22	0.136
690	A	4	3	1.	22	0.136
691	A	4	3	1.	22	0.136
692	A	4	3	1.	22	0.136
693	A	4	3	1.	22	0.136
694	A	4	3	1.	22	0.136
695	A	16	10	1.	24	0.417
696	A	14	10	1.	24	0.417
697	A	12	10	1.	24	0.417
698	A	10	10	1.	24	0.417
699	A	8	8	1.	24	0.333
700	A	6	6	1.	24	0.25
701	A	6	6	1.	24	0.25
702	A	8	7	1.	24	0.292
703	A	10	7	1.	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
704	A	12	7	1.	24	0.292
705	A	4	3	1.	24	0.125
706	A	4	3	1.	24	0.125
707	A	4	3	1.	24	0.125
708	A	4	3	1.	24	0.125
709	A	4	3	1.	24	0.125
710	A	4	3	1.	24	0.125
711	A	4	3	1.	24	0.125
712	A	4	3	1.	24	0.125
713	A	4	3	1.	24	0.125
714	A	4	3	1.	24	0.125
715	A	4	3	1.	24	0.125
716	A	4	3	1.	24	0.125
717	A	4	3	1.	24	0.125
718	A	4	3	1.	24	0.125
719	A	4	3	1.	24	0.125
720	A	4	3	1.	24	0.125
721	A	4	3	1.	24	0.125
722	A	4	3	1.	24	0.125
723	A	16	10	1.	24	0.417
724	A	14	10	1.	24	0.417
725	A	12	10	1.	24	0.417
726	A	10	10	1.	24	0.417
727	A	8	8	1.	24	0.333
728	A	6	6	1.	24	0.25
729	A	6	6	1.	24	0.25
730	A	8	7	1.	24	0.292
731	A	10	7	1.	24	0.292
732	A	4	3	1.	24	0.125
733	A	4	3	1.	24	0.125
734	A	4	3	1.	24	0.125
735	A	4	3	1.	24	0.125
736	A	4	3	1.	24	0.125
737	A	4	3	1.	24	0.125
738	A	4	3	1.	24	0.125
739	A	4	3	1.	24	0.125
740	A	4	3	1.	24	0.125
741	A	4	3	1.	25	0.12
742	A	4	3	1.	25	0.12
743	A	3	2	1.	23	0.087
744	A	4	3	1.	22	0.136
745	A	4	3	1.	25	0.12
746	A	3	3	1.	25	0.12
747	A	8	7	1.	27	0.259

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
748	A	7	6	1.	27	0.222
749	A	6	5	1.	25	0.2
750	A	8	8	1.	24	0.333
751	A	8	8	1.	27	0.296
752	A	6	5	1.	27	0.185
753	A	7	6	1.	27	0.222
754	A	9	7	1.	27	0.259
755	A	10	7	1.	27	0.259
756	A	4	3	1.	27	0.111
757	A	4	3	1.	27	0.111
758	A	4	3	1.	25	0.12
759	A	4	3	1.	24	0.125
760	A	4	3	1.	27	0.111
761	A	4	3	1.	27	0.111
762	A	4	3	1.	27	0.111
763	A	4	3	1.	27	0.111
764	A	4	3	1.	27	0.111
765	A	4	3	1.	27	0.111
766	A	4	3	1.	27	0.111
767	A	3	2	1.	25	0.08
768	A	4	3	1.	24	0.125
769	A	4	3	1.	27	0.111
770	A	3	3	1.	27	0.111
771	A	8	7	1.	27	0.259
772	A	7	6	1.	27	0.222
773	A	6	5	1.	25	0.2
774	A	8	8	1.	24	0.333
775	A	8	8	1.	27	0.296
776	A	6	5	1.	27	0.185
777	A	7	6	1.	27	0.222
778	A	9	7	1.	27	0.259
779	A	10	7	1.	27	0.259
780	A	4	3	1.	27	0.111
781	A	4	3	1.	27	0.111
782	A	4	3	1.	25	0.12
783	A	4	3	1.	24	0.125
784	A	4	3	1.	27	0.111
785	A	4	3	1.	27	0.111
786	A	4	3	1.	27	0.111
787	A	4	3	1.	27	0.111
788	A	4	3	1.	27	0.111
789	A	3	3	1.	23	0.13
790	A	3	3	1.	23	0.13
791	C	3	3	0.21	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
792	C	3	3	0.51	20	0.15
793	A	5	5	1.	22	0.227
794	A	11	10	1.	22	0.454
795	C	3	3	0.24	24	0.125
796	A	7	5	1.11	24	0.208
797	A	4	4	1.	24	0.167
798	A	5	5	1.	24	0.208
799	A	18	11	1.	24	0.458
800	A	3	3	1.	22	0.136
801	A	13	6	1.	22	0.273
802	A	7	6	1.	22	0.273
803	A	10	5	1.	22	0.227
804	A	5	4	1.	20	0.2
805	A	5	4	1.	22	0.182
806	A	10	5	1.	22	0.227
807	A	7	6	1.	22	0.273
808	A	16	8	1.	15	0.533
809	A	11	7	1.	14	0.5
810	A	13	8	1.	17	0.471
811	A	7	7	1.	16	0.438
812	A	22	8	1.	15	0.533
813	A	16	8	1.	14	0.571
814	A	19	8	1.	17	0.471
815	A	13	8	1.	16	0.5
816	A	11	7	1.	15	0.467
817	A	6	6	1.	14	0.429
818	A	7	7	1.	12	0.583
819	A	7	7	1.	12	0.583
820	A	6	6	1.	10	0.6
821	A	5	5	1.	8	0.625
822	A	7	7	1.	12	0.583
823	A	4	4	1.	12	0.333
824	A	5	5	1.	12	0.417
825	A	7	6	1.	12	0.5
826	A	3	2	1.	14	0.143
827	A	3	2	1.	14	0.143
828	A	3	2	1.	14	0.143
829	A	3	2	1.	12	0.167
830	A	3	2	1.	10	0.2
831	A	3	2	1.	14	0.143
832	A	3	2	1.	14	0.143
833	A	3	2	1.	14	0.143
834	A	3	2	1.	14	0.143
835	A	8	8	1.	14	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
836	A	8	7	1.	14	0.5
837	A	7	6	1.	12	0.5
838	A	6	6	1.	10	0.6
839	A	8	8	1.	14	0.571
840	A	5	4	1.	14	0.286
841	A	6	5	1.	14	0.357
842	A	8	7	1.	14	0.5
843	A	7	7	1.	14	0.5
844	A	7	7	1.	14	0.5
845	A	6	6	1.	12	0.5
846	A	5	5	1.	10	0.5
847	A	7	7	1.	14	0.5
848	A	4	4	1.	14	0.286
849	A	5	5	1.	14	0.357
850	A	7	6	1.	14	0.429
851	A	3	2	1.	14	0.143
852	A	3	2	1.	14	0.143
853	A	3	2	1.	14	0.143
854	A	3	2	1.	12	0.167
855	A	3	2	1.	10	0.2
856	A	3	2	1.	14	0.143
857	A	3	2	1.	14	0.143
858	A	3	2	1.	14	0.143
859	A	3	2	1.	14	0.143
860	A	8	8	1.	14	0.571
861	A	8	7	1.	14	0.5
862	A	7	6	1.	12	0.5
863	A	6	6	1.	10	0.6
864	A	8	8	1.	14	0.571
865	A	5	4	1.	14	0.286
866	A	6	5	1.	14	0.357
867	A	8	7	1.	14	0.5
868	A	4	4	1.	16	0.25
869	A	6	6	1.	34	0.176
870	A	6	6	1.	34	0.176
871	A	5	5	1.	32	0.156
872	A	2	2	1.	31	0.065
873	A	4	4	1.	34	0.118
874	A	6	6	1.	34	0.176
875	A	4	3	1.	14	0.214
876	A	4	4	1.	14	0.286
877	A	4	4	1.	14	0.286
878	A	3	3	1.	12	0.25
879	A	2	2	1.	10	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
880	A	4	4	1.	14	0.286
881	A	2	2	1.	14	0.143
882	A	3	3	1.	14	0.214
883	A	7	4	1.	20	0.2
884	A	6	4	1.	20	0.2
885	A	5	4	1.	20	0.2
886	A	4	4	1.	18	0.222
887	A	5	5	1.	23	0.217
888	A	4	4	1.	23	0.174
889	A	5	5	1.	23	0.217
890	A	3	3	1.	21	0.143
891	A	1	1	1.	20	0.05
892	A	6	6	1.	23	0.261
893	A	6	6	1.	23	0.261
894	A	7	7	1.	23	0.304
895	A	8	7	1.	23	0.304
896	A	6	5	1.	23	0.217
897	A	5	4	1.	23	0.174
898	A	5	4	1.	23	0.174
899	A	4	4	1.	23	0.174
900	A	4	4	1.	23	0.174
901	A	3	3	1.	21	0.143
902	A	3	3	1.	20	0.15
903	A	7	6	1.	23	0.261
904	A	7	6	1.	23	0.261
905	A	8	7	1.	23	0.304
906	A	9	7	1.	23	0.304
907	A	6	4	1.	23	0.174
908	A	6	4	1.	23	0.174
909	A	5	4	1.	23	0.174
910	A	5	4	1.	23	0.174
911	A	4	4	1.19	23	0.174
912	A	4	4	1.	23	0.174
913	A	4	4	1.	21	0.19
914	A	4	4	1.	20	0.2
915	A	8	6	1.	23	0.261
916	A	8	6	1.	23	0.261
917	A	9	7	1.	23	0.304
918	A	5	4	1.	20	0.2
919	A	6	4	1.	20	0.2
920	A	3	2	1.	24	0.083
921	A	3	2	1.	24	0.083
922	A	3	2	1.	24	0.083
923	A	3	2	1.	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
924	A	2	2	1.	21	0.095
925	A	4	4	1.	24	0.167
926	A	3	2	1.	24	0.083
927	A	3	2	1.	24	0.083
928	A	3	2	1.	24	0.083
929	A	3	2	1.	24	0.083
930	A	3	2	1.	24	0.083
931	A	3	2	1.	24	0.083
932	A	4	3	1.	22	0.136
933	A	4	3	1.	21	0.143
934	A	3	2	1.	24	0.083
935	A	3	2	1.	24	0.083
936	A	3	2	1.	24	0.083
937	A	3	2	1.	24	0.083
938	A	3	2	1.	24	0.083
939	A	3	2	1.	24	0.083
940	A	3	2	1.	24	0.083
941	A	4	3	1.	24	0.125
942	A	4	3	1.	24	0.125
943	A	4	3	1.	22	0.136
944	A	4	3	1.	21	0.143
945	A	3	2	1.	24	0.083
946	A	3	2	1.	24	0.083
947	A	3	2	1.	24	0.083
948	A	3	2	1.	24	0.083
949	A	4	3	1.	25	0.12
950	A	4	3	1.	23	0.13
951	A	3	2	1.	22	0.091
952	A	4	3	1.	25	0.12
953	A	4	3	1.	25	0.12
954	A	4	3	1.	22	0.136
955	A	4	3	1.	22	0.136
956	A	4	3	1.	22	0.136
957	A	4	3	1.	25	0.12
958	A	4	3	1.	25	0.12
959	A	4	3	1.	25	0.12
960	A	4	3	1.	23	0.13
961	A	3	3	1.	22	0.136
962	A	5	5	1.	25	0.2
963	A	4	3	1.	25	0.12
964	A	4	3	1.	25	0.12
965	A	4	3	1.	25	0.12
966	A	4	3	1.	25	0.12
967	A	4	3	1.	25	0.12

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
968	A	4	3	1.	25	0.12
969	A	4	3	1.	25	0.12
970	A	5	4	1.	23	0.174
971	A	5	4	1.	22	0.182
972	A	4	3	1.	25	0.12
973	A	4	3	1.	25	0.12
974	A	4	3	1.	25	0.12
975	A	4	3	1.	25	0.12
976	A	4	3	1.	25	0.12
977	A	4	3	1.	25	0.12
978	A	4	3	1.	25	0.12
979	A	5	4	1.	25	0.16
980	A	5	4	1.	25	0.16
981	A	5	4	1.	23	0.174
982	A	5	4	1.	22	0.182
983	A	4	3	1.	25	0.12
984	A	4	3	1.	25	0.12
985	A	4	3	1.	25	0.12
986	A	5	4	1.	22	0.182
987	A	4	3	1.	23	0.13
988	A	4	3	1.	21	0.143
989	A	4	3	1.	23	0.13
990	A	4	3	1.	23	0.13
991	A	4	3	1.	23	0.13
992	A	3	2	1.	24	0.083
993	A	3	2	1.	24	0.083
994	A	3	2	1.	24	0.083
995	A	2	2	1.	24	0.083
996	A	6	4	1.	24	0.167
997	A	6	4	1.	24	0.167
998	A	4	3	1.	25	0.12
999	A	4	3	1.	25	0.12
1000	A	4	3	1.	25	0.12
1001	A	3	3	1.	25	0.12
1002	A	7	5	1.	25	0.2
1003	A	7	5	1.	25	0.2
1004	A	5	4	1.	23	0.174
1005	A	6	5	1.	22	0.227
1006	A	6	5	1.	22	0.227
1007	A	4	4	1.	20	0.2
1008	A	2	2	1.	19	0.105
1009	A	5	5	1.	22	0.227
1010	A	5	5	1.	22	0.227
1011	A	5	5	1.	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1012	A	7	6	1.	23	0.261
1013	A	7	6	1.	23	0.261
1014	A	5	5	1.	21	0.238
1015	A	3	3	1.	20	0.15
1016	A	6	6	1.	23	0.261
1017	A	6	6	1.	23	0.261
1018	A	6	6	1.	23	0.261
1019	A	3	2	1.	23	0.087
1020	A	3	2	1.	23	0.087
1021	A	3	2	1.	23	0.087
1022	A	3	2	1.	21	0.095
1023	A	2	2	1.	20	0.1
1024	A	3	2	1.	23	0.087
1025	A	3	2	1.	23	0.087
1026	A	3	2	1.	23	0.087
1027	A	2	2	1.	23	0.087
1028	A	3	2	1.	25	0.08
1029	A	3	2	1.	25	0.08
1030	A	3	2	1.	25	0.08
1031	A	3	2	1.	23	0.087
1032	A	3	2	1.	22	0.091
1033	A	3	2	1.	25	0.08
1034	A	3	2	1.	25	0.08
1035	A	2	2	1.	25	0.08
1036	A	3	2	1.	25	0.08
1037	A	3	2	1.	25	0.08
1038	A	3	2	1.	25	0.08
1039	A	3	2	1.	25	0.08
1040	A	3	2	1.	25	0.08
1041	A	3	2	1.	25	0.08
1042	A	3	2	1.	23	0.087
1043	A	3	2	1.	22	0.091
1044	A	3	2	1.	25	0.08
1045	A	3	2	1.	25	0.08
1046	A	3	2	1.	25	0.08
1047	A	3	2	1.	25	0.08
1048	A	3	2	1.	22	0.091
1049	A	3	2	1.	25	0.08
1050	A	3	2	1.	25	0.08
1051	A	3	2	1.	25	0.08
1052	A	3	2	1.	23	0.087
1053	A	2	2	1.	22	0.091
1054	A	3	2	1.	25	0.08
1055	A	3	2	1.	25	0.08

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1056	A	3	2	1.	25	0.08
1057	A	3	2	1.	25	0.08
1058	A	3	2	1.	25	0.08
1059	A	3	2	1.	25	0.08
1060	A	4	3	1.	25	0.12
1061	A	4	3	1.	23	0.13
1062	A	4	3	1.	22	0.136
1063	A	3	2	1.	25	0.08
1064	A	3	2	1.	25	0.08
1065	A	3	2	1.	25	0.08
1066	A	3	2	1.	25	0.08
1067	A	3	2	1.	25	0.08
1068	A	4	3	1.	25	0.12
1069	A	4	3	1.	25	0.12
1070	A	2	2	1.	25	0.08
1071	A	4	3	1.	23	0.13
1072	A	4	3	1.	22	0.136
1073	A	3	2	1.	25	0.08
1074	A	3	2	1.	25	0.08
1075	A	3	2	1.	25	0.08
1076	A	4	3	1.	22	0.136
1077	A	7	6	1.	27	0.222
1078	A	6	6	1.	27	0.222
1079	A	5	5	1.	25	0.2
1080	A	5	5	1.	24	0.208
1081	A	8	8	1.	27	0.296
1082	A	8	8	1.	27	0.296
1083	A	6	6	1.	27	0.222
1084	A	7	7	1.	27	0.259
1085	A	8	7	1.	27	0.259
1086	A	8	7	1.	27	0.259
1087	A	7	7	1.	27	0.259
1088	A	6	6	1.	25	0.24
1089	A	6	6	1.	24	0.25
1090	A	9	9	1.	27	0.333
1091	A	9	9	1.	27	0.333
1092	A	9	9	1.	27	0.333
1093	A	9	9	1.	27	0.333
1094	A	7	7	1.	27	0.259
1095	A	8	8	1.	27	0.296
1096	A	9	8	1.	27	0.296
1097	A	10	8	1.	27	0.296
1098	A	9	7	1.	27	0.259
1099	A	8	7	1.	27	0.259

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1100	A	7	6	1.	25	0.24
1101	A	7	6	1.	24	0.25
1102	A	10	9	1.	27	0.333
1103	A	10	9	1.	27	0.333
1104	A	10	10	1.	27	0.37
1105	A	10	9	1.	27	0.333
1106	A	10	10	1.	27	0.37
1107	A	8	6	1.	24	0.25
1108	A	7	6	1.	27	0.222
1109	A	5	5	1.	27	0.185
1110	A	5	5	1.	25	0.2
1111	A	4	4	1.	24	0.167
1112	A	5	5	1.	27	0.185
1113	A	6	6	1.	27	0.222
1114	A	7	7	1.	27	0.259
1115	A	8	7	1.	27	0.259
1116	A	6	5	1.	27	0.185
1117	A	5	5	1.	27	0.185
1118	A	3	3	1.	25	0.12
1119	A	3	3	1.	24	0.125
1120	A	7	7	1.	27	0.259
1121	A	7	6	1.	27	0.222
1122	A	8	7	1.	27	0.259
1123	A	4	4	1.	24	0.167
1124	A	5	4	1.	24	0.167
1125	A	3	2	1.	25	0.08
1126	A	3	2	1.	25	0.08
1127	A	3	2	1.	23	0.087
1128	A	2	2	1.	25	0.08
1129	A	6	5	1.	25	0.2
1130	A	8	5	1.	25	0.2
1131	A	7	5	1.	27	0.185
1132	A	7	5	1.	27	0.185
1133	A	7	5	1.	27	0.185
1134	A	7	5	1.	27	0.185
1135	A	7	5	1.	27	0.185
1136	A	3	3	1.	22	0.136
1137	A	7	5	1.	23	0.217
1138	A	6	5	1.	23	0.217
1139	A	6	5	1.	21	0.238
1140	A	5	4	1.	20	0.2
1141	A	8	7	1.	23	0.304
1142	A	8	8	1.	23	0.348
1143	A	8	7	1.	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1144	A	7	6	1.	23	0.261
1145	A	8	7	1.	23	0.304
1146	A	9	7	1.	23	0.304
1147	A	6	5	1.	22	0.227
1148	A	7	5	1.	22	0.227
1149	A	8	5	1.	22	0.227
1150	A	6	6	1.	25	0.24
1151	A	4	4	1.	23	0.174
1152	A	1	1	1.	22	0.045
1153	A	5	4	1.	22	0.182
1154	A	6	4	1.	22	0.182
1155	A	7	5	1.	22	0.227
1156	A	4	3	1.	27	0.111
1157	A	4	3	1.	27	0.111
1158	A	4	3	1.	25	0.12
1159	A	4	3	1.	24	0.125
1160	A	4	3	1.	27	0.111
1161	A	4	3	1.	27	0.111
1162	A	4	3	1.	27	0.111
1163	A	4	3	1.	27	0.111
1164	A	4	3	1.	27	0.111
1165	A	3	3	1.	24	0.125
1166	A	4	3	1.	24	0.125
1167	A	4	3	1.	24	0.125
1168	A	4	3	1.	24	0.125
1169	A	4	3	1.	24	0.125
1170	A	3	3	1.	24	0.125
1171	A	5	4	1.	24	0.167
1172	A	5	4	1.	24	0.167
1173	A	5	4	1.	27	0.148
1174	A	7	5	1.	25	0.2
1175	A	8	7	1.	25	0.28
1176	A	8	7	1.	25	0.28
1177	A	5	5	1.	23	0.217
1178	A	3	3	1.	22	0.136
1179	A	8	8	1.	25	0.32
1180	A	9	9	1.	25	0.36
1181	A	8	7	1.	25	0.28
1182	A	3	2	1.	22	0.091
1183	A	3	2	1.	22	0.091
1184	A	3	2	1.	22	0.091
1185	A	2	2	1.	22	0.091
1186	A	3	2	1.28	20	0.1
1187	A	2	2	1.	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1188	A	2	2	1.	22	0.091
1189	A	4	3	1.	22	0.136
1190	A	4	3	1.	22	0.136
1191	A	3	3	1.	22	0.136
1192	A	7	4	1.	22	0.182
1193	A	6	4	1.	22	0.182
1194	A	5	4	1.	22	0.182
1195	A	4	4	1.	20	0.2
1196	A	1	1	1.	22	0.045
1197	A	3	3	1.	22	0.136
1198	A	4	4	1.	22	0.182
1199	A	5	4	1.	22	0.182
1200	A	6	4	1.	22	0.182
1201	A	4	3	1.	27	0.111
1202	A	4	3	1.	27	0.111
1203	A	4	3	1.	25	0.12
1204	A	3	2	1.	24	0.083
1205	A	4	3	1.	27	0.111
1206	A	4	3	1.	27	0.111
1207	A	4	3	1.	24	0.125
1208	A	4	3	1.	24	0.125
1209	A	4	3	1.	24	0.125
1210	A	4	3	1.	24	0.125
1211	A	3	3	1.	24	0.125
1212	A	5	4	1.	24	0.167
1213	A	5	4	1.	24	0.167
1214	A	5	4	1.	24	0.167
1215	A	5	4	1.	25	0.16
1216	A	6	5	1.	24	0.208
1217	A	6	5	1.	24	0.208
1218	A	4	4	1.	22	0.182
1219	A	2	2	1.	21	0.095
1220	A	5	5	1.	24	0.208
1221	A	5	5	1.	24	0.208
1222	A	7	6	1.	25	0.24
1223	A	7	6	1.	25	0.24
1224	A	5	5	1.	23	0.217
1225	A	3	3	1.	22	0.136
1226	A	6	6	1.	25	0.24
1227	A	6	6	1.	25	0.24
1228	A	3	2	1.	22	0.091
1229	A	3	2	1.	22	0.091
1230	A	3	2	1.	22	0.091
1231	A	2	2	1.	20	0.1

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1232	A	2	2	1.	22	0.091
1233	A	4	3	1.	22	0.136
1234	A	4	3	1.	22	0.136
1235	A	4	3	1.	22	0.136
1236	A	7	6	1.	27	0.222
1237	A	6	6	1.	27	0.222
1238	A	5	5	1.	25	0.2
1239	A	5	5	1.	24	0.208
1240	A	8	8	1.	27	0.296
1241	A	8	8	1.	27	0.296
1242	A	6	6	1.	27	0.222
1243	A	7	7	1.	27	0.259
1244	A	8	7	1.	27	0.259
1245	A	6	6	1.	24	0.25
1246	A	7	6	1.	24	0.25
1247	A	8	6	1.	24	0.25
1248	A	4	4	1.	24	0.167
1249	A	3	3	1.	24	0.125
1250	A	4	4	1.	24	0.167
1251	A	5	4	1.	24	0.167
1252	A	7	5	1.	27	0.185
1253	A	3	3	1.	22	0.136
1254	A	8	5	1.	22	0.227
1255	A	7	5	1.	22	0.227
1256	A	6	5	1.	22	0.227
1257	A	5	4	1.	20	0.2
1258	A	1	1	1.	22	0.045
1259	A	5	4	1.	22	0.182
1260	A	6	4	1.	22	0.182
1261	A	7	5	1.	22	0.227
1262	A	4	3	1.	27	0.111
1263	A	4	3	1.	27	0.111
1264	A	4	3	1.	25	0.12
1265	A	4	3	1.	24	0.125
1266	A	4	3	1.	27	0.111
1267	A	4	3	1.	27	0.111
1268	A	4	3	1.	27	0.111
1269	A	4	3	1.	27	0.111
1270	A	4	3	1.	27	0.111
1271	A	4	3	1.	24	0.125
1272	A	4	3	1.	24	0.125
1273	A	4	3	1.	24	0.125
1274	A	3	3	1.	24	0.125
1275	A	4	3	1.	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1276	A	3	3	1.	24	0.125
1277	A	5	4	1.	24	0.167
1278	A	5	4	1.	24	0.167
1279	A	5	4	1.	27	0.148
1280	A	3	3	1.	22	0.136
1281	A	18	10	1.	25	0.4
1282	A	16	10	1.	25	0.4
1283	A	14	10	1.	25	0.4
1284	A	12	9	1.	25	0.36
1285	A	1	1	1.	25	0.04
1286	A	2	2	1.	25	0.08
1287	A	3	2	1.	25	0.08
1288	A	4	2	1.	25	0.08
1289	A	19	11	1.	26	0.423
1290	A	17	11	1.	26	0.423
1291	A	15	11	1.	26	0.423
1292	A	13	10	1.	26	0.385
1293	A	1	1	1.	26	0.038
1294	A	2	2	1.	26	0.077
1295	A	3	2	1.	26	0.077
1296	A	4	2	1.	26	0.077
1297	A	10	8	1.	29	0.276
1298	A	8	7	1.	29	0.241
1299	A	5	5	1.	27	0.185
1300	A	5	5	1.	26	0.192
1301	A	8	7	1.	29	0.241
1302	A	9	8	1.	29	0.276
1303	A	5	5	1.	29	0.172
1304	A	1	1	1.	29	0.034
1305	A	4	4	1.	27	0.148
1306	A	3	3	1.	26	0.115
1307	A	3	3	1.	29	0.103
1308	A	2	2	1.	20	0.1
1309	A	2	2	1.	22	0.091
1310	A	2	2	1.	22	0.091
1311	A	5	5	1.	25	0.2
1312	A	4	4	1.	25	0.16
1313	A	4	4	1.48	25	0.16
1314	A	3	3	1.	23	0.13
1315	A	1	1	1.	22	0.045
1316	A	3	3	1.11	25	0.12
1317	A	5	5	1.11	25	0.2
1318	A	10	9	1.	25	0.36
1319	A	10	5	1.	25	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1320	A	2	2	1.	25	0.08
1321	A	3	3	1.48	23	0.13
1322	A	2	2	1.	22	0.091
1323	A	6	5	1.05	25	0.2
1324	A	7	5	1.06	25	0.2
1325	A	3	2	1.	22	0.091
1326	A	4	2	1.	22	0.091
1327	A	5	5	1.	27	0.185
1328	A	5	5	1.	27	0.185
1329	A	4	4	1.	25	0.16
1330	A	3	3	1.	24	0.125
1331	A	7	5	1.11	27	0.185
1332	C	3	3	0.36	27	0.111
1333	A	3	3	1.	24	0.125
1334	A	5	5	1.	27	0.185
1335	A	5	5	1.	27	0.185
1336	A	4	4	1.	25	0.16
1337	A	3	3	1.	24	0.125
1338	A	3	3	1.	27	0.111
1339	A	4	4	1.	27	0.148
1340	A	6	6	1.	27	0.222
1341	A	5	5	1.	27	0.185
1342	A	4	4	1.	27	0.148
1343	A	1	1	1.	25	0.04
1344	A	1	1	1.	24	0.042
1345	A	6	6	1.02	27	0.222
1346	A	7	6	1.01	27	0.222
1347	A	8	7	1.01	27	0.259
1348	A	7	7	1.	27	0.259
1349	A	2	2	1.	27	0.074
1350	A	3	3	1.	25	0.12
1351	A	2	2	1.	24	0.083
1352	A	8	6	1.01	27	0.222
1353	A	9	6	1.01	27	0.222
1354	A	10	7	1.01	27	0.259
1355	A	3	2	1.	24	0.083
1356	A	2	2	1.	25	0.08
1357	A	2	2	1.	23	0.087
1358	A	2	2	1.	25	0.08
1359	A	2	2	1.	25	0.08
1360	A	3	3	1.	25	0.12
1361	A	4	4	1.	23	0.174
1362	A	3	3	1.	22	0.136
1363	A	3	2	1.	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1364	A	4	3	1.	27	0.111
1365	A	3	3	1.	23	0.13
1366	A	3	3	1.	23	0.13
1367	A	1	1	1.	33	0.03
1368	A	2	2	1.	25	0.08
1369	A	2	2	1.	25	0.08
1370	A	2	2	1.	25	0.08
1371	A	2	2	1.	25	0.08
1372	A	2	2	1.	25	0.08
1373	A	3	3	1.	27	0.111
1374	A	3	3	1.	27	0.111
1375	A	4	3	1.	25	0.12
1376	A	4	3	1.	27	0.111
1377	A	3	3	1.	27	0.111
1378	A	3	3	1.	27	0.111

Chapter 3

Listing of integrals

3.1 $\int e^{\tanh^{-1}(ax)} x^4 \, dx$

Optimal. Leaf size=111

$$-\frac{x^4\sqrt{1-a^2x^2}}{5a}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}-\frac{4x^2\sqrt{1-a^2x^2}}{15a^3}-\frac{(45ax+64)\sqrt{1-a^2x^2}}{120a^5}+\frac{3\sin^{-1}(ax)}{8a^5}$$

```
[Out] (-4*x^2*Sqrt[1 - a^2*x^2])/(15*a^3) - (x^3*Sqrt[1 - a^2*x^2])/(4*a^2) - (x^
4*Sqrt[1 - a^2*x^2])/(5*a) - ((64 + 45*a*x)*Sqrt[1 - a^2*x^2])/(120*a^5) +
(3*ArcSin[a*x])/(8*a^5)
```

Rubi [A] time = 0.10464, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6124, 833, 780, 216}

$$-\frac{x^4\sqrt{1-a^2x^2}}{5a}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}-\frac{4x^2\sqrt{1-a^2x^2}}{15a^3}-\frac{(45ax+64)\sqrt{1-a^2x^2}}{120a^5}+\frac{3\sin^{-1}(ax)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^4,x]
```

```
[Out] (-4*x^2*Sqrt[1 - a^2*x^2])/(15*a^3) - (x^3*Sqrt[1 - a^2*x^2])/(4*a^2) - (x^
4*Sqrt[1 - a^2*x^2])/(5*a) - ((64 + 45*a*x)*Sqrt[1 - a^2*x^2])/(120*a^5) +
(3*ArcSin[a*x])/(8*a^5)
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}x^4\,dx &= \int \frac{x^4(1+ax)}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{x^4\sqrt{1-a^2x^2}}{5a}-\frac{\int \frac{x^3\left(-4a-5a^2x\right)}{\sqrt{1-a^2x^2}}\,dx}{5a^2}\\
&= -\frac{x^3\sqrt{1-a^2x^2}}{4a^2}-\frac{x^4\sqrt{1-a^2x^2}}{5a}+\frac{\int \frac{x^2\left(15a^2+16a^3x\right)}{\sqrt{1-a^2x^2}}\,dx}{20a^4}\\
&= -\frac{4x^2\sqrt{1-a^2x^2}}{15a^3}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}-\frac{x^4\sqrt{1-a^2x^2}}{5a}-\frac{\int \frac{x\left(-32a^3-45a^4x\right)}{\sqrt{1-a^2x^2}}\,dx}{60a^6}\\
&= -\frac{4x^2\sqrt{1-a^2x^2}}{15a^3}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}-\frac{x^4\sqrt{1-a^2x^2}}{5a}-\frac{(64+45ax)\sqrt{1-a^2x^2}}{120a^5}+\frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{8a^4}\\
&= -\frac{4x^2\sqrt{1-a^2x^2}}{15a^3}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}-\frac{x^4\sqrt{1-a^2x^2}}{5a}-\frac{(64+45ax)\sqrt{1-a^2x^2}}{120a^5}+\frac{3\sin^{-1}(ax)}{8a^5}
\end{aligned}$$

Mathematica [A] time = 0.0521778, size = 60, normalized size = 0.54

$$\frac{45\sin^{-1}(ax)-\sqrt{1-a^2x^2}\left(24a^4x^4+30a^3x^3+32a^2x^2+45ax+64\right)}{120a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^4,x]
```

```
[Out] (-(Sqrt[1 - a^2*x^2]*(64 + 45*a*x + 32*a^2*x^2 + 30*a^3*x^3 + 24*a^4*x^4))
+ 45*ArcSin[a*x])/(120*a^5)
```

Maple [A] time = 0.039, size = 127, normalized size = 1.1

$$-\frac{x^4}{5\,a}\sqrt{-a^2x^2+1}-\frac{4\,x^2}{15\,a^3}\sqrt{-a^2x^2+1}-\frac{8}{15\,a^5}\sqrt{-a^2x^2+1}-\frac{x^3}{4\,a^2}\sqrt{-a^2x^2+1}-\frac{3\,x}{8\,a^4}\sqrt{-a^2x^2+1}+\frac{3}{8\,a^4}\arctan\left(x\sqrt{a^2}\frac{}{\sqrt{-a^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4,x)
```

```
[Out] -1/5*x^4*(-a^2*x^2+1)^(1/2)/a-4/15*x^2*(-a^2*x^2+1)^(1/2)/a^3-8/15/a^5*(-a^
2*x^2+1)^(1/2)-1/4*x^3*(-a^2*x^2+1)^(1/2)/a^2-3/8/a^4*x*(-a^2*x^2+1)^(1/2)+
```

```
3/8/a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.45737, size = 158, normalized size = 1.42

$$-\frac{\sqrt{-a^2x^2+1}x^4}{5\,a}-\frac{\sqrt{-a^2x^2+1}x^3}{4\,a^2}-\frac{4\,\sqrt{-a^2x^2+1}x^2}{15\,a^3}-\frac{3\,\sqrt{-a^2x^2+1}x}{8\,a^4}+\frac{3\,\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\,\sqrt{a^2}a^4}-\frac{8\,\sqrt{-a^2x^2+1}}{15\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4,x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-a^2*x^2 + 1)*x^4/a - 1/4*sqrt(-a^2*x^2 + 1)*x^3/a^2 - 4/15*sqrt(
-a^2*x^2 + 1)*x^2/a^3 - 3/8*sqrt(-a^2*x^2 + 1)*x/a^4 + 3/8*arcsin(a^2*x/sqr
t(a^2))/(sqrt(a^2)*a^4) - 8/15*sqrt(-a^2*x^2 + 1)/a^5
```

Fricas [A] time = 2.3653, size = 176, normalized size = 1.59

$$-\frac{\left(24\,a^4x^4+30\,a^3x^3+32\,a^2x^2+45\,ax+64\right)\sqrt{-a^2x^2+1}+90\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{120\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4,x, algorithm="fricas")
```

```
[Out] -1/120*((24*a^4*x^4 + 30*a^3*x^3 + 32*a^2*x^2 + 45*a*x + 64)*sqrt(-a^2*x^2
+ 1) + 90*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^5
```

Sympy [A] time = 5.94124, size = 221, normalized size = 1.99

$$a\left(\begin{cases}-\frac{x^4\sqrt{-a^2x^2+1}}{5a^2}-\frac{4x^2\sqrt{-a^2x^2+1}}{15a^4}-\frac{8\sqrt{-a^2x^2+1}}{15a^6} & \text{for } a\neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{ix^5}{4\sqrt{a^2x^2-1}}-\frac{ix^3}{8a^2\sqrt{a^2x^2-1}}+\frac{3ix}{8a^4\sqrt{a^2x^2-1}}-\frac{3i\operatorname{acosh}(ax)}{8a^5} & \text{for } |a\\ \frac{x^5}{4\sqrt{-a^2x^2+1}}+\frac{x^3}{8a^2\sqrt{-a^2x^2+1}}-\frac{3x}{8a^4\sqrt{-a^2x^2+1}}+\frac{3\operatorname{asin}(ax)}{8a^5} & \text{other}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4,x)
```

```
[Out] a*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/(5*a**2) - 4*x**2*sqrt(-a**2*x**2 +
 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(15*a**6), Ne(a, 0)), (x**6/6, True)
) + Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(a**2*x
**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a**5), A
bs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqrt(-a**
2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**5), Tr
ue))
```

Giac [A] time = 1.26046, size = 92, normalized size = 0.83

$$-\frac{1}{120}\sqrt{-a^2x^2+1}\left(\left(2\left(3\,x\left(\frac{4\,x}{a}+\frac{5}{a^2}\right)+\frac{16}{a^3}\right)x+\frac{45}{a^4}\right)x+\frac{64}{a^5}\right)+\frac{3\,\arcsin(ax)\operatorname{sgn}(a)}{8\,a^4|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4,x, algorithm="giac")
```

```
[Out] -1/120*sqrt(-a^2*x^2 + 1)*((2*(3*x*(4*x/a + 5/a^2) + 16/a^3)*x + 45/a^4)*x
+ 64/a^5) + 3/8*arcsin(a*x)*sgn(a)/(a^4*abs(a))
```

3.2 $\int e^{\tanh^{-1}(ax)} x^3 \, dx$

Optimal. Leaf size=87

$$-\frac{x^3\sqrt{1-a^2x^2}}{4a}-\frac{x^2\sqrt{1-a^2x^2}}{3a^2}-\frac{(9ax+16)\sqrt{1-a^2x^2}}{24a^4}+\frac{3\sin^{-1}(ax)}{8a^4}$$

```
[Out] -(x^2*Sqrt[1 - a^2*x^2])/(3*a^2) - (x^3*Sqrt[1 - a^2*x^2])/(4*a) - ((16 + 9
*a*x)*Sqrt[1 - a^2*x^2])/(24*a^4) + (3*ArcSin[a*x])/(8*a^4)
```

Rubi [A] time = 0.0754742, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6124, 833, 780, 216}

$$-\frac{x^3\sqrt{1-a^2x^2}}{4a}-\frac{x^2\sqrt{1-a^2x^2}}{3a^2}-\frac{(9ax+16)\sqrt{1-a^2x^2}}{24a^4}+\frac{3\sin^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^3,x]
```

```
[Out] -(x^2*Sqrt[1 - a^2*x^2])/(3*a^2) - (x^3*Sqrt[1 - a^2*x^2])/(4*a) - ((16 + 9
*a*x)*Sqrt[1 - a^2*x^2])/(24*a^4) + (3*ArcSin[a*x])/(8*a^4)
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^3\,dx = \int \frac{x^3(1+ax)}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{\int \frac{x^2(-3a-4a^2x)}{\sqrt{1-a^2x^2}}\,dx}{4a^2}$$
$$= -\frac{x^2\sqrt{1-a^2x^2}}{3a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} + \frac{\int \frac{x(8a^2+9a^3x)}{\sqrt{1-a^2x^2}}\,dx}{12a^4}$$
$$= -\frac{x^2\sqrt{1-a^2x^2}}{3a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{(16+9ax)\sqrt{1-a^2x^2}}{24a^4} + \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{8a^3}$$
$$= -\frac{x^2\sqrt{1-a^2x^2}}{3a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{(16+9ax)\sqrt{1-a^2x^2}}{24a^4} + \frac{3\sin^{-1}(ax)}{8a^4}$$

Mathematica [A] time = 0.0371607, size = 52, normalized size = 0.6

$$\frac{9\sin^{-1}(ax) - \sqrt{1-a^2x^2}\left(6a^3x^3+8a^2x^2+9ax+16\right)}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^3,x]
```

```
[Out] (-(Sqrt[1 - a^2*x^2]*(16 + 9*a*x + 8*a^2*x^2 + 6*a^3*x^3)) + 9*ArcSin[a*x])
/(24*a^4)
```

Maple [A] time = 0.037, size = 107, normalized size = 1.2

$$-\frac{x^3}{4a}\sqrt{-a^2x^2+1} - \frac{3x}{8a^3}\sqrt{-a^2x^2+1} + \frac{3}{8a^3}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} - \frac{x^2}{3a^2}\sqrt{-a^2x^2+1} - \frac{2}{3a^4}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3,x)
```

```
[Out] -1/4*x^3*(-a^2*x^2+1)^(1/2)/a-3/8/a^3*x*(-a^2*x^2+1)^(1/2)+3/8/a^3/(a^2)^(1
/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-1/3*x^2*(-a^2*x^2+1)^(1/2)/a^2
-2/3*(-a^2*x^2+1)^(1/2)/a^4
```

Maxima [A] time = 1.4249, size = 131, normalized size = 1.51

$$-\frac{\sqrt{-a^2x^2+1}x^3}{4a} - \frac{\sqrt{-a^2x^2+1}x^2}{3a^2} - \frac{3\sqrt{-a^2x^2+1}x}{8a^3} + \frac{3\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\sqrt{a^2}a^3} - \frac{2\sqrt{-a^2x^2+1}}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-a^2*x^2 + 1)*x^3/a - 1/3*sqrt(-a^2*x^2 + 1)*x^2/a^2 - 3/8*sqrt(-
a^2*x^2 + 1)*x/a^3 + 3/8*arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^3) - 2/3*sqrt
```

```
(-a^2*x^2 + 1)/a^4
```

Fricas [A] time = 2.12157, size = 153, normalized size = 1.76

$$-\frac{\left(6\,a^3x^3+8\,a^2x^2+9\,ax+16\right)\sqrt{-a^2x^2+1}+18\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/24*((6*a^3*x^3 + 8*a^2*x^2 + 9*a*x + 16)*sqrt(-a^2*x^2 + 1) + 18*arctan(
(sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^4
```

Sympy [C] time = 5.7756, size = 199, normalized size = 2.29

$$a\left(\begin{cases}-\frac{ix^5}{4\sqrt{a^2x^2-1}}-\frac{ix^3}{8a^2\sqrt{a^2x^2-1}}+\frac{3ix}{8a^4\sqrt{a^2x^2-1}}-\frac{3i\,\mathrm{acosh}\,(ax)}{8a^5} & \text{for } \left|a^2x^2\right|>1\\ \frac{x^5}{4\sqrt{-a^2x^2+1}}+\frac{x^3}{8a^2\sqrt{-a^2x^2+1}}-\frac{3x}{8a^4\sqrt{-a^2x^2+1}}+\frac{3\,\mathrm{asin}\,(ax)}{8a^5} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{x^2\sqrt{-a^2x^2+1}}{3a^2}-\frac{2\sqrt{-a^2x^2+1}}{3a^4} & \text{for } a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3,x)
```

```
[Out] a*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(a**2*x**
2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a**5), Abs
(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqrt(-a**2*
x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**5), True
)) + Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1
)/(3*a**4), Ne(a, 0)), (x**4/4, True))
```

Giac [A] time = 1.22701, size = 80, normalized size = 0.92

$$-\frac{1}{24}\sqrt{-a^2x^2+1}\left(\left(2\,x\left(\frac{3\,x}{a}+\frac{4}{a^2}\right)+\frac{9}{a^3}\right)x+\frac{16}{a^4}\right)+\frac{3\,\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{8\,a^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3,x, algorithm="giac")
```

```
[Out] -1/24*sqrt(-a^2*x^2 + 1)*((2*x*(3*x/a + 4/a^2) + 9/a^3)*x + 16/a^4) + 3/8*a
rcsin(a*x)*sgn(a)/(a^3*abs(a))
```

3.3 $\int e^{\tanh^{-1}(ax)} x^2 \, dx$

Optimal. Leaf size=74

$$\frac{\left(1-a^2x^2\right)^{3/2}}{3a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}-\frac{\sqrt{1-a^2x^2}}{a^3}+\frac{\sin^{-1}(ax)}{2a^3}$$

```
[Out] -(Sqrt[1 - a^2*x^2]/a^3) - (x*Sqrt[1 - a^2*x^2])/(2*a^2) + (1 - a^2*x^2)^(3
/2)/(3*a^3) + ArcSin[a*x]/(2*a^3)
```

Rubi [A] time = 0.0536717, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6124, 797, 641, 195, 216}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{3a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}-\frac{\sqrt{1-a^2x^2}}{a^3}+\frac{\sin^{-1}(ax)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2,x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/a^3) - (x*Sqrt[1 - a^2*x^2])/(2*a^2) + (1 - a^2*x^2)^(3
/2)/(3*a^3) + ArcSin[a*x]/(2*a^3)
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1+ax)}{\sqrt{1-a^2x^2}}\,dx \\
&= \frac{\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx}{a^2} - \frac{\int (1+ax)\sqrt{1-a^2x^2}\,dx}{a^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{a^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{3a^3} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2} - \frac{\int \sqrt{1-a^2x^2}\,dx}{a^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2} + \frac{\left(1-a^2x^2\right)^{3/2}}{3a^3} + \frac{\sin^{-1}(ax)}{a^3} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2} + \frac{\left(1-a^2x^2\right)^{3/2}}{3a^3} + \frac{\sin^{-1}(ax)}{2a^3}
\end{aligned}
$$

Mathematica [A] time = 0.0317354, size = 44, normalized size = 0.59

$$\frac{3\sin^{-1}(ax) - \sqrt{1-a^2x^2}\left(2a^2x^2+3ax+4\right)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^2,x]
```

```
[Out] (-(Sqrt[1 - a^2*x^2]*(4 + 3*a*x + 2*a^2*x^2)) + 3*ArcSin[a*x])/(6*a^3)
```

Maple [A] time = 0.036, size = 87, normalized size = 1.2

$$-\frac{x^2}{3a}\sqrt{-a^2x^2+1} - \frac{2}{3a^3}\sqrt{-a^2x^2+1} - \frac{x}{2a^2}\sqrt{-a^2x^2+1} + \frac{1}{2a^2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2,x)
```

```
[Out] -1/3*x^2/a*(-a^2*x^2+1)^(1/2)-2/3*(-a^2*x^2+1)^(1/2)/a^3-1/2*x*(-a^2*x^2+1)
^(1/2)/a^2+1/2/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.43816, size = 104, normalized size = 1.41

$$-\frac{\sqrt{-a^2x^2+1}x^2}{3a} - \frac{\sqrt{-a^2x^2+1}x}{2a^2} + \frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}a^2} - \frac{2\sqrt{-a^2x^2+1}}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-a^2*x^2 + 1)*x^2/a - 1/2*sqrt(-a^2*x^2 + 1)*x/a^2 + 1/2*arcsin(a
^2*x/sqrt(a^2))/(sqrt(a^2)*a^2) - 2/3*sqrt(-a^2*x^2 + 1)/a^3
```

Fricas [A] time = 2.13418, size = 132, normalized size = 1.78

$$-\frac{\left(2\,a^{2}x^{2}+3\,ax+4\right)\sqrt{-a^{2}x^{2}+1}+6\,\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)}{6\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/6*((2*a^2*x^2 + 3*a*x + 4)*sqrt(-a^2*x^2 + 1) + 6*arctan((sqrt(-a^2*x^2
+ 1) - 1)/(a*x)))/a^3
```

Sympy [A] time = 4.08993, size = 133, normalized size = 1.8

$$a\left(\begin{cases}-\frac{x^{2}\sqrt{-a^{2}x^{2}+1}}{3a^{2}}-\frac{2\sqrt{-a^{2}x^{2}+1}}{3a^{4}} & \text{for } a\neq 0\\ \frac{x^{4}}{4} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{ix\sqrt{a^{2}x^{2}-1}}{2a^{2}}-\frac{i\operatorname{acosh}(ax)}{2a^{3}} & \text{for } \left|a^{2}x^{2}\right|>1\\ \frac{x^{3}}{2\sqrt{-a^{2}x^{2}+1}}-\frac{x}{2a^{2}\sqrt{-a^{2}x^{2}+1}}+\frac{\operatorname{asin}(ax)}{2a^{3}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2,x)
```

```
[Out] a*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(
3*a**4), Ne(a, 0)), (x**4/4, True)) + Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(
2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x
**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True))
```

Giac [A] time = 1.21182, size = 68, normalized size = 0.92

$$-\frac{1}{6}\sqrt{-a^{2}x^{2}+1}\left(x\left(\frac{2\,x}{a}+\frac{3}{a^{2}}\right)+\frac{4}{a^{3}}\right)+\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{2\,a^{2}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-a^2*x^2 + 1)*(x*(2*x/a + 3/a^2) + 4/a^3) + 1/2*arcsin(a*x)*sgn(a
)/(a^2*abs(a))
```

3.4 $\int e^{\tanh^{-1}(ax)} x \, dx$

Optimal. Leaf size=38

$$\frac{\sin^{-1}(ax)}{2a^2} - \frac{(ax+2)\sqrt{1-a^2x^2}}{2a^2}$$

```
[Out] -((2 + a*x)*Sqrt[1 - a^2*x^2])/(2*a^2) + ArcSin[a*x]/(2*a^2)
```

Rubi [A] time = 0.0217802, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6124, 780, 216}

$$\frac{\sin^{-1}(ax)}{2a^2} - \frac{(ax+2)\sqrt{1-a^2x^2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x,x]
```

```
[Out] -((2 + a*x)*Sqrt[1 - a^2*x^2])/(2*a^2) + ArcSin[a*x]/(2*a^2)
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)} x \, dx &= \int \frac{x(1+ax)}{\sqrt{1-a^2x^2}} \, dx \\
&= -\frac{(2+ax)\sqrt{1-a^2x^2}}{2a^2} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}} \, dx}{2a} \\
&= -\frac{(2+ax)\sqrt{1-a^2x^2}}{2a^2} + \frac{\sin^{-1}(ax)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0245613, size = 33, normalized size = 0.87

$$\frac{\sin^{-1}(ax) - (ax+2)\sqrt{1-a^2x^2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x,x]
```

```
[Out] (-((2 + a*x)*Sqrt[1 - a^2*x^2]) + ArcSin[a*x])/(2*a^2)
```

Maple [B] time = 0.036, size = 67, normalized size = 1.8

$$-\frac{x}{2a}\sqrt{-a^2x^2+1}+\frac{1}{2a}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{1}{a^2}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x,x)
```

```
[Out] -1/2*x/a*(-a^2*x^2+1)^(1/2)+1/2/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^
2+1)^(1/2))-(-a^2*x^2+1)^(1/2)/a^2
```

Maxima [A] time = 1.43191, size = 77, normalized size = 2.03

$$-\frac{\sqrt{-a^2x^2+1}x}{2a}+\frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}a}-\frac{\sqrt{-a^2x^2+1}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x/a + 1/2*arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a) - s
qrt(-a^2*x^2 + 1)/a^2
```

Fricas [A] time = 2.08853, size = 113, normalized size = 2.97

$$-\frac{\sqrt{-a^2x^2+1}(ax+2)+2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(-a^2*x^2 + 1)*(a*x + 2) + 2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x
)))/a^2
```

Sympy [C] time = 4.00632, size = 110, normalized size = 2.89

$$a\left(\begin{cases}-\frac{ix\sqrt{a^2x^2-1}}{2a^2}-\frac{i\operatorname{acosh}(ax)}{2a^3} & \text{for } \left|a^2x^2\right|>1\\ \frac{x^3}{2\sqrt{-a^2x^2+1}}-\frac{x}{2a^2\sqrt{-a^2x^2+1}}+\frac{\operatorname{asin}(ax)}{2a^3} & \text{otherwise}\end{cases}\right)+\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x,x)
```

```
[Out] a*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs
(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**
2 + 1)) + asin(a*x)/(2*a**3), True)) + Piecewise((x**2/2, Eq(a**2, 0)), (-s
qrt(-a**2*x**2 + 1)/a**2, True))
```

Giac [A] time = 1.21536, size = 55, normalized size = 1.45

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\left(\frac{x}{a}+\frac{2}{a^2}\right)+\frac{\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{2\,a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(x/a + 2/a^2) + 1/2*arcsin(a*x)*sgn(a)/(a*abs(a))
```

3.5 $\int e^{\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=28

$$\frac{\sin^{-1}(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}$$

```
[Out] -(Sqrt[1 - a^2*x^2]/a) + ArcSin[a*x]/a
```

Rubi [A] time = 0.0109539, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6123, 641, 216}

$$\frac{\sin^{-1}(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x],x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/a) + ArcSin[a*x]/a
```

Rule 6123

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 + a*x)^((n + 1)/2)/(
(1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2]), x] /; FreeQ[a, x] && IntegerQ[(n
- 1)/2]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}\,dx &= \int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{a}+\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{a}+\frac{\sin^{-1}(ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0103446, size = 25, normalized size = 0.89

$$\frac{\sin^{-1}(ax)-\sqrt{1-a^2x^2}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x],x]
```

```
[Out] (-Sqrt[1 - a^2*x^2] + ArcSin[a*x])/a
```

Maple [A] time = 0.032, size = 45, normalized size = 1.6

$$-\frac{1}{a}\sqrt{-a^2x^2+1}+\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a+1/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2)
)
```

Maxima [A] time = 1.41809, size = 47, normalized size = 1.68

$$\frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}-\frac{\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - sqrt(-a^2*x^2 + 1)/a
```

Fricas [A] time = 2.09674, size = 92, normalized size = 3.29

$$-\frac{\sqrt{-a^2x^2+1}+2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(-a^2*x^2 + 1) + 2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a
```

Sympy [A] time = 1.62957, size = 19, normalized size = 0.68

$$\begin{cases}\frac{-\sqrt{-a^2x^2+1}+\operatorname{asin}(ax)}{a} & \text{for } a \neq 0\\ x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise(((-sqrt(-a**2*x**2 + 1) + asin(a*x))/a, Ne(a, 0)), (x, True))
```

Giac [A] time = 1.17587, size = 39, normalized size = 1.39

$$\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{|a|} - \frac{\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/abs(a) - sqrt(-a^2*x^2 + 1)/a
```

3.6 $\int \frac{e^{\tanh^{-1}(ax)}}{x} dx$

Optimal. Leaf size=22

$$\sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.0423695, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6124, 844, 216, 266, 63, 208}

$$\sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/x,x]
```

```
[Out] ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{1+ax}{x\sqrt{1-a^2x^2}}\,dx \\
&= a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx + \int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= \sin^{-1}(ax) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \sin^{-1}(ax) - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2} \\
&= \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.014277, size = 26, normalized size = 1.18

$$-\log\left(\sqrt{1-a^2x^2}+1\right)+\sin^{-1}(ax)+\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/x,x]
```

```
[Out] ArcSin[a*x] + Log[x] - Log[1 + Sqrt[1 - a^2*x^2]]
```

Maple [B] time = 0.033, size = 44, normalized size = 2.

$$a\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-arctanh(1/(-a^2*x^2+
1)^(1/2))
```

Maxima [B] time = 1.44288, size = 63, normalized size = 2.86

$$\frac{a\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}-\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/a
bs(x))
```

Fricas [B] time = 2.02858, size = 104, normalized size = 4.73

$$-2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + log((sqrt(-a^2*x^2 + 1) - 1)/x)
```

Sympy [B] time = 9.42463, size = 70, normalized size = 3.18

$$a\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)+\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] a*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*as
inh(x*sqrt(-a**2)), a**2 < 0)) + Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**
2) > 1), (I*asin(1/(a*x)), True))
```

Giac [B] time = 1.20805, size = 69, normalized size = 3.14

$$\frac{a\arcsin(ax)\operatorname{sgn}(a)}{|a|}-\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] a*arcsin(a*x)*sgn(a)/abs(a) - a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
2*a)/(a^2*abs(x)))/abs(a)
```

3.7 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2} \, dx$

Optimal. Leaf size=38

$$-\frac{\sqrt{1-a^2x^2}}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -(Sqrt[1 - a^2*x^2]/x) - a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.0444022, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6124, 807, 266, 63, 208}

$$-\frac{\sqrt{1-a^2x^2}}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/x^2,x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/x) - a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{1+ax}{x^2\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{x} + a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{x} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1-a^2x^2}}{x} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a} \\
&= -\frac{\sqrt{1-a^2x^2}}{x} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0294072, size = 44, normalized size = 1.16

$$-\frac{\sqrt{1-a^2x^2}}{x} - a\log\left(\sqrt{1-a^2x^2}+1\right) + a\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/x^2,x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/x) + a*Log[x] - a*Log[1 + Sqrt[1 - a^2*x^2]]
```

Maple [A] time = 0.035, size = 35, normalized size = 0.9

$$-\frac{1}{x}\sqrt{-a^2x^2+1} - a\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/x-a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.42873, size = 63, normalized size = 1.66

$$-a\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{\sqrt{-a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -a*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2*x^2 + 1)/x
```

Fricas [A] time = 1.95132, size = 84, normalized size = 2.21

$$\frac{ax\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (a*x*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1))/x
```

Sympy [C] time = 6.42878, size = 65, normalized size = 1.71

$$a\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\ |a^2x^2|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] a*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True
)) + Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2
*x**2 + 1)/x, True))
```

Giac [B] time = 1.17853, size = 130, normalized size = 3.42

$$\frac{a^4x}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|}-\frac{a^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|}-\frac{\sqrt{-a^2x^2+1}|a|+a}{2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - a^2*log(1/2*abs(-2*sqr
t(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/2*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)/(x*abs(a))
```

3.8 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3} \, dx$

Optimal. Leaf size=64

$$-\frac{a\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -Sqrt[1 - a^2*x^2]/(2*x^2) - (a*Sqrt[1 - a^2*x^2])/x - (a^2*ArcTanh[Sqrt[1
- a^2*x^2]])/2
```

Rubi [A] time = 0.0626748, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6124, 835, 807, 266, 63, 208}

$$-\frac{a\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/x^3,x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*x^2) - (a*Sqrt[1 - a^2*x^2])/x - (a^2*ArcTanh[Sqrt[1
- a^2*x^2]])/2
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^3}\,dx &= \int \frac{1+ax}{x^3\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{1}{2}\int \frac{-2a-a^2x}{x^2\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{x}+\frac{1}{2}a^2\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{x}+\frac{1}{4}a^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{x}-\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)\\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{x}-\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0399536, size = 58, normalized size = 0.91

$$\frac{1}{2}\left(-\frac{(2ax+1)\sqrt{1-a^2x^2}}{x^2}-a^2\log\left(\sqrt{1-a^2x^2}+1\right)+a^2\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/x^3,x]
```

```
[Out] (-(((1 + 2*a*x)*Sqrt[1 - a^2*x^2])/x^2) + a^2*Log[x] - a^2*Log[1 + Sqrt[1 -
 a^2*x^2]])/2
```

Maple [A] time = 0.037, size = 55, normalized size = 0.9

$$-\frac{a}{x}\sqrt{-a^2x^2+1}-\frac{1}{2\,x^2}\sqrt{-a^2x^2+1}-\frac{a^2}{2}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3,x)
```

```
[Out] -a*(-a^2*x^2+1)^(1/2)/x-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*
x^2+1)^(1/2))
```

Maxima [A] time = 1.4343, size = 90, normalized size = 1.41

$$-\frac{1}{2}\,a^2\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\sqrt{-a^2x^2+1}a}{x}-\frac{\sqrt{-a^2x^2+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2*x^2 + 1)*a
/x - 1/2*sqrt(-a^2*x^2 + 1)/x^2
```

Fricas [A] time = 1.73341, size = 113, normalized size = 1.77

$$\frac{a^2x^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}(2\,ax+1)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1)*(2*a*x +
1))/x^2
```

Sympy [C] time = 4.39093, size = 136, normalized size = 2.12

$$a\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for } \left|a^2x^2\right|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} & \text{for } \frac{1}{|a^2x^2|}>1\\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] a*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x*
*2 + 1)/x, True)) + Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2
*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sq
rt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))
```

Giac [B] time = 1.2075, size = 213, normalized size = 3.33

$$\frac{\left(a^3+\frac{4\left(\sqrt{-a^2x^2+1}|a|+a\right)a}{x}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}|a|+a\right)^2|a|}-\frac{a^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{\frac{4\left(\sqrt{-a^2x^2+1}|a|+a\right)a|a|}{x}+\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)^2|a|}{ax^2}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3 + 4*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x)*a^4*x^2/((sqrt(-a^2*x^2 +
 1)*abs(a) + a)^2*abs(a)) - 1/2*a^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a
) - 2*a)/(a^2*abs(x)))/abs(a) - 1/8*(4*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a*ab
s(a)/x + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)/(a*x^2))/a^2
```

3.9 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4} \, dx$

Optimal. Leaf size=90

$$-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{a\sqrt{1-a^2x^2}}{2x^2}-\frac{\sqrt{1-a^2x^2}}{3x^3}-\frac{1}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -Sqrt[1 - a^2*x^2]/(3*x^3) - (a*Sqrt[1 - a^2*x^2])/(2*x^2) - (2*a^2*Sqrt[1
- a^2*x^2])/(3*x) - (a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rubi [A] time = 0.0851134, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6124, 835, 807, 266, 63, 208}

$$-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{a\sqrt{1-a^2x^2}}{2x^2}-\frac{\sqrt{1-a^2x^2}}{3x^3}-\frac{1}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/x^4,x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(3*x^3) - (a*Sqrt[1 - a^2*x^2])/(2*x^2) - (2*a^2*Sqrt[1
- a^2*x^2])/(3*x) - (a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^4} dx &= \int \frac{1+ax}{x^4\sqrt{1-a^2x^2}} dx \\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{1}{3}\int \frac{-3a-2a^2x}{x^3\sqrt{1-a^2x^2}} dx \\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{a\sqrt{1-a^2x^2}}{2x^2} + \frac{1}{6}\int \frac{4a^2+3a^3x}{x^2\sqrt{1-a^2x^2}} dx \\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{a\sqrt{1-a^2x^2}}{2x^2} - \frac{2a^2\sqrt{1-a^2x^2}}{3x} + \frac{1}{2}a^3\int \frac{1}{x\sqrt{1-a^2x^2}} dx \\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{a\sqrt{1-a^2x^2}}{2x^2} - \frac{2a^2\sqrt{1-a^2x^2}}{3x} + \frac{1}{4}a^3 \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}} dx, x, x^2\right) \\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{a\sqrt{1-a^2x^2}}{2x^2} - \frac{2a^2\sqrt{1-a^2x^2}}{3x} - \frac{1}{2}a \operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}} dx, x, \sqrt{1-a^2x^2}\right) \\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{a\sqrt{1-a^2x^2}}{2x^2} - \frac{2a^2\sqrt{1-a^2x^2}}{3x} - \frac{1}{2}a^3 \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0523851, size = 67, normalized size = 0.74

$$\frac{1}{6}\left(-\frac{\sqrt{1-a^2x^2}\left(4a^2x^2+3ax+2\right)}{x^3} - 3a^3\log\left(\sqrt{1-a^2x^2}+1\right) + 3a^3\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/x^4,x]
```

```
[Out] (-((Sqrt[1 - a^2*x^2]*(2 + 3*a*x + 4*a^2*x^2))/x^3) + 3*a^3*Log[x] - 3*a^3*
Log[1 + Sqrt[1 - a^2*x^2]])/6
```

Maple [A] time = 0.037, size = 77, normalized size = 0.9

$$a\left(-\frac{1}{2x^2}\sqrt{-a^2x^2+1} - \frac{a^2}{2}\operatorname{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right) - \frac{1}{3x^3}\sqrt{-a^2x^2+1} - \frac{2a^2}{3x}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4,x)
```

```
[Out] a*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2)))-1/3*(
-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2+1)^(1/2)/x
```

Maxima [A] time = 1.42523, size = 117, normalized size = 1.3

$$-\frac{1}{2}a^3\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{2\sqrt{-a^2x^2+1}a^2}{3x}-\frac{\sqrt{-a^2x^2+1}a}{2x^2}-\frac{\sqrt{-a^2x^2+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*a^3*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 2/3*sqrt(-a^2*x^2 +
1)*a^2/x - 1/2*sqrt(-a^2*x^2 + 1)*a/x^2 - 1/3*sqrt(-a^2*x^2 + 1)/x^3
```

Fricas [A] time = 1.60205, size = 132, normalized size = 1.47

$$\frac{3a^3x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(4a^2x^2+3ax+2\right)\sqrt{-a^2x^2+1}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^3*x^3*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (4*a^2*x^2 + 3*a*x + 2)*sq
rt(-a^2*x^2 + 1))/x^3
```

Sympy [C] time = 6.05506, size = 185, normalized size = 2.06

$$a\left(\begin{cases}-\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} & \text{for } \frac{1}{|a^2x^2|}>1\\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{2ia^2\sqrt{a^2x^2-1}}{3x}-\frac{i\sqrt{a^2x^2-1}}{3x^3} & \text{for } |a^2x^2|>1\\ -\frac{2a^2\sqrt{-a^2x^2+1}}{3x}-\frac{\sqrt{-a^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] a*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/A
bs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**
2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True)) + Piecewise((-2*I*a**2*
sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2)
> 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3),
True))
```

Giac [B] time = 1.22321, size = 284, normalized size = 3.16

$$\frac{\left(a^4+\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)a^2}{x}+\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|}-\frac{a^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{2|a|}-\frac{\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4}{x}+\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)^2a^2}{x^2}+\frac{\left(\sqrt{\right.}}{}}{24a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4 + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2/x + 9*(sqrt(-a^2*x^2 + 1)
*abs(a) + a)^2/x^2)*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*abs(a)) - 1/
2*a^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a)
- 1/24*(9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4/x + 3*(sqrt(-a^2*x^2 + 1)*abs
(a) + a)^2*a^2/x^2 + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/x^3)/(a^2*abs(a))
```

3.10 $\int \frac{e^{\tanh^{-1}(ax)}}{x^5} dx$

Optimal. Leaf size=114

$$-\frac{2a^3\sqrt{1-a^2x^2}}{3x}-\frac{3a^2\sqrt{1-a^2x^2}}{8x^2}-\frac{a\sqrt{1-a^2x^2}}{3x^3}-\frac{\sqrt{1-a^2x^2}}{4x^4}-\frac{3}{8}a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -Sqrt[1 - a^2*x^2]/(4*x^4) - (a*Sqrt[1 - a^2*x^2])/(3*x^3) - (3*a^2*Sqrt[1
- a^2*x^2])/(8*x^2) - (2*a^3*Sqrt[1 - a^2*x^2])/(3*x) - (3*a^4*ArcTanh[Sqrt
[1 - a^2*x^2]])/8
```

Rubi [A] time = 0.107458, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6124, 835, 807, 266, 63, 208}

$$-\frac{2a^3\sqrt{1-a^2x^2}}{3x}-\frac{3a^2\sqrt{1-a^2x^2}}{8x^2}-\frac{a\sqrt{1-a^2x^2}}{3x^3}-\frac{\sqrt{1-a^2x^2}}{4x^4}-\frac{3}{8}a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/x^5,x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(4*x^4) - (a*Sqrt[1 - a^2*x^2])/(3*x^3) - (3*a^2*Sqrt[1
- a^2*x^2])/(8*x^2) - (2*a^3*Sqrt[1 - a^2*x^2])/(3*x) - (3*a^4*ArcTanh[Sqrt
[1 - a^2*x^2]])/8
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^5}\,dx &= \int \frac{1+ax}{x^5\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{1}{4}\int \frac{-4a-3a^2x}{x^4\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{a\sqrt{1-a^2x^2}}{3x^3} + \frac{1}{12}\int \frac{9a^2+8a^3x}{x^3\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} - \frac{1}{24}\int \frac{-16a^3-9a^4x}{x^2\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} - \frac{2a^3\sqrt{1-a^2x^2}}{3x} + \frac{1}{8}\left(3a^4\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} - \frac{2a^3\sqrt{1-a^2x^2}}{3x} + \frac{1}{16}\left(3a^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right.\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} - \frac{2a^3\sqrt{1-a^2x^2}}{3x} - \frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a\right.\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} - \frac{2a^3\sqrt{1-a^2x^2}}{3x} - \frac{3}{8}a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0600546, size = 75, normalized size = 0.66

$$\frac{1}{24}\left(-\frac{\sqrt{1-a^2x^2}\left(16a^3x^3+9a^2x^2+8ax+6\right)}{x^4}-9a^4\log\left(\sqrt{1-a^2x^2}+1\right)+9a^4\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/x^5,x]
```

```
[Out] (-((Sqrt[1 - a^2*x^2]*(6 + 8*a*x + 9*a^2*x^2 + 16*a^3*x^3))/x^4) + 9*a^4*Lo
g[x] - 9*a^4*Log[1 + Sqrt[1 - a^2*x^2]])/24
```

Maple [A] time = 0.038, size = 100, normalized size = 0.9

$$-\frac{1}{4x^4}\sqrt{-a^2x^2+1}+\frac{3a^2}{4}\left(-\frac{1}{2x^2}\sqrt{-a^2x^2+1}-\frac{a^2}{2}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)+a\left(-\frac{1}{3x^3}\sqrt{-a^2x^2+1}-\frac{2a^2}{3x}\sqrt{-a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^5,x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)/x^4+3/4*a^2*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*ar
ctanh(1/(-a^2*x^2+1)^(1/2)))+a*(-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x
^2+1)^(1/2)/x)
```

Maxima [A] time = 1.42476, size = 144, normalized size = 1.26

$$-\frac{3}{8}\,a^{4}\log\left(\frac{2\sqrt{-a^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)-\frac{2\sqrt{-a^{2}x^{2}+1}a^{3}}{3\,x}-\frac{3\sqrt{-a^{2}x^{2}+1}a^{2}}{8\,x^{2}}-\frac{\sqrt{-a^{2}x^{2}+1}a}{3\,x^{3}}-\frac{\sqrt{-a^{2}x^{2}+1}}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -3/8*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 2/3*sqrt(-a^2*x^2 +
1)*a^3/x - 3/8*sqrt(-a^2*x^2 + 1)*a^2/x^2 - 1/3*sqrt(-a^2*x^2 + 1)*a/x^3 -
1/4*sqrt(-a^2*x^2 + 1)/x^4
```

Fricas [A] time = 1.72512, size = 151, normalized size = 1.32

$$\frac{9\,a^{4}x^{4}\log\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{x}\right)-\left(16\,a^{3}x^{3}+9\,a^{2}x^{2}+8\,ax+6\right)\sqrt{-a^{2}x^{2}+1}}{24\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(9*a^4*x^4*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (16*a^3*x^3 + 9*a^2*x^2 +
 8*a*x + 6)*sqrt(-a^2*x^2 + 1))/x^4
```

Sympy [C] time = 9.57736, size = 258, normalized size = 2.26

$$a\left(\begin{cases}-\frac{2ia^{2}\sqrt{a^{2}x^{2}-1}}{3x}-\frac{i\sqrt{a^{2}x^{2}-1}}{3x^{3}} & \text{for}\ \left|a^{2}x^{2}\right|>1\\ -\frac{2a^{2}\sqrt{-a^{2}x^{2}+1}}{3x}-\frac{\sqrt{-a^{2}x^{2}+1}}{3x^{3}} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{3a^{4}\operatorname{acosh}\left(\frac{1}{ax}\right)}{8}+\frac{3a^{3}}{8x\sqrt{-1+\frac{1}{a^{2}x^{2}}}}-\frac{a}{8x^{3}\sqrt{-1+\frac{1}{a^{2}x^{2}}}}-\frac{1}{4ax^{5}\sqrt{-1+\frac{1}{a^{2}x^{2}}}} & \text{for}\ \frac{1}{\left|a^{2}x^{2}\right|\ldots}\\ \frac{3ia^{4}\operatorname{asin}\left(\frac{1}{ax}\right)}{8}-\frac{3ia^{3}}{8x\sqrt{1-\frac{1}{a^{2}x^{2}}}}+\frac{ia}{8x^{3}\sqrt{1-\frac{1}{a^{2}x^{2}}}}+\frac{i}{4ax^{5}\sqrt{1-\frac{1}{a^{2}x^{2}}}} & \text{otherw}\ldots\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**5,x)
```

```
[Out] a*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3
*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a*
*2*x**2 + 1)/(3*x**3), True)) + Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*a**
3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - 1/
(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*asin(
1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1 -
```

```
1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True))
```

Giac [B] time = 1.19903, size = 369, normalized size = 3.24

$$\frac{\left(3\,a^5+\frac{8\left(\sqrt{-a^2x^2+1}|a|+a\right)a^3}{x}+\frac{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^2a}{x^2}+\frac{72\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{ax^3}\right)a^8x^4}{192\left(\sqrt{-a^2x^2+1}|a|+a\right)^4|a|}-\frac{3\,a^5\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{8\,|a|}-\frac{\frac{72\left(\sqrt{-a^2x^2+1}|a|+a\right)a^5}{x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^5 + 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^3/x + 24*(sqrt(-a^2*x^2
+ 1)*abs(a) + a)^2*a/x^2 + 72*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a*x^3))*a^
8*x^4/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*abs(a)) - 3/8*a^5*log(1/2*abs(-2*s
qrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/192*(72*(sqrt(-a^2
*x^2 + 1)*abs(a) + a)*a^5*abs(a)/x + 24*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a
^3*abs(a)/x^2 + 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a*abs(a)/x^3 + 3*(sqrt(
-a^2*x^2 + 1)*abs(a) + a)^4*abs(a)/(a*x^4))/a^4
```

3.11 $\int e^{2\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=44

$$-\frac{x^2}{a^2}-\frac{2x}{a^3}-\frac{2\log(1-ax)}{a^4}-\frac{2x^3}{3a}-\frac{x^4}{4}$$

```
[Out] (-2*x)/a^3 - x^2/a^2 - (2*x^3)/(3*a) - x^4/4 - (2*Log[1 - a*x])/a^4
```

Rubi [A] time = 0.0423027, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$-\frac{x^2}{a^2}-\frac{2x}{a^3}-\frac{2\log(1-ax)}{a^4}-\frac{2x^3}{3a}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^3,x]
```

```
[Out] (-2*x)/a^3 - x^2/a^2 - (2*x^3)/(3*a) - x^4/4 - (2*Log[1 - a*x])/a^4
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^3\,dx &= \int \frac{x^3(1+ax)}{1-ax}\,dx\\ &= \int \left(-\frac{2}{a^3}-\frac{2x}{a^2}-\frac{2x^2}{a}-x^3-\frac{2}{a^3(-1+ax)}\right)dx\\ &= -\frac{2x}{a^3}-\frac{x^2}{a^2}-\frac{2x^3}{3a}-\frac{x^4}{4}-\frac{2\log(1-ax)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.0183638, size = 44, normalized size = 1.

$$-\frac{x^2}{a^2}-\frac{2x}{a^3}-\frac{2\log(1-ax)}{a^4}-\frac{2x^3}{3a}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^3,x]
```

```
[Out] (-2*x)/a^3 - x^2/a^2 - (2*x^3)/(3*a) - x^4/4 - (2*Log[1 - a*x])/a^4
```

Maple [A] time = 0.027, size = 40, normalized size = 0.9

$$-\frac{x^4}{4} - \frac{2x^3}{3a} - \frac{x^2}{a^2} - 2\frac{x}{a^3} - 2\frac{\ln(ax-1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3,x)
```

```
[Out] -1/4*x^4-2/3*x^3/a-x^2/a^2-2*x/a^3-2/a^4*ln(a*x-1)
```

Maxima [A] time = 0.939016, size = 58, normalized size = 1.32

$$-\frac{3a^3x^4 + 8a^2x^3 + 12ax^2 + 24x}{12a^3} - \frac{2\log(ax-1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^3*x^4 + 8*a^2*x^3 + 12*a*x^2 + 24*x)/a^3 - 2*log(a*x - 1)/a^4
```

Fricas [A] time = 1.6383, size = 101, normalized size = 2.3

$$-\frac{3a^4x^4 + 8a^3x^3 + 12a^2x^2 + 24ax + 24\log(ax-1)}{12a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*x^4 + 8*a^3*x^3 + 12*a^2*x^2 + 24*a*x + 24*log(a*x - 1))/a^4
```

Sympy [A] time = 0.429389, size = 39, normalized size = 0.89

$$-\frac{x^4}{4} - \frac{2x^3}{3a} - \frac{x^2}{a^2} - \frac{2x}{a^3} - \frac{2\log(ax-1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3,x)
```

```
[Out] -x**4/4 - 2*x**3/(3*a) - x**2/a**2 - 2*x/a**3 - 2*log(a*x - 1)/a**4
```

Giac [A] time = 1.15617, size = 63, normalized size = 1.43

$$-\frac{3a^4x^4 + 8a^3x^3 + 12a^2x^2 + 24ax}{12a^4} - \frac{2\log(|ax-1|)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*a^4*x^4 + 8*a^3*x^3 + 12*a^2*x^2 + 24*a*x)/a^4 - 2*log(abs(a*x - 1
))/a^4
```

3.12 $\int e^{2\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=34

$$-\frac{2x}{a^2}-\frac{2\log(1-ax)}{a^3}-\frac{x^2}{a}-\frac{x^3}{3}$$

```
[Out] (-2*x)/a^2 - x^2/a - x^3/3 - (2*Log[1 - a*x])/a^3
```

Rubi [A] time = 0.0339557, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$-\frac{2x}{a^2}-\frac{2\log(1-ax)}{a^3}-\frac{x^2}{a}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^2,x]
```

```
[Out] (-2*x)/a^2 - x^2/a - x^3/3 - (2*Log[1 - a*x])/a^3
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1+ax)}{1-ax}\,dx\\ &= \int \left(-\frac{2}{a^2}-\frac{2x}{a}-x^2-\frac{2}{a^2(-1+ax)}\right)dx\\ &= -\frac{2x}{a^2}-\frac{x^2}{a}-\frac{x^3}{3}-\frac{2\log(1-ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.0134466, size = 34, normalized size = 1.

$$-\frac{2x}{a^2}-\frac{2\log(1-ax)}{a^3}-\frac{x^2}{a}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^2,x]
```

```
[Out] (-2*x)/a^2 - x^2/a - x^3/3 - (2*Log[1 - a*x])/a^3
```

Maple [A] time = 0.027, size = 32, normalized size = 0.9

$$-\frac{x^3}{3}-\frac{x^2}{a}-2\,\frac{x}{a^2}-2\,\frac{\ln(ax-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2,x)
```

```
[Out] -1/3*x^3-x^2/a-2*x/a^2-2/a^3*ln(a*x-1)
```

Maxima [A] time = 0.960327, size = 46, normalized size = 1.35

$$-\frac{a^2x^3+3\,ax^2+6\,x}{3\,a^2}-\frac{2\,\log(ax-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*x^3 + 3*a*x^2 + 6*x)/a^2 - 2*log(a*x - 1)/a^3
```

Fricas [A] time = 1.68955, size = 77, normalized size = 2.26

$$-\frac{a^3x^3+3\,a^2x^2+6\,ax+6\,\log(ax-1)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(a^3*x^3 + 3*a^2*x^2 + 6*a*x + 6*log(a*x - 1))/a^3
```

Sympy [A] time = 0.38483, size = 29, normalized size = 0.85

$$-\frac{x^3}{3}-\frac{x^2}{a}-\frac{2x}{a^2}-\frac{2\log(ax-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2,x)
```

```
[Out] -x**3/3 - x**2/a - 2*x/a**2 - 2*log(a*x - 1)/a**3
```

Giac [A] time = 1.17959, size = 51, normalized size = 1.5

$$-\frac{a^3x^3+3\,a^2x^2+6\,ax}{3\,a^3}-\frac{2\,\log(|ax-1|)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2,x, algorithm="giac")
```

```
[Out] -1/3*(a^3*x^3 + 3*a^2*x^2 + 6*a*x)/a^3 - 2*log(abs(a*x - 1))/a^3
```

3.13 $\int e^{2\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=26

$$-\frac{2\log(1-ax)}{a^2}-\frac{2x}{a}-\frac{x^2}{2}$$

```
[Out] (-2*x)/a - x^2/2 - (2*Log[1 - a*x])/a^2
```

Rubi [A] time = 0.0287868, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6126, 77}

$$-\frac{2\log(1-ax)}{a^2}-\frac{2x}{a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x,x]
```

```
[Out] (-2*x)/a - x^2/2 - (2*Log[1 - a*x])/a^2
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x\,dx &= \int \frac{x(1+ax)}{1-ax}\,dx\\ &= \int\left(-\frac{2}{a}-x-\frac{2}{a(-1+ax)}\right)dx\\ &= -\frac{2x}{a}-\frac{x^2}{2}-\frac{2\log(1-ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0107103, size = 26, normalized size = 1.

$$-\frac{2\log(1-ax)}{a^2}-\frac{2x}{a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x,x]
```

```
[Out] (-2*x)/a - x^2/2 - (2*Log[1 - a*x])/a^2
```

Maple [A] time = 0.032, size = 24, normalized size = 0.9

$$-\frac{x^2}{2} - 2\frac{x}{a} - 2\frac{\ln(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x,x)
```

```
[Out] -1/2*x^2-2*x/a-2/a^2*ln(a*x-1)
```

Maxima [A] time = 0.963384, size = 35, normalized size = 1.35

$$-\frac{ax^2 + 4x}{2a} - \frac{2\log(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x,x, algorithm="maxima")
```

```
[Out] -1/2*(a*x^2 + 4*x)/a - 2*log(a*x - 1)/a^2
```

Fricas [A] time = 1.61819, size = 61, normalized size = 2.35

$$-\frac{a^2x^2 + 4ax + 4\log(ax-1)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*x^2 + 4*a*x + 4*log(a*x - 1))/a^2
```

Sympy [A] time = 0.38652, size = 22, normalized size = 0.85

$$-\frac{x^2}{2} - \frac{2x}{a} - \frac{2\log(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x,x)
```

```
[Out] -x**2/2 - 2*x/a - 2*log(a*x - 1)/a**2
```

Giac [A] time = 1.14283, size = 41, normalized size = 1.58

$$-\frac{a^2x^2 + 4ax}{2a^2} - \frac{2\log(|ax-1|)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*x^2 + 4*a*x)/a^2 - 2*log(abs(a*x - 1))/a^2
```

3.14 $\int e^{2\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=16

$$-\frac{2\log(1-ax)}{a}-x$$

```
[Out] -x - (2*Log[1 - a*x])/a
```

Rubi [A] time = 0.0120185, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6125, 43}

$$-\frac{2\log(1-ax)}{a}-x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x]),x]

[Out] -x - (2*Log[1 - a*x])/a
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] && !IntegerQ[(n - 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}\,dx &= \int \frac{1+ax}{1-ax}\,dx\\ &= \int \left(-1-\frac{2}{-1+ax}\right)dx\\ &= -x-\frac{2\log(1-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0094114, size = 16, normalized size = 1.

$$-\frac{2\log(1-ax)}{a}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x]),x]

[Out] -x - (2*Log[1 - a*x])/a
```

Maple [A] time = 0.027, size = 16, normalized size = 1.

$$-x-2\,\frac{\ln\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1),x)
```

```
[Out] -x-2/a*ln(a*x-1)
```

Maxima [A] time = 0.952724, size = 20, normalized size = 1.25

$$-x-\frac{2\,\log\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -x - 2*log(a*x - 1)/a
```

Fricas [A] time = 1.74374, size = 36, normalized size = 2.25

$$-\frac{ax+2\,\log\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -(a*x + 2*log(a*x - 1))/a
```

Sympy [A] time = 0.187183, size = 12, normalized size = 0.75

$$-x-\frac{2\log\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1),x)
```

```
[Out] -x - 2*log(a*x - 1)/a
```

Giac [A] time = 1.17178, size = 22, normalized size = 1.38

$$-x-\frac{2\,\log\left(|ax-1|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -x - 2*log(abs(a*x - 1))/a
```

3.15 $\int \frac{e^{2\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=12

$$\log(x) - 2\log(1 - ax)$$

```
[Out] Log[x] - 2*Log[1 - a*x]
```

Rubi [A] time = 0.0253327, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 72}

$$\log(x) - 2\log(1 - ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/x,x]

[Out] Log[x] - 2*Log[1 - a*x]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{1+ax}{x(1-ax)}\,dx\\ &= \int \left(\frac{1}{x} - \frac{2a}{-1+ax}\right)dx\\ &= \log(x) - 2\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.0065252, size = 12, normalized size = 1.

$$\log(x) - 2\log(1 - ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/x,x]

[Out] Log[x] - 2*Log[1 - a*x]
```

Maple [A] time = 0.033, size = 12, normalized size = 1.

$$\ln(x) - 2\ln(ax - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x,x)
```

```
[Out] ln(x)-2*ln(a*x-1)
```

Maxima [A] time = 0.951546, size = 15, normalized size = 1.25

$$-2\log(ax - 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x,x, algorithm="maxima")
```

```
[Out] -2*log(a*x - 1) + log(x)
```

Fricas [A] time = 1.76841, size = 35, normalized size = 2.92

$$-2\log(ax - 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x,x, algorithm="fricas")
```

```
[Out] -2*log(a*x - 1) + log(x)
```

Sympy [A] time = 0.2531, size = 10, normalized size = 0.83

$$\log(x) - 2\log\left(x - \frac{1}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x,x)
```

```
[Out] log(x) - 2*log(x - 1/a)
```

Giac [A] time = 1.20251, size = 18, normalized size = 1.5

$$-2\log(|ax - 1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x,x, algorithm="giac")
```

```
[Out] -2*log(abs(a*x - 1)) + log(abs(x))
```

3.16 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=21

$$2a\log(x) - 2a\log(1-ax) - \frac{1}{x}$$

```
[Out] -x^(-1) + 2*a*Log[x] - 2*a*Log[1 - a*x]
```

Rubi [A] time = 0.0283413, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$2a\log(x) - 2a\log(1-ax) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/x^2,x]
```

```
[Out] -x^(-1) + 2*a*Log[x] - 2*a*Log[1 - a*x]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{1+ax}{x^2(1-ax)}\,dx\\
&= \int \left(\frac{1}{x^2}+\frac{2a}{x}-\frac{2a^2}{-1+ax}\right)dx\\
&= -\frac{1}{x}+2a\log(x)-2a\log(1-ax)
\end{aligned}$$

Mathematica [A] time = 0.0098314, size = 21, normalized size = 1.

$$2a\log(x) - 2a\log(1-ax) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/x^2,x]

[Out] -x^(-1) + 2*a*Log[x] - 2*a*Log[1 - a*x]
```

Maple [A] time = 0.052, size = 21, normalized size = 1.

$$-x^{-1} + 2\,a\ln(x) - 2\,a\ln(ax - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^2,x)

[Out] -1/x+2*a*ln(x)-2*a*ln(a*x-1)
```

Maxima [A] time = 0.950452, size = 27, normalized size = 1.29

$$-2\,a\log(ax - 1) + 2\,a\log(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2,x, algorithm="maxima")

[Out] -2*a*log(a*x - 1) + 2*a*log(x) - 1/x
```

Fricas [A] time = 1.7354, size = 59, normalized size = 2.81

$$-\frac{2\,ax\log(ax - 1) - 2\,ax\log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2,x, algorithm="fricas")

[Out] -(2*a*x*log(a*x - 1) - 2*a*x*log(x) + 1)/x
```

Sympy [A] time = 0.553719, size = 17, normalized size = 0.81

$$-2a\left(-\log(x) + \log\left(x - \frac{1}{a}\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**2,x)

[Out] -2*a*(-log(x) + log(x - 1/a)) - 1/x
```

Giac [A] time = 1.16747, size = 30, normalized size = 1.43

$$-2\,a\log\left(|ax-1|\right)+2\,a\log\left(|x|\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2,x, algorithm="giac")
```

```
[Out] -2*a*log(abs(a*x - 1)) + 2*a*log(abs(x)) - 1/x
```

3.17 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=33

$$2a^2\log(x) - 2a^2\log(1-ax) - \frac{2a}{x} - \frac{1}{2x^2}$$

```
[Out] -1/(2*x^2) - (2*a)/x + 2*a^2*Log[x] - 2*a^2*Log[1 - a*x]
```

Rubi [A] time = 0.0322523, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$2a^2\log(x) - 2a^2\log(1-ax) - \frac{2a}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/x^3,x]

[Out] -1/(2*x^2) - (2*a)/x + 2*a^2*Log[x] - 2*a^2*Log[1 - a*x]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x^3}\,dx &= \int \frac{1+ax}{x^3(1-ax)}\,dx \\
&= \int \left(\frac{1}{x^3} + \frac{2a}{x^2} + \frac{2a^2}{x} - \frac{2a^3}{-1+ax}\right)dx \\
&= -\frac{1}{2x^2} - \frac{2a}{x} + 2a^2\log(x) - 2a^2\log(1-ax)
\end{aligned}$$

Mathematica [A] time = 0.0109985, size = 33, normalized size = 1.

$$2a^2\log(x) - 2a^2\log(1-ax) - \frac{2a}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/x^3,x]

[Out] -1/(2*x^2) - (2*a)/x + 2*a^2*Log[x] - 2*a^2*Log[1 - a*x]
```

Maple [A] time = 0.033, size = 31, normalized size = 0.9

$$-\frac{1}{2\,x^2}-2\,\frac{a}{x}+2\,a^2\ln(x)-2\,a^2\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^3,x)

[Out] -1/2/x^2-2*a/x+2*a^2*ln(x)-2*a^2*ln(a*x-1)
```

Maxima [A] time = 0.94655, size = 41, normalized size = 1.24

$$-2\,a^2\log(ax-1)+2\,a^2\log(x)-\frac{4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3,x, algorithm="maxima")

[Out] -2*a^2*log(a*x - 1) + 2*a^2*log(x) - 1/2*(4*a*x + 1)/x^2
```

Fricas [A] time = 1.67456, size = 89, normalized size = 2.7

$$-\frac{4\,a^2x^2\log(ax-1)-4\,a^2x^2\log(x)+4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3,x, algorithm="fricas")

[Out] -1/2*(4*a^2*x^2*log(a*x - 1) - 4*a^2*x^2*log(x) + 4*a*x + 1)/x^2
```

Sympy [A] time = 0.489527, size = 27, normalized size = 0.82

$$-2a^2\left(-\log(x)+\log\left(x-\frac{1}{a}\right)\right)-\frac{4ax+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**3,x)

[Out] -2*a**2*(-log(x) + log(x - 1/a)) - (4*a*x + 1)/(2*x**2)
```

Giac [A] time = 1.15007, size = 43, normalized size = 1.3

$$-2\,a^2\log\left(|ax-1|\right)+2\,a^2\log\left(|x|\right)-\frac{4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3,x, algorithm="giac")
```

```
[Out] -2*a^2*log(abs(a*x - 1)) + 2*a^2*log(abs(x)) - 1/2*(4*a*x + 1)/x^2
```

3.18 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=41

$$-\frac{2a^2}{x}+2a^3\log(x)-2a^3\log(1-ax)-\frac{a}{x^2}-\frac{1}{3x^3}$$

```
[Out] -1/(3*x^3) - a/x^2 - (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 - a*x]
```

Rubi [A] time = 0.0356681, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$-\frac{2a^2}{x}+2a^3\log(x)-2a^3\log(1-ax)-\frac{a}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/(3*x^3) - a/x^2 - (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 - a*x]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{x^4}\,dx &= \int \frac{1+ax}{x^4(1-ax)}\,dx\\ &= \int \left(\frac{1}{x^4}+\frac{2a}{x^3}+\frac{2a^2}{x^2}+\frac{2a^3}{x}-\frac{2a^4}{-1+ax}\right)dx\\ &= -\frac{1}{3x^3}-\frac{a}{x^2}-\frac{2a^2}{x}+2a^3\log(x)-2a^3\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.0131947, size = 41, normalized size = 1.

$$-\frac{2a^2}{x}+2a^3\log(x)-2a^3\log(1-ax)-\frac{a}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/(3*x^3) - a/x^2 - (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 - a*x]
```

Maple [A] time = 0.037, size = 39, normalized size = 1.

$$-\frac{1}{3x^3}-\frac{a}{x^2}-2\frac{a^2}{x}+2a^3\ln(x)-2a^3\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^4,x)
```

```
[Out] -1/3/x^3-a/x^2-2*a^2/x+2*a^3*ln(x)-2*a^3*ln(a*x-1)
```

Maxima [A] time = 0.951531, size = 51, normalized size = 1.24

$$-2a^3\log(ax-1)+2a^3\log(x)-\frac{6a^2x^2+3ax+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4,x, algorithm="maxima")
```

```
[Out] -2*a^3*log(a*x - 1) + 2*a^3*log(x) - 1/3*(6*a^2*x^2 + 3*a*x + 1)/x^3
```

Fricas [A] time = 1.69405, size = 105, normalized size = 2.56

$$-\frac{6a^3x^3\log(ax-1)-6a^3x^3\log(x)+6a^2x^2+3ax+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^3*x^3*log(a*x - 1) - 6*a^3*x^3*log(x) + 6*a^2*x^2 + 3*a*x + 1)/x^
3
```

Sympy [A] time = 0.485669, size = 36, normalized size = 0.88

$$-2a^3\left(-\log(x)+\log\left(x-\frac{1}{a}\right)\right)-\frac{6a^2x^2+3ax+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**4,x)
```

```
[Out] -2*a**3*(-log(x) + log(x - 1/a)) - (6*a**2*x**2 + 3*a*x + 1)/(3*x**3)
```

Giac [A] time = 1.14824, size = 54, normalized size = 1.32

$$-2\,a^3\log\left(|ax-1|\right)+2\,a^3\log\left(|x|\right)-\frac{6\,a^2x^2+3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4,x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(a*x - 1)) + 2*a^3*log(abs(x)) - 1/3*(6*a^2*x^2 + 3*a*x + 1)/
x^3
```

3.19 $\int e^{3\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=92

$$\frac{(ax+1)^3}{a^3\sqrt{1-a^2x^2}}+\frac{(ax+3)^2\sqrt{1-a^2x^2}}{3a^3}+\frac{(3ax+28)\sqrt{1-a^2x^2}}{6a^3}-\frac{11\sin^{-1}(ax)}{2a^3}$$

```
[Out] (1 + a*x)^3/(a^3*Sqrt[1 - a^2*x^2]) + ((3 + a*x)^2*Sqrt[1 - a^2*x^2])/(3*a^
3) + ((28 + 3*a*x)*Sqrt[1 - a^2*x^2])/(6*a^3) - (11*ArcSin[a*x])/(2*a^3)
```

Rubi [A] time = 0.649267, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6124, 1633, 1593, 12, 852, 1635, 1654, 780, 216}

$$\frac{(ax+1)^3}{a^3\sqrt{1-a^2x^2}}+\frac{(ax+3)^2\sqrt{1-a^2x^2}}{3a^3}+\frac{(3ax+28)\sqrt{1-a^2x^2}}{6a^3}-\frac{11\sin^{-1}(ax)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^2,x]
```

```
[Out] (1 + a*x)^3/(a^3*Sqrt[1 - a^2*x^2]) + ((3 + a*x)^2*Sqrt[1 - a^2*x^2])/(3*a^
3) + ((28 + 3*a*x)*Sqrt[1 - a^2*x^2])/(6*a^3) - (11*ArcSin[a*x])/(2*a^3)
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1+ax)^2}{(1-ax)\sqrt{1-a^2x^2}}\,dx \\
&= -\left(a\int \frac{\sqrt{1-a^2x^2}\left(-\frac{x^2}{a}-x^3\right)}{(1-ax)^2}\,dx\right) \\
&= -\left(a\int \frac{\left(-\frac{1}{a}-x\right)x^2\sqrt{1-a^2x^2}}{(1-ax)^2}\,dx\right) \\
&= a^2\int \frac{x^2\left(1-a^2x^2\right)^{3/2}}{a^2(1-ax)^3}\,dx \\
&= \int \frac{x^2\left(1-a^2x^2\right)^{3/2}}{(1-ax)^3}\,dx \\
&= \int \frac{x^2(1+ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx \\
&= \frac{(1+ax)^3}{a^3\sqrt{1-a^2x^2}} - \int \frac{\left(\frac{3}{a^2}+\frac{x}{a}\right)(1+ax)^2}{\sqrt{1-a^2x^2}}\,dx \\
&= \frac{(1+ax)^3}{a^3\sqrt{1-a^2x^2}} + \frac{(3+ax)^2\sqrt{1-a^2x^2}}{3a^3} + \frac{1}{3}\int \frac{\left(\frac{3}{a^2}+\frac{x}{a}\right)(-5-3ax)}{\sqrt{1-a^2x^2}}\,dx \\
&= \frac{(1+ax)^3}{a^3\sqrt{1-a^2x^2}} + \frac{(3+ax)^2\sqrt{1-a^2x^2}}{3a^3} + \frac{(28+3ax)\sqrt{1-a^2x^2}}{6a^3} - \frac{11\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^2} \\
&= \frac{(1+ax)^3}{a^3\sqrt{1-a^2x^2}} + \frac{(3+ax)^2\sqrt{1-a^2x^2}}{3a^3} + \frac{(28+3ax)\sqrt{1-a^2x^2}}{6a^3} - \frac{11\sin^{-1}(ax)}{2a^3}
\end{aligned}
$$

Mathematica [A] time = 0.0552105, size = 58, normalized size = 0.63

$$
\frac{\frac{\sqrt{1-a^2x^2}\left(2a^3x^3+7a^2x^2+19ax-52\right)}{ax-1} - 33\sin^{-1}(ax)}{6a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^2,x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(-52 + 19*a*x + 7*a^2*x^2 + 2*a^3*x^3))/(-1 + a*x) - 33
*ArcSin[a*x])/(6*a^3)
```

Maple [A] time = 0.047, size = 122, normalized size = 1.3

$$
-\frac{x^4a}{3}\frac{1}{\sqrt{-a^2x^2+1}} - \frac{13x^2}{3a}\frac{1}{\sqrt{-a^2x^2+1}} + \frac{26}{3a^3}\frac{1}{\sqrt{-a^2x^2+1}} - \frac{3x^3}{2}\frac{1}{\sqrt{-a^2x^2+1}} + \frac{11x}{2a^2}\frac{1}{\sqrt{-a^2x^2+1}} - \frac{11}{2a^2}\arctan\left(x\sqrt{a^2}\frac{}{\sqrt{-}}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2,x)
```

```
[Out] -1/3*x^4*a/(-a^2*x^2+1)^(1/2)-13/3/a*x^2/(-a^2*x^2+1)^(1/2)+26/3/a^3/(-a^2*
x^2+1)^(1/2)-3/2*x^3/(-a^2*x^2+1)^(1/2)+11/2*x/a^2/(-a^2*x^2+1)^(1/2)-11/2/
```

```
a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.44558, size = 151, normalized size = 1.64

$$-\frac{ax^4}{3\sqrt{-a^2x^2+1}}-\frac{3x^3}{2\sqrt{-a^2x^2+1}}-\frac{13x^2}{3\sqrt{-a^2x^2+1}a}+\frac{11x}{2\sqrt{-a^2x^2+1}a^2}-\frac{11\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}a^2}+\frac{26}{3\sqrt{-a^2x^2+1}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/3*a*x^4/sqrt(-a^2*x^2 + 1) - 3/2*x^3/sqrt(-a^2*x^2 + 1) - 13/3*x^2/(sqrt
(-a^2*x^2 + 1)*a) + 11/2*x/(sqrt(-a^2*x^2 + 1)*a^2) - 11/2*arcsin(a^2*x/sqr
t(a^2))/(sqrt(a^2)*a^2) + 26/3/(sqrt(-a^2*x^2 + 1)*a^3)
```

Fricas [A] time = 1.79943, size = 197, normalized size = 2.14

$$\frac{52\,ax+66\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^3x^3+7\,a^2x^2+19\,ax-52\right)\sqrt{-a^2x^2+1}-52}{6\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(52*a*x + 66*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^3*
x^3 + 7*a^2*x^2 + 19*a*x - 52)*sqrt(-a^2*x^2 + 1) - 52)/(a^4*x - a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\,(ax+1)^3}{\left(-(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2,x)
```

```
[Out] Integral(x**2*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A] time = 1.1822, size = 117, normalized size = 1.27

$$\frac{1}{6}\sqrt{-a^2x^2+1}\left(x\left(\frac{2x}{a}+\frac{9}{a^2}\right)+\frac{28}{a^3}\right)-\frac{11\arcsin(ax)\,\mathrm{sgn}(a)}{2\,a^2|a|}+\frac{8}{a^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(-a^2*x^2 + 1)*(x*(2*x/a + 9/a^2) + 28/a^3) - 11/2*arcsin(a*x)*sgn(
a)/(a^2*abs(a)) + 8/(a^2*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(
a))
```

3.20 $\int e^{3\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=88

$$\frac{(1-a^2x^2)^{5/2}}{a^2(1-ax)^3}+\frac{3(1-a^2x^2)^{3/2}}{2a^2(1-ax)}+\frac{9\sqrt{1-a^2x^2}}{2a^2}-\frac{9\sin^{-1}(ax)}{2a^2}$$

```
[Out] (9*Sqrt[1 - a^2*x^2])/(2*a^2) + (3*(1 - a^2*x^2)^(3/2))/(2*a^2*(1 - a*x)) +
 (1 - a^2*x^2)^(5/2)/(a^2*(1 - a*x)^3) - (9*ArcSin[a*x])/(2*a^2)
```

Rubi [A] time = 0.382688, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {6124, 1633, 1593, 12, 793, 665, 216}

$$\frac{(1-a^2x^2)^{5/2}}{a^2(1-ax)^3}+\frac{3(1-a^2x^2)^{3/2}}{2a^2(1-ax)}+\frac{9\sqrt{1-a^2x^2}}{2a^2}-\frac{9\sin^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x,x]
```

```
[Out] (9*Sqrt[1 - a^2*x^2])/(2*a^2) + (3*(1 - a^2*x^2)^(3/2))/(2*a^2*(1 - a*x)) +
 (1 - a^2*x^2)^(5/2)/(a^2*(1 - a*x)^3) - (9*ArcSin[a*x])/(2*a^2)
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
```

```
+ 1, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{3\tanh^{-1}(ax)}x\,dx &= \int \frac{x(1+ax)^2}{(1-ax)\sqrt{1-a^2x^2}}\,dx\\
&= -\left(a\int \frac{\left(-\frac{x}{a}-x^2\right)\sqrt{1-a^2x^2}}{(1-ax)^2}\,dx\right)\\
&= -\left(a\int \frac{\left(-\frac{1}{a}-x\right)x\sqrt{1-a^2x^2}}{(1-ax)^2}\,dx\right)\\
&= a^2\int \frac{x\left(1-a^2x^2\right)^{3/2}}{a^2(1-ax)^3}\,dx\\
&= \int \frac{x\left(1-a^2x^2\right)^{3/2}}{(1-ax)^3}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1-ax)^3}-\frac{3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(1-ax)^2}\,dx}{a}\\
&= \frac{3\left(1-a^2x^2\right)^{3/2}}{2a^2(1-ax)}+\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1-ax)^3}-\frac{9\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx}{2a}\\
&= \frac{9\sqrt{1-a^2x^2}}{2a^2}+\frac{3\left(1-a^2x^2\right)^{3/2}}{2a^2(1-ax)}+\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1-ax)^3}-\frac{9\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a}\\
&= \frac{9\sqrt{1-a^2x^2}}{2a^2}+\frac{3\left(1-a^2x^2\right)^{3/2}}{2a^2(1-ax)}+\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1-ax)^3}-\frac{9\sin^{-1}(ax)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0362377, size = 53, normalized size = 0.6

$$\sqrt{1-a^2x^2}\left(-\frac{4}{a^2(ax-1)}+\frac{3}{a^2}+\frac{x}{2a}\right)-\frac{9\sin^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x,x]
```

```
[Out] Sqrt[1 - a^2*x^2]*(3/a^2 + x/(2*a) - 4/(a^2*(-1 + a*x))) - (9*ArcSin[a*x])/
(2*a^2)
```

Maple [A] time = 0.04, size = 102, normalized size = 1.2

$$-\frac{x^3a}{2}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{9x}{2a}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{9}{2a}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-3\frac{x^2}{\sqrt{-a^2x^2+1}}+7\frac{1}{a^2\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x,x)
```

```
[Out] -1/2*a*x^3/(-a^2*x^2+1)^(1/2)+9/2/a*x/(-a^2*x^2+1)^(1/2)-9/2/a/(a^2)^(1/2)*
arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-3*x^2/(-a^2*x^2+1)^(1/2)+7/a^2/(-a
^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.4337, size = 124, normalized size = 1.41

$$-\frac{ax^3}{2\sqrt{-a^2x^2+1}}-\frac{3x^2}{\sqrt{-a^2x^2+1}}+\frac{9x}{2\sqrt{-a^2x^2+1}a}-\frac{9\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}a}+\frac{7}{\sqrt{-a^2x^2+1}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] -1/2*a*x^3/sqrt(-a^2*x^2 + 1) - 3*x^2/sqrt(-a^2*x^2 + 1) + 9/2*x/(sqrt(-a^2
*x^2 + 1)*a) - 9/2*arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a) + 7/(sqrt(-a^2*x^2
 + 1)*a^2)
```

Fricas [A] time = 1.69398, size = 177, normalized size = 2.01

$$\frac{14\,ax+18\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(a^2x^2+5\,ax-14\right)\sqrt{-a^2x^2+1}-14}{2\left(a^3x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(14*a*x + 18*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a^2*x^
2 + 5*a*x - 14)*sqrt(-a^2*x^2 + 1) - 14)/(a^3*x - a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\,(ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x,x)
```

```
[Out] Integral(x*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A] time = 1.20726, size = 105, normalized size = 1.19

$$\frac{1}{2}\sqrt{-a^2x^2+1}\left(\frac{x}{a}+\frac{6}{a^2}\right)-\frac{9\arcsin(ax)\operatorname{sgn}(a)}{2a|a|}+\frac{8}{a\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*(x/a + 6/a^2) - 9/2*arcsin(a*x)*sgn(a)/(a*abs(a)) +
8/(a*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

3.21 $\int e^{3\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=55

$$\frac{2(ax+1)^2}{a\sqrt{1-a^2x^2}}+\frac{3\sqrt{1-a^2x^2}}{a}-\frac{3\sin^{-1}(ax)}{a}$$

```
[Out] (2*(1 + a*x)^2)/(a*Sqrt[1 - a^2*x^2]) + (3*Sqrt[1 - a^2*x^2])/a - (3*ArcSin
[a*x])/a
```

Rubi [A] time = 0.0491881, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6123, 853, 669, 641, 216}

$$\frac{2(ax+1)^2}{a\sqrt{1-a^2x^2}}+\frac{3\sqrt{1-a^2x^2}}{a}-\frac{3\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x]),x]
```

```
[Out] (2*(1 + a*x)^2)/(a*Sqrt[1 - a^2*x^2]) + (3*Sqrt[1 - a^2*x^2])/a - (3*ArcSin
[a*x])/a
```

Rule 6123

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 + a*x)^((n + 1)/2)/(
(1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2]), x] /; FreeQ[a, x] && IntegerQ[(n
- 1)/2]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\,dx &= \int \frac{(1+ax)^2}{(1-ax)\sqrt{1-a^2x^2}}\,dx\\
&= \int \frac{(1+ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx\\
&= \frac{2(1+ax)^2}{a\sqrt{1-a^2x^2}} - 3\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{2(1+ax)^2}{a\sqrt{1-a^2x^2}} + \frac{3\sqrt{1-a^2x^2}}{a} - 3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{2(1+ax)^2}{a\sqrt{1-a^2x^2}} + \frac{3\sqrt{1-a^2x^2}}{a} - \frac{3\sin^{-1}(ax)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0291828, size = 39, normalized size = 0.71

$$
\frac{\sqrt{1-a^2x^2}\left(1-\frac{4}{ax-1}\right)}{a} - \frac{3\sin^{-1}(ax)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1 - 4/(-1 + a*x)))/a - (3*ArcSin[a*x])/a
```

Maple [A] time = 0.037, size = 79, normalized size = 1.4

$$
-ax^2\frac{1}{\sqrt{-a^2x^2+1}} + 5\frac{1}{a\sqrt{-a^2x^2+1}} + 4\frac{x}{\sqrt{-a^2x^2+1}} - 3\frac{1}{\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -a*x^2/(-a^2*x^2+1)^(1/2)+5/a/(-a^2*x^2+1)^(1/2)+4*x/(-a^2*x^2+1)^(1/2)-3/(
a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.43676, size = 93, normalized size = 1.69

$$
-\frac{ax^2}{\sqrt{-a^2x^2+1}} + \frac{4x}{\sqrt{-a^2x^2+1}} - \frac{3\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}} + \frac{5}{\sqrt{-a^2x^2+1}a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*x^2/sqrt(-a^2*x^2 + 1) + 4*x/sqrt(-a^2*x^2 + 1) - 3*arcsin(a^2*x/sqrt(a^
2))/sqrt(a^2) + 5/(sqrt(-a^2*x^2 + 1)*a)
```

Fricas [A] time = 1.71195, size = 147, normalized size = 2.67

$$\frac{5\,ax + 6\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2+1}(ax-5) - 5}{a^2x - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (5*a*x + 6*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2
 + 1)*(a*x - 5) - 5)/(a^2*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A] time = 1.21633, size = 85, normalized size = 1.55

$$-\frac{3\arcsin(ax)\,\mathrm{sgn}(a)}{|a|} + \frac{\sqrt{-a^2x^2+1}}{a} + \frac{8}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -3*arcsin(a*x)*sgn(a)/abs(a) + sqrt(-a^2*x^2 + 1)/a + 8/(((sqrt(-a^2*x^2 +
1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

3.22 $\int \frac{e^{3\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=48

$$\frac{4\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\sin^{-1}(ax)$$

```
[Out] (4*Sqrt[1 - a^2*x^2])/(1 - a*x) - ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.786747, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6124, 6742, 216, 266, 63, 208, 651}

$$\frac{4\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/x,x]
```

```
[Out] (4*Sqrt[1 - a^2*x^2])/(1 - a*x) - ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{(1+ax)^2}{x(1-ax)\sqrt{1-a^2x^2}}\,dx\\
&= \int \left(-\frac{a}{\sqrt{1-a^2x^2}}+\frac{1}{x\sqrt{1-a^2x^2}}-\frac{4a}{(-1+ax)\sqrt{1-a^2x^2}}\right)dx\\
&= -\left(a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\right)-(4a)\int \frac{1}{(-1+ax)\sqrt{1-a^2x^2}}\,dx+\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= \frac{4\sqrt{1-a^2x^2}}{1-ax}-\sin^{-1}(ax)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= \frac{4\sqrt{1-a^2x^2}}{1-ax}-\sin^{-1}(ax)-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2}\\
&= \frac{4\sqrt{1-a^2x^2}}{1-ax}-\sin^{-1}(ax)-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0402435, size = 51, normalized size = 1.06

$$-\frac{4\sqrt{1-a^2x^2}}{ax-1}-\log\left(\sqrt{1-a^2x^2}+1\right)-\sin^{-1}(ax)+\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/x,x]
```

```
[Out] (-4*Sqrt[1 - a^2*x^2])/(-1 + a*x) - ArcSin[a*x] + Log[x] - Log[1 + Sqrt[1 -
 a^2*x^2]]
```

Maple [A] time = 0.037, size = 75, normalized size = 1.6

$$4\frac{ax}{\sqrt{-a^2x^2+1}}-a\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+4\frac{1}{\sqrt{-a^2x^2+1}}-\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x,x)
```

```
[Out] 4*a*x/(-a^2*x^2+1)^(1/2)-a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1
/2))+4/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.43973, size = 105, normalized size = 2.19

$$\frac{4\,ax}{\sqrt{-a^2x^2+1}}-\frac{a\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}+\frac{4}{\sqrt{-a^2x^2+1}}-\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 4*a*x/sqrt(-a^2*x^2 + 1) - a*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + 4/sqrt(-a^
2*x^2 + 1) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A] time = 1.85225, size = 193, normalized size = 4.02

$$\frac{4\,ax + 2\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + (ax-1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - 4\,\sqrt{-a^2x^2+1} - 4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (4*a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a*x - 1)*log
((sqrt(-a^2*x^2 + 1) - 1)/x) - 4*sqrt(-a^2*x^2 + 1) - 4)/(a*x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{x\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] Integral((a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [B] time = 1.1919, size = 117, normalized size = 2.44

$$-\frac{a\arcsin(ax)\,\mathrm{sgn}(a)}{|a|} - \frac{a\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|} + \frac{8\,a}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -a*arcsin(a*x)*sgn(a)/abs(a) - a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
 2*a)/(a^2*abs(x)))/abs(a) + 8*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x)
- 1)*abs(a))
```

3.23 $\int \frac{e^{3\tanh^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=63

$$\frac{4a\sqrt{1-a^2x^2}}{1-ax} - \frac{\sqrt{1-a^2x^2}}{x} - 3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -(Sqrt[1 - a^2*x^2]/x) + (4*a*Sqrt[1 - a^2*x^2])/(1 - a*x) - 3*a*ArcTanh[Sq
rt[1 - a^2*x^2]]
```

Rubi [A] time = 0.696511, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6124, 6742, 264, 266, 63, 208, 651}

$$\frac{4a\sqrt{1-a^2x^2}}{1-ax} - \frac{\sqrt{1-a^2x^2}}{x} - 3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/x^2,x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/x) + (4*a*Sqrt[1 - a^2*x^2])/(1 - a*x) - 3*a*ArcTanh[Sq
rt[1 - a^2*x^2]]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1+ax)^2}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^2\sqrt{1-a^2x^2}} + \frac{3a}{x\sqrt{1-a^2x^2}} - \frac{4a^2}{(-1+ax)\sqrt{1-a^2x^2}}\right) dx \\
&= (3a)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - \left(4a^2\right)\int \frac{1}{(-1+ax)\sqrt{1-a^2x^2}}\,dx + \int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{x} + \frac{4a\sqrt{1-a^2x^2}}{1-ax} + \frac{1}{2}(3a)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1-a^2x^2}}{x} + \frac{4a\sqrt{1-a^2x^2}}{1-ax} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a} \\
&= -\frac{\sqrt{1-a^2x^2}}{x} + \frac{4a\sqrt{1-a^2x^2}}{1-ax} - 3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0530598, size = 57, normalized size = 0.9

$$\sqrt{1-a^2x^2}\left(-\frac{4a}{ax-1}-\frac{1}{x}\right) - 3a\log\left(\sqrt{1-a^2x^2}+1\right) + 3a\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/x^2,x]
```

```
[Out] Sqrt[1 - a^2*x^2]*(-x^(-1) - (4*a)/(-1 + a*x)) + 3*a*Log[x] - 3*a*Log[1 + S
qrt[1 - a^2*x^2]]
```

Maple [A] time = 0.041, size = 82, normalized size = 1.3

$$a\frac{1}{\sqrt{-a^2x^2+1}} + 5\frac{a^2x}{\sqrt{-a^2x^2+1}} - \frac{1}{x}\frac{1}{\sqrt{-a^2x^2+1}} + 3a\left(\frac{1}{\sqrt{-a^2x^2+1}} - \text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^2,x)
```

```
[Out] a/(-a^2*x^2+1)^(1/2)+5*a^2*x/(-a^2*x^2+1)^(1/2)-1/x/(-a^2*x^2+1)^(1/2)+3*a*
(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2)))
```

Maxima [A] time = 0.953605, size = 108, normalized size = 1.71

$$\frac{5\,a^2x}{\sqrt{-a^2x^2+1}} - 3\,a\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{4\,a}{\sqrt{-a^2x^2+1}} - \frac{1}{\sqrt{-a^2x^2+1}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5*a^2*x/sqrt(-a^2*x^2 + 1) - 3*a*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)
) + 4*a/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*x)
```

Fricas [A] time = 1.85595, size = 159, normalized size = 2.52

$$\frac{4\,a^2x^2 - 4\,ax + 3\left(a^2x^2 - ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \sqrt{-a^2x^2+1}(5\,ax-1)}{ax^2 - x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] (4*a^2*x^2 - 4*a*x + 3*(a^2*x^2 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sq
rt(-a^2*x^2 + 1)*(5*a*x - 1))/(a*x^2 - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{x^2\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] Integral((a*x + 1)**3/(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [B] time = 1.18698, size = 203, normalized size = 3.22

$$-\frac{3\,a^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|} - \frac{\left(a^2 - \frac{17\left(\sqrt{-a^2x^2+1}|a|+a\right)}{x}\right)a^2x}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)|a|} - \frac{\sqrt{-a^2x^2+1}|a|+a}{2\,x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -3*a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a)
 - 1/2*(a^2 - 17*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/x)*a^2*x/((sqrt(-a^2*x^2 +
 1)*abs(a) + a)*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) - 1/2
*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(x*abs(a))
```

3.24 $\int \frac{e^{3\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=91

$$\frac{4a^2\sqrt{1-a^2x^2}}{1-ax}-\frac{3a\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -Sqrt[1 - a^2*x^2]/(2*x^2) - (3*a*Sqrt[1 - a^2*x^2])/x + (4*a^2*Sqrt[1 - a^
2*x^2])/(1 - a*x) - (9*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rubi [A] time = 0.741374, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6124, 6742, 266, 51, 63, 208, 264, 651}

$$\frac{4a^2\sqrt{1-a^2x^2}}{1-ax}-\frac{3a\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/x^3,x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*x^2) - (3*a*Sqrt[1 - a^2*x^2])/x + (4*a^2*Sqrt[1 - a^
2*x^2])/(1 - a*x) - (9*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1+ax)^2}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^3\sqrt{1-a^2x^2}} + \frac{3a}{x^2\sqrt{1-a^2x^2}} + \frac{4a^2}{x\sqrt{1-a^2x^2}} - \frac{4a^3}{(-1+ax)\sqrt{1-a^2x^2}}\right)dx \\
&= (3a)\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx + \left(4a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - \left(4a^3\right)\int \frac{1}{(-1+ax)\sqrt{1-a^2x^2}}\,dx + \int \frac{1}{x^3\sqrt{1-}} \\
&= -\frac{3a\sqrt{1-a^2x^2}}{x} + \frac{4a^2\sqrt{1-a^2x^2}}{1-ax} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}}\,dx, x, x^2\right) + \left(2a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2}}\right. \\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2} - \frac{3a\sqrt{1-a^2x^2}}{x} + \frac{4a^2\sqrt{1-a^2x^2}}{1-ax} - 4\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right) + \frac{1}{4}a^2\,\text{Subs} \\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2} - \frac{3a\sqrt{1-a^2x^2}}{x} + \frac{4a^2\sqrt{1-a^2x^2}}{1-ax} - 4a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx\right. \\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2} - \frac{3a\sqrt{1-a^2x^2}}{x} + \frac{4a^2\sqrt{1-a^2x^2}}{1-ax} - \frac{9}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0703125, size = 75, normalized size = 0.82

$$\sqrt{1-a^2x^2}\left(-\frac{4a^2}{ax-1} - \frac{3a}{x} - \frac{1}{2x^2}\right) - \frac{9}{2}a^2\log\left(\sqrt{1-a^2x^2}+1\right) + \frac{9}{2}a^2\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/x^3,x]
```

```
[Out] Sqrt[1 - a^2*x^2]*(-1/(2*x^2) - (3*a)/x - (4*a^2)/(-1 + a*x)) + (9*a^2*Log[
x])/2 - (9*a^2*Log[1 + Sqrt[1 - a^2*x^2]])/2
```

Maple [A] time = 0.043, size = 108, normalized size = 1.2

$$xa^{3}\frac{1}{\sqrt{-a^{2}x^{2}+1}}+3\,a\left(-\frac{1}{x\sqrt{-a^{2}x^{2}+1}}+2\,\frac{a^{2}x}{\sqrt{-a^{2}x^{2}+1}}\right)+\frac{9\,a^{2}}{2}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}-\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)\right)-\frac{1}{2\,x^{2}}\frac{1}{\sqrt{-a^{2}x^{2}+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^3,x)
```

```
[Out] a^3*x/(-a^2*x^2+1)^(1/2)+3*a*(-1/x/(-a^2*x^2+1)^(1/2)+2*a^2*x/(-a^2*x^2+1)^
(1/2))+9/2*a^2*(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2)))-1/2/x^2
/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 0.952971, size = 138, normalized size = 1.52

$$\frac{7\,a^{3}x}{\sqrt{-a^{2}x^{2}+1}}-\frac{9}{2}\,a^{2}\log\left(\frac{2\,\sqrt{-a^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)+\frac{9\,a^{2}}{2\,\sqrt{-a^{2}x^{2}+1}}-\frac{3\,a}{\sqrt{-a^{2}x^{2}+1}x}-\frac{1}{2\,\sqrt{-a^{2}x^{2}+1}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 7*a^3*x/sqrt(-a^2*x^2 + 1) - 9/2*a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/ab
s(x)) + 9/2*a^2/sqrt(-a^2*x^2 + 1) - 3*a/(sqrt(-a^2*x^2 + 1)*x) - 1/2/(sqrt
(-a^2*x^2 + 1)*x^2)
```

Fricas [A] time = 2.01856, size = 196, normalized size = 2.15

$$\frac{8\,a^{3}x^{3}-8\,a^{2}x^{2}+9\left(a^{3}x^{3}-a^{2}x^{2}\right)\log\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{x}\right)-\left(14\,a^{2}x^{2}-5\,ax-1\right)\sqrt{-a^{2}x^{2}+1}}{2\left(ax^{3}-x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(8*a^3*x^3 - 8*a^2*x^2 + 9*(a^3*x^3 - a^2*x^2)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) - (14*a^2*x^2 - 5*a*x - 1)*sqrt(-a^2*x^2 + 1))/(a*x^3 - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)^{3}}{x^{3}\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] Integral((a*x + 1)**3/(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [B] time = 1.21473, size = 288, normalized size = 3.16

$$-\frac{\left(a^3+\frac{11\left(\sqrt{-a^2x^2+1}|a|+a\right)a}{x}-\frac{76\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{ax^2}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}|a|+a\right)^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}-\frac{9\,a^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{\frac{12\left(\sqrt{-a^2x^2+1}|a|+a\right)a|a|}{x}+\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)}{ax^2}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(a^3 + 11*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 76*(sqrt(-a^2*x^2 + 1)
*abs(a) + a)^2/(a*x^2))*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*((sqrt(-
a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) - 9/2*a^3*log(1/2*abs(-2*sqrt
(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/8*(12*(sqrt(-a^2*x^2
+ 1)*abs(a) + a)*a*abs(a)/x + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)/(a*x
^2))/a^2
```

3.25 $\int \frac{e^{3\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=117

$$\frac{4a^3\sqrt{1-a^2x^2}}{1-ax}-\frac{14a^2\sqrt{1-a^2x^2}}{3x}-\frac{3a\sqrt{1-a^2x^2}}{2x^2}-\frac{\sqrt{1-a^2x^2}}{3x^3}-\frac{11}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -Sqrt[1 - a^2*x^2]/(3*x^3) - (3*a*Sqrt[1 - a^2*x^2])/(2*x^2) - (14*a^2*Sqrt
[1 - a^2*x^2])/(3*x) + (4*a^3*Sqrt[1 - a^2*x^2])/(1 - a*x) - (11*a^3*ArcTan
h[Sqrt[1 - a^2*x^2]])/2
```

Rubi [A] time = 0.737876, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6124, 6742, 271, 264, 266, 51, 63, 208, 651}

$$\frac{4a^3\sqrt{1-a^2x^2}}{1-ax}-\frac{14a^2\sqrt{1-a^2x^2}}{3x}-\frac{3a\sqrt{1-a^2x^2}}{2x^2}-\frac{\sqrt{1-a^2x^2}}{3x^3}-\frac{11}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/x^4,x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(3*x^3) - (3*a*Sqrt[1 - a^2*x^2])/(2*x^2) - (14*a^2*Sqrt
[1 - a^2*x^2])/(3*x) + (4*a^3*Sqrt[1 - a^2*x^2])/(1 - a*x) - (11*a^3*ArcTan
h[Sqrt[1 - a^2*x^2]])/2
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{x^4}\,dx &= \int \frac{(1+ax)^2}{x^4(1-ax)\sqrt{1-a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^4\sqrt{1-a^2x^2}} + \frac{3a}{x^3\sqrt{1-a^2x^2}} + \frac{4a^2}{x^2\sqrt{1-a^2x^2}} + \frac{4a^3}{x\sqrt{1-a^2x^2}} - \frac{4a^4}{(-1+ax)\sqrt{1-a^2x^2}}\right) dx \\
&= (3a)\int \frac{1}{x^3\sqrt{1-a^2x^2}}\,dx + \left(4a^2\right)\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx + \left(4a^3\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - \left(4a^4\right)\int \frac{}{(-1+ax}\\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{4a^2\sqrt{1-a^2x^2}}{x} + \frac{4a^3\sqrt{1-a^2x^2}}{1-ax} + \frac{1}{2}(3a)\,\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}}\,dx, x, x^2\right) + \frac{1}{3}\left(2a^2\right)\\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{3a\sqrt{1-a^2x^2}}{2x^2} - \frac{14a^2\sqrt{1-a^2x^2}}{3x} + \frac{4a^3\sqrt{1-a^2x^2}}{1-ax} - (4a)\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1}\right.\\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{3a\sqrt{1-a^2x^2}}{2x^2} - \frac{14a^2\sqrt{1-a^2x^2}}{3x} + \frac{4a^3\sqrt{1-a^2x^2}}{1-ax} - 4a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \frac{1}{2}(3a)\\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{3a\sqrt{1-a^2x^2}}{2x^2} - \frac{14a^2\sqrt{1-a^2x^2}}{3x} + \frac{4a^3\sqrt{1-a^2x^2}}{1-ax} - \frac{11}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0803808, size = 81, normalized size = 0.69

$$\frac{1}{6}\left(\frac{\sqrt{1-a^2x^2}\left(-52a^3x^3+19a^2x^2+7ax+2\right)}{x^3(ax-1)} - 33a^3\log\left(\sqrt{1-a^2x^2}+1\right) + 33a^3\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/x^4,x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(2 + 7*a*x + 19*a^2*x^2 - 52*a^3*x^3))/(x^3*(-1 + a*x))
 + 33*a^3*Log[x] - 33*a^3*Log[1 + Sqrt[1 - a^2*x^2]])/6
```

Maple [A] time = 0.045, size = 146, normalized size = 1.3

$$\frac{13a^2}{3}\left(-\frac{1}{x}\frac{1}{\sqrt{-a^2x^2+1}}+2\frac{a^2x}{\sqrt{-a^2x^2+1}}\right)+a^3\left(\frac{1}{\sqrt{-a^2x^2+1}}-\operatorname{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)+3a\left(-1/2\frac{1}{x^2\sqrt{-a^2x^2+1}}+3/2a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^4,x)
```

```
[Out] 13/3*a^2*(-1/x/(-a^2*x^2+1)^(1/2)+2*a^2*x/(-a^2*x^2+1)^(1/2))+a^3*(1/(-a^2*
x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2)))+3*a*(-1/2/x^2/(-a^2*x^2+1)^(1/2
)+3/2*a^2*(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2))))-1/3/x^3/(-a
^2*x^2+1)^(1/2)
```

Maxima [A] time = 0.955518, size = 165, normalized size = 1.41

$$\frac{26a^4x}{3\sqrt{-a^2x^2+1}}-\frac{11}{2}a^3\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{11a^3}{2\sqrt{-a^2x^2+1}}-\frac{13a^2}{3\sqrt{-a^2x^2+1}x}-\frac{3a}{2\sqrt{-a^2x^2+1}x^2}-\frac{1}{3\sqrt{-a^2x^2+1}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 26/3*a^4*x/sqrt(-a^2*x^2 + 1) - 11/2*a^3*log(2*sqrt(-a^2*x^2 + 1)/abs(x) +
2/abs(x)) + 11/2*a^3/sqrt(-a^2*x^2 + 1) - 13/3*a^2/(sqrt(-a^2*x^2 + 1)*x) -
 3/2*a/(sqrt(-a^2*x^2 + 1)*x^2) - 1/3/(sqrt(-a^2*x^2 + 1)*x^3)
```

Fricas [A] time = 1.88863, size = 217, normalized size = 1.85

$$\frac{24a^4x^4-24a^3x^3+33\left(a^4x^4-a^3x^3\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(52a^3x^3-19a^2x^2-7ax-2\right)\sqrt{-a^2x^2+1}}{6\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(24*a^4*x^4 - 24*a^3*x^3 + 33*(a^4*x^4 - a^3*x^3)*log((sqrt(-a^2*x^2 +
1) - 1)/x) - (52*a^3*x^3 - 19*a^2*x^2 - 7*a*x - 2)*sqrt(-a^2*x^2 + 1))/(a*x
^4 - x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)^3}{x^4\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] Integral((a*x + 1)**3/(x**4*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [B] time = 1.23796, size = 358, normalized size = 3.06

$$-\frac{\left(a^4+\frac{8\left(\sqrt{-a^2x^2+1}|a|+a\right)a^2}{x}+\frac{48\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{x^2}-\frac{249\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^2x^3}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}-\frac{11\,a^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{57\left(\sqrt{-a^2x^2+1}|a|+\cdots\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/24*(a^4 + 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2/x + 48*(sqrt(-a^2*x^2 +
1)*abs(a) + a)^2/x^2 - 249*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^2*x^3))*a^6
*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a
^2*x) - 1)*abs(a)) - 11/2*a^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*
a)/(a^2*abs(x)))/abs(a) - 1/24*(57*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4/x +
9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2/x^2 + (sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3/x^3)/(a^2*abs(a))
```

3.26 $\int e^{4\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=57

$$\frac{4x^2}{a^2}+\frac{12x}{a^3}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}+\frac{4x^3}{3a}+\frac{x^4}{4}$$

```
[Out] (12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16*
Log[1 - a*x])/a^4
```

Rubi [A] time = 0.0495242, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 88}

$$\frac{4x^2}{a^2}+\frac{12x}{a^3}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}+\frac{4x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*x^3,x]
```

```
[Out] (12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16*
Log[1 - a*x])/a^4
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}x^3\,dx&=\int\frac{x^3(1+ax)^2}{(1-ax)^2}\,dx\\&=\int\left(\frac{12}{a^3}+\frac{8x}{a^2}+\frac{4x^2}{a}+x^3+\frac{4}{a^3(-1+ax)^2}+\frac{16}{a^3(-1+ax)}\right)dx\\&=\frac{12x}{a^3}+\frac{4x^2}{a^2}+\frac{4x^3}{3a}+\frac{x^4}{4}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.0450642, size = 57, normalized size = 1.

$$\frac{4x^2}{a^2}+\frac{12x}{a^3}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}+\frac{4x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*x^3,x]
```

```
[Out] (12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16*
Log[1 - a*x])/a^4
```

Maple [A] time = 0.039, size = 52, normalized size = 0.9

$$\frac{x^4}{4}+\frac{4\,x^3}{3\,a}+4\,\frac{x^2}{a^2}+12\,\frac{x}{a^3}-4\,\frac{1}{a^4\,(ax-1)}+16\,\frac{\ln(ax-1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*x^3,x)
```

```
[Out] 1/4*x^4+4/3*x^3/a+4*x^2/a^2+12*x/a^3-4/a^4/(a*x-1)+16/a^4*ln(a*x-1)
```

Maxima [A] time = 0.94969, size = 78, normalized size = 1.37

$$-\frac{4}{a^5x-a^4}+\frac{3\,a^3x^4+16\,a^2x^3+48\,ax^2+144\,x}{12\,a^3}+\frac{16\,\log(ax-1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^3,x, algorithm="maxima")
```

```
[Out] -4/(a^5*x - a^4) + 1/12*(3*a^3*x^4 + 16*a^2*x^3 + 48*a*x^2 + 144*x)/a^3 + 1
6*log(a*x - 1)/a^4
```

Fricas [A] time = 1.87873, size = 155, normalized size = 2.72

$$\frac{3\,a^5x^5+13\,a^4x^4+32\,a^3x^3+96\,a^2x^2-144\,ax+192\,(ax-1)\log(ax-1)-48}{12\left(a^5x-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^5*x^5 + 13*a^4*x^4 + 32*a^3*x^3 + 96*a^2*x^2 - 144*a*x + 192*(a*x
 - 1)*log(a*x - 1) - 48)/(a^5*x - a^4)
```

Sympy [A] time = 0.442445, size = 49, normalized size = 0.86

$$\frac{x^4}{4}-\frac{4}{a^5x-a^4}+\frac{4x^3}{3a}+\frac{4x^2}{a^2}+\frac{12x}{a^3}+\frac{16\log(ax-1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*x**3,x)
```

```
[Out] x**4/4 - 4/(a**5*x - a**4) + 4*x**3/(3*a) + 4*x**2/a**2 + 12*x/a**3 + 16*lo
g(a*x - 1)/a**4
```

Giac [A] time = 1.17888, size = 82, normalized size = 1.44

$$\frac{16 \log\left(|ax - 1|\right)}{a^4} - \frac{4}{(ax - 1)a^4} + \frac{3\,a^8x^4 + 16\,a^7x^3 + 48\,a^6x^2 + 144\,a^5x}{12\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^3,x, algorithm="giac")
```

```
[Out] 16*log(abs(a*x - 1))/a^4 - 4/((a*x - 1)*a^4) + 1/12*(3*a^8*x^4 + 16*a^7*x^3
 + 48*a^6*x^2 + 144*a^5*x)/a^8
```

3.27 $\int e^{4\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=47

$$\frac{8x}{a^2}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}+\frac{2x^2}{a}+\frac{x^3}{3}$$

```
[Out] (8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

Rubi [A] time = 0.0437176, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 88}

$$\frac{8x}{a^2}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}+\frac{2x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*x^2,x]
```

```
[Out] (8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1+ax)^2}{(1-ax)^2}\,dx\\ &= \int \left(\frac{8}{a^2}+\frac{4x}{a}+x^2+\frac{4}{a^2(-1+ax)^2}+\frac{12}{a^2(-1+ax)}\right)dx\\ &= \frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.0368859, size = 47, normalized size = 1.

$$\frac{8x}{a^2}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}+\frac{2x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*x^2,x]
```

```
[Out] (8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

Maple [A] time = 0.037, size = 44, normalized size = 0.9

$$\frac{x^3}{3}+2\,\frac{x^2}{a}+8\,\frac{x}{a^2}-4\,\frac{1}{a^3\,(ax-1)}+12\,\frac{\ln(ax-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*x^2,x)
```

```
[Out] 1/3*x^3+2*x^2/a+8*x/a^2-4/a^3/(a*x-1)+12/a^3*ln(a*x-1)
```

Maxima [A] time = 0.946584, size = 66, normalized size = 1.4

$$-\frac{4}{a^4x-a^3}+\frac{a^2x^3+6\,ax^2+24\,x}{3\,a^2}+\frac{12\,\log(ax-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^2,x, algorithm="maxima")
```

```
[Out] -4/(a^4*x - a^3) + 1/3*(a^2*x^3 + 6*a*x^2 + 24*x)/a^2 + 12*log(a*x - 1)/a^3
```

Fricas [A] time = 1.81206, size = 130, normalized size = 2.77

$$\frac{a^4x^4+5\,a^3x^3+18\,a^2x^2-24\,ax+36\,(ax-1)\log(ax-1)-12}{3\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^4*x^4 + 5*a^3*x^3 + 18*a^2*x^2 - 24*a*x + 36*(a*x - 1)*log(a*x - 1)
- 12)/(a^4*x - a^3)
```

Sympy [A] time = 0.516896, size = 39, normalized size = 0.83

$$\frac{x^3}{3}-\frac{4}{a^4x-a^3}+\frac{2x^2}{a}+\frac{8x}{a^2}+\frac{12\log(ax-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*x**2,x)
```

```
[Out] x**3/3 - 4/(a**4*x - a**3) + 2*x**2/a + 8*x/a**2 + 12*log(a*x - 1)/a**3
```

Giac [A] time = 1.1634, size = 70, normalized size = 1.49

$$\frac{12\,\log\left(|ax-1|\right)}{a^3}-\frac{4}{(ax-1)a^3}+\frac{a^6x^3+6\,a^5x^2+24\,a^4x}{3\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^2,x, algorithm="giac")
```

```
[Out] 12*log(abs(a*x - 1))/a^3 - 4/((a*x - 1)*a^3) + 1/3*(a^6*x^3 + 6*a^5*x^2 + 2
4*a^4*x)/a^6
```

3.28 $\int e^{4\tanh^{-1}(ax)} x\,dx$

Optimal. Leaf size=39

$$\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}+\frac{4x}{a}+\frac{x^2}{2}$$

```
[Out] (4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

Rubi [A] time = 0.0285469, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6126, 77}

$$\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}+\frac{4x}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*x,x]
```

```
[Out] (4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int e^{4\tanh^{-1}(ax)} x\,dx &= \int \frac{x(1+ax)^2}{(1-ax)^2}\,dx \\
&= \int \left(\frac{4}{a}+x+\frac{4}{a(-1+ax)^2}+\frac{8}{a(-1+ax)}\right)dx \\
&= \frac{4x}{a}+\frac{x^2}{2}+\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.0291175, size = 39, normalized size = 1.

$$\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}+\frac{4x}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*x,x]
```

```
[Out] (4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

Maple [A] time = 0.036, size = 36, normalized size = 0.9

$$\frac{x^2}{2} + 4\,\frac{x}{a} - 4\,\frac{1}{a^2\,(ax-1)} + 8\,\frac{\ln(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*x,x)
```

```
[Out] 1/2*x^2+4*x/a-4/a^2/(a*x-1)+8/a^2*ln(a*x-1)
```

Maxima [A] time = 0.967439, size = 55, normalized size = 1.41

$$\frac{ax^2 + 8\,x}{2\,a} - \frac{4}{a^3x - a^2} + \frac{8\,\log(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x,x, algorithm="maxima")
```

```
[Out] 1/2*(a*x^2 + 8*x)/a - 4/(a^3*x - a^2) + 8*log(a*x - 1)/a^2
```

Fricas [A] time = 1.83649, size = 109, normalized size = 2.79

$$\frac{a^3x^3 + 7\,a^2x^2 - 8\,ax + 16\,(ax-1)\log(ax-1) - 8}{2\left(a^3x - a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*x^3 + 7*a^2*x^2 - 8*a*x + 16*(a*x - 1)*log(a*x - 1) - 8)/(a^3*x -
a^2)
```

Sympy [A] time = 0.389765, size = 31, normalized size = 0.79

$$\frac{x^2}{2} - \frac{4}{a^3x - a^2} + \frac{4x}{a} + \frac{8\log(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*x,x)
```

```
[Out] x**2/2 - 4/(a**3*x - a**2) + 4*x/a + 8*log(a*x - 1)/a**2
```

Giac [A] time = 1.17703, size = 59, normalized size = 1.51

$$\frac{8 \log(|ax-1|)}{a^2} + \frac{a^4x^2 + 8a^3x}{2a^4} - \frac{4}{(ax-1)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x,x, algorithm="giac")
```

```
[Out] 8*log(abs(a*x - 1))/a^2 + 1/2*(a^4*x^2 + 8*a^3*x)/a^4 - 4/((a*x - 1)*a^2)
```

3.29 $\int e^{4\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{4}{a(1-ax)}+\frac{4\log(1-ax)}{a}+x$$

```
[Out] x + 4/(a*(1 - a*x)) + (4*Log[1 - a*x])/a
```

Rubi [A] time = 0.0143055, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6125, 43}

$$\frac{4}{a(1-ax)}+\frac{4\log(1-ax)}{a}+x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x]),x]
```

```
[Out] x + 4/(a*(1 - a*x)) + (4*Log[1 - a*x])/a
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}\,dx &= \int \frac{(1+ax)^2}{(1-ax)^2}\,dx\\ &= \int \left(1+\frac{4}{(-1+ax)^2}+\frac{4}{-1+ax}\right)dx\\ &= x+\frac{4}{a(1-ax)}+\frac{4\log(1-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.016691, size = 26, normalized size = 0.96

$$-\frac{4}{a(ax-1)}+\frac{4\log(1-ax)}{a}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x]),x]
```

```
[Out] x - 4/(a*(-1 + a*x)) + (4*Log[1 - a*x])/a
```

Maple [A] time = 0.032, size = 26, normalized size = 1.

$$x-4\frac{1}{a\left(ax-1\right)}+4\frac{\ln\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2,x)
```

```
[Out] x-4/a/(a*x-1)+4/a*ln(a*x-1)
```

Maxima [A] time = 0.941763, size = 35, normalized size = 1.3

$$x+\frac{4\log\left(ax-1\right)}{a}-\frac{4}{a^2x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] x + 4*log(a*x - 1)/a - 4/(a^2*x - a)
```

Fricas [A] time = 1.80605, size = 81, normalized size = 3.

$$\frac{a^2x^2-ax+4\left(ax-1\right)\log\left(ax-1\right)-4}{a^2x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] (a^2*x^2 - a*x + 4*(a*x - 1)*log(a*x - 1) - 4)/(a^2*x - a)
```

Sympy [A] time = 0.492423, size = 19, normalized size = 0.7

$$x-\frac{4}{a^2x-a}+\frac{4\log\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2,x)
```

```
[Out] x - 4/(a**2*x - a) + 4*log(a*x - 1)/a
```

Giac [A] time = 1.17458, size = 35, normalized size = 1.3

$$x+\frac{4\log\left(|ax-1|\right)}{a}-\frac{4}{(ax-1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] x + 4*log(abs(a*x - 1))/a - 4/((a*x - 1)*a)
```

3.30 $\int \frac{e^{4\tanh^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=13

$$\frac{4}{1-ax}+\log(x)$$

```
[Out] 4/(1 - a*x) + Log[x]
```

Rubi [A] time = 0.0251725, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 88}

$$\frac{4}{1-ax}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/x,x]
```

```
[Out] 4/(1 - a*x) + Log[x]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\tanh^{-1}(ax)}}{x} \, dx &= \int \frac{(1+ax)^2}{x(1-ax)^2} \, dx \\
&= \int \left(\frac{1}{x}+\frac{4a}{(-1+ax)^2}\right) dx \\
&= \frac{4}{1-ax}+\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0091648, size = 13, normalized size = 1.

$$\frac{4}{1-ax}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/x,x]
```

```
[Out] 4/(1 - a*x) + Log[x]
```

Maple [A] time = 0.033, size = 13, normalized size = 1.

$$\ln(x) - 4(ax-1)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/x,x)
```

```
[Out] ln(x)-4/(a*x-1)
```

Maxima [A] time = 0.983844, size = 16, normalized size = 1.23

$$-\frac{4}{ax-1} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x,x, algorithm="maxima")
```

```
[Out] -4/(a*x - 1) + log(x)
```

Fricas [A] time = 1.98258, size = 46, normalized size = 3.54

$$\frac{(ax-1)\log(x) - 4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x,x, algorithm="fricas")
```

```
[Out] ((a*x - 1)*log(x) - 4)/(a*x - 1)
```

Sympy [A] time = 0.47497, size = 8, normalized size = 0.62

$$\log(x) - \frac{4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/x,x)
```

```
[Out] log(x) - 4/(a*x - 1)
```

Giac [A] time = 1.24653, size = 18, normalized size = 1.38

$$-\frac{4}{ax-1} + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x,x, algorithm="giac")
```

```
[Out] -4/(a*x - 1) + log(abs(x))
```

3.31 $\int \frac{e^{4\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=32

$$\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)-\frac{1}{x}$$

```
[Out] -x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

Rubi [A] time = 0.0331429, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 88}

$$\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/x^2,x]
```

```
[Out] -x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1+ax)^2}{x^2(1-ax)^2}\,dx\\ &= \int \left(\frac{1}{x^2}+\frac{4a}{x}+\frac{4a^2}{(-1+ax)^2}-\frac{4a^2}{-1+ax}\right)dx\\ &= -\frac{1}{x}+\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.0220597, size = 32, normalized size = 1.

$$\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/x^2,x]
```

```
[Out] -x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

Maple [A] time = 0.038, size = 31, normalized size = 1.

$$-x^{-1}+4\,a\ln(x)-4\,\frac{a}{ax-1}-4\,a\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/x^2,x)
```

```
[Out] -1/x+4*a*ln(x)-4*a/(a*x-1)-4*a*ln(a*x-1)
```

Maxima [A] time = 0.948432, size = 46, normalized size = 1.44

$$-4\,a\log(ax-1)+4\,a\log(x)-\frac{5\,ax-1}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^2,x, algorithm="maxima")
```

```
[Out] -4*a*log(a*x - 1) + 4*a*log(x) - (5*a*x - 1)/(a*x^2 - x)
```

Fricas [A] time = 2.03712, size = 116, normalized size = 3.62

$$-\frac{5\,ax+4\left(a^2x^2-ax\right)\log(ax-1)-4\left(a^2x^2-ax\right)\log(x)-1}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^2,x, algorithm="fricas")
```

```
[Out] -(5*a*x + 4*(a^2*x^2 - a*x)*log(a*x - 1) - 4*(a^2*x^2 - a*x)*log(x) - 1)/(a
*x^2 - x)
```

Sympy [A] time = 0.60999, size = 26, normalized size = 0.81

$$4a\left(\log(x)-\log\left(x-\frac{1}{a}\right)\right)-\frac{5ax-1}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/x**2,x)
```

```
[Out] 4*a*(log(x) - log(x - 1/a)) - (5*a*x - 1)/(a*x**2 - x)
```

Giac [A] time = 1.1761, size = 49, normalized size = 1.53

$$-4 a \log(|ax-1|) + 4 a \log(|x|) - \frac{5 ax - 1}{ax^2 - x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^2,x, algorithm="giac")
```

```
[Out] -4*a*log(abs(a*x - 1)) + 4*a*log(abs(x)) - (5*a*x - 1)/(a*x^2 - x)
```

3.32 $\int \frac{e^{4\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=46

$$\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)-\frac{4a}{x}-\frac{1}{2x^2}$$

```
[Out] -1/(2*x^2) - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*x
]
```

Rubi [A] time = 0.0371758, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 88}

$$\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)-\frac{4a}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/x^3,x]
```

```
[Out] -1/(2*x^2) - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*x
]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\tanh^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1+ax)^2}{x^3(1-ax)^2}\,dx\\ &= \int \left(\frac{1}{x^3}+\frac{4a}{x^2}+\frac{8a^2}{x}+\frac{4a^3}{(-1+ax)^2}-\frac{8a^3}{-1+ax}\right)dx\\ &= -\frac{1}{2x^2}-\frac{4a}{x}+\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.0276315, size = 46, normalized size = 1.

$$\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)-\frac{4a}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/x^3,x]
```

```
[Out] -1/(2*x^2) - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*x
]
```

Maple [A] time = 0.037, size = 43, normalized size = 0.9

$$-\frac{1}{2\,x^{2}}-4\,\frac{a}{x}+8\,a^{2}\ln\left(x\right)-4\,\frac{a^{2}}{ax-1}-8\,a^{2}\ln\left(ax-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/x^3,x)
```

```
[Out] -1/2/x^2-4*a/x+8*a^2*ln(x)-4*a^2/(a*x-1)-8*a^2*ln(a*x-1)
```

Maxima [A] time = 0.958038, size = 65, normalized size = 1.41

$$-8\,a^{2}\log\left(ax-1\right)+8\,a^{2}\log\left(x\right)-\frac{16\,a^{2}x^{2}-7\,ax-1}{2\left(ax^{3}-x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^3,x, algorithm="maxima")
```

```
[Out] -8*a^2*log(a*x - 1) + 8*a^2*log(x) - 1/2*(16*a^2*x^2 - 7*a*x - 1)/(a*x^3 -
x^2)
```

Fricas [A] time = 1.85459, size = 155, normalized size = 3.37

$$-\frac{16\,a^{2}x^{2}-7\,ax+16\left(a^{3}x^{3}-a^{2}x^{2}\right)\log\left(ax-1\right)-16\left(a^{3}x^{3}-a^{2}x^{2}\right)\log\left(x\right)-1}{2\left(ax^{3}-x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(16*a^2*x^2 - 7*a*x + 16*(a^3*x^3 - a^2*x^2)*log(a*x - 1) - 16*(a^3*x^
3 - a^2*x^2)*log(x) - 1)/(a*x^3 - x^2)
```

Sympy [A] time = 0.679465, size = 41, normalized size = 0.89

$$8a^{2}\left(\log\left(x\right)-\log\left(x-\frac{1}{a}\right)\right)-\frac{16a^{2}x^{2}-7ax-1}{2ax^{3}-2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/x**3,x)
```

```
[Out] 8*a**2*(log(x) - log(x - 1/a)) - (16*a**2*x**2 - 7*a*x - 1)/(2*a*x**3 - 2*x
**2)
```

Giac [A] time = 1.18404, size = 63, normalized size = 1.37

$$-8\,a^2\log\left(|ax-1|\right)+8\,a^2\log\left(|x|\right)-\frac{16\,a^2x^2-7\,ax-1}{2\,(ax-1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^3,x, algorithm="giac")
```

```
[Out] -8*a^2*log(abs(a*x - 1)) + 8*a^2*log(abs(x)) - 1/2*(16*a^2*x^2 - 7*a*x - 1)
/((a*x - 1)*x^2)
```

3.33 $\int \frac{e^{4\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=54

$$\frac{4a^3}{1-ax}-\frac{8a^2}{x}+12a^3\log(x)-12a^3\log(1-ax)-\frac{2a}{x^2}-\frac{1}{3x^3}$$

```
[Out] -1/(3*x^3) - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 12
*a^3*Log[1 - a*x]
```

Rubi [A] time = 0.0449314, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 88}

$$\frac{4a^3}{1-ax}-\frac{8a^2}{x}+12a^3\log(x)-12a^3\log(1-ax)-\frac{2a}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/(3*x^3) - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 12
*a^3*Log[1 - a*x]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\tanh^{-1}(ax)}}{x^4}\,dx &= \int \frac{(1+ax)^2}{x^4(1-ax)^2}\,dx\\ &= \int \left(\frac{1}{x^4}+\frac{4a}{x^3}+\frac{8a^2}{x^2}+\frac{12a^3}{x}+\frac{4a^4}{(-1+ax)^2}-\frac{12a^4}{-1+ax}\right)dx\\ &= -\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{8a^2}{x}+\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.0427096, size = 54, normalized size = 1.

$$\frac{4a^3}{1-ax}-\frac{8a^2}{x}+12a^3\log(x)-12a^3\log(1-ax)-\frac{2a}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/(3*x^3) - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 12
*a^3*Log[1 - a*x]
```

Maple [A] time = 0.037, size = 51, normalized size = 0.9

$$-\frac{1}{3\,x^3}-2\,\frac{a}{x^2}-8\,\frac{a^2}{x}+12\,a^3\ln(x)-4\,\frac{a^3}{ax-1}-12\,a^3\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/x^4,x)
```

```
[Out] -1/3/x^3-2*a/x^2-8*a^2/x+12*a^3*ln(x)-4*a^3/(a*x-1)-12*a^3*ln(a*x-1)
```

Maxima [A] time = 0.946822, size = 76, normalized size = 1.41

$$-12\,a^3\log(ax-1)+12\,a^3\log(x)-\frac{36\,a^3x^3-18\,a^2x^2-5\,ax-1}{3\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^4,x, algorithm="maxima")
```

```
[Out] -12*a^3*log(a*x - 1) + 12*a^3*log(x) - 1/3*(36*a^3*x^3 - 18*a^2*x^2 - 5*a*x
 - 1)/(a*x^4 - x^3)
```

Fricas [A] time = 1.91817, size = 173, normalized size = 3.2

$$-\frac{36\,a^3x^3-18\,a^2x^2-5\,ax+36\left(a^4x^4-a^3x^3\right)\log(ax-1)-36\left(a^4x^4-a^3x^3\right)\log(x)-1}{3\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(36*a^3*x^3 - 18*a^2*x^2 - 5*a*x + 36*(a^4*x^4 - a^3*x^3)*log(a*x - 1)
 - 36*(a^4*x^4 - a^3*x^3)*log(x) - 1)/(a*x^4 - x^3)
```

Sympy [A] time = 0.689214, size = 49, normalized size = 0.91

$$12a^3\left(\log(x)-\log\left(x-\frac{1}{a}\right)\right)-\frac{36a^3x^3-18a^2x^2-5ax-1}{3ax^4-3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/x**4,x)
```

```
[Out] 12*a**3*(log(x) - log(x - 1/a)) - (36*a**3*x**3 - 18*a**2*x**2 - 5*a*x - 1)
/(3*a*x**4 - 3*x**3)
```

Giac [A] time = 1.2206, size = 74, normalized size = 1.37

$$-12\,a^3\log\left(|ax-1|\right)+12\,a^3\log\left(|x|\right)-\frac{36\,a^3x^3-18\,a^2x^2-5\,ax-1}{3\,(ax-1)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^4,x, algorithm="giac")
```

```
[Out] -12*a^3*log(abs(a*x - 1)) + 12*a^3*log(abs(x)) - 1/3*(36*a^3*x^3 - 18*a^2*x
^2 - 5*a*x - 1)/((a*x - 1)*x^3)
```

3.34 $\int e^{-\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=87

$$\frac{x^3\sqrt{1-a^2x^2}}{4a}-\frac{x^2\sqrt{1-a^2x^2}}{3a^2}-\frac{(16-9ax)\sqrt{1-a^2x^2}}{24a^4}-\frac{3\sin^{-1}(ax)}{8a^4}$$

```
[Out] -(x^2*Sqrt[1 - a^2*x^2])/(3*a^2) + (x^3*Sqrt[1 - a^2*x^2])/(4*a) - ((16 - 9
*a*x)*Sqrt[1 - a^2*x^2])/(24*a^4) - (3*ArcSin[a*x])/(8*a^4)
```

Rubi [A] time = 0.076842, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6124, 833, 780, 216}

$$\frac{x^3\sqrt{1-a^2x^2}}{4a}-\frac{x^2\sqrt{1-a^2x^2}}{3a^2}-\frac{(16-9ax)\sqrt{1-a^2x^2}}{24a^4}-\frac{3\sin^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^ArcTanh[a*x],x]
```

```
[Out] -(x^2*Sqrt[1 - a^2*x^2])/(3*a^2) + (x^3*Sqrt[1 - a^2*x^2])/(4*a) - ((16 - 9
*a*x)*Sqrt[1 - a^2*x^2])/(24*a^4) - (3*ArcSin[a*x])/(8*a^4)
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}x^3\,dx = \int \frac{x^3(1-ax)}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{\int \frac{x^2(3a-4a^2x)}{\sqrt{1-a^2x^2}}\,dx}{4a^2}$$
$$= -\frac{x^2\sqrt{1-a^2x^2}}{3a^2} + \frac{x^3\sqrt{1-a^2x^2}}{4a} + \frac{\int \frac{x(8a^2-9a^3x)}{\sqrt{1-a^2x^2}}\,dx}{12a^4}$$
$$= -\frac{x^2\sqrt{1-a^2x^2}}{3a^2} + \frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{(16-9ax)\sqrt{1-a^2x^2}}{24a^4} - \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{8a^3}$$
$$= -\frac{x^2\sqrt{1-a^2x^2}}{3a^2} + \frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{(16-9ax)\sqrt{1-a^2x^2}}{24a^4} - \frac{3\sin^{-1}(ax)}{8a^4}$$

Mathematica [A] time = 0.0413456, size = 51, normalized size = 0.59

$$\frac{\sqrt{1-a^2x^2}\left(6a^3x^3-8a^2x^2+9ax-16\right)-9\sin^{-1}(ax)}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-16 + 9*a*x - 8*a^2*x^2 + 6*a^3*x^3) - 9*ArcSin[a*x])/(
24*a^4)
```

Maple [B] time = 0.043, size = 154, normalized size = 1.8

$$-\frac{x}{4\,a^3}\left(-a^2x^2+1\right)^{\frac{3}{2}} + \frac{5\,x}{8\,a^3}\sqrt{-a^2x^2+1} + \frac{5}{8\,a^3}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} + \frac{1}{3\,a^4}\left(-a^2x^2+1\right)^{\frac{3}{2}} - \frac{1}{a^4}\sqrt{-a^2\left(x+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/4/a^3*x*(-a^2*x^2+1)^(3/2)+5/8/a^3*x*(-a^2*x^2+1)^(1/2)+5/8/a^3/(a^2)^(1
/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/3/a^4*(-a^2*x^2+1)^(3/2)-1/a
^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)-1/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*
x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

Maxima [A] time = 1.43909, size = 108, normalized size = 1.24

$$-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}{4\,a^3} + \frac{5\,\sqrt{-a^2x^2+1}x}{8\,a^3} + \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3\,a^4} - \frac{3\,\arcsin\left(ax\right)}{8\,a^4} - \frac{\sqrt{-a^2x^2+1}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-a^2*x^2 + 1)^(3/2)*x/a^3 + 5/8*sqrt(-a^2*x^2 + 1)*x/a^3 + 1/3*(-a^2*
x^2 + 1)^(3/2)/a^4 - 3/8*arcsin(a*x)/a^4 - sqrt(-a^2*x^2 + 1)/a^4
```

Fricas [A] time = 1.95591, size = 151, normalized size = 1.74

$$\frac{\left(6\,a^3x^3-8\,a^2x^2+9\,ax-16\right)\sqrt{-a^2x^2+1}+18\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*((6*a^3*x^3 - 8*a^2*x^2 + 9*a*x - 16)*sqrt(-a^2*x^2 + 1) + 18*arctan((
sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [A] time = 1.19229, size = 80, normalized size = 0.92

$$\frac{1}{24}\,\sqrt{-a^2x^2+1}\left(\left(2\,x\left(\frac{3\,x}{a}-\frac{4}{a^2}\right)+\frac{9}{a^3}\right)x-\frac{16}{a^4}\right)-\frac{3\,\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{8\,a^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(-a^2*x^2 + 1)*((2*x*(3*x/a - 4/a^2) + 9/a^3)*x - 16/a^4) - 3/8*ar
csin(a*x)*sgn(a)/(a^3*abs(a))
```

3.35 $\int e^{-\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=73

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{3a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}+\frac{\sqrt{1-a^2x^2}}{a^3}+\frac{\sin^{-1}(ax)}{2a^3}$$

```
[Out] Sqrt[1 - a^2*x^2]/a^3 - (x*Sqrt[1 - a^2*x^2])/(2*a^2) - (1 - a^2*x^2)^(3/2)
/(3*a^3) + ArcSin[a*x]/(2*a^3)
```

Rubi [A] time = 0.055245, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6124, 797, 641, 195, 216}

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{3a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}+\frac{\sqrt{1-a^2x^2}}{a^3}+\frac{\sin^{-1}(ax)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^ArcTanh[a*x],x]
```

```
[Out] Sqrt[1 - a^2*x^2]/a^3 - (x*Sqrt[1 - a^2*x^2])/(2*a^2) - (1 - a^2*x^2)^(3/2)
/(3*a^3) + ArcSin[a*x]/(2*a^3)
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}x^2\,dx = \int \frac{x^2(1-ax)}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx}{a^2} - \frac{\int (1-ax)\sqrt{1-a^2x^2}\,dx}{a^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\left(1-a^2x^2\right)^{3/2}}{3a^3} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2} - \frac{\int \sqrt{1-a^2x^2}\,dx}{a^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2} - \frac{\left(1-a^2x^2\right)^{3/2}}{3a^3} + \frac{\sin^{-1}(ax)}{a^3} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2} - \frac{\left(1-a^2x^2\right)^{3/2}}{3a^3} + \frac{\sin^{-1}(ax)}{2a^3}$$

Mathematica [A] time = 0.0316323, size = 43, normalized size = 0.59

$$\frac{\sqrt{1-a^2x^2}\left(2a^2x^2-3ax+4\right)+3\sin^{-1}(ax)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(4 - 3*a*x + 2*a^2*x^2) + 3*ArcSin[a*x])/(6*a^3)
```

Maple [B] time = 0.038, size = 134, normalized size = 1.8

$$-\frac{1}{3\,a^3}\left(-a^2x^2+1\right)^{\frac{3}{2}} - \frac{x}{2\,a^2}\sqrt{-a^2x^2+1} - \frac{1}{2\,a^2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} + \frac{1}{a^3}\sqrt{-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)} + \frac{1}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/3*(-a^2*x^2+1)^(3/2)/a^3-1/2*x*(-a^2*x^2+1)^(1/2)/a^2-1/2/a^2/(a^2)^(1/2
)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/a^3*(-a^2*(x+1/a)^2+2*a*(x+1/a
))^(1/2)+1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a)
)^(1/2))
```

Maxima [A] time = 1.43345, size = 82, normalized size = 1.12

$$-\frac{\sqrt{-a^2x^2+1}x}{2\,a^2} - \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3\,a^3} + \frac{\arcsin(ax)}{2\,a^3} + \frac{\sqrt{-a^2x^2+1}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x/a^2 - 1/3*(-a^2*x^2 + 1)^(3/2)/a^3 + 1/2*arcsin(a
*x)/a^3 + sqrt(-a^2*x^2 + 1)/a^3
```

Fricas [A] time = 1.85855, size = 131, normalized size = 1.79

$$\frac{\left(2\,a^2x^2 - 3\,ax + 4\right)\sqrt{-a^2x^2 + 1} - 6\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^2*x^2 - 3*a*x + 4)*sqrt(-a^2*x^2 + 1) - 6*arctan((sqrt(-a^2*x^2 +
 1) - 1)/(a*x)))/a^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [A] time = 1.17102, size = 68, normalized size = 0.93

$$\frac{1}{6}\,\sqrt{-a^2x^2 + 1}\left(x\left(\frac{2\,x}{a} - \frac{3}{a^2}\right) + \frac{4}{a^3}\right) + \frac{\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{2\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(-a^2*x^2 + 1)*(x*(2*x/a - 3/a^2) + 4/a^3) + 1/2*arcsin(a*x)*sgn(a)
/(a^2*abs(a))
```

3.36 $\int e^{-\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{1-a^2x^2}(2-ax)}{2a^2}-\frac{\sin^{-1}(ax)}{2a^2}$$

```
[Out] -((2 - a*x)*Sqrt[1 - a^2*x^2])/(2*a^2) - ArcSin[a*x]/(2*a^2)
```

Rubi [A] time = 0.0231506, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6124, 780, 216}

$$-\frac{\sqrt{1-a^2x^2}(2-ax)}{2a^2}-\frac{\sin^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^ArcTanh[a*x],x]
```

```
[Out] -((2 - a*x)*Sqrt[1 - a^2*x^2])/(2*a^2) - ArcSin[a*x]/(2*a^2)
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}x\,dx &= \int \frac{x(1-ax)}{\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{(2-ax)\sqrt{1-a^2x^2}}{2a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a}\\ &= -\frac{(2-ax)\sqrt{1-a^2x^2}}{2a^2}-\frac{\sin^{-1}(ax)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.0253898, size = 34, normalized size = 0.87

$$\frac{(ax-2)\sqrt{1-a^2x^2}-\sin^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^ArcTanh[a*x],x]
```

```
[Out] ((-2 + a*x)*Sqrt[1 - a^2*x^2] - ArcSin[a*x])/(2*a^2)
```

Maple [B] time = 0.04, size = 119, normalized size = 3.1

$$\frac{x}{2a}\sqrt{-a^2x^2+1}+\frac{1}{2a}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{1}{a^2}\sqrt{-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)}-\frac{1}{a}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2\left(\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*x/a*(-a^2*x^2+1)^(1/2)+1/2/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2
+1)^(1/2))-1/a^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)-1/a/(a^2)^(1/2)*arctan(
(a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

Maxima [A] time = 1.42151, size = 61, normalized size = 1.56

$$\frac{\sqrt{-a^2x^2+1}x}{2a}-\frac{\arcsin(ax)}{2a^2}-\frac{\sqrt{-a^2x^2+1}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*x/a - 1/2*arcsin(a*x)/a^2 - sqrt(-a^2*x^2 + 1)/a^2
```

Fricas [A] time = 1.9756, size = 112, normalized size = 2.87

$$\frac{\sqrt{-a^2x^2+1}(ax-2)+2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-a^2*x^2 + 1)*(a*x - 2) + 2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)
))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [A] time = 1.1623, size = 55, normalized size = 1.41

$$\frac{1}{2}\sqrt{-a^2x^2+1}\left(\frac{x}{a}-\frac{2}{a^2}\right)-\frac{\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{2\,a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*(x/a - 2/a^2) - 1/2*arcsin(a*x)*sgn(a)/(a*abs(a))
```

3.37 $\int e^{-\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{\sqrt{1-a^2x^2}}{a}+\frac{\sin^{-1}(ax)}{a}$$

```
[Out] Sqrt[1 - a^2*x^2]/a + ArcSin[a*x]/a
```

Rubi [A] time = 0.0108152, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6123, 641, 216}

$$\frac{\sqrt{1-a^2x^2}}{a}+\frac{\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-ArcTanh[a*x]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/a + ArcSin[a*x]/a
```

Rule 6123

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 + a*x)^((n + 1)/2)/(
(1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2]), x] /; FreeQ[a, x] && IntegerQ[(n
- 1)/2]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}\,dx &= \int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{\sqrt{1-a^2x^2}}{a}+\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{\sqrt{1-a^2x^2}}{a}+\frac{\sin^{-1}(ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0123919, size = 23, normalized size = 0.85

$$\frac{\sqrt{1-a^2x^2}+\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2] + ArcSin[a*x])/a
```

Maple [B] time = 0.031, size = 66, normalized size = 2.4

$$\frac{1}{a}\sqrt{-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)}+\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)}}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(
-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

Maxima [A] time = 1.44107, size = 34, normalized size = 1.26

$$\frac{\arcsin\left(ax\right)}{a}+\frac{\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(a*x)/a + sqrt(-a^2*x^2 + 1)/a
```

Fricas [A] time = 1.9401, size = 90, normalized size = 3.33

$$\frac{\sqrt{-a^2x^2+1}-2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-a^2*x^2 + 1) - 2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [A] time = 1.17369, size = 38, normalized size = 1.41

$$\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{|a|} + \frac{\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/abs(a) + sqrt(-a^2*x^2 + 1)/a
```

3.38 $\int \frac{e^{-\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=24

$$-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\sin^{-1}(ax)$$

```
[Out] -ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.0416582, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6124, 844, 216, 266, 63, 208}

$$-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*x),x]
```

```
[Out] -ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{1-ax}{x\sqrt{1-a^2x^2}}\,dx\\ &= -\left(a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\right)+\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\ &= -\sin^{-1}(ax)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\ &= -\sin^{-1}(ax)-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2}\\ &= -\sin^{-1}(ax)-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0146738, size = 28, normalized size = 1.17

$$-\log\left(\sqrt{1-a^2x^2}+1\right)-\sin^{-1}(ax)+\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*x),x]
```

```
[Out] -ArcSin[a*x] + Log[x] - Log[1 + Sqrt[1 - a^2*x^2]]
```

Maple [B] time = 0.041, size = 93, normalized size = 3.9

$$\sqrt{-a^2x^2+1}-\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\sqrt{-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)}-a\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2\left(x+a^{-1}\right)^2+2a\left(x+\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] (-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2))-(-a^2*(x+1/a)^2+2*a*(x+1/a
))^(1/2)-a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1
/2))
```

Maxima [A] time = 1.44148, size = 57, normalized size = 2.38

$$-a\left(\frac{\arcsin(ax)}{a}+\frac{\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -a*(arcsin(a*x)/a + log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))/a)
```

Fricas [A] time = 1.99666, size = 103, normalized size = 4.29

$$2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + log((sqrt(-a^2*x^2 + 1) - 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{x(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(x*(a*x + 1)), x)
```

Giac [B] time = 1.21772, size = 70, normalized size = 2.92

$$-\frac{a\arcsin(ax)\operatorname{sgn}(a)}{|a|}-\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -a*arcsin(a*x)*sgn(a)/abs(a) - a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
 2*a)/(a^2*abs(x)))/abs(a)
```

3.39 $\int \frac{e^{-\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=37

$$a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}}{x}$$

```
[Out] -(Sqrt[1 - a^2*x^2]/x) + a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.0420877, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6124, 807, 266, 63, 208}

$$a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/x) + a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{1-ax}{x^2\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{\sqrt{1-a^2x^2}}{x} - a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{\sqrt{1-a^2x^2}}{x} - \frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{1-a^2x^2}}{x} + \frac{\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a}\\ &= -\frac{\sqrt{1-a^2x^2}}{x} + a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0257737, size = 44, normalized size = 1.19

$$-\frac{\sqrt{1-a^2x^2}}{x} + a\log\left(\sqrt{1-a^2x^2}+1\right) - a\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/x) - a*Log[x] + a*Log[1 + Sqrt[1 - a^2*x^2]]
```

Maple [B] time = 0.059, size = 162, normalized size = 4.4

$$-\frac{1}{x}\left(-a^2x^2+1\right)^{\frac{3}{2}} - a^2x\sqrt{-a^2x^2+1} - a^2\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} + a\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) - a\sqrt{-a^2x^2+1} + a\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] -1/x*(-a^2*x^2+1)^(3/2)-a^2*x*(-a^2*x^2+1)^(1/2)-a^2/(a^2)^(1/2)*arctan((a^
2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+a*arctanh(1/(-a^2*x^2+1)^(1/2))-a*(-a^2*x^2+
1)^(1/2)+a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+a^2/(a^2)^(1/2)*arctan((a^2)^
(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*x^2), x)
```

Fricas [A] time = 1.89839, size = 85, normalized size = 2.3

$$-\frac{ax\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*x*log((sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{x^2(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(x**2*(a*x + 1)), x)
```

Giac [B] time = 1.1928, size = 128, normalized size = 3.46

$$\frac{a^4x}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|}+\frac{a^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|}-\frac{\sqrt{-a^2x^2+1}|a|+a}{2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) + a^2*log(1/2*abs(-2*sqr
t(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/2*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)/(x*abs(a))
```

3.40 $\int \frac{e^{-\tanh^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=63

$$\frac{a\sqrt{1-a^2x^2}}{x} - \frac{\sqrt{1-a^2x^2}}{2x^2} - \frac{1}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -Sqrt[1 - a^2*x^2]/(2*x^2) + (a*Sqrt[1 - a^2*x^2])/x - (a^2*ArcTanh[Sqrt[1
- a^2*x^2]])/2
```

Rubi [A] time = 0.0604718, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6124, 835, 807, 266, 63, 208}

$$\frac{a\sqrt{1-a^2x^2}}{x} - \frac{\sqrt{1-a^2x^2}}{2x^2} - \frac{1}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*x^3),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*x^2) + (a*Sqrt[1 - a^2*x^2])/x - (a^2*ArcTanh[Sqrt[1
- a^2*x^2]])/2
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{x^3}\,dx &= \int \frac{1-ax}{x^3\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2} - \frac{1}{2}\int \frac{2a-a^2x}{x^2\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2} + \frac{a\sqrt{1-a^2x^2}}{x} + \frac{1}{2}a^2\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2} + \frac{a\sqrt{1-a^2x^2}}{x} + \frac{1}{4}a^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2} + \frac{a\sqrt{1-a^2x^2}}{x} - \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right) \\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2} + \frac{a\sqrt{1-a^2x^2}}{x} - \frac{1}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0419927, size = 57, normalized size = 0.9

$$
\frac{1}{2}\left(\frac{(2ax-1)\sqrt{1-a^2x^2}}{x^2} - a^2\log\left(\sqrt{1-a^2x^2}+1\right) + a^2\log(x)\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*x^3),x]
```

```
[Out] (((-1 + 2*a*x)*Sqrt[1 - a^2*x^2])/x^2 + a^2*Log[x] - a^2*Log[1 + Sqrt[1 - a
^2*x^2]])/2
```

Maple [B] time = 0.043, size = 186, normalized size = 3.

$$
\frac{a}{x}\left(-a^2x^2+1\right)^{\frac{3}{2}} + a^3x\sqrt{-a^2x^2+1} + a^3\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} + \frac{a^2}{2}\sqrt{-a^2x^2+1} - \frac{a^2}{2}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x)
```

```
[Out] a/x*(-a^2*x^2+1)^(3/2)+a^3*x*(-a^2*x^2+1)^(1/2)+a^3/(a^2)^(1/2)*arctan((a^2
)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/2*a^2*(-a^2*x^2+1)^(1/2)-1/2*a^2*arctanh(1/
```

```
(-a^2*x^2+1)^(1/2))-a^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)-a^3/(a^2)^(1/2)*
arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-1/2/x^2*(-a^2*x^2+
1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*x^3), x)
```

Fricas [A] time = 1.93545, size = 113, normalized size = 1.79

$$\frac{a^2x^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}(2\,ax-1)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2*log((sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1)*(2*a*x -
1))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{x^3(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(x**3*(a*x + 1)), x)
```

Giac [B] time = 1.21876, size = 215, normalized size = 3.41

$$\frac{\left(a^3-\frac{4\left(\sqrt{-a^2x^2+1}|a|+a\right)a}{x}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}|a|+a\right)^2|a|}-\frac{a^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}+\frac{\frac{4\left(\sqrt{-a^2x^2+1}|a|+a\right)a|a|}{x}-\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)^2|a|}{ax^2}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3 - 4*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x)*a^4*x^2/((sqrt(-a^2*x^2 +
 1)*abs(a) + a)^2*abs(a)) - 1/2*a^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a
) - 2*a)/(a^2*abs(x)))/abs(a) + 1/8*(4*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a*ab
s(a)/x - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)/(a*x^2))/a^2
```

3.41 $\int \frac{e^{-\tanh^{-1}(ax)}}{x^4} dx$

Optimal. Leaf size=90

$$-\frac{2a^2\sqrt{1-a^2x^2}}{3x}+\frac{a\sqrt{1-a^2x^2}}{2x^2}-\frac{\sqrt{1-a^2x^2}}{3x^3}+\frac{1}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -Sqrt[1 - a^2*x^2]/(3*x^3) + (a*Sqrt[1 - a^2*x^2])/(2*x^2) - (2*a^2*Sqrt[1
- a^2*x^2])/(3*x) + (a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rubi [A] time = 0.082619, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6124, 835, 807, 266, 63, 208}

$$-\frac{2a^2\sqrt{1-a^2x^2}}{3x}+\frac{a\sqrt{1-a^2x^2}}{2x^2}-\frac{\sqrt{1-a^2x^2}}{3x^3}+\frac{1}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*x^4),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(3*x^3) + (a*Sqrt[1 - a^2*x^2])/(2*x^2) - (2*a^2*Sqrt[1
- a^2*x^2])/(3*x) + (a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{x^4}\,dx &= \int \frac{1-ax}{x^4\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3}-\frac{1}{3}\int \frac{3a-2a^2x}{x^3\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3}+\frac{a\sqrt{1-a^2x^2}}{2x^2}+\frac{1}{6}\int \frac{4a^2-3a^3x}{x^2\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3}+\frac{a\sqrt{1-a^2x^2}}{2x^2}-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{1}{2}a^3\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3}+\frac{a\sqrt{1-a^2x^2}}{2x^2}-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{1}{4}a^3\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3}+\frac{a\sqrt{1-a^2x^2}}{2x^2}-\frac{2a^2\sqrt{1-a^2x^2}}{3x}+\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)\\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3}+\frac{a\sqrt{1-a^2x^2}}{2x^2}-\frac{2a^2\sqrt{1-a^2x^2}}{3x}+\frac{1}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0531421, size = 66, normalized size = 0.73

$$\frac{1}{6}\left(\frac{\left(-4a^2x^2+3ax-2\right)\sqrt{1-a^2x^2}}{x^3}+3a^3\log\left(\sqrt{1-a^2x^2}+1\right)-3a^3\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*x^4),x]
```

```
[Out] (((-2 + 3*a*x - 4*a^2*x^2)*Sqrt[1 - a^2*x^2])/x^3 - 3*a^3*Log[x] + 3*a^3*Lo
g[1 + Sqrt[1 - a^2*x^2]])/6
```

Maple [B] time = 0.055, size = 207, normalized size = 2.3

$$-\frac{a^2}{x}\left(-a^2x^2+1\right)^{\frac{3}{2}}-a^4x\sqrt{-a^2x^2+1}-a^4\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{a^3}{2}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{a^3}{2}\sqrt{-a^2x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x)
```

```
[Out] -a^2/x*(-a^2*x^2+1)^(3/2)-a^4*x*(-a^2*x^2+1)^(1/2)-a^4/(a^2)^(1/2)*arctan((
a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/2*a^3*arctanh(1/(-a^2*x^2+1)^(1/2))-1/2*
a^3*(-a^2*x^2+1)^(1/2)+a^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+a^4/(a^2)^(1/
2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))+1/2*a/x^2*(-a^2
*x^2+1)^(3/2)-1/3/x^3*(-a^2*x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*x^4), x)
```

Fricas [A] time = 1.91862, size = 134, normalized size = 1.49

$$-\frac{3\,a^3x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(4\,a^2x^2-3\,ax+2\right)\sqrt{-a^2x^2+1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*a^3*x^3*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (4*a^2*x^2 - 3*a*x + 2)*s
qrt(-a^2*x^2 + 1))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{x^4\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(x**4*(a*x + 1)), x)
```

Giac [B] time = 1.2316, size = 284, normalized size = 3.16

$$\frac{\left(a^4-\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)a^2}{x}+\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|}+\frac{a^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4}{x}-\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)^2a^2}{x^2}+\frac{(\sqrt{\cdots}}{\cdots}}{24\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4 - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2/x + 9*(sqrt(-a^2*x^2 + 1)
*abs(a) + a)^2/x^2)*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*abs(a)) + 1/
2*a^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a)
- 1/24*(9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4/x - 3*(sqrt(-a^2*x^2 + 1)*abs
(a) + a)^2*a^2/x^2 + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/x^3)/(a^2*abs(a))
```

3.42 $\int \frac{e^{-\tanh^{-1}(ax)}}{x^5} dx$

Optimal. Leaf size=114

$$\frac{2a^3\sqrt{1-a^2x^2}}{3x} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} + \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{3}{8}a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -Sqrt[1 - a^2*x^2]/(4*x^4) + (a*Sqrt[1 - a^2*x^2])/(3*x^3) - (3*a^2*Sqrt[1
- a^2*x^2])/(8*x^2) + (2*a^3*Sqrt[1 - a^2*x^2])/(3*x) - (3*a^4*ArcTanh[Sqrt
[1 - a^2*x^2]])/8
```

Rubi [A] time = 0.103138, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6124, 835, 807, 266, 63, 208}

$$\frac{2a^3\sqrt{1-a^2x^2}}{3x} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} + \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{3}{8}a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*x^5),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(4*x^4) + (a*Sqrt[1 - a^2*x^2])/(3*x^3) - (3*a^2*Sqrt[1
- a^2*x^2])/(8*x^2) + (2*a^3*Sqrt[1 - a^2*x^2])/(3*x) - (3*a^4*ArcTanh[Sqrt
[1 - a^2*x^2]])/8
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{x^5}\,dx &= \int \frac{1-ax}{x^5\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4}-\frac{1}{4}\int \frac{4a-3a^2x}{x^4\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4}+\frac{a\sqrt{1-a^2x^2}}{3x^3}+\frac{1}{12}\int \frac{9a^2-8a^3x}{x^3\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4}+\frac{a\sqrt{1-a^2x^2}}{3x^3}-\frac{3a^2\sqrt{1-a^2x^2}}{8x^2}-\frac{1}{24}\int \frac{16a^3-9a^4x}{x^2\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4}+\frac{a\sqrt{1-a^2x^2}}{3x^3}-\frac{3a^2\sqrt{1-a^2x^2}}{8x^2}+\frac{2a^3\sqrt{1-a^2x^2}}{3x}+\frac{1}{8}\left(3a^4\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4}+\frac{a\sqrt{1-a^2x^2}}{3x^3}-\frac{3a^2\sqrt{1-a^2x^2}}{8x^2}+\frac{2a^3\sqrt{1-a^2x^2}}{3x}+\frac{1}{16}\left(3a^4\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,\right.\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4}+\frac{a\sqrt{1-a^2x^2}}{3x^3}-\frac{3a^2\sqrt{1-a^2x^2}}{8x^2}+\frac{2a^3\sqrt{1-a^2x^2}}{3x}-\frac{1}{8}\left(3a^2\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{\right.\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4}+\frac{a\sqrt{1-a^2x^2}}{3x^3}-\frac{3a^2\sqrt{1-a^2x^2}}{8x^2}+\frac{2a^3\sqrt{1-a^2x^2}}{3x}-\frac{3}{8}a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.057365, size = 74, normalized size = 0.65

$$\frac{1}{24}\left(\frac{\sqrt{1-a^2x^2}\left(16a^3x^3-9a^2x^2+8ax-6\right)}{x^4}-9a^4\log\left(\sqrt{1-a^2x^2}+1\right)+9a^4\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*x^5),x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(-6 + 8*a*x - 9*a^2*x^2 + 16*a^3*x^3))/x^4 + 9*a^4*Log[
x] - 9*a^4*Log[1 + Sqrt[1 - a^2*x^2]])/24
```

Maple [B] time = 0.054, size = 226, normalized size = 2.

$$-\frac{1}{4x^4}\left(-a^2x^2+1\right)^{\frac{3}{2}}-\frac{5a^2}{8x^2}\left(-a^2x^2+1\right)^{\frac{3}{2}}-\frac{3a^4}{8}\operatorname{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{3a^4}{8}\sqrt{-a^2x^2+1}+\frac{a^3}{x}\left(-a^2x^2+1\right)^{\frac{3}{2}}+a^5x\sqrt{-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^5,x)
```

```
[Out] -1/4/x^4*(-a^2*x^2+1)^(3/2)-5/8*a^2/x^2*(-a^2*x^2+1)^(3/2)-3/8*a^4*arctanh(
1/(-a^2*x^2+1)^(1/2))+3/8*a^4*(-a^2*x^2+1)^(1/2)+a^3/x*(-a^2*x^2+1)^(3/2)+a
^5*x*(-a^2*x^2+1)^(1/2)+a^5/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(
1/2))-a^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)-a^5/(a^2)^(1/2)*arctan((a^2)^(
1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))+1/3*a/x^3*(-a^2*x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*x^5), x)
```

Fricas [A] time = 1.98605, size = 151, normalized size = 1.32

$$\frac{9\,a^4x^4\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(16\,a^3x^3-9\,a^2x^2+8\,ax-6\right)\sqrt{-a^2x^2+1}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(9*a^4*x^4*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (16*a^3*x^3 - 9*a^2*x^2 +
 8*a*x - 6)*sqrt(-a^2*x^2 + 1))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{x^5(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(x**5*(a*x + 1)), x)
```

Giac [B] time = 1.20479, size = 369, normalized size = 3.24

$$\frac{\left(3\,a^5-\frac{8\left(\sqrt{-a^2x^2+1}|a|+a\right)a^3}{x}+\frac{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^2a}{x^2}-\frac{72\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{ax^3}\right)a^8x^4}{192\left(\sqrt{-a^2x^2+1}|a|+a\right)^4|a|}-\frac{3\,a^5\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{8\,|a|}+\frac{72\left(\sqrt{-a^2x^2+1}|a|+a\right)a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^5 - 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^3/x + 24*(sqrt(-a^2*x^2
+ 1)*abs(a) + a)^2*a/x^2 - 72*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a*x^3))*a^
8*x^4/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*abs(a)) - 3/8*a^5*log(1/2*abs(-2*s
qrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 1/192*(72*(sqrt(-a^2
*x^2 + 1)*abs(a) + a)*a^5*abs(a)/x - 24*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a
^3*abs(a)/x^2 + 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a*abs(a)/x^3 - 3*(sqrt(
-a^2*x^2 + 1)*abs(a) + a)^4*abs(a)/(a*x^4))/a^4
```

3.43 $\int e^{-2\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=43

$$-\frac{x^2}{a^2}+\frac{2x}{a^3}-\frac{2\log(ax+1)}{a^4}+\frac{2x^3}{3a}-\frac{x^4}{4}$$

```
[Out] (2*x)/a^3 - x^2/a^2 + (2*x^3)/(3*a) - x^4/4 - (2*Log[1 + a*x])/a^4
```

Rubi [A] time = 0.0392676, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$-\frac{x^2}{a^2}+\frac{2x}{a^3}-\frac{2\log(ax+1)}{a^4}+\frac{2x^3}{3a}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (2*x)/a^3 - x^2/a^2 + (2*x^3)/(3*a) - x^4/4 - (2*Log[1 + a*x])/a^4
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}x^3\,dx&=\int\frac{x^3(1-ax)}{1+ax}\,dx\\&=\int\left(\frac{2}{a^3}-\frac{2x}{a^2}+\frac{2x^2}{a}-x^3-\frac{2}{a^3(1+ax)}\right)dx\\&=\frac{2x}{a^3}-\frac{x^2}{a^2}+\frac{2x^3}{3a}-\frac{x^4}{4}-\frac{2\log(1+ax)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.0187701, size = 43, normalized size = 1.

$$-\frac{x^2}{a^2}+\frac{2x}{a^3}-\frac{2\log(ax+1)}{a^4}+\frac{2x^3}{3a}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (2*x)/a^3 - x^2/a^2 + (2*x^3)/(3*a) - x^4/4 - (2*Log[1 + a*x])/a^4
```

Maple [A] time = 0.033, size = 40, normalized size = 0.9

$$2\,{\frac{x}{{a}^{3}}}-{\frac{{x}^{2}}{{a}^{2}}}+{\frac{2\,{x}^{3}}{3\,a}}-{\frac{{x}^{4}}{4}}-2\,{\frac{\ln \left(ax+1 \right) }{{a}^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 2*x/a^3-x^2/a^2+2/3*x^3/a-1/4*x^4-2/a^4*ln(a*x+1)
```

Maxima [A] time = 0.945287, size = 58, normalized size = 1.35

$$-\frac{3\,a^{3}x^{4}-8\,a^{2}x^{3}+12\,ax^{2}-24\,x}{12\,a^{3}}-\frac{2\,\log\left(ax+1\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^3*x^4 - 8*a^2*x^3 + 12*a*x^2 - 24*x)/a^3 - 2*log(a*x + 1)/a^4
```

Fricas [A] time = 1.76151, size = 101, normalized size = 2.35

$$-\frac{3\,a^{4}x^{4}-8\,a^{3}x^{3}+12\,a^{2}x^{2}-24\,ax+24\,\log\left(ax+1\right)}{12\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 24*a*x + 24*log(a*x + 1))/a^4
```

Sympy [A] time = 0.338888, size = 37, normalized size = 0.86

$$-\frac{x^{4}}{4}+\frac{2x^{3}}{3a}-\frac{x^{2}}{a^{2}}+\frac{2x}{a^{3}}-\frac{2\log\left(ax+1\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -x**4/4 + 2*x**3/(3*a) - x**2/a**2 + 2*x/a**3 - 2*log(a*x + 1)/a**4
```

Giac [A] time = 1.23158, size = 89, normalized size = 2.07

$$\frac{(ax+1)^4\left(\frac{20}{ax+1} - \frac{54}{(ax+1)^2} + \frac{84}{(ax+1)^3} - 3\right)}{12\,a^4} + \frac{2\,\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*(a*x + 1)^4*(20/(a*x + 1) - 54/(a*x + 1)^2 + 84/(a*x + 1)^3 - 3)/a^4 +
 2*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a^4
```

3.44 $\int e^{-2\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=32

$$-\frac{2x}{a^2}+\frac{2\log(ax+1)}{a^3}+\frac{x^2}{a}-\frac{x^3}{3}$$

```
[Out] (-2*x)/a^2 + x^2/a - x^3/3 + (2*Log[1 + a*x])/a^3
```

Rubi [A] time = 0.0331909, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$-\frac{2x}{a^2}+\frac{2\log(ax+1)}{a^3}+\frac{x^2}{a}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-2*x)/a^2 + x^2/a - x^3/3 + (2*Log[1 + a*x])/a^3
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}x^2\,dx&=\int\frac{x^2(1-ax)}{1+ax}\,dx\\&=\int\left(-\frac{2}{a^2}+\frac{2x}{a}-x^2+\frac{2}{a^2(1+ax)}\right)dx\\&=-\frac{2x}{a^2}+\frac{x^2}{a}-\frac{x^3}{3}+\frac{2\log(1+ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.0138062, size = 32, normalized size = 1.

$$-\frac{2x}{a^2}+\frac{2\log(ax+1)}{a^3}+\frac{x^2}{a}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-2*x)/a^2 + x^2/a - x^3/3 + (2*Log[1 + a*x])/a^3
```

Maple [A] time = 0.03, size = 31, normalized size = 1.

$$-2\frac{x}{a^2}+\frac{x^2}{a}-\frac{x^3}{3}+2\frac{\ln(ax+1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -2*x/a^2+x^2/a-1/3*x^3+2*ln(a*x+1)/a^3
```

Maxima [A] time = 0.974067, size = 46, normalized size = 1.44

$$-\frac{a^2x^3-3\,ax^2+6\,x}{3\,a^2}+\frac{2\,\log(ax+1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*x^3 - 3*a*x^2 + 6*x)/a^2 + 2*log(a*x + 1)/a^3
```

Fricas [A] time = 1.91128, size = 77, normalized size = 2.41

$$-\frac{a^3x^3-3\,a^2x^2+6\,ax-6\,\log(ax+1)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/3*(a^3*x^3 - 3*a^2*x^2 + 6*a*x - 6*log(a*x + 1))/a^3
```

Sympy [A] time = 0.356007, size = 27, normalized size = 0.84

$$-\frac{x^3}{3}+\frac{x^2}{a}-\frac{2x}{a^2}+\frac{2\log(ax+1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -x**3/3 + x**2/a - 2*x/a**2 + 2*log(a*x + 1)/a**3
```

Giac [A] time = 1.17283, size = 77, normalized size = 2.41

$$\frac{(ax+1)^3\left(\frac{6}{ax+1}-\frac{15}{(ax+1)^2}-1\right)}{3a^3}-\frac{2\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*(a*x + 1)^3*(6/(a*x + 1) - 15/(a*x + 1)^2 - 1)/a^3 - 2*log(abs(a*x + 1)
/((a*x + 1)^2*abs(a)))/a^3
```

3.45 $\int e^{-2\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=25

$$-\frac{2\log(ax+1)}{a^2}+\frac{2x}{a}-\frac{x^2}{2}$$

```
[Out] (2*x)/a - x^2/2 - (2*Log[1 + a*x])/a^2
```

Rubi [A] time = 0.0239714, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6126, 77}

$$-\frac{2\log(ax+1)}{a^2}+\frac{2x}{a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (2*x)/a - x^2/2 - (2*Log[1 + a*x])/a^2
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}x\,dx &= \int \frac{x(1-ax)}{1+ax}\,dx\\ &= \int \left(\frac{2}{a}-x-\frac{2}{a(1+ax)}\right)dx\\ &= \frac{2x}{a}-\frac{x^2}{2}-\frac{2\log(1+ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0108768, size = 25, normalized size = 1.

$$-\frac{2\log(ax+1)}{a^2}+\frac{2x}{a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (2*x)/a - x^2/2 - (2*Log[1 + a*x])/a^2
```

Maple [A] time = 0.032, size = 24, normalized size = 1.

$$2\,{\frac {x}{a}}-{\frac {{x}^{2}}{2}}-2\,{\frac {\ln \left(ax+1 \right) }{{a}^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 2*x/a-1/2*x^2-2/a^2*ln(a*x+1)
```

Maxima [A] time = 0.947156, size = 35, normalized size = 1.4

$$-\frac {a x^{2} - 4 \, x}{2 \, a} - \frac {2 \, \log \left (a x + 1\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*(a*x^2 - 4*x)/a - 2*log(a*x + 1)/a^2
```

Fricas [A] time = 1.84173, size = 61, normalized size = 2.44

$$-\frac {a^{2} x^{2} - 4 \, a x + 4 \, \log \left (a x + 1\right)}{2 \, a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*x^2 - 4*a*x + 4*log(a*x + 1))/a^2
```

Sympy [A] time = 0.286735, size = 20, normalized size = 0.8

$$- \frac {x^{2}}{2} + \frac {2 x}{a} - \frac {2 \log {\left (a x + 1 \right)}}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -x**2/2 + 2*x/a - 2*log(a*x + 1)/a**2
```

Giac [B] time = 1.16934, size = 70, normalized size = 2.8

$$\frac{\frac{(ax+1)^2\left(\frac{6}{ax+1}-1\right)}{a}+\frac{4\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*((a*x + 1)^2*(6/(a*x + 1) - 1)/a + 4*log(abs(a*x + 1)/((a*x + 1)^2*abs(
a)))/a)/a
```

3.46 $\int e^{-2\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=15

$$\frac{2\log(ax+1)}{a}-x$$

```
[Out] -x + (2*Log[1 + a*x])/a
```

Rubi [A] time = 0.0095735, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6125, 43}

$$\frac{2\log(ax+1)}{a}-x$$

Antiderivative was successfully verified.

```
[In] Int[E^(-2*ArcTanh[a*x]),x]
```

```
[Out] -x + (2*Log[1 + a*x])/a
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}\,dx &= \int \frac{1-ax}{1+ax}\,dx\\ &= \int \left(-1+\frac{2}{1+ax}\right)dx\\ &= -x+\frac{2\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0096169, size = 15, normalized size = 1.

$$\frac{2\log(ax+1)}{a}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-2*ArcTanh[a*x]),x]
```

```
[Out] -x + (2*Log[1 + a*x])/a
```

Maple [A] time = 0.026, size = 16, normalized size = 1.1

$$-x+2\,\frac{\ln\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -x+2*ln(a*x+1)/a
```

Maxima [A] time = 0.95785, size = 20, normalized size = 1.33

$$-x+\frac{2\,\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -x + 2*log(a*x + 1)/a
```

Fricas [A] time = 1.871, size = 36, normalized size = 2.4

$$-\frac{ax-2\,\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -(a*x - 2*log(a*x + 1))/a
```

Sympy [A] time = 0.136363, size = 10, normalized size = 0.67

$$-x+\frac{2\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -x + 2*log(a*x + 1)/a
```

Giac [B] time = 1.17427, size = 86, normalized size = 5.73

$$-a^{2}\left(\frac{ax+1}{a^{3}}+\frac{2\,\log\left(\frac{|ax+1|}{(ax+1)^{2}|a|}\right)}{a^{3}}-\frac{1}{(ax+1)a^{3}}\right)-\frac{1}{(ax+1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -a^2*((a*x + 1)/a^3 + 2*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a^3 - 1/((a*
x + 1)*a^3)) - 1/((a*x + 1)*a)
```

3.47 $\int \frac{e^{-2\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=11

$$\log(x) - 2\log(ax+1)$$

[Out] Log[x] - 2*Log[1 + a*x]

Rubi [A] time = 0.023803, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 72}

$$\log(x) - 2\log(ax+1)$$

Antiderivative was successfully verified.

[In] Int[1/(E^(2*ArcTanh[a*x])*x),x]

[Out] Log[x] - 2*Log[1 + a*x]

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{1-ax}{x(1+ax)}\,dx\\ &= \int \left(\frac{1}{x} - \frac{2a}{1+ax}\right)dx\\ &= \log(x) - 2\log(1+ax)\end{aligned}$$

Mathematica [A] time = 0.0061687, size = 11, normalized size = 1.

$$\log(x) - 2\log(ax+1)$$

Antiderivative was successfully verified.

[In] Integrate[1/(E^(2*ArcTanh[a*x])*x),x]

[Out] Log[x] - 2*Log[1 + a*x]

Maple [A] time = 0.031, size = 12, normalized size = 1.1

$$\ln(x) - 2\ln(ax + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/x,x)
```

```
[Out] ln(x)-2*ln(a*x+1)
```

Maxima [A] time = 0.957287, size = 15, normalized size = 1.36

$$-2\log(ax + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="maxima")
```

```
[Out] -2*log(a*x + 1) + log(x)
```

Fricas [A] time = 1.75575, size = 35, normalized size = 3.18

$$-2\log(ax + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="fricas")
```

```
[Out] -2*log(a*x + 1) + log(x)
```

Sympy [A] time = 0.153514, size = 10, normalized size = 0.91

$$\log(x) - 2\log\left(x + \frac{1}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/x,x)
```

```
[Out] log(x) - 2*log(x + 1/a)
```

Giac [B] time = 1.14313, size = 58, normalized size = 5.27

$$a\left(\frac{\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a} + \frac{\log\left(\left|-\frac{1}{ax+1} + 1\right|\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="giac")
```

```
[Out] a*(log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a + log(abs(-1/(a*x + 1) + 1))/a)
```

3.48 $\int \frac{e^{-2\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=20

$$-2a\log(x)+2a\log(ax+1)-\frac{1}{x}$$

```
[Out] -x^(-1) - 2*a*Log[x] + 2*a*Log[1 + a*x]
```

Rubi [A] time = 0.0270994, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$-2a\log(x)+2a\log(ax+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] -x^(-1) - 2*a*Log[x] + 2*a*Log[1 + a*x]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{1-ax}{x^2(1+ax)}\,dx\\ &= \int \left(\frac{1}{x^2}-\frac{2a}{x}+\frac{2a^2}{1+ax}\right)dx\\ &= -\frac{1}{x}-2a\log(x)+2a\log(1+ax)\end{aligned}$$

Mathematica [A] time = 0.0086829, size = 20, normalized size = 1.

$$-2a\log(x)+2a\log(ax+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] -x^(-1) - 2*a*Log[x] + 2*a*Log[1 + a*x]
```

Maple [A] time = 0.034, size = 21, normalized size = 1.1

$$-x^{-1} - 2\,a\ln(x) + 2\,a\ln(ax+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/x^2,x)
```

```
[Out] -1/x-2*a*ln(x)+2*a*ln(a*x+1)
```

Maxima [A] time = 0.946294, size = 27, normalized size = 1.35

$$2\,a\log(ax+1) - 2\,a\log(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="maxima")
```

```
[Out] 2*a*log(a*x + 1) - 2*a*log(x) - 1/x
```

Fricas [A] time = 1.96922, size = 58, normalized size = 2.9

$$\frac{2\,ax\log(ax+1) - 2\,ax\log(x) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*log(a*x + 1) - 2*a*x*log(x) - 1)/x
```

Sympy [A] time = 1.07266, size = 17, normalized size = 0.85

$$-2a\left(\log(x) - \log\left(x + \frac{1}{a}\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/x**2,x)
```

```
[Out] -2*a*(log(x) - log(x + 1/a)) - 1/x
```

Giac [A] time = 1.19509, size = 41, normalized size = 2.05

$$-2\,a\log\left(\left|-\frac{1}{ax+1}+1\right|\right)+\frac{a}{\frac{1}{ax+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="giac")
```

```
[Out] -2*a*log(abs(-1/(a*x + 1) + 1)) + a/(1/(a*x + 1) - 1)
```

3.49 $\int \frac{e^{-2\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=32

$$2a^2\log(x) - 2a^2\log(ax+1) + \frac{2a}{x} - \frac{1}{2x^2}$$

```
[Out] -1/(2*x^2) + (2*a)/x + 2*a^2*Log[x] - 2*a^2*Log[1 + a*x]
```

Rubi [A] time = 0.0297747, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$2a^2\log(x) - 2a^2\log(ax+1) + \frac{2a}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] -1/(2*x^2) + (2*a)/x + 2*a^2*Log[x] - 2*a^2*Log[1 + a*x]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{x^3}\,dx &= \int \frac{1-ax}{x^3(1+ax)}\,dx \\
&= \int \left(\frac{1}{x^3} - \frac{2a}{x^2} + \frac{2a^2}{x} - \frac{2a^3}{1+ax}\right) dx \\
&= -\frac{1}{2x^2} + \frac{2a}{x} + 2a^2\log(x) - 2a^2\log(1+ax)
\end{aligned}$$

Mathematica [A] time = 0.0101006, size = 32, normalized size = 1.

$$2a^2\log(x) - 2a^2\log(ax+1) + \frac{2a}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] -1/(2*x^2) + (2*a)/x + 2*a^2*Log[x] - 2*a^2*Log[1 + a*x]
```

Maple [A] time = 0.035, size = 31, normalized size = 1.

$$-\frac{1}{2\,x^2}+2\,\frac{a}{x}+2\,a^2\ln(x)-2\,a^2\ln(ax+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/x^3,x)
```

```
[Out] -1/2/x^2+2*a/x+2*a^2*ln(x)-2*a^2*ln(a*x+1)
```

Maxima [A] time = 0.991817, size = 41, normalized size = 1.28

$$-2\,a^2\log(ax+1)+2\,a^2\log(x)+\frac{4\,ax-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="maxima")
```

```
[Out] -2*a^2*log(a*x + 1) + 2*a^2*log(x) + 1/2*(4*a*x - 1)/x^2
```

Fricas [A] time = 1.8204, size = 89, normalized size = 2.78

$$-\frac{4\,a^2x^2\log(ax+1)-4\,a^2x^2\log(x)-4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^2*x^2*log(a*x + 1) - 4*a^2*x^2*log(x) - 4*a*x + 1)/x^2
```

Sympy [A] time = 0.669226, size = 26, normalized size = 0.81

$$-2a^2\left(-\log(x)+\log\left(x+\frac{1}{a}\right)\right)+\frac{4ax-1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/x**3,x)
```

```
[Out] -2*a**2*(-log(x) + log(x + 1/a)) + (4*a*x - 1)/(2*x**2)
```

<u>Giac</u> [A] time = 1.15121, size = 68, normalized size = 2.12

$$2\,a^2\log\left(\left|-\frac{1}{ax+1}+1\right|\right)+\frac{5\,a^2-\frac{6\,a^2}{ax+1}}{2\left(\frac{1}{ax+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="giac")
```

```
[Out] 2*a^2*log(abs(-1/(a*x + 1) + 1)) + 1/2*(5*a^2 - 6*a^2/(a*x + 1))/(1/(a*x +
1) - 1)^2
```

3.50 $\int \frac{e^{-2\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=39

$$-\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(ax+1)+\frac{a}{x^2}-\frac{1}{3x^3}$$

```
[Out] -1/(3*x^3) + a/x^2 - (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 + a*x]
```

Rubi [A] time = 0.0317947, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$-\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(ax+1)+\frac{a}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] -1/(3*x^3) + a/x^2 - (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 + a*x]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(ax)}}{x^4}\,dx &= \int \frac{1-ax}{x^4(1+ax)}\,dx\\ &= \int \left(\frac{1}{x^4}-\frac{2a}{x^3}+\frac{2a^2}{x^2}-\frac{2a^3}{x}+\frac{2a^4}{1+ax}\right)dx\\ &= -\frac{1}{3x^3}+\frac{a}{x^2}-\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(1+ax)\end{aligned}$$

Mathematica [A] time = 0.0117499, size = 39, normalized size = 1.

$$-\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(ax+1)+\frac{a}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] -1/(3*x^3) + a/x^2 - (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 + a*x]
```

Maple [A] time = 0.038, size = 38, normalized size = 1.

$$-\frac{1}{3\,x^{3}}+\frac{a}{x^{2}}-2\,\frac{a^{2}}{x}-2\,a^{3}\ln\left(x\right)+2\,a^{3}\ln\left(ax+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/x^4,x)
```

```
[Out] -1/3/x^3+a/x^2-2*a^2/x-2*a^3*ln(x)+2*a^3*ln(a*x+1)
```

Maxima [A] time = 0.949299, size = 51, normalized size = 1.31

$$2\,a^{3}\log\left(ax+1\right)-2\,a^{3}\log\left(x\right)-\frac{6\,a^{2}x^{2}-3\,ax+1}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="maxima")
```

```
[Out] 2*a^3*log(a*x + 1) - 2*a^3*log(x) - 1/3*(6*a^2*x^2 - 3*a*x + 1)/x^3
```

Fricas [A] time = 1.82843, size = 104, normalized size = 2.67

$$\frac{6\,a^{3}x^{3}\log\left(ax+1\right)-6\,a^{3}x^{3}\log\left(x\right)-6\,a^{2}x^{2}+3\,ax-1}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(6*a^3*x^3*log(a*x + 1) - 6*a^3*x^3*log(x) - 6*a^2*x^2 + 3*a*x - 1)/x^3
```

Sympy [A] time = 0.603, size = 36, normalized size = 0.92

$$-2a^{3}\left(\log\left(x\right)-\log\left(x+\frac{1}{a}\right)\right)-\frac{6a^{2}x^{2}-3ax+1}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/x**4,x)
```

```
[Out] -2*a**3*(log(x) - log(x + 1/a)) - (6*a**2*x**2 - 3*a*x + 1)/(3*x**3)
```

Giac [A] time = 1.19352, size = 84, normalized size = 2.15

$$-2\,a^{3}\log\left(\left|-\frac{1}{ax+1}+1\right|\right)+\frac{10\,a^{3}-\frac{24\,a^{3}}{ax+1}+\frac{15\,a^{3}}{(ax+1)^{2}}}{3\left(\frac{1}{ax+1}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(-1/(a*x + 1) + 1)) + 1/3*(10*a^3 - 24*a^3/(a*x + 1) + 15*a^3
/(a*x + 1)^2)/(1/(a*x + 1) - 1)^3
```

3.51 $\int e^{-3\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=131

$$-\frac{x^3\sqrt{1-a^2x^2}}{4a}+\frac{x^2\sqrt{1-a^2x^2}}{a^2}+\frac{9(2-3ax)\sqrt{1-a^2x^2}}{8a^4}+\frac{27\sqrt{1-a^2x^2}}{4a^4}+\frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}}+\frac{51\sin^{-1}(ax)}{8a^4}$$

```
[Out] (1 - a*x)^3/(a^4*Sqrt[1 - a^2*x^2]) + (27*Sqrt[1 - a^2*x^2])/(4*a^4) + (x^2
*Sqrt[1 - a^2*x^2])/a^2 - (x^3*Sqrt[1 - a^2*x^2])/(4*a) + (9*(2 - 3*a*x)*Sq
rt[1 - a^2*x^2])/(8*a^4) + (51*ArcSin[a*x])/(8*a^4)
```

Rubi [A] time = 0.689497, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6124, 1633, 1593, 12, 852, 1635, 1815, 27, 743, 641, 216}

$$-\frac{x^3\sqrt{1-a^2x^2}}{4a}+\frac{x^2\sqrt{1-a^2x^2}}{a^2}+\frac{9(2-3ax)\sqrt{1-a^2x^2}}{8a^4}+\frac{27\sqrt{1-a^2x^2}}{4a^4}+\frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}}+\frac{51\sin^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (1 - a*x)^3/(a^4*Sqrt[1 - a^2*x^2]) + (27*Sqrt[1 - a^2*x^2])/(4*a^4) + (x^2
*Sqrt[1 - a^2*x^2])/a^2 - (x^3*Sqrt[1 - a^2*x^2])/(4*a) + (9*(2 - 3*a*x)*Sq
rt[1 - a^2*x^2])/(8*a^4) + (51*ArcSin[a*x])/(8*a^4)
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}x^3\,dx = \int \frac{x^3(1-ax)^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx$$
$$= a\int \frac{\sqrt{1-a^2x^2}\left(\frac{x^3}{a}-x^4\right)}{(1+ax)^2}\,dx$$
$$= a\int \frac{\left(\frac{1}{a}-x\right)x^3\sqrt{1-a^2x^2}}{(1+ax)^2}\,dx$$
$$= a^2\int \frac{x^3\left(1-a^2x^2\right)^{3/2}}{a^2(1+ax)^3}\,dx$$
$$= \int \frac{x^3\left(1-a^2x^2\right)^{3/2}}{(1+ax)^3}\,dx$$
$$= \int \frac{x^3(1-ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx$$
$$= \frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}} - \int \frac{(1-ax)^2\left(-\frac{3}{a^3}+\frac{x}{a^2}-\frac{x^2}{a}\right)}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}} - \frac{x^3\sqrt{1-a^2x^2}}{4a} + \frac{\int \frac{\frac{12}{a}-28x+27ax^2-12a^2x^3}{\sqrt{1-a^2x^2}}\,dx}{4a^2}$$
$$= \frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{x^2\sqrt{1-a^2x^2}}{a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{\int \frac{-36a+108a^2x-81a^3x^2}{\sqrt{1-a^2x^2}}\,dx}{12a^4}$$
$$= \frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{x^2\sqrt{1-a^2x^2}}{a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{\int -\frac{9a(-2+3ax)^2}{\sqrt{1-a^2x^2}}\,dx}{12a^4}$$
$$= \frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{x^2\sqrt{1-a^2x^2}}{a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} + \frac{3\int \frac{(-2+3ax)^2}{\sqrt{1-a^2x^2}}\,dx}{4a^3}$$
$$= \frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{x^2\sqrt{1-a^2x^2}}{a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} + \frac{9(2-3ax)\sqrt{1-a^2x^2}}{8a^4} - \frac{3\int \frac{-17a^2+18a^3x}{\sqrt{1-a^2x^2}}\,dx}{8a^5}$$
$$= \frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{27\sqrt{1-a^2x^2}}{4a^4} + \frac{x^2\sqrt{1-a^2x^2}}{a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} + \frac{9(2-3ax)\sqrt{1-a^2x^2}}{8a^4} + \frac{51\int \frac{1}{\sqrt{1-a^2x}}$$
$$= \frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{27\sqrt{1-a^2x^2}}{4a^4} + \frac{x^2\sqrt{1-a^2x^2}}{a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} + \frac{9(2-3ax)\sqrt{1-a^2x^2}}{8a^4} + \frac{51\sin^{-1}(ax}{8a^4}$$

Mathematica [A] time = 0.0554135, size = 70, normalized size = 0.53

$$\sqrt{1-a^2x^2}\left(\frac{x^2}{a^2}-\frac{19x}{8a^3}+\frac{4}{a^4(ax+1)}+\frac{6}{a^4}-\frac{x^3}{4a}\right)+\frac{51\sin^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]*(6/a^4 - (19*x)/(8*a^3) + x^2/a^2 - x^3/(4*a) + 4/(a^4*(1
 + a*x))) + (51*ArcSin[a*x])/(8*a^4)
```

Maple [B] time = 0.054, size = 235, normalized size = 1.8

$$\frac{x}{4a^3}\left(-a^2x^2+1\right)^{\frac{3}{2}}+\frac{3x}{8a^3}\sqrt{-a^2x^2+1}+\frac{3}{8a^3}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+5\frac{\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)\right)^{5/2}}{a^6\left(x+a^{-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/4/a^3*x*(-a^2*x^2+1)^(3/2)+3/8/a^3*x*(-a^2*x^2+1)^(1/2)+3/8/a^3/(a^2)^(1/
2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+5/a^6/(x+1/a)^2*(-a^2*(x+1/a)^2
+2*a*(x+1/a))^(5/2)+4/a^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+6/a^3*(-a^2*(x
+1/a)^2+2*a*(x+1/a))^(1/2)*x+6/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(
x+1/a)^2+2*a*(x+1/a))^(1/2))+1/a^7/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(
5/2)
```

Maxima [C] time = 1.48586, size = 290, normalized size = 2.21

$$-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{a^6x^2+2a^5x+a^4}+\frac{3\left(-a^2x^2+1\right)^{\frac{3}{2}}}{2\left(a^5x+a^4\right)}+\frac{6\sqrt{-a^2x^2+1}}{a^5x+a^4}+\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}{4a^3}-\frac{3\sqrt{a^2x^2+4ax+3}x}{2a^3}+\frac{3\sqrt{-a^2x^2+1}x}{8a^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -(-a^2*x^2 + 1)^(3/2)/(a^6*x^2 + 2*a^5*x + a^4) + 3/2*(-a^2*x^2 + 1)^(3/2)/
(a^5*x + a^4) + 6*sqrt(-a^2*x^2 + 1)/(a^5*x + a^4) + 1/4*(-a^2*x^2 + 1)^(3/
2)*x/a^3 - 3/2*sqrt(a^2*x^2 + 4*a*x + 3)*x/a^3 + 3/8*sqrt(-a^2*x^2 + 1)*x/a
^3 - (-a^2*x^2 + 1)^(3/2)/a^4 + 3/2*I*arcsin(a*x + 2)/a^4 + 63/8*arcsin(a*x
)/a^4 - 3*sqrt(a^2*x^2 + 4*a*x + 3)/a^4 + 9/2*sqrt(-a^2*x^2 + 1)/a^4
```

Fricas [A] time = 2.05419, size = 216, normalized size = 1.65

$$\frac{80ax-102(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(2a^4x^4-6a^3x^3+11a^2x^2-29ax-80\right)\sqrt{-a^2x^2+1}+80}{8\left(a^5x+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(80*a*x - 102*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (2*a^4
*x^4 - 6*a^3*x^3 + 11*a^2*x^2 - 29*a*x - 80)*sqrt(-a^2*x^2 + 1) + 80)/(a^5*
x + a^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)
```

Giac [A] time = 1.22591, size = 128, normalized size = 0.98

$$-\frac{1}{8}\sqrt{-a^2x^2+1}\left(\left(2x\left(\frac{x}{a}-\frac{4}{a^2}\right)+\frac{19}{a^3}\right)x-\frac{48}{a^4}\right)+\frac{51\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{8a^3|a|}-\frac{8}{a^3\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-a^2*x^2 + 1)*((2*x*(x/a - 4/a^2) + 19/a^3)*x - 48/a^4) + 51/8*ar
csin(a*x)*sgn(a)/(a^3*abs(a)) - 8/(a^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^
2*x) + 1)*abs(a))
```

3.52 $\int e^{-3\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=95

$$-\frac{(1-ax)^3}{a^3\sqrt{1-a^2x^2}}-\frac{(3-ax)^2\sqrt{1-a^2x^2}}{3a^3}-\frac{(28-3ax)\sqrt{1-a^2x^2}}{6a^3}-\frac{11\sin^{-1}(ax)}{2a^3}$$

```
[Out] -((1 - a*x)^3/(a^3*Sqrt[1 - a^2*x^2])) - ((28 - 3*a*x)*Sqrt[1 - a^2*x^2])/(
6*a^3) - ((3 - a*x)^2*Sqrt[1 - a^2*x^2])/(3*a^3) - (11*ArcSin[a*x])/(2*a^3)
```

Rubi [A] time = 0.637627, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6124, 1633, 1593, 12, 852, 1635, 1654, 780, 216}

$$-\frac{(1-ax)^3}{a^3\sqrt{1-a^2x^2}}-\frac{(3-ax)^2\sqrt{1-a^2x^2}}{3a^3}-\frac{(28-3ax)\sqrt{1-a^2x^2}}{6a^3}-\frac{11\sin^{-1}(ax)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((1 - a*x)^3/(a^3*Sqrt[1 - a^2*x^2])) - ((28 - 3*a*x)*Sqrt[1 - a^2*x^2])/(
6*a^3) - ((3 - a*x)^2*Sqrt[1 - a^2*x^2])/(3*a^3) - (11*ArcSin[a*x])/(2*a^3)
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1-ax)^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx\\
&= a\int \frac{\sqrt{1-a^2x^2}\left(\frac{x^2}{a}-x^3\right)}{(1+ax)^2}\,dx\\
&= a\int \frac{\left(\frac{1}{a}-x\right)x^2\sqrt{1-a^2x^2}}{(1+ax)^2}\,dx\\
&= a^2\int \frac{x^2\left(1-a^2x^2\right)^{3/2}}{a^2(1+ax)^3}\,dx\\
&= \int \frac{x^2\left(1-a^2x^2\right)^{3/2}}{(1+ax)^3}\,dx\\
&= \int \frac{x^2(1-ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx\\
&= -\frac{(1-ax)^3}{a^3\sqrt{1-a^2x^2}} - \int \frac{\left(\frac{3}{a^2}-\frac{x}{a}\right)(1-ax)^2}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{(1-ax)^3}{a^3\sqrt{1-a^2x^2}} - \frac{(3-ax)^2\sqrt{1-a^2x^2}}{3a^3} + \frac{1}{3}\int \frac{\left(\frac{3}{a^2}-\frac{x}{a}\right)(-5+3ax)}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{(1-ax)^3}{a^3\sqrt{1-a^2x^2}} - \frac{(28-3ax)\sqrt{1-a^2x^2}}{6a^3} - \frac{(3-ax)^2\sqrt{1-a^2x^2}}{3a^3} - \frac{11\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^2}\\
&= -\frac{(1-ax)^3}{a^3\sqrt{1-a^2x^2}} - \frac{(28-3ax)\sqrt{1-a^2x^2}}{6a^3} - \frac{(3-ax)^2\sqrt{1-a^2x^2}}{3a^3} - \frac{11\sin^{-1}(ax)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.0688988, size = 58, normalized size = 0.61

$$-\frac{\frac{\sqrt{1-a^2x^2}\left(2a^3x^3-7a^2x^2+19ax+52\right)}{ax+1}+33\sin^{-1}(ax)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(52 + 19*a*x - 7*a^2*x^2 + 2*a^3*x^3))/(1 + a*x) + 33*
ArcSin[a*x])/(6*a^3)
```

Maple [B] time = 0.048, size = 170, normalized size = 1.8

$$-4\,\frac{\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right)^{5/2}}{a^5\left(x+a^{-1}\right)^2}-\frac{11}{3\,a^3}\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right)^{\frac{3}{2}}-\frac{11\,x}{2\,a^2}\sqrt{-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -4/a^5/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-11/3/a^3*(-a^2*(x+1/a)^
2+2*a*(x+1/a))^(3/2)-11/2/a^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-11/2/a^2
```

```
/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-1/a^6
/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)
```

Maxima [C] time = 1.49219, size = 239, normalized size = 2.52

$$\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{a^5x^2+2\,a^4x+a^3}-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{a^4x+a^3}-\frac{6\,\sqrt{-a^2x^2+1}}{a^4x+a^3}+\frac{\sqrt{a^2x^2+4\,ax+3}x}{2\,a^2}+\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3\,a^3}-\frac{i\,\arcsin\left(ax+2\right)}{2\,a^3}-\frac{6\,\arcsin}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] (-a^2*x^2 + 1)^(3/2)/(a^5*x^2 + 2*a^4*x + a^3) - (-a^2*x^2 + 1)^(3/2)/(a^4*
x + a^3) - 6*sqrt(-a^2*x^2 + 1)/(a^4*x + a^3) + 1/2*sqrt(a^2*x^2 + 4*a*x +
3)*x/a^2 + 1/3*(-a^2*x^2 + 1)^(3/2)/a^3 - 1/2*I*arcsin(a*x + 2)/a^3 - 6*arc
sin(a*x)/a^3 + sqrt(a^2*x^2 + 4*a*x + 3)/a^3 - 3*sqrt(-a^2*x^2 + 1)/a^3
```

Fricas [A] time = 1.87613, size = 198, normalized size = 2.08

$$-\frac{52\,ax-66\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^3x^3-7\,a^2x^2+19\,ax+52\right)\sqrt{-a^2x^2+1}+52}{6\left(a^4x+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(52*a*x - 66*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^3
*x^3 - 7*a^2*x^2 + 19*a*x + 52)*sqrt(-a^2*x^2 + 1) + 52)/(a^4*x + a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)
```

Giac [A] time = 1.24343, size = 117, normalized size = 1.23

$$-\frac{1}{6}\sqrt{-a^2x^2+1}\left(x\left(\frac{2\,x}{a}-\frac{9}{a^2}\right)+\frac{28}{a^3}\right)-\frac{11\,\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{2\,a^2|a|}+\frac{8}{a^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-a^2*x^2 + 1)*(x*(2*x/a - 9/a^2) + 28/a^3) - 11/2*arcsin(a*x)*sgn
(a)/(a^2*abs(a)) + 8/(a^2*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs
(a))
```

3.53 $\int e^{-3\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=86

$$\frac{(1-a^2x^2)^{5/2}}{a^2(ax+1)^3}+\frac{3(1-a^2x^2)^{3/2}}{2a^2(ax+1)}+\frac{9\sqrt{1-a^2x^2}}{2a^2}+\frac{9\sin^{-1}(ax)}{2a^2}$$

```
[Out] (9*Sqrt[1 - a^2*x^2])/(2*a^2) + (3*(1 - a^2*x^2)^(3/2))/(2*a^2*(1 + a*x)) +
 (1 - a^2*x^2)^(5/2)/(a^2*(1 + a*x)^3) + (9*ArcSin[a*x])/(2*a^2)
```

Rubi [A] time = 0.364178, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {6124, 1633, 1593, 12, 793, 665, 216}

$$\frac{(1-a^2x^2)^{5/2}}{a^2(ax+1)^3}+\frac{3(1-a^2x^2)^{3/2}}{2a^2(ax+1)}+\frac{9\sqrt{1-a^2x^2}}{2a^2}+\frac{9\sin^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (9*Sqrt[1 - a^2*x^2])/(2*a^2) + (3*(1 - a^2*x^2)^(3/2))/(2*a^2*(1 + a*x)) +
 (1 - a^2*x^2)^(5/2)/(a^2*(1 + a*x)^3) + (9*ArcSin[a*x])/(2*a^2)
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
```

```
+ 1, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}x\,dx &= \int \frac{x(1-ax)^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx\\
&= a\int \frac{\left(\frac{x}{a}-x^2\right)\sqrt{1-a^2x^2}}{(1+ax)^2}\,dx\\
&= a\int \frac{\left(\frac{1}{a}-x\right)x\sqrt{1-a^2x^2}}{(1+ax)^2}\,dx\\
&= a^2\int \frac{x\left(1-a^2x^2\right)^{3/2}}{a^2(1+ax)^3}\,dx\\
&= \int \frac{x\left(1-a^2x^2\right)^{3/2}}{(1+ax)^3}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1+ax)^3}+\frac{3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(1+ax)^2}\,dx}{a}\\
&= \frac{3\left(1-a^2x^2\right)^{3/2}}{2a^2(1+ax)}+\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1+ax)^3}+\frac{9\int \frac{\sqrt{1-a^2x^2}}{1+ax}\,dx}{2a}\\
&= \frac{9\sqrt{1-a^2x^2}}{2a^2}+\frac{3\left(1-a^2x^2\right)^{3/2}}{2a^2(1+ax)}+\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1+ax)^3}+\frac{9\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a}\\
&= \frac{9\sqrt{1-a^2x^2}}{2a^2}+\frac{3\left(1-a^2x^2\right)^{3/2}}{2a^2(1+ax)}+\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1+ax)^3}+\frac{9\sin^{-1}(ax)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.0533774, size = 44, normalized size = 0.51

$$\frac{\sqrt{1-a^2x^2}\left(-ax+\frac{8}{ax+1}+6\right)+9\sin^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(6 - a*x + 8/(1 + a*x)) + 9*ArcSin[a*x])/(2*a^2)
```

Maple [B] time = 0.043, size = 169, normalized size = 2.

$$3\frac{\left(-a^{2}\left(x+a^{-1}\right)^{2}+2a\left(x+a^{-1}\right)\right)^{5/2}}{a^{4}\left(x+a^{-1}\right)^{2}}+3\frac{\left(-a^{2}\left(x+a^{-1}\right)^{2}+2a\left(x+a^{-1}\right)\right)^{3/2}}{a^{2}}+\frac{9x}{2a}\sqrt{-a^{2}\left(x+a^{-1}\right)^{2}+2a\left(x+a^{-1}\right)}+\frac{9}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 3/a^4/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)+3/a^2*(-a^2*(x+1/a)^2+2*
a*(x+1/a))^(3/2)+9/2/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x+9/2/a/(a^2)^(1/
2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))+1/a^5/(x+1/a)^3
*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)
```

Maxima [A] time = 1.46216, size = 149, normalized size = 1.73

$$-\frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{a^{4}x^{2}+2a^{3}x+a^{2}}+\frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{2\left(a^{3}x+a^{2}\right)}+\frac{6\sqrt{-a^{2}x^{2}+1}}{a^{3}x+a^{2}}+\frac{9\arcsin\left(ax\right)}{2a^{2}}+\frac{3\sqrt{-a^{2}x^{2}+1}}{2a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -(-a^2*x^2 + 1)^(3/2)/(a^4*x^2 + 2*a^3*x + a^2) + 1/2*(-a^2*x^2 + 1)^(3/2)/
(a^3*x + a^2) + 6*sqrt(-a^2*x^2 + 1)/(a^3*x + a^2) + 9/2*arcsin(a*x)/a^2 +
3/2*sqrt(-a^2*x^2 + 1)/a^2
```

Fricas [A] time = 1.92971, size = 177, normalized size = 2.06

$$\frac{14ax-18(ax+1)\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)-\left(a^{2}x^{2}-5ax-14\right)\sqrt{-a^{2}x^{2}+1}+14}{2\left(a^{3}x+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(14*a*x - 18*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (a^2*x^
2 - 5*a*x - 14)*sqrt(-a^2*x^2 + 1) + 14)/(a^3*x + a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)
```

Giac [A] time = 1.17709, size = 105, normalized size = 1.22

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\left(\frac{x}{a}-\frac{6}{a^2}\right)+\frac{9\arcsin(ax)\operatorname{sgn}(a)}{2a|a|}-\frac{8}{a\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(x/a - 6/a^2) + 9/2*arcsin(a*x)*sgn(a)/(a*abs(a)) -
 8/(a*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

3.54 $\int e^{-3\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=56

$$-\frac{2(1-ax)^2}{a\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{a}-\frac{3\sin^{-1}(ax)}{a}$$

```
[Out] (-2*(1 - a*x)^2)/(a*Sqrt[1 - a^2*x^2]) - (3*Sqrt[1 - a^2*x^2])/a - (3*ArcSi
n[a*x])/a
```

Rubi [A] time = 0.0492489, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6123, 853, 669, 641, 216}

$$-\frac{2(1-ax)^2}{a\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{a}-\frac{3\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-3*ArcTanh[a*x]),x]
```

```
[Out] (-2*(1 - a*x)^2)/(a*Sqrt[1 - a^2*x^2]) - (3*Sqrt[1 - a^2*x^2])/a - (3*ArcSi
n[a*x])/a
```

Rule 6123

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 + a*x)^((n + 1)/2)/(
(1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2]), x] /; FreeQ[a, x] && IntegerQ[(n
- 1)/2]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}\,dx &= \int \frac{(1-ax)^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx \\
&= \int \frac{(1-ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx \\
&= -\frac{2(1-ax)^2}{a\sqrt{1-a^2x^2}} - 3\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{2(1-ax)^2}{a\sqrt{1-a^2x^2}} - \frac{3\sqrt{1-a^2x^2}}{a} - 3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{2(1-ax)^2}{a\sqrt{1-a^2x^2}} - \frac{3\sqrt{1-a^2x^2}}{a} - \frac{3\sin^{-1}(ax)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0298534, size = 39, normalized size = 0.7

$$
\frac{\sqrt{1-a^2x^2}\left(-\frac{4}{ax+1}-1\right)}{a} - \frac{3\sin^{-1}(ax)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-1 - 4/(1 + a*x)))/a - (3*ArcSin[a*x])/a
```

Maple [B] time = 0.041, size = 164, normalized size = 2.9

$$
-\frac{1}{a^4\left(x+a^{-1}\right)^3}\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)\right)^{\frac{5}{2}} - 2\frac{\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)\right)^{5/2}}{a^3\left(x+a^{-1}\right)^2} - 2\frac{\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x-\right.\right.}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/a^4/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-2/a^3/(x+1/a)^2*(-a^2*(
x+1/a)^2+2*a*(x+1/a))^(5/2)-2/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)-3*(-a^2*
(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+
1/a)^2+2*a*(x+1/a))^(1/2))
```

Maxima [A] time = 1.43039, size = 85, normalized size = 1.52

$$
\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{a^3x^2+2\,a^2x+a} - \frac{3\arcsin(ax)}{a} - \frac{6\sqrt{-a^2x^2+1}}{a^2x+a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] (-a^2*x^2 + 1)^(3/2)/(a^3*x^2 + 2*a^2*x + a) - 3*arcsin(a*x)/a - 6*sqrt(-a^
2*x^2 + 1)/(a^2*x + a)
```

Fricas [A] time = 2.01886, size = 149, normalized size = 2.66

$$-\frac{5\,ax - 6\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2+1}(ax+5) + 5}{a^2x + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(5*a*x - 6*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x + 5) + 5)/(a^2*x + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)
```

Giac [A] time = 1.21332, size = 86, normalized size = 1.54

$$-\frac{3\arcsin(ax)\,\mathrm{sgn}(a)}{|a|} - \frac{\sqrt{-a^2x^2+1}}{a} + \frac{8}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} + 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -3*arcsin(a*x)*sgn(a)/abs(a) - sqrt(-a^2*x^2 + 1)/a + 8/(((sqrt(-a^2*x^2 +
1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

3.55 $\int \frac{e^{-3\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=45

$$\frac{4\sqrt{1-a^2x^2}}{ax+1}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\sin^{-1}(ax)$$

```
[Out] (4*Sqrt[1 - a^2*x^2])/(1 + a*x) + ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.708101, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6124, 6742, 216, 266, 63, 208, 651}

$$\frac{4\sqrt{1-a^2x^2}}{ax+1}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] (4*Sqrt[1 - a^2*x^2])/(1 + a*x) + ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{(1-ax)^2}{x(1+ax)\sqrt{1-a^2x^2}}\,dx \\
&= \int \left(\frac{a}{\sqrt{1-a^2x^2}} + \frac{1}{x\sqrt{1-a^2x^2}} - \frac{4a}{(1+ax)\sqrt{1-a^2x^2}}\right)dx \\
&= a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx - (4a)\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx + \int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= \frac{4\sqrt{1-a^2x^2}}{1+ax} + \sin^{-1}(ax) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \frac{4\sqrt{1-a^2x^2}}{1+ax} + \sin^{-1}(ax) - \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2} \\
&= \frac{4\sqrt{1-a^2x^2}}{1+ax} + \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0369465, size = 49, normalized size = 1.09

$$\frac{4\sqrt{1-a^2x^2}}{ax+1} - \log\left(\sqrt{1-a^2x^2}+1\right) + \sin^{-1}(ax) + \log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] (4*Sqrt[1 - a^2*x^2])/(1 + a*x) + ArcSin[a*x] + Log[x] - Log[1 + Sqrt[1 - a
^2*x^2]]
```

Maple [B] time = 0.049, size = 200, normalized size = 4.4

$$\frac{1}{a^2\left(x+a^{-1}\right)^2}\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right)^{\frac{5}{2}} + \frac{2}{3}\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right)^{\frac{3}{2}} + a\sqrt{-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x)
```

```
[Out] 1/a^2/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)+2/3*(-a^2*(x+1/a)^2+2*a*
(x+1/a))^(3/2)+a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x+a/(a^2)^(1/2)*arctan(
(a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))+1/3*(-a^2*x^2+1)^(3/2)+(-
a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2))+1/a^3/(x+1/a)^3*(-a^2*(x+1/a
)^2+2*a*(x+1/a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x), x)
```

Fricas [B] time = 2.06162, size = 193, normalized size = 4.29

$$\frac{4\,ax-2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+(ax+1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+4\,\sqrt{-a^2x^2+1}+4}{ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (4*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a*x + 1)*log
((sqrt(-a^2*x^2 + 1) - 1)/x) + 4*sqrt(-a^2*x^2 + 1) + 4)/(a*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}{x\,(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(x*(a*x + 1)**3), x)
```

Giac [B] time = 1.16404, size = 116, normalized size = 2.58

$$\frac{a\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{|a|}-\frac{a\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\frac{8\,a}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="giac")
```

```
[Out] a*arcsin(a*x)*sgn(a)/abs(a) - a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
2*a)/(a^2*abs(x)))/abs(a) - 8*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) +
 1)*abs(a))
```

3.56 $\int \frac{e^{-3\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=62

$$-\frac{4a\sqrt{1-a^2x^2}}{ax+1}-\frac{\sqrt{1-a^2x^2}}{x}+3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -(Sqrt[1 - a^2*x^2]/x) - (4*a*Sqrt[1 - a^2*x^2])/(1 + a*x) + 3*a*ArcTanh[Sq
rt[1 - a^2*x^2]]
```

Rubi [A] time = 0.68961, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6124, 6742, 264, 266, 63, 208, 651}

$$-\frac{4a\sqrt{1-a^2x^2}}{ax+1}-\frac{\sqrt{1-a^2x^2}}{x}+3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/x) - (4*a*Sqrt[1 - a^2*x^2])/(1 + a*x) + 3*a*ArcTanh[Sq
rt[1 - a^2*x^2]]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1-ax)^2}{x^2(1+ax)\sqrt{1-a^2x^2}}\,dx\\
&= \int \left(\frac{1}{x^2\sqrt{1-a^2x^2}}-\frac{3a}{x\sqrt{1-a^2x^2}}+\frac{4a^2}{(1+ax)\sqrt{1-a^2x^2}}\right)dx\\
&= -\left((3a)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\right)+\left(4a^2\right)\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx+\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{x}-\frac{4a\sqrt{1-a^2x^2}}{1+ax}-\frac{1}{2}(3a)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-a^2x^2}}{x}-\frac{4a\sqrt{1-a^2x^2}}{1+ax}+\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a}\\
&= -\frac{\sqrt{1-a^2x^2}}{x}-\frac{4a\sqrt{1-a^2x^2}}{1+ax}+3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0517184, size = 57, normalized size = 0.92

$$\sqrt{1-a^2x^2}\left(-\frac{4a}{ax+1}-\frac{1}{x}\right)+3a\log\left(\sqrt{1-a^2x^2}+1\right)-3a\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]*(-x^(-1) - (4*a)/(1 + a*x)) - 3*a*Log[x] + 3*a*Log[1 + Sq
rt[1 - a^2*x^2]]
```

Maple [B] time = 0.052, size = 261, normalized size = 4.2

$$-\frac{1}{x}\left(-a^2x^2+1\right)^{\frac{5}{2}}-a^2x\left(-a^2x^2+1\right)^{\frac{3}{2}}-\frac{3\,a^2x}{2}\sqrt{-a^2x^2+1}-\frac{3\,a^2}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+a\left(-a^2\left(x+a^{-1}\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x)
```

```
[Out] -1/x*(-a^2*x^2+1)^(5/2)-a^2*x*(-a^2*x^2+1)^(3/2)-3/2*a^2*x*(-a^2*x^2+1)^(1/
2)-3/2*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+a*(-a^2*(x+
```

```
1/a)^2+2*a*(x+1/a))^(3/2)+3/2*a^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x+3/2*
a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-a*
(-a^2*x^2+1)^(3/2)+3*a*arctanh(1/(-a^2*x^2+1)^(1/2))-3*a*(-a^2*x^2+1)^(1/2)
-1/a^2/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^2), x)
```

Fricas [A] time = 1.94304, size = 161, normalized size = 2.6

$$-\frac{4\,a^2x^2+4\,ax+3\left(a^2x^2+ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}(5\,ax+1)}{ax^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(4*a^2*x^2 + 4*a*x + 3*(a^2*x^2 + a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) + s
qrt(-a^2*x^2 + 1)*(5*a*x + 1))/(a*x^2 + x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}{x^2(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(x**2*(a*x + 1)**3), x)
```

Giac [B] time = 1.16566, size = 203, normalized size = 3.27

$$\frac{3\,a^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\frac{\left(a^2+\frac{17\left(\sqrt{-a^2x^2+1}|a|+a\right)}{x}\right)a^2x}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}-\frac{\sqrt{-a^2x^2+1}|a|+a}{2\,x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 3*a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a)
+ 1/2*(a^2 + 17*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/x)*a^2*x/((sqrt(-a^2*x^2 +
1)*abs(a) + a)*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a)) - 1/2*
(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(x*abs(a))
```

3.57 $\int \frac{e^{-3\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=90

$$\frac{4a^2\sqrt{1-a^2x^2}}{ax+1}+\frac{3a\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -Sqrt[1 - a^2*x^2]/(2*x^2) + (3*a*Sqrt[1 - a^2*x^2])/x + (4*a^2*Sqrt[1 - a^
2*x^2])/(1 + a*x) - (9*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rubi [A] time = 0.735463, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6124, 6742, 266, 51, 63, 208, 264, 651}

$$\frac{4a^2\sqrt{1-a^2x^2}}{ax+1}+\frac{3a\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*x^2) + (3*a*Sqrt[1 - a^2*x^2])/x + (4*a^2*Sqrt[1 - a^
2*x^2])/(1 + a*x) - (9*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1-ax)^2}{x^3(1+ax)\sqrt{1-a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^3\sqrt{1-a^2x^2}} - \frac{3a}{x^2\sqrt{1-a^2x^2}} + \frac{4a^2}{x\sqrt{1-a^2x^2}} - \frac{4a^3}{(1+ax)\sqrt{1-a^2x^2}}\right) dx \\
&= -\left((3a)\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx\right) + \left(4a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - \left(4a^3\right)\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx + \int \frac{1}{x^3\sqrt{\ldots}} \\
&= \frac{3a\sqrt{1-a^2x^2}}{x} + \frac{4a^2\sqrt{1-a^2x^2}}{1+ax} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}}\,dx, x, x^2\right) + \left(2a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2\ldots}}\right. \\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2} + \frac{3a\sqrt{1-a^2x^2}}{x} + \frac{4a^2\sqrt{1-a^2x^2}}{1+ax} - 4\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right) + \frac{1}{4}a^2\,\text{Sub}\ldots \\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2} + \frac{3a\sqrt{1-a^2x^2}}{x} + \frac{4a^2\sqrt{1-a^2x^2}}{1+ax} - 4a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,d\ldots\right. \\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2} + \frac{3a\sqrt{1-a^2x^2}}{x} + \frac{4a^2\sqrt{1-a^2x^2}}{1+ax} - \frac{9}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0609339, size = 75, normalized size = 0.83

$$\sqrt{1-a^2x^2}\left(\frac{4a^2}{ax+1}+\frac{3a}{x}-\frac{1}{2x^2}\right)-\frac{9}{2}a^2\log\left(\sqrt{1-a^2x^2}+1\right)+\frac{9}{2}a^2\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] Sqrt[1 - a^2*x^2]*(-1/(2*x^2) + (3*a)/x + (4*a^2)/(1 + a*x)) + (9*a^2*Log[x
])/2 - (9*a^2*Log[1 + Sqrt[1 - a^2*x^2]])/2
```

Maple [B] time = 0.066, size = 319, normalized size = 3.5

$$3\frac{a\left(-a^{2}x^{2}+1\right)^{5/2}}{x}+3a^{3}x\left(-a^{2}x^{2}+1\right)^{3/2}+\frac{9xa^{3}}{2}\sqrt{-a^{2}x^{2}+1}+\frac{9a^{3}}{2}\arctan\left(x\sqrt{a^{2}}\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)\frac{1}{\sqrt{a^{2}}}-\frac{1}{\left(x+a^{-1}\right)^{2}}\left(-a^{2}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x)
```

```
[Out] 3*a/x*(-a^2*x^2+1)^(5/2)+3*a^3*x*(-a^2*x^2+1)^(3/2)+9/2*a^3*x*(-a^2*x^2+1)^
(1/2)+9/2*a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-1/(x+1/a
)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-3*a^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(
3/2)-9/2*a^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-9/2*a^3/(a^2)^(1/2)*arcta
n((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))+3/2*a^2*(-a^2*x^2+1)^(3
/2)+9/2*a^2*(-a^2*x^2+1)^(1/2)-9/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))-1/2/x^
2*(-a^2*x^2+1)^(5/2)+1/a/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{(ax+1)^{3}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^3), x)
```

Fricas [A] time = 1.95905, size = 196, normalized size = 2.18

$$\frac{8a^{3}x^{3}+8a^{2}x^{2}+9\left(a^{3}x^{3}+a^{2}x^{2}\right)\log\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{x}\right)+\left(14a^{2}x^{2}+5ax-1\right)\sqrt{-a^{2}x^{2}+1}}{2\left(ax^{3}+x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(8*a^3*x^3 + 8*a^2*x^2 + 9*(a^3*x^3 + a^2*x^2)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) + (14*a^2*x^2 + 5*a*x - 1)*sqrt(-a^2*x^2 + 1))/(a*x^3 + x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}{x^{3}(ax+1)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(x**3*(a*x + 1)**3), x)
```

Giac [B] time = 1.21042, size = 289, normalized size = 3.21

$$\frac{\left(a^3 - \frac{11\left(\sqrt{-a^2x^2+1}|a|+a\right)a}{x} - \frac{76\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{ax^2}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}|a|+a\right)^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|} - \frac{9\,a^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|} + \frac{\frac{12\left(\sqrt{-a^2x^2+1}|a|+a\right)a|a|}{x} - \frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)^2|a|}{ax^2}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3 - 11*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 76*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^2/(a*x^2))*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*((sqrt(-a
^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a)) - 9/2*a^3*log(1/2*abs(-2*sqrt(
-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 1/8*(12*(sqrt(-a^2*x^2 +
 1)*abs(a) + a)*a*abs(a)/x - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)/(a*x^
2))/a^2
```

3.58 $\int \frac{e^{-3\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=116

$$-\frac{4a^3\sqrt{1-a^2x^2}}{ax+1}-\frac{14a^2\sqrt{1-a^2x^2}}{3x}+\frac{3a\sqrt{1-a^2x^2}}{2x^2}-\frac{\sqrt{1-a^2x^2}}{3x^3}+\frac{11}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -Sqrt[1 - a^2*x^2]/(3*x^3) + (3*a*Sqrt[1 - a^2*x^2])/(2*x^2) - (14*a^2*Sqrt
[1 - a^2*x^2])/(3*x) - (4*a^3*Sqrt[1 - a^2*x^2])/(1 + a*x) + (11*a^3*ArcTan
h[Sqrt[1 - a^2*x^2]])/2
```

Rubi [A] time = 0.747871, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6124, 6742, 271, 264, 266, 51, 63, 208, 651}

$$-\frac{4a^3\sqrt{1-a^2x^2}}{ax+1}-\frac{14a^2\sqrt{1-a^2x^2}}{3x}+\frac{3a\sqrt{1-a^2x^2}}{2x^2}-\frac{\sqrt{1-a^2x^2}}{3x^3}+\frac{11}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(3*x^3) + (3*a*Sqrt[1 - a^2*x^2])/(2*x^2) - (14*a^2*Sqrt
[1 - a^2*x^2])/(3*x) - (4*a^3*Sqrt[1 - a^2*x^2])/(1 + a*x) + (11*a^3*ArcTan
h[Sqrt[1 - a^2*x^2]])/2
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{x^4}\,dx &= \int \frac{(1-ax)^2}{x^4(1+ax)\sqrt{1-a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^4\sqrt{1-a^2x^2}} - \frac{3a}{x^3\sqrt{1-a^2x^2}} + \frac{4a^2}{x^2\sqrt{1-a^2x^2}} - \frac{4a^3}{x\sqrt{1-a^2x^2}} + \frac{4a^4}{(1+ax)\sqrt{1-a^2x^2}}\right) dx \\
&= -\left((3a)\int \frac{1}{x^3\sqrt{1-a^2x^2}}\,dx\right) + \left(4a^2\right)\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx - \left(4a^3\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx + \left(4a^4\right)\int \frac{}{(1+ \\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{4a^2\sqrt{1-a^2x^2}}{x} - \frac{4a^3\sqrt{1-a^2x^2}}{1+ax} - \frac{1}{2}(3a)\,\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}}\,dx, x, x^2\right) + \frac{1}{3}\left(2a^2\right. \\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} + \frac{3a\sqrt{1-a^2x^2}}{2x^2} - \frac{14a^2\sqrt{1-a^2x^2}}{3x} - \frac{4a^3\sqrt{1-a^2x^2}}{1+ax} + (4a)\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{\right. \\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} + \frac{3a\sqrt{1-a^2x^2}}{2x^2} - \frac{14a^2\sqrt{1-a^2x^2}}{3x} - \frac{4a^3\sqrt{1-a^2x^2}}{1+ax} + 4a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) + \frac{1}{2}(3 \\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} + \frac{3a\sqrt{1-a^2x^2}}{2x^2} - \frac{14a^2\sqrt{1-a^2x^2}}{3x} - \frac{4a^3\sqrt{1-a^2x^2}}{1+ax} + \frac{11}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.075407, size = 82, normalized size = 0.71

$$\frac{1}{6}\left(-\frac{\sqrt{1-a^2x^2}\left(52a^3x^3+19a^2x^2-7ax+2\right)}{x^3(ax+1)} + 33a^3\log\left(\sqrt{1-a^2x^2}+1\right) - 33a^3\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] (-((Sqrt[1 - a^2*x^2]*(2 - 7*a*x + 19*a^2*x^2 + 52*a^3*x^3))/(x^3*(1 + a*x)
)) - 33*a^3*Log[x] + 33*a^3*Log[1 + Sqrt[1 - a^2*x^2]])/6
```

Maple [B] time = 0.057, size = 338, normalized size = 2.9

$$-\frac{16a^2}{3x}\left(-a^2x^2+1\right)^{\frac{5}{2}}-\frac{16a^4x}{3}\left(-a^2x^2+1\right)^{\frac{3}{2}}-8a^4x\sqrt{-a^2x^2+1}-8\frac{a^4}{\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+2\frac{a\left(-a^2\left(x+a^{-1}\right)^2+2\right.}{\left(x+a^{-1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x)
```

```
[Out] -16/3*a^2/x*(-a^2*x^2+1)^(5/2)-16/3*a^4*x*(-a^2*x^2+1)^(3/2)-8*a^4*x*(-a^2*
x^2+1)^(1/2)-8*a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+2*a
/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)+16/3*a^3*(-a^2*(x+1/a)^2+2*a*
(x+1/a))^(3/2)+8*a^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x+8*a^4/(a^2)^(1/2)
*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-11/6*a^3*(-a^2*x^
2+1)^(3/2)-11/2*a^3*(-a^2*x^2+1)^(1/2)+11/2*a^3*arctanh(1/(-a^2*x^2+1)^(1/2
))+3/2*a/x^2*(-a^2*x^2+1)^(5/2)-1/3/x^3*(-a^2*x^2+1)^(5/2)-1/(x+1/a)^3*(-a^
2*(x+1/a)^2+2*a*(x+1/a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^4), x)
```

Fricas [A] time = 1.86297, size = 219, normalized size = 1.89

$$-\frac{24a^4x^4+24a^3x^3+33\left(a^4x^4+a^3x^3\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(52a^3x^3+19a^2x^2-7ax+2\right)\sqrt{-a^2x^2+1}}{6\left(ax^4+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(24*a^4*x^4 + 24*a^3*x^3 + 33*(a^4*x^4 + a^3*x^3)*log((sqrt(-a^2*x^2 +
 1) - 1)/x) + (52*a^3*x^3 + 19*a^2*x^2 - 7*a*x + 2)*sqrt(-a^2*x^2 + 1))/(a*
x^4 + x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}}{x^4(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(x**4*(a*x + 1)**3), x)
```

Giac [B] time = 1.21526, size = 358, normalized size = 3.09

$$\frac{\left(a^4-\frac{8\left(\sqrt{-a^2x^2+1}|a|+a\right)a^2}{x}+\frac{48\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{x^2}+\frac{249\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^2x^3}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}+\frac{11\,a^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{57\left(\sqrt{-a^2x^2+1}|a|+a\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4 - 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2/x + 48*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)^2/x^2 + 249*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^2*x^3))*a^6*
x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^
2*x) + 1)*abs(a)) + 11/2*a^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a
)/(a^2*abs(x)))/abs(a) - 1/24*(57*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4/x - 9
*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2/x^2 + (sqrt(-a^2*x^2 + 1)*abs(a) + a
)^3/x^3)/(a^2*abs(a))
```

3.59 $\int \frac{e^{-3\tanh^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=135

$$\frac{4a^4\sqrt{1-a^2x^2}}{ax+1}+\frac{6a^3\sqrt{1-a^2x^2}}{x}-\frac{19a^2\sqrt{1-a^2x^2}}{8x^2}+\frac{a\sqrt{1-a^2x^2}}{x^3}-\frac{\sqrt{1-a^2x^2}}{4x^4}-\frac{51}{8}a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -Sqrt[1 - a^2*x^2]/(4*x^4) + (a*Sqrt[1 - a^2*x^2])/x^3 - (19*a^2*Sqrt[1 - a
^2*x^2])/(8*x^2) + (6*a^3*Sqrt[1 - a^2*x^2])/x + (4*a^4*Sqrt[1 - a^2*x^2])/
(1 + a*x) - (51*a^4*ArcTanh[Sqrt[1 - a^2*x^2]])/8
```

Rubi [A] time = 0.822491, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6124, 6742, 266, 51, 63, 208, 271, 264, 651}

$$\frac{4a^4\sqrt{1-a^2x^2}}{ax+1}+\frac{6a^3\sqrt{1-a^2x^2}}{x}-\frac{19a^2\sqrt{1-a^2x^2}}{8x^2}+\frac{a\sqrt{1-a^2x^2}}{x^3}-\frac{\sqrt{1-a^2x^2}}{4x^4}-\frac{51}{8}a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(4*x^4) + (a*Sqrt[1 - a^2*x^2])/x^3 - (19*a^2*Sqrt[1 - a
^2*x^2])/(8*x^2) + (6*a^3*Sqrt[1 - a^2*x^2])/x + (4*a^4*Sqrt[1 - a^2*x^2])/
(1 + a*x) - (51*a^4*ArcTanh[Sqrt[1 - a^2*x^2]])/8
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{x^5}\,dx = \int \frac{(1-ax)^2}{x^5(1+ax)\sqrt{1-a^2x^2}}\,dx$$

$$= \int \left(\frac{1}{x^5\sqrt{1-a^2x^2}} - \frac{3a}{x^4\sqrt{1-a^2x^2}} + \frac{4a^2}{x^3\sqrt{1-a^2x^2}} - \frac{4a^3}{x^2\sqrt{1-a^2x^2}} + \frac{4a^4}{x\sqrt{1-a^2x^2}} - \frac{4a^5}{(1+ax)\sqrt{1-a^2 \ldots}}\right.$$

$$= -\left((3a)\int \frac{1}{x^4\sqrt{1-a^2x^2}}\,dx\right) + \left(4a^2\right)\int \frac{1}{x^3\sqrt{1-a^2x^2}}\,dx - \left(4a^3\right)\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx + \left(4a^4\right)\int \frac{\ldots}{x\sqrt{\ldots}}$$

$$= \frac{a\sqrt{1-a^2x^2}}{x^3} + \frac{4a^3\sqrt{1-a^2x^2}}{x} + \frac{4a^4\sqrt{1-a^2x^2}}{1+ax} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-a^2x}}\,dx, x, x^2\right) + \left(2a^2\right)\text{Subs}\ldots$$

$$= -\frac{\sqrt{1-a^2x^2}}{4x^4} + \frac{a\sqrt{1-a^2x^2}}{x^3} - \frac{2a^2\sqrt{1-a^2x^2}}{x^2} + \frac{6a^3\sqrt{1-a^2x^2}}{x} + \frac{4a^4\sqrt{1-a^2x^2}}{1+ax} + \frac{1}{8}\left(3a^2\right)\text{Subst}\left(\ldots\right.$$

$$= -\frac{\sqrt{1-a^2x^2}}{4x^4} + \frac{a\sqrt{1-a^2x^2}}{x^3} - \frac{19a^2\sqrt{1-a^2x^2}}{8x^2} + \frac{6a^3\sqrt{1-a^2x^2}}{x} + \frac{4a^4\sqrt{1-a^2x^2}}{1+ax} - 4a^4\tanh^{-1}\left(\sqrt{1\ldots}\right.$$

$$= -\frac{\sqrt{1-a^2x^2}}{4x^4} + \frac{a\sqrt{1-a^2x^2}}{x^3} - \frac{19a^2\sqrt{1-a^2x^2}}{8x^2} + \frac{6a^3\sqrt{1-a^2x^2}}{x} + \frac{4a^4\sqrt{1-a^2x^2}}{1+ax} - 6a^4\tanh^{-1}\left(\sqrt{1\ldots}\right.$$

$$= -\frac{\sqrt{1-a^2x^2}}{4x^4} + \frac{a\sqrt{1-a^2x^2}}{x^3} - \frac{19a^2\sqrt{1-a^2x^2}}{8x^2} + \frac{6a^3\sqrt{1-a^2x^2}}{x} + \frac{4a^4\sqrt{1-a^2x^2}}{1+ax} - \frac{51}{8}a^4\tanh^{-1}\left(\sqrt{\ldots}\right.$$

Mathematica [A] time = 0.0787101, size = 89, normalized size = 0.66

$$\frac{1}{8}\left(\frac{\sqrt{1-a^2x^2}\left(80a^4x^4+29a^3x^3-11a^2x^2+6ax-2\right)}{x^4(ax+1)} - 51a^4\log\left(\sqrt{1-a^2x^2}+1\right) + 51a^4\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(-2 + 6*a*x - 11*a^2*x^2 + 29*a^3*x^3 + 80*a^4*x^4))/(x
^4*(1 + a*x)) + 51*a^4*Log[x] - 51*a^4*Log[1 + Sqrt[1 - a^2*x^2]])/8
```

Maple [B] time = 0.066, size = 359, normalized size = 2.7

$$\frac{a}{\left(x+a^{-1}\right)^{3}}\left(-a^{2}\left(x+a^{-1}\right)^{2}+2\,a\left(x+a^{-1}\right)\right)^{\frac{5}{2}}+\frac{a}{x^{3}}\left(-a^{2}x^{2}+1\right)^{\frac{5}{2}}-3\,\frac{a^{2}\left(-a^{2}\left(x+a^{-1}\right)^{2}+2\,a\left(x+a^{-1}\right)\right)^{5/2}}{\left(x+a^{-1}\right)^{2}}-12\,a^{5}\sqrt{-a^{2}(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x)
```

```
[Out] a/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)+a/x^3*(-a^2*x^2+1)^(5/2)-3*a
^2/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-12*a^5*(-a^2*(x+1/a)^2+2*a*
(x+1/a))^(1/2)*x-12*a^5/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*
a*(x+1/a))^(1/2))+8*a^3/x*(-a^2*x^2+1)^(5/2)+8*a^5*x*(-a^2*x^2+1)^(3/2)+12*
a^5*x*(-a^2*x^2+1)^(1/2)+12*a^5/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))-23/8*a^2/x^2*(-a^2*x^2+1)^(5/2)-1/4/x^4*(-a^2*x^2+1)^(5/2)+17/8*a
^4*(-a^2*x^2+1)^(3/2)+51/8*a^4*(-a^2*x^2+1)^(1/2)-51/8*a^4*arctanh(1/(-a^2*
x^2+1)^(1/2))-8*a^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^{3}x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^5), x)
```

Fricas [A] time = 1.78333, size = 235, normalized size = 1.74

$$\frac{32\,a^{5}x^{5}+32\,a^{4}x^{4}+51\left(a^{5}x^{5}+a^{4}x^{4}\right)\log\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{x}\right)+\left(80\,a^{4}x^{4}+29\,a^{3}x^{3}-11\,a^{2}x^{2}+6\,ax-2\right)\sqrt{-a^{2}x^{2}+1}}{8\left(ax^{5}+x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(32*a^5*x^5 + 32*a^4*x^4 + 51*(a^5*x^5 + a^4*x^4)*log((sqrt(-a^2*x^2 +
1) - 1)/x) + (80*a^4*x^4 + 29*a^3*x^3 - 11*a^2*x^2 + 6*a*x - 2)*sqrt(-a^2*x
```

```
^2 + 1))/(a*x^5 + x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}}{x^5(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**5,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(x**5*(a*x + 1)**3), x)
```

Giac [B] time = 1.20077, size = 440, normalized size = 3.26

$$\frac{\left(a^5-\frac{7\left(\sqrt{-a^2x^2+1}|a|+a\right)a^3}{x}+\frac{32\left(\sqrt{-a^2x^2+1}|a|+a\right)^2a}{x^2}-\frac{160\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{ax^3}-\frac{712\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^3x^4}\right)a^8x^4}{64\left(\sqrt{-a^2x^2+1}|a|+a\right)^4\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}-\frac{51\,a^5\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\right.}{2\,a^2|x|}\right.}{8\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/64*(a^5 - 7*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^3/x + 32*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)^2*a/x^2 - 160*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a*x^3) - 712
*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^3*x^4))*a^8*x^4/((sqrt(-a^2*x^2 + 1)*
abs(a) + a)^4*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a)) - 51/8*
a^5*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) +
1/64*(200*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^5*abs(a)/x - 40*(sqrt(-a^2*x^2
+ 1)*abs(a) + a)^2*a^3*abs(a)/x^2 + 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a*a
bs(a)/x^3 - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*abs(a)/(a*x^4))/a^4
```

3.60 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=31

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{4},-\frac{1}{4};m+2;ax,-ax\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/4, -1/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rubi [A] time = 0.0272853, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6126, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{4},-\frac{1}{4};m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/4, -1/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{2}\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{1}{4},-\frac{1}{4};2+m;ax,-ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.299125, size = 0, normalized size = 0.

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(ArcTanh[a*x]/2)*x^m,x]
```

```
[Out] Integrate[E^(ArcTanh[a*x]/2)*x^m, x]
```

Maple [F] time = 0.126, size = 0, normalized size = 0.

$$\int \sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\,x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^m,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

3.61 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=282

$$-\frac{x(1-ax)^{3/4}(ax+1)^{5/4}}{3a^2}-\frac{(1-ax)^{3/4}(ax+1)^{5/4}}{12a^3}-\frac{3(1-ax)^{3/4}\sqrt[4]{ax+1}}{8a^3}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}a^3}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right.}{16}$$

```
[Out] (-3*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(8*a^3) - ((1 - a*x)^(3/4)*(1 + a*x)^(
5/4))/(12*a^3) - (x*(1 - a*x)^(3/4)*(1 + a*x)^(5/4))/(3*a^2) + (3*ArcTan[1
- (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (3*ArcTan[1
 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (3*Log[1 +
 Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/
(16*Sqrt[2]*a^3) + (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a
*x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3)
```

Rubi [A] time = 0.214073, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 90, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x(1-ax)^{3/4}(ax+1)^{5/4}}{3a^2}-\frac{(1-ax)^{3/4}(ax+1)^{5/4}}{12a^3}-\frac{3(1-ax)^{3/4}\sqrt[4]{ax+1}}{8a^3}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}a^3}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right.}{16}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*x^2,x]
```

```
[Out] (-3*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(8*a^3) - ((1 - a*x)^(3/4)*(1 + a*x)^(
5/4))/(12*a^3) - (x*(1 - a*x)^(3/4)*(1 + a*x)^(5/4))/(3*a^2) + (3*ArcTan[1
- (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (3*ArcTan[1
 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (3*Log[1 +
 Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/
(16*Sqrt[2]*a^3) + (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a
*x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}x^2\,dx = \int \frac{x^2\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx$$

$$= -\frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} - \frac{\int \frac{\left(-1-\frac{ax}{2}\right)\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx}{3a^2}$$

$$= -\frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} + \frac{3\int \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx}{8a^2}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} + \frac{3\int \frac{1}{\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx}{16a^2}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx\right.}{4a^3}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x\right.}{4a^3}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} + \frac{3\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x\right.}{8a^3}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\right.}{16a^3}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} - \frac{3\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}}{}\right.}{16\sqrt{2}a^3}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} + \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right.}{8\sqrt{2}a^3}$$

Mathematica [C] time = 0.0364549, size = 69, normalized size = 0.24

$$-\frac{(1-ax)^{3/4}\left(6\sqrt[4]{2}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-ax)\right)+\sqrt[4]{ax+1}\left(4a^2x^2+5ax+1\right)\right)}{12a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)*x^2,x]
```

```
[Out] -((1 - a*x)^(3/4)*((1 + a*x)^(1/4)*(1 + 5*a*x + 4*a^2*x^2) + 6*2^(1/4)*Hype
rgeometric2F1[-1/4, 3/4, 7/4, (1 - a*x)/2]))/(12*a^3)
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int \sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

Fricas [B] time = 1.88041, size = 1350, normalized size = 4.79

$$-\frac{36\sqrt{2}a^3\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^9\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}+\left(a^7x-a^6\right)\sqrt{\frac{1}{a^{12}}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}-\sqrt{2}a^9\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}-1\right)+36\sqrt{2}a\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/96*(36*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^9*sqrt((sqrt(2)*(a^4
*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) + (a^7*x - a^
6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(3/4) - sqrt(2)
*a^9*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) - 1) + 36*sqrt(2)*
a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^9*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(-s
qrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - (a^7*x - a^6)*sqrt(a^(-12))
+ sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(3/4) - sqrt(2)*a^9*sqrt(-sqrt(-
a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) + 1) - 9*sqrt(2)*a^3*(a^(-12))^(1/4
)*log((sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^
(1/4) + (a^7*x - a^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 9*sq
rt(2)*a^3*(a^(-12))^(1/4)*log(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 +
 1)/(a*x - 1))*(a^(-12))^(1/4) - (a^7*x - a^6)*sqrt(a^(-12)) + sqrt(-a^2*x^
2 + 1))/(a*x - 1)) - 4*(8*a^3*x^3 + 2*a^2*x^2 + a*x - 11)*sqrt(-sqrt(-a^2*x
^2 + 1)/(a*x - 1)))/a^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x**2,x)
```

```
[Out] Integral(x**2*sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

3.62 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=255

$$-\frac{(1-ax)^{3/4}(ax+1)^{5/4}}{2a^2}-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{4a^2}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right.}{4\sqrt{2}a^2}$$

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*a^2) - ((1 - a*x)^(3/4)*(1 + a*x)^(5/
4))/(2*a^2) + ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(4*Sqrt
[2]*a^2) - ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(4*Sqrt[2]
*a^2) - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1
+ a*x)^(1/4)]/(8*Sqrt[2]*a^2) + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt
[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(8*Sqrt[2]*a^2)
```

Rubi [A] time = 0.173493, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6126, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(1-ax)^{3/4}(ax+1)^{5/4}}{2a^2}-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{4a^2}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right.}{4\sqrt{2}a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*x,x]
```

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*a^2) - ((1 - a*x)^(3/4)*(1 + a*x)^(5/
4))/(2*a^2) + ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(4*Sqrt
[2]*a^2) - ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(4*Sqrt[2]
*a^2) - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1
+ a*x)^(1/4)]/(8*Sqrt[2]*a^2) + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt
[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(8*Sqrt[2]*a^2)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}x\,dx = \int \frac{x\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx$$

$$= -\frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{\int \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx}{4a}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{\int \frac{1}{\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx}{8a}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} - \frac{\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{2a^2}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} - \frac{\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a^2}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4a^2} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x,\right.}{4a^2}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8a^2} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+\right.}{8a}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8\sqrt{2}a^2} + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1+}}\right.}{8\sqrt{2}a^2}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}a^2} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}a^2} - \frac{\log\left(\right.}{}$$

Mathematica [C] time = 0.020352, size = 56, normalized size = 0.22

$$-\frac{(1-ax)^{3/4}\left(2\sqrt[4]{2}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-ax)\right)+3(ax+1)^{5/4}\right)}{6a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)*x,x]
```

```
[Out] -((1 - a*x)^(3/4)*(3*(1 + a*x)^(5/4) + 2*2^(1/4)*Hypergeometric2F1[-1/4, 3/
4, 7/4, (1 - a*x)/2]))/(6*a^2)
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int \sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

Fricas [B] time = 1.98196, size = 1303, normalized size = 5.11

$$-\frac{4\sqrt{2}a^2\left(\frac{1}{a^8}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^6\sqrt{\frac{\sqrt{2}\left(a^3x-a^2\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^8}\right)^{\frac{1}{4}}+\left(a^5x-a^4\right)\sqrt{\frac{1}{a^8}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^8}\right)^{\frac{3}{4}}-\sqrt{2}a^6\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^8}\right)^{\frac{3}{4}}-1\right)+4\sqrt{2}a^2\left(\frac{1}{a^8}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*a^2*(a^(-8))^(1/4)*arctan(sqrt(2)*a^6*sqrt((sqrt(2)*(a^3*x
 - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) + (a^5*x - a^4)*
sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(3/4) - sqrt(2)*a^6*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) - 1) + 4*sqrt(2)*a^2*(a^
(-8))^(1/4)*arctan(sqrt(2)*a^6*sqrt(-(sqrt(2)*(a^3*x - a^2)*sqrt(-sqrt(-a^2
*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-a^
2*x^2 + 1))/(a*x - 1))*(a^(-8))^(3/4) - sqrt(2)*a^6*sqrt(-sqrt(-a^2*x^2 + 1
)/(a*x - 1))*(a^(-8))^(3/4) + 1) - sqrt(2)*a^2*(a^(-8))^(1/4)*log((sqrt(2)*
(a^3*x - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) + (a^5*x -
 a^4)*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + sqrt(2)*a^2*(a^(-8))^
(1/4)*log(-(sqrt(2)*(a^3*x - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-
8))^(1/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 4
*(2*a^2*x^2 + a*x - 3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x,x)
```

```
[Out] Integral(x*sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="giac")
```

```
[Out] integrate(x*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

3.63 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=222

$$-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{\sqrt{2}a}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\ldots}{\sqrt[4]{\ldots}}\right.}{\sqrt{2}\ldots}$$

```
[Out] -(((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/a) + ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4
))/(1 + a*x)^(1/4)]/(Sqrt[2]*a) - ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 +
 a*x)^(1/4)]/(Sqrt[2]*a) - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(
1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(2*Sqrt[2]*a) + Log[1 + Sqrt[1 - a*x]/Sqrt
[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(2*Sqrt[2]*a)
```

Rubi [A] time = 0.146268, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6125, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{\sqrt{2}a}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\ldots}{\sqrt[4]{\ldots}}\right.}{\sqrt{2}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2),x]
```

```
[Out] -(((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/a) + ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4
))/(1 + a*x)^(1/4)]/(Sqrt[2]*a) - ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 +
 a*x)^(1/4)]/(Sqrt[2]*a) - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(
1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(2*Sqrt[2]*a) + Log[1 + Sqrt[1 - a*x]/Sqrt
[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(2*Sqrt[2]*a)
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}\,dx = \int \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{1}{2}\int \frac{1}{\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a} - \frac{\text{Su}}{}$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}a} + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}a} - \frac{\text{Subst}\left(\int \frac{1}{-1-x}\right.}{}$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}a}$$

Mathematica [A] time = 0.201769, size = 149, normalized size = 0.67

$$\frac{-\frac{8e^{\frac{1}{2}\tanh^{-1}(ax)}}{e^{2\tanh^{-1}(ax)}+1} - \sqrt{2}\log\left(-\sqrt{2}e^{\frac{1}{2}\tanh^{-1}(ax)} + e^{\tanh^{-1}(ax)} + 1\right) + \sqrt{2}\log\left(\sqrt{2}e^{\frac{1}{2}\tanh^{-1}(ax)} + e^{\tanh^{-1}(ax)} + 1\right) - 2\sqrt{2}\tan^{-1}\left(1\right.}{4a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2),x]
```

```
[Out] ((-8*E^(ArcTanh[a*x]/2))/(1 + E^(2*ArcTanh[a*x])) - 2*Sqrt[2]*ArcTan[1 - Sq
rt[2]*E^(ArcTanh[a*x]/2)] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcTanh[a*x]/2)
] - Sqrt[2]*Log[1 - Sqrt[2]*E^(ArcTanh[a*x]/2) + E^ArcTanh[a*x]] + Sqrt[2]*
Log[1 + Sqrt[2]*E^(ArcTanh[a*x]/2) + E^ArcTanh[a*x]])/(4*a)
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int \sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

Fricas [B] time = 1.77107, size = 1261, normalized size = 5.68

$$-\frac{4\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^3\sqrt{\frac{\sqrt{2}(a^2x-a)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}+(a^3x-a^2)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-\sqrt{2}a^3\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-1\right)+4\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*a^3*sqrt((sqrt(2)*(a^2*x -
a)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) + (a^3*x - a^2)*sqrt(
a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(3/4) - sqrt(2)*a^3*sqrt(
-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) - 1) + 4*sqrt(2)*a*(a^(-4))^(
1/4)*arctan(sqrt(2)*a^3*sqrt(-(sqrt(2)*(a^2*x - a)*sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1))*(a^(-4))^(1/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1
))/(a*x - 1))*(a^(-4))^(3/4) - sqrt(2)*a^3*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x -
1))*(a^(-4))^(3/4) + 1) - sqrt(2)*a*(a^(-4))^(1/4)*log((sqrt(2)*(a^2*x - a)
*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) + (a^3*x - a^2)*sqrt(a^
(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + sqrt(2)*a*(a^(-4))^(1/4)*log(-(sqr
t(2)*(a^2*x - a)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) - (a^3*
x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 4*(a*x - 1)*sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

3.64 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=227

$$-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)-\sqrt{2}\tan$$

```
[Out] -2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1
 - a*x)^(1/4))/(1 + a*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/
4))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] - Log[1 +
 Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/S
qrt[2] + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1
 + a*x)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.163354, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6126, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)-\sqrt{2}\tan$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/x,x]
```

```
[Out] -2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1
 - a*x)^(1/4))/(1 + a*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/
4))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] - Log[1 +
 Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/S
qrt[2] + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1
 + a*x)^(1/4)]/Sqrt[2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{\sqrt[4]{1+ax}}{x\sqrt[4]{1-ax}}\,dx\\
&= a\int \frac{1}{\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx + \int \frac{1}{x\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx\\
&= -\left(4\,\mathrm{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)\right) + 4\,\mathrm{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= -\left(2\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\right) - 2\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 4\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,d\right.\\
&= -2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + 2\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - 2\,\mathrm{Subst}\left(\int \frac{1+}{1+}\right.\\
&= -2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{\mathrm{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} - \frac{\mathrm{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-}\right.}{\sqrt{}}\\
&= -2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+ax}}\right.}{\sqrt{2}}\\
&= -2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.
\end{aligned}
$$

Mathematica [C] time = 0.0298875, size = 83, normalized size = 0.37

$$
-\frac{2(1-ax)^{3/4}\left(\sqrt[4]{2}(ax+1)^{3/4}\mathrm{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-ax)\right)+2\mathrm{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1-ax}{ax+1}\right)\right)}{3(ax+1)^{3/4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/x,x]
```

```
[Out] (-2*(1 - a*x)^(3/4)*(2^(1/4)*(1 + a*x)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/
4, (1 - a*x)/2] + 2*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)]))/(
```

```
3*(1 + a*x)^(3/4))
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x, x)
```

Fricas [B] time = 1.76652, size = 999, normalized size = 4.4

$$-2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{ax+\sqrt{2}(ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-\sqrt{-a^2x^2+1}-1}{ax-1}}-\sqrt{2}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2
 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt(2)*sqrt(-sqrt(
-a^2*x^2 + 1)/(a*x - 1)) - 1) - 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x - sqrt(2
)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(
a*x - 1)) - sqrt(2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 1/2*sqrt(2)*
log(4*(a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-
a^2*x^2 + 1) - 1)/(a*x - 1)) - 1/2*sqrt(2)*log(4*(a*x - sqrt(2)*(a*x - 1)*s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - 2
*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - log(sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1)) + 1) + log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x, x)
```

3.65 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=73

$$-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{x}-a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

```
[Out] -(((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/x) - a*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)
^(1/4)] - a*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)]
```

Rubi [A] time = 0.0337581, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6126, 94, 93, 212, 206, 203}

$$-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{x}-a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/x^2,x]
```

```
[Out] -(((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/x) - a*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)
^(1/4)] - a*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{\sqrt[4]{1+ax}}{x^2\sqrt[4]{1-ax}}\,dx \\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{x} + \frac{1}{2}a\int \frac{1}{x\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx \\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{x} + (2a)\,\mathrm{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) \\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{x} - a\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - a\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) \\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{x} - a\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - a\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0142849, size = 58, normalized size = 0.79

$$-\frac{(1-ax)^{3/4}\left(2ax\,\mathrm{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1-ax}{ax+1}\right)+3ax+3\right)}{3x(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/x^2,x]
```

```
[Out] -((1 - a*x)^(3/4)*(3 + 3*a*x + 2*a*x*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*
x)/(1 + a*x)]))/(3*x*(1 + a*x)^(3/4))
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^2, x)
```

Fricas [B] time = 1.8026, size = 286, normalized size = 3.92

$$-\frac{2\,ax\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\,(ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + a*x*log(sqrt(-sqr
t(-a^2*x^2 + 1)/(a*x - 1)) + 1) - a*x*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1
)) - 1) - 2*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^2, x)
```

3.66 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=110

$$-\frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(1-ax)^{3/4}(ax+1)^{5/4}}{2x^2}-\frac{a(1-ax)^{3/4}\sqrt[4]{ax+1}}{4x}$$

```
[Out] -(a*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*x) - ((1 - a*x)^(3/4)*(1 + a*x)^(5/
4))/(2*x^2) - (a^2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (a^2*ArcTan
h[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4
```

Rubi [A] time = 0.0438452, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6126, 96, 94, 93, 212, 206, 203}

$$-\frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(1-ax)^{3/4}(ax+1)^{5/4}}{2x^2}-\frac{a(1-ax)^{3/4}\sqrt[4]{ax+1}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/x^3,x]
```

```
[Out] -(a*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*x) - ((1 - a*x)^(3/4)*(1 + a*x)^(5/
4))/(2*x^2) - (a^2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (a^2*ArcTan
h[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^3}\,dx &= \int \frac{\sqrt[4]{1+ax}}{x^3\sqrt[4]{1-ax}}\,dx\\
&= -\frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2x^2}+\frac{1}{4}a\int \frac{\sqrt[4]{1+ax}}{x^2\sqrt[4]{1-ax}}\,dx\\
&= -\frac{a(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x}-\frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2x^2}+\frac{1}{8}a^2\int \frac{1}{x\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx\\
&= -\frac{a(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x}-\frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2x^2}+\frac{1}{2}a^2\,\mathrm{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= -\frac{a(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x}-\frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2x^2}-\frac{1}{4}a^2\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-\frac{1}{4}a^2\,\mathrm{Sub}\\
&= -\frac{a(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x}-\frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2x^2}-\frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-\frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0172729, size = 70, normalized size = 0.64

$$
-\frac{(1-ax)^{3/4}\left(2a^2x^2\,\mathrm{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1-ax}{ax+1}\right)+9a^2x^2+15ax+6\right)}{12x^2(ax+1)^{3/4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/x^3,x]
```

```
[Out] -((1 - a*x)^(3/4)*(6 + 15*a*x + 9*a^2*x^2 + 2*a^2*x^2*Hypergeometric2F1[3/4
, 1, 7/4, (1 - a*x)/(1 + a*x)]))/(12*x^2*(1 + a*x)^(3/4))
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^3, x)
```

Fricas [A] time = 1.84158, size = 321, normalized size = 2.92

$$-\frac{2\,a^2x^2\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+a^2x^2\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-a^2x^2\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\left(3\,a^2x^2-ax-2\right)\sqrt{-\frac{\sqrt{-a^2x}}{ax-}}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a^2*x^2*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + a^2*x^2*log(s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - a^2*x^2*log(sqrt(-sqrt(-a^2*x^2 +
 1)/(a*x - 1)) - 1) - 2*(3*a^2*x^2 - a*x - 2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
 - 1)))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^3, x)
```

3.67 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=139

$$-\frac{11a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{24x}-\frac{3}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{3}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{5a(1-ax)^{3/4}\sqrt[4]{ax+1}}{12x^2}-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{3x^3}$$

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(3*x^3) - (5*a*(1 - a*x)^(3/4)*(1 + a*x)
^(1/4))/(12*x^2) - (11*a^2*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(24*x) - (3*a^3
*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8 - (3*a^3*ArcTanh[(1 + a*x)^(1/4
)/(1 - a*x)^(1/4)])/8
```

Rubi [A] time = 0.0620214, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 212, 206, 203}

$$-\frac{11a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{24x}-\frac{3}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{3}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{5a(1-ax)^{3/4}\sqrt[4]{ax+1}}{12x^2}-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/x^4,x]
```

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(3*x^3) - (5*a*(1 - a*x)^(3/4)*(1 + a*x)
^(1/4))/(12*x^2) - (11*a^2*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(24*x) - (3*a^3
*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8 - (3*a^3*ArcTanh[(1 + a*x)^(1/4
)/(1 - a*x)^(1/4)])/8
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^4}\,dx &= \int \frac{\sqrt[4]{1+ax}}{x^4\sqrt[4]{1-ax}}\,dx\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3}+\frac{1}{3}\int \frac{\frac{5a}{2}+2a^2x}{x^3\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3}-\frac{5a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2}-\frac{1}{6}\int \frac{-\frac{11a^2}{4}-\frac{5a^3x}{2}}{x^2\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3}-\frac{5a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2}-\frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x}+\frac{1}{6}\int \frac{9a^3}{8x\sqrt[4]{1-ax}(1+ax)}\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3}-\frac{5a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2}-\frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x}+\frac{1}{16}\left(3a^3\right)\int \frac{1}{x\sqrt[4]{1-ax}(1}\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3}-\frac{5a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2}-\frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x}+\frac{1}{4}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{-1+}\right.\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3}-\frac{5a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2}-\frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x}-\frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{1-}\right.\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3}-\frac{5a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2}-\frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x}-\frac{3}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-
\end{aligned}$$

Mathematica [C] time = 0.0216139, size = 78, normalized size = 0.56

$$-\frac{(1-ax)^{3/4}\left(6a^3x^3\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1-ax}{ax+1}\right)+11a^3x^3+21a^2x^2+18ax+8\right)}{24x^3(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/x^4,x]
```

```
[Out] -((1 - a*x)^(3/4)*(8 + 18*a*x + 21*a^2*x^2 + 11*a^3*x^3 + 6*a^3*x^3*Hyperge
ometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)]))/(24*x^3*(1 + a*x)^(3/4))
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^4, x)
```

Fricas [A] time = 1.7291, size = 347, normalized size = 2.5

$$-\frac{18\,a^3x^3\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+9\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-9\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\left(11\,a^3x^3-a^2x^2-2\,ax\right.}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(18*a^3*x^3*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 9*a^3*x^3*l
og(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 9*a^3*x^3*log(sqrt(-sqrt(-a^2
*x^2 + 1)/(a*x - 1)) - 1) - 2*(11*a^3*x^3 - a^2*x^2 - 2*a*x - 8)*sqrt(-sqrt
(-a^2*x^2 + 1)/(a*x - 1)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^4, x)
```

3.68 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=168

$$-\frac{29a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{96x^2}-\frac{83a^3(1-ax)^{3/4}\sqrt[4]{ax+1}}{192x}-\frac{11}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{11}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{7a(1-ax)^{3/4}}{24x^3}$$

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*x^4) - (7*a*(1 - a*x)^(3/4)*(1 + a*x)
^(1/4))/(24*x^3) - (29*a^2*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(96*x^2) - (83*
a^3*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(192*x) - (11*a^4*ArcTan[(1 + a*x)^(1/
4)/(1 - a*x)^(1/4)])/64 - (11*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])
/64
```

Rubi [A] time = 0.0794604, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 212, 206, 203}

$$-\frac{29a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{96x^2}-\frac{83a^3(1-ax)^{3/4}\sqrt[4]{ax+1}}{192x}-\frac{11}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{11}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{7a(1-ax)^{3/4}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/x^5,x]
```


```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*x^4) - (7*a*(1 - a*x)^(3/4)*(1 + a*x)
^(1/4))/(24*x^3) - (29*a^2*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(96*x^2) - (83*
a^3*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(192*x) - (11*a^4*ArcTan[(1 + a*x)^(1/
4)/(1 - a*x)^(1/4)])/64 - (11*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])
/64
```


Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^5}dx = \int \frac{\sqrt[4]{1+ax}}{x^5\sqrt[4]{1-ax}}dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} + \frac{1}{4}\int \frac{\frac{7a}{2}+3a^2x}{x^4\sqrt[4]{1-ax}(1+ax)^{3/4}}dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} - \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x^3} - \frac{1}{12}\int \frac{-\frac{29a^2}{4}-7a^3x}{x^3\sqrt[4]{1-ax}(1+ax)^{3/4}}dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} - \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x^3} - \frac{29a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{96x^2} + \frac{1}{24}\int \frac{\frac{83a^3}{8}+\frac{29a^4x}{4}}{x^2\sqrt[4]{1-ax}(1+ax)^{3/}}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} - \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x^3} - \frac{29a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{96x^2} - \frac{83a^3(1-ax)^{3/4}\sqrt[4]{1+ax}}{192x} -$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} - \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x^3} - \frac{29a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{96x^2} - \frac{83a^3(1-ax)^{3/4}\sqrt[4]{1+ax}}{192x} +$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} - \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x^3} - \frac{29a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{96x^2} - \frac{83a^3(1-ax)^{3/4}\sqrt[4]{1+ax}}{192x} +$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} - \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x^3} - \frac{29a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{96x^2} - \frac{83a^3(1-ax)^{3/4}\sqrt[4]{1+ax}}{192x} -$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} - \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x^3} - \frac{29a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{96x^2} - \frac{83a^3(1-ax)^{3/4}\sqrt[4]{1+ax}}{192x} -$$

Mathematica [C] time = 0.0257589, size = 86, normalized size = 0.51

$$-\frac{(1-ax)^{3/4}\left(22a^4x^4\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1-ax}{ax+1}\right)+83a^4x^4+141a^3x^3+114a^2x^2+104ax+48\right)}{192x^4(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/x^5,x]
```

```
[Out] -((1 - a*x)^(3/4)*(48 + 104*a*x + 114*a^2*x^2 + 141*a^3*x^3 + 83*a^4*x^4 +
22*a^4*x^4*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)]))/(192*x^4*(
1 + a*x)^(3/4))
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^5, x)
```

Fricas [A] time = 1.78073, size = 373, normalized size = 2.22

$$-\frac{66\,a^4x^4\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+33\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-33\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\left(83\,a^4x^4-25\,a^3\right.}{384\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/384*(66*a^4*x^4*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 33*a^4*x^4
*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 33*a^4*x^4*log(sqrt(-sqrt(-
a^2*x^2 + 1)/(a*x - 1)) - 1) - 2*(83*a^4*x^4 - 25*a^3*x^3 - 2*a^2*x^2 - 8*a
*x - 48)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^5, x)
```

3.69 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^6}\,dx$

Optimal. Leaf size=197

$$-\frac{269a^3(1-ax)^{3/4}\sqrt[4]{ax+1}}{960x^2}-\frac{11a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{48x^3}-\frac{611a^4(1-ax)^{3/4}\sqrt[4]{ax+1}}{1920x}-\frac{31}{128}a^5\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{31}{128}a^5\tanh$$

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(5*x^5) - (9*a*(1 - a*x)^(3/4)*(1 + a*x)
^(1/4))/(40*x^4) - (11*a^2*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(48*x^3) - (269
*a^3*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(960*x^2) - (611*a^4*(1 - a*x)^(3/4)*
(1 + a*x)^(1/4))/(1920*x) - (31*a^5*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)]
)/128 - (31*a^5*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/128
```

Rubi [A] time = 0.0955458, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 212, 206, 203}

$$-\frac{269a^3(1-ax)^{3/4}\sqrt[4]{ax+1}}{960x^2}-\frac{11a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{48x^3}-\frac{611a^4(1-ax)^{3/4}\sqrt[4]{ax+1}}{1920x}-\frac{31}{128}a^5\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{31}{128}a^5\tanh$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/x^6,x]
```

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(5*x^5) - (9*a*(1 - a*x)^(3/4)*(1 + a*x)
^(1/4))/(40*x^4) - (11*a^2*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(48*x^3) - (269
*a^3*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(960*x^2) - (611*a^4*(1 - a*x)^(3/4)*
(1 + a*x)^(1/4))/(1920*x) - (31*a^5*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)]
)/128 - (31*a^5*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/128
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

erQ[m]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^6}\,dx = \int \frac{\sqrt[4]{1+ax}}{x^6\sqrt[4]{1-ax}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} + \frac{1}{5}\int \frac{\frac{9a}{2}+4a^2x}{x^5\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} - \frac{9a(1-ax)^{3/4}\sqrt[4]{1+ax}}{40x^4} - \frac{1}{20}\int \frac{-\frac{55a^2}{4}-\frac{27a^3x}{2}}{x^4\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} - \frac{9a(1-ax)^{3/4}\sqrt[4]{1+ax}}{40x^4} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{48x^3} + \frac{1}{60}\int \frac{\frac{269a^3}{8}+\frac{55a^4x}{2}}{x^3\sqrt[4]{1-ax}(1+ax)^{3/4}}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} - \frac{9a(1-ax)^{3/4}\sqrt[4]{1+ax}}{40x^4} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{48x^3} - \frac{269a^3(1-ax)^{3/4}\sqrt[4]{1+ax}}{960x^2} -$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} - \frac{9a(1-ax)^{3/4}\sqrt[4]{1+ax}}{40x^4} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{48x^3} - \frac{269a^3(1-ax)^{3/4}\sqrt[4]{1+ax}}{960x^2} -$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} - \frac{9a(1-ax)^{3/4}\sqrt[4]{1+ax}}{40x^4} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{48x^3} - \frac{269a^3(1-ax)^{3/4}\sqrt[4]{1+ax}}{960x^2} -$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} - \frac{9a(1-ax)^{3/4}\sqrt[4]{1+ax}}{40x^4} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{48x^3} - \frac{269a^3(1-ax)^{3/4}\sqrt[4]{1+ax}}{960x^2} -$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} - \frac{9a(1-ax)^{3/4}\sqrt[4]{1+ax}}{40x^4} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{48x^3} - \frac{269a^3(1-ax)^{3/4}\sqrt[4]{1+ax}}{960x^2} -$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} - \frac{9a(1-ax)^{3/4}\sqrt[4]{1+ax}}{40x^4} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{48x^3} - \frac{269a^3(1-ax)^{3/4}\sqrt[4]{1+ax}}{960x^2} -$$

Mathematica [C] time = 0.0295945, size = 94, normalized size = 0.48

$$-\frac{(1-ax)^{3/4}\left(310a^5x^5\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1-ax}{ax+1}\right)+611a^5x^5+1149a^4x^4+978a^3x^3+872a^2x^2+816ax+384\right)}{1920x^5(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/x^6,x]
```

```
[Out] -((1 - a*x)^(3/4)*(384 + 816*a*x + 872*a^2*x^2 + 978*a^3*x^3 + 1149*a^4*x^4
 + 611*a^5*x^5 + 310*a^5*x^5*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 +
a*x)]))/(1920*x^5*(1 + a*x)^(3/4))
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int \frac{1}{x^6}\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^6,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^6, x)
```

Fricas [A] time = 1.77041, size = 400, normalized size = 2.03

$$-\frac{930\,a^5x^5\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+465\,a^5x^5\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-465\,a^5x^5\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\left(611\,a^5x^5-\right.}{3840\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/3840*(930*a^5*x^5*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 465*a^5*
x^5*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 465*a^5*x^5*log(sqrt(-sq
rt(-a^2*x^2 + 1)/(a*x - 1)) - 1) - 2*(611*a^5*x^5 - 73*a^4*x^4 - 98*a^3*x^3
 - 8*a^2*x^2 - 48*a*x - 384)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^6, x)
```

3.70 $\int e^{\frac{3}{2}\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=31

$$\frac{x^{m+1}F_1\left(m+1;\frac{3}{4},-\frac{3}{4};m+2;ax,-ax\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, 3/4, -3/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rubi [A] time = 0.0278198, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6126, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{3}{4},-\frac{3}{4};m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 3/4, -3/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\frac{3}{2}\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{3}{4},-\frac{3}{4};2+m;ax,-ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.30296, size = 0, normalized size = 0.

$$\int e^{\frac{3}{2}\tanh^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)*x^m,x]
```

```
[Out] Integrate[E^((3*ArcTanh[a*x])/2)*x^m, x]
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int \left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^m,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}x^m\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^m*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))/(a*x -
 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

3.71 $\int e^{\frac{3}{2}\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=290

$$-\frac{x^2\sqrt[4]{1-ax}(ax+1)^{7/4}}{4a^2}-\frac{\sqrt[4]{1-ax}(ax+1)^{7/4}(4ax+11)}{32a^4}-\frac{41\sqrt[4]{1-ax}(ax+1)^{3/4}}{64a^4}+\frac{123\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{128\sqrt{2}a^4}-\cdots$$

```
[Out] (-41*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(64*a^4) - (x^2*(1 - a*x)^(1/4)*(1 +
a*x)^(7/4))/(4*a^2) - ((1 - a*x)^(1/4)*(1 + a*x)^(7/4)*(11 + 4*a*x))/(32*a^
4) + (123*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2
]*a^4) - (123*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sq
rt[2]*a^4) + (123*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^
(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4) - (123*Log[1 + Sqrt[1 - a*x]/Sqr
t[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rubi [A] time = 0.20398, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 100, 147, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^2\sqrt[4]{1-ax}(ax+1)^{7/4}}{4a^2}-\frac{\sqrt[4]{1-ax}(ax+1)^{7/4}(4ax+11)}{32a^4}-\frac{41\sqrt[4]{1-ax}(ax+1)^{3/4}}{64a^4}+\frac{123\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{128\sqrt{2}a^4}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)*x^3,x]
```

```
[Out] (-41*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(64*a^4) - (x^2*(1 - a*x)^(1/4)*(1 +
a*x)^(7/4))/(4*a^2) - ((1 - a*x)^(1/4)*(1 + a*x)^(7/4)*(11 + 4*a*x))/(32*a^
4) + (123*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2
]*a^4) - (123*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sq
rt[2]*a^4) + (123*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^
(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4) - (123*Log[1 + Sqrt[1 - a*x]/Sqr
t[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
```

```
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{3}{2}\tanh^{-1}(ax)}x^3\,dx = \int \frac{x^3(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx$$

$$= -\frac{x^2\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^2} - \frac{\int \frac{x\left(-2-\frac{3ax}{2}\right)(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx}{4a^2}$$

$$= -\frac{x^2\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}(11+4ax)}{32a^4} + \frac{41\int \frac{(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx}{64a^3}$$

$$= -\frac{41\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} - \frac{x^2\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}(11+4ax)}{32a^4} + \frac{123\int \frac{1}{(1-ax)^{3/4}\cdots}}{128a\cdots}$$

$$= -\frac{41\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} - \frac{x^2\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}(11+4ax)}{32a^4} - \frac{123\,\mathrm{Subst}\left(\int\cdots\right.}{\cdots}$$

$$= -\frac{41\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} - \frac{x^2\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}(11+4ax)}{32a^4} - \frac{123\,\mathrm{Subst}\left(\int\cdots\right.}{\cdots}$$

$$= -\frac{41\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} - \frac{x^2\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}(11+4ax)}{32a^4} - \frac{123\,\mathrm{Subst}\left(\int\cdots\right.}{\cdots}$$

$$= -\frac{41\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} - \frac{x^2\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}(11+4ax)}{32a^4} - \frac{123\,\mathrm{Subst}\left(\int\cdots\right.}{\cdots}$$

$$= -\frac{41\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} - \frac{x^2\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}(11+4ax)}{32a^4} + \frac{123\log\left(1+\cdots\right.}{12\cdots}$$

$$= -\frac{41\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} - \frac{x^2\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}(11+4ax)}{32a^4} + \frac{123\tan^{-1}\left(1\cdots\right.}{64\sqrt{\cdots}}$$

Mathematica [C] time = 0.102758, size = 131, normalized size = 0.45

$$-\frac{\sqrt[4]{1-ax}\left(24\ 2^{3/4}\mathrm{Hypergeometric2F1}\left(-\frac{11}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-ax)\right)-8\ 2^{3/4}\mathrm{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-ax)\right)-\cdots\right.}{4a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)*x^3,x]
```

```
[Out] -((1 - a*x)^(1/4)*(a^2*x^2*(1 + a*x)^(3/4) + a^3*x^3*(1 + a*x)^(3/4) + 24*2
^(3/4)*Hypergeometric2F1[-11/4, 1/4, 5/4, (1 - a*x)/2] - 8*2^(3/4)*Hypergeo
metric2F1[-7/4, 1/4, 5/4, (1 - a*x)/2] - 2*2^(3/4)*Hypergeometric2F1[-3/4,
1/4, 5/4, (1 - a*x)/2]))/(4*a^4)
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int \left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^3,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

Fricas [B] time = 1.91593, size = 1403, normalized size = 4.84

$$-\frac{492\sqrt{2}a^4\left(\frac{1}{a^{16}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^4\sqrt{\frac{\sqrt{2}\left(a^{13}x-a^{12}\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{3}{4}}+\left(a^9x-a^8\right)\sqrt{\frac{1}{a^{16}}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{1}{4}}-\sqrt{2}a^4\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{1}{4}}-1\right)+492\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/256*(492*sqrt(2)*a^4*(a^(-16))^(1/4)*arctan(sqrt(2)*a^4*sqrt((sqrt(2)*(a
^13*x - a^12)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) + (a^9*x
- a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(1/4) - sqr
t(2)*a^4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) - 1) + 492*sqr
t(2)*a^4*(a^(-16))^(1/4)*arctan(sqrt(2)*a^4*sqrt(-(sqrt(2)*(a^13*x - a^12)*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) - (a^9*x - a^8)*sqrt(a^
(-16)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(1/4) - sqrt(2)*a^4*sqrt(
-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) + 1) + 123*sqrt(2)*a^4*(a^(-
16))^(1/4)*log((sqrt(2)*(a^13*x - a^12)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))
*(a^(-16))^(3/4) + (a^9*x - a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x -
 1)) - 123*sqrt(2)*a^4*(a^(-16))^(1/4)*log(-(sqrt(2)*(a^13*x - a^12)*sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) - (a^9*x - a^8)*sqrt(a^(-16))
```

```
 + sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 4*(16*a^3*x^3 + 24*a^2*x^2 + 30*a*x + 6
3)*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)*x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

3.72 $\int e^{\frac{3}{2}\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=282

$$-\frac{x\sqrt[4]{1-ax}(ax+1)^{7/4}}{3a^2}-\frac{\sqrt[4]{1-ax}(ax+1)^{7/4}}{4a^3}-\frac{17\sqrt[4]{1-ax}(ax+1)^{3/4}}{24a^3}+\frac{17\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}a^3}-\frac{17\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{}}{}\right.}{16\sqrt{2}a^{}}$$

```
[Out] (-17*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(24*a^3) - ((1 - a*x)^(1/4)*(1 + a*x)
^(7/4))/(4*a^3) - (x*(1 - a*x)^(1/4)*(1 + a*x)^(7/4))/(3*a^2) + (17*ArcTan[
1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (17*ArcTa
n[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) + (17*Log
[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4
)])/(16*Sqrt[2]*a^3) - (17*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(
1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3)
```

Rubi [A] time = 0.199776, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 90, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x\sqrt[4]{1-ax}(ax+1)^{7/4}}{3a^2}-\frac{\sqrt[4]{1-ax}(ax+1)^{7/4}}{4a^3}-\frac{17\sqrt[4]{1-ax}(ax+1)^{3/4}}{24a^3}+\frac{17\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}a^3}-\frac{17\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{}}{}\right.}{16\sqrt{2}a^{}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)*x^2,x]
```

```
[Out] (-17*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(24*a^3) - ((1 - a*x)^(1/4)*(1 + a*x)
^(7/4))/(4*a^3) - (x*(1 - a*x)^(1/4)*(1 + a*x)^(7/4))/(3*a^2) + (17*ArcTan[
1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (17*ArcTa
n[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) + (17*Log
[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4
)])/(16*Sqrt[2]*a^3) - (17*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(
1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{3}{2}\tanh^{-1}(ax)}x^2\,dx = \int \frac{x^2(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx$$

$$= -\frac{x\sqrt[4]{1-ax}(1+ax)^{7/4}}{3a^2} - \frac{\int \frac{\left(-1-\frac{3ax}{2}\right)(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx}{3a^2}$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^3} - \frac{x\sqrt[4]{1-ax}(1+ax)^{7/4}}{3a^2} + \frac{17\int \frac{(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx}{24a^2}$$

$$= -\frac{17\sqrt[4]{1-ax}(1+ax)^{3/4}}{24a^3} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^3} - \frac{x\sqrt[4]{1-ax}(1+ax)^{7/4}}{3a^2} + \frac{17\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx}{16a^2}$$

$$= -\frac{17\sqrt[4]{1-ax}(1+ax)^{3/4}}{24a^3} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^3} - \frac{x\sqrt[4]{1-ax}(1+ax)^{7/4}}{3a^2} - \frac{17\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1\ldots}\right.}{4a^3}$$

$$= -\frac{17\sqrt[4]{1-ax}(1+ax)^{3/4}}{24a^3} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^3} - \frac{x\sqrt[4]{1-ax}(1+ax)^{7/4}}{3a^2} - \frac{17\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\ldots}}{\sqrt[4]{1+\ldots}}\right.}{4a^3}$$

$$= -\frac{17\sqrt[4]{1-ax}(1+ax)^{3/4}}{24a^3} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^3} - \frac{x\sqrt[4]{1-ax}(1+ax)^{7/4}}{3a^2} - \frac{17\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\ldots}}{\sqrt[4]{1+\ldots}}\right.}{8a^3}$$

$$= -\frac{17\sqrt[4]{1-ax}(1+ax)^{3/4}}{24a^3} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^3} - \frac{x\sqrt[4]{1-ax}(1+ax)^{7/4}}{3a^2} - \frac{17\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \ldots\right.}{16a^3}$$

$$= -\frac{17\sqrt[4]{1-ax}(1+ax)^{3/4}}{24a^3} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^3} - \frac{x\sqrt[4]{1-ax}(1+ax)^{7/4}}{3a^2} + \frac{17\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\ldots\right.}{16\sqrt{2}a^3}$$

$$= -\frac{17\sqrt[4]{1-ax}(1+ax)^{3/4}}{24a^3} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{4a^3} - \frac{x\sqrt[4]{1-ax}(1+ax)^{7/4}}{3a^2} + \frac{17\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8\sqrt{2}a^3} - \ldots$$

Mathematica [C] time = 0.0319156, size = 69, normalized size = 0.24

$$-\frac{\sqrt[4]{1-ax}\left(34\,2^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-ax)\right)+(ax+1)^{3/4}\left(4a^2x^2+7ax+3\right)\right)}{12a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)*x^2,x]
```

```
[Out] -((1 - a*x)^(1/4)*((1 + a*x)^(3/4)*(3 + 7*a*x + 4*a^2*x^2) + 34*2^(3/4)*Hyp
ergeometric2F1[-3/4, 1/4, 5/4, (1 - a*x)/2]))/(12*a^3)
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^2,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

Fricas [B] time = 1.86407, size = 1374, normalized size = 4.87

$$-\frac{204\sqrt{2}a^3\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^3\sqrt{\frac{\sqrt{2}\left(a^{10}x-a^9\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}+\left(a^7x-a^6\right)\sqrt{\frac{1}{a^{12}}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}-\sqrt{2}a^3\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}-1\right)+2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/96*(204*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^3*sqrt((sqrt(2)*(a^
10*x - a^9)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) + (a^7*x -
a^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(1/4) - sqrt(
2)*a^3*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - 1) + 204*sqrt(
2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^3*sqrt(-(sqrt(2)*(a^10*x - a^9)*sqr
t(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) - (a^7*x - a^6)*sqrt(a^(-1
2)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(1/4) - sqrt(2)*a^3*sqrt(-sq
rt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) + 1) + 51*sqrt(2)*a^3*(a^(-12))
^(1/4)*log((sqrt(2)*(a^10*x - a^9)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(
-12))^(3/4) + (a^7*x - a^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))
- 51*sqrt(2)*a^3*(a^(-12))^(1/4)*log(-(sqrt(2)*(a^10*x - a^9)*sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) - (a^7*x - a^6)*sqrt(a^(-12)) + sqrt
(-a^2*x^2 + 1))/(a*x - 1)) + 4*(8*a^2*x^2 + 14*a*x + 23)*sqrt(-a^2*x^2 + 1)
*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)*x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

3.73 $\int e^{\frac{3}{2}\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=255

$$-\frac{\sqrt[4]{1-ax}(ax+1)^{7/4}}{2a^2}-\frac{3\sqrt[4]{1-ax}(ax+1)^{3/4}}{4a^2}+\frac{9\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}-\frac{9\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}+\frac{9\tan^{-1}}{}$$

```
[Out] (-3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*a^2) - ((1 - a*x)^(1/4)*(1 + a*x)^(
7/4))/(2*a^2) + (9*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(
4*Sqrt[2]*a^2) - (9*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/
(4*Sqrt[2]*a^2) + (9*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*
x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2) - (9*Log[1 + Sqrt[1 - a*x]/Sqrt
[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rubi [A] time = 0.168539, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6126, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{1-ax}(ax+1)^{7/4}}{2a^2}-\frac{3\sqrt[4]{1-ax}(ax+1)^{3/4}}{4a^2}+\frac{9\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}-\frac{9\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}+\frac{9\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)*x,x]
```

```
[Out] (-3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*a^2) - ((1 - a*x)^(1/4)*(1 + a*x)^(
7/4))/(2*a^2) + (9*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(
4*Sqrt[2]*a^2) - (9*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/
(4*Sqrt[2]*a^2) + (9*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*
x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2) - (9*Log[1 + Sqrt[1 - a*x]/Sqrt
[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{3}{2}\tanh^{-1}(ax)}x\,dx = \int \frac{x(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{2a^2} + \frac{3\int \frac{(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx}{4a}$$
$$= -\frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{2a^2} + \frac{9\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx}{8a}$$
$$= -\frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{2a^2} - \frac{9\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)}{2a^2}$$
$$= -\frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{2a^2} - \frac{9\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a^2}$$
$$= -\frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{2a^2} - \frac{9\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4a^2} - \frac{9\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\right.}{4a^2}$$
$$= -\frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{2a^2} - \frac{9\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8a^2} - \frac{9\,\text{Subst}\left(\int \right.}{}$$
$$= -\frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{2a^2} + \frac{9\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8\sqrt{2}a^2} - \frac{9\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} +\right.}{8\sqrt{2}a^2}$$
$$= -\frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{2a^2} + \frac{9\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}a^2} - \frac{9\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}a^2} +$$

Mathematica [C] time = 0.0173623, size = 54, normalized size = 0.21

$$-\frac{\sqrt[4]{1-ax}\left(6\,2^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-ax)\right)+(ax+1)^{7/4}\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)*x,x]
```

```
[Out] -((1 - a*x)^(1/4)*((1 + a*x)^(7/4) + 6*2^(3/4)*Hypergeometric2F1[-3/4, 1/4,
 5/4, (1 - a*x)/2]))/(2*a^2)
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int \left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

Fricas [B] time = 1.8909, size = 1323, normalized size = 5.19

$$-\frac{36\sqrt{2}a^2\left(\frac{1}{a^8}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^2\sqrt{\frac{\sqrt{2}\left(a^7x-a^6\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^8}\right)^{\frac{3}{4}}+\left(a^5x-a^4\right)\sqrt{\frac{1}{a^8}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^8}\right)^{\frac{1}{4}}-\sqrt{2}a^2\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^8}\right)^{\frac{1}{4}}-1\right)+36\sqrt{2}a^2\frac{1}{a^8}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="fricas")
```

```
[Out] -1/16*(36*sqrt(2)*a^2*(a^(-8))^(1/4)*arctan(sqrt(2)*a^2*sqrt((sqrt(2)*(a^7*
x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) + (a^5*x - a^4)
*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(1/4) - sqrt(2)*a^2
*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) - 1) + 36*sqrt(2)*a^2*(
a^(-8))^(1/4)*arctan(sqrt(2)*a^2*sqrt(-(sqrt(2)*(a^7*x - a^6)*sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-
a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(1/4) - sqrt(2)*a^2*sqrt(-sqrt(-a^2*x^2 +
 1)/(a*x - 1))*(a^(-8))^(1/4) + 1) + 9*sqrt(2)*a^2*(a^(-8))^(1/4)*log((sqrt
(2)*(a^7*x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) + (a^5
*x - a^4)*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 9*sqrt(2)*a^2*(a^
(-8))^(1/4)*log(-(sqrt(2)*(a^7*x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))
*(a^(-8))^(3/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-a^2*x^2 + 1))/(a*x - 1
)) + 4*sqrt(-a^2*x^2 + 1)*(2*a*x + 5)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/
a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)*x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="giac")
```

```
[Out] integrate(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

3.74 $\int e^{\frac{3}{2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=223

$$-\frac{\sqrt[4]{1-ax}(ax+1)^{3/4}}{a}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{\sqrt{2}a}-\frac{3\tan^{-1}(}{}$$

```
[Out] -(((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/a) + (3*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(
1/4))/(1 + a*x)^(1/4)])/(Sqrt[2]*a) - (3*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4
))/(1 + a*x)^(1/4)])/(Sqrt[2]*a) + (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] -
 (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*Sqrt[2]*a) - (3*Log[1 + Sqr
t[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*S
qrt[2]*a)
```

Rubi [A] time = 0.134412, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6125, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{1-ax}(ax+1)^{3/4}}{a}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{\sqrt{2}a}-\frac{3\tan^{-1}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2),x]
```


```
[Out] -(((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/a) + (3*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(
1/4))/(1 + a*x)^(1/4)])/(Sqrt[2]*a) - (3*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4
))/(1 + a*x)^(1/4)])/(Sqrt[2]*a) + (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] -
 (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*Sqrt[2]*a) - (3*Log[1 + Sqr
t[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*S
qrt[2]*a)
```


Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{\frac{3}{2}\tanh^{-1}(ax)}\,dx &= \int \frac{(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} + \frac{3}{2}\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx \\
&= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} - \frac{6\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a} \\
&= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} - \frac{6\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} \\
&= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} - \frac{3\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{3\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} \\
&= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a} + \frac{3\,\mathrm{S}}{} \\
&= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} + \frac{3\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}a} - \frac{3\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}a} - \frac{3\,\mathrm{Subst}\left(\int \frac{}{-1}\right.}{} \\
&= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} + \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} - \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} + \frac{3\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-a}}{\sqrt[4]{1+ax}}\right.}{2\sqrt{2}a}
\end{aligned}
$$

Mathematica [C] time = 0.0594765, size = 48, normalized size = 0.22

$$
\frac{8e^{\frac{3}{2}\tanh^{-1}(ax)}\left(\mathrm{Hypergeometric2F1}\left(\frac{3}{4}, 2, \frac{7}{4}, -e^{2\tanh^{-1}(ax)}\right) - \frac{1}{e^{2\tanh^{-1}(ax)}+1}\right)}{a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2),x]
```

```
[Out] (8*E^((3*ArcTanh[a*x])/2)*(-(1 + E^(2*ArcTanh[a*x]))^(-1) + Hypergeometric2
F1[3/4, 2, 7/4, -E^(2*ArcTanh[a*x])]))/a
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$
\int \left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

Fricas [B] time = 1.87931, size = 1281, normalized size = 5.74

$$-\frac{12\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\sqrt{2}a\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-1\right)+12\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(12*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt((sqrt(2)*(a^4*x - a
^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*sqrt
(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) - 1) + 12*sqrt(2)*a*(a^(-4))^(
1/4)*arctan(sqrt(2)*a*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1))*(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1
))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)
)*(a^(-4))^(1/4) + 1) + 3*sqrt(2)*a*(a^(-4))^(1/4)*log((sqrt(2)*(a^4*x - a^
3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*sqrt(
a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 3*sqrt(2)*a*(a^(-4))^(1/4)*log(-
(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) -
 (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 4*sqrt(-a^2*
x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

3.75 $\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=227

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)-\sqrt{2}\tan^{-1}$$

```
[Out] 2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1
- a*x)^(1/4))/(1 + a*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4
))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] + Log[1 +
Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sq
rt[2] - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1
+ a*x)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.15811, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6126, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)-\sqrt{2}\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)/x,x]
```

```
[Out] 2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1
- a*x)^(1/4))/(1 + a*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4
))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] + Log[1 +
Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sq
rt[2] - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1
+ a*x)^(1/4)]/Sqrt[2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{(1+ax)^{3/4}}{x(1-ax)^{3/4}}\,dx \\
&= a\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx + \int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx \\
&= -\left(4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)\right) + 4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) \\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 4\,\text{Subst}\left(\int \frac{1}{1+x^4}\,d \right. \\
&= 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - 2\,\text{Subst}\left(\int \frac{1+x}{1+x}\right. \\
&= 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x}\right.}{\sqrt{2}} \\
&= 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right.}{\sqrt{2}} \\
&= 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+a}}{\sqrt[4]{1-a}}\right.
\end{aligned}
$$

Mathematica [C] time = 0.0291235, size = 83, normalized size = 0.37

$$
-2\,2^{3/4}\sqrt[4]{1-ax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-ax)\right) - \frac{4\sqrt[4]{1-ax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},-\frac{1-ax}{-ax-1}\right)}{\sqrt[4]{ax+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)/x,x]
```

```
[Out] -2*2^(3/4)*(1 - a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (1 - a*x)/2] -
(4*(1 - a*x)^(1/4)*Hypergeometric2F1[1/4, 1, 5/4, -((1 - a*x)/(-1 - a*x))])
/(1 + a*x)^(1/4)
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x, x)
```

Fricas [B] time = 1.8377, size = 999, normalized size = 4.4

$$-2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{ax+\sqrt{2}(ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-\sqrt{-a^2x^2+1}-1}{ax-1}}-\sqrt{2}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2
 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt(2)*sqrt(-sqrt(
-a^2*x^2 + 1)/(a*x - 1)) - 1) - 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x - sqrt(2
)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(
a*x - 1)) - sqrt(2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 1/2*sqrt(2)*
log(4*(a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-
a^2*x^2 + 1) - 1)/(a*x - 1)) + 1/2*sqrt(2)*log(4*(a*x - sqrt(2)*(a*x - 1)*s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) + 2
*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - log(sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1)) + 1) + log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x, x)
```

3.76 $\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=73

$$-\frac{\sqrt[4]{1-ax}(ax+1)^{3/4}}{x}+3a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-3a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

```
[Out] -(((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/x) + 3*a*ArcTan[(1 + a*x)^(1/4)/(1 - a*
x)^(1/4)] - 3*a*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)]
```

Rubi [A] time = 0.0327228, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6126, 94, 93, 298, 203, 206}

$$-\frac{\sqrt[4]{1-ax}(ax+1)^{3/4}}{x}+3a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-3a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)/x^2,x]
```

```
[Out] -(((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/x) + 3*a*ArcTan[(1 + a*x)^(1/4)/(1 - a*
x)^(1/4)] - 3*a*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1+ax)^{3/4}}{x^2(1-ax)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{x} + \frac{1}{2}(3a)\int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx\\
&= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{x} + (6a)\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{x} - (3a)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + (3a)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{x} + 3a\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 3a\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0132688, size = 55, normalized size = 0.75

$$
-\frac{\sqrt[4]{1-ax}\left(6ax\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1-ax}{ax+1}\right)+ax+1\right)}{x\sqrt[4]{ax+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)/x^2,x]
```

```
[Out] -(((1 - a*x)^(1/4)*(1 + a*x + 6*a*x*Hypergeometric2F1[1/4, 1, 5/4, (1 - a*x
)/(1 + a*x)]))/(x*(1 + a*x)^(1/4)))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$
\int \frac{1}{x^2}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x^2, x)
```

Fricas [B] time = 1.75448, size = 302, normalized size = 4.14

$$\frac{6\,ax\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-3\,ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)+3\,ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\,\sqrt{-a^2x^2+1}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*x*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 3*a*x*log(sqrt(-sq
rt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 3*a*x*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
- 1)) - 1) - 2*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x^2, x)
```

3.77 $\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=110

$$\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{\sqrt[4]{1-ax}(ax+1)^{7/4}}{2x^2}-\frac{3a\sqrt[4]{1-ax}(ax+1)^{3/4}}{4x}$$

```
[Out] (-3*a*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*x) - ((1 - a*x)^(1/4)*(1 + a*x)^(
7/4))/(2*x^2) + (9*a^2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (9*a^2*
ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4
```

Rubi [A] time = 0.0435755, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6126, 96, 94, 93, 298, 203, 206}

$$\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{\sqrt[4]{1-ax}(ax+1)^{7/4}}{2x^2}-\frac{3a\sqrt[4]{1-ax}(ax+1)^{3/4}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)/x^3,x]
```

```
[Out] (-3*a*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*x) - ((1 - a*x)^(1/4)*(1 + a*x)^(
7/4))/(2*x^2) + (9*a^2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (9*a^2*
ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1+ax)^{3/4}}{x^3(1-ax)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{2x^2} + \frac{1}{4}(3a)\int \frac{(1+ax)^{3/4}}{x^2(1-ax)^{3/4}}\,dx\\
&= -\frac{3a\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{2x^2} + \frac{1}{8}\left(9a^2\right)\int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx\\
&= -\frac{3a\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{2x^2} + \frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= -\frac{3a\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{2x^2} - \frac{1}{4}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{1}{4}\left(9a\right.\\
&= -\frac{3a\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x} - \frac{\sqrt[4]{1-ax}(1+ax)^{7/4}}{2x^2} + \frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0161923, size = 70, normalized size = 0.64

$$-\frac{\sqrt[4]{1-ax}\left(18a^2x^2\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1-ax}{ax+1}\right)+5a^2x^2+7ax+2\right)}{4x^2\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)/x^3,x]
```

```
[Out] -((1 - a*x)^(1/4)*(2 + 7*a*x + 5*a^2*x^2 + 18*a^2*x^2*Hypergeometric2F1[1/4
, 1, 5/4, (1 - a*x)/(1 + a*x)]))/(4*x^2*(1 + a*x)^(1/4))
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}} {\left((ax+1)\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)}^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{3}{2}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x^3, x)
```

Fricas [A] time = 1.6901, size = 339, normalized size = 3.08

$$\frac{18\, a^{2}x^{2} \arctan\left(\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}\right) - 9\, a^{2}x^{2} \log\left(\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}+1\right) + 9\, a^{2}x^{2} \log\left(\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}-1\right) - 2\, \sqrt{-a^{2}x^{2}+1}(5\, ax+2)\sqrt{}}{8\, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(18*a^2*x^2*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 9*a^2*x^2*log
(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 9*a^2*x^2*log(sqrt(-sqrt(-a^2*x
^2 + 1)/(a*x - 1)) - 1) - 2*sqrt(-a^2*x^2 + 1)*(5*a*x + 2)*sqrt(-sqrt(-a^2*
x^2 + 1)/(a*x - 1)))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x^3, x)
```

3.78 $\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=139

$$-\frac{23a^2\sqrt[4]{1-ax}(ax+1)^{3/4}}{24x}+\frac{17}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{17}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{7a\sqrt[4]{1-ax}(ax+1)^{3/4}}{12x^2}-\frac{\sqrt[4]{1-ax}(ax+1)^{3/4}}{3x^3}$$

```
[Out] -((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(3*x^3) - (7*a*(1 - a*x)^(1/4)*(1 + a*x)
^(3/4))/(12*x^2) - (23*a^2*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(24*x) + (17*a^
3*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8 - (17*a^3*ArcTanh[(1 + a*x)^(1
/4)/(1 - a*x)^(1/4)])/8
```

Rubi [A] time = 0.0612806, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 298, 203, 206}

$$-\frac{23a^2\sqrt[4]{1-ax}(ax+1)^{3/4}}{24x}+\frac{17}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{17}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{7a\sqrt[4]{1-ax}(ax+1)^{3/4}}{12x^2}-\frac{\sqrt[4]{1-ax}(ax+1)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)/x^4,x]
```

```
[Out] -((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(3*x^3) - (7*a*(1 - a*x)^(1/4)*(1 + a*x)
^(3/4))/(12*x^2) - (23*a^2*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(24*x) + (17*a^
3*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8 - (17*a^3*ArcTanh[(1 + a*x)^(1
/4)/(1 - a*x)^(1/4)])/8
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{(1+ax)^{3/4}}{x^4(1-ax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{3x^3} + \frac{1}{3}\int \frac{\frac{7a}{2}+2a^2x}{x^3(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{3x^3} - \frac{7a\sqrt[4]{1-ax}(1+ax)^{3/4}}{12x^2} - \frac{1}{6}\int \frac{-\frac{23a^2}{4}-\frac{7a^3x}{2}}{x^2(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{3x^3} - \frac{7a\sqrt[4]{1-ax}(1+ax)^{3/4}}{12x^2} - \frac{23a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x} + \frac{1}{6}\int \frac{51a^3}{8x(1-ax)^{3/4}\sqrt[4]{1+\cdots}}$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{3x^3} - \frac{7a\sqrt[4]{1-ax}(1+ax)^{3/4}}{12x^2} - \frac{23a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x} + \frac{1}{16}\left(17a^3\right)\int \frac{1\cdots}{x(1-ax)^{3\cdots}}$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{3x^3} - \frac{7a\sqrt[4]{1-ax}(1+ax)^{3/4}}{12x^2} - \frac{23a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x} + \frac{1}{4}\left(17a^3\right)\mathrm{Subst}\left(\int \frac{\cdots}{-1\cdots}\right.$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{3x^3} - \frac{7a\sqrt[4]{1-ax}(1+ax)^{3/4}}{12x^2} - \frac{23a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x} - \frac{1}{8}\left(17a^3\right)\mathrm{Subst}\left(\int \frac{1\cdots}{1-\cdots}\right.$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{3x^3} - \frac{7a\sqrt[4]{1-ax}(1+ax)^{3/4}}{12x^2} - \frac{23a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x} + \frac{17}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$

Mathematica [C] time = 0.0237815, size = 78, normalized size = 0.56

$$-\frac{\sqrt[4]{1-ax}\left(102a^3x^3\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1-ax}{ax+1}\right)+23a^3x^3+37a^2x^2+22ax+8\right)}{24x^3\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)/x^4,x]
```

```
[Out] -((1 - a*x)^(1/4)*(8 + 22*a*x + 37*a^2*x^2 + 23*a^3*x^3 + 102*a^3*x^3*Hyper
geometric2F1[1/4, 1, 5/4, (1 - a*x)/(1 + a*x)]))/(24*x^3*(1 + a*x)^(1/4))
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x^4, x)
```

Fricas [A] time = 1.74006, size = 363, normalized size = 2.61

$$\frac{102\,a^3x^3\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-51\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)+51\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\left(23\,a^2x^2+14\,ax+8\right.}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(102*a^3*x^3*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 51*a^3*x^3*
log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 51*a^3*x^3*log(sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1)) - 1) - 2*(23*a^2*x^2 + 14*a*x + 8)*sqrt(-a^2*x^2 + 1
)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x^4, x)
```

3.79 $\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=168

$$-\frac{15a^2\sqrt[4]{1-ax}(ax+1)^{3/4}}{32x^2}-\frac{63a^3\sqrt[4]{1-ax}(ax+1)^{3/4}}{64x}+\frac{123}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{123}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{3a\sqrt[4]{1-ax}(a}{8x^3}$$

```
[Out] -((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*x^4) - (3*a*(1 - a*x)^(1/4)*(1 + a*x)
^(3/4))/(8*x^3) - (15*a^2*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(32*x^2) - (63*a
^3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(64*x) + (123*a^4*ArcTan[(1 + a*x)^(1/4
)/(1 - a*x)^(1/4)])/64 - (123*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])
/64
```

Rubi [A] time = 0.0780924, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 298, 203, 206}

$$-\frac{15a^2\sqrt[4]{1-ax}(ax+1)^{3/4}}{32x^2}-\frac{63a^3\sqrt[4]{1-ax}(ax+1)^{3/4}}{64x}+\frac{123}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{123}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{3a\sqrt[4]{1-ax}(a}{8x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)/x^5,x]
```

```
[Out] -((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*x^4) - (3*a*(1 - a*x)^(1/4)*(1 + a*x)
^(3/4))/(8*x^3) - (15*a^2*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(32*x^2) - (63*a
^3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(64*x) + (123*a^4*ArcTan[(1 + a*x)^(1/4
)/(1 - a*x)^(1/4)])/64 - (123*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])
/64
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x^5}\,dx = \int \frac{(1+ax)^{3/4}}{x^5(1-ax)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x^4} + \frac{1}{4}\int \frac{\frac{9a}{2}+3a^2x}{x^4(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$
$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x^4} - \frac{3a\sqrt[4]{1-ax}(1+ax)^{3/4}}{8x^3} - \frac{1}{12}\int \frac{-\frac{45a^2}{4}-9a^3x}{x^3(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$
$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x^4} - \frac{3a\sqrt[4]{1-ax}(1+ax)^{3/4}}{8x^3} - \frac{15a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{32x^2} + \frac{1}{24}\int \frac{\frac{189a^3}{8}+\frac{45a^4x}{4}}{x^2(1-ax)^{3/4}\sqrt[4]{1+a}}$$
$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x^4} - \frac{3a\sqrt[4]{1-ax}(1+ax)^{3/4}}{8x^3} - \frac{15a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{32x^2} - \frac{63a^3\sqrt[4]{1-ax}(1+ax)^{3/4}}{64x} -$$
$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x^4} - \frac{3a\sqrt[4]{1-ax}(1+ax)^{3/4}}{8x^3} - \frac{15a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{32x^2} - \frac{63a^3\sqrt[4]{1-ax}(1+ax)^{3/4}}{64x} +$$
$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x^4} - \frac{3a\sqrt[4]{1-ax}(1+ax)^{3/4}}{8x^3} - \frac{15a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{32x^2} - \frac{63a^3\sqrt[4]{1-ax}(1+ax)^{3/4}}{64x} +$$
$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x^4} - \frac{3a\sqrt[4]{1-ax}(1+ax)^{3/4}}{8x^3} - \frac{15a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{32x^2} - \frac{63a^3\sqrt[4]{1-ax}(1+ax)^{3/4}}{64x} -$$
$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x^4} - \frac{3a\sqrt[4]{1-ax}(1+ax)^{3/4}}{8x^3} - \frac{15a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{32x^2} - \frac{63a^3\sqrt[4]{1-ax}(1+ax)^{3/4}}{64x} +$$

Mathematica [C] time = 0.0272951, size = 86, normalized size = 0.51

$$-\frac{\sqrt[4]{1-ax}\left(246a^4x^4\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1-ax}{ax+1}\right)+63a^4x^4+93a^3x^3+54a^2x^2+40ax+16\right)}{64x^4\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)/x^5,x]
```

```
[Out] -((1 - a*x)^(1/4)*(16 + 40*a*x + 54*a^2*x^2 + 93*a^3*x^3 + 63*a^4*x^4 + 246
*a^4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (1 - a*x)/(1 + a*x)]))/(64*x^4*(1 +
 a*x)^(1/4))
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x^5, x)
```

Fricas [A] time = 1.7373, size = 386, normalized size = 2.3

$$\frac{246\,a^4x^4\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-123\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)+123\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\left(63\,a^3x^3+30\right.}{128\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/128*(246*a^4*x^4*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 123*a^4*x^
4*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 123*a^4*x^4*log(sqrt(-sqrt
(-a^2*x^2 + 1)/(a*x - 1)) - 1) - 2*(63*a^3*x^3 + 30*a^2*x^2 + 24*a*x + 16)*
sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x^5, x)
```

3.80 $\int e^{\frac{5}{2}\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=31

$$\frac{x^{m+1}F_1\left(m+1;\frac{5}{4},-\frac{5}{4};m+2;ax,-ax\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, 5/4, -5/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rubi [A] time = 0.025512, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6126, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{5}{4},-\frac{5}{4};m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 5/4, -5/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\frac{5}{2}\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1+ax)^{5/4}}{(1-ax)^{5/4}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{5}{4},-\frac{5}{4};2+m;ax,-ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.332336, size = 0, normalized size = 0.

$$\int e^{\frac{5}{2}\tanh^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)*x^m,x]
```

```
[Out] Integrate[E^((5*ArcTanh[a*x])/2)*x^m, x]
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int \left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^m,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ax+1)x^m\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(-(a*x + 1)*x^m*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))/(a*x - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

3.81 $\int e^{\frac{5}{2}\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=317

$$\frac{17x^2(1-ax)^{3/4}(ax+1)^{5/4}}{4a^2}+\frac{(1-ax)^{3/4}(ax+1)^{5/4}(452ax+521)}{96a^4}+\frac{475(1-ax)^{3/4}\sqrt[4]{ax+1}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right.}{128\sqrt{2}a^4}$$

```
[Out] (475*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(64*a^4) + (4*x^3*(1 + a*x)^(5/4))/(a
*(1 - a*x)^(1/4)) + (17*x^2*(1 - a*x)^(3/4)*(1 + a*x)^(5/4))/(4*a^2) + ((1
- a*x)^(3/4)*(1 + a*x)^(5/4)*(521 + 452*a*x))/(96*a^4) - (475*ArcTan[1 - (S
qrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (475*ArcTan[1
+ (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (475*Log[1
 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]
)/(128*Sqrt[2]*a^4) - (475*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(
1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rubi [A] time = 0.24137, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {6126, 97, 153, 147, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{17x^2(1-ax)^{3/4}(ax+1)^{5/4}}{4a^2}+\frac{(1-ax)^{3/4}(ax+1)^{5/4}(452ax+521)}{96a^4}+\frac{475(1-ax)^{3/4}\sqrt[4]{ax+1}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right.}{128\sqrt{2}a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)*x^3,x]
```

```
[Out] (475*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(64*a^4) + (4*x^3*(1 + a*x)^(5/4))/(a
*(1 - a*x)^(1/4)) + (17*x^2*(1 - a*x)^(3/4)*(1 + a*x)^(5/4))/(4*a^2) + ((1
- a*x)^(3/4)*(1 + a*x)^(5/4)*(521 + 452*a*x))/(96*a^4) - (475*ArcTan[1 - (S
qrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (475*ArcTan[1
+ (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (475*Log[1
 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]
)/(128*Sqrt[2]*a^4) - (475*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(
1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
```

```
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{5}{2}\tanh^{-1}(ax)}x^3\,dx = \int \frac{x^3(1+ax)^{5/4}}{(1-ax)^{5/4}}\,dx$$

$$= \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} - \frac{4\int \frac{x^2\sqrt[4]{1+ax}\left(3+\frac{17ax}{4}\right)}{\sqrt[4]{1-ax}}\,dx}{a}$$

$$= \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{\int \frac{x\sqrt[4]{1+ax}\left(-\frac{17a}{2}-\frac{113a^2x}{8}\right)}{\sqrt[4]{1-ax}}\,dx}{a^3}$$

$$= \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{(1-ax)^{3/4}(1+ax)^{5/4}(521+452ax)}{96a^4} - \frac{475\int \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,d}{64a^3}$$

$$= \frac{475(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} + \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{(1-ax)^{3/4}(1+ax)^{5/4}(521+}{96a^4}$$

$$= \frac{475(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} + \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{(1-ax)^{3/4}(1+ax)^{5/4}(521+}{96a^4}$$

$$= \frac{475(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} + \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{(1-ax)^{3/4}(1+ax)^{5/4}(521+}{96a^4}$$

$$= \frac{475(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} + \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{(1-ax)^{3/4}(1+ax)^{5/4}(521+}{96a^4}$$

$$= \frac{475(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} + \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{(1-ax)^{3/4}(1+ax)^{5/4}(521+}{96a^4}$$

$$= \frac{475(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} + \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{(1-ax)^{3/4}(1+ax)^{5/4}(521+}{96a^4}$$

$$= \frac{475(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} + \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{(1-ax)^{3/4}(1+ax)^{5/4}(521+}{96a^4}$$

Mathematica [C] time = 0.0455598, size = 74, normalized size = 0.23

$$\frac{(ax+1)^{9/4}\left(-6a^2x^2-5ax+59\right)-380\sqrt[4]{2}(ax-1)\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-ax)\right)}{24a^4\sqrt[4]{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)*x^3,x]
```

```
[Out] ((1 + a*x)^(9/4)*(59 - 5*a*x - 6*a^2*x^2) - 380*2^(1/4)*(-1 + a*x)*Hypergeo
metric2F1[-5/4, 3/4, 7/4, (1 - a*x)/2])/(24*a^4*(1 - a*x)^(1/4))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^3,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

Fricas [B] time = 1.83958, size = 1400, normalized size = 4.42

$$5700\sqrt{2}a^4\left(\frac{1}{a^{16}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^{12}\sqrt{\frac{\sqrt{2}\left(a^5x-a^4\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{1}{4}}+\left(a^9x-a^8\right)\sqrt{\frac{1}{a^{16}}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{3}{4}}-\sqrt{2}a^{12}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{3}{4}}-1\right)+5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/768*(5700*sqrt(2)*a^4*(a^(-16))^(1/4)*arctan(sqrt(2)*a^12*sqrt((sqrt(2)*(
a^5*x - a^4)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) + (a^9*x -
 a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(3/4) - sqrt
(2)*a^12*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) - 1) + 5700*sq
rt(2)*a^4*(a^(-16))^(1/4)*arctan(sqrt(2)*a^12*sqrt(-(sqrt(2)*(a^5*x - a^4)*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) - (a^9*x - a^8)*sqrt(a^
(-16)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(3/4) - sqrt(2)*a^12*sqrt
(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) + 1) - 1425*sqrt(2)*a^4*(a^
(-16))^(1/4)*log((sqrt(2)*(a^5*x - a^4)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))
*(a^(-16))^(1/4) + (a^9*x - a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x -
 1)) + 1425*sqrt(2)*a^4*(a^(-16))^(1/4)*log(-(sqrt(2)*(a^5*x - a^4)*sqrt(-s
qrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) - (a^9*x - a^8)*sqrt(a^(-16))
+ sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 4*(48*a^4*x^4 + 136*a^3*x^3 + 226*a^2*x^
2 + 521*a*x - 2467)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)*x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

3.82 $\int e^{\frac{5}{2}\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=305

$$\frac{(1-ax)^{3/4}(ax+1)^{9/4}}{3a^3}+\frac{2(ax+1)^{9/4}}{a^3\sqrt[4]{1-ax}}+\frac{11(1-ax)^{3/4}(ax+1)^{5/4}}{4a^3}+\frac{55(1-ax)^{3/4}\sqrt[4]{ax+1}}{8a^3}+\frac{55\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right.}{16\sqrt{2}a^3}$$

```
[Out] (55*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(8*a^3) + (11*(1 - a*x)^(3/4)*(1 + a*x
)^(5/4))/(4*a^3) + (2*(1 + a*x)^(9/4))/(a^3*(1 - a*x)^(1/4)) + ((1 - a*x)^(
3/4)*(1 + a*x)^(9/4))/(3*a^3) - (55*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1
 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) + (55*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/
(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) + (55*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x]
 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3) - (55*Log[1
 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]
)/(16*Sqrt[2]*a^3)
```

Rubi [A] time = 0.225932, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 89, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{(1-ax)^{3/4}(ax+1)^{9/4}}{3a^3}+\frac{2(ax+1)^{9/4}}{a^3\sqrt[4]{1-ax}}+\frac{11(1-ax)^{3/4}(ax+1)^{5/4}}{4a^3}+\frac{55(1-ax)^{3/4}\sqrt[4]{ax+1}}{8a^3}+\frac{55\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right.}{16\sqrt{2}a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)*x^2,x]
```

```
[Out] (55*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(8*a^3) + (11*(1 - a*x)^(3/4)*(1 + a*x
)^(5/4))/(4*a^3) + (2*(1 + a*x)^(9/4))/(a^3*(1 - a*x)^(1/4)) + ((1 - a*x)^(
3/4)*(1 + a*x)^(9/4))/(3*a^3) - (55*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1
 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) + (55*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/
(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) + (55*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x]
 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3) - (55*Log[1
 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]
)/(16*Sqrt[2]*a^3)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{5}{2}\tanh^{-1}(ax)}x^2\,dx = \int \frac{x^2(1+ax)^{5/4}}{(1-ax)^{5/4}}\,dx$$

$$= \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} - \frac{2\int \frac{(1+ax)^{5/4}\left(\frac{5a}{2}+\frac{a^2x}{2}\right)}{\sqrt[4]{1-ax}}\,dx}{a^3}$$

$$= \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} - \frac{11\int \frac{(1+ax)^{5/4}}{\sqrt[4]{1-ax}}\,dx}{2a^2}$$

$$= \frac{11(1-ax)^{3/4}(1+ax)^{5/4}}{4a^3} + \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} - \frac{55\int \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx}{8a^2}$$

$$= \frac{55(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} + \frac{11(1-ax)^{3/4}(1+ax)^{5/4}}{4a^3} + \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} - \frac{55\int}{}$$

$$= \frac{55(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} + \frac{11(1-ax)^{3/4}(1+ax)^{5/4}}{4a^3} + \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} + \frac{55\,\mathrm{Su}}{}$$

$$= \frac{55(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} + \frac{11(1-ax)^{3/4}(1+ax)^{5/4}}{4a^3} + \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} + \frac{55\,\mathrm{Su}}{}$$

$$= \frac{55(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} + \frac{11(1-ax)^{3/4}(1+ax)^{5/4}}{4a^3} + \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} - \frac{55\,\mathrm{Su}}{}$$

$$= \frac{55(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} + \frac{11(1-ax)^{3/4}(1+ax)^{5/4}}{4a^3} + \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} + \frac{55\,\mathrm{Su}}{}$$

$$= \frac{55(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} + \frac{11(1-ax)^{3/4}(1+ax)^{5/4}}{4a^3} + \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} + \frac{55\log}{}$$

$$= \frac{55(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} + \frac{11(1-ax)^{3/4}(1+ax)^{5/4}}{4a^3} + \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} - \frac{55\tan}{}$$

Mathematica [C] time = 0.0311002, size = 66, normalized size = 0.22

$$\frac{(7-ax)(ax+1)^{9/4} - 44\sqrt[4]{2}(ax-1)\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-ax)\right)}{3a^3\sqrt[4]{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)*x^2,x]
```

```
[Out] ((7 - a*x)*(1 + a*x)^(9/4) - 44*2^(1/4)*(-1 + a*x)*Hypergeometric2F1[-5/4,
3/4, 7/4, (1 - a*x)/2])/(3*a^3*(1 - a*x)^(1/4))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^2,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

Fricas [B] time = 1.87446, size = 1364, normalized size = 4.47

$$660\,\sqrt{2}a^3\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^9\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}+\left(a^7x-a^6\right)\sqrt{\frac{1}{a^{12}}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}-\sqrt{2}a^9\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}-1\right)+660\,\sqrt{2}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/96*(660*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^9*sqrt((sqrt(2)*(a^4
*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) + (a^7*x - a^
6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(3/4) - sqrt(2)
*a^9*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) - 1) + 660*sqrt(2)
*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^9*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - (a^7*x - a^6)*sqrt(a^(-12))
 + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(3/4) - sqrt(2)*a^9*sqrt(-sqrt(
-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) + 1) - 165*sqrt(2)*a^3*(a^(-12))^(
1/4)*log((sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12
))^(1/4) + (a^7*x - a^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 1
65*sqrt(2)*a^3*(a^(-12))^(1/4)*log(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*
x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - (a^7*x - a^6)*sqrt(a^(-12)) + sqrt(-a
```

```
^2*x^2 + 1))/(a*x - 1)) - 4*(8*a^3*x^3 + 26*a^2*x^2 + 61*a*x - 287)*sqrt(-s
qrt(-a^2*x^2 + 1)/(a*x - 1)))/a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)*x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

3.83 $\int e^{\frac{5}{2}\tanh^{-1}(ax)} x\,dx$

Optimal. Leaf size=279

$$\frac{2(ax+1)^{9/4}}{a^2\sqrt[4]{1-ax}}+\frac{5(1-ax)^{3/4}(ax+1)^{5/4}}{2a^2}+\frac{25(1-ax)^{3/4}\sqrt[4]{ax+1}}{4a^2}+\frac{25\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}-\frac{25\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-a}}{\sqrt[4]{ax+1}}\right.}{8\sqrt{2}a^2}$$

```
[Out] (25*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*a^2) + (5*(1 - a*x)^(3/4)*(1 + a*x)
^(5/4))/(2*a^2) + (2*(1 + a*x)^(9/4))/(a^2*(1 - a*x)^(1/4)) - (25*ArcTan[1
- (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt[2]*a^2) + (25*ArcTan[
1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt[2]*a^2) + (25*Log[1
 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]
)/(8*Sqrt[2]*a^2) - (25*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 -
 a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rubi [A] time = 0.193707, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6126, 78, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{2(ax+1)^{9/4}}{a^2\sqrt[4]{1-ax}}+\frac{5(1-ax)^{3/4}(ax+1)^{5/4}}{2a^2}+\frac{25(1-ax)^{3/4}\sqrt[4]{ax+1}}{4a^2}+\frac{25\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}-\frac{25\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-a}}{\sqrt[4]{ax+1}}\right.}{8\sqrt{2}a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)*x,x]
```

```
[Out] (25*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*a^2) + (5*(1 - a*x)^(3/4)*(1 + a*x)
^(5/4))/(2*a^2) + (2*(1 + a*x)^(9/4))/(a^2*(1 - a*x)^(1/4)) - (25*ArcTan[1
- (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt[2]*a^2) + (25*ArcTan[
1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt[2]*a^2) + (25*Log[1
 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]
)/(8*Sqrt[2]*a^2) - (25*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 -
 a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{5}{2}\tanh^{-1}(ax)}x\,dx = \int \frac{x(1+ax)^{5/4}}{(1-ax)^{5/4}}\,dx$$

$$= \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} - \frac{5\int \frac{(1+ax)^{5/4}}{\sqrt[4]{1-ax}}\,dx}{a}$$

$$= \frac{5(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} - \frac{25\int \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx}{4a}$$

$$= \frac{25(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} + \frac{5(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} - \frac{25\int \frac{1}{\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx}{8a}$$

$$= \frac{25(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} + \frac{5(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} + \frac{25\,\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right.}{2a^2}$$

$$= \frac{25(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} + \frac{5(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} + \frac{25\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a^2}$$

$$= \frac{25(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} + \frac{5(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} - \frac{25\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4a^2} + \cdots$$

$$= \frac{25(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} + \frac{5(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} + \frac{25\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8a^2} \cdots$$

$$= \frac{25(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} + \frac{5(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} + \frac{25\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8\sqrt{2}a^2} - \cdots$$

$$= \frac{25(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} + \frac{5(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} - \frac{25\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}a^2} + \frac{25\tan^{-1}\cdots}{\cdots}$$

Mathematica [C] time = 0.0252343, size = 61, normalized size = 0.22

$$\frac{6(ax+1)^{9/4} - 40\sqrt[4]{2}(ax-1)\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-ax)\right)}{3a^2\sqrt[4]{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)*x,x]
```

```
[Out] (6*(1 + a*x)^(9/4) - 40*2^(1/4)*(-1 + a*x)*Hypergeometric2F1[-5/4, 3/4, 7/4
, (1 - a*x)/2])/(3*a^2*(1 - a*x)^(1/4))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

Fricas [B] time = 1.87972, size = 1319, normalized size = 4.73

$$100\sqrt{2}a^2\frac{1}{a^8}^{\frac{1}{4}}\arctan\left(\sqrt{2}a^6\sqrt{\frac{\sqrt{2}\left(a^3x-a^2\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^8}^{\frac{1}{4}}+\left(a^5x-a^4\right)\sqrt{\frac{1}{a^8}}-\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^8}^{\frac{3}{4}}-\sqrt{2}a^6\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^8}^{\frac{3}{4}}-1\right)+100\sqrt{2}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="fricas")
```

```
[Out] 1/16*(100*sqrt(2)*a^2*(a^(-8))^(1/4)*arctan(sqrt(2)*a^6*sqrt((sqrt(2)*(a^3*
x - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) + (a^5*x - a^4)
*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(3/4) - sqrt(2)*a^6
*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) - 1) + 100*sqrt(2)*a^2*
(a^(-8))^(1/4)*arctan(sqrt(2)*a^6*sqrt(-(sqrt(2)*(a^3*x - a^2)*sqrt(-sqrt(-
a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(
-a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(3/4) - sqrt(2)*a^6*sqrt(-sqrt(-a^2*x^2
+ 1)/(a*x - 1))*(a^(-8))^(3/4) + 1) - 25*sqrt(2)*a^2*(a^(-8))^(1/4)*log((sq
rt(2)*(a^3*x - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) + (a
^5*x - a^4)*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 25*sqrt(2)*a^2*
(a^(-8))^(1/4)*log(-(sqrt(2)*(a^3*x - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x -
1))*(a^(-8))^(1/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-a^2*x^2 + 1))/(a*x
- 1)) - 4*(2*a^2*x^2 + 9*a*x - 43)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)*x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="giac")
```

```
[Out] integrate(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

3.84 $\int e^{\frac{5}{2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=247

$$\frac{4(ax+1)^{5/4}}{a\sqrt[4]{1-ax}}+\frac{5(1-ax)^{3/4}\sqrt[4]{ax+1}}{a}+\frac{5\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}-\frac{5\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\cdots}{\sqrt[4]{a\cdots}}\right.}{\sqrt{2}a}$$

```
[Out] (5*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/a + (4*(1 + a*x)^(5/4))/(a*(1 - a*x)^(1
/4)) - (5*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(Sqrt[2]*a
) + (5*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(Sqrt[2]*a) +
 (5*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*
x)^(1/4)])/(2*Sqrt[2]*a) - (5*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2
]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*Sqrt[2]*a)
```

Rubi [A] time = 0.165459, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.1, Rules used = {6125, 47, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{4(ax+1)^{5/4}}{a\sqrt[4]{1-ax}}+\frac{5(1-ax)^{3/4}\sqrt[4]{ax+1}}{a}+\frac{5\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}-\frac{5\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\cdots}{\sqrt[4]{a\cdots}}\right.}{\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] (5*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/a + (4*(1 + a*x)^(5/4))/(a*(1 - a*x)^(1
/4)) - (5*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(Sqrt[2]*a
) + (5*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(Sqrt[2]*a) +
 (5*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*
x)^(1/4)])/(2*Sqrt[2]*a) - (5*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2
]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*Sqrt[2]*a)
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{5}{2}\tanh^{-1}(ax)}dx = \int \frac{(1+ax)^{5/4}}{(1-ax)^{5/4}}dx$$
$$= \frac{4(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} - 5\int \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}dx$$
$$= \frac{5(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{4(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} - \frac{5}{2}\int \frac{1}{\sqrt[4]{1-ax}(1+ax)^{3/4}}dx$$
$$= \frac{5(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{4(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{10\,\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}dx, x, \sqrt[4]{1-ax}\right)}{a}$$
$$= \frac{5(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{4(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{10\,\text{Subst}\left(\int \frac{x^2}{1+x^4}dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= \frac{5(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{4(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} - \frac{5\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} + \frac{5\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}dx, x, \frac{\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.}{a}$$
$$= \frac{5(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{4(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{5\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a} + \frac{5\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\right.}{2a}$$
$$= \frac{5(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{4(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{5\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}a} - \frac{5\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}a}$$
$$= \frac{5(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{4(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} - \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} + \frac{5\log\left(1\right.}{}$$

Mathematica [A] time = 0.220073, size = 174, normalized size = 0.7

$$\frac{\frac{40e^{\frac{1}{2}\tanh^{-1}(ax)}}{e^{2\tanh^{-1}(ax)}+1} + \frac{32e^{\frac{5}{2}\tanh^{-1}(ax)}}{e^{2\tanh^{-1}(ax)}+1} + 5\sqrt{2}\log\left(-\sqrt{2}e^{\frac{1}{2}\tanh^{-1}(ax)} + e^{\tanh^{-1}(ax)}+1\right) - 5\sqrt{2}\log\left(\sqrt{2}e^{\frac{1}{2}\tanh^{-1}(ax)} + e^{\tanh^{-1}(ax)}+1\right)}{4a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] ((40*E^(ArcTanh[a*x]/2))/(1 + E^(2*ArcTanh[a*x])) + (32*E^((5*ArcTanh[a*x])
/2))/(1 + E^(2*ArcTanh[a*x])) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^(ArcTanh[a*
x]/2)] - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcTanh[a*x]/2)] + 5*Sqrt[2]*Log[
1 - Sqrt[2]*E^(ArcTanh[a*x]/2) + E^ArcTanh[a*x]] - 5*Sqrt[2]*Log[1 + Sqrt[2
]*E^(ArcTanh[a*x]/2) + E^ArcTanh[a*x]])/(4*a)
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

Fricas [B] time = 1.78938, size = 1268, normalized size = 5.13

$$20\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^3\sqrt{\frac{\sqrt{2}(a^2x-a)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}+(a^3x-a^2)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-\sqrt{2}a^3\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-1\right)+20\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(20*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*a^3*sqrt((sqrt(2)*(a^2*x -
a)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) + (a^3*x - a^2)*sqrt(
a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(3/4) - sqrt(2)*a^3*sqrt(
-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) - 1) + 20*sqrt(2)*a*(a^(-4))^
(1/4)*arctan(sqrt(2)*a^3*sqrt(-(sqrt(2)*(a^2*x - a)*sqrt(-sqrt(-a^2*x^2 + 1
)/(a*x - 1))*(a^(-4))^(1/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 +
1))/(a*x - 1))*(a^(-4))^(3/4) - sqrt(2)*a^3*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x -
 1))*(a^(-4))^(3/4) + 1) - 5*sqrt(2)*a*(a^(-4))^(1/4)*log((sqrt(2)*(a^2*x -
 a)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) + (a^3*x - a^2)*sqrt
(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 5*sqrt(2)*a*(a^(-4))^(1/4)*log(
-(sqrt(2)*(a^2*x - a)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) -
(a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 4*(a*x - 9)*s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

3.85 $\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=248

$$\frac{8\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)$$

```
[Out] (8*(1 + a*x)^(1/4))/(1 - a*x)^(1/4) - 2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1
/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)] + Sqrt
[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 +
a*x)^(1/4)/(1 - a*x)^(1/4)] + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2
]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - a*x]/Sqrt[1
+ a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.205454, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 16, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6126, 98, 21, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$\frac{8\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)/x,x]
```

```
[Out] (8*(1 + a*x)^(1/4))/(1 - a*x)^(1/4) - 2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1
/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)] + Sqrt
[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 +
a*x)^(1/4)/(1 - a*x)^(1/4)] + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2
]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - a*x]/Sqrt[1
+ a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sqrt[2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x}\,dx = \int \frac{(1+ax)^{5/4}}{x(1-ax)^{5/4}}\,dx$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - \frac{4\int \frac{-\frac{a}{4}+\frac{a^2x}{4}}{x\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx}{a}$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} + \int \frac{(1-ax)^{3/4}}{x(1+ax)^{3/4}}\,dx$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - a\int \frac{1}{\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx + \int \frac{1}{x\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} + 4\,\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right) + 4\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + 4\,\text{Subst}$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) + 2$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} + \text{Su}$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\right.}{}$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - 2$$

Mathematica [C] time = 0.03583, size = 93, normalized size = 0.38

$$\frac{4\left(3\sqrt[4]{2}(ax+1)^{3/4}\text{Hypergeometric2F1}\left(-\frac{1}{4},-\frac{1}{4},\frac{3}{4},\frac{1}{2}(1-ax)\right) + (ax-1)\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1-ax}{ax+1}\right) + 3ax\right.}{3\sqrt[4]{1-ax}(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)/x,x]
```

```
[Out] (4*(3 + 3*a*x + 3*2^(1/4)*(1 + a*x)^(3/4)*Hypergeometric2F1[-1/4, -1/4, 3/4
, (1 - a*x)/2] + (-1 + a*x)*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a
*x)]))/(3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="maxima")

[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)/x, x)

Fricas [B] time = 1.81951, size = 1052, normalized size = 4.24

$$2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{ax+\sqrt{2}(ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-\sqrt{-a^2x^2+1}-1}{ax-1}}-\sqrt{2}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{ax-}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="fricas")

[Out] 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt(2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1) + 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x - sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt(2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 1/2*sqrt(2)*log(4*(a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) + 1/2*sqrt(2)*log(4*(a*x - sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) + 8*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 2*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)/x, x)
```

3.86 $\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=95

$$-\frac{(ax+1)^{5/4}}{x\sqrt[4]{1-ax}}+\frac{10a\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}-5a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-5a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

```
[Out] (10*a*(1 + a*x)^(1/4))/(1 - a*x)^(1/4) - (1 + a*x)^(5/4)/(x*(1 - a*x)^(1/4)
) - 5*a*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] - 5*a*ArcTanh[(1 + a*x)^(1/
4)/(1 - a*x)^(1/4)]
```

Rubi [A] time = 0.0386483, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6126, 94, 93, 212, 206, 203}

$$-\frac{(ax+1)^{5/4}}{x\sqrt[4]{1-ax}}+\frac{10a\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}-5a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-5a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)/x^2,x]
```


```
[Out] (10*a*(1 + a*x)^(1/4))/(1 - a*x)^(1/4) - (1 + a*x)^(5/4)/(x*(1 - a*x)^(1/4)
) - 5*a*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] - 5*a*ArcTanh[(1 + a*x)^(1/
4)/(1 - a*x)^(1/4)]
```


Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1+ax)^{5/4}}{x^2(1-ax)^{5/4}}\,dx\\
&= -\frac{(1+ax)^{5/4}}{x\sqrt[4]{1-ax}}+\frac{1}{2}(5a)\int \frac{\sqrt[4]{1+ax}}{x(1-ax)^{5/4}}\,dx\\
&= \frac{10a\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}-\frac{(1+ax)^{5/4}}{x\sqrt[4]{1-ax}}+\frac{1}{2}(5a)\int \frac{1}{x\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx\\
&= \frac{10a\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}-\frac{(1+ax)^{5/4}}{x\sqrt[4]{1-ax}}+(10a)\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= \frac{10a\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}-\frac{(1+ax)^{5/4}}{x\sqrt[4]{1-ax}}-(5a)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-(5a)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\right.\\
&= \frac{10a\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}-\frac{(1+ax)^{5/4}}{x\sqrt[4]{1-ax}}-5a\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-5a\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.018959, size = 74, normalized size = 0.78

$$\frac{10ax(ax-1)\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1-ax}{ax+1}\right)+3\left(9a^2x^2+8ax-1\right)}{3x\sqrt[4]{1-ax}(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)/x^2,x]
```

```
[Out] (3*(-1 + 8*a*x + 9*a^2*x^2) + 10*a*x*(-1 + a*x)*Hypergeometric2F1[3/4, 1, 7
/4, (1 - a*x)/(1 + a*x)])/(3*x*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)/x^2, x)
```

Fricas [A] time = 1.69999, size = 296, normalized size = 3.12

$$-\frac{10\,ax\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+5\,ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-5\,ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\,(9\,ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(10*a*x*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 5*a*x*log(sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 5*a*x*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*
x - 1)) - 1) - 2*(9*a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)/x^2, x)
```

3.87 $\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=136

$$\frac{25a^2\sqrt[4]{ax+1}}{2\sqrt[4]{1-ax}}-\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(ax+1)^{9/4}}{2x^2\sqrt[4]{1-ax}}-\frac{5a(ax+1)^{5/4}}{4x\sqrt[4]{1-ax}}$$

```
[Out] (25*a^2*(1 + a*x)^(1/4))/(2*(1 - a*x)^(1/4)) - (5*a*(1 + a*x)^(5/4))/(4*x*(
1 - a*x)^(1/4)) - (1 + a*x)^(9/4)/(2*x^2*(1 - a*x)^(1/4)) - (25*a^2*ArcTan[
(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (25*a^2*ArcTanh[(1 + a*x)^(1/4)/(1 -
a*x)^(1/4)])/4
```

Rubi [A] time = 0.0485649, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6126, 96, 94, 93, 212, 206, 203}

$$\frac{25a^2\sqrt[4]{ax+1}}{2\sqrt[4]{1-ax}}-\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(ax+1)^{9/4}}{2x^2\sqrt[4]{1-ax}}-\frac{5a(ax+1)^{5/4}}{4x\sqrt[4]{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)/x^3,x]
```


```
[Out] (25*a^2*(1 + a*x)^(1/4))/(2*(1 - a*x)^(1/4)) - (5*a*(1 + a*x)^(5/4))/(4*x*(
1 - a*x)^(1/4)) - (1 + a*x)^(9/4)/(2*x^2*(1 - a*x)^(1/4)) - (25*a^2*ArcTan[
(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (25*a^2*ArcTanh[(1 + a*x)^(1/4)/(1 -
a*x)^(1/4)])/4
```


Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1+ax)^{5/4}}{x^3(1-ax)^{5/4}}\,dx\\
&= -\frac{(1+ax)^{9/4}}{2x^2\sqrt[4]{1-ax}} + \frac{1}{4}(5a)\int \frac{(1+ax)^{5/4}}{x^2(1-ax)^{5/4}}\,dx\\
&= -\frac{5a(1+ax)^{5/4}}{4x\sqrt[4]{1-ax}} - \frac{(1+ax)^{9/4}}{2x^2\sqrt[4]{1-ax}} + \frac{1}{8}\left(25a^2\right)\int \frac{\sqrt[4]{1+ax}}{x(1-ax)^{5/4}}\,dx\\
&= \frac{25a^2\sqrt[4]{1+ax}}{2\sqrt[4]{1-ax}} - \frac{5a(1+ax)^{5/4}}{4x\sqrt[4]{1-ax}} - \frac{(1+ax)^{9/4}}{2x^2\sqrt[4]{1-ax}} + \frac{1}{8}\left(25a^2\right)\int \frac{1}{x\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx\\
&= \frac{25a^2\sqrt[4]{1+ax}}{2\sqrt[4]{1-ax}} - \frac{5a(1+ax)^{5/4}}{4x\sqrt[4]{1-ax}} - \frac{(1+ax)^{9/4}}{2x^2\sqrt[4]{1-ax}} + \frac{1}{2}\left(25a^2\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= \frac{25a^2\sqrt[4]{1+ax}}{2\sqrt[4]{1-ax}} - \frac{5a(1+ax)^{5/4}}{4x\sqrt[4]{1-ax}} - \frac{(1+ax)^{9/4}}{2x^2\sqrt[4]{1-ax}} - \frac{1}{4}\left(25a^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{1}{4}\left(25a^2\right.\\
&= \frac{25a^2\sqrt[4]{1+ax}}{2\sqrt[4]{1-ax}} - \frac{5a(1+ax)^{5/4}}{4x\sqrt[4]{1-ax}} - \frac{(1+ax)^{9/4}}{2x^2\sqrt[4]{1-ax}} - \frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0235652, size = 86, normalized size = 0.63

$$\frac{50a^2x^2(ax-1)\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1-ax}{ax+1}\right)+3\left(43a^3x^3+34a^2x^2-11ax-2\right)}{12x^2\sqrt[4]{1-ax}(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)/x^3,x]
```

```
[Out] (3*(-2 - 11*a*x + 34*a^2*x^2 + 43*a^3*x^3) + 50*a^2*x^2*(-1 + a*x)*Hypergeo
metric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)])/(12*x^2*(1 - a*x)^(1/4)*(1 + a
```

```
*x)^(3/4))
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}} \left((ax+1) \frac{1}{\sqrt{-a^{2}x^{2}+1}} \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)/x^3, x)
```

Fricas [A] time = 2.12449, size = 335, normalized size = 2.46

$$-\frac{50\, a^{2}x^{2} \arctan\left(\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}\right) + 25\, a^{2}x^{2} \log\left(\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}+1\right) - 25\, a^{2}x^{2} \log\left(\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}-1\right) - 2\left(43\, a^{2}x^{2} - 9\, ax\right.}{8\, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(50*a^2*x^2*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 25*a^2*x^2*l
og(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 25*a^2*x^2*log(sqrt(-sqrt(-a^
2*x^2 + 1)/(a*x - 1)) - 1) - 2*(43*a^2*x^2 - 9*a*x - 2)*sqrt(-sqrt(-a^2*x^2
 + 1)/(a*x - 1)))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)/x^3, x)
```

3.88 $\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=165

$$\frac{287a^3\sqrt[4]{ax+1}}{24\sqrt[4]{1-ax}}-\frac{61a^2\sqrt[4]{ax+1}}{24x\sqrt[4]{1-ax}}-\frac{55}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{55}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{13a\sqrt[4]{ax+1}}{12x^2\sqrt[4]{1-ax}}-\frac{\sqrt[4]{ax+1}}{3x^3\sqrt[4]{1-ax}}$$

```
[Out] (287*a^3*(1 + a*x)^(1/4))/(24*(1 - a*x)^(1/4)) - (1 + a*x)^(1/4)/(3*x^3*(1
- a*x)^(1/4)) - (13*a*(1 + a*x)^(1/4))/(12*x^2*(1 - a*x)^(1/4)) - (61*a^2*(
1 + a*x)^(1/4))/(24*x*(1 - a*x)^(1/4)) - (55*a^3*ArcTan[(1 + a*x)^(1/4)/(1
- a*x)^(1/4)])/8 - (55*a^3*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8
```

Rubi [A] time = 0.0787501, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6126, 98, 151, 155, 12, 93, 212, 206, 203}

$$\frac{287a^3\sqrt[4]{ax+1}}{24\sqrt[4]{1-ax}}-\frac{61a^2\sqrt[4]{ax+1}}{24x\sqrt[4]{1-ax}}-\frac{55}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{55}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{13a\sqrt[4]{ax+1}}{12x^2\sqrt[4]{1-ax}}-\frac{\sqrt[4]{ax+1}}{3x^3\sqrt[4]{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)/x^4,x]
```

```
[Out] (287*a^3*(1 + a*x)^(1/4))/(24*(1 - a*x)^(1/4)) - (1 + a*x)^(1/4)/(3*x^3*(1
- a*x)^(1/4)) - (13*a*(1 + a*x)^(1/4))/(12*x^2*(1 - a*x)^(1/4)) - (61*a^2*(
1 + a*x)^(1/4))/(24*x*(1 - a*x)^(1/4)) - (55*a^3*ArcTan[(1 + a*x)^(1/4)/(1
- a*x)^(1/4)])/8 - (55*a^3*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

erQ[m]

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{(1+ax)^{5/4}}{x^4(1-ax)^{5/4}}\,dx$$
$$= -\frac{\sqrt[4]{1+ax}}{3x^3\sqrt[4]{1-ax}} - \frac{1}{3}\int \frac{-\frac{13a}{2}-6a^2x}{x^3(1-ax)^{5/4}(1+ax)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{1+ax}}{3x^3\sqrt[4]{1-ax}} - \frac{13a\sqrt[4]{1+ax}}{12x^2\sqrt[4]{1-ax}} + \frac{1}{6}\int \frac{\frac{61a^2}{4}+13a^3x}{x^2(1-ax)^{5/4}(1+ax)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{1+ax}}{3x^3\sqrt[4]{1-ax}} - \frac{13a\sqrt[4]{1+ax}}{12x^2\sqrt[4]{1-ax}} - \frac{61a^2\sqrt[4]{1+ax}}{24x\sqrt[4]{1-ax}} - \frac{1}{6}\int \frac{-\frac{165a^3}{8}-\frac{61a^4x}{4}}{x(1-ax)^{5/4}(1+ax)^{3/4}}\,dx$$
$$= \frac{287a^3\sqrt[4]{1+ax}}{24\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{3x^3\sqrt[4]{1-ax}} - \frac{13a\sqrt[4]{1+ax}}{12x^2\sqrt[4]{1-ax}} - \frac{61a^2\sqrt[4]{1+ax}}{24x\sqrt[4]{1-ax}} + \frac{\int \frac{165a^4}{16x\sqrt[4]{1-ax}(1+ax)^{3/4}}dx}{3a}$$
$$= \frac{287a^3\sqrt[4]{1+ax}}{24\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{3x^3\sqrt[4]{1-ax}} - \frac{13a\sqrt[4]{1+ax}}{12x^2\sqrt[4]{1-ax}} - \frac{61a^2\sqrt[4]{1+ax}}{24x\sqrt[4]{1-ax}} + \frac{1}{16}\left(55a^3\right)\int \frac{1}{x\sqrt[4]{1-ax}(1+ax)^{3/4}}$$
$$= \frac{287a^3\sqrt[4]{1+ax}}{24\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{3x^3\sqrt[4]{1-ax}} - \frac{13a\sqrt[4]{1+ax}}{12x^2\sqrt[4]{1-ax}} - \frac{61a^2\sqrt[4]{1+ax}}{24x\sqrt[4]{1-ax}} + \frac{1}{4}\left(55a^3\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,$$
$$= \frac{287a^3\sqrt[4]{1+ax}}{24\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{3x^3\sqrt[4]{1-ax}} - \frac{13a\sqrt[4]{1+ax}}{12x^2\sqrt[4]{1-ax}} - \frac{61a^2\sqrt[4]{1+ax}}{24x\sqrt[4]{1-ax}} - \frac{1}{8}\left(55a^3\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x,\right.$$
$$= \frac{287a^3\sqrt[4]{1+ax}}{24\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{3x^3\sqrt[4]{1-ax}} - \frac{13a\sqrt[4]{1+ax}}{12x^2\sqrt[4]{1-ax}} - \frac{61a^2\sqrt[4]{1+ax}}{24x\sqrt[4]{1-ax}} - \frac{55}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{55}{8}a^3\,\text{ta}$$

Mathematica [C] time = 0.0282137, size = 91, normalized size = 0.55

$$\frac{110a^3x^3(ax-1)\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1-ax}{ax+1}\right)+287a^4x^4+226a^3x^3-87a^2x^2-34ax-8}{24x^3\sqrt[4]{1-ax}(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)/x^4,x]
```

```
[Out] (-8 - 34*a*x - 87*a^2*x^2 + 226*a^3*x^3 + 287*a^4*x^4 + 110*a^3*x^3*(-1 + a
*x)*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)])/(24*x^3*(1 - a*x)^
(1/4)*(1 + a*x)^(3/4))
```

Maple [F] time = 0.117, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)/x^4, x)
```

Fricas [A] time = 2.0049, size = 360, normalized size = 2.18

$$-\frac{330\,a^3x^3\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+165\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-165\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\left(287\,a^3x^3-61\,a\right.}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(330*a^3*x^3*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 165*a^3*x^
3*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 165*a^3*x^3*log(sqrt(-sqrt
(-a^2*x^2 + 1)/(a*x - 1)) - 1) - 2*(287*a^3*x^3 - 61*a^2*x^2 - 26*a*x - 8)*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)/x^4, x)
```

3.89 $\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=194

$$-\frac{113a^2\sqrt[4]{ax+1}}{96x^2\sqrt[4]{1-ax}}+\frac{2467a^4\sqrt[4]{ax+1}}{192\sqrt[4]{1-ax}}-\frac{521a^3\sqrt[4]{ax+1}}{192x\sqrt[4]{1-ax}}-\frac{475}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{475}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{17a\sqrt[4]{ax+1}}{24x^3\sqrt[4]{1-ax}}$$

```
[Out] (2467*a^4*(1 + a*x)^(1/4))/(192*(1 - a*x)^(1/4)) - (1 + a*x)^(1/4)/(4*x^4*(
1 - a*x)^(1/4)) - (17*a*(1 + a*x)^(1/4))/(24*x^3*(1 - a*x)^(1/4)) - (113*a^
2*(1 + a*x)^(1/4))/(96*x^2*(1 - a*x)^(1/4)) - (521*a^3*(1 + a*x)^(1/4))/(19
2*x*(1 - a*x)^(1/4)) - (475*a^4*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/64
 - (475*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/64
```

Rubi [A] time = 0.0940809, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6126, 98, 151, 155, 12, 93, 212, 206, 203}

$$-\frac{113a^2\sqrt[4]{ax+1}}{96x^2\sqrt[4]{1-ax}}+\frac{2467a^4\sqrt[4]{ax+1}}{192\sqrt[4]{1-ax}}-\frac{521a^3\sqrt[4]{ax+1}}{192x\sqrt[4]{1-ax}}-\frac{475}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{475}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{17a\sqrt[4]{ax+1}}{24x^3\sqrt[4]{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)/x^5,x]
```

```
[Out] (2467*a^4*(1 + a*x)^(1/4))/(192*(1 - a*x)^(1/4)) - (1 + a*x)^(1/4)/(4*x^4*(
1 - a*x)^(1/4)) - (17*a*(1 + a*x)^(1/4))/(24*x^3*(1 - a*x)^(1/4)) - (113*a^
2*(1 + a*x)^(1/4))/(96*x^2*(1 - a*x)^(1/4)) - (521*a^3*(1 + a*x)^(1/4))/(19
2*x*(1 - a*x)^(1/4)) - (475*a^4*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/64
 - (475*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/64
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x^5}\,dx = \int \frac{(1+ax)^{5/4}}{x^5(1-ax)^{5/4}}\,dx$$

$$= -\frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{1}{4}\int \frac{-\frac{17a}{2} - 8a^2x}{x^4(1-ax)^{5/4}(1+ax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{17a\sqrt[4]{1+ax}}{24x^3\sqrt[4]{1-ax}} + \frac{1}{12}\int \frac{\frac{113a^2}{4} + \frac{51a^3x}{2}}{x^3(1-ax)^{5/4}(1+ax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{17a\sqrt[4]{1+ax}}{24x^3\sqrt[4]{1-ax}} - \frac{113a^2\sqrt[4]{1+ax}}{96x^2\sqrt[4]{1-ax}} - \frac{1}{24}\int \frac{-\frac{521a^3}{8} - \frac{113a^4x}{2}}{x^2(1-ax)^{5/4}(1+ax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{17a\sqrt[4]{1+ax}}{24x^3\sqrt[4]{1-ax}} - \frac{113a^2\sqrt[4]{1+ax}}{96x^2\sqrt[4]{1-ax}} - \frac{521a^3\sqrt[4]{1+ax}}{192x\sqrt[4]{1-ax}} + \frac{1}{24}\int \frac{\frac{1425a^4}{16} + \frac{521a^5x}{8}}{x(1-ax)^{5/4}(1+ax)^{3/4}}\,d$$

$$= \frac{2467a^4\sqrt[4]{1+ax}}{192\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{17a\sqrt[4]{1+ax}}{24x^3\sqrt[4]{1-ax}} - \frac{113a^2\sqrt[4]{1+ax}}{96x^2\sqrt[4]{1-ax}} - \frac{521a^3\sqrt[4]{1+ax}}{192x\sqrt[4]{1-ax}} - \frac{\int -\frac{1425}{32x\sqrt[4]{1-ax}(}}{12a}$$

$$= \frac{2467a^4\sqrt[4]{1+ax}}{192\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{17a\sqrt[4]{1+ax}}{24x^3\sqrt[4]{1-ax}} - \frac{113a^2\sqrt[4]{1+ax}}{96x^2\sqrt[4]{1-ax}} - \frac{521a^3\sqrt[4]{1+ax}}{192x\sqrt[4]{1-ax}} + \frac{1}{128}\left(475a^4\right)$$

$$= \frac{2467a^4\sqrt[4]{1+ax}}{192\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{17a\sqrt[4]{1+ax}}{24x^3\sqrt[4]{1-ax}} - \frac{113a^2\sqrt[4]{1+ax}}{96x^2\sqrt[4]{1-ax}} - \frac{521a^3\sqrt[4]{1+ax}}{192x\sqrt[4]{1-ax}} + \frac{1}{32}\left(475a^4\right)\mathrm{S}$$

$$= \frac{2467a^4\sqrt[4]{1+ax}}{192\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{17a\sqrt[4]{1+ax}}{24x^3\sqrt[4]{1-ax}} - \frac{113a^2\sqrt[4]{1+ax}}{96x^2\sqrt[4]{1-ax}} - \frac{521a^3\sqrt[4]{1+ax}}{192x\sqrt[4]{1-ax}} - \frac{1}{64}\left(475a^4\right)\mathrm{S}$$

$$= \frac{2467a^4\sqrt[4]{1+ax}}{192\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{17a\sqrt[4]{1+ax}}{24x^3\sqrt[4]{1-ax}} - \frac{113a^2\sqrt[4]{1+ax}}{96x^2\sqrt[4]{1-ax}} - \frac{521a^3\sqrt[4]{1+ax}}{192x\sqrt[4]{1-ax}} - \frac{475}{64}a^4\tan^{-1}$$

Mathematica [C] time = 0.0322788, size = 99, normalized size = 0.51

$$\frac{950a^4x^4(ax-1)\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1-ax}{ax+1}\right) + 2467a^5x^5 + 1946a^4x^4 - 747a^3x^3 - 362a^2x^2 - 184ax - 48}{192x^4\sqrt[4]{1-ax}(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)/x^5,x]
```

```
[Out] (-48 - 184*a*x - 362*a^2*x^2 - 747*a^3*x^3 + 1946*a^4*x^4 + 2467*a^5*x^5 +
950*a^4*x^4*(-1 + a*x)*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)])
/(192*x^4*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)/x^5, x)
```

Fricas [A] time = 2.00803, size = 390, normalized size = 2.01

$$-\frac{2850\,a^4x^4\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+1425\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-1425\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\left(2467\,a^4x^4 -\right.}{384\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/384*(2850*a^4*x^4*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 1425*a^4
*x^4*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 1425*a^4*x^4*log(sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1) - 2*(2467*a^4*x^4 - 521*a^3*x^3 - 226*a^
2*x^2 - 136*a*x - 48)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)/x^5, x)
```

3.90 $\int e^{-\frac{1}{2}\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=31

$$\frac{x^{m+1}F_1\left(m+1;-\frac{1}{4},\frac{1}{4};m+2;ax,-ax\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, -1/4, 1/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rubi [A] time = 0.0247624, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6126, 133}

$$\frac{x^{m+1}F_1\left(m+1;-\frac{1}{4},\frac{1}{4};m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(ArcTanh[a*x]/2),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -1/4, 1/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{1}{2}\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;-\frac{1}{4},\frac{1}{4};2+m;ax,-ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.29763, size = 0, normalized size = 0.

$$\int e^{-\frac{1}{2}\tanh^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^(ArcTanh[a*x]/2),x]
```

```
[Out] Integrate[x^m/E^(ArcTanh[a*x]/2), x]
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int x^m \frac{1}{\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}x^m\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))/(a*x +
1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**m/sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

3.91 $\int e^{-\frac{1}{2}\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=290

$$-\frac{x^2(1-ax)^{5/4}(ax+1)^{3/4}}{4a^2}-\frac{(25-4ax)(1-ax)^{5/4}(ax+1)^{3/4}}{96a^4}-\frac{11\sqrt[4]{1-ax}(ax+1)^{3/4}}{64a^4}-\frac{11\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{128\sqrt{2}a^4}+\frac{11}{}$$

```
[Out] (-11*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(64*a^4) - (x^2*(1 - a*x)^(5/4)*(1 +
a*x)^(3/4))/(4*a^2) - ((25 - 4*a*x)*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(96*a^
4) - (11*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]
*a^4) + (11*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt
[2]*a^4) - (11*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/
4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4) + (11*Log[1 + Sqrt[1 - a*x]/Sqrt[1
+ a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rubi [A] time = 0.212892, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 100, 147, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^2(1-ax)^{5/4}(ax+1)^{3/4}}{4a^2}-\frac{(25-4ax)(1-ax)^{5/4}(ax+1)^{3/4}}{96a^4}-\frac{11\sqrt[4]{1-ax}(ax+1)^{3/4}}{64a^4}-\frac{11\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{128\sqrt{2}a^4}+\frac{11}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(ArcTanh[a*x]/2),x]
```

```
[Out] (-11*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(64*a^4) - (x^2*(1 - a*x)^(5/4)*(1 +
a*x)^(3/4))/(4*a^2) - ((25 - 4*a*x)*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(96*a^
4) - (11*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]
*a^4) + (11*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt
[2]*a^4) - (11*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/
4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4) + (11*Log[1 + Sqrt[1 - a*x]/Sqrt[1
+ a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
```

```
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}\tanh^{-1}(ax)}x^3\,dx = \int \frac{x^3\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} - \frac{\int \frac{x\sqrt[4]{1-ax}\left(-2+\frac{ax}{2}\right)}{\sqrt[4]{1+ax}}\,dx}{4a^2}$$

$$= -\frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} - \frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4} - \frac{11\int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx}{64a^3}$$

$$= -\frac{11\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} - \frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} - \frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4} - \frac{11\int \frac{1}{(1-ax)^{3/\ldots}}\ldots}{128\ldots}$$

$$= -\frac{11\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} - \frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} - \frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4} + \frac{11\,\mathrm{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{11\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} - \frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} - \frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4} + \frac{11\,\mathrm{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{11\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} - \frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} - \frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4} + \frac{11\,\mathrm{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{11\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} - \frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} - \frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4} + \frac{11\,\mathrm{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{11\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} - \frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} - \frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4} - \frac{11\log\left(1+\ldots\right.}{1\ldots}$$

$$= -\frac{11\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} - \frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} - \frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4} - \frac{11\tan^{-1}\left(1\ldots\right.}{64\sqrt{\ldots}}$$

Mathematica [C] time = 0.103508, size = 116, normalized size = 0.4

$$\frac{(1-ax)^{5/4}\left(4\ 2^{3/4}\mathrm{Hypergeometric2F1}\left(-\frac{7}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-ax)\right)-12\ 2^{3/4}\mathrm{Hypergeometric2F1}\left(-\frac{3}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-ax)\right)+5\ 2^{3}\ldots\right.}{20a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^(ArcTanh[a*x]/2),x]
```

```
[Out] ((1 - a*x)^(5/4)*(-5*a^2*x^2*(1 + a*x)^(3/4) + 4*2^(3/4)*Hypergeometric2F1[
-7/4, 5/4, 9/4, (1 - a*x)/2] - 12*2^(3/4)*Hypergeometric2F1[-3/4, 5/4, 9/4,
 (1 - a*x)/2] + 5*2^(3/4)*Hypergeometric2F1[1/4, 5/4, 9/4, (1 - a*x)/2]))/(
20*a^4)
```

Maple [F] time = 0.11, size = 0, normalized size = 0.

$$\int x^{3} \frac{1}{\sqrt{(ax+1)\frac{1}{\sqrt{-a^{2}x^{2}+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

Fricas [B] time = 2.28416, size = 1399, normalized size = 4.82

$$132\sqrt{2}a^{4}\left(\frac{1}{a^{16}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^{4}\sqrt{\frac{\sqrt{2}\left(a^{13}x-a^{12}\right)\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{3}{4}}+\left(a^{9}x-a^{8}\right)\sqrt{\frac{1}{a^{16}}}-\sqrt{-a^{2}x^{2}+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{1}{4}}-\sqrt{2}a^{4}\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{1}{4}}-1\right)+13$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(132*sqrt(2)*a^4*(a^(-16))^(1/4)*arctan(sqrt(2)*a^4*sqrt((sqrt(2)*(a^
13*x - a^12)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) + (a^9*x -
 a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(1/4) - sqrt
(2)*a^4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) - 1) + 132*sqrt
(2)*a^4*(a^(-16))^(1/4)*arctan(sqrt(2)*a^4*sqrt(-(sqrt(2)*(a^13*x - a^12)*s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) - (a^9*x - a^8)*sqrt(a^(
-16)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(1/4) - sqrt(2)*a^4*sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) + 1) + 33*sqrt(2)*a^4*(a^(-16
))^(1/4)*log((sqrt(2)*(a^13*x - a^12)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(
a^(-16))^(3/4) + (a^9*x - a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x - 1
)) - 33*sqrt(2)*a^4*(a^(-16))^(1/4)*log(-(sqrt(2)*(a^13*x - a^12)*sqrt(-sqr
```

```
t(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) - (a^9*x - a^8)*sqrt(a^(-16)) +
sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 4*(48*a^3*x^3 - 56*a^2*x^2 + 58*a*x - 83)*
sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2),x)

[Out] Integral(x**3/sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(x^3/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

3.92 $\int e^{-\frac{1}{2}\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=282

$$-\frac{x(ax+1)^{3/4}(1-ax)^{5/4}}{3a^2}+\frac{(ax+1)^{3/4}(1-ax)^{5/4}}{12a^3}+\frac{3(ax+1)^{3/4}\sqrt[4]{1-ax}}{8a^3}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}a^3}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right.}{16}$$

```
[Out] (3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(8*a^3) + ((1 - a*x)^(5/4)*(1 + a*x)^(3
/4))/(12*a^3) - (x*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(3*a^2) + (3*ArcTan[1 -
 (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (3*ArcTan[1
+ (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) + (3*Log[1 +
Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(
16*Sqrt[2]*a^3) - (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*
x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3)
```

Rubi [A] time = 0.198589, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 90, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x(ax+1)^{3/4}(1-ax)^{5/4}}{3a^2}+\frac{(ax+1)^{3/4}(1-ax)^{5/4}}{12a^3}+\frac{3(ax+1)^{3/4}\sqrt[4]{1-ax}}{8a^3}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}a^3}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right.}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(ArcTanh[a*x]/2),x]
```

```
[Out] (3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(8*a^3) + ((1 - a*x)^(5/4)*(1 + a*x)^(3
/4))/(12*a^3) - (x*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(3*a^2) + (3*ArcTan[1 -
 (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (3*ArcTan[1
+ (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) + (3*Log[1 +
Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(
16*Sqrt[2]*a^3) - (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*
x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}\tanh^{-1}(ax)}x^2\,dx = \int \frac{x^2\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} - \frac{\int \frac{\sqrt[4]{1-ax}\left(-1+\frac{ax}{2}\right)}{\sqrt[4]{1+ax}}\,dx}{3a^2}$$

$$= \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} + \frac{3\int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx}{8a^2}$$

$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{8a^3} + \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} + \frac{3\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx}{16a^2}$$

$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{8a^3} + \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, \ldots\right.}{4a^3}$$

$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{8a^3} + \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \ldots\right.}{4a^3}$$

$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{8a^3} + \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} - \frac{3\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \ldots\right.}{8a^3}$$

$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{8a^3} + \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,\ldots\right.}{16a^3}$$

$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{8a^3} + \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} + \frac{3\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \ldots\right.}{16\sqrt{2}a^3}$$

$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{8a^3} + \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} + \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8\sqrt{2}a^3}$$

Mathematica [C] time = 0.0260234, size = 62, normalized size = 0.22

$$-\frac{(1-ax)^{5/4}\left(9\ 2^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-ax)\right)+5(ax+1)^{3/4}(4ax-1)\right)}{60a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(ArcTanh[a*x]/2),x]
```

```
[Out] -((1 - a*x)^(5/4)*(5*(1 + a*x)^(3/4)*(-1 + 4*a*x) + 9*2^(3/4)*Hypergeometri
c2F1[1/4, 5/4, 9/4, (1 - a*x)/2]))/(60*a^3)
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

Fricas [B] time = 2.18141, size = 1369, normalized size = 4.85

$$-\frac{36\sqrt{2}a^3\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^3\sqrt{\frac{\sqrt{2}\left(a^{10}x-a^9\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}+\left(a^7x-a^6\right)\sqrt{\frac{1}{a^{12}}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}-\sqrt{2}a^3\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}-1\right)+36\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/96*(36*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^3*sqrt((sqrt(2)*(a^1
0*x - a^9)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) + (a^7*x - a
^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(1/4) - sqrt(2
)*a^3*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - 1) + 36*sqrt(2)
*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^3*sqrt(-(sqrt(2)*(a^10*x - a^9)*sqrt(
-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) - (a^7*x - a^6)*sqrt(a^(-12)
) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(1/4) - sqrt(2)*a^3*sqrt(-sqrt
(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) + 1) + 9*sqrt(2)*a^3*(a^(-12))^(1
/4)*log((sqrt(2)*(a^10*x - a^9)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12
))^(3/4) + (a^7*x - a^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 9
*sqrt(2)*a^3*(a^(-12))^(1/4)*log(-(sqrt(2)*(a^10*x - a^9)*sqrt(-sqrt(-a^2*x
^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) - (a^7*x - a^6)*sqrt(a^(-12)) + sqrt(-a^
2*x^2 + 1))/(a*x - 1)) - 4*(8*a^2*x^2 - 10*a*x + 11)*sqrt(-a^2*x^2 + 1)*sqr
t(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

3.93 $\int e^{-\frac{1}{2}\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=255

$$-\frac{(ax+1)^{3/4}(1-ax)^{5/4}}{2a^2}-\frac{(ax+1)^{3/4}\sqrt[4]{1-ax}}{4a^2}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{4\sqrt{2}a^2}$$

```
[Out] -((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*a^2) - ((1 - a*x)^(5/4)*(1 + a*x)^(3/
4))/(2*a^2) - ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(4*Sqrt
[2]*a^2) + ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(4*Sqrt[2]
*a^2) - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1
+ a*x)^(1/4)]/(8*Sqrt[2]*a^2) + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt
[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(8*Sqrt[2]*a^2)
```

Rubi [A] time = 0.168347, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6126, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(ax+1)^{3/4}(1-ax)^{5/4}}{2a^2}-\frac{(ax+1)^{3/4}\sqrt[4]{1-ax}}{4a^2}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{4\sqrt{2}a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(ArcTanh[a*x]/2),x]
```

```
[Out] -((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*a^2) - ((1 - a*x)^(5/4)*(1 + a*x)^(3/
4))/(2*a^2) - ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(4*Sqrt
[2]*a^2) + ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(4*Sqrt[2]
*a^2) - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1
+ a*x)^(1/4)]/(8*Sqrt[2]*a^2) + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt
[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(8*Sqrt[2]*a^2)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}\tanh^{-1}(ax)}x\,dx = \int \frac{x\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx$$
$$= -\frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{\int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx}{4a}$$
$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx}{8a}$$
$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)}{2a^2}$$
$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a^2}$$
$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4a^2} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x\right.}{4a^2}$$
$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8a^2} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x\right.}{8}$$
$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8\sqrt{2}a^2} + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{1}}\right.}{8\sqrt{2}a^2}$$
$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}a^2} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}a^2} - \frac{\log}{}$$

Mathematica [C] time = 0.018661, size = 55, normalized size = 0.22

$$\frac{(1-ax)^{5/4}\left(2^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-ax)\right)-5(ax+1)^{3/4}\right)}{10a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(ArcTanh[a*x]/2),x]
```

```
[Out] ((1 - a*x)^(5/4)*(-5*(1 + a*x)^(3/4) + 2^(3/4)*Hypergeometric2F1[1/4, 5/4,
9/4, (1 - a*x)/2]))/(10*a^2)
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

Fricas [B] time = 2.17513, size = 1314, normalized size = 5.15

$$4\sqrt{2}a^2\frac{1}{a^8}^{\frac{1}{4}}\arctan\left(\sqrt{2}a^2\sqrt{\frac{\sqrt{2}\left(a^7x-a^6\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^8}^{\frac{3}{4}}+\left(a^5x-a^4\right)\sqrt{\frac{1}{a^8}}-\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^8}^{\frac{1}{4}}-\sqrt{2}a^2\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^8}^{\frac{1}{4}}-1\right)+4\sqrt{2}a^2\frac{1}{a^8}^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(4*sqrt(2)*a^2*(a^(-8))^(1/4)*arctan(sqrt(2)*a^2*sqrt((sqrt(2)*(a^7*x
- a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) + (a^5*x - a^4)*s
qrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(1/4) - sqrt(2)*a^2*s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) - 1) + 4*sqrt(2)*a^2*(a^(
-8))^(1/4)*arctan(sqrt(2)*a^2*sqrt(-(sqrt(2)*(a^7*x - a^6)*sqrt(-sqrt(-a^2*
x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-a^2
*x^2 + 1))/(a*x - 1))*(a^(-8))^(1/4) - sqrt(2)*a^2*sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1))*(a^(-8))^(1/4) + 1) + sqrt(2)*a^2*(a^(-8))^(1/4)*log((sqrt(2)*(
a^7*x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) + (a^5*x -
a^4)*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - sqrt(2)*a^2*(a^(-8))^(
1/4)*log(-(sqrt(2)*(a^7*x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8
))^(3/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 4*
sqrt(-a^2*x^2 + 1)*(2*a*x - 3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(x/sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

3.94 $\int e^{-\frac{1}{2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=221

$$\frac{\sqrt[4]{1-ax}(ax+1)^{3/4}}{a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{\sqrt{2}a}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a}$$

```
[Out] ((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/a + ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/
(1 + a*x)^(1/4)]/(Sqrt[2]*a) - ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*
x)^(1/4)]/(Sqrt[2]*a) + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 -
 a*x)^(1/4))/(1 + a*x)^(1/4)]/(2*Sqrt[2]*a) - Log[1 + Sqrt[1 - a*x]/Sqrt[1
+ a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(2*Sqrt[2]*a)
```

Rubi [A] time = 0.138131, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6125, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{1-ax}(ax+1)^{3/4}}{a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{\sqrt{2}a}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-ArcTanh[a*x]/2),x]
```

```
[Out] ((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/a + ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/
(1 + a*x)^(1/4)]/(Sqrt[2]*a) - ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*
x)^(1/4)]/(Sqrt[2]*a) + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 -
 a*x)^(1/4))/(1 + a*x)^(1/4)]/(2*Sqrt[2]*a) - Log[1 + Sqrt[1 - a*x]/Sqrt[1
+ a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(2*Sqrt[2]*a)
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}\tanh^{-1}(ax)}\,dx = \int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx$$
$$= \frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} + \frac{1}{2}\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$
$$= \frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$
$$= \frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} - \frac{2\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= \frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= \frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a} + \cdots$$
$$= \frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}a} - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}a} - \cdots$$
$$= \frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} + \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}a} \cdots$$

Mathematica [C] time = 0.0346025, size = 35, normalized size = 0.16

$$\frac{8e^{\frac{3}{2}\tanh^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\tanh^{-1}(ax)}\right)}{3a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-ArcTanh[a*x]/2),x]
```

```
[Out] (8*E^((3*ArcTanh[a*x])/2)*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcTanh[a*x]
)])/(3*a)
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

Fricas [B] time = 1.82559, size = 1273, normalized size = 5.76

$$-\frac{4\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\sqrt{2}a\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-1\right)+4\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\text{arct}\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt((sqrt(2)*(a^4*x - a^
3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*sqrt(
a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-s
qrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) - 1) + 4*sqrt(2)*a*(a^(-4))^(1/
4)*arctan(sqrt(2)*a*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(
a*x - 1))*(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))
/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*
(a^(-4))^(1/4) + 1) + sqrt(2)*a*(a^(-4))^(1/4)*log((sqrt(2)*(a^4*x - a^3)*s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*sqrt(a^(-
4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - sqrt(2)*a*(a^(-4))^(1/4)*log(-(sqrt(
2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) - (a^3*
x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 4*sqrt(-a^2*x^2 +
1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

3.95 $\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=227

$$-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)+\sqrt{2}\tan$$

```
[Out] 2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1
- a*x)^(1/4))/(1 + a*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4
))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] - Log[1 +
Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sq
rt[2] + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1
+ a*x)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.161261, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6126, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)+\sqrt{2}\tan$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcTanh[a*x]/2)*x),x]
```

```
[Out] 2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1
- a*x)^(1/4))/(1 + a*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4
))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] - Log[1 +
Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sq
rt[2] + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1
+ a*x)^(1/4)]/Sqrt[2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{\sqrt[4]{1-ax}}{x\sqrt[4]{1+ax}}\,dx \\
&= -\left(a\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx\right)+\int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx \\
&= 4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx,x,\sqrt[4]{1-ax}\right)+4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) \\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\right)+2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)+4\,\text{Subst}\left(\int \frac{1}{1+x^4}\right. \\
&= 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)+2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)+2\,\text{Subst}\left(\int \frac{1+}{1+}\right. \\
&= 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-\frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-}\right.}{\sqrt{2}} \\
&= 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-\frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}}+\frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}\sqrt[4]{1-a}}{\sqrt[4]{1+ax}}\right.}{\sqrt{2}} \\
&= 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)+\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)-2\tanh^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.
\end{aligned}
$$

Mathematica [C] time = 0.0258194, size = 83, normalized size = 0.37

$$
2\,2^{3/4}\sqrt[4]{1-ax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-ax)\right)-\frac{4\sqrt[4]{1-ax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},-\frac{1-ax}{-ax-1}\right)}{\sqrt[4]{ax+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcTanh[a*x]/2)*x),x]
```

```
[Out] 2*2^(3/4)*(1 - a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (1 - a*x)/2] - (
4*(1 - a*x)^(1/4)*Hypergeometric2F1[1/4, 1, 5/4, -((1 - a*x)/(-1 - a*x))])/
(1 + a*x)^(1/4)
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

Fricas [B] time = 1.80956, size = 998, normalized size = 4.4

$$2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{ax+\sqrt{2}(ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-\sqrt{-a^2x^2+1}-1}{ax-1}}-\sqrt{2}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{a}{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2
+ 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt(2)*sqrt(-sqrt(-
a^2*x^2 + 1)/(a*x - 1)) - 1) + 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x - sqrt(2)
*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a
*x - 1)) - sqrt(2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 1/2*sqrt(2)*l
og(4*(a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a
^2*x^2 + 1) - 1)/(a*x - 1)) - 1/2*sqrt(2)*log(4*(a*x - sqrt(2)*(a*x - 1)*sq
rt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) + 2*
arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - log(sqrt(-sqrt(-a^2*x^2 + 1)/
(a*x - 1)) + 1) + log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(1/(x*sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

3.96 $\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{\sqrt[4]{1-ax}(ax+1)^{3/4}}{x}-a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

```
[Out] -(((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/x) - a*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)
^(1/4)] + a*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)]
```

Rubi [A] time = 0.0363453, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6126, 94, 93, 298, 203, 206}

$$-\frac{\sqrt[4]{1-ax}(ax+1)^{3/4}}{x}-a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcTanh[a*x]/2)*x^2),x]
```

```
[Out] -(((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/x) - a*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)
^(1/4)] + a*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{\sqrt[4]{1-ax}}{x^2\sqrt[4]{1+ax}}\,dx\\
&= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{x} - \frac{1}{2}a\int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx\\
&= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{x} - (2a)\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{x} + a\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{x} - a\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + a\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0136235, size = 55, normalized size = 0.76

$$
\frac{\sqrt[4]{1-ax}\left(2ax\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1-ax}{ax+1}\right)-ax-1\right)}{x\sqrt[4]{ax+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcTanh[a*x]/2)*x^2),x]
```

```
[Out] ((1 - a*x)^(1/4)*(-1 - a*x + 2*a*x*Hypergeometric2F1[1/4, 1, 5/4, (1 - a*x)
/(1 + a*x)]))/(x*(1 + a*x)^(1/4))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$
\int \frac{1}{x^2}\frac{1}{\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

Fricas [B] time = 1.74347, size = 298, normalized size = 4.14

$$-\frac{2\,ax\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)+ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\sqrt{-a^2x^2+1}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - a*x*log(sqrt(-sqr
t(-a^2*x^2 + 1)/(a*x - 1)) + 1) + a*x*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1
)) - 1) + 2*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(1/(x**2*sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

3.97 $\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=110

$$\frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(1-ax)^{5/4}(ax+1)^{3/4}}{2x^2}+\frac{a\sqrt[4]{1-ax}(ax+1)^{3/4}}{4x}$$

```
[Out] (a*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*x) - ((1 - a*x)^(5/4)*(1 + a*x)^(3/4
))/(2*x^2) + (a^2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (a^2*ArcTanh
[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4
```

Rubi [A] time = 0.0455312, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6126, 96, 94, 93, 298, 203, 206}

$$\frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(1-ax)^{5/4}(ax+1)^{3/4}}{2x^2}+\frac{a\sqrt[4]{1-ax}(ax+1)^{3/4}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcTanh[a*x]/2)*x^3),x]
```

```
[Out] (a*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*x) - ((1 - a*x)^(5/4)*(1 + a*x)^(3/4
))/(2*x^2) + (a^2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (a^2*ArcTanh
[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x^3}\,dx &= \int \frac{\sqrt[4]{1-ax}}{x^3\sqrt[4]{1+ax}}\,dx\\
&= -\frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2x^2} - \frac{1}{4}a\int \frac{\sqrt[4]{1-ax}}{x^2\sqrt[4]{1+ax}}\,dx\\
&= \frac{a\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2x^2} + \frac{1}{8}a^2\int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx\\
&= \frac{a\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2x^2} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= \frac{a\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2x^2} - \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{1}{4}a^2\,\text{Sub}\\
&= \frac{a\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2x^2} + \frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0205784, size = 69, normalized size = 0.63

$$\frac{\sqrt[4]{1-ax}\left(-2a^2x^2\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1-ax}{ax+1}\right)+3a^2x^2+ax-2\right)}{4x^2\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcTanh[a*x]/2)*x^3),x]
```

```
[Out] ((1 - a*x)^(1/4)*(-2 + a*x + 3*a^2*x^2 - 2*a^2*x^2*Hypergeometric2F1[1/4, 1
, 5/4, (1 - a*x)/(1 + a*x)]))/(4*x^2*(1 + a*x)^(1/4))
```

Maple [F] time = 0.107, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}} \frac{1}{\sqrt{(ax+1)\frac{1}{\sqrt{-a^{2}x^{2}+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

Fricas [A] time = 1.74879, size = 332, normalized size = 3.02

$$\frac{2\,a^{2}x^{2}\arctan\left(\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}\right) - a^{2}x^{2}\log\left(\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}+1\right) + a^{2}x^{2}\log\left(\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}-1\right) + 2\sqrt{-a^{2}x^{2}+1}(3\,ax-2)\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}}{8\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a^2*x^2*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - a^2*x^2*log(sq
rt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + a^2*x^2*log(sqrt(-sqrt(-a^2*x^2 +
1)/(a*x - 1)) - 1) + 2*sqrt(-a^2*x^2 + 1)*(3*a*x - 2)*sqrt(-sqrt(-a^2*x^2 +
 1)/(a*x - 1)))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**3,x)
```

```
[Out] Integral(1/(x**3*sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

3.98 $\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x^4} dx$

Optimal. Leaf size=139

$$-\frac{11a^2\sqrt[4]{1-ax}(ax+1)^{3/4}}{24x}-\frac{3}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{3}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{5a\sqrt[4]{1-ax}(ax+1)^{3/4}}{12x^2}-\frac{\sqrt[4]{1-ax}(ax+1)^{3/4}}{3x^3}$$

```
[Out] -((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(3*x^3) + (5*a*(1 - a*x)^(1/4)*(1 + a*x)
^(3/4))/(12*x^2) - (11*a^2*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(24*x) - (3*a^3
*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8 + (3*a^3*ArcTanh[(1 + a*x)^(1/4
)/(1 - a*x)^(1/4)])/8
```

Rubi [A] time = 0.0619941, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 298, 203, 206}

$$-\frac{11a^2\sqrt[4]{1-ax}(ax+1)^{3/4}}{24x}-\frac{3}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{3}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{5a\sqrt[4]{1-ax}(ax+1)^{3/4}}{12x^2}-\frac{\sqrt[4]{1-ax}(ax+1)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcTanh[a*x]/2)*x^4),x]
```

```
[Out] -((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(3*x^3) + (5*a*(1 - a*x)^(1/4)*(1 + a*x)
^(3/4))/(12*x^2) - (11*a^2*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(24*x) - (3*a^3
*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8 + (3*a^3*ArcTanh[(1 + a*x)^(1/4
)/(1 - a*x)^(1/4)])/8
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{\sqrt[4]{1-ax}}{x^4\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{3x^3} + \frac{1}{3}\int \frac{-\frac{5a}{2}+2a^2x}{x^3(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{3x^3} + \frac{5a\sqrt[4]{1-ax}(1+ax)^{3/4}}{12x^2} - \frac{1}{6}\int \frac{-\frac{11a^2}{4}+\frac{5a^3x}{2}}{x^2(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{3x^3} + \frac{5a\sqrt[4]{1-ax}(1+ax)^{3/4}}{12x^2} - \frac{11a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x} + \frac{1}{6}\int -\frac{9a^3}{8x(1-ax)^{3/4}\sqrt[4]{1}}$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{3x^3} + \frac{5a\sqrt[4]{1-ax}(1+ax)^{3/4}}{12x^2} - \frac{11a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x} - \frac{1}{16}\left(3a^3\right)\int \frac{1}{x(1-ax)^{3}}$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{3x^3} + \frac{5a\sqrt[4]{1-ax}(1+ax)^{3/4}}{12x^2} - \frac{11a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x} - \frac{1}{4}\left(3a^3\right)\text{Subst}\left(\int \frac{}{-1}\right.$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{3x^3} + \frac{5a\sqrt[4]{1-ax}(1+ax)^{3/4}}{12x^2} - \frac{11a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x} + \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{1-}\right.$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{3x^3} + \frac{5a\sqrt[4]{1-ax}(1+ax)^{3/4}}{12x^2} - \frac{11a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x} - \frac{3}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$

Mathematica [C] time = 0.0219663, size = 78, normalized size = 0.56

$$\frac{\sqrt[4]{1-ax}\left(18a^3x^3\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1-ax}{ax+1}\right)-11a^3x^3-a^2x^2+2ax-8\right)}{24x^3\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcTanh[a*x]/2)*x^4),x]
```

```
[Out] ((1 - a*x)^(1/4)*(-8 + 2*a*x - a^2*x^2 - 11*a^3*x^3 + 18*a^3*x^3*Hypergeome
tric2F1[1/4, 1, 5/4, (1 - a*x)/(1 + a*x)]))/(24*x^3*(1 + a*x)^(1/4))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\frac{1}{\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

Fricas [A] time = 1.82132, size = 360, normalized size = 2.59

$$-\frac{18\,a^3x^3\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-9\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)+9\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\left(11\,a^2x^2-10\,ax+8\right)\cdots}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(18*a^3*x^3*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 9*a^3*x^3*l
og(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 9*a^3*x^3*log(sqrt(-sqrt(-a^2
*x^2 + 1)/(a*x - 1)) - 1) + 2*(11*a^2*x^2 - 10*a*x + 8)*sqrt(-a^2*x^2 + 1)*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x^4*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

3.99 $\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x^5} dx$

Optimal. Leaf size=168

$$-\frac{29a^2\sqrt[4]{1-ax}(ax+1)^{3/4}}{96x^2}+\frac{83a^3\sqrt[4]{1-ax}(ax+1)^{3/4}}{192x}+\frac{11}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{11}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{7a\sqrt[4]{1-ax}(ax}{24x^3}$$

```
[Out] -((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*x^4) + (7*a*(1 - a*x)^(1/4)*(1 + a*x)
^(3/4))/(24*x^3) - (29*a^2*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(96*x^2) + (83*
a^3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(192*x) + (11*a^4*ArcTan[(1 + a*x)^(1/
4)/(1 - a*x)^(1/4)])/64 - (11*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])
/64
```

Rubi [A] time = 0.0785711, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 298, 203, 206}

$$-\frac{29a^2\sqrt[4]{1-ax}(ax+1)^{3/4}}{96x^2}+\frac{83a^3\sqrt[4]{1-ax}(ax+1)^{3/4}}{192x}+\frac{11}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{11}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{7a\sqrt[4]{1-ax}(ax}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcTanh[a*x]/2)*x^5),x]
```

```
[Out] -((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*x^4) + (7*a*(1 - a*x)^(1/4)*(1 + a*x)
^(3/4))/(24*x^3) - (29*a^2*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(96*x^2) + (83*
a^3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(192*x) + (11*a^4*ArcTan[(1 + a*x)^(1/
4)/(1 - a*x)^(1/4)])/64 - (11*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])
/64
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x^5}\,dx = \int \frac{\sqrt[4]{1-ax}}{x^5\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x^4} + \frac{1}{4}\int \frac{-\frac{7a}{2}+3a^2x}{x^4(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x^4} + \frac{7a\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x^3} - \frac{1}{12}\int \frac{-\frac{29a^2}{4}+7a^3x}{x^3(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x^4} + \frac{7a\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x^3} - \frac{29a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{96x^2} + \frac{1}{24}\int \frac{-\frac{83a^3}{8}+\frac{29a^4x}{4}}{x^2(1-ax)^{3/4}\sqrt[4]{1+}}$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x^4} + \frac{7a\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x^3} - \frac{29a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{96x^2} + \frac{83a^3\sqrt[4]{1-ax}(1+ax)^{3/4}}{192x} -$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x^4} + \frac{7a\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x^3} - \frac{29a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{96x^2} + \frac{83a^3\sqrt[4]{1-ax}(1+ax)^{3/4}}{192x} -$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x^4} + \frac{7a\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x^3} - \frac{29a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{96x^2} + \frac{83a^3\sqrt[4]{1-ax}(1+ax)^{3/4}}{192x} -$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x^4} + \frac{7a\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x^3} - \frac{29a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{96x^2} + \frac{83a^3\sqrt[4]{1-ax}(1+ax)^{3/4}}{192x} -$$

$$= -\frac{\sqrt[4]{1-ax}(1+ax)^{3/4}}{4x^4} + \frac{7a\sqrt[4]{1-ax}(1+ax)^{3/4}}{24x^3} - \frac{29a^2\sqrt[4]{1-ax}(1+ax)^{3/4}}{96x^2} + \frac{83a^3\sqrt[4]{1-ax}(1+ax)^{3/4}}{192x} -$$

Mathematica [C] time = 0.0281805, size = 86, normalized size = 0.51

$$\frac{\sqrt[4]{1-ax}\left(-66a^4x^4\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1-ax}{ax+1}\right)+83a^4x^4+25a^3x^3-2a^2x^2+8ax-48\right)}{192x^4\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcTanh[a*x]/2)*x^5),x]
```

```
[Out] ((1 - a*x)^(1/4)*(-48 + 8*a*x - 2*a^2*x^2 + 25*a^3*x^3 + 83*a^4*x^4 - 66*a^
4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (1 - a*x)/(1 + a*x)]))/(192*x^4*(1 + a
*x)^(1/4))
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\frac{1}{\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^5*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

Fricas [A] time = 1.74222, size = 382, normalized size = 2.27

$$\frac{66\,a^4x^4\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right) - 33\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right) + 33\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right) + 2\left(83\,a^3x^3 - 58\,a^2x\right.}{384\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/384*(66*a^4*x^4*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 33*a^4*x^4*
log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 33*a^4*x^4*log(sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1)) - 1) + 2*(83*a^3*x^3 - 58*a^2*x^2 + 56*a*x - 48)*sqr
t(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(1/(x^5*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

3.100 $\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=31

$$\frac{x^{m+1}F_1\left(m+1;-\frac{3}{4},\frac{3}{4};m+2;ax,-ax\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, -3/4, 3/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rubi [A] time = 0.027221, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6126, 133}

$$\frac{x^{m+1}F_1\left(m+1;-\frac{3}{4},\frac{3}{4};m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((3*ArcTanh[a*x])/2),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -3/4, 3/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1-ax)^{3/4}}{(1+ax)^{3/4}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;-\frac{3}{4},\frac{3}{4};2+m;ax,-ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.317077, size = 0, normalized size = 0.

$$\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^((3*ArcTanh[a*x])/2),x]
```

```
[Out] Integrate[x^m/E^((3*ArcTanh[a*x])/2), x]
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int x^m \left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ax-1)x^m\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*x - 1)*x^m*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))/(a*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

3.101 $\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=290

$$-\frac{x^2(1-ax)^{7/4}\sqrt[4]{ax+1}}{4a^2}-\frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{ax+1}}{32a^4}-\frac{41(1-ax)^{3/4}\sqrt[4]{ax+1}}{64a^4}+\frac{123\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{128\sqrt{2}a^4}\ 12\ --$$

```
[Out] (-41*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(64*a^4) - (x^2*(1 - a*x)^(7/4)*(1 +
a*x)^(1/4))/(4*a^2) - ((11 - 4*a*x)*(1 - a*x)^(7/4)*(1 + a*x)^(1/4))/(32*a^
4) - (123*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2
]*a^4) + (123*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sq
rt[2]*a^4) + (123*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^
(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4) - (123*Log[1 + Sqrt[1 - a*x]/Sqr
t[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rubi [A] time = 0.20771, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 100, 147, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^2(1-ax)^{7/4}\sqrt[4]{ax+1}}{4a^2}-\frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{ax+1}}{32a^4}-\frac{41(1-ax)^{3/4}\sqrt[4]{ax+1}}{64a^4}+\frac{123\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{128\sqrt{2}a^4}\ 12\ --$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((3*ArcTanh[a*x])/2),x]
```


```
[Out] (-41*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(64*a^4) - (x^2*(1 - a*x)^(7/4)*(1 +
a*x)^(1/4))/(4*a^2) - ((11 - 4*a*x)*(1 - a*x)^(7/4)*(1 + a*x)^(1/4))/(32*a^
4) - (123*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2
]*a^4) + (123*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sq
rt[2]*a^4) + (123*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^
(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4) - (123*Log[1 + Sqrt[1 - a*x]/Sqr
t[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```


Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
```

```
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x^3\,dx = \int \frac{x^3(1-ax)^{3/4}}{(1+ax)^{3/4}}\,dx$$

$$= -\frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{\int \frac{x(1-ax)^{3/4}\left(-2+\frac{3ax}{2}\right)}{(1+ax)^{3/4}}\,dx}{4a^2}$$

$$= -\frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{1+ax}}{32a^4} - \frac{41\int \frac{(1-ax)^{3/4}}{(1+ax)^{3/4}}\,dx}{64a^3}$$

$$= -\frac{41(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} - \frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{1+ax}}{32a^4} - \frac{123\int \frac{}{\sqrt[4]{1-ax}(}}{128a}$$

$$= -\frac{41(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} - \frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{1+ax}}{32a^4} + \frac{123\,\mathrm{Subst}\Big(}{}$$

$$= -\frac{41(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} - \frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{1+ax}}{32a^4} + \frac{123\,\mathrm{Subst}\Big(}{}$$

$$= -\frac{41(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} - \frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{1+ax}}{32a^4} - \frac{123\,\mathrm{Subst}\Big(}{}$$

$$= -\frac{41(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} - \frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{1+ax}}{32a^4} + \frac{123\,\mathrm{Subst}\Big(}{}$$

$$= -\frac{41(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} - \frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{1+ax}}{32a^4} + \frac{123\log\Big(1+}{12}$$

$$= -\frac{41(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} - \frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{1+ax}}{32a^4} - \frac{123\tan^{-1}\Big(1}{64\sqrt{}}$$

Mathematica [C] time = 0.103745, size = 116, normalized size = 0.4

$$\frac{(1-ax)^{7/4}\left(12\sqrt[4]{2}\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}(1-ax)\right)-20\sqrt[4]{2}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}(1-ax)\right)+\right.}{28a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^((3*ArcTanh[a*x])/2),x]
```

```
[Out] ((1 - a*x)^(7/4)*(-7*a^2*x^2*(1 + a*x)^(1/4) + 12*2^(1/4)*Hypergeometric2F1
[-5/4, 7/4, 11/4, (1 - a*x)/2] - 20*2^(1/4)*Hypergeometric2F1[-1/4, 7/4, 11
/4, (1 - a*x)/2] + 7*2^(1/4)*Hypergeometric2F1[3/4, 7/4, 11/4, (1 - a*x)/2]
))/(28*a^4)
```

Maple [F] time = 0.107, size = 0, normalized size = 0.

$$\int x^3 \left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

Fricas [B] time = 1.95388, size = 1388, normalized size = 4.79

$$492\,\sqrt{2}a^4\frac{1}{a^{16}}^{\frac{1}{4}}\arctan\left(\sqrt{2}a^{12}\sqrt{\frac{\sqrt{2}\left(a^5x-a^4\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^{16}}^{\frac{1}{4}}+\left(a^9x-a^8\right)\sqrt{\frac{1}{a^{16}}}-\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^{16}}^{\frac{3}{4}}-\sqrt{2}a^{12}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^{16}}^{\frac{3}{4}}-1\right)+492\,\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/256*(492*sqrt(2)*a^4*(a^(-16))^(1/4)*arctan(sqrt(2)*a^12*sqrt((sqrt(2)*(a
^5*x - a^4)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) + (a^9*x -
a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(3/4) - sqrt(
2)*a^12*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) - 1) + 492*sqrt
(2)*a^4*(a^(-16))^(1/4)*arctan(sqrt(2)*a^12*sqrt(-(sqrt(2)*(a^5*x - a^4)*sq
rt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) - (a^9*x - a^8)*sqrt(a^(-
16)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(3/4) - sqrt(2)*a^12*sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) + 1) - 123*sqrt(2)*a^4*(a^(-1
6))^(1/4)*log((sqrt(2)*(a^5*x - a^4)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a
^(-16))^(1/4) + (a^9*x - a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)
) + 123*sqrt(2)*a^4*(a^(-16))^(1/4)*log(-(sqrt(2)*(a^5*x - a^4)*sqrt(-sqrt(
```

```
-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) - (a^9*x - a^8)*sqrt(a^(-16)) + sq
rt(-a^2*x^2 + 1))/(a*x - 1)) - 4*(16*a^4*x^4 - 40*a^3*x^3 + 54*a^2*x^2 - 93
*a*x + 63)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Integral(x**3/((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

3.102 $\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=282

$$-\frac{x\sqrt[4]{ax+1}(1-ax)^{7/4}}{3a^2}+\frac{\sqrt[4]{ax+1}(1-ax)^{7/4}}{4a^3}+\frac{17\sqrt[4]{ax+1}(1-ax)^{3/4}}{24a^3}-\frac{17\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}a^3}+\frac{17\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\right.}{16\sqrt{2}a}$$

```
[Out] (17*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(24*a^3) + ((1 - a*x)^(7/4)*(1 + a*x)^
(1/4))/(4*a^3) - (x*(1 - a*x)^(7/4)*(1 + a*x)^(1/4))/(3*a^2) + (17*ArcTan[1
 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (17*ArcTan
[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (17*Log[
1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)
])/(16*Sqrt[2]*a^3) + (17*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1
 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3)
```

Rubi [A] time = 0.207409, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 90, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x\sqrt[4]{ax+1}(1-ax)^{7/4}}{3a^2}+\frac{\sqrt[4]{ax+1}(1-ax)^{7/4}}{4a^3}+\frac{17\sqrt[4]{ax+1}(1-ax)^{3/4}}{24a^3}-\frac{17\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}a^3}+\frac{17\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\right.}{16\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((3*ArcTanh[a*x])/2),x]
```

```
[Out] (17*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(24*a^3) + ((1 - a*x)^(7/4)*(1 + a*x)^
(1/4))/(4*a^3) - (x*(1 - a*x)^(7/4)*(1 + a*x)^(1/4))/(3*a^2) + (17*ArcTan[1
 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (17*ArcTan
[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (17*Log[
1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)
])/(16*Sqrt[2]*a^3) + (17*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1
 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x^2\,dx = \int \frac{x^2(1-ax)^{3/4}}{(1+ax)^{3/4}}\,dx$$

$$= -\frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} - \frac{\int \frac{(1-ax)^{3/4}\left(-1+\frac{3ax}{2}\right)}{(1+ax)^{3/4}}\,dx}{3a^2}$$

$$= \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^3} - \frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} + \frac{17\int \frac{(1-ax)^{3/4}}{(1+ax)^{3/4}}\,dx}{24a^2}$$

$$= \frac{17(1-ax)^{3/4}\sqrt[4]{1+ax}}{24a^3} + \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^3} - \frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} + \frac{17\int \frac{1}{\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx}{16a^2}$$

$$= \frac{17(1-ax)^{3/4}\sqrt[4]{1+ax}}{24a^3} + \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^3} - \frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} - \frac{17\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{\ldots}\right)}{4a^3}$$

$$= \frac{17(1-ax)^{3/4}\sqrt[4]{1+ax}}{24a^3} + \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^3} - \frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} - \frac{17\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-a\ldots}}{\sqrt[4]{1+a\ldots}}\right)}{4a^3}$$

$$= \frac{17(1-ax)^{3/4}\sqrt[4]{1+ax}}{24a^3} + \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^3} - \frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} + \frac{17\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\ldots}}{\sqrt[4]{1+\ldots}}\right)}{8a^3}$$

$$= \frac{17(1-ax)^{3/4}\sqrt[4]{1+ax}}{24a^3} + \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^3} - \frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} - \frac{17\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \ldots\right)}{16a^3}$$

$$= \frac{17(1-ax)^{3/4}\sqrt[4]{1+ax}}{24a^3} + \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^3} - \frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} - \frac{17\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\ldots\right)}{16\sqrt{2}a^3}$$

$$= \frac{17(1-ax)^{3/4}\sqrt[4]{1+ax}}{24a^3} + \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^3} - \frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} + \frac{17\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8\sqrt{2}a^3} - \ldots$$

Mathematica [C] time = 0.0275494, size = 62, normalized size = 0.22

$$-\frac{(1-ax)^{7/4}\left(17\sqrt[4]{2}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}(1-ax)\right)+7\sqrt[4]{ax+1}(4ax-3)\right)}{84a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^((3*ArcTanh[a*x])/2),x]
```

```
[Out] -((1 - a*x)^(7/4)*(7*(1 + a*x)^(1/4)*(-3 + 4*a*x) + 17*2^(1/4)*Hypergeometr
ic2F1[3/4, 7/4, 11/4, (1 - a*x)/2]))/(84*a^3)
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int x^2 \left((ax+1) \frac{1}{\sqrt{-a^2x^2+1}} \right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

Fricas [B] time = 1.74827, size = 1361, normalized size = 4.83

$$-\frac{204\sqrt{2}a^3\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^9\sqrt{\frac{\sqrt{2}(a^4x-a^3)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}+(a^7x-a^6)\sqrt{\frac{1}{a^{12}}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}-\sqrt{2}a^9\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}-1\right)+20}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/96*(204*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^9*sqrt((sqrt(2)*(a^
4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) + (a^7*x - a
^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(3/4) - sqrt(2
)*a^9*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) - 1) + 204*sqrt(2
)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^9*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(
-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - (a^7*x - a^6)*sqrt(a^(-12)
) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(3/4) - sqrt(2)*a^9*sqrt(-sqrt
(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) + 1) - 51*sqrt(2)*a^3*(a^(-12))^(
1/4)*log((sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12
))^(1/4) + (a^7*x - a^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 5
1*sqrt(2)*a^3*(a^(-12))^(1/4)*log(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x
^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - (a^7*x - a^6)*sqrt(a^(-12)) + sqrt(-a^
2*x^2 + 1))/(a*x - 1)) + 4*(8*a^3*x^3 - 22*a^2*x^2 + 37*a*x - 23)*sqrt(-sqr
t(-a^2*x^2 + 1)/(a*x - 1)))/a^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Integral(x**2/((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

3.103 $\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=255

$$-\frac{\sqrt[4]{ax+1}(1-ax)^{7/4}}{2a^2}-\frac{3\sqrt[4]{ax+1}(1-ax)^{3/4}}{4a^2}+\frac{9\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}-\frac{9\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}-\frac{9\tan^{-1}(}{4}$$

```
[Out] (-3*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*a^2) - ((1 - a*x)^(7/4)*(1 + a*x)^(
1/4))/(2*a^2) - (9*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(
4*Sqrt[2]*a^2) + (9*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/
(4*Sqrt[2]*a^2) + (9*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*
x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2) - (9*Log[1 + Sqrt[1 - a*x]/Sqrt
[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rubi [A] time = 0.169892, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6126, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{ax+1}(1-ax)^{7/4}}{2a^2}-\frac{3\sqrt[4]{ax+1}(1-ax)^{3/4}}{4a^2}+\frac{9\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}-\frac{9\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}-\frac{9\tan^{-1}(}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((3*ArcTanh[a*x])/2),x]
```


```
[Out] (-3*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*a^2) - ((1 - a*x)^(7/4)*(1 + a*x)^(
1/4))/(2*a^2) - (9*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(
4*Sqrt[2]*a^2) + (9*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/
(4*Sqrt[2]*a^2) + (9*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*
x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2) - (9*Log[1 + Sqrt[1 - a*x]/Sqrt
[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2)
```


Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x\,dx = \int \frac{x(1-ax)^{3/4}}{(1+ax)^{3/4}}\,dx$$

$$= -\frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2a^2} - \frac{3\int \frac{(1-ax)^{3/4}}{(1+ax)^{3/4}}\,dx}{4a}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2a^2} - \frac{9\int \frac{1}{\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx}{8a}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2a^2} + \frac{9\,\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{2a^2}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2a^2} + \frac{9\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a^2}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2a^2} - \frac{9\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4a^2} + \frac{9\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\right.}{4a^2}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2a^2} + \frac{9\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8a^2} + \frac{9\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2a^2} + \frac{9\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8\sqrt{2}a^2} - \frac{9\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right.}{8\sqrt{2}a^2}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2a^2} - \frac{9\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}a^2} + \frac{9\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}a^2}$$

Mathematica [C] time = 0.0202954, size = 56, normalized size = 0.22

$$\frac{(1-ax)^{7/4}\left(3\sqrt[4]{2}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}(1-ax)\right)-7\sqrt[4]{ax+1}\right)}{14a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^((3*ArcTanh[a*x])/2),x]
```

```
[Out] ((1 - a*x)^(7/4)*(-7*(1 + a*x)^(1/4) + 3*2^(1/4)*Hypergeometric2F1[3/4, 7/4
, 11/4, (1 - a*x)/2]))/(14*a^2)
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int x\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

Fricas [B] time = 1.84449, size = 1312, normalized size = 5.15

$$36\sqrt{2}a^2\frac{1}{a^8}^{\frac{1}{4}}\arctan\left(\sqrt{2}a^6\sqrt{\frac{\sqrt{2}(a^3x-a^2)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^8}^{\frac{1}{4}}+(a^5x-a^4)\sqrt{\frac{1}{a^8}}-\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^8}^{\frac{3}{4}}-\sqrt{2}a^6\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^8}^{\frac{3}{4}}-1\right)+36\sqrt{2}a^2\frac{1}{a^8}^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*(36*sqrt(2)*a^2*(a^(-8))^(1/4)*arctan(sqrt(2)*a^6*sqrt((sqrt(2)*(a^3*x
 - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) + (a^5*x - a^4)*
sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(3/4) - sqrt(2)*a^6*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) - 1) + 36*sqrt(2)*a^2*(a
^(-8))^(1/4)*arctan(sqrt(2)*a^6*sqrt(-(sqrt(2)*(a^3*x - a^2)*sqrt(-sqrt(-a^
2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-a
^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(3/4) - sqrt(2)*a^6*sqrt(-sqrt(-a^2*x^2 +
1)/(a*x - 1))*(a^(-8))^(3/4) + 1) - 9*sqrt(2)*a^2*(a^(-8))^(1/4)*log((sqrt(
2)*(a^3*x - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) + (a^5*
x - a^4)*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 9*sqrt(2)*a^2*(a^(
-8))^(1/4)*log(-(sqrt(2)*(a^3*x - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*
(a^(-8))^(1/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)
) - 4*(2*a^2*x^2 - 7*a*x + 5)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Integral(x/((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

3.104 $\int e^{-\frac{3}{2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=222

$$\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{a}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{\sqrt{2}a}-\frac{3\tan^{-1}\left(\cdots\right.}{\sqrt{\cdots}}$$

```
[Out] ((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/a + (3*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4
))/(1 + a*x)^(1/4)])/(Sqrt[2]*a) - (3*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/
(1 + a*x)^(1/4)])/(Sqrt[2]*a) - (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (S
qrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*Sqrt[2]*a) + (3*Log[1 + Sqrt[1
 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*Sqrt
[2]*a)
```

Rubi [A] time = 0.140097, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6125, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{a}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{\sqrt{2}a}-\frac{3\tan^{-1}\left(\cdots\right.}{\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-3*ArcTanh[a*x])/2),x]
```

```
[Out] ((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/a + (3*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4
))/(1 + a*x)^(1/4)])/(Sqrt[2]*a) - (3*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/
(1 + a*x)^(1/4)])/(Sqrt[2]*a) - (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (S
qrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*Sqrt[2]*a) + (3*Log[1 + Sqrt[1
 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*Sqrt
[2]*a)
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{-\frac{3}{2}\tanh^{-1}(ax)}dx = \int \frac{(1-ax)^{3/4}}{(1+ax)^{3/4}}dx$$
$$= \frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{3}{2}\int \frac{1}{\sqrt[4]{1-ax}(1+ax)^{3/4}}dx$$
$$= \frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} - \frac{6\,\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}dx, x, \sqrt[4]{1-ax}\right)}{a}$$
$$= \frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} - \frac{6\,\text{Subst}\left(\int \frac{x^2}{1+x^4}dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= \frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{3\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{3\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= \frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a} - \frac{3\,\text{S}}{}$$
$$= \frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} - \frac{3\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}a} + \frac{3\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}a} - \frac{3\,\text{Subst}\left(\int \frac{1}{-1-}\right.}{}$$
$$= \frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} - \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} - \frac{3\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right.}{2\sqrt{2}a}$$

Mathematica [A] time = 0.227285, size = 150, normalized size = 0.68

$$\frac{\frac{8e^{\frac{1}{2}\tanh^{-1}(ax)}}{e^{2\tanh^{-1}(ax)}+1} - 3\sqrt{2}\log\left(-\sqrt{2}e^{\frac{1}{2}\tanh^{-1}(ax)} + e^{\tanh^{-1}(ax)} + 1\right) + 3\sqrt{2}\log\left(\sqrt{2}e^{\frac{1}{2}\tanh^{-1}(ax)} + e^{\tanh^{-1}(ax)} + 1\right) - 6\sqrt{2}\tan^{-1}\left(1-\right.}{4a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-3*ArcTanh[a*x])/2),x]
```

```
[Out] ((8*E^(ArcTanh[a*x]/2))/(1 + E^(2*ArcTanh[a*x])) - 6*Sqrt[2]*ArcTan[1 - Sqr
t[2]*E^(ArcTanh[a*x]/2)] + 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcTanh[a*x]/2)]
 - 3*Sqrt[2]*Log[1 - Sqrt[2]*E^(ArcTanh[a*x]/2) + E^ArcTanh[a*x]] + 3*Sqrt[
2]*Log[1 + Sqrt[2]*E^(ArcTanh[a*x]/2) + E^ArcTanh[a*x]])/(4*a)
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int \left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(-3/2), x)
```

Fricas [B] time = 1.7879, size = 1269, normalized size = 5.72

$$-\frac{12\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^3\sqrt{\frac{\sqrt{2}\left(a^2x-a\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-\sqrt{2}a^3\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-1\right)+12\sqrt{2}a\left(\frac{1}{a^4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(12*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*a^3*sqrt((sqrt(2)*(a^2*x -
 a)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) + (a^3*x - a^2)*sqrt
(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(3/4) - sqrt(2)*a^3*sqrt
(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) - 1) + 12*sqrt(2)*a*(a^(-4))
^(1/4)*arctan(sqrt(2)*a^3*sqrt(-(sqrt(2)*(a^2*x - a)*sqrt(-sqrt(-a^2*x^2 +
1)/(a*x - 1))*(a^(-4))^(1/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 +
 1))/(a*x - 1))*(a^(-4))^(3/4) - sqrt(2)*a^3*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
- 1))*(a^(-4))^(3/4) + 1) - 3*sqrt(2)*a*(a^(-4))^(1/4)*log((sqrt(2)*(a^2*x
- a)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) + (a^3*x - a^2)*sqr
t(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 3*sqrt(2)*a*(a^(-4))^(1/4)*log
(-(sqrt(2)*(a^2*x - a)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) -
 (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 4*(a*x - 1)*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(-3/2), x)
```

3.105 $\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=227

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)+\sqrt{2}\tan$$

```
[Out] -2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1
 - a*x)^(1/4))/(1 + a*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/
4))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] + Log[1 +
 Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/S
qrt[2] - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1
 + a*x)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.162551, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6126, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)+\sqrt{2}\tan$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcTanh[a*x])/2)*x),x]
```

```
[Out] -2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1
 - a*x)^(1/4))/(1 + a*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/
4))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] + Log[1 +
 Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/S
qrt[2] - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1
 + a*x)^(1/4)]/Sqrt[2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{(1-ax)^{3/4}}{x(1+ax)^{3/4}}\,dx \\
&= -\left(a\int \frac{1}{\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx\right)+\int \frac{1}{x\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx \\
&= 4\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx,x,\sqrt[4]{1-ax}\right)+4\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) \\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\right)-2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)+4\,\text{Subst}\left(\int \frac{x^{\cdots}}{1+\cdots}\right. \\
&= -2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)+2\,\text{Subst}\left(\int\cdots\right. \\
&= -2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)+\frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}}+\frac{\text{Subst}\left(\int \cdots\right.}{\cdots} \\
&= -2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)+\frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}}-\frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\cdots\right)}{\sqrt{2}} \\
&= -2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)+\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)-2\tanh^{-1}\left(\cdots\right.
\end{aligned}$$

Mathematica [C] time = 0.028336, size = 83, normalized size = 0.37

$$\frac{2(1-ax)^{3/4}\left(\sqrt[4]{2}(ax+1)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-ax)\right)-2\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1-ax}{ax+1}\right)\right)}{3(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcTanh[a*x])/2)*x),x]
```

```
[Out] (2*(1 - a*x)^(3/4)*(2^(1/4)*(1 + a*x)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4
, (1 - a*x)/2] - 2*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)]))/(3
```

```
*(1 + a*x)^(3/4))
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int \frac{1}{x} \left((ax+1) \frac{1}{\sqrt{-a^2x^2+1}} \right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \left(\frac{ax+1}{\sqrt{-a^2x^2+1}} \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

Fricas [B] time = 1.87684, size = 998, normalized size = 4.4

$$2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{ax+\sqrt{2}(ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-\sqrt{-a^2x^2+1}-1}{ax-1}}-\sqrt{2}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{ax-}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2
+ 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt(2)*sqrt(-sqrt(-
a^2*x^2 + 1)/(a*x - 1)) - 1) + 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x - sqrt(2)
*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a
*x - 1)) - sqrt(2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 1/2*sqrt(2)*l
og(4*(a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a
^2*x^2 + 1) - 1)/(a*x - 1)) + 1/2*sqrt(2)*log(4*(a*x - sqrt(2)*(a*x - 1)*sq
rt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - 2*
arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - log(sqrt(-sqrt(-a^2*x^2 + 1)/
(a*x - 1)) + 1) + log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x,x)
```

```
[Out] Integral(1/(x*((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(1/(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

3.106 $\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=73

$$-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{x}+3a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+3a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

```
[Out] -(((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/x) + 3*a*ArcTan[(1 + a*x)^(1/4)/(1 - a*
x)^(1/4)] + 3*a*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)]
```

Rubi [A] time = 0.032562, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6126, 94, 93, 212, 206, 203}

$$-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{x}+3a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+3a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcTanh[a*x])/2)*x^2),x]
```

```
[Out] -(((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/x) + 3*a*ArcTan[(1 + a*x)^(1/4)/(1 - a*
x)^(1/4)] + 3*a*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1-ax)^{3/4}}{x^2(1+ax)^{3/4}}\,dx\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{x} - \frac{1}{2}(3a)\int \frac{1}{x\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{x} - (6a)\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{x} + (3a)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + (3a)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{x} + 3a\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + 3a\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\end{aligned}$$

Mathematica [C] time = 0.0141001, size = 55, normalized size = 0.75

$$\frac{(1-ax)^{3/4}\left(2ax\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1-ax}{ax+1}\right)-ax-1\right)}{x(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcTanh[a*x])/2)*x^2),x]
```

```
[Out] ((1 - a*x)^(3/4)*(-1 - a*x + 2*a*x*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)
/(1 + a*x)]))/(x*(1 + a*x)^(3/4))
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

Fricas [B] time = 1.69651, size = 290, normalized size = 3.97

$$\frac{6\,ax\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+3\,ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-3\,ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\,(ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*x*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 3*a*x*log(sqrt(-sq
rt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 3*a*x*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
- 1)) - 1) + 2*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

3.107 $\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=110

$$-\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(1-ax)^{7/4}\sqrt[4]{ax+1}}{2x^2}+\frac{3a(1-ax)^{3/4}\sqrt[4]{ax+1}}{4x}$$

```
[Out] (3*a*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*x) - ((1 - a*x)^(7/4)*(1 + a*x)^(1
/4))/(2*x^2) - (9*a^2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (9*a^2*A
rcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4
```

Rubi [A] time = 0.0421458, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6126, 96, 94, 93, 212, 206, 203}

$$-\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(1-ax)^{7/4}\sqrt[4]{ax+1}}{2x^2}+\frac{3a(1-ax)^{3/4}\sqrt[4]{ax+1}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcTanh[a*x])/2)*x^3),x]
```

```
[Out] (3*a*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*x) - ((1 - a*x)^(7/4)*(1 + a*x)^(1
/4))/(2*x^2) - (9*a^2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (9*a^2*A
rcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1-ax)^{3/4}}{x^3(1+ax)^{3/4}}\,dx\\
&= -\frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2x^2} - \frac{1}{4}(3a)\int \frac{(1-ax)^{3/4}}{x^2(1+ax)^{3/4}}\,dx\\
&= \frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2x^2} + \frac{1}{8}\left(9a^2\right)\int \frac{1}{x\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx\\
&= \frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2x^2} + \frac{1}{2}\left(9a^2\right)\operatorname{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= \frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2x^2} - \frac{1}{4}\left(9a^2\right)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{1}{4}\left(9a^2\right)\mathrm{S}\\
&= \frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2x^2} - \frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0205215, size = 70, normalized size = 0.64

$$\frac{(1-ax)^{3/4}\left(-6a^2x^2\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1-ax}{ax+1}\right)+5a^2x^2+3ax-2\right)}{4x^2(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcTanh[a*x])/2)*x^3),x]
```

```
[Out] ((1 - a*x)^(3/4)*(-2 + 3*a*x + 5*a^2*x^2 - 6*a^2*x^2*Hypergeometric2F1[3/4,
 1, 7/4, (1 - a*x)/(1 + a*x)]))/(4*x^2*(1 + a*x)^(3/4))
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}} \left((ax+1)\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

Fricas [A] time = 1.67503, size = 331, normalized size = 3.01

$$-\frac{18\,a^{2}x^{2}\arctan\left(\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}\right)+9\,a^{2}x^{2}\log\left(\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}+1\right)-9\,a^{2}x^{2}\log\left(\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}-1\right)+2\left(5\,a^{2}x^{2}-7\,ax+2\right)}{8\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(18*a^2*x^2*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 9*a^2*x^2*lo
g(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 9*a^2*x^2*log(sqrt(-sqrt(-a^2*
x^2 + 1)/(a*x - 1)) - 1) + 2*(5*a^2*x^2 - 7*a*x + 2)*sqrt(-sqrt(-a^2*x^2 +
1)/(a*x - 1)))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

3.108 $\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=139

$$-\frac{23a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{24x}+\frac{17}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{17}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{7a(1-ax)^{3/4}\sqrt[4]{ax+1}}{12x^2}-\frac{(1-ax)^{3/4}\sqrt[4]{a}}{3x^3}$$

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(3*x^3) + (7*a*(1 - a*x)^(3/4)*(1 + a*x)
^(1/4))/(12*x^2) - (23*a^2*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(24*x) + (17*a^
3*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8 + (17*a^3*ArcTanh[(1 + a*x)^(1
/4)/(1 - a*x)^(1/4)])/8
```

Rubi [A] time = 0.0614669, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 212, 206, 203}

$$-\frac{23a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{24x}+\frac{17}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{17}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{7a(1-ax)^{3/4}\sqrt[4]{ax+1}}{12x^2}-\frac{(1-ax)^{3/4}\sqrt[4]{a}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcTanh[a*x])/2)*x^4),x]
```

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(3*x^3) + (7*a*(1 - a*x)^(3/4)*(1 + a*x)
^(1/4))/(12*x^2) - (23*a^2*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(24*x) + (17*a^
3*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8 + (17*a^3*ArcTanh[(1 + a*x)^(1
/4)/(1 - a*x)^(1/4)])/8
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{(1-ax)^{3/4}}{x^4(1+ax)^{3/4}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} + \frac{1}{3}\int \frac{-\frac{7a}{2}+2a^2x}{x^3\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} + \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{1}{6}\int \frac{-\frac{23a^2}{4}+\frac{7a^3x}{2}}{x^2\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} + \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{23a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x} + \frac{1}{6}\int -\frac{51a^3}{8x\sqrt[4]{1-ax}(1+ax)}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} + \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{23a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x} - \frac{1}{16}\left(17a^3\right)\int \frac{1}{x\sqrt[4]{1-ax}(1}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} + \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{23a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x} - \frac{1}{4}\left(17a^3\right)\text{Subst}\left(\int \frac{1}{-1+}\right.$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} + \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{23a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x} + \frac{1}{8}\left(17a^3\right)\text{Subst}\left(\int \frac{1}{1-x}\right.$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} + \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{23a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x} + \frac{17}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) +$$

Mathematica [C] time = 0.0218255, size = 78, normalized size = 0.56

$$\frac{(1-ax)^{3/4}\left(34a^3x^3\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1-ax}{ax+1}\right)-23a^3x^3-9a^2x^2+6ax-8\right)}{24x^3(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcTanh[a*x])/2)*x^4),x]
```

```
[Out] ((1 - a*x)^(3/4)*(-8 + 6*a*x - 9*a^2*x^2 - 23*a^3*x^3 + 34*a^3*x^3*Hypergeo
metric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)]))/(24*x^3*(1 + a*x)^(3/4))
```

Maple [F] time = 0.107, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

Fricas [A] time = 1.77509, size = 355, normalized size = 2.55

$$\frac{102\,a^3x^3\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+51\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-51\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\left(23\,a^3x^3-37\,a^2\right.}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(102*a^3*x^3*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 51*a^3*x^3*
log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 51*a^3*x^3*log(sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1)) - 1) + 2*(23*a^3*x^3 - 37*a^2*x^2 + 22*a*x - 8)*sqrt
(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x^4*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

3.109 $\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=168

$$-\frac{15a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{32x^2}+\frac{63a^3(1-ax)^{3/4}\sqrt[4]{ax+1}}{64x}-\frac{123}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{123}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{3a(1-a}{}$$

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*x^4) + (3*a*(1 - a*x)^(3/4)*(1 + a*x)
^(1/4))/(8*x^3) - (15*a^2*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(32*x^2) + (63*a
^3*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(64*x) - (123*a^4*ArcTan[(1 + a*x)^(1/4
)/(1 - a*x)^(1/4)])/64 - (123*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])
/64
```

Rubi [A] time = 0.0790486, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 212, 206, 203}

$$-\frac{15a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{32x^2}+\frac{63a^3(1-ax)^{3/4}\sqrt[4]{ax+1}}{64x}-\frac{123}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{123}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{3a(1-a}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcTanh[a*x])/2)*x^5),x]
```

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*x^4) + (3*a*(1 - a*x)^(3/4)*(1 + a*x)
^(1/4))/(8*x^3) - (15*a^2*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(32*x^2) + (63*a
^3*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(64*x) - (123*a^4*ArcTan[(1 + a*x)^(1/4
)/(1 - a*x)^(1/4)])/64 - (123*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])
/64
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x^5}dx = \int \frac{(1-ax)^{3/4}}{x^5(1+ax)^{3/4}}dx$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} + \frac{1}{4}\int \frac{-\frac{9a}{2}+3a^2x}{x^4\sqrt[4]{1-ax}(1+ax)^{3/4}}dx$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} + \frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{8x^3} - \frac{1}{12}\int \frac{-\frac{45a^2}{4}+9a^3x}{x^3\sqrt[4]{1-ax}(1+ax)^{3/4}}dx$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} + \frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{8x^3} - \frac{15a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{32x^2} + \frac{1}{24}\int \frac{-\frac{189a^3}{8}+\frac{45a}{4}}{x^2\sqrt[4]{1-ax}(1+}$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} + \frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{8x^3} - \frac{15a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{32x^2} + \frac{63a^3(1-ax)^{3/4}\sqrt[4]{1+a}}{64x}$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} + \frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{8x^3} - \frac{15a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{32x^2} + \frac{63a^3(1-ax)^{3/4}\sqrt[4]{1+a}}{64x}$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} + \frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{8x^3} - \frac{15a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{32x^2} + \frac{63a^3(1-ax)^{3/4}\sqrt[4]{1+a}}{64x}$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} + \frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{8x^3} - \frac{15a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{32x^2} + \frac{63a^3(1-ax)^{3/4}\sqrt[4]{1+a}}{64x}$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} + \frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{8x^3} - \frac{15a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{32x^2} + \frac{63a^3(1-ax)^{3/4}\sqrt[4]{1+a}}{64x}$$

Mathematica [C] time = 0.0290486, size = 86, normalized size = 0.51

$$\frac{(1-ax)^{3/4}\left(-82a^4x^4\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1-ax}{ax+1}\right)+63a^4x^4+33a^3x^3-6a^2x^2+8ax-16\right)}{64x^4(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcTanh[a*x])/2)*x^5),x]
```

```
[Out] ((1 - a*x)^(3/4)*(-16 + 8*a*x - 6*a^2*x^2 + 33*a^3*x^3 + 63*a^4*x^4 - 82*a^
4*x^4*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)]))/(64*x^4*(1 + a*
x)^(3/4))
```

Maple [F] time = 0.114, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^5*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

Fricas [A] time = 2.04378, size = 379, normalized size = 2.26

$$-\frac{246\,a^4x^4\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+123\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-123\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\left(63\,a^4x^4-93\,a^3\right.}{128\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/128*(246*a^4*x^4*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 123*a^4*x
^4*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 123*a^4*x^4*log(sqrt(-sqr
t(-a^2*x^2 + 1)/(a*x - 1)) - 1) + 2*(63*a^4*x^4 - 93*a^3*x^3 + 54*a^2*x^2 -
 40*a*x + 16)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(1/(x^5*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

3.110 $\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=31

$$\frac{x^{m+1}F_1\left(m+1;-\frac{5}{4},\frac{5}{4};m+2;ax,-ax\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, -5/4, 5/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rubi [A] time = 0.026762, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6126, 133}

$$\frac{x^{m+1}F_1\left(m+1;-\frac{5}{4},\frac{5}{4};m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((5*ArcTanh[a*x])/2),x]

[Out] (x^(1 + m)*AppellF1[1 + m, -5/4, 5/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1-ax)^{5/4}}{(1+ax)^{5/4}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;-\frac{5}{4},\frac{5}{4};2+m;ax,-ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.531764, size = 0, normalized size = 0.

$$\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] Integrate[x^m/E^((5*ArcTanh[a*x])/2), x]
```

Maple [F] time = 0.14, size = 0, normalized size = 0.

$$\int x^{m}\left((ax+1)\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^{2}x^{2}+1}(ax-1)x^{m}\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}}{a^{2}x^{2}+2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^m*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x -
1))/(a^2*x^2 + 2*a*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

3.111 $\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=317

$$\frac{17x^2(1-ax)^{5/4}(ax+1)^{3/4}}{4a^2}+\frac{(521-452ax)(1-ax)^{5/4}(ax+1)^{3/4}}{96a^4}+\frac{475\sqrt[4]{1-ax}(ax+1)^{3/4}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\right.}{128\sqrt{2}a^4}$$

```
[Out] (-4*x^3*(1 - a*x)^(5/4))/(a*(1 + a*x)^(1/4)) + (475*(1 - a*x)^(1/4)*(1 + a*
x)^(3/4))/(64*a^4) + (17*x^2*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(4*a^2) + ((5
21 - 452*a*x)*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(96*a^4) + (475*ArcTan[1 - (
Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]*a^4) - (475*ArcTan[1
 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (475*Log[
1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)
])/(128*Sqrt[2]*a^4) - (475*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*
(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rubi [A] time = 0.237861, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {6126, 97, 153, 147, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{17x^2(1-ax)^{5/4}(ax+1)^{3/4}}{4a^2}+\frac{(521-452ax)(1-ax)^{5/4}(ax+1)^{3/4}}{96a^4}+\frac{475\sqrt[4]{1-ax}(ax+1)^{3/4}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\right.}{128\sqrt{2}a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((5*ArcTanh[a*x])/2),x]
```


```
[Out] (-4*x^3*(1 - a*x)^(5/4))/(a*(1 + a*x)^(1/4)) + (475*(1 - a*x)^(1/4)*(1 + a*
x)^(3/4))/(64*a^4) + (17*x^2*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(4*a^2) + ((5
21 - 452*a*x)*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(96*a^4) + (475*ArcTan[1 - (
Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]*a^4) - (475*ArcTan[1
 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (475*Log[
1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)
])/(128*Sqrt[2]*a^4) - (475*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*
(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```


Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
```

```
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x^3\,dx = \int \frac{x^3(1-ax)^{5/4}}{(1+ax)^{5/4}}\,dx$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{4\int \frac{x^2\left(3-\frac{17ax}{4}\right)\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx}{a}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} - \frac{\int \frac{x\sqrt[4]{1-ax}\left(\frac{17a}{2}-\frac{113a^2x}{8}\right)}{\sqrt[4]{1+ax}}\,dx}{a^3}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} + \frac{(521-452ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4} + \frac{475\int \ldots}{64a \ldots}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{475\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} + \frac{(521-452ax)(1-ax \ldots}{96a^4}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{475\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} + \frac{(521-452ax)(1-ax \ldots}{96a^4}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{475\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} + \frac{(521-452ax)(1-ax \ldots}{96a^4}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{475\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} + \frac{(521-452ax)(1-ax \ldots}{96a^4}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{475\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} + \frac{(521-452ax)(1-ax \ldots}{96a^4}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{475\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} + \frac{(521-452ax)(1-ax \ldots}{96a^4}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{475\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a^4} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} + \frac{(521-452ax)(1-ax \ldots}{96a^4}$$

Mathematica [C] time = 0.0361382, size = 79, normalized size = 0.25

$$-\frac{(1-ax)^{9/4}\left(95\,2^{3/4}\sqrt[4]{ax+1}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}(1-ax)\right)+3\left(6a^2x^2-5ax-59\right)\right)}{72a^4\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] -((1 - a*x)^(9/4)*(3*(-59 - 5*a*x + 6*a^2*x^2) + 95*2^(3/4)*(1 + a*x)^(1/4)
*Hypergeometric2F1[1/4, 9/4, 13/4, (1 - a*x)/2]))/(72*a^4*(1 + a*x)^(1/4))
```

Maple [F] time = 0.147, size = 0, normalized size = 0.

$$\int x^3\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

Fricas [B] time = 2.29605, size = 1500, normalized size = 4.73

$$-\frac{5700\sqrt{2}\left(a^5x+a^4\right)\left(\frac{1}{a^{16}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^4\sqrt{\frac{\sqrt{2}\left(a^{13}x-a^{12}\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{3}{4}}+\left(a^9x-a^8\right)\sqrt{\frac{1}{a^{16}}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{1}{4}}-\sqrt{2}a^4\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{1}{4}}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/768*(5700*sqrt(2)*(a^5*x + a^4)*(a^(-16))^(1/4)*arctan(sqrt(2)*a^4*sqrt(
(sqrt(2)*(a^13*x - a^12)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4
) + (a^9*x - a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^
(1/4) - sqrt(2)*a^4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) - 1
) + 5700*sqrt(2)*(a^5*x + a^4)*(a^(-16))^(1/4)*arctan(sqrt(2)*a^4*sqrt(-(sq
rt(2)*(a^13*x - a^12)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) -
 (a^9*x - a^8)*sqrt(a^(-16)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(1/
4) - sqrt(2)*a^4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) + 1) +
 1425*sqrt(2)*(a^5*x + a^4)*(a^(-16))^(1/4)*log((sqrt(2)*(a^13*x - a^12)*sq
rt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) + (a^9*x - a^8)*sqrt(a^(-
16)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 1425*sqrt(2)*(a^5*x + a^4)*(a^(-16)
)^(1/4)*log(-(sqrt(2)*(a^13*x - a^12)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(
a^(-16))^(3/4) - (a^9*x - a^8)*sqrt(a^(-16)) + sqrt(-a^2*x^2 + 1))/(a*x - 1
)) + 4*(48*a^4*x^4 - 136*a^3*x^3 + 226*a^2*x^2 - 521*a*x - 2467)*sqrt(-a^2*
x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/(a^5*x + a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

3.112 $\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=305

$$-\frac{(ax+1)^{3/4}(1-ax)^{9/4}}{3a^3}-\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{ax+1}}-\frac{11(ax+1)^{3/4}(1-ax)^{5/4}}{4a^3}-\frac{55(ax+1)^{3/4}\sqrt[4]{1-ax}}{8a^3}-\frac{55\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}a^3}$$

```
[Out] (-2*(1 - a*x)^(9/4))/(a^3*(1 + a*x)^(1/4)) - (55*(1 - a*x)^(1/4)*(1 + a*x)^
(3/4))/(8*a^3) - (11*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(4*a^3) - ((1 - a*x)^
(9/4)*(1 + a*x)^(3/4))/(3*a^3) - (55*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(
1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) + (55*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))
/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (55*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x
] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3) + (55*Log[
1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)
])/(16*Sqrt[2]*a^3)
```

Rubi [A] time = 0.225252, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 89, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(ax+1)^{3/4}(1-ax)^{9/4}}{3a^3}-\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{ax+1}}-\frac{11(ax+1)^{3/4}(1-ax)^{5/4}}{4a^3}-\frac{55(ax+1)^{3/4}\sqrt[4]{1-ax}}{8a^3}-\frac{55\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] (-2*(1 - a*x)^(9/4))/(a^3*(1 + a*x)^(1/4)) - (55*(1 - a*x)^(1/4)*(1 + a*x)^
(3/4))/(8*a^3) - (11*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(4*a^3) - ((1 - a*x)^
(9/4)*(1 + a*x)^(3/4))/(3*a^3) - (55*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(
1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) + (55*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))
/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (55*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x
] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3) + (55*Log[
1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)
])/(16*Sqrt[2]*a^3)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x^2\,dx = \int \frac{x^2(1-ax)^{5/4}}{(1+ax)^{5/4}}\,dx$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} + \frac{2\int \frac{(1-ax)^{5/4}\left(-\frac{5a}{2}+\frac{a^2x}{2}\right)}{\sqrt[4]{1+ax}}\,dx}{a^3}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3} - \frac{11\int \frac{(1-ax)^{5/4}}{\sqrt[4]{1+ax}}\,dx}{2a^2}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3} - \frac{55\int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx}{8a^2}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{55\sqrt[4]{1-ax}(1+ax)^{3/4}}{8a^3} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3} - \frac{55\int \frac{\cdots}{(1\cdots}}{\cdots}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{55\sqrt[4]{1-ax}(1+ax)^{3/4}}{8a^3} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3} + \frac{55\,\mathrm{Sub}\cdots}{\cdots}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{55\sqrt[4]{1-ax}(1+ax)^{3/4}}{8a^3} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3} + \frac{55\,\mathrm{Sub}\cdots}{\cdots}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{55\sqrt[4]{1-ax}(1+ax)^{3/4}}{8a^3} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3} + \frac{55\,\mathrm{Sub}\cdots}{\cdots}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{55\sqrt[4]{1-ax}(1+ax)^{3/4}}{8a^3} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3} + \frac{55\,\mathrm{Sub}\cdots}{\cdots}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{55\sqrt[4]{1-ax}(1+ax)^{3/4}}{8a^3} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3} - \frac{55\log\cdots}{\cdots}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{55\sqrt[4]{1-ax}(1+ax)^{3/4}}{8a^3} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3} - \frac{55\tan\cdots}{\cdots}$$

Mathematica [C] time = 0.0347669, size = 70, normalized size = 0.23

$$\frac{(1-ax)^{9/4}\left(11\,2^{3/4}\sqrt[4]{ax+1}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}(1-ax)\right)-3(ax+7)\right)}{9a^3\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] ((1 - a*x)^(9/4)*(-3*(7 + a*x) + 11*2^(3/4)*(1 + a*x)^(1/4)*Hypergeometric2
F1[1/4, 9/4, 13/4, (1 - a*x)/2]))/(9*a^3*(1 + a*x)^(1/4))
```

Maple [F] time = 0.129, size = 0, normalized size = 0.

$$\int x^{2} \left((ax+1)\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

Fricas [B] time = 2.09669, size = 1462, normalized size = 4.79

$$660\,\sqrt{2}\left(a^{4}x+a^{3}\right)\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^{3}\sqrt{\frac{\sqrt{2}\left(a^{10}x-a^{9}\right)\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}+\left(a^{7}x-a^{6}\right)\sqrt{\frac{1}{a^{12}}}-\sqrt{-a^{2}x^{2}+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}-\sqrt{2}a^{3}\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/96*(660*sqrt(2)*(a^4*x + a^3)*(a^(-12))^(1/4)*arctan(sqrt(2)*a^3*sqrt((sq
rt(2)*(a^10*x - a^9)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) +
(a^7*x - a^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(1/4
) - sqrt(2)*a^3*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - 1) +
660*sqrt(2)*(a^4*x + a^3)*(a^(-12))^(1/4)*arctan(sqrt(2)*a^3*sqrt(-(sqrt(2)
*(a^10*x - a^9)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) - (a^7*
x - a^6)*sqrt(a^(-12)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(1/4) - s
qrt(2)*a^3*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) + 1) + 165*s
qrt(2)*(a^4*x + a^3)*(a^(-12))^(1/4)*log((sqrt(2)*(a^10*x - a^9)*sqrt(-sqrt
(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) + (a^7*x - a^6)*sqrt(a^(-12)) - s
qrt(-a^2*x^2 + 1))/(a*x - 1)) - 165*sqrt(2)*(a^4*x + a^3)*(a^(-12))^(1/4)*l
```

```
og(-(sqrt(2)*(a^10*x - a^9)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(
3/4) - (a^7*x - a^6)*sqrt(a^(-12)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 4*(8*
a^3*x^3 - 26*a^2*x^2 + 61*a*x + 287)*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2
 + 1)/(a*x - 1)))/(a^4*x + a^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

3.113 $\int e^{-\frac{5}{2}\tanh^{-1}(ax)} x\,dx$

Optimal. Leaf size=279

$$\frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{ax+1}}+\frac{5(ax+1)^{3/4}(1-ax)^{5/4}}{2a^2}+\frac{25(ax+1)^{3/4}\sqrt[4]{1-ax}}{4a^2}+\frac{25\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}-\frac{25\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\cdots\right.}{8\sqrt{2}a^2}$$

```
[Out] (2*(1 - a*x)^(9/4))/(a^2*(1 + a*x)^(1/4)) + (25*(1 - a*x)^(1/4)*(1 + a*x)^(
3/4))/(4*a^2) + (5*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(2*a^2) + (25*ArcTan[1
- (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt[2]*a^2) - (25*ArcTan[
1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt[2]*a^2) + (25*Log[1
 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]
)/(8*Sqrt[2]*a^2) - (25*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 -
 a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rubi [A] time = 0.192671, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6126, 78, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{ax+1}}+\frac{5(ax+1)^{3/4}(1-ax)^{5/4}}{2a^2}+\frac{25(ax+1)^{3/4}\sqrt[4]{1-ax}}{4a^2}+\frac{25\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}-\frac{25\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\cdots\right.}{8\sqrt{2}a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] (2*(1 - a*x)^(9/4))/(a^2*(1 + a*x)^(1/4)) + (25*(1 - a*x)^(1/4)*(1 + a*x)^(
3/4))/(4*a^2) + (5*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(2*a^2) + (25*ArcTan[1
- (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt[2]*a^2) - (25*ArcTan[
1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt[2]*a^2) + (25*Log[1
 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]
)/(8*Sqrt[2]*a^2) - (25*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 -
 a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x\,dx = \int \frac{x(1-ax)^{5/4}}{(1+ax)^{5/4}}\,dx$$
$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{5\int \frac{(1-ax)^{5/4}}{\sqrt[4]{1+ax}}\,dx}{a}$$
$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{25\int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx}{4a}$$
$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{25\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{25\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx}{8a}$$
$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{25\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{25\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\right.}{2a^2}$$
$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{25\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{25\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a^2}$$
$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{25\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{25\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4a^2}$$
$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{25\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{25\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.}{8a^2}$$
$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{25\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{25\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8\sqrt{2}a^2}$$
$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{25\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a^2} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{25\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}a^2} - \frac{25\,\mathrm{t}}{}$$

Mathematica [C] time = 0.0241381, size = 64, normalized size = 0.23

$$-\frac{2(1-ax)^{9/4}\left(5\ 2^{3/4}\sqrt[4]{ax+1}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}(1-ax)\right)-9\right)}{9a^2\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] (-2*(1 - a*x)^(9/4)*(-9 + 5*2^(3/4)*(1 + a*x)^(1/4)*Hypergeometric2F1[1/4,
9/4, 13/4, (1 - a*x)/2]))/(9*a^2*(1 + a*x)^(1/4))
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int x\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

Fricas [B] time = 1.83541, size = 1413, normalized size = 5.06

$$-\frac{100\sqrt{2}\left(a^3x+a^2\right)\left(\frac{1}{a^8}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^2\sqrt{\frac{\sqrt{2}\left(a^7x-a^6\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^8}\right)^{\frac{3}{4}}+\left(a^5x-a^4\right)\sqrt{\frac{1}{a^8}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^8}\right)^{\frac{1}{4}}-\sqrt{2}a^2\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^8}\right)^{\frac{1}{4}}-1\right)+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(100*sqrt(2)*(a^3*x + a^2)*(a^(-8))^(1/4)*arctan(sqrt(2)*a^2*sqrt((sq
rt(2)*(a^7*x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) + (a
^5*x - a^4)*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(1/4) -
sqrt(2)*a^2*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) - 1) + 100*s
qrt(2)*(a^3*x + a^2)*(a^(-8))^(1/4)*arctan(sqrt(2)*a^2*sqrt(-(sqrt(2)*(a^7*
x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) - (a^5*x - a^4)
*sqrt(a^(-8)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(1/4) - sqrt(2)*a^2
*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) + 1) + 25*sqrt(2)*(a^3*
x + a^2)*(a^(-8))^(1/4)*log((sqrt(2)*(a^7*x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1))*(a^(-8))^(3/4) + (a^5*x - a^4)*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1
))/(a*x - 1)) - 25*sqrt(2)*(a^3*x + a^2)*(a^(-8))^(1/4)*log(-(sqrt(2)*(a^7*
x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) - (a^5*x - a^4)
*sqrt(a^(-8)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 4*(2*a^2*x^2 - 9*a*x - 43)
*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/(a^3*x + a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

3.114 $\int e^{-\frac{5}{2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=247

$$-\frac{4(1-ax)^{5/4}}{a\sqrt[4]{ax+1}}-\frac{5(ax+1)^{3/4}\sqrt[4]{1-ax}}{a}-\frac{5\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{5\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{ax+}}\right.}{\sqrt{2}a}$$

```
[Out] (-4*(1 - a*x)^(5/4))/(a*(1 + a*x)^(1/4)) - (5*(1 - a*x)^(1/4)*(1 + a*x)^(3/
4))/a - (5*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(Sqrt[2]*
a) + (5*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(Sqrt[2]*a)
- (5*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a
*x)^(1/4)])/(2*Sqrt[2]*a) + (5*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[
2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*Sqrt[2]*a)
```

Rubi [A] time = 0.157708, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.1, Rules used = {6125, 47, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{4(1-ax)^{5/4}}{a\sqrt[4]{ax+1}}-\frac{5(ax+1)^{3/4}\sqrt[4]{1-ax}}{a}-\frac{5\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{5\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{ax+}}\right.}{\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-5*ArcTanh[a*x])/2),x]
```


```
[Out] (-4*(1 - a*x)^(5/4))/(a*(1 + a*x)^(1/4)) - (5*(1 - a*x)^(1/4)*(1 + a*x)^(3/
4))/a - (5*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(Sqrt[2]*
a) + (5*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(Sqrt[2]*a)
- (5*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a
*x)^(1/4)])/(2*Sqrt[2]*a) + (5*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[
2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*Sqrt[2]*a)
```


Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{5}{2}\tanh^{-1}(ax)}\,dx = \int \frac{(1-ax)^{5/4}}{(1+ax)^{5/4}}\,dx$$
$$= -\frac{4(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} - 5\int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx$$
$$= -\frac{4(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} - \frac{5\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} - \frac{5}{2}\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$
$$= -\frac{4(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} - \frac{5\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} + \frac{10\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$
$$= -\frac{4(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} - \frac{5\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} + \frac{10\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= -\frac{4(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} - \frac{5\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} + \frac{5\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} + \frac{5\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-a}}{\sqrt[4]{1+a}}\right.}{a}$$
$$= -\frac{4(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} - \frac{5\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} + \frac{5\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a} + \frac{5\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx\right.}{2a}$$
$$= -\frac{4(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} - \frac{5\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} - \frac{5\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}a} + \frac{5\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}a}$$
$$= -\frac{4(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} - \frac{5\sqrt[4]{1-ax}(1+ax)^{3/4}}{a} - \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} - \frac{5\log\left(1+\right.}{}$$

Mathematica [C] time = 0.0537654, size = 33, normalized size = 0.13

$$-\frac{8e^{-\frac{1}{2}\tanh^{-1}(ax)}\text{Hypergeometric2F1}\left(-\frac{1}{4}, 2, \frac{3}{4}, -e^{2\tanh^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-5*ArcTanh[a*x])/2),x]
```

```
[Out] (-8*Hypergeometric2F1[-1/4, 2, 3/4, -E^(2*ArcTanh[a*x])])/(a*E^(ArcTanh[a*x
]/2))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(-5/2), x)
```

Fricas [B] time = 1.97635, size = 1361, normalized size = 5.51

$$\frac{20\sqrt{2}\left(a^2x+a\right)\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\sqrt{2}a\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-1\right)+20\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(20*sqrt(2)*(a^2*x + a)*(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt((sqrt(2)*(
a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x -
a^2)*sqrt(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)
*a*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) - 1) + 20*sqrt(2)*(a^
2*x + a)*(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(
-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4))
+ sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-sqrt(-a^2
*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) + 1) + 5*sqrt(2)*(a^2*x + a)*(a^(-4))^(
1/4)*log((sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4)
)^(3/4) + (a^3*x - a^2)*sqrt(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 5*s
qrt(2)*(a^2*x + a)*(a^(-4))^(1/4)*log(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-
a^2*x^2 + 1))/(a*x - 1)) - 4*sqrt(-a^2*x^2 + 1)*(a*x + 9)*sqrt(-sqrt(-a^2*x
^2 + 1)/(a*x - 1)))/(a^2*x + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(-5/2), x)
```

3.115 $\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=248

$$\frac{8\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{ax+1}}\right.$$

```
[Out] (8*(1 - a*x)^(1/4))/(1 + a*x)^(1/4) + 2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1
/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)] - Sqrt
[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 +
a*x)^(1/4)/(1 - a*x)^(1/4)] + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2
]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - a*x]/Sqrt[1
+ a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.204206, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 16, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6126, 98, 21, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$\frac{8\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{ax+}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcTanh[a*x])/2)*x),x]
```

```
[Out] (8*(1 - a*x)^(1/4))/(1 + a*x)^(1/4) + 2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1
/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)] - Sqrt
[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 +
a*x)^(1/4)/(1 - a*x)^(1/4)] + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2
]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - a*x]/Sqrt[1
+ a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sqrt[2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x}\,dx = \int \frac{(1-ax)^{5/4}}{x(1+ax)^{5/4}}\,dx$$
$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} + \frac{4\int \frac{\frac{a}{4}+\frac{a^2x}{4}}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx}{a}$$
$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} + \int \frac{(1+ax)^{3/4}}{x(1-ax)^{3/4}}\,dx$$
$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} + a\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx + \int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$
$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} - 4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right) + 4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$
$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} - 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 4\,\text{Subst}\Bigg(\int$$
$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - 2\,\text{Su}$$
$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} + \frac{\text{Subs}}{}$$
$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-}}{\sqrt{1-}}\right.}{}$$
$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - 2\,\text{ta}$$

Mathematica [C] time = 0.0632869, size = 90, normalized size = 0.36

$$\frac{\sqrt[4]{1-ax}\left(-20\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1-ax}{ax+1}\right) + 2^{3/4}(1-ax)\sqrt[4]{ax+1}\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-ax)\right) + 20\right)}{5\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcTanh[a*x])/2)*x),x]
```

```
[Out] ((1 - a*x)^(1/4)*(20 - 20*Hypergeometric2F1[1/4, 1, 5/4, (1 - a*x)/(1 + a*x
)] + 2^(3/4)*(1 - a*x)*(1 + a*x)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, (1
- a*x)/2]))/(5*(1 + a*x)^(1/4))
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

Fricas [B] time = 1.9718, size = 1191, normalized size = 4.8

$$-\frac{4\sqrt{2}(ax+1)\arctan\left(\sqrt{2}\sqrt{\frac{ax+\sqrt{2}(ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-\sqrt{-a^2x^2+1}-1}{ax-1}}-\sqrt{2}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+4\sqrt{2}(ax+1)\arctan\left(\sqrt{2}\sqrt{\frac{ax-}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(2)*(a*x + 1)*arctan(sqrt(2)*sqrt((a*x + sqrt(2)*(a*x - 1)*sqrt
(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt
(2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1) + 4*sqrt(2)*(a*x + 1)*arctan(s
qrt(2)*sqrt((a*x - sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) -
sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt(2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
- 1)) + 1) + sqrt(2)*(a*x + 1)*log(4*(a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt(2)*(a*x
+ 1)*log(4*(a*x - sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - s
qrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - 4*(a*x + 1)*arctan(sqrt(-sqrt(-a^2*x^2
+ 1)/(a*x - 1))) + 2*(a*x + 1)*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1)
 - 2*(a*x + 1)*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1) - 16*sqrt(-a^2*
x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/(a*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(1/(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

3.116 $\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=95

$$-\frac{(1-ax)^{5/4}}{x\sqrt[4]{ax+1}}-\frac{10a\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-5a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+5a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

```
[Out] (-10*a*(1 - a*x)^(1/4))/(1 + a*x)^(1/4) - (1 - a*x)^(5/4)/(x*(1 + a*x)^(1/4
)) - 5*a*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] + 5*a*ArcTanh[(1 + a*x)^(1
/4)/(1 - a*x)^(1/4)]
```

Rubi [A] time = 0.039102, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6126, 94, 93, 298, 203, 206}

$$-\frac{(1-ax)^{5/4}}{x\sqrt[4]{ax+1}}-\frac{10a\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-5a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+5a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcTanh[a*x])/2)*x^2),x]
```

```
[Out] (-10*a*(1 - a*x)^(1/4))/(1 + a*x)^(1/4) - (1 - a*x)^(5/4)/(x*(1 + a*x)^(1/4
)) - 5*a*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] + 5*a*ArcTanh[(1 + a*x)^(1
/4)/(1 - a*x)^(1/4)]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1-ax)^{5/4}}{x^2(1+ax)^{5/4}}\,dx\\
&= -\frac{(1-ax)^{5/4}}{x\sqrt[4]{1+ax}} - \frac{1}{2}(5a)\int \frac{\sqrt[4]{1-ax}}{x(1+ax)^{5/4}}\,dx\\
&= -\frac{10a\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} - \frac{(1-ax)^{5/4}}{x\sqrt[4]{1+ax}} - \frac{1}{2}(5a)\int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx\\
&= -\frac{10a\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} - \frac{(1-ax)^{5/4}}{x\sqrt[4]{1+ax}} - (10a)\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= -\frac{10a\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} - \frac{(1-ax)^{5/4}}{x\sqrt[4]{1+ax}} + (5a)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - (5a)\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \right.\\
&= -\frac{10a\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} - \frac{(1-ax)^{5/4}}{x\sqrt[4]{1+ax}} - 5a\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + 5a\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0175106, size = 55, normalized size = 0.58

$$\frac{\sqrt[4]{1-ax}\left(10ax\,\mathrm{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1-ax}{ax+1}\right)-9ax-1\right)}{x\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcTanh[a*x])/2)*x^2),x]
```

```
[Out] ((1 - a*x)^(1/4)*(-1 - 9*a*x + 10*a*x*Hypergeometric2F1[1/4, 1, 5/4, (1 - a
*x)/(1 + a*x)]))/(x*(1 + a*x)^(1/4))
```

Maple [F] time = 0.117, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

Fricas [B] time = 1.73732, size = 383, normalized size = 4.03

$$-\frac{2\sqrt{-a^2x^2+1}(9\,ax+1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+10\left(a^2x^2+ax\right)\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-5\left(a^2x^2+ax\right)\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)+}{2\left(ax^2+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(-a^2*x^2 + 1)*(9*a*x + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))
+ 10*(a^2*x^2 + a*x)*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 5*(a^2*x
^2 + a*x)*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 5*(a^2*x^2 + a*x)*
log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1))/(a*x^2 + x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

3.117 $\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=136

$$\frac{25a^2\sqrt[4]{1-ax}}{2\sqrt[4]{ax+1}}+\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(1-ax)^{9/4}}{2x^2\sqrt[4]{ax+1}}+\frac{5a(1-ax)^{5/4}}{4x\sqrt[4]{ax+1}}$$

```
[Out] (25*a^2*(1 - a*x)^(1/4))/(2*(1 + a*x)^(1/4)) + (5*a*(1 - a*x)^(5/4))/(4*x*(
1 + a*x)^(1/4)) - (1 - a*x)^(9/4)/(2*x^2*(1 + a*x)^(1/4)) + (25*a^2*ArcTan[
(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (25*a^2*ArcTanh[(1 + a*x)^(1/4)/(1 -
a*x)^(1/4)])/4
```

Rubi [A] time = 0.0536025, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6126, 96, 94, 93, 298, 203, 206}

$$\frac{25a^2\sqrt[4]{1-ax}}{2\sqrt[4]{ax+1}}+\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(1-ax)^{9/4}}{2x^2\sqrt[4]{ax+1}}+\frac{5a(1-ax)^{5/4}}{4x\sqrt[4]{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcTanh[a*x])/2)*x^3),x]
```

```
[Out] (25*a^2*(1 - a*x)^(1/4))/(2*(1 + a*x)^(1/4)) + (5*a*(1 - a*x)^(5/4))/(4*x*(
1 + a*x)^(1/4)) - (1 - a*x)^(9/4)/(2*x^2*(1 + a*x)^(1/4)) + (25*a^2*ArcTan[
(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (25*a^2*ArcTanh[(1 + a*x)^(1/4)/(1 -
a*x)^(1/4)])/4
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1-ax)^{5/4}}{x^3(1+ax)^{5/4}}\,dx\\
&= -\frac{(1-ax)^{9/4}}{2x^2\sqrt[4]{1+ax}} - \frac{1}{4}(5a)\int \frac{(1-ax)^{5/4}}{x^2(1+ax)^{5/4}}\,dx\\
&= \frac{5a(1-ax)^{5/4}}{4x\sqrt[4]{1+ax}} - \frac{(1-ax)^{9/4}}{2x^2\sqrt[4]{1+ax}} + \frac{1}{8}\left(25a^2\right)\int \frac{\sqrt[4]{1-ax}}{x(1+ax)^{5/4}}\,dx\\
&= \frac{25a^2\sqrt[4]{1-ax}}{2\sqrt[4]{1+ax}} + \frac{5a(1-ax)^{5/4}}{4x\sqrt[4]{1+ax}} - \frac{(1-ax)^{9/4}}{2x^2\sqrt[4]{1+ax}} + \frac{1}{8}\left(25a^2\right)\int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx\\
&= \frac{25a^2\sqrt[4]{1-ax}}{2\sqrt[4]{1+ax}} + \frac{5a(1-ax)^{5/4}}{4x\sqrt[4]{1+ax}} - \frac{(1-ax)^{9/4}}{2x^2\sqrt[4]{1+ax}} + \frac{1}{2}\left(25a^2\right)\mathrm{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= \frac{25a^2\sqrt[4]{1-ax}}{2\sqrt[4]{1+ax}} + \frac{5a(1-ax)^{5/4}}{4x\sqrt[4]{1+ax}} - \frac{(1-ax)^{9/4}}{2x^2\sqrt[4]{1+ax}} - \frac{1}{4}\left(25a^2\right)\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{1}{4}\left(\right.\\
&= \frac{25a^2\sqrt[4]{1-ax}}{2\sqrt[4]{1+ax}} + \frac{5a(1-ax)^{5/4}}{4x\sqrt[4]{1+ax}} - \frac{(1-ax)^{9/4}}{2x^2\sqrt[4]{1+ax}} + \frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right.
\end{aligned}
$$

Mathematica [C] time = 0.0226725, size = 70, normalized size = 0.51

$$
\frac{\sqrt[4]{1-ax}\left(-50a^2x^2\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1-ax}{ax+1}\right)+43a^2x^2+9ax-2\right)}{4x^2\sqrt[4]{ax+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcTanh[a*x])/2)*x^3),x]
```

```
[Out] ((1 - a*x)^(1/4)*(-2 + 9*a*x + 43*a^2*x^2 - 50*a^2*x^2*Hypergeometric2F1[1/
4, 1, 5/4, (1 - a*x)/(1 + a*x)]))/(4*x^2*(1 + a*x)^(1/4))
```

Maple [F] time = 0.117, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

Fricas [A] time = 1.76074, size = 421, normalized size = 3.1

$$\frac{2\left(43\,a^2x^2+9\,ax-2\right)\sqrt{-a^2x^2+1}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+50\left(a^3x^3+a^2x^2\right)\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-25\left(a^3x^3+a^2x^2\right)\log\left(\sqrt{-\frac{\sqrt{-a^2}}{ax}}\right.}{8\left(ax^3+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(43*a^2*x^2 + 9*a*x - 2)*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1)) + 50*(a^3*x^3 + a^2*x^2)*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x -
 1))) - 25*(a^3*x^3 + a^2*x^2)*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1)
 + 25*(a^3*x^3 + a^2*x^2)*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1))/(a*
x^3 + x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

3.118 $\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=165

$$-\frac{287a^3\sqrt[4]{1-ax}}{24\sqrt[4]{ax+1}}-\frac{61a^2\sqrt[4]{1-ax}}{24x\sqrt[4]{ax+1}}-\frac{55}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{55}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{ax+1}}-\frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{ax+1}}$$

```
[Out] (-287*a^3*(1 - a*x)^(1/4))/(24*(1 + a*x)^(1/4)) - (1 - a*x)^(1/4)/(3*x^3*(1
 + a*x)^(1/4)) + (13*a*(1 - a*x)^(1/4))/(12*x^2*(1 + a*x)^(1/4)) - (61*a^2*
(1 - a*x)^(1/4))/(24*x*(1 + a*x)^(1/4)) - (55*a^3*ArcTan[(1 + a*x)^(1/4)/(1
 - a*x)^(1/4)])/8 + (55*a^3*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8
```

Rubi [A] time = 0.0754489, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6126, 98, 151, 155, 12, 93, 298, 203, 206}

$$-\frac{287a^3\sqrt[4]{1-ax}}{24\sqrt[4]{ax+1}}-\frac{61a^2\sqrt[4]{1-ax}}{24x\sqrt[4]{ax+1}}-\frac{55}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{55}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{ax+1}}-\frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcTanh[a*x])/2)*x^4),x]
```

```
[Out] (-287*a^3*(1 - a*x)^(1/4))/(24*(1 + a*x)^(1/4)) - (1 - a*x)^(1/4)/(3*x^3*(1
 + a*x)^(1/4)) + (13*a*(1 - a*x)^(1/4))/(12*x^2*(1 + a*x)^(1/4)) - (61*a^2*
(1 - a*x)^(1/4))/(24*x*(1 + a*x)^(1/4)) - (55*a^3*ArcTan[(1 + a*x)^(1/4)/(1
 - a*x)^(1/4)])/8 + (55*a^3*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{(1-ax)^{5/4}}{x^4(1+ax)^{5/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{1+ax}} - \frac{1}{3}\int \frac{\frac{13a}{2}-6a^2x}{x^3(1-ax)^{3/4}(1+ax)^{5/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{1+ax}} + \frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{1+ax}} + \frac{1}{6}\int \frac{\frac{61a^2}{4}-13a^3x}{x^2(1-ax)^{3/4}(1+ax)^{5/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{1+ax}} + \frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{1+ax}} - \frac{61a^2\sqrt[4]{1-ax}}{24x\sqrt[4]{1+ax}} - \frac{1}{6}\int \frac{\frac{165a^3}{8}-\frac{61a^4x}{4}}{x(1-ax)^{3/4}(1+ax)^{5/4}}\,dx$$

$$= -\frac{287a^3\sqrt[4]{1-ax}}{24\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{1+ax}} + \frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{1+ax}} - \frac{61a^2\sqrt[4]{1-ax}}{24x\sqrt[4]{1+ax}} - \frac{\int \frac{165a^4}{16x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx}{3a}$$

$$= -\frac{287a^3\sqrt[4]{1-ax}}{24\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{1+ax}} + \frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{1+ax}} - \frac{61a^2\sqrt[4]{1-ax}}{24x\sqrt[4]{1+ax}} - \frac{1}{16}\left(55a^3\right)\int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}$$

$$= -\frac{287a^3\sqrt[4]{1-ax}}{24\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{1+ax}} + \frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{1+ax}} - \frac{61a^2\sqrt[4]{1-ax}}{24x\sqrt[4]{1+ax}} - \frac{1}{4}\left(55a^3\right)\mathrm{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x\right.$$

$$= -\frac{287a^3\sqrt[4]{1-ax}}{24\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{1+ax}} + \frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{1+ax}} - \frac{61a^2\sqrt[4]{1-ax}}{24x\sqrt[4]{1+ax}} + \frac{1}{8}\left(55a^3\right)\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x,\right.$$

$$= -\frac{287a^3\sqrt[4]{1-ax}}{24\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{1+ax}} + \frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{1+ax}} - \frac{61a^2\sqrt[4]{1-ax}}{24x\sqrt[4]{1+ax}} - \frac{55}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{55}{8}a^3\,\mathrm{ta}$$

Mathematica [C] time = 0.0231751, size = 78, normalized size = 0.47

$$\frac{\sqrt[4]{1-ax}\left(330a^3x^3\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1-ax}{ax+1}\right)-287a^3x^3-61a^2x^2+26ax-8\right)}{24x^3\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcTanh[a*x])/2)*x^4),x]
```

```
[Out] ((1 - a*x)^(1/4)*(-8 + 26*a*x - 61*a^2*x^2 - 287*a^3*x^3 + 330*a^3*x^3*Hype
rgeometric2F1[1/4, 1, 5/4, (1 - a*x)/(1 + a*x)]))/(24*x^3*(1 + a*x)^(1/4))
```

Maple [F] time = 0.127, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

Fricas [A] time = 1.79842, size = 448, normalized size = 2.72

$$-\frac{2\left(287\,a^3x^3+61\,a^2x^2-26\,ax+8\right)\sqrt{-a^2x^2+1}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+330\left(a^4x^4+a^3x^3\right)\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-165\left(a^4x^4+\right.}{48\left(ax^4+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(2*(287*a^3*x^3 + 61*a^2*x^2 - 26*a*x + 8)*sqrt(-a^2*x^2 + 1)*sqrt(-s
qrt(-a^2*x^2 + 1)/(a*x - 1)) + 330*(a^4*x^4 + a^3*x^3)*arctan(sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1))) - 165*(a^4*x^4 + a^3*x^3)*log(sqrt(-sqrt(-a^2*x^2 +
 1)/(a*x - 1)) + 1) + 165*(a^4*x^4 + a^3*x^3)*log(sqrt(-sqrt(-a^2*x^2 + 1)/
(a*x - 1)) - 1))/(a*x^4 + x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x^4*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

3.119 $\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=194

$$-\frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{ax+1}}+\frac{2467a^4\sqrt[4]{1-ax}}{192\sqrt[4]{ax+1}}+\frac{521a^3\sqrt[4]{1-ax}}{192x\sqrt[4]{ax+1}}+\frac{475}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{475}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{17a\sqrt[4]{1-a}}{24x^3\sqrt[4]{ax+1}}$$

```
[Out] (2467*a^4*(1 - a*x)^(1/4))/(192*(1 + a*x)^(1/4)) - (1 - a*x)^(1/4)/(4*x^4*(
1 + a*x)^(1/4)) + (17*a*(1 - a*x)^(1/4))/(24*x^3*(1 + a*x)^(1/4)) - (113*a^
2*(1 - a*x)^(1/4))/(96*x^2*(1 + a*x)^(1/4)) + (521*a^3*(1 - a*x)^(1/4))/(19
2*x*(1 + a*x)^(1/4)) + (475*a^4*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/64
 - (475*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/64
```

Rubi [A] time = 0.0983551, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6126, 98, 151, 155, 12, 93, 298, 203, 206}

$$-\frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{ax+1}}+\frac{2467a^4\sqrt[4]{1-ax}}{192\sqrt[4]{ax+1}}+\frac{521a^3\sqrt[4]{1-ax}}{192x\sqrt[4]{ax+1}}+\frac{475}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{475}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{17a\sqrt[4]{1-a}}{24x^3\sqrt[4]{ax+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcTanh[a*x])/2)*x^5),x]
```

```
[Out] (2467*a^4*(1 - a*x)^(1/4))/(192*(1 + a*x)^(1/4)) - (1 - a*x)^(1/4)/(4*x^4*(
1 + a*x)^(1/4)) + (17*a*(1 - a*x)^(1/4))/(24*x^3*(1 + a*x)^(1/4)) - (113*a^
2*(1 - a*x)^(1/4))/(96*x^2*(1 + a*x)^(1/4)) + (521*a^3*(1 - a*x)^(1/4))/(19
2*x*(1 + a*x)^(1/4)) + (475*a^4*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/64
 - (475*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/64
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x^5}dx = \int \frac{(1-ax)^{5/4}}{x^5(1+ax)^{5/4}}dx$$
$$= -\frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} - \frac{1}{4}\int \frac{\frac{17a}{2}-8a^2x}{x^4(1-ax)^{3/4}(1+ax)^{5/4}}dx$$
$$= -\frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} + \frac{17a\sqrt[4]{1-ax}}{24x^3\sqrt[4]{1+ax}} + \frac{1}{12}\int \frac{\frac{113a^2}{4}-\frac{51a^3x}{2}}{x^3(1-ax)^{3/4}(1+ax)^{5/4}}dx$$
$$= -\frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} + \frac{17a\sqrt[4]{1-ax}}{24x^3\sqrt[4]{1+ax}} - \frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{1+ax}} - \frac{1}{24}\int \frac{\frac{521a^3}{8}-\frac{113a^4x}{2}}{x^2(1-ax)^{3/4}(1+ax)^{5/4}}dx$$
$$= -\frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} + \frac{17a\sqrt[4]{1-ax}}{24x^3\sqrt[4]{1+ax}} - \frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{1+ax}} + \frac{521a^3\sqrt[4]{1-ax}}{192x\sqrt[4]{1+ax}} + \frac{1}{24}\int \frac{\frac{1425a^4}{16}-\frac{521a^5x}{8}}{x(1-ax)^{3/4}(1+ax)^{5/4}}dx$$
$$= \frac{2467a^4\sqrt[4]{1-ax}}{192\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} + \frac{17a\sqrt[4]{1-ax}}{24x^3\sqrt[4]{1+ax}} - \frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{1+ax}} + \frac{521a^3\sqrt[4]{1-ax}}{192x\sqrt[4]{1+ax}} + \frac{\int \frac{1425a^5}{32x(1-ax)^{3/4}\sqrt[4]{1}}}{12a}$$
$$= \frac{2467a^4\sqrt[4]{1-ax}}{192\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} + \frac{17a\sqrt[4]{1-ax}}{24x^3\sqrt[4]{1+ax}} - \frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{1+ax}} + \frac{521a^3\sqrt[4]{1-ax}}{192x\sqrt[4]{1+ax}} + \frac{1}{128}\left(475a^4\right)\int$$
$$= \frac{2467a^4\sqrt[4]{1-ax}}{192\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} + \frac{17a\sqrt[4]{1-ax}}{24x^3\sqrt[4]{1+ax}} - \frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{1+ax}} + \frac{521a^3\sqrt[4]{1-ax}}{192x\sqrt[4]{1+ax}} + \frac{1}{32}\left(475a^4\right)\text{Su}$$
$$= \frac{2467a^4\sqrt[4]{1-ax}}{192\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} + \frac{17a\sqrt[4]{1-ax}}{24x^3\sqrt[4]{1+ax}} - \frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{1+ax}} + \frac{521a^3\sqrt[4]{1-ax}}{192x\sqrt[4]{1+ax}} - \frac{1}{64}\left(475a^4\right)\text{Su}$$
$$= \frac{2467a^4\sqrt[4]{1-ax}}{192\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} + \frac{17a\sqrt[4]{1-ax}}{24x^3\sqrt[4]{1+ax}} - \frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{1+ax}} + \frac{521a^3\sqrt[4]{1-ax}}{192x\sqrt[4]{1+ax}} + \frac{475}{64}a^4\tan^{-1}\Bigg($$

Mathematica [C] time = 0.0319339, size = 86, normalized size = 0.44

$$\frac{\sqrt[4]{1-ax}\left(-2850a^4x^4\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1-ax}{ax+1}\right)+2467a^4x^4+521a^3x^3-226a^2x^2+136ax-48\right)}{192x^4\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcTanh[a*x])/2)*x^5),x]
```

```
[Out] ((1 - a*x)^(1/4)*(-48 + 136*a*x - 226*a^2*x^2 + 521*a^3*x^3 + 2467*a^4*x^4
- 2850*a^4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (1 - a*x)/(1 + a*x)]))/(192*x
^4*(1 + a*x)^(1/4))
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^5*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

Fricas [A] time = 1.79816, size = 477, normalized size = 2.46

$$\frac{2\left(2467\,a^4x^4+521\,a^3x^3-226\,a^2x^2+136\,ax-48\right)\sqrt{-a^2x^2+1}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+2850\left(a^5x^5+a^4x^4\right)\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right.}{384\left(ax^5+x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/384*(2*(2467*a^4*x^4 + 521*a^3*x^3 - 226*a^2*x^2 + 136*a*x - 48)*sqrt(-a^
2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 2850*(a^5*x^5 + a^4*x^4)*a
rctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 1425*(a^5*x^5 + a^4*x^4)*log(s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 1425*(a^5*x^5 + a^4*x^4)*log(sqrt
(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1))/(a*x^5 + x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(1/(x^5*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

3.120 $\int e^{\frac{1}{3}\tanh^{-1}(x)}x^m\,dx$

Optimal. Leaf size=28

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{6},-\frac{1}{6};m+2;x,-x\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/6, -1/6, 2 + m, x, -x])/(1 + m)
```

Rubi [A] time = 0.0238578, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{6},-\frac{1}{6};m+2;x,-x\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[x]/3)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/6, -1/6, 2 + m, x, -x])/(1 + m)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{3}\tanh^{-1}(x)}x^m\,dx &= \int \frac{x^m\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{1}{6},-\frac{1}{6};2+m;x,-x\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.231273, size = 0, normalized size = 0.

$$\int e^{\frac{1}{3}\tanh^{-1}(x)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(ArcTanh[x]/3)*x^m,x]
```

```
[Out] Integrate[E^(ArcTanh[x]/3)*x^m, x]
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \sqrt[3]{(1+x)\frac{1}{\sqrt{-x^2+1}}}\,x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^m,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*((x + 1)/sqrt(-x^2 + 1))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^m \left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*(-sqrt(-x^2 + 1)/(x - 1))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(1/3)*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*((x + 1)/sqrt(-x^2 + 1))^(1/3), x)
```

3.121 $\int e^{\frac{1}{3}\tanh^{-1}(x)}x^2\,dx$

Optimal. Leaf size=245

$$-\frac{1}{3}(1-x)^{5/6}x(x+1)^{7/6}-\frac{1}{18}(1-x)^{5/6}(x+1)^{7/6}-\frac{19}{54}(1-x)^{5/6}\sqrt[6]{x+1}-\frac{19\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{108\sqrt{3}}+\frac{19\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+\right.}{108\sqrt{}}$$

```
[Out] (-19*(1 - x)^(5/6)*(1 + x)^(1/6))/54 - ((1 - x)^(5/6)*(1 + x)^(7/6))/18 - (
(1 - x)^(5/6)*x*(1 + x)^(7/6))/3 - (19*ArcTan[(1 - x)^(1/6)/(1 + x)^(1/6)])
/81 + (19*ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))/(1 + x)^(1/6)])/162 - (19*ArcT
an[Sqrt[3] + (2*(1 - x)^(1/6))/(1 + x)^(1/6)])/162 - (19*Log[1 + (1 - x)^(1
/3)/(1 + x)^(1/3) - (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)])/(108*Sqrt[3]) +
 (19*Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) + (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^
(1/6)])/(108*Sqrt[3])
```

Rubi [A] time = 0.385404, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6126, 90, 80, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$-\frac{1}{3}(1-x)^{5/6}x(x+1)^{7/6}-\frac{1}{18}(1-x)^{5/6}(x+1)^{7/6}-\frac{19}{54}(1-x)^{5/6}\sqrt[6]{x+1}-\frac{19\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{108\sqrt{3}}+\frac{19\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+\right.}{108\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[x]/3)*x^2,x]
```


```
[Out] (-19*(1 - x)^(5/6)*(1 + x)^(1/6))/54 - ((1 - x)^(5/6)*(1 + x)^(7/6))/18 - (
(1 - x)^(5/6)*x*(1 + x)^(7/6))/3 - (19*ArcTan[(1 - x)^(1/6)/(1 + x)^(1/6)])
/81 + (19*ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))/(1 + x)^(1/6)])/162 - (19*ArcT
an[Sqrt[3] + (2*(1 - x)^(1/6))/(1 + x)^(1/6)])/162 - (19*Log[1 + (1 - x)^(1
/3)/(1 + x)^(1/3) - (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)])/(108*Sqrt[3]) +
 (19*Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) + (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^
(1/6)])/(108*Sqrt[3])
```


Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{3}\tanh^{-1}(x)}x^2\,dx = \int \frac{x^2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx$$

$$= -\frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6} - \frac{1}{3}\int \frac{\left(-1-\frac{x}{3}\right)\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx$$

$$= -\frac{1}{18}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6} + \frac{19}{54}\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx$$

$$= -\frac{19}{54}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{18}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6} + \frac{19}{162}\int \frac{1}{\sqrt[6]{1-x}(1+x)^{5/6}}$$

$$= -\frac{19}{54}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{18}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6} - \frac{19}{27}\text{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\right.$$

$$= -\frac{19}{54}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{18}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6} - \frac{19}{27}\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx,\right.$$

$$= -\frac{19}{54}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{18}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6} - \frac{19}{81}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,\right.$$

$$= -\frac{19}{54}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{18}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6} - \frac{19}{81}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - \frac{19}{32}$$

$$= -\frac{19}{54}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{18}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6} - \frac{19}{81}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - \frac{19}{}$$

$$= -\frac{19}{54}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{18}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6} - \frac{19}{81}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) + \frac{19}{16}$$

Mathematica [C] time = 0.0272581, size = 59, normalized size = 0.24

$$-\frac{1}{90}(1-x)^{5/6}\left(38\sqrt[6]{2}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{6},\frac{11}{6},\frac{1-x}{2}\right)+5\sqrt[6]{x+1}\left(6x^2+7x+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[x]/3)*x^2,x]
```

```
[Out] -((1 - x)^(5/6)*(5*(1 + x)^(1/6)*(1 + 7*x + 6*x^2) + 38*2^(1/6)*Hypergeomet
ric2F1[-1/6, 5/6, 11/6, (1 - x)/2]))/90
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \sqrt[3]{(1+x)\frac{1}{\sqrt{-x^2+1}}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^2,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((x + 1)/sqrt(-x^2 + 1))^(1/3), x)
```

Fricas [A] time = 1.93252, size = 882, normalized size = 3.6

$$\frac{19}{324}\sqrt{3}\log\left(1444\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+1444\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+1444\right)-\frac{19}{324}\sqrt{3}\log\left(-1444\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+1444\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="fricas")
```

```
[Out] 19/324*sqrt(3)*log(1444*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1444*(-sq
rt(-x^2 + 1)/(x - 1))^(2/3) + 1444) - 19/324*sqrt(3)*log(-1444*sqrt(3)*(-sq
rt(-x^2 + 1)/(x - 1))^(1/3) + 1444*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1444)
+ 1/54*(18*x^3 + 3*x^2 + x - 22)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) - 19/81*ar
ctan(sqrt(3) + 1/19*sqrt(-1444*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 14
44*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1444) - 2*(-sqrt(-x^2 + 1)/(x - 1))^(1
/3)) - 19/81*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/
3) + (-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1) - 2*(-sqrt(-x^2 + 1)/(x - 1))^(1/
3)) + 19/81*arctan((-sqrt(-x^2 + 1)/(x - 1))^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(1/3)*x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((x + 1)/sqrt(-x^2 + 1))^(1/3), x)
```

3.122 $\int e^{\frac{1}{3}\tanh^{-1}(x)} x\,dx$

Optimal. Leaf size=224

$$-\frac{1}{2}(1-x)^{5/6}(x+1)^{7/6}-\frac{1}{6}(1-x)^{5/6}\sqrt[6]{x+1}-\frac{\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{12\sqrt{3}}+\frac{\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{12\sqrt{3}}-\frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{x+1}}\right)$$

```
[Out] -((1 - x)^(5/6)*(1 + x)^(1/6))/6 - ((1 - x)^(5/6)*(1 + x)^(7/6))/2 - ArcTan
[(1 - x)^(1/6)/(1 + x)^(1/6)]/9 + ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))/(1 + x
)^(1/6)]/18 - ArcTan[Sqrt[3] + (2*(1 - x)^(1/6))/(1 + x)^(1/6)]/18 - Log[1
+ (1 - x)^(1/3)/(1 + x)^(1/3) - (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(12*
Sqrt[3]) + Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) + (Sqrt[3]*(1 - x)^(1/6))/(1
 + x)^(1/6)]/(12*Sqrt[3])
```

Rubi [A] time = 0.34011, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.1, Rules used = {6126, 80, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$-\frac{1}{2}(1-x)^{5/6}(x+1)^{7/6}-\frac{1}{6}(1-x)^{5/6}\sqrt[6]{x+1}-\frac{\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{12\sqrt{3}}+\frac{\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{12\sqrt{3}}-\frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[x]/3)*x,x]
```

```
[Out] -((1 - x)^(5/6)*(1 + x)^(1/6))/6 - ((1 - x)^(5/6)*(1 + x)^(7/6))/2 - ArcTan
[(1 - x)^(1/6)/(1 + x)^(1/6)]/9 + ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))/(1 + x
)^(1/6)]/18 - ArcTan[Sqrt[3] + (2*(1 - x)^(1/6))/(1 + x)^(1/6)]/18 - Log[1
+ (1 - x)^(1/3)/(1 + x)^(1/3) - (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(12*
Sqrt[3]) + Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) + (Sqrt[3]*(1 - x)^(1/6))/(1
 + x)^(1/6)]/(12*Sqrt[3])
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{3}\tanh^{-1}(x)}x\,dx = \int \frac{x\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx$$
$$= -\frac{1}{2}(1-x)^{5/6}(1+x)^{7/6} + \frac{1}{6}\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx$$
$$= -\frac{1}{6}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{2}(1-x)^{5/6}(1+x)^{7/6} + \frac{1}{18}\int \frac{1}{\sqrt[6]{1-x}(1+x)^{5/6}}\,dx$$
$$= -\frac{1}{6}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{2}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{3}\,\text{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\,dx, x, \sqrt[6]{1-x}\right)$$
$$= -\frac{1}{6}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{2}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{3}\,\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right)$$
$$= -\frac{1}{6}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{2}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{9}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - \frac{1}{9}\,\text{Subst}\left(\int \frac{-\frac{1}{2}}{1-\sqrt{}}\right.$$
$$= -\frac{1}{6}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{2}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - \frac{1}{36}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \right.$$
$$= -\frac{1}{6}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{2}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - \frac{\log\left(1+\frac{\sqrt[3]{1-x}}{\sqrt[3]{1+x}} - \frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right)}{12\sqrt{3}} + \frac{\log\left(1\right.}{}$$
$$= -\frac{1}{6}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{2}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) + \frac{1}{18}\tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - \frac{1}{18}\,\text{ta}$$

Mathematica [C] time = 0.0132472, size = 49, normalized size = 0.22

$$-\frac{1}{10}(1-x)^{5/6}\left(2\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{6},\frac{11}{6},\frac{1-x}{2}\right) + 5(x+1)^{7/6}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[x]/3)*x,x]
```

```
[Out] -((1 - x)^(5/6)*(5*(1 + x)^(7/6) + 2*2^(1/6)*Hypergeometric2F1[-1/6, 5/6, 1
1/6, (1 - x)/2]))/10
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \sqrt[3]{(1+x)\frac{1}{\sqrt{-x^2+1}}}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(1/3)*x,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(1/3)*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((x + 1)/sqrt(-x^2 + 1))^(1/3), x)
```

Fricas [A] time = 1.73803, size = 810, normalized size = 3.62

$$\frac{1}{36}\sqrt{3}\log\left(4\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+4\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+4\right)-\frac{1}{36}\sqrt{3}\log\left(-4\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+4\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x,x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(3)*log(4*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 4*(-sqrt(-x^2
+ 1)/(x - 1))^(2/3) + 4) - 1/36*sqrt(3)*log(-4*sqrt(3)*(-sqrt(-x^2 + 1)/(x
- 1))^(1/3) + 4*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 4) + 1/6*(3*x^2 + x - 4)*
(-sqrt(-x^2 + 1)/(x - 1))^(1/3) - 1/9*arctan(sqrt(3) + sqrt(-4*sqrt(3)*(-sq
rt(-x^2 + 1)/(x - 1))^(1/3) + 4*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 4) - 2*(-
sqrt(-x^2 + 1)/(x - 1))^(1/3)) - 1/9*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*(-sqr
t(-x^2 + 1)/(x - 1))^(1/3) + (-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1) - 2*(-sqr
t(-x^2 + 1)/(x - 1))^(1/3)) + 1/9*arctan((-sqrt(-x^2 + 1)/(x - 1))^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(1/3)*x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x,x, algorithm="giac")
```

```
[Out] integrate(x*((x + 1)/sqrt(-x^2 + 1))^(1/3), x)
```

3.123 $\int e^{\frac{1}{3}\tanh^{-1}(x)}\,dx$

Optimal. Leaf size=202

$$-(1-x)^{5/6}\sqrt[6]{x+1}-\frac{\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{2\sqrt{3}}+\frac{\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{2\sqrt{3}}-\frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{x+1}}\right)+\frac{1}{3}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{1-x}}{\sqrt[6]{x+1}}\right)$$

```
[Out] -((1 - x)^(5/6)*(1 + x)^(1/6)) - (2*ArcTan[(1 - x)^(1/6)/(1 + x)^(1/6)])/3
+ ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))/(1 + x)^(1/6)]/3 - ArcTan[Sqrt[3] + (2
*(1 - x)^(1/6))/(1 + x)^(1/6)]/3 - Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) - (S
qrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(2*Sqrt[3]) + Log[1 + (1 - x)^(1/3)/(1
 + x)^(1/3) + (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(2*Sqrt[3])
```

Rubi [A] time = 0.32911, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.25, Rules used = {6125, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$-(1-x)^{5/6}\sqrt[6]{x+1}-\frac{\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{2\sqrt{3}}+\frac{\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{2\sqrt{3}}-\frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{x+1}}\right)+\frac{1}{3}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{1-x}}{\sqrt[6]{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[x]/3),x]
```

```
[Out] -((1 - x)^(5/6)*(1 + x)^(1/6)) - (2*ArcTan[(1 - x)^(1/6)/(1 + x)^(1/6)])/3
+ ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))/(1 + x)^(1/6)]/3 - ArcTan[Sqrt[3] + (2
*(1 - x)^(1/6))/(1 + x)^(1/6)]/3 - Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) - (S
qrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(2*Sqrt[3]) + Log[1 + (1 - x)^(1/3)/(1
 + x)^(1/3) + (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(2*Sqrt[3])
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{\frac{1}{3}\tanh^{-1}(x)}\,dx &= \int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx \\
&= -(1-x)^{5/6}\sqrt[6]{1+x}+\frac{1}{3}\int \frac{1}{\sqrt[6]{1-x}(1+x)^{5/6}}\,dx \\
&= -(1-x)^{5/6}\sqrt[6]{1+x}-2\,\text{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\,dx,x,\sqrt[6]{1-x}\right) \\
&= -(1-x)^{5/6}\sqrt[6]{1+x}-2\,\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx,x,\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) \\
&= -(1-x)^{5/6}\sqrt[6]{1+x}-\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right)-\frac{2}{3}\,\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx,x,\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right)- \\
&= -(1-x)^{5/6}\sqrt[6]{1+x}-\frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right)-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx,x,\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right)-\frac{1}{6}\,\text{Subst}\left(\int \frac{}{1+}\right. \\
&= -(1-x)^{5/6}\sqrt[6]{1+x}-\frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right)-\frac{\log\left(1+\frac{\sqrt[3]{1-x}}{\sqrt[3]{1+x}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right)}{2\sqrt{3}}+\frac{\log\left(1+\frac{\sqrt[3]{1-x}}{\sqrt[3]{1+x}}+\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right)}{2\sqrt{3}}+\frac{1}{3}\,\text{S} \\
&= -(1-x)^{5/6}\sqrt[6]{1+x}-\frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right)+\frac{1}{3}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right)-\frac{1}{3}\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right)-\frac{\log(}{}
\end{aligned}
$$

Mathematica [C] time = 0.0433318, size = 39, normalized size = 0.19

$$
2e^{\frac{1}{3}\tanh^{-1}(x)}\left(\text{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},-e^{2\tanh^{-1}(x)}\right)-\frac{1}{e^{2\tanh^{-1}(x)}+1}\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[x]/3),x]
```

```
[Out] 2*E^(ArcTanh[x]/3)*(-(1 + E^(2*ArcTanh[x]))^(-1) + Hypergeometric2F1[1/6, 1
, 7/6, -E^(2*ArcTanh[x])])
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$
\int \sqrt[3]{(1+x)\frac{1}{\sqrt{-x^2+1}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(1/3),x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(1/3), x)
```

Fricas [A] time = 1.79535, size = 791, normalized size = 3.92

$$\frac{1}{6}\sqrt{3}\log\left(4\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+4\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+4\right)-\frac{1}{6}\sqrt{3}\log\left(-4\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+4\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+4\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(4*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 4*(-sqrt(-x^2 +
 1)/(x - 1))^(2/3) + 4) - 1/6*sqrt(3)*log(-4*sqrt(3)*(-sqrt(-x^2 + 1)/(x -
1))^(1/3) + 4*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 4) + (x - 1)*(-sqrt(-x^2 +
1)/(x - 1))^(1/3) - 2/3*arctan(sqrt(3) + sqrt(-4*sqrt(3)*(-sqrt(-x^2 + 1)/(
x - 1))^(1/3) + 4*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 4) - 2*(-sqrt(-x^2 + 1)
/(x - 1))^(1/3)) - 2/3*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*(-sqrt(-x^2 + 1)/(x
 - 1))^(1/3) + (-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1) - 2*(-sqrt(-x^2 + 1)/(x
 - 1))^(1/3)) + 2/3*arctan((-sqrt(-x^2 + 1)/(x - 1))^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{\frac{x+1}{\sqrt{1-x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(1/3),x)
```

```
[Out] Integral(((x + 1)/sqrt(1 - x**2))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(1/3), x)
```

3.124 $\int \frac{e^{\frac{1}{3}\tanh^{-1}(x)}}{x}\,dx$

Optimal. Leaf size=346

$$-\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)+\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}-\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)-\frac{1}{2}\log\left(\ldots\right.$$

```
[Out] -2*ArcTan[(1 - x)^(1/6)/(1 + x)^(1/6)] + ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))
/(1 + x)^(1/6)] - ArcTan[Sqrt[3] + (2*(1 - x)^(1/6))/(1 + x)^(1/6)] + Sqrt[
3]*ArcTan[(1 - (2*(1 + x)^(1/6))/(1 - x)^(1/6))/Sqrt[3]] - Sqrt[3]*ArcTan[(
1 + (2*(1 + x)^(1/6))/(1 - x)^(1/6))/Sqrt[3]] - 2*ArcTanh[(1 + x)^(1/6)/(1
- x)^(1/6)] - (Sqrt[3]*Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) - (Sqrt[3]*(1 -
x)^(1/6))/(1 + x)^(1/6)])/2 + (Sqrt[3]*Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3)
+ (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)])/2 + Log[1 - (1 + x)^(1/6)/(1 - x)
^(1/6) + (1 + x)^(1/3)/(1 - x)^(1/3)]/2 - Log[1 + (1 + x)^(1/6)/(1 - x)^(1/
6) + (1 + x)^(1/3)/(1 - x)^(1/3)]/2
```

Rubi [A] time = 0.454681, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 13, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {6126, 105, 63, 331, 295, 634, 618, 204, 628, 203, 93, 210, 206}

$$-\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)+\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}-\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)-\frac{1}{2}\log\left(\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[x]/3)/x,x]
```


```
[Out] -2*ArcTan[(1 - x)^(1/6)/(1 + x)^(1/6)] + ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))
/(1 + x)^(1/6)] - ArcTan[Sqrt[3] + (2*(1 - x)^(1/6))/(1 + x)^(1/6)] + Sqrt[
3]*ArcTan[(1 - (2*(1 + x)^(1/6))/(1 - x)^(1/6))/Sqrt[3]] - Sqrt[3]*ArcTan[(
1 + (2*(1 + x)^(1/6))/(1 - x)^(1/6))/Sqrt[3]] - 2*ArcTanh[(1 + x)^(1/6)/(1
- x)^(1/6)] - (Sqrt[3]*Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) - (Sqrt[3]*(1 -
x)^(1/6))/(1 + x)^(1/6)])/2 + (Sqrt[3]*Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3)
+ (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)])/2 + Log[1 - (1 + x)^(1/6)/(1 - x)
^(1/6) + (1 + x)^(1/3)/(1 - x)^(1/3)]/2 - Log[1 + (1 + x)^(1/6)/(1 - x)^(1/
6) + (1 + x)^(1/3)/(1 - x)^(1/3)]/2
```


Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\tanh^{-1}(x)}}{x}\,dx = \int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}\,x}\,dx$$

$$= \int \frac{1}{\sqrt[6]{1-x}(1+x)^{5/6}}\,dx + \int \frac{1}{\sqrt[6]{1-x}\,x(1+x)^{5/6}}\,dx$$

$$= -\left(6\,\text{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\,dx, x, \sqrt[6]{1-x}\right)\right) + 6\,\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right)$$

$$= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right)\right) - 2\,\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) - 2\,\text{Subst}\left(\int \frac{1+\frac{x}{2}}{1+x+\cdots}\right.$$

$$= -2\tanh^{-1}\left(\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sqrt[6]{1+\cdots}}{\sqrt[6]{1-\cdots}}\right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) + \frac{1}{2}\log\left(1-\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}+\frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\right) - \frac{1}{2}\log\left(1+\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}+\frac{\sqrt[3]{1\cdots}}{\sqrt[3]{1\cdots}}\right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) + \sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right) - \sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) - \frac{1}{2}\sqrt{3}\log\cdots$$

$$= -2\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) + \tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - \tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) + \sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right) - \sqrt{3}\cdots$$

Mathematica [C] time = 0.0255055, size = 74, normalized size = 0.21

$$-\frac{3(1-x)^{5/6}\left(\sqrt[6]{2}(x+1)^{5/6}\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{6},\frac{11}{6},\frac{1-x}{2}\right)+2\text{Hypergeometric2F1}\left(\frac{5}{6},1,\frac{11}{6},\frac{1-x}{x+1}\right)\right)}{5(x+1)^{5/6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[x]/3)/x,x]
```

```
[Out] (-3*(1 - x)^(5/6)*(2^(1/6)*(1 + x)^(5/6)*Hypergeometric2F1[5/6, 5/6, 11/6,
(1 - x)/2] + 2*Hypergeometric2F1[5/6, 1, 11/6, (1 - x)/(1 + x)]))/(5*(1 + x
)^(5/6))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt[3]{(1+x)\frac{1}{\sqrt{-x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(1/3)/x,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(1/3)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(1/3)/x, x)
```

Fricas [A] time = 1.9226, size = 1268, normalized size = 3.66

$$-\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+\frac{1}{2}\sqrt{3}\log\left(4\sqrt{3}\left(-\frac{\sqrt{-x^2+}}{x-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1/3*sqrt(3))
- sqrt(3)*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) - 1/3*sqrt(3))
 + 1/2*sqrt(3)*log(4*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 4*(-sqrt(-x^
2 + 1)/(x - 1))^(2/3) + 4) - 1/2*sqrt(3)*log(-4*sqrt(3)*(-sqrt(-x^2 + 1)/(x
 - 1))^(1/3) + 4*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 4) - 2*arctan(sqrt(3) +
sqrt(-4*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 4*(-sqrt(-x^2 + 1)/(x - 1
))^(2/3) + 4) - 2*(-sqrt(-x^2 + 1)/(x - 1))^(1/3)) - 2*arctan(-sqrt(3) + 2*
sqrt(sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + (-sqrt(-x^2 + 1)/(x - 1))^(2
/3) + 1) - 2*(-sqrt(-x^2 + 1)/(x - 1))^(1/3)) + 2*arctan((-sqrt(-x^2 + 1)/(
x - 1))^(1/3)) - 1/2*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) + (-sqrt(-x^2 + 1)
/(x - 1))^(1/3) + 1) + 1/2*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) - (-sqrt(-x^
2 + 1)/(x - 1))^(1/3) + 1) - log((-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1) + log
((-sqrt(-x^2 + 1)/(x - 1))^(1/3) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{\frac{x+1}{\sqrt{1-x^2}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(1/3)/x,x)
```

```
[Out] Integral(((x + 1)/sqrt(1 - x**2))**(1/3)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(1/3)/x, x)
```

3.125 $\int \frac{e^{\frac{1}{3}\tanh^{-1}(x)}}{x^2}\,dx$

Optimal. Leaf size=194

$$-\frac{(1-x)^{5/6}\sqrt[6]{x+1}}{x}+\frac{1}{6}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}-\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)-\frac{1}{6}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}+\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{x+1}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[6]{x+1}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(((1 - x)^(5/6)*(1 + x)^(1/6))/x) + ArcTan[(1 - (2*(1 + x)^(1/6))/(1 - x)^
(1/6))/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + (2*(1 + x)^(1/6))/(1 - x)^(1/6))/Sqrt
[3]]/Sqrt[3] - (2*ArcTanh[(1 + x)^(1/6)/(1 - x)^(1/6)])/3 + Log[1 - (1 + x)
^(1/6)/(1 - x)^(1/6) + (1 + x)^(1/3)/(1 - x)^(1/3)]/6 - Log[1 + (1 + x)^(1/
6)/(1 - x)^(1/6) + (1 + x)^(1/3)/(1 - x)^(1/3)]/6
```

Rubi [A] time = 0.157203, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6126, 94, 93, 210, 634, 618, 204, 628, 206}

$$-\frac{(1-x)^{5/6}\sqrt[6]{x+1}}{x}+\frac{1}{6}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}-\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)-\frac{1}{6}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}+\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{x+1}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[6]{x+1}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[x]/3)/x^2,x]
```

```
[Out] -(((1 - x)^(5/6)*(1 + x)^(1/6))/x) + ArcTan[(1 - (2*(1 + x)^(1/6))/(1 - x)^
(1/6))/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + (2*(1 + x)^(1/6))/(1 - x)^(1/6))/Sqrt
[3]]/Sqrt[3] - (2*ArcTanh[(1 + x)^(1/6)/(1 - x)^(1/6)])/3 + Log[1 - (1 + x)
^(1/6)/(1 - x)^(1/6) + (1 + x)^(1/3)/(1 - x)^(1/3)]/6 - Log[1 + (1 + x)^(1/
6)/(1 - x)^(1/6) + (1 + x)^(1/3)/(1 - x)^(1/3)]/6
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\tanh^{-1}(x)}}{x^2}\,dx = \int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}x^2}\,dx$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{x} + \frac{1}{3}\int \frac{1}{\sqrt[6]{1-x}x(1+x)^{5/6}}\,dx$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{x} + 2\,\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right)$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{x} - \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) - \frac{2}{3}\,\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) -$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{x} - \frac{2}{3}\tanh^{-1}\left(\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) + \frac{1}{6}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) - \frac{1}{6}\,\text{Subst}\left(\int\right.$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{x} - \frac{2}{3}\tanh^{-1}\left(\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) + \frac{1}{6}\log\left(1 - \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}} + \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\right) - \frac{1}{6}\log\left(1 + \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}} + \right.$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{x} - \frac{\tan^{-1}\left(\frac{-1+\frac{2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\tanh^{-1}\left(\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) + \frac{1}{6}\log\left(1 - \frac{\sqrt[6]{1}}{\sqrt[6]{1}}\right.$$

Mathematica [C] time = 0.0112355, size = 50, normalized size = 0.26

$$-\frac{(1-x)^{5/6}\left(2x\text{Hypergeometric2F1}\left(\frac{5}{6},1,\frac{11}{6},\frac{1-x}{x+1}\right)+5x+5\right)}{5x(x+1)^{5/6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[x]/3)/x^2,x]
```

```
[Out] -((1 - x)^(5/6)*(5 + 5*x + 2*x*Hypergeometric2F1[5/6, 1, 11/6, (1 - x)/(1 +
 x)]))/(5*x*(1 + x)^(5/6))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt[3]{(1+x)\frac{1}{\sqrt{-x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^2,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(1/3)/x^2, x)
```

Fricas [A] time = 1.73773, size = 626, normalized size = 3.23

$$-\frac{2\sqrt{3}x\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+2\sqrt{3}x\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\log\left(\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+\left(-\frac{\sqrt{-x^2+1}}{x-1}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1/3*
sqrt(3)) + 2*sqrt(3)*x*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) -
 1/3*sqrt(3)) + x*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) + (-sqrt(-x^2 + 1)/(x
 - 1))^(1/3) + 1) - x*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) - (-sqrt(-x^2 + 1
)/(x - 1))^(1/3) + 1) + 2*x*log((-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1) - 2*x*
log((-sqrt(-x^2 + 1)/(x - 1))^(1/3) - 1) - 6*(x - 1)*(-sqrt(-x^2 + 1)/(x -
1))^(1/3))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(1/3)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(1/3)/x^2, x)
```

3.126 $\int \frac{e^{\frac{1}{3}\tanh^{-1}(x)}}{x^3}\,dx$

Optimal. Leaf size=224

$$-\frac{(1-x)^{5/6}(x+1)^{7/6}}{2x^2}-\frac{(1-x)^{5/6}\sqrt[6]{x+1}}{6x}+\frac{1}{36}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}-\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)-\frac{1}{36}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}+\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)+\frac{\tan^{-1}\left(\right.}{6}$$

```
[Out] -((1 - x)^(5/6)*(1 + x)^(1/6))/(6*x) - ((1 - x)^(5/6)*(1 + x)^(7/6))/(2*x^2
) + ArcTan[(1 - (2*(1 + x)^(1/6))/(1 - x)^(1/6))/Sqrt[3]]/(6*Sqrt[3]) - Arc
Tan[(1 + (2*(1 + x)^(1/6))/(1 - x)^(1/6))/Sqrt[3]]/(6*Sqrt[3]) - ArcTanh[(1
 + x)^(1/6)/(1 - x)^(1/6)]/9 + Log[1 - (1 + x)^(1/6)/(1 - x)^(1/6) + (1 + x
)^(1/3)/(1 - x)^(1/3)]/36 - Log[1 + (1 + x)^(1/6)/(1 - x)^(1/6) + (1 + x)^(
1/3)/(1 - x)^(1/3)]/36
```

Rubi [A] time = 0.17509, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6126, 96, 94, 93, 210, 634, 618, 204, 628, 206}

$$-\frac{(1-x)^{5/6}(x+1)^{7/6}}{2x^2}-\frac{(1-x)^{5/6}\sqrt[6]{x+1}}{6x}+\frac{1}{36}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}-\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)-\frac{1}{36}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}+\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)+\frac{\tan^{-1}\left(\right.}{6}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[x]/3)/x^3,x]
```

```
[Out] -((1 - x)^(5/6)*(1 + x)^(1/6))/(6*x) - ((1 - x)^(5/6)*(1 + x)^(7/6))/(2*x^2
) + ArcTan[(1 - (2*(1 + x)^(1/6))/(1 - x)^(1/6))/Sqrt[3]]/(6*Sqrt[3]) - Arc
Tan[(1 + (2*(1 + x)^(1/6))/(1 - x)^(1/6))/Sqrt[3]]/(6*Sqrt[3]) - ArcTanh[(1
 + x)^(1/6)/(1 - x)^(1/6)]/9 + Log[1 - (1 + x)^(1/6)/(1 - x)^(1/6) + (1 + x
)^(1/3)/(1 - x)^(1/3)]/36 - Log[1 + (1 + x)^(1/6)/(1 - x)^(1/6) + (1 + x)^(
1/3)/(1 - x)^(1/3)]/36
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\tanh^{-1}(x)}}{x^3}\,dx = \int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}x^3}\,dx$$
$$= -\frac{(1-x)^{5/6}(1+x)^{7/6}}{2x^2} + \frac{1}{6}\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}x^2}\,dx$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{6x} - \frac{(1-x)^{5/6}(1+x)^{7/6}}{2x^2} + \frac{1}{18}\int \frac{1}{\sqrt[6]{1-x}x(1+x)^{5/6}}\,dx$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{6x} - \frac{(1-x)^{5/6}(1+x)^{7/6}}{2x^2} + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right)$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{6x} - \frac{(1-x)^{5/6}(1+x)^{7/6}}{2x^2} - \frac{1}{9}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) - \frac{1}{9}\,\text{Subst}\left(\int \frac{1-}{1-x}\right.$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{6x} - \frac{(1-x)^{5/6}(1+x)^{7/6}}{2x^2} - \frac{1}{9}\tanh^{-1}\left(\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) + \frac{1}{36}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \right.$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{6x} - \frac{(1-x)^{5/6}(1+x)^{7/6}}{2x^2} - \frac{1}{9}\tanh^{-1}\left(\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) + \frac{1}{36}\log\left(1 - \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}} + \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\right) -$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{6x} - \frac{(1-x)^{5/6}(1+x)^{7/6}}{2x^2} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{1}{9}\tanh^{-1}\left(\frac{\sqrt[6]{1+}}{\sqrt[6]{1-}}\right.$$

Mathematica [C] time = 0.0145863, size = 60, normalized size = 0.27

$$-\frac{(1-x)^{5/6}\left(2x^2\text{Hypergeometric2F1}\left(\frac{5}{6},1,\frac{11}{6},\frac{1-x}{x+1}\right)+5\left(4x^2+7x+3\right)\right)}{30x^2(x+1)^{5/6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[x]/3)/x^3,x]
```

```
[Out] -((1 - x)^(5/6)*(5*(3 + 7*x + 4*x^2) + 2*x^2*Hypergeometric2F1[5/6, 1, 11/6
, (1 - x)/(1 + x)]))/(30*x^2*(1 + x)^(5/6))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt[3]{(1+x)\frac{1}{\sqrt{-x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^3,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(1/3)/x^3, x)
```

Fricas [A] time = 1.73425, size = 657, normalized size = 2.93

$$-\frac{2\sqrt{3}x^2\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+2\sqrt{3}x^2\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x^2\log\left(\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+\left(-\frac{\sqrt{-x^2\cdots}}{x-\cdots}\right.\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/36*(2*sqrt(3)*x^2*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1
/3*sqrt(3)) + 2*sqrt(3)*x^2*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1
/3) - 1/3*sqrt(3)) + x^2*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) + (-sqrt(-x^2
+ 1)/(x - 1))^(1/3) + 1) - x^2*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) - (-sqrt
(-x^2 + 1)/(x - 1))^(1/3) + 1) + 2*x^2*log((-sqrt(-x^2 + 1)/(x - 1))^(1/3)
+ 1) - 2*x^2*log((-sqrt(-x^2 + 1)/(x - 1))^(1/3) - 1) - 6*(4*x^2 - x - 3)*(
-sqrt(-x^2 + 1)/(x - 1))^(1/3))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(1/3)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(1/3)/x^3, x)
```

3.127 $\int e^{\frac{2}{3}\tanh^{-1}(x)} x^m \, dx$

Optimal. Leaf size=28

$$\frac{x^{m+1} F_1\left(m+1;\frac{1}{3},-\frac{1}{3};m+2;x,-x\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/3, -1/3, 2 + m, x, -x])/(1 + m)
```

Rubi [A] time = 0.0263824, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 133}

$$\frac{x^{m+1} F_1\left(m+1;\frac{1}{3},-\frac{1}{3};m+2;x,-x\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcTanh[x])/3)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/3, -1/3, 2 + m, x, -x])/(1 + m)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{\frac{2}{3}\tanh^{-1}(x)} x^m \, dx &= \int \frac{x^m \sqrt[3]{1+x}}{\sqrt[3]{1-x}} \, dx \\
&= \frac{x^{1+m} F_1\left(1+m;\frac{1}{3},-\frac{1}{3};2+m;x,-x\right)}{1+m}
\end{aligned}$$

Mathematica [F] time = 0.235992, size = 0, normalized size = 0.

$$\int e^{\frac{2}{3}\tanh^{-1}(x)} x^m \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^((2*ArcTanh[x])/3)*x^m,x]
```

```
[Out] Integrate[E^((2*ArcTanh[x])/3)*x^m, x]
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \left((1+x)\frac{1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^m,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*((x + 1)/sqrt(-x^2 + 1))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m \left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*(-sqrt(-x^2 + 1)/(x - 1))^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(2/3)*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*((x + 1)/sqrt(-x^2 + 1))^(2/3), x)
```

3.128 $\int e^{\frac{2}{3}\tanh^{-1}(x)}x^2\,dx$

Optimal. Leaf size=133

$$-\frac{1}{3}(1-x)^{2/3}x(x+1)^{4/3}-\frac{1}{9}(1-x)^{2/3}(x+1)^{4/3}-\frac{11}{27}(1-x)^{2/3}\sqrt[3]{x+1}+\frac{11}{81}\log(x+1)+\frac{11}{27}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right)+\frac{22\tan^{-1}\left(\right.}{2}$$

```
[Out] (-11*(1 - x)^(2/3)*(1 + x)^(1/3))/27 - ((1 - x)^(2/3)*(1 + x)^(4/3))/9 - ((
1 - x)^(2/3)*x*(1 + x)^(4/3))/3 + (22*ArcTan[1/Sqrt[3] - (2*(1 - x)^(1/3))/
(Sqrt[3]*(1 + x)^(1/3))])/(27*Sqrt[3]) + (11*Log[1 + x])/81 + (11*Log[1 + (
1 - x)^(1/3)/(1 + x)^(1/3)])/27
```

Rubi [A] time = 0.049726, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6126, 90, 80, 50, 60}

$$-\frac{1}{3}(1-x)^{2/3}x(x+1)^{4/3}-\frac{1}{9}(1-x)^{2/3}(x+1)^{4/3}-\frac{11}{27}(1-x)^{2/3}\sqrt[3]{x+1}+\frac{11}{81}\log(x+1)+\frac{11}{27}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right)+\frac{22\tan^{-1}\left(\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcTanh[x])/3)*x^2,x]
```

```
[Out] (-11*(1 - x)^(2/3)*(1 + x)^(1/3))/27 - ((1 - x)^(2/3)*(1 + x)^(4/3))/9 - ((
1 - x)^(2/3)*x*(1 + x)^(4/3))/3 + (22*ArcTan[1/Sqrt[3] - (2*(1 - x)^(1/3))/
(Sqrt[3]*(1 + x)^(1/3))])/(27*Sqrt[3]) + (11*Log[1 + x])/81 + (11*Log[1 + (
1 - x)^(1/3)/(1 + x)^(1/3)])/27
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rubi steps

$$\begin{aligned}
\int e^{\frac{2}{3}\tanh^{-1}(x)}x^2\,dx &= \int \frac{x^2\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx\\
&= -\frac{1}{3}(1-x)^{2/3}x(1+x)^{4/3}-\frac{1}{3}\int \frac{\left(-1-\frac{2x}{3}\right)\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx\\
&= -\frac{1}{9}(1-x)^{2/3}(1+x)^{4/3}-\frac{1}{3}(1-x)^{2/3}x(1+x)^{4/3}+\frac{11}{27}\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx\\
&= -\frac{11}{27}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{9}(1-x)^{2/3}(1+x)^{4/3}-\frac{1}{3}(1-x)^{2/3}x(1+x)^{4/3}+\frac{22}{81}\int \frac{1}{\sqrt[3]{1-x}(1+x)^{2/3}}\,dx\\
&= -\frac{11}{27}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{9}(1-x)^{2/3}(1+x)^{4/3}-\frac{1}{3}(1-x)^{2/3}x(1+x)^{4/3}+\frac{22\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{27\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.026558, size = 59, normalized size = 0.44

$$-\frac{1}{18}(1-x)^{2/3}\left(11\sqrt[3]{2}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{1-x}{2}\right)+2\sqrt[3]{x+1}\left(3x^2+4x+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcTanh[x])/3)*x^2,x]
```

```
[Out] -((1 - x)^(2/3)*(2*(1 + x)^(1/3)*(1 + 4*x + 3*x^2) + 11*2^(1/3)*Hypergeomet
ric2F1[-1/3, 2/3, 5/3, (1 - x)/2]))/18
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \left((1+x)\frac{1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^2,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((x + 1)/sqrt(-x^2 + 1))^(2/3), x)
```

Fricas [A] time = 1.68016, size = 428, normalized size = 3.22

$$\frac{1}{27}\left(9x^3+3x^2+2x-14\right)\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+\frac{22}{81}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}-\frac{1}{3}\sqrt{3}\right)+\frac{22}{81}\log\left(\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*x^3 + 3*x^2 + 2*x - 14)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 22/81*sqr
t(3)*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) - 1/3*sqrt(3)) + 22
/81*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1) - 11/81*log(-((x - 1)*(-sqrt(-
x^2 + 1)/(x - 1))^(2/3) - x + sqrt(-x^2 + 1)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3
) + 1)/(x - 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(2/3)*x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((x + 1)/sqrt(-x^2 + 1))^(2/3), x)
```

3.129 $\int e^{\frac{2}{3}\tanh^{-1}(x)}x\,dx$

Optimal. Leaf size=112

$$-\frac{1}{2}(1-x)^{2/3}(x+1)^{4/3}-\frac{1}{3}(1-x)^{2/3}\sqrt[3]{x+1}+\frac{1}{9}\log(x+1)+\frac{1}{3}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right)+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{3\sqrt{3}}$$

```
[Out] -((1 - x)^(2/3)*(1 + x)^(1/3))/3 - ((1 - x)^(2/3)*(1 + x)^(4/3))/2 + (2*Arc
Tan[1/Sqrt[3] - (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))])/(3*Sqrt[3]) + L
og[1 + x]/9 + Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3)]/3
```

Rubi [A] time = 0.0319433, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6126, 80, 50, 60}

$$-\frac{1}{2}(1-x)^{2/3}(x+1)^{4/3}-\frac{1}{3}(1-x)^{2/3}\sqrt[3]{x+1}+\frac{1}{9}\log(x+1)+\frac{1}{3}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right)+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcTanh[x])/3)*x,x]
```

```
[Out] -((1 - x)^(2/3)*(1 + x)^(1/3))/3 - ((1 - x)^(2/3)*(1 + x)^(4/3))/2 + (2*Arc
Tan[1/Sqrt[3] - (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))])/(3*Sqrt[3]) + L
og[1 + x]/9 + Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3)]/3
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
```

```
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{2}{3}\tanh^{-1}(x)}x\,dx &= \int \frac{x\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx\\ &= -\frac{1}{2}(1-x)^{2/3}(1+x)^{4/3}+\frac{1}{3}\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx\\ &= -\frac{1}{3}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{2}(1-x)^{2/3}(1+x)^{4/3}+\frac{2}{9}\int \frac{1}{\sqrt[3]{1-x}(1+x)^{2/3}}\,dx\\ &= -\frac{1}{3}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{2}(1-x)^{2/3}(1+x)^{4/3}+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{3\sqrt{3}}+\frac{1}{9}\log(1+x)+\frac{1}{3}\log\left(1+\frac{\sqrt[3]{1}}{\sqrt[3]{1}}\right.\end{aligned}$$

Mathematica [C] time = 0.0130997, size = 46, normalized size = 0.41

$$-\frac{1}{2}(1-x)^{2/3}\left(\sqrt[3]{2}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{1-x}{2}\right)+(x+1)^{4/3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcTanh[x])/3)*x,x]
```

```
[Out] -((1 - x)^(2/3)*((1 + x)^(4/3) + 2^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/3,
(1 - x)/2]))/2
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \left((1+x)\frac{1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(2/3)*x,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(2/3)*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((x + 1)/sqrt(-x^2 + 1))^(2/3), x)
```

Fricas [A] time = 1.63883, size = 406, normalized size = 3.62

$$\frac{1}{6}\left(3x^2+2x-5\right)\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+\frac{2}{9}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}-\frac{1}{3}\sqrt{3}\right)+\frac{2}{9}\log\left(\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+1\right)-\frac{1}{9}\text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x,x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 + 2*x - 5)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 2/9*sqrt(3)*arctan(
2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) - 1/3*sqrt(3)) + 2/9*log((-sqrt
(-x^2 + 1)/(x - 1))^(2/3) + 1) - 1/9*log(-((x - 1)*(-sqrt(-x^2 + 1)/(x - 1)
)^(2/3) - x + sqrt(-x^2 + 1)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1)/(x - 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(2/3)*x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x,x, algorithm="giac")
```

```
[Out] integrate(x*((x + 1)/sqrt(-x^2 + 1))^(2/3), x)
```

3.130 $\int e^{\frac{2}{3}\tanh^{-1}(x)}\,dx$

Optimal. Leaf size=84

$$-(1-x)^{2/3}\sqrt[3]{x+1}+\frac{1}{3}\log(x+1)+\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right)+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{\sqrt{3}}$$

```
[Out] -((1 - x)^(2/3)*(1 + x)^(1/3)) + (2*ArcTan[1/Sqrt[3] - (2*(1 - x)^(1/3))/(S
qrt[3]*(1 + x)^(1/3))])/Sqrt[3] + Log[1 + x]/3 + Log[1 + (1 - x)^(1/3)/(1 +
 x)^(1/3)]
```

Rubi [A] time = 0.0171154, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6125, 50, 60}

$$-(1-x)^{2/3}\sqrt[3]{x+1}+\frac{1}{3}\log(x+1)+\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right)+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcTanh[x])/3),x]
```

```
[Out] -((1 - x)^(2/3)*(1 + x)^(1/3)) + (2*ArcTan[1/Sqrt[3] - (2*(1 - x)^(1/3))/(S
qrt[3]*(1 + x)^(1/3))])/Sqrt[3] + Log[1 + x]/3 + Log[1 + (1 - x)^(1/3)/(1 +
 x)^(1/3)]
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rubi steps

$$\int e^{\frac{2}{3}\tanh^{-1}(x)}\,dx = \int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx$$
$$= -(1-x)^{2/3}\sqrt[3]{1+x} + \frac{2}{3}\int \frac{1}{\sqrt[3]{1-x}(1+x)^{2/3}}\,dx$$
$$= -(1-x)^{2/3}\sqrt[3]{1+x} + \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1+x) + \log\left(1 + \frac{\sqrt[3]{1-x}}{\sqrt[3]{1+x}}\right)$$

Mathematica [A] time = 0.135186, size = 87, normalized size = 1.04

$$-\frac{2e^{\frac{2}{3}\tanh^{-1}(x)}}{e^{2\tanh^{-1}(x)}+1} + \frac{2}{3}\log\left(e^{\frac{2}{3}\tanh^{-1}(x)}+1\right) - \frac{1}{3}\log\left(-e^{\frac{2}{3}\tanh^{-1}(x)} + e^{\frac{4}{3}\tanh^{-1}(x)}+1\right) + \frac{2\tan^{-1}\left(\frac{2e^{\frac{2}{3}\tanh^{-1}(x)}-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcTanh[x])/3),x]
```

```
[Out] (-2*E^((2*ArcTanh[x])/3))/(1 + E^(2*ArcTanh[x])) + (2*ArcTan[(-1 + 2*E^((2*
ArcTanh[x])/3))/Sqrt[3]])/Sqrt[3] + (2*Log[1 + E^((2*ArcTanh[x])/3)])/3 - L
og[1 - E^((2*ArcTanh[x])/3) + E^((4*ArcTanh[x])/3)]/3
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \left((1+x)\frac{1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(2/3),x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(2/3), x)
```

Fricas [B] time = 1.67409, size = 387, normalized size = 4.61

$$(x-1)\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+\frac{2}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}-\frac{1}{3}\sqrt{3}\right)+\frac{2}{3}\log\left(\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+1\right)-\frac{1}{3}\log\left(-\frac{(x-1)\left(-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3),x, algorithm="fricas")
```

```
[Out] (x - 1)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 2/3*sqrt(3)*arctan(2/3*sqrt(3)*(-
sqrt(-x^2 + 1)/(x - 1))^(2/3) - 1/3*sqrt(3)) + 2/3*log((-sqrt(-x^2 + 1)/(x
- 1))^(2/3) + 1) - 1/3*log(-((x - 1)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) - x +
sqrt(-x^2 + 1)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1)/(x - 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(2/3), x)
```

3.131 $\int \frac{e^{\frac{2}{3}\tanh^{-1}(x)}}{x} dx$

Optimal. Leaf size=135

$$-\frac{\log(x)}{2} + \frac{1}{2}\log(x+1) + \frac{3}{2}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right) + \frac{3}{2}\log\left(\sqrt[3]{1-x} - \sqrt[3]{x+1}\right) + \sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}\right) + \sqrt{3}\tan^{-1}$$

```
[Out] Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))] + Sqr
t[3]*ArcTan[1/Sqrt[3] + (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))] - Log[x]
/2 + Log[1 + x]/2 + (3*Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3)])/2 + (3*Log[(1
- x)^(1/3) - (1 + x)^(1/3)])/2
```

Rubi [A] time = 0.0317323, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6126, 105, 60, 91}

$$-\frac{\log(x)}{2} + \frac{1}{2}\log(x+1) + \frac{3}{2}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right) + \frac{3}{2}\log\left(\sqrt[3]{1-x} - \sqrt[3]{x+1}\right) + \sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}\right) + \sqrt{3}\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcTanh[x])/3)/x,x]
```

```
[Out] Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))] + Sqr
t[3]*ArcTan[1/Sqrt[3] + (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))] - Log[x]
/2 + Log[1 + x]/2 + (3*Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3)])/2 + (3*Log[(1
- x)^(1/3) - (1 + x)^(1/3)])/2
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
```

```
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\frac{2}{3}\tanh^{-1}(x)}}{x}\,dx &= \int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}x}\,dx\\ &= \int \frac{1}{\sqrt[3]{1-x}(1+x)^{2/3}}\,dx + \int \frac{1}{\sqrt[3]{1-x}x(1+x)^{2/3}}\,dx\\ &= \sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{1+x}}\right) + \sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{1+x}}\right) - \frac{\log(x)}{2} + \frac{1}{2}\log(1+x) + \frac{3}{2}\log\left(1+\right.\end{aligned}$$

Mathematica [C] time = 0.0227641, size = 74, normalized size = 0.55

$$-\frac{3(1-x)^{2/3}\left(\sqrt[3]{2}(x+1)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{1-x}{2}\right)+2\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},\frac{1-x}{x+1}\right)\right)}{4(x+1)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcTanh[x])/3)/x,x]
```

```
[Out] (-3*(1 - x)^(2/3)*(2^(1/3)*(1 + x)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (
1 - x)/2] + 2*Hypergeometric2F1[2/3, 1, 5/3, (1 - x)/(1 + x)]))/(4*(1 + x)^
(2/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left((1+x)\frac{1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(2/3)/x,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(2/3)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x,x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(2/3)/x, x)
```

Fricas [A] time = 1.6711, size = 405, normalized size = 3.

$$-\sqrt{3}\arctan\left(-\frac{\sqrt{3}(x-1)-2\sqrt{3}\sqrt{-x^2+1}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}}{3(x-1)}\right)-\frac{1}{2}\log\left(-\frac{(x+1)\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}-x+\sqrt{-x^2+1}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+}{x-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(-1/3*(sqrt(3)*(x - 1) - 2*sqrt(3)*sqrt(-x^2 + 1)*(-sqrt(-x^
2 + 1)/(x - 1))^(1/3))/(x - 1)) - 1/2*log(-((x + 1)*(-sqrt(-x^2 + 1)/(x - 1
))^(2/3) - x + sqrt(-x^2 + 1)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1)/(x - 1))
 + log(-(x + sqrt(-x^2 + 1)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) - 1)/(x - 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(2/3)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x,x, algorithm="giac")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(2/3)/x, x)
```

3.132 $\int \frac{e^{\frac{2}{3}\tanh^{-1}(x)}}{x^2}\,dx$

Optimal. Leaf size=85

$$-\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{x}-\frac{\log(x)}{3}+\log\left(\sqrt[3]{1-x}-\sqrt[3]{x+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(((1 - x)^(2/3)*(1 + x)^(1/3))/x) + (2*ArcTan[1/Sqrt[3] + (2*(1 - x)^(1/3)
)/(Sqrt[3]*(1 + x)^(1/3))])/Sqrt[3] - Log[x]/3 + Log[(1 - x)^(1/3) - (1 + x
)^(1/3)]
```

Rubi [A] time = 0.0314888, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6126, 94, 91}

$$-\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{x}-\frac{\log(x)}{3}+\log\left(\sqrt[3]{1-x}-\sqrt[3]{x+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcTanh[x])/3)/x^2,x]
```

```
[Out] -(((1 - x)^(2/3)*(1 + x)^(1/3))/x) + (2*ArcTan[1/Sqrt[3] + (2*(1 - x)^(1/3)
)/(Sqrt[3]*(1 + x)^(1/3))])/Sqrt[3] - Log[x]/3 + Log[(1 - x)^(1/3) - (1 + x
)^(1/3)]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rubi steps

$$\int \frac{e^{\frac{2}{3}\tanh^{-1}(x)}}{x^2}\,dx = \int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}x^2}\,dx$$
$$= -\frac{(1-x)^{2/3}\sqrt[3]{1+x}}{x} + \frac{2}{3}\int \frac{1}{\sqrt[3]{1-x}x(1+x)^{2/3}}\,dx$$
$$= -\frac{(1-x)^{2/3}\sqrt[3]{1+x}}{x} + \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{\sqrt{3}} - \frac{\log(x)}{3} + \log\left(\sqrt[3]{1-x}-\sqrt[3]{1+x}\right)$$

Mathematica [C] time = 0.0109406, size = 45, normalized size = 0.53

$$-\frac{(1-x)^{2/3}\left(x\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},\frac{1-x}{x+1}\right)+x+1\right)}{x(x+1)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcTanh[x])/3)/x^2,x]
```

```
[Out] -(((1 - x)^(2/3)*(1 + x + x*Hypergeometric2F1[2/3, 1, 5/3, (1 - x)/(1 + x)]
))/(x*(1 + x)^(2/3)))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left((1+x)\frac{1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^2,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(2/3)/x^2, x)
```

Fricas [B] time = 1.6858, size = 398, normalized size = 4.68

$$-\frac{2\sqrt{3}x\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right) - 2x\log\left(\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}-1\right) + x\log\left(\frac{(x-1)\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+x-\sqrt{-x^2+1}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}-1}{x-1}\right)}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(3)*x*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1/3*
sqrt(3)) - 2*x*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) - 1) + x*log(((x - 1)*(-
sqrt(-x^2 + 1)/(x - 1))^(2/3) + x - sqrt(-x^2 + 1)*(-sqrt(-x^2 + 1)/(x - 1)
)^(1/3) - 1)/(x - 1)) - 3*(x - 1)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(2/3)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(2/3)/x^2, x)
```

3.133 $\int \frac{e^{\frac{2}{3}\tanh^{-1}(x)}}{x^3}\,dx$

Optimal. Leaf size=116

$$-\frac{(1-x)^{2/3}(x+1)^{4/3}}{2x^2}-\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{3x}-\frac{\log(x)}{9}+\frac{1}{3}\log\left(\sqrt[3]{1-x}-\sqrt[3]{x+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -((1 - x)^(2/3)*(1 + x)^(1/3))/(3*x) - ((1 - x)^(2/3)*(1 + x)^(4/3))/(2*x^2
) + (2*ArcTan[1/Sqrt[3] + (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))])/(3*Sq
rt[3]) - Log[x]/9 + Log[(1 - x)^(1/3) - (1 + x)^(1/3)]/3
```

Rubi [A] time = 0.0377013, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6126, 96, 94, 91}

$$-\frac{(1-x)^{2/3}(x+1)^{4/3}}{2x^2}-\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{3x}-\frac{\log(x)}{9}+\frac{1}{3}\log\left(\sqrt[3]{1-x}-\sqrt[3]{x+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcTanh[x])/3)/x^3,x]
```

```
[Out] -((1 - x)^(2/3)*(1 + x)^(1/3))/(3*x) - ((1 - x)^(2/3)*(1 + x)^(4/3))/(2*x^2
) + (2*ArcTan[1/Sqrt[3] + (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))])/(3*Sq
rt[3]) - Log[x]/9 + Log[(1 - x)^(1/3) - (1 + x)^(1/3)]/3
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\frac{2}{3}\tanh^{-1}(x)}}{x^3}\,dx &= \int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}x^3}\,dx\\ &= -\frac{(1-x)^{2/3}(1+x)^{4/3}}{2x^2}+\frac{1}{3}\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}x^2}\,dx\\ &= -\frac{(1-x)^{2/3}\sqrt[3]{1+x}}{3x}-\frac{(1-x)^{2/3}(1+x)^{4/3}}{2x^2}+\frac{2}{9}\int \frac{1}{\sqrt[3]{1-x}x(1+x)^{2/3}}\,dx\\ &= -\frac{(1-x)^{2/3}\sqrt[3]{1+x}}{3x}-\frac{(1-x)^{2/3}(1+x)^{4/3}}{2x^2}+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{3\sqrt{3}}-\frac{\log(x)}{9}+\frac{1}{3}\log\left(\sqrt[3]{1-x}-\sqrt[3]{1+\right.\end{aligned}$$

Mathematica [C] time = 0.0128138, size = 57, normalized size = 0.49

$$-\frac{(1-x)^{2/3}\left(2x^2\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},\frac{1-x}{x+1}\right)+5x^2+8x+3\right)}{6x^2(x+1)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcTanh[x])/3)/x^3,x]
```

```
[Out] -((1 - x)^(2/3)*(3 + 8*x + 5*x^2 + 2*x^2*Hypergeometric2F1[2/3, 1, 5/3, (1
- x)/(1 + x)]))/(6*x^2*(1 + x)^(2/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left((1+x)\frac{1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^3,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(2/3)/x^3, x)
```

Fricas [A] time = 1.72863, size = 427, normalized size = 3.68

$$-\frac{4\sqrt{3}x^2\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)-4x^2\log\left(\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}-1\right)+2x^2\log\left(\frac{(x-1)\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+x-\sqrt{-x^2+1}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)}{x-1}\right.}{18x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/18*(4*sqrt(3)*x^2*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1
/3*sqrt(3)) - 4*x^2*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) - 1) + 2*x^2*log(((
x - 1)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + x - sqrt(-x^2 + 1)*(-sqrt(-x^2 + 1
)/(x - 1))^(1/3) - 1)/(x - 1)) - 3*(5*x^2 - 2*x - 3)*(-sqrt(-x^2 + 1)/(x -
1))^(2/3))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(2/3)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(2/3)/x^3, x)
```

3.134 $\int e^{\frac{1}{4}\tanh^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=31

$$\frac{x^{m+1} F_1\left(m+1; \frac{1}{8}, -\frac{1}{8}; m+2; ax, -ax\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/8, -1/8, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rubi [A] time = 0.0275486, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6126, 133}

$$\frac{x^{m+1} F_1\left(m+1; \frac{1}{8}, -\frac{1}{8}; m+2; ax, -ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/4)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/8, -1/8, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{4}\tanh^{-1}(ax)} x^m \, dx &= \int \frac{x^m \sqrt[8]{1+ax}}{\sqrt[8]{1-ax}} \, dx\\ &= \frac{x^{1+m} F_1\left(1+m; \frac{1}{8}, -\frac{1}{8}; 2+m; ax, -ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.302039, size = 0, normalized size = 0.

$$\int e^{\frac{1}{4}\tanh^{-1}(ax)} x^m \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(ArcTanh[a*x]/4)*x^m,x]
```

```
[Out] Integrate[E^(ArcTanh[a*x]/4)*x^m, x]
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \sqrt[4]{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\, x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^m,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m \left(-\frac{\sqrt{-a^2x^2+1}}{ax-1}\right)^{\frac{1}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/4)*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4), x)
```

3.135 $\int e^{\frac{1}{4}\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=646

$$-\frac{x(1-ax)^{7/8}(ax+1)^{9/8}}{3a^2}-\frac{(1-ax)^{7/8}(ax+1)^{9/8}}{24a^3}-\frac{11(1-ax)^{7/8}\sqrt[8]{ax+1}}{32a^3}-\frac{11\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)}{256a^3}$$

```
[Out] (-11*(1 - a*x)^(7/8)*(1 + a*x)^(1/8))/(32*a^3) - ((1 - a*x)^(7/8)*(1 + a*x)
^(9/8))/(24*a^3) - (x*(1 - a*x)^(7/8)*(1 + a*x)^(9/8))/(3*a^2) + (11*Sqrt[2
 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)
)/Sqrt[2 + Sqrt[2]]])/(128*a^3) + (11*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sq
rt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/(128*a^3)
 - (11*Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1
 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(128*a^3) - (11*Sqrt[2 - Sqrt[2]]*ArcTan
[(Sqrt[2 + Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 - Sqrt[2]
]])/(128*a^3) - (11*Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/
4) - (Sqrt[2 - Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(256*a^3) + (11*
Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) + (Sqrt[2 - Sqrt[
2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(256*a^3) - (11*Sqrt[2 + Sqrt[2]]*Lo
g[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))
/(1 + a*x)^(1/8)])/(256*a^3) + (11*Sqrt[2 + Sqrt[2]]*Log[1 + (1 - a*x)^(1/4
)/(1 + a*x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(
256*a^3)
```

Rubi [A] time = 0.701821, antiderivative size = 646, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {6126, 90, 80, 50, 63, 331, 299, 1122, 1169, 634, 618, 204, 628}

$$-\frac{x(1-ax)^{7/8}(ax+1)^{9/8}}{3a^2}-\frac{(1-ax)^{7/8}(ax+1)^{9/8}}{24a^3}-\frac{11(1-ax)^{7/8}\sqrt[8]{ax+1}}{32a^3}-\frac{11\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)}{256a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/4)*x^2,x]
```


```
[Out] (-11*(1 - a*x)^(7/8)*(1 + a*x)^(1/8))/(32*a^3) - ((1 - a*x)^(7/8)*(1 + a*x)
^(9/8))/(24*a^3) - (x*(1 - a*x)^(7/8)*(1 + a*x)^(9/8))/(3*a^2) + (11*Sqrt[2
 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)
)/Sqrt[2 + Sqrt[2]]])/(128*a^3) + (11*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sq
rt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/(128*a^3)
 - (11*Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1
 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(128*a^3) - (11*Sqrt[2 - Sqrt[2]]*ArcTan
[(Sqrt[2 + Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 - Sqrt[2]
]])/(128*a^3) - (11*Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/
4) - (Sqrt[2 - Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(256*a^3) + (11*
Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) + (Sqrt[2 - Sqrt[
2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(256*a^3) - (11*Sqrt[2 + Sqrt[2]]*Lo
g[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))
/(1 + a*x)^(1/8)])/(256*a^3) + (11*Sqrt[2 + Sqrt[2]]*Log[1 + (1 - a*x)^(1/4
)/(1 + a*x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(
256*a^3)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
```

```
x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{1}{4}\tanh^{-1}(ax)}x^2\,dx = \int \frac{x^2\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\,dx$$

$$= -\frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} - \frac{\int \frac{\left(-1-\frac{ax}{4}\right)\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\,dx}{3a^2}$$

$$= -\frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} + \frac{11\int \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\,dx}{32a^2}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} + \frac{11\int \frac{1}{\sqrt[8]{1-ax}(1+ax)^{7/8}}\,dx}{128a^2}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} - \frac{11\,\text{Subst}\left(\int \frac{x^6}{\left(2-x^8\right)^{7/8}}\,dx,\right.}{16a^3}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} - \frac{11\,\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx,x,\right.}{16a^3}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} - \frac{11\,\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,d\right.}{32\sqrt{2}a^3}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} + \frac{11\,\text{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,d\right.}{32\sqrt{2}a^3}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} - \frac{11\,\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-(1-\sqrt{}}{1-\sqrt{2-\sqrt{2}}x+}\right.}{64\sqrt{2\left(2-\sqrt{}\right.}}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} - \frac{\left(11\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\right)\text{Subs}}{}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} - \frac{11\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-}}{\sqrt[4]{1-}}\right.}{256a^3}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} + \frac{11\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-}}{}\right.}{128a^3}$$

Mathematica [C] time = 0.0353602, size = 70, normalized size = 0.11

$$-\frac{(1-ax)^{7/8}\left(66\sqrt[8]{2}\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{7}{8},\frac{15}{8},\frac{1}{2}(1-ax)\right)+7\sqrt[8]{ax+1}\left(8a^2x^2+9ax+1\right)\right)}{168a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/4)*x^2,x]
```

```
[Out] -((1 - a*x)^(7/8)*(7*(1 + a*x)^(1/8)*(1 + 9*a*x + 8*a^2*x^2) + 66*2^(1/8)*H
ypergeometric2F1[-1/8, 7/8, 15/8, (1 - a*x)/2]))/(168*a^3)
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \sqrt[4]{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^2,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4), x)
```

Fricas [B] time = 2.39818, size = 6882, normalized size = 10.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="fricas")
```

```
[Out] -1/3072*(264*a^3*sqrt(sqrt(2) + 2)*(a^(-24))^(1/8)*arctan((2*sqrt(a^6*(a^(-
24))^(1/4) + a^3*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(
a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^21*(a^(-24))^(7/8)
- 2*a^21*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(7/8) - sqrt(-sqrt
(2) + 2))/sqrt(sqrt(2) + 2)) + 264*a^3*sqrt(sqrt(2) + 2)*(a^(-24))^(1/8)*ar
ctan((2*sqrt(a^6*(a^(-24))^(1/4) - a^3*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 +
 1)/(a*x - 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))
*a^21*(a^(-24))^(7/8) - 2*a^21*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-2
4))^(7/8) + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 264*a^3*sqrt(-sqrt(2)
+ 2)*(a^(-24))^(1/8)*arctan((2*sqrt(a^6*(a^(-24))^(1/4) + a^3*sqrt(sqrt(2)
+ 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^
2*x^2 + 1)/(a*x - 1)))*a^21*(a^(-24))^(7/8) - 2*a^21*(-sqrt(-a^2*x^2 + 1)/(
a*x - 1))^(1/4)*(a^(-24))^(7/8) - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) +
264*a^3*sqrt(-sqrt(2) + 2)*(a^(-24))^(1/8)*arctan((2*sqrt(a^6*(a^(-24))^(1/
4) - a^3*sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^
(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^21*(a^(-24))^(7/8) - 2*a^21*
(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(7/8) + sqrt(sqrt(2) + 2))/
sqrt(-sqrt(2) + 2)) - 66*a^3*sqrt(sqrt(2) + 2)*(a^(-24))^(1/8)*log(a^6*(a^(
-24))^(1/4) + a^3*sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(
a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 66*a^3*sqrt(sqrt(2)
 + 2)*(a^(-24))^(1/8)*log(a^6*(a^(-24))^(1/4) - a^3*sqrt(sqrt(2) + 2)*(-sqr
t(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1))) - 66*a^3*sqrt(-sqrt(2) + 2)*(a^(-24))^(1/8)*log(a^6*(a^(-24))^
```

```
(1/4) + a^3*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-2
4))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 66*a^3*sqrt(-sqrt(2) + 2
)*(a^(-24))^(1/8)*log(a^6*(a^(-24))^(1/4) - a^3*sqrt(-sqrt(2) + 2)*(-sqrt(-
a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a
*x - 1))) + 132*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) + sqrt(2)*a^3*sqrt(-sqrt(2)
+ 2))*(a^(-24))^(1/8)*arctan((2*sqrt(2)*sqrt(a^6*(a^(-24))^(1/4) + 1/2*(sqr
t(2)*a^3*sqrt(sqrt(2) + 2) - sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^
2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1
)))*a^21*(a^(-24))^(7/8) - 2*sqrt(2)*a^21*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(
1/4)*(a^(-24))^(7/8) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2
) + 2) + sqrt(-sqrt(2) + 2))) + 132*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) + sqrt(2
)*a^3*sqrt(-sqrt(2) + 2))*(a^(-24))^(1/8)*arctan((2*sqrt(2)*sqrt(a^6*(a^(-2
4))^(1/4) - 1/2*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) - sqrt(2)*a^3*sqrt(-sqrt(2)
+ 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1)))*a^21*(a^(-24))^(7/8) - 2*sqrt(2)*a^21*(-sqrt(-a^2*x
^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(7/8) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2
) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 132*(sqrt(2)*a^3*sqrt(s
qrt(2) + 2) - sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(a^(-24))^(1/8)*arctan(-(2*sq
rt(2)*sqrt(a^6*(a^(-24))^(1/4) + 1/2*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) + sqrt(
2)*a^3*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^
(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^21*(a^(-24))^(7/8) - 2*sqrt(
2)*a^21*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(7/8) - sqrt(sqrt(2
) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) - 13
2*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) - sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(a^(-24)
)^(1/8)*arctan(-(2*sqrt(2)*sqrt(a^6*(a^(-24))^(1/4) - 1/2*(sqrt(2)*a^3*sqrt
(sqrt(2) + 2) + sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x -
 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^21*(a^(
-24))^(7/8) - 2*sqrt(2)*a^21*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24)
)^(7/8) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt
(-sqrt(2) + 2))) - 33*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) + sqrt(2)*a^3*sqrt(-sq
rt(2) + 2))*(a^(-24))^(1/8)*log(a^6*(a^(-24))^(1/4) + 1/2*(sqrt(2)*a^3*sqrt
(sqrt(2) + 2) + sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x -
 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 33*(sqr
t(2)*a^3*sqrt(sqrt(2) + 2) + sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(a^(-24))^(1/8
)*log(a^6*(a^(-24))^(1/4) - 1/2*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) + sqrt(2)*a^
3*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(1/8)
 + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 33*(sqrt(2)*a^3*sqrt(sqrt(2) + 2)
 - sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(a^(-24))^(1/8)*log(a^6*(a^(-24))^(1/4)
+ 1/2*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) - sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(-sq
rt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1
)/(a*x - 1))) + 33*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) - sqrt(2)*a^3*sqrt(-sqrt(
2) + 2))*(a^(-24))^(1/8)*log(a^6*(a^(-24))^(1/4) - 1/2*(sqrt(2)*a^3*sqrt(sq
rt(2) + 2) - sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1)
)^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 32*(32*a^3
*x^3 + 4*a^2*x^2 + a*x - 37)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4))/a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/4)*x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4), x)
```

3.136 $\int e^{\frac{1}{4}\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=619

$$-\frac{(1-ax)^{7/8}(ax+1)^{9/8}}{2a^2}-\frac{(1-ax)^{7/8}\sqrt[8]{ax+1}}{8a^2}-\frac{\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)}{64a^2}+\frac{\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}\right.}{64a^2}$$

```
[Out] -((1 - a*x)^(7/8)*(1 + a*x)^(1/8))/(8*a^2) - ((1 - a*x)^(7/8)*(1 + a*x)^(9/
8))/(2*a^2) + (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 - a*x)^(
1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(32*a^2) + (Sqrt[2 - Sqrt[2]]*Ar
cTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 - Sqr
t[2]]])/(32*a^2) - (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - a
*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(32*a^2) - (Sqrt[2 - Sqrt[2
]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2
- Sqrt[2]]])/(32*a^2) - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x
)^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(64*a^2) +
(Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) + (Sqrt[2 - Sqrt
[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(64*a^2) - (Sqrt[2 + Sqrt[2]]*Log[1
 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1
 + a*x)^(1/8)])/(64*a^2) + (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 +
a*x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(64*a^2)
```

Rubi [A] time = 0.481502, antiderivative size = 619, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6126, 80, 50, 63, 331, 299, 1122, 1169, 634, 618, 204, 628}

$$-\frac{(1-ax)^{7/8}(ax+1)^{9/8}}{2a^2}-\frac{(1-ax)^{7/8}\sqrt[8]{ax+1}}{8a^2}-\frac{\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)}{64a^2}+\frac{\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}\right.}{64a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/4)*x,x]
```


```
[Out] -((1 - a*x)^(7/8)*(1 + a*x)^(1/8))/(8*a^2) - ((1 - a*x)^(7/8)*(1 + a*x)^(9/
8))/(2*a^2) + (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 - a*x)^(
1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(32*a^2) + (Sqrt[2 - Sqrt[2]]*Ar
cTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 - Sqr
t[2]]])/(32*a^2) - (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - a
*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(32*a^2) - (Sqrt[2 - Sqrt[2
]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2
- Sqrt[2]]])/(32*a^2) - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x
)^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(64*a^2) +
(Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) + (Sqrt[2 - Sqrt
[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(64*a^2) - (Sqrt[2 + Sqrt[2]]*Log[1
 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1
 + a*x)^(1/8)])/(64*a^2) + (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 +
a*x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(64*a^2)
```


Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
```

]

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{1}{4}\tanh^{-1}(ax)}x\,dx = \int \frac{x\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\,dx$$

$$= -\frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} + \frac{\int \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\,dx}{8a}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} + \frac{\int \frac{1}{\sqrt[8]{1-ax}(1+ax)^{7/8}}\,dx}{32a}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} - \frac{\text{Subst}\left(\int \frac{x^6}{(2-x^8)^{7/8}}\,dx, x, \sqrt[8]{1-ax}\right)}{4a^2}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} - \frac{\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{4a^2}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} - \frac{\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{8\sqrt{2}a^2} + \frac{\text{Subst}\left(\int \frac{}{1+}\right.}{}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} + \frac{\text{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{8\sqrt{2}a^2} - \frac{\text{Subst}\left(\int \frac{1}{1+}\right.}{}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} - \frac{\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-(1-\sqrt{2})x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{16\sqrt{2\left(2-\sqrt{2}\right)}a^2} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} - \frac{\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right.}{32a^2}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} - \frac{\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{64a^2} + \frac{\sqrt{2-}}{}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} + \frac{\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}}{\sqrt{2+\sqrt{2}}}\right)}{32a^2} + \frac{\sqrt{2-\sqrt{2}}\tan^{-1}}{3}$$

Mathematica [C] time = 0.0213972, size = 56, normalized size = 0.09

$$-\frac{(1-ax)^{7/8}\left(2\sqrt[8]{2}\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{7}{8},\frac{15}{8},\frac{1}{2}(1-ax)\right)+7(ax+1)^{9/8}\right)}{14a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/4)*x,x]
```

```
[Out] -((1 - a*x)^(7/8)*(7*(1 + a*x)^(9/8) + 2*2^(1/8)*Hypergeometric2F1[-1/8, 7/
8, 15/8, (1 - a*x)/2]))/(14*a^2)
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \sqrt[4]{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4), x)
```

Fricas [B] time = 2.28637, size = 6819, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="fricas")
```

```
[Out] -1/256*(8*a^2*sqrt(sqrt(2) + 2)*(a^(-16))^(1/8)*arctan((2*sqrt(a^4*(a^(-16)
)^(1/4) + a^2*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(
-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^14*(a^(-16))^(7/8) - 2
*a^14*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(7/8) - sqrt(-sqrt(2)
 + 2))/sqrt(sqrt(2) + 2)) + 8*a^2*sqrt(sqrt(2) + 2)*(a^(-16))^(1/8)*arctan(
(2*sqrt(a^4*(a^(-16))^(1/4) - a^2*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(
a*x - 1))^(1/4)*(a^(-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^14
*(a^(-16))^(7/8) - 2*a^14*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(
7/8) + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 8*a^2*sqrt(-sqrt(2) + 2)*(a
^(-16))^(1/8)*arctan((2*sqrt(a^4*(a^(-16))^(1/4) + a^2*sqrt(sqrt(2) + 2)*(-
sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 +
 1)/(a*x - 1)))*a^14*(a^(-16))^(7/8) - 2*a^14*(-sqrt(-a^2*x^2 + 1)/(a*x - 1
))^(1/4)*(a^(-16))^(7/8) - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 8*a^2*s
qrt(-sqrt(2) + 2)*(a^(-16))^(1/8)*arctan((2*sqrt(a^4*(a^(-16))^(1/4) - a^2*
sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/8) + s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^14*(a^(-16))^(7/8) - 2*a^14*(-sqrt(-a
^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(7/8) + sqrt(sqrt(2) + 2))/sqrt(-sqr
t(2) + 2)) - 2*a^2*sqrt(sqrt(2) + 2)*(a^(-16))^(1/8)*log(a^4*(a^(-16))^(1/4
) + a^2*sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(
1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 2*a^2*sqrt(sqrt(2) + 2)*(a^(-
16))^(1/8)*log(a^4*(a^(-16))^(1/4) - a^2*sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2
+ 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))
) - 2*a^2*sqrt(-sqrt(2) + 2)*(a^(-16))^(1/8)*log(a^4*(a^(-16))^(1/4) + a^2*
sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/8) +
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 2*a^2*sqrt(-sqrt(2) + 2)*(a^(-16))^(
1/8)*log(a^4*(a^(-16))^(1/4) - a^2*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/
(a*x - 1))^(1/4)*(a^(-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 4
*(sqrt(2)*a^2*sqrt(sqrt(2) + 2) + sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(a^(-16))
^(1/8)*arctan((2*sqrt(2)*sqrt(a^4*(a^(-16))^(1/4) + 1/2*(sqrt(2)*a^2*sqrt(s
```

```
qrt(2) + 2) - sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1
))^(1/4)*(a^(-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^14*(a^(-1
6))^(7/8) - 2*sqrt(2)*a^14*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^
(7/8) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-
sqrt(2) + 2))) + 4*(sqrt(2)*a^2*sqrt(sqrt(2) + 2) + sqrt(2)*a^2*sqrt(-sqrt(
2) + 2))*(a^(-16))^(1/8)*arctan((2*sqrt(2)*sqrt(a^4*(a^(-16))^(1/4) - 1/2*(
sqrt(2)*a^2*sqrt(sqrt(2) + 2) - sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2
*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
- 1)))*a^14*(a^(-16))^(7/8) - 2*sqrt(2)*a^14*(-sqrt(-a^2*x^2 + 1)/(a*x - 1)
)^(1/4)*(a^(-16))^(7/8) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqr
t(2) + 2) + sqrt(-sqrt(2) + 2))) - 4*(sqrt(2)*a^2*sqrt(sqrt(2) + 2) - sqrt(
2)*a^2*sqrt(-sqrt(2) + 2))*(a^(-16))^(1/8)*arctan(-(2*sqrt(2)*sqrt(a^4*(a^(
-16))^(1/4) + 1/2*(sqrt(2)*a^2*sqrt(sqrt(2) + 2) + sqrt(2)*a^2*sqrt(-sqrt(2
) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/8) + sqrt(-sqrt(
-a^2*x^2 + 1)/(a*x - 1)))*a^14*(a^(-16))^(7/8) - 2*sqrt(2)*a^14*(-sqrt(-a^2
*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(7/8) - sqrt(sqrt(2) + 2) - sqrt(-sqrt
(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) - 4*(sqrt(2)*a^2*sqrt(s
qrt(2) + 2) - sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(a^(-16))^(1/8)*arctan(-(2*sq
rt(2)*sqrt(a^4*(a^(-16))^(1/4) - 1/2*(sqrt(2)*a^2*sqrt(sqrt(2) + 2) + sqrt(
2)*a^2*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^
(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^14*(a^(-16))^(7/8) - 2*sqrt(
2)*a^14*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(7/8) + sqrt(sqrt(2
) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) - (s
qrt(2)*a^2*sqrt(sqrt(2) + 2) + sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(a^(-16))^(1
/8)*log(a^4*(a^(-16))^(1/4) + 1/2*(sqrt(2)*a^2*sqrt(sqrt(2) + 2) + sqrt(2)*
a^2*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/
8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + (sqrt(2)*a^2*sqrt(sqrt(2) + 2)
+ sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(a^(-16))^(1/8)*log(a^4*(a^(-16))^(1/4) -
 1/2*(sqrt(2)*a^2*sqrt(sqrt(2) + 2) + sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(-sqr
t(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1))) - (sqrt(2)*a^2*sqrt(sqrt(2) + 2) - sqrt(2)*a^2*sqrt(-sqrt(2) +
 2))*(a^(-16))^(1/8)*log(a^4*(a^(-16))^(1/4) + 1/2*(sqrt(2)*a^2*sqrt(sqrt(2
) + 2) - sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1
/4)*(a^(-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + (sqrt(2)*a^2*s
qrt(sqrt(2) + 2) - sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(a^(-16))^(1/8)*log(a^4*
(a^(-16))^(1/4) - 1/2*(sqrt(2)*a^2*sqrt(sqrt(2) + 2) - sqrt(2)*a^2*sqrt(-sq
rt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/8) + sqrt(-s
qrt(-a^2*x^2 + 1)/(a*x - 1))) - 32*(4*a^2*x^2 + a*x - 5)*(-sqrt(-a^2*x^2 +
1)/(a*x - 1))^(1/4))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt[4]{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/4)*x,x)
```

```
[Out] Integral(x*((a*x + 1)/sqrt(-a**2*x**2 + 1))**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="giac")
```

```
[Out] integrate(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4), x)
```

3.137 $\int e^{\frac{1}{4}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=591

$$-\frac{(1-ax)^{7/8}\sqrt[8]{ax+1}}{a}-\frac{\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)}{8a}+\frac{\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)}{8a}-\sqrt{2+\sqrt{\cdots}}$$

```
[Out] -(((1 - a*x)^(7/8)*(1 + a*x)^(1/8))/a) + (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2
- Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(4*a)
 + (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 +
a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/(4*a) - (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 -
 Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(4*a)
- (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1 + a
*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/(4*a) - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)
^(1/4)/(1 + a*x)^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8
)])/(8*a) + (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) + (S
qrt[2 - Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(8*a) - (Sqrt[2 + Sqrt[
2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - a*x)^
(1/8))/(1 + a*x)^(1/8)])/(8*a) + (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)
/(1 + a*x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(8
*a)
```

Rubi [A] time = 0.436628, antiderivative size = 591, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.1, Rules used = {6125, 50, 63, 331, 299, 1122, 1169, 634, 618, 204, 628}

$$-\frac{(1-ax)^{7/8}\sqrt[8]{ax+1}}{a}-\frac{\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)}{8a}+\frac{\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)}{8a}-\sqrt{2+\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/4),x]
```

```
[Out] -(((1 - a*x)^(7/8)*(1 + a*x)^(1/8))/a) + (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2
- Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(4*a)
 + (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 +
a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/(4*a) - (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 -
 Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(4*a)
- (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1 + a
*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/(4*a) - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)
^(1/4)/(1 + a*x)^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8
)])/(8*a) + (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) + (S
qrt[2 - Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(8*a) - (Sqrt[2 + Sqrt[
2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - a*x)^
(1/8))/(1 + a*x)^(1/8)])/(8*a) + (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)
/(1 + a*x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(8
*a)
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{1}{4}\tanh^{-1}(ax)}\,dx = \int \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\,dx$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a} + \frac{1}{4}\int \frac{1}{\sqrt[8]{1-ax}(1+ax)^{7/8}}\,dx$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a} - \frac{2\,\mathrm{Subst}\left(\int \frac{x^6}{(2-x^8)^{7/8}}\,dx, x, \sqrt[8]{1-ax}\right)}{a}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a} - \frac{2\,\mathrm{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{a}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a} - \frac{\mathrm{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{\sqrt{2}a} + \frac{\mathrm{Subst}\left(\int \frac{x^4}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{\sqrt{2}a}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a} + \frac{\mathrm{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{\sqrt{2}a} - \frac{\mathrm{Subst}\left(\int \frac{1+\sqrt{2}x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{\sqrt{2}a}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a} - \frac{\mathrm{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}a} - \frac{\mathrm{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \cdots\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}a}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a} - \frac{\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{4a} - \frac{\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\mathrm{Subst}\cdots}{\cdots}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a} - \frac{\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{8a} + \frac{\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}+\cdots\right)}{8a}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a} + \frac{\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}}{\sqrt{2+\sqrt{2}}}\right)}{4a} + \frac{\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}}{\sqrt{2-\sqrt{2}}}\right)}{4a} - \frac{\sqrt{2\cdots}}{\cdots}$$

Mathematica [C] time = 0.0528874, size = 48, normalized size = 0.08

$$\frac{2e^{\frac{1}{4}\tanh^{-1}(ax)}\left(\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},-e^{2\tanh^{-1}(ax)}\right)-\frac{1}{e^{2\tanh^{-1}(ax)}+1}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/4),x]
```

```
[Out] (2*E^(ArcTanh[a*x]/4)*(-(1 + E^(2*ArcTanh[a*x]))^(-1) + Hypergeometric2F1[1
/8, 1, 9/8, -E^(2*ArcTanh[a*x])]))/a
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \sqrt[4]{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4), x)
```

Fricas [B] time = 2.24933, size = 6561, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4),x, algorithm="fricas")
```

```
[Out] -1/32*(8*a*sqrt(sqrt(2) + 2)*(a^(-8))^(1/8)*arctan((2*sqrt(a^2*(a^(-8))^(1/
4) + a*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(1
/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^7*(a^(-8))^(7/8) - 2*a^7*(-sqr
t(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(7/8) - sqrt(-sqrt(2) + 2))/sqrt(
sqrt(2) + 2)) + 8*a*sqrt(sqrt(2) + 2)*(a^(-8))^(1/8)*arctan((2*sqrt(a^2*(a^
(-8))^(1/4) - a*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a
^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^7*(a^(-8))^(7/8) - 2*
a^7*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(7/8) + sqrt(-sqrt(2) +
2))/sqrt(sqrt(2) + 2)) + 8*a*sqrt(-sqrt(2) + 2)*(a^(-8))^(1/8)*arctan((2*sq
rt(a^2*(a^(-8))^(1/4) + a*sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))
^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^7*(a^(-8))^(
7/8) - 2*a^7*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(7/8) - sqrt(sq
rt(2) + 2))/sqrt(-sqrt(2) + 2)) + 8*a*sqrt(-sqrt(2) + 2)*(a^(-8))^(1/8)*arc
```

```
tan((2*sqrt(a^2*(a^(-8))^(1/4) - a*sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(
a*x - 1))^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^7*(
a^(-8))^(7/8) - 2*a^7*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(7/8)
+ sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 2*a*sqrt(sqrt(2) + 2)*(a^(-8))^(
1/8)*log(a^2*(a^(-8))^(1/4) + a*sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x
 - 1))^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 2*a*sq
rt(sqrt(2) + 2)*(a^(-8))^(1/8)*log(a^2*(a^(-8))^(1/4) - a*sqrt(sqrt(2) + 2)
*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2
 + 1)/(a*x - 1))) - 2*a*sqrt(-sqrt(2) + 2)*(a^(-8))^(1/8)*log(a^2*(a^(-8))^
(1/4) + a*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))
^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 2*a*sqrt(-sqrt(2) + 2)*(a^(
-8))^(1/8)*log(a^2*(a^(-8))^(1/4) - a*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 +
1)/(a*x - 1))^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) +
 4*(sqrt(2)*a*sqrt(sqrt(2) + 2) + sqrt(2)*a*sqrt(-sqrt(2) + 2))*(a^(-8))^(1
/8)*arctan((2*sqrt(2)*sqrt(a^2*(a^(-8))^(1/4) + 1/2*(sqrt(2)*a*sqrt(sqrt(2)
 + 2) - sqrt(2)*a*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)
*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^7*(a^(-8))^(7/8) -
 2*sqrt(2)*a^7*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(7/8) - sqrt(
sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))
) + 4*(sqrt(2)*a*sqrt(sqrt(2) + 2) + sqrt(2)*a*sqrt(-sqrt(2) + 2))*(a^(-8))
^(1/8)*arctan((2*sqrt(2)*sqrt(a^2*(a^(-8))^(1/4) - 1/2*(sqrt(2)*a*sqrt(sqrt
(2) + 2) - sqrt(2)*a*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1
/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^7*(a^(-8))^(7/8
) - 2*sqrt(2)*a^7*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(7/8) + sq
rt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) +
2))) - 4*(sqrt(2)*a*sqrt(sqrt(2) + 2) - sqrt(2)*a*sqrt(-sqrt(2) + 2))*(a^(-
8))^(1/8)*arctan(-(2*sqrt(2)*sqrt(a^2*(a^(-8))^(1/4) + 1/2*(sqrt(2)*a*sqrt(
sqrt(2) + 2) + sqrt(2)*a*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1)
)^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^7*(a^(-8))^
(7/8) - 2*sqrt(2)*a^7*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(7/8)
- sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2
) + 2))) - 4*(sqrt(2)*a*sqrt(sqrt(2) + 2) - sqrt(2)*a*sqrt(-sqrt(2) + 2))*(
a^(-8))^(1/8)*arctan(-(2*sqrt(2)*sqrt(a^2*(a^(-8))^(1/4) - 1/2*(sqrt(2)*a*s
qrt(sqrt(2) + 2) + sqrt(2)*a*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x
- 1))^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^7*(a^(-
8))^(7/8) - 2*sqrt(2)*a^7*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(7
/8) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sq
rt(2) + 2))) - (sqrt(2)*a*sqrt(sqrt(2) + 2) + sqrt(2)*a*sqrt(-sqrt(2) + 2))
*(a^(-8))^(1/8)*log(a^2*(a^(-8))^(1/4) + 1/2*(sqrt(2)*a*sqrt(sqrt(2) + 2) +
 sqrt(2)*a*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8
))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + (sqrt(2)*a*sqrt(sqrt(2) +
 2) + sqrt(2)*a*sqrt(-sqrt(2) + 2))*(a^(-8))^(1/8)*log(a^2*(a^(-8))^(1/4) -
 1/2*(sqrt(2)*a*sqrt(sqrt(2) + 2) + sqrt(2)*a*sqrt(-sqrt(2) + 2))*(-sqrt(-a
^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
 - 1))) - (sqrt(2)*a*sqrt(sqrt(2) + 2) - sqrt(2)*a*sqrt(-sqrt(2) + 2))*(a^(
-8))^(1/8)*log(a^2*(a^(-8))^(1/4) + 1/2*(sqrt(2)*a*sqrt(sqrt(2) + 2) - sqrt
(2)*a*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(1
/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + (sqrt(2)*a*sqrt(sqrt(2) + 2) -
 sqrt(2)*a*sqrt(-sqrt(2) + 2))*(a^(-8))^(1/8)*log(a^2*(a^(-8))^(1/4) - 1/2*
(sqrt(2)*a*sqrt(sqrt(2) + 2) - sqrt(2)*a*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^
2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)
)) - 32*(a*x - 1)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[4]{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/4),x)
```

```
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4), x)
```

3.138 $\int \frac{e^{\frac{1}{4}\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=759

$$-\frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)+\frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)-\frac{1}{2}\sqrt{2+\sqrt{}}$$

```
[Out] -2*ArcTan[(1 + a*x)^(1/8)/(1 - a*x)^(1/8)] + Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt
[2 - Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]] + S
qrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^
(1/8))/Sqrt[2 - Sqrt[2]]] - Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (
2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]] - Sqrt[2 - Sqrt[2]]*
ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 - S
qrt[2]]] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)] -
Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)] - 2*ArcTanh[(
1 + a*x)^(1/8)/(1 - a*x)^(1/8)] - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4
)/(1 + a*x)^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/2
 + (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) + (Sqrt[2 - S
qrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/2 - (Sqrt[2 + Sqrt[2]]*Log[1 + (
1 - a*x)^(1/4)/(1 + a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a
*x)^(1/8)])/2 + (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4)
+ (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/2 + Log[1 - (Sqrt[2
]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8) + (1 + a*x)^(1/4)/(1 - a*x)^(1/4)]/Sqrt[
2] - Log[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8) + (1 + a*x)^(1/4)/(1
 - a*x)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.527173, antiderivative size = 759, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 20, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.429, Rules used = {6126, 105, 63, 331, 299, 1122, 1169, 634, 618, 204, 628, 93, 214, 212, 206, 203, 211, 1165, 1162, 617}

$$-\frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)+\frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)-\frac{1}{2}\sqrt{2+\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/4)/x,x]
```

```
[Out] -2*ArcTan[(1 + a*x)^(1/8)/(1 - a*x)^(1/8)] + Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt
[2 - Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]] + S
qrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^
(1/8))/Sqrt[2 - Sqrt[2]]] - Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (
2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]] - Sqrt[2 - Sqrt[2]]*
ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 - S
qrt[2]]] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)] -
Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)] - 2*ArcTanh[(
1 + a*x)^(1/8)/(1 - a*x)^(1/8)] - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4
)/(1 + a*x)^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/2
 + (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) + (Sqrt[2 - S
qrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/2 - (Sqrt[2 + Sqrt[2]]*Log[1 + (
1 - a*x)^(1/4)/(1 + a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a
*x)^(1/8)])/2 + (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4)
```

```
+ (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/2 + Log[1 - (Sqrt[2
]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8) + (1 + a*x)^(1/4)/(1 - a*x)^(1/4)]/Sqrt[
2] - Log[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8) + (1 + a*x)^(1/4)/(1
 - a*x)^(1/4)]/Sqrt[2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}\tanh^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt[8]{1+ax}}{x\sqrt[8]{1-ax}}\,dx$$
$$= a\int \frac{1}{\sqrt[8]{1-ax}(1+ax)^{7/8}}\,dx + \int \frac{1}{x\sqrt[8]{1-ax}(1+ax)^{7/8}}\,dx$$
$$= -\left(8\,\text{Subst}\left(\int \frac{x^6}{(2-x^8)^{7/8}}\,dx, x, \sqrt[8]{1-ax}\right)\right) + 8\,\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)$$
$$= -\left(4\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)\right) - 4\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - 8\,\text{Subst}\left(\int \frac{x^6}{1+x \cdots}\right.$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - 2\,\text{Subst}\left(\int \frac{1-x \cdots}{1+x \cdots}\right.$$
$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{\cdots}}{-1+\cdots}\right.}{\cdots}$$
$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) + \frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}+\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}+\cdots\right.}{\sqrt{2}}$$
$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[8]{\cdots}}{\sqrt[8]{\cdots}}\right.$$
$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[8]{\cdots}}{\sqrt[8]{\cdots}}\right.$$
$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) + \sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}}{\sqrt{2+\sqrt{2}}}\right) + \sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\cdots}}{\sqrt[8]{1+\cdots}}}{\sqrt{2-\sqrt{2}}}\right.$$

Mathematica [C] time = 0.0295594, size = 83, normalized size = 0.11

$$-\frac{4(1-ax)^{7/8}\left(\sqrt[8]{2}(ax+1)^{7/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{7}{8},\frac{15}{8},\frac{1}{2}(1-ax)\right)+2\text{Hypergeometric2F1}\left(\frac{7}{8},1,\frac{15}{8},\frac{1-ax}{ax+1}\right)\right)}{7(ax+1)^{7/8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/4)/x,x]
```

```
[Out] (-4*(1 - a*x)^(7/8)*(2^(1/8)*(1 + a*x)^(7/8)*Hypergeometric2F1[7/8, 7/8, 15
/8, (1 - a*x)/2] + 2*Hypergeometric2F1[7/8, 1, 15/8, (1 - a*x)/(1 + a*x)]))
/(7*(1 + a*x)^(7/8))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt[4]{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x,x)

[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="maxima")

[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4)/x, x)
```

Fricas [B] time = 2.24658, size = 5937, normalized size = 7.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="fricas")

[Out] -1/2*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((sqrt(
2)*sqrt(2*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*(-sqrt(-
a^2*x^2 + 1)/(a*x - 1))^(1/4) + 4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 4)
- 2*sqrt(2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) - sqrt(sqrt(2) + 2) + sqr
t(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/2*(sqrt(2)*s
qrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((sqrt(2)*sqrt(-2*(sqr
t(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(
a*x - 1))^(1/4) + 4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 4) - 2*sqrt(2)*(-
sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2
))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) + 1/2*(sqrt(2)*sqrt(sqrt(2) +
2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(sqrt(2)*sqrt(2*(sqrt(2)*sqrt(sqrt
(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4
) + 4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 4) - 2*sqrt(2)*(-sqrt(-a^2*x^2
+ 1)/(a*x - 1))^(1/4) - sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(
2) + 2) - sqrt(-sqrt(2) + 2))) + 1/2*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*s
qrt(-sqrt(2) + 2))*arctan(-(sqrt(2)*sqrt(-2*(sqrt(2)*sqrt(sqrt(2) + 2) + sq
rt(2)*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + 4*sqrt(-s
qrt(-a^2*x^2 + 1)/(a*x - 1)) + 4) - 2*sqrt(2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1
))^(1/4) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqr
t(-sqrt(2) + 2))) + 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2)
+ 2))*log(2*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*(-sqrt
(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + 4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 4
) - 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(-2*(sq
rt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/
(a*x - 1))^(1/4) + 4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 4) + 1/8*(sqrt(2
)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(2*(sqrt(2)*sqrt(sqrt(
2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)
 + 4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 4) - 1/8*(sqrt(2)*sqrt(sqrt(2) +
 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(-2*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(
2)*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + 4*sqrt(-sqrt
```

```
(-a^2*x^2 + 1)/(a*x - 1)) + 4) + 2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*(-sq
rt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1
) - sqrt(2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) - 1) + 2*sqrt(2)*arctan(1
/2*sqrt(2)*sqrt(-4*sqrt(2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + 4*sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 4) - sqrt(2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1)
)^(1/4) + 1) - sqrt(-sqrt(2) + 2)*arctan((2*sqrt(sqrt(sqrt(2) + 2)*(-sqrt(-
a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) -
sqrt(sqrt(2) + 2) - 2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4))/sqrt(-sqrt(2)
+ 2)) - sqrt(-sqrt(2) + 2)*arctan((2*sqrt(-sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^
2 + 1)/(a*x - 1))^(1/4) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + sqrt(s
qrt(2) + 2) - 2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4))/sqrt(-sqrt(2) + 2))
- sqrt(sqrt(2) + 2)*arctan((2*sqrt(sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/
(a*x - 1))^(1/4) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - sqrt(-sqrt(2)
 + 2) - 2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4))/sqrt(sqrt(2) + 2)) - sqrt(
sqrt(2) + 2)*arctan((2*sqrt(-sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x -
 1))^(1/4) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + sqrt(-sqrt(2) + 2)
- 2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4))/sqrt(sqrt(2) + 2)) - 1/2*sqrt(2)
*log(4*sqrt(2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + 4*sqrt(-sqrt(-a^2*x^
2 + 1)/(a*x - 1)) + 4) + 1/2*sqrt(2)*log(-4*sqrt(2)*(-sqrt(-a^2*x^2 + 1)/(a
*x - 1))^(1/4) + 4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 4) + 1/4*sqrt(sqrt
(2) + 2)*log(sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt
(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 1/4*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt
(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(-sqrt(-a^2*x^2 + 1)/(
a*x - 1)) + 1) + 1/4*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*
x^2 + 1)/(a*x - 1))^(1/4) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 1/4*
sqrt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^
(1/4) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 2*arctan((-sqrt(-a^2*x^2
 + 1)/(a*x - 1))^(1/4)) - log((-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + 1) +
log((-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/4)/x,x)
```

```
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(1/4)/x, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

3.139 $\int \frac{e^{\frac{1}{4}\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=271

$$-\frac{(1-ax)^{7/8}\sqrt[8]{ax+1}}{x}+\frac{a\log\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}-\frac{\sqrt{2}\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}+1\right)}{4\sqrt{2}}-\frac{a\log\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}+\frac{\sqrt{2}\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}+1\right)}{4\sqrt{2}}-\frac{1}{2}a\tan^{-1}\left(\frac{\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}\right)+\frac{a\tan^{-1}\left(1\right.}{2}$$

```
[Out] -(((1 - a*x)^(7/8)*(1 + a*x)^(1/8))/x) - (a*ArcTan[(1 + a*x)^(1/8)/(1 - a*x
)^(1/8)])/2 + (a*ArcTan[1 - (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)])/(2*
Sqrt[2]) - (a*ArcTan[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)])/(2*Sqr
t[2]) - (a*ArcTanh[(1 + a*x)^(1/8)/(1 - a*x)^(1/8)])/2 + (a*Log[1 - (Sqrt[2
]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8) + (1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/(4*S
qrt[2]) - (a*Log[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8) + (1 + a*x)^
(1/4)/(1 - a*x)^(1/4)])/(4*Sqrt[2])
```

Rubi [A] time = 0.129204, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {6126, 94, 93, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(1-ax)^{7/8}\sqrt[8]{ax+1}}{x}+\frac{a\log\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}-\frac{\sqrt{2}\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}+1\right)}{4\sqrt{2}}-\frac{a\log\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}+\frac{\sqrt{2}\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}+1\right)}{4\sqrt{2}}-\frac{1}{2}a\tan^{-1}\left(\frac{\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}\right)+\frac{a\tan^{-1}\left(1\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/4)/x^2,x]
```

```
[Out] -(((1 - a*x)^(7/8)*(1 + a*x)^(1/8))/x) - (a*ArcTan[(1 + a*x)^(1/8)/(1 - a*x
)^(1/8)])/2 + (a*ArcTan[1 - (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)])/(2*
Sqrt[2]) - (a*ArcTan[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)])/(2*Sqr
t[2]) - (a*ArcTanh[(1 + a*x)^(1/8)/(1 - a*x)^(1/8)])/2 + (a*Log[1 - (Sqrt[2
]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8) + (1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/(4*S
qrt[2]) - (a*Log[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8) + (1 + a*x)^
(1/4)/(1 - a*x)^(1/4)])/(4*Sqrt[2])
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}\tanh^{-1}(ax)}}{x^2}\,dx = \int \frac{\sqrt[8]{1+ax}}{x^2\sqrt[8]{1-ax}}\,dx$$
$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{x} + \frac{1}{4}a\int \frac{1}{x\sqrt[8]{1-ax}(1+ax)^{7/8}}\,dx$$
$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{x} + (2a)\,\mathrm{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)$$
$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{x} - a\,\mathrm{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - a\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)$$
$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{x} - \frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)$$
$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{x} - \frac{1}{2}a\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \frac{1}{2}a\tanh^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \frac{1}{4}a\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\right.$$
$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{x} - \frac{1}{2}a\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \frac{1}{2}a\tanh^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) + \frac{a\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}+\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)}{4\sqrt{2}}$$
$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{x} - \frac{1}{2}a\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) + \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)}{2\sqrt{2}} - \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)}{2\sqrt{2}} - \frac{1}{2}a$$

Mathematica [C] time = 0.016594, size = 58, normalized size = 0.21

$$-\frac{(1-ax)^{7/8}\left(2ax\,\mathrm{Hypergeometric2F1}\left(\frac{7}{8},1,\frac{15}{8},\frac{1-ax}{ax+1}\right)+7ax+7\right)}{7x(ax+1)^{7/8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/4)/x^2,x]
```

```
[Out] -((1 - a*x)^(7/8)*(7 + 7*a*x + 2*a*x*Hypergeometric2F1[7/8, 1, 15/8, (1 - a
*x)/(1 + a*x)]))/(7*x*(1 + a*x)^(7/8))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt[4]{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^2,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4)/x^2, x)
```

Fricas [B] time = 1.92928, size = 1359, normalized size = 5.01

$$-\frac{4\,ax\arctan\left(\left(-\frac{\sqrt{-a^2x^2+1}}{ax-1}\right)^{\frac{1}{4}}\right)+2\,ax\log\left(\left(-\frac{\sqrt{-a^2x^2+1}}{ax-1}\right)^{\frac{1}{4}}+1\right)-2\,ax\log\left(\left(-\frac{\sqrt{-a^2x^2+1}}{ax-1}\right)^{\frac{1}{4}}-1\right)-4\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(-\frac{a^4}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*x*arctan((-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)) + 2*a*x*log((-sqr
t(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + 1) - 2*a*x*log((-sqrt(-a^2*x^2 + 1)/(a*x
 - 1))^(1/4) - 1) - 4*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(3
/4)*a*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) - sqrt(2)*(a^4)^(3/4)*sqrt(a^2*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + sqrt(2)*(a^4)^(1/4)*a*(-sqrt(-a^2*x^2
 + 1)/(a*x - 1))^(1/4) + sqrt(a^4)))/a^4) - 4*sqrt(2)*(a^4)^(1/4)*x*arctan(
(a^4 - sqrt(2)*(a^4)^(3/4)*a*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(2
)*(a^4)^(3/4)*sqrt(a^2*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(2)*(a^4)^
(1/4)*a*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(a^4)))/a^4) + sqrt(2)*
(a^4)^(1/4)*x*log(a^2*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + sqrt(2)*(a^4)^(
1/4)*a*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(a^4)) - sqrt(2)*(a^4)^(
1/4)*x*log(a^2*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(2)*(a^4)^(1/4)*a*
(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(a^4)) - 8*(a*x - 1)*(-sqrt(-a^
2*x^2 + 1)/(a*x - 1))^(1/4))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/4)/x**2,x)
```

```
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(1/4)/x**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.140 $\int \frac{e^{\frac{1}{4}\tanh^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=312

$$\frac{a^2 \log\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}} - \frac{\sqrt{2}\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}} + 1\right)}{32\sqrt{2}} - \frac{a^2 \log\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}} + \frac{\sqrt{2}\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}} + 1\right)}{32\sqrt{2}} - \frac{1}{16}a^2 \tan^{-1}\left(\frac{\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}\right) + \frac{a^2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}\right)}{16\sqrt{2}} -$$

```
[Out] -(a*(1 - a*x)^(7/8)*(1 + a*x)^(1/8))/(8*x) - ((1 - a*x)^(7/8)*(1 + a*x)^(9/
8))/(2*x^2) - (a^2*ArcTan[(1 + a*x)^(1/8)/(1 - a*x)^(1/8)])/16 + (a^2*ArcTa
n[1 - (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)])/(16*Sqrt[2]) - (a^2*ArcTa
n[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)])/(16*Sqrt[2]) - (a^2*ArcTa
nh[(1 + a*x)^(1/8)/(1 - a*x)^(1/8)])/16 + (a^2*Log[1 - (Sqrt[2]*(1 + a*x)^(
1/8))/(1 - a*x)^(1/8) + (1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/(32*Sqrt[2]) - (a
^2*Log[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8) + (1 + a*x)^(1/4)/(1 -
 a*x)^(1/4)])/(32*Sqrt[2])
```

Rubi [A] time = 0.152656, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6126, 96, 94, 93, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$\frac{a^2 \log\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}} - \frac{\sqrt{2}\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}} + 1\right)}{32\sqrt{2}} - \frac{a^2 \log\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}} + \frac{\sqrt{2}\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}} + 1\right)}{32\sqrt{2}} - \frac{1}{16}a^2 \tan^{-1}\left(\frac{\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}\right) + \frac{a^2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}\right)}{16\sqrt{2}} -$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/4)/x^3,x]
```

```
[Out] -(a*(1 - a*x)^(7/8)*(1 + a*x)^(1/8))/(8*x) - ((1 - a*x)^(7/8)*(1 + a*x)^(9/
8))/(2*x^2) - (a^2*ArcTan[(1 + a*x)^(1/8)/(1 - a*x)^(1/8)])/16 + (a^2*ArcTa
n[1 - (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)])/(16*Sqrt[2]) - (a^2*ArcTa
n[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)])/(16*Sqrt[2]) - (a^2*ArcTa
nh[(1 + a*x)^(1/8)/(1 - a*x)^(1/8)])/16 + (a^2*Log[1 - (Sqrt[2]*(1 + a*x)^(
1/8))/(1 - a*x)^(1/8) + (1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/(32*Sqrt[2]) - (a
^2*Log[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8) + (1 + a*x)^(1/4)/(1 -
 a*x)^(1/4)])/(32*Sqrt[2])
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}\tanh^{-1}(ax)}}{x^3}\,dx = \int \frac{\sqrt[8]{1+ax}}{x^3\sqrt[8]{1-ax}}\,dx$$

$$= -\frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} + \frac{1}{8}a\int \frac{\sqrt[8]{1+ax}}{x^2\sqrt[8]{1-ax}}\,dx$$

$$= -\frac{a(1-ax)^{7/8}\sqrt[8]{1+ax}}{8x} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} + \frac{1}{32}a^2\int \frac{1}{x\sqrt[8]{1-ax}(1+ax)^{7/8}}\,dx$$

$$= -\frac{a(1-ax)^{7/8}\sqrt[8]{1+ax}}{8x} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} + \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)$$

$$= -\frac{a(1-ax)^{7/8}\sqrt[8]{1+ax}}{8x} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} - \frac{1}{8}a^2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \frac{1}{8}a^2\,\text{Sub}$$

$$= -\frac{a(1-ax)^{7/8}\sqrt[8]{1+ax}}{8x} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} - \frac{1}{16}a^2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \frac{1}{16}a^2\,\text{S}$$

$$= -\frac{a(1-ax)^{7/8}\sqrt[8]{1+ax}}{8x} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} - \frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \frac{1}{16}a^2\tanh^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right.$$

$$= -\frac{a(1-ax)^{7/8}\sqrt[8]{1+ax}}{8x} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} - \frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \frac{1}{16}a^2\tanh^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right.$$

$$= -\frac{a(1-ax)^{7/8}\sqrt[8]{1+ax}}{8x} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} - \frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) + \frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)}{16\sqrt{2}}$$

Mathematica [C] time = 0.0209067, size = 73, normalized size = 0.23

$$-\frac{(1-ax)^{7/8}\left(2a^2x^2\text{Hypergeometric2F1}\left(\frac{7}{8},1,\frac{15}{8},\frac{1-ax}{ax+1}\right)+7\left(5a^2x^2+9ax+4\right)\right)}{56x^2(ax+1)^{7/8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/4)/x^3,x]
```

```
[Out] -((1 - a*x)^(7/8)*(7*(4 + 9*a*x + 5*a^2*x^2) + 2*a^2*x^2*Hypergeometric2F1[
7/8, 1, 15/8, (1 - a*x)/(1 + a*x)]))/(56*x^2*(1 + a*x)^(7/8))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt[4]{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^3,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4)/x^3, x)
```

Fricas [B] time = 2.37574, size = 1423, normalized size = 4.56

$$-\frac{4\,a^2x^2\arctan\left(\left(-\frac{\sqrt{-a^2x^2+1}}{ax-1}\right)^{\frac{1}{4}}\right)+2\,a^2x^2\log\left(\left(-\frac{\sqrt{-a^2x^2+1}}{ax-1}\right)^{\frac{1}{4}}+1\right)-2\,a^2x^2\log\left(\left(-\frac{\sqrt{-a^2x^2+1}}{ax-1}\right)^{\frac{1}{4}}-1\right)-4\,\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\Bigg(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*a^2*x^2*arctan((-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)) + 2*a^2*x^2*
log((-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + 1) - 2*a^2*x^2*log((-sqrt(-a^2*
x^2 + 1)/(a*x - 1))^(1/4) - 1) - 4*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + s
qrt(2)*(a^8)^(3/4)*a^2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) - sqrt(2)*(a^8
)^(3/4)*sqrt(a^4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + sqrt(2)*(a^8)^(1/4)*
a^2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(a^8)))/a^8) - 4*sqrt(2)*(a
^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(3/4)*a^2*(-sqrt(-a^2*x^2 + 1)/(a
*x - 1))^(1/4) + sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
 - 1)) - sqrt(2)*(a^8)^(1/4)*a^2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sq
```

```
rt(a^8)))/a^8) + sqrt(2)*(a^8)^(1/4)*x^2*log(a^4*sqrt(-sqrt(-a^2*x^2 + 1)/(
a*x - 1)) + sqrt(2)*(a^8)^(1/4)*a^2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) +
 sqrt(a^8)) - sqrt(2)*(a^8)^(1/4)*x^2*log(a^4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
 - 1)) - sqrt(2)*(a^8)^(1/4)*a^2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sq
rt(a^8)) - 8*(5*a^2*x^2 - a*x - 4)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4))/x
^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/4)/x**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.141 $\int e^{4\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=45

$$-4x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,ax)+\frac{4x^{m+1}}{1-ax}+\frac{x^{m+1}}{m+1}$$

```
[Out] x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 - a*x) - 4*x^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, a*x]
```

Rubi [A] time = 0.0431906, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6126, 89, 80, 64}

$$-4x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)+\frac{4x^{m+1}}{1-ax}+\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*x^m,x]
```

```
[Out] x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 - a*x) - 4*x^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, a*x]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}x^m\,dx = \int \frac{x^m(1+ax)^2}{(1-ax)^2}\,dx$$
$$= \frac{4x^{1+m}}{1-ax} - \frac{\int \frac{x^m\left(a^2(3+4m)+a^3x\right)}{1-ax}dx}{a^2}$$
$$= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1-ax} - (4(1+m))\int \frac{x^m}{1-ax}\,dx$$
$$= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1-ax} - 4x^{1+m}\,{}_2F_1(1,1+m;2+m;ax)$$

Mathematica [A] time = 0.0227577, size = 47, normalized size = 1.04

$$\frac{x^{m+1}(-4(m+1)(ax-1)\text{Hypergeometric2F1}(1,m+1,m+2,ax)+ax-4m-5)}{(m+1)(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(-5 - 4*m + a*x - 4*(1 + m)*(-1 + a*x)*Hypergeometric2F1[1, 1 +
m, 2 + m, a*x]))/((1 + m)*(-1 + a*x))
```

Maple [C] time = 0.444, size = 461, normalized size = 10.2

$$\frac{1}{2}\left(-a^2\right)^{-\frac{1}{2}-\frac{m}{2}}\left(-\frac{x^{1+m}\left(2a^2x^2-m-3\right)}{\left(-a^2x^2+1\right)a^4(1+m)}\left(-a^2\right)^{\frac{5}{2}+\frac{m}{2}}-\frac{x^{1+m}(3+m)}{2a^4}\left(-a^2\right)^{\frac{5}{2}+\frac{m}{2}}\text{LerchPhi}\left(a^2x^2,1,\frac{1}{2}+\frac{m}{2}\right)\right)+2\frac{\left(-a^2\right)^{-m}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*x^m,x)
```

```
[Out] 1/2*(-a^2)^(-1/2-1/2*m)*(-x^(1+m)*(-a^2)^(5/2+1/2*m)*(2*a^2*x^2-m-3)/(-a^2*
x^2+1)/a^4/(1+m)-1/2*x^(1+m)*(-a^2)^(5/2+1/2*m)*(3+m)/a^4*LerchPhi(a^2*x^2,
1,1/2+1/2*m))+2/a*(-a^2)^(-1/2*m)*(-x^m*(-a^2)^(1/2*m)*(2*a^2*x^2-m-2)/(-a^
2*x^2+1)/m-1/2*x^m*(-a^2)^(1/2*m)*(2+m)*LerchPhi(a^2*x^2,1,1/2*m))-3*(-a^2)
^(-1/2-1/2*m)*(-1/(3+m)*x^(1+m)*(-a^2)^(3/2+1/2*m)*(-3-m)/a^2/(-a^2*x^2+1)-
1/2*x^(1+m)*(-a^2)^(3/2+1/2*m)*(1+m)/a^2*LerchPhi(a^2*x^2,1,1/2+1/2*m))-2/a
*(-a^2)^(-1/2*m)*(1/(2+m)*x^m*(-a^2)^(1/2*m)*(-m-2)/(-a^2*x^2+1)+1/2*x^m*(-
a^2)^(1/2*m)*m*LerchPhi(a^2*x^2,1,1/2*m))+1/2*(-a^2)^(-1/2-1/2*m)*(-2/(1+m)
*x^(1+m)*(-a^2)^(1/2+1/2*m)*(-1-m)/(-2*a^2*x^2+2)+2/(1+m)*x^(1+m)*(-a^2)^(1
/2+1/2*m)*(-1/4*m^2+1/4)*LerchPhi(a^2*x^2,1,1/2+1/2*m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^4x^m}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^m,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^4*x^m/(a^2*x^2 - 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2x^2+2\,ax+1\right)x^m}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*x^m/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\,(ax+1)^2}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*x**m,x)
```

```
[Out] Integral(x**m*(a*x + 1)**2/(a*x - 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^4x^m}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^m,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^4*x^m/(a^2*x^2 - 1)^2, x)
```

3.142 $\int e^{3\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=151

$$-\frac{3x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{m+1}+\frac{4x^{m+1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{m+1}-\frac{ax^{m+2}\text{Hype}}{}$$

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m
) - (a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2
+ m) + (4*x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/
(1 + m) + (4*a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, a^2*x
^2])/(2 + m)
```

Rubi [A] time = 0.881088, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6124, 6742, 364, 850, 808}

$$-\frac{3x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{4x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}-\frac{ax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}+\frac{4ax^{m+2}\,{}_2F_1\left(\frac{3}{2},\right.}{m+}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^m,x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m
) - (a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2
+ m) + (4*x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/
(1 + m) + (4*a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, a^2*x
^2])/(2 + m)
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1+ax)^2}{(1-ax)\sqrt{1-a^2x^2}}\,dx\\
&= \int \left(-\frac{3x^m}{\sqrt{1-a^2x^2}}-\frac{ax^{1+m}}{\sqrt{1-a^2x^2}}+\frac{4x^m}{(1-ax)\sqrt{1-a^2x^2}}\right)dx\\
&= -\left(3\int \frac{x^m}{\sqrt{1-a^2x^2}}\,dx\right)+4\int \frac{x^m}{(1-ax)\sqrt{1-a^2x^2}}\,dx-a\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m}-\frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}+4\int \frac{x^m(1+ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx\\
&= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m}-\frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}+4\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}}\,dx+(4a)\int\\
&= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m}-\frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}+\frac{4x^{1+m}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m}
\end{aligned}
$$

Mathematica [C] time = 0.0787062, size = 92, normalized size = 0.61

$$
\frac{\sqrt{-ax-1}\sqrt{1-ax}x^{m+1}\left(F_1\left(m+1;-\frac{1}{2},\frac{1}{2};m+2;-ax,ax\right)-2F_1\left(m+1;-\frac{1}{2},\frac{3}{2};m+2;-ax,ax\right)\right)}{(m+1)\sqrt{ax-1}\sqrt{ax+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*Sqrt[-1 - a*x]*Sqrt[1 - a*x]*(AppellF1[1 + m, -1/2, 1/2, 2 + m,
-(a*x), a*x] - 2*AppellF1[1 + m, -1/2, 3/2, 2 + m, -(a*x), a*x]))/((1 + m)*
Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Maple [A] time = 0.343, size = 139, normalized size = 0.9

$$
\frac{x^{1+m}}{1+m}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{1}{2}+\frac{m}{2};\frac{3}{2}+\frac{m}{2};a^2x^2\right)+\frac{a^3x^{4+m}}{4+m}{}_2\mathrm{F}_1\left(\frac{3}{2},2+\frac{m}{2};3+\frac{m}{2};a^2x^2\right)+3\,\frac{a^2x^{3+m}{}_2\mathrm{F}_1(3/2,3/2+m/2;\,5/2+m/2;\,a^2x}{3+m}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m,x)
```

```
[Out] x^(1+m)*hypergeom([3/2,1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)+a^3/(4+m)*x^(4
+m)*hypergeom([3/2,2+1/2*m],[3+1/2*m],a^2*x^2)+3*a^2/(3+m)*x^(3+m)*hypergeo
m([3/2,3/2+1/2*m],[5/2+1/2*m],a^2*x^2)+3*a*x^(2+m)*hypergeom([3/2,1+1/2*m],
[2+1/2*m],a^2*x^2)/(2+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3 x^m}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^3*x^m/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}(ax+1)x^m}{a^2x^2-2\,ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*x^m/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\,(ax+1)^3}{(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**m,x)
```

```
[Out] Integral(x**m*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3 x^m}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3*x^m/(-a^2*x^2 + 1)^(3/2), x)
```

3.143 $\int e^{2\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{2x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{m+1}-\frac{x^{m+1}}{m+1}$$

```
[Out] -(x^(1 + m)/(1 + m)) + (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x]
)/(1 + m)
```

Rubi [A] time = 0.0245886, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6126, 80, 64}

$$\frac{2x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{m+1}-\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^m,x]
```

```
[Out] -(x^(1 + m)/(1 + m)) + (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x]
)/(1 + m)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^m\,dx&=\int\frac{x^m(1+ax)}{1-ax}\,dx\\&=-\frac{x^{1+m}}{1+m}+2\int\frac{x^m}{1-ax}\,dx\\&=-\frac{x^{1+m}}{1+m}+\frac{2x^{1+m}\,{}_2F_1(1,1+m;2+m;ax)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.0068547, size = 26, normalized size = 0.72

$$\frac{x^{m+1}(2\text{Hypergeometric2F1}(1,m+1,m+2,ax)-1)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(-1 + 2*Hypergeometric2F1[1, 1 + m, 2 + m, a*x]))/(1 + m)
```

Maple [C] time = 0.359, size = 184, normalized size = 5.1

$$-\frac{1}{2}\left(-a^2\right)^{-\frac{1}{2}-\frac{m}{2}}\left(2\frac{x^{1+m}\left(-a^2\right)^{3/2+m/2}(-3-m)}{(1+m)(3+m)a^2}+\frac{x^{1+m}}{a^2}\left(-a^2\right)^{\frac{3}{2}+\frac{m}{2}}\text{LerchPhi}\left(a^2x^2,1,\frac{1}{2}+\frac{m}{2}\right)\right)-\frac{1}{a}\left(-a^2\right)^{-\frac{m}{2}}\left(-2\frac{x^m\left(-a\right.}{r}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m,x)
```

```
[Out] -1/2*(-a^2)^(-1/2-1/2*m)*(2/(3+m)*x^(1+m)*(-a^2)^(3/2+1/2*m)*(-3-m)/(1+m)/a
^2+x^(1+m)*(-a^2)^(3/2+1/2*m)/a^2*LerchPhi(a^2*x^2,1,1/2+1/2*m))-1/a*(-a^2)
^(-1/2*m)*(-2/(2+m)*x^m*(-a^2)^(1/2*m)*(-m-2)/m-x^m*(-a^2)^(1/2*m)*LerchPhi
(a^2*x^2,1,1/2*m))+1/(1+m)*x^(1+m)*(1/2+1/2*m)*LerchPhi(a^2*x^2,1,1/2+1/2*m
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(ax+1)^2x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ax+1)x^m}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m,x, algorithm="fricas")
```

```
[Out] integral(-(a*x + 1)*x^m/(a*x - 1), x)
```

Sympy [B] time = 3.26814, size = 99, normalized size = 2.75

$$\frac{amx^2x^m\Phi\left(ax,1,m+2\right)\Gamma\left(m+2\right)}{\Gamma\left(m+3\right)}+\frac{2ax^2x^m\Phi\left(ax,1,m+2\right)\Gamma\left(m+2\right)}{\Gamma\left(m+3\right)}+\frac{mxx^m\Phi\left(ax,1,m+1\right)\Gamma\left(m+1\right)}{\Gamma\left(m+2\right)}+\frac{xx^m\Phi\left(ax,1,\right.}{\Gamma\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m,x)
```

```
[Out] a*m*x**2*x**m*lerchphi(a*x, 1, m + 2)*gamma(m + 2)/gamma(m + 3) + 2*a*x**2*
x**m*lerchphi(a*x, 1, m + 2)*gamma(m + 2)/gamma(m + 3) + m*x*x**m*lerchphi(
a*x, 1, m + 1)*gamma(m + 1)/gamma(m + 2) + x*x**m*lerchphi(a*x, 1, m + 1)*g
amma(m + 1)/gamma(m + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)^2x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

3.144 $\int e^{\tanh^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=74

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{m+1}+\frac{ax^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{m+2}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m) +
 (a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2 + m
)
```

Rubi [A] time = 0.0419184, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6124, 808, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m) +
 (a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2 + m
)
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^m\,dx = \int \frac{x^m(1+ax)}{\sqrt{1-a^2x^2}}\,dx$$
$$= a\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx + \int \frac{x^m}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m} + \frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}$$

Mathematica [C] time = 0.0299778, size = 70, normalized size = 0.95

$$-\frac{\sqrt{-ax-1}\sqrt{1-ax}x^{m+1}F_1\left(m+1;-\frac{1}{2},\frac{1}{2};m+2;-ax,ax\right)}{(m+1)\sqrt{ax-1}\sqrt{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^m,x]
```

```
[Out] -((x^(1 + m)*Sqrt[-1 - a*x]*Sqrt[1 - a*x]*AppellF1[1 + m, -1/2, 1/2, 2 + m,
 -(a*x), a*x])/((1 + m)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]))
```

Maple [A] time = 0.234, size = 67, normalized size = 0.9

$$\frac{x^{1+m}}{1+m}{}_2F_1\left(\frac{1}{2},\frac{1}{2}+\frac{m}{2};\frac{3}{2}+\frac{m}{2};a^2x^2\right)+\frac{ax^{2+m}}{2+m}{}_2F_1\left(\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};a^2x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m,x)
```

```
[Out] x^(1+m)*hypergeom([1/2,1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)+a*x^(2+m)*hype
rgeom([1/2,1+1/2*m],[2+1/2*m],a^2*x^2)/(2+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}x^m}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^m/(a*x - 1), x)
```

Sympy [C] time = 3.7963, size = 97, normalized size = 1.31

$$\frac{ax^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{xx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m,x)
```

```
[Out] a*x**2*x**m*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,), a**2*x**2*exp_
polar(2*I*pi))/(2*gamma(m/2 + 2)) + x*x**m*gamma(m/2 + 1/2)*hyper((1/2, m/2
 + 1/2), (m/2 + 3/2,), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

3.145 $\int e^{-\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=75

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{m+1}-\frac{ax^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{m+2}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m) -
 (a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2 + m
)
```

Rubi [A] time = 0.0445418, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6124, 808, 364}

$$\frac{x^{m+1}{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}-\frac{ax^{m+2}{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^ArcTanh[a*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m) -
 (a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2 + m
)
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}x^m\,dx = \int \frac{x^m(1-ax)}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\left(a\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx\right) + \int \frac{x^m}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m} - \frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}$$

Mathematica [C] time = 0.0271574, size = 31, normalized size = 0.41

$$\frac{x^{m+1}F_1\left(m+1;-\frac{1}{2},\frac{1}{2};m+2;ax,-ax\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/E^ArcTanh[a*x],x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -1/2, 1/2, 2 + m, a*x, -(a*x)])/(1 + m)
```

Maple [F] time = 0.381, size = 0, normalized size = 0.

$$\int \frac{x^m}{ax+1}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\,x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^m/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^m}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^m/(a*x + 1), x)
```

3.146 $\int e^{-2\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=37

$$\frac{2x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-ax)}{m+1}-\frac{x^{m+1}}{m+1}$$

```
[Out] -(x^(1 + m)/(1 + m)) + (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*
x)])/(1 + m)
```

Rubi [A] time = 0.0249729, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6126, 80, 64}

$$\frac{2x^{m+1}\,{}_2F_1(1,m+1;m+2;-ax)}{m+1}-\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(x^(1 + m)/(1 + m)) + (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*
x)])/(1 + m)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1-ax)}{1+ax}\,dx\\ &= -\frac{x^{1+m}}{1+m}+2\int \frac{x^m}{1+ax}\,dx\\ &= -\frac{x^{1+m}}{1+m}+\frac{2x^{1+m}\,{}_2F_1(1,1+m;2+m;-ax)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.0074615, size = 27, normalized size = 0.73

$$\frac{x^{m+1}(2\text{Hypergeometric2F1}(1,m+1,m+2,-ax)-1)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (x^(1 + m)*(-1 + 2*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)]))/(1 + m)
```

Maple [C] time = 0.36, size = 126, normalized size = 3.4

$$-a^{-1-m}\left(\frac{x^m a^m \left(a^2 m x^2-a m x-2\,a x-m^2-3\,m-2\right)}{m\,(1+m)\,(ax+1)}+x^m a^m\,(2+m)\,\mathrm{LerchPhi}\,(-ax,1,m)\right)+a^{-1-m}\left(\frac{x^m a^m\,(-1-m)}{(1+m)\,(ax+1)}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -a^(-1-m)*(x^m*a^m*(a^2*m*x^2-a*m*x-2*a*x-m^2-3*m-2)/(1+m)/m/(a*x+1)+x^m*a^
m*(2+m)*LerchPhi(-a*x,1,m))+a^(-1-m)*(1/(1+m)*x^m*a^m*(-1-m)/(a*x+1)+x^m*a^
m*m*LerchPhi(-a*x,1,m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(a^2x^2-1\right)x^m}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*x^m/(a*x + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ax-1)x^m}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-(a*x - 1)*x^m/(a*x + 1), x)
```

Sympy [C] time = 3.69069, size = 119, normalized size = 3.22

$$-\frac{amx^2x^m\Phi\left(axe^{i\pi},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)}-\frac{2ax^2x^m\Phi\left(axe^{i\pi},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)}+\frac{mxx^m\Phi\left(axe^{i\pi},1,m+1\right)\Gamma(m+1)}{\Gamma(m+2)}+\frac{xx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -a*m*x**2*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 2)*gamma(m + 2)/gamma(m
 + 3) - 2*a*x**2*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 2)*gamma(m + 2)/
gamma(m + 3) + m*x*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 1)*gamma(m + 1
)/gamma(m + 2) + x*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 1)*gamma(m + 1
)/gamma(m + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(a^2x^2-1)x^m}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*x^m/(a*x + 1)^2, x)
```

3.147 $\int e^{-3\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=150

$$-\frac{3x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{m+1}+\frac{4x^{m+1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{m+1}+\frac{ax^{m+2}\text{Hyperge}}{}$$

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m
) + (a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2
+ m) + (4*x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/
(1 + m) - (4*a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, a^2*x
^2])/(2 + m)
```

Rubi [A] time = 0.813987, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6124, 6742, 364, 850, 808}

$$-\frac{3x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{4x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}-\frac{4ax^{m+2}\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\right.}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m
) + (a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2
+ m) + (4*x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/
(1 + m) - (4*a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, a^2*x
^2])/(2 + m)
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1-ax)^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx\\
&= \int \left(-\frac{3x^m}{\sqrt{1-a^2x^2}}+\frac{ax^{1+m}}{\sqrt{1-a^2x^2}}+\frac{4x^m}{(1+ax)\sqrt{1-a^2x^2}}\right)dx\\
&= -\left(3\int \frac{x^m}{\sqrt{1-a^2x^2}}\,dx\right)+4\int \frac{x^m}{(1+ax)\sqrt{1-a^2x^2}}\,dx+a\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m}+\frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}+4\int \frac{x^m(1-ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx\\
&= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m}+\frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}+4\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}}\,dx-(4a\\
&= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m}+\frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}+\frac{4x^{1+m}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};a^2\right.}{1+m}
\end{aligned}$$

Mathematica [C] time = 0.0517339, size = 55, normalized size = 0.37

$$-\frac{x^{m+1}\left(F_1\left(m+1;-\frac{1}{2},\frac{1}{2};m+2;ax,-ax\right)-2F_1\left(m+1;-\frac{1}{2},\frac{3}{2};m+2;ax,-ax\right)\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((x^(1 + m)*(AppellF1[1 + m, -1/2, 1/2, 2 + m, a*x, -(a*x)] - 2*AppellF1[1
 + m, -1/2, 3/2, 2 + m, a*x, -(a*x)]))/(1 + m))
```

Maple [F] time = 0.332, size = 0, normalized size = 0.

$$\int \frac{x^m}{(ax+1)^3}\left(-a^2x^2+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^m/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^m}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^m/(a*x + 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}(ax-1)x^m}{a^2x^2+2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^m/(a^2*x^2 + 2*a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**m*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^m}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^m/(a*x + 1)^3, x)
```

3.148 $\int e^{n \tanh^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=35

$$\frac{x^{m+1} F_1\left(m+1; \frac{n}{2}, -\frac{n}{2}; m+2; ax, -ax\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, n/2, -n/2, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rubi [A] time = 0.0295276, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 133}

$$\frac{x^{m+1} F_1\left(m+1; \frac{n}{2}, -\frac{n}{2}; m+2; ax, -ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, n/2, -n/2, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{n \tanh^{-1}(ax)} x^m \, dx &= \int x^m (1-ax)^{-n/2} (1+ax)^{n/2} \, dx \\ &= \frac{x^{1+m} F_1\left(1+m; \frac{n}{2}, -\frac{n}{2}; 2+m; ax, -ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.224708, size = 0, normalized size = 0.

$$\int e^{n \tanh^{-1}(ax)} x^m \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^m,x]
```

```
[Out] Integrate[E^(n*ArcTanh[a*x])*x^m, x]
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)} x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^m,x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m e^{n \operatorname{atanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**m,x)
```

```
[Out] Integral(x**m*exp(n*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.149 $\int e^{n \tanh^{-1}(ax)} x^3 \, dx$

Optimal. Leaf size=155

$$-\frac{2^{\frac{n}{2}-2} n \left(n^2+8\right) (1-ax)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{3a^4(2-n)} - \frac{(ax+1)^{\frac{n+2}{2}} \left(2anx+n^2+6\right) (1-ax)^{1-}}{24a^4}$$

```
[Out] -(x^2*(1 - a*x)^(1 - n/2)*(1 + a*x)^((2 + n)/2))/(4*a^2) - ((1 - a*x)^(1 -
n/2)*(1 + a*x)^((2 + n)/2)*(6 + n^2 + 2*a*n*x))/(24*a^4) - (2^(-2 + n/2)*n*
(8 + n^2)*(1 - a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1
- a*x)/2])/(3*a^4*(2 - n))
```

Rubi [A] time = 0.117522, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6126, 100, 147, 69}

$$-\frac{2^{\frac{n}{2}-2} n \left(n^2+8\right) (1-ax)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{3a^4(2-n)} - \frac{(ax+1)^{\frac{n+2}{2}} \left(2anx+n^2+6\right) (1-ax)^{1-\frac{n}{2}}}{24a^4} - \frac{x^2(ax+1)^{\frac{n+2}{2}} (}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^3,x]
```

```
[Out] -(x^2*(1 - a*x)^(1 - n/2)*(1 + a*x)^((2 + n)/2))/(4*a^2) - ((1 - a*x)^(1 -
n/2)*(1 + a*x)^((2 + n)/2)*(6 + n^2 + 2*a*n*x))/(24*a^4) - (2^(-2 + n/2)*n*
(8 + n^2)*(1 - a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1
- a*x)/2])/(3*a^4*(2 - n))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}x^3\,dx = \int x^3(1-ax)^{-n/2}(1+ax)^{n/2}\,dx$$
$$= -\frac{x^2(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{4a^2} - \frac{\int x(1-ax)^{-n/2}(1+ax)^{n/2}(-2-anx)\,dx}{4a^2}$$
$$= -\frac{x^2(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{4a^2} - \frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}\left(6+n^2+2anx\right)}{24a^4} + \frac{\left(n\left(8+n^2\right)\right)\int(1-ax)^{-}}{24a^3}$$
$$= -\frac{x^2(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{4a^2} - \frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}\left(6+n^2+2anx\right)}{24a^4} - \frac{2^{-2+\frac{n}{2}}n\left(8+n^2\right)(1-ax}{}$$

Mathematica [A] time = 0.1721, size = 182, normalized size = 1.17

$$-\frac{(1-ax)^{1-\frac{n}{2}}\left((n-2)\left(a^2x^2(ax+1)^{\frac{n}{2}+1}-2^{\frac{n}{2}+1}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right)\right)-2^{\frac{n}{2}+3}n\text{Hyperge}}{4a^4(n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^3,x]
```

```
[Out] -((1 - a*x)^(1 - n/2)*(-(2^(3 + n/2)*n*Hypergeometric2F1[-2 - n/2, 1 - n/2,
 2 - n/2, (1 - a*x)/2]) + 2^(3 + n/2)*(-1 + n)*Hypergeometric2F1[-1 - n/2,
1 - n/2, 2 - n/2, (1 - a*x)/2] + (-2 + n)*(a^2*x^2*(1 + a*x)^(1 + n/2) - 2^
(1 + n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2])))/(4*a^4*
(-2 + n))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^3,x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 e^{n \operatorname{atanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**3,x)
```

```
[Out] Integral(x**3*exp(n*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.150 $\int e^{n\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=141

$$-\frac{2^{n/2}\left(n^2+2\right)(1-ax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{3a^3(2-n)}-\frac{n(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{6a^3}-\frac{x(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{3a^2}$$

```
[Out] -(n*(1 - a*x)^(1 - n/2)*(1 + a*x)^((2 + n)/2))/(6*a^3) - (x*(1 - a*x)^(1 -
n/2)*(1 + a*x)^((2 + n)/2))/(3*a^2) - (2^(n/2)*(2 + n^2)*(1 - a*x)^(1 - n/2
)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2])/(3*a^3*(2 - n))
```

Rubi [A] time = 0.0948837, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6126, 90, 80, 69}

$$-\frac{2^{n/2}\left(n^2+2\right)(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{3a^3(2-n)}-\frac{n(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{6a^3}-\frac{x(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^2,x]
```


```
[Out] -(n*(1 - a*x)^(1 - n/2)*(1 + a*x)^((2 + n)/2))/(6*a^3) - (x*(1 - a*x)^(1 -
n/2)*(1 + a*x)^((2 + n)/2))/(3*a^2) - (2^(n/2)*(2 + n^2)*(1 - a*x)^(1 - n/2
)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2])/(3*a^3*(2 - n))
```


Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int e^{n\tanh^{-1}(ax)}x^2\,dx &= \int x^2(1-ax)^{-n/2}(1+ax)^{n/2}\,dx\\
&= -\frac{x(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{3a^2}-\frac{\int (1-ax)^{-n/2}(1+ax)^{n/2}(-1-anx)\,dx}{3a^2}\\
&= -\frac{n(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{6a^3}-\frac{x(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{3a^2}+\frac{\left(2+n^2\right)\int (1-ax)^{-n/2}(1+ax)^{n/2}\,dx}{6a^2}\\
&= -\frac{n(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{6a^3}-\frac{x(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{3a^2}-\frac{2^{n/2}\left(2+n^2\right)(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\right.}{3a^3(2-n)}
\end{aligned}$$

Mathematica [A] time = 0.051358, size = 96, normalized size = 0.68

$$-\frac{(1-ax)^{1-\frac{n}{2}}\left((n-2)(ax+1)^{\frac{n}{2}+1}(2ax+n)-2^{\frac{n}{2}+1}\left(n^2+2\right)\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right)\right)}{6a^3(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^2,x]
```

```
[Out] -((1 - a*x)^(1 - n/2)*((-2 + n)*(1 + a*x)^(1 + n/2)*(n + 2*a*x) - 2^(1 + n/
2)*(2 + n^2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2]))/(6*a^
3*(-2 + n))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 e^{n \operatorname{atanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**2,x)
```

```
[Out] Integral(x**2*exp(n*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.151 $\int e^{n\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=99

$$-\frac{2^{n/2}n(1-ax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a^2(2-n)}-\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{2a^2}$$

```
[Out] -((1 - a*x)^(1 - n/2)*(1 + a*x)^((2 + n)/2))/(2*a^2) - (2^(n/2)*n*(1 - a*x)
^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2])/(a^2*(2
- n))
```

Rubi [A] time = 0.0414173, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6126, 80, 69}

$$-\frac{2^{n/2}n(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^2(2-n)}-\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x,x]
```

```
[Out] -((1 - a*x)^(1 - n/2)*(1 + a*x)^((2 + n)/2))/(2*a^2) - (2^(n/2)*n*(1 - a*x)
^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2])/(a^2*(2
- n))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}x\,dx = \int x(1-ax)^{-n/2}(1+ax)^{n/2}\,dx$$
$$= -\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{2a^2} + \frac{n\int(1-ax)^{-n/2}(1+ax)^{n/2}\,dx}{2a}$$
$$= -\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{2a^2} - \frac{2^{n/2}n(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^2(2-n)}$$

Mathematica [A] time = 0.0242738, size = 86, normalized size = 0.87

$$-\frac{(1-ax)^{1-\frac{n}{2}}\left((n-2)(ax+1)^{\frac{n}{2}+1}-2^{\frac{n}{2}+1}n\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right)\right)}{2a^2(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x,x]
```

```
[Out] -((1 - a*x)^(1 - n/2)*((-2 + n)*(1 + a*x)^(1 + n/2) - 2^(1 + n/2)*n*Hyperge
ometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2]))/(2*a^2*(-2 + n))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x,x)
```

```
[Out] int(exp(n*arctanh(a*x))*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x,x, algorithm="fricas")
```

```
[Out] integral(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{n\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x,x)
```

```
[Out] Integral(x*exp(n*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x,x, algorithm="giac")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.152 $\int e^{n\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=65

$$-\frac{2^{\frac{n}{2}+1}(1-ax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a(2-n)}$$

```
[Out] -((2^(1 + n/2)*(1 - a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2
, (1 - a*x)/2])/(a*(2 - n)))
```

Rubi [A] time = 0.0116714, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6125, 69}

$$-\frac{2^{\frac{n}{2}+1}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x]),x]
```

```
[Out] -((2^(1 + n/2)*(1 - a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2
, (1 - a*x)/2])/(a*(2 - n)))
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}\,dx &= \int (1-ax)^{-n/2}(1+ax)^{n/2}\,dx\\ &= -\frac{2^{1+\frac{n}{2}}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(2-n)}\end{aligned}$$

Mathematica [A] time = 0.0295074, size = 46, normalized size = 0.71

$$\frac{4e^{(n+2)\tanh^{-1}(ax)}\text{Hypergeometric2F1}\left(2,\frac{n}{2}+1,\frac{n}{2}+2,-e^{2\tanh^{-1}(ax)}\right)}{a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x]),x]
```

```
[Out] (4*E^((2 + n)*ArcTanh[a*x])*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E^(2*Ar
cTanh[a*x])])/(a*(2 + n))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int e^{n\operatorname{Artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x)),x)
```

```
[Out] int(exp(n*arctanh(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x)),x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x)),x)
```

```
[Out] Integral(exp(n*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x)),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.153 $\int \frac{e^{n\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=111

$$\frac{2(1-ax)^{-n/2}(ax+1)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{1-ax}{ax+1}\right)}{n}-\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\right.}{n}$$

```
[Out] (2*(1 + a*x)^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (1 - a*x)/(1 + a*x)]
)/(n*(1 - a*x)^(n/2)) - (2^(1 + n/2)*Hypergeometric2F1[-n/2, -n/2, 1 - n/2,
 (1 - a*x)/2])/(n*(1 - a*x)^(n/2))
```

Rubi [A] time = 0.0509592, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6126, 105, 69, 131}

$$\frac{2(1-ax)^{-n/2}(ax+1)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{n}-\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/x,x]
```

```
[Out] (2*(1 + a*x)^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (1 - a*x)/(1 + a*x)]
)/(n*(1 - a*x)^(n/2)) - (2^(1 + n/2)*Hypergeometric2F1[-n/2, -n/2, 1 - n/2,
 (1 - a*x)/2])/(n*(1 - a*x)^(n/2))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
```

```
f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x}\,dx = \int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{x}\,dx$$
$$= -\left(a\int (1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}\,dx\right)+\int \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}}{x}\,dx$$
$$= \frac{2(1-ax)^{-n/2}(1+ax)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{1-ax}{1+ax}\right)}{n}-\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{n}$$

Mathematica [A] time = 0.0202705, size = 95, normalized size = 0.86

$$\frac{2(1-ax)^{-n/2}\left((ax+1)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{1-ax}{ax+1}\right)-2^{n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}(1-\right.\right.}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/x,x]
```

```
[Out] (2*((1 + a*x)^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (1 - a*x)/(1 + a*x)
] - 2^(n/2)*Hypergeometric2F1[-n/2, -n/2, 1 - n/2, (1 - a*x)/2]))/(n*(1 - a
*x)^(n/2))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{e^{n\mathrm{Artanh}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x,x)
```

```
[Out] int(exp(n*arctanh(a*x))/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n\,\text{atanh}\,(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x,x)
```

```
[Out] Integral(exp(n*atanh(a*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

3.154 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=67

$$-\frac{4a(1-ax)^{1-\frac{n}{2}}(ax+1)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-ax}{ax+1}\right)}{2-n}$$

```
[Out] (-4*a*(1 - a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n
/2, 2 - n/2, (1 - a*x)/(1 + a*x)])/(2 - n)
```

Rubi [A] time = 0.0311097, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 131}

$$-\frac{4a(1-ax)^{1-\frac{n}{2}}(ax+1)^{\frac{n-2}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{2-n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/x^2,x]
```

```
[Out] (-4*a*(1 - a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n
/2, 2 - n/2, (1 - a*x)/(1 + a*x)])/(2 - n)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{x^2}\,dx\\ &= -\frac{4a(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{1+ax}\right)}{2-n}\end{aligned}$$

Mathematica [A] time = 0.0163086, size = 65, normalized size = 0.97

$$\frac{4a(1-ax)^{1-\frac{n}{2}}(ax+1)^{\frac{n}{2}-1}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-ax}{ax+1}\right)}{n-2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/x^2,x]
```

```
[Out] (4*a*(1 - a*x)^(1 - n/2)*(1 + a*x)^(-1 + n/2)*Hypergeometric2F1[2, 1 - n/2,
 2 - n/2, (1 - a*x)/(1 + a*x)])/(-2 + n)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{e^{n\operatorname{Artanh}(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n\operatorname{atanh}(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**2,x)
```

```
[Out] Integral(exp(n*atanh(a*x))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

3.155 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=105

$$-\frac{2a^2n(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-ax}{ax+1}\right)}{2-n}-\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{2x^2}$$

```
[Out] -((1 - a*x)^(1 - n/2)*(1 + a*x)^((2 + n)/2))/(2*x^2) - (2*a^2*n*(1 - a*x)^(
1 - n/2)*(1 + a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, (1 -
 a*x)/(1 + a*x)])/(2 - n)
```

Rubi [A] time = 0.0489941, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6126, 96, 131}

$$-\frac{2a^2n(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{2-n}-\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/x^3,x]
```

```
[Out] -((1 - a*x)^(1 - n/2)*(1 + a*x)^((2 + n)/2))/(2*x^2) - (2*a^2*n*(1 - a*x)^(
1 - n/2)*(1 + a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, (1 -
 a*x)/(1 + a*x)])/(2 - n)
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^3}\,dx = \int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{x^3}\,dx$$
$$= -\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{2x^2} + \frac{1}{2}(an)\int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{x^2}\,dx$$
$$= -\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{2x^2} - \frac{2a^2n(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{1+ax}\right)}{2-n}$$

Mathematica [A] time = 0.029136, size = 91, normalized size = 0.87

$$\frac{(1-ax)^{1-\frac{n}{2}}(ax+1)^{\frac{n}{2}-1}\left(4a^2nx^2\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-ax}{ax+1}\right)-(n-2)(ax+1)^2\right)}{2(n-2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/x^3,x]
```

```
[Out] ((1 - a*x)^(1 - n/2)*(1 + a*x)^(-1 + n/2)*(-((-2 + n)*(1 + a*x)^2) + 4*a^2*
n*x^2*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, (1 - a*x)/(1 + a*x)]))/(2*(-2
+ n)*x^2)
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{e^{n\text{Artanh}(ax)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^3,x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{atanh}(ax)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**3,x)
```

```
[Out] Integral(exp(n*atanh(a*x))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)
```

3.156 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=147

$$-\frac{2a^3\left(n^2+2\right)(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-ax}{ax+1}\right)}{3(2-n)}-\frac{an(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{6x^2}-\frac{(ax+$$

```
[Out] -((1 - a*x)^(1 - n/2)*(1 + a*x)^((2 + n)/2))/(3*x^3) - (a*n*(1 - a*x)^(1 -
n/2)*(1 + a*x)^((2 + n)/2))/(6*x^2) - (2*a^3*(2 + n^2)*(1 - a*x)^(1 - n/2)*
(1 + a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, (1 - a*x)/(1
+ a*x)])/(3*(2 - n))
```

Rubi [A] time = 0.0676119, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6126, 129, 151, 12, 131}

$$-\frac{2a^3\left(n^2+2\right)(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{3(2-n)}-\frac{an(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{6x^2}-\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/x^4,x]
```

```
[Out] -((1 - a*x)^(1 - n/2)*(1 + a*x)^((2 + n)/2))/(3*x^3) - (a*n*(1 - a*x)^(1 -
n/2)*(1 + a*x)^((2 + n)/2))/(6*x^2) - (2*a^3*(2 + n^2)*(1 - a*x)^(1 - n/2)*
(1 + a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, (1 - a*x)/(1
+ a*x)])/(3*(2 - n))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\tanh^{-1}(ax)}}{x^4}\,dx &= \int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{x^4}\,dx \\
&= -\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{3x^3} - \frac{1}{3}\int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}\left(-an-a^2x\right)}{x^3}\,dx \\
&= -\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{3x^3} - \frac{an(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{6x^2} + \frac{1}{6}\int \frac{a^2\left(2+n^2\right)(1-ax)^{-n/2}(1+ax)^{n/2}}{x^2}\,dx \\
&= -\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{3x^3} - \frac{an(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{6x^2} + \frac{1}{6}\left(a^2\left(2+n^2\right)\right)\int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{x^2}\,dx \\
&= -\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{3x^3} - \frac{an(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{6x^2} - \frac{2a^3\left(2+n^2\right)(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(\right.}{3(2-n)}
\end{aligned}$$

Mathematica [A] time = 0.0437709, size = 101, normalized size = 0.69

$$\frac{(1-ax)^{1-\frac{n}{2}}(ax+1)^{\frac{n}{2}-1}\left(4a^3\left(n^2+2\right)x^3\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-ax}{ax+1}\right)-(n-2)(ax+1)^2(anx+2)\right)}{6(n-2)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/x^4,x]
```

```
[Out] ((1 - a*x)^(1 - n/2)*(1 + a*x)^(-1 + n/2)*(-((-2 + n)*(1 + a*x)^2*(2 + a*n*
x)) + 4*a^3*(2 + n^2)*x^3*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, (1 - a*x)/
(1 + a*x)]))/(6*(-2 + n)*x^3)
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \frac{e^{n\text{Artanh}(ax)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^4,x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n\,\mathrm{atanh}\,(ax)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**4,x)
```

```
[Out] Integral(exp(n*atanh(a*x))/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

3.157 $\int e^{\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=65

$$-\frac{2\sqrt{2}(c-acx)^{p+1}\text{Hypergeometric2F1}\left(-\frac{1}{2},p+\frac{1}{2},p+\frac{3}{2},\frac{1}{2}(1-ax)\right)}{ac(2p+1)\sqrt{1-ax}}$$

```
[Out] (-2*Sqrt[2]*(c - a*c*x)^(1 + p)*Hypergeometric2F1[-1/2, 1/2 + p, 3/2 + p, (
1 - a*x)/2])/(a*c*(1 + 2*p)*Sqrt[1 - a*x])
```

Rubi [A] time = 0.0490697, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6130, 23, 69}

$$-\frac{2\sqrt{2}(c-acx)^{p+1}\,{}_2F_1\left(-\frac{1}{2},p+\frac{1}{2};p+\frac{3}{2};\frac{1}{2}(1-ax)\right)}{ac(2p+1)\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^p,x]
```

```
[Out] (-2*Sqrt[2]*(c - a*c*x)^(1 + p)*Hypergeometric2F1[-1/2, 1/2 + p, 3/2 + p, (
1 - a*x)/2])/(a*c*(1 + 2*p)*Sqrt[1 - a*x])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}(c-acx)^p\,dx = \int \frac{\sqrt{1+ax}(c-acx)^p}{\sqrt{1-ax}}\,dx$$
$$= \frac{\sqrt{c-acx}\int \sqrt{1+ax}(c-acx)^{-\frac{1}{2}+p}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{2}(c-acx)^{1+p}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}+p;\frac{3}{2}+p;\frac{1}{2}(1-ax)\right)}{ac(1+2p)\sqrt{1-ax}}$$

Mathematica [A] time = 0.0307093, size = 53, normalized size = 0.82

$$-\frac{2\sqrt{2-2ax}(c-acx)^p\text{Hypergeometric2F1}\left(-\frac{1}{2},p+\frac{1}{2},p+\frac{3}{2},\frac{1}{2}-\frac{ax}{2}\right)}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^p,x]
```

```
[Out] (-2*Sqrt[2 - 2*a*x]*(c - a*c*x)^p*Hypergeometric2F1[-1/2, 1/2 + p, 3/2 + p,
 1/2 - (a*x)/2])/(a + 2*a*p)
```

Maple [F] time = 0.378, size = 0, normalized size = 0.

$$\int (ax+1)\,(-acx+c)^p\,\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^p,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)(-acx+c)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(-a*c*x + c)^p/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}(-acx+c)^p}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^p/(a*x - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c(ax-1))^p (ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1))**p*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)(-acx+c)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(-a*c*x + c)^p/sqrt(-a^2*x^2 + 1), x)
```

3.158 $\int e^{\tanh^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=123

$$\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7}{8}c^4x\sqrt{1-a^2x^2}+\frac{7c^4\sin^{-1}(ax)}{8a}$$

```
[Out] (7*c^4*x*Sqrt[1 - a^2*x^2])/8 + (7*c^4*(1 - a^2*x^2)^(3/2))/(12*a) + (7*c^4
*(1 - a*x)*(1 - a^2*x^2)^(3/2))/(20*a) + (c^4*(1 - a*x)^2*(1 - a^2*x^2)^(3/
2))/(5*a) + (7*c^4*ArcSin[a*x])/(8*a)
```

Rubi [A] time = 0.0804764, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6127, 671, 641, 195, 216}

$$\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7}{8}c^4x\sqrt{1-a^2x^2}+\frac{7c^4\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^4,x]
```

```
[Out] (7*c^4*x*Sqrt[1 - a^2*x^2])/8 + (7*c^4*(1 - a^2*x^2)^(3/2))/(12*a) + (7*c^4
*(1 - a*x)*(1 - a^2*x^2)^(3/2))/(20*a) + (c^4*(1 - a*x)^2*(1 - a^2*x^2)^(3/
2))/(5*a) + (7*c^4*ArcSin[a*x])/(8*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}(c-acx)^4\,dx = c\int (c-acx)^3\sqrt{1-a^2x^2}\,dx$$
$$= \frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a} + \frac{1}{5}\left(7c^2\right)\int (c-acx)^2\sqrt{1-a^2x^2}\,dx$$
$$= \frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a} + \frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a} + \frac{1}{4}\left(7c^3\right)\int (c-acx)\sqrt{1-a^2x^2}\,dx$$
$$= \frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a} + \frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a} + \frac{1}{4}\left(7c^4\right)\int \sqrt{1-a^2}$$
$$= \frac{7}{8}c^4x\sqrt{1-a^2x^2} + \frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a} + \frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a} +$$
$$= \frac{7}{8}c^4x\sqrt{1-a^2x^2} + \frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a} + \frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a} +$$

Mathematica [A] time = 0.106434, size = 75, normalized size = 0.61

$$-\frac{c^4\left(\sqrt{1-a^2x^2}\left(24a^4x^4-90a^3x^3+112a^2x^2-15ax-136\right)+210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^4,x]
```

```
[Out] -(c^4*(Sqrt[1 - a^2*x^2]*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3 + 24*a^4
*x^4) + 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(120*a)
```

Maple [A] time = 0.045, size = 137, normalized size = 1.1

$$-\frac{c^4a^3x^4}{5}\sqrt{-a^2x^2+1} - \frac{14\,c^4ax^2}{15}\sqrt{-a^2x^2+1} + \frac{17\,c^4}{15\,a}\sqrt{-a^2x^2+1} + \frac{3\,a^2c^4x^3}{4}\sqrt{-a^2x^2+1} + \frac{c^4x}{8}\sqrt{-a^2x^2+1} + \frac{7\,c^4}{8}\arctan\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4,x)
```

```
[Out] -1/5*c^4*a^3*x^4*(-a^2*x^2+1)^(1/2)-14/15*c^4*a*x^2*(-a^2*x^2+1)^(1/2)+17/1
5*c^4*(-a^2*x^2+1)^(1/2)/a+3/4*c^4*a^2*x^3*(-a^2*x^2+1)^(1/2)+1/8*c^4*x*(-a
^2*x^2+1)^(1/2)+7/8*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2)
)
```

Maxima [A] time = 1.43655, size = 171, normalized size = 1.39

$$-\frac{1}{5}\sqrt{-a^2x^2+1}a^3c^4x^4 + \frac{3}{4}\sqrt{-a^2x^2+1}a^2c^4x^3 - \frac{14}{15}\sqrt{-a^2x^2+1}ac^4x^2 + \frac{1}{8}\sqrt{-a^2x^2+1}c^4x + \frac{7\,c^4\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\sqrt{a^2}} + \frac{17\sqrt{-a^2x}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-a^2*x^2 + 1)*a^3*c^4*x^4 + 3/4*sqrt(-a^2*x^2 + 1)*a^2*c^4*x^3 -
14/15*sqrt(-a^2*x^2 + 1)*a*c^4*x^2 + 1/8*sqrt(-a^2*x^2 + 1)*c^4*x + 7/8*c^4
*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + 17/15*sqrt(-a^2*x^2 + 1)*c^4/a
```

Fricas [A] time = 1.57251, size = 209, normalized size = 1.7

$$-\frac{210\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(24\,a^4c^4x^4-90\,a^3c^4x^3+112\,a^2c^4x^2-15\,ac^4x-136\,c^4\right)\sqrt{-a^2x^2+1}}{120\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/120*(210*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (24*a^4*c^4*x^4 -
90*a^3*c^4*x^3 + 112*a^2*c^4*x^2 - 15*a*c^4*x - 136*c^4)*sqrt(-a^2*x^2 + 1)
)/a
```

Sympy [A] time = 8.06879, size = 226, normalized size = 1.84

$$\begin{cases}\frac{3c^4\sqrt{-a^2x^2+1}+2c^4\left(\begin{cases}-\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\operatorname{asin}(ax)}{2} & \text{for } ax>-1\wedge ax<1\end{cases}\right)+2c^4\left(\begin{cases}\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3}-\sqrt{-a^2x^2+1} & \text{for } ax>-1\wedge ax<\end{cases}\right.}{} \\ c^4x\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4,x)
```

```
[Out] Piecewise(((3*c**4*sqrt(-a**2*x**2 + 1) + 2*c**4*Piecewise((-a*x*sqrt(-a**2
*x**2 + 1)/2 + asin(a*x)/2, (a*x > -1) & (a*x < 1))) + 2*c**4*Piecewise(((-
a**2*x**2 + 1)**(3/2)/3 - sqrt(-a**2*x**2 + 1), (a*x > -1) & (a*x < 1))) -
3*c**4*Piecewise((a*x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/8 - a*x*sqrt(
-a**2*x**2 + 1)/2 + 3*asin(a*x)/8, (a*x > -1) & (a*x < 1))) + c**4*Piecewis
e((-(-a**2*x**2 + 1)**(5/2)/5 + 2*(-a**2*x**2 + 1)**(3/2)/3 - sqrt(-a**2*x*
*2 + 1), (a*x > -1) & (a*x < 1))) + c**4*asin(a*x))/a, Ne(a, 0)), (c**4*x,
True))
```

Giac [A] time = 1.23868, size = 105, normalized size = 0.85

$$\frac{7\,c^4\arcsin(ax)\operatorname{sgn}(a)}{8\,|a|}+\frac{1}{120}\,\sqrt{-a^2x^2+1}\left(\frac{136\,c^4}{a}+\left(15\,c^4-2\left(56\,ac^4+3\left(4\,a^3c^4x-15\,a^2c^4\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 7/8*c^4*arcsin(a*x)*sgn(a)/abs(a) + 1/120*sqrt(-a^2*x^2 + 1)*(136*c^4/a + (
15*c^4 - 2*(56*a*c^4 + 3*(4*a^3*c^4*x - 15*a^2*c^4)*x)*x)*x)
```

3.159 $\int e^{\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=91

$$\frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a}+\frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{5}{8}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\sin^{-1}(ax)}{8a}$$

```
[Out] (5*c^3*x*Sqrt[1 - a^2*x^2])/8 + (5*c^3*(1 - a^2*x^2)^(3/2))/(12*a) + (c^3*(
1 - a*x)*(1 - a^2*x^2)^(3/2))/(4*a) + (5*c^3*ArcSin[a*x])/(8*a)
```

Rubi [A] time = 0.0559282, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6127, 671, 641, 195, 216}

$$\frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a}+\frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{5}{8}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^3,x]
```

```
[Out] (5*c^3*x*Sqrt[1 - a^2*x^2])/8 + (5*c^3*(1 - a^2*x^2)^(3/2))/(12*a) + (c^3*(
1 - a*x)*(1 - a^2*x^2)^(3/2))/(4*a) + (5*c^3*ArcSin[a*x])/(8*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(ax)}(c-acx)^3\,dx &= c\int (c-acx)^2\sqrt{1-a^2x^2}\,dx\\
&= \frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a}+\frac{1}{4}\left(5c^2\right)\int (c-acx)\sqrt{1-a^2x^2}\,dx\\
&= \frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a}+\frac{1}{4}\left(5c^3\right)\int \sqrt{1-a^2x^2}\,dx\\
&= \frac{5}{8}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a}+\frac{1}{8}\left(5c^3\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{5}{8}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a}+\frac{5c^3\sin^{-1}(ax)}{8a}
\end{aligned}
$$

Mathematica [A] time = 0.0878642, size = 67, normalized size = 0.74

$$
\frac{c^3\left(\sqrt{1-a^2x^2}\left(6a^3x^3-16a^2x^2+9ax+16\right)-30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(Sqrt[1 - a^2*x^2]*(16 + 9*a*x - 16*a^2*x^2 + 6*a^3*x^3) - 30*ArcSin[S
qrt[1 - a*x]/Sqrt[2]]))/(24*a)
```

Maple [A] time = 0.039, size = 114, normalized size = 1.3

$$
\frac{c^3a^2x^3}{4}\sqrt{-a^2x^2+1}+\frac{3\,c^3x}{8}\sqrt{-a^2x^2+1}+\frac{5\,c^3}{8}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{2\,c^3ax^2}{3}\sqrt{-a^2x^2+1}+\frac{2\,c^3}{3\,a}\sqrt{-a^2x^2+}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3,x)
```

```
[Out] 1/4*c^3*a^2*x^3*(-a^2*x^2+1)^(1/2)+3/8*c^3*x*(-a^2*x^2+1)^(1/2)+5/8*c^3/(a^
2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-2/3*c^3*a*x^2*(-a^2*x^2+1
)^(1/2)+2/3*c^3*(-a^2*x^2+1)^(1/2)/a
```

Maxima [A] time = 1.43166, size = 140, normalized size = 1.54

$$
\frac{1}{4}\sqrt{-a^2x^2+1}a^2c^3x^3-\frac{2}{3}\sqrt{-a^2x^2+1}ac^3x^2+\frac{3}{8}\sqrt{-a^2x^2+1}c^3x+\frac{5\,c^3\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\sqrt{a^2}}+\frac{2\sqrt{-a^2x^2+1}c^3}{3\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(-a^2*x^2 + 1)*a^2*c^3*x^3 - 2/3*sqrt(-a^2*x^2 + 1)*a*c^3*x^2 + 3/8
*sqrt(-a^2*x^2 + 1)*c^3*x + 5/8*c^3*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + 2/3
```

```
*sqrt(-a^2*x^2 + 1)*c^3/a
```

Fricas [A] time = 1.64638, size = 178, normalized size = 1.96

$$-\frac{30\,c^{3}\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)-\left(6\,a^{3}c^{3}x^{3}-16\,a^{2}c^{3}x^{2}+9\,ac^{3}x+16\,c^{3}\right)\sqrt{-a^{2}x^{2}+1}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(30*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (6*a^3*c^3*x^3 - 16*
a^2*c^3*x^2 + 9*a*c^3*x + 16*c^3)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [A] time = 5.60757, size = 136, normalized size = 1.49

$$\begin{cases} -\frac{-2c^{3}\sqrt{-a^{2}x^{2}+1}-2c^{3}\left(\begin{cases}\frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{3}-\sqrt{-a^{2}x^{2}+1} & \text{for}\: ax > -1 \wedge ax < 1\end{cases}\right)+c^{3}\left(\begin{cases}\frac{ax\left(-2a^{2}x^{2}+1\right)\sqrt{-a^{2}x^{2}+1}}{8}-\frac{ax\sqrt{-a^{2}x^{2}+1}}{2}+\frac{3\operatorname{asin}(ax)}{8} & \ldots\end{cases}\right)\ldots}{a} & \ldots \\ c^{3}x & \ldots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**3,x)
```

```
[Out] Piecewise((-(-2*c**3*sqrt(-a**2*x**2 + 1) - 2*c**3*Piecewise(((-a**2*x**2 +
 1)**(3/2)/3 - sqrt(-a**2*x**2 + 1), (a*x > -1) & (a*x < 1))) + c**3*Piecew
ise((a*x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/8 - a*x*sqrt(-a**2*x**2 +
1)/2 + 3*asin(a*x)/8, (a*x > -1) & (a*x < 1))) - c**3*asin(a*x))/a, Ne(a, 0
)), (c**3*x, True))
```

Giac [A] time = 1.17741, size = 89, normalized size = 0.98

$$\frac{5\,c^{3}\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{8\,|a|}+\frac{1}{24}\,\sqrt{-a^{2}x^{2}+1}\left(\frac{16\,c^{3}}{a}+\left(9\,c^{3}+2\left(3\,a^{2}c^{3}x-8\,ac^{3}\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 5/8*c^3*arcsin(a*x)*sgn(a)/abs(a) + 1/24*sqrt(-a^2*x^2 + 1)*(16*c^3/a + (9*
c^3 + 2*(3*a^2*c^3*x - 8*a*c^3)*x)*x)
```

3.160 $\int e^{\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=61

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c^2x\sqrt{1-a^2x^2}+\frac{c^2\sin^{-1}(ax)}{2a}$$

```
[Out] (c^2*x*Sqrt[1 - a^2*x^2])/2 + (c^2*(1 - a^2*x^2)^(3/2))/(3*a) + (c^2*ArcSin
[a*x])/(2*a)
```

Rubi [A] time = 0.0374053, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6127, 641, 195, 216}

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c^2x\sqrt{1-a^2x^2}+\frac{c^2\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^2,x]
```

```
[Out] (c^2*x*Sqrt[1 - a^2*x^2])/2 + (c^2*(1 - a^2*x^2)^(3/2))/(3*a) + (c^2*ArcSin
[a*x])/(2*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}(c-acx)^2\,dx = c\int (c-acx)\sqrt{1-a^2x^2}\,dx$$
$$= \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a} + c^2\int \sqrt{1-a^2x^2}\,dx$$
$$= \frac{1}{2}c^2x\sqrt{1-a^2x^2} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a} + \frac{1}{2}c^2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{1}{2}c^2x\sqrt{1-a^2x^2} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a} + \frac{c^2\sin^{-1}(ax)}{2a}$$

Mathematica [A] time = 0.0803272, size = 59, normalized size = 0.97

$$-\frac{c^2\left(\sqrt{1-a^2x^2}\left(2a^2x^2-3ax-2\right)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^2,x]
```

```
[Out] -(c^2*(Sqrt[1 - a^2*x^2]*(-2 - 3*a*x + 2*a^2*x^2) + 6*ArcSin[Sqrt[1 - a*x]/
Sqrt[2]]))/(6*a)
```

Maple [A] time = 0.036, size = 91, normalized size = 1.5

$$-\frac{ac^2x^2}{3}\sqrt{-a^2x^2+1}+\frac{c^2}{3\,a}\sqrt{-a^2x^2+1}+\frac{xc^2}{2}\sqrt{-a^2x^2+1}+\frac{c^2}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2,x)
```

```
[Out] -1/3*c^2*a*x^2*(-a^2*x^2+1)^(1/2)+1/3*c^2*(-a^2*x^2+1)^(1/2)/a+1/2*c^2*x*(-
a^2*x^2+1)^(1/2)+1/2*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2
))
```

Maxima [A] time = 1.47717, size = 109, normalized size = 1.79

$$-\frac{1}{3}\sqrt{-a^2x^2+1}ac^2x^2+\frac{1}{2}\sqrt{-a^2x^2+1}c^2x+\frac{c^2\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}}+\frac{\sqrt{-a^2x^2+1}c^2}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-a^2*x^2 + 1)*a*c^2*x^2 + 1/2*sqrt(-a^2*x^2 + 1)*c^2*x + 1/2*c^2*
arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + 1/3*sqrt(-a^2*x^2 + 1)*c^2/a
```

Fricas [A] time = 1.64093, size = 151, normalized size = 2.48

$$-\frac{6\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^2c^2x^2-3\,ac^2x-2\,c^2\right)\sqrt{-a^2x^2+1}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^2*c^2*x^2 - 3*a*c
^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [A] time = 4.66622, size = 102, normalized size = 1.67

$$\begin{cases}\frac{c^2\sqrt{-a^2x^2+1}-c^2\left(\begin{cases}-\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\operatorname{asin}(ax)}{2} & \text{for}\ ax>-1\wedge ax<1\end{cases}\right)+c^2\left(\begin{cases}\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3}-\sqrt{-a^2x^2+1} & \text{for}\ ax>-1\wedge ax<1\end{cases}\right)+\dots}{a} \\ c^2x\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2,x)
```

```
[Out] Piecewise(((c**2*sqrt(-a**2*x**2 + 1) - c**2*Piecewise((-a*x*sqrt(-a**2*x**
2 + 1)/2 + asin(a*x)/2, (a*x > -1) & (a*x < 1))) + c**2*Piecewise(((-a**2*x
**2 + 1)**(3/2)/3 - sqrt(-a**2*x**2 + 1), (a*x > -1) & (a*x < 1))) + c**2*a
sin(a*x))/a, Ne(a, 0)), (c**2*x, True))
```

Giac [A] time = 1.20193, size = 73, normalized size = 1.2

$$\frac{c^2\arcsin(ax)\operatorname{sgn}(a)}{2\,|a|}-\frac{1}{6}\sqrt{-a^2x^2+1}\left(\left(2\,ac^2x-3\,c^2\right)x-\frac{2\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^2*arcsin(a*x)*sgn(a)/abs(a) - 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c^2*x - 3*
c^2)*x - 2*c^2/a)
```

3.161 $\int e^{\tanh^{-1}(ax)}(c - acx)\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}$$

```
[Out] (c*x*Sqrt[1 - a^2*x^2])/2 + (c*ArcSin[a*x])/(2*a)
```

Rubi [A] time = 0.0191962, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6127, 195, 216}

$$\frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x),x]
```

```
[Out] (c*x*Sqrt[1 - a^2*x^2])/2 + (c*ArcSin[a*x])/(2*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}(c-acx)\,dx &= c\int\sqrt{1-a^2x^2}\,dx\\ &= \frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{1}{2}c\int\frac{1}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}\end{aligned}$$

Mathematica [A] time = 0.0120922, size = 30, normalized size = 0.91

$$\frac{c\left(ax\sqrt{1-a^2x^2}+\sin^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x),x]
```

```
[Out] (c*(a*x*Sqrt[1 - a^2*x^2] + ArcSin[a*x]))/(2*a)
```

Maple [A] time = 0.032, size = 46, normalized size = 1.4

$$\frac{cx}{2}\sqrt{-a^2x^2+1}+\frac{c}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c),x)
```

```
[Out] 1/2*c*x*(-a^2*x^2+1)^(1/2)+1/2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2
+1)^(1/2))
```

Maxima [A] time = 1.43426, size = 49, normalized size = 1.48

$$\frac{1}{2}\sqrt{-a^2x^2+1}cx+\frac{c\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*c*x + 1/2*c*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2)
```

Fricas [A] time = 1.62335, size = 107, normalized size = 3.24

$$\frac{\sqrt{-a^2x^2+1}acx-2\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-a^2*x^2 + 1)*a*c*x - 2*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))
/a
```

Sympy [A] time = 2.66111, size = 46, normalized size = 1.39

$$\begin{cases} -\frac{c\left(\begin{cases} -\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\operatorname{asin}(ax)}{2} & \text{for}\: ax > -1 \wedge ax < 1 \end{cases}\right)-c\operatorname{asin}(ax)}{a} & \text{for}\: a \neq 0 \\ cx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c),x)
```

```
[Out] Piecewise((-(c*Piecewise((-a*x*sqrt(-a**2*x**2 + 1)/2 + asin(a*x)/2, (a*x >
 -1) & (a*x < 1))) - c*asin(a*x))/a, Ne(a, 0)), (c*x, True))
```

Giac [A] time = 1.19223, size = 41, normalized size = 1.24

$$\frac{1}{2}\sqrt{-a^2x^2+1}cx+\frac{c\arcsin(ax)\operatorname{sgn}(a)}{2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*c*x + 1/2*c*arcsin(a*x)*sgn(a)/abs(a)
```

3.162 $\int \frac{e^{\tanh^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{1-a^2x^2}}{ac(1-ax)}-\frac{\sin^{-1}(ax)}{ac}$$

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*c*(1 - a*x)) - ArcSin[a*x]/(a*c)
```

Rubi [A] time = 0.0407308, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6127, 663, 216}

$$\frac{2\sqrt{1-a^2x^2}}{ac(1-ax)}-\frac{\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*c*(1 - a*x)) - ArcSin[a*x]/(a*c)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{c-acx}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx\\ &= \frac{2\sqrt{1-a^2x^2}}{ac(1-ax)}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\ &= \frac{2\sqrt{1-a^2x^2}}{ac(1-ax)}-\frac{\sin^{-1}(ax)}{ac}\end{aligned}$$

Mathematica [A] time = 0.0244408, size = 46, normalized size = 1.07

$$\frac{2\left(\frac{\sqrt{ax+1}}{\sqrt{1-ax}}+\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x),x]
```

```
[Out] (2*(Sqrt[1 + a*x]/Sqrt[1 - a*x] + ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*c)
```

Maple [A] time = 0.036, size = 76, normalized size = 1.8

$$-\frac{1}{c}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-2\frac{1}{a^2c}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x)
```

```
[Out] -1/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-2/c/a^2/(x-1/a)*(
-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66591, size = 136, normalized size = 3.16

$$\frac{2\left(ax+(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\sqrt{-a^2x^2+1}-1\right)}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 2*(a*x + (a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 +
 1) - 1)/(a^2*c*x - a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{ax}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int\frac{1}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c),x)
```

```
[Out] -(Integral(a*x/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Inte
gral(1/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

Giac [A] time = 1.23907, size = 72, normalized size = 1.67

$$-\frac{\arcsin(ax)\operatorname{sgn}(a)}{c|a|}+\frac{4}{c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(c*abs(a)) + 4/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2
*x) - 1)*abs(a))
```

3.163 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^2} dx$

Optimal. Leaf size=32

$$\frac{(1-a^2x^2)^{3/2}}{3ac^2(1-ax)^3}$$

```
[Out] (1 - a^2*x^2)^(3/2)/(3*a*c^2*(1 - a*x)^3)
```

Rubi [A] time = 0.0333797, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6127, 651}

$$\frac{(1-a^2x^2)^{3/2}}{3ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^2,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(3*a*c^2*(1 - a*x)^3)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^2} dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3} dx\\ &= \frac{(1-a^2x^2)^{3/2}}{3ac^2(1-ax)^3}\end{aligned}$$

Mathematica [A] time = 0.010643, size = 29, normalized size = 0.91

$$\frac{(ax+1)^{3/2}}{3ac^2(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^(3/2)/(3*a*c^2*(1 - a*x)^(3/2))
```

Maple [A] time = 0.032, size = 35, normalized size = 1.1

$$-\frac{(ax+1)^2}{(3\,ax-3)\,c^2a}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x)
```

```
[Out] -1/3*(a*x+1)^2/(a*x-1)/c^2/(-a^2*x^2+1)^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.535, size = 127, normalized size = 3.97

$$\frac{a^2x^2-2\,ax+\sqrt{-a^2x^2+1}(ax+1)+1}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*x^2 - 2*a*x + sqrt(-a^2*x^2 + 1)*(a*x + 1) + 1)/(a^3*c^2*x^2 - 2*a
^2*c^2*x + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{ax}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int\frac{1}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(a*x/(a**2*x**2*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1)
+ sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**2*x**2*sqrt(-a**2*x**2 + 1) -
2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2), x)
```

3.164 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=65

$$\frac{(1-a^2x^2)^{3/2}}{15ac^3(1-ax)^3}+\frac{(1-a^2x^2)^{3/2}}{5ac^3(1-ax)^4}$$

```
[Out] (1 - a^2*x^2)^(3/2)/(5*a*c^3*(1 - a*x)^4) + (1 - a^2*x^2)^(3/2)/(15*a*c^3*(
1 - a*x)^3)
```

Rubi [A] time = 0.0495264, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6127, 659, 651}

$$\frac{(1-a^2x^2)^{3/2}}{15ac^3(1-ax)^3}+\frac{(1-a^2x^2)^{3/2}}{5ac^3(1-ax)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^3,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(5*a*c^3*(1 - a*x)^4) + (1 - a^2*x^2)^(3/2)/(15*a*c^3*(
1 - a*x)^3)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^3}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}}{5ac^3(1-ax)^4} + \frac{1}{5}\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}}{5ac^3(1-ax)^4} + \frac{\left(1-a^2x^2\right)^{3/2}}{15ac^3(1-ax)^3}$$

Mathematica [A] time = 0.0173418, size = 35, normalized size = 0.54

$$\frac{(4-ax)(ax+1)^{3/2}}{15ac^3(1-ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^3,x]
```

```
[Out] ((4 - a*x)*(1 + a*x)^(3/2))/(15*a*c^3*(1 - a*x)^(5/2))
```

Maple [A] time = 0.033, size = 40, normalized size = 0.6

$$-\frac{(ax-4)(ax+1)^2}{15c^3(ax-1)^2 a}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x)
```

```
[Out] -1/15*(a*x-4)*(a*x+1)^2/(a*x-1)^2/c^3/(-a^2*x^2+1)^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62949, size = 188, normalized size = 2.89

$$\frac{4a^3x^3 - 12a^2x^2 + 12ax + \left(a^2x^2 - 3ax - 4\right)\sqrt{-a^2x^2+1} - 4}{15\left(a^4c^3x^3 - 3a^3c^3x^2 + 3a^2c^3x - ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(4*a^3*x^3 - 12*a^2*x^2 + 12*a*x + (a^2*x^2 - 3*a*x - 4)*sqrt(-a^2*x^2
 + 1) - 4)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(a*x/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**
2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(
1/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + 3*a*
x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [B] time = 1.2062, size = 196, normalized size = 3.02

$$-\frac{2\left(\frac{5\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{25\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}-\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4}-4\right)}{15\,c^3\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -2/15*(5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 25*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)^2/(a^4*x^2) + 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 1
5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 4)/(c^3*((sqrt(-a^2*x^2 + 1
)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

3.165 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=97

$$\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^4(1-ax)^3}+\frac{2\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}$$

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a*c^4*(1 - a*x)^5) + (2*(1 - a^2*x^2)^(3/2))/(35*a*c
^4*(1 - a*x)^4) + (2*(1 - a^2*x^2)^(3/2))/(105*a*c^4*(1 - a*x)^3)
```

Rubi [A] time = 0.0685209, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6127, 659, 651}

$$\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^4(1-ax)^3}+\frac{2\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^4,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a*c^4*(1 - a*x)^5) + (2*(1 - a^2*x^2)^(3/2))/(35*a*c
^4*(1 - a*x)^4) + (2*(1 - a^2*x^2)^(3/2))/(105*a*c^4*(1 - a*x)^3)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^4}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}+\frac{2}{7}\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx}{35c}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^4(1-ax)^3}
\end{aligned}
$$

Mathematica [A] time = 0.0213528, size = 43, normalized size = 0.44

$$
-\frac{(ax+1)^{3/2}\left(-2a^2x^2+10ax-23\right)}{105ac^4(1-ax)^{7/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^4,x]
```

```
[Out] -((1 + a*x)^(3/2)*(-23 + 10*a*x - 2*a^2*x^2))/(105*a*c^4*(1 - a*x)^(7/2))
```

Maple [A] time = 0.03, size = 49, normalized size = 0.5

$$
-\frac{\left(2a^2x^2-10ax+23\right)(ax+1)^2}{105c^4(ax-1)^3a}\frac{1}{\sqrt{-a^2x^2+1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x)
```

```
[Out] -1/105*(2*a^2*x^2-10*a*x+23)*(a*x+1)^2/(a*x-1)^3/c^4/(-a^2*x^2+1)^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66848, size = 254, normalized size = 2.62

$$
\frac{23a^4x^4-92a^3x^3+138a^2x^2-92ax+\left(2a^3x^3-8a^2x^2+13ax+23\right)\sqrt{-a^2x^2+1}+23}{105\left(a^5c^4x^4-4a^4c^4x^3+6a^3c^4x^2-4a^2c^4x+ac^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(23*a^4*x^4 - 92*a^3*x^3 + 138*a^2*x^2 - 92*a*x + (2*a^3*x^3 - 8*a^2*
x^2 + 13*a*x + 23)*sqrt(-a^2*x^2 + 1) + 23)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 +
6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(a*x/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*x**3*sqrt(-a**2*x**2
 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sqr
t(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**
3*x**3*sqrt(-a**2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt
(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

Giac [B] time = 1.22944, size = 269, normalized size = 2.77

$$-\frac{2\left(\frac{56\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{273\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{350\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}-\frac{455\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4}+\frac{210\left(\sqrt{-a^2x^2+1}|a|+a\right)^5}{a^{10}x^5}-\frac{105\left(\sqrt{-a^2x^2+1}|a|+a\right)^6}{a^{12}x^6}-23\right)}{105\,c^4\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -2/105*(56*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 273*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)^2/(a^4*x^2) + 350*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3)
 - 455*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 210*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)^5/(a^10*x^5) - 105*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a^12*x^
6) - 23)/(c^4*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^7*abs(a))
```

3.166 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=129

$$\frac{2(1-a^2x^2)^{3/2}}{315ac^5(1-ax)^3}+\frac{2(1-a^2x^2)^{3/2}}{105ac^5(1-ax)^4}+\frac{(1-a^2x^2)^{3/2}}{21ac^5(1-ax)^5}+\frac{(1-a^2x^2)^{3/2}}{9ac^5(1-ax)^6}$$

```
[Out] (1 - a^2*x^2)^(3/2)/(9*a*c^5*(1 - a*x)^6) + (1 - a^2*x^2)^(3/2)/(21*a*c^5*(
1 - a*x)^5) + (2*(1 - a^2*x^2)^(3/2))/(105*a*c^5*(1 - a*x)^4) + (2*(1 - a^2
*x^2)^(3/2))/(315*a*c^5*(1 - a*x)^3)
```

Rubi [A] time = 0.0910225, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6127, 659, 651}

$$\frac{2(1-a^2x^2)^{3/2}}{315ac^5(1-ax)^3}+\frac{2(1-a^2x^2)^{3/2}}{105ac^5(1-ax)^4}+\frac{(1-a^2x^2)^{3/2}}{21ac^5(1-ax)^5}+\frac{(1-a^2x^2)^{3/2}}{9ac^5(1-ax)^6}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^5,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(9*a*c^5*(1 - a*x)^6) + (1 - a^2*x^2)^(3/2)/(21*a*c^5*(
1 - a*x)^5) + (2*(1 - a^2*x^2)^(3/2))/(105*a*c^5*(1 - a*x)^4) + (2*(1 - a^2
*x^2)^(3/2))/(315*a*c^5*(1 - a*x)^3)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^5}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^6}\,dx \\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{9ac^5(1-ax)^6}+\frac{1}{3}\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx \\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{9ac^5(1-ax)^6}+\frac{\left(1-a^2x^2\right)^{3/2}}{21ac^5(1-ax)^5}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx}{21c} \\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{9ac^5(1-ax)^6}+\frac{\left(1-a^2x^2\right)^{3/2}}{21ac^5(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^5(1-ax)^4}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx}{105c^2} \\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{9ac^5(1-ax)^6}+\frac{\left(1-a^2x^2\right)^{3/2}}{21ac^5(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^5(1-ax)^4}+\frac{2\left(1-a^2x^2\right)^{3/2}}{315ac^5(1-ax)^3}
\end{aligned}
$$

Mathematica [A] time = 0.0246279, size = 51, normalized size = 0.4

$$
\frac{(ax+1)^{3/2}\left(-2a^3x^3+12a^2x^2-33ax+58\right)}{315ac^5(1-ax)^{9/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^5,x]
```

```
[Out] ((1 + a*x)^(3/2)*(58 - 33*a*x + 12*a^2*x^2 - 2*a^3*x^3))/(315*a*c^5*(1 - a*
x)^(9/2))
```

Maple [A] time = 0.035, size = 57, normalized size = 0.4

$$
-\frac{\left(2\,x^3a^3-12\,a^2x^2+33\,ax-58\right)(ax+1)^2}{315\,c^5\,(ax-1)^4\,a}\frac{1}{\sqrt{-a^2x^2+1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^5,x)
```

```
[Out] -1/315*(2*a^3*x^3-12*a^2*x^2+33*a*x-58)*(a*x+1)^2/(a*x-1)^4/c^5/(-a^2*x^2+1
)^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70595, size = 319, normalized size = 2.47

$$\frac{58\,a^{5}x^{5}-290\,a^{4}x^{4}+580\,a^{3}x^{3}-580\,a^{2}x^{2}+290\,ax+\left(2\,a^{4}x^{4}-10\,a^{3}x^{3}+21\,a^{2}x^{2}-25\,ax-58\right)\sqrt{-a^{2}x^{2}+1}-58}{315\left(a^{6}c^{5}x^{5}-5\,a^{5}c^{5}x^{4}+10\,a^{4}c^{5}x^{3}-10\,a^{3}c^{5}x^{2}+5\,a^{2}c^{5}x-ac^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/315*(58*a^5*x^5 - 290*a^4*x^4 + 580*a^3*x^3 - 580*a^2*x^2 + 290*a*x + (2*
a^4*x^4 - 10*a^3*x^3 + 21*a^2*x^2 - 25*a*x - 58)*sqrt(-a^2*x^2 + 1) - 58)/(
a^6*c^5*x^5 - 5*a^5*c^5*x^4 + 10*a^4*c^5*x^3 - 10*a^3*c^5*x^2 + 5*a^2*c^5*x
 - a*c^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax}{a^{5}x^{5}\sqrt{-a^{2}x^{2}+1}-5a^{4}x^{4}\sqrt{-a^{2}x^{2}+1}+10a^{3}x^{3}\sqrt{-a^{2}x^{2}+1}-10a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}+5ax\sqrt{-a^{2}x^{2}+1}-\sqrt{-a^{2}x^{2}+1}}\,dx+\int \frac{}{a^{5}x^{5}\sqrt{-a^{2}x^{2}+1}-5a^{4}x^{4}\sqrt{-a^{2}x^{2}+1}+10a^{3}x \cdots}}{c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**5,x)
```

```
[Out] -(Integral(a*x/(a**5*x**5*sqrt(-a**2*x**2 + 1) - 5*a**4*x**4*sqrt(-a**2*x**
2 + 1) + 10*a**3*x**3*sqrt(-a**2*x**2 + 1) - 10*a**2*x**2*sqrt(-a**2*x**2 +
 1) + 5*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(1/(
a**5*x**5*sqrt(-a**2*x**2 + 1) - 5*a**4*x**4*sqrt(-a**2*x**2 + 1) + 10*a**3
*x**3*sqrt(-a**2*x**2 + 1) - 10*a**2*x**2*sqrt(-a**2*x**2 + 1) + 5*a*x*sqrt
(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}(acx-c)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^5), x)
```

3.167 $\int e^{2\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=41

$$\frac{(c-acx)^{p+1}}{ac(p+1)}-\frac{2(c-acx)^p}{ap}$$

```
[Out] (-2*(c - a*c*x)^p)/(a*p) + (c - a*c*x)^(1 + p)/(a*c*(1 + p))
```

Rubi [A] time = 0.0441538, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6130, 21, 43}

$$\frac{(c-acx)^{p+1}}{ac(p+1)}-\frac{2(c-acx)^p}{ap}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (-2*(c - a*c*x)^p)/(a*p) + (c - a*c*x)^(1 + p)/(a*c*(1 + p))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}(c-acx)^p\,dx &= \int \frac{(1+ax)(c-acx)^p}{1-ax}\,dx\\ &= c\int (1+ax)(c-acx)^{-1+p}\,dx\\ &= c\int \left(2(c-acx)^{-1+p}-\frac{(c-acx)^p}{c}\right)dx\\ &= -\frac{2(c-acx)^p}{ap}+\frac{(c-acx)^{1+p}}{ac(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0214986, size = 29, normalized size = 0.71

$$-\frac{(apx+p+2)(c-acx)^p}{ap(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] -(((c - a*c*x)^p*(2 + p + a*p*x))/(a*p*(1 + p)))
```

Maple [A] time = 0.033, size = 30, normalized size = 0.7

$$-\frac{(-acx+c)^p\left(apx+p+2\right)}{ap\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^p,x)
```

```
[Out] -(-a*c*x+c)^p*(a*p*x+p+2)/a/p/(1+p)
```

Maxima [A] time = 1.13554, size = 47, normalized size = 1.15

$$-\frac{\left(ac^ppx+c^p\left(p+2\right)\right)(-ax+1)^p}{\left(p^2+p\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] -(a*c^p*p*x + c^p*(p + 2))*(-a*x + 1)^p/((p^2 + p)*a)
```

Fricas [A] time = 1.68245, size = 63, normalized size = 1.54

$$-\frac{\left(apx+p+2\right)(-acx+c)^p}{ap^2+ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] -(a*p*x + p + 2)*(-a*c*x + c)^p/(a*p^2 + a*p)
```

Sympy [A] time = 1.13121, size = 126, normalized size = 3.07

$$\begin{cases} c^p x & \text{for } a = 0 \\ \frac{ax\log\left(x-\frac{1}{a}\right)}{a^2cx-ac} - \frac{\log\left(x-\frac{1}{a}\right)}{a^2cx-ac} - \frac{2}{a^2cx-ac} & \text{for } p = -1 \\ -x - \frac{2\log\left(x-\frac{1}{a}\right)}{a} & \text{for } p = 0 \\ -\frac{apx(-acx+c)^p}{ap^2+ap} - \frac{p(-acx+c)^p}{ap^2+ap} - \frac{2(-acx+c)^p}{ap^2+ap} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**p,x)
```

```
[Out] Piecewise((c**p*x, Eq(a, 0)), (a*x*log(x - 1/a)/(a**2*c*x - a*c) - log(x -
1/a)/(a**2*c*x - a*c) - 2/(a**2*c*x - a*c), Eq(p, -1)), (-x - 2*log(x - 1/a
)/a, Eq(p, 0)), (-a*p*x*(-a*c*x + c)**p/(a*p**2 + a*p) - p*(-a*c*x + c)**p/
(a*p**2 + a*p) - 2*(-a*c*x + c)**p/(a*p**2 + a*p), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)^2(-acx+c)^p}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2*(-a*c*x + c)^p/(a^2*x^2 - 1), x)
```

3.168 $\int e^{2\tanh^{-1}(ax)}(c-acx)^5\,dx$

Optimal. Leaf size=37

$$\frac{c^5(1-ax)^6}{6a}-\frac{2c^5(1-ax)^5}{5a}$$

```
[Out] (-2*c^5*(1 - a*x)^5)/(5*a) + (c^5*(1 - a*x)^6)/(6*a)
```

Rubi [A] time = 0.0339982, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{c^5(1-ax)^6}{6a}-\frac{2c^5(1-ax)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x)^5,x]
```

```
[Out] (-2*c^5*(1 - a*x)^5)/(5*a) + (c^5*(1 - a*x)^6)/(6*a)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}(c-acx)^5\,dx &= c^5\int (1-ax)^4(1+ax)\,dx\\ &= c^5\int \left(2(1-ax)^4-(1-ax)^5\right)dx\\ &= -\frac{2c^5(1-ax)^5}{5a}+\frac{c^5(1-ax)^6}{6a}\end{aligned}$$

Mathematica [A] time = 0.018634, size = 23, normalized size = 0.62

$$\frac{c^5(ax-1)^5(5ax+7)}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x)^5,x]
```

```
[Out] (c^5*(-1 + a*x)^5*(7 + 5*a*x))/(30*a)
```

Maple [A] time = 0.028, size = 45, normalized size = 1.2

$$c^{5}\left(\frac{x^{6}a^{5}}{6}-\frac{3\,x^{5}a^{4}}{5}+\frac{x^{4}a^{3}}{2}+\frac{2\,x^{3}a^{2}}{3}-\frac{3\,a x^{2}}{2}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^5,x)
```

```
[Out] c^5*(1/6*x^6*a^5-3/5*x^5*a^4+1/2*x^4*a^3+2/3*x^3*a^2-3/2*a*x^2+x)
```

Maxima [A] time = 0.946944, size = 80, normalized size = 2.16

$$\frac{1}{6}\,a^{5}c^{5}x^{6}-\frac{3}{5}\,a^{4}c^{5}x^{5}+\frac{1}{2}\,a^{3}c^{5}x^{4}+\frac{2}{3}\,a^{2}c^{5}x^{3}-\frac{3}{2}\,ac^{5}x^{2}+c^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/6*a^5*c^5*x^6 - 3/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 + 2/3*a^2*c^5*x^3 - 3/2
*a*c^5*x^2 + c^5*x
```

Fricas [A] time = 1.49136, size = 128, normalized size = 3.46

$$\frac{1}{6}\,a^{5}c^{5}x^{6}-\frac{3}{5}\,a^{4}c^{5}x^{5}+\frac{1}{2}\,a^{3}c^{5}x^{4}+\frac{2}{3}\,a^{2}c^{5}x^{3}-\frac{3}{2}\,ac^{5}x^{2}+c^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/6*a^5*c^5*x^6 - 3/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 + 2/3*a^2*c^5*x^3 - 3/2
*a*c^5*x^2 + c^5*x
```

Sympy [B] time = 0.104863, size = 66, normalized size = 1.78

$$\frac{a^{5}c^{5}x^{6}}{6}-\frac{3a^{4}c^{5}x^{5}}{5}+\frac{a^{3}c^{5}x^{4}}{2}+\frac{2a^{2}c^{5}x^{3}}{3}-\frac{3ac^{5}x^{2}}{2}+c^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**5,x)
```

```
[Out] a**5*c**5*x**6/6 - 3*a**4*c**5*x**5/5 + a**3*c**5*x**4/2 + 2*a**2*c**5*x**3
/3 - 3*a*c**5*x**2/2 + c**5*x
```

Giac [A] time = 1.24932, size = 80, normalized size = 2.16

$$\frac{1}{6}a^5c^5x^6 - \frac{3}{5}a^4c^5x^5 + \frac{1}{2}a^3c^5x^4 + \frac{2}{3}a^2c^5x^3 - \frac{3}{2}ac^5x^2 + c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/6*a^5*c^5*x^6 - 3/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 + 2/3*a^2*c^5*x^3 - 3/2
*a*c^5*x^2 + c^5*x
```

3.169 $\int e^{2\tanh^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=37

$$\frac{c^4(1-ax)^5}{5a}-\frac{c^4(1-ax)^4}{2a}$$

```
[Out] -(c^4*(1 - a*x)^4)/(2*a) + (c^4*(1 - a*x)^5)/(5*a)
```

Rubi [A] time = 0.0323223, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{c^4(1-ax)^5}{5a}-\frac{c^4(1-ax)^4}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x)^4,x]
```

```
[Out] -(c^4*(1 - a*x)^4)/(2*a) + (c^4*(1 - a*x)^5)/(5*a)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}(c-acx)^4\,dx &= c^4\int(1-ax)^3(1+ax)\,dx\\ &= c^4\int\left(2(1-ax)^3-(1-ax)^4\right)dx\\ &= -\frac{c^4(1-ax)^4}{2a}+\frac{c^4(1-ax)^5}{5a}\end{aligned}$$

Mathematica [A] time = 0.0155135, size = 32, normalized size = 0.86

$$c^4\left(-\frac{1}{5}a^4x^5+\frac{a^3x^4}{2}-ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x)^4,x]
```

```
[Out] c^4*(x - a*x^2 + (a^3*x^4)/2 - (a^4*x^5)/5)
```

Maple [A] time = 0.025, size = 29, normalized size = 0.8

$$c^4\left(-\frac{x^5a^4}{5}+\frac{x^4a^3}{2}-ax^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^4,x)
```

```
[Out] c^4*(-1/5*x^5*a^4+1/2*x^4*a^3-a*x^2+x)
```

Maxima [A] time = 0.947139, size = 50, normalized size = 1.35

$$-\frac{1}{5}a^4c^4x^5+\frac{1}{2}a^3c^4x^4-ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/5*a^4*c^4*x^5 + 1/2*a^3*c^4*x^4 - a*c^4*x^2 + c^4*x
```

Fricas [A] time = 1.54512, size = 76, normalized size = 2.05

$$-\frac{1}{5}a^4c^4x^5+\frac{1}{2}a^3c^4x^4-ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/5*a^4*c^4*x^5 + 1/2*a^3*c^4*x^4 - a*c^4*x^2 + c^4*x
```

Sympy [A] time = 0.091998, size = 36, normalized size = 0.97

$$-\frac{a^4c^4x^5}{5}+\frac{a^3c^4x^4}{2}-ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**4,x)
```

```
[Out] -a**4*c**4*x**5/5 + a**3*c**4*x**4/2 - a*c**4*x**2 + c**4*x
```

Giac [A] time = 1.20767, size = 50, normalized size = 1.35

$$-\frac{1}{5}a^4c^4x^5+\frac{1}{2}a^3c^4x^4-ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/5*a^4*c^4*x^5 + 1/2*a^3*c^4*x^4 - a*c^4*x^2 + c^4*x
```

3.170 $\int e^{2\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=37

$$\frac{c^3(1-ax)^4}{4a}-\frac{2c^3(1-ax)^3}{3a}$$

```
[Out] (-2*c^3*(1 - a*x)^3)/(3*a) + (c^3*(1 - a*x)^4)/(4*a)
```

Rubi [A] time = 0.0347904, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{c^3(1-ax)^4}{4a}-\frac{2c^3(1-ax)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (-2*c^3*(1 - a*x)^3)/(3*a) + (c^3*(1 - a*x)^4)/(4*a)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}(c-acx)^3\,dx &= c^3\int (1-ax)^2(1+ax)\,dx\\ &= c^3\int \left(2(1-ax)^2-(1-ax)^3\right)dx\\ &= -\frac{2c^3(1-ax)^3}{3a}+\frac{c^3(1-ax)^4}{4a}\end{aligned}$$

Mathematica [A] time = 0.0135797, size = 30, normalized size = 0.81

$$\frac{1}{12}c^3x\left(3a^3x^3-4a^2x^2-6ax+12\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (c^3*x*(12 - 6*a*x - 4*a^2*x^2 + 3*a^3*x^3))/12
```

Maple [A] time = 0.032, size = 29, normalized size = 0.8

$$c^3\left(\frac{x^4a^3}{4}-\frac{x^3a^2}{3}-\frac{ax^2}{2}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^3,x)
```

```
[Out] c^3*(1/4*x^4*a^3-1/3*x^3*a^2-1/2*a*x^2+x)
```

Maxima [A] time = 0.957431, size = 50, normalized size = 1.35

$$\frac{1}{4}\,a^3c^3x^4-\frac{1}{3}\,a^2c^3x^3-\frac{1}{2}\,ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 - 1/2*a*c^3*x^2 + c^3*x
```

Fricas [A] time = 1.56118, size = 80, normalized size = 2.16

$$\frac{1}{4}\,a^3c^3x^4-\frac{1}{3}\,a^2c^3x^3-\frac{1}{2}\,ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 - 1/2*a*c^3*x^2 + c^3*x
```

Sympy [A] time = 0.089358, size = 37, normalized size = 1.

$$\frac{a^3c^3x^4}{4}-\frac{a^2c^3x^3}{3}-\frac{ac^3x^2}{2}+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**3,x)
```

```
[Out] a**3*c**3*x**4/4 - a**2*c**3*x**3/3 - a*c**3*x**2/2 + c**3*x
```

Giac [A] time = 1.18426, size = 50, normalized size = 1.35

$$\frac{1}{4}\,a^3c^3x^4-\frac{1}{3}\,a^2c^3x^3-\frac{1}{2}\,ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 - 1/2*a*c^3*x^2 + c^3*x
```

3.171 $\int e^{2\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=19

$$c^2x-\frac{1}{3}a^2c^2x^3$$

```
[Out] c^2*x - (a^2*c^2*x^3)/3
```

Rubi [A] time = 0.0255281, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 41}

$$c^2x-\frac{1}{3}a^2c^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x)^2,x]
```

```
[Out] c^2*x - (a^2*c^2*x^3)/3
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}(c-acx)^2\,dx &= c^2\int (1-ax)(1+ax)\,dx\\ &= c^2\int \left(1-a^2x^2\right)\,dx\\ &= c^2x-\frac{1}{3}a^2c^2x^3\end{aligned}$$

Mathematica [A] time = 0.0082351, size = 16, normalized size = 0.84

$$c^2\left(x-\frac{a^2x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x)^2,x]
```

```
[Out] c^2*(x - (a^2*x^3)/3)
```

Maple [A] time = 0.026, size = 15, normalized size = 0.8

$$c^{2}\left(-\frac{x^{3}a^{2}}{3}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^2,x)
```

```
[Out] c^2*(-1/3*x^3*a^2+x)
```

Maxima [A] time = 0.959621, size = 23, normalized size = 1.21

$$-\frac{1}{3}\,a^{2}c^{2}x^{3}+c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3*a^2*c^2*x^3 + c^2*x
```

Fricas [A] time = 1.54131, size = 35, normalized size = 1.84

$$-\frac{1}{3}\,a^{2}c^{2}x^{3}+c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/3*a^2*c^2*x^3 + c^2*x
```

Sympy [A] time = 0.085084, size = 15, normalized size = 0.79

$$-\frac{a^{2}c^{2}x^{3}}{3}+c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**2,x)
```

```
[Out] -a**2*c**2*x**3/3 + c**2*x
```

Giac [A] time = 1.23071, size = 23, normalized size = 1.21

$$-\frac{1}{3}\,a^{2}c^{2}x^{3}+c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/3*a^2*c^2*x^3 + c^2*x
```

3.172 $\int e^{2\tanh^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=13

$$\frac{1}{2}acx^2+cx$$

```
[Out] c*x + (a*c*x^2)/2
```

Rubi [C] time = 0.011943, antiderivative size = 26, normalized size of antiderivative = 2., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2288}

$$\frac{c\left(1-a^2x^2\right)e^{2\tanh^{-1}(ax)}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x),x]
```

```
[Out] (c*E^(2*ArcTanh[a*x])*(1 - a^2*x^2))/(2*a)
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}(c-acx)\,dx=\frac{ce^{2\tanh^{-1}(ax)}\left(1-a^2x^2\right)}{2a}$$

Mathematica [C] time = 0.0077832, size = 26, normalized size = 2.

$$\frac{c\left(1-a^2x^2\right)e^{2\tanh^{-1}(ax)}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x),x]
```

```
[Out] (c*E^(2*ArcTanh[a*x])*(1 - a^2*x^2))/(2*a)
```

Maple [A] time = 0.027, size = 11, normalized size = 0.9

$$c\left(\frac{ax^2}{2}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c),x)
```

```
[Out] c*(1/2*a*x^2+x)
```

Maxima [A] time = 0.943945, size = 15, normalized size = 1.15

$$\frac{1}{2}acx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a*c*x^2 + c*x
```

Fricas [A] time = 1.51341, size = 26, normalized size = 2.

$$\frac{1}{2}acx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*a*c*x^2 + c*x
```

Sympy [A] time = 0.078766, size = 10, normalized size = 0.77

$$\frac{acx^2}{2} + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c),x)
```

```
[Out] a*c*x**2/2 + c*x
```

Giac [A] time = 1.17431, size = 15, normalized size = 1.15

$$\frac{1}{2}acx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 1/2*a*c*x^2 + c*x
```

3.173 $\int \frac{e^{2\tanh^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=31

$$\frac{2}{ac(1-ax)}+\frac{\log(1-ax)}{ac}$$

```
[Out] 2/(a*c*(1 - a*x)) + Log[1 - a*x]/(a*c)
```

Rubi [A] time = 0.0381136, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{2}{ac(1-ax)}+\frac{\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a*c*x),x]
```

```
[Out] 2/(a*c*(1 - a*x)) + Log[1 - a*x]/(a*c)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{c-acx}\,dx &= \frac{\int \frac{1+ax}{(1-ax)^2}\,dx}{c}\\ &= \frac{\int \left(\frac{2}{(-1+ax)^2}+\frac{1}{-1+ax}\right)dx}{c}\\ &= \frac{2}{ac(1-ax)}+\frac{\log(1-ax)}{ac}\end{aligned}$$

Mathematica [A] time = 0.0162467, size = 25, normalized size = 0.81

$$\frac{\frac{2}{1-ax}+\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a*c*x),x]
```

```
[Out] (2/(1 - a*x) + Log[1 - a*x])/(a*c)
```

Maple [A] time = 0.036, size = 30, normalized size = 1.

$$-2\frac{1}{ac(ax-1)}+\frac{\ln(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c),x)
```

```
[Out] -2/c/a/(a*x-1)+1/c/a*ln(a*x-1)
```

Maxima [A] time = 0.934844, size = 39, normalized size = 1.26

$$-\frac{2}{a^2cx-ac}+\frac{\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -2/(a^2*c*x - a*c) + log(a*x - 1)/(a*c)
```

Fricas [A] time = 1.68026, size = 62, normalized size = 2.

$$\frac{(ax-1)\log(ax-1)-2}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] ((a*x - 1)*log(a*x - 1) - 2)/(a^2*c*x - a*c)
```

Sympy [A] time = 0.333816, size = 20, normalized size = 0.65

$$-\frac{2}{a^2cx-ac}+\frac{\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a*c*x+c),x)
```

```
[Out] -2/(a**2*c*x - a*c) + log(a*x - 1)/(a*c)
```

Giac [A] time = 1.2026, size = 41, normalized size = 1.32

$$\frac{\log\left(|ax-1|\right)}{ac}-\frac{2}{(ax-1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] log(abs(a*x - 1))/(a*c) - 2/((a*x - 1)*a*c)
```

3.174 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=13

$$\frac{x}{c^2(1-ax)^2}$$

```
[Out] x/(c^2*(1 - a*x)^2)
```

Rubi [A] time = 0.0261538, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 34}

$$\frac{x}{c^2(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a*c*x)^2,x]
```

```
[Out] x/(c^2*(1 - a*x)^2)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^2}\,dx &= \frac{\int \frac{1+ax}{(1-ax)^3}\,dx}{c^2}\\ &= \frac{x}{c^2(1-ax)^2}\end{aligned}$$

Mathematica [A] time = 0.0069741, size = 25, normalized size = 1.92

$$\frac{(ax+1)^2}{4ac^2(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^2/(4*a*c^2*(1 - a*x)^2)
```

Maple [B] time = 0.039, size = 28, normalized size = 2.2

$$\frac{1}{c^2}\left(\frac{1}{a(ax-1)}+\frac{1}{a(ax-1)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^2,x)
```

```
[Out] 1/c^2*(1/a/(a*x-1)+1/a/(a*x-1)^2)
```

Maxima [B] time = 0.938602, size = 34, normalized size = 2.62

$$\frac{x}{a^2c^2x^2-2\,ac^2x+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] x/(a^2*c^2*x^2 - 2*a*c^2*x + c^2)
```

Fricas [B] time = 1.68245, size = 47, normalized size = 3.62

$$\frac{x}{a^2c^2x^2-2\,ac^2x+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] x/(a^2*c^2*x^2 - 2*a*c^2*x + c^2)
```

Sympy [B] time = 0.377952, size = 22, normalized size = 1.69

$$\frac{x}{a^2c^2x^2-2ac^2x+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a*c*x+c)**2,x)
```

```
[Out] x/(a**2*c**2*x**2 - 2*a*c**2*x + c**2)
```

Giac [B] time = 1.2299, size = 43, normalized size = 3.31

$$\frac{1}{(acx-c)^2a}+\frac{1}{(acx-c)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/((a*c*x - c)^2*a) + 1/((a*c*x - c)*a*c)
```

3.175 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=37

$$\frac{2}{3ac^3(1-ax)^3}-\frac{1}{2ac^3(1-ax)^2}$$

```
[Out] 2/(3*a*c^3*(1 - a*x)^3) - 1/(2*a*c^3*(1 - a*x)^2)
```

Rubi [A] time = 0.0391528, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{2}{3ac^3(1-ax)^3}-\frac{1}{2ac^3(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a*c*x)^3,x]
```

```
[Out] 2/(3*a*c^3*(1 - a*x)^3) - 1/(2*a*c^3*(1 - a*x)^2)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^3}\,dx &= \frac{\int \frac{1+ax}{(1-ax)^4}\,dx}{c^3}\\ &= \frac{\int \left(\frac{2}{(-1+ax)^4}+\frac{1}{(-1+ax)^3}\right)dx}{c^3}\\ &= \frac{2}{3ac^3(1-ax)^3}-\frac{1}{2ac^3(1-ax)^2}\end{aligned}$$

Mathematica [A] time = 0.0137785, size = 23, normalized size = 0.62

$$-\frac{3ax+1}{6ac^3(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a*c*x)^3,x]
```

```
[Out] -(1 + 3*a*x)/(6*a*c^3*(-1 + a*x)^3)
```

Maple [A] time = 0.035, size = 30, normalized size = 0.8

$$\frac{1}{c^3}\left(-\frac{1}{2\,a\,(ax-1)^2}-\frac{2}{3\,a\,(ax-1)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^3,x)
```

```
[Out] 1/c^3*(-1/2/a/(a*x-1)^2-2/3/a/(a*x-1)^3)
```

Maxima [A] time = 0.948002, size = 63, normalized size = 1.7

$$-\frac{3\,ax+1}{6\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Fricas [A] time = 1.9254, size = 95, normalized size = 2.57

$$-\frac{3\,ax+1}{6\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(3*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Sympy [A] time = 0.490163, size = 48, normalized size = 1.3

$$-\frac{3ax+1}{6a^4c^3x^3-18a^3c^3x^2+18a^2c^3x-6ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a*c*x+c)**3,x)
```

```
[Out] -(3*a*x + 1)/(6*a**4*c**3*x**3 - 18*a**3*c**3*x**2 + 18*a**2*c**3*x - 6*a*c
**3)
```

Giac [A] time = 1.20623, size = 28, normalized size = 0.76

$$-\frac{3\,ax+1}{6\,(ax-1)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/6*(3*a*x + 1)/((a*x - 1)^3*a*c^3)
```

3.176 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^4} \, dx$

Optimal. Leaf size=37

$$\frac{1}{2ac^4(1-ax)^4}-\frac{1}{3ac^4(1-ax)^3}$$

```
[Out] 1/(2*a*c^4*(1 - a*x)^4) - 1/(3*a*c^4*(1 - a*x)^3)
```

Rubi [A] time = 0.0388345, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{1}{2ac^4(1-ax)^4}-\frac{1}{3ac^4(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a*c*x)^4,x]
```

```
[Out] 1/(2*a*c^4*(1 - a*x)^4) - 1/(3*a*c^4*(1 - a*x)^3)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^4} \, dx &= \frac{\int \frac{1+ax}{(1-ax)^5} \, dx}{c^4}\\
&= \frac{\int \left(-\frac{2}{(-1+ax)^5}-\frac{1}{(-1+ax)^4}\right) \, dx}{c^4}\\
&= \frac{1}{2ac^4(1-ax)^4}-\frac{1}{3ac^4(1-ax)^3}
\end{aligned}$$

Mathematica [A] time = 0.0140367, size = 23, normalized size = 0.62

$$\frac{2ax+1}{6ac^4(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a*c*x)^4,x]
```

```
[Out] (1 + 2*a*x)/(6*a*c^4*(-1 + a*x)^4)
```

Maple [A] time = 0.033, size = 30, normalized size = 0.8

$$\frac{1}{c^4}\left(\frac{1}{3\,a\,(ax-1)^3}+\frac{1}{2\,a\,(ax-1)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^4,x)
```

```
[Out] 1/c^4*(1/3/a/(a*x-1)^3+1/2/a/(a*x-1)^4)
```

Maxima [A] time = 0.968887, size = 77, normalized size = 2.08

$$\frac{2\,ax+1}{6\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a*x + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x
+ a*c^4)
```

Fricas [A] time = 1.78093, size = 115, normalized size = 3.11

$$\frac{2\,ax+1}{6\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*x + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x
+ a*c^4)
```

Sympy [A] time = 0.525027, size = 58, normalized size = 1.57

$$\frac{2ax+1}{6a^5c^4x^4-24a^4c^4x^3+36a^3c^4x^2-24a^2c^4x+6ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a*c*x+c)**4,x)
```

```
[Out] (2*a*x + 1)/(6*a**5*c**4*x**4 - 24*a**4*c**4*x**3 + 36*a**3*c**4*x**2 - 24*
a**2*c**4*x + 6*a*c**4)
```

Giac [A] time = 1.17071, size = 28, normalized size = 0.76

$$\frac{2\,ax+1}{6\,(ax-1)^4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*a*x + 1)/((a*x - 1)^4*a*c^4)
```

3.177 $\int e^{3\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=65

$$\frac{4\sqrt{2}(c-acx)^{p+1}\text{Hypergeometric2F1}\left(-\frac{3}{2},p-\frac{1}{2},p+\frac{1}{2},\frac{1}{2}(1-ax)\right)}{ac(1-2p)(1-ax)^{3/2}}$$

```
[Out] (4*Sqrt[2]*(c - a*c*x)^(1 + p)*Hypergeometric2F1[-3/2, -1/2 + p, 1/2 + p, (
1 - a*x)/2])/(a*c*(1 - 2*p)*(1 - a*x)^(3/2))
```

Rubi [A] time = 0.0518962, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6130, 23, 69}

$$\frac{4\sqrt{2}(c-acx)^{p+1}\,{}_2F_1\left(-\frac{3}{2},p-\frac{1}{2};p+\frac{1}{2};\frac{1}{2}(1-ax)\right)}{ac(1-2p)(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (4*Sqrt[2]*(c - a*c*x)^(1 + p)*Hypergeometric2F1[-3/2, -1/2 + p, 1/2 + p, (
1 - a*x)/2])/(a*c*(1 - 2*p)*(1 - a*x)^(3/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}(c-acx)^p\,dx = \int \frac{(1+ax)^{3/2}(c-acx)^p}{(1-ax)^{3/2}}\,dx$$
$$= \frac{(c-acx)^{3/2}\int (1+ax)^{3/2}(c-acx)^{-\frac{3}{2}+p}\,dx}{(1-ax)^{3/2}}$$
$$= \frac{4\sqrt{2}(c-acx)^{1+p}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2}+p;\frac{1}{2}+p;\frac{1}{2}(1-ax)\right)}{ac(1-2p)(1-ax)^{3/2}}$$

Mathematica [A] time = 0.0235826, size = 58, normalized size = 0.89

$$\frac{4\sqrt{2}(c-acx)^p\text{Hypergeometric2F1}\left(-\frac{3}{2},p-\frac{1}{2},p+\frac{1}{2},\frac{1}{2}-\frac{ax}{2}\right)}{(a-2ap)\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (4*Sqrt[2]*(c - a*c*x)^p*Hypergeometric2F1[-3/2, -1/2 + p, 1/2 + p, 1/2 - (
a*x)/2])/((a - 2*a*p)*Sqrt[1 - a*x])
```

Maple [F] time = 0.47, size = 0, normalized size = 0.

$$\int (ax+1)^3(-acx+c)^p\left(-a^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^p,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3(-acx+c)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^3*(-a*c*x + c)^p/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}(ax+1)(-acx+c)^p}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*(-a*c*x + c)^p/(a^2*x^2 - 2*a*x + 1),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c(ax-1))^p (ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1))**p*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3(-acx+c)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3*(-a*c*x + c)^p/(-a^2*x^2 + 1)^(3/2), x)
```

3.178 $\int e^{3\tanh^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=83

$$\frac{c^4\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{4}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{3}{8}c^4x\sqrt{1-a^2x^2}+\frac{3c^4\sin^{-1}(ax)}{8a}$$

```
[Out] (3*c^4*x*Sqrt[1 - a^2*x^2])/8 + (c^4*x*(1 - a^2*x^2)^(3/2))/4 + (c^4*(1 - a
^2*x^2)^(5/2))/(5*a) + (3*c^4*ArcSin[a*x])/(8*a)
```

Rubi [A] time = 0.0485722, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6127, 641, 195, 216}

$$\frac{c^4\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{4}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{3}{8}c^4x\sqrt{1-a^2x^2}+\frac{3c^4\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x)^4,x]
```

```
[Out] (3*c^4*x*Sqrt[1 - a^2*x^2])/8 + (c^4*x*(1 - a^2*x^2)^(3/2))/4 + (c^4*(1 - a
^2*x^2)^(5/2))/(5*a) + (3*c^4*ArcSin[a*x])/(8*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}(c-acx)^4\,dx = c^3\int (c-acx)\left(1-a^2x^2\right)^{3/2}\,dx$$
$$= \frac{c^4\left(1-a^2x^2\right)^{5/2}}{5a} + c^4\int \left(1-a^2x^2\right)^{3/2}\,dx$$
$$= \frac{1}{4}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{c^4\left(1-a^2x^2\right)^{5/2}}{5a} + \frac{1}{4}\left(3c^4\right)\int \sqrt{1-a^2x^2}\,dx$$
$$= \frac{3}{8}c^4x\sqrt{1-a^2x^2} + \frac{1}{4}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{c^4\left(1-a^2x^2\right)^{5/2}}{5a} + \frac{1}{8}\left(3c^4\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{3}{8}c^4x\sqrt{1-a^2x^2} + \frac{1}{4}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{c^4\left(1-a^2x^2\right)^{5/2}}{5a} + \frac{3c^4\sin^{-1}(ax)}{8a}$$

Mathematica [A] time = 0.0933112, size = 75, normalized size = 0.9

$$\frac{c^4\left(\sqrt{1-a^2x^2}\left(8a^4x^4-10a^3x^3-16a^2x^2+25ax+8\right)-30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{40a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x)^4,x]
```

```
[Out] (c^4*(Sqrt[1 - a^2*x^2]*(8 + 25*a*x - 16*a^2*x^2 - 10*a^3*x^3 + 8*a^4*x^4)
- 30*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(40*a)
```

Maple [B] time = 0.069, size = 183, normalized size = 2.2

$$-\frac{c^4a^5x^6}{5}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{3\,c^4a^3x^4}{5}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{3\,c^4ax^2}{5}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{c^4}{5\,a}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{a^4c^4x^5}{4}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{7\,a^2c^4x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^4,x)
```

```
[Out] -1/5*c^4*a^5*x^6/(-a^2*x^2+1)^(1/2)+3/5*c^4*a^3*x^4/(-a^2*x^2+1)^(1/2)-3/5*
c^4*a*x^2/(-a^2*x^2+1)^(1/2)+1/5*c^4/a/(-a^2*x^2+1)^(1/2)+1/4*c^4*a^4*x^5/(
-a^2*x^2+1)^(1/2)-7/8*c^4*a^2*x^3/(-a^2*x^2+1)^(1/2)+5/8*c^4*x/(-a^2*x^2+1)
^(1/2)+3/8*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [B] time = 1.45437, size = 234, normalized size = 2.82

$$-\frac{a^5c^4x^6}{5\,\sqrt{-a^2x^2+1}}+\frac{a^4c^4x^5}{4\,\sqrt{-a^2x^2+1}}+\frac{3\,a^3c^4x^4}{5\,\sqrt{-a^2x^2+1}}-\frac{7\,a^2c^4x^3}{8\,\sqrt{-a^2x^2+1}}-\frac{3\,ac^4x^2}{5\,\sqrt{-a^2x^2+1}}+\frac{5\,c^4x}{8\,\sqrt{-a^2x^2+1}}+\frac{3\,c^4\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right.}{8\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/5*a^5*c^4*x^6/sqrt(-a^2*x^2 + 1) + 1/4*a^4*c^4*x^5/sqrt(-a^2*x^2 + 1) +
3/5*a^3*c^4*x^4/sqrt(-a^2*x^2 + 1) - 7/8*a^2*c^4*x^3/sqrt(-a^2*x^2 + 1) - 3
```

```
/5*a*c^4*x^2/sqrt(-a^2*x^2 + 1) + 5/8*c^4*x/sqrt(-a^2*x^2 + 1) + 3/8*c^4*ar
csin(a^2*x/sqrt(a^2))/sqrt(a^2) + 1/5*c^4/(sqrt(-a^2*x^2 + 1)*a)
```

Fricas [A] time = 1.93317, size = 201, normalized size = 2.42

$$-\frac{30\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(8\,a^4c^4x^4-10\,a^3c^4x^3-16\,a^2c^4x^2+25\,ac^4x+8\,c^4\right)\sqrt{-a^2x^2+1}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/40*(30*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (8*a^4*c^4*x^4 - 10*
a^3*c^4*x^3 - 16*a^2*c^4*x^2 + 25*a*c^4*x + 8*c^4)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [A] time = 20.2233, size = 459, normalized size = 5.53

$$-a^5c^4\left(\begin{cases}-\frac{x^4\sqrt{-a^2x^2+1}}{5a^2}-\frac{4x^2\sqrt{-a^2x^2+1}}{15a^4}-\frac{8\sqrt{-a^2x^2+1}}{15a^6} & \text{for } a\neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)+a^4c^4\left(\begin{cases}-\frac{ix^5}{4\sqrt{a^2x^2-1}}-\frac{ix^3}{8a^2\sqrt{a^2x^2-1}}+\frac{3ix}{8a^4\sqrt{a^2x^2-1}}-\frac{3i\operatorname{acosh}(ax)}{8a^5}\\ \frac{x^5}{4\sqrt{-a^2x^2+1}}+\frac{x^3}{8a^2\sqrt{-a^2x^2+1}}-\frac{3x}{8a^4\sqrt{-a^2x^2+1}}+\frac{3\operatorname{asin}(ax)}{8a^5}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**4,x)
```

```
[Out] -a**5*c**4*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/(5*a**2) - 4*x**2*sqrt(-a*
*2*x**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(15*a**6), Ne(a, 0)), (x**6
/6, True)) + a**4*c**4*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/
(8*a**2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*aco
sh(a*x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**
3/(8*a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asi
n(a*x)/(8*a**5), True)) + 2*a**3*c**4*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)
/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) - 2
*a**2*c**4*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a
**3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(
-a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) - a*c**4*Piecewise((x**2/2, E
q(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) + c**4*Piecewise((sqrt(a**
(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a
**2 < 0))
```

Giac [A] time = 1.28475, size = 105, normalized size = 1.27

$$\frac{3\,c^4\arcsin(ax)\operatorname{sgn}(a)}{8\,|a|}+\frac{1}{40}\sqrt{-a^2x^2+1}\left(\frac{8\,c^4}{a}+\left(25\,c^4-2\left(8\,ac^4-\left(4\,a^3c^4x-5\,a^2c^4\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 3/8*c^4*arcsin(a*x)*sgn(a)/abs(a) + 1/40*sqrt(-a^2*x^2 + 1)*(8*c^4/a + (25*
c^4 - 2*(8*a*c^4 - (4*a^3*c^4*x - 5*a^2*c^4)*x)*x)*x)
```

3.179 $\int e^{3\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=59

$$\frac{1}{4}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{3}{8}c^3x\sqrt{1-a^2x^2}+\frac{3c^3\sin^{-1}(ax)}{8a}$$

```
[Out] (3*c^3*x*Sqrt[1 - a^2*x^2])/8 + (c^3*x*(1 - a^2*x^2)^(3/2))/4 + (3*c^3*ArcS
in[a*x])/(8*a)
```

Rubi [A] time = 0.0356936, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 195, 216}

$$\frac{1}{4}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{3}{8}c^3x\sqrt{1-a^2x^2}+\frac{3c^3\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (3*c^3*x*Sqrt[1 - a^2*x^2])/8 + (c^3*x*(1 - a^2*x^2)^(3/2))/4 + (3*c^3*ArcS
in[a*x])/(8*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{3\tanh^{-1}(ax)}(c-acx)^3\,dx&=c^3\int\left(1-a^2x^2\right)^{3/2}\,dx\\&=\frac{1}{4}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{1}{4}\left(3c^3\right)\int\sqrt{1-a^2x^2}\,dx\\&=\frac{3}{8}c^3x\sqrt{1-a^2x^2}+\frac{1}{4}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{1}{8}\left(3c^3\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx\\&=\frac{3}{8}c^3x\sqrt{1-a^2x^2}+\frac{1}{4}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{3c^3\sin^{-1}(ax)}{8a}\end{aligned}$$

Mathematica [A] time = 0.0364763, size = 44, normalized size = 0.75

$$\frac{c^3\left(ax\sqrt{1-a^2x^2}\left(5-2a^2x^2\right)+3\sin^{-1}(ax)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(a*x*(5 - 2*a^2*x^2)*Sqrt[1 - a^2*x^2] + 3*ArcSin[a*x]))/(8*a)
```

Maple [A] time = 0.044, size = 96, normalized size = 1.6

$$\frac{a^4c^3x^5}{4}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{7\,c^3a^2x^3}{8}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{5\,c^3x}{8}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{3\,c^3}{8}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^3,x)
```

```
[Out] 1/4*c^3*a^4*x^5/(-a^2*x^2+1)^(1/2)-7/8*c^3*a^2*x^3/(-a^2*x^2+1)^(1/2)+5/8*c
^3*x/(-a^2*x^2+1)^(1/2)+3/8*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))
```

Maxima [A] time = 1.47573, size = 116, normalized size = 1.97

$$\frac{a^4c^3x^5}{4\sqrt{-a^2x^2+1}}-\frac{7\,a^2c^3x^3}{8\sqrt{-a^2x^2+1}}+\frac{5\,c^3x}{8\sqrt{-a^2x^2+1}}+\frac{3\,c^3\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^4*c^3*x^5/sqrt(-a^2*x^2 + 1) - 7/8*a^2*c^3*x^3/sqrt(-a^2*x^2 + 1) + 5
/8*c^3*x/sqrt(-a^2*x^2 + 1) + 3/8*c^3*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2)
```

Fricas [A] time = 1.89182, size = 140, normalized size = 2.37

$$-\frac{6\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^3c^3x^3-5\,ac^3x\right)\sqrt{-a^2x^2+1}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^3*c^3*x^3 - 5*a*c
^3*x)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [A] time = 8.58381, size = 301, normalized size = 5.1

$$a^4c^3\left(\begin{cases} -\frac{ix^5}{4\sqrt{a^2x^2-1}} - \frac{ix^3}{8a^2\sqrt{a^2x^2-1}} + \frac{3ix}{8a^4\sqrt{a^2x^2-1}} - \frac{3i\operatorname{acosh}(ax)}{8a^5} & \text{for } \left|a^2x^2\right| > 1 \\ \frac{x^5}{4\sqrt{-a^2x^2+1}} + \frac{x^3}{8a^2\sqrt{-a^2x^2+1}} - \frac{3x}{8a^4\sqrt{-a^2x^2+1}} + \frac{3\operatorname{asin}(ax)}{8a^5} & \text{otherwise} \end{cases}\right) - 2a^2c^3\left(\begin{cases} -\frac{ix\sqrt{a^2x^2-1}}{2a^2} - \frac{i\operatorname{acosh}(ax)}{2a^3} \\ \frac{x^3}{2\sqrt{-a^2x^2+1}} - \frac{x}{2a^2\sqrt{-a^2x^2+1}} + \frac{\operatorname{asin}(ax)}{2a^3} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**3,x)
```

```
[Out] a**4*c**3*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(
a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a*
*5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqr
t(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**
5), True)) - 2*a**2*c**3*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*a
cosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x
/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) + c**3*Piecewis
e((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt
(-a**2)), a**2 < 0))
```

Giac [A] time = 1.2034, size = 65, normalized size = 1.1

$$\frac{3\,c^3\arcsin(ax)\operatorname{sgn}(a)}{8\,|a|} - \frac{1}{8}\left(2\,a^2c^3x^2 - 5\,c^3\right)\sqrt{-a^2x^2+1}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 3/8*c^3*arcsin(a*x)*sgn(a)/abs(a) - 1/8*(2*a^2*c^3*x^2 - 5*c^3)*sqrt(-a^2*x
^2 + 1)*x
```

3.180 $\int e^{3 \tanh^{-1}(ax)}(c - acx)^2 \, dx$

Optimal. Leaf size=61

$$-\frac{c^2 \left(1-a^2 x^2\right)^{3/2}}{3 a}+\frac{1}{2} c^2 x \sqrt{1-a^2 x^2}+\frac{c^2 \sin^{-1}(a x)}{2 a}$$

```
[Out] (c^2*x*Sqrt[1 - a^2*x^2])/2 - (c^2*(1 - a^2*x^2)^(3/2))/(3*a) + (c^2*ArcSin
[a*x])/(2*a)
```

Rubi [A] time = 0.0458223, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6127, 665, 195, 216}

$$-\frac{c^2 \left(1-a^2 x^2\right)^{3/2}}{3 a}+\frac{1}{2} c^2 x \sqrt{1-a^2 x^2}+\frac{c^2 \sin^{-1}(a x)}{2 a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (c^2*x*Sqrt[1 - a^2*x^2])/2 - (c^2*(1 - a^2*x^2)^(3/2))/(3*a) + (c^2*ArcSin
[a*x])/(2*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}(c-acx)^2\,dx = c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{c-acx}\,dx$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a} + c^2\int \sqrt{1-a^2x^2}\,dx$$
$$= \frac{1}{2}c^2x\sqrt{1-a^2x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a} + \frac{1}{2}c^2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{1}{2}c^2x\sqrt{1-a^2x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a} + \frac{c^2\sin^{-1}(ax)}{2a}$$

Mathematica [A] time = 0.0735729, size = 59, normalized size = 0.97

$$\frac{c^2\left(\sqrt{1-a^2x^2}\left(2a^2x^2+3ax-2\right)-6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (c^2*(Sqrt[1 - a^2*x^2]*(-2 + 3*a*x + 2*a^2*x^2) - 6*ArcSin[Sqrt[1 - a*x]/S
qrt[2]]))/(6*a)
```

Maple [B] time = 0.047, size = 137, normalized size = 2.3

$$-\frac{c^2a^3x^4}{3}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{2\,ac^2x^2}{3}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{c^2}{3\,a}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{a^2c^2x^3}{2}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{xc^2}{2}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{c^2}{2}\arctan\Bigg(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^2,x)
```

```
[Out] -1/3*c^2*a^3*x^4/(-a^2*x^2+1)^(1/2)+2/3*c^2*a*x^2/(-a^2*x^2+1)^(1/2)-1/3*c^
2/a/(-a^2*x^2+1)^(1/2)-1/2*c^2*a^2*x^3/(-a^2*x^2+1)^(1/2)+1/2*c^2*x/(-a^2*x
^2+1)^(1/2)+1/2*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [B] time = 1.44349, size = 171, normalized size = 2.8

$$-\frac{a^3c^2x^4}{3\sqrt{-a^2x^2+1}}-\frac{a^2c^2x^3}{2\sqrt{-a^2x^2+1}}+\frac{2\,ac^2x^2}{3\sqrt{-a^2x^2+1}}+\frac{c^2x}{2\sqrt{-a^2x^2+1}}+\frac{c^2\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}}-\frac{c^2}{3\sqrt{-a^2x^2+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3*a^3*c^2*x^4/sqrt(-a^2*x^2 + 1) - 1/2*a^2*c^2*x^3/sqrt(-a^2*x^2 + 1) +
2/3*a*c^2*x^2/sqrt(-a^2*x^2 + 1) + 1/2*c^2*x/sqrt(-a^2*x^2 + 1) + 1/2*c^2*a
rcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - 1/3*c^2/(sqrt(-a^2*x^2 + 1)*a)
```

Fricas [A] time = 1.82295, size = 151, normalized size = 2.48

$$-\frac{6 c^2 \arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - \left(2 a^2c^2x^2 + 3 ac^2x - 2 c^2\right)\sqrt{-a^2x^2+1}}{6 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (2*a^2*c^2*x^2 + 3*a*c
^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [A] time = 12.4624, size = 221, normalized size = 3.62

$$-a^3c^2\left(\begin{cases} -\frac{x^2\sqrt{-a^2x^2+1}}{3a^2} - \frac{2\sqrt{-a^2x^2+1}}{3a^4} & \text{for } a \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}\right) - a^2c^2\left(\begin{cases} -\frac{ix\sqrt{a^2x^2-1}}{2a^2} - \frac{i\operatorname{acosh}(ax)}{2a^3} & \text{for } \left|a^2x^2\right| > 1 \\ \frac{x^3}{2\sqrt{-a^2x^2+1}} - \frac{x}{2a^2\sqrt{-a^2x^2+1}} + \frac{\operatorname{asin}(ax)}{2a^3} & \text{otherwise} \end{cases}\right) + ac^2\left(\begin{cases} \frac{x^2}{2} \\ -\frac{\sqrt{}}{} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**2,x)
```

```
[Out] -a**3*c**2*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x*
*2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) - a**2*c**2*Piecewise((-I*x*sq
rt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1), (x
**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/
(2*a**3), True)) + a*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**
2 + 1)/a**2, True)) + c**2*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**
2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0))
```

Giac [A] time = 1.22594, size = 73, normalized size = 1.2

$$\frac{c^2 \arcsin(ax)\operatorname{sgn}(a)}{2|a|} + \frac{1}{6}\sqrt{-a^2x^2+1}\left(\left(2 ac^2x + 3 c^2\right)x - \frac{2 c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^2*arcsin(a*x)*sgn(a)/abs(a) + 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c^2*x + 3*
c^2)*x - 2*c^2/a)
```

3.181 $\int e^{3\tanh^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=65

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}-\frac{3c\sqrt{1-a^2x^2}}{2a}+\frac{3c\sin^{-1}(ax)}{2a}$$

```
[Out] (-3*c*Sqrt[1 - a^2*x^2])/(2*a) - (c*(1 - a^2*x^2)^(3/2))/(2*a*(1 - a*x)) +
(3*c*ArcSin[a*x])/(2*a)
```

Rubi [A] time = 0.047755, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6127, 665, 216}

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}-\frac{3c\sqrt{1-a^2x^2}}{2a}+\frac{3c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x),x]
```

```
[Out] (-3*c*Sqrt[1 - a^2*x^2])/(2*a) - (c*(1 - a^2*x^2)^(3/2))/(2*a*(1 - a*x)) +
(3*c*ArcSin[a*x])/(2*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}(c-acx)\,dx = c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^2}\,dx$$
$$= -\frac{c\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)} + \frac{1}{2}\left(3c^2\right)\int \frac{\sqrt{1-a^2x^2}}{c-acx}\,dx$$
$$= -\frac{3c\sqrt{1-a^2x^2}}{2a} - \frac{c\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)} + \frac{1}{2}(3c)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{3c\sqrt{1-a^2x^2}}{2a} - \frac{c\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)} + \frac{3c\sin^{-1}(ax)}{2a}$$

Mathematica [A] time = 0.0333457, size = 48, normalized size = 0.74

$$-\frac{c\left(\sqrt{1-a^2x^2}(ax+4)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x),x]
```

```
[Out] -(c*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a)
```

Maple [A] time = 0.039, size = 104, normalized size = 1.6

$$\frac{a^2cx^3}{2}\frac{1}{\sqrt{-a^2x^2+1}} - \frac{cx}{2}\frac{1}{\sqrt{-a^2x^2+1}} + \frac{3c}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} + 2\frac{acx^2}{\sqrt{-a^2x^2+1}} - 2\frac{c}{a\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c),x)
```

```
[Out] 1/2*c*a^2*x^3/(-a^2*x^2+1)^(1/2)-1/2*c*x/(-a^2*x^2+1)^(1/2)+3/2*c/(a^2)^(1/
2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+2*c*a*x^2/(-a^2*x^2+1)^(1/2)-2*
c/a/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.44198, size = 127, normalized size = 1.95

$$\frac{a^2cx^3}{2\sqrt{-a^2x^2+1}} + \frac{2\,acx^2}{\sqrt{-a^2x^2+1}} - \frac{cx}{2\sqrt{-a^2x^2+1}} + \frac{3\,c\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}} - \frac{2\,c}{\sqrt{-a^2x^2+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c*x^3/sqrt(-a^2*x^2 + 1) + 2*a*c*x^2/sqrt(-a^2*x^2 + 1) - 1/2*c*x/s
qrt(-a^2*x^2 + 1) + 3/2*c*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - 2*c/(sqrt(-a^
2*x^2 + 1)*a)
```

Fricas [A] time = 1.96266, size = 119, normalized size = 1.83

$$-\frac{6\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}(acx+4\,c)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(6*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*(a*c*
x + 4*c))/a
```

Sympy [A] time = 8.49674, size = 165, normalized size = 2.54

$$a^2c\left(\begin{cases}-\frac{ix\sqrt{a^2x^2-1}}{2a^2}-\frac{i\operatorname{acosh}(ax)}{2a^3} & \text{for } \left|a^2x^2\right|>1\\ \frac{x^3}{2\sqrt{-a^2x^2+1}}-\frac{x}{2a^2\sqrt{-a^2x^2+1}}+\frac{\operatorname{asin}(ax)}{2a^3} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)+c\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right)\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a\right.}\right.\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c),x)
```

```
[Out] a**2*c*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3)
, Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**
2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) + 2*a*c*Piecewise((x**2/2, Eq(a**
2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) + c*Piecewise((sqrt(a**(-2))*as
in(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)
)
```

Giac [A] time = 1.20616, size = 51, normalized size = 0.78

$$\frac{3\,c\arcsin(ax)\operatorname{sgn}(a)}{2\,|a|}-\frac{1}{2}\sqrt{-a^2x^2+1}\left(cx+\frac{4\,c}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 3/2*c*arcsin(a*x)*sgn(a)/abs(a) - 1/2*sqrt(-a^2*x^2 + 1)*(c*x + 4*c/a)
```

3.182 $\int \frac{e^{3\tanh^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=74

$$\frac{2\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^3}-\frac{2\sqrt{1-a^2x^2}}{ac(1-ax)}+\frac{\sin^{-1}(ax)}{ac}$$

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a*c*(1 - a*x)) + (2*(1 - a^2*x^2)^(3/2))/(3*a*c*(1
- a*x)^3) + ArcSin[a*x]/(a*c)
```

Rubi [A] time = 0.0589498, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 663, 216}

$$\frac{2\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^3}-\frac{2\sqrt{1-a^2x^2}}{ac(1-ax)}+\frac{\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a*c*x),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a*c*(1 - a*x)) + (2*(1 - a^2*x^2)^(3/2))/(3*a*c*(1
- a*x)^3) + ArcSin[a*x]/(a*c)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3\tanh^{-1}(ax)}}{c-acx}\,dx &= c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^4}\,dx\\ &= \frac{2\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^3} - c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx\\ &= -\frac{2\sqrt{1-a^2x^2}}{ac(1-ax)} + \frac{2\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^3} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\ &= -\frac{2\sqrt{1-a^2x^2}}{ac(1-ax)} + \frac{2\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^3} + \frac{\sin^{-1}(ax)}{ac}\end{aligned}$$

Mathematica [C] time = 0.0122167, size = 45, normalized size = 0.61

$$\frac{4\sqrt{2}\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1-ax)\right)}{3ac(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a*c*x),x]
```

```
[Out] (4*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - a*x)/2])/(3*a*c*(1 - a*
x)^(3/2))
```

Maple [B] time = 0.039, size = 146, normalized size = 2.

$$-8\,\frac{x}{c\sqrt{-a^2x^2+1}}+\frac{1}{c}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-4\,\frac{1}{ac\sqrt{-a^2x^2+1}}-\frac{8}{3\,a^2c}\left(x-a^{-1}\right)^{-1}\frac{1}{\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c),x)
```

```
[Out] -8/c*x/(-a^2*x^2+1)^(1/2)+1/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)
^(1/2))-4/c/a/(-a^2*x^2+1)^(1/2)-8/3/c/a^2/(x-1/a)/(-a^2*(x-1/a)^2-2*a*(x-1
/a))^(1/2)+16/3/c/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00288, size = 216, normalized size = 2.92

$$-\frac{2\left(2\,a^2x^2-4\,ax+3\left(a^2x^2-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-2\,\sqrt{-a^2x^2+1}(2\,ax-1)+2\right)}{3\left(a^3cx^2-2\,a^2cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -2/3*(2*a^2*x^2 - 4*a*x + 3*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2 + 1
) - 1)/(a*x)) - 2*sqrt(-a^2*x^2 + 1)*(2*a*x - 1) + 2)/(a^3*c*x^2 - 2*a^2*c*
x + a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{3ax}{-a^3x^3\sqrt{-a^2x^2+1}+a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int\frac{3a^2x^2}{-a^3x^3\sqrt{-a^2x^2+1}+a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int\frac{}{-a^3x^3\sqrt{-a^2}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c),x)
```

```
[Out] -(Integral(3*a*x/(-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x*
*2 + 1) + a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(3
*a**2*x**2/(-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1
) + a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x*
*3/(-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1) + a*x*
sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(1/(-a**3*x**3*s
qrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1) + a*x*sqrt(-a**2*x**2
+ 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

Giac [A] time = 1.23573, size = 107, normalized size = 1.45

$$\frac{\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{c|a|}+\frac{8\left(\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-1\right)}{3\,c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/(c*abs(a)) + 8/3*(3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2
*x) - 1)/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^3*abs(a))
```

3.183 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=32

$$\frac{(1-a^2x^2)^{5/2}}{5ac^2(1-ax)^5}$$

```
[Out] (1 - a^2*x^2)^(5/2)/(5*a*c^2*(1 - a*x)^5)
```

Rubi [A] time = 0.0367227, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6127, 651}

$$\frac{(1-a^2x^2)^{5/2}}{5ac^2(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a*c*x)^2,x]
```

```
[Out] (1 - a^2*x^2)^(5/2)/(5*a*c^2*(1 - a*x)^5)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^2}\,dx &= c^3\int \frac{(1-a^2x^2)^{3/2}}{(c-acx)^5}\,dx\\ &= \frac{(1-a^2x^2)^{5/2}}{5ac^2(1-ax)^5}\end{aligned}$$

Mathematica [A] time = 0.0103136, size = 29, normalized size = 0.91

$$\frac{(ax+1)^{5/2}}{5ac^2(1-ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^(5/2)/(5*a*c^2*(1 - a*x)^(5/2))
```

Maple [A] time = 0.031, size = 35, normalized size = 1.1

$$-\frac{(ax+1)^4}{(5\,ax-5)\,c^2a}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^2,x)
```

```
[Out] -1/5*(a*x+1)^4/(a*x-1)/c^2/(-a^2*x^2+1)^(3/2)/a
```

Maxima [B] time = 1.02232, size = 197, normalized size = 6.16

$$\frac{8}{5\left(\sqrt{-a^2x^2+1}a^3c^2x^2-2\sqrt{-a^2x^2+1}a^2c^2x+\sqrt{-a^2x^2+1}ac^2\right)}+\frac{12}{5\left(\sqrt{-a^2x^2+1}a^2c^2x-\sqrt{-a^2x^2+1}ac^2\right)}+\frac{x}{5\sqrt{-a^2x^2+1}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] 8/5/(sqrt(-a^2*x^2 + 1)*a^3*c^2*x^2 - 2*sqrt(-a^2*x^2 + 1)*a^2*c^2*x + sqrt
(-a^2*x^2 + 1)*a*c^2) + 12/5/(sqrt(-a^2*x^2 + 1)*a^2*c^2*x - sqrt(-a^2*x^2
+ 1)*a*c^2) + 1/5*x/(sqrt(-a^2*x^2 + 1)*c^2) + 1/(sqrt(-a^2*x^2 + 1)*a*c^2)
```

Fricas [B] time = 1.92553, size = 181, normalized size = 5.66

$$\frac{a^3x^3-3\,a^2x^2+3\,ax-\left(a^2x^2+2\,ax+1\right)\sqrt{-a^2x^2+1}-1}{5\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/5*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - (a^2*x^2 + 2*a*x + 1)*sqrt(-a^2*x^2 + 1)
 - 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{3ax}{-a^4x^4\sqrt{-a^2x^2+1}+2a^3x^3\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{3a^2x^2}{-a^4x^4\sqrt{-a^2x^2+1}+2a^3x^3\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{}{-a^4x^4\sqrt{-}}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(3*a*x/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x
**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integra
l(3*a**2*x**2/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x**
2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(
a**3*x**3/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x**2 +
1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(1/(-
a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a*x*s
qrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}(acx-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(a*c*x - c)^2), x)
```

3.184 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=65

$$\frac{(1-a^2x^2)^{5/2}}{35ac^3(1-ax)^5}+\frac{(1-a^2x^2)^{5/2}}{7ac^3(1-ax)^6}$$

```
[Out] (1 - a^2*x^2)^(5/2)/(7*a*c^3*(1 - a*x)^6) + (1 - a^2*x^2)^(5/2)/(35*a*c^3*(
1 - a*x)^5)
```

Rubi [A] time = 0.0514531, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 651}

$$\frac{(1-a^2x^2)^{5/2}}{35ac^3(1-ax)^5}+\frac{(1-a^2x^2)^{5/2}}{7ac^3(1-ax)^6}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (1 - a^2*x^2)^(5/2)/(7*a*c^3*(1 - a*x)^6) + (1 - a^2*x^2)^(5/2)/(35*a*c^3*(
1 - a*x)^5)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^3}\,dx = c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^6}\,dx$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}}{7ac^3(1-ax)^6} + \frac{1}{7}c^2\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^5}\,dx$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}}{7ac^3(1-ax)^6} + \frac{\left(1-a^2x^2\right)^{5/2}}{35ac^3(1-ax)^5}$$

Mathematica [A] time = 0.0183458, size = 34, normalized size = 0.52

$$-\frac{(ax-6)(ax+1)^{5/2}}{35ac^3(1-ax)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a*c*x)^3,x]
```

```
[Out] -((-6 + a*x)*(1 + a*x)^(5/2))/(35*a*c^3*(1 - a*x)^(7/2))
```

Maple [A] time = 0.033, size = 40, normalized size = 0.6

$$-\frac{(ax-6)\,(ax+1)^4}{35\,c^3\,(ax-1)^2\,a}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^3,x)
```

```
[Out] -1/35*(a*x-6)*(a*x+1)^4/(a*x-1)^2/c^3/(-a^2*x^2+1)^(3/2)/a
```

Maxima [B] time = 0.994151, size = 292, normalized size = 4.49

$$-\frac{8}{7\left(\sqrt{-a^2x^2+1}a^4c^3x^3 - 3\sqrt{-a^2x^2+1}a^3c^3x^2 + 3\sqrt{-a^2x^2+1}a^2c^3x - \sqrt{-a^2x^2+1}ac^3\right)} - \frac{}{35\left(\sqrt{-a^2x^2+1}a^3c^3x^2 - 2\sqrt{-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -8/7/(sqrt(-a^2*x^2 + 1)*a^4*c^3*x^3 - 3*sqrt(-a^2*x^2 + 1)*a^3*c^3*x^2 + 3
*sqrt(-a^2*x^2 + 1)*a^2*c^3*x - sqrt(-a^2*x^2 + 1)*a*c^3) - 52/35/(sqrt(-a^
2*x^2 + 1)*a^3*c^3*x^2 - 2*sqrt(-a^2*x^2 + 1)*a^2*c^3*x + sqrt(-a^2*x^2 + 1
)*a*c^3) - 18/35/(sqrt(-a^2*x^2 + 1)*a^2*c^3*x - sqrt(-a^2*x^2 + 1)*a*c^3)
+ 1/35*x/(sqrt(-a^2*x^2 + 1)*c^3)
```

Fricas [B] time = 1.81045, size = 244, normalized size = 3.75

$$\frac{6\,a^4x^4 - 24\,a^3x^3 + 36\,a^2x^2 - 24\,ax - \left(a^3x^3 - 4\,a^2x^2 - 11\,ax - 6\right)\sqrt{-a^2x^2+1} + 6}{35\left(a^5c^3x^4 - 4\,a^4c^3x^3 + 6\,a^3c^3x^2 - 4\,a^2c^3x + ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/35*(6*a^4*x^4 - 24*a^3*x^3 + 36*a^2*x^2 - 24*a*x - (a^3*x^3 - 4*a^2*x^2 -
 11*a*x - 6)*sqrt(-a^2*x^2 + 1) + 6)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c
^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{3ax}{-a^5x^5\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-2a^3x^3\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{}{-a^5x^5\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-2a^3x^3\sqrt{-}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(3*a*x/(-a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*
x**2 + 1) - 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2
+ 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(3*
a**2*x**2/(-a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 +
1) - 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) +
3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**3
/(-a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a*
*3*x**3*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + 3*a*x*sqr
t(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(1/(-a**5*x**5*sqrt
(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**3*x**3*sqrt(-a**
2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1)
 - sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [B] time = 1.27725, size = 269, normalized size = 4.14

$$-\frac{2\left(\frac{7\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{91\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{70\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}-\frac{140\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4}+\frac{35\left(\sqrt{-a^2x^2+1}|a|+a\right)^5}{a^{10}x^5}-\frac{35\left(\sqrt{-a^2x^2+1}|a|+a\right)^6}{a^{12}x^6}-6\right.}{35\,c^3\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -2/35*(7*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 91*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)^2/(a^4*x^2) + 70*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 1
40*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 35*(sqrt(-a^2*x^2 + 1)*abs
(a) + a)^5/(a^10*x^5) - 35*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a^12*x^6) - 6
)/(c^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^7*abs(a))
```

3.185 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=97

$$\frac{2\left(1-a^2x^2\right)^{5/2}}{315ac^4(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{5/2}}{63ac^4(1-ax)^6}+\frac{\left(1-a^2x^2\right)^{5/2}}{9ac^4(1-ax)^7}$$

```
[Out] (1 - a^2*x^2)^(5/2)/(9*a*c^4*(1 - a*x)^7) + (2*(1 - a^2*x^2)^(5/2))/(63*a*c
^4*(1 - a*x)^6) + (2*(1 - a^2*x^2)^(5/2))/(315*a*c^4*(1 - a*x)^5)
```

Rubi [A] time = 0.069434, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 651}

$$\frac{2\left(1-a^2x^2\right)^{5/2}}{315ac^4(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{5/2}}{63ac^4(1-ax)^6}+\frac{\left(1-a^2x^2\right)^{5/2}}{9ac^4(1-ax)^7}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a*c*x)^4,x]
```

```
[Out] (1 - a^2*x^2)^(5/2)/(9*a*c^4*(1 - a*x)^7) + (2*(1 - a^2*x^2)^(5/2))/(63*a*c
^4*(1 - a*x)^6) + (2*(1 - a^2*x^2)^(5/2))/(315*a*c^4*(1 - a*x)^5)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^4}\,dx = c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^7}\,dx$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}}{9ac^4(1-ax)^7} + \frac{1}{9}\left(2c^2\right)\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^6}\,dx$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}}{9ac^4(1-ax)^7} + \frac{2\left(1-a^2x^2\right)^{5/2}}{63ac^4(1-ax)^6} + \frac{1}{63}(2c)\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^5}\,dx$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}}{9ac^4(1-ax)^7} + \frac{2\left(1-a^2x^2\right)^{5/2}}{63ac^4(1-ax)^6} + \frac{2\left(1-a^2x^2\right)^{5/2}}{315ac^4(1-ax)^5}$$

Mathematica [A] time = 0.0244797, size = 43, normalized size = 0.44

$$\frac{(ax+1)^{5/2}\left(2a^2x^2-14ax+47\right)}{315ac^4(1-ax)^{9/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a*c*x)^4,x]
```

```
[Out] ((1 + a*x)^(5/2)*(47 - 14*a*x + 2*a^2*x^2))/(315*a*c^4*(1 - a*x)^(9/2))
```

Maple [A] time = 0.033, size = 49, normalized size = 0.5

$$-\frac{\left(2\,a^2x^2-14\,ax+47\right)(ax+1)^4}{315\,c^4\,(ax-1)^3\,a}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^4,x)
```

```
[Out] -1/315*(2*a^2*x^2-14*a*x+47)*(a*x+1)^4/(a*x-1)^3/c^4/(-a^2*x^2+1)^(3/2)/a
```

Maxima [B] time = 1.0056, size = 441, normalized size = 4.55

$$\frac{8}{9\left(\sqrt{-a^2x^2+1}a^5c^4x^4-4\sqrt{-a^2x^2+1}a^4c^4x^3+6\sqrt{-a^2x^2+1}a^3c^4x^2-4\sqrt{-a^2x^2+1}a^2c^4x+\sqrt{-a^2x^2+1}ac^4\right)}+\frac{}{63\left(\sqrt{-a^2x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 8/9/(sqrt(-a^2*x^2 + 1)*a^5*c^4*x^4 - 4*sqrt(-a^2*x^2 + 1)*a^4*c^4*x^3 + 6*
sqrt(-a^2*x^2 + 1)*a^3*c^4*x^2 - 4*sqrt(-a^2*x^2 + 1)*a^2*c^4*x + sqrt(-a^2
*x^2 + 1)*a*c^4) + 68/63/(sqrt(-a^2*x^2 + 1)*a^4*c^4*x^3 - 3*sqrt(-a^2*x^2
+ 1)*a^3*c^4*x^2 + 3*sqrt(-a^2*x^2 + 1)*a^2*c^4*x - sqrt(-a^2*x^2 + 1)*a*c^
4) + 106/315/(sqrt(-a^2*x^2 + 1)*a^3*c^4*x^2 - 2*sqrt(-a^2*x^2 + 1)*a^2*c^4
*x + sqrt(-a^2*x^2 + 1)*a*c^4) - 1/315/(sqrt(-a^2*x^2 + 1)*a^2*c^4*x - sqrt
(-a^2*x^2 + 1)*a*c^4) + 2/315*x/(sqrt(-a^2*x^2 + 1)*c^4)
```

Fricas [A] time = 1.76256, size = 319, normalized size = 3.29

$$\frac{47\,a^5x^5 - 235\,a^4x^4 + 470\,a^3x^3 - 470\,a^2x^2 + 235\,ax - \left(2\,a^4x^4 - 10\,a^3x^3 + 21\,a^2x^2 + 80\,ax + 47\right)\sqrt{-a^2x^2+1} - 47}{315\left(a^6c^4x^5 - 5\,a^5c^4x^4 + 10\,a^4c^4x^3 - 10\,a^3c^4x^2 + 5\,a^2c^4x - ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(47*a^5*x^5 - 235*a^4*x^4 + 470*a^3*x^3 - 470*a^2*x^2 + 235*a*x - (2*
a^4*x^4 - 10*a^3*x^3 + 21*a^2*x^2 + 80*a*x + 47)*sqrt(-a^2*x^2 + 1) - 47)/(
a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x
 - a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3ax}{-a^6x^6\sqrt{-a^2x^2+1}+4a^5x^5\sqrt{-a^2x^2+1}-5a^4x^4\sqrt{-a^2x^2+1}+5a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx + \int \frac{}{-a^6x^6\sqrt{-a^2x^2+1}+4a^5x^5\sqrt{-a^2x^2+1}-5a^4x^4 \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(3*a*x/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 4*a**5*x**5*sqrt(-a**2*x
**2 + 1) - 5*a**4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**2*x**2*sqrt(-a**2*x**2 +
 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(3*a
**2*x**2/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 4*a**5*x**5*sqrt(-a**2*x**2 + 1
) - 5*a**4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4
*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**3/
(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 4*a**5*x**5*sqrt(-a**2*x**2 + 1) - 5*a**
4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt
(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(1/(-a**6*x**6*sqrt(
-a**2*x**2 + 1) + 4*a**5*x**5*sqrt(-a**2*x**2 + 1) - 5*a**4*x**4*sqrt(-a**2
*x**2 + 1) + 5*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1)
+ sqrt(-a**2*x**2 + 1)), x))/c**4
```

Giac [B] time = 1.20365, size = 342, normalized size = 3.53

$$-\frac{2\left(\frac{108\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x} - \frac{1062\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2} + \frac{1638\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3} - \frac{3402\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4} + \frac{2520\left(\sqrt{-a^2x^2+1}|a|+a\right)^5}{a^{10}x^5} - \frac{2310\left(\sqrt{-a \ldots\right.}{\ldots} \ldots\right)}{315\,c^4\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)^9|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -2/315*(108*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1062*(sqrt(-a^2*x^2 +
 1)*abs(a) + a)^2/(a^4*x^2) + 1638*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x
^3) - 3402*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 2520*(sqrt(-a^2*x^
```

```
2 + 1)*abs(a) + a)^5/(a^10*x^5) - 2310*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a
^12*x^6) + 630*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7/(a^14*x^7) - 315*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)^8/(a^16*x^8) - 47)/(c^4*((sqrt(-a^2*x^2 + 1)*abs(a)
+ a)/(a^2*x) - 1)^9*abs(a))
```

3.186 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=129

$$\frac{2\left(1-a^2x^2\right)^{5/2}}{1155ac^5(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{5/2}}{231ac^5(1-ax)^6}+\frac{\left(1-a^2x^2\right)^{5/2}}{33ac^5(1-ax)^7}+\frac{\left(1-a^2x^2\right)^{5/2}}{11ac^5(1-ax)^8}$$

```
[Out] (1 - a^2*x^2)^(5/2)/(11*a*c^5*(1 - a*x)^8) + (1 - a^2*x^2)^(5/2)/(33*a*c^5*
(1 - a*x)^7) + (2*(1 - a^2*x^2)^(5/2))/(231*a*c^5*(1 - a*x)^6) + (2*(1 - a^
2*x^2)^(5/2))/(1155*a*c^5*(1 - a*x)^5)
```

Rubi [A] time = 0.0917504, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 651}

$$\frac{2\left(1-a^2x^2\right)^{5/2}}{1155ac^5(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{5/2}}{231ac^5(1-ax)^6}+\frac{\left(1-a^2x^2\right)^{5/2}}{33ac^5(1-ax)^7}+\frac{\left(1-a^2x^2\right)^{5/2}}{11ac^5(1-ax)^8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a*c*x)^5,x]
```

```
[Out] (1 - a^2*x^2)^(5/2)/(11*a*c^5*(1 - a*x)^8) + (1 - a^2*x^2)^(5/2)/(33*a*c^5*
(1 - a*x)^7) + (2*(1 - a^2*x^2)^(5/2))/(231*a*c^5*(1 - a*x)^6) + (2*(1 - a^
2*x^2)^(5/2))/(1155*a*c^5*(1 - a*x)^5)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^5}\,dx = c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^8}\,dx$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}}{11ac^5(1-ax)^8} + \frac{1}{11}\left(3c^2\right)\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^7}\,dx$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}}{11ac^5(1-ax)^8} + \frac{\left(1-a^2x^2\right)^{5/2}}{33ac^5(1-ax)^7} + \frac{1}{33}(2c)\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^6}\,dx$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}}{11ac^5(1-ax)^8} + \frac{\left(1-a^2x^2\right)^{5/2}}{33ac^5(1-ax)^7} + \frac{2\left(1-a^2x^2\right)^{5/2}}{231ac^5(1-ax)^6} + \frac{2}{231}\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^5}\,dx$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}}{11ac^5(1-ax)^8} + \frac{\left(1-a^2x^2\right)^{5/2}}{33ac^5(1-ax)^7} + \frac{2\left(1-a^2x^2\right)^{5/2}}{231ac^5(1-ax)^6} + \frac{2\left(1-a^2x^2\right)^{5/2}}{1155ac^5(1-ax)^5}$$

Mathematica [A] time = 0.0274414, size = 51, normalized size = 0.4

$$-\frac{(ax+1)^{5/2}\left(2a^3x^3-16a^2x^2+61ax-152\right)}{1155ac^5(1-ax)^{11/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a*c*x)^5,x]
```

```
[Out] -((1 + a*x)^(5/2)*(-152 + 61*a*x - 16*a^2*x^2 + 2*a^3*x^3))/(1155*a*c^5*(1
- a*x)^(11/2))
```

Maple [A] time = 0.036, size = 57, normalized size = 0.4

$$-\frac{\left(2\,x^3a^3-16\,a^2x^2+61\,ax-152\right)(ax+1)^4}{1155\,c^5\,(ax-1)^4\,a}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^5,x)
```

```
[Out] -1/1155*(2*a^3*x^3-16*a^2*x^2+61*a*x-152)*(a*x+1)^4/(a*x-1)^4/c^5/(-a^2*x^2
+1)^(3/2)/a
```

Maxima [B] time = 1.02939, size = 624, normalized size = 4.84

$$-\frac{8}{11\left(\sqrt{-a^2x^2+1}a^6c^5x^5-5\sqrt{-a^2x^2+1}a^5c^5x^4+10\sqrt{-a^2x^2+1}a^4c^5x^3-10\sqrt{-a^2x^2+1}a^3c^5x^2+5\sqrt{-a^2x^2+1}a^2c^5x-\sqrt{-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -8/11/(sqrt(-a^2*x^2 + 1)*a^6*c^5*x^5 - 5*sqrt(-a^2*x^2 + 1)*a^5*c^5*x^4 +
10*sqrt(-a^2*x^2 + 1)*a^4*c^5*x^3 - 10*sqrt(-a^2*x^2 + 1)*a^3*c^5*x^2 + 5*s
qrt(-a^2*x^2 + 1)*a^2*c^5*x - sqrt(-a^2*x^2 + 1)*a*c^5) - 28/33/(sqrt(-a^2*
```

```
x^2 + 1)*a^5*c^5*x^4 - 4*sqrt(-a^2*x^2 + 1)*a^4*c^5*x^3 + 6*sqrt(-a^2*x^2 +
 1)*a^3*c^5*x^2 - 4*sqrt(-a^2*x^2 + 1)*a^2*c^5*x + sqrt(-a^2*x^2 + 1)*a*c^5
) - 58/231/(sqrt(-a^2*x^2 + 1)*a^4*c^5*x^3 - 3*sqrt(-a^2*x^2 + 1)*a^3*c^5*x
^2 + 3*sqrt(-a^2*x^2 + 1)*a^2*c^5*x - sqrt(-a^2*x^2 + 1)*a*c^5) + 1/1155/(s
qrt(-a^2*x^2 + 1)*a^3*c^5*x^2 - 2*sqrt(-a^2*x^2 + 1)*a^2*c^5*x + sqrt(-a^2*
x^2 + 1)*a*c^5) - 1/1155/(sqrt(-a^2*x^2 + 1)*a^2*c^5*x - sqrt(-a^2*x^2 + 1)
*a*c^5) + 2/1155*x/(sqrt(-a^2*x^2 + 1)*c^5)
```

Fricas [A] time = 1.7893, size = 389, normalized size = 3.02

$$\frac{152\,a^6x^6-912\,a^5x^5+2280\,a^4x^4-3040\,a^3x^3+2280\,a^2x^2-912\,ax-\left(2\,a^5x^5-12\,a^4x^4+31\,a^3x^3-46\,a^2x^2-243\,ax-\right.}{1155\left(a^7c^5x^6-6\,a^6c^5x^5+15\,a^5c^5x^4-20\,a^4c^5x^3+15\,a^3c^5x^2-6\,a^2c^5x+ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/1155*(152*a^6*x^6 - 912*a^5*x^5 + 2280*a^4*x^4 - 3040*a^3*x^3 + 2280*a^2*
x^2 - 912*a*x - (2*a^5*x^5 - 12*a^4*x^4 + 31*a^3*x^3 - 46*a^2*x^2 - 243*a*x
 - 152)*sqrt(-a^2*x^2 + 1) + 152)/(a^7*c^5*x^6 - 6*a^6*c^5*x^5 + 15*a^5*c^5
*x^4 - 20*a^4*c^5*x^3 + 15*a^3*c^5*x^2 - 6*a^2*c^5*x + a*c^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{3ax}{-a^7x^7\sqrt{-a^2x^2+1}+5a^6x^6\sqrt{-a^2x^2+1}-9a^5x^5\sqrt{-a^2x^2+1}+5a^4x^4\sqrt{-a^2x^2+1}+5a^3x^3\sqrt{-a^2x^2+1}-9a^2x^2\sqrt{-a^2x^2+1}+5ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{}{-a^7x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**5,x)
```

```
[Out] -(Integral(3*a*x/(-a**7*x**7*sqrt(-a**2*x**2 + 1) + 5*a**6*x**6*sqrt(-a**2*
x**2 + 1) - 9*a**5*x**5*sqrt(-a**2*x**2 + 1) + 5*a**4*x**4*sqrt(-a**2*x**2
+ 1) + 5*a**3*x**3*sqrt(-a**2*x**2 + 1) - 9*a**2*x**2*sqrt(-a**2*x**2 + 1)
+ 5*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(3*a**2*
x**2/(-a**7*x**7*sqrt(-a**2*x**2 + 1) + 5*a**6*x**6*sqrt(-a**2*x**2 + 1) -
9*a**5*x**5*sqrt(-a**2*x**2 + 1) + 5*a**4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**
3*x**3*sqrt(-a**2*x**2 + 1) - 9*a**2*x**2*sqrt(-a**2*x**2 + 1) + 5*a*x*sqrt
(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**3/(-a**7*x*
*7*sqrt(-a**2*x**2 + 1) + 5*a**6*x**6*sqrt(-a**2*x**2 + 1) - 9*a**5*x**5*sq
rt(-a**2*x**2 + 1) + 5*a**4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**3*x**3*sqrt(-a
**2*x**2 + 1) - 9*a**2*x**2*sqrt(-a**2*x**2 + 1) + 5*a*x*sqrt(-a**2*x**2 +
1) - sqrt(-a**2*x**2 + 1)), x) + Integral(1/(-a**7*x**7*sqrt(-a**2*x**2 + 1
) + 5*a**6*x**6*sqrt(-a**2*x**2 + 1) - 9*a**5*x**5*sqrt(-a**2*x**2 + 1) + 5
*a**4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**3*x**3*sqrt(-a**2*x**2 + 1) - 9*a**2
*x**2*sqrt(-a**2*x**2 + 1) + 5*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 +
 1)), x))/c**5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}(acx-c)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(a*c*x - c)^5), x)
```

3.187 $\int e^{4\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=66

$$\frac{4c(c-acx)^{p-1}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{p+1}}{ac(p+1)}$$

```
[Out] (4*c*(c - a*c*x)^(-1 + p))/(a*(1 - p)) + (4*(c - a*c*x)^p)/(a*p) - (c - a*c
*x)^(1 + p)/(a*c*(1 + p))
```

Rubi [A] time = 0.0583784, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6130, 21, 43}

$$\frac{4c(c-acx)^{p-1}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{p+1}}{ac(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (4*c*(c - a*c*x)^(-1 + p))/(a*(1 - p)) + (4*(c - a*c*x)^p)/(a*p) - (c - a*c
*x)^(1 + p)/(a*c*(1 + p))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{4\tanh^{-1}(ax)}(c-acx)^p\,dx &= \int \frac{(1+ax)^2(c-acx)^p}{(1-ax)^2}\,dx\\
&= c^2\int (1+ax)^2(c-acx)^{-2+p}\,dx\\
&= c^2\int \left(4(c-acx)^{-2+p}-\frac{4(c-acx)^{-1+p}}{c}+\frac{(c-acx)^p}{c^2}\right)dx\\
&= \frac{4c(c-acx)^{-1+p}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{1+p}}{ac(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.0746383, size = 50, normalized size = 0.76

$$\frac{\left(\frac{ax}{p+1}+\frac{4}{(p-1)(ax-1)}+\frac{3p+4}{p(p+1)}\right)(c-acx)^p}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] ((c - a*c*x)^p*((4 + 3*p)/(p*(1 + p)) + (a*x)/(1 + p) + 4/((-1 + p)*(-1 + a
*x))))/a
```

Maple [A] time = 0.035, size = 74, normalized size = 1.1

$$\frac{\left(a^2p^2x^2-a^2x^2p+2\,ap^2x+2\,apx-4\,ax+p^2+3\,p+4\right)(-acx+c)^p}{(ax-1)\,ap\left(p^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^p,x)
```

```
[Out] (a^2*p^2*x^2-a^2*p*x^2+2*a*p^2*x+2*a*p*x-4*a*x+p^2+3*p+4)*(-a*c*x+c)^p/(a*x
-1)/a/p/(p^2-1)
```

Maxima [A] time = 1.1761, size = 104, normalized size = 1.58

$$\frac{\left(\left(p^2-p\right)a^2c^px^2+2\left(p^2+p-2\right)ac^px+\left(p^2+3\,p+4\right)c^p\right)(-ax+1)^p}{\left(p^3-p\right)a^2x-\left(p^3-p\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] ((p^2 - p)*a^2*c^p*x^2 + 2*(p^2 + p - 2)*a*c^p*x + (p^2 + 3*p + 4)*c^p)*(-a
*x + 1)^p/((p^3 - p)*a^2*x - (p^3 - p)*a)
```

Fricas [A] time = 1.73217, size = 161, normalized size = 2.44

$$-\frac{\left(\left(a^2p^2-a^2p\right)x^2+p^2+2\left(ap^2+ap-2\,a\right)x+3\,p+4\right)(-acx+c)^p}{ap^3-ap-\left(a^2p^3-a^2p\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] -((a^2*p^2 - a^2*p)*x^2 + p^2 + 2*(a*p^2 + a*p - 2*a)*x + 3*p + 4)*(-a*c*x
+ c)^p/(a*p^3 - a*p - (a^2*p^3 - a^2*p)*x)
```

Sympy [A] time = 3.00102, size = 570, normalized size = 8.64

$$\begin{cases} c^p x \\ -\frac{a^2x^2\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac} + \frac{4a^2x^2}{a^3cx^2-2a^2cx+ac} + \frac{2ax\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac} - \frac{4ax}{a^3cx^2-2a^2cx+ac} - \frac{\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac} + \frac{2}{a^3cx^2-2a^2cx+ac} \\ \frac{a^2x^2}{a^2x-a} + \frac{4ax\log\left(x-\frac{1}{a}\right)}{a^2x-a} - \frac{ax}{a^2x-a} - \frac{4\log\left(x-\frac{1}{a}\right)}{a^2x-a} - \frac{4}{a^2x-a} \\ -\frac{acx^2}{2} - 3cx - \frac{4c\log\left(x-\frac{1}{a}\right)}{a} \\ \frac{a^2p^2x^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} - \frac{a^2px^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} + \frac{2ap^2x(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} + \frac{2apx(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} - \frac{4ax(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} + \frac{p^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} + \frac{}{a^2p} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a*c*x+c)**p,x)
```

```
[Out] Piecewise((c**p*x, Eq(a, 0)), (-a**2*x**2*log(x - 1/a)/(a**3*c*x**2 - 2*a**
2*c*x + a*c) + 4*a**2*x**2/(a**3*c*x**2 - 2*a**2*c*x + a*c) + 2*a*x*log(x -
 1/a)/(a**3*c*x**2 - 2*a**2*c*x + a*c) - 4*a*x/(a**3*c*x**2 - 2*a**2*c*x +
a*c) - log(x - 1/a)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + 2/(a**3*c*x**2 - 2*a
**2*c*x + a*c), Eq(p, -1)), (a**2*x**2/(a**2*x - a) + 4*a*x*log(x - 1/a)/(a
**2*x - a) - a*x/(a**2*x - a) - 4*log(x - 1/a)/(a**2*x - a) - 4/(a**2*x - a
), Eq(p, 0)), (-a*c*x**2/2 - 3*c*x - 4*c*log(x - 1/a)/a, Eq(p, 1)), (a**2*p
**2*x**2*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) - a**2*p*x
**2*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + 2*a*p**2*x*(-
a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + 2*a*p*x*(-a*c*x + c
)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) - 4*a*x*(-a*c*x + c)**p/(a**2*
p**3*x - a**2*p*x - a*p**3 + a*p) + p**2*(-a*c*x + c)**p/(a**2*p**3*x - a**
2*p*x - a*p**3 + a*p) + 3*p*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**
3 + a*p) + 4*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p), True)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^4(-acx+c)^p}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^4*(-a*c*x + c)^p/(a^2*x^2 - 1)^2, x)
```

3.188 $\int e^{4\tanh^{-1}(ax)}(c-acx)^5\,dx$

Optimal. Leaf size=53

$$-\frac{c^5(1-ax)^6}{6a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^4}{a}$$

```
[Out] -((c^5*(1 - a*x)^4)/a) + (4*c^5*(1 - a*x)^5)/(5*a) - (c^5*(1 - a*x)^6)/(6*a
)
```

Rubi [A] time = 0.0437818, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$-\frac{c^5(1-ax)^6}{6a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^4}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a*c*x)^5,x]
```

```
[Out] -((c^5*(1 - a*x)^4)/a) + (4*c^5*(1 - a*x)^5)/(5*a) - (c^5*(1 - a*x)^6)/(6*a
)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}(c-acx)^5\,dx &= c^5\int (1-ax)^3(1+ax)^2\,dx\\ &= c^5\int \left(4(1-ax)^3-4(1-ax)^4+(1-ax)^5\right)dx\\ &= -\frac{c^5(1-ax)^4}{a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}\end{aligned}$$

Mathematica [A] time = 0.0199559, size = 31, normalized size = 0.58

$$-\frac{c^5(ax-1)^4\left(5a^2x^2+14ax+11\right)}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a*c*x)^5,x]

[Out] -(c^5*(-1 + a*x)^4*(11 + 14*a*x + 5*a^2*x^2))/(30*a)
```

Maple [A] time = 0.029, size = 45, normalized size = 0.9

$$c^5\left(-\frac{x^6a^5}{6}+\frac{x^5a^4}{5}+\frac{x^4a^3}{2}-\frac{2\,x^3a^2}{3}-\frac{ax^2}{2}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^5,x)

[Out] c^5*(-1/6*x^6*a^5+1/5*x^5*a^4+1/2*x^4*a^3-2/3*x^3*a^2-1/2*a*x^2+x)
```

Maxima [A] time = 1.02925, size = 80, normalized size = 1.51

$$-\frac{1}{6}\,a^5c^5x^6+\frac{1}{5}\,a^4c^5x^5+\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3-\frac{1}{2}\,ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^5,x, algorithm="maxima")

[Out] -1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1/
2*a*c^5*x^2 + c^5*x
```

Fricas [A] time = 1.51472, size = 130, normalized size = 2.45

$$-\frac{1}{6}\,a^5c^5x^6+\frac{1}{5}\,a^4c^5x^5+\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3-\frac{1}{2}\,ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^5,x, algorithm="fricas")

[Out] -1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1/
2*a*c^5*x^2 + c^5*x
```

Sympy [A] time = 0.113358, size = 63, normalized size = 1.19

$$-\frac{a^5c^5x^6}{6}+\frac{a^4c^5x^5}{5}+\frac{a^3c^5x^4}{2}-\frac{2a^2c^5x^3}{3}-\frac{ac^5x^2}{2}+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a*c*x+c)**5,x)

[Out] -a**5*c**5*x**6/6 + a**4*c**5*x**5/5 + a**3*c**5*x**4/2 - 2*a**2*c**5*x**3/
3 - a*c**5*x**2/2 + c**5*x
```

Giac [A] time = 1.23673, size = 80, normalized size = 1.51

$$-\frac{1}{6}\,a^{5}c^{5}x^{6}+\frac{1}{5}\,a^{4}c^{5}x^{5}+\frac{1}{2}\,a^{3}c^{5}x^{4}-\frac{2}{3}\,a^{2}c^{5}x^{3}-\frac{1}{2}\,ac^{5}x^{2}+c^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1/
2*a*c^5*x^2 + c^5*x
```

3.189 $\int e^{4\tanh^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=32

$$\frac{1}{5}a^4c^4x^5-\frac{2}{3}a^2c^4x^3+c^4x$$

```
[Out] c^4*x - (2*a^2*c^4*x^3)/3 + (a^4*c^4*x^5)/5
```

Rubi [A] time = 0.0327282, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 41, 194}

$$\frac{1}{5}a^4c^4x^5-\frac{2}{3}a^2c^4x^3+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a*c*x)^4,x]
```

```
[Out] c^4*x - (2*a^2*c^4*x^3)/3 + (a^4*c^4*x^5)/5
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}(c-acx)^4\,dx &= c^4\int(1-ax)^2(1+ax)^2\,dx\\ &= c^4\int\left(1-a^2x^2\right)^2\,dx\\ &= c^4\int\left(1-2a^2x^2+a^4x^4\right)\,dx\\ &= c^4x-\frac{2}{3}a^2c^4x^3+\frac{1}{5}a^4c^4x^5\end{aligned}$$

Mathematica [A] time = 0.0141597, size = 26, normalized size = 0.81

$$c^4\left(\frac{a^4x^5}{5}-\frac{2a^2x^3}{3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a*c*x)^4,x]
```

```
[Out] c^4*(x - (2*a^2*x^3)/3 + (a^4*x^5)/5)
```

Maple [A] time = 0.026, size = 23, normalized size = 0.7

$$c^4\left(\frac{x^5a^4}{5}-\frac{2\,x^3a^2}{3}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^4,x)
```

```
[Out] c^4*(1/5*x^5*a^4-2/3*x^3*a^2+x)
```

Maxima [A] time = 0.958906, size = 38, normalized size = 1.19

$$\frac{1}{5}\,a^4c^4x^5-\frac{2}{3}\,a^2c^4x^3+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

Fricas [A] time = 1.42513, size = 58, normalized size = 1.81

$$\frac{1}{5}\,a^4c^4x^5-\frac{2}{3}\,a^2c^4x^3+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

Sympy [A] time = 0.095848, size = 29, normalized size = 0.91

$$\frac{a^4c^4x^5}{5}-\frac{2a^2c^4x^3}{3}+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a*c*x+c)**4,x)
```

```
[Out] a**4*c**4*x**5/5 - 2*a**2*c**4*x**3/3 + c**4*x
```

Giac [A] time = 1.21997, size = 38, normalized size = 1.19

$$\frac{1}{5}a^4c^4x^5 - \frac{2}{3}a^2c^4x^3 + c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

3.190 $\int e^{4\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=35

$$\frac{2c^3(ax+1)^3}{3a}-\frac{c^3(ax+1)^4}{4a}$$

```
[Out] (2*c^3*(1 + a*x)^3)/(3*a) - (c^3*(1 + a*x)^4)/(4*a)
```

Rubi [A] time = 0.0375389, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{2c^3(ax+1)^3}{3a}-\frac{c^3(ax+1)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (2*c^3*(1 + a*x)^3)/(3*a) - (c^3*(1 + a*x)^4)/(4*a)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}(c-acx)^3\,dx &= c^3\int (1-ax)(1+ax)^2\,dx\\ &= c^3\int \left(2(1+ax)^2-(1+ax)^3\right)dx\\ &= \frac{2c^3(1+ax)^3}{3a}-\frac{c^3(1+ax)^4}{4a}\end{aligned}$$

Mathematica [A] time = 0.0143999, size = 30, normalized size = 0.86

$$-\frac{1}{12}c^3x\left(3a^3x^3+4a^2x^2-6ax-12\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a*c*x)^3,x]
```

```
[Out] -(c^3*x*(-12 - 6*a*x + 4*a^2*x^2 + 3*a^3*x^3))/12
```

Maple [A] time = 0.032, size = 29, normalized size = 0.8

$$c^{3}\left(-\frac{x^{4}a^{3}}{4}-\frac{x^{3}a^{2}}{3}+\frac{ax^{2}}{2}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^3,x)
```

```
[Out] c^3*(-1/4*x^4*a^3-1/3*x^3*a^2+1/2*a*x^2+x)
```

Maxima [A] time = 0.947361, size = 50, normalized size = 1.43

$$-\frac{1}{4}a^{3}c^{3}x^{4}-\frac{1}{3}a^{2}c^{3}x^{3}+\frac{1}{2}ac^{3}x^{2}+c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

Fricas [A] time = 1.46329, size = 81, normalized size = 2.31

$$-\frac{1}{4}a^{3}c^{3}x^{4}-\frac{1}{3}a^{2}c^{3}x^{3}+\frac{1}{2}ac^{3}x^{2}+c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

Sympy [A] time = 0.096224, size = 37, normalized size = 1.06

$$-\frac{a^{3}c^{3}x^{4}}{4}-\frac{a^{2}c^{3}x^{3}}{3}+\frac{ac^{3}x^{2}}{2}+c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a*c*x+c)**3,x)
```

```
[Out] -a**3*c**3*x**4/4 - a**2*c**3*x**3/3 + a*c**3*x**2/2 + c**3*x
```

Giac [A] time = 1.22087, size = 50, normalized size = 1.43

$$-\frac{1}{4}a^{3}c^{3}x^{4}-\frac{1}{3}a^{2}c^{3}x^{3}+\frac{1}{2}ac^{3}x^{2}+c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

3.191 $\int e^{4\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=17

$$\frac{c^2(ax+1)^3}{3a}$$

```
[Out] (c^2*(1 + a*x)^3)/(3*a)
```

Rubi [A] time = 0.0246137, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 32}

$$\frac{c^2(ax+1)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (c^2*(1 + a*x)^3)/(3*a)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}(c-acx)^2\,dx &= c^2\int (1+ax)^2\,dx\\ &= \frac{c^2(1+ax)^3}{3a}\end{aligned}$$

Mathematica [A] time = 0.0118375, size = 21, normalized size = 1.24

$$c^2\left(\frac{a^2x^3}{3}+ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a*c*x)^2,x]
```

```
[Out] c^2*(x + a*x^2 + (a^2*x^3)/3)
```

Maple [A] time = 0.027, size = 16, normalized size = 0.9

$$\frac{c^2 (ax+1)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^2,x)
```

```
[Out] 1/3*c^2*(a*x+1)^3/a
```

Maxima [A] time = 0.949741, size = 34, normalized size = 2.

$$\frac{1}{3}a^2c^2x^3 + ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

Fricas [A] time = 1.4816, size = 50, normalized size = 2.94

$$\frac{1}{3}a^2c^2x^3 + ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

Sympy [A] time = 0.100328, size = 24, normalized size = 1.41

$$\frac{a^2c^2x^3}{3} + ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a*c*x+c)**2,x)
```

```
[Out] a**2*c**2*x**3/3 + a*c**2*x**2 + c**2*x
```

Giac [A] time = 1.14613, size = 34, normalized size = 2.

$$\frac{1}{3}a^2c^2x^3 + ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

3.192 $\int e^{4\tanh^{-1}(ax)}(c - acx)\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}acx^2 - \frac{4c\log(1-ax)}{a} - 3cx$$

```
[Out] -3*c*x - (a*c*x^2)/2 - (4*c*Log[1 - a*x])/a
```

Rubi [A] time = 0.0225509, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6129, 43}

$$-\frac{1}{2}acx^2 - \frac{4c\log(1-ax)}{a} - 3cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a*c*x),x]
```

```
[Out] -3*c*x - (a*c*x^2)/2 - (4*c*Log[1 - a*x])/a
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{4\tanh^{-1}(ax)}(c - acx)\,dx &= c\int \frac{(1+ax)^2}{1-ax}\,dx \\
&= c\int \left(-3 - ax + \frac{4}{1-ax}\right)dx \\
&= -3cx - \frac{1}{2}acx^2 - \frac{4c\log(1-ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0104497, size = 26, normalized size = 0.96

$$c\left(-\frac{ax^2}{2} - \frac{4\log(1-ax)}{a} - 3x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a*c*x),x]
```

```
[Out] c*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a)
```

Maple [A] time = 0.031, size = 25, normalized size = 0.9

$$-\frac{acx^2}{2} - 3\,cx - 4\,\frac{c\ln(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c),x)
```

```
[Out] -1/2*a*c*x^2-3*c*x-4*c/a*ln(a*x-1)
```

Maxima [A] time = 0.949466, size = 32, normalized size = 1.19

$$-\frac{1}{2}\,acx^2 - 3\,cx - \frac{4\,c\log(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*c*x^2 - 3*c*x - 4*c*log(a*x - 1)/a
```

Fricas [A] time = 1.54496, size = 66, normalized size = 2.44

$$-\frac{a^2cx^2 + 6\,acx + 8\,c\log(ax-1)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c*x^2 + 6*a*c*x + 8*c*log(a*x - 1))/a
```

Sympy [A] time = 0.299975, size = 26, normalized size = 0.96

$$-\frac{acx^2}{2} - 3cx - \frac{4c\log(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a*c*x+c),x)
```

```
[Out] -a*c*x**2/2 - 3*c*x - 4*c*log(a*x - 1)/a
```

Giac [A] time = 1.19612, size = 47, normalized size = 1.74

$$-\frac{4\,c\log(|ax-1|)}{a} - \frac{a^3cx^2 + 6\,a^2cx}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -4*c*log(abs(a*x - 1))/a - 1/2*(a^3*c*x^2 + 6*a^2*c*x)/a^2
```

3.193 $\int \frac{e^{4\tanh^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=48

$$-\frac{4}{ac(1-ax)}+\frac{2}{ac(1-ax)^2}-\frac{\log(1-ax)}{ac}$$

```
[Out] 2/(a*c*(1 - a*x)^2) - 4/(a*c*(1 - a*x)) - Log[1 - a*x]/(a*c)
```

Rubi [A] time = 0.0449021, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$-\frac{4}{ac(1-ax)}+\frac{2}{ac(1-ax)^2}-\frac{\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - a*c*x),x]
```

```
[Out] 2/(a*c*(1 - a*x)^2) - 4/(a*c*(1 - a*x)) - Log[1 - a*x]/(a*c)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\tanh^{-1}(ax)}}{c-acx}\,dx &= \frac{\int \frac{(1+ax)^2}{(1-ax)^3}\,dx}{c}\\
&= \frac{\int \left(\frac{1}{1-ax}-\frac{4}{(-1+ax)^3}-\frac{4}{(-1+ax)^2}\right)dx}{c}\\
&= \frac{2}{ac(1-ax)^2}-\frac{4}{ac(1-ax)}-\frac{\log(1-ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.0184713, size = 36, normalized size = 0.75

$$\frac{4ax+(ax-1)^2(-\log(1-ax))-2}{ac(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - a*c*x),x]
```

```
[Out] (-2 + 4*a*x - (-1 + a*x)^2*Log[1 - a*x])/(a*c*(-1 + a*x)^2)
```

Maple [A] time = 0.035, size = 46, normalized size = 1.

$$4\,\frac{1}{ac\,(ax-1)}+2\,\frac{1}{ac\,(ax-1)^{2}}-\frac{\ln\,(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c),x)
```

```
[Out] 4/c/a/(a*x-1)+2/c/a/(a*x-1)^2-1/c/a*ln(a*x-1)
```

Maxima [A] time = 0.952757, size = 59, normalized size = 1.23

$$\frac{2\,(2\,ax-1)}{a^{3}cx^{2}-2\,a^{2}cx+ac}-\frac{\log\,(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 2*(2*a*x - 1)/(a^3*c*x^2 - 2*a^2*c*x + a*c) - log(a*x - 1)/(a*c)
```

Fricas [A] time = 1.64862, size = 108, normalized size = 2.25

$$\frac{4\,ax-\left(a^{2}x^{2}-2\,ax+1\right)\log\,(ax-1)-2}{a^{3}cx^{2}-2\,a^{2}cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] (4*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x - 1) - 2)/(a^3*c*x^2 - 2*a^2*c*x + a
*c)
```

Sympy [A] time = 0.435039, size = 36, normalized size = 0.75

$$\frac{4ax-2}{a^{3}cx^{2}-2a^{2}cx+ac}-\frac{\log\,(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(-a*c*x+c),x)
```

```
[Out] (4*a*x - 2)/(a**3*c*x**2 - 2*a**2*c*x + a*c) - log(a*x - 1)/(a*c)
```

Giac [A] time = 1.18496, size = 50, normalized size = 1.04

$$-\frac{\log\left(|ax-1|\right)}{ac}+\frac{2\,(2\,ax-1)}{(ax-1)^2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -log(abs(a*x - 1))/(a*c) + 2*(2*a*x - 1)/((a*x - 1)^2*a*c)
```

3.194 $\int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=25

$$\frac{(ax+1)^3}{6ac^2(1-ax)^3}$$

```
[Out] (1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

Rubi [A] time = 0.0276597, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 37}

$$\frac{(ax+1)^3}{6ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^2}\,dx &= \frac{\int \frac{(1+ax)^2}{(1-ax)^4}\,dx}{c^2}\\ &= \frac{(1+ax)^3}{6ac^2(1-ax)^3}\end{aligned}$$

Mathematica [A] time = 0.0081627, size = 25, normalized size = 1.

$$\frac{(ax+1)^3}{6ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

Maple [A] time = 0.036, size = 42, normalized size = 1.7

$$\frac{1}{c^2}\left(-\frac{1}{a\,(ax-1)}-2\,\frac{1}{a\,(ax-1)^2}-\frac{4}{3\,a\,(ax-1)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^2,x)
```

```
[Out] 1/c^2*(-1/a/(a*x-1)-2/a/(a*x-1)^2-4/3/a/(a*x-1)^3)
```

Maxima [B] time = 0.952293, size = 69, normalized size = 2.76

$$-\frac{3\,a^2x^2+1}{3\,\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a^2*x^2 + 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Fricas [B] time = 1.69147, size = 100, normalized size = 4.

$$-\frac{3\,a^2x^2+1}{3\,\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*x^2 + 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [B] time = 0.476108, size = 51, normalized size = 2.04

$$-\frac{3a^2x^2+1}{3a^4c^2x^3-9a^3c^2x^2+9a^2c^2x-3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(-a*c*x+c)**2,x)
```

```
[Out] -(3*a**2*x**2 + 1)/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2*c**2*x - 3
*a*c**2)
```

Giac [B] time = 1.178, size = 68, normalized size = 2.72

$$-\frac{2}{(acx-c)^2a}-\frac{1}{(acx-c)ac}-\frac{4c}{3(acx-c)^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -2/((a*c*x - c)^2*a) - 1/((a*c*x - c)*a*c) - 4/3*c/((a*c*x - c)^3*a)
```

3.195 $\int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=52

$$\frac{1}{2ac^3(1-ax)^2}-\frac{4}{3ac^3(1-ax)^3}+\frac{1}{ac^3(1-ax)^4}$$

```
[Out] 1/(a*c^3*(1 - a*x)^4) - 4/(3*a*c^3*(1 - a*x)^3) + 1/(2*a*c^3*(1 - a*x)^2)
```

Rubi [A] time = 0.0431774, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{1}{2ac^3(1-ax)^2}-\frac{4}{3ac^3(1-ax)^3}+\frac{1}{ac^3(1-ax)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - a*c*x)^3,x]
```

```
[Out] 1/(a*c^3*(1 - a*x)^4) - 4/(3*a*c^3*(1 - a*x)^3) + 1/(2*a*c^3*(1 - a*x)^2)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^3}\,dx &= \frac{\int \frac{(1+ax)^2}{(1-ax)^5}\,dx}{c^3}\\
&= \frac{\int \left(-\frac{4}{(-1+ax)^5}-\frac{4}{(-1+ax)^4}-\frac{1}{(-1+ax)^3}\right)dx}{c^3}\\
&= \frac{1}{ac^3(1-ax)^4}-\frac{4}{3ac^3(1-ax)^3}+\frac{1}{2ac^3(1-ax)^2}
\end{aligned}$$

Mathematica [A] time = 0.0169185, size = 31, normalized size = 0.6

$$\frac{3a^2x^2+2ax+1}{6ac^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (1 + 2*a*x + 3*a^2*x^2)/(6*a*c^3*(-1 + a*x)^4)
```

Maple [A] time = 0.032, size = 41, normalized size = 0.8

$$\frac{1}{c^3}\left(\frac{1}{2\,a\,(ax-1)^2}+\frac{4}{3\,a\,(ax-1)^3}+\frac{1}{a\,(ax-1)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^3,x)
```

```
[Out] 1/c^3*(1/2/a/(a*x-1)^2+4/3/a/(a*x-1)^3+1/a/(a*x-1)^4)
```

Maxima [A] time = 0.951407, size = 88, normalized size = 1.69

$$\frac{3\,a^2x^2+2\,ax+1}{6\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*x^2 + 2*a*x + 1)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 -
4*a^2*c^3*x + a*c^3)
```

Fricas [A] time = 1.55492, size = 131, normalized size = 2.52

$$\frac{3\,a^2x^2+2\,ax+1}{6\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*x^2 + 2*a*x + 1)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 -
4*a^2*c^3*x + a*c^3)
```

Sympy [A] time = 0.5357, size = 66, normalized size = 1.27

$$\frac{3a^2x^2+2ax+1}{6a^5c^3x^4-24a^4c^3x^3+36a^3c^3x^2-24a^2c^3x+6ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(-a*c*x+c)**3,x)
```

```
[Out] (3*a**2*x**2 + 2*a*x + 1)/(6*a**5*c**3*x**4 - 24*a**4*c**3*x**3 + 36*a**3*c
**3*x**2 - 24*a**2*c**3*x + 6*a*c**3)
```

Giac [A] time = 1.17137, size = 39, normalized size = 0.75

$$\frac{3\,a^2x^2+2\,ax+1}{6\,(ax-1)^4ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*a^2*x^2 + 2*a*x + 1)/((a*x - 1)^4*a*c^3)
```

3.196 $\int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=53

$$\frac{1}{3ac^4(1-ax)^3}-\frac{1}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}$$

```
[Out] 4/(5*a*c^4*(1 - a*x)^5) - 1/(a*c^4*(1 - a*x)^4) + 1/(3*a*c^4*(1 - a*x)^3)
```

Rubi [A] time = 0.0420262, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{1}{3ac^4(1-ax)^3}-\frac{1}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - a*c*x)^4,x]
```

```
[Out] 4/(5*a*c^4*(1 - a*x)^5) - 1/(a*c^4*(1 - a*x)^4) + 1/(3*a*c^4*(1 - a*x)^3)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^4}\,dx &= \frac{\int \frac{(1+ax)^2}{(1-ax)^6}\,dx}{c^4}\\ &= \frac{\int \left(\frac{4}{(-1+ax)^6}+\frac{4}{(-1+ax)^5}+\frac{1}{(-1+ax)^4}\right)dx}{c^4}\\ &= \frac{4}{5ac^4(1-ax)^5}-\frac{1}{ac^4(1-ax)^4}+\frac{1}{3ac^4(1-ax)^3}\end{aligned}$$

Mathematica [A] time = 0.0179795, size = 31, normalized size = 0.58

$$-\frac{5a^2x^2+5ax+2}{15ac^4(ax-1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -(2 + 5*a*x + 5*a^2*x^2)/(15*a*c^4*(-1 + a*x)^5)
```

Maple [A] time = 0.033, size = 42, normalized size = 0.8

$$\frac{1}{c^4}\left(-\frac{1}{3\,a\,(ax-1)^3}-\frac{1}{a\,(ax-1)^4}-\frac{4}{5\,a\,(ax-1)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^4,x)
```

```
[Out] 1/c^4*(-1/3/a/(a*x-1)^3-1/a/(a*x-1)^4-4/5/a/(a*x-1)^5)
```

Maxima [A] time = 0.965371, size = 104, normalized size = 1.96

$$-\frac{5\,a^2x^2+5\,ax+2}{15\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/15*(5*a^2*x^2 + 5*a*x + 2)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3
 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Fricas [A] time = 1.5278, size = 158, normalized size = 2.98

$$-\frac{5\,a^2x^2+5\,ax+2}{15\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(5*a^2*x^2 + 5*a*x + 2)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3
 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [A] time = 0.652361, size = 80, normalized size = 1.51

$$-\frac{5a^2x^2+5ax+2}{15a^6c^4x^5-75a^5c^4x^4+150a^4c^4x^3-150a^3c^4x^2+75a^2c^4x-15ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(-a*c*x+c)**4,x)
```

```
[Out] -(5*a**2*x**2 + 5*a*x + 2)/(15*a**6*c**4*x**5 - 75*a**5*c**4*x**4 + 150*a**
4*c**4*x**3 - 150*a**3*c**4*x**2 + 75*a**2*c**4*x - 15*a*c**4)
```

Giac [A] time = 1.17879, size = 39, normalized size = 0.74

$$-\frac{5\,a^2x^2+5\,ax+2}{15\,(ax-1)^5ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/15*(5*a^2*x^2 + 5*a*x + 2)/((a*x - 1)^5*a*c^4)
```

3.197 $\int e^{-\tanh^{-1}(ax)}(c - acx)^p \, dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{2}\sqrt{1-ax}(c-acx)^{p+1}\text{Hypergeometric2F1}\left(\frac{1}{2},p+\frac{3}{2},p+\frac{5}{2},\frac{1}{2}(1-ax)\right)}{ac(2p+3)}$$

```
[Out] -((Sqrt[2]*Sqrt[1 - a*x]*(c - a*c*x)^(1 + p)*Hypergeometric2F1[1/2, 3/2 + p
, 5/2 + p, (1 - a*x)/2])/(a*c*(3 + 2*p)))
```

Rubi [A] time = 0.0493803, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6130, 23, 69}

$$-\frac{\sqrt{2}\sqrt{1-ax}(c-acx)^{p+1}\,{}_2F_1\left(\frac{1}{2},p+\frac{3}{2};p+\frac{5}{2};\frac{1}{2}(1-ax)\right)}{ac(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^p/E^ArcTanh[a*x],x]
```

```
[Out] -((Sqrt[2]*Sqrt[1 - a*x]*(c - a*c*x)^(1 + p)*Hypergeometric2F1[1/2, 3/2 + p
, 5/2 + p, (1 - a*x)/2])/(a*c*(3 + 2*p)))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}(c-acx)^p\,dx=\int\frac{\sqrt{1-ax}(c-acx)^p}{\sqrt{1+ax}}\,dx$$
$$=\frac{\sqrt{1-ax}\int\frac{(c-acx)^{\frac{1}{2}+p}}{\sqrt{1+ax}}\,dx}{\sqrt{c-acx}}$$
$$=-\frac{\sqrt{2}\sqrt{1-ax}(c-acx)^{1+p}\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+p;\frac{5}{2}+p;\frac{1}{2}(1-ax)\right)}{ac(3+2p)}$$

Mathematica [A] time = 0.0232447, size = 59, normalized size = 0.91

$$\frac{\sqrt{2-2ax}(ax-1)(c-acx)^p\text{Hypergeometric2F1}\left(\frac{1}{2},p+\frac{3}{2},p+\frac{5}{2},\frac{1}{2}-\frac{ax}{2}\right)}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^p/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[2 - 2*a*x]*(-1 + a*x)*(c - a*c*x)^p*Hypergeometric2F1[1/2, 3/2 + p, 5
/2 + p, 1/2 - (a*x)/2])/(a*(3 + 2*p))
```

Maple [F] time = 0.451, size = 0, normalized size = 0.

$$\int\frac{(-acx+c)^p}{ax+1}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int((-a*c*x+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}(-acx+c)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}(-acx+c)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c(ax-1))^p \sqrt{-(ax-1)(ax+1)}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**p*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}(-acx+c)^p}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

3.198 $\int e^{-\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=133

$$\frac{c^3(1-ax)^3\sqrt{1-a^2x^2}}{4a}+\frac{7c^3(1-ax)^2\sqrt{1-a^2x^2}}{12a}+\frac{35c^3(1-ax)\sqrt{1-a^2x^2}}{24a}+\frac{35c^3\sqrt{1-a^2x^2}}{8a}+\frac{35c^3\sin^{-1}(ax)}{8a}$$

```
[Out] (35*c^3*Sqrt[1 - a^2*x^2])/(8*a) + (35*c^3*(1 - a*x)*Sqrt[1 - a^2*x^2])/(24
*a) + (7*c^3*(1 - a*x)^2*Sqrt[1 - a^2*x^2])/(12*a) + (c^3*(1 - a*x)^3*Sqrt[
1 - a^2*x^2])/(4*a) + (35*c^3*ArcSin[a*x])/(8*a)
```

Rubi [A] time = 0.0955488, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6127, 671, 641, 216}

$$\frac{c^3(1-ax)^3\sqrt{1-a^2x^2}}{4a}+\frac{7c^3(1-ax)^2\sqrt{1-a^2x^2}}{12a}+\frac{35c^3(1-ax)\sqrt{1-a^2x^2}}{24a}+\frac{35c^3\sqrt{1-a^2x^2}}{8a}+\frac{35c^3\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^3/E^ArcTanh[a*x],x]
```

```
[Out] (35*c^3*Sqrt[1 - a^2*x^2])/(8*a) + (35*c^3*(1 - a*x)*Sqrt[1 - a^2*x^2])/(24
*a) + (7*c^3*(1 - a*x)^2*Sqrt[1 - a^2*x^2])/(12*a) + (c^3*(1 - a*x)^3*Sqrt[
1 - a^2*x^2])/(4*a) + (35*c^3*ArcSin[a*x])/(8*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}(c-acx)^3\,dx = \frac{\int \frac{(c-acx)^4}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{c^3(1-ax)^3\sqrt{1-a^2x^2}}{4a} + \frac{7}{4}\int \frac{(c-acx)^3}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{7c^3(1-ax)^2\sqrt{1-a^2x^2}}{12a} + \frac{c^3(1-ax)^3\sqrt{1-a^2x^2}}{4a} + \frac{1}{12}(35c)\int \frac{(c-acx)^2}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{35c^3(1-ax)\sqrt{1-a^2x^2}}{24a} + \frac{7c^3(1-ax)^2\sqrt{1-a^2x^2}}{12a} + \frac{c^3(1-ax)^3\sqrt{1-a^2x^2}}{4a} + \frac{1}{8}\left(35c^2\right)\int$$
$$= \frac{35c^3\sqrt{1-a^2x^2}}{8a} + \frac{35c^3(1-ax)\sqrt{1-a^2x^2}}{24a} + \frac{7c^3(1-ax)^2\sqrt{1-a^2x^2}}{12a} + \frac{c^3(1-ax)^3\sqrt{1-a^2}}{4a}$$
$$= \frac{35c^3\sqrt{1-a^2x^2}}{8a} + \frac{35c^3(1-ax)\sqrt{1-a^2x^2}}{24a} + \frac{7c^3(1-ax)^2\sqrt{1-a^2x^2}}{12a} + \frac{c^3(1-ax)^3\sqrt{1-a^2}}{4a}$$

Mathematica [A] time = 0.0693521, size = 80, normalized size = 0.6

$$\frac{c^3\left(\frac{\sqrt{ax+1}\left(6a^4x^4-38a^3x^3+113a^2x^2-241ax+160\right)}{\sqrt{1-ax}}-210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^3/E^ArcTanh[a*x],x]
```

```
[Out] (c^3*((Sqrt[1 + a*x]*(160 - 241*a*x + 113*a^2*x^2 - 38*a^3*x^3 + 6*a^4*x^4)
)/Sqrt[1 - a*x] - 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(24*a)
```

Maple [A] time = 0.039, size = 160, normalized size = 1.2

$$\frac{c^3x}{4}\left(-a^2x^2+1\right)^{\frac{3}{2}}-\frac{29\,c^3x}{8}\sqrt{-a^2x^2+1}-\frac{29\,c^3}{8}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{4\,c^3}{3\,a}\left(-a^2x^2+1\right)^{\frac{3}{2}}+8\frac{c^3\sqrt{-a^2\left(x+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/4*c^3*x*(-a^2*x^2+1)^(3/2)-29/8*c^3*x*(-a^2*x^2+1)^(1/2)-29/8*c^3/(a^2)^(
1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-4/3*c^3*(-a^2*x^2+1)^(3/2)/a+
8*c^3/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+8*c^3/(a^2)^(1/2)*arctan((a^2)^(
1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

Maxima [A] time = 1.4716, size = 120, normalized size = 0.9

$$\frac{1}{4}\left(-a^2x^2+1\right)^{\frac{3}{2}}c^3x-\frac{29}{8}\sqrt{-a^2x^2+1}c^3x-\frac{4\left(-a^2x^2+1\right)^{\frac{3}{2}}c^3}{3\,a}+\frac{35\,c^3\arcsin\left(ax\right)}{8\,a}+\frac{8\sqrt{-a^2x^2+1}c^3}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-a^2*x^2 + 1)^(3/2)*c^3*x - 29/8*sqrt(-a^2*x^2 + 1)*c^3*x - 4/3*(-a^2*
x^2 + 1)^(3/2)*c^3/a + 35/8*c^3*arcsin(a*x)/a + 8*sqrt(-a^2*x^2 + 1)*c^3/a
```

Fricas [A] time = 1.62902, size = 182, normalized size = 1.37

$$-\frac{210\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(6\,a^3c^3x^3-32\,a^2c^3x^2+81\,ac^3x-160\,c^3\right)\sqrt{-a^2x^2+1}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(210*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (6*a^3*c^3*x^3 - 32
*a^2*c^3*x^2 + 81*a*c^3*x - 160*c^3)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^3\left(\int -\frac{\sqrt{-a^2x^2+1}}{ax+1}\,dx+\int \frac{3ax\sqrt{-a^2x^2+1}}{ax+1}\,dx+\int -\frac{3a^2x^2\sqrt{-a^2x^2+1}}{ax+1}\,dx+\int \frac{a^3x^3\sqrt{-a^2x^2+1}}{ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**3/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -c**3*(Integral(-sqrt(-a**2*x**2 + 1)/(a*x + 1), x) + Integral(3*a*x*sqrt(-
a**2*x**2 + 1)/(a*x + 1), x) + Integral(-3*a**2*x**2*sqrt(-a**2*x**2 + 1)/(
a*x + 1), x) + Integral(a**3*x**3*sqrt(-a**2*x**2 + 1)/(a*x + 1), x))
```

Giac [A] time = 1.189, size = 90, normalized size = 0.68

$$\frac{35\,c^3\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{8\,|a|}+\frac{1}{24}\,\sqrt{-a^2x^2+1}\left(\frac{160\,c^3}{a}-\left(81\,c^3+2\left(3\,a^2c^3x-16\,ac^3\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 35/8*c^3*arcsin(a*x)*sgn(a)/abs(a) + 1/24*sqrt(-a^2*x^2 + 1)*(160*c^3/a - (
81*c^3 + 2*(3*a^2*c^3*x - 16*a*c^3)*x)*x)
```

3.199 $\int e^{-\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=101

$$\frac{c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{5c^2(1-ax)\sqrt{1-a^2x^2}}{6a}+\frac{5c^2\sqrt{1-a^2x^2}}{2a}+\frac{5c^2\sin^{-1}(ax)}{2a}$$

```
[Out] (5*c^2*Sqrt[1 - a^2*x^2])/(2*a) + (5*c^2*(1 - a*x)*Sqrt[1 - a^2*x^2])/(6*a)
 + (c^2*(1 - a*x)^2*Sqrt[1 - a^2*x^2])/(3*a) + (5*c^2*ArcSin[a*x])/(2*a)
```

Rubi [A] time = 0.0699396, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6127, 671, 641, 216}

$$\frac{c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{5c^2(1-ax)\sqrt{1-a^2x^2}}{6a}+\frac{5c^2\sqrt{1-a^2x^2}}{2a}+\frac{5c^2\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^2/E^ArcTanh[a*x],x]
```


```
[Out] (5*c^2*Sqrt[1 - a^2*x^2])/(2*a) + (5*c^2*(1 - a*x)*Sqrt[1 - a^2*x^2])/(6*a)
 + (c^2*(1 - a*x)^2*Sqrt[1 - a^2*x^2])/(3*a) + (5*c^2*ArcSin[a*x])/(2*a)
```


Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}(c-acx)^2\,dx = \frac{\int \frac{(c-acx)^3}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a} + \frac{5}{3}\int \frac{(c-acx)^2}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{5c^2(1-ax)\sqrt{1-a^2x^2}}{6a} + \frac{c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a} + \frac{1}{2}(5c)\int \frac{c-acx}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{5c^2\sqrt{1-a^2x^2}}{2a} + \frac{5c^2(1-ax)\sqrt{1-a^2x^2}}{6a} + \frac{c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a} + \frac{1}{2}\left(5c^2\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{5c^2\sqrt{1-a^2x^2}}{2a} + \frac{5c^2(1-ax)\sqrt{1-a^2x^2}}{6a} + \frac{c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a} + \frac{5c^2\sin^{-1}(ax)}{2a}$$

Mathematica [A] time = 0.0822894, size = 72, normalized size = 0.71

$$\frac{c^2\left(\frac{\sqrt{ax+1}\left(-2a^3x^3+11a^2x^2-31ax+22\right)}{\sqrt{1-ax}} - 30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^2/E^ArcTanh[a*x],x]
```

```
[Out] (c^2*((Sqrt[1 + a*x]*(22 - 31*a*x + 11*a^2*x^2 - 2*a^3*x^3))/Sqrt[1 - a*x]
- 30*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(6*a)
```

Maple [A] time = 0.041, size = 142, normalized size = 1.4

$$-\frac{c^2}{3\,a}\left(-a^2x^2+1\right)^{\frac{3}{2}} - \frac{3\,xc^2}{2}\sqrt{-a^2x^2+1} - \frac{3\,c^2}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} + 4\,\frac{c^2\sqrt{-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)}}{a} + 4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/3*c^2*(-a^2*x^2+1)^(3/2)/a-3/2*c^2*x*(-a^2*x^2+1)^(1/2)-3/2*c^2/(a^2)^(1
/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+4*c^2/a*(-a^2*(x+1/a)^2+2*a*(x
+1/a))^(1/2)+4*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+
1/a))^(1/2))
```

Maxima [A] time = 1.44836, size = 96, normalized size = 0.95

$$-\frac{3}{2}\sqrt{-a^2x^2+1}c^2x - \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}{3\,a} + \frac{5\,c^2\arcsin(ax)}{2\,a} + \frac{4\sqrt{-a^2x^2+1}c^2}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(-a^2*x^2 + 1)*c^2*x - 1/3*(-a^2*x^2 + 1)^(3/2)*c^2/a + 5/2*c^2*ar
csin(a*x)/a + 4*sqrt(-a^2*x^2 + 1)*c^2/a
```

Fricas [A] time = 1.84439, size = 154, normalized size = 1.52

$$-\frac{30\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(2\,a^2c^2x^2-9\,ac^2x+22\,c^2\right)\sqrt{-a^2x^2+1}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(30*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (2*a^2*c^2*x^2 - 9*a*
c^2*x + 22*c^2)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int\frac{\sqrt{-a^2x^2+1}}{ax+1}\,dx+\int-\frac{2ax\sqrt{-a^2x^2+1}}{ax+1}\,dx+\int\frac{a^2x^2\sqrt{-a^2x^2+1}}{ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**2/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c**2*(Integral(sqrt(-a**2*x**2 + 1)/(a*x + 1), x) + Integral(-2*a*x*sqrt(-a
**2*x**2 + 1)/(a*x + 1), x) + Integral(a**2*x**2*sqrt(-a**2*x**2 + 1)/(a*x
+ 1), x))
```

Giac [A] time = 1.20531, size = 73, normalized size = 0.72

$$\frac{5\,c^2\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{2\,|a|}+\frac{1}{6}\,\sqrt{-a^2x^2+1}\left(\left(2\,ac^2x-9\,c^2\right)x+\frac{22\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 5/2*c^2*arcsin(a*x)*sgn(a)/abs(a) + 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c^2*x - 9*
c^2)*x + 22*c^2/a)
```

3.200 $\int e^{-\tanh^{-1}(ax)}(c - acx)\,dx$

Optimal. Leaf size=63

$$\frac{c(1-ax)\sqrt{1-a^2x^2}}{2a}+\frac{3c\sqrt{1-a^2x^2}}{2a}+\frac{3c\sin^{-1}(ax)}{2a}$$

```
[Out] (3*c*Sqrt[1 - a^2*x^2])/(2*a) + (c*(1 - a*x)*Sqrt[1 - a^2*x^2])/(2*a) + (3*
c*ArcSin[a*x])/(2*a)
```

Rubi [A] time = 0.0428937, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6127, 671, 641, 216}

$$\frac{c(1-ax)\sqrt{1-a^2x^2}}{2a}+\frac{3c\sqrt{1-a^2x^2}}{2a}+\frac{3c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)/E^ArcTanh[a*x],x]
```

```
[Out] (3*c*Sqrt[1 - a^2*x^2])/(2*a) + (c*(1 - a*x)*Sqrt[1 - a^2*x^2])/(2*a) + (3*
c*ArcSin[a*x])/(2*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}(c-acx)\,dx=\frac{\int\frac{(c-acx)^2}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$=\frac{c(1-ax)\sqrt{1-a^2x^2}}{2a}+\frac{3}{2}\int\frac{c-acx}{\sqrt{1-a^2x^2}}\,dx$$
$$=\frac{3c\sqrt{1-a^2x^2}}{2a}+\frac{c(1-ax)\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}(3c)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$=\frac{3c\sqrt{1-a^2x^2}}{2a}+\frac{c(1-ax)\sqrt{1-a^2x^2}}{2a}+\frac{3c\sin^{-1}(ax)}{2a}$$

Mathematica [A] time = 0.0494209, size = 61, normalized size = 0.97

$$\frac{c\left(\frac{\sqrt{ax+1}(a^2x^2-5ax+4)}{\sqrt{1-ax}}-6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)/E^ArcTanh[a*x],x]
```

```
[Out] (c*((Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2))/Sqrt[1 - a*x] - 6*ArcSin[Sqrt[1 -
 a*x]/Sqrt[2]]))/(2*a)
```

Maple [B] time = 0.036, size = 114, normalized size = 1.8

$$-\frac{cx}{2}\sqrt{-a^2x^2+1}-\frac{c}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+2\frac{c\sqrt{-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)}}{a}+2\frac{c}{\sqrt{a^2}}\arctan\left(\frac{}{\sqrt{-a^2\left(\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*c*x*(-a^2*x^2+1)^(1/2)-1/2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^
2+1)^(1/2))+2*c/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+2*c/(a^2)^(1/2)*arctan
((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

Maxima [A] time = 1.43728, size = 61, normalized size = 0.97

$$-\frac{1}{2}\sqrt{-a^2x^2+1}cx+\frac{3\,c\arcsin(ax)}{2\,a}+\frac{2\sqrt{-a^2x^2+1}c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*c*x + 3/2*c*arcsin(a*x)/a + 2*sqrt(-a^2*x^2 + 1)*c/
a
```

Fricas [A] time = 1.65818, size = 119, normalized size = 1.89

$$-\frac{6\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}(acx-4\,c)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(6*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*(a*c*
x - 4*c))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int -\frac{\sqrt{-a^2x^2+1}}{ax+1}\,dx+\int \frac{ax\sqrt{-a^2x^2+1}}{ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -c*(Integral(-sqrt(-a**2*x**2 + 1)/(a*x + 1), x) + Integral(a*x*sqrt(-a**2*
x**2 + 1)/(a*x + 1), x))
```

Giac [A] time = 1.20465, size = 51, normalized size = 0.81

$$\frac{3\,c\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{2\,|a|}-\frac{1}{2}\,\sqrt{-a^2x^2+1}\left(cx-\frac{4\,c}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*c*arcsin(a*x)*sgn(a)/abs(a) - 1/2*sqrt(-a^2*x^2 + 1)*(c*x - 4*c/a)
```

3.201 $\int \frac{e^{-\tanh^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=11

$$\frac{\sin^{-1}(ax)}{ac}$$

[Out] ArcSin[a*x]/(a*c)

Rubi [A] time = 0.0279087, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6127, 216}

$$\frac{\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

[In] Int[1/(E^ArcTanh[a*x]*(c - a*c*x)),x]

[Out] ArcSin[a*x]/(a*c)

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tanh^{-1}(ax)}}{c-acx}\,dx &= \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\ &= \frac{\sin^{-1}(ax)}{ac}\end{aligned}$$

Mathematica [A] time = 0.0068999, size = 11, normalized size = 1.

$$\frac{\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

[In] Integrate[1/(E^ArcTanh[a*x]*(c - a*c*x)),x]

[Out] ArcSin[a*x]/(a*c)

Maple [B] time = 0.041, size = 154, normalized size = 14.

$$\frac{1}{2ac}\sqrt{-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)}+\frac{1}{2c}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)}}\right)\frac{1}{\sqrt{a^2}}-\frac{1}{2ac}\sqrt{-a^2\left(x-a^{-1}\right)^2-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x)
```

```
[Out] 1/2/c/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1/2/c/(a^2)^(1/2)*arctan((a^2)^(
1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-1/2/c/a*(-a^2*(x-1/a)^2-2*a*(x-1
/a))^(1/2)+1/2/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/
a))^(1/2))
```

Maxima [B] time = 1.47068, size = 42, normalized size = 3.82

$$\frac{\arcsin\left(\frac{x}{c\sqrt{\frac{1}{a^2c^2}}}\right)}{a^2c^2\sqrt{\frac{1}{a^2c^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] arcsin(x/(c*sqrt(1/(a^2*c^2))))/(a^2*c^2*sqrt(1/(a^2*c^2)))
```

Fricas [B] time = 1.58867, size = 66, normalized size = 6.

$$-\frac{2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x))/(a*c)
```

Sympy [A] time = 4.22909, size = 44, normalized size = 4.

$$\frac{\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c),x)
```

```
[Out] Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asin
h(x*sqrt(-a**2)), a**2 < 0))/c
```

Giac [A] time = 1.19563, size = 19, normalized size = 1.73

$$\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{c|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/(c*abs(a))
```

3.202 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=29

$$\frac{\sqrt{1-a^2x^2}}{ac^2(1-ax)}$$

```
[Out] Sqrt[1 - a^2*x^2]/(a*c^2*(1 - a*x))
```

Rubi [A] time = 0.0356859, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6127, 651}

$$\frac{\sqrt{1-a^2x^2}}{ac^2(1-ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a*c*x)^2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a*c^2*(1 - a*x))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{1}{(c-acx)\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{\sqrt{1-a^2x^2}}{ac^2(1-ax)}$$

Mathematica [A] time = 0.0089311, size = 26, normalized size = 0.9

$$\frac{\sqrt{ax+1}}{ac^2\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a*c*x)^2),x]
```

```
[Out] Sqrt[1 + a*x]/(a*c^2*Sqrt[1 - a*x])
```

Maple [A] time = 0.031, size = 28, normalized size = 1.

$$-\frac{1}{(ax-1)\,ac^2}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/(a*x-1)/a/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(acx-c)^2(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*c*x - c)^2*(a*x + 1)), x)
```

Fricas [A] time = 1.66866, size = 70, normalized size = 2.41

$$\frac{ax-\sqrt{-a^2x^2+1}-1}{a^2c^2x-ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] (a*x - sqrt(-a^2*x^2 + 1) - 1)/(a^2*c^2*x - a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^3x^3-a^2x^2-ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**2,x)
```

```
[Out] Integral(sqrt(-a**2*x**2 + 1)/(a**3*x**3 - a**2*x**2 - a*x + 1), x)/c**2
```

Giac [C] time = 1.19765, size = 95, normalized size = 3.28

$$-c^2\left(\frac{\sqrt{-\frac{2c}{acx-c}-1}\operatorname{sgn}\left(\frac{1}{acx-c}\right)\operatorname{sgn}(a)\operatorname{sgn}(c)}{a^2c^4}-\frac{i\operatorname{sgn}\left(\frac{1}{acx-c}\right)\operatorname{sgn}(a)\operatorname{sgn}(c)}{a^2c^4}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -c^2*(sqrt(-2*c/(a*c*x - c) - 1)*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)/(a^2*c^4)
 - I*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)/(a^2*c^4))*abs(a)
```

3.203 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^3} dx$

Optimal. Leaf size=65

$$\frac{\sqrt{1-a^2x^2}}{3ac^3(1-ax)}+\frac{\sqrt{1-a^2x^2}}{3ac^3(1-ax)^2}$$

```
[Out] Sqrt[1 - a^2*x^2]/(3*a*c^3*(1 - a*x)^2) + Sqrt[1 - a^2*x^2]/(3*a*c^3*(1 - a
*x))
```

Rubi [A] time = 0.0516709, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 651}

$$\frac{\sqrt{1-a^2x^2}}{3ac^3(1-ax)}+\frac{\sqrt{1-a^2x^2}}{3ac^3(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a*c*x)^3),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(3*a*c^3*(1 - a*x)^2) + Sqrt[1 - a^2*x^2]/(3*a*c^3*(1 - a
*x))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\int \frac{1}{(c-acx)^2\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{\sqrt{1-a^2x^2}}{3ac^3(1-ax)^2} + \frac{\int \frac{1}{(c-acx)\sqrt{1-a^2x^2}}\,dx}{3c^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{3ac^3(1-ax)^2} + \frac{\sqrt{1-a^2x^2}}{3ac^3(1-ax)}$$

Mathematica [A] time = 0.0168789, size = 34, normalized size = 0.52

$$-\frac{(ax-2)\sqrt{ax+1}}{3ac^3(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a*c*x)^3),x]
```

```
[Out] -((-2 + a*x)*Sqrt[1 + a*x])/(3*a*c^3*(1 - a*x)^(3/2))
```

Maple [A] time = 0.032, size = 33, normalized size = 0.5

$$-\frac{ax-2}{3\,(ax-1)^2\,c^3a}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x)
```

```
[Out] -1/3*(-a^2*x^2+1)^(1/2)*(a*x-2)/(a*x-1)^2/c^3/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-a^2x^2+1}}{(acx-c)^3(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-a^2*x^2 + 1)/((a*c*x - c)^3*(a*x + 1)), x)
```

Fricas [A] time = 1.61314, size = 130, normalized size = 2.

$$\frac{2\,a^2x^2 - 4\,ax - \sqrt{-a^2x^2+1}(ax-2) + 2}{3\left(a^3c^3x^2 - 2\,a^2c^3x + ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(2*a^2*x^2 - 4*a*x - sqrt(-a^2*x^2 + 1)*(a*x - 2) + 2)/(a^3*c^3*x^2 - 2
*a^2*c^3*x + a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^4x^4-2a^3x^3+2ax-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**3,x)
```

```
[Out] -Integral(sqrt(-a**2*x**2 + 1)/(a**4*x**4 - 2*a**3*x**3 + 2*a*x - 1), x)/c*
*3
```

Giac [A] time = 1.23481, size = 123, normalized size = 1.89

$$-\frac{2\left(\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}-2\right)}{3\,c^3\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 3*(sqrt(-a^2*x^2 + 1)*abs
(a) + a)^2/(a^4*x^2) - 2)/(c^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1
)^3*abs(a))
```

3.204 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^4} dx$

Optimal. Leaf size=97

$$\frac{2\sqrt{1-a^2x^2}}{15ac^4(1-ax)}+\frac{2\sqrt{1-a^2x^2}}{15ac^4(1-ax)^2}+\frac{\sqrt{1-a^2x^2}}{5ac^4(1-ax)^3}$$

```
[Out] Sqrt[1 - a^2*x^2]/(5*a*c^4*(1 - a*x)^3) + (2*Sqrt[1 - a^2*x^2])/(15*a*c^4*(
1 - a*x)^2) + (2*Sqrt[1 - a^2*x^2])/(15*a*c^4*(1 - a*x))
```

Rubi [A] time = 0.0681872, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 651}

$$\frac{2\sqrt{1-a^2x^2}}{15ac^4(1-ax)}+\frac{2\sqrt{1-a^2x^2}}{15ac^4(1-ax)^2}+\frac{\sqrt{1-a^2x^2}}{5ac^4(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a*c*x)^4),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(5*a*c^4*(1 - a*x)^3) + (2*Sqrt[1 - a^2*x^2])/(15*a*c^4*(
1 - a*x)^2) + (2*Sqrt[1 - a^2*x^2])/(15*a*c^4*(1 - a*x))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{1}{(c-acx)^3\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{\sqrt{1-a^2x^2}}{5ac^4(1-ax)^3} + \frac{2\int \frac{1}{(c-acx)^2\sqrt{1-a^2x^2}}\,dx}{5c^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{5ac^4(1-ax)^3} + \frac{2\sqrt{1-a^2x^2}}{15ac^4(1-ax)^2} + \frac{2\int \frac{1}{(c-acx)\sqrt{1-a^2x^2}}\,dx}{15c^3}$$
$$= \frac{\sqrt{1-a^2x^2}}{5ac^4(1-ax)^3} + \frac{2\sqrt{1-a^2x^2}}{15ac^4(1-ax)^2} + \frac{2\sqrt{1-a^2x^2}}{15ac^4(1-ax)}$$

Mathematica [A] time = 0.0200372, size = 43, normalized size = 0.44

$$\frac{\sqrt{ax+1}\left(2a^2x^2-6ax+7\right)}{15ac^4(1-ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a*c*x)^4),x]
```

```
[Out] (Sqrt[1 + a*x]*(7 - 6*a*x + 2*a^2*x^2))/(15*a*c^4*(1 - a*x)^(5/2))
```

Maple [A] time = 0.03, size = 42, normalized size = 0.4

$$-\frac{2\,a^2x^2-6\,ax+7}{15\,(ax-1)^3\,c^4a}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x)
```

```
[Out] -1/15*(-a^2*x^2+1)^(1/2)*(2*a^2*x^2-6*a*x+7)/(a*x-1)^3/c^4/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(acx-c)^4(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*c*x - c)^4*(a*x + 1)), x)
```

Fricas [A] time = 1.70404, size = 190, normalized size = 1.96

$$\frac{7\,a^3x^3-21\,a^2x^2+21\,ax-\left(2\,a^2x^2-6\,ax+7\right)\sqrt{-a^2x^2+1}-7}{15\left(a^4c^4x^3-3\,a^3c^4x^2+3\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(7*a^3*x^3 - 21*a^2*x^2 + 21*a*x - (2*a^2*x^2 - 6*a*x + 7)*sqrt(-a^2*x
^2 + 1) - 7)/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**4,x)
```

```
[Out] Integral(sqrt(-a**2*x**2 + 1)/(a**5*x**5 - 3*a**4*x**4 + 2*a**3*x**3 + 2*a*
*2*x**2 - 3*a*x + 1), x)/c**4
```

Giac [A] time = 1.2111, size = 196, normalized size = 2.02

$$-\frac{2\left(\frac{20\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{40\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{30\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}-\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4}-7\right)}{15\,c^4\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -2/15*(20*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 40*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^2/(a^4*x^2) + 30*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) -
15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 7)/(c^4*((sqrt(-a^2*x^2 +
1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

3.205 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^5} dx$

Optimal. Leaf size=129

$$\frac{2\sqrt{1-a^2x^2}}{35ac^5(1-ax)}+\frac{2\sqrt{1-a^2x^2}}{35ac^5(1-ax)^2}+\frac{3\sqrt{1-a^2x^2}}{35ac^5(1-ax)^3}+\frac{\sqrt{1-a^2x^2}}{7ac^5(1-ax)^4}$$

```
[Out] Sqrt[1 - a^2*x^2]/(7*a*c^5*(1 - a*x)^4) + (3*Sqrt[1 - a^2*x^2])/(35*a*c^5*(
1 - a*x)^3) + (2*Sqrt[1 - a^2*x^2])/(35*a*c^5*(1 - a*x)^2) + (2*Sqrt[1 - a^
2*x^2])/(35*a*c^5*(1 - a*x))
```

Rubi [A] time = 0.0899088, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 651}

$$\frac{2\sqrt{1-a^2x^2}}{35ac^5(1-ax)}+\frac{2\sqrt{1-a^2x^2}}{35ac^5(1-ax)^2}+\frac{3\sqrt{1-a^2x^2}}{35ac^5(1-ax)^3}+\frac{\sqrt{1-a^2x^2}}{7ac^5(1-ax)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a*c*x)^5),x]
```


```
[Out] Sqrt[1 - a^2*x^2]/(7*a*c^5*(1 - a*x)^4) + (3*Sqrt[1 - a^2*x^2])/(35*a*c^5*(
1 - a*x)^3) + (2*Sqrt[1 - a^2*x^2])/(35*a*c^5*(1 - a*x)^2) + (2*Sqrt[1 - a^
2*x^2])/(35*a*c^5*(1 - a*x))
```


Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\int \frac{1}{(c-acx)^4\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{\sqrt{1-a^2x^2}}{7ac^5(1-ax)^4} + \frac{3\int \frac{1}{(c-acx)^3\sqrt{1-a^2x^2}}\,dx}{7c^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{7ac^5(1-ax)^4} + \frac{3\sqrt{1-a^2x^2}}{35ac^5(1-ax)^3} + \frac{6\int \frac{1}{(c-acx)^2\sqrt{1-a^2x^2}}\,dx}{35c^3}$$
$$= \frac{\sqrt{1-a^2x^2}}{7ac^5(1-ax)^4} + \frac{3\sqrt{1-a^2x^2}}{35ac^5(1-ax)^3} + \frac{2\sqrt{1-a^2x^2}}{35ac^5(1-ax)^2} + \frac{2\int \frac{1}{(c-acx)\sqrt{1-a^2x^2}}\,dx}{35c^4}$$
$$= \frac{\sqrt{1-a^2x^2}}{7ac^5(1-ax)^4} + \frac{3\sqrt{1-a^2x^2}}{35ac^5(1-ax)^3} + \frac{2\sqrt{1-a^2x^2}}{35ac^5(1-ax)^2} + \frac{2\sqrt{1-a^2x^2}}{35ac^5(1-ax)}$$

Mathematica [A] time = 0.0243125, size = 51, normalized size = 0.4

$$-\frac{\sqrt{ax+1}\left(2a^3x^3-8a^2x^2+13ax-12\right)}{35ac^5(1-ax)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a*c*x)^5),x]
```

```
[Out] -(Sqrt[1 + a*x]*(-12 + 13*a*x - 8*a^2*x^2 + 2*a^3*x^3))/(35*a*c^5*(1 - a*x)
^(7/2))
```

Maple [A] time = 0.03, size = 50, normalized size = 0.4

$$-\frac{2x^3a^3-8a^2x^2+13ax-12}{35(ax-1)^4c^5a}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^5,x)
```

```
[Out] -1/35*(-a^2*x^2+1)^(1/2)*(2*a^3*x^3-8*a^2*x^2+13*a*x-12)/(a*x-1)^4/c^5/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-a^2x^2+1}}{(acx-c)^5(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-a^2*x^2 + 1)/((a*c*x - c)^5*(a*x + 1)), x)
```

Fricas [A] time = 1.59161, size = 251, normalized size = 1.95

$$\frac{12\,a^4x^4 - 48\,a^3x^3 + 72\,a^2x^2 - 48\,ax - \left(2\,a^3x^3 - 8\,a^2x^2 + 13\,ax - 12\right)\sqrt{-a^2x^2 + 1} + 12}{35\left(a^5c^5x^4 - 4\,a^4c^5x^3 + 6\,a^3c^5x^2 - 4\,a^2c^5x + ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/35*(12*a^4*x^4 - 48*a^3*x^3 + 72*a^2*x^2 - 48*a*x - (2*a^3*x^3 - 8*a^2*x^
2 + 13*a*x - 12)*sqrt(-a^2*x^2 + 1) + 12)/(a^5*c^5*x^4 - 4*a^4*c^5*x^3 + 6*
a^3*c^5*x^2 - 4*a^2*c^5*x + a*c^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\,dx}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**5,x)
```

```
[Out] -Integral(sqrt(-a**2*x**2 + 1)/(a**6*x**6 - 4*a**5*x**5 + 5*a**4*x**4 - 5*a
**2*x**2 + 4*a*x - 1), x)/c**5
```

Giac [C] time = 1.25206, size = 221, normalized size = 1.71

$$\frac{1}{280}c^2\left(\frac{\left(5\left(\frac{2c}{acx-c}+1\right)^3\sqrt{-\frac{2c}{acx-c}-1}-21\left(\frac{2c}{acx-c}+1\right)^2\sqrt{-\frac{2c}{acx-c}-1}-35\left(-\frac{2c}{acx-c}-1\right)^{\frac{3}{2}}-35\sqrt{-\frac{2c}{acx-c}-1}\right)\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}}{a^2c^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/280*c^2*((5*(2*c/(a*c*x - c) + 1)^3*sqrt(-2*c/(a*c*x - c) - 1) - 21*(2*c/
(a*c*x - c) + 1)^2*sqrt(-2*c/(a*c*x - c) - 1) - 35*(-2*c/(a*c*x - c) - 1)^(
3/2) - 35*sqrt(-2*c/(a*c*x - c) - 1))*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)/(a^2
*c^7) + 16*I*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)/(a^2*c^7))*abs(a)
```

3.206 $\int e^{-2\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=44

$$-\frac{(c-acx)^{p+2}\text{Hypergeometric2F1}\left(1,p+2,p+3,\frac{1}{2}(1-ax)\right)}{2ac^2(p+2)}$$

```
[Out] -((c - a*c*x)^(2 + p)*Hypergeometric2F1[1, 2 + p, 3 + p, (1 - a*x)/2])/(2*a
*c^2*(2 + p))
```

Rubi [A] time = 0.0417842, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6130, 21, 68}

$$-\frac{(c-acx)^{p+2}\,{}_2F_1\left(1,p+2;p+3;\frac{1}{2}(1-ax)\right)}{2ac^2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^p/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -((c - a*c*x)^(2 + p)*Hypergeometric2F1[1, 2 + p, 3 + p, (1 - a*x)/2])/(2*a
*c^2*(2 + p))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}(c-acx)^p\,dx &= \int \frac{(1-ax)(c-acx)^p}{1+ax}\,dx\\ &= \frac{\int \frac{(c-acx)^{1+p}}{1+ax}\,dx}{c}\\ &= -\frac{(c-acx)^{2+p}\,{}_2F_1\left(1,2+p;3+p;\frac{1}{2}(1-ax)\right)}{2ac^2(2+p)}\end{aligned}$$

Mathematica [A] time = 0.014607, size = 43, normalized size = 0.98

$$\frac{(ax-1)(c-acx)^p\left(\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{1}{2}(1-ax)\right)-1\right)}{a(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^p/E^(2*ArcTanh[a*x]),x]
```

```
[Out] ((-1 + a*x)*(c - a*c*x)^p*(-1 + Hypergeometric2F1[1, 1 + p, 2 + p, (1 - a*x
)/2]))/(a*(1 + p))
```

Maple [F] time = 0.359, size = 0, normalized size = 0.

$$\int \frac{(-acx+c)^p\left(-a^2x^2+1\right)}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] int((-a*c*x+c)^p/(a*x+1)^2*(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)(-acx+c)^p}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(-a*c*x + c)^p/(a*x + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ax-1)(-acx+c)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-(a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{(-acx+c)^p}{ax+1}\,dx-\int \frac{ax\,(-acx+c)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**p/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-(-a*c*x + c)**p/(a*x + 1), x) - Integral(a*x*(-a*c*x + c)**p/(a*
x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(a^2x^2 - 1\right)(-acx + c)^p}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*(-a*c*x + c)^p/(a*x + 1)^2, x)
```

3.207 $\int e^{-2\tanh^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=91

$$\frac{c^4(1-ax)^5}{5a}+\frac{c^4(1-ax)^4}{2a}+\frac{4c^4(1-ax)^3}{3a}+\frac{4c^4(1-ax)^2}{a}+\frac{32c^4\log(ax+1)}{a}-16c^4x$$

```
[Out] -16*c^4*x + (4*c^4*(1 - a*x)^2)/a + (4*c^4*(1 - a*x)^3)/(3*a) + (c^4*(1 - a
*x)^4)/(2*a) + (c^5*(1 - a*x)^5)/(5*a) + (32*c^4*Log[1 + a*x])/a
```

Rubi [A] time = 0.0487565, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{c^4(1-ax)^5}{5a}+\frac{c^4(1-ax)^4}{2a}+\frac{4c^4(1-ax)^3}{3a}+\frac{4c^4(1-ax)^2}{a}+\frac{32c^4\log(ax+1)}{a}-16c^4x$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^4/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -16*c^4*x + (4*c^4*(1 - a*x)^2)/a + (4*c^4*(1 - a*x)^3)/(3*a) + (c^4*(1 - a
*x)^4)/(2*a) + (c^4*(1 - a*x)^5)/(5*a) + (32*c^4*Log[1 + a*x])/a
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}(c-acx)^4\,dx &= c^4\int\frac{(1-ax)^5}{1+ax}\,dx\\
&= c^4\int\left(-16-8(1-ax)-4(1-ax)^2-2(1-ax)^3-(1-ax)^4+\frac{32}{1+ax}\right)dx\\
&= -16c^4x+\frac{4c^4(1-ax)^2}{a}+\frac{4c^4(1-ax)^3}{3a}+\frac{c^4(1-ax)^4}{2a}+\frac{c^4(1-ax)^5}{5a}+\frac{32c^4\log(1+ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0191269, size = 56, normalized size = 0.62

$$-\frac{c^4\left(6a^5x^5-45a^4x^4+160a^3x^3-390a^2x^2+930ax-960\log(ax+1)-181\right)}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^4/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c^4*(-181 + 930*a*x - 390*a^2*x^2 + 160*a^3*x^3 - 45*a^4*x^4 + 6*a^5*x^5
- 960*Log[1 + a*x]))/(30*a)
```

Maple [A] time = 0.028, size = 64, normalized size = 0.7

$$-\frac{a^4c^4x^5}{5}+\frac{3\,c^4x^4a^3}{2}-\frac{16\,a^2c^4x^3}{3}+13\,c^4x^2a-31\,c^4x+32\,\frac{c^4\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^4/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/5*a^4*c^4*x^5+3/2*c^4*x^4*a^3-16/3*a^2*c^4*x^3+13*c^4*x^2*a-31*c^4*x+32*
c^4*ln(a*x+1)/a
```

Maxima [A] time = 0.941401, size = 85, normalized size = 0.93

$$-\frac{1}{5}\,a^4c^4x^5+\frac{3}{2}\,a^3c^4x^4-\frac{16}{3}\,a^2c^4x^3+13\,ac^4x^2-31\,c^4x+\frac{32\,c^4\log(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/5*a^4*c^4*x^5 + 3/2*a^3*c^4*x^4 - 16/3*a^2*c^4*x^3 + 13*a*c^4*x^2 - 31*c
^4*x + 32*c^4*log(a*x + 1)/a
```

Fricas [A] time = 1.54748, size = 155, normalized size = 1.7

$$-\frac{6\,a^5c^4x^5-45\,a^4c^4x^4+160\,a^3c^4x^3-390\,a^2c^4x^2+930\,ac^4x-960\,c^4\log(ax+1)}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/30*(6*a^5*c^4*x^5 - 45*a^4*c^4*x^4 + 160*a^3*c^4*x^3 - 390*a^2*c^4*x^2 +
 930*a*c^4*x - 960*c^4*log(a*x + 1))/a
```

Sympy [A] time = 0.346247, size = 68, normalized size = 0.75

$$-\frac{a^4c^4x^5}{5}+\frac{3a^3c^4x^4}{2}-\frac{16a^2c^4x^3}{3}+13ac^4x^2-31c^4x+\frac{32c^4\log(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**4/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -a**4*c**4*x**5/5 + 3*a**3*c**4*x**4/2 - 16*a**2*c**4*x**3/3 + 13*a*c**4*x*
*2 - 31*c**4*x + 32*c**4*log(a*x + 1)/a
```

Giac [A] time = 1.19463, size = 127, normalized size = 1.4

$$-\frac{\left(6\,c^4 - \frac{75\,c^4}{ax+1} + \frac{400\,c^4}{(ax+1)^2} - \frac{1200\,c^4}{(ax+1)^3} + \frac{2400\,c^4}{(ax+1)^4}\right)(ax+1)^5}{30\,a} - \frac{32\,c^4\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/30*(6*c^4 - 75*c^4/(a*x + 1) + 400*c^4/(a*x + 1)^2 - 1200*c^4/(a*x + 1)^
3 + 2400*c^4/(a*x + 1)^4)*(a*x + 1)^5/a - 32*c^4*log(abs(a*x + 1)/((a*x + 1
)^2*abs(a)))/a
```

3.208 $\int e^{-2\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=73

$$\frac{c^3(1-ax)^4}{4a}+\frac{2c^3(1-ax)^3}{3a}+\frac{2c^3(1-ax)^2}{a}+\frac{16c^3\log(ax+1)}{a}-8c^3x$$

```
[Out] -8*c^3*x + (2*c^3*(1 - a*x)^2)/a + (2*c^3*(1 - a*x)^3)/(3*a) + (c^3*(1 - a*
x)^4)/(4*a) + (16*c^3*Log[1 + a*x])/a
```

Rubi [A] time = 0.0434497, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{c^3(1-ax)^4}{4a}+\frac{2c^3(1-ax)^3}{3a}+\frac{2c^3(1-ax)^2}{a}+\frac{16c^3\log(ax+1)}{a}-8c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -8*c^3*x + (2*c^3*(1 - a*x)^2)/a + (2*c^3*(1 - a*x)^3)/(3*a) + (c^3*(1 - a*
x)^4)/(4*a) + (16*c^3*Log[1 + a*x])/a
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}(c-acx)^3\,dx&=c^3\int\frac{(1-ax)^4}{1+ax}\,dx\\&=c^3\int\left(-8-4(1-ax)-2(1-ax)^2-(1-ax)^3+\frac{16}{1+ax}\right)dx\\&=-8c^3x+\frac{2c^3(1-ax)^2}{a}+\frac{2c^3(1-ax)^3}{3a}+\frac{c^3(1-ax)^4}{4a}+\frac{16c^3\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0166093, size = 48, normalized size = 0.66

$$\frac{c^3\left(3a^4x^4-20a^3x^3+66a^2x^2-180ax+192\log(ax+1)+35\right)}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (c^3*(35 - 180*a*x + 66*a^2*x^2 - 20*a^3*x^3 + 3*a^4*x^4 + 192*Log[1 + a*x]
))/(12*a)
```

Maple [A] time = 0.029, size = 53, normalized size = 0.7

$$\frac{c^3x^4a^3}{4} - \frac{5\,c^3x^3a^2}{3} + \frac{11\,c^3x^2a}{2} - 15\,c^3x + 16\,\frac{c^3\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^3/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/4*c^3*x^4*a^3-5/3*c^3*x^3*a^2+11/2*c^3*x^2*a-15*c^3*x+16*c^3*ln(a*x+1)/a
```

Maxima [A] time = 0.945315, size = 70, normalized size = 0.96

$$\frac{1}{4}\,a^3c^3x^4 - \frac{5}{3}\,a^2c^3x^3 + \frac{11}{2}\,ac^3x^2 - 15\,c^3x + \frac{16\,c^3\log(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/4*a^3*c^3*x^4 - 5/3*a^2*c^3*x^3 + 11/2*a*c^3*x^2 - 15*c^3*x + 16*c^3*log(
a*x + 1)/a
```

Fricas [A] time = 1.66449, size = 128, normalized size = 1.75

$$\frac{3\,a^4c^3x^4 - 20\,a^3c^3x^3 + 66\,a^2c^3x^2 - 180\,ac^3x + 192\,c^3\log(ax+1)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^4*c^3*x^4 - 20*a^3*c^3*x^3 + 66*a^2*c^3*x^2 - 180*a*c^3*x + 192*c
^3*log(a*x + 1))/a
```

Sympy [A] time = 0.334794, size = 56, normalized size = 0.77

$$\frac{a^3c^3x^4}{4} - \frac{5a^2c^3x^3}{3} + \frac{11ac^3x^2}{2} - 15c^3x + \frac{16c^3\log(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**3/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] a**3*c**3*x**4/4 - 5*a**2*c**3*x**3/3 + 11*a*c**3*x**2/2 - 15*c**3*x + 16*c
**3*log(a*x + 1)/a
```

Giac [A] time = 1.18512, size = 111, normalized size = 1.52

$$\frac{\left(3\,c^3-\frac{32\,c^3}{ax+1}+\frac{144\,c^3}{(ax+1)^2}-\frac{384\,c^3}{(ax+1)^3}\right)(ax+1)^4}{12\,a}-\frac{16\,c^3\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*(3*c^3 - 32*c^3/(a*x + 1) + 144*c^3/(a*x + 1)^2 - 384*c^3/(a*x + 1)^3)
*(a*x + 1)^4/a - 16*c^3*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a
```

3.209 $\int e^{-2\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=54

$$\frac{c^2(1-ax)^3}{3a}+\frac{c^2(1-ax)^2}{a}+\frac{8c^2\log(ax+1)}{a}-4c^2x$$

```
[Out] -4*c^2*x + (c^2*(1 - a*x)^2)/a + (c^2*(1 - a*x)^3)/(3*a) + (8*c^2*Log[1 + a
*x])/a
```

Rubi [A] time = 0.0363179, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{c^2(1-ax)^3}{3a}+\frac{c^2(1-ax)^2}{a}+\frac{8c^2\log(ax+1)}{a}-4c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^2/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -4*c^2*x + (c^2*(1 - a*x)^2)/a + (c^2*(1 - a*x)^3)/(3*a) + (8*c^2*Log[1 + a
*x])/a
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}(c-acx)^2\,dx &= c^2\int\frac{(1-ax)^3}{1+ax}\,dx\\
&= c^2\int\left(-4-2(1-ax)-(1-ax)^2+\frac{8}{1+ax}\right)dx\\
&= -4c^2x+\frac{c^2(1-ax)^2}{a}+\frac{c^2(1-ax)^3}{3a}+\frac{8c^2\log(1+ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0134649, size = 39, normalized size = 0.72

$$-\frac{c^2\left(a^3x^3-6a^2x^2+21ax-24\log(ax+1)-4\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^2/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c^2*(-4 + 21*a*x - 6*a^2*x^2 + a^3*x^3 - 24*Log[1 + a*x]))/(3*a)
```

Maple [A] time = 0.029, size = 42, normalized size = 0.8

$$-\frac{a^2c^2x^3}{3}+2\,c^2x^2a-7\,xc^2+8\,\frac{c^2\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^2/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/3*a^2*c^2*x^3+2*c^2*x^2*a-7*x*c^2+8*c^2*ln(a*x+1)/a
```

Maxima [A] time = 0.944377, size = 55, normalized size = 1.02

$$-\frac{1}{3}\,a^2c^2x^3+2\,ac^2x^2-7\,c^2x+\frac{8\,c^2\log(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/3*a^2*c^2*x^3 + 2*a*c^2*x^2 - 7*c^2*x + 8*c^2*log(a*x + 1)/a
```

Fricas [A] time = 1.66764, size = 99, normalized size = 1.83

$$-\frac{a^3c^2x^3-6\,a^2c^2x^2+21\,ac^2x-24\,c^2\log(ax+1)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/3*(a^3*c^2*x^3 - 6*a^2*c^2*x^2 + 21*a*c^2*x - 24*c^2*log(a*x + 1))/a
```

Sympy [A] time = 0.3118, size = 41, normalized size = 0.76

$$-\frac{a^2c^2x^3}{3}+2ac^2x^2-7c^2x+\frac{8c^2\log(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**2/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -a**2*c**2*x**3/3 + 2*a*c**2*x**2 - 7*c**2*x + 8*c**2*log(a*x + 1)/a
```

Giac [A] time = 1.17303, size = 92, normalized size = 1.7

$$-\frac{\left(c^2-\frac{9\,c^2}{ax+1}+\frac{36\,c^2}{(ax+1)^2}\right)(ax+1)^3}{3\,a}-\frac{8\,c^2\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/3*(c^2 - 9*c^2/(a*x + 1) + 36*c^2/(a*x + 1)^2)*(a*x + 1)^3/a - 8*c^2*log
(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a
```

3.210 $\int e^{-2\tanh^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=26

$$\frac{1}{2}acx^2+\frac{4c\log(ax+1)}{a}-3cx$$

```
[Out] -3*c*x + (a*c*x^2)/2 + (4*c*Log[1 + a*x])/a
```

Rubi [A] time = 0.0230231, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6129, 43}

$$\frac{1}{2}acx^2+\frac{4c\log(ax+1)}{a}-3cx$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -3*c*x + (a*c*x^2)/2 + (4*c*Log[1 + a*x])/a
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}(c-acx)\,dx&=c\int\frac{(1-ax)^2}{1+ax}\,dx\\&=c\int\left(-3+ax+\frac{4}{1+ax}\right)dx\\&=-3cx+\frac{1}{2}acx^2+\frac{4c\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0134928, size = 25, normalized size = 0.96

$$c\left(\frac{ax^2}{2}+\frac{4\log(ax+1)}{a}-3x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] c*(-3*x + (a*x^2)/2 + (4*Log[1 + a*x])/a)
```

Maple [A] time = 0.036, size = 25, normalized size = 1.

$$-3\,cx+\frac{acx^2}{2}+4\,\frac{c\ln\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -3*c*x+1/2*a*c*x^2+4*c*ln(a*x+1)/a
```

Maxima [A] time = 0.94396, size = 32, normalized size = 1.23

$$\frac{1}{2}\,acx^2-3\,cx+\frac{4\,c\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*a*c*x^2 - 3*c*x + 4*c*log(a*x + 1)/a
```

Fricas [A] time = 1.60623, size = 65, normalized size = 2.5

$$\frac{a^2cx^2-6\,acx+8\,c\log\left(ax+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*c*x^2 - 6*a*c*x + 8*c*log(a*x + 1))/a
```

Sympy [A] time = 0.286534, size = 24, normalized size = 0.92

$$\frac{acx^2}{2}-3cx+\frac{4c\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] a*c*x**2/2 - 3*c*x + 4*c*log(a*x + 1)/a
```

Giac [B] time = 1.23473, size = 68, normalized size = 2.62

$$\frac{(ax+1)^2\left(c-\frac{8c}{ax+1}\right)}{2a}-\frac{4c\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*(a*x + 1)^2*(c - 8*c/(a*x + 1))/a - 4*c*log(abs(a*x + 1)/((a*x + 1)^2*a
bs(a)))/a
```

3.211 $\int \frac{e^{-2\tanh^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=13

$$\frac{\log(ax+1)}{ac}$$

```
[Out] Log[1 + a*x]/(a*c)
```

Rubi [A] time = 0.0256917, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 31}

$$\frac{\log(ax+1)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)),x]
```

```
[Out] Log[1 + a*x]/(a*c)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(ax)}}{c-acx}\,dx &= \frac{\int \frac{1}{1+ax}\,dx}{c}\\ &= \frac{\log(1+ax)}{ac}\end{aligned}$$

Mathematica [A] time = 0.0059627, size = 13, normalized size = 1.

$$\frac{\log(ax+1)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)),x]
```

```
[Out] Log[1 + a*x]/(a*c)
```

Maple [A] time = 0.03, size = 14, normalized size = 1.1

$$\frac{\ln(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c),x)
```

```
[Out] ln(a*x+1)/a/c
```

Maxima [A] time = 0.943681, size = 18, normalized size = 1.38

$$\frac{\log(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] log(a*x + 1)/(a*c)
```

Fricas [A] time = 1.57094, size = 27, normalized size = 2.08

$$\frac{\log(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] log(a*x + 1)/(a*c)
```

Sympy [A] time = 0.084929, size = 10, normalized size = 0.77

$$\frac{\log(acx+c)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c),x)
```

```
[Out] log(a*c*x + c)/(a*c)
```

Giac [A] time = 1.23518, size = 36, normalized size = 2.77

$$-\frac{\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c),x, algorithm="giac")

[Out] -log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/(a*c)
```

3.212 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=11

$$\frac{\tanh^{-1}(ax)}{ac^2}$$

```
[Out] ArcTanh[a*x]/(a*c^2)
```

Rubi [A] time = 0.0268221, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 35, 206}

$$\frac{\tanh^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^2),x]
```

```
[Out] ArcTanh[a*x]/(a*c^2)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^2}\,dx &= \frac{\int \frac{1}{(1-ax)(1+ax)}\,dx}{c^2}\\
&= \frac{\int \frac{1}{1-a^2x^2}\,dx}{c^2}\\
&= \frac{\tanh^{-1}(ax)}{ac^2}
\end{aligned}$$

Mathematica [A] time = 0.007319, size = 11, normalized size = 1.

$$\frac{\tanh^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^2),x]
```

```
[Out] ArcTanh[a*x]/(a*c^2)
```

Maple [B] time = 0.032, size = 30, normalized size = 2.7

$$\frac{\ln(ax+1)}{2\,ac^2}-\frac{\ln(ax-1)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^2,x)
```

```
[Out] 1/2*ln(a*x+1)/a/c^2-1/2/c^2/a*ln(a*x-1)
```

Maxima [B] time = 0.945176, size = 39, normalized size = 3.55

$$\frac{\log(ax+1)}{2\,ac^2}-\frac{\log(ax-1)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*log(a*x + 1)/(a*c^2) - 1/2*log(a*x - 1)/(a*c^2)
```

Fricas [B] time = 1.57252, size = 58, normalized size = 5.27

$$\frac{\log(ax+1)-\log(ax-1)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(log(a*x + 1) - log(a*x - 1))/(a*c^2)
```

Sympy [B] time = 0.158413, size = 22, normalized size = 2.

$$-\frac{\frac{\log\left(x-\frac{1}{a}\right)}{2}-\frac{\log\left(x+\frac{1}{a}\right)}{2}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**2,x)
```

```
[Out] -(log(x - 1/a)/2 - log(x + 1/a)/2)/(a*c**2)
```

Giac [B] time = 1.27475, size = 34, normalized size = 3.09

$$\frac{\log\left(\left|-\frac{2c}{acx-c}-1\right|\right)}{2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(-2*c/(a*c*x - c) - 1))/(a*c^2)
```

3.213 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=33

$$\frac{1}{2ac^3(1-ax)}+\frac{\tanh^{-1}(ax)}{2ac^3}$$

```
[Out] 1/(2*a*c^3*(1 - a*x)) + ArcTanh[a*x]/(2*a*c^3)
```

Rubi [A] time = 0.0396926, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 44, 207}

$$\frac{1}{2ac^3(1-ax)}+\frac{\tanh^{-1}(ax)}{2ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^3),x]
```

```
[Out] 1/(2*a*c^3*(1 - a*x)) + ArcTanh[a*x]/(2*a*c^3)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^3}\,dx &= \frac{\int \frac{1}{(1-ax)^2(1+ax)}\,dx}{c^3}\\
&= \frac{\int \left(\frac{1}{2(-1+ax)^2}-\frac{1}{2\left(-1+a^2x^2\right)}\right)dx}{c^3}\\
&= \frac{1}{2ac^3(1-ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{2c^3}\\
&= \frac{1}{2ac^3(1-ax)}+\frac{\tanh^{-1}(ax)}{2ac^3}
\end{aligned}$$

Mathematica [A] time = 0.0180625, size = 31, normalized size = 0.94

$$\frac{\frac{1}{2a(1-ax)}+\frac{\tanh^{-1}(ax)}{2a}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^3),x]
```

```
[Out] (1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a))/c^3
```

Maple [A] time = 0.036, size = 45, normalized size = 1.4

$$\frac{\ln(ax+1)}{4\,ac^3}-\frac{1}{2\,ac^3\,(ax-1)}-\frac{\ln(ax-1)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^3,x)
```

```
[Out] 1/4*ln(a*x+1)/a/c^3-1/2/c^3/a/(a*x-1)-1/4/c^3/a*ln(a*x-1)
```

Maxima [A] time = 0.940455, size = 65, normalized size = 1.97

$$-\frac{1}{2\left(a^2c^3x-ac^3\right)}+\frac{\log(ax+1)}{4\,ac^3}-\frac{\log(ax-1)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(a^2*c^3*x - a*c^3) + 1/4*log(a*x + 1)/(a*c^3) - 1/4*log(a*x - 1)/(a*c
^3)
```

Fricas [A] time = 1.58162, size = 107, normalized size = 3.24

$$\frac{(ax-1)\log(ax+1)-(ax-1)\log(ax-1)-2}{4\left(a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a*x - 1)*log(a*x + 1) - (a*x - 1)*log(a*x - 1) - 2)/(a^2*c^3*x - a*c^
3)
```

Sympy [A] time = 0.423233, size = 39, normalized size = 1.18

$$-\frac{1}{2a^2c^3x-2ac^3}+\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{4}+\frac{\log\left(x+\frac{1}{a}\right)}{4}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**3,x)
```

```
[Out] -1/(2*a**2*c**3*x - 2*a*c**3) + (-log(x - 1/a)/4 + log(x + 1/a)/4)/(a*c**3)
```

Giac [A] time = 1.20039, size = 58, normalized size = 1.76

$$-\frac{\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{4\,ac^3}+\frac{1}{4\,ac^3\left(\frac{2}{ax+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*log(abs(-2/(a*x + 1) + 1))/(a*c^3) + 1/4/(a*c^3*(2/(a*x + 1) - 1))
```

3.214 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=51

$$\frac{1}{4ac^4(1-ax)}+\frac{1}{4ac^4(1-ax)^2}+\frac{\tanh^{-1}(ax)}{4ac^4}$$

```
[Out] 1/(4*a*c^4*(1 - a*x)^2) + 1/(4*a*c^4*(1 - a*x)) + ArcTanh[a*x]/(4*a*c^4)
```

Rubi [A] time = 0.0471335, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 44, 207}

$$\frac{1}{4ac^4(1-ax)}+\frac{1}{4ac^4(1-ax)^2}+\frac{\tanh^{-1}(ax)}{4ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^4),x]
```

```
[Out] 1/(4*a*c^4*(1 - a*x)^2) + 1/(4*a*c^4*(1 - a*x)) + ArcTanh[a*x]/(4*a*c^4)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^4}\,dx &= \frac{\int \frac{1}{(1-ax)^3(1+ax)}\,dx}{c^4}\\
&= \frac{\int \left(-\frac{1}{2(-1+ax)^3}+\frac{1}{4(-1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^4}\\
&= \frac{1}{4ac^4(1-ax)^2}+\frac{1}{4ac^4(1-ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^4}\\
&= \frac{1}{4ac^4(1-ax)^2}+\frac{1}{4ac^4(1-ax)}+\frac{\tanh^{-1}(ax)}{4ac^4}
\end{aligned}$$

Mathematica [A] time = 0.0224491, size = 35, normalized size = 0.69

$$\frac{-ax+(ax-1)^2\tanh^{-1}(ax)+2}{4ac^4(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^4),x]
```

```
[Out] (2 - a*x + (-1 + a*x)^2*ArcTanh[a*x])/(4*a*c^4*(-1 + a*x)^2)
```

Maple [A] time = 0.035, size = 60, normalized size = 1.2

$$\frac{\ln(ax+1)}{8\,ac^4}+\frac{1}{4\,ac^4\,(ax-1)^2}-\frac{1}{4\,ac^4\,(ax-1)}-\frac{\ln(ax-1)}{8\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^4,x)
```

```
[Out] 1/8*ln(a*x+1)/a/c^4+1/4/c^4/a/(a*x-1)^2-1/4/c^4/a/(a*x-1)-1/8/c^4/a*ln(a*x-
1)
```

Maxima [A] time = 0.956351, size = 85, normalized size = 1.67

$$-\frac{ax-2}{4\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}+\frac{\log(ax+1)}{8\,ac^4}-\frac{\log(ax-1)}{8\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/4*(a*x - 2)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4) + 1/8*log(a*x + 1)/(a*c^
4) - 1/8*log(a*x - 1)/(a*c^4)
```

Fricas [A] time = 1.60785, size = 173, normalized size = 3.39

$$-\frac{2\,ax-\left(a^2x^2-2\,ax+1\right)\log(ax+1)+\left(a^2x^2-2\,ax+1\right)\log(ax-1)-4}{8\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*lo
g(a*x - 1) - 4)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [A] time = 0.499936, size = 56, normalized size = 1.1

$$-\frac{ax-2}{4a^3c^4x^2-8a^2c^4x+4ac^4}-\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**4,x)
```

```
[Out] -(a*x - 2)/(4*a**3*c**4*x**2 - 8*a**2*c**4*x + 4*a*c**4) - (log(x - 1/a)/8
- log(x + 1/a)/8)/(a*c**4)
```

Giac [A] time = 1.20275, size = 78, normalized size = 1.53

$$-\frac{\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{8\,ac^4}-\frac{\frac{3}{a}-\frac{8}{(ax+1)a}}{16\,c^4\left(\frac{2}{ax+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(-2/(a*x + 1) + 1))/(a*c^4) - 1/16*(3/a - 8/((a*x + 1)*a))/(c^4
*(2/(a*x + 1) - 1)^2)
```

3.215 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=69

$$\frac{1}{8ac^5(1-ax)}+\frac{1}{8ac^5(1-ax)^2}+\frac{1}{6ac^5(1-ax)^3}+\frac{\tanh^{-1}(ax)}{8ac^5}$$

```
[Out] 1/(6*a*c^5*(1 - a*x)^3) + 1/(8*a*c^5*(1 - a*x)^2) + 1/(8*a*c^5*(1 - a*x)) +
 ArcTanh[a*x]/(8*a*c^5)
```

Rubi [A] time = 0.0533353, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 44, 207}

$$\frac{1}{8ac^5(1-ax)}+\frac{1}{8ac^5(1-ax)^2}+\frac{1}{6ac^5(1-ax)^3}+\frac{\tanh^{-1}(ax)}{8ac^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^5),x]
```

```
[Out] 1/(6*a*c^5*(1 - a*x)^3) + 1/(8*a*c^5*(1 - a*x)^2) + 1/(8*a*c^5*(1 - a*x)) +
 ArcTanh[a*x]/(8*a*c^5)
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\int \frac{1}{(1-ax)^4(1+ax)}\,dx}{c^5}$$

$$= \frac{\int \left(\frac{1}{2(-1+ax)^4} - \frac{1}{4(-1+ax)^3} + \frac{1}{8(-1+ax)^2} - \frac{1}{8\left(-1+a^2x^2\right)}\right)dx}{c^5}$$

$$= \frac{1}{6ac^5(1-ax)^3} + \frac{1}{8ac^5(1-ax)^2} + \frac{1}{8ac^5(1-ax)} - \frac{\int \frac{1}{-1+a^2x^2}\,dx}{8c^5}$$

$$= \frac{1}{6ac^5(1-ax)^3} + \frac{1}{8ac^5(1-ax)^2} + \frac{1}{8ac^5(1-ax)} + \frac{\tanh^{-1}(ax)}{8ac^5}$$

Mathematica [A] time = 0.0285637, size = 44, normalized size = 0.64

$$\frac{-3a^2x^2 + 9ax + 3(ax-1)^3\tanh^{-1}(ax) - 10}{24ac^5(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^5),x]
```

```
[Out] (-10 + 9*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^3*ArcTanh[a*x])/(24*a*c^5*(-1 + a*x
)^3)
```

Maple [A] time = 0.039, size = 75, normalized size = 1.1

$$\frac{\ln(ax+1)}{16c^5a} - \frac{1}{6c^5a(ax-1)^3} + \frac{1}{8c^5a(ax-1)^2} - \frac{1}{8c^5a(ax-1)} - \frac{\ln(ax-1)}{16c^5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^5,x)
```

```
[Out] 1/16/c^5/a*ln(a*x+1)-1/6/c^5/a/(a*x-1)^3+1/8/c^5/a/(a*x-1)^2-1/8/c^5/a/(a*x
-1)-1/16/c^5/a*ln(a*x-1)
```

Maxima [A] time = 0.97298, size = 113, normalized size = 1.64

$$-\frac{3a^2x^2 - 9ax + 10}{24\left(a^4c^5x^3 - 3a^3c^5x^2 + 3a^2c^5x - ac^5\right)} + \frac{\log(ax+1)}{16ac^5} - \frac{\log(ax-1)}{16ac^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(3*a^2*x^2 - 9*a*x + 10)/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x -
 a*c^5) + 1/16*log(a*x + 1)/(a*c^5) - 1/16*log(a*x - 1)/(a*c^5)
```

Fricas [A] time = 1.6729, size = 252, normalized size = 3.65

$$-\frac{6\,a^{2}x^{2}-18\,ax-3\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(ax+1\right)+3\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(ax-1\right)+20}{48\left(a^{4}c^{5}x^{3}-3\,a^{3}c^{5}x^{2}+3\,a^{2}c^{5}x-ac^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/48*(6*a^2*x^2 - 18*a*x - 3*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a*x + 1
) + 3*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a*x - 1) + 20)/(a^4*c^5*x^3 - 3
*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

Sympy [A] time = 0.614269, size = 76, normalized size = 1.1

$$-\frac{3a^{2}x^{2}-9ax+10}{24a^{4}c^{5}x^{3}-72a^{3}c^{5}x^{2}+72a^{2}c^{5}x-24ac^{5}}+\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{16}+\frac{\log\left(x+\frac{1}{a}\right)}{16}}{ac^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**5,x)
```

```
[Out] -(3*a**2*x**2 - 9*a*x + 10)/(24*a**4*c**5*x**3 - 72*a**3*c**5*x**2 + 72*a**
2*c**5*x - 24*a*c**5) + (-log(x - 1/a)/16 + log(x + 1/a)/16)/(a*c**5)
```

Giac [A] time = 1.22605, size = 120, normalized size = 1.74

$$\frac{\log\left(\left|-\frac{2\,c}{acx-c}-1\right|\right)}{16\,ac^{5}}-\frac{\frac{3\,a^{2}c^{2}}{acx-c}-\frac{3\,a^{2}c^{3}}{(acx-c)^{2}}+\frac{4\,a^{2}c^{4}}{(acx-c)^{3}}}{24\,a^{3}c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/16*log(abs(-2*c/(a*c*x - c) - 1))/(a*c^5) - 1/24*(3*a^2*c^2/(a*c*x - c) -
 3*a^2*c^3/(a*c*x - c)^2 + 4*a^2*c^4/(a*c*x - c)^3)/(a^3*c^6)
```

3.216 $\int e^{-3\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=65

$$-\frac{(1-ax)^{3/2}(c-acx)^{p+1}\text{Hypergeometric2F1}\left(\frac{3}{2},p+\frac{5}{2},p+\frac{7}{2},\frac{1}{2}(1-ax)\right)}{\sqrt{2}ac(2p+5)}$$

```
[Out] -(((1 - a*x)^(3/2)*(c - a*c*x)^(1 + p)*Hypergeometric2F1[3/2, 5/2 + p, 7/2
+ p, (1 - a*x)/2])/(Sqrt[2]*a*c*(5 + 2*p)))
```

Rubi [A] time = 0.0521735, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6130, 23, 69}

$$-\frac{(1-ax)^{3/2}(c-acx)^{p+1}\,{}_2F_1\left(\frac{3}{2},p+\frac{5}{2};p+\frac{7}{2};\frac{1}{2}(1-ax)\right)}{\sqrt{2}ac(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^p/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(((1 - a*x)^(3/2)*(c - a*c*x)^(1 + p)*Hypergeometric2F1[3/2, 5/2 + p, 7/2
+ p, (1 - a*x)/2])/(Sqrt[2]*a*c*(5 + 2*p)))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(1-ax)^{3/2}(c-acx)^p}{(1+ax)^{3/2}}\,dx$$
$$=\frac{(1-ax)^{3/2}\int\frac{(c-acx)^{\frac{3}{2}+p}}{(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}$$
$$=-\frac{(1-ax)^{3/2}(c-acx)^{1+p}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2}+p;\frac{7}{2}+p;\frac{1}{2}(1-ax)\right)}{\sqrt{2}ac(5+2p)}$$

Mathematica [A] time = 0.0275208, size = 60, normalized size = 0.92

$$-\frac{(1-ax)^{5/2}(c-acx)^p\text{Hypergeometric2F1}\left(\frac{3}{2},p+\frac{5}{2},p+\frac{7}{2},\frac{1}{2}-\frac{ax}{2}\right)}{\sqrt{2}a(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^p/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(((1 - a*x)^(5/2)*(c - a*c*x)^p*Hypergeometric2F1[3/2, 5/2 + p, 7/2 + p, 1
/2 - (a*x)/2])/(Sqrt[2]*a*(5 + 2*p)))
```

Maple [F] time = 0.474, size = 0, normalized size = 0.

$$\int\frac{(-acx+c)^p}{(ax+1)^3}\left(-a^2x^2+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int((-a*c*x+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}(-acx+c)^p}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(-a*c*x + c)^p/(a*x + 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}(ax-1)(-acx+c)^p}{a^2x^2+2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(a*x - 1)*(-a*c*x + c)^p/(a^2*x^2 + 2*a*x + 1)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c(ax-1))^p (-(ax-1)(ax+1))^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**p/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**p*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}(-acx+c)^p}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(-a*c*x + c)^p/(a*x + 1)^3, x)
```

3.217 $\int e^{-3\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=163

$$-\frac{2c^3(1-ax)^5}{a\sqrt{1-a^2x^2}}-\frac{9c^3\sqrt{1-a^2x^2}(1-ax)^3}{4a}-\frac{21c^3\sqrt{1-a^2x^2}(1-ax)^2}{4a}-\frac{105c^3\sqrt{1-a^2x^2}(1-ax)}{8a}-\frac{315c^3\sqrt{1-a^2x^2}}{8a}-\frac{315}{}$$

```
[Out] (-2*c^3*(1 - a*x)^5)/(a*Sqrt[1 - a^2*x^2]) - (315*c^3*Sqrt[1 - a^2*x^2])/(8
*a) - (105*c^3*(1 - a*x)*Sqrt[1 - a^2*x^2])/(8*a) - (21*c^3*(1 - a*x)^2*Sqr
t[1 - a^2*x^2])/(4*a) - (9*c^3*(1 - a*x)^3*Sqrt[1 - a^2*x^2])/(4*a) - (315*
c^3*ArcSin[a*x])/(8*a)
```

Rubi [A] time = 0.125341, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6127, 669, 671, 641, 216}

$$-\frac{2c^3(1-ax)^5}{a\sqrt{1-a^2x^2}}-\frac{9c^3\sqrt{1-a^2x^2}(1-ax)^3}{4a}-\frac{21c^3\sqrt{1-a^2x^2}(1-ax)^2}{4a}-\frac{105c^3\sqrt{1-a^2x^2}(1-ax)}{8a}-\frac{315c^3\sqrt{1-a^2x^2}}{8a}-\frac{315}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-2*c^3*(1 - a*x)^5)/(a*Sqrt[1 - a^2*x^2]) - (315*c^3*Sqrt[1 - a^2*x^2])/(8
*a) - (105*c^3*(1 - a*x)*Sqrt[1 - a^2*x^2])/(8*a) - (21*c^3*(1 - a*x)^2*Sqr
t[1 - a^2*x^2])/(4*a) - (9*c^3*(1 - a*x)^3*Sqrt[1 - a^2*x^2])/(4*a) - (315*
c^3*ArcSin[a*x])/(8*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}(c-acx)^3\,dx = \frac{\int \frac{(c-acx)^6}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$
$$= -\frac{2c^3(1-ax)^5}{a\sqrt{1-a^2x^2}} - \frac{9\int \frac{(c-acx)^4}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= -\frac{2c^3(1-ax)^5}{a\sqrt{1-a^2x^2}} - \frac{9c^3(1-ax)^3\sqrt{1-a^2x^2}}{4a} - \frac{63}{4}\int \frac{(c-acx)^3}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{2c^3(1-ax)^5}{a\sqrt{1-a^2x^2}} - \frac{21c^3(1-ax)^2\sqrt{1-a^2x^2}}{4a} - \frac{9c^3(1-ax)^3\sqrt{1-a^2x^2}}{4a} - \frac{1}{4}(105c)\int \frac{(c-acx)^2}{\sqrt{1-a^2x^2}}$$
$$= -\frac{2c^3(1-ax)^5}{a\sqrt{1-a^2x^2}} - \frac{105c^3(1-ax)\sqrt{1-a^2x^2}}{8a} - \frac{21c^3(1-ax)^2\sqrt{1-a^2x^2}}{4a} - \frac{9c^3(1-ax)^3\sqrt{1-a^2}}{4a}$$
$$= -\frac{2c^3(1-ax)^5}{a\sqrt{1-a^2x^2}} - \frac{315c^3\sqrt{1-a^2x^2}}{8a} - \frac{105c^3(1-ax)\sqrt{1-a^2x^2}}{8a} - \frac{21c^3(1-ax)^2\sqrt{1-a^2x^2}}{4a} - 9$$
$$= -\frac{2c^3(1-ax)^5}{a\sqrt{1-a^2x^2}} - \frac{315c^3\sqrt{1-a^2x^2}}{8a} - \frac{105c^3(1-ax)\sqrt{1-a^2x^2}}{8a} - \frac{21c^3(1-ax)^2\sqrt{1-a^2x^2}}{4a} - 9$$

Mathematica [C] time = 0.0196711, size = 45, normalized size = 0.28

$$-\frac{c^3(1-ax)^{11/2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{11}{2},\frac{13}{2},\frac{1}{2}(1-ax)\right)}{11\sqrt{2}a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(c^3*(1 - a*x)^(11/2)*Hypergeometric2F1[3/2, 11/2, 13/2, (1 - a*x)/2])/(11
*Sqrt[2]*a)
```

Maple [A] time = 0.049, size = 245, normalized size = 1.5

$$-\frac{c^3x}{4}\left(-a^2x^2+1\right)^{\frac{3}{2}} - \frac{3c^3x}{8}\sqrt{-a^2x^2+1} - \frac{3c^3}{8}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} - 28\frac{c^3\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)\right)^{5/2}}{a^3\left(x+a^{-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/4*c^3*x*(-a^2*x^2+1)^(3/2)-3/8*c^3*x*(-a^2*x^2+1)^(1/2)-3/8*c^3/(a^2)^(1
/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-28*c^3/a^3/(x+1/a)^2*(-a^2*(x+
1/a)^2+2*a*(x+1/a))^(5/2)-26*c^3/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)-39*c^
3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-39*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2
```

```
)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-8*c^3/a^4/(x+1/a)^3*(-a^2*(x+1/a)^2
+2*a*(x+1/a))^(5/2)
```

Maxima [C] time = 1.49614, size = 315, normalized size = 1.93

$$-\frac{1}{4}\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}c^{3}x+3\sqrt{a^{2}x^{2}+4\,ax+3}c^{3}x-\frac{3}{8}\sqrt{-a^{2}x^{2}+1}c^{3}x+\frac{8\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}c^{3}}{a^{3}x^{2}+2\,a^{2}x+a}-\frac{6\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}c^{3}}{a^{2}x+a}+\frac{2\left(-a^{2}x^{2}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-a^2*x^2 + 1)^(3/2)*c^3*x + 3*sqrt(a^2*x^2 + 4*a*x + 3)*c^3*x - 3/8*s
qrt(-a^2*x^2 + 1)*c^3*x + 8*(-a^2*x^2 + 1)^(3/2)*c^3/(a^3*x^2 + 2*a^2*x + a
) - 6*(-a^2*x^2 + 1)^(3/2)*c^3/(a^2*x + a) + 2*(-a^2*x^2 + 1)^(3/2)*c^3/a -
 3*I*c^3*arcsin(a*x + 2)/a - 339/8*c^3*arcsin(a*x)/a - 48*sqrt(-a^2*x^2 + 1
)*c^3/(a^2*x + a) + 6*sqrt(a^2*x^2 + 4*a*x + 3)*c^3/a - 18*sqrt(-a^2*x^2 +
1)*c^3/a
```

Fricas [A] time = 1.67829, size = 267, normalized size = 1.64

$$-\frac{496\,ac^{3}x+496\,c^{3}-630\left(ac^{3}x+c^{3}\right)\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)-\left(2\,a^{4}c^{3}x^{4}-14\,a^{3}c^{3}x^{3}+51\,a^{2}c^{3}x^{2}-173\,ac^{3}x-496\,c^{3}\right)\sqrt{-}}{8\left(a^{2}x+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(496*a*c^3*x + 496*c^3 - 630*(a*c^3*x + c^3)*arctan((sqrt(-a^2*x^2 + 1
) - 1)/(a*x)) - (2*a^4*c^3*x^4 - 14*a^3*c^3*x^3 + 51*a^2*c^3*x^2 - 173*a*c^
3*x - 496*c^3)*sqrt(-a^2*x^2 + 1))/(a^2*x + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^{3}\left(\int-\frac{\sqrt{-a^{2}x^{2}+1}}{a^{3}x^{3}+3a^{2}x^{2}+3ax+1}\,dx+\int\frac{3ax\sqrt{-a^{2}x^{2}+1}}{a^{3}x^{3}+3a^{2}x^{2}+3ax+1}\,dx+\int-\frac{2a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}}{a^{3}x^{3}+3a^{2}x^{2}+3ax+1}\,dx+\int-\frac{2a^{3}x}{a^{3}x^{3}+3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**3/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -c**3*(Integral(-sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1)
, x) + Integral(3*a*x*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x
 + 1), x) + Integral(-2*a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*
x**2 + 3*a*x + 1), x) + Integral(-2*a**3*x**3*sqrt(-a**2*x**2 + 1)/(a**3*x*
*3 + 3*a**2*x**2 + 3*a*x + 1), x) + Integral(3*a**4*x**4*sqrt(-a**2*x**2 +
1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1), x) + Integral(-a**5*x**5*sqrt(-a*
*2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1), x))
```

Giac [A] time = 1.28941, size = 139, normalized size = 0.85

$$-\frac{315\,c^3\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{8\,|a|}-\frac{1}{8}\,\sqrt{-a^2x^2+1}\left(\frac{240\,c^3}{a}-\left(67\,c^3+2\,\left(a^2c^3x-8\,ac^3\right)x\right)x\right)+\frac{64\,c^3}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -315/8*c^3*arcsin(a*x)*sgn(a)/abs(a) - 1/8*sqrt(-a^2*x^2 + 1)*(240*c^3/a -
(67*c^3 + 2*(a^2*c^3*x - 8*a*c^3)*x)*x) + 64*c^3/(((sqrt(-a^2*x^2 + 1)*abs(
a) + a)/(a^2*x) + 1)*abs(a))
```

3.218 $\int e^{-3\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=131

$$-\frac{2c^2(1-ax)^4}{a\sqrt{1-a^2x^2}}-\frac{7c^2\sqrt{1-a^2x^2}(1-ax)^2}{3a}-\frac{35c^2\sqrt{1-a^2x^2}(1-ax)}{6a}-\frac{35c^2\sqrt{1-a^2x^2}}{2a}-\frac{35c^2\sin^{-1}(ax)}{2a}$$

```
[Out] (-2*c^2*(1 - a*x)^4)/(a*Sqrt[1 - a^2*x^2]) - (35*c^2*Sqrt[1 - a^2*x^2])/(2*
a) - (35*c^2*(1 - a*x)*Sqrt[1 - a^2*x^2])/(6*a) - (7*c^2*(1 - a*x)^2*Sqrt[1
 - a^2*x^2])/(3*a) - (35*c^2*ArcSin[a*x])/(2*a)
```

Rubi [A] time = 0.0941339, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6127, 669, 671, 641, 216}

$$-\frac{2c^2(1-ax)^4}{a\sqrt{1-a^2x^2}}-\frac{7c^2\sqrt{1-a^2x^2}(1-ax)^2}{3a}-\frac{35c^2\sqrt{1-a^2x^2}(1-ax)}{6a}-\frac{35c^2\sqrt{1-a^2x^2}}{2a}-\frac{35c^2\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-2*c^2*(1 - a*x)^4)/(a*Sqrt[1 - a^2*x^2]) - (35*c^2*Sqrt[1 - a^2*x^2])/(2*
a) - (35*c^2*(1 - a*x)*Sqrt[1 - a^2*x^2])/(6*a) - (7*c^2*(1 - a*x)^2*Sqrt[1
 - a^2*x^2])/(3*a) - (35*c^2*ArcSin[a*x])/(2*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}(c-acx)^2\,dx = \frac{\int \frac{(c-acx)^5}{(1-a^2x^2)^{3/2}}\,dx}{c^3}$$
$$= -\frac{2c^2(1-ax)^4}{a\sqrt{1-a^2x^2}} - \frac{7\int \frac{(c-acx)^3}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= -\frac{2c^2(1-ax)^4}{a\sqrt{1-a^2x^2}} - \frac{7c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a} - \frac{35}{3}\int \frac{(c-acx)^2}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{2c^2(1-ax)^4}{a\sqrt{1-a^2x^2}} - \frac{35c^2(1-ax)\sqrt{1-a^2x^2}}{6a} - \frac{7c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a} - \frac{1}{2}(35c)\int \frac{c-acx}{\sqrt{1-a^2x^2}}\,d$$
$$= -\frac{2c^2(1-ax)^4}{a\sqrt{1-a^2x^2}} - \frac{35c^2\sqrt{1-a^2x^2}}{2a} - \frac{35c^2(1-ax)\sqrt{1-a^2x^2}}{6a} - \frac{7c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a} - \frac{1}{2}(3$$
$$= -\frac{2c^2(1-ax)^4}{a\sqrt{1-a^2x^2}} - \frac{35c^2\sqrt{1-a^2x^2}}{2a} - \frac{35c^2(1-ax)\sqrt{1-a^2x^2}}{6a} - \frac{7c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a} - 35c^2$$

Mathematica [C] time = 0.0165767, size = 45, normalized size = 0.34

$$-\frac{c^2(1-ax)^{9/2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{9}{2},\frac{11}{2},\frac{1}{2}(1-ax)\right)}{9\sqrt{2}a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(c^2*(1 - a*x)^(9/2)*Hypergeometric2F1[3/2, 9/2, 11/2, (1 - a*x)/2])/(9*Sq
rt[2]*a)
```

Maple [A] time = 0.045, size = 179, normalized size = 1.4

$$-12\,\frac{c^2\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right)^{5/2}}{a^3\left(x+a^{-1}\right)^2} - \frac{35\,c^2}{3\,a}\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right)^{\frac{3}{2}} - \frac{35\,xc^2}{2}\sqrt{-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -12*c^2/a^3/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-35/3*c^2/a*(-a^2*(
x+1/a)^2+2*a*(x+1/a))^(3/2)-35/2*c^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-3
5/2*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)
)-4*c^2/a^4/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)
```

Maxima [C] time = 1.46234, size = 265, normalized size = 2.02

$$\frac{1}{2}\sqrt{a^2x^2+4\,ax+3}c^2x+\frac{4\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}{a^3x^2+2\,a^2x+a}-\frac{2\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}{a^2x+a}+\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}{3\,a}-\frac{i\,c^2\arcsin\left(ax+2\right)}{2\,a}-\frac{18\,c^2\arcsi}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a^2*x^2 + 4*a*x + 3)*c^2*x + 4*(-a^2*x^2 + 1)^(3/2)*c^2/(a^3*x^2 +
 2*a^2*x + a) - 2*(-a^2*x^2 + 1)^(3/2)*c^2/(a^2*x + a) + 1/3*(-a^2*x^2 + 1)
^(3/2)*c^2/a - 1/2*I*c^2*arcsin(a*x + 2)/a - 18*c^2*arcsin(a*x)/a - 24*sqrt
(-a^2*x^2 + 1)*c^2/(a^2*x + a) + sqrt(a^2*x^2 + 4*a*x + 3)*c^2/a - 6*sqrt(-
a^2*x^2 + 1)*c^2/a
```

Fricas [A] time = 1.68004, size = 243, normalized size = 1.85

$$-\frac{166\,ac^2x+166\,c^2-210\left(ac^2x+c^2\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^3c^2x^3-13\,a^2c^2x^2+55\,ac^2x+166\,c^2\right)\sqrt{-a^2x^2+1}}{6\left(a^2x+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(166*a*c^2*x + 166*c^2 - 210*(a*c^2*x + c^2)*arctan((sqrt(-a^2*x^2 + 1
) - 1)/(a*x)) + (2*a^3*c^2*x^3 - 13*a^2*c^2*x^2 + 55*a*c^2*x + 166*c^2)*sqr
t(-a^2*x^2 + 1))/(a^2*x + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int\frac{\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx+\int-\frac{2ax\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx+\int\frac{2a^3x^3\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx+\int-\frac{a^4x^4\sqrt{-}}{a^3x^3+3a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**2/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] c**2*(Integral(sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1),
x) + Integral(-2*a*x*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x
+ 1), x) + Integral(2*a**3*x**3*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x*
*2 + 3*a*x + 1), x) + Integral(-a**4*x**4*sqrt(-a**2*x**2 + 1)/(a**3*x**3 +
 3*a**2*x**2 + 3*a*x + 1), x))
```

Giac [A] time = 1.29119, size = 123, normalized size = 0.94

$$-\frac{35\,c^2\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{2\,|a|}-\frac{1}{6}\sqrt{-a^2x^2+1}\left(\left(2\,ac^2x-15\,c^2\right)x+\frac{70\,c^2}{a}\right)+\frac{32\,c^2}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -35/2*c^2*arcsin(a*x)*sgn(a)/abs(a) - 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c^2*x -
15*c^2)*x + 70*c^2/a) + 32*c^2/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) +
1)*abs(a))
```

3.219 $\int e^{-3\tanh^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=91

$$-\frac{2c(1-ax)^3}{a\sqrt{1-a^2x^2}}-\frac{5c\sqrt{1-a^2x^2}(1-ax)}{2a}-\frac{15c\sqrt{1-a^2x^2}}{2a}-\frac{15c\sin^{-1}(ax)}{2a}$$

```
[Out] (-2*c*(1 - a*x)^3)/(a*Sqrt[1 - a^2*x^2]) - (15*c*Sqrt[1 - a^2*x^2])/(2*a) -
 (5*c*(1 - a*x)*Sqrt[1 - a^2*x^2])/(2*a) - (15*c*ArcSin[a*x])/(2*a)
```

Rubi [A] time = 0.0641629, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6127, 669, 671, 641, 216}

$$-\frac{2c(1-ax)^3}{a\sqrt{1-a^2x^2}}-\frac{5c\sqrt{1-a^2x^2}(1-ax)}{2a}-\frac{15c\sqrt{1-a^2x^2}}{2a}-\frac{15c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-2*c*(1 - a*x)^3)/(a*Sqrt[1 - a^2*x^2]) - (15*c*Sqrt[1 - a^2*x^2])/(2*a) -
 (5*c*(1 - a*x)*Sqrt[1 - a^2*x^2])/(2*a) - (15*c*ArcSin[a*x])/(2*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}(c-acx)\,dx = \frac{\int \frac{(c-acx)^4}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$
$$= -\frac{2c(1-ax)^3}{a\sqrt{1-a^2x^2}} - \frac{5\int \frac{(c-acx)^2}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= -\frac{2c(1-ax)^3}{a\sqrt{1-a^2x^2}} - \frac{5c(1-ax)\sqrt{1-a^2x^2}}{2a} - \frac{15}{2}\int \frac{c-acx}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{2c(1-ax)^3}{a\sqrt{1-a^2x^2}} - \frac{15c\sqrt{1-a^2x^2}}{2a} - \frac{5c(1-ax)\sqrt{1-a^2x^2}}{2a} - \frac{1}{2}(15c)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{2c(1-ax)^3}{a\sqrt{1-a^2x^2}} - \frac{15c\sqrt{1-a^2x^2}}{2a} - \frac{5c(1-ax)\sqrt{1-a^2x^2}}{2a} - \frac{15c\sin^{-1}(ax)}{2a}$$

Mathematica [C] time = 0.0148091, size = 43, normalized size = 0.47

$$-\frac{c(1-ax)^{7/2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{7}{2},\frac{9}{2},\frac{1}{2}(1-ax)\right)}{7\sqrt{2}a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(c*(1 - a*x)^(7/2)*Hypergeometric2F1[3/2, 7/2, 9/2, (1 - a*x)/2])/(7*Sqrt[
2]*a)
```

Maple [B] time = 0.043, size = 169, normalized size = 1.9

$$-5\,\frac{c\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right)^{5/2}}{a^3\left(x+a^{-1}\right)^2}-5\,\frac{c\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right)^{3/2}}{a}-\frac{15\,cx}{2}\sqrt{-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -5*c/a^3/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-5*c/a*(-a^2*(x+1/a)^2
+2*a*(x+1/a))^(3/2)-15/2*c*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-15/2*c/(a^2
)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-2*c/a^4/(x
+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)
```

Maxima [A] time = 1.44679, size = 147, normalized size = 1.62

$$\frac{2\left(-a^2x^2+1\right)^{\frac{3}{2}}c}{a^3x^2+2\,a^2x+a}-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}c}{2\left(a^2x+a\right)}-\frac{15\,c\arcsin\left(ax\right)}{2\,a}-\frac{12\,\sqrt{-a^2x^2+1}c}{a^2x+a}-\frac{3\,\sqrt{-a^2x^2+1}c}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(-a^2*x^2 + 1)^(3/2)*c/(a^3*x^2 + 2*a^2*x + a) - 1/2*(-a^2*x^2 + 1)^(3/2)
*c/(a^2*x + a) - 15/2*c*arcsin(a*x)/a - 12*sqrt(-a^2*x^2 + 1)*c/(a^2*x + a)
 - 3/2*sqrt(-a^2*x^2 + 1)*c/a
```

Fricas [A] time = 1.64923, size = 192, normalized size = 2.11

$$-\frac{24\,acx - 30\,(acx + c)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - \left(a^2cx^2 - 7\,acx - 24\,c\right)\sqrt{-a^2x^2+1} + 24\,c}{2\left(a^2x + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(24*a*c*x - 30*(a*c*x + c)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (a
^2*c*x^2 - 7*a*c*x - 24*c)*sqrt(-a^2*x^2 + 1) + 24*c)/(a^2*x + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int -\frac{\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx + \int \frac{ax\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx + \int \frac{a^2x^2\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx + \int -\frac{a^3x^3\sqrt{\cdot}}{a^3x^3+3a^2\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -c*(Integral(-sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1), x
) + Integral(a*x*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1)
, x) + Integral(a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3
*a*x + 1), x) + Integral(-a**3*x**3*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**
2*x**2 + 3*a*x + 1), x))
```

Giac [A] time = 1.24556, size = 99, normalized size = 1.09

$$-\frac{15\,c\arcsin(ax)\,\mathrm{sgn}(a)}{2\,|a|} + \frac{1}{2}\sqrt{-a^2x^2+1}\left(cx - \frac{8\,c}{a}\right) + \frac{16\,c}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} + 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -15/2*c*arcsin(a*x)*sgn(a)/abs(a) + 1/2*sqrt(-a^2*x^2 + 1)*(c*x - 8*c/a) +
16*c/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

3.220 $\int \frac{e^{-3\tanh^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=41

$$-\frac{2(1-ax)}{ac\sqrt{1-a^2x^2}}-\frac{\sin^{-1}(ax)}{ac}$$

```
[Out] (-2*(1 - a*x))/(a*c*Sqrt[1 - a^2*x^2]) - ArcSin[a*x]/(a*c)
```

Rubi [A] time = 0.04145, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 653, 216}

$$-\frac{2(1-ax)}{ac\sqrt{1-a^2x^2}}-\frac{\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)),x]
```

```
[Out] (-2*(1 - a*x))/(a*c*Sqrt[1 - a^2*x^2]) - ArcSin[a*x]/(a*c)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\tanh^{-1}(ax)}}{c-acx}\,dx &= \frac{\int \frac{(c-acx)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}\\ &= -\frac{2(1-ax)}{ac\sqrt{1-a^2x^2}}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\ &= -\frac{2(1-ax)}{ac\sqrt{1-a^2x^2}}-\frac{\sin^{-1}(ax)}{ac}\end{aligned}$$

Mathematica [A] time = 0.0521901, size = 59, normalized size = 1.44

$$\frac{2\left(\sqrt{1-a^2x^2}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)+ax-1\right)}{ac\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)),x]
```

```
[Out] (2*(-1 + a*x + Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*c*Sqrt[
1 - a^2*x^2])
```

Maple [B] time = 0.052, size = 292, normalized size = 7.1

$$-\frac{3}{4a^3c\left(x+a^{-1}\right)^2}\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)\right)^{\frac{5}{2}}-\frac{17}{24ac}\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)\right)^{\frac{3}{2}}-\frac{17x}{16c}\sqrt{-a^2\left(x+a^{-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c),x)
```

```
[Out] -3/4/c/a^3/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-17/24/c/a*(-a^2*(x+
1/a)^2+2*a*(x+1/a))^(3/2)-17/16/c*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-17/1
6/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-1/
24/c/a*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+1/16/c*(-a^2*(x-1/a)^2-2*a*(x-1/a
))^(1/2)*x+1/16/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1
/a))^(1/2))-1/2/c/a^4/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(acx-c)(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -integrate((-a^2*x^2 + 1)^(3/2)/((a*c*x - c)*(a*x + 1)^3), x)
```

Fricas [A] time = 1.67339, size = 138, normalized size = 3.37

$$-\frac{2\left(ax-(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}+1\right)}{a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -2*(a*x - (a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2
+ 1) + 1)/(a^2*c*x + a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^4x^4+2a^3x^3-2ax-1}\,dx + \int -\frac{a^2x^2\sqrt{-a^2x^2+1}}{a^4x^4+2a^3x^3-2ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c),x)
```

```
[Out] -(Integral(sqrt(-a**2*x**2 + 1)/(a**4*x**4 + 2*a**3*x**3 - 2*a*x - 1), x) +
 Integral(-a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**4*x**4 + 2*a**3*x**3 - 2*a*x
- 1), x))/c
```

Giac [A] time = 1.23199, size = 72, normalized size = 1.76

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{c|a|} + \frac{4}{c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(c*abs(a)) + 4/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2
*x) + 1)*abs(a))
```

3.221 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=28

$$-\frac{1-ax}{ac^2\sqrt{1-a^2x^2}}$$

```
[Out] -((1 - a*x)/(a*c^2*Sqrt[1 - a^2*x^2]))
```

Rubi [A] time = 0.0329932, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6127, 637}

$$-\frac{1-ax}{ac^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^2),x]

[Out] -((1 - a*x)/(a*c^2*Sqrt[1 - a^2*x^2]))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^2}\,dx &= \frac{\int \frac{c-acx}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}\\ &= -\frac{1-ax}{ac^2\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0104067, size = 27, normalized size = 0.96

$$-\frac{\sqrt{1-ax}}{ac^2\sqrt{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^2),x]

[Out] -(Sqrt[1 - a*x]/(a*c^2*Sqrt[1 + a*x]))
```

Maple [A] time = 0.03, size = 34, normalized size = 1.2

$$\frac{1}{(ax-1)\,c^2a\,(ax+1)^2}\left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^2,x)
```

```
[Out] (-a^2*x^2+1)^(3/2)/(a*x-1)/c^2/a/(a*x+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(acx-c)^2(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*c*x - c)^2*(a*x + 1)^3), x)
```

Fricas [A] time = 1.65454, size = 72, normalized size = 2.57

$$-\frac{ax+\sqrt{-a^2x^2+1}+1}{a^2c^2x+ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] -(a*x + sqrt(-a^2*x^2 + 1) + 1)/(a^2*c^2*x + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\,dx+\int -\frac{a^2x^2\sqrt{-a^2x^2+1}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(sqrt(-a**2*x**2 + 1)/(a**5*x**5 + a**4*x**4 - 2*a**3*x**3 - 2*a**
2*x**2 + a*x + 1), x) + Integral(-a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**5*x**5
 + a**4*x**4 - 2*a**3*x**3 - 2*a**2*x**2 + a*x + 1), x))/c**2
```

Giac [C] time = 1.22409, size = 93, normalized size = 3.32

$$c^2\left(\frac{i\,\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}\left(a\right)\mathrm{sgn}\left(c\right)}{a^2c^4}+\frac{\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}\left(a\right)\mathrm{sgn}\left(c\right)}{a^2c^4\sqrt{-\frac{2\,c}{acx-c}-1}}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] c^2*(I*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)/(a^2*c^4) + sgn(1/(a*c*x - c))*sgn(
a)*sgn(c)/(a^2*c^4*sqrt(-2*c/(a*c*x - c) - 1)))*abs(a)
```

3.222 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=19

$$\frac{x}{c^3\sqrt{1-a^2x^2}}$$

```
[Out] x/(c^3*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0278557, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6127, 191}

$$\frac{x}{c^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^3),x]
```

```
[Out] x/(c^3*Sqrt[1 - a^2*x^2])
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^3}\,dx &= \frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}\\ &= \frac{x}{c^3\sqrt{1-a^2x^2}} \end{aligned}$$

Mathematica [A] time = 0.0111319, size = 19, normalized size = 1.

$$\frac{x}{c^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^3),x]
```

```
[Out] x/(c^3*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.03, size = 32, normalized size = 1.7

$$\frac{x}{(ax-1)^2 c^3 (ax+1)^2} \left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^3,x)
```

```
[Out] (-a^2*x^2+1)^(3/2)*x/(a*x-1)^2/c^3/(a*x+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(acx-c)^3(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] -integrate((-a^2*x^2 + 1)^(3/2)/((a*c*x - c)^3*(a*x + 1)^3), x)
```

Fricas [A] time = 1.65895, size = 58, normalized size = 3.05

$$-\frac{\sqrt{-a^2x^2+1}x}{a^2c^3x^2-c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] -sqrt(-a^2*x^2 + 1)*x/(a^2*c^3*x^2 - c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx + \int -\frac{a^2x^2\sqrt{-a^2x^2+1}}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(sqrt(-a**2*x**2 + 1)/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1)
, x) + Integral(-a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**6*x**6 - 3*a**4*x**4 +
3*a**2*x**2 - 1), x))/c**3
```

Giac [A] time = 1.21393, size = 39, normalized size = 2.05

$$-\frac{\sqrt{-a^2x^2+1}x}{\left(a^2x^2-1\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -sqrt(-a^2*x^2 + 1)*x/((a^2*x^2 - 1)*c^3)
```

3.223 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=55

$$\frac{2x}{3c^4\sqrt{1-a^2x^2}}+\frac{1}{3ac^4(1-ax)\sqrt{1-a^2x^2}}$$

```
[Out] (2*x)/(3*c^4*Sqrt[1 - a^2*x^2]) + 1/(3*a*c^4*(1 - a*x)*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0445643, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 191}

$$\frac{2x}{3c^4\sqrt{1-a^2x^2}}+\frac{1}{3ac^4(1-ax)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^4),x]
```

```
[Out] (2*x)/(3*c^4*Sqrt[1 - a^2*x^2]) + 1/(3*a*c^4*(1 - a*x)*Sqrt[1 - a^2*x^2])
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^4}\,dx &= \frac{\int \frac{1}{(c-acx)\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}\\ &= \frac{1}{3ac^4(1-ax)\sqrt{1-a^2x^2}}+\frac{2\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^4}\\ &= \frac{2x}{3c^4\sqrt{1-a^2x^2}}+\frac{1}{3ac^4(1-ax)\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0168939, size = 45, normalized size = 0.82

$$\frac{2a^2x^2-2ax-1}{3ac^4(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^4),x]
```

```
[Out] (-1 - 2*a*x + 2*a^2*x^2)/(3*a*c^4*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.031, size = 49, normalized size = 0.9

$$\frac{2\,a^2x^2-2\,ax-1}{3\,(ax-1)^3\,c^4a\,(ax+1)^2}\left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^4,x)
```

```
[Out] 1/3*(-a^2*x^2+1)^(3/2)*(2*a^2*x^2-2*a*x-1)/(a*x-1)^3/c^4/a/(a*x+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(acx-c)^4(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*c*x - c)^4*(a*x + 1)^3), x)
```

Fricas [A] time = 1.62778, size = 173, normalized size = 3.15

$$\frac{a^3x^3-a^2x^2-ax-\left(2\,a^2x^2-2\,ax-1\right)\sqrt{-a^2x^2+1}+1}{3\left(a^4c^4x^3-a^3c^4x^2-a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] 1/3*(a^3*x^3 - a^2*x^2 - a*x - (2*a^2*x^2 - 2*a*x - 1)*sqrt(-a^2*x^2 + 1) +
 1)/(a^4*c^4*x^3 - a^3*c^4*x^2 - a^2*c^4*x + a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^7x^7-a^6x^6-3a^5x^5+3a^4x^4+3a^3x^3-3a^2x^2-ax+1}\,dx+\int -\frac{a^2x^2\sqrt{-a^2x^2+1}}{a^7x^7-a^6x^6-3a^5x^5+3a^4x^4+3a^3x^3-3a^2x^2-ax+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(sqrt(-a**2*x**2 + 1)/(a**7*x**7 - a**6*x**6 - 3*a**5*x**5 + 3*a**
4*x**4 + 3*a**3*x**3 - 3*a**2*x**2 - a*x + 1), x) + Integral(-a**2*x**2*sqr
t(-a**2*x**2 + 1)/(a**7*x**7 - a**6*x**6 - 3*a**5*x**5 + 3*a**4*x**4 + 3*a*
*3*x**3 - 3*a**2*x**2 - a*x + 1), x))/c**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(acx-c)^4(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*c*x - c)^4*(a*x + 1)^3), x)
```

3.224 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=87

$$\frac{2x}{5c^5\sqrt{1-a^2x^2}}+\frac{1}{5ac^5(1-ax)\sqrt{1-a^2x^2}}+\frac{1}{5ac^5(1-ax)^2\sqrt{1-a^2x^2}}$$

```
[Out] (2*x)/(5*c^5*Sqrt[1 - a^2*x^2]) + 1/(5*a*c^5*(1 - a*x)^2*Sqrt[1 - a^2*x^2])
 + 1/(5*a*c^5*(1 - a*x)*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0615994, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 191}

$$\frac{2x}{5c^5\sqrt{1-a^2x^2}}+\frac{1}{5ac^5(1-ax)\sqrt{1-a^2x^2}}+\frac{1}{5ac^5(1-ax)^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^5),x]
```

```
[Out] (2*x)/(5*c^5*Sqrt[1 - a^2*x^2]) + 1/(5*a*c^5*(1 - a*x)^2*Sqrt[1 - a^2*x^2])
 + 1/(5*a*c^5*(1 - a*x)*Sqrt[1 - a^2*x^2])
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\int \frac{1}{(c-acx)^2(1-a^2x^2)^{3/2}}\,dx}{c^3}$$
$$= \frac{1}{5ac^5(1-ax)^2\sqrt{1-a^2x^2}} + \frac{3\int \frac{1}{(c-acx)(1-a^2x^2)^{3/2}}\,dx}{5c^4}$$
$$= \frac{1}{5ac^5(1-ax)^2\sqrt{1-a^2x^2}} + \frac{1}{5ac^5(1-ax)\sqrt{1-a^2x^2}} + \frac{2\int \frac{1}{(1-a^2x^2)^{3/2}}\,dx}{5c^5}$$
$$= \frac{2x}{5c^5\sqrt{1-a^2x^2}} + \frac{1}{5ac^5(1-ax)^2\sqrt{1-a^2x^2}} + \frac{1}{5ac^5(1-ax)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0193726, size = 52, normalized size = 0.6

$$\frac{2a^3x^3-4a^2x^2+ax+2}{5ac^5(ax-1)^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^5),x]
```

```
[Out] (2 + a*x - 4*a^2*x^2 + 2*a^3*x^3)/(5*a*c^5*(-1 + a*x)^2*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.033, size = 56, normalized size = 0.6

$$\frac{2\,x^3a^3-4\,a^2x^2+ax+2}{5\,(ax-1)^4\,c^5a\,(ax+1)^2}\left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^5,x)
```

```
[Out] 1/5*(-a^2*x^2+1)^(3/2)*(2*a^3*x^3-4*a^2*x^2+a*x+2)/(a*x-1)^4/c^5/a/(a*x+1)^
2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(acx-c)^5(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^5,x, algorithm="maxima"
)
```

```
[Out] -integrate((-a^2*x^2 + 1)^(3/2)/((a*c*x - c)^5*(a*x + 1)^3), x)
```

Fricas [A] time = 1.61503, size = 200, normalized size = 2.3

$$\frac{2\,a^4x^4 - 4\,a^3x^3 + 4\,ax - \left(2\,a^3x^3 - 4\,a^2x^2 + ax + 2\right)\sqrt{-a^2x^2 + 1} - 2}{5\left(a^5c^5x^4 - 2\,a^4c^5x^3 + 2\,a^2c^5x - ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^5,x, algorithm="fricas"
)
```

```
[Out] 1/5*(2*a^4*x^4 - 4*a^3*x^3 + 4*a*x - (2*a^3*x^3 - 4*a^2*x^2 + a*x + 2)*sqrt
(-a^2*x^2 + 1) - 2)/(a^5*c^5*x^4 - 2*a^4*c^5*x^3 + 2*a^2*c^5*x - a*c^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^8x^8-2a^7x^7-2a^6x^6+6a^5x^5-6a^3x^3+2a^2x^2+2ax-1}\,dx + \int -\frac{a^2x^2\sqrt{-a^2x^2+1}}{a^8x^8-2a^7x^7-2a^6x^6+6a^5x^5-6a^3x^3+2a^2x^2+2ax-1}\,dx}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**5,x)
```

```
[Out] -(Integral(sqrt(-a**2*x**2 + 1)/(a**8*x**8 - 2*a**7*x**7 - 2*a**6*x**6 + 6*
a**5*x**5 - 6*a**3*x**3 + 2*a**2*x**2 + 2*a*x - 1), x) + Integral(-a**2*x**
2*sqrt(-a**2*x**2 + 1)/(a**8*x**8 - 2*a**7*x**7 - 2*a**6*x**6 + 6*a**5*x**5
 - 6*a**3*x**3 + 2*a**2*x**2 + 2*a*x - 1), x))/c**5
```

Giac [C] time = 1.38634, size = 235, normalized size = 2.7

$$\frac{1}{40}\left(\left(a\left(\frac{5}{a^3c^7\sqrt{-\frac{2\,c}{acx-c}-1}} - \frac{a^{12}c^{28}\left(\frac{2\,c}{acx-c}+1\right)^2\sqrt{-\frac{2\,c}{acx-c}-1} + 5\,a^{12}c^{28}\left(-\frac{2\,c}{acx-c}-1\right)^{\frac{3}{2}} + 15\,a^{12}c^{28}\sqrt{-\frac{2\,c}{acx-c}-1}}{a^{15}c^{35}}\right)\mathrm{sgn}\left(\frac{1}{acx-c}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/40*(a*(5/(a^3*c^7*sqrt(-2*c/(a*c*x - c) - 1)) - (a^12*c^28*(2*c/(a*c*x -
c) + 1)^2*sqrt(-2*c/(a*c*x - c) - 1) + 5*a^12*c^28*(-2*c/(a*c*x - c) - 1)^(
3/2) + 15*a^12*c^28*sqrt(-2*c/(a*c*x - c) - 1))/(a^15*c^35))*sgn(1/(a*c*x -
 c))*sgn(a)*sgn(c) + 16*I*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)/(a^2*c^7))*c^2*a
bs(a)
```

3.225 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^6}\,dx$

Optimal. Leaf size=119

$$\frac{8x}{35c^6\sqrt{1-a^2x^2}}+\frac{4}{35ac^6(1-ax)\sqrt{1-a^2x^2}}+\frac{4}{35ac^6(1-ax)^2\sqrt{1-a^2x^2}}+\frac{1}{7ac^6(1-ax)^3\sqrt{1-a^2x^2}}$$

```
[Out] (8*x)/(35*c^6*Sqrt[1 - a^2*x^2]) + 1/(7*a*c^6*(1 - a*x)^3*Sqrt[1 - a^2*x^2]
) + 4/(35*a*c^6*(1 - a*x)^2*Sqrt[1 - a^2*x^2]) + 4/(35*a*c^6*(1 - a*x)*Sqrt
[1 - a^2*x^2])
```

Rubi [A] time = 0.0762582, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 191}

$$\frac{8x}{35c^6\sqrt{1-a^2x^2}}+\frac{4}{35ac^6(1-ax)\sqrt{1-a^2x^2}}+\frac{4}{35ac^6(1-ax)^2\sqrt{1-a^2x^2}}+\frac{1}{7ac^6(1-ax)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^6),x]
```


```
[Out] (8*x)/(35*c^6*Sqrt[1 - a^2*x^2]) + 1/(7*a*c^6*(1 - a*x)^3*Sqrt[1 - a^2*x^2]
) + 4/(35*a*c^6*(1 - a*x)^2*Sqrt[1 - a^2*x^2]) + 4/(35*a*c^6*(1 - a*x)*Sqrt
[1 - a^2*x^2])
```


Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^6}\,dx = \frac{\int \frac{1}{(c-acx)^3(1-a^2x^2)^{3/2}}\,dx}{c^3}$$

$$= \frac{1}{7ac^6(1-ax)^3\sqrt{1-a^2x^2}} + \frac{4\int \frac{1}{(c-acx)^2(1-a^2x^2)^{3/2}}\,dx}{7c^4}$$

$$= \frac{1}{7ac^6(1-ax)^3\sqrt{1-a^2x^2}} + \frac{4}{35ac^6(1-ax)^2\sqrt{1-a^2x^2}} + \frac{12\int \frac{1}{(c-acx)(1-a^2x^2)^{3/2}}\,dx}{35c^5}$$

$$= \frac{1}{7ac^6(1-ax)^3\sqrt{1-a^2x^2}} + \frac{4}{35ac^6(1-ax)^2\sqrt{1-a^2x^2}} + \frac{4}{35ac^6(1-ax)\sqrt{1-a^2x^2}} + \frac{8\int \frac{1}{(1-a^2x^2)^{3/2}}\,dx}{35c^6}$$

$$= \frac{8x}{35c^6\sqrt{1-a^2x^2}} + \frac{1}{7ac^6(1-ax)^3\sqrt{1-a^2x^2}} + \frac{4}{35ac^6(1-ax)^2\sqrt{1-a^2x^2}} + \frac{4}{35ac^6(1-ax)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0234367, size = 61, normalized size = 0.51

$$\frac{8a^4x^4-24a^3x^3+20a^2x^2+4ax-13}{35ac^6(ax-1)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^6),x]
```

```
[Out] (-13 + 4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4)/(35*a*c^6*(-1 + a*x)^3*
Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.031, size = 65, normalized size = 0.6

$$\frac{8\,x^4a^4-24\,x^3a^3+20\,a^2x^2+4\,ax-13}{35\,(ax-1)^5\,c^6a\,(ax+1)^2}\left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^6,x)
```

```
[Out] 1/35*(-a^2*x^2+1)^(3/2)*(8*a^4*x^4-24*a^3*x^3+20*a^2*x^2+4*a*x-13)/(a*x-1)^
5/c^6/a/(a*x+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(acx-c)^6(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^6,x, algorithm="maxima"
)
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*c*x - c)^6*(a*x + 1)^3), x)
```

Fricas [A] time = 1.6598, size = 308, normalized size = 2.59

$$\frac{13\,a^5x^5-39\,a^4x^4+26\,a^3x^3+26\,a^2x^2-39\,ax-\left(8\,a^4x^4-24\,a^3x^3+20\,a^2x^2+4\,ax-13\right)\sqrt{-a^2x^2+1}+13}{35\left(a^6c^6x^5-3\,a^5c^6x^4+2\,a^4c^6x^3+2\,a^3c^6x^2-3\,a^2c^6x+ac^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^6,x, algorithm="fricas"
)
```

```
[Out] 1/35*(13*a^5*x^5 - 39*a^4*x^4 + 26*a^3*x^3 + 26*a^2*x^2 - 39*a*x - (8*a^4*x
^4 - 24*a^3*x^3 + 20*a^2*x^2 + 4*a*x - 13)*sqrt(-a^2*x^2 + 1) + 13)/(a^6*c^
6*x^5 - 3*a^5*c^6*x^4 + 2*a^4*c^6*x^3 + 2*a^3*c^6*x^2 - 3*a^2*c^6*x + a*c^6
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^9x^9-3a^8x^8+8a^6x^6-6a^5x^5-6a^4x^4+8a^3x^3-3ax+1}\,dx+\int -\frac{a^2x^2\sqrt{-a^2x^2+1}}{a^9x^9-3a^8x^8+8a^6x^6-6a^5x^5-6a^4x^4+8a^3x^3-3ax+1}\,dx}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**6,x)
```

```
[Out] (Integral(sqrt(-a**2*x**2 + 1)/(a**9*x**9 - 3*a**8*x**8 + 8*a**6*x**6 - 6*a
**5*x**5 - 6*a**4*x**4 + 8*a**3*x**3 - 3*a*x + 1), x) + Integral(-a**2*x**2
*sqrt(-a**2*x**2 + 1)/(a**9*x**9 - 3*a**8*x**8 + 8*a**6*x**6 - 6*a**5*x**5
- 6*a**4*x**4 + 8*a**3*x**3 - 3*a*x + 1), x))/c**6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(acx-c)^6(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^6,x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*c*x - c)^6*(a*x + 1)^3), x)
```

3.226 $\int e^{\tanh^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=176

$$\frac{4096c^6\left(1-a^2x^2\right)^{3/2}}{3465a(c-acx)^{3/2}}+\frac{1024c^5\left(1-a^2x^2\right)^{3/2}}{1155a\sqrt{c-acx}}+\frac{128c^4\left(1-a^2x^2\right)^{3/2}\sqrt{c-acx}}{231a}+\frac{32c^3\left(1-a^2x^2\right)^{3/2}(c-acx)^{3/2}}{99a}+\frac{2c^2\left(1-a^2\right.}{}$$

```
[Out] (4096*c^6*(1 - a^2*x^2)^(3/2))/(3465*a*(c - a*c*x)^(3/2)) + (1024*c^5*(1 -
a^2*x^2)^(3/2))/(1155*a*Sqrt[c - a*c*x]) + (128*c^4*Sqrt[c - a*c*x]*(1 - a^
2*x^2)^(3/2))/(231*a) + (32*c^3*(c - a*c*x)^(3/2)*(1 - a^2*x^2)^(3/2))/(99*
a) + (2*c^2*(c - a*c*x)^(5/2)*(1 - a^2*x^2)^(3/2))/(11*a)
```

Rubi [A] time = 0.128885, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 657, 649}

$$\frac{4096c^6\left(1-a^2x^2\right)^{3/2}}{3465a(c-acx)^{3/2}}+\frac{1024c^5\left(1-a^2x^2\right)^{3/2}}{1155a\sqrt{c-acx}}+\frac{128c^4\left(1-a^2x^2\right)^{3/2}\sqrt{c-acx}}{231a}+\frac{32c^3\left(1-a^2x^2\right)^{3/2}(c-acx)^{3/2}}{99a}+\frac{2c^2\left(1-a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^(9/2),x]
```

```
[Out] (4096*c^6*(1 - a^2*x^2)^(3/2))/(3465*a*(c - a*c*x)^(3/2)) + (1024*c^5*(1 -
a^2*x^2)^(3/2))/(1155*a*Sqrt[c - a*c*x]) + (128*c^4*Sqrt[c - a*c*x]*(1 - a^
2*x^2)^(3/2))/(231*a) + (32*c^3*(c - a*c*x)^(3/2)*(1 - a^2*x^2)^(3/2))/(99*
a) + (2*c^2*(c - a*c*x)^(5/2)*(1 - a^2*x^2)^(3/2))/(11*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}(c-acx)^{9/2}\,dx = c\int (c-acx)^{7/2}\sqrt{1-a^2x^2}\,dx$$
$$= \frac{2c^2(c-acx)^{5/2}\left(1-a^2x^2\right)^{3/2}}{11a} + \frac{1}{11}\left(16c^2\right)\int (c-acx)^{5/2}\sqrt{1-a^2x^2}\,dx$$
$$= \frac{32c^3(c-acx)^{3/2}\left(1-a^2x^2\right)^{3/2}}{99a} + \frac{2c^2(c-acx)^{5/2}\left(1-a^2x^2\right)^{3/2}}{11a} + \frac{1}{33}\left(64c^3\right)\int (c-acx)^{3/2}$$
$$= \frac{128c^4\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{231a} + \frac{32c^3(c-acx)^{3/2}\left(1-a^2x^2\right)^{3/2}}{99a} + \frac{2c^2(c-acx)^{5/2}\left(1-a^2x^2\right)}{11a}$$
$$= \frac{1024c^5\left(1-a^2x^2\right)^{3/2}}{1155a\sqrt{c-acx}} + \frac{128c^4\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{231a} + \frac{32c^3(c-acx)^{3/2}\left(1-a^2x^2\right)^{3/2}}{99a} + \frac{2}{}$$
$$= \frac{4096c^6\left(1-a^2x^2\right)^{3/2}}{3465a(c-acx)^{3/2}} + \frac{1024c^5\left(1-a^2x^2\right)^{3/2}}{1155a\sqrt{c-acx}} + \frac{128c^4\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{231a} + \frac{32c^3(c-a}{}$$

Mathematica [A] time = 0.0500722, size = 70, normalized size = 0.4

$$\frac{2c^4(ax+1)^{3/2}\left(315a^4x^4-1820a^3x^3+4530a^2x^2-6396ax+5419\right)\sqrt{c-acx}}{3465a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^(9/2),x]
```

```
[Out] (2*c^4*(1 + a*x)^(3/2)*Sqrt[c - a*c*x]*(5419 - 6396*a*x + 4530*a^2*x^2 - 18
20*a^3*x^3 + 315*a^4*x^4))/(3465*a*Sqrt[1 - a*x])
```

Maple [A] time = 0.038, size = 71, normalized size = 0.4

$$\frac{2\left(315x^4a^4-1820x^3a^3+4530a^2x^2-6396ax+5419\right)(ax+1)^2}{3465(ax-1)^4a}(-acx+c)^{\frac{9}{2}}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(9/2),x)
```

```
[Out] 2/3465*(a*x+1)^2*(315*a^4*x^4-1820*a^3*x^3+4530*a^2*x^2-6396*a*x+5419)*(-a*
c*x+c)^(9/2)/a/(a*x-1)^4/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.02965, size = 203, normalized size = 1.15

$$\frac{2\left(35a^6c^{\frac{9}{2}}x^6-175a^5c^{\frac{9}{2}}x^5+360a^4c^{\frac{9}{2}}x^4-422a^3c^{\frac{9}{2}}x^3+459a^2c^{\frac{9}{2}}x^2-1451ac^{\frac{9}{2}}x-2902c^{\frac{9}{2}}\right)}{385\sqrt{ax+1}a} + \frac{2\left(35a^5c^{\frac{9}{2}}x^5-185a^4c^{\frac{9}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="maxima"
)
```

```
[Out] 2/385*(35*a^6*c^(9/2)*x^6 - 175*a^5*c^(9/2)*x^5 + 360*a^4*c^(9/2)*x^4 - 422
*a^3*c^(9/2)*x^3 + 459*a^2*c^(9/2)*x^2 - 1451*a*c^(9/2)*x - 2902*c^(9/2))/(
sqrt(a*x + 1)*a) + 2/315*(35*a^5*c^(9/2)*x^5 - 185*a^4*c^(9/2)*x^4 + 422*a^
3*c^(9/2)*x^3 - 634*a^2*c^(9/2)*x^2 + 1591*a*c^(9/2)*x + 2867*c^(9/2))/(sqr
t(a*x + 1)*a)
```

Fricas [A] time = 1.70215, size = 212, normalized size = 1.2

$$-\frac{2\left(315\,a^{5}c^{4}x^{5}-1505\,a^{4}c^{4}x^{4}+2710\,a^{3}c^{4}x^{3}-1866\,a^{2}c^{4}x^{2}-977\,ac^{4}x+5419\,c^{4}\right)\sqrt{-a^{2}x^{2}+1}\sqrt{-acx+c}}{3465\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="fricas"
)
```

```
[Out] -2/3465*(315*a^5*c^4*x^5 - 1505*a^4*c^4*x^4 + 2710*a^3*c^4*x^3 - 1866*a^2*c
^4*x^2 - 977*a*c^4*x + 5419*c^4)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2*x
 - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48155, size = 117, normalized size = 0.66

$$-\frac{2\left(4096\,\sqrt{2}c^{\frac{7}{2}}-\frac{315\,(acx+c)^{\frac{11}{2}}-3080\,(acx+c)^{\frac{9}{2}}c+11880\,(acx+c)^{\frac{7}{2}}c^{2}-22176\,(acx+c)^{\frac{5}{2}}c^{3}+18480\,(acx+c)^{\frac{3}{2}}c^{4}}{c^{2}}\right)c^{2}}{3465\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] -2/3465*(4096*sqrt(2)*c^(7/2) - (315*(a*c*x + c)^(11/2) - 3080*(a*c*x + c)^
(9/2)*c + 11880*(a*c*x + c)^(7/2)*c^2 - 22176*(a*c*x + c)^(5/2)*c^3 + 18480
*(a*c*x + c)^(3/2)*c^4)/c^2)*c^2/(a*abs(c))
```

3.227 $\int e^{\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=141

$$\frac{256c^5\left(1-a^2x^2\right)^{3/2}}{315a(c-acx)^{3/2}}+\frac{64c^4\left(1-a^2x^2\right)^{3/2}}{105a\sqrt{c-acx}}+\frac{8c^3\left(1-a^2x^2\right)^{3/2}\sqrt{c-acx}}{21a}+\frac{2c^2\left(1-a^2x^2\right)^{3/2}(c-acx)^{3/2}}{9a}$$

```
[Out] (256*c^5*(1 - a^2*x^2)^(3/2))/(315*a*(c - a*c*x)^(3/2)) + (64*c^4*(1 - a^2*
x^2)^(3/2))/(105*a*Sqrt[c - a*c*x]) + (8*c^3*Sqrt[c - a*c*x]*(1 - a^2*x^2)^
(3/2))/(21*a) + (2*c^2*(c - a*c*x)^(3/2)*(1 - a^2*x^2)^(3/2))/(9*a)
```

Rubi [A] time = 0.104202, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 657, 649}

$$\frac{256c^5\left(1-a^2x^2\right)^{3/2}}{315a(c-acx)^{3/2}}+\frac{64c^4\left(1-a^2x^2\right)^{3/2}}{105a\sqrt{c-acx}}+\frac{8c^3\left(1-a^2x^2\right)^{3/2}\sqrt{c-acx}}{21a}+\frac{2c^2\left(1-a^2x^2\right)^{3/2}(c-acx)^{3/2}}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^(7/2),x]
```

```
[Out] (256*c^5*(1 - a^2*x^2)^(3/2))/(315*a*(c - a*c*x)^(3/2)) + (64*c^4*(1 - a^2*
x^2)^(3/2))/(105*a*Sqrt[c - a*c*x]) + (8*c^3*Sqrt[c - a*c*x]*(1 - a^2*x^2)^
(3/2))/(21*a) + (2*c^2*(c - a*c*x)^(3/2)*(1 - a^2*x^2)^(3/2))/(9*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx = c\int (c-acx)^{5/2}\sqrt{1-a^2x^2}\,dx$$
$$= \frac{2c^2(c-acx)^{3/2}\left(1-a^2x^2\right)^{3/2}}{9a} + \frac{1}{3}\left(4c^2\right)\int (c-acx)^{3/2}\sqrt{1-a^2x^2}\,dx$$
$$= \frac{8c^3\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{21a} + \frac{2c^2(c-acx)^{3/2}\left(1-a^2x^2\right)^{3/2}}{9a} + \frac{1}{21}\left(32c^3\right)\int \sqrt{c-acx}\sqrt{1-a^2x^2}$$
$$= \frac{64c^4\left(1-a^2x^2\right)^{3/2}}{105a\sqrt{c-acx}} + \frac{8c^3\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{21a} + \frac{2c^2(c-acx)^{3/2}\left(1-a^2x^2\right)^{3/2}}{9a} + \frac{1}{105}\left(128c^4\right.$$
$$= \frac{256c^5\left(1-a^2x^2\right)^{3/2}}{315a(c-acx)^{3/2}} + \frac{64c^4\left(1-a^2x^2\right)^{3/2}}{105a\sqrt{c-acx}} + \frac{8c^3\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{21a} + \frac{2c^2(c-acx)^{3/2}\left(1-\right.}{9a}$$

Mathematica [A] time = 0.0413035, size = 62, normalized size = 0.44

$$-\frac{2c^3(ax+1)^{3/2}\left(35a^3x^3-165a^2x^2+321ax-319\right)\sqrt{c-acx}}{315a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^(7/2),x]
```

```
[Out] (-2*c^3*(1 + a*x)^(3/2)*Sqrt[c - a*c*x]*(-319 + 321*a*x - 165*a^2*x^2 + 35*
a^3*x^3))/(315*a*Sqrt[1 - a*x])
```

Maple [A] time = 0.033, size = 63, normalized size = 0.5

$$\frac{2\left(35x^3a^3-165a^2x^2+321ax-319\right)(ax+1)^2}{315(ax-1)^3a}(-acx+c)^{\frac{7}{2}}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(7/2),x)
```

```
[Out] 2/315*(a*x+1)^2*(35*a^3*x^3-165*a^2*x^2+321*a*x-319)*(-a*c*x+c)^(7/2)/a/(a*
x-1)^3/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.02646, size = 173, normalized size = 1.23

$$-\frac{2\left(5a^5c^{\frac{7}{2}}x^5-20a^4c^{\frac{7}{2}}x^4+32a^3c^{\frac{7}{2}}x^3-34a^2c^{\frac{7}{2}}x^2+91ac^{\frac{7}{2}}x+182c^{\frac{7}{2}}\right)}{45\sqrt{ax+1}a} - \frac{2\left(5a^4c^{\frac{7}{2}}x^4-22a^3c^{\frac{7}{2}}x^3+44a^2c^{\frac{7}{2}}x^2-106ac^{\frac{7}{2}}\right.}{35\sqrt{ax+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="maxima"
)
```

```
[Out] -2/45*(5*a^5*c^(7/2)*x^5 - 20*a^4*c^(7/2)*x^4 + 32*a^3*c^(7/2)*x^3 - 34*a^2
*c^(7/2)*x^2 + 91*a*c^(7/2)*x + 182*c^(7/2))/(sqrt(a*x + 1)*a) - 2/35*(5*a^
```

```
4*c^(7/2)*x^4 - 22*a^3*c^(7/2)*x^3 + 44*a^2*c^(7/2)*x^2 - 106*a*c^(7/2)*x -
 177*c^(7/2))/(sqrt(a*x + 1)*a)
```

Fricas [A] time = 1.58907, size = 176, normalized size = 1.25

$$\frac{2\left(35\,a^4c^3x^4-130\,a^3c^3x^3+156\,a^2c^3x^2+2\,ac^3x-319\,c^3\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{315\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="fricas"
)
```

```
[Out] 2/315*(35*a^4*c^3*x^4 - 130*a^3*c^3*x^3 + 156*a^2*c^3*x^2 + 2*a*c^3*x - 319
*c^3)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3117, size = 99, normalized size = 0.7

$$-\frac{2\left(256\,\sqrt{2}c^{\frac{5}{2}}+\frac{35\,(acx+c)^{\frac{9}{2}}-270\,(acx+c)^{\frac{7}{2}}c+756\,(acx+c)^{\frac{5}{2}}c^2-840\,(acx+c)^{\frac{3}{2}}c^3}{c^2}\right)c^2}{315\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -2/315*(256*sqrt(2)*c^(5/2) + (35*(a*c*x + c)^(9/2) - 270*(a*c*x + c)^(7/2)
*c + 756*(a*c*x + c)^(5/2)*c^2 - 840*(a*c*x + c)^(3/2)*c^3)/c^2)*c^2/(a*abs
(c))
```

3.228 $\int e^{\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=106

$$\frac{64c^4\left(1-a^2x^2\right)^{3/2}}{105a(c-acx)^{3/2}}+\frac{16c^3\left(1-a^2x^2\right)^{3/2}}{35a\sqrt{c-acx}}+\frac{2c^2\left(1-a^2x^2\right)^{3/2}\sqrt{c-acx}}{7a}$$

```
[Out] (64*c^4*(1 - a^2*x^2)^(3/2))/(105*a*(c - a*c*x)^(3/2)) + (16*c^3*(1 - a^2*x
^2)^(3/2))/(35*a*Sqrt[c - a*c*x]) + (2*c^2*Sqrt[c - a*c*x]*(1 - a^2*x^2)^(3
/2))/(7*a)
```

Rubi [A] time = 0.0817861, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 657, 649}

$$\frac{64c^4\left(1-a^2x^2\right)^{3/2}}{105a(c-acx)^{3/2}}+\frac{16c^3\left(1-a^2x^2\right)^{3/2}}{35a\sqrt{c-acx}}+\frac{2c^2\left(1-a^2x^2\right)^{3/2}\sqrt{c-acx}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^(5/2),x]
```

```
[Out] (64*c^4*(1 - a^2*x^2)^(3/2))/(105*a*(c - a*c*x)^(3/2)) + (16*c^3*(1 - a^2*x
^2)^(3/2))/(35*a*Sqrt[c - a*c*x]) + (2*c^2*Sqrt[c - a*c*x]*(1 - a^2*x^2)^(3
/2))/(7*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx = c\int (c-acx)^{3/2}\sqrt{1-a^2x^2}\,dx$$
$$= \frac{2c^2\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{7a} + \frac{1}{7}\left(8c^2\right)\int \sqrt{c-acx}\sqrt{1-a^2x^2}\,dx$$
$$= \frac{16c^3\left(1-a^2x^2\right)^{3/2}}{35a\sqrt{c-acx}} + \frac{2c^2\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{7a} + \frac{1}{35}\left(32c^3\right)\int \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx$$
$$= \frac{64c^4\left(1-a^2x^2\right)^{3/2}}{105a(c-acx)^{3/2}} + \frac{16c^3\left(1-a^2x^2\right)^{3/2}}{35a\sqrt{c-acx}} + \frac{2c^2\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{7a}$$

Mathematica [A] time = 0.0343594, size = 54, normalized size = 0.51

$$\frac{2c^2(ax+1)^{3/2}\left(15a^2x^2-54ax+71\right)\sqrt{c-acx}}{105a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*c^2*(1 + a*x)^(3/2)*Sqrt[c - a*c*x]*(71 - 54*a*x + 15*a^2*x^2))/(105*a*S
qrt[1 - a*x])
```

Maple [A] time = 0.031, size = 55, normalized size = 0.5

$$\frac{2\left(15\,a^2x^2-54\,ax+71\right)(ax+1)^2}{105\,(ax-1)^2\,a}(-acx+c)^{\frac{5}{2}}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(5/2),x)
```

```
[Out] 2/105*(a*x+1)^2*(15*a^2*x^2-54*a*x+71)*(-a*c*x+c)^(5/2)/a/(a*x-1)^2/(-a^2*x
^2+1)^(1/2)
```

Maxima [A] time = 1.02198, size = 143, normalized size = 1.35

$$\frac{2\left(3\,a^4c^{\frac{5}{2}}x^4-9\,a^3c^{\frac{5}{2}}x^3+11\,a^2c^{\frac{5}{2}}x^2-23\,ac^{\frac{5}{2}}x-46\,c^{\frac{5}{2}}\right)}{21\,\sqrt{ax+1}a}+\frac{2\left(3\,a^3c^{\frac{5}{2}}x^3-11\,a^2c^{\frac{5}{2}}x^2+29\,ac^{\frac{5}{2}}x+43\,c^{\frac{5}{2}}\right)}{15\,\sqrt{ax+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 2/21*(3*a^4*c^(5/2)*x^4 - 9*a^3*c^(5/2)*x^3 + 11*a^2*c^(5/2)*x^2 - 23*a*c^(
5/2)*x - 46*c^(5/2))/(sqrt(a*x + 1)*a) + 2/15*(3*a^3*c^(5/2)*x^3 - 11*a^2*c
^(5/2)*x^2 + 29*a*c^(5/2)*x + 43*c^(5/2))/(sqrt(a*x + 1)*a)
```

Fricas [A] time = 1.68112, size = 151, normalized size = 1.42

$$-\frac{2\left(15\,a^{3}c^{2}x^{3}-39\,a^{2}c^{2}x^{2}+17\,ac^{2}x+71\,c^{2}\right)\sqrt{-a^{2}x^{2}+1}\sqrt{-acx+c}}{105\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="fricas"
)
```

```
[Out] -2/105*(15*a^3*c^2*x^3 - 39*a^2*c^2*x^2 + 17*a*c^2*x + 71*c^2)*sqrt(-a^2*x^
2 + 1)*sqrt(-a*c*x + c)/(a^2*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-c\left(ax-1\right)\right)^{\frac{5}{2}}\left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**(5/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(5/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [A] time = 1.29471, size = 82, normalized size = 0.77

$$-\frac{2\left(64\,\sqrt{2}c^{\frac{3}{2}}-\frac{15\left(acx+c\right)^{\frac{7}{2}}-84\left(acx+c\right)^{\frac{5}{2}}c+140\left(acx+c\right)^{\frac{3}{2}}c^{2}}{c^{2}}\right)c^{2}}{105\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/105*(64*sqrt(2)*c^(3/2) - (15*(a*c*x + c)^(7/2) - 84*(a*c*x + c)^(5/2)*c
 + 140*(a*c*x + c)^(3/2)*c^2)/c^2)*c^2/(a*abs(c))
```

3.229 $\int e^{\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=71

$$\frac{8c^3\left(1-a^2x^2\right)^{3/2}}{15a(c-acx)^{3/2}}+\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{5a\sqrt{c-acx}}$$

```
[Out] (8*c^3*(1 - a^2*x^2)^(3/2))/(15*a*(c - a*c*x)^(3/2)) + (2*c^2*(1 - a^2*x^2)
^(3/2))/(5*a*Sqrt[c - a*c*x])
```

Rubi [A] time = 0.0607667, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 657, 649}

$$\frac{8c^3\left(1-a^2x^2\right)^{3/2}}{15a(c-acx)^{3/2}}+\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{5a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^(3/2),x]
```

```
[Out] (8*c^3*(1 - a^2*x^2)^(3/2))/(15*a*(c - a*c*x)^(3/2)) + (2*c^2*(1 - a^2*x^2)
^(3/2))/(5*a*Sqrt[c - a*c*x])
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx &= c\int \sqrt{c-acx}\sqrt{1-a^2x^2}\,dx\\
&= \frac{2c^2\left(1-a^2x^2\right)^{3/2}}{5a\sqrt{c-acx}}+\frac{1}{5}\left(4c^2\right)\int \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx\\
&= \frac{8c^3\left(1-a^2x^2\right)^{3/2}}{15a(c-acx)^{3/2}}+\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{5a\sqrt{c-acx}}
\end{aligned}$$

Mathematica [A] time = 0.0253229, size = 44, normalized size = 0.62

$$-\frac{2c(ax+1)^{3/2}(3ax-7)\sqrt{c-acx}}{15a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^(3/2),x]
```

```
[Out] (-2*c*(1 + a*x)^(3/2)*(-7 + 3*a*x)*Sqrt[c - a*c*x])/(15*a*Sqrt[1 - a*x])
```

Maple [A] time = 0.03, size = 47, normalized size = 0.7

$$\frac{2\,(3\,ax-7)\,(ax+1)^2}{15\,(ax-1)\,a}(-acx+c)^{\frac{3}{2}}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(3/2),x)
```

```
[Out] 2/15*(a*x+1)^2*(3*a*x-7)*(-a*c*x+c)^(3/2)/a/(a*x-1)/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.03068, size = 111, normalized size = 1.56

$$-\frac{2\left(a^3c^{\frac{3}{2}}x^3-2\,a^2c^{\frac{3}{2}}x^2+3\,ac^{\frac{3}{2}}x+6\,c^{\frac{3}{2}}\right)}{5\,\sqrt{ax+1}a}-\frac{2\left(a^2c^{\frac{3}{2}}x^2-4\,ac^{\frac{3}{2}}x-5\,c^{\frac{3}{2}}\right)}{3\,\sqrt{ax+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -2/5*(a^3*c^(3/2)*x^3 - 2*a^2*c^(3/2)*x^2 + 3*a*c^(3/2)*x + 6*c^(3/2))/(sqr
t(a*x + 1)*a) - 2/3*(a^2*c^(3/2)*x^2 - 4*a*c^(3/2)*x - 5*c^(3/2))/(sqrt(a*x
 + 1)*a)
```

Fricas [A] time = 1.78191, size = 113, normalized size = 1.59

$$\frac{2\left(3\,a^2cx^2-4\,acx-7\,c\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{15\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 2/15*(3*a^2*c*x^2 - 4*a*c*x - 7*c)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2
*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c\,(ax-1))^{\frac{3}{2}}\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(3/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [A] time = 1.22043, size = 63, normalized size = 0.89

$$-\frac{2\left(8\,\sqrt{2}\sqrt{c}+\frac{3\,(acx+c)^{\frac{5}{2}}-10\,(acx+c)^{\frac{3}{2}}c}{c^2}\right)c^2}{15\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/15*(8*sqrt(2)*sqrt(c) + (3*(a*c*x + c)^(5/2) - 10*(a*c*x + c)^(3/2)*c)/c
^2)*c^2/(a*abs(c))
```

3.230 $\int e^{\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=35

$$\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}$$

```
[Out] (2*c^2*(1 - a^2*x^2)^(3/2))/(3*a*(c - a*c*x)^(3/2))
```

Rubi [A] time = 0.0410775, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6127, 649}

$$\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - a*c*x],x]

[Out] (2*c^2*(1 - a^2*x^2)^(3/2))/(3*a*(c - a*c*x)^(3/2))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}\sqrt{c-acx}\,dx&=c\int\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx\\&=\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0131721, size = 37, normalized size = 1.06

$$\frac{2(ax+1)^{3/2}\sqrt{c-acx}}{3a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - a*c*x],x]

[Out] (2*(1 + a*x)^(3/2)*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - a*x])
```

Maple [A] time = 0.027, size = 34, normalized size = 1.

$$\frac{2 (ax+1)^2}{3a}\sqrt{-acx+c}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/3*(a*x+1)^2*(-a*c*x+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.03323, size = 78, normalized size = 2.23

$$\frac{2\left(a^2\sqrt{c}x^2 - a\sqrt{c}x - 2\sqrt{c}\right)}{3\sqrt{ax+1}a} + \frac{2\left(a\sqrt{c}x + \sqrt{c}\right)}{\sqrt{ax+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 2/3*(a^2*sqrt(c)*x^2 - a*sqrt(c)*x - 2*sqrt(c))/(sqrt(a*x + 1)*a) + 2*(a*sq
rt(c)*x + sqrt(c))/(sqrt(a*x + 1)*a)
```

Fricas [A] time = 1.86509, size = 86, normalized size = 2.46

$$-\frac{2\sqrt{-a^2x^2+1}\sqrt{-acx+c}(ax+1)}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/3*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x + 1)/(a^2*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c(ax-1)}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [A] time = 1.18932, size = 43, normalized size = 1.23

$$-\frac{2\left(2\sqrt{2}\sqrt{c}-\frac{(acx+c)^{\frac{3}{2}}}{c}\right)c}{3a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(2*sqrt(2)*sqrt(c) - (a*c*x + c)^(3/2)/c)*c/(a*abs(c))
```

3.231 $\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=83

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a\sqrt{c}}-\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}$$

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x]) + (2*Sqrt[2]*ArcTanh[(Sqrt[c]*Sq
rt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/(a*Sqrt[c])
```

Rubi [A] time = 0.0816572, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6127, 665, 661, 208}

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a\sqrt{c}}-\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x]) + (2*Sqrt[2]*ArcTanh[(Sqrt[c]*Sq
rt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/(a*Sqrt[c])
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{3/2}}\,dx \\
&= -\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}} + 2\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}} - (4ac)\,\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right) \\
&= -\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}} + \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.0248053, size = 62, normalized size = 0.75

$$-\frac{2\sqrt{c-acx}\left(\sqrt{ax+1}-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{ac\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(Sqrt[1 + a*x] - Sqrt[2]*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]]
))/(a*c*Sqrt[1 - a*x])
```

Maple [A] time = 0.11, size = 84, normalized size = 1.

$$-2\,\frac{\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}}{(ax-1)\sqrt{c\,(ax+1)}ca}\left(\sqrt{c}\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)-\sqrt{c\,(ax+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2),x)
```

```
[Out] -2*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(c^(1/2)*2^(1/2)*arctanh(1/2*(c*(a
*x+1))^(1/2)*2^(1/2)/c^(1/2))-(c*(a*x+1))^(1/2))/(a*x-1)/(c*(a*x+1))^(1/2)/
c/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\sqrt{-acx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)), x)
```

Fricas [A] time = 1.94142, size = 491, normalized size = 5.92

$$\left[\frac{\frac{\sqrt{2}(acx-c)\log\left(-\frac{a^2x^2+2\,ax-\frac{2\,\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{\sqrt{c}}-3}{a^2x^2-2\,ax+1}\right)}{\sqrt{c}}+2\,\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{a^2cx-ac}, \frac{2\left(\sqrt{2}(acx-c)\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{\ldots}}{a^2x^2-1}\right.\right.}{a^2cx-ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [(sqrt(2)*(a*c*x - c)*log(-(a^2*x^2 + 2*a*x - 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*
sqrt(-a*c*x + c)/sqrt(c) - 3)/(a^2*x^2 - 2*a*x + 1))/sqrt(c) + 2*sqrt(-a^2*
x^2 + 1)*sqrt(-a*c*x + c))/(a^2*c*x - a*c), 2*(sqrt(2)*(a*c*x - c)*sqrt(-1/
c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-1/c)/(a^2*x^2 -
 1)) + sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^2*c*x - a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-c\,(ax-1)}\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [A] time = 1.29129, size = 123, normalized size = 1.48

$$-\frac{2\,c\left(\frac{\frac{\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\,\sqrt{-c}}\right)}{\sqrt{-c}}+\sqrt{acx+c}}{c}-\frac{\sqrt{2}c\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)+\sqrt{2}\sqrt{-c}\sqrt{c}}{\sqrt{-c}c}\right)}{a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*c*((sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) + sq
rt(a*c*x + c))/c - (sqrt(2)*c*arctan(sqrt(c)/sqrt(-c)) + sqrt(2)*sqrt(-c)*s
qrt(c))/(sqrt(-c)*c))/(a*abs(c))
```

3.232 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{1-a^2x^2}}{a(c-acx)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{\sqrt{2}ac^{3/2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(a*(c - a*c*x)^(3/2)) - ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2
])/(Sqrt[2]*Sqrt[c - a*c*x])]/(Sqrt[2]*a*c^(3/2))
```

Rubi [A] time = 0.0801186, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6127, 663, 661, 208}

$$\frac{\sqrt{1-a^2x^2}}{a(c-acx)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{\sqrt{2}ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^(3/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a*(c - a*c*x)^(3/2)) - ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2
])/(Sqrt[2]*Sqrt[c - a*c*x])]/(Sqrt[2]*a*c^(3/2))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{5/2}}\,dx$$
$$= \frac{\sqrt{1-a^2x^2}}{a(c-acx)^{3/2}} - \frac{\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{2c}$$
$$= \frac{\sqrt{1-a^2x^2}}{a(c-acx)^{3/2}} + a\,\mathrm{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$
$$= \frac{\sqrt{1-a^2x^2}}{a(c-acx)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{\sqrt{2}ac^{3/2}}$$

Mathematica [A] time = 0.0532543, size = 80, normalized size = 0.98

$$-\frac{\sqrt{c-acx}\left(2ax+(ax-1)\sqrt{2ax+2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)+2\right)}{2ac^2(ax-1)\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^(3/2),x]
```

```
[Out] -(Sqrt[c - a*c*x]*(2 + 2*a*x + (-1 + a*x)*Sqrt[2 + 2*a*x]*ArcTanh[Sqrt[1 +
a*x]/Sqrt[2]]))/(2*a*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.102, size = 111, normalized size = 1.4

$$\frac{1}{2\,(ax-1)^2\,a}\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}\left(\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{c\,(ax+1)}\frac{1}{\sqrt{c}}\right)xac-\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{c\,(ax+1)}\frac{1}{\sqrt{c}}\right)c+2\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(3/2),x)
```

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(2^(1/2)*arctanh(1/2*(c*(a*x+1))^
(1/2)*2^(1/2)/c^(1/2))*x*a*c-2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/
c^(1/2))*c+2*(c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)^2/(c*(a*x+1))^(1/2)/c^(5/2)
/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}(-acx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^(3/2)), x)
```

Fricas [A] time = 1.97195, size = 590, normalized size = 7.2

$$\left[\frac{\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+4\,\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{4\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)},-\frac{\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{-c}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*s
qrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x
+ 1)) + 4*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^3*c^2*x^2 - 2*a^2*c^2*x +
 a*c^2), -1/2*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-
a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*x^2 +
 1)*sqrt(-a*c*x + c))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-c\left(ax-1\right)\right)^{\frac{3}{2}}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(a*x - 1))**(3/2)*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [A] time = 1.27361, size = 78, normalized size = 0.95

$$\frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\,\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{2\,\sqrt{acx+c}}{acx-c}}{2\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) - 2*sqrt
(a*c*x + c)/(a*c*x - c))/(a*abs(c))
```

3.233 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^{5/2}} \, dx$

Optimal. Leaf size=122

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{8\sqrt{2}ac^{5/2}}-\frac{\sqrt{1-a^2x^2}}{8ac(c-acx)^{3/2}}+\frac{\sqrt{1-a^2x^2}}{2a(c-acx)^{5/2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(2*a*(c - a*c*x)^(5/2)) - Sqrt[1 - a^2*x^2]/(8*a*c*(c - a
*c*x)^(3/2)) - ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x]
)]/(8*Sqrt[2]*a*c^(5/2))
```

Rubi [A] time = 0.106377, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6127, 663, 673, 661, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{8\sqrt{2}ac^{5/2}}-\frac{\sqrt{1-a^2x^2}}{8ac(c-acx)^{3/2}}+\frac{\sqrt{1-a^2x^2}}{2a(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^(5/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a*(c - a*c*x)^(5/2)) - Sqrt[1 - a^2*x^2]/(8*a*c*(c - a
*c*x)^(3/2)) - ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x]
)]/(8*Sqrt[2]*a*c^(5/2))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{7/2}}\,dx\\ &= \frac{\sqrt{1-a^2x^2}}{2a(c-acx)^{5/2}}-\frac{\int \frac{1}{(c-acx)^{3/2}\sqrt{1-a^2x^2}}\,dx}{4c}\\ &= \frac{\sqrt{1-a^2x^2}}{2a(c-acx)^{5/2}}-\frac{\sqrt{1-a^2x^2}}{8ac(c-acx)^{3/2}}-\frac{\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{16c^2}\\ &= \frac{\sqrt{1-a^2x^2}}{2a(c-acx)^{5/2}}-\frac{\sqrt{1-a^2x^2}}{8ac(c-acx)^{3/2}}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)}{8c}\\ &= \frac{\sqrt{1-a^2x^2}}{2a(c-acx)^{5/2}}-\frac{\sqrt{1-a^2x^2}}{8ac(c-acx)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{8\sqrt{2}ac^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.0184426, size = 57, normalized size = 0.47

$$\frac{(ax+1)^{3/2}\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(\frac{3}{2},3,\frac{5}{2},\frac{1}{2}(ax+1)\right)}{12ac^3\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^(5/2),x]
```

```
[Out] ((1 + a*x)^(3/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[3/2, 3, 5/2, (1 + a*x)/2
])/(12*a*c^3*Sqrt[1 - a*x])
```

Maple [A] time = 0.102, size = 156, normalized size = 1.3

$$\frac{1}{16\,(ax-1)^3\,a}\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}\left(\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{c\,(ax+1)}\frac{1}{\sqrt{c}}\right)x^2a^2c-2\,\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)xac-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(5/2),x)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(7/2)*(2^(1/2)*arctanh(1/2*(c*
(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-2*2^(1/2)*arctanh(1/2*(c*(a*x+1))
^(1/2)*2^(1/2)/c^(1/2))*x*a*c-2*x*a*(c*(a*x+1))^(1/2)*c^(1/2)+2^(1/2)*arcta
nh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-6*(c*(a*x+1))^(1/2)*c^(1/2))/(a
*x-1)^3/(c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}(-acx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^(5/2)), x)
```

Fricas [A] time = 2.02664, size = 695, normalized size = 5.7

$$\left[\frac{\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2acx+2\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3c}{a^2x^2-2ax+1}\right)-4\sqrt{-a^2x^2+1}\sqrt{-acx+c}(ax+3)}{32\left(a^4c^3x^3-3a^3c^3x^2+3a^2c^3x-ac^3\right)}, -\frac{\sqrt{2}(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas"
)
```

```
[Out] [1/32*(sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 +
2*a*c*x + 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2
*x^2 - 2*a*x + 1)) - 4*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x + 3))/(a^4*
c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3), -1/16*(sqrt(2)*(a^3*x^3 - 3
*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*
x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a
*x + 3))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-c\left(ax-1\right)\right)^{\frac{5}{2}}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(a*x - 1))**(5/2)*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [A] time = 1.30308, size = 103, normalized size = 0.84

$$\frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}c}+\frac{2\left((acx+c)^{\frac{3}{2}}+2\sqrt{acx+c}c\right)}{(acx-c)^2c}}{16a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c) + 2
*((a*c*x + c)^(3/2) + 2*sqrt(a*c*x + c)*c)/((a*c*x - c)^2*c))/(a*abs(c))
```

3.234 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{\sqrt{1-a^2x^2}}{32ac^2(c-acx)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{32\sqrt{2}ac^{7/2}}-\frac{\sqrt{1-a^2x^2}}{24ac(c-acx)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{3a(c-acx)^{7/2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(3*a*(c - a*c*x)^(7/2)) - Sqrt[1 - a^2*x^2]/(24*a*c*(c -
a*c*x)^(5/2)) - Sqrt[1 - a^2*x^2]/(32*a*c^2*(c - a*c*x)^(3/2)) - ArcTanh[(S
qrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])]/(32*Sqrt[2]*a*c^(7/2))
```

Rubi [A] time = 0.130479, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6127, 663, 673, 661, 208}

$$-\frac{\sqrt{1-a^2x^2}}{32ac^2(c-acx)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{32\sqrt{2}ac^{7/2}}-\frac{\sqrt{1-a^2x^2}}{24ac(c-acx)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{3a(c-acx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^(7/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(3*a*(c - a*c*x)^(7/2)) - Sqrt[1 - a^2*x^2]/(24*a*c*(c -
a*c*x)^(5/2)) - Sqrt[1 - a^2*x^2]/(32*a*c^2*(c - a*c*x)^(3/2)) - ArcTanh[(S
qrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])]/(32*Sqrt[2]*a*c^(7/2))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{9/2}}\,dx \\
&= \frac{\sqrt{1-a^2x^2}}{3a(c-acx)^{7/2}} - \frac{\int \frac{1}{(c-acx)^{5/2}\sqrt{1-a^2x^2}}\,dx}{6c} \\
&= \frac{\sqrt{1-a^2x^2}}{3a(c-acx)^{7/2}} - \frac{\sqrt{1-a^2x^2}}{24ac(c-acx)^{5/2}} - \frac{\int \frac{1}{(c-acx)^{3/2}\sqrt{1-a^2x^2}}\,dx}{16c^2} \\
&= \frac{\sqrt{1-a^2x^2}}{3a(c-acx)^{7/2}} - \frac{\sqrt{1-a^2x^2}}{24ac(c-acx)^{5/2}} - \frac{\sqrt{1-a^2x^2}}{32ac^2(c-acx)^{3/2}} - \frac{\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{64c^3} \\
&= \frac{\sqrt{1-a^2x^2}}{3a(c-acx)^{7/2}} - \frac{\sqrt{1-a^2x^2}}{24ac(c-acx)^{5/2}} - \frac{\sqrt{1-a^2x^2}}{32ac^2(c-acx)^{3/2}} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)}{32c^2} \\
&= \frac{\sqrt{1-a^2x^2}}{3a(c-acx)^{7/2}} - \frac{\sqrt{1-a^2x^2}}{24ac(c-acx)^{5/2}} - \frac{\sqrt{1-a^2x^2}}{32ac^2(c-acx)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{32\sqrt{2}ac^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.01961, size = 57, normalized size = 0.36

$$\frac{(ax+1)^{3/2}\sqrt{c-acx}\,\mathrm{Hypergeometric2F1}\left(\frac{3}{2},4,\frac{5}{2},\frac{1}{2}(ax+1)\right)}{24ac^4\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^(7/2),x]
```

```
[Out] ((1 + a*x)^(3/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[3/2, 4, 5/2, (1 + a*x)/2
])/(24*a*c^4*Sqrt[1 - a*x])
```

Maple [A] time = 0.105, size = 208, normalized size = 1.3

$$\frac{1}{192\,(ax-1)^4\,a}\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}\left(3\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^3a^3c-9\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(7/2),x)
```

```
[Out] 1/192*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(9/2)*(3*2^(1/2)*arctanh(1/2*
(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^3*a^3*c-9*2^(1/2)*arctanh(1/2*(c*(a*x+
1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-6*x^2*a^2*(c*(a*x+1))^(1/2)*c^(1/2)+9*
2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c+20*x*a*(c*(a*x
+1))^(1/2)*c^(1/2)-3*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))
*c+50*(c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)^4/(c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}(-acx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^(7/2)), x)
```

Fricas [A] time = 2.14033, size = 822, normalized size = 5.24

$$\left[\frac{3\sqrt{2}\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)-4\left(3\,a^2x^2-10\,ax-25\right)\sqrt{-a^2x}}{384\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas"
)
```

```
[Out] [1/384*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*log
(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt
(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^2*x^2 - 10*a*x - 25)*sqrt(-a^2*x
^2 + 1)*sqrt(-a*c*x + c))/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*
a^2*c^4*x + a*c^4), -1/192*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*
a*x + 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-
c)/(a^2*c*x^2 - c)) + 2*(3*a^2*x^2 - 10*a*x - 25)*sqrt(-a^2*x^2 + 1)*sqrt(-
a*c*x + c))/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*
c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-c\left(ax-1\right)\right)^{\frac{7}{2}}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(a*x - 1))**(7/2)*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [A] time = 1.31744, size = 124, normalized size = 0.79

$$\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}c^2}+\frac{2\left(3\left(acx+c\right)^{\frac{5}{2}}-16\left(acx+c\right)^{\frac{3}{2}}c-12\sqrt{acx+c}c^2\right)}{\left(acx-c\right)^3c^2}}{192\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/192*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c^2
) + 2*(3*(a*c*x + c)^(5/2) - 16*(a*c*x + c)^(3/2)*c - 12*sqrt(a*c*x + c)*c^
2)/((a*c*x - c)^3*c^2))/(a*abs(c))
```

3.235 $\int e^{2\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=40

$$\frac{2(c-acx)^{9/2}}{9ac}-\frac{4(c-acx)^{7/2}}{7a}$$

```
[Out] (-4*(c - a*c*x)^(7/2))/(7*a) + (2*(c - a*c*x)^(9/2))/(9*a*c)
```

Rubi [A] time = 0.0477566, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 21, 43}

$$\frac{2(c-acx)^{9/2}}{9ac}-\frac{4(c-acx)^{7/2}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (-4*(c - a*c*x)^(7/2))/(7*a) + (2*(c - a*c*x)^(9/2))/(9*a*c)
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx&=\int\frac{(1+ax)(c-acx)^{7/2}}{1-ax}\,dx\\&=c\int(1+ax)(c-acx)^{5/2}\,dx\\&=c\int\left(2(c-acx)^{5/2}-\frac{(c-acx)^{7/2}}{c}\right)dx\\&=-\frac{4(c-acx)^{7/2}}{7a}+\frac{2(c-acx)^{9/2}}{9ac}\end{aligned}$$

Mathematica [A] time = 0.0401507, size = 34, normalized size = 0.85

$$\frac{2c^3(ax-1)^3(7ax+11)\sqrt{c-acx}}{63a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (2*c^3*(-1 + a*x)^3*(11 + 7*a*x)*Sqrt[c - a*c*x])/(63*a)
```

Maple [A] time = 0.033, size = 21, normalized size = 0.5

$$-\frac{14\,ax+22}{63\,a}(-acx+c)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(7/2),x)
```

```
[Out] -2/63*(-a*c*x+c)^(7/2)*(7*a*x+11)/a
```

Maxima [A] time = 0.95692, size = 43, normalized size = 1.08

$$\frac{2\left(7\,(-acx+c)^{\frac{9}{2}}-18\,(-acx+c)^{\frac{7}{2}}c\right)}{63\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/63*(7*(-a*c*x + c)^(9/2) - 18*(-a*c*x + c)^(7/2)*c)/(a*c)
```

Fricas [A] time = 2.26179, size = 131, normalized size = 3.28

$$\frac{2\left(7\,a^4c^3x^4-10\,a^3c^3x^3-12\,a^2c^3x^2+26\,ac^3x-11\,c^3\right)\sqrt{-acx+c}}{63\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*a^4*c^3*x^4 - 10*a^3*c^3*x^3 - 12*a^2*c^3*x^2 + 26*a*c^3*x - 11*c^3
)*sqrt(-a*c*x + c)/a
```

Sympy [A] time = 29.9103, size = 172, normalized size = 4.3

$$c^3\left(\begin{cases}\sqrt{c}x & \text{for } a=0\\ 0 & \text{for } c=0\\ -\frac{2(-acx+c)^{\frac{3}{2}}}{3ac} & \text{otherwise}\end{cases}\right)-\frac{2c\left(-\frac{c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a}+\frac{2\left(\frac{c^2(-acx+c)^{\frac{3}{2}}}{3}-\frac{2c(-acx+c)^{\frac{5}{2}}}{5}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a}+\frac{2\left(-\frac{c^3(-acx+c)^{\frac{3}{2}}}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(7/2),x)
```

```
[Out] c**3*Piecewise((sqrt(c)*x, Eq(a, 0)), (0, Eq(c, 0)), (-2*(-a*c*x + c)**(3/2
)/(3*a*c), True)) - 2*c*(-c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**(5/2)/5)/
a + 2*(c**2*(-a*c*x + c)**(3/2)/3 - 2*c*(-a*c*x + c)**(5/2)/5 + (-a*c*x + c
)**(7/2)/7)/a + 2*(-c**3*(-a*c*x + c)**(3/2)/3 + 3*c**2*(-a*c*x + c)**(5/2)
/5 - 3*c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/(a*c)
```

Giac [B] time = 1.18852, size = 234, normalized size = 5.85

$$-\frac{2\left(45\,(acx-c)^3\sqrt{-acx+c}+126\,(acx-c)^2\sqrt{-acx+c}c+21\left(3\,(acx-c)^2\sqrt{-acx+c}-5\,(-acx+c)^{\frac{3}{2}}c\right)c-\frac{35\,(acx-c)^4\sqrt{-}}{}\right.}{315\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -2/315*(45*(a*c*x - c)^3*sqrt(-a*c*x + c) + 126*(a*c*x - c)^2*sqrt(-a*c*x +
 c)*c + 21*(3*(a*c*x - c)^2*sqrt(-a*c*x + c) - 5*(-a*c*x + c)^(3/2)*c)*c -
(35*(a*c*x - c)^4*sqrt(-a*c*x + c) + 135*(a*c*x - c)^3*sqrt(-a*c*x + c)*c +
 189*(a*c*x - c)^2*sqrt(-a*c*x + c)*c^2 - 105*(-a*c*x + c)^(3/2)*c^3)/c)/a
```

3.236 $\int e^{2\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=40

$$\frac{2(c-acx)^{7/2}}{7ac}-\frac{4(c-acx)^{5/2}}{5a}$$

```
[Out] (-4*(c - a*c*x)^(5/2))/(5*a) + (2*(c - a*c*x)^(7/2))/(7*a*c)
```

Rubi [A] time = 0.0472998, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 21, 43}

$$\frac{2(c-acx)^{7/2}}{7ac}-\frac{4(c-acx)^{5/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (-4*(c - a*c*x)^(5/2))/(5*a) + (2*(c - a*c*x)^(7/2))/(7*a*c)
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx&=\int\frac{(1+ax)(c-acx)^{5/2}}{1-ax}\,dx\\&=c\int(1+ax)(c-acx)^{3/2}\,dx\\&=c\int\left(2(c-acx)^{3/2}-\frac{(c-acx)^{5/2}}{c}\right)dx\\&=-\frac{4(c-acx)^{5/2}}{5a}+\frac{2(c-acx)^{7/2}}{7ac}\end{aligned}$$

Mathematica [A] time = 0.0366126, size = 34, normalized size = 0.85

$$-\frac{2c^2(ax-1)^2(5ax+9)\sqrt{c-acx}}{35a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (-2*c^2*(-1 + a*x)^2*(9 + 5*a*x)*Sqrt[c - a*c*x])/(35*a)
```

Maple [A] time = 0.028, size = 21, normalized size = 0.5

$$-\frac{10\,ax+18}{35\,a}(-acx+c)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(5/2),x)
```

```
[Out] -2/35*(-a*c*x+c)^(5/2)*(5*a*x+9)/a
```

Maxima [A] time = 0.947635, size = 43, normalized size = 1.08

$$\frac{2\left(5\,(-acx+c)^{\frac{7}{2}}-14\,(-acx+c)^{\frac{5}{2}}c\right)}{35\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*(-a*c*x + c)^(7/2) - 14*(-a*c*x + c)^(5/2)*c)/(a*c)
```

Fricas [A] time = 2.22103, size = 104, normalized size = 2.6

$$-\frac{2\left(5\,a^3c^2x^3-a^2c^2x^2-13\,ac^2x+9\,c^2\right)\sqrt{-acx+c}}{35\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/35*(5*a^3*c^2*x^3 - a^2*c^2*x^2 - 13*a*c^2*x + 9*c^2)*sqrt(-a*c*x + c)/a
```

Sympy [A] time = 12.5871, size = 76, normalized size = 1.9

$$c^2\left(\begin{cases}\sqrt{c}x & \text{for } a=0\\ 0 & \text{for } c=0\\ -\frac{2(-acx+c)^{\frac{3}{2}}}{3ac} & \text{otherwise}\end{cases}\right)+\frac{2\left(\frac{c^2(-acx+c)^{\frac{3}{2}}}{3}-\frac{2c(-acx+c)^{\frac{5}{2}}}{5}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(5/2),x)
```

```
[Out] c**2*Piecewise((sqrt(c)*x, Eq(a, 0)), (0, Eq(c, 0)), (-2*(-a*c*x + c)**(3/2
)/(3*a*c), True)) + 2*(c**2*(-a*c*x + c)**(3/2)/3 - 2*c*(-a*c*x + c)**(5/2)
/5 + (-a*c*x + c)**(7/2)/7)/(a*c)
```

Giac [B] time = 1.19518, size = 108, normalized size = 2.7

$$-\frac{2\left(35\,(-acx+c)^{\frac{3}{2}}c+\frac{15\,(acx-c)^{3}\sqrt{-acx+c}+42\,(acx-c)^{2}\sqrt{-acx+c}c-35\,(-acx+c)^{\frac{3}{2}}c^{2}}{c}\right)}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/105*(35*(-a*c*x + c)^(3/2)*c + (15*(a*c*x - c)^3*sqrt(-a*c*x + c) + 42*(
a*c*x - c)^2*sqrt(-a*c*x + c)*c - 35*(-a*c*x + c)^(3/2)*c^2)/c)/a
```

3.237 $\int e^{2\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=40

$$\frac{2(c-acx)^{5/2}}{5ac}-\frac{4(c-acx)^{3/2}}{3a}$$

```
[Out] (-4*(c - a*c*x)^(3/2))/(3*a) + (2*(c - a*c*x)^(5/2))/(5*a*c)
```

Rubi [A] time = 0.0491684, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 21, 43}

$$\frac{2(c-acx)^{5/2}}{5ac}-\frac{4(c-acx)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (-4*(c - a*c*x)^(3/2))/(3*a) + (2*(c - a*c*x)^(5/2))/(5*a*c)
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx &= \int \frac{(1+ax)(c-acx)^{3/2}}{1-ax}\,dx\\ &= c\int (1+ax)\sqrt{c-acx}\,dx\\ &= c\int \left(2\sqrt{c-acx}-\frac{(c-acx)^{3/2}}{c}\right)dx\\ &= -\frac{4(c-acx)^{3/2}}{3a}+\frac{2(c-acx)^{5/2}}{5ac}\end{aligned}$$

Mathematica [A] time = 0.0363297, size = 30, normalized size = 0.75

$$\frac{2c(ax-1)(3ax+7)\sqrt{c-acx}}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (2*c*(-1 + a*x)*(7 + 3*a*x)*Sqrt[c - a*c*x])/(15*a)
```

Maple [A] time = 0.035, size = 21, normalized size = 0.5

$$-\frac{6\,ax+14}{15\,a}\,(-acx+c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(3/2),x)
```

```
[Out] -2/15*(-a*c*x+c)^(3/2)*(3*a*x+7)/a
```

Maxima [A] time = 0.94798, size = 43, normalized size = 1.08

$$\frac{2\left(3\,(-acx+c)^{\frac{5}{2}}-10\,(-acx+c)^{\frac{3}{2}}c\right)}{15\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*(-a*c*x + c)^(5/2) - 10*(-a*c*x + c)^(3/2)*c)/(a*c)
```

Fricas [A] time = 2.20487, size = 74, normalized size = 1.85

$$\frac{2\left(3\,a^2cx^2+4\,acx-7\,c\right)\sqrt{-acx+c}}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*c*x^2 + 4*a*c*x - 7*c)*sqrt(-a*c*x + c)/a
```

Sympy [A] time = 12.3217, size = 58, normalized size = 1.45

$$c\left(\begin{cases}\sqrt{c}x & \text{for}\ a=0\\ 0 & \text{for}\ c=0\\ -\frac{2(-acx+c)^{\frac{3}{2}}}{3ac} & \text{otherwise}\end{cases}\right)+\frac{2\left(-\frac{c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(3/2),x)
```

```
[Out] c*Piecewise((sqrt(c)*x, Eq(a, 0)), (0, Eq(c, 0)), (-2*(-a*c*x + c)**(3/2)/(
3*a*c), True)) + 2*(-c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**(5/2)/5)/(a*c)
```

Giac [A] time = 1.19049, size = 76, normalized size = 1.9

$$-\frac{2\left(5\,(-acx+c)^{\frac{3}{2}}-\frac{3\,(acx-c)^{2}\sqrt{-acx+c}-5\,(-acx+c)^{\frac{3}{2}}c}{c}\right)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/15*(5*(-a*c*x + c)^(3/2) - (3*(a*c*x - c)^2*sqrt(-a*c*x + c) - 5*(-a*c*x
 + c)^(3/2)*c)/c)/a
```

3.238 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=38

$$\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}$$

```
[Out] (-4*Sqrt[c - a*c*x])/a + (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Rubi [A] time = 0.0428272, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 21, 43}

$$\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x],x]

[Out] (-4*Sqrt[c - a*c*x])/a + (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx &= \int \frac{(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\
&= c\int \frac{1+ax}{\sqrt{c-acx}}\,dx\\
&= c\int \left(\frac{2}{\sqrt{c-acx}}-\frac{\sqrt{c-acx}}{c}\right)dx\\
&= -\frac{4\sqrt{c-acx}}{a}+\frac{2(c-acx)^{3/2}}{3ac}
\end{aligned}$$

Mathematica [A] time = 0.0258122, size = 23, normalized size = 0.61

$$-\frac{2(ax+5)\sqrt{c-acx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (-2*(5 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Maple [A] time = 0.038, size = 20, normalized size = 0.5

$$-\frac{2\,ax+10}{3\,a}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/3*(-a*c*x+c)^(1/2)*(a*x+5)/a
```

Maxima [A] time = 0.952638, size = 41, normalized size = 1.08

$$\frac{2\left((-acx+c)^{\frac{3}{2}}-6\,\sqrt{-acx+c}c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

Fricas [A] time = 2.16869, size = 47, normalized size = 1.24

$$-\frac{2\,\sqrt{-acx+c}(ax+5)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-a*c*x + c)*(a*x + 5)/a
```

Sympy [A] time = 3.80534, size = 31, normalized size = 0.82

$$-\frac{2\left(2c\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(1/2),x)
```

```
[Out] -2*(2*c*sqrt(-a*c*x + c) - (-a*c*x + c)**(3/2)/3)/(a*c)
```

Giac [A] time = 1.19106, size = 41, normalized size = 1.08

$$\frac{2\left((-acx+c)^{\frac{3}{2}} - 6\sqrt{-acx+c}c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

3.239 $\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=36

$$\frac{2\sqrt{c-acx}}{ac}+\frac{4}{a\sqrt{c-acx}}$$

```
[Out] 4/(a*Sqrt[c - a*c*x]) + (2*Sqrt[c - a*c*x])/(a*c)
```

Rubi [A] time = 0.0467426, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 21, 43}

$$\frac{2\sqrt{c-acx}}{ac}+\frac{4}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] 4/(a*Sqrt[c - a*c*x]) + (2*Sqrt[c - a*c*x])/(a*c)
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= \int \frac{1+ax}{(1-ax)\sqrt{c-acx}}\,dx\\
&= c\int \frac{1+ax}{(c-acx)^{3/2}}\,dx\\
&= c\int \left(\frac{2}{(c-acx)^{3/2}}-\frac{1}{c\sqrt{c-acx}}\right)dx\\
&= \frac{4}{a\sqrt{c-acx}}+\frac{2\sqrt{c-acx}}{ac}
\end{aligned}$$

Mathematica [A] time = 0.0231416, size = 21, normalized size = 0.58

$$\frac{6-2ax}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (6 - 2*a*x)/(a*Sqrt[c - a*c*x])
```

Maple [A] time = 0.033, size = 20, normalized size = 0.6

$$-2\,\frac{ax-3}{\sqrt{-acx+c}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(1/2),x)
```

```
[Out] -2*(a*x-3)/(-a*c*x+c)^(1/2)/a
```

Maxima [A] time = 0.949968, size = 41, normalized size = 1.14

$$\frac{2\left(\sqrt{-acx+c}+\frac{2c}{\sqrt{-acx+c}}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(-a*c*x + c) + 2*c/sqrt(-a*c*x + c))/(a*c)
```

Fricas [A] time = 2.16841, size = 62, normalized size = 1.72

$$\frac{2\sqrt{-acx+c}(ax-3)}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-a*c*x + c)*(a*x - 3)/(a^2*c*x - a*c)
```

Sympy [A] time = 29.8365, size = 48, normalized size = 1.33

$$\begin{cases} \frac{\frac{2}{\sqrt{-acx+c}}-\frac{2\left(-\frac{c}{\sqrt{-acx+c}}-\sqrt{-acx+c}\right)}{c}}{a} & \text{for}\ a\neq 0 \\ \frac{x}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Piecewise(((2/sqrt(-a*c*x + c) - 2*(-c/sqrt(-a*c*x + c) - sqrt(-a*c*x + c))
/c)/a, Ne(a, 0)), (x/sqrt(c), True))
```

Giac [A] time = 1.1747, size = 43, normalized size = 1.19

$$\frac{4}{\sqrt{-acx + c}a} + \frac{2\sqrt{-acx + c}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 4/(sqrt(-a*c*x + c)*a) + 2*sqrt(-a*c*x + c)/(a*c)
```

3.240 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=38

$$\frac{4}{3a(c-acx)^{3/2}}-\frac{2}{ac\sqrt{c-acx}}$$

```
[Out] 4/(3*a*(c - a*c*x)^(3/2)) - 2/(a*c*Sqrt[c - a*c*x])
```

Rubi [A] time = 0.0477366, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 21, 43}

$$\frac{4}{3a(c-acx)^{3/2}}-\frac{2}{ac\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] 4/(3*a*(c - a*c*x)^(3/2)) - 2/(a*c*Sqrt[c - a*c*x])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx &= \int \frac{1+ax}{(1-ax)(c-acx)^{3/2}}\,dx\\
&= c\int \frac{1+ax}{(c-acx)^{5/2}}\,dx\\
&= c\int \left(\frac{2}{(c-acx)^{5/2}}-\frac{1}{c(c-acx)^{3/2}}\right)dx\\
&= \frac{4}{3a(c-acx)^{3/2}}-\frac{2}{ac\sqrt{c-acx}}
\end{aligned}$$

Mathematica [A] time = 0.0434728, size = 34, normalized size = 0.89

$$\frac{2(3ax-1)\sqrt{c-acx}}{3ac^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (2*(-1 + 3*a*x)*Sqrt[c - a*c*x])/(3*a*c^2*(-1 + a*x)^2)
```

Maple [A] time = 0.037, size = 21, normalized size = 0.6

$$\frac{6\,ax-2}{3\,a}\left(-acx+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(3/2),x)
```

```
[Out] 2/3*(3*a*x-1)/(-a*c*x+c)^(3/2)/a
```

Maxima [A] time = 0.94244, size = 35, normalized size = 0.92

$$\frac{2\,(3\,acx-c)}{3\,(-acx+c)^{\frac{3}{2}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*a*c*x - c)/((-a*c*x + c)^(3/2)*a*c)
```

Fricas [A] time = 2.20809, size = 95, normalized size = 2.5

$$\frac{2\,\sqrt{-acx+c}(3\,ax-1)}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-a*c*x + c)*(3*a*x - 1)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [A] time = 32.4306, size = 29, normalized size = 0.76

$$\frac{4}{3a\,(-acx+c)^{\frac{3}{2}}}-\frac{2}{ac\sqrt{-acx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a*c*x+c)**(3/2),x)
```

```
[Out] 4/(3*a*(-a*c*x + c)**(3/2)) - 2/(a*c*sqrt(-a*c*x + c))
```

Giac [A] time = 1.20962, size = 49, normalized size = 1.29

$$-\frac{2\,(3\,acx - c)}{3\,(acx - c)\sqrt{-acx + c}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*a*c*x - c)/((a*c*x - c)*sqrt(-a*c*x + c)*a*c)
```

3.241 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=40

$$\frac{4}{5a(c-acx)^{5/2}}-\frac{2}{3ac(c-acx)^{3/2}}$$

```
[Out] 4/(5*a*(c - a*c*x)^(5/2)) - 2/(3*a*c*(c - a*c*x)^(3/2))
```

Rubi [A] time = 0.0491552, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 21, 43}

$$\frac{4}{5a(c-acx)^{5/2}}-\frac{2}{3ac(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] 4/(5*a*(c - a*c*x)^(5/2)) - 2/(3*a*c*(c - a*c*x)^(3/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx &= \int \frac{1+ax}{(1-ax)(c-acx)^{5/2}}\,dx\\ &= c\int \frac{1+ax}{(c-acx)^{7/2}}\,dx\\ &= c\int \left(\frac{2}{(c-acx)^{7/2}}-\frac{1}{c(c-acx)^{5/2}}\right)dx\\ &= \frac{4}{5a(c-acx)^{5/2}}-\frac{2}{3ac(c-acx)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0542111, size = 34, normalized size = 0.85

$$-\frac{2(5ax+1)\sqrt{c-acx}}{15ac^3(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] (-2*(1 + 5*a*x)*Sqrt[c - a*c*x])/(15*a*c^3*(-1 + a*x)^3)
```

Maple [A] time = 0.033, size = 21, normalized size = 0.5

$$\frac{10\,ax+2}{15\,a}(-acx+c)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(5/2),x)
```

```
[Out] 2/15*(5*a*x+1)/(-a*c*x+c)^(5/2)/a
```

Maxima [A] time = 0.945715, size = 32, normalized size = 0.8

$$\frac{2\,(5\,acx+c)}{15\,(-acx+c)^{\frac{5}{2}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*a*c*x + c)/((-a*c*x + c)^(5/2)*a*c)
```

Fricas [A] time = 2.08208, size = 119, normalized size = 2.98

$$-\frac{2\sqrt{-acx+c}(5\,ax+1)}{15\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*sqrt(-a*c*x + c)*(5*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3
*x - a*c^3)
```

Sympy [A] time = 73.4389, size = 31, normalized size = 0.78

$$\frac{4}{5a\,(-acx+c)^{\frac{5}{2}}}-\frac{2}{3ac\,(-acx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a*c*x+c)**(5/2),x)
```

```
[Out] 4/(5*a*(-a*c*x + c)**(5/2)) - 2/(3*a*c*(-a*c*x + c)**(3/2))
```

Giac [A] time = 1.20757, size = 46, normalized size = 1.15

$$\frac{2\,(5\,acx + c)}{15\,(acx - c)^2\sqrt{-acx + c}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*(5*a*c*x + c)/((a*c*x - c)^2*sqrt(-a*c*x + c)*a*c)
```

3.242 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=40

$$\frac{4}{7a(c-acx)^{7/2}}-\frac{2}{5ac(c-acx)^{5/2}}$$

```
[Out] 4/(7*a*(c - a*c*x)^(7/2)) - 2/(5*a*c*(c - a*c*x)^(5/2))
```

Rubi [A] time = 0.0517986, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 21, 43}

$$\frac{4}{7a(c-acx)^{7/2}}-\frac{2}{5ac(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] 4/(7*a*(c - a*c*x)^(7/2)) - 2/(5*a*c*(c - a*c*x)^(5/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx &= \int \frac{1+ax}{(1-ax)(c-acx)^{7/2}}\,dx\\
&= c\int \frac{1+ax}{(c-acx)^{9/2}}\,dx\\
&= c\int \left(\frac{2}{(c-acx)^{9/2}}-\frac{1}{c(c-acx)^{7/2}}\right)dx\\
&= \frac{4}{7a(c-acx)^{7/2}}-\frac{2}{5ac(c-acx)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0524085, size = 34, normalized size = 0.85

$$\frac{2(7ax+3)\sqrt{c-acx}}{35ac^4(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] (2*(3 + 7*a*x)*Sqrt[c - a*c*x])/(35*a*c^4*(-1 + a*x)^4)
```

Maple [A] time = 0.031, size = 21, normalized size = 0.5

$$\frac{14\,ax+6}{35\,a}(-acx+c)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(7/2),x)
```

```
[Out] 2/35*(7*a*x+3)/(-a*c*x+c)^(7/2)/a
```

Maxima [A] time = 0.954384, size = 35, normalized size = 0.88

$$\frac{2\,(7\,acx+3\,c)}{35\,(-acx+c)^{\frac{7}{2}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/35*(7*a*c*x + 3*c)/((-a*c*x + c)^(7/2)*a*c)
```

Fricas [B] time = 2.15597, size = 139, normalized size = 3.48

$$\frac{2\sqrt{-acx+c}(7\,ax+3)}{35\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*sqrt(-a*c*x + c)*(7*a*x + 3)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*
x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [A] time = 54.7369, size = 31, normalized size = 0.78

$$\frac{4}{7a\,(-acx+c)^{\frac{7}{2}}}-\frac{2}{5ac\,(-acx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a*c*x+c)**(7/2),x)
```

```
[Out] 4/(7*a*(-a*c*x + c)**(7/2)) - 2/(5*a*c*(-a*c*x + c)**(5/2))
```

Giac [A] time = 1.16364, size = 49, normalized size = 1.22

$$-\frac{2\,(7\,acx+3\,c)}{35\,(acx-c)^3\sqrt{-acx+c}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -2/35*(7*a*c*x + 3*c)/((a*c*x - c)^3*sqrt(-a*c*x + c)*a*c)
```

3.243 $\int e^{3\tanh^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=141

$$\frac{256c^7\left(1-a^2x^2\right)^{5/2}}{1155a(c-acx)^{5/2}}+\frac{64c^6\left(1-a^2x^2\right)^{5/2}}{231a(c-acx)^{3/2}}+\frac{8c^5\left(1-a^2x^2\right)^{5/2}}{33a\sqrt{c-acx}}+\frac{2c^4\left(1-a^2x^2\right)^{5/2}\sqrt{c-acx}}{11a}$$

```
[Out] (256*c^7*(1 - a^2*x^2)^(5/2))/(1155*a*(c - a*c*x)^(5/2)) + (64*c^6*(1 - a^2
*x^2)^(5/2))/(231*a*(c - a*c*x)^(3/2)) + (8*c^5*(1 - a^2*x^2)^(5/2))/(33*a*
Sqrt[c - a*c*x]) + (2*c^4*Sqrt[c - a*c*x]*(1 - a^2*x^2)^(5/2))/(11*a)
```

Rubi [A] time = 0.108989, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6127, 657, 649}

$$\frac{256c^7\left(1-a^2x^2\right)^{5/2}}{1155a(c-acx)^{5/2}}+\frac{64c^6\left(1-a^2x^2\right)^{5/2}}{231a(c-acx)^{3/2}}+\frac{8c^5\left(1-a^2x^2\right)^{5/2}}{33a\sqrt{c-acx}}+\frac{2c^4\left(1-a^2x^2\right)^{5/2}\sqrt{c-acx}}{11a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x)^(9/2),x]
```


```
[Out] (256*c^7*(1 - a^2*x^2)^(5/2))/(1155*a*(c - a*c*x)^(5/2)) + (64*c^6*(1 - a^2
*x^2)^(5/2))/(231*a*(c - a*c*x)^(3/2)) + (8*c^5*(1 - a^2*x^2)^(5/2))/(33*a*
Sqrt[c - a*c*x]) + (2*c^4*Sqrt[c - a*c*x]*(1 - a^2*x^2)^(5/2))/(11*a)
```


Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}(c-acx)^{9/2}\,dx = c^3\int (c-acx)^{3/2}\left(1-a^2x^2\right)^{3/2}\,dx$$
$$= \frac{2c^4\sqrt{c-acx}\left(1-a^2x^2\right)^{5/2}}{11a} + \frac{1}{11}\left(12c^4\right)\int \sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}\,dx$$
$$= \frac{8c^5\left(1-a^2x^2\right)^{5/2}}{33a\sqrt{c-acx}} + \frac{2c^4\sqrt{c-acx}\left(1-a^2x^2\right)^{5/2}}{11a} + \frac{1}{33}\left(32c^5\right)\int \frac{\left(1-a^2x^2\right)^{3/2}}{\sqrt{c-acx}}\,dx$$
$$= \frac{64c^6\left(1-a^2x^2\right)^{5/2}}{231a(c-acx)^{3/2}} + \frac{8c^5\left(1-a^2x^2\right)^{5/2}}{33a\sqrt{c-acx}} + \frac{2c^4\sqrt{c-acx}\left(1-a^2x^2\right)^{5/2}}{11a} + \frac{1}{231}\left(128c^6\right)\int \frac{(1-}{(c-}$$
$$= \frac{256c^7\left(1-a^2x^2\right)^{5/2}}{1155a(c-acx)^{5/2}} + \frac{64c^6\left(1-a^2x^2\right)^{5/2}}{231a(c-acx)^{3/2}} + \frac{8c^5\left(1-a^2x^2\right)^{5/2}}{33a\sqrt{c-acx}} + \frac{2c^4\sqrt{c-acx}\left(1-a^2x^2\right)^{5/2}}{11a}$$

Mathematica [A] time = 0.0450786, size = 62, normalized size = 0.44

$$-\frac{2c^4(ax+1)^{5/2}\left(105a^3x^3-455a^2x^2+755ax-533\right)\sqrt{c-acx}}{1155a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x)^(9/2),x]
```

```
[Out] (-2*c^4*(1 + a*x)^(5/2)*Sqrt[c - a*c*x]*(-533 + 755*a*x - 455*a^2*x^2 + 105
*a^3*x^3))/(1155*a*Sqrt[1 - a*x])
```

Maple [A] time = 0.039, size = 63, normalized size = 0.5

$$\frac{2\left(105\,x^3a^3-455\,a^2x^2+755\,ax-533\right)(ax+1)^4}{1155\,(ax-1)^3\,a}(-acx+c)^{\frac{9}{2}}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(9/2),x)
```

```
[Out] 2/1155*(a*x+1)^4*(105*a^3*x^3-455*a^2*x^2+755*a*x-533)*(-a*c*x+c)^(9/2)/a/(
a*x-1)^3/(-a^2*x^2+1)^(3/2)
```

Maxima [B] time = 1.15545, size = 343, normalized size = 2.43

$$-\frac{2\left(35\,a^6c^{\frac{9}{2}}x^6-175\,a^5c^{\frac{9}{2}}x^5+415\,a^4c^{\frac{9}{2}}x^4-741\,a^3c^{\frac{9}{2}}x^3+1482\,a^2c^{\frac{9}{2}}x^2-5928\,ac^{\frac{9}{2}}x-11856\,c^{\frac{9}{2}}\right)}{385\sqrt{ax+1}a} - \frac{2\left(7\,a^5c^{\frac{9}{2}}x^5-37\,a^4c^{\frac{9}{2}}x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="maxim
a")
```

```
[Out] -2/385*(35*a^6*c^(9/2)*x^6 - 175*a^5*c^(9/2)*x^5 + 415*a^4*c^(9/2)*x^4 - 74
1*a^3*c^(9/2)*x^3 + 1482*a^2*c^(9/2)*x^2 - 5928*a*c^(9/2)*x - 11856*c^(9/2)
```

```
)/(sqrt(a*x + 1)*a) - 2/21*(7*a^5*c^(9/2)*x^5 - 37*a^4*c^(9/2)*x^4 + 97*a^3
*c^(9/2)*x^3 - 215*a^2*c^(9/2)*x^2 + 860*a*c^(9/2)*x + 1720*c^(9/2))/(sqrt(
a*x + 1)*a) - 6/35*(5*a^4*c^(9/2)*x^4 - 29*a^3*c^(9/2)*x^3 + 93*a^2*c^(9/2)
*x^2 - 407*a*c^(9/2)*x - 814*c^(9/2))/(sqrt(a*x + 1)*a) - 2/5*(a^3*c^(9/2)*
x^3 - 7*a^2*c^(9/2)*x^2 + 43*a*c^(9/2)*x + 91*c^(9/2))/(sqrt(a*x + 1)*a)
```

Fricas [A] time = 2.21952, size = 204, normalized size = 1.45

$$\frac{2\left(105\,a^5c^4x^5-245\,a^4c^4x^4-50\,a^3c^4x^3+522\,a^2c^4x^2-311\,ac^4x-533\,c^4\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{1155\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="frica
s")
```

```
[Out] 2/1155*(105*a^5*c^4*x^5 - 245*a^4*c^4*x^4 - 50*a^3*c^4*x^3 + 522*a^2*c^4*x^
2 - 311*a*c^4*x - 533*c^4)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30233, size = 99, normalized size = 0.7

$$-\frac{2\left(512\,\sqrt{2}c^{\frac{7}{2}}+\frac{105\,(acx+c)^{\frac{11}{2}}-770\,(acx+c)^{\frac{9}{2}}c+1980\,(acx+c)^{\frac{7}{2}}c^2-1848\,(acx+c)^{\frac{5}{2}}c^3}{c^2}\right)c^2}{1155\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="giac"
)
```

```
[Out] -2/1155*(512*sqrt(2)*c^(7/2) + (105*(a*c*x + c)^(11/2) - 770*(a*c*x + c)^(9
/2)*c + 1980*(a*c*x + c)^(7/2)*c^2 - 1848*(a*c*x + c)^(5/2)*c^3)/c^2)*c^2/(
a*abs(c))
```

3.244 $\int e^{3\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=106

$$\frac{64c^6\left(1-a^2x^2\right)^{5/2}}{315a(c-acx)^{5/2}}+\frac{16c^5\left(1-a^2x^2\right)^{5/2}}{63a(c-acx)^{3/2}}+\frac{2c^4\left(1-a^2x^2\right)^{5/2}}{9a\sqrt{c-acx}}$$

```
[Out] (64*c^6*(1 - a^2*x^2)^(5/2))/(315*a*(c - a*c*x)^(5/2)) + (16*c^5*(1 - a^2*x
^2)^(5/2))/(63*a*(c - a*c*x)^(3/2)) + (2*c^4*(1 - a^2*x^2)^(5/2))/(9*a*Sqrt
[c - a*c*x])
```

Rubi [A] time = 0.087507, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6127, 657, 649}

$$\frac{64c^6\left(1-a^2x^2\right)^{5/2}}{315a(c-acx)^{5/2}}+\frac{16c^5\left(1-a^2x^2\right)^{5/2}}{63a(c-acx)^{3/2}}+\frac{2c^4\left(1-a^2x^2\right)^{5/2}}{9a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (64*c^6*(1 - a^2*x^2)^(5/2))/(315*a*(c - a*c*x)^(5/2)) + (16*c^5*(1 - a^2*x
^2)^(5/2))/(63*a*(c - a*c*x)^(3/2)) + (2*c^4*(1 - a^2*x^2)^(5/2))/(9*a*Sqrt
[c - a*c*x])
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx = c^3\int \sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}\,dx$$
$$= \frac{2c^4\left(1-a^2x^2\right)^{5/2}}{9a\sqrt{c-acx}} + \frac{1}{9}\left(8c^4\right)\int \frac{\left(1-a^2x^2\right)^{3/2}}{\sqrt{c-acx}}\,dx$$
$$= \frac{16c^5\left(1-a^2x^2\right)^{5/2}}{63a(c-acx)^{3/2}} + \frac{2c^4\left(1-a^2x^2\right)^{5/2}}{9a\sqrt{c-acx}} + \frac{1}{63}\left(32c^5\right)\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{3/2}}\,dx$$
$$= \frac{64c^6\left(1-a^2x^2\right)^{5/2}}{315a(c-acx)^{5/2}} + \frac{16c^5\left(1-a^2x^2\right)^{5/2}}{63a(c-acx)^{3/2}} + \frac{2c^4\left(1-a^2x^2\right)^{5/2}}{9a\sqrt{c-acx}}$$

Mathematica [A] time = 0.0358382, size = 54, normalized size = 0.51

$$\frac{2c^3(ax+1)^{5/2}\left(35a^2x^2-110ax+107\right)\sqrt{c-acx}}{315a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (2*c^3*(1 + a*x)^(5/2)*Sqrt[c - a*c*x]*(107 - 110*a*x + 35*a^2*x^2))/(315*a
*Sqrt[1 - a*x])
```

Maple [A] time = 0.032, size = 55, normalized size = 0.5

$$\frac{2\left(35\,a^2x^2-110\,ax+107\right)(ax+1)^4}{315\,(ax-1)^2\,a}(-acx+c)^{\frac{7}{2}}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(7/2),x)
```

```
[Out] 2/315*(a*x+1)^4*(35*a^2*x^2-110*a*x+107)*(-a*c*x+c)^(7/2)/a/(a*x-1)^2/(-a^2
*x^2+1)^(3/2)
```

Maxima [B] time = 1.13235, size = 284, normalized size = 2.68

$$\frac{2\left(5\,a^5c^{\frac{7}{2}}x^5-20\,a^4c^{\frac{7}{2}}x^4+41\,a^3c^{\frac{7}{2}}x^3-82\,a^2c^{\frac{7}{2}}x^2+328\,ac^{\frac{7}{2}}x+656\,c^{\frac{7}{2}}\right)}{45\sqrt{ax+1}a} + \frac{2\left(15\,a^4c^{\frac{7}{2}}x^4-66\,a^3c^{\frac{7}{2}}x^3+167\,a^2c^{\frac{7}{2}}x^2-66\right.}{35\sqrt{ax+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="maxim
a")
```

```
[Out] 2/45*(5*a^5*c^(7/2)*x^5 - 20*a^4*c^(7/2)*x^4 + 41*a^3*c^(7/2)*x^3 - 82*a^2*
c^(7/2)*x^2 + 328*a*c^(7/2)*x + 656*c^(7/2))/(sqrt(a*x + 1)*a) + 2/35*(15*a
^4*c^(7/2)*x^4 - 66*a^3*c^(7/2)*x^3 + 167*a^2*c^(7/2)*x^2 - 668*a*c^(7/2)*x
 - 1336*c^(7/2))/(sqrt(a*x + 1)*a) + 2/5*(3*a^3*c^(7/2)*x^3 - 16*a^2*c^(7/2
```

```
)*x^2 + 79*a*c^(7/2)*x + 158*c^(7/2))/(sqrt(a*x + 1)*a) + 2/3*(a^2*c^(7/2)*
x^2 - 10*a*c^(7/2)*x - 23*c^(7/2))/(sqrt(a*x + 1)*a)
```

Fricas [A] time = 2.22248, size = 177, normalized size = 1.67

$$-\frac{2\left(35\,a^4c^3x^4-40\,a^3c^3x^3-78\,a^2c^3x^2+104\,ac^3x+107\,c^3\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{315\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="frica
s")
```

```
[Out] -2/315*(35*a^4*c^3*x^4 - 40*a^3*c^3*x^3 - 78*a^2*c^3*x^2 + 104*a*c^3*x + 10
7*c^3)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27557, size = 82, normalized size = 0.77

$$-\frac{2\left(128\,\sqrt{2}c^{\frac{5}{2}}-\frac{35\,(acx+c)^{\frac{9}{2}}-180\,(acx+c)^{\frac{7}{2}}c+252\,(acx+c)^{\frac{5}{2}}c^2}{c^2}\right)c^2}{315\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="giac"
)
```

```
[Out] -2/315*(128*sqrt(2)*c^(5/2) - (35*(a*c*x + c)^(9/2) - 180*(a*c*x + c)^(7/2)
*c + 252*(a*c*x + c)^(5/2)*c^2)/c^2)*c^2/(a*abs(c))
```

3.245 $\int e^{3\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=71

$$\frac{8c^5\left(1-a^2x^2\right)^{5/2}}{35a(c-acx)^{5/2}}+\frac{2c^4\left(1-a^2x^2\right)^{5/2}}{7a(c-acx)^{3/2}}$$

```
[Out] (8*c^5*(1 - a^2*x^2)^(5/2))/(35*a*(c - a*c*x)^(5/2)) + (2*c^4*(1 - a^2*x^2)
^(5/2))/(7*a*(c - a*c*x)^(3/2))
```

Rubi [A] time = 0.0665109, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6127, 657, 649}

$$\frac{8c^5\left(1-a^2x^2\right)^{5/2}}{35a(c-acx)^{5/2}}+\frac{2c^4\left(1-a^2x^2\right)^{5/2}}{7a(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (8*c^5*(1 - a^2*x^2)^(5/2))/(35*a*(c - a*c*x)^(5/2)) + (2*c^4*(1 - a^2*x^2)
^(5/2))/(7*a*(c - a*c*x)^(3/2))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int e^{3\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx &= c^3\int\frac{\left(1-a^2x^2\right)^{3/2}}{\sqrt{c-acx}}\,dx\\ &= \frac{2c^4\left(1-a^2x^2\right)^{5/2}}{7a(c-acx)^{3/2}}+\frac{1}{7}\left(4c^4\right)\int\frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{3/2}}\,dx\\ &= \frac{8c^5\left(1-a^2x^2\right)^{5/2}}{35a(c-acx)^{5/2}}+\frac{2c^4\left(1-a^2x^2\right)^{5/2}}{7a(c-acx)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0300175, size = 46, normalized size = 0.65

$$-\frac{2c^2(ax+1)^{5/2}(5ax-9)\sqrt{c-acx}}{35a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (-2*c^2*(1 + a*x)^(5/2)*(-9 + 5*a*x)*Sqrt[c - a*c*x])/(35*a*Sqrt[1 - a*x])
```

Maple [A] time = 0.028, size = 47, normalized size = 0.7

$$\frac{2\,(5\,ax-9)\,(ax+1)^4}{35\,(ax-1)\,a}(-acx+c)^{\frac{5}{2}}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(5/2),x)
```

```
[Out] 2/35*(a*x+1)^4*(5*a*x-9)*(-a*c*x+c)^(5/2)/a/(a*x-1)/(-a^2*x^2+1)^(3/2)
```

Maxima [B] time = 1.14323, size = 221, normalized size = 3.11

$$-\frac{2\left(a^4c^{\frac{5}{2}}x^4-3\,a^3c^{\frac{5}{2}}x^3+6\,a^2c^{\frac{5}{2}}x^2-24\,ac^{\frac{5}{2}}x-48\,c^{\frac{5}{2}}\right)}{7\,\sqrt{ax+1}a}-\frac{2\left(3\,a^3c^{\frac{5}{2}}x^3-11\,a^2c^{\frac{5}{2}}x^2+44\,ac^{\frac{5}{2}}x+88\,c^{\frac{5}{2}}\right)}{5\,\sqrt{ax+1}a}-\frac{2\left(a^2c^{\frac{5}{2}}x^2-7\,ac^{\frac{5}{2}}x\right.}{\sqrt{ax+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/7*(a^4*c^(5/2)*x^4 - 3*a^3*c^(5/2)*x^3 + 6*a^2*c^(5/2)*x^2 - 24*a*c^(5/2
)*x - 48*c^(5/2))/(sqrt(a*x + 1)*a) - 2/5*(3*a^3*c^(5/2)*x^3 - 11*a^2*c^(5/
2)*x^2 + 44*a*c^(5/2)*x + 88*c^(5/2))/(sqrt(a*x + 1)*a) - 2*(a^2*c^(5/2)*x^
2 - 7*a*c^(5/2)*x - 14*c^(5/2))/(sqrt(a*x + 1)*a) - 2*(a*c^(5/2)*x + 3*c^(5
/2))/(sqrt(a*x + 1)*a)
```

Fricas [A] time = 2.21482, size = 142, normalized size = 2.

$$\frac{2\left(5\,a^3c^2x^3+a^2c^2x^2-13\,ac^2x-9\,c^2\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{35\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="frica
s")
```

```
[Out] 2/35*(5*a^3*c^2*x^3 + a^2*c^2*x^2 - 13*a*c^2*x - 9*c^2)*sqrt(-a^2*x^2 + 1)*
sqrt(-a*c*x + c)/(a^2*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(ax-1\right)\right)^{\frac{5}{2}}\left(ax+1\right)^{3}}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(5/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(5/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2),
x)
```

Giac [A] time = 1.23897, size = 63, normalized size = 0.89

$$-\frac{2\left(16\sqrt{2}c^{\frac{3}{2}}+\frac{5\left(acx+c\right)^{\frac{7}{2}}-14\left(acx+c\right)^{\frac{5}{2}}c}{c^{2}}\right)c^{2}}{35\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="giac"
)
```

```
[Out] -2/35*(16*sqrt(2)*c^(3/2) + (5*(a*c*x + c)^(7/2) - 14*(a*c*x + c)^(5/2)*c)/
c^2)*c^2/(a*abs(c))
```

3.246 $\int e^{3\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=35

$$\frac{2c^4\left(1-a^2x^2\right)^{5/2}}{5a(c-acx)^{5/2}}$$

```
[Out] (2*c^4*(1 - a^2*x^2)^(5/2))/(5*a*(c - a*c*x)^(5/2))
```

Rubi [A] time = 0.0498722, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6127, 649}

$$\frac{2c^4\left(1-a^2x^2\right)^{5/2}}{5a(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (2*c^4*(1 - a^2*x^2)^(5/2))/(5*a*(c - a*c*x)^(5/2))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int e^{3\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx &= c^3\int\frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{3/2}}\,dx\\ &= \frac{2c^4\left(1-a^2x^2\right)^{5/2}}{5a(c-acx)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0224341, size = 37, normalized size = 1.06

$$\frac{2(ax+1)^{5/2}(c-acx)^{3/2}}{5a(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (2*(1 + a*x)^(5/2)*(c - a*c*x)^(3/2))/(5*a*(1 - a*x)^(3/2))
```

Maple [A] time = 0.034, size = 34, normalized size = 1.

$$\frac{2 (ax+1)^4}{5a} (-acx+c)^{\frac{3}{2}} \left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(3/2),x)
```

```
[Out] 2/5*(a*x+1)^4*(-a*c*x+c)^(3/2)/a/(-a^2*x^2+1)^(3/2)
```

Maxima [B] time = 1.11083, size = 163, normalized size = 4.66

$$-\frac{2 c^{\frac{3}{2}}}{\sqrt{ax+1}a} + \frac{2 \left(a^3c^{\frac{3}{2}}x^3 - 2 a^2c^{\frac{3}{2}}x^2 + 8 ac^{\frac{3}{2}}x + 16 c^{\frac{3}{2}}\right)}{5 \sqrt{ax+1}a} + \frac{2 \left(a^2c^{\frac{3}{2}}x^2 - 4 ac^{\frac{3}{2}}x - 8 c^{\frac{3}{2}}\right)}{\sqrt{ax+1}a} + \frac{6 \left(ac^{\frac{3}{2}}x + 2 c^{\frac{3}{2}}\right)}{\sqrt{ax+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -2*c^(3/2)/(sqrt(a*x + 1)*a) + 2/5*(a^3*c^(3/2)*x^3 - 2*a^2*c^(3/2)*x^2 + 8
*a*c^(3/2)*x + 16*c^(3/2))/(sqrt(a*x + 1)*a) + 2*(a^2*c^(3/2)*x^2 - 4*a*c^(
3/2)*x - 8*c^(3/2))/(sqrt(a*x + 1)*a) + 6*(a*c^(3/2)*x + 2*c^(3/2))/(sqrt(a
*x + 1)*a)
```

Fricas [A] time = 2.24996, size = 108, normalized size = 3.09

$$-\frac{2 \left(a^2cx^2 + 2 acx + c\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{5 \left(a^2x - a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] -2/5*(a^2*c*x^2 + 2*a*c*x + c)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2*x -
 a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c (ax-1))^{\frac{3}{2}} (ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(3/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2),
x)
```

Giac [A] time = 1.17194, size = 46, normalized size = 1.31

$$-\frac{2\left(4\sqrt{2}\sqrt{c}-\frac{(acx+c)^{\frac{5}{2}}}{c^2}\right)c^2}{5a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] -2/5*(4*sqrt(2)*sqrt(c) - (a*c*x + c)^(5/2)/c^2)*c^2/(a*abs(c))
```

3.247 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=119

$$-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}-\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a}$$

```
[Out] (-4*c*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x]) - (2*c^2*(1 - a^2*x^2)^(3/2))/
(3*a*(c - a*c*x)^(3/2)) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*
x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/a
```

Rubi [A] time = 0.103845, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6127, 665, 661, 208}

$$-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}-\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*c*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x]) - (2*c^2*(1 - a^2*x^2)^(3/2))/
(3*a*(c - a*c*x)^(3/2)) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*
x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/a
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\sqrt{c-acx}\,dx &= c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{5/2}}\,dx\\
&= -\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}+\left(2c^2\right)\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{3/2}}\,dx\\
&= -\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}+(4c)\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}-\left(8ac^2\right)\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\
&= -\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0326463, size = 67, normalized size = 0.56

$$
-\frac{2\sqrt{c-acx}\left(\sqrt{ax+1}(ax+7)-6\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{3a\sqrt{1-ax}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(Sqrt[1 + a*x]*(7 + a*x) - 6*Sqrt[2]*ArcTanh[Sqrt[1 + a
*x]/Sqrt[2]]))/(3*a*Sqrt[1 - a*x])
```

Maple [A] time = 0.096, size = 95, normalized size = 0.8

$$
-\frac{2}{(3\,ax-3)\,a}\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}\left(6\,\sqrt{c}\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)-xa\sqrt{c\,(ax+1)}-7\,\sqrt{c\,(ax+1)}\right)\frac{1}{\sqrt{c\,(ax+1)}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/3*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(6*c^(1/2)*2^(1/2)*arctanh(1/2*(
c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-x*a*(c*(a*x+1))^(1/2)-7*(c*(a*x+1))^(1/2)
)/(a*x-1)/(c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{-acx+c}(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 2.2336, size = 518, normalized size = 4.35

$$\left[\frac{2\left(3\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,acx-2\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+\sqrt{-a^2x^2+1}\sqrt{-acx+c}(ax+7)\right)}{3\left(a^2x-a\right)}, \frac{2\left(6\sqrt{2}(ax-1)\sqrt{-c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] [2/3*(3*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqr
t(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + sq
rt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x + 7))/(a^2*x - a), 2/3*(6*sqrt(2)*(a
*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c
)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x + 7))/(a^2*x
- a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(ax-1\right)}\left(ax+1\right)^3}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A] time = 1.26187, size = 142, normalized size = 1.19

$$-\frac{2\left(\frac{6\sqrt{2}c^2\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+(acx+c)^{\frac{3}{2}}+6\sqrt{acx+c}c\right)}{3\,a|c|}+\frac{4\sqrt{2}\left(3\,c^2\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)+4\sqrt{-c}c^{\frac{3}{2}}\right)}{3\,a\sqrt{-c}|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] -2/3*(6*sqrt(2)*c^2*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) +
 (a*c*x + c)^(3/2) + 6*sqrt(a*c*x + c)*c)/(a*abs(c)) + 4/3*sqrt(2)*(3*c^2*a
rctan(sqrt(c)/sqrt(-c)) + 4*sqrt(-c)*c^(3/2))/(a*sqrt(-c)*abs(c))
```

3.248 $\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=115

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{a(c-acx)^{5/2}}+\frac{3\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}-\frac{3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a\sqrt{c}}$$

```
[Out] (3*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x]) + (c^2*(1 - a^2*x^2)^(3/2))/(a*(c
 - a*c*x)^(5/2)) - (3*Sqrt[2]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*
Sqrt[c - a*c*x])])/(a*Sqrt[c])
```

Rubi [A] time = 0.100717, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6127, 663, 665, 661, 208}

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{a(c-acx)^{5/2}}+\frac{3\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}-\frac{3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (3*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x]) + (c^2*(1 - a^2*x^2)^(3/2))/(a*(c
 - a*c*x)^(5/2)) - (3*Sqrt[2]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*
Sqrt[c - a*c*x])])/(a*Sqrt[c])
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx = c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{7/2}}\,dx$$
$$= \frac{c^2\left(1-a^2x^2\right)^{3/2}}{a(c-acx)^{5/2}} - \frac{1}{2}(3c)\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{3/2}}\,dx$$
$$= \frac{3\sqrt{1-a^2x^2}}{a\sqrt{c-acx}} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{a(c-acx)^{5/2}} - 3\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{3\sqrt{1-a^2x^2}}{a\sqrt{c-acx}} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{a(c-acx)^{5/2}} + (6ac)\,\mathrm{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$
$$= \frac{3\sqrt{1-a^2x^2}}{a\sqrt{c-acx}} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{a(c-acx)^{5/2}} - \frac{3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a\sqrt{c}}$$

Mathematica [C] time = 0.022388, size = 57, normalized size = 0.5

$$\frac{(ax+1)^{5/2}(c-acx)^{3/2}\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},\frac{1}{2}(ax+1)\right)}{10ac^2(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] ((1 + a*x)^(5/2)*(c - a*c*x)^(3/2)*Hypergeometric2F1[2, 5/2, 7/2, (1 + a*x)
/2])/(10*a*c^2*(1 - a*x)^(3/2))
```

Maple [A] time = 0.105, size = 127, normalized size = 1.1

$$\frac{1}{(ax-1)^2a}\sqrt{-a^2x^2+1}\sqrt{-c(ax-1)}\left(3\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c(ax+1)}\sqrt{2}}{\sqrt{c}}\right)xac - 2\,xa\sqrt{c(ax+1)}\sqrt{c} - 3\sqrt{2}\mathrm{Artanh}\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(1/2),x)
```

```
[Out] (-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(3*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1
/2)*2^(1/2)/c^(1/2))*x*a*c-2*x*a*(c*(a*x+1))^(1/2)*c^(1/2)-3*2^(1/2)*arctan
h(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+4*(c*(a*x+1))^(1/2)*c^(1/2))/c^(
3/2)/(a*x-1)^2/(c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*sqrt(-a*c*x + c)), x)
```

Fricas [A] time = 2.2702, size = 608, normalized size = 5.29

$$\left[-\frac{4\sqrt{-a^2x^2+1}\sqrt{-acx+c}(ax-2)-\frac{3\sqrt{2}\left(a^2cx^2-2\,acx+c\right)\log\left(-\frac{a^2x^2+2\,ax+\frac{2\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{\sqrt{c}}-3}{a^2x^2-2\,ax+1}\right)}{\sqrt{c}}}{2\left(a^3cx^2-2\,a^2cx+ac\right)},-\frac{3\sqrt{2}\left(a^2cx^2-2\,acx+c\right)\sqrt{-\frac{1}{c}}\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/2*(4*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x - 2) - 3*sqrt(2)*(a^2*c*x
^2 - 2*a*c*x + c)*log(-(a^2*x^2 + 2*a*x + 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt
(-a*c*x + c)/sqrt(c) - 3)/(a^2*x^2 - 2*a*x + 1))/sqrt(c))/(a^3*c*x^2 - 2*a^
2*c*x + a*c), -(3*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*sqrt(-1/c)*arctan(sqrt(
2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-1/c)/(a^2*x^2 - 1)) + 2*sqrt(-
a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x - 2))/(a^3*c*x^2 - 2*a^2*c*x + a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\sqrt{-c\,(ax-1)}\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)**3/(sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)), x
)
```

Giac [A] time = 1.26486, size = 95, normalized size = 0.83

$$\frac{\frac{3\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{acx+c}-\frac{2\sqrt{acx+c}c}{acx-c}}{a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] (3*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*sqrt
(a*c*x + c) - 2*sqrt(a*c*x + c)*c/(a*c*x - c))/(a*abs(c))
```

3.249 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=122

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{2a(c-acx)^{7/2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{4\sqrt{2}ac^{3/2}}-\frac{3\sqrt{1-a^2x^2}}{4a(c-acx)^{3/2}}$$

```
[Out] (-3*Sqrt[1 - a^2*x^2])/(4*a*(c - a*c*x)^(3/2)) + (c^2*(1 - a^2*x^2)^(3/2))/
(2*a*(c - a*c*x)^(7/2)) + (3*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*S
qrt[c - a*c*x])])/(4*Sqrt[2]*a*c^(3/2))
```

Rubi [A] time = 0.104767, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6127, 663, 661, 208}

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{2a(c-acx)^{7/2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{4\sqrt{2}ac^{3/2}}-\frac{3\sqrt{1-a^2x^2}}{4a(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a*c*x)^(3/2),x]
```


```
[Out] (-3*Sqrt[1 - a^2*x^2])/(4*a*(c - a*c*x)^(3/2)) + (c^2*(1 - a^2*x^2)^(3/2))/
(2*a*(c - a*c*x)^(7/2)) + (3*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*S
qrt[c - a*c*x])])/(4*Sqrt[2]*a*c^(3/2))
```


Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx &= c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{9/2}}\,dx\\
&= \frac{c^2\left(1-a^2x^2\right)^{3/2}}{2a(c-acx)^{7/2}}-\frac{1}{4}(3c)\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{5/2}}\,dx\\
&= -\frac{3\sqrt{1-a^2x^2}}{4a(c-acx)^{3/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{2a(c-acx)^{7/2}}+\frac{3\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{8c}\\
&= -\frac{3\sqrt{1-a^2x^2}}{4a(c-acx)^{3/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{2a(c-acx)^{7/2}}-\frac{1}{4}(3a)\,\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\
&= -\frac{3\sqrt{1-a^2x^2}}{4a(c-acx)^{3/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{2a(c-acx)^{7/2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{4\sqrt{2}ac^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.075281, size = 91, normalized size = 0.75

$$
\frac{\sqrt{c-acx}\left(10a^2x^2+8ax+3(ax-1)^2\sqrt{2ax+2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)-2\right)}{8ac^2(ax-1)^2\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (Sqrt[c - a*c*x]*(-2 + 8*a*x + 10*a^2*x^2 + 3*(-1 + a*x)^2*Sqrt[2 + 2*a*x]*
ArcTanh[Sqrt[1 + a*x]/Sqrt[2]]))/(8*a*c^2*(-1 + a*x)^2*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.109, size = 158, normalized size = 1.3

$$
-\frac{1}{8\,(ax-1)^3\,a}\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}\left(3\,\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^2a^2c-6\,\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2),x)
```

```
[Out] -1/8*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(5/2)*(3*2^(1/2)*arctanh(1/2*(
c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-6*2^(1/2)*arctanh(1/2*(c*(a*x+1
))^(1/2)*2^(1/2)/c^(1/2))*x*a*c+10*x*a*(c*(a*x+1))^(1/2)*c^(1/2)+3*2^(1/2)*
arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-2*(c*(a*x+1))^(1/2)*c^(1/2
))/(a*x-1)^3/(c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}(-acx+c)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(-a*c*x + c)^(3/2)), x)
```

Fricas [A] time = 2.34228, size = 703, normalized size = 5.76

$$\left[\frac{3\sqrt{2}\left(a^3x^3 - 3a^2x^2 + 3ax - 1\right)\sqrt{c}\log\left(-\frac{a^2cx^2 + 2acx - 2\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c} - 3c}{a^2x^2 - 2ax + 1}\right) - 4\sqrt{-a^2x^2+1}\sqrt{-acx+c}(5ax-1)}{16\left(a^4c^2x^3 - 3a^3c^2x^2 + 3a^2c^2x - ac^2\right)}, 3\sqrt{2}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/16*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(a^2*c*x^2
+ 2*a*c*x - 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a
^2*x^2 - 2*a*x + 1)) - 4*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(5*a*x - 1))/(
a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2), 1/8*(3*sqrt(2)*(a^3*x^3
 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-
a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c
)*(5*a*x - 1))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{(-c(ax-1))^{\frac{3}{2}}\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-c*(a*x - 1))**(3/2)*(-(a*x - 1)*(a*x + 1))**(3/2))
, x)
```

Giac [A] time = 1.37443, size = 99, normalized size = 0.81

$$-\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{2\left(5(acx+c)^{\frac{3}{2}} - 6\sqrt{acx+c}c\right)}{(acx-c)^2}}{8a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] -1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) - 2*(
5*(a*c*x + c)^(3/2) - 6*sqrt(a*c*x + c)*c)/(a*c*x - c)^2)/(a*abs(c))
```

3.250 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{9/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{16\sqrt{2}ac^{5/2}}-\frac{\sqrt{1-a^2x^2}}{4a(c-acx)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{16ac(c-acx)^{3/2}}$$

```
[Out] -Sqrt[1 - a^2*x^2]/(4*a*(c - a*c*x)^(5/2)) + Sqrt[1 - a^2*x^2]/(16*a*c*(c -
 a*c*x)^(3/2)) + (c^2*(1 - a^2*x^2)^(3/2))/(3*a*(c - a*c*x)^(9/2)) + ArcTan
h[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])]/(16*Sqrt[2]*a*c^(5
/2))
```

Rubi [A] time = 0.133522, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6127, 663, 673, 661, 208}

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{9/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{16\sqrt{2}ac^{5/2}}-\frac{\sqrt{1-a^2x^2}}{4a(c-acx)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{16ac(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a*c*x)^(5/2),x]
```


```
[Out] -Sqrt[1 - a^2*x^2]/(4*a*(c - a*c*x)^(5/2)) + Sqrt[1 - a^2*x^2]/(16*a*c*(c -
 a*c*x)^(3/2)) + (c^2*(1 - a^2*x^2)^(3/2))/(3*a*(c - a*c*x)^(9/2)) + ArcTan
h[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])]/(16*Sqrt[2]*a*c^(5
/2))
```


Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
```

```
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx &= c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{11/2}}\,dx\\
&= \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{9/2}}-\frac{1}{2}c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{7/2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4a(c-acx)^{5/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{9/2}}+\frac{\int \frac{1}{(c-acx)^{3/2}\sqrt{1-a^2x^2}}\,dx}{8c}\\
&= -\frac{\sqrt{1-a^2x^2}}{4a(c-acx)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{16ac(c-acx)^{3/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{9/2}}+\frac{\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{32c^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{4a(c-acx)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{16ac(c-acx)^{3/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{9/2}}-\frac{a\,\mathrm{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)}{16c}\\
&= -\frac{\sqrt{1-a^2x^2}}{4a(c-acx)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{16ac(c-acx)^{3/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{9/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{16\sqrt{2}ac^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0242673, size = 57, normalized size = 0.36

$$\frac{(ax+1)^{5/2}(c-acx)^{3/2}\mathrm{Hypergeometric2F1}\left(\frac{5}{2},4,\frac{7}{2},\frac{1}{2}(ax+1)\right)}{40ac^4(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] ((1 + a*x)^(5/2)*(c - a*c*x)^(3/2)*Hypergeometric2F1[5/2, 4, 7/2, (1 + a*x)
/2])/(40*a*c^4*(1 - a*x)^(3/2))
```

Maple [A] time = 0.112, size = 208, normalized size = 1.3

$$-\frac{1}{96\,(ax-1)^4\,a}\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}\left(3\,\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^3a^3c-9\,\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(5/2),x)
```

```
[Out] -1/96*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(7/2)*(3*2^(1/2)*arctanh(1/2*
(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^3*a^3*c-9*2^(1/2)*arctanh(1/2*(c*(a*x+
1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-6*x^2*a^2*(c*(a*x+1))^(1/2)*c^(1/2)+9*
2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c-44*x*a*(c*(a*x
```

```
+1))^(1/2)*c^(1/2)-3*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))
*c-14*(c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)^4/(c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}(-acx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(-a*c*x + c)^(5/2)), x)
```

Fricas [A] time = 2.27386, size = 817, normalized size = 5.2

$$\left[\frac{3\sqrt{2}\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3c}{a^2x^2-2ax+1}\right)+4\left(3a^2x^2+22ax+7\right)\sqrt{-a^2x^2}}{192\left(a^5c^3x^4-4a^4c^3x^3+6a^3c^3x^2-4a^2c^3x+ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/192*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*log
(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt
(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*(3*a^2*x^2 + 22*a*x + 7)*sqrt(-a^2*x^
2 + 1)*sqrt(-a*c*x + c))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a
^2*c^3*x + a*c^3), 1/96*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x
 + 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/
(a^2*c*x^2 - c)) + 2*(3*a^2*x^2 + 22*a*x + 7)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*
x + c))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{(-c(ax-1))^{\frac{5}{2}}\,(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-c*(a*x - 1))**(5/2)*(-(a*x - 1)*(a*x + 1))**(3/2))
, x)
```

Giac [A] time = 1.38714, size = 124, normalized size = 0.79

$$-\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}c}+\frac{2\left(3\,(acx+c)^{\frac{5}{2}}+16\,(acx+c)^{\frac{3}{2}}c-12\,\sqrt{acx+c}c^{2}\right)}{(acx-c)^{3}c}}{96\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/96*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c)
+ 2*(3*(a*c*x + c)^(5/2) + 16*(a*c*x + c)^(3/2)*c - 12*sqrt(a*c*x + c)*c^2)
/((a*c*x - c)^3*c))/(a*abs(c))
```

3.251 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=192

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4a(c-acx)^{11/2}}+\frac{3\sqrt{1-a^2x^2}}{256ac^2(c-acx)^{3/2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{256\sqrt{2}ac^{7/2}}-\frac{\sqrt{1-a^2x^2}}{8a(c-acx)^{7/2}}+\frac{\sqrt{1-a^2x^2}}{64ac(c-acx)^{5/2}}$$

```
[Out] -Sqrt[1 - a^2*x^2]/(8*a*(c - a*c*x)^(7/2)) + Sqrt[1 - a^2*x^2]/(64*a*c*(c -
 a*c*x)^(5/2)) + (3*Sqrt[1 - a^2*x^2])/(256*a*c^2*(c - a*c*x)^(3/2)) + (c^2
*(1 - a^2*x^2)^(3/2))/(4*a*(c - a*c*x)^(11/2)) + (3*ArcTanh[(Sqrt[c]*Sqrt[1
 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/(256*Sqrt[2]*a*c^(7/2))
```

Rubi [A] time = 0.159156, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6127, 663, 673, 661, 208}

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4a(c-acx)^{11/2}}+\frac{3\sqrt{1-a^2x^2}}{256ac^2(c-acx)^{3/2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{256\sqrt{2}ac^{7/2}}-\frac{\sqrt{1-a^2x^2}}{8a(c-acx)^{7/2}}+\frac{\sqrt{1-a^2x^2}}{64ac(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(8*a*(c - a*c*x)^(7/2)) + Sqrt[1 - a^2*x^2]/(64*a*c*(c -
 a*c*x)^(5/2)) + (3*Sqrt[1 - a^2*x^2])/(256*a*c^2*(c - a*c*x)^(3/2)) + (c^2
*(1 - a^2*x^2)^(3/2))/(4*a*(c - a*c*x)^(11/2)) + (3*ArcTanh[(Sqrt[c]*Sqrt[1
 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/(256*Sqrt[2]*a*c^(7/2))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
```

```
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx &= c^3 \int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{13/2}}\,dx \\
&= \frac{c^2\left(1-a^2x^2\right)^{3/2}}{4a(c-acx)^{11/2}} - \frac{1}{8}(3c)\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{9/2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{8a(c-acx)^{7/2}} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{4a(c-acx)^{11/2}} + \frac{\int \frac{1}{(c-acx)^{5/2}\sqrt{1-a^2x^2}}\,dx}{16c} \\
&= -\frac{\sqrt{1-a^2x^2}}{8a(c-acx)^{7/2}} + \frac{\sqrt{1-a^2x^2}}{64ac(c-acx)^{5/2}} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{4a(c-acx)^{11/2}} + \frac{3\int \frac{1}{(c-acx)^{3/2}\sqrt{1-a^2x^2}}\,dx}{128c^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{8a(c-acx)^{7/2}} + \frac{\sqrt{1-a^2x^2}}{64ac(c-acx)^{5/2}} + \frac{3\sqrt{1-a^2x^2}}{256ac^2(c-acx)^{3/2}} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{4a(c-acx)^{11/2}} + \frac{3\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{512c^3} \\
&= -\frac{\sqrt{1-a^2x^2}}{8a(c-acx)^{7/2}} + \frac{\sqrt{1-a^2x^2}}{64ac(c-acx)^{5/2}} + \frac{3\sqrt{1-a^2x^2}}{256ac^2(c-acx)^{3/2}} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{4a(c-acx)^{11/2}} - \frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{-2a^2c+a^2c}\right.}{256c^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{8a(c-acx)^{7/2}} + \frac{\sqrt{1-a^2x^2}}{64ac(c-acx)^{5/2}} + \frac{3\sqrt{1-a^2x^2}}{256ac^2(c-acx)^{3/2}} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{4a(c-acx)^{11/2}} + \frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{256\sqrt{2}ac^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0238101, size = 57, normalized size = 0.3

$$\frac{(ax+1)^{5/2}(c-acx)^{3/2}\,\text{Hypergeometric2F1}\left(\frac{5}{2},5,\frac{7}{2},\frac{1}{2}(ax+1)\right)}{80ac^5(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] ((1 + a*x)^(5/2)*(c - a*c*x)^(3/2)*Hypergeometric2F1[5/2, 5, 7/2, (1 + a*x)
/2])/(80*a*c^5*(1 - a*x)^(3/2))
```

Maple [A] time = 0.109, size = 258, normalized size = 1.3

$$-\frac{1}{512\,(ax-1)^5\,a}\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}\left(3\,\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^4a^4c-12\,\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(7/2),x)
```

```
[Out] -1/512*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(9/2)*(3*2^(1/2)*arctanh(1/2
*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^4*a^4*c-12*2^(1/2)*arctanh(1/2*(c*(a*
```

```
x+1))^(1/2)*2^(1/2)/c^(1/2))*x^3*a^3*c-6*x^3*a^3*(c*(a*x+1))^(1/2)*c^(1/2)+
18*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c+26*x^2*
a^2*(c*(a*x+1))^(1/2)*c^(1/2)-12*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1
/2)/c^(1/2))*x*a*c+158*x*a*(c*(a*x+1))^(1/2)*c^(1/2)+3*2^(1/2)*arctanh(1/2*
(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+78*(c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)^
5/(c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}(-acx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(-a*c*x + c)^(7/2)), x)
```

Fricas [A] time = 2.33018, size = 944, normalized size = 4.92

$$\left[\frac{3\sqrt{2}\left(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+5\,ax-1\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+4\left(3\,a^3x^3-13\,a^2x^2\right.}{1024\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="frica
s")
```

```
[Out] [1/1024*(3*sqrt(2)*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x -
 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-
a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*(3*a^3*x^3 - 13*a^2*x^
2 - 79*a*x - 39)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^6*c^4*x^5 - 5*a^5*
c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4), 1/512*(3*
sqrt(2)*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*sqrt(-c
)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 -
c)) + 2*(3*a^3*x^3 - 13*a^2*x^2 - 79*a*x - 39)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c
*x + c))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5
*a^2*c^4*x - a*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{(-c\,(ax-1))^{\frac{7}{2}}\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-c*(a*x - 1))**(7/2)*(-(a*x - 1)*(a*x + 1))**(3/2))
, x)
```

Giac [A] time = 1.4621, size = 142, normalized size = 0.74

$$-\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}c^2}+\frac{2\left(3(acx+c)^{\frac{7}{2}}-22(acx+c)^{\frac{5}{2}}c-44(acx+c)^{\frac{3}{2}}c^2+24\sqrt{acx+c}c^3\right)}{(acx-c)^4c^2}}{512a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="giac"
)
```

```
[Out] -1/512*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c^
2) + 2*(3*(a*c*x + c)^(7/2) - 22*(a*c*x + c)^(5/2)*c - 44*(a*c*x + c)^(3/2)
*c^2 + 24*sqrt(a*c*x + c)*c^3)/((a*c*x - c)^4*c^2))/(a*abs(c))
```

3.252 $\int e^{-\tanh^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=206

$$\frac{16384c^5\sqrt{1-a^2x^2}}{693a\sqrt{c-acx}}+\frac{4096c^4\sqrt{1-a^2x^2}\sqrt{c-acx}}{693a}+\frac{512c^3\sqrt{1-a^2x^2}(c-acx)^{3/2}}{231a}+\frac{640c^2\sqrt{1-a^2x^2}(c-acx)^{5/2}}{693a}+\frac{40c\sqrt{1}}{}$$

```
[Out] (16384*c^5*Sqrt[1 - a^2*x^2])/(693*a*Sqrt[c - a*c*x]) + (4096*c^4*Sqrt[c -
a*c*x]*Sqrt[1 - a^2*x^2])/(693*a) + (512*c^3*(c - a*c*x)^(3/2)*Sqrt[1 - a^2
*x^2])/(231*a) + (640*c^2*(c - a*c*x)^(5/2)*Sqrt[1 - a^2*x^2])/(693*a) + (4
0*c*(c - a*c*x)^(7/2)*Sqrt[1 - a^2*x^2])/(99*a) + (2*(c - a*c*x)^(9/2)*Sqrt
[1 - a^2*x^2])/(11*a)
```

Rubi [A] time = 0.161547, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6127, 657, 649}

$$\frac{16384c^5\sqrt{1-a^2x^2}}{693a\sqrt{c-acx}}+\frac{4096c^4\sqrt{1-a^2x^2}\sqrt{c-acx}}{693a}+\frac{512c^3\sqrt{1-a^2x^2}(c-acx)^{3/2}}{231a}+\frac{640c^2\sqrt{1-a^2x^2}(c-acx)^{5/2}}{693a}+\frac{40c\sqrt{1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(9/2)/E^ArcTanh[a*x],x]
```


```
[Out] (16384*c^5*Sqrt[1 - a^2*x^2])/(693*a*Sqrt[c - a*c*x]) + (4096*c^4*Sqrt[c -
a*c*x]*Sqrt[1 - a^2*x^2])/(693*a) + (512*c^3*(c - a*c*x)^(3/2)*Sqrt[1 - a^2
*x^2])/(231*a) + (640*c^2*(c - a*c*x)^(5/2)*Sqrt[1 - a^2*x^2])/(693*a) + (4
0*c*(c - a*c*x)^(7/2)*Sqrt[1 - a^2*x^2])/(99*a) + (2*(c - a*c*x)^(9/2)*Sqrt
[1 - a^2*x^2])/(11*a)
```


Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{\int \frac{(c-acx)^{11/2}}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{2(c-acx)^{9/2}\sqrt{1-a^2x^2}}{11a} + \frac{20}{11}\int \frac{(c-acx)^{9/2}}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{40c(c-acx)^{7/2}\sqrt{1-a^2x^2}}{99a} + \frac{2(c-acx)^{9/2}\sqrt{1-a^2x^2}}{11a} + \frac{1}{99}(320c)\int \frac{(c-acx)^{7/2}}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{640c^2(c-acx)^{5/2}\sqrt{1-a^2x^2}}{693a} + \frac{40c(c-acx)^{7/2}\sqrt{1-a^2x^2}}{99a} + \frac{2(c-acx)^{9/2}\sqrt{1-a^2x^2}}{11a} + \frac{1}{231}($$
$$= \frac{512c^3(c-acx)^{3/2}\sqrt{1-a^2x^2}}{231a} + \frac{640c^2(c-acx)^{5/2}\sqrt{1-a^2x^2}}{693a} + \frac{40c(c-acx)^{7/2}\sqrt{1-a^2x^2}}{99a} + $$
$$= \frac{4096c^4\sqrt{c-acx}\sqrt{1-a^2x^2}}{693a} + \frac{512c^3(c-acx)^{3/2}\sqrt{1-a^2x^2}}{231a} + \frac{640c^2(c-acx)^{5/2}\sqrt{1-a^2x^2}}{693a} + $$
$$= \frac{16384c^5\sqrt{1-a^2x^2}}{693a\sqrt{c-acx}} + \frac{4096c^4\sqrt{c-acx}\sqrt{1-a^2x^2}}{693a} + \frac{512c^3(c-acx)^{3/2}\sqrt{1-a^2x^2}}{231a} + \frac{640c^2(c}{}$$

Mathematica [A] time = 0.0553282, size = 73, normalized size = 0.35

$$-\frac{2c^5\sqrt{1-a^2x^2}\left(63a^5x^5-455a^4x^4+1510a^3x^3-3198a^2x^2+5419ax-11531\right)}{693a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(9/2)/E^ArcTanh[a*x],x]
```

```
[Out] (-2*c^5*Sqrt[1 - a^2*x^2]*(-11531 + 5419*a*x - 3198*a^2*x^2 + 1510*a^3*x^3
- 455*a^4*x^4 + 63*a^5*x^5))/(693*a*Sqrt[c - a*c*x])
```

Maple [A] time = 0.031, size = 72, normalized size = 0.4

$$\frac{126\,x^5a^5-910\,x^4a^4+3020\,x^3a^3-6396\,a^2x^2+10838\,ax-23062}{693\,(ax-1)^5\,a}\sqrt{-a^2x^2+1}\,(-acx+c)^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2/693*(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(9/2)*(63*a^5*x^5-455*a^4*x^4+1510*a^3*
x^3-3198*a^2*x^2+5419*a*x-11531)/(a*x-1)^5/a
```

Maxima [A] time = 1.01987, size = 111, normalized size = 0.54

$$-\frac{2\left(63\,a^5c^{\frac{9}{2}}x^5-455\,a^4c^{\frac{9}{2}}x^4+1510\,a^3c^{\frac{9}{2}}x^3-3198\,a^2c^{\frac{9}{2}}x^2+5419\,ac^{\frac{9}{2}}x-11531\,c^{\frac{9}{2}}\right)\sqrt{ax+1}(ax-1)}{693\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -2/693*(63*a^5*c^(9/2)*x^5 - 455*a^4*c^(9/2)*x^4 + 1510*a^3*c^(9/2)*x^3 - 3
198*a^2*c^(9/2)*x^2 + 5419*a*c^(9/2)*x - 11531*c^(9/2))*sqrt(a*x + 1)*(a*x
- 1)/(a^2*x - a)
```

Fricas [A] time = 2.19806, size = 209, normalized size = 1.01

$$\frac{2\left(63\,a^5c^4x^5-455\,a^4c^4x^4+1510\,a^3c^4x^3-3198\,a^2c^4x^2+5419\,ac^4x-11531\,c^4\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{693\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/693*(63*a^5*c^4*x^5 - 455*a^4*c^4*x^4 + 1510*a^3*c^4*x^3 - 3198*a^2*c^4*x
^2 + 5419*a*c^4*x - 11531*c^4)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2*x -
 a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(9/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32767, size = 132, normalized size = 0.64

$$-\frac{16384\,\sqrt{2}c^{\frac{7}{2}}|c|}{693\,a}-\frac{2\left(63\,(acx+c)^{\frac{11}{2}}-770\,(acx+c)^{\frac{9}{2}}c+3960\,(acx+c)^{\frac{7}{2}}c^2-11088\,(acx+c)^{\frac{5}{2}}c^3+18480\,(acx+c)^{\frac{3}{2}}c^4}{693\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -16384/693*sqrt(2)*c^(7/2)*abs(c)/a - 2/693*(63*(a*c*x + c)^(11/2) - 770*(a
*c*x + c)^(9/2)*c + 3960*(a*c*x + c)^(7/2)*c^2 - 11088*(a*c*x + c)^(5/2)*c^
3 + 18480*(a*c*x + c)^(3/2)*c^4 - 22176*sqrt(a*c*x + c)*c^5)*abs(c)/(a*c^2)
```

3.253 $\int e^{-\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=171

$$\frac{4096c^4\sqrt{1-a^2x^2}}{315a\sqrt{c-acx}}+\frac{1024c^3\sqrt{1-a^2x^2}\sqrt{c-acx}}{315a}+\frac{128c^2\sqrt{1-a^2x^2}(c-acx)^{3/2}}{105a}+\frac{32c\sqrt{1-a^2x^2}(c-acx)^{5/2}}{63a}+\frac{2\sqrt{1-a^2x^2}(c-acx)^{7/2}}{9a}$$

```
[Out] (4096*c^4*Sqrt[1 - a^2*x^2])/(315*a*Sqrt[c - a*c*x]) + (1024*c^3*Sqrt[c - a
*c*x]*Sqrt[1 - a^2*x^2])/(315*a) + (128*c^2*(c - a*c*x)^(3/2)*Sqrt[1 - a^2*
x^2])/(105*a) + (32*c*(c - a*c*x)^(5/2)*Sqrt[1 - a^2*x^2])/(63*a) + (2*(c -
 a*c*x)^(7/2)*Sqrt[1 - a^2*x^2])/(9*a)
```

Rubi [A] time = 0.133945, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6127, 657, 649}

$$\frac{4096c^4\sqrt{1-a^2x^2}}{315a\sqrt{c-acx}}+\frac{1024c^3\sqrt{1-a^2x^2}\sqrt{c-acx}}{315a}+\frac{128c^2\sqrt{1-a^2x^2}(c-acx)^{3/2}}{105a}+\frac{32c\sqrt{1-a^2x^2}(c-acx)^{5/2}}{63a}+\frac{2\sqrt{1-a^2x^2}(c-acx)^{7/2}}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(7/2)/E^ArcTanh[a*x],x]
```


```
[Out] (4096*c^4*Sqrt[1 - a^2*x^2])/(315*a*Sqrt[c - a*c*x]) + (1024*c^3*Sqrt[c - a
*c*x]*Sqrt[1 - a^2*x^2])/(315*a) + (128*c^2*(c - a*c*x)^(3/2)*Sqrt[1 - a^2*
x^2])/(105*a) + (32*c*(c - a*c*x)^(5/2)*Sqrt[1 - a^2*x^2])/(63*a) + (2*(c -
 a*c*x)^(7/2)*Sqrt[1 - a^2*x^2])/(9*a)
```


Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{\int \frac{(c-acx)^{9/2}}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{2(c-acx)^{7/2}\sqrt{1-a^2x^2}}{9a} + \frac{16}{9}\int \frac{(c-acx)^{7/2}}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{32c(c-acx)^{5/2}\sqrt{1-a^2x^2}}{63a} + \frac{2(c-acx)^{7/2}\sqrt{1-a^2x^2}}{9a} + \frac{1}{21}(64c)\int \frac{(c-acx)^{5/2}}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{128c^2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{105a} + \frac{32c(c-acx)^{5/2}\sqrt{1-a^2x^2}}{63a} + \frac{2(c-acx)^{7/2}\sqrt{1-a^2x^2}}{9a} + \frac{1}{10}$$
$$= \frac{1024c^3\sqrt{c-acx}\sqrt{1-a^2x^2}}{315a} + \frac{128c^2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{105a} + \frac{32c(c-acx)^{5/2}\sqrt{1-a^2x^2}}{63a} +$$
$$= \frac{4096c^4\sqrt{1-a^2x^2}}{315a\sqrt{c-acx}} + \frac{1024c^3\sqrt{c-acx}\sqrt{1-a^2x^2}}{315a} + \frac{128c^2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{105a} + 32c(c$$

Mathematica [A] time = 0.0395576, size = 65, normalized size = 0.38

$$\frac{2c^4\sqrt{1-a^2x^2}\left(35a^4x^4-220a^3x^3+642a^2x^2-1276ax+2867\right)}{315a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(7/2)/E^ArcTanh[a*x],x]
```

```
[Out] (2*c^4*Sqrt[1 - a^2*x^2]*(2867 - 1276*a*x + 642*a^2*x^2 - 220*a^3*x^3 + 35*
a^4*x^4))/(315*a*Sqrt[c - a*c*x])
```

Maple [A] time = 0.034, size = 64, normalized size = 0.4

$$\frac{70\,x^4a^4-440\,x^3a^3+1284\,a^2x^2-2552\,ax+5734}{315\,(ax-1)^4\,a}\sqrt{-a^2x^2+1}\,(-acx+c)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2/315*(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(7/2)*(35*a^4*x^4-220*a^3*x^3+642*a^2*x
^2-1276*a*x+2867)/(a*x-1)^4/a
```

Maxima [A] time = 1.01109, size = 96, normalized size = 0.56

$$\frac{2\left(35\,a^4c^{\frac{7}{2}}x^4-220\,a^3c^{\frac{7}{2}}x^3+642\,a^2c^{\frac{7}{2}}x^2-1276\,ac^{\frac{7}{2}}x+2867\,c^{\frac{7}{2}}\right)\sqrt{ax+1}(ax-1)}{315\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 2/315*(35*a^4*c^(7/2)*x^4 - 220*a^3*c^(7/2)*x^3 + 642*a^2*c^(7/2)*x^2 - 127
6*a*c^(7/2)*x + 2867*c^(7/2))*sqrt(a*x + 1)*(a*x - 1)/(a^2*x - a)
```

Fricas [A] time = 2.2511, size = 182, normalized size = 1.06

$$-\frac{2\left(35\,a^4c^3x^4-220\,a^3c^3x^3+642\,a^2c^3x^2-1276\,ac^3x+2867\,c^3\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{315\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/315*(35*a^4*c^3*x^4 - 220*a^3*c^3*x^3 + 642*a^2*c^3*x^2 - 1276*a*c^3*x +
 2867*c^3)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(7/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30607, size = 115, normalized size = 0.67

$$-\frac{4096\,\sqrt{2}c^{\frac{5}{2}}|c|}{315\,a}+\frac{2\left(35\,(acx+c)^{\frac{9}{2}}-360\,(acx+c)^{\frac{7}{2}}c+1512\,(acx+c)^{\frac{5}{2}}c^2-3360\,(acx+c)^{\frac{3}{2}}c^3+5040\,\sqrt{acx+c}c^4\right)|c|}{315\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -4096/315*sqrt(2)*c^(5/2)*abs(c)/a + 2/315*(35*(a*c*x + c)^(9/2) - 360*(a*c
*x + c)^(7/2)*c + 1512*(a*c*x + c)^(5/2)*c^2 - 3360*(a*c*x + c)^(3/2)*c^3 +
 5040*sqrt(a*c*x + c)*c^4)*abs(c)/(a*c^2)
```

3.254 $\int e^{-\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=136

$$\frac{256c^3\sqrt{1-a^2x^2}}{35a\sqrt{c-acx}}+\frac{64c^2\sqrt{1-a^2x^2}\sqrt{c-acx}}{35a}+\frac{24c\sqrt{1-a^2x^2}(c-acx)^{3/2}}{35a}+\frac{2\sqrt{1-a^2x^2}(c-acx)^{5/2}}{7a}$$

```
[Out] (256*c^3*Sqrt[1 - a^2*x^2])/(35*a*Sqrt[c - a*c*x]) + (64*c^2*Sqrt[c - a*c*x
]*Sqrt[1 - a^2*x^2])/(35*a) + (24*c*(c - a*c*x)^(3/2)*Sqrt[1 - a^2*x^2])/(3
5*a) + (2*(c - a*c*x)^(5/2)*Sqrt[1 - a^2*x^2])/(7*a)
```

Rubi [A] time = 0.106038, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6127, 657, 649}

$$\frac{256c^3\sqrt{1-a^2x^2}}{35a\sqrt{c-acx}}+\frac{64c^2\sqrt{1-a^2x^2}\sqrt{c-acx}}{35a}+\frac{24c\sqrt{1-a^2x^2}(c-acx)^{3/2}}{35a}+\frac{2\sqrt{1-a^2x^2}(c-acx)^{5/2}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(5/2)/E^ArcTanh[a*x],x]
```

```
[Out] (256*c^3*Sqrt[1 - a^2*x^2])/(35*a*Sqrt[c - a*c*x]) + (64*c^2*Sqrt[c - a*c*x
]*Sqrt[1 - a^2*x^2])/(35*a) + (24*c*(c - a*c*x)^(3/2)*Sqrt[1 - a^2*x^2])/(3
5*a) + (2*(c - a*c*x)^(5/2)*Sqrt[1 - a^2*x^2])/(7*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{\int \frac{(c-acx)^{7/2}}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{2(c-acx)^{5/2}\sqrt{1-a^2x^2}}{7a} + \frac{12}{7}\int \frac{(c-acx)^{5/2}}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{24c(c-acx)^{3/2}\sqrt{1-a^2x^2}}{35a} + \frac{2(c-acx)^{5/2}\sqrt{1-a^2x^2}}{7a} + \frac{1}{35}(96c)\int \frac{(c-acx)^{3/2}}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{64c^2\sqrt{c-acx}\sqrt{1-a^2x^2}}{35a} + \frac{24c(c-acx)^{3/2}\sqrt{1-a^2x^2}}{35a} + \frac{2(c-acx)^{5/2}\sqrt{1-a^2x^2}}{7a} + \frac{1}{35}\left(128c\right.$$
$$= \frac{256c^3\sqrt{1-a^2x^2}}{35a\sqrt{c-acx}} + \frac{64c^2\sqrt{c-acx}\sqrt{1-a^2x^2}}{35a} + \frac{24c(c-acx)^{3/2}\sqrt{1-a^2x^2}}{35a} + \frac{2(c-acx)^{5/2}\sqrt{1}}{7a}$$

Mathematica [A] time = 0.034936, size = 57, normalized size = 0.42

$$-\frac{2c^3\sqrt{1-a^2x^2}\left(5a^3x^3-27a^2x^2+71ax-177\right)}{35a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(5/2)/E^ArcTanh[a*x],x]
```

```
[Out] (-2*c^3*Sqrt[1 - a^2*x^2]*(-177 + 71*a*x - 27*a^2*x^2 + 5*a^3*x^3))/(35*a*S
qrt[c - a*c*x])
```

Maple [A] time = 0.03, size = 56, normalized size = 0.4

$$\frac{10\,x^3a^3-54\,a^2x^2+142\,ax-354}{35\,(ax-1)^3\,a}\sqrt{-a^2x^2+1}\,(-acx+c)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2/35*(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(5/2)*(5*a^3*x^3-27*a^2*x^2+71*a*x-177)/
(a*x-1)^3/a
```

Maxima [A] time = 1.01309, size = 81, normalized size = 0.6

$$-\frac{2\left(5\,a^3c^{\frac{5}{2}}x^3-27\,a^2c^{\frac{5}{2}}x^2+71\,ac^{\frac{5}{2}}x-177\,c^{\frac{5}{2}}\right)\sqrt{ax+1}(ax-1)}{35\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -2/35*(5*a^3*c^(5/2)*x^3 - 27*a^2*c^(5/2)*x^2 + 71*a*c^(5/2)*x - 177*c^(5/2
))*sqrt(a*x + 1)*(a*x - 1)/(a^2*x - a)
```

Fricas [A] time = 2.11936, size = 149, normalized size = 1.1

$$\frac{2\left(5a^{3}c^{2}x^{3}-27a^{2}c^{2}x^{2}+71ac^{2}x-177c^{2}\right)\sqrt{-a^{2}x^{2}+1}\sqrt{-acx+c}}{35\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/35*(5*a^3*c^2*x^3 - 27*a^2*c^2*x^2 + 71*a*c^2*x - 177*c^2)*sqrt(-a^2*x^2
+ 1)*sqrt(-a*c*x + c)/(a^2*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-c\left(ax-1\right)\right)^{\frac{5}{2}}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(5/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(5/2)*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [A] time = 1.26309, size = 97, normalized size = 0.71

$$-\frac{256\sqrt{2}c^{\frac{3}{2}}|c|}{35a}-\frac{2\left(5\left(acx+c\right)^{\frac{7}{2}}-42\left(acx+c\right)^{\frac{5}{2}}c+140\left(acx+c\right)^{\frac{3}{2}}c^{2}-280\sqrt{acx+c}c^{3}\right)|c|}{35ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -256/35*sqrt(2)*c^(3/2)*abs(c)/a - 2/35*(5*(a*c*x + c)^(7/2) - 42*(a*c*x +
c)^(5/2)*c + 140*(a*c*x + c)^(3/2)*c^2 - 280*sqrt(a*c*x + c)*c^3)*abs(c)/(a
*c^2)
```

3.255 $\int e^{-\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=101

$$\frac{64c^2\sqrt{1-a^2x^2}}{15a\sqrt{c-acx}}+\frac{16c\sqrt{1-a^2x^2}\sqrt{c-acx}}{15a}+\frac{2\sqrt{1-a^2x^2}(c-acx)^{3/2}}{5a}$$

```
[Out] (64*c^2*Sqrt[1 - a^2*x^2])/(15*a*Sqrt[c - a*c*x]) + (16*c*Sqrt[c - a*c*x]*S
qrt[1 - a^2*x^2])/(15*a) + (2*(c - a*c*x)^(3/2)*Sqrt[1 - a^2*x^2])/(5*a)
```

Rubi [A] time = 0.0867293, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6127, 657, 649}

$$\frac{64c^2\sqrt{1-a^2x^2}}{15a\sqrt{c-acx}}+\frac{16c\sqrt{1-a^2x^2}\sqrt{c-acx}}{15a}+\frac{2\sqrt{1-a^2x^2}(c-acx)^{3/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(3/2)/E^ArcTanh[a*x],x]
```

```
[Out] (64*c^2*Sqrt[1 - a^2*x^2])/(15*a*Sqrt[c - a*c*x]) + (16*c*Sqrt[c - a*c*x]*S
qrt[1 - a^2*x^2])/(15*a) + (2*(c - a*c*x)^(3/2)*Sqrt[1 - a^2*x^2])/(5*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{\int \frac{(c-acx)^{5/2}}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{5a} + \frac{8}{5}\int \frac{(c-acx)^{3/2}}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{16c\sqrt{c-acx}\sqrt{1-a^2x^2}}{15a} + \frac{2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{5a} + \frac{1}{15}(32c)\int \frac{\sqrt{c-acx}}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{64c^2\sqrt{1-a^2x^2}}{15a\sqrt{c-acx}} + \frac{16c\sqrt{c-acx}\sqrt{1-a^2x^2}}{15a} + \frac{2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{5a}$$

Mathematica [A] time = 0.028437, size = 49, normalized size = 0.49

$$\frac{2c^2\sqrt{1-a^2x^2}\left(3a^2x^2-14ax+43\right)}{15a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(3/2)/E^ArcTanh[a*x],x]
```

```
[Out] (2*c^2*Sqrt[1 - a^2*x^2]*(43 - 14*a*x + 3*a^2*x^2))/(15*a*Sqrt[c - a*c*x])
```

Maple [A] time = 0.03, size = 48, normalized size = 0.5

$$\frac{6\,a^2x^2-28\,ax+86}{15\,(ax-1)^2\,a}\sqrt{-a^2x^2+1}\,(-acx+c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2/15*(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(3/2)*(3*a^2*x^2-14*a*x+43)/(a*x-1)^2/a
```

Maxima [A] time = 0.997573, size = 66, normalized size = 0.65

$$\frac{2\left(3\,a^2c^{\frac{3}{2}}x^2-14\,ac^{\frac{3}{2}}x+43\,c^{\frac{3}{2}}\right)\sqrt{ax+1}(ax-1)}{15\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 2/15*(3*a^2*c^(3/2)*x^2 - 14*a*c^(3/2)*x + 43*c^(3/2))*sqrt(a*x + 1)*(a*x -
 1)/(a^2*x - a)
```

Fricas [A] time = 2.23512, size = 117, normalized size = 1.16

$$-\frac{2\left(3a^{2}cx^{2}-14acx+43c\right)\sqrt{-a^{2}x^{2}+1}\sqrt{-acx+c}}{15\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/15*(3*a^2*c*x^2 - 14*a*c*x + 43*c)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(
a^2*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-c\left(ax-1\right)\right)^{\frac{3}{2}}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(3/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(3/2)*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [A] time = 1.22211, size = 80, normalized size = 0.79

$$-\frac{64\sqrt{2}\sqrt{c}|c|}{15a}+\frac{2\left(3\left(acx+c\right)^{\frac{5}{2}}-20\left(acx+c\right)^{\frac{3}{2}}c+60\sqrt{acx+c}c^{2}\right)|c|}{15ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -64/15*sqrt(2)*sqrt(c)*abs(c)/a + 2/15*(3*(a*c*x + c)^(5/2) - 20*(a*c*x + c
)^(3/2)*c + 60*sqrt(a*c*x + c)*c^2)*abs(c)/(a*c^2)
```

3.256 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=66

$$\frac{8c\sqrt{1-a^2x^2}}{3a\sqrt{c-acx}}+\frac{2\sqrt{1-a^2x^2}\sqrt{c-acx}}{3a}$$

```
[Out] (8*c*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[c - a*c*x]) + (2*Sqrt[c - a*c*x]*Sqrt[1 -
 a^2*x^2])/(3*a)
```

Rubi [A] time = 0.0622191, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6127, 657, 649}

$$\frac{8c\sqrt{1-a^2x^2}}{3a\sqrt{c-acx}}+\frac{2\sqrt{1-a^2x^2}\sqrt{c-acx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^ArcTanh[a*x],x]
```

```
[Out] (8*c*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[c - a*c*x]) + (2*Sqrt[c - a*c*x]*Sqrt[1 -
 a^2*x^2])/(3*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}\sqrt{c-acx}\,dx &= \frac{\int \frac{(c-acx)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{c}\\ &= \frac{2\sqrt{c-acx}\sqrt{1-a^2x^2}}{3a}+\frac{4}{3}\int \frac{\sqrt{c-acx}}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{8c\sqrt{1-a^2x^2}}{3a\sqrt{c-acx}}+\frac{2\sqrt{c-acx}\sqrt{1-a^2x^2}}{3a}\end{aligned}$$

Mathematica [A] time = 0.0197502, size = 38, normalized size = 0.58

$$-\frac{2c(ax-5)\sqrt{1-a^2x^2}}{3a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^ArcTanh[a*x],x]
```

```
[Out] (-2*c*(-5 + a*x)*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[c - a*c*x])
```

Maple [A] time = 0.029, size = 39, normalized size = 0.6

$$\frac{2\,ax-10}{(3\,ax-3)\,a}\sqrt{-a^2x^2+1}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2/3*(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)*(a*x-5)/(a*x-1)/a
```

Maxima [A] time = 1.00005, size = 50, normalized size = 0.76

$$-\frac{2\left(a\sqrt{c}x-5\sqrt{c}\right)\sqrt{ax+1}(ax-1)}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -2/3*(a*sqrt(c)*x - 5*sqrt(c))*sqrt(a*x + 1)*(a*x - 1)/(a^2*x - a)
```

Fricas [A] time = 2.13332, size = 85, normalized size = 1.29

$$\frac{2\sqrt{-a^2x^2+1}\sqrt{-acx+c}(ax-5)}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/3*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x - 5)/(a^2*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\,(ax-1)}\sqrt{-\,(ax-1)\,(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [A] time = 1.17445, size = 58, normalized size = 0.88

$$-\frac{2\left(\frac{4\sqrt{2}c^{\frac{3}{2}}}{a}+\frac{(acx+c)^{\frac{3}{2}}-6\sqrt{acx+c}c}{a}\right)|c|}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(4*sqrt(2)*c^(3/2)/a + ((a*c*x + c)^(3/2) - 6*sqrt(a*c*x + c)*c)/a)*ab
s(c)/c^2
```

3.257 $\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=30

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}$$

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x])
```

Rubi [A] time = 0.0473599, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6127, 649}

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*Sqrt[c - a*c*x]),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x])
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= \frac{\int \frac{\sqrt{c-acx}}{\sqrt{1-a^2x^2}}\,dx}{c}\\ &= \frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}\end{aligned}$$

Mathematica [A] time = 0.015685, size = 30, normalized size = 1.

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*Sqrt[c - a*c*x]),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x])
```

Maple [A] time = 0.028, size = 27, normalized size = 0.9

$$2\,\frac{\sqrt{-a^2x^2+1}}{a\sqrt{-acx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2),x)
```

```
[Out] 2*(-a^2*x^2+1)^(1/2)/a/(-a*c*x+c)^(1/2)
```

Maxima [A] time = 0.980754, size = 20, normalized size = 0.67

$$\frac{2\sqrt{ax+1}}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2*sqrt(a*x + 1)/(a*sqrt(c))
```

Fricas [A] time = 2.1293, size = 76, normalized size = 2.53

$$-\frac{2\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] -2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2*c*x - a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{-c(ax-1)}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(-c*(a*x - 1))*(a*x + 1)), x)
```

Giac [A] time = 1.23812, size = 42, normalized size = 1.4

$$-\frac{2\left(\frac{\sqrt{2}\sqrt{c}}{a}-\frac{\sqrt{acx+c}}{a}\right)|c|}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] -2*(sqrt(2)*sqrt(c)/a - sqrt(a*c*x + c)/a)*abs(c)/c^2
```

3.258 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{ac^{3/2}}$$

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/(a
*c^(3/2))
```

Rubi [A] time = 0.0622823, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6127, 661, 208}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a*c*x)^(3/2)),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/(a
*c^(3/2))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx &= \frac{\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\left((2a)\,\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\right)\\
&= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{ac^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.024151, size = 44, normalized size = 0.86

$$\frac{\sqrt{2-2ax}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{ac\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a*c*x)^(3/2)),x]
```

```
[Out] (Sqrt[2 - 2*a*x]*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(a*c*Sqrt[c - a*c*x])
```

Maple [A] time = 0.096, size = 68, normalized size = 1.3

$$\frac{\sqrt{2}}{(-ax+1)\,a}\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{c\,(ax+1)}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c\,(ax+1)}}c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(3/2),x)
```

```
[Out] 1/(-a*x+1)/(c*(a*x+1))^(1/2)/c^(3/2)/a*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2
)*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(-acx+c)^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((-a*c*x + c)^(3/2)*(a*x + 1)), x)
```

Fricas [A] time = 2.16435, size = 321, normalized size = 6.29

$$\left[\frac{\sqrt{2}\log\left(-\frac{a^2x^2+2\,ax-\frac{2\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{\sqrt{c}}-3}{a^2x^2-2\,ax+1}\right)}{2\,ac^{\frac{3}{2}}},\ \frac{\sqrt{2}\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{-\frac{1}{c}}}{a^2x^2-1}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/2*sqrt(2)*log(-(a^2*x^2 + 2*a*x - 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c
*x + c)/sqrt(c) - 3)/(a^2*x^2 - 2*a*x + 1))/(a*c^(3/2)), sqrt(2)*sqrt(-1/c)
*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-1/c)/(a^2*x^2 - 1
))/(a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{(-c(ax-1))^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(a*x - 1))**(3/2)*(a*x + 1)), x)
```

Giac [A] time = 1.22171, size = 84, normalized size = 1.65

$$-\frac{\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)}{a\sqrt{-c}}\right)|c|}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] -(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) - sqrt(
2)*arctan(sqrt(c)/sqrt(-c))/(a*sqrt(-c)))*abs(c)/c^2
```

3.259 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=90

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{2\sqrt{2}ac^{5/2}}+\frac{\sqrt{1-a^2x^2}}{2ac(c-acx)^{3/2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(2*a*c*(c - a*c*x)^(3/2)) + ArcTanh[(Sqrt[c]*Sqrt[1 - a^2
*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])]/(2*Sqrt[2]*a*c^(5/2))
```

Rubi [A] time = 0.0822888, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6127, 673, 661, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{2\sqrt{2}ac^{5/2}}+\frac{\sqrt{1-a^2x^2}}{2ac(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a*c*x)^(5/2)),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a*c*(c - a*c*x)^(3/2)) + ArcTanh[(Sqrt[c]*Sqrt[1 - a^2
*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])]/(2*Sqrt[2]*a*c^(5/2))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^{5/2}}dx = \frac{\int \frac{1}{(c-acx)^{3/2}\sqrt{1-a^2x^2}}dx}{c}$$
$$= \frac{\sqrt{1-a^2x^2}}{2ac(c-acx)^{3/2}} + \frac{\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}dx}{4c^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{2ac(c-acx)^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)}{2c}$$
$$= \frac{\sqrt{1-a^2x^2}}{2ac(c-acx)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{2\sqrt{2}ac^{5/2}}$$

Mathematica [A] time = 0.0584424, size = 70, normalized size = 0.78

$$-\frac{\sqrt{2}(ax-1)\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)-2\sqrt{ax+1}}{4ac^2\sqrt{1-ax}\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a*c*x)^(5/2)),x]
```

```
[Out] -(-2*Sqrt[1 + a*x] + Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(4*
a*c^2*Sqrt[1 - a*x]*Sqrt[c - a*c*x])
```

Maple [A] time = 0.103, size = 111, normalized size = 1.2

$$-\frac{1}{4(ax-1)^2 a}\sqrt{-a^2x^2+1}\sqrt{-c(ax-1)}\left(\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{c(ax+1)}\frac{1}{\sqrt{c}}\right)xac-\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{c(ax+1)}\frac{1}{\sqrt{c}}\right)c-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(5/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(7/2)*(2^(1/2)*arctanh(1/2*(c*
(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c-2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)
*2^(1/2)/c^(1/2))*c-2*(c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)^2/(c*(a*x+1))^(1/2
)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(-acx+c)^{\frac{5}{2}}(ax+1)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((-a*c*x + c)^(5/2)*(a*x + 1)), x)
```

Fricas [A] time = 2.24517, size = 589, normalized size = 6.54

$$\left[\frac{\sqrt{2}\left(a^{2}x^{2}-2\,ax+1\right)\sqrt{c}\log\left(-\frac{a^{2}cx^{2}+2\,acx-2\,\sqrt{2}\sqrt{-a^{2}x^{2}+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^{2}x^{2}-2\,ax+1}\right)+4\,\sqrt{-a^{2}x^{2}+1}\sqrt{-acx+c}}{8\left(a^{3}c^{3}x^{2}-2\,a^{2}c^{3}x+ac^{3}\right)},\frac{\sqrt{2}\left(a^{2}x^{2}-2\,ax+1\right)\sqrt{-c}\,a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/8*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*s
qrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x
+ 1)) + 4*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^3*c^3*x^2 - 2*a^2*c^3*x +
 a*c^3), 1/4*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a
^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 2*sqrt(-a^2*x^2 +
1)*sqrt(-a*c*x + c))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{(-c(ax-1))^{\frac{5}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(a*x - 1))**(5/2)*(a*x + 1)), x)
```

Giac [A] time = 1.51033, size = 92, normalized size = 1.02

$$-\frac{\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}c}+\frac{2\sqrt{acx+c}}{(acx-c)ac}\right)|c|}{4\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/4*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c) +
 2*sqrt(a*c*x + c)/((a*c*x - c)*a*c))*abs(c)/c^2
```

3.260 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=125

$$\frac{3\sqrt{1-a^2x^2}}{16ac^2(c-acx)^{3/2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{16\sqrt{2}ac^{7/2}}+\frac{\sqrt{1-a^2x^2}}{4ac(c-acx)^{5/2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(4*a*c*(c - a*c*x)^(5/2)) + (3*Sqrt[1 - a^2*x^2])/(16*a*c
^2*(c - a*c*x)^(3/2)) + (3*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqr
t[c - a*c*x])])/(16*Sqrt[2]*a*c^(7/2))
```

Rubi [A] time = 0.104941, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6127, 673, 661, 208}

$$\frac{3\sqrt{1-a^2x^2}}{16ac^2(c-acx)^{3/2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{16\sqrt{2}ac^{7/2}}+\frac{\sqrt{1-a^2x^2}}{4ac(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a*c*x)^(7/2)),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(4*a*c*(c - a*c*x)^(5/2)) + (3*Sqrt[1 - a^2*x^2])/(16*a*c
^2*(c - a*c*x)^(3/2)) + (3*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqr
t[c - a*c*x])])/(16*Sqrt[2]*a*c^(7/2))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\int \frac{1}{(c-acx)^{5/2}\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{\sqrt{1-a^2x^2}}{4ac(c-acx)^{5/2}} + \frac{3\int \frac{1}{(c-acx)^{3/2}\sqrt{1-a^2x^2}}\,dx}{8c^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{4ac(c-acx)^{5/2}} + \frac{3\sqrt{1-a^2x^2}}{16ac^2(c-acx)^{3/2}} + \frac{3\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{32c^3}$$
$$= \frac{\sqrt{1-a^2x^2}}{4ac(c-acx)^{5/2}} + \frac{3\sqrt{1-a^2x^2}}{16ac^2(c-acx)^{3/2}} - \frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)}{16c^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{4ac(c-acx)^{5/2}} + \frac{3\sqrt{1-a^2x^2}}{16ac^2(c-acx)^{3/2}} + \frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{16\sqrt{2}ac^{7/2}}$$

Mathematica [C] time = 0.0237634, size = 52, normalized size = 0.42

$$\frac{\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},\frac{1}{2}(ax+1)\right)}{4ac^3\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a*c*x)^(7/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1/2, 3, 3/2, (1 + a*x)/2])/(4*a*c^3*Sq
rt[c - a*c*x])
```

Maple [A] time = 0.106, size = 158, normalized size = 1.3

$$-\frac{1}{32\,(ax-1)^3\,a}\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}\left(3\,\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^2a^2c-6\,\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)xa\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(7/2),x)
```

```
[Out] -1/32*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(9/2)*(3*2^(1/2)*arctanh(1/2*
(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-6*2^(1/2)*arctanh(1/2*(c*(a*x+
1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c-6*x*a*(c*(a*x+1))^(1/2)*c^(1/2)+3*2^(1/2)*
arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+14*(c*(a*x+1))^(1/2)*c^(1/
2))/(a*x-1)^3/(c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(-acx+c)^{\frac{7}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((-a*c*x + c)^(7/2)*(a*x + 1)), x)
```

Fricas [A] time = 2.20508, size = 705, normalized size = 5.64

$$\left[\frac{3\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3c}{a^2x^2-2ax+1}\right)+4\sqrt{-a^2x^2+1}\sqrt{-acx+c}(3ax-7)}{64\left(a^4c^4x^3-3a^3c^4x^2+3a^2c^4x-ac^4\right)}, \ 3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="frica
s")
```

```
[Out] [1/64*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(a^2*c*x^2
+ 2*a*c*x - 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a
^2*x^2 - 2*a*x + 1)) + 4*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(3*a*x - 7))/(
a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4), 1/32*(3*sqrt(2)*(a^3*x^
3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(
-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x +
c)*(3*a*x - 7))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{(-c(ax-1))^{\frac{7}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(a*x - 1))**(7/2)*(a*x + 1)), x)
```

Giac [A] time = 1.46088, size = 112, normalized size = 0.9

$$-\frac{\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}c^2}+\frac{2\left(3(acx+c)^{\frac{3}{2}}-10\sqrt{acx+c}c\right)}{(acx-c)^2ac^2}\right)|c|}{32c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="giac"
)
```

```
[Out] -1/32*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c
^2) + 2*(3*(a*c*x + c)^(3/2) - 10*sqrt(a*c*x + c)*c)/((a*c*x - c)^2*a*c^2))
*abs(c)/c^2
```

3.261 $\int e^{-2\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=137

$$\frac{16c^2(c-acx)^{3/2}}{3a}+\frac{32c^3\sqrt{c-acx}}{a}-\frac{32\sqrt{2}c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}+\frac{2(c-acx)^{9/2}}{9ac}+\frac{4(c-acx)^{7/2}}{7a}+\frac{8c(c-acx)^{5/2}}{5a}$$

```
[Out] (32*c^3*Sqrt[c - a*c*x])/a + (16*c^2*(c - a*c*x)^(3/2))/(3*a) + (8*c*(c - a
*c*x)^(5/2))/(5*a) + (4*(c - a*c*x)^(7/2))/(7*a) + (2*(c - a*c*x)^(9/2))/(9
*a*c) - (32*Sqrt[2]*c^(7/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.110809, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6130, 21, 50, 63, 206}

$$\frac{16c^2(c-acx)^{3/2}}{3a}+\frac{32c^3\sqrt{c-acx}}{a}-\frac{32\sqrt{2}c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}+\frac{2(c-acx)^{9/2}}{9ac}+\frac{4(c-acx)^{7/2}}{7a}+\frac{8c(c-acx)^{5/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(7/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (32*c^3*Sqrt[c - a*c*x])/a + (16*c^2*(c - a*c*x)^(3/2))/(3*a) + (8*c*(c - a
*c*x)^(5/2))/(5*a) + (4*(c - a*c*x)^(7/2))/(7*a) + (2*(c - a*c*x)^(9/2))/(9
*a*c) - (32*Sqrt[2]*c^(7/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx = \int \frac{(1-ax)(c-acx)^{7/2}}{1+ax}\,dx$$
$$= \frac{\int \frac{(c-acx)^{9/2}}{1+ax}\,dx}{c}$$
$$= \frac{2(c-acx)^{9/2}}{9ac} + 2\int \frac{(c-acx)^{7/2}}{1+ax}\,dx$$
$$= \frac{4(c-acx)^{7/2}}{7a} + \frac{2(c-acx)^{9/2}}{9ac} + (4c)\int \frac{(c-acx)^{5/2}}{1+ax}\,dx$$
$$= \frac{8c(c-acx)^{5/2}}{5a} + \frac{4(c-acx)^{7/2}}{7a} + \frac{2(c-acx)^{9/2}}{9ac} + \left(8c^2\right)\int \frac{(c-acx)^{3/2}}{1+ax}\,dx$$
$$= \frac{16c^2(c-acx)^{3/2}}{3a} + \frac{8c(c-acx)^{5/2}}{5a} + \frac{4(c-acx)^{7/2}}{7a} + \frac{2(c-acx)^{9/2}}{9ac} + \left(16c^3\right)\int \frac{\sqrt{c-acx}}{1+ax}$$
$$= \frac{32c^3\sqrt{c-acx}}{a} + \frac{16c^2(c-acx)^{3/2}}{3a} + \frac{8c(c-acx)^{5/2}}{5a} + \frac{4(c-acx)^{7/2}}{7a} + \frac{2(c-acx)^{9/2}}{9ac} + (3$$
$$= \frac{32c^3\sqrt{c-acx}}{a} + \frac{16c^2(c-acx)^{3/2}}{3a} + \frac{8c(c-acx)^{5/2}}{5a} + \frac{4(c-acx)^{7/2}}{7a} + \frac{2(c-acx)^{9/2}}{9ac} - (6$$
$$= \frac{32c^3\sqrt{c-acx}}{a} + \frac{16c^2(c-acx)^{3/2}}{3a} + \frac{8c(c-acx)^{5/2}}{5a} + \frac{4(c-acx)^{7/2}}{7a} + \frac{2(c-acx)^{9/2}}{9ac} - 3$$

Mathematica [A] time = 0.0736412, size = 88, normalized size = 0.64

$$\frac{2c^3\left(\left(35a^4x^4-230a^3x^3+732a^2x^2-1754ax+6257\right)\sqrt{c-acx}-5040\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{315a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(7/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (2*c^3*(Sqrt[c - a*c*x]*(6257 - 1754*a*x + 732*a^2*x^2 - 230*a^3*x^3 + 35*a
^4*x^4) - 5040*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]))
/(315*a)
```

Maple [A] time = 0.035, size = 101, normalized size = 0.7

$$2\frac{1}{ac}\left(1/9\,(-acx+c)^{9/2}+2/7\,(-acx+c)^{7/2}c+4/5\,(-acx+c)^{5/2}c^2+8/3\,c^3\,(-acx+c)^{3/2}+16\,\sqrt{-acx+c}c^4-16\,c^{9/2}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 2/c/a*(1/9*(-a*c*x+c)^(9/2)+2/7*(-a*c*x+c)^(7/2)*c+4/5*(-a*c*x+c)^(5/2)*c^2
+8/3*c^3*(-a*c*x+c)^(3/2)+16*(-a*c*x+c)^(1/2)*c^4-16*c^(9/2)*2^(1/2)*arctan
h(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.2586, size = 517, normalized size = 3.77

$$\left[\frac{2\left(2520\sqrt{2}c^{\frac{7}{2}}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\left(35a^4c^3x^4-230a^3c^3x^3+732a^2c^3x^2-1754ac^3x+6257c^3\right)\sqrt{-acx+c}\right)}{315a}, \frac{2(\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [2/315*(2520*sqrt(2)*c^(7/2)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c
) - 3*c)/(a*x + 1)) + (35*a^4*c^3*x^4 - 230*a^3*c^3*x^3 + 732*a^2*c^3*x^2 -
 1754*a*c^3*x + 6257*c^3)*sqrt(-a*c*x + c))/a, 2/315*(5040*sqrt(2)*sqrt(-c)
*c^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) + (35*a^4*c^3*x^4 - 23
0*a^3*c^3*x^3 + 732*a^2*c^3*x^2 - 1754*a*c^3*x + 6257*c^3)*sqrt(-a*c*x + c)
)/a]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(7/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25569, size = 217, normalized size = 1.58

$$\frac{32\sqrt{2}c^4\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{2\left(35(acx-c)^4\sqrt{-acx+c}a^8c^8-90(acx-c)^3\sqrt{-acx+c}a^8c^9+252(acx-c)^2\sqrt{-acx+c}a^{\cdots}\right.}{315a^9c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 32*sqrt(2)*c^4*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) +
 2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c)*a^8*c^8 - 90*(a*c*x - c)^3*sqrt(-
a*c*x + c)*a^8*c^9 + 252*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^8*c^10 + 840*(-a*
c*x + c)^(3/2)*a^8*c^11 + 5040*sqrt(-a*c*x + c)*a^8*c^12)/(a^9*c^9)
```

3.262 $\int e^{-2\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=116

$$\frac{16c^2\sqrt{c-acx}}{a}-\frac{16\sqrt{2}c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}+\frac{2(c-acx)^{7/2}}{7ac}+\frac{4(c-acx)^{5/2}}{5a}+\frac{8c(c-acx)^{3/2}}{3a}$$

```
[Out] (16*c^2*Sqrt[c - a*c*x])/a + (8*c*(c - a*c*x)^(3/2))/(3*a) + (4*(c - a*c*x)
^(5/2))/(5*a) + (2*(c - a*c*x)^(7/2))/(7*a*c) - (16*Sqrt[2]*c^(5/2)*ArcTanh
[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.0983316, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6130, 21, 50, 63, 206}

$$\frac{16c^2\sqrt{c-acx}}{a}-\frac{16\sqrt{2}c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}+\frac{2(c-acx)^{7/2}}{7ac}+\frac{4(c-acx)^{5/2}}{5a}+\frac{8c(c-acx)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(5/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (16*c^2*Sqrt[c - a*c*x])/a + (8*c*(c - a*c*x)^(3/2))/(3*a) + (4*(c - a*c*x)
^(5/2))/(5*a) + (2*(c - a*c*x)^(7/2))/(7*a*c) - (16*Sqrt[2]*c^(5/2)*ArcTanh
[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx &= \int \frac{(1-ax)(c-acx)^{5/2}}{1+ax}\,dx \\
&= \frac{\int \frac{(c-acx)^{7/2}}{1+ax}\,dx}{c} \\
&= \frac{2(c-acx)^{7/2}}{7ac} + 2\int \frac{(c-acx)^{5/2}}{1+ax}\,dx \\
&= \frac{4(c-acx)^{5/2}}{5a} + \frac{2(c-acx)^{7/2}}{7ac} + (4c)\int \frac{(c-acx)^{3/2}}{1+ax}\,dx \\
&= \frac{8c(c-acx)^{3/2}}{3a} + \frac{4(c-acx)^{5/2}}{5a} + \frac{2(c-acx)^{7/2}}{7ac} + \left(8c^2\right)\int \frac{\sqrt{c-acx}}{1+ax}\,dx \\
&= \frac{16c^2\sqrt{c-acx}}{a} + \frac{8c(c-acx)^{3/2}}{3a} + \frac{4(c-acx)^{5/2}}{5a} + \frac{2(c-acx)^{7/2}}{7ac} + \left(16c^3\right)\int \frac{1}{(1+ax)\sqrt{c}} \\
&= \frac{16c^2\sqrt{c-acx}}{a} + \frac{8c(c-acx)^{3/2}}{3a} + \frac{4(c-acx)^{5/2}}{5a} + \frac{2(c-acx)^{7/2}}{7ac} - \frac{\left(32c^2\right)\operatorname{Subst}\left(\int \frac{1}{2-\frac{x}{c}}\right.}{a} \\
&= \frac{16c^2\sqrt{c-acx}}{a} + \frac{8c(c-acx)^{3/2}}{3a} + \frac{4(c-acx)^{5/2}}{5a} + \frac{2(c-acx)^{7/2}}{7ac} - \frac{16\sqrt{2}c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{}}\right.}{a}
\end{aligned}$$

Mathematica [A] time = 0.0563871, size = 80, normalized size = 0.69

$$-\frac{2c^2\left(\left(15a^3x^3-87a^2x^2+269ax-1037\right)\sqrt{c-acx}+840\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(5/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-2*c^2*(Sqrt[c - a*c*x]*(-1037 + 269*a*x - 87*a^2*x^2 + 15*a^3*x^3) + 840*
Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]))/(105*a)
```

Maple [A] time = 0.034, size = 87, normalized size = 0.8

$$2\frac{1}{ac}\left(1/7\,(-acx+c)^{7/2}+2/5\,c\,(-acx+c)^{5/2}+4/3\,(-acx+c)^{3/2}\,c^2+8\,\sqrt{-acx+c}c^3-8\,c^{7/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 2/c/a*(1/7*(-a*c*x+c)^(7/2)+2/5*c*(-a*c*x+c)^(5/2)+4/3*(-a*c*x+c)^(3/2)*c^2
+8*(-a*c*x+c)^(1/2)*c^3-8*c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1
/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.34143, size = 460, normalized size = 3.97

$$\left[\frac{2\left(420\sqrt{2}c^{\frac{5}{2}}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-\left(15a^3c^2x^3-87a^2c^2x^2+269ac^2x-1037c^2\right)\sqrt{-acx+c}\right)}{105a}, \frac{2\left(840\sqrt{2}\sqrt{-c}c^2\,\mathrm{arc}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [2/105*(420*sqrt(2)*c^(5/2)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) - (15*a^3*c^2*x^3 - 87*a^2*c^2*x^2 + 269*a*c^2*x - 1037*
c^2)*sqrt(-a*c*x + c))/a, 2/105*(840*sqrt(2)*sqrt(-c)*c^2*arctan(1/2*sqrt(2
)*sqrt(-a*c*x + c)*sqrt(-c)/c) - (15*a^3*c^2*x^3 - 87*a^2*c^2*x^2 + 269*a*c
^2*x - 1037*c^2)*sqrt(-a*c*x + c))/a]
```

Sympy [A] time = 55.4094, size = 109, normalized size = 0.94

$$\frac{16\sqrt{2}c^3\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{16c^2\sqrt{-acx+c}}{a}+\frac{8c\left(-acx+c\right)^{\frac{3}{2}}}{3a}+\frac{4\left(-acx+c\right)^{\frac{5}{2}}}{5a}+\frac{2\left(-acx+c\right)^{\frac{7}{2}}}{7ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(5/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] 16*sqrt(2)*c**3*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*sqrt(-c)) +
16*c**2*sqrt(-a*c*x + c)/a + 8*c*(-a*c*x + c)**(3/2)/(3*a) + 4*(-a*c*x + c)
**(5/2)/(5*a) + 2*(-a*c*x + c)**(7/2)/(7*a*c)
```

Giac [A] time = 1.21418, size = 181, normalized size = 1.56

$$\frac{16\sqrt{2}c^3\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\left(15\left(acx-c\right)^3\sqrt{-acx+c}a^6c^6-42\left(acx-c\right)^2\sqrt{-acx+c}a^6c^7-140\left(-acx+c\right)^{\frac{3}{2}}a^6c^8-840\right.}{105a^7c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 16*sqrt(2)*c^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) -
 2/105*(15*(a*c*x - c)^3*sqrt(-a*c*x + c)*a^6*c^6 - 42*(a*c*x - c)^2*sqrt(-
a*c*x + c)*a^6*c^7 - 140*(-a*c*x + c)^(3/2)*a^6*c^8 - 840*sqrt(-a*c*x + c)*
a^6*c^9)/(a^7*c^7)
```

3.263 $\int e^{-2\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=95

$$-\frac{8\sqrt{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}+\frac{2(c-acx)^{5/2}}{5ac}+\frac{4(c-acx)^{3/2}}{3a}+\frac{8c\sqrt{c-acx}}{a}$$

```
[Out] (8*c*Sqrt[c - a*c*x])/a + (4*(c - a*c*x)^(3/2))/(3*a) + (2*(c - a*c*x)^(5/2
))/(5*a*c) - (8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])
/a
```

Rubi [A] time = 0.0792074, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6130, 21, 50, 63, 206}

$$-\frac{8\sqrt{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}+\frac{2(c-acx)^{5/2}}{5ac}+\frac{4(c-acx)^{3/2}}{3a}+\frac{8c\sqrt{c-acx}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(3/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (8*c*Sqrt[c - a*c*x])/a + (4*(c - a*c*x)^(3/2))/(3*a) + (2*(c - a*c*x)^(5/2
))/(5*a*c) - (8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])
/a
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx &= \int \frac{(1-ax)(c-acx)^{3/2}}{1+ax}\,dx \\
&= \frac{\int \frac{(c-acx)^{5/2}}{1+ax}\,dx}{c} \\
&= \frac{2(c-acx)^{5/2}}{5ac} + 2\int \frac{(c-acx)^{3/2}}{1+ax}\,dx \\
&= \frac{4(c-acx)^{3/2}}{3a} + \frac{2(c-acx)^{5/2}}{5ac} + (4c)\int \frac{\sqrt{c-acx}}{1+ax}\,dx \\
&= \frac{8c\sqrt{c-acx}}{a} + \frac{4(c-acx)^{3/2}}{3a} + \frac{2(c-acx)^{5/2}}{5ac} + \left(8c^2\right)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx \\
&= \frac{8c\sqrt{c-acx}}{a} + \frac{4(c-acx)^{3/2}}{3a} + \frac{2(c-acx)^{5/2}}{5ac} - \frac{(16c)\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{a} \\
&= \frac{8c\sqrt{c-acx}}{a} + \frac{4(c-acx)^{3/2}}{3a} + \frac{2(c-acx)^{5/2}}{5ac} - \frac{8\sqrt{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0417121, size = 71, normalized size = 0.75

$$\frac{2c\left(3a^2x^2-16ax+73\right)\sqrt{c-acx}-120\sqrt{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(3/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (2*c*Sqrt[c - a*c*x]*(73 - 16*a*x + 3*a^2*x^2) - 120*Sqrt[2]*c^(3/2)*ArcTan
h[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(15*a)
```

Maple [A] time = 0.033, size = 73, normalized size = 0.8

$$2\,\frac{1}{ac}\left(1/5\,(-acx+c)^{5/2}+2/3\,c\,(-acx+c)^{3/2}+4\,\sqrt{-acx+c}c^2-4\,c^{5/2}\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 2/c/a*(1/5*(-a*c*x+c)^(5/2)+2/3*c*(-a*c*x+c)^(3/2)+4*(-a*c*x+c)^(1/2)*c^2-4
*c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.37417, size = 379, normalized size = 3.99

$$\left[\frac{2\left(30\sqrt{2}c^{\frac{3}{2}}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\left(3a^2cx^2-16acx+73c\right)\sqrt{-acx+c}\right)}{15a}, \frac{2\left(60\sqrt{2}\sqrt{-c}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)+\left(3\right.\right.}{15a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [2/15*(30*sqrt(2)*c^(3/2)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) + (3*a^2*c*x^2 - 16*a*c*x + 73*c)*sqrt(-a*c*x + c))/a, 2/1
5*(60*sqrt(2)*sqrt(-c)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) +
(3*a^2*c*x^2 - 16*a*c*x + 73*c)*sqrt(-a*c*x + c))/a]
```

Sympy [A] time = 42.4353, size = 90, normalized size = 0.95

$$\frac{8\sqrt{2}c^2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{8c\sqrt{-acx+c}}{a}+\frac{4\left(-acx+c\right)^{\frac{3}{2}}}{3a}+\frac{2\left(-acx+c\right)^{\frac{5}{2}}}{5ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(3/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] 8*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*sqrt(-c)) + 8
*c*sqrt(-a*c*x + c)/a + 4*(-a*c*x + c)**(3/2)/(3*a) + 2*(-a*c*x + c)**(5/2)
/(5*a*c)
```

Giac [A] time = 1.25342, size = 144, normalized size = 1.52

$$\frac{8\sqrt{2}c^2\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{2\left(3\left(acx-c\right)^2\sqrt{-acx+c}a^4c^4+10\left(-acx+c\right)^{\frac{3}{2}}a^4c^5+60\sqrt{-acx+c}a^4c^6\right)}{15a^5c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 8*sqrt(2)*c^2*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) +
2/15*(3*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c^4 + 10*(-a*c*x + c)^(3/2)*a^4*
c^5 + 60*sqrt(-a*c*x + c)*a^4*c^6)/(a^5*c^5)
```

3.264 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=76

$$\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{c-acx}}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] (4*Sqrt[c - a*c*x])/a + (2*(c - a*c*x)^(3/2))/(3*a*c) - (4*Sqrt[2]*Sqrt[c]*
ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.0661381, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6130, 21, 50, 63, 206}

$$\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{c-acx}}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a + (2*(c - a*c*x)^(3/2))/(3*a*c) - (4*Sqrt[2]*Sqrt[c]*
ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx &= \int \frac{(1-ax)\sqrt{c-acx}}{1+ax}\,dx\\
&= \frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{c}\\
&= \frac{2(c-acx)^{3/2}}{3ac} + 2\int \frac{\sqrt{c-acx}}{1+ax}\,dx\\
&= \frac{4\sqrt{c-acx}}{a} + \frac{2(c-acx)^{3/2}}{3ac} + (4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx\\
&= \frac{4\sqrt{c-acx}}{a} + \frac{2(c-acx)^{3/2}}{3ac} - \frac{8\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{a}\\
&= \frac{4\sqrt{c-acx}}{a} + \frac{2(c-acx)^{3/2}}{3ac} - \frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0398144, size = 61, normalized size = 0.8

$$
-\frac{2(ax-7)\sqrt{c-acx}+12\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{3a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(2*(-7 + a*x)*Sqrt[c - a*c*x] + 12*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]
/(Sqrt[2]*Sqrt[c])])/(3*a)
```

Maple [A] time = 0.032, size = 59, normalized size = 0.8

$$
2\,\frac{1}{ac}\left(1/3\,(-acx+c)^{3/2}+2\,c\sqrt{-acx+c}-2\,c^{3/2}\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 2/c/a*(1/3*(-a*c*x+c)^(3/2)+2*c*(-a*c*x+c)^(1/2)-2*c^(3/2)*2^(1/2)*arctanh(
1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.58972, size = 312, normalized size = 4.11

$$\left[\frac{2\left(3\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-\sqrt{-acx+c}(ax-7)\right)}{3a}, \frac{2\left(6\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-\sqrt{-acx+c}(ax-7)\right)}{3a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3
*c)/(a*x + 1)) - sqrt(-a*c*x + c)*(a*x - 7))/a, 2/3*(6*sqrt(2)*sqrt(-c)*arc
tan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(a*x - 7))/
a]
```

Sympy [A] time = 6.77008, size = 75, normalized size = 0.99

$$-\frac{2\left(-\frac{2\sqrt{2}c^2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-2c\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -2*(-2*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) -
2*c*sqrt(-a*c*x + c) - (-a*c*x + c)**(3/2)/3)/(a*c)
```

Giac [A] time = 1.27303, size = 104, normalized size = 1.37

$$\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{2\left((-acx+c)^{\frac{3}{2}}a^2c^2+6\sqrt{-acx+c}a^2c^3\right)}{3a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) + 2/
3*((-a*c*x + c)^(3/2)*a^2*c^2 + 6*sqrt(-a*c*x + c)*a^2*c^3)/(a^3*c^3)
```

3.265 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{c-acx}}{ac}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] (2*Sqrt[c - a*c*x])/(a*c) - (2*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqr
t[c])])/(a*Sqrt[c])
```

Rubi [A] time = 0.057621, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6130, 21, 50, 63, 206}

$$\frac{2\sqrt{c-acx}}{ac}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x])/(a*c) - (2*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqr
t[c])])/(a*Sqrt[c])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= \int \frac{1-ax}{(1+ax)\sqrt{c-acx}}\,dx \\
&= \frac{\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{c} \\
&= \frac{2\sqrt{c-acx}}{ac} + 2\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx \\
&= \frac{2\sqrt{c-acx}}{ac} - \frac{4\,\mathrm{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac} \\
&= \frac{2\sqrt{c-acx}}{ac} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.0245004, size = 58, normalized size = 1.

$$\frac{2\sqrt{c-acx}}{ac} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x])/(a*c) - (2*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqr
t[c])])/(a*Sqrt[c])
```

Maple [A] time = 0.033, size = 45, normalized size = 0.8

$$2\,\frac{1}{ac}\left(\sqrt{-acx+c} - \mathrm{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\sqrt{2}\sqrt{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/c/a*((-a*c*x+c)^(1/2)-arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/
2)*c^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.57227, size = 294, normalized size = 5.07

$$\left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{ax+\frac{2\sqrt{2}\sqrt{-acx+c}}{\sqrt{c}}-3}{ax+1}\right)+2\sqrt{-acx+c}}{ac}, -\frac{2\left(\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-\frac{1}{c}}}{ax-1}\right)-\sqrt{-acx+c}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*sqrt(c)*log((a*x + 2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(c) - 3)/(a*x +
 1)) + 2*sqrt(-a*c*x + c))/(a*c), -2*(sqrt(2)*c*sqrt(-1/c)*arctan(sqrt(2)*s
qrt(-a*c*x + c)*sqrt(-1/c)/(a*x - 1)) - sqrt(-a*c*x + c))/(a*c)]
```

Sympy [A] time = 23.6234, size = 58, normalized size = 1.

$$\frac{2\sqrt{-acx+c}}{ac}+\frac{2\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}}{\sqrt{-\frac{1}{c}}\sqrt{-acx+c}}\right)}{ac\sqrt{-\frac{1}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**(1/2),x)
```

```
[Out] 2*sqrt(-a*c*x + c)/(a*c) + 2*sqrt(2)*atan(sqrt(2)/(sqrt(-1/c)*sqrt(-a*c*x +
 c)))/(a*c*sqrt(-1/c))
```

Giac [A] time = 1.22109, size = 69, normalized size = 1.19

$$\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{2\sqrt{-acx+c}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) + 2*sq
rt(-a*c*x + c)/(a*c)
```

3.266 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

```
[Out] -((Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2)))
```

Rubi [A] time = 0.0511705, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6130, 21, 63, 206}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2)))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{1-ax}{(1+ax)(c-acx)^{3/2}}\,dx$$
$$= \frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{c}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac^2}$$
$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Mathematica [A] time = 0.0167521, size = 38, normalized size = 1.

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2)))
```

Maple [A] time = 0.032, size = 30, normalized size = 0.8

$$-\frac{\sqrt{2}}{a}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-acx+c}\frac{1}{\sqrt{c}}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(3/2),x)
```

```
[Out] -arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.49753, size = 232, normalized size = 6.11

$$\left[\frac{\sqrt{2}\log\left(\frac{ax+\frac{2\sqrt{2}\sqrt{-acx+c}}{\sqrt{c}}-3}{ax+1}\right)}{2\,ac^{\frac{3}{2}}}, -\frac{\sqrt{2}\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-\frac{1}{c}}}{ax-1}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log((a*x + 2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(c) - 3)/(a*x + 1))/
(a*c^(3/2)), -sqrt(2)*sqrt(-1/c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-1/c)
/(a*x - 1))/(a*c)]
```

Sympy [A] time = 32.3983, size = 39, normalized size = 1.03

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{ac\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**(3/2),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*c*sqrt(-c))
```

Giac [A] time = 1.19686, size = 47, normalized size = 1.24

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c)
```

3.267 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=57

$$\frac{1}{ac^2\sqrt{c-acx}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

```
[Out] 1/(a*c^2*Sqrt[c - a*c*x]) - ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(Sqr
t[2]*a*c^(5/2))
```

Rubi [A] time = 0.0604231, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6130, 21, 51, 63, 206}

$$\frac{1}{ac^2\sqrt{c-acx}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] 1/(a*c^2*Sqrt[c - a*c*x]) - ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(Sqr
t[2]*a*c^(5/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx &= \int \frac{1-ax}{(1+ax)(c-acx)^{5/2}}\,dx \\
&= \frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{c} \\
&= \frac{1}{ac^2\sqrt{c-acx}} + \frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{2c^2} \\
&= \frac{1}{ac^2\sqrt{c-acx}} - \frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac^3} \\
&= \frac{1}{ac^2\sqrt{c-acx}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0215347, size = 36, normalized size = 0.63

$$\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-ax)\right)}{ac^2\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, (1 - a*x)/2]/(a*c^2*Sqrt[c - a*c*x])
```

Maple [A] time = 0.035, size = 50, normalized size = 0.9

$$2\frac{1}{ac}\left(-1/4\frac{\sqrt{2}}{c^{3/2}}\text{Artanh}\left(1/2\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)+1/2\frac{1}{c\sqrt{-acx+c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(5/2),x)
```

```
[Out] 2/c/a*(-1/4/c^(3/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))+1
/2/c/(-a*c*x+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.54433, size = 360, normalized size = 6.32

$$\left[\frac{\sqrt{2}(ax-1)\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-4\sqrt{-acx+c}}{4\left(a^2c^3x-ac^3\right)}, \frac{\sqrt{2}(ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-2\sqrt{-acx+c}}{2\left(a^2c^3x-ac^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*(a*x - 1)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqr
t(c) - 3*c)/(a*x + 1)) - 4*sqrt(-a*c*x + c))/(a^2*c^3*x - a*c^3), 1/2*(sqrt
(2)*(a*x - 1)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*
sqrt(-a*c*x + c))/(a^2*c^3*x - a*c^3)]
```

Sympy [A] time = 21.7705, size = 60, normalized size = 1.05

$$\frac{1}{ac^2\sqrt{-acx+c}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{2ac^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**(5/2),x)
```

```
[Out] 1/(a*c**2*sqrt(-a*c*x + c)) + sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt
(-c)))/(2*a*c**2*sqrt(-c))
```

Giac [A] time = 1.25676, size = 72, normalized size = 1.26

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{2a\sqrt{-c}c^2}+\frac{1}{\sqrt{-acx+c}ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^2)
+ 1/(sqrt(-a*c*x + c)*a*c^2)
```

3.268 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=83

$$\frac{1}{2ac^3\sqrt{c-acx}}+\frac{1}{3ac^2(c-acx)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

```
[Out] 1/(3*a*c^2*(c - a*c*x)^(3/2)) + 1/(2*a*c^3*Sqrt[c - a*c*x]) - ArcTanh[Sqrt[
c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(2*Sqrt[2]*a*c^(7/2))
```

Rubi [A] time = 0.0696084, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6130, 21, 51, 63, 206}

$$\frac{1}{2ac^3\sqrt{c-acx}}+\frac{1}{3ac^2(c-acx)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] 1/(3*a*c^2*(c - a*c*x)^(3/2)) + 1/(2*a*c^3*Sqrt[c - a*c*x]) - ArcTanh[Sqrt[
c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(2*Sqrt[2]*a*c^(7/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx &= \int \frac{1-ax}{(1+ax)(c-acx)^{7/2}}\,dx\\ &= \frac{\int \frac{1}{(1+ax)(c-acx)^{5/2}}\,dx}{c}\\ &= \frac{1}{3ac^2(c-acx)^{3/2}}+\frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{2c^2}\\ &= \frac{1}{3ac^2(c-acx)^{3/2}}+\frac{1}{2ac^3\sqrt{c-acx}}+\frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{4c^3}\\ &= \frac{1}{3ac^2(c-acx)^{3/2}}+\frac{1}{2ac^3\sqrt{c-acx}}-\frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{2ac^4}\\ &= \frac{1}{3ac^2(c-acx)^{3/2}}+\frac{1}{2ac^3\sqrt{c-acx}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}\end{aligned}$$

Mathematica [C] time = 0.0245637, size = 39, normalized size = 0.47

$$\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1-ax)\right)}{3ac^2(c-acx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, (1 - a*x)/2]/(3*a*c^2*(c - a*c*x)^(3/2))
```

Maple [A] time = 0.043, size = 64, normalized size = 0.8

$$2\,\frac{1}{ac}\left(-1/8\,\frac{\sqrt{2}}{c^{5/2}}\text{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)+1/4\,\frac{1}{\sqrt{-acx+c}c^2}+1/6\,\frac{1}{c\,(-acx+c)^{3/2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(7/2),x)
```

```
[Out] 2/c/a*(-1/8/c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))+1
/4/c^2/(-a*c*x+c)^(1/2)+1/6/c/(-a*c*x+c)^(3/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.32513, size = 477, normalized size = 5.75

$$\left[\frac{3\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)-4\sqrt{-acx+c}(3\,ax-5)}{24\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}, \frac{3\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{\cdots}}{\cdots}\right)\cdots}{12\left(a^3c^4x^2-2\,a\cdots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) - 4*sqrt(-a*c*x + c)*(3*a*x - 5))/(a^
3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), 1/12*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqr
t(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*sqrt(-a*c*x + c)*
(3*a*x - 5))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [A] time = 58.3661, size = 80, normalized size = 0.96

$$\frac{1}{3ac^2\left(-acx+c\right)^{\frac{3}{2}}}+\frac{1}{2ac^3\sqrt{-acx+c}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{4ac^3\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**(7/2),x)
```

```
[Out] 1/(3*a*c**2*(-a*c*x + c)**(3/2)) + 1/(2*a*c**3*sqrt(-a*c*x + c)) + sqrt(2)*
atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(4*a*c**3*sqrt(-c))
```

Giac [A] time = 1.19311, size = 99, normalized size = 1.19

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{4\,a\sqrt{-c}c^3}+\frac{3\,acx-5\,c}{6\,(acx-c)\sqrt{-acx+c}ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^3)
+ 1/6*(3*a*c*x - 5*c)/((a*c*x - c)*sqrt(-a*c*x + c)*a*c^3)
```

3.269 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{9/2}}\,dx$

Optimal. Leaf size=104

$$\frac{1}{4ac^4\sqrt{c-acx}}+\frac{1}{6ac^3(c-acx)^{3/2}}+\frac{1}{5ac^2(c-acx)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

```
[Out] 1/(5*a*c^2*(c - a*c*x)^(5/2)) + 1/(6*a*c^3*(c - a*c*x)^(3/2)) + 1/(4*a*c^4*
Sqrt[c - a*c*x]) - ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(4*Sqrt[2]*a*
c^(9/2))
```

Rubi [A] time = 0.0817663, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6130, 21, 51, 63, 206}

$$\frac{1}{4ac^4\sqrt{c-acx}}+\frac{1}{6ac^3(c-acx)^{3/2}}+\frac{1}{5ac^2(c-acx)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^(9/2)),x]
```

```
[Out] 1/(5*a*c^2*(c - a*c*x)^(5/2)) + 1/(6*a*c^3*(c - a*c*x)^(3/2)) + 1/(4*a*c^4*
Sqrt[c - a*c*x]) - ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(4*Sqrt[2]*a*
c^(9/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{9/2}}\,dx &= \int \frac{1-ax}{(1+ax)(c-acx)^{9/2}}\,dx \\
&= \frac{\int \frac{1}{(1+ax)(c-acx)^{7/2}}\,dx}{c} \\
&= \frac{1}{5ac^2(c-acx)^{5/2}} + \frac{\int \frac{1}{(1+ax)(c-acx)^{5/2}}\,dx}{2c^2} \\
&= \frac{1}{5ac^2(c-acx)^{5/2}} + \frac{1}{6ac^3(c-acx)^{3/2}} + \frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{4c^3} \\
&= \frac{1}{5ac^2(c-acx)^{5/2}} + \frac{1}{6ac^3(c-acx)^{3/2}} + \frac{1}{4ac^4\sqrt{c-acx}} + \frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{8c^4} \\
&= \frac{1}{5ac^2(c-acx)^{5/2}} + \frac{1}{6ac^3(c-acx)^{3/2}} + \frac{1}{4ac^4\sqrt{c-acx}} - \frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{4ac^5} \\
&= \frac{1}{5ac^2(c-acx)^{5/2}} + \frac{1}{6ac^3(c-acx)^{3/2}} + \frac{1}{4ac^4\sqrt{c-acx}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}
\end{aligned}$$

Mathematica [C] time = 0.0299514, size = 39, normalized size = 0.38

$$\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1-ax)\right)}{5ac^2(c-acx)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^(9/2)),x]
```

```
[Out] Hypergeometric2F1[-5/2, 1, -3/2, (1 - a*x)/2]/(5*a*c^2*(c - a*c*x)^(5/2))
```

Maple [A] time = 0.044, size = 78, normalized size = 0.8

$$2\,\frac{1}{ac}\left(-1/16\,\frac{\sqrt{2}}{c^{7/2}}\text{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)+1/8\,\frac{1}{c^3\sqrt{-acx+c}}+1/12\,\frac{1}{c^2\,(-acx+c)^{3/2}}+1/10\,\frac{1}{c\,(-acx+c)^{5/2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(9/2),x)
```

```
[Out] 2/c/a*(-1/16/c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))+
1/8/c^3/(-a*c*x+c)^(1/2)+1/12/c^2/(-a*c*x+c)^(3/2)+1/10/c/(-a*c*x+c)^(5/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.32154, size = 598, normalized size = 5.75

$$\left[\frac{15\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-4\left(15a^2x^2-40ax+37\right)\sqrt{-acx+c}}{240\left(a^4c^5x^3-3a^3c^5x^2+3a^2c^5x-ac^5\right)}, \frac{15\sqrt{2}\left(a^3x^3-3a^2x^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log((a*c*x + 2
*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) - 4*(15*a^2*x^2 - 40*a*
x + 37)*sqrt(-a*c*x + c))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^
5), 1/120*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(1/2
*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*(15*a^2*x^2 - 40*a*x + 37)*sqrt(-
a*c*x + c))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15584, size = 126, normalized size = 1.21

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{8a\sqrt{-c}c^4}+\frac{15(acx-c)^2-10(acx-c)c+12c^2}{60(acx-c)^2\sqrt{-acx+c}ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^4)
+ 1/60*(15*(a*c*x - c)^2 - 10*(a*c*x - c)*c + 12*c^2)/((a*c*x - c)^2*sqrt(-
a*c*x + c)*a*c^4)
```

3.270 $\int e^{-3\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=171

$$\frac{2(c-acx)^{9/2}}{7ac^2\sqrt{1-a^2x^2}}-\frac{4096c^2\sqrt{c-acx}}{35a\sqrt{1-a^2x^2}}+\frac{32(c-acx)^{7/2}}{35ac\sqrt{1-a^2x^2}}+\frac{128(c-acx)^{5/2}}{35a\sqrt{1-a^2x^2}}+\frac{1024c(c-acx)^{3/2}}{35a\sqrt{1-a^2x^2}}$$

```
[Out] (-4096*c^2*Sqrt[c - a*c*x])/(35*a*Sqrt[1 - a^2*x^2]) + (1024*c*(c - a*c*x)^
(3/2))/(35*a*Sqrt[1 - a^2*x^2]) + (128*(c - a*c*x)^(5/2))/(35*a*Sqrt[1 - a^
2*x^2]) + (32*(c - a*c*x)^(7/2))/(35*a*c*Sqrt[1 - a^2*x^2]) + (2*(c - a*c*x
)^(9/2))/(7*a*c^2*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.136879, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6127, 657, 649}

$$\frac{2(c-acx)^{9/2}}{7ac^2\sqrt{1-a^2x^2}}-\frac{4096c^2\sqrt{c-acx}}{35a\sqrt{1-a^2x^2}}+\frac{32(c-acx)^{7/2}}{35ac\sqrt{1-a^2x^2}}+\frac{128(c-acx)^{5/2}}{35a\sqrt{1-a^2x^2}}+\frac{1024c(c-acx)^{3/2}}{35a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(5/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-4096*c^2*Sqrt[c - a*c*x])/(35*a*Sqrt[1 - a^2*x^2]) + (1024*c*(c - a*c*x)^
(3/2))/(35*a*Sqrt[1 - a^2*x^2]) + (128*(c - a*c*x)^(5/2))/(35*a*Sqrt[1 - a^
2*x^2]) + (32*(c - a*c*x)^(7/2))/(35*a*c*Sqrt[1 - a^2*x^2]) + (2*(c - a*c*x
)^(9/2))/(7*a*c^2*Sqrt[1 - a^2*x^2])
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{\int \frac{(c-acx)^{11/2}}{(1-a^2x^2)^{3/2}}dx}{c^3}$$

$$= \frac{2(c-acx)^{9/2}}{7ac^2\sqrt{1-a^2x^2}} + \frac{16\int \frac{(c-acx)^{9/2}}{(1-a^2x^2)^{3/2}}dx}{7c^2}$$

$$= \frac{32(c-acx)^{7/2}}{35ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{9/2}}{7ac^2\sqrt{1-a^2x^2}} + \frac{192\int \frac{(c-acx)^{7/2}}{(1-a^2x^2)^{3/2}}dx}{35c}$$

$$= \frac{128(c-acx)^{5/2}}{35a\sqrt{1-a^2x^2}} + \frac{32(c-acx)^{7/2}}{35ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{9/2}}{7ac^2\sqrt{1-a^2x^2}} + \frac{512}{35}\int \frac{(c-acx)^{5/2}}{(1-a^2x^2)^{3/2}}dx$$

$$= \frac{1024c(c-acx)^{3/2}}{35a\sqrt{1-a^2x^2}} + \frac{128(c-acx)^{5/2}}{35a\sqrt{1-a^2x^2}} + \frac{32(c-acx)^{7/2}}{35ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{9/2}}{7ac^2\sqrt{1-a^2x^2}} + \frac{1}{35}(2048c)\int$$

$$= -\frac{4096c^2\sqrt{c-acx}}{35a\sqrt{1-a^2x^2}} + \frac{1024c(c-acx)^{3/2}}{35a\sqrt{1-a^2x^2}} + \frac{128(c-acx)^{5/2}}{35a\sqrt{1-a^2x^2}} + \frac{32(c-acx)^{7/2}}{35ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)}{7ac^2\sqrt{1-a}}$$

Mathematica [A] time = 0.0389899, size = 70, normalized size = 0.41

$$\frac{2c^3\sqrt{1-ax}\left(5a^4x^4-36a^3x^3+142a^2x^2-708ax-1451\right)}{35a\sqrt{ax+1}\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^(5/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (2*c^3*Sqrt[1 - a*x]*(-1451 - 708*a*x + 142*a^2*x^2 - 36*a^3*x^3 + 5*a^4*x^
4))/(35*a*Sqrt[1 + a*x]*Sqrt[c - a*c*x])
```

Maple [A] time = 0.032, size = 71, normalized size = 0.4

$$\frac{10\,x^4a^4-72\,x^3a^3+284\,a^2x^2-1416\,ax-2902}{35\,(ax+1)^2\,(ax-1)^4\,a}\left(-a^2x^2+1\right)^{\frac{3}{2}}(-acx+c)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 2/35*(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(5/2)*(5*a^4*x^4-36*a^3*x^3+142*a^2*x^2-
708*a*x-1451)/(a*x+1)^2/(a*x-1)^4/a
```

Maxima [A] time = 1.0306, size = 99, normalized size = 0.58

$$\frac{2\left(5\,a^4c^{\frac{5}{2}}x^4-36\,a^3c^{\frac{5}{2}}x^3+142\,a^2c^{\frac{5}{2}}x^2-708\,ac^{\frac{5}{2}}x-1451\,c^{\frac{5}{2}}\right)\sqrt{ax+1}(ax-1)}{35\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/35*(5*a^4*c^(5/2)*x^4 - 36*a^3*c^(5/2)*x^3 + 142*a^2*c^(5/2)*x^2 - 708*a*
c^(5/2)*x - 1451*c^(5/2))*sqrt(a*x + 1)*(a*x - 1)/(a^3*x^2 - a)
```

Fricas [A] time = 1.98562, size = 180, normalized size = 1.05

$$-\frac{2\left(5\,a^4c^2x^4 - 36\,a^3c^2x^3 + 142\,a^2c^2x^2 - 708\,ac^2x - 1451\,c^2\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{35\left(a^3x^2 - a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="frica
s")
```

```
[Out] -2/35*(5*a^4*c^2*x^4 - 36*a^3*c^2*x^3 + 142*a^2*c^2*x^2 - 708*a*c^2*x - 145
1*c^2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^3*x^2 - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(5/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32841, size = 115, normalized size = 0.67

$$\frac{2048\,\sqrt{2}c^{\frac{3}{2}}|c|}{35\,a} + \frac{2\left(5\,(acx+c)^{\frac{7}{2}} - 56\,(acx+c)^{\frac{5}{2}}c + 280\,(acx+c)^{\frac{3}{2}}c^2 - 1120\,\sqrt{acx+c}c^3 - \frac{560\,c^4}{\sqrt{acx+c}}\right)|c|}{35\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac"
)
```

```
[Out] 2048/35*sqrt(2)*c^(3/2)*abs(c)/a + 2/35*(5*(a*c*x + c)^(7/2) - 56*(a*c*x +
c)^(5/2)*c + 280*(a*c*x + c)^(3/2)*c^2 - 1120*sqrt(a*c*x + c)*c^3 - 560*c^4
/sqrt(a*c*x + c))*abs(c)/(a*c^2)
```

3.271 $\int e^{-3\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=136

$$\frac{2(c-acx)^{7/2}}{5ac^2\sqrt{1-a^2x^2}}+\frac{8(c-acx)^{5/2}}{5ac\sqrt{1-a^2x^2}}+\frac{64(c-acx)^{3/2}}{5a\sqrt{1-a^2x^2}}-\frac{256c\sqrt{c-acx}}{5a\sqrt{1-a^2x^2}}$$

```
[Out] (-256*c*Sqrt[c - a*c*x])/(5*a*Sqrt[1 - a^2*x^2]) + (64*(c - a*c*x)^(3/2))/(
5*a*Sqrt[1 - a^2*x^2]) + (8*(c - a*c*x)^(5/2))/(5*a*c*Sqrt[1 - a^2*x^2]) +
(2*(c - a*c*x)^(7/2))/(5*a*c^2*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.112988, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6127, 657, 649}

$$\frac{2(c-acx)^{7/2}}{5ac^2\sqrt{1-a^2x^2}}+\frac{8(c-acx)^{5/2}}{5ac\sqrt{1-a^2x^2}}+\frac{64(c-acx)^{3/2}}{5a\sqrt{1-a^2x^2}}-\frac{256c\sqrt{c-acx}}{5a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(3/2)/E^(3*ArcTanh[a*x]),x]
```


```
[Out] (-256*c*Sqrt[c - a*c*x])/(5*a*Sqrt[1 - a^2*x^2]) + (64*(c - a*c*x)^(3/2))/(
5*a*Sqrt[1 - a^2*x^2]) + (8*(c - a*c*x)^(5/2))/(5*a*c*Sqrt[1 - a^2*x^2]) +
(2*(c - a*c*x)^(7/2))/(5*a*c^2*Sqrt[1 - a^2*x^2])
```


Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{\int \frac{(c-acx)^{9/2}}{\left(1-a^2x^2\right)^{3/2}}dx}{c^3}$$
$$= \frac{2(c-acx)^{7/2}}{5ac^2\sqrt{1-a^2x^2}} + \frac{12\int \frac{(c-acx)^{7/2}}{\left(1-a^2x^2\right)^{3/2}}dx}{5c^2}$$
$$= \frac{8(c-acx)^{5/2}}{5ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{7/2}}{5ac^2\sqrt{1-a^2x^2}} + \frac{32\int \frac{(c-acx)^{5/2}}{\left(1-a^2x^2\right)^{3/2}}dx}{5c}$$
$$= \frac{64(c-acx)^{3/2}}{5a\sqrt{1-a^2x^2}} + \frac{8(c-acx)^{5/2}}{5ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{7/2}}{5ac^2\sqrt{1-a^2x^2}} + \frac{128}{5}\int \frac{(c-acx)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}dx$$
$$= -\frac{256c\sqrt{c-acx}}{5a\sqrt{1-a^2x^2}} + \frac{64(c-acx)^{3/2}}{5a\sqrt{1-a^2x^2}} + \frac{8(c-acx)^{5/2}}{5ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{7/2}}{5ac^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0341686, size = 61, normalized size = 0.45

$$-\frac{2c^2\sqrt{1-ax}\left(a^3x^3-7a^2x^2+43ax+91\right)}{5a\sqrt{ax+1}\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^(3/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-2*c^2*Sqrt[1 - a*x]*(91 + 43*a*x - 7*a^2*x^2 + a^3*x^3))/(5*a*Sqrt[1 + a*
x]*Sqrt[c - a*c*x])
```

Maple [A] time = 0.033, size = 62, normalized size = 0.5

$$\frac{2\,x^3a^3-14\,a^2x^2+86\,ax+182}{5\,(ax+1)^2\,(ax-1)^3\,a}\left(-a^2x^2+1\right)^{\frac{3}{2}}(-acx+c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 2/5*(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(3/2)*(a^3*x^3-7*a^2*x^2+43*a*x+91)/(a*x+
1)^2/(a*x-1)^3/a
```

Maxima [A] time = 1.02732, size = 82, normalized size = 0.6

$$-\frac{2\left(a^3c^{\frac{3}{2}}x^3-7\,a^2c^{\frac{3}{2}}x^2+43\,ac^{\frac{3}{2}}x+91\,c^{\frac{3}{2}}\right)\sqrt{ax+1}(ax-1)}{5\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -2/5*(a^3*c^(3/2)*x^3 - 7*a^2*c^(3/2)*x^2 + 43*a*c^(3/2)*x + 91*c^(3/2))*sq
rt(a*x + 1)*(a*x - 1)/(a^3*x^2 - a)
```

Fricas [A] time = 1.99046, size = 134, normalized size = 0.99

$$\frac{2\left(a^3cx^3 - 7\,a^2cx^2 + 43\,acx + 91\,c\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{5\left(a^3x^2 - a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="frica
s")
```

```
[Out] 2/5*(a^3*c*x^3 - 7*a^2*c*x^2 + 43*a*c*x + 91*c)*sqrt(-a^2*x^2 + 1)*sqrt(-a*
c*x + c)/(a^3*x^2 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(ax-1\right)\right)^{\frac{3}{2}}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(3/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(3/2)*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3,
x)
```

Giac [A] time = 1.2811, size = 95, normalized size = 0.7

$$\frac{128\,\sqrt{2}\sqrt{c}|c|}{5\,a} - \frac{2\left(\left(acx+c\right)^{\frac{5}{2}} - 10\left(acx+c\right)^{\frac{3}{2}}c + 60\,\sqrt{acx+c}c^2 + \frac{40\,c^3}{\sqrt{acx+c}}\right)|c|}{5\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac"
)
```

```
[Out] 128/5*sqrt(2)*sqrt(c)*abs(c)/a - 2/5*((a*c*x + c)^(5/2) - 10*(a*c*x + c)^(3
/2)*c + 60*sqrt(a*c*x + c)*c^2 + 40*c^3/sqrt(a*c*x + c))*abs(c)/(a*c^2)
```

3.272 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=103

$$\frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}}+\frac{16(c-acx)^{3/2}}{3ac\sqrt{1-a^2x^2}}-\frac{64\sqrt{c-acx}}{3a\sqrt{1-a^2x^2}}$$

```
[Out] (-64*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - a^2*x^2]) + (16*(c - a*c*x)^(3/2))/(3*a
*c*Sqrt[1 - a^2*x^2]) + (2*(c - a*c*x)^(5/2))/(3*a*c^2*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.083532, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6127, 657, 649}

$$\frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}}+\frac{16(c-acx)^{3/2}}{3ac\sqrt{1-a^2x^2}}-\frac{64\sqrt{c-acx}}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-64*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - a^2*x^2]) + (16*(c - a*c*x)^(3/2))/(3*a
*c*Sqrt[1 - a^2*x^2]) + (2*(c - a*c*x)^(5/2))/(3*a*c^2*Sqrt[1 - a^2*x^2])
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\int \frac{(c-acx)^{7/2}}{(1-a^2x^2)^{3/2}}dx}{c^3}$$
$$= \frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}} + \frac{8\int \frac{(c-acx)^{5/2}}{(1-a^2x^2)^{3/2}}dx}{3c^2}$$
$$= \frac{16(c-acx)^{3/2}}{3ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}} + \frac{32\int \frac{(c-acx)^{3/2}}{(1-a^2x^2)^{3/2}}dx}{3c}$$
$$= -\frac{64\sqrt{c-acx}}{3a\sqrt{1-a^2x^2}} + \frac{16(c-acx)^{3/2}}{3ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0281885, size = 51, normalized size = 0.5

$$\frac{2c\sqrt{1-ax}\left(a^2x^2-10ax-23\right)}{3a\sqrt{ax+1}\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (2*c*Sqrt[1 - a*x]*(-23 - 10*a*x + a^2*x^2))/(3*a*Sqrt[1 + a*x]*Sqrt[c - a*
c*x])
```

Maple [A] time = 0.031, size = 54, normalized size = 0.5

$$\frac{2\,a^2x^2-20\,ax-46}{3\,(ax+1)^2\,(ax-1)^2\,a}\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 2/3*(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)*(a^2*x^2-10*a*x-23)/(a*x+1)^2/(a*x-
1)^2/a
```

Maxima [A] time = 1.04341, size = 68, normalized size = 0.66

$$\frac{2\left(a^2\sqrt{c}x^2-10\,a\sqrt{c}x-23\,\sqrt{c}\right)\sqrt{ax+1}(ax-1)}{3\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/3*(a^2*sqrt(c)*x^2 - 10*a*sqrt(c)*x - 23*sqrt(c))*sqrt(a*x + 1)*(a*x - 1)
/(a^3*x^2 - a)
```

Fricas [A] time = 1.59835, size = 108, normalized size = 1.05

$$-\frac{2\left(a^2x^2-10\,ax-23\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{3\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="frica
s")
```

```
[Out] -2/3*(a^2*x^2 - 10*a*x - 23)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^3*x^2 -
 a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\,(ax-1)}\,(-\,(ax-1)\,(ax+1))^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)
```

Giac [A] time = 1.25188, size = 77, normalized size = 0.75

$$\frac{32\,\sqrt{2}|c|}{3\,a\sqrt{c}}+\frac{2\left((acx+c)^{\frac{3}{2}}-12\,\sqrt{acx+c}c-\frac{12\,c^2}{\sqrt{acx+c}}\right)|c|}{3\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac"
)
```

```
[Out] 32/3*sqrt(2)*abs(c)/(a*sqrt(c)) + 2/3*((a*c*x + c)^(3/2) - 12*sqrt(a*c*x +
c)*c - 12*c^2/sqrt(a*c*x + c))*abs(c)/(a*c^2)
```

3.273 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=67

$$\frac{2(c-acx)^{3/2}}{ac^2\sqrt{1-a^2x^2}}-\frac{8\sqrt{c-acx}}{ac\sqrt{1-a^2x^2}}$$

```
[Out] (-8*Sqrt[c - a*c*x])/(a*c*Sqrt[1 - a^2*x^2]) + (2*(c - a*c*x)^(3/2))/(a*c^2
*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0639084, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6127, 657, 649}

$$\frac{2(c-acx)^{3/2}}{ac^2\sqrt{1-a^2x^2}}-\frac{8\sqrt{c-acx}}{ac\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (-8*Sqrt[c - a*c*x])/(a*c*Sqrt[1 - a^2*x^2]) + (2*(c - a*c*x)^(3/2))/(a*c^2
*Sqrt[1 - a^2*x^2])
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\int \frac{(c-acx)^{5/2}}{(1-a^2x^2)^{3/2}}\,dx}{c^3}$$

$$= \frac{2(c-acx)^{3/2}}{ac^2\sqrt{1-a^2x^2}} + \frac{4\int \frac{(c-acx)^{3/2}}{(1-a^2x^2)^{3/2}}\,dx}{c^2}$$

$$= -\frac{8\sqrt{c-acx}}{ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{3/2}}{ac^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0237497, size = 40, normalized size = 0.6

$$-\frac{2\sqrt{1-ax}(ax+3)}{a\sqrt{ax+1}\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (-2*Sqrt[1 - a*x]*(3 + a*x))/(a*Sqrt[1 + a*x]*Sqrt[c - a*c*x])
```

Maple [A] time = 0.029, size = 46, normalized size = 0.7

$$2\,\frac{\left(-a^2x^2+1\right)^{3/2}(ax+3)}{\sqrt{-acx+c}\,(ax+1)^2\,(ax-1)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(1/2),x)
```

```
[Out] 2*(-a^2*x^2+1)^(3/2)*(a*x+3)/(-a*c*x+c)^(1/2)/(a*x+1)^2/(a*x-1)/a
```

Maxima [A] time = 0.987793, size = 41, normalized size = 0.61

$$-\frac{2\,(ax+3)\sqrt{ax+1}}{a^2\sqrt{c}x+a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -2*(a*x + 3)*sqrt(a*x + 1)/(a^2*sqrt(c)*x + a*sqrt(c))
```

Fricas [A] time = 1.57468, size = 90, normalized size = 1.34

$$\frac{2\sqrt{-a^2x^2+1}\sqrt{-acx+c}(ax+3)}{a^3cx^2-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x + 3)/(a^3*c*x^2 - a*c)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.21528, size = 59, normalized size = 0.88

$$\frac{4\sqrt{2}|c|}{ac^{\frac{3}{2}}} - \frac{2\left(\sqrt{acx+c} + \frac{2c}{\sqrt{acx+c}}\right)|c|}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] 4*sqrt(2)*abs(c)/(a*c^(3/2)) - 2*(sqrt(a*c*x + c) + 2*c/sqrt(a*c*x + c))*ab
s(c)/(a*c^2)
```

3.274 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=33

$$-\frac{2\sqrt{c-acx}}{ac^2\sqrt{1-a^2x^2}}$$

```
[Out] (-2*Sqrt[c - a*c*x])/(a*c^2*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0504776, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6127, 649}

$$-\frac{2\sqrt{c-acx}}{ac^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^(3/2)),x]

[Out] (-2*Sqrt[c - a*c*x])/(a*c^2*Sqrt[1 - a^2*x^2])
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx &= \frac{\int \frac{(c-acx)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}\\ &= -\frac{2\sqrt{c-acx}}{ac^2\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0235171, size = 35, normalized size = 1.06

$$-\frac{2(1-ax)^{3/2}}{a\sqrt{ax+1}(c-acx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] (-2*(1 - a*x)^(3/2))/(a*Sqrt[1 + a*x]*(c - a*c*x)^(3/2))
```

Maple [A] time = 0.031, size = 34, normalized size = 1.

$$-2\,{\frac {\left(-{a}^{2}{x}^{2}+1\right)^{3/2}}{\left(-acx+c\right)^{3/2}\left(ax+1\right)^{2}a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2),x)
```

```
[Out] -2*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2)/(a*x+1)^2/a
```

Maxima [A] time = 0.984977, size = 38, normalized size = 1.15

$$-\frac{2\,\sqrt{ax+1}\sqrt{c}}{a^2c^2x+ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] -2*sqrt(a*x + 1)*sqrt(c)/(a^2*c^2*x + a*c^2)
```

Fricas [A] time = 1.57041, size = 82, normalized size = 2.48

$$\frac{2\,\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{a^3c^2x^2-ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^3*c^2*x^2 - a*c^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.19356, size = 41, normalized size = 1.24

$$\frac{\left(\frac{\sqrt{2}}{a\sqrt{c}} - \frac{2}{\sqrt{acx+ca}}\right)|c|}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] (sqrt(2)/(a*sqrt(c)) - 2/(sqrt(a*c*x + c)*a))*abs(c)/c^2
```

3.275 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{\sqrt{2}ac^{5/2}}-\frac{\sqrt{c-acx}}{ac^3\sqrt{1-a^2x^2}}$$

```
[Out] -(Sqrt[c - a*c*x]/(a*c^3*Sqrt[1 - a^2*x^2])) + ArcTanh[(Sqrt[c]*Sqrt[1 - a^
2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])]/(Sqrt[2]*a*c^(5/2))
```

Rubi [A] time = 0.0810259, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6127, 667, 661, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{\sqrt{2}ac^{5/2}}-\frac{\sqrt{c-acx}}{ac^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] -(Sqrt[c - a*c*x]/(a*c^3*Sqrt[1 - a^2*x^2])) + ArcTanh[(Sqrt[c]*Sqrt[1 - a^
2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])]/(Sqrt[2]*a*c^(5/2))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(d*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(p + 1)), x] + Dist[(d*(m + 2*p
 + 2))/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && LtQ[0, m,
1] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\int \frac{\sqrt{c-acx}}{(1-a^2x^2)^{3/2}}\,dx}{c^3}$$
$$= -\frac{\sqrt{c-acx}}{ac^3\sqrt{1-a^2x^2}} + \frac{\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{2c^2}$$
$$= -\frac{\sqrt{c-acx}}{ac^3\sqrt{1-a^2x^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)}{c}$$
$$= -\frac{\sqrt{c-acx}}{ac^3\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{\sqrt{2}ac^{5/2}}$$

Mathematica [C] time = 0.0243918, size = 55, normalized size = 0.65

$$-\frac{(1-ax)^{3/2}\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(ax+1)\right)}{ac\sqrt{ax+1}(c-acx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] -(((1 - a*x)^(3/2)*Hypergeometric2F1[-1/2, 1, 1/2, (1 + a*x)/2])/(a*c*Sqrt[
1 + a*x]*(c - a*c*x)^(3/2)))
```

Maple [A] time = 0.102, size = 82, normalized size = 1.

$$-\frac{1}{(2\,ax-2)\,(ax+1)\,a}\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}\left(\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{c\,(ax+1)}\frac{1}{\sqrt{c}}\right)\sqrt{2}\sqrt{c\,(ax+1)}-2\,\sqrt{c}\right)c^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(5/2),x)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(7/2)*(arctanh(1/2*(c*(a*x+1))
^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(c*(a*x+1))^(1/2)-2*c^(1/2))/(a*x-1)/(a*x+1
)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(-acx+c)^{\frac{5}{2}}(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((-a*c*x + c)^(5/2)*(a*x + 1)^3), x)
```

Fricas [A] time = 1.63432, size = 529, normalized size = 6.22

$$\left[\frac{\sqrt{2}\left(a^2x^2-1\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+4\,\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{4\left(a^3c^3x^2-ac^3\right)},\frac{\sqrt{2}\left(a^2x^2-1\right)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-a}}{}\right.}{2\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*(sqrt(2)*(a^2*x^2 - 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*s
qrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) +
4*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^3*c^3*x^2 - a*c^3), 1/2*(sqrt(2)*
(a^2*x^2 - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*s
qrt(-c)/(a^2*c*x^2 - c)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^3*c^3*
x^2 - a*c^3)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.19903, size = 127, normalized size = 1.49

$$-\frac{\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}c}-\frac{\sqrt{2}\left(\sqrt{c}\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)+\sqrt{-c}\right)}{a\sqrt{-c}c^{\frac{3}{2}}}+\frac{2}{\sqrt{acx+c}ac}\right)|c|}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="gia
c")
```

```
[Out] -1/2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c) -
 sqrt(2)*(sqrt(c)*arctan(sqrt(c)/sqrt(-c)) + sqrt(-c))/(a*sqrt(-c)*c^(3/2))
 + 2/(sqrt(a*c*x + c)*a*c))*abs(c)/c^2
```

3.276 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{3\sqrt{c-acx}}{4ac^4\sqrt{1-a^2x^2}}+\frac{1}{2ac^3\sqrt{1-a^2x^2}\sqrt{c-acx}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{4\sqrt{2}ac^{7/2}}$$

```
[Out] 1/(2*a*c^3*Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2]) - (3*Sqrt[c - a*c*x])/(4*a*c^
4*Sqrt[1 - a^2*x^2]) + (3*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt
[c - a*c*x])])/(4*Sqrt[2]*a*c^(7/2))
```

Rubi [A] time = 0.102302, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6127, 673, 667, 661, 208}

$$-\frac{3\sqrt{c-acx}}{4ac^4\sqrt{1-a^2x^2}}+\frac{1}{2ac^3\sqrt{1-a^2x^2}\sqrt{c-acx}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{4\sqrt{2}ac^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^(7/2)),x]
```


```
[Out] 1/(2*a*c^3*Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2]) - (3*Sqrt[c - a*c*x])/(4*a*c^
4*Sqrt[1 - a^2*x^2]) + (3*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt
[c - a*c*x])])/(4*Sqrt[2]*a*c^(7/2))
```


Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(d*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(p + 1)), x] + Dist[(d*(m + 2*p
 + 2))/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && LtQ[0, m,
1] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
```

```
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx &= \frac{\int \frac{1}{\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}\\
&= \frac{1}{2ac^3\sqrt{c-acx}\sqrt{1-a^2x^2}}+\frac{3\int \frac{\sqrt{c-acx}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{4c^4}\\
&= \frac{1}{2ac^3\sqrt{c-acx}\sqrt{1-a^2x^2}}-\frac{3\sqrt{c-acx}}{4ac^4\sqrt{1-a^2x^2}}+\frac{3\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{8c^3}\\
&= \frac{1}{2ac^3\sqrt{c-acx}\sqrt{1-a^2x^2}}-\frac{3\sqrt{c-acx}}{4ac^4\sqrt{1-a^2x^2}}-\frac{(3a)\,\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)}{4c^2}\\
&= \frac{1}{2ac^3\sqrt{c-acx}\sqrt{1-a^2x^2}}-\frac{3\sqrt{c-acx}}{4ac^4\sqrt{1-a^2x^2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{4\sqrt{2}ac^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.0292282, size = 57, normalized size = 0.46

$$-\frac{(1-ax)^{3/2}\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},\frac{1}{2}(ax+1)\right)}{2ac^2\sqrt{ax+1}(c-acx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] -((1 - a*x)^(3/2)*Hypergeometric2F1[-1/2, 2, 1/2, (1 + a*x)/2])/(2*a*c^2*Sq
rt[1 + a*x]*(c - a*c*x)^(3/2))
```

Maple [A] time = 0.118, size = 124, normalized size = 1.

$$-\frac{1}{8\,(ax-1)^2\,(ax+1)\,a}\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}\left(3\,\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)xa\sqrt{c\,(ax+1)}-3\,\text{Artanh}\left(1/2\,\frac{\sqrt{c\,(a}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(7/2),x)
```

```
[Out] -1/8*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(9/2)*(3*2^(1/2)*arctanh(1/2*(
c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*(c*(a*x+1))^(1/2)-3*arctanh(1/2*(c*(a
*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(c*(a*x+1))^(1/2)-6*x*a*c^(1/2)+2*c^(
1/2))/(a*x-1)^2/(a*x+1)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(-acx+c\right)^{\frac{7}{2}}\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((-a*c*x + c)^(7/2)*(a*x + 1)^3), x)
```

Fricas [A] time = 1.70856, size = 682, normalized size = 5.46

$$\left[\frac{3\sqrt{2}\left(a^3x^3-a^2x^2-ax+1\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+4\,\sqrt{-a^2x^2+1}\sqrt{-acx+c}(3\,ax-1)}{16\left(a^4c^4x^3-a^3c^4x^2-a^2c^4x+ac^4\right)},\ 3\sqrt{2}(\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="fri
cas")
```

```
[Out] [1/16*(3*sqrt(2)*(a^3*x^3 - a^2*x^2 - a*x + 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*
a*c*x - 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x
^2 - 2*a*x + 1)) + 4*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(3*a*x - 1))/(a^4*
c^4*x^3 - a^3*c^4*x^2 - a^2*c^4*x + a*c^4), 1/8*(3*sqrt(2)*(a^3*x^3 - a^2*x
^2 - a*x + 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*s
qrt(-c)/(a^2*c*x^2 - c)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(3*a*x - 1
))/(a^4*c^4*x^3 - a^3*c^4*x^2 - a^2*c^4*x + a*c^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31478, size = 111, normalized size = 0.89

$$-\frac{\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}c^2}+\frac{2\,(3\,acx-c)}{\left((acx+c)^{\frac{3}{2}}-2\,\sqrt{acx+c}c\right)ac^2}\right)|c|}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="gia
c")
```

```
[Out] -1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^
2) + 2*(3*a*c*x - c)/((((a*c*x + c)^(3/2) - 2*sqrt(a*c*x + c)*c)*a*c^2))*abs
(c)/c^2
```

3.277 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^{9/2}}\,dx$

Optimal. Leaf size=160

$$-\frac{15\sqrt{c-acx}}{32ac^5\sqrt{1-a^2x^2}}+\frac{5}{16ac^4\sqrt{1-a^2x^2}\sqrt{c-acx}}+\frac{1}{4ac^3\sqrt{1-a^2x^2}(c-acx)^{3/2}}+\frac{15\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{32\sqrt{2}ac^{9/2}}$$

```
[Out] 1/(4*a*c^3*(c - a*c*x)^(3/2)*Sqrt[1 - a^2*x^2]) + 5/(16*a*c^4*Sqrt[c - a*c*
x]*Sqrt[1 - a^2*x^2]) - (15*Sqrt[c - a*c*x])/(32*a*c^5*Sqrt[1 - a^2*x^2]) +
 (15*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/(32*Sq
rt[2]*a*c^(9/2))
```

Rubi [A] time = 0.129878, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6127, 673, 667, 661, 208}

$$-\frac{15\sqrt{c-acx}}{32ac^5\sqrt{1-a^2x^2}}+\frac{5}{16ac^4\sqrt{1-a^2x^2}\sqrt{c-acx}}+\frac{1}{4ac^3\sqrt{1-a^2x^2}(c-acx)^{3/2}}+\frac{15\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{32\sqrt{2}ac^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^(9/2)),x]
```


```
[Out] 1/(4*a*c^3*(c - a*c*x)^(3/2)*Sqrt[1 - a^2*x^2]) + 5/(16*a*c^4*Sqrt[c - a*c*
x]*Sqrt[1 - a^2*x^2]) - (15*Sqrt[c - a*c*x])/(32*a*c^5*Sqrt[1 - a^2*x^2]) +
 (15*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/(32*Sq
rt[2]*a*c^(9/2))
```


Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(d*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(p + 1)), x] + Dist[(d*(m + 2*p
 + 2))/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && LtQ[0, m,
1] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
```

```
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \frac{\int \frac{1}{(c-acx)^{3/2}\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$

$$= \frac{1}{4ac^3(c-acx)^{3/2}\sqrt{1-a^2x^2}} + \frac{5\int \frac{1}{\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}\,dx}{8c^4}$$

$$= \frac{1}{4ac^3(c-acx)^{3/2}\sqrt{1-a^2x^2}} + \frac{5}{16ac^4\sqrt{c-acx}\sqrt{1-a^2x^2}} + \frac{15\int \frac{\sqrt{c-acx}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{32c^5}$$

$$= \frac{1}{4ac^3(c-acx)^{3/2}\sqrt{1-a^2x^2}} + \frac{5}{16ac^4\sqrt{c-acx}\sqrt{1-a^2x^2}} - \frac{15\sqrt{c-acx}}{32ac^5\sqrt{1-a^2x^2}} + \frac{15\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{64c^4}$$

$$= \frac{1}{4ac^3(c-acx)^{3/2}\sqrt{1-a^2x^2}} + \frac{5}{16ac^4\sqrt{c-acx}\sqrt{1-a^2x^2}} - \frac{15\sqrt{c-acx}}{32ac^5\sqrt{1-a^2x^2}} - \frac{(15a)\,\mathrm{Subst}\left(\int \frac{1}{-2a^2c+a^2}\right.}{32c^3}$$

$$= \frac{1}{4ac^3(c-acx)^{3/2}\sqrt{1-a^2x^2}} + \frac{5}{16ac^4\sqrt{c-acx}\sqrt{1-a^2x^2}} - \frac{15\sqrt{c-acx}}{32ac^5\sqrt{1-a^2x^2}} + \frac{15\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{32\sqrt{2}ac^{9/2}}$$

Mathematica [C] time = 0.0299716, size = 57, normalized size = 0.36

$$-\frac{(1-ax)^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},3,\frac{1}{2},\frac{1}{2}(ax+1)\right)}{4ac^3\sqrt{ax+1}(c-acx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^(9/2)),x]
```

```
[Out] -((1 - a*x)^(3/2)*Hypergeometric2F1[-1/2, 3, 1/2, (1 + a*x)/2])/(4*a*c^3*Sq
rt[1 + a*x]*(c - a*c*x)^(3/2))
```

Maple [A] time = 0.119, size = 173, normalized size = 1.1

$$-\frac{1}{64\,(ax-1)^3\,(ax+1)\,a}\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}\left(15\,\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)\sqrt{2}x^2a^2\sqrt{c\,(ax+1)}-30\,\sqrt{2}\mathrm{Artanh}\left(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(9/2),x)
```

```
[Out] -1/64*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(11/2)*(15*arctanh(1/2*(c*(a*
x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*x^2*a^2*(c*(a*x+1))^(1/2)-30*2^(1/2)*a
```

```
rctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*(c*(a*x+1))^(1/2)-30*x^2*
a^2*c^(1/2)+15*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(c*(a
*x+1))^(1/2)+40*x*a*c^(1/2)+6*c^(1/2))/(a*x-1)^3/(a*x+1)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(-acx+c\right)^{\frac{9}{2}}\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(9/2),x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((-a*c*x + c)^(9/2)*(a*x + 1)^3), x)
```

Fricas [A] time = 1.63633, size = 747, normalized size = 4.67

$$\left[\frac{15\sqrt{2}\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+4\left(15\,a^2x^2-20\,ax-3\right)\sqrt{-a^2x^2+1}\sqrt{\cdot}}{128\left(a^5c^5x^4-2\,a^4c^5x^3+2\,a^2c^5x-ac^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(9/2),x, algorithm="fri
cas")
```

```
[Out] [1/128*(15*sqrt(2)*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*sqrt(c)*log(-(a^2*c*x^
2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/
(a^2*x^2 - 2*a*x + 1)) + 4*(15*a^2*x^2 - 20*a*x - 3)*sqrt(-a^2*x^2 + 1)*sqr
t(-a*c*x + c))/(a^5*c^5*x^4 - 2*a^4*c^5*x^3 + 2*a^2*c^5*x - a*c^5), 1/64*(1
5*sqrt(2)*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a
^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 2*(15*a^2*x^2 - 20
*a*x - 3)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^5*c^5*x^4 - 2*a^4*c^5*x^3
 + 2*a^2*c^5*x - a*c^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30077, size = 134, normalized size = 0.84

$$-\frac{\left(\frac{15\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}c^3}+\frac{16}{\sqrt{acx+c}ac^3}+\frac{2\left(7(acx+c)^{\frac{3}{2}}-18\sqrt{acx+c}c\right)}{(acx-c)^2ac^3}\right)|c|}{64c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(9/2),x, algorithm="gia
c")
```

```
[Out] -1/64*(15*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*
c^3) + 16/(sqrt(a*c*x + c)*a*c^3) + 2*(7*(a*c*x + c)^(3/2) - 18*sqrt(a*c*x
+ c)*c)/((a*c*x - c)^2*a*c^3))*abs(c)/c^2
```

3.278 $\int e^{n\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=81

$$-\frac{2^{\frac{n}{2}+1}(c-acx)^{9/2}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(\frac{9-n}{2},-\frac{n}{2},\frac{11-n}{2},\frac{1}{2}(1-ax)\right)}{ac(9-n)}$$

```
[Out] -((2^(1 + n/2)*(c - a*c*x)^(9/2)*Hypergeometric2F1[(9 - n)/2, -n/2, (11 - n
)/2, (1 - a*x)/2])/(a*c*(9 - n)*(1 - a*x)^(n/2)))
```

Rubi [A] time = 0.0667769, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 23, 69}

$$-\frac{2^{\frac{n}{2}+1}(c-acx)^{9/2}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{9-n}{2},-\frac{n}{2};\frac{11-n}{2};\frac{1}{2}(1-ax)\right)}{ac(9-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] -((2^(1 + n/2)*(c - a*c*x)^(9/2)*Hypergeometric2F1[(9 - n)/2, -n/2, (11 - n
)/2, (1 - a*x)/2])/(a*c*(9 - n)*(1 - a*x)^(n/2)))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int e^{n\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx &= \int (1-ax)^{-n/2}(1+ax)^{n/2}(c-acx)^{7/2}\,dx\\
&= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{\frac{7}{2}-\frac{n}{2}}\,dx\\
&= -\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}(c-acx)^{9/2}\,{}_2F_1\left(\frac{9-n}{2},-\frac{n}{2};\frac{11-n}{2};\frac{1}{2}(1-ax)\right)}{ac(9-n)}
\end{aligned}$$

Mathematica [A] time = 0.0535116, size = 80, normalized size = 0.99

$$\frac{c^3 2^{\frac{n}{2}+1}\sqrt{c-acx}(1-ax)^{4-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{9}{2}-\frac{n}{2},-\frac{n}{2},\frac{11}{2}-\frac{n}{2},\frac{1}{2}-\frac{ax}{2}\right)}{a(n-9)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (2^(1 + n/2)*c^3*(1 - a*x)^(4 - n/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[9/2
- n/2, -n/2, 11/2 - n/2, 1/2 - (a*x)/2])/(a*(-9 + n))
```

Maple [F] time = 0.246, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}(-acx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a*c*x+c)^(7/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a*c*x+c)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx+c)^{\frac{7}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(7/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^3c^3x^3-3\,a^2c^3x^2+3\,ac^3x-c^3\right)\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^3*c^3*x^3 - 3*a^2*c^3*x^2 + 3*a*c^3*x - c^3)*sqrt(-a*c*x + c)*
((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx+c)^{\frac{7}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(7/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.279 $\int e^{n\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=81

$$-\frac{2^{\frac{n}{2}+1}(c-acx)^{7/2}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(\frac{7-n}{2},-\frac{n}{2},\frac{9-n}{2},\frac{1}{2}(1-ax)\right)}{ac(7-n)}$$

```
[Out] -((2^(1 + n/2)*(c - a*c*x)^(7/2)*Hypergeometric2F1[(7 - n)/2, -n/2, (9 - n)
/2, (1 - a*x)/2])/(a*c*(7 - n)*(1 - a*x)^(n/2)))
```

Rubi [A] time = 0.0702035, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 23, 69}

$$-\frac{2^{\frac{n}{2}+1}(c-acx)^{7/2}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{7-n}{2},-\frac{n}{2};\frac{9-n}{2};\frac{1}{2}(1-ax)\right)}{ac(7-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] -((2^(1 + n/2)*(c - a*c*x)^(7/2)*Hypergeometric2F1[(7 - n)/2, -n/2, (9 - n)
/2, (1 - a*x)/2])/(a*c*(7 - n)*(1 - a*x)^(n/2)))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int e^{n\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx &= \int (1-ax)^{-n/2}(1+ax)^{n/2}(c-acx)^{5/2}\,dx\\
&= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{\frac{5}{2}-\frac{n}{2}}\,dx\\
&= -\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}(c-acx)^{7/2}\,{}_2F_1\left(\frac{7-n}{2},-\frac{n}{2};\frac{9-n}{2};\frac{1}{2}(1-ax)\right)}{ac(7-n)}
\end{aligned}$$

Mathematica [A] time = 0.0432954, size = 80, normalized size = 0.99

$$\frac{c^2 2^{\frac{n}{2}+1}\sqrt{c-acx}(1-ax)^{3-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{7}{2}-\frac{n}{2},-\frac{n}{2},\frac{9}{2}-\frac{n}{2},\frac{1}{2}-\frac{ax}{2}\right)}{a(n-7)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (2^(1 + n/2)*c^2*(1 - a*x)^(3 - n/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[7/2
- n/2, -n/2, 9/2 - n/2, 1/2 - (a*x)/2])/(a*(-7 + n))
```

Maple [F] time = 0.233, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}(-acx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a*c*x+c)^(5/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a*c*x+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(-acx+c)^{\frac{5}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(5/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2c^2x^2-2\,ac^2x+c^2\right)\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*sqrt(-a*c*x + c)*((a*x + 1)/(a*x -
 1))^(1/2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx+c)^{\frac{5}{2}} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(5/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.280 $\int e^{n\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=81

$$-\frac{2^{\frac{n}{2}+1}(c-acx)^{5/2}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(\frac{5-n}{2},-\frac{n}{2},\frac{7-n}{2},\frac{1}{2}(1-ax)\right)}{ac(5-n)}$$

```
[Out] -((2^(1 + n/2)*(c - a*c*x)^(5/2)*Hypergeometric2F1[(5 - n)/2, -n/2, (7 - n)
/2, (1 - a*x)/2])/(a*c*(5 - n)*(1 - a*x)^(n/2)))
```

Rubi [A] time = 0.0679431, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 23, 69}

$$-\frac{2^{\frac{n}{2}+1}(c-acx)^{5/2}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{5-n}{2},-\frac{n}{2};\frac{7-n}{2};\frac{1}{2}(1-ax)\right)}{ac(5-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] -((2^(1 + n/2)*(c - a*c*x)^(5/2)*Hypergeometric2F1[(5 - n)/2, -n/2, (7 - n)
/2, (1 - a*x)/2])/(a*c*(5 - n)*(1 - a*x)^(n/2)))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx &= \int (1-ax)^{-n/2}(1+ax)^{n/2}(c-acx)^{3/2}\,dx\\ &= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{\frac{3}{2}-\frac{n}{2}}\,dx\\ &= -\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}(c-acx)^{5/2}\,{}_2F_1\left(\frac{5-n}{2},-\frac{n}{2};\frac{7-n}{2};\frac{1}{2}(1-ax)\right)}{ac(5-n)}\end{aligned}$$

Mathematica [A] time = 0.0353, size = 78, normalized size = 0.96

$$\frac{c2^{\frac{n}{2}+1}\sqrt{c-acx}(1-ax)^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{5}{2}-\frac{n}{2},-\frac{n}{2},\frac{7}{2}-\frac{n}{2},\frac{1}{2}-\frac{ax}{2}\right)}{a(n-5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (2^(1 + n/2)*c*(1 - a*x)^(2 - n/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[5/2 -
n/2, -n/2, 7/2 - n/2, 1/2 - (a*x)/2])/(a*(-5 + n))
```

Maple [F] time = 0.23, size = 0, normalized size = 0.

$$\int e^{n\operatorname{Artanh}(ax)}(-acx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a*c*x+c)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a*c*x+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx+c)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-(acx-c)\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*c*x - c)*sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a*c*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx+c)^{\frac{3}{2}} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.281 $\int e^{n\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=81

$$-\frac{2^{\frac{n}{2}+1}(c-acx)^{3/2}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(\frac{3-n}{2},-\frac{n}{2},\frac{5-n}{2},\frac{1}{2}(1-ax)\right)}{ac(3-n)}$$

```
[Out] -((2^(1 + n/2)*(c - a*c*x)^(3/2)*Hypergeometric2F1[(3 - n)/2, -n/2, (5 - n)
/2, (1 - a*x)/2])/(a*c*(3 - n)*(1 - a*x)^(n/2)))
```

Rubi [A] time = 0.0633918, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 23, 69}

$$-\frac{2^{\frac{n}{2}+1}(c-acx)^{3/2}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{3-n}{2},-\frac{n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{ac(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] -((2^(1 + n/2)*(c - a*c*x)^(3/2)*Hypergeometric2F1[(3 - n)/2, -n/2, (5 - n)
/2, (1 - a*x)/2])/(a*c*(3 - n)*(1 - a*x)^(n/2)))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}\sqrt{c-acx}\,dx &= \int (1-ax)^{-n/2}(1+ax)^{n/2}\sqrt{c-acx}\,dx\\ &= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{\frac{1}{2}-\frac{n}{2}}\,dx\\ &= -\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}(c-acx)^{3/2}\,{}_2F_1\left(\frac{3-n}{2},-\frac{n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{ac(3-n)}\end{aligned}$$

Mathematica [A] time = 0.0306401, size = 77, normalized size = 0.95

$$\frac{2^{\frac{n}{2}+1}\sqrt{c-acx}(1-ax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{3}{2}-\frac{n}{2},-\frac{n}{2},\frac{5}{2}-\frac{n}{2},\frac{1}{2}-\frac{ax}{2}\right)}{a(n-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2^(1 + n/2)*(1 - a*x)^(1 - n/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[3/2 - n/
2, -n/2, 5/2 - n/2, 1/2 - (a*x)/2])/(a*(-3 + n))
```

Maple [F] time = 0.224, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}\sqrt{-acx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a*c*x+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a*c*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.282 $\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=81

$$-\frac{2^{\frac{n}{2}+1}\sqrt{c-acx}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(\frac{1-n}{2},-\frac{n}{2},\frac{3-n}{2},\frac{1}{2}(1-ax)\right)}{ac(1-n)}$$

```
[Out] -((2^(1 + n/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[(1 - n)/2, -n/2, (3 - n)/2
, (1 - a*x)/2])/(a*c*(1 - n)*(1 - a*x)^(n/2)))
```

Rubi [A] time = 0.0648541, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 23, 69}

$$-\frac{2^{\frac{n}{2}+1}\sqrt{c-acx}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1-n}{2},-\frac{n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{ac(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] -((2^(1 + n/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[(1 - n)/2, -n/2, (3 - n)/2
, (1 - a*x)/2])/(a*c*(1 - n)*(1 - a*x)^(n/2)))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{\sqrt{c-acx}}\,dx$$
$$= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{-\frac{1}{2}-\frac{n}{2}}\,dx$$
$$= -\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\sqrt{c-acx}\,{}_2F_1\left(\frac{1-n}{2},-\frac{n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{ac(1-n)}$$

Mathematica [A] time = 0.0256864, size = 78, normalized size = 0.96

$$\frac{2^{\frac{n}{2}+1}\sqrt{c-acx}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{n}{2},-\frac{n}{2},\frac{3}{2}-\frac{n}{2},\frac{1}{2}-\frac{ax}{2}\right)}{ac(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (2^(1 + n/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[1/2 - n/2, -n/2, 3/2 - n/2,
1/2 - (a*x)/2])/(a*c*(-1 + n)*(1 - a*x)^(n/2))
```

Maple [F] time = 0.231, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}\frac{1}{\sqrt{-acx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a*c*x+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/(-a*c*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-acx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a*c*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{atanh}(ax)}}{\sqrt{-c(ax-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/sqrt(-c*(a*x - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-acx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a*c*x + c), x)
```

3.283 $\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-1),-\frac{n}{2},\frac{1-n}{2},\frac{1}{2}(1-ax)\right)}{ac(n+1)\sqrt{c-acx}}$$

```
[Out] (2^(1 + n/2)*Hypergeometric2F1[(-1 - n)/2, -n/2, (1 - n)/2, (1 - a*x)/2])/(
a*c*(1 + n)*(1 - a*x)^(n/2)*Sqrt[c - a*c*x])
```

Rubi [A] time = 0.0682523, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 23, 69}

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1}{2}(-n-1),-\frac{n}{2};\frac{1-n}{2};\frac{1}{2}(1-ax)\right)}{ac(n+1)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (2^(1 + n/2)*Hypergeometric2F1[(-1 - n)/2, -n/2, (1 - n)/2, (1 - a*x)/2])/(
a*c*(1 + n)*(1 - a*x)^(n/2)*Sqrt[c - a*c*x])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{(c-acx)^{3/2}}\,dx$$
$$= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{-\frac{3}{2}-\frac{n}{2}}\,dx$$
$$= \frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1}{2}(-1-n),-\frac{n}{2};\frac{1-n}{2};\frac{1}{2}(1-ax)\right)}{ac(1+n)\sqrt{c-acx}}$$

Mathematica [A] time = 0.0243085, size = 78, normalized size = 1.

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2}-\frac{1}{2},-\frac{n}{2},\frac{1}{2}-\frac{n}{2},\frac{1}{2}-\frac{ax}{2}\right)}{ac(n+1)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (2^(1 + n/2)*Hypergeometric2F1[-1/2 - n/2, -n/2, 1/2 - n/2, 1/2 - (a*x)/2])
/(a*c*(1 + n)*(1 - a*x)^(n/2)*Sqrt[c - a*c*x])
```

Maple [F] time = 0.212, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}(-acx+c)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a*c*x+c)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/(-a*c*x+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2c^2x^2-2\,ac^2x+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*x^2 - 2*a*
c^2*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{atanh}(ax)}}{(-c(ax-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/(-c*(a*x - 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(3/2), x)
```

3.284 $\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-3),-\frac{n}{2},\frac{1}{2}(-n-1),\frac{1}{2}(1-ax)\right)}{ac(n+3)(c-acx)^{3/2}}$$

```
[Out] (2^(1 + n/2)*Hypergeometric2F1[(-3 - n)/2, -n/2, (-1 - n)/2, (1 - a*x)/2])/
(a*c*(3 + n)*(1 - a*x)^(n/2)*(c - a*c*x)^(3/2))
```

Rubi [A] time = 0.0671078, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 23, 69}

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1}{2}(-n-3),-\frac{n}{2};\frac{1}{2}(-n-1);\frac{1}{2}(1-ax)\right)}{ac(n+3)(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] (2^(1 + n/2)*Hypergeometric2F1[(-3 - n)/2, -n/2, (-1 - n)/2, (1 - a*x)/2])/
(a*c*(3 + n)*(1 - a*x)^(n/2)*(c - a*c*x)^(3/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{(c-acx)^{5/2}}\,dx$$
$$= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{-\frac{5}{2}-\frac{n}{2}}\,dx$$
$$= \frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1}{2}(-3-n),-\frac{n}{2};\frac{1}{2}(-1-n);\frac{1}{2}(1-ax)\right)}{ac(3+n)(c-acx)^{3/2}}$$

Mathematica [A] time = 0.0316857, size = 80, normalized size = 1.03

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-\frac{n}{2}-1}\text{Hypergeometric2F1}\left(-\frac{n}{2}-\frac{3}{2},-\frac{n}{2},-\frac{n}{2}-\frac{1}{2},\frac{1}{2}-\frac{ax}{2}\right)}{ac^2(n+3)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] (2^(1 + n/2)*(1 - a*x)^(-1 - n/2)*Hypergeometric2F1[-3/2 - n/2, -n/2, -1/2
- n/2, 1/2 - (a*x)/2])/(a*c^2*(3 + n)*Sqrt[c - a*c*x])
```

Maple [F] time = 0.219, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}(-acx+c)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a*c*x+c)^(5/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/(-a*c*x+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^3c^3x^3-3\,a^2c^3x^2+3\,ac^3x-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^3*c^3*x^3 - 3*a
^2*c^3*x^2 + 3*a*c^3*x - c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(5/2), x)
```

3.285 $\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=78

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-5),-\frac{n}{2},\frac{1}{2}(-n-3),\frac{1}{2}(1-ax)\right)}{ac(n+5)(c-acx)^{5/2}}$$

```
[Out] (2^(1 + n/2)*Hypergeometric2F1[(-5 - n)/2, -n/2, (-3 - n)/2, (1 - a*x)/2])/
(a*c*(5 + n)*(1 - a*x)^(n/2)*(c - a*c*x)^(5/2))
```

Rubi [A] time = 0.0672785, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 23, 69}

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1}{2}(-n-5),-\frac{n}{2};\frac{1}{2}(-n-3);\frac{1}{2}(1-ax)\right)}{ac(n+5)(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] (2^(1 + n/2)*Hypergeometric2F1[(-5 - n)/2, -n/2, (-3 - n)/2, (1 - a*x)/2])/
(a*c*(5 + n)*(1 - a*x)^(n/2)*(c - a*c*x)^(5/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{(c-acx)^{7/2}}\,dx$$
$$= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{-\frac{7}{2}-\frac{n}{2}}\,dx$$
$$= \frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1}{2}(-5-n),-\frac{n}{2};\frac{1}{2}(-3-n);\frac{1}{2}(1-ax)\right)}{ac(5+n)(c-acx)^{5/2}}$$

Mathematica [A] time = 0.0423045, size = 80, normalized size = 1.03

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-\frac{n}{2}-2}\text{Hypergeometric2F1}\left(-\frac{n}{2}-\frac{5}{2},-\frac{n}{2},-\frac{n}{2}-\frac{3}{2},\frac{1}{2}-\frac{ax}{2}\right)}{ac^3(n+5)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] (2^(1 + n/2)*(1 - a*x)^(-2 - n/2)*Hypergeometric2F1[-5/2 - n/2, -n/2, -3/2
- n/2, 1/2 - (a*x)/2])/(a*c^3*(5 + n)*Sqrt[c - a*c*x])
```

Maple [F] time = 0.228, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}(-acx+c)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a*c*x+c)^(7/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/(-a*c*x+c)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^4x^4-4a^3c^4x^3+6a^2c^4x^2-4ac^4x+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^4*x^4 - 4*a^
3*c^4*x^3 + 6*a^2*c^4*x^2 - 4*a*c^4*x + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(7/2), x)
```

3.286 $\int e^{\tanh^{-1}(ax)}x^4(c-acx)\,dx$

Optimal. Leaf size=83

$$\frac{1}{6}cx^5\sqrt{1-a^2x^2}-\frac{cx^3\sqrt{1-a^2x^2}}{24a^2}-\frac{cx\sqrt{1-a^2x^2}}{16a^4}+\frac{c\sin^{-1}(ax)}{16a^5}$$

```
[Out] -(c*x*Sqrt[1 - a^2*x^2])/(16*a^4) - (c*x^3*Sqrt[1 - a^2*x^2])/(24*a^2) + (c
*x^5*Sqrt[1 - a^2*x^2])/6 + (c*ArcSin[a*x])/(16*a^5)
```

Rubi [A] time = 0.0708394, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6128, 279, 321, 216}

$$\frac{1}{6}cx^5\sqrt{1-a^2x^2}-\frac{cx^3\sqrt{1-a^2x^2}}{24a^2}-\frac{cx\sqrt{1-a^2x^2}}{16a^4}+\frac{c\sin^{-1}(ax)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^4*(c - a*c*x),x]
```

```
[Out] -(c*x*Sqrt[1 - a^2*x^2])/(16*a^4) - (c*x^3*Sqrt[1 - a^2*x^2])/(24*a^2) + (c
*x^5*Sqrt[1 - a^2*x^2])/6 + (c*ArcSin[a*x])/(16*a^5)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^4(c-acx)\,dx = c\int x^4\sqrt{1-a^2x^2}\,dx$$
$$= \frac{1}{6}cx^5\sqrt{1-a^2x^2}+\frac{1}{6}c\int\frac{x^4}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{cx^3\sqrt{1-a^2x^2}}{24a^2}+\frac{1}{6}cx^5\sqrt{1-a^2x^2}+\frac{c\int\frac{x^2}{\sqrt{1-a^2x^2}}\,dx}{8a^2}$$
$$= -\frac{cx\sqrt{1-a^2x^2}}{16a^4}-\frac{cx^3\sqrt{1-a^2x^2}}{24a^2}+\frac{1}{6}cx^5\sqrt{1-a^2x^2}+\frac{c\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{16a^4}$$
$$= -\frac{cx\sqrt{1-a^2x^2}}{16a^4}-\frac{cx^3\sqrt{1-a^2x^2}}{24a^2}+\frac{1}{6}cx^5\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{16a^5}$$

Mathematica [A] time = 0.0570437, size = 50, normalized size = 0.6

$$\frac{c\left(ax\sqrt{1-a^2x^2}\left(8a^4x^4-2a^2x^2-3\right)+3\sin^{-1}(ax)\right)}{48a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^4*(c - a*c*x),x]
```

```
[Out] (c*(a*x*Sqrt[1 - a^2*x^2]*(-3 - 2*a^2*x^2 + 8*a^4*x^4) + 3*ArcSin[a*x]))/(4
8*a^5)
```

Maple [A] time = 0.041, size = 91, normalized size = 1.1

$$\frac{cx^5}{6}\sqrt{-a^2x^2+1}-\frac{cx^3}{24\,a^2}\sqrt{-a^2x^2+1}-\frac{cx}{16\,a^4}\sqrt{-a^2x^2+1}+\frac{c}{16\,a^4}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4*(-a*c*x+c),x)
```

```
[Out] 1/6*c*x^5*(-a^2*x^2+1)^(1/2)-1/24*c*x^3*(-a^2*x^2+1)^(1/2)/a^2-1/16*c*x*(-a
^2*x^2+1)^(1/2)/a^4+1/16*c/a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1
)^(1/2))
```

Maxima [A] time = 1.43528, size = 109, normalized size = 1.31

$$\frac{1}{6}\sqrt{-a^2x^2+1}cx^5-\frac{\sqrt{-a^2x^2+1}cx^3}{24\,a^2}-\frac{\sqrt{-a^2x^2+1}cx}{16\,a^4}+\frac{c\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{16\sqrt{a^2}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(-a^2*x^2 + 1)*c*x^5 - 1/24*sqrt(-a^2*x^2 + 1)*c*x^3/a^2 - 1/16*sqr
t(-a^2*x^2 + 1)*c*x/a^4 + 1/16*c*arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^4)
```

Fricas [A] time = 1.72732, size = 155, normalized size = 1.87

$$-\frac{6\,c\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)-\left(8\,a^{5}cx^{5}-2\,a^{3}cx^{3}-3\,acx\right)\sqrt{-a^{2}x^{2}+1}}{48\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/48*(6*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (8*a^5*c*x^5 - 2*a^3*c*
x^3 - 3*a*c*x)*sqrt(-a^2*x^2 + 1))/a^5
```

Sympy [C] time = 11.942, size = 357, normalized size = 4.3

$$-a^{2}c\left(\begin{cases}-\frac{ix^{7}}{6\sqrt{a^{2}x^{2}-1}}-\frac{ix^{5}}{24a^{2}\sqrt{a^{2}x^{2}-1}}-\frac{5ix^{3}}{48a^{4}\sqrt{a^{2}x^{2}-1}}+\frac{5ix}{16a^{6}\sqrt{a^{2}x^{2}-1}}-\frac{5i\operatorname{acosh}(ax)}{16a^{7}} & \text{for } \left|a^{2}x^{2}\right|>1\\ \frac{x^{7}}{6\sqrt{-a^{2}x^{2}+1}}+\frac{x^{5}}{24a^{2}\sqrt{-a^{2}x^{2}+1}}+\frac{5x^{3}}{48a^{4}\sqrt{-a^{2}x^{2}+1}}-\frac{5x}{16a^{6}\sqrt{-a^{2}x^{2}+1}}+\frac{5\operatorname{asin}(ax)}{16a^{7}} & \text{otherwise}\end{cases}\right)+c\left(\begin{cases}-\frac{ix^{5}}{4\sqrt{a^{2}x^{2}-1}}-\frac{ix^{3}}{8a^{2}\sqrt{a^{2}x^{2}-1}} \\ \frac{x^{5}}{4\sqrt{-a^{2}x^{2}+1}}+\frac{x^{3}}{8a^{2}\sqrt{-a^{2}x^{2}+1}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4*(-a*c*x+c),x)
```

```
[Out] -a**2*c*Piecewise((-I*x**7/(6*sqrt(a**2*x**2 - 1)) - I*x**5/(24*a**2*sqrt(a
**2*x**2 - 1)) - 5*I*x**3/(48*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(16*a**6*sq
rt(a**2*x**2 - 1)) - 5*I*acosh(a*x)/(16*a**7), Abs(a**2*x**2) > 1), (x**7/(
6*sqrt(-a**2*x**2 + 1)) + x**5/(24*a**2*sqrt(-a**2*x**2 + 1)) + 5*x**3/(48*
a**4*sqrt(-a**2*x**2 + 1)) - 5*x/(16*a**6*sqrt(-a**2*x**2 + 1)) + 5*asin(a*
x)/(16*a**7), True)) + c*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**
3/(8*a**2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*a
cosh(a*x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x
**3/(8*a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*a
sin(a*x)/(8*a**5), True))
```

Giac [A] time = 1.19992, size = 77, normalized size = 0.93

$$\frac{1}{48}\,\sqrt{-a^{2}x^{2}+1}\left(2\left(4\,cx^{2}-\frac{c}{a^{2}}\right)x^{2}-\frac{3\,c}{a^{4}}\right)x+\frac{c\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{16\,a^{4}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(-a^2*x^2 + 1)*(2*(4*c*x^2 - c/a^2)*x^2 - 3*c/a^4)*x + 1/16*c*arcs
in(a*x)*sgn(a)/(a^4*abs(a))
```

3.287 $\int e^{\tanh^{-1}(ax)}x^3(c-acx)\,dx$

Optimal. Leaf size=45

$$\frac{c\left(1-a^2x^2\right)^{5/2}}{5a^4}-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a^4}$$

[Out] -(c*(1 - a^2*x^2)^(3/2))/(3*a^4) + (c*(1 - a^2*x^2)^(5/2))/(5*a^4)

Rubi [A] time = 0.0649259, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6128, 266, 43}

$$\frac{c\left(1-a^2x^2\right)^{5/2}}{5a^4}-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a^4}$$

Antiderivative was successfully verified.

[In] Int[E^ArcTanh[a*x]*x^3*(c - a*c*x),x]

[Out] -(c*(1 - a^2*x^2)^(3/2))/(3*a^4) + (c*(1 - a^2*x^2)^(5/2))/(5*a^4)

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x^3(c-acx)\,dx &= c\int x^3\sqrt{1-a^2x^2}\,dx\\ &= \frac{1}{2}c\,\text{Subst}\left(\int x\sqrt{1-a^2x}\,dx,x,x^2\right)\\ &= \frac{1}{2}c\,\text{Subst}\left(\int\left(\frac{\sqrt{1-a^2x}}{a^2}-\frac{\left(1-a^2x\right)^{3/2}}{a^2}\right)dx,x,x^2\right)\\ &= -\frac{c\left(1-a^2x^2\right)^{3/2}}{3a^4}+\frac{c\left(1-a^2x^2\right)^{5/2}}{5a^4}\end{aligned}$$

Mathematica [A] time = 0.0241801, size = 32, normalized size = 0.71

$$-\frac{c\left(1-a^2x^2\right)^{3/2}\left(3a^2x^2+2\right)}{15a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^3*(c - a*c*x),x]
```

```
[Out] -(c*(1 - a^2*x^2)^(3/2)*(2 + 3*a^2*x^2))/(15*a^4)
```

Maple [A] time = 0.033, size = 43, normalized size = 1.

$$-\frac{(ax-1)^2(ax+1)^2\left(3a^2x^2+2\right)c}{15a^4}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c),x)
```

```
[Out] -1/15*(a*x-1)^2*(a*x+1)^2*(3*a^2*x^2+2)*c/a^4/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.42526, size = 78, normalized size = 1.73

$$\frac{1}{5}\sqrt{-a^2x^2+1}cx^4-\frac{\sqrt{-a^2x^2+1}cx^2}{15a^2}-\frac{2\sqrt{-a^2x^2+1}c}{15a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(-a^2*x^2 + 1)*c*x^4 - 1/15*sqrt(-a^2*x^2 + 1)*c*x^2/a^2 - 2/15*sqr
t(-a^2*x^2 + 1)*c/a^4
```

Fricas [A] time = 1.5817, size = 82, normalized size = 1.82

$$\frac{\left(3a^4cx^4-a^2cx^2-2c\right)\sqrt{-a^2x^2+1}}{15a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a^4*c*x^4 - a^2*c*x^2 - 2*c)*sqrt(-a^2*x^2 + 1)/a^4
```

Sympy [A] time = 1.00177, size = 66, normalized size = 1.47

$$\begin{cases} \frac{cx^4\sqrt{-a^2x^2+1}}{5}-\frac{cx^2\sqrt{-a^2x^2+1}}{15a^2}-\frac{2c\sqrt{-a^2x^2+1}}{15a^4} & \text{for } a\neq 0 \\ \frac{cx^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3*(-a*c*x+c),x)
```

```
[Out] Piecewise((c*x**4*sqrt(-a**2*x**2 + 1)/5 - c*x**2*sqrt(-a**2*x**2 + 1)/(15*
a**2) - 2*c*sqrt(-a**2*x**2 + 1)/(15*a**4), Ne(a, 0)), (c*x**4/4, True))
```

Giac [A] time = 1.20941, size = 63, normalized size = 1.4

$$\frac{3\left(a^2x^2-1\right)^2\sqrt{-a^2x^2+1}c-5\left(-a^2x^2+1\right)^{\frac{3}{2}}c}{15\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 1/15*(3*(a^2*x^2 - 1)^2*sqrt(-a^2*x^2 + 1)*c - 5*(-a^2*x^2 + 1)^(3/2)*c)/a^
4
```

3.288 $\int e^{\tanh^{-1}(ax)} x^2 (c - acx) \, dx$

Optimal. Leaf size=58

$$\frac{1}{4} c x^3 \sqrt{1 - a^2 x^2} - \frac{c x \sqrt{1 - a^2 x^2}}{8 a^2} + \frac{c \sin^{-1}(ax)}{8 a^3}$$

```
[Out] -(c*x*Sqrt[1 - a^2*x^2])/(8*a^2) + (c*x^3*Sqrt[1 - a^2*x^2])/4 + (c*ArcSin[
a*x])/(8*a^3)
```

Rubi [A] time = 0.0568359, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6128, 279, 321, 216}

$$\frac{1}{4} c x^3 \sqrt{1 - a^2 x^2} - \frac{c x \sqrt{1 - a^2 x^2}}{8 a^2} + \frac{c \sin^{-1}(ax)}{8 a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2*(c - a*c*x),x]
```

```
[Out] -(c*x*Sqrt[1 - a^2*x^2])/(8*a^2) + (c*x^3*Sqrt[1 - a^2*x^2])/4 + (c*ArcSin[
a*x])/(8*a^3)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^2(c-acx)\,dx = c\int x^2\sqrt{1-a^2x^2}\,dx$$
$$= \frac{1}{4}cx^3\sqrt{1-a^2x^2} + \frac{1}{4}c\int \frac{x^2}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{cx\sqrt{1-a^2x^2}}{8a^2} + \frac{1}{4}cx^3\sqrt{1-a^2x^2} + \frac{c\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{8a^2}$$
$$= -\frac{cx\sqrt{1-a^2x^2}}{8a^2} + \frac{1}{4}cx^3\sqrt{1-a^2x^2} + \frac{c\sin^{-1}(ax)}{8a^3}$$

Mathematica [A] time = 0.0412941, size = 40, normalized size = 0.69

$$\frac{c\left(ax\sqrt{1-a^2x^2}\left(2a^2x^2-1\right)+\sin^{-1}(ax)\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^2*(c - a*c*x),x]
```

```
[Out] (c*(a*x*Sqrt[1 - a^2*x^2]*(-1 + 2*a^2*x^2) + ArcSin[a*x]))/(8*a^3)
```

Maple [A] time = 0.039, size = 70, normalized size = 1.2

$$\frac{cx^3}{4}\sqrt{-a^2x^2+1}-\frac{cx}{8\,a^2}\sqrt{-a^2x^2+1}+\frac{c}{8\,a^2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c),x)
```

```
[Out] 1/4*c*x^3*(-a^2*x^2+1)^(1/2)-1/8*c*x*(-a^2*x^2+1)^(1/2)/a^2+1/8*c/a^2/(a^2)
^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.43469, size = 81, normalized size = 1.4

$$\frac{1}{4}\sqrt{-a^2x^2+1}cx^3-\frac{\sqrt{-a^2x^2+1}cx}{8\,a^2}+\frac{c\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\,\sqrt{a^2}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(-a^2*x^2 + 1)*c*x^3 - 1/8*sqrt(-a^2*x^2 + 1)*c*x/a^2 + 1/8*c*arcsi
n(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2)
```

Fricas [A] time = 1.60507, size = 132, normalized size = 2.28

$$-\frac{2\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(2\,a^3cx^3-acx\right)\sqrt{-a^2x^2+1}}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/8*(2*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (2*a^3*c*x^3 - a*c*x)*sq
rt(-a^2*x^2 + 1))/a^3
```

Sympy [C] time = 7.4772, size = 245, normalized size = 4.22

$$-a^{2}c\left(\begin{cases} -\frac{ix^{5}}{4\sqrt{a^{2}x^{2}-1}} - \frac{ix^{3}}{8a^{2}\sqrt{a^{2}x^{2}-1}} + \frac{3ix}{8a^{4}\sqrt{a^{2}x^{2}-1}} - \frac{3i\operatorname{acosh}(ax)}{8a^{5}} & \text{for}\ \left|a^{2}x^{2}\right| > 1 \\ \frac{x^{5}}{4\sqrt{-a^{2}x^{2}+1}} + \frac{x^{3}}{8a^{2}\sqrt{-a^{2}x^{2}+1}} - \frac{3x}{8a^{4}\sqrt{-a^{2}x^{2}+1}} + \frac{3\operatorname{asin}(ax)}{8a^{5}} & \text{otherwise} \end{cases}\right) + c\left(\begin{cases} -\frac{ix\sqrt{a^{2}x^{2}-1}}{2a^{2}} - \frac{i\operatorname{acosh}(ax)}{2a^{3}} & \text{f} \\ \frac{x^{3}}{2\sqrt{-a^{2}x^{2}+1}} - \frac{x}{2a^{2}\sqrt{-a^{2}x^{2}+1}} + \frac{\operatorname{asin}(ax)}{2a^{3}} & \text{o} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(-a*c*x+c),x)
```

```
[Out] -a**2*c*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(a*
*2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a**5
), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqrt(
-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**5)
, True)) + c*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2
*a**3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqr
t(-a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True))
```

Giac [A] time = 1.22126, size = 61, normalized size = 1.05

$$\frac{1}{8}\sqrt{-a^{2}x^{2}+1}\left(2\,cx^{2}-\frac{c}{a^{2}}\right)x+\frac{c\arcsin(ax)\operatorname{sgn}(a)}{8\,a^{2}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(-a^2*x^2 + 1)*(2*c*x^2 - c/a^2)*x + 1/8*c*arcsin(a*x)*sgn(a)/(a^2*
abs(a))
```

3.289 $\int e^{\tanh^{-1}(ax)}x(c-acx)\,dx$

Optimal. Leaf size=22

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a^2}$$

```
[Out] -(c*(1 - a^2*x^2)^(3/2))/(3*a^2)
```

Rubi [A] time = 0.0282274, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6128, 261}

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x*(c - a*c*x),x]
```

```
[Out] -(c*(1 - a^2*x^2)^(3/2))/(3*a^2)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x(c-acx)\,dx &= c\int x\sqrt{1-a^2x^2}\,dx\\ &= -\frac{c\left(1-a^2x^2\right)^{3/2}}{3a^2}\end{aligned}$$

Mathematica [A] time = 0.0127704, size = 22, normalized size = 1.

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x*(c - a*c*x),x]
```

```
[Out] -(c*(1 - a^2*x^2)^(3/2))/(3*a^2)
```

Maple [A] time = 0.029, size = 33, normalized size = 1.5

$$-\frac{(ax-1)^2(ax+1)^2c}{3a^2}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c),x)
```

```
[Out] -1/3*(a*x-1)^2*(a*x+1)^2*c/a^2/(-a^2*x^2+1)^(1/2)
```

Maxima [B] time = 1.4168, size = 50, normalized size = 2.27

$$\frac{1}{3}\sqrt{-a^2x^2+1}cx^2-\frac{\sqrt{-a^2x^2+1}c}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-a^2*x^2 + 1)*c*x^2 - 1/3*sqrt(-a^2*x^2 + 1)*c/a^2
```

Fricas [A] time = 1.51222, size = 59, normalized size = 2.68

$$\frac{\left(a^2cx^2-c\right)\sqrt{-a^2x^2+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*c*x^2 - c)*sqrt(-a^2*x^2 + 1)/a^2
```

Sympy [A] time = 0.580131, size = 42, normalized size = 1.91

$$\begin{cases} \frac{cx^2\sqrt{-a^2x^2+1}}{3}-\frac{c\sqrt{-a^2x^2+1}}{3a^2} & \text{for } a\neq 0 \\ \frac{cx^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(-a*c*x+c),x)
```

```
[Out] Piecewise((c*x**2*sqrt(-a**2*x**2 + 1)/3 - c*sqrt(-a**2*x**2 + 1)/(3*a**2),
 Ne(a, 0)), (c*x**2/2, True))
```

Giac [A] time = 1.22562, size = 24, normalized size = 1.09

$$-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}c}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/3*(-a^2*x^2 + 1)^(3/2)*c/a^2
```

3.290 $\int e^{\tanh^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}$$

```
[Out] (c*x*Sqrt[1 - a^2*x^2])/2 + (c*ArcSin[a*x])/(2*a)
```

Rubi [A] time = 0.0198065, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6127, 195, 216}

$$\frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x),x]
```

```
[Out] (c*x*Sqrt[1 - a^2*x^2])/2 + (c*ArcSin[a*x])/(2*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}(c-acx)\,dx &= c\int\sqrt{1-a^2x^2}\,dx\\ &= \frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{1}{2}c\int\frac{1}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}\end{aligned}$$

Mathematica [A] time = 0.0093686, size = 30, normalized size = 0.91

$$\frac{c\left(ax\sqrt{1-a^2x^2}+\sin^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x),x]
```

```
[Out] (c*(a*x*Sqrt[1 - a^2*x^2] + ArcSin[a*x]))/(2*a)
```

Maple [A] time = 0.043, size = 46, normalized size = 1.4

$$\frac{cx}{2}\sqrt{-a^2x^2+1}+\frac{c}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c),x)
```

```
[Out] 1/2*c*x*(-a^2*x^2+1)^(1/2)+1/2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2
+1)^(1/2))
```

Maxima [A] time = 1.41016, size = 49, normalized size = 1.48

$$\frac{1}{2}\sqrt{-a^2x^2+1}cx+\frac{c\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*c*x + 1/2*c*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2)
```

Fricas [A] time = 1.75386, size = 107, normalized size = 3.24

$$\frac{\sqrt{-a^2x^2+1}acx-2c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-a^2*x^2 + 1)*a*c*x - 2*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))
/a
```

Sympy [A] time = 3.39108, size = 46, normalized size = 1.39

$$\begin{cases} -\frac{c\left(\begin{cases} -\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\operatorname{asin}(ax)}{2} & \text{for } ax>-1 \wedge ax<1 \end{cases}\right)-c\operatorname{asin}(ax)}{a} & \text{for } a\neq 0 \\ cx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c),x)
```

```
[Out] Piecewise((-(c*Piecewise((-a*x*sqrt(-a**2*x**2 + 1)/2 + asin(a*x)/2, (a*x >
 -1) & (a*x < 1))) - c*asin(a*x))/a, Ne(a, 0)), (c*x, True))
```

Giac [A] time = 1.20023, size = 41, normalized size = 1.24

$$\frac{1}{2}\sqrt{-a^2x^2+1}cx+\frac{c\arcsin(ax)\,\mathrm{sgn}(a)}{2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*c*x + 1/2*c*arcsin(a*x)*sgn(a)/abs(a)
```

3.291 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)}{x} dx$

Optimal. Leaf size=35

$$c\sqrt{1-a^2x^2} - c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] c*Sqrt[1 - a^2*x^2] - c*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.0530402, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6128, 266, 50, 63, 208}

$$c\sqrt{1-a^2x^2} - c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x))/x,x]
```

```
[Out] c*Sqrt[1 - a^2*x^2] - c*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}(c-acx)}{x}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x}\,dx\\
&= \frac{1}{2}c\,\text{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x}\,dx, x, x^2\right)\\
&= c\sqrt{1-a^2x^2}+\frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)\\
&= c\sqrt{1-a^2x^2}-\frac{c\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2}\\
&= c\sqrt{1-a^2x^2}-c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [B] time = 0.0778286, size = 79, normalized size = 2.26

$$c\left(-\frac{a^2x^2}{\sqrt{1-a^2x^2}}+\frac{1}{\sqrt{1-a^2x^2}}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\sin^{-1}(ax)+2\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x))/x,x]
```

```
[Out] c*(1/Sqrt[1 - a^2*x^2] - (a^2*x^2)/Sqrt[1 - a^2*x^2] + ArcSin[a*x] + 2*ArcS
in[Sqrt[1 - a*x]/Sqrt[2]] - ArcTanh[Sqrt[1 - a^2*x^2]])
```

Maple [A] time = 0.035, size = 32, normalized size = 0.9

$$-c\left(-\sqrt{-a^2x^2+1}+\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x,x)
```

```
[Out] -c*(-(-a^2*x^2+1)^(1/2)+arctanh(1/(-a^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.41313, size = 59, normalized size = 1.69

$$-c\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\sqrt{-a^2x^2+1}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x,x, algorithm="maxima")
```

```
[Out] -c*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*x^2 + 1)*c
```

Fricas [A] time = 1.72293, size = 78, normalized size = 2.23

$$c \log \left(\frac{\sqrt{-a^2x^2+1}-1}{x} \right) + \sqrt{-a^2x^2+1}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x,x, algorithm="fricas")
```

```
[Out] c*log((sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1)*c
```

Sympy [A] time = 20.4257, size = 66, normalized size = 1.89

$$\frac{a^2c\left(\begin{cases} -x^2 & \text{for } a^2=0 \\ \frac{2\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise} \end{cases}\right)}{2} - \frac{c\left(-\log\left(-1+\frac{1}{\sqrt{-a^2x^2+1}}\right)+\log\left(1+\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)/x,x)
```

```
[Out] a**2*c*Piecewise((-x**2, Eq(a**2, 0)), (2*sqrt(-a**2*x**2 + 1)/a**2, True))
/2 - c*(-log(-1 + 1/sqrt(-a**2*x**2 + 1)) + log(1 + 1/sqrt(-a**2*x**2 + 1))
)/2
```

Giac [A] time = 1.14362, size = 70, normalized size = 2.

$$\frac{1}{2}c\left(2\sqrt{-a^2x^2+1} - \log\left(\sqrt{-a^2x^2+1}+1\right) + \log\left(-\sqrt{-a^2x^2+1}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*c*(2*sqrt(-a^2*x^2 + 1) - log(sqrt(-a^2*x^2 + 1) + 1) + log(-sqrt(-a^2*
x^2 + 1) + 1))
```

3.292 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)}{x^2}\,dx$

Optimal. Leaf size=29

$$-\frac{c\sqrt{1-a^2x^2}}{x}-ac\sin^{-1}(ax)$$

```
[Out] -((c*Sqrt[1 - a^2*x^2])/x) - a*c*ArcSin[a*x]
```

Rubi [A] time = 0.0429045, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6128, 277, 216}

$$-\frac{c\sqrt{1-a^2x^2}}{x}-ac\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x))/x^2,x]
```

```
[Out] -((c*Sqrt[1 - a^2*x^2])/x) - a*c*ArcSin[a*x]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}(c-acx)}{x^2}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x^2}\,dx\\ &= -\frac{c\sqrt{1-a^2x^2}}{x}-\left(a^2c\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{c\sqrt{1-a^2x^2}}{x}-ac\sin^{-1}(ax)\end{aligned}$$

Mathematica [A] time = 0.0285526, size = 28, normalized size = 0.97

$$-\frac{c\left(\sqrt{1-a^2x^2}+ax\sin^{-1}(ax)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x))/x^2,x]
```

```
[Out] -((c*(Sqrt[1 - a^2*x^2] + a*x*ArcSin[a*x]))/x)
```

Maple [A] time = 0.039, size = 51, normalized size = 1.8

$$-a^2c\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{c}{x}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^2,x)
```

```
[Out] -c*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-c*(-a^2*x^2+1)^
(1/2)/x
```

Maxima [A] time = 1.44362, size = 55, normalized size = 1.9

$$-\frac{a^2c\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}-\frac{\sqrt{-a^2x^2+1}c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -a^2*c*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - sqrt(-a^2*x^2 + 1)*c/x
```

Fricas [A] time = 1.68843, size = 101, normalized size = 3.48

$$\frac{2\,acx\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\sqrt{-a^2x^2+1}c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*c*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*c)/x
```

Sympy [C] time = 2.96254, size = 88, normalized size = 3.03

$$-a^2c\left(\begin{cases}\sqrt{\frac{1}{a^2}}\,\mathrm{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\\ \sqrt{-\frac{1}{a^2}}\,\mathrm{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\end{cases}\right)+c\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for } \left|a^2x^2\right|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)/x**2,x)
```

```
[Out] -a**2*c*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a*
*2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + c*Piecewise((-I*sqrt(a**2*x**2 - 1)/
x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))
```

Giac [B] time = 1.15102, size = 100, normalized size = 3.45

$$\frac{a^4cx}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|}-\frac{a^2c\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{|a|}-\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)c}{2\,x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a^4*c*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - a^2*c*arcsin(a*x)*sg
n(a)/abs(a) - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c/(x*abs(a))
```

3.293 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)}{x^3}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}a^2c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{c\sqrt{1-a^2x^2}}{2x^2}$$

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(2*x^2) + (a^2*c*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rubi [A] time = 0.0578259, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6128, 266, 47, 63, 208}

$$\frac{1}{2}a^2c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{c\sqrt{1-a^2x^2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x))/x^3,x]
```

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(2*x^2) + (a^2*c*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}(c-acx)}{x^3}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x^3}\,dx\\
&= \frac{1}{2}c\,\text{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2}-\frac{1}{4}\left(a^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2}+\frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)\\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2}+\frac{1}{2}a^2c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0228295, size = 67, normalized size = 1.46

$$\frac{c\left(a^2x^2+a^2x^2\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-1\right)}{2x^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x))/x^3,x]
```

```
[Out] (c*(-1 + a^2*x^2 + a^2*x^2*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(
2*x^2*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.039, size = 40, normalized size = 0.9

$$-c\left(-\frac{a^2}{2}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{1}{2\,x^2}\sqrt{-a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^3,x)
```

```
[Out] -c*(-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))+1/2*(-a^2*x^2+1)^(1/2)/x^2)
```

Maxima [A] time = 1.42605, size = 69, normalized size = 1.5

$$\frac{1}{2}\,a^2c\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\sqrt{-a^2x^2+1}c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 1/2*sqrt(-a^2*x^2 +
 1)*c/x^2
```

Fricas [A] time = 1.68195, size = 104, normalized size = 2.26

$$-\frac{a^2cx^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c*x^2*log((sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1)*c)/x^2
```

Sympy [B] time = 21.6313, size = 78, normalized size = 1.7

$$\frac{a^2\left(-\frac{c\log\left(-1+\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{c\log\left(1+\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}-\frac{c}{2\left(1+\frac{1}{\sqrt{-a^2x^2+1}}\right)}-\frac{c}{2\left(-1+\frac{1}{\sqrt{-a^2x^2+1}}\right)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)/x**3,x)
```

```
[Out] a**2*(-c*log(-1 + 1/sqrt(-a**2*x**2 + 1))/2 + c*log(1 + 1/sqrt(-a**2*x**2 +
 1))/2 - c/(2*(1 + 1/sqrt(-a**2*x**2 + 1))) - c/(2*(-1 + 1/sqrt(-a**2*x**2
+ 1))))/2
```

Giac [A] time = 1.28067, size = 82, normalized size = 1.78

$$-\frac{1}{4}\,a^2c\left(\frac{2\,\sqrt{-a^2x^2+1}}{a^2x^2}-\log\left(\sqrt{-a^2x^2+1}+1\right)+\log\left(-\sqrt{-a^2x^2+1}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*a^2*c*(2*sqrt(-a^2*x^2 + 1)/(a^2*x^2) - log(sqrt(-a^2*x^2 + 1) + 1) +
log(-sqrt(-a^2*x^2 + 1) + 1))
```

3.294 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)}{x^4}\,dx$

Optimal. Leaf size=22

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{3x^3}$$

```
[Out] -(c*(1 - a^2*x^2)^(3/2))/(3*x^3)
```

Rubi [A] time = 0.0392372, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6128, 264}

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x))/x^4,x]
```

```
[Out] -(c*(1 - a^2*x^2)^(3/2))/(3*x^3)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}(c-acx)}{x^4}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x^4}\,dx\\ &= -\frac{c\left(1-a^2x^2\right)^{3/2}}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.0141034, size = 22, normalized size = 1.

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x))/x^4,x]
```

```
[Out] -(c*(1 - a^2*x^2)^(3/2))/(3*x^3)
```

Maple [A] time = 0.03, size = 33, normalized size = 1.5

$$-\frac{(ax+1)^2 (ax-1)^2 c}{3x^3} \frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^4,x)
```

```
[Out] -1/3*(a*x+1)^2*(a*x-1)^2*c/x^3/(-a^2*x^2+1)^(1/2)
```

Maxima [B] time = 1.43693, size = 54, normalized size = 2.45

$$\frac{\sqrt{-a^2x^2+1}a^2c}{3x} - \frac{\sqrt{-a^2x^2+1}c}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-a^2*x^2 + 1)*a^2*c/x - 1/3*sqrt(-a^2*x^2 + 1)*c/x^3
```

Fricas [A] time = 1.6253, size = 59, normalized size = 2.68

$$\frac{\left(a^2cx^2-c\right)\sqrt{-a^2x^2+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*c*x^2 - c)*sqrt(-a^2*x^2 + 1)/x^3
```

Sympy [C] time = 3.35489, size = 133, normalized size = 6.05

$$-a^2c\left(\begin{cases} -\frac{i\sqrt{a^2x^2-1}}{x} & \text{for } |a^2x^2|>1 \\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise} \end{cases}\right) + c\left(\begin{cases} -\frac{2ia^2\sqrt{a^2x^2-1}}{3x} - \frac{i\sqrt{a^2x^2-1}}{3x^3} & \text{for } |a^2x^2|>1 \\ -\frac{2a^2\sqrt{-a^2x^2+1}}{3x} - \frac{\sqrt{-a^2x^2+1}}{3x^3} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)/x**4,x)
```

```
[Out] -a**2*c*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a
**2*x**2 + 1)/x, True)) + c*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x)
- I*sqrt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*
```

```
x**2 + 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), True))
```

Giac [B] time = 1.29068, size = 167, normalized size = 7.59

$$\frac{\left(a^4c - \frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c}{x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|} + \frac{\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4c}{x} - \frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c}{x^3}}{24\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4*c - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c/x^2)*a^6*x^3/((sqrt(-a^
2*x^2 + 1)*abs(a) + a)^3*abs(a)) + 1/24*(3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*
a^4*c/x - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c/x^3)/(a^2*abs(a))
```

3.295 $\int e^{\tanh^{-1}(ax)}x^3(c-acx)^2\,dx$

Optimal. Leaf size=124

$$\frac{c^2x^3\left(1-a^2x^2\right)^{3/2}}{6a}-\frac{c^2x^2\left(1-a^2x^2\right)^{3/2}}{5a^2}-\frac{c^2x\sqrt{1-a^2x^2}}{16a^3}-\frac{c^2(16-15ax)\left(1-a^2x^2\right)^{3/2}}{120a^4}-\frac{c^2\sin^{-1}(ax)}{16a^4}$$

```
[Out] -(c^2*x*Sqrt[1 - a^2*x^2])/(16*a^3) - (c^2*x^2*(1 - a^2*x^2)^(3/2))/(5*a^2)
 + (c^2*x^3*(1 - a^2*x^2)^(3/2))/(6*a) - (c^2*(16 - 15*a*x)*(1 - a^2*x^2)^(
3/2))/(120*a^4) - (c^2*ArcSin[a*x])/(16*a^4)
```

Rubi [A] time = 0.135755, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6128, 833, 780, 195, 216}

$$\frac{c^2x^3\left(1-a^2x^2\right)^{3/2}}{6a}-\frac{c^2x^2\left(1-a^2x^2\right)^{3/2}}{5a^2}-\frac{c^2x\sqrt{1-a^2x^2}}{16a^3}-\frac{c^2(16-15ax)\left(1-a^2x^2\right)^{3/2}}{120a^4}-\frac{c^2\sin^{-1}(ax)}{16a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^3*(c - a*c*x)^2,x]
```

```
[Out] -(c^2*x*Sqrt[1 - a^2*x^2])/(16*a^3) - (c^2*x^2*(1 - a^2*x^2)^(3/2))/(5*a^2)
 + (c^2*x^3*(1 - a^2*x^2)^(3/2))/(6*a) - (c^2*(16 - 15*a*x)*(1 - a^2*x^2)^(
3/2))/(120*a^4) - (c^2*ArcSin[a*x])/(16*a^4)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^3(c-acx)^2\,dx = c\int x^3(c-acx)\sqrt{1-a^2x^2}\,dx$$
$$= \frac{c^2x^3\left(1-a^2x^2\right)^{3/2}}{6a} - \frac{c\int x^2\left(3ac-6a^2cx\right)\sqrt{1-a^2x^2}\,dx}{6a^2}$$
$$= -\frac{c^2x^2\left(1-a^2x^2\right)^{3/2}}{5a^2} + \frac{c^2x^3\left(1-a^2x^2\right)^{3/2}}{6a} + \frac{c\int x\left(12a^2c-15a^3cx\right)\sqrt{1-a^2x^2}\,dx}{30a^4}$$
$$= -\frac{c^2x^2\left(1-a^2x^2\right)^{3/2}}{5a^2} + \frac{c^2x^3\left(1-a^2x^2\right)^{3/2}}{6a} - \frac{c^2(16-15ax)\left(1-a^2x^2\right)^{3/2}}{120a^4} - \frac{c^2\int\sqrt{1-a^2x^2}}{8a^3}$$
$$= -\frac{c^2x\sqrt{1-a^2x^2}}{16a^3} - \frac{c^2x^2\left(1-a^2x^2\right)^{3/2}}{5a^2} + \frac{c^2x^3\left(1-a^2x^2\right)^{3/2}}{6a} - \frac{c^2(16-15ax)\left(1-a^2x^2\right)^{3/2}}{120a^4}$$
$$= -\frac{c^2x\sqrt{1-a^2x^2}}{16a^3} - \frac{c^2x^2\left(1-a^2x^2\right)^{3/2}}{5a^2} + \frac{c^2x^3\left(1-a^2x^2\right)^{3/2}}{6a} - \frac{c^2(16-15ax)\left(1-a^2x^2\right)^{3/2}}{120a^4}$$

Mathematica [A] time = 0.19242, size = 89, normalized size = 0.72

$$-\frac{c^2\left(\sqrt{1-a^2x^2}\left(40a^5x^5-48a^4x^4-10a^3x^3+16a^2x^2-15ax+32\right)-60\sin^{-1}(ax)-150\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{240a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^3*(c - a*c*x)^2,x]
```

```
[Out] -(c^2*(Sqrt[1 - a^2*x^2]*(32 - 15*a*x + 16*a^2*x^2 - 10*a^3*x^3 - 48*a^4*x^
4 + 40*a^5*x^5) - 60*ArcSin[a*x] - 150*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(240
*a^4)
```

Maple [A] time = 0.05, size = 163, normalized size = 1.3

$$-\frac{ac^2x^5}{6}\sqrt{-a^2x^2+1} + \frac{c^2x^3}{24\,a}\sqrt{-a^2x^2+1} + \frac{xc^2}{16\,a^3}\sqrt{-a^2x^2+1} - \frac{c^2}{16\,a^3}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} + \frac{c^2x^4}{5}\sqrt{-a^2x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^2,x)
```

```
[Out] -1/6*c^2*a*x^5*(-a^2*x^2+1)^(1/2)+1/24*c^2/a*x^3*(-a^2*x^2+1)^(1/2)+1/16*c^
2*x*(-a^2*x^2+1)^(1/2)/a^3-1/16*c^2/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-
a^2*x^2+1)^(1/2))+1/5*c^2*x^4*(-a^2*x^2+1)^(1/2)-1/15*c^2*x^2/a^2*(-a^2*x^2
+1)^(1/2)-2/15*c^2/a^4*(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.43174, size = 207, normalized size = 1.67

$$-\frac{1}{6}\sqrt{-a^2x^2+1}ac^2x^5+\frac{1}{5}\sqrt{-a^2x^2+1}c^2x^4+\frac{\sqrt{-a^2x^2+1}c^2x^3}{24\,a}-\frac{\sqrt{-a^2x^2+1}c^2x^2}{15\,a^2}+\frac{\sqrt{-a^2x^2+1}c^2x}{16\,a^3}-\frac{c^2\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{16\,\sqrt{a^2}a^3}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] -1/6*sqrt(-a^2*x^2 + 1)*a*c^2*x^5 + 1/5*sqrt(-a^2*x^2 + 1)*c^2*x^4 + 1/24*s
qrt(-a^2*x^2 + 1)*c^2*x^3/a - 1/15*sqrt(-a^2*x^2 + 1)*c^2*x^2/a^2 + 1/16*sq
rt(-a^2*x^2 + 1)*c^2*x/a^3 - 1/16*c^2*arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^
3) - 2/15*sqrt(-a^2*x^2 + 1)*c^2/a^4
```

Fricas [A] time = 1.61169, size = 230, normalized size = 1.85

$$\frac{30\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(40\,a^5c^2x^5-48\,a^4c^2x^4-10\,a^3c^2x^3+16\,a^2c^2x^2-15\,ac^2x+32\,c^2\right)\sqrt{-a^2x^2+1}}{240\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] 1/240*(30*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (40*a^5*c^2*x^5 - 48
*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 16*a^2*c^2*x^2 - 15*a*c^2*x + 32*c^2)*sqrt(
-a^2*x^2 + 1))/a^4
```

Sympy [A] time = 14.6914, size = 486, normalized size = 3.92

$$a^3c^2\left(\begin{cases}-\frac{ix^7}{6\sqrt{a^2x^2-1}}-\frac{ix^5}{24a^2\sqrt{a^2x^2-1}}-\frac{5ix^3}{48a^4\sqrt{a^2x^2-1}}+\frac{5ix}{16a^6\sqrt{a^2x^2-1}}-\frac{5i\operatorname{acosh}(ax)}{16a^7} & \text{for } \left|a^2x^2\right|>1\\ \frac{x^7}{6\sqrt{-a^2x^2+1}}+\frac{x^5}{24a^2\sqrt{-a^2x^2+1}}+\frac{5x^3}{48a^4\sqrt{-a^2x^2+1}}-\frac{5x}{16a^6\sqrt{-a^2x^2+1}}+\frac{5\operatorname{asin}(ax)}{16a^7} & \text{otherwise}\end{cases}\right)-a^2c^2\left(\begin{cases}-\frac{x^4\sqrt{-a^2x^2+1}}{5a^2}-\frac{4x^2\sqrt{-a}}{15a} \\ \frac{x^6}{6}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3*(-a*c*x+c)**2,x)
```

```
[Out] a**3*c**2*Piecewise((-I*x**7/(6*sqrt(a**2*x**2 - 1)) - I*x**5/(24*a**2*sqrt
(a**2*x**2 - 1)) - 5*I*x**3/(48*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(16*a**6*
sqrt(a**2*x**2 - 1)) - 5*I*acosh(a*x)/(16*a**7), Abs(a**2*x**2) > 1), (x**7
/(6*sqrt(-a**2*x**2 + 1)) + x**5/(24*a**2*sqrt(-a**2*x**2 + 1)) + 5*x**3/(4
8*a**4*sqrt(-a**2*x**2 + 1)) - 5*x/(16*a**6*sqrt(-a**2*x**2 + 1)) + 5*asin(
a*x)/(16*a**7), True)) - a**2*c**2*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/(5
*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(15
*a**6), Ne(a, 0)), (x**6/6, True)) - a*c**2*Piecewise((-I*x**5/(4*sqrt(a**2
*x**2 - 1)) - I*x**3/(8*a**2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2
*x**2 - 1)) - 3*I*acosh(a*x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-
a**2*x**2 + 1)) + x**3/(8*a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a*
*2*x**2 + 1)) + 3*asin(a*x)/(8*a**5), True)) + c**2*Piecewise((-x**2*sqrt(-
```

```
a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4
/4, True))
```

Giac [A] time = 1.36787, size = 124, normalized size = 1.

$$-\frac{1}{240}\sqrt{-a^2x^2+1}\left(\left(2\left(\left(4\left(5\,ac^2x-6\,c^2\right)x-\frac{5\,c^2}{a}\right)x+\frac{8\,c^2}{a^2}\right)x-\frac{15\,c^2}{a^3}\right)x+\frac{32\,c^2}{a^4}\right)-\frac{c^2\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{16\,a^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/240*sqrt(-a^2*x^2 + 1)*((2*((4*(5*a*c^2*x - 6*c^2)*x - 5*c^2/a)*x + 8*c^
2/a^2)*x - 15*c^2/a^3)*x + 32*c^2/a^4) - 1/16*c^2*arcsin(a*x)*sgn(a)/(a^3*a
bs(a))
```

3.296 $\int e^{\tanh^{-1}(ax)} x^2 (c - acx)^2 \, dx$

Optimal. Leaf size=113

$$-\frac{c^2 \left(1-a^2x^2\right)^{5/2}}{5a^3}+\frac{c^2 \left(1-a^2x^2\right)^{3/2}}{3a^3}-\frac{c^2x \left(1-a^2x^2\right)^{3/2}}{4a^2}+\frac{c^2x\sqrt{1-a^2x^2}}{8a^2}+\frac{c^2\sin^{-1}(ax)}{8a^3}$$

```
[Out] (c^2*x*Sqrt[1 - a^2*x^2])/(8*a^2) + (c^2*(1 - a^2*x^2)^(3/2))/(3*a^3) - (c^
2*x*(1 - a^2*x^2)^(3/2))/(4*a^2) - (c^2*(1 - a^2*x^2)^(5/2))/(5*a^3) + (c^2
*ArcSin[a*x])/(8*a^3)
```

Rubi [A] time = 0.120825, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6128, 797, 641, 195, 216}

$$-\frac{c^2 \left(1-a^2x^2\right)^{5/2}}{5a^3}+\frac{c^2 \left(1-a^2x^2\right)^{3/2}}{3a^3}-\frac{c^2x \left(1-a^2x^2\right)^{3/2}}{4a^2}+\frac{c^2x\sqrt{1-a^2x^2}}{8a^2}+\frac{c^2\sin^{-1}(ax)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2*(c - a*c*x)^2,x]
```

```
[Out] (c^2*x*Sqrt[1 - a^2*x^2])/(8*a^2) + (c^2*(1 - a^2*x^2)^(3/2))/(3*a^3) - (c^
2*x*(1 - a^2*x^2)^(3/2))/(4*a^2) - (c^2*(1 - a^2*x^2)^(5/2))/(5*a^3) + (c^2
*ArcSin[a*x])/(8*a^3)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^2(c-acx)^2\,dx = c\int x^2(c-acx)\sqrt{1-a^2x^2}\,dx$$
$$= \frac{c\int (c-acx)\sqrt{1-a^2x^2}\,dx}{a^2} - \frac{c\int (c-acx)\left(1-a^2x^2\right)^{3/2}\,dx}{a^2}$$
$$= \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^3} - \frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a^3} + \frac{c^2\int \sqrt{1-a^2x^2}\,dx}{a^2} - \frac{c^2\int \left(1-a^2x^2\right)^{3/2}\,dx}{a^2}$$
$$= \frac{c^2x\sqrt{1-a^2x^2}}{2a^2} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^3} - \frac{c^2x\left(1-a^2x^2\right)^{3/2}}{4a^2} - \frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a^3} + \frac{c^2\int \frac{1}{\sqrt{1-a^2x^2}}\,d}{2a^2}$$
$$= \frac{c^2x\sqrt{1-a^2x^2}}{8a^2} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^3} - \frac{c^2x\left(1-a^2x^2\right)^{3/2}}{4a^2} - \frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a^3} + \frac{c^2\sin^{-1}(ax)}{2a^3} -$$
$$= \frac{c^2x\sqrt{1-a^2x^2}}{8a^2} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^3} - \frac{c^2x\left(1-a^2x^2\right)^{3/2}}{4a^2} - \frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a^3} + \frac{c^2\sin^{-1}(ax)}{8a^3}$$

Mathematica [A] time = 0.079369, size = 75, normalized size = 0.66

$$-\frac{c^2\left(\sqrt{1-a^2x^2}\left(24a^4x^4-30a^3x^3-8a^2x^2+15ax-16\right)+30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{120a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^2*(c - a*c*x)^2,x]
```

```
[Out] -(c^2*(Sqrt[1 - a^2*x^2]*(-16 + 15*a*x - 8*a^2*x^2 - 30*a^3*x^3 + 24*a^4*x^
4) + 30*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(120*a^3)
```

Maple [A] time = 0.044, size = 140, normalized size = 1.2

$$-\frac{ac^2x^4}{5}\sqrt{-a^2x^2+1}+\frac{c^2x^2}{15\,a}\sqrt{-a^2x^2+1}+\frac{2\,c^2}{15\,a^3}\sqrt{-a^2x^2+1}+\frac{c^2x^3}{4}\sqrt{-a^2x^2+1}-\frac{xc^2}{8\,a^2}\sqrt{-a^2x^2+1}+\frac{c^2}{8\,a^2}\arctan\left(x\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^2,x)
```

```
[Out] -1/5*c^2*a*x^4*(-a^2*x^2+1)^(1/2)+1/15*c^2/a*x^2*(-a^2*x^2+1)^(1/2)+2/15*c^
2/a^3*(-a^2*x^2+1)^(1/2)+1/4*c^2*x^3*(-a^2*x^2+1)^(1/2)-1/8*c^2*x*(-a^2*x^2
+1)^(1/2)/a^2+1/8*c^2/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/
2))
```

Maxima [A] time = 1.42927, size = 176, normalized size = 1.56

$$-\frac{1}{5}\sqrt{-a^2x^2+1}ac^2x^4+\frac{1}{4}\sqrt{-a^2x^2+1}c^2x^3+\frac{\sqrt{-a^2x^2+1}c^2x^2}{15\,a}-\frac{\sqrt{-a^2x^2+1}c^2x}{8\,a^2}+\frac{c^2\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\,\sqrt{a^2}a^2}+\frac{2\,\sqrt{-a^2x^2+1}c^2}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] -1/5*sqrt(-a^2*x^2 + 1)*a*c^2*x^4 + 1/4*sqrt(-a^2*x^2 + 1)*c^2*x^3 + 1/15*s
qrt(-a^2*x^2 + 1)*c^2*x^2/a - 1/8*sqrt(-a^2*x^2 + 1)*c^2*x/a^2 + 1/8*c^2*ar
csin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2) + 2/15*sqrt(-a^2*x^2 + 1)*c^2/a^3
```

Fricas [A] time = 1.64557, size = 207, normalized size = 1.83

$$-\frac{30\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(24\,a^4c^2x^4-30\,a^3c^2x^3-8\,a^2c^2x^2+15\,ac^2x-16\,c^2\right)\sqrt{-a^2x^2+1}}{120\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/120*(30*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (24*a^4*c^2*x^4 - 3
0*a^3*c^2*x^3 - 8*a^2*c^2*x^2 + 15*a*c^2*x - 16*c^2)*sqrt(-a^2*x^2 + 1))/a^
3
```

Sympy [C] time = 10.218, size = 374, normalized size = 3.31

$$a^3c^2\left(\begin{cases}-\frac{x^4\sqrt{-a^2x^2+1}}{5a^2}-\frac{4x^2\sqrt{-a^2x^2+1}}{15a^4}-\frac{8\sqrt{-a^2x^2+1}}{15a^6} & \text{for}\ a\neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)-a^2c^2\left(\begin{cases}-\frac{ix^5}{4\sqrt{a^2x^2-1}}-\frac{ix^3}{8a^2\sqrt{a^2x^2-1}}+\frac{3ix}{8a^4\sqrt{a^2x^2-1}}-\frac{3i\operatorname{acosh}(ax)}{8a^5}\\ \frac{x^5}{4\sqrt{-a^2x^2+1}}+\frac{x^3}{8a^2\sqrt{-a^2x^2+1}}-\frac{3x}{8a^4\sqrt{-a^2x^2+1}}+\frac{3\operatorname{asin}(ax)}{8a^5}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(-a*c*x+c)**2,x)
```

```
[Out] a**3*c**2*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/(5*a**2) - 4*x**2*sqrt(-a**
2*x**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(15*a**6), Ne(a, 0)), (x**6/
6, True)) - a**2*c**2*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(
8*a**2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acos
h(a*x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3
/(8*a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin
(a*x)/(8*a**5), True)) - a*c**2*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a*
*2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) + c**2*Pi
ecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**
2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 +
1)) + asin(a*x)/(2*a**3), True))
```

Giac [A] time = 1.39847, size = 109, normalized size = 0.96

$$-\frac{1}{120}\sqrt{-a^2x^2+1}\left(\left(2\left(3\left(4\,ac^2x-5\,c^2\right)x-\frac{4\,c^2}{a}\right)x+\frac{15\,c^2}{a^2}\right)x-\frac{16\,c^2}{a^3}\right)+\frac{c^2\arcsin(ax)\operatorname{sgn}(a)}{8\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/120*sqrt(-a^2*x^2 + 1)*((2*(3*(4*a*c^2*x - 5*c^2)*x - 4*c^2/a)*x + 15*c^
2/a^2)*x - 16*c^2/a^3) + 1/8*c^2*arcsin(a*x)*sgn(a)/(a^2*abs(a))
```

3.297 $\int e^{\tanh^{-1}(ax)}x(c-acx)^2\,dx$

Optimal. Leaf size=70

$$-\frac{c^2(4-3ax)\left(1-a^2x^2\right)^{3/2}}{12a^2}-\frac{c^2x\sqrt{1-a^2x^2}}{8a}-\frac{c^2\sin^{-1}(ax)}{8a^2}$$

```
[Out] -(c^2*x*Sqrt[1 - a^2*x^2])/(8*a) - (c^2*(4 - 3*a*x)*(1 - a^2*x^2)^(3/2))/(1
2*a^2) - (c^2*ArcSin[a*x])/(8*a^2)
```

Rubi [A] time = 0.0620288, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6128, 780, 195, 216}

$$-\frac{c^2(4-3ax)\left(1-a^2x^2\right)^{3/2}}{12a^2}-\frac{c^2x\sqrt{1-a^2x^2}}{8a}-\frac{c^2\sin^{-1}(ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x*(c - a*c*x)^2,x]
```


```
[Out] -(c^2*x*Sqrt[1 - a^2*x^2])/(8*a) - (c^2*(4 - 3*a*x)*(1 - a^2*x^2)^(3/2))/(1
2*a^2) - (c^2*ArcSin[a*x])/(8*a^2)
```


Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x(c-acx)^2\,dx = c\int x(c-acx)\sqrt{1-a^2x^2}\,dx$$
$$= -\frac{c^2(4-3ax)\left(1-a^2x^2\right)^{3/2}}{12a^2} - \frac{c^2\int\sqrt{1-a^2x^2}\,dx}{4a}$$
$$= -\frac{c^2x\sqrt{1-a^2x^2}}{8a} - \frac{c^2(4-3ax)\left(1-a^2x^2\right)^{3/2}}{12a^2} - \frac{c^2\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{8a}$$
$$= -\frac{c^2x\sqrt{1-a^2x^2}}{8a} - \frac{c^2(4-3ax)\left(1-a^2x^2\right)^{3/2}}{12a^2} - \frac{c^2\sin^{-1}(ax)}{8a^2}$$

Mathematica [A] time = 0.0962106, size = 67, normalized size = 0.96

$$-\frac{c^2\left(\sqrt{1-a^2x^2}\left(6a^3x^3-8a^2x^2-3ax+8\right)-6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x*(c - a*c*x)^2,x]
```

```
[Out] -(c^2*(Sqrt[1 - a^2*x^2]*(8 - 3*a*x - 8*a^2*x^2 + 6*a^3*x^3) - 6*ArcSin[Sqr
t[1 - a*x]/Sqrt[2]]))/(24*a^2)
```

Maple [A] time = 0.041, size = 117, normalized size = 1.7

$$-\frac{ac^2x^3}{4}\sqrt{-a^2x^2+1}+\frac{xc^2}{8\,a}\sqrt{-a^2x^2+1}-\frac{c^2}{8\,a}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{c^2x^2}{3}\sqrt{-a^2x^2+1}-\frac{c^2}{3\,a^2}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^2,x)
```

```
[Out] -1/4*c^2*a*x^3*(-a^2*x^2+1)^(1/2)+1/8*c^2*x*(-a^2*x^2+1)^(1/2)/a-1/8*c^2/a/
(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/3*c^2*x^2*(-a^2*x^2+
1)^(1/2)-1/3*c^2/a^2*(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.43219, size = 144, normalized size = 2.06

$$-\frac{1}{4}\sqrt{-a^2x^2+1}ac^2x^3+\frac{1}{3}\sqrt{-a^2x^2+1}c^2x^2+\frac{\sqrt{-a^2x^2+1}c^2x}{8\,a}-\frac{c^2\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\sqrt{a^2}a}-\frac{\sqrt{-a^2x^2+1}c^2}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-a^2*x^2 + 1)*a*c^2*x^3 + 1/3*sqrt(-a^2*x^2 + 1)*c^2*x^2 + 1/8*sq
rt(-a^2*x^2 + 1)*c^2*x/a - 1/8*c^2*arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a) -
1/3*sqrt(-a^2*x^2 + 1)*c^2/a^2
```

Fricas [A] time = 1.59973, size = 176, normalized size = 2.51

$$\frac{6\,c^{2}\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)-\left(6\,a^{3}c^{2}x^{3}-8\,a^{2}c^{2}x^{2}-3\,ac^{2}x+8\,c^{2}\right)\sqrt{-a^{2}x^{2}+1}}{24\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(6*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (6*a^3*c^2*x^3 - 8*a^2
*c^2*x^2 - 3*a*c^2*x + 8*c^2)*sqrt(-a^2*x^2 + 1))/a^2
```

Sympy [A] time = 9.61017, size = 330, normalized size = 4.71

$$a^{3}c^{2}\left(\begin{cases}-\frac{ix^{5}}{4\sqrt{a^{2}x^{2}-1}}-\frac{ix^{3}}{8a^{2}\sqrt{a^{2}x^{2}-1}}+\frac{3ix}{8a^{4}\sqrt{a^{2}x^{2}-1}}-\frac{3i\operatorname{acosh}(ax)}{8a^{5}} & \text{for}\ \left|a^{2}x^{2}\right|>1\\ \frac{x^{5}}{4\sqrt{-a^{2}x^{2}+1}}+\frac{x^{3}}{8a^{2}\sqrt{-a^{2}x^{2}+1}}-\frac{3x}{8a^{4}\sqrt{-a^{2}x^{2}+1}}+\frac{3\operatorname{asin}(ax)}{8a^{5}} & \text{otherwise}\end{cases}\right)-a^{2}c^{2}\left(\begin{cases}-\frac{x^{2}\sqrt{-a^{2}x^{2}+1}}{3a^{2}}-\frac{2\sqrt{-a^{2}x^{2}+1}}{3a^{4}} & \text{for}\ a\neq 0\\ \frac{x^{4}}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(-a*c*x+c)**2,x)
```

```
[Out] a**3*c**2*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(
a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a*
*5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqr
t(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**
5), True)) - a**2*c**2*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*s
qrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) - a*c**2*Piecewise
((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2)
 > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + a
sin(a*x)/(2*a**3), True)) + c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a
**2*x**2 + 1)/a**2, True))
```

Giac [A] time = 1.35527, size = 93, normalized size = 1.33

$$-\frac{c^{2}\arcsin(ax)\operatorname{sgn}(a)}{8\,a|a|}-\frac{1}{24}\sqrt{-a^{2}x^{2}+1}\left(\left(2\left(3\,ac^{2}x-4\,c^{2}\right)x-\frac{3\,c^{2}}{a}\right)x+\frac{8\,c^{2}}{a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*c^2*arcsin(a*x)*sgn(a)/(a*abs(a)) - 1/24*sqrt(-a^2*x^2 + 1)*((2*(3*a*c
^2*x - 4*c^2)*x - 3*c^2/a)*x + 8*c^2/a^2)
```

3.298 $\int e^{\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=61

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c^2x\sqrt{1-a^2x^2}+\frac{c^2\sin^{-1}(ax)}{2a}$$

```
[Out] (c^2*x*Sqrt[1 - a^2*x^2])/2 + (c^2*(1 - a^2*x^2)^(3/2))/(3*a) + (c^2*ArcSin
[a*x])/(2*a)
```

Rubi [A] time = 0.0372357, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6127, 641, 195, 216}

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c^2x\sqrt{1-a^2x^2}+\frac{c^2\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^2,x]
```


```
[Out] (c^2*x*Sqrt[1 - a^2*x^2])/2 + (c^2*(1 - a^2*x^2)^(3/2))/(3*a) + (c^2*ArcSin
[a*x])/(2*a)
```


Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(ax)}(c-acx)^2\,dx &= c\int (c-acx)\sqrt{1-a^2x^2}\,dx\\
&= \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+c^2\int \sqrt{1-a^2x^2}\,dx\\
&= \frac{1}{2}c^2x\sqrt{1-a^2x^2}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c^2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{1}{2}c^2x\sqrt{1-a^2x^2}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{c^2\sin^{-1}(ax)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.0740159, size = 59, normalized size = 0.97

$$
-\frac{c^2\left(\sqrt{1-a^2x^2}\left(2a^2x^2-3ax-2\right)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{6a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^2,x]
```

```
[Out] -(c^2*(Sqrt[1 - a^2*x^2]*(-2 - 3*a*x + 2*a^2*x^2) + 6*ArcSin[Sqrt[1 - a*x]/
Sqrt[2]]))/(6*a)
```

Maple [A] time = 0.039, size = 91, normalized size = 1.5

$$
-\frac{c^2x^2a}{3}\sqrt{-a^2x^2+1}+\frac{c^2}{3\,a}\sqrt{-a^2x^2+1}+\frac{xc^2}{2}\sqrt{-a^2x^2+1}+\frac{c^2}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2,x)
```

```
[Out] -1/3*c^2*a*x^2*(-a^2*x^2+1)^(1/2)+1/3*c^2*(-a^2*x^2+1)^(1/2)/a+1/2*c^2*x*(-
a^2*x^2+1)^(1/2)+1/2*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2
))
```

Maxima [A] time = 1.42586, size = 109, normalized size = 1.79

$$
-\frac{1}{3}\sqrt{-a^2x^2+1}ac^2x^2+\frac{1}{2}\sqrt{-a^2x^2+1}c^2x+\frac{c^2\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}}+\frac{\sqrt{-a^2x^2+1}c^2}{3\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-a^2*x^2 + 1)*a*c^2*x^2 + 1/2*sqrt(-a^2*x^2 + 1)*c^2*x + 1/2*c^2*
arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + 1/3*sqrt(-a^2*x^2 + 1)*c^2/a
```

Fricas [A] time = 1.62567, size = 151, normalized size = 2.48

$$-\frac{6\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^2c^2x^2-3\,ac^2x-2\,c^2\right)\sqrt{-a^2x^2+1}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^2*c^2*x^2 - 3*a*c
^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [A] time = 5.30452, size = 102, normalized size = 1.67

$$\begin{cases}\frac{c^2\sqrt{-a^2x^2+1}-c^2\left(\begin{cases}-\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\operatorname{asin}(ax)}{2} & \text{for}\ ax>-1\wedge ax<1\end{cases}\right)+c^2\left(\begin{cases}\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3}-\sqrt{-a^2x^2+1} & \text{for}\ ax>-1\wedge ax<1\end{cases}\right)+}{a} \\ c^2x\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2,x)
```

```
[Out] Piecewise(((c**2*sqrt(-a**2*x**2 + 1) - c**2*Piecewise((-a*x*sqrt(-a**2*x**
2 + 1)/2 + asin(a*x)/2, (a*x > -1) & (a*x < 1))) + c**2*Piecewise(((-a**2*x
**2 + 1)**(3/2)/3 - sqrt(-a**2*x**2 + 1), (a*x > -1) & (a*x < 1))) + c**2*a
sin(a*x))/a, Ne(a, 0)), (c**2*x, True))
```

Giac [A] time = 1.31044, size = 73, normalized size = 1.2

$$\frac{c^2\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{2\,|a|}-\frac{1}{6}\,\sqrt{-a^2x^2+1}\left(\left(2\,ac^2x-3\,c^2\right)x-\frac{2\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^2*arcsin(a*x)*sgn(a)/abs(a) - 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c^2*x - 3*
c^2)*x - 2*c^2/a)
```

3.299 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x}\,dx$

Optimal. Leaf size=59

$$\frac{1}{2}c^2(2-ax)\sqrt{1-a^2x^2}-c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{1}{2}c^2\sin^{-1}(ax)$$

```
[Out] (c^2*(2 - a*x)*Sqrt[1 - a^2*x^2])/2 - (c^2*ArcSin[a*x])/2 - c^2*ArcTanh[Sqr
t[1 - a^2*x^2]]
```

Rubi [A] time = 0.101381, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 815, 844, 216, 266, 63, 208}

$$\frac{1}{2}c^2(2-ax)\sqrt{1-a^2x^2}-c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{1}{2}c^2\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x,x]
```

```
[Out] (c^2*(2 - a*x)*Sqrt[1 - a^2*x^2])/2 - (c^2*ArcSin[a*x])/2 - c^2*ArcTanh[Sqr
t[1 - a^2*x^2]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x}\,dx &= c\int \frac{(c-acx)\sqrt{1-a^2x^2}}{x}\,dx\\
&= \frac{1}{2}c^2(2-ax)\sqrt{1-a^2x^2}-\frac{c\int \frac{-2a^2c+a^3cx}{x\sqrt{1-a^2x^2}}\,dx}{2a^2}\\
&= \frac{1}{2}c^2(2-ax)\sqrt{1-a^2x^2}+c^2\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx-\frac{1}{2}\left(ac^2\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{1}{2}c^2(2-ax)\sqrt{1-a^2x^2}-\frac{1}{2}c^2\sin^{-1}(ax)+\frac{1}{2}c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= \frac{1}{2}c^2(2-ax)\sqrt{1-a^2x^2}-\frac{1}{2}c^2\sin^{-1}(ax)-\frac{c^2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2}\\
&= \frac{1}{2}c^2(2-ax)\sqrt{1-a^2x^2}-\frac{1}{2}c^2\sin^{-1}(ax)-c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [B] time = 0.0777885, size = 125, normalized size = 2.12

$$
\frac{c^2\left(a^3x^3-2a^2x^2+\sqrt{1-a^2x^2}\sin^{-1}(ax)+4\sqrt{1-a^2x^2}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-2\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-ax+2\right)}{2\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x,x]
```

```
[Out] (c^2*(2 - a*x - 2*a^2*x^2 + a^3*x^3 + Sqrt[1 - a^2*x^2]*ArcSin[a*x] + 4*Sqr
t[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]] - 2*Sqrt[1 - a^2*x^2]*ArcTanh[
Sqrt[1 - a^2*x^2]]))/(2*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.037, size = 86, normalized size = 1.5

$$
-\frac{ac^2x}{2}\sqrt{-a^2x^2+1}-\frac{ac^2}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+c^2\sqrt{-a^2x^2+1}-c^2\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x,x)
```

```
[Out] -1/2*c^2*a*x*(-a^2*x^2+1)^(1/2)-1/2*c^2*a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/
(-a^2*x^2+1)^(1/2))+c^2*(-a^2*x^2+1)^(1/2)-c^2*arctanh(1/(-a^2*x^2+1)^(1/2)
)
```

Maxima [A] time = 1.42381, size = 120, normalized size = 2.03

$$-\frac{1}{2}\sqrt{-a^2x^2+1}ac^2x-\frac{ac^2\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}}-c^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\sqrt{-a^2x^2+1}c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*a*c^2*x - 1/2*a*c^2*arcsin(a^2*x/sqrt(a^2))/sqrt(a^
2) - c^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*x^2 + 1)*c
^2
```

Fricas [A] time = 1.70968, size = 169, normalized size = 2.86

$$c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+c^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\frac{1}{2}\left(ac^2x-2c^2\right)\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x,x, algorithm="fricas")
```

```
[Out] c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + c^2*log((sqrt(-a^2*x^2 + 1) -
1)/x) - 1/2*(a*c^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1)
```

Sympy [C] time = 12.0665, size = 201, normalized size = 3.41

$$a^3c^2\left(\begin{cases}-\frac{ix\sqrt{a^2x^2-1}}{2a^2}-\frac{i\operatorname{acosh}(ax)}{2a^3} & \text{for } |a^2x^2|>1\\ \frac{x^3}{2\sqrt{-a^2x^2+1}}-\frac{x}{2a^2\sqrt{-a^2x^2+1}}+\frac{\operatorname{asin}(ax)}{2a^3} & \text{otherwise}\end{cases}\right)-a^2c^2\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-ac^2\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right)\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right.\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2/x,x)
```

```
[Out] a**3*c**2*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a*
*3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-
a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) - a**2*c**2*Piecewise((x**2/2,
 Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) - a*c**2*Piecewise((sqrt
(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt(-a**2)
), a**2 < 0)) + c**2*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*
```

```
asin(1/(a*x)), True))
```

Giac [A] time = 1.44031, size = 113, normalized size = 1.92

$$-\frac{ac^2 \arcsin(ax)\operatorname{sgn}(a)}{2|a|} - \frac{ac^2 \log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|} - \frac{1}{2}\left(ac^2x - 2c^2\right)\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x,x, algorithm="giac")
```

```
[Out] -1/2*a*c^2*arcsin(a*x)*sgn(a)/abs(a) - a*c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 +
 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/2*(a*c^2*x - 2*c^2)*sqrt(-a^2*x^
2 + 1)
```

3.300 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{c^2(ax+1)\sqrt{1-a^2x^2}}{x}+ac^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-ac^2\sin^{-1}(ax)$$

```
[Out] -((c^2*(1 + a*x)*Sqrt[1 - a^2*x^2])/x) - a*c^2*ArcSin[a*x] + a*c^2*ArcTanh[
Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.103498, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 813, 844, 216, 266, 63, 208}

$$-\frac{c^2(ax+1)\sqrt{1-a^2x^2}}{x}+ac^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-ac^2\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^2,x]
```

```
[Out] -((c^2*(1 + a*x)*Sqrt[1 - a^2*x^2])/x) - a*c^2*ArcSin[a*x] + a*c^2*ArcTanh[
Sqrt[1 - a^2*x^2]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^2}\,dx &= c\int \frac{(c-acx)\sqrt{1-a^2x^2}}{x^2}\,dx\\
&= -\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{x}-\frac{1}{2}c\int \frac{2ac+2a^2cx}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{x}-\left(ac^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx-\left(a^2c^2\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{x}-ac^2\sin^{-1}(ax)-\frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= -\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{x}-ac^2\sin^{-1}(ax)+\frac{c^2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a}\\
&= -\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{x}-ac^2\sin^{-1}(ax)+ac^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.17488, size = 84, normalized size = 1.45

$$\frac{1}{2}c^2\left(\frac{2(ax-1)(ax+1)^2}{x\sqrt{1-a^2x^2}}+2a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-a\sin^{-1}(ax)+2a\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^2,x]
```

```
[Out] (c^2*((2*(-1 + a*x)*(1 + a*x)^2)/(x*Sqrt[1 - a^2*x^2]) - a*ArcSin[a*x] + 2*
a*ArcSin[Sqrt[1 - a*x]/Sqrt[2]] + 2*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2
```

Maple [A] time = 0.039, size = 91, normalized size = 1.6

$$-c^2a\sqrt{-a^2x^2+1}-a^2c^2\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{c^2}{x}\sqrt{-a^2x^2+1}+c^2a\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^2,x)
```

```
[Out] -c^2*a*(-a^2*x^2+1)^(1/2)-c^2*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^
2+1)^(1/2))-c^2/x*(-a^2*x^2+1)^(1/2)+c^2*a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.43454, size = 127, normalized size = 2.19

$$-\frac{a^2c^2\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}+ac^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\sqrt{-a^2x^2+1}ac^2-\frac{\sqrt{-a^2x^2+1}c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^2,x, algorithm="maxima"
)
```

```
[Out] -a^2*c^2*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + a*c^2*log(2*sqrt(-a^2*x^2 + 1)
/abs(x) + 2/abs(x)) - sqrt(-a^2*x^2 + 1)*a*c^2 - sqrt(-a^2*x^2 + 1)*c^2/x
```

Fricas [A] time = 1.59955, size = 193, normalized size = 3.33

$$\frac{2\,ac^2x\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-ac^2x\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-ac^2x-\left(ac^2x+c^2\right)\sqrt{-a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^2,x, algorithm="fricas"
)
```

```
[Out] (2*a*c^2*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - a*c^2*x*log((sqrt(-a^2*
x^2 + 1) - 1)/x) - a*c^2*x - (a*c^2*x + c^2)*sqrt(-a^2*x^2 + 1))/x
```

Sympy [C] time = 4.60682, size = 153, normalized size = 2.64

$$a^3c^2\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-a^2c^2\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)-ac^2\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)+c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2/x**2,x)
```

```
[Out] a**3*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, Tru
e)) - a**2*c**2*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sq
rt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - a*c**2*Piecewise((-acosh(1/(
a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True)) + c**2*Piecewise((-I
*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True
))
```

Giac [B] time = 1.25461, size = 189, normalized size = 3.26

$$\frac{a^4c^2x}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|}-\frac{a^2c^2\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{|a|}+\frac{a^2c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|}-\sqrt{-a^2x^2+1}ac^2-\frac{\left(\sqrt{-a^2x^2+1}|a|\right.}{2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a^4*c^2*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - a^2*c^2*arcsin(a*x
)*sgn(a)/abs(a) + a^2*c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(
a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*a*c^2 - 1/2*(sqrt(-a^2*x^2 + 1)*ab
s(a) + a)*c^2/(x*abs(a))
```

3.301 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^3} dx$

Optimal. Leaf size=67

$$-\frac{c^2(1-2ax)\sqrt{1-a^2x^2}}{2x^2}+\frac{1}{2}a^2c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+a^2c^2\sin^{-1}(ax)$$

```
[Out] -(c^2*(1 - 2*a*x)*Sqrt[1 - a^2*x^2])/(2*x^2) + a^2*c^2*ArcSin[a*x] + (a^2*c
^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rubi [A] time = 0.104605, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 811, 844, 216, 266, 63, 208}

$$-\frac{c^2(1-2ax)\sqrt{1-a^2x^2}}{2x^2}+\frac{1}{2}a^2c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+a^2c^2\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^3,x]
```


```
[Out] -(c^2*(1 - 2*a*x)*Sqrt[1 - a^2*x^2])/(2*x^2) + a^2*c^2*ArcSin[a*x] + (a^2*c
^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```


Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^3}\,dx &= c\int \frac{(c-acx)\sqrt{1-a^2x^2}}{x^3}\,dx\\
&= -\frac{c^2(1-2ax)\sqrt{1-a^2x^2}}{2x^2}-\frac{1}{4}c\int \frac{2a^2c-4a^3cx}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c^2(1-2ax)\sqrt{1-a^2x^2}}{2x^2}-\frac{1}{2}\left(a^2c^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx+\left(a^3c^2\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c^2(1-2ax)\sqrt{1-a^2x^2}}{2x^2}+a^2c^2\sin^{-1}(ax)-\frac{1}{4}\left(a^2c^2\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= -\frac{c^2(1-2ax)\sqrt{1-a^2x^2}}{2x^2}+a^2c^2\sin^{-1}(ax)+\frac{1}{2}c^2\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)\\
&= -\frac{c^2(1-2ax)\sqrt{1-a^2x^2}}{2x^2}+a^2c^2\sin^{-1}(ax)+\frac{1}{2}a^2c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [B] time = 0.100961, size = 147, normalized size = 2.19

$$
-\frac{c^2\left(4a^3x^3-2a^2x^2+a^2x^2\sqrt{1-a^2x^2}\sin^{-1}(ax)+10a^2x^2\sqrt{1-a^2x^2}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-2a^2x^2\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right.}{4x^2\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^3,x]
```

```
[Out] -(c^2*(2 - 4*a*x - 2*a^2*x^2 + 4*a^3*x^3 + a^2*x^2*Sqrt[1 - a^2*x^2]*ArcSin
[a*x] + 10*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]] - 2*a^2*
x^2*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(4*x^2*Sqrt[1 - a^2*x^2]
)
```

Maple [A] time = 0.048, size = 95, normalized size = 1.4

$$
c^2a^3\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{ac^2}{x}\sqrt{-a^2x^2+1}+\frac{a^2c^2}{2}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{c^2}{2x^2}\sqrt{-a^2x^2+1}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^3,x)
```

```
[Out] c^2*a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+c^2*a/x*(-a^2*
x^2+1)^(1/2)+1/2*c^2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))-1/2*c^2/x^2*(-a^2*x^
2+1)^(1/2)
```

Maxima [A] time = 1.42484, size = 132, normalized size = 1.97

$$\frac{a^3c^2\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}+\frac{1}{2}a^2c^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{\sqrt{-a^2x^2+1}ac^2}{x}-\frac{\sqrt{-a^2x^2+1}c^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^3,x, algorithm="maxima"
)
```

```
[Out] a^3*c^2*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + 1/2*a^2*c^2*log(2*sqrt(-a^2*x^2
 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*x^2 + 1)*a*c^2/x - 1/2*sqrt(-a^2*x^2 +
 1)*c^2/x^2
```

Fricas [A] time = 1.64576, size = 203, normalized size = 3.03

$$-\frac{4a^2c^2x^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+a^2c^2x^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(2ac^2x-c^2\right)\sqrt{-a^2x^2+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^3,x, algorithm="fricas"
)
```

```
[Out] -1/2*(4*a^2*c^2*x^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + a^2*c^2*x^2*lo
g((sqrt(-a^2*x^2 + 1) - 1)/x) - (2*a*c^2*x - c^2)*sqrt(-a^2*x^2 + 1))/x^2
```

Sympy [C] time = 7.36336, size = 226, normalized size = 3.37

$$a^3c^2\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)-a^2c^2\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)-ac^2\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\ \left|a^2x^2\right|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2/x**3,x)
```

```
[Out] a**3*c**2*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/
a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - a**2*c**2*Piecewise((-acosh(1/(a*x
```

```
)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True)) - a*c**2*Piecewise((-I*
sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True)
) + c**2*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*
x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a
**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))
```

Giac [B] time = 1.43579, size = 259, normalized size = 3.87

$$\frac{a^3c^2\arcsin(ax)\operatorname{sgn}(a)}{|a|}+\frac{a^3c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{2|a|}+\frac{\left(a^3c^2-\frac{4\left(\sqrt{-a^2x^2+1}|a|+a\right)ac^2}{x}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}|a|+a\right)^2|a|}+\frac{\frac{4\left(\sqrt{-a^2x^2+1}|a|+a\right)ac^2|a|}{x}-\frac{\left(\sqrt{-a\right.}}{}}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^3,x, algorithm="giac")
```

```
[Out] a^3*c^2*arcsin(a*x)*sgn(a)/abs(a) + 1/2*a^3*c^2*log(1/2*abs(-2*sqrt(-a^2*x^
2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 1/8*(a^3*c^2 - 4*(sqrt(-a^2*x^2
 + 1)*abs(a) + a)*a*c^2/x)*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a
)) + 1/8*(4*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a*c^2*abs(a)/x - (sqrt(-a^2*x^2
 + 1)*abs(a) + a)^2*c^2*abs(a)/(a*x^2))/a^2
```

3.302 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^4} \, dx$

Optimal. Leaf size=75

$$\frac{ac^2\sqrt{1-a^2x^2}}{2x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3x^3}-\frac{1}{2}a^3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] (a*c^2*Sqrt[1 - a^2*x^2])/(2*x^2) - (c^2*(1 - a^2*x^2)^(3/2))/(3*x^3) - (a^
3*c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rubi [A] time = 0.0927444, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 807, 266, 47, 63, 208}

$$\frac{ac^2\sqrt{1-a^2x^2}}{2x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3x^3}-\frac{1}{2}a^3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^4,x]
```

```
[Out] (a*c^2*Sqrt[1 - a^2*x^2])/(2*x^2) - (c^2*(1 - a^2*x^2)^(3/2))/(3*x^3) - (a^
3*c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^4}\,dx &= c\int \frac{(c-acx)\sqrt{1-a^2x^2}}{x^4}\,dx\\
&= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3x^3} - \left(ac^2\right)\int \frac{\sqrt{1-a^2x^2}}{x^3}\,dx\\
&= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3x^3} - \frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x^2}\,dx, x, x^2\right)\\
&= \frac{ac^2\sqrt{1-a^2x^2}}{2x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{1}{4}\left(a^3c^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)\\
&= \frac{ac^2\sqrt{1-a^2x^2}}{2x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3x^3} - \frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)\\
&= \frac{ac^2\sqrt{1-a^2x^2}}{2x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3x^3} - \frac{1}{2}a^3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0285044, size = 91, normalized size = 1.21

$$-\frac{c^2\left(2a^4x^4+3a^3x^3-4a^2x^2+3a^3x^3\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-3ax+2\right)}{6x^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^4,x]
```

```
[Out] -(c^2*(2 - 3*a*x - 4*a^2*x^2 + 3*a^3*x^3 + 2*a^4*x^4 + 3*a^3*x^3*Sqrt[1 - a
^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(6*x^3*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.039, size = 100, normalized size = 1.3

$$c^2\left(\frac{a^2}{3x}\sqrt{-a^2x^2+1}-a^3\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-a\left(-\frac{1}{2x^2}\sqrt{-a^2x^2+1}-\frac{a^2}{2}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)-\frac{1}{3x^3}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^4,x)
```

```
[Out] c^2*(1/3*a^2*(-a^2*x^2+1)^(1/2)/x-a^3*arctanh(1/(-a^2*x^2+1)^(1/2))-a*(-1/2
*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2)))-1/3*(-a^2*x^
2+1)^(1/2)/x^3)
```

Maxima [A] time = 1.44382, size = 134, normalized size = 1.79

$$-\frac{1}{2}a^3c^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{\sqrt{-a^2x^2+1}a^2c^2}{3x}+\frac{\sqrt{-a^2x^2+1}ac^2}{2x^2}-\frac{\sqrt{-a^2x^2+1}c^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^4,x, algorithm="maxima"
)
```

```
[Out] -1/2*a^3*c^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 1/3*sqrt(-a^2*x^
2 + 1)*a^2*c^2/x + 1/2*sqrt(-a^2*x^2 + 1)*a*c^2/x^2 - 1/3*sqrt(-a^2*x^2 + 1
)*c^2/x^3
```

Fricas [A] time = 1.51902, size = 154, normalized size = 2.05

$$\frac{3a^3c^2x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(2a^2c^2x^2+3ac^2x-2c^2\right)\sqrt{-a^2x^2+1}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^4,x, algorithm="fricas"
)
```

```
[Out] 1/6*(3*a^3*c^2*x^3*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (2*a^2*c^2*x^2 + 3*a*c
^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1))/x^3
```

Sympy [C] time = 7.48187, size = 270, normalized size = 3.6

$$a^3c^2\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)-a^2c^2\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\ \left|a^2x^2\right|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)-ac^2\left(\begin{cases}-\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x}\\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2/x**4,x)
```

```
[Out] a**3*c**2*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x
)), True)) - a**2*c**2*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2)
> 1), (-sqrt(-a**2*x**2 + 1)/x, True)) - a*c**2*Piecewise((-a**2*acosh(1/(a
*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*a
sin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1
/(a**2*x**2))), True)) + c**2*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x
) - I*sqrt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**
```

```
2*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), True))
```

Giac [B] time = 1.38843, size = 315, normalized size = 4.2

$$\frac{\left(a^4c^2 - \frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)a^2c^2}{x} - \frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^2}{x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|} - \frac{a^4c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{2|a|} + \frac{\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4c^2}{x} + \frac{3\left(\sqrt{-a^2x^2+1}|\right.}{x^2}}{24a^2|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4*c^2 - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2*c^2/x - 3*(sqrt(-a^2*
x^2 + 1)*abs(a) + a)^2*c^2/x^2)*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*
abs(a)) - 1/2*a^4*c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*
abs(x)))/abs(a) + 1/24*(3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^2/x + 3*(sq
rt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^2/x^2 - (sqrt(-a^2*x^2 + 1)*abs(a) + a
)^3*c^2/x^3)/(a^2*abs(a))
```

3.303 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^5} dx$

Optimal. Leaf size=102

$$-\frac{a^2c^2\sqrt{1-a^2x^2}}{8x^2}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{3x^3}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{1}{8}a^4c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -(a^2*c^2*Sqrt[1 - a^2*x^2])/(8*x^2) - (c^2*(1 - a^2*x^2)^(3/2))/(4*x^4) +
(a*c^2*(1 - a^2*x^2)^(3/2))/(3*x^3) + (a^4*c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/
8
```

Rubi [A] time = 0.118751, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 835, 807, 266, 47, 63, 208}

$$-\frac{a^2c^2\sqrt{1-a^2x^2}}{8x^2}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{3x^3}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{1}{8}a^4c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^5,x]
```

```
[Out] -(a^2*c^2*Sqrt[1 - a^2*x^2])/(8*x^2) - (c^2*(1 - a^2*x^2)^(3/2))/(4*x^4) +
(a*c^2*(1 - a^2*x^2)^(3/2))/(3*x^3) + (a^4*c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/
8
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^5}\,dx &= c\int \frac{(c-acx)\sqrt{1-a^2x^2}}{x^5}\,dx\\
&= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{1}{4}c\int \frac{\left(4ac-a^2cx\right)\sqrt{1-a^2x^2}}{x^4}\,dx\\
&= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{1}{4}\left(a^2c^2\right)\int \frac{\sqrt{1-a^2x^2}}{x^3}\,dx\\
&= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{1}{8}\left(a^2c^2\right)\mathrm{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{a^2c^2\sqrt{1-a^2x^2}}{8x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{3x^3}-\frac{1}{16}\left(a^4c^2\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\right.\\
&= -\frac{a^2c^2\sqrt{1-a^2x^2}}{8x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{1}{8}\left(a^2c^2\right)\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,\right.\\
&= -\frac{a^2c^2\sqrt{1-a^2x^2}}{8x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{1}{8}a^4c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0299276, size = 99, normalized size = 0.97

$$\frac{c^2\left(8a^5x^5-3a^4x^4-16a^3x^3+9a^2x^2+3a^4x^4\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+8ax-6\right)}{24x^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^5,x]
```

```
[Out] (c^2*(-6 + 8*a*x + 9*a^2*x^2 - 16*a^3*x^3 - 3*a^4*x^4 + 8*a^5*x^5 + 3*a^4*x
^4*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(24*x^4*Sqrt[1 - a^2*x^2]
```

```
)
```

Maple [A] time = 0.041, size = 125, normalized size = 1.2

$$c^{2}\left(-\frac{1}{4x^{4}}\sqrt{-a^{2}x^{2}+1}-\frac{a^{2}}{4}\left(-\frac{1}{2x^{2}}\sqrt{-a^{2}x^{2}+1}-\frac{a^{2}}{2}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)\right)-\frac{a^{3}}{x}\sqrt{-a^{2}x^{2}+1}-a\left(-\frac{1}{3x^{3}}\sqrt{-a^{2}x^{2}+1}-\frac{2a}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^5,x)
```

```
[Out] c^2*(-1/4*(-a^2*x^2+1)^(1/2)/x^4-1/4*a^2*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a
^2*arctanh(1/(-a^2*x^2+1)^(1/2)))-a^3*(-a^2*x^2+1)^(1/2)/x-a*(-1/3*(-a^2*x^
2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2+1)^(1/2)/x))
```

Maxima [A] time = 1.46087, size = 165, normalized size = 1.62

$$\frac{1}{8}a^{4}c^{2}\log\left(\frac{2\sqrt{-a^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\sqrt{-a^{2}x^{2}+1}a^{3}c^{2}}{3x}+\frac{\sqrt{-a^{2}x^{2}+1}a^{2}c^{2}}{8x^{2}}+\frac{\sqrt{-a^{2}x^{2}+1}ac^{2}}{3x^{3}}-\frac{\sqrt{-a^{2}x^{2}+1}c^{2}}{4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^5,x, algorithm="maxima"
)
```

```
[Out] 1/8*a^4*c^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 1/3*sqrt(-a^2*x^2
 + 1)*a^3*c^2/x + 1/8*sqrt(-a^2*x^2 + 1)*a^2*c^2/x^2 + 1/3*sqrt(-a^2*x^2 +
1)*a*c^2/x^3 - 1/4*sqrt(-a^2*x^2 + 1)*c^2/x^4
```

Fricas [A] time = 1.55657, size = 178, normalized size = 1.75

$$-\frac{3a^{4}c^{2}x^{4}\log\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{x}\right)+\left(8a^{3}c^{2}x^{3}-3a^{2}c^{2}x^{2}-8ac^{2}x+6c^{2}\right)\sqrt{-a^{2}x^{2}+1}}{24x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^5,x, algorithm="fricas"
)
```

```
[Out] -1/24*(3*a^4*c^2*x^4*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (8*a^3*c^2*x^3 - 3*a
^2*c^2*x^2 - 8*a*c^2*x + 6*c^2)*sqrt(-a^2*x^2 + 1))/x^4
```

Sympy [C] time = 10.6173, size = 415, normalized size = 4.07

$$a^{3}c^{2}\left(\begin{cases}-\frac{i\sqrt{a^{2}x^{2}-1}}{x} & \text{for } |a^{2}x^{2}|>1\\ -\frac{\sqrt{-a^{2}x^{2}+1}}{x} & \text{otherwise}\end{cases}\right)-a^{2}c^{2}\left(\begin{cases}-\frac{a^{2}\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^{2}x^{2}}}}{2x} & \text{for } \frac{1}{|a^{2}x^{2}|}>1\\ \frac{ia^{2}\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^{2}x^{2}}}}+\frac{i}{2ax^{3}\sqrt{1-\frac{1}{a^{2}x^{2}}}} & \text{otherwise}\end{cases}\right)-ac^{2}\left(\begin{cases}-\frac{2ia^{2}\sqrt{a^{2}x^{2}-1}}{3x}- \\ -\frac{2a^{2}\sqrt{-a^{2}x^{2}+1}}{3x}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2/x**5,x)
```

```
[Out] a**3*c**2*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(
-a**2*x**2 + 1)/x, True)) - a**2*c**2*Piecewise((-a**2*acosh(1/(a*x))/2 - a
*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x
))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**
2))), True)) - a*c**2*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sq
rt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 +
 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), True)) + c**2*Piecewise((-3*a**4
*acosh(1/(a*x))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(
-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x*
*2) > 1), (3*I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))
) + I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**
2))), True))
```

Giac [B] time = 1.40846, size = 324, normalized size = 3.18

$$\frac{\left(3\,a^5c^2 - \frac{8\left(\sqrt{-a^2x^2+1}|a|+a\right)a^3c^2}{x} + \frac{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c^2}{ax^3}\right)a^8x^4}{192\left(\sqrt{-a^2x^2+1}|a|+a\right)^4|a|} + \frac{a^5c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{8\,|a|} - \frac{\frac{24\left(\sqrt{-a^2x^2+1}|a|+a\right)a^5c^2|a|}{x} - \frac{8\left(\sqrt{-a\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^5*c^2 - 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^3*c^2/x + 24*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^3*c^2/(a*x^3))*a^8*x^4/((sqrt(-a^2*x^2 + 1)*abs(a)
 + a)^4*abs(a)) + 1/8*a^5*c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*
a)/(a^2*abs(x)))/abs(a) - 1/192*(24*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^5*c^2
*abs(a)/x - 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a*c^2*abs(a)/x^3 + 3*(sqrt(
-a^2*x^2 + 1)*abs(a) + a)^4*c^2*abs(a)/(a*x^4))/a^4
```

3.304 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^6} dx$

Optimal. Leaf size=129

$$\frac{a^3c^2\sqrt{1-a^2x^2}}{8x^2}-\frac{2a^2c^2\left(1-a^2x^2\right)^{3/2}}{15x^3}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{1}{8}a^5c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] (a^3*c^2*Sqrt[1 - a^2*x^2])/(8*x^2) - (c^2*(1 - a^2*x^2)^(3/2))/(5*x^5) + (
a*c^2*(1 - a^2*x^2)^(3/2))/(4*x^4) - (2*a^2*c^2*(1 - a^2*x^2)^(3/2))/(15*x^
3) - (a^5*c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/8
```

Rubi [A] time = 0.140495, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 835, 807, 266, 47, 63, 208}

$$\frac{a^3c^2\sqrt{1-a^2x^2}}{8x^2}-\frac{2a^2c^2\left(1-a^2x^2\right)^{3/2}}{15x^3}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{1}{8}a^5c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^6,x]
```

```
[Out] (a^3*c^2*Sqrt[1 - a^2*x^2])/(8*x^2) - (c^2*(1 - a^2*x^2)^(3/2))/(5*x^5) + (
a*c^2*(1 - a^2*x^2)^(3/2))/(4*x^4) - (2*a^2*c^2*(1 - a^2*x^2)^(3/2))/(15*x^
3) - (a^5*c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/8
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^6}\,dx &= c\int \frac{(c-acx)\sqrt{1-a^2x^2}}{x^6}\,dx\\
&= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{1}{5}c\int \frac{\left(5ac-2a^2cx\right)\sqrt{1-a^2x^2}}{x^5}\,dx\\
&= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{1}{20}c\int \frac{\left(8a^2c-5a^3cx\right)\sqrt{1-a^2x^2}}{x^4}\,dx\\
&= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{2a^2c^2\left(1-a^2x^2\right)^{3/2}}{15x^3}-\frac{1}{4}\left(a^3c^2\right)\int \frac{\sqrt{1-a^2x^2}}{x^3}\,dx\\
&= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{2a^2c^2\left(1-a^2x^2\right)^{3/2}}{15x^3}-\frac{1}{8}\left(a^3c^2\right)\mathrm{Subst}\left(\int \frac{\sqrt{1-a^2}}{x^2}\right.\\
&= \frac{a^3c^2\sqrt{1-a^2x^2}}{8x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{2a^2c^2\left(1-a^2x^2\right)^{3/2}}{15x^3}+\frac{1}{16}\left(a^5c^2\right)\mathrm{S}\\
&= \frac{a^3c^2\sqrt{1-a^2x^2}}{8x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{2a^2c^2\left(1-a^2x^2\right)^{3/2}}{15x^3}-\frac{1}{8}\left(a^3c^2\right)\mathrm{Su}\\
&= \frac{a^3c^2\sqrt{1-a^2x^2}}{8x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{2a^2c^2\left(1-a^2x^2\right)^{3/2}}{15x^3}-\frac{1}{8}a^5c^2\tanh
\end{aligned}$$

Mathematica [A] time = 0.0346876, size = 107, normalized size = 0.83

$$-\frac{c^2\left(16a^6x^6-15a^5x^5-8a^4x^4+45a^3x^3-32a^2x^2+15a^5x^5\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-30ax+24\right)}{120x^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^6,x]
```

```
[Out] -(c^2*(24 - 30*a*x - 32*a^2*x^2 + 45*a^3*x^3 - 8*a^4*x^4 - 15*a^5*x^5 + 16*
a^6*x^6 + 15*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(120*x^
5*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.043, size = 168, normalized size = 1.3

$$c^{2}\left(-a\left(-\frac{1}{4x^{4}}\sqrt{-a^{2}x^{2}+1}+\frac{3a^{2}}{4}\left(-\frac{1}{2x^{2}}\sqrt{-a^{2}x^{2}+1}-\frac{a^{2}}{2}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)\right)\right)-\frac{1}{5x^{5}}\sqrt{-a^{2}x^{2}+1}-\frac{a^{2}}{5}\left(-\frac{1}{3x^{3}}\sqrt{-a^{2}x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^6,x)
```

```
[Out] c^2*(-a*(-1/4*(-a^2*x^2+1)^(1/2)/x^4+3/4*a^2*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1
/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))))-1/5/x^5*(-a^2*x^2+1)^(1/2)-1/5*a^2*(
-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2+1)^(1/2)/x)+a^3*(-1/2*(-a^2*x
^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))))
```

Maxima [A] time = 1.43017, size = 196, normalized size = 1.52

$$-\frac{1}{8}a^{5}c^{2}\log\left(\frac{2\sqrt{-a^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)+\frac{2\sqrt{-a^{2}x^{2}+1}a^{4}c^{2}}{15x}-\frac{\sqrt{-a^{2}x^{2}+1}a^{3}c^{2}}{8x^{2}}+\frac{\sqrt{-a^{2}x^{2}+1}a^{2}c^{2}}{15x^{3}}+\frac{\sqrt{-a^{2}x^{2}+1}ac^{2}}{4x^{4}}-\frac{\sqrt{-a^{2}x}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^6,x, algorithm="maxima"
)
```

```
[Out] -1/8*a^5*c^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 2/15*sqrt(-a^2*x
^2 + 1)*a^4*c^2/x - 1/8*sqrt(-a^2*x^2 + 1)*a^3*c^2/x^2 + 1/15*sqrt(-a^2*x^2
 + 1)*a^2*c^2/x^3 + 1/4*sqrt(-a^2*x^2 + 1)*a*c^2/x^4 - 1/5*sqrt(-a^2*x^2 +
1)*c^2/x^5
```

Fricas [A] time = 1.69478, size = 207, normalized size = 1.6

$$\frac{15a^{5}c^{2}x^{5}\log\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{x}\right)+\left(16a^{4}c^{2}x^{4}-15a^{3}c^{2}x^{3}+8a^{2}c^{2}x^{2}+30ac^{2}x-24c^{2}\right)\sqrt{-a^{2}x^{2}+1}}{120x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^6,x, algorithm="fricas"
)
```

```
[Out] 1/120*(15*a^5*c^2*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (16*a^4*c^2*x^4 - 1
5*a^3*c^2*x^3 + 8*a^2*c^2*x^2 + 30*a*c^2*x - 24*c^2)*sqrt(-a^2*x^2 + 1))/x^
5
```

Sympy [C] time = 11.0082, size = 522, normalized size = 4.05

$$a^3c^2\left(\begin{cases} -\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2} - \frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} & \text{for } \frac{1}{|a^2x^2|} > 1 \\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2} - \frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}} + \frac{i}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise} \end{cases}\right) - a^2c^2\left(\begin{cases} -\frac{2ia^2\sqrt{a^2x^2-1}}{3x} - \frac{i\sqrt{a^2x^2-1}}{3x^3} & \text{for } |a^2x^2| > 1 \\ -\frac{2a^2\sqrt{-a^2x^2+1}}{3x} - \frac{\sqrt{-a^2x^2+1}}{3x^3} & \text{otherwise} \end{cases}\right) - ac^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2/x**6,x)
```

```
[Out] a**3*c**2*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2
*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(
a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True)) - a**2*c**2*Pie
cewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**3
), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x*
*2 + 1)/(3*x**3), True)) - a*c**2*Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*a
**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) -
1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*asi
n(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1
- 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True)) + c**2*Pie
cewise((-8*a**5*sqrt(-1 + 1/(a**2*x**2))/15 - 4*a**3*sqrt(-1 + 1/(a**2*x**2
))/(15*x**2) - a*sqrt(-1 + 1/(a**2*x**2))/(5*x**4), 1/Abs(a**2*x**2) > 1),
(-8*I*a**5*sqrt(1 - 1/(a**2*x**2))/15 - 4*I*a**3*sqrt(1 - 1/(a**2*x**2))/(1
5*x**2) - I*a*sqrt(1 - 1/(a**2*x**2))/(5*x**4), True))
```

Giac [B] time = 1.36511, size = 401, normalized size = 3.11

$$\frac{\left(6a^6c^2 - \frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4c^2}{x} + \frac{10\left(\sqrt{-a^2x^2+1}|a|+a\right)^2a^2c^2}{x^2} - \frac{60\left(\sqrt{-a^2x^2+1}|a|+a\right)^4c^2}{a^2x^4}\right)a^{10}x^5}{960\left(\sqrt{-a^2x^2+1}|a|+a\right)^5|a|} - \frac{a^6c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{8|a|} + \frac{60(\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^6*c^2 - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^2/x + 10*(sqrt(
-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^2/x^2 - 60*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)^4*c^2/(a^2*x^4))*a^10*x^5/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*abs(a)) - 1/
8*a^6*c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs
(a) + 1/960*(60*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^8*c^2/x - 10*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^3*a^4*c^2/x^3 + 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*a^
2*c^2/x^4 - 6*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^2/x^5)/(a^4*abs(a))
```

3.305 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^7}\,dx$

Optimal. Leaf size=156

$$-\frac{a^4c^2\sqrt{1-a^2x^2}}{16x^2}+\frac{2a^3c^2\left(1-a^2x^2\right)^{3/2}}{15x^3}-\frac{a^2c^2\left(1-a^2x^2\right)^{3/2}}{8x^4}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6}+\frac{1}{16}a^6c^2\tanh^{-1}\left(\sqrt{1-\right.$$

```
[Out] -(a^4*c^2*Sqrt[1 - a^2*x^2])/(16*x^2) - (c^2*(1 - a^2*x^2)^(3/2))/(6*x^6) +
 (a*c^2*(1 - a^2*x^2)^(3/2))/(5*x^5) - (a^2*c^2*(1 - a^2*x^2)^(3/2))/(8*x^4
) + (2*a^3*c^2*(1 - a^2*x^2)^(3/2))/(15*x^3) + (a^6*c^2*ArcTanh[Sqrt[1 - a^
2*x^2]])/16
```

Rubi [A] time = 0.172594, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 835, 807, 266, 47, 63, 208}

$$-\frac{a^4c^2\sqrt{1-a^2x^2}}{16x^2}+\frac{2a^3c^2\left(1-a^2x^2\right)^{3/2}}{15x^3}-\frac{a^2c^2\left(1-a^2x^2\right)^{3/2}}{8x^4}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6}+\frac{1}{16}a^6c^2\tanh^{-1}\left(\sqrt{1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^7,x]
```

```
[Out] -(a^4*c^2*Sqrt[1 - a^2*x^2])/(16*x^2) - (c^2*(1 - a^2*x^2)^(3/2))/(6*x^6) +
 (a*c^2*(1 - a^2*x^2)^(3/2))/(5*x^5) - (a^2*c^2*(1 - a^2*x^2)^(3/2))/(8*x^4
) + (2*a^3*c^2*(1 - a^2*x^2)^(3/2))/(15*x^3) + (a^6*c^2*ArcTanh[Sqrt[1 - a^
2*x^2]])/16
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^7}\,dx = c\int \frac{(c-acx)\sqrt{1-a^2x^2}}{x^7}\,dx$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6} - \frac{1}{6}c\int \frac{\left(6ac-3a^2cx\right)\sqrt{1-a^2x^2}}{x^6}\,dx$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{1}{30}c\int \frac{\left(15a^2c-12a^3cx\right)\sqrt{1-a^2x^2}}{x^5}\,dx$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{a^2c^2\left(1-a^2x^2\right)^{3/2}}{8x^4} - \frac{1}{120}c\int \frac{\left(48a^3c-15a^4cx\right)\sqrt{1}}{x^4}$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{a^2c^2\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{2a^3c^2\left(1-a^2x^2\right)^{3/2}}{15x^3} + \frac{1}{8}\left(a^4c^2\right.$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{a^2c^2\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{2a^3c^2\left(1-a^2x^2\right)^{3/2}}{15x^3} + \frac{1}{16}\left(a^4c\right.$$
$$= -\frac{a^4c^2\sqrt{1-a^2x^2}}{16x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{a^2c^2\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{2a^3c^2\left(1-a\right.}{15x^3}$$
$$= -\frac{a^4c^2\sqrt{1-a^2x^2}}{16x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{a^2c^2\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{2a^3c^2\left(1-a\right.}{15x^3}$$
$$= -\frac{a^4c^2\sqrt{1-a^2x^2}}{16x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{a^2c^2\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{2a^3c^2\left(1-a\right.}{15x^3}$$

Mathematica [A] time = 0.0388684, size = 115, normalized size = 0.74

$$\frac{c^2\left(32a^7x^7-15a^6x^6-16a^5x^5+5a^4x^4-64a^3x^3+50a^2x^2+15a^6x^6\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+48ax-40\right)}{240x^6\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^7,x]
```

```
[Out] (c^2*(-40 + 48*a*x + 50*a^2*x^2 - 64*a^3*x^3 + 5*a^4*x^4 - 16*a^5*x^5 - 15*
a^6*x^6 + 32*a^7*x^7 + 15*a^6*x^6*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^
2]]))/(240*x^6*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.049, size = 193, normalized size = 1.2

$$c^2\left(-\frac{a^2}{6}\left(-\frac{1}{4x^4}\sqrt{-a^2x^2+1}+\frac{3a^2}{4}\left(-\frac{1}{2x^2}\sqrt{-a^2x^2+1}-\frac{a^2}{2}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)\right)-a\left(-\frac{1}{5x^5}\sqrt{-a^2x^2+1}+\frac{4a^2}{5}\left(-\frac{1}{3x^3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^7,x)
```

```
[Out] c^2*(-1/6*a^2*(-1/4*(-a^2*x^2+1)^(1/2)/x^4+3/4*a^2*(-1/2*(-a^2*x^2+1)^(1/2)
/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))))-a*(-1/5/x^5*(-a^2*x^2+1)^(1/2)
+4/5*a^2*(-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2+1)^(1/2)/x))-1/6/x^
6*(-a^2*x^2+1)^(1/2)+a^3*(-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2+1)^
(1/2)/x))
```

Maxima [A] time = 1.4252, size = 227, normalized size = 1.46

$$\frac{1}{16}a^6c^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{2\sqrt{-a^2x^2+1}a^5c^2}{15x}+\frac{\sqrt{-a^2x^2+1}a^4c^2}{16x^2}-\frac{\sqrt{-a^2x^2+1}a^3c^2}{15x^3}+\frac{\sqrt{-a^2x^2+1}a^2c^2}{24x^4}+\frac{\sqrt{-a^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^7,x, algorithm="maxima"
)
```

```
[Out] 1/16*a^6*c^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 2/15*sqrt(-a^2*x
^2 + 1)*a^5*c^2/x + 1/16*sqrt(-a^2*x^2 + 1)*a^4*c^2/x^2 - 1/15*sqrt(-a^2*x^
2 + 1)*a^3*c^2/x^3 + 1/24*sqrt(-a^2*x^2 + 1)*a^2*c^2/x^4 + 1/5*sqrt(-a^2*x^
2 + 1)*a*c^2/x^5 - 1/6*sqrt(-a^2*x^2 + 1)*c^2/x^6
```

Fricas [A] time = 1.70295, size = 232, normalized size = 1.49

$$-\frac{15a^6c^2x^6\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(32a^5c^2x^5-15a^4c^2x^4+16a^3c^2x^3-10a^2c^2x^2-48ac^2x+40c^2\right)\sqrt{-a^2x^2+1}}{240x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^7,x, algorithm="fricas"
)
```

```
[Out] -1/240*(15*a^6*c^2*x^6*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (32*a^5*c^2*x^5 -
15*a^4*c^2*x^4 + 16*a^3*c^2*x^3 - 10*a^2*c^2*x^2 - 48*a*c^2*x + 40*c^2)*sqr
```

```
t(-a^2*x^2 + 1))/x^6
```

Sympy [C] time = 15.382, size = 644, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2/x**7,x)
```

```
[Out] a**3*c**2*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2
 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) -
sqrt(-a**2*x**2 + 1)/(3*x**3), True)) - a**2*c**2*Piecewise((-3*a**4*acosh(
1/(a*x))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/
(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1
), (3*I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a
/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), T
rue)) - a*c**2*Piecewise((-8*a**5*sqrt(-1 + 1/(a**2*x**2))/15 - 4*a**3*sqrt
(-1 + 1/(a**2*x**2))/(15*x**2) - a*sqrt(-1 + 1/(a**2*x**2))/(5*x**4), 1/Abs
(a**2*x**2) > 1), (-8*I*a**5*sqrt(1 - 1/(a**2*x**2))/15 - 4*I*a**3*sqrt(1 -
 1/(a**2*x**2))/(15*x**2) - I*a*sqrt(1 - 1/(a**2*x**2))/(5*x**4), True)) +
c**2*Piecewise((-5*a**6*acosh(1/(a*x))/16 + 5*a**5/(16*x*sqrt(-1 + 1/(a**2*
x**2))) - 5*a**3/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) - a/(24*x**5*sqrt(-1 +
1/(a**2*x**2))) - 1/(6*a*x**7*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) >
 1), (5*I*a**6*asin(1/(a*x))/16 - 5*I*a**5/(16*x*sqrt(1 - 1/(a**2*x**2))) +
 5*I*a**3/(48*x**3*sqrt(1 - 1/(a**2*x**2))) + I*a/(24*x**5*sqrt(1 - 1/(a**2
*x**2))) + I/(6*a*x**7*sqrt(1 - 1/(a**2*x**2))), True))
```

Giac [B] time = 1.25303, size = 572, normalized size = 3.67

$$\frac{\left(5\,a^7c^2-\frac{12\left(\sqrt{-a^2x^2+1}|a|+a\right)a^5c^2}{x}+\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^2a^3c^2}{x^2}-\frac{20\left(\sqrt{-a^2x^2+1}|a|+a\right)^3ac^2}{x^3}-\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^4c^2}{ax^4}+\frac{120\left(\sqrt{-a^2x^2+1}|a|+a\right)^5c^2}{a^3x^5}\right)}{1920\left(\sqrt{-a^2x^2+1}|a|+a\right)^6|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^7*c^2 - 12*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^5*c^2/x + 15*(sqrt
(-a^2*x^2 + 1)*abs(a) + a)^2*a^3*c^2/x^2 - 20*(sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*a*c^2/x^3 - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^2/(a*x^4) + 120*(sq
rt(-a^2*x^2 + 1)*abs(a) + a)^5*c^2/(a^3*x^5))*a^12*x^6/((sqrt(-a^2*x^2 + 1)
*abs(a) + a)^6*abs(a)) + 1/16*a^7*c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs
(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/1920*(120*(sqrt(-a^2*x^2 + 1)*abs(a) +
a)*a^9*c^2*abs(a)/x - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^7*c^2*abs(a)/x
^2 - 20*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a^5*c^2*abs(a)/x^3 + 15*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)^4*a^3*c^2*abs(a)/x^4 - 12*(sqrt(-a^2*x^2 + 1)*abs(a)
 + a)^5*a*c^2*abs(a)/x^5 + 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6*c^2*abs(a)/(
a*x^6))/a^6
```

3.306 $\int e^{\tanh^{-1}(ax)}x^3(c-acx)^3\,dx$

Optimal. Leaf size=148

$$-\frac{1}{7}c^3x^4\left(1-a^2x^2\right)^{3/2}+\frac{c^3x^3\left(1-a^2x^2\right)^{3/2}}{3a}-\frac{11c^3x^2\left(1-a^2x^2\right)^{3/2}}{35a^2}-\frac{c^3x\sqrt{1-a^2x^2}}{8a^3}-\frac{c^3(88-105ax)\left(1-a^2x^2\right)^{3/2}}{420a^4}-\frac{c^3\sin^{-}}{8a}$$

```
[Out] -(c^3*x*Sqrt[1 - a^2*x^2])/(8*a^3) - (11*c^3*x^2*(1 - a^2*x^2)^(3/2))/(35*a
^2) + (c^3*x^3*(1 - a^2*x^2)^(3/2))/(3*a) - (c^3*x^4*(1 - a^2*x^2)^(3/2))/7
 - (c^3*(88 - 105*a*x)*(1 - a^2*x^2)^(3/2))/(420*a^4) - (c^3*ArcSin[a*x])/(
8*a^4)
```

Rubi [A] time = 0.237505, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 1809, 833, 780, 195, 216}

$$-\frac{1}{7}c^3x^4\left(1-a^2x^2\right)^{3/2}+\frac{c^3x^3\left(1-a^2x^2\right)^{3/2}}{3a}-\frac{11c^3x^2\left(1-a^2x^2\right)^{3/2}}{35a^2}-\frac{c^3x\sqrt{1-a^2x^2}}{8a^3}-\frac{c^3(88-105ax)\left(1-a^2x^2\right)^{3/2}}{420a^4}-\frac{c^3\sin^{-}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^3*(c - a*c*x)^3,x]
```

```
[Out] -(c^3*x*Sqrt[1 - a^2*x^2])/(8*a^3) - (11*c^3*x^2*(1 - a^2*x^2)^(3/2))/(35*a
^2) + (c^3*x^3*(1 - a^2*x^2)^(3/2))/(3*a) - (c^3*x^4*(1 - a^2*x^2)^(3/2))/7
 - (c^3*(88 - 105*a*x)*(1 - a^2*x^2)^(3/2))/(420*a^4) - (c^3*ArcSin[a*x])/(
8*a^4)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^3(c-acx)^3\,dx = c\int x^3(c-acx)^2\sqrt{1-a^2x^2}\,dx$$

$$= -\frac{1}{7}c^3x^4\left(1-a^2x^2\right)^{3/2} - \frac{c\int x^3\left(-11a^2c^2+14a^3c^2x\right)\sqrt{1-a^2x^2}\,dx}{7a^2}$$

$$= \frac{c^3x^3\left(1-a^2x^2\right)^{3/2}}{3a} - \frac{1}{7}c^3x^4\left(1-a^2x^2\right)^{3/2} + \frac{c\int x^2\left(-42a^3c^2+66a^4c^2x\right)\sqrt{1-a^2x^2}\,dx}{42a^4}$$

$$= -\frac{11c^3x^2\left(1-a^2x^2\right)^{3/2}}{35a^2} + \frac{c^3x^3\left(1-a^2x^2\right)^{3/2}}{3a} - \frac{1}{7}c^3x^4\left(1-a^2x^2\right)^{3/2} - \frac{c\int x\left(-132a^4c^2+21\cdots\right.}{21\cdots}$$

$$= -\frac{11c^3x^2\left(1-a^2x^2\right)^{3/2}}{35a^2} + \frac{c^3x^3\left(1-a^2x^2\right)^{3/2}}{3a} - \frac{1}{7}c^3x^4\left(1-a^2x^2\right)^{3/2} - \frac{c^3(88-105ax)\left(1-\cdots\right.}{420a^4}$$

$$= -\frac{c^3x\sqrt{1-a^2x^2}}{8a^3} - \frac{11c^3x^2\left(1-a^2x^2\right)^{3/2}}{35a^2} + \frac{c^3x^3\left(1-a^2x^2\right)^{3/2}}{3a} - \frac{1}{7}c^3x^4\left(1-a^2x^2\right)^{3/2} - \frac{c^3(8\cdots}{\cdots}$$

$$= -\frac{c^3x\sqrt{1-a^2x^2}}{8a^3} - \frac{11c^3x^2\left(1-a^2x^2\right)^{3/2}}{35a^2} + \frac{c^3x^3\left(1-a^2x^2\right)^{3/2}}{3a} - \frac{1}{7}c^3x^4\left(1-a^2x^2\right)^{3/2} - \frac{c^3(8\cdots}{\cdots}$$

Mathematica [A] time = 0.123037, size = 91, normalized size = 0.61

$$\frac{c^3\left(\sqrt{1-a^2x^2}\left(120a^6x^6-280a^5x^5+144a^4x^4+70a^3x^3-88a^2x^2+105ax-176\right)+210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{840a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^3*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(Sqrt[1 - a^2*x^2]*(-176 + 105*a*x - 88*a^2*x^2 + 70*a^3*x^3 + 144*a^4
*x^4 - 280*a^5*x^5 + 120*a^6*x^6) + 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(84
0*a^4)
```

Maple [A] time = 0.055, size = 186, normalized size = 1.3

$$\frac{a^2c^3x^6}{7}\sqrt{-a^2x^2+1}+\frac{6\,c^3x^4}{35}\sqrt{-a^2x^2+1}-\frac{11\,c^3x^2}{105\,a^2}\sqrt{-a^2x^2+1}-\frac{22\,c^3}{105\,a^4}\sqrt{-a^2x^2+1}-\frac{ac^3x^5}{3}\sqrt{-a^2x^2+1}+\frac{c^3x^3}{12\,a}\sqrt{-a^2x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^3,x)
```

```
[Out] 1/7*c^3*a^2*x^6*(-a^2*x^2+1)^(1/2)+6/35*c^3*x^4*(-a^2*x^2+1)^(1/2)-11/105*c
^3*x^2/a^2*(-a^2*x^2+1)^(1/2)-22/105*c^3/a^4*(-a^2*x^2+1)^(1/2)-1/3*c^3*a*x
^5*(-a^2*x^2+1)^(1/2)+1/12*c^3/a*x^3*(-a^2*x^2+1)^(1/2)+1/8*c^3*x*(-a^2*x^2
+1)^(1/2)/a^3-1/8*c^3/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/
2))
```

Maxima [A] time = 1.44087, size = 238, normalized size = 1.61

$$\frac{1}{7}\sqrt{-a^2x^2+1}a^2c^3x^6-\frac{1}{3}\sqrt{-a^2x^2+1}ac^3x^5+\frac{6}{35}\sqrt{-a^2x^2+1}c^3x^4+\frac{\sqrt{-a^2x^2+1}c^3x^3}{12\,a}-\frac{11\sqrt{-a^2x^2+1}c^3x^2}{105\,a^2}+\frac{\sqrt{-a^2x^2+1}}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] 1/7*sqrt(-a^2*x^2 + 1)*a^2*c^3*x^6 - 1/3*sqrt(-a^2*x^2 + 1)*a*c^3*x^5 + 6/3
5*sqrt(-a^2*x^2 + 1)*c^3*x^4 + 1/12*sqrt(-a^2*x^2 + 1)*c^3*x^3/a - 11/105*s
qrt(-a^2*x^2 + 1)*c^3*x^2/a^2 + 1/8*sqrt(-a^2*x^2 + 1)*c^3*x/a^3 - 1/8*c^3*
arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^3) - 22/105*sqrt(-a^2*x^2 + 1)*c^3/a^4
```

Fricas [A] time = 1.67375, size = 261, normalized size = 1.76

$$\frac{210\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(120\,a^6c^3x^6-280\,a^5c^3x^5+144\,a^4c^3x^4+70\,a^3c^3x^3-88\,a^2c^3x^2+105\,ac^3x-176\,c^3\right)\sqrt{-a^2x^2}}{840\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/840*(210*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (120*a^6*c^3*x^6 -
280*a^5*c^3*x^5 + 144*a^4*c^3*x^4 + 70*a^3*c^3*x^3 - 88*a^2*c^3*x^2 + 105*a
*c^3*x - 176*c^3)*sqrt(-a^2*x^2 + 1))/a^4
```

Sympy [A] time = 14.4931, size = 512, normalized size = 3.46

$$-a^4c^3\left(\begin{cases}-\frac{x^6\sqrt{-a^2x^2+1}}{7a^2}-\frac{6x^4\sqrt{-a^2x^2+1}}{35a^4}-\frac{8x^2\sqrt{-a^2x^2+1}}{35a^6}-\frac{16\sqrt{-a^2x^2+1}}{35a^8} & \text{for } a\neq 0\\ \frac{x^8}{8} & \text{otherwise}\end{cases}\right)+2a^3c^3\left(\begin{cases}-\frac{ix^7}{6\sqrt{a^2x^2-1}}-\frac{ix^5}{24a^2\sqrt{a^2x^2-1}}-\frac{5ix^3}{48a^4\sqrt{a^2x}}\\ \frac{x^7}{6\sqrt{-a^2x^2+1}}+\frac{x^5}{24a^2\sqrt{-a^2x^2+1}}+\frac{5x}{48a^4\sqrt{-}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3*(-a*c*x+c)**3,x)
```

```
[Out] -a**4*c**3*Piecewise((-x**6*sqrt(-a**2*x**2 + 1)/(7*a**2) - 6*x**4*sqrt(-a*
*2*x**2 + 1)/(35*a**4) - 8*x**2*sqrt(-a**2*x**2 + 1)/(35*a**6) - 16*sqrt(-a
**2*x**2 + 1)/(35*a**8), Ne(a, 0)), (x**8/8, True)) + 2*a**3*c**3*Piecewise
((-I*x**7/(6*sqrt(a**2*x**2 - 1)) - I*x**5/(24*a**2*sqrt(a**2*x**2 - 1)) -
5*I*x**3/(48*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(16*a**6*sqrt(a**2*x**2 - 1)
) - 5*I*acosh(a*x)/(16*a**7), Abs(a**2*x**2) > 1), (x**7/(6*sqrt(-a**2*x**2
 + 1)) + x**5/(24*a**2*sqrt(-a**2*x**2 + 1)) + 5*x**3/(48*a**4*sqrt(-a**2*x
**2 + 1)) - 5*x/(16*a**6*sqrt(-a**2*x**2 + 1)) + 5*asin(a*x)/(16*a**7), Tru
e)) - 2*a*c**3*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*
sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/
(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**
2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(
8*a**5), True)) + c**3*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*s
qrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True))
```

Giac [A] time = 1.2967, size = 140, normalized size = 0.95

$$\frac{1}{840}\sqrt{-a^2x^2+1}\left(\left(2\left(\left(\frac{35\,c^3}{a}+4\left(18\,c^3+5\left(3\,a^2c^3x-7\,ac^3\right)x\right)x\right)x-\frac{44\,c^3}{a^2}\right)x+\frac{105\,c^3}{a^3}\right)x-\frac{176\,c^3}{a^4}\right)-\frac{c^3\arcsin\left(ax\right)\mathrm{sgn}}{8\,a^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/840*sqrt(-a^2*x^2 + 1)*((2*((35*c^3/a + 4*(18*c^3 + 5*(3*a^2*c^3*x - 7*a*
c^3)*x)*x)*x - 44*c^3/a^2)*x + 105*c^3/a^3)*x - 176*c^3/a^4) - 1/8*c^3*arcs
in(a*x)*sgn(a)/(a^3*abs(a))
```

3.307 $\int e^{\tanh^{-1}(ax)}x^2(c-acx)^3\,dx$

Optimal. Leaf size=121

$$-\frac{1}{6}c^3x^3\left(1-a^2x^2\right)^{3/2}+\frac{2c^3x^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{c^3(32-45ax)\left(1-a^2x^2\right)^{3/2}}{120a^3}+\frac{3c^3x\sqrt{1-a^2x^2}}{16a^2}+\frac{3c^3\sin^{-1}(ax)}{16a^3}$$

```
[Out] (3*c^3*x*Sqrt[1 - a^2*x^2])/(16*a^2) + (2*c^3*x^2*(1 - a^2*x^2)^(3/2))/(5*a
) - (c^3*x^3*(1 - a^2*x^2)^(3/2))/6 + (c^3*(32 - 45*a*x)*(1 - a^2*x^2)^(3/2
))/(120*a^3) + (3*c^3*ArcSin[a*x])/(16*a^3)
```

Rubi [A] time = 0.200639, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 1809, 833, 780, 195, 216}

$$-\frac{1}{6}c^3x^3\left(1-a^2x^2\right)^{3/2}+\frac{2c^3x^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{c^3(32-45ax)\left(1-a^2x^2\right)^{3/2}}{120a^3}+\frac{3c^3x\sqrt{1-a^2x^2}}{16a^2}+\frac{3c^3\sin^{-1}(ax)}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2*(c - a*c*x)^3,x]
```

```
[Out] (3*c^3*x*Sqrt[1 - a^2*x^2])/(16*a^2) + (2*c^3*x^2*(1 - a^2*x^2)^(3/2))/(5*a
) - (c^3*x^3*(1 - a^2*x^2)^(3/2))/6 + (c^3*(32 - 45*a*x)*(1 - a^2*x^2)^(3/2
))/(120*a^3) + (3*c^3*ArcSin[a*x])/(16*a^3)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x^2(c-acx)^3\,dx &= c\int x^2(c-acx)^2\sqrt{1-a^2x^2}\,dx\\ &= -\frac{1}{6}c^3x^3\left(1-a^2x^2\right)^{3/2}-\frac{c\int x^2\left(-9a^2c^2+12a^3c^2x\right)\sqrt{1-a^2x^2}\,dx}{6a^2}\\ &= \frac{2c^3x^2\left(1-a^2x^2\right)^{3/2}}{5a}-\frac{1}{6}c^3x^3\left(1-a^2x^2\right)^{3/2}+\frac{c\int x\left(-24a^3c^2+45a^4c^2x\right)\sqrt{1-a^2x^2}\,dx}{30a^4}\\ &= \frac{2c^3x^2\left(1-a^2x^2\right)^{3/2}}{5a}-\frac{1}{6}c^3x^3\left(1-a^2x^2\right)^{3/2}+\frac{c^3(32-45ax)\left(1-a^2x^2\right)^{3/2}}{120a^3}+\frac{\left(3c^3\right)\int\sqrt{1-}}{8a^2}\\ &= \frac{3c^3x\sqrt{1-a^2x^2}}{16a^2}+\frac{2c^3x^2\left(1-a^2x^2\right)^{3/2}}{5a}-\frac{1}{6}c^3x^3\left(1-a^2x^2\right)^{3/2}+\frac{c^3(32-45ax)\left(1-a^2x^2\right)^{3}}{120a^3}\\ &= \frac{3c^3x\sqrt{1-a^2x^2}}{16a^2}+\frac{2c^3x^2\left(1-a^2x^2\right)^{3/2}}{5a}-\frac{1}{6}c^3x^3\left(1-a^2x^2\right)^{3/2}+\frac{c^3(32-45ax)\left(1-a^2x^2\right)^{3}}{120a^3}\end{aligned}$$

Mathematica [A] time = 0.0856895, size = 83, normalized size = 0.69

$$\frac{c^3\left(\sqrt{1-a^2x^2}\left(40a^5x^5-96a^4x^4+50a^3x^3+32a^2x^2-45ax+64\right)-90\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{240a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^2*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(Sqrt[1 - a^2*x^2]*(64 - 45*a*x + 32*a^2*x^2 + 50*a^3*x^3 - 96*a^4*x^4
 + 40*a^5*x^5) - 90*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(240*a^3)
```

Maple [A] time = 0.046, size = 163, normalized size = 1.4

$$\frac{c^3a^2x^5}{6}\sqrt{-a^2x^2+1}+\frac{5\,c^3x^3}{24}\sqrt{-a^2x^2+1}-\frac{3\,c^3x}{16\,a^2}\sqrt{-a^2x^2+1}+\frac{3\,c^3}{16\,a^2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{2\,c^3ax^4}{5}\sqrt{-a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^3,x)
```

```
[Out] 1/6*c^3*a^2*x^5*(-a^2*x^2+1)^(1/2)+5/24*c^3*x^3*(-a^2*x^2+1)^(1/2)-3/16*c^3
*x*(-a^2*x^2+1)^(1/2)/a^2+3/16*c^3/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a
^2*x^2+1)^(1/2))-2/5*c^3*a*x^4*(-a^2*x^2+1)^(1/2)+2/15*c^3/a*x^2*(-a^2*x^2+
1)^(1/2)+4/15*c^3/a^3*(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.4305, size = 207, normalized size = 1.71

$$\frac{1}{6}\sqrt{-a^2x^2+1}a^2c^3x^5 - \frac{2}{5}\sqrt{-a^2x^2+1}ac^3x^4 + \frac{5}{24}\sqrt{-a^2x^2+1}c^3x^3 + \frac{2\sqrt{-a^2x^2+1}c^3x^2}{15\,a} - \frac{3\sqrt{-a^2x^2+1}c^3x}{16\,a^2} + \frac{3\,c^3\arcsin\left(\right.}{16\sqrt{a^2}a^{\cdot}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] 1/6*sqrt(-a^2*x^2 + 1)*a^2*c^3*x^5 - 2/5*sqrt(-a^2*x^2 + 1)*a*c^3*x^4 + 5/2
4*sqrt(-a^2*x^2 + 1)*c^3*x^3 + 2/15*sqrt(-a^2*x^2 + 1)*c^3*x^2/a - 3/16*sqr
t(-a^2*x^2 + 1)*c^3*x/a^2 + 3/16*c^3*arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2
) + 4/15*sqrt(-a^2*x^2 + 1)*c^3/a^3
```

Fricas [A] time = 1.64863, size = 231, normalized size = 1.91

$$-\frac{90\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - \left(40\,a^5c^3x^5 - 96\,a^4c^3x^4 + 50\,a^3c^3x^3 + 32\,a^2c^3x^2 - 45\,ac^3x + 64\,c^3\right)\sqrt{-a^2x^2+1}}{240\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/240*(90*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (40*a^5*c^3*x^5 - 9
6*a^4*c^3*x^4 + 50*a^3*c^3*x^3 + 32*a^2*c^3*x^2 - 45*a*c^3*x + 64*c^3)*sqrt
(-a^2*x^2 + 1))/a^3
```

Sympy [C] time = 12.1477, size = 423, normalized size = 3.5

$$-a^4c^3\left(\begin{cases} -\frac{ix^7}{6\sqrt{a^2x^2-1}} - \frac{ix^5}{24a^2\sqrt{a^2x^2-1}} - \frac{5ix^3}{48a^4\sqrt{a^2x^2-1}} + \frac{5ix}{16a^6\sqrt{a^2x^2-1}} - \frac{5i\operatorname{acosh}(ax)}{16a^7} & \text{for } \left|a^2x^2\right| > 1 \\ \frac{x^7}{6\sqrt{-a^2x^2+1}} + \frac{x^5}{24a^2\sqrt{-a^2x^2+1}} + \frac{5x^3}{48a^4\sqrt{-a^2x^2+1}} - \frac{5x}{16a^6\sqrt{-a^2x^2+1}} + \frac{5\operatorname{asin}(ax)}{16a^7} & \text{otherwise} \end{cases}\right) + 2a^3c^3\left(\begin{cases} -\frac{x^4\sqrt{-a^2x^2+1}}{5a^2} - \frac{4x^2}{} \\ \frac{x^6}{6} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(-a*c*x+c)**3,x)
```

```
[Out] -a**4*c**3*Piecewise((-I*x**7/(6*sqrt(a**2*x**2 - 1)) - I*x**5/(24*a**2*sqr
t(a**2*x**2 - 1)) - 5*I*x**3/(48*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(16*a**6
*sqrt(a**2*x**2 - 1)) - 5*I*acosh(a*x)/(16*a**7), Abs(a**2*x**2) > 1), (x**
7/(6*sqrt(-a**2*x**2 + 1)) + x**5/(24*a**2*sqrt(-a**2*x**2 + 1)) + 5*x**3/(
```

```
48*a**4*sqrt(-a**2*x**2 + 1)) - 5*x/(16*a**6*sqrt(-a**2*x**2 + 1)) + 5*asin
(a*x)/(16*a**7), True)) + 2*a**3*c**3*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)
/(5*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/
(15*a**6), Ne(a, 0)), (x**6/6, True)) - 2*a*c**3*Piecewise((-x**2*sqrt(-a**
2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4,
 True)) + c**3*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/
(2*a**3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*s
qrt(-a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True))
```

Giac [A] time = 1.19496, size = 124, normalized size = 1.02

$$\frac{3\,c^3\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{16\,a^2|a|}+\frac{1}{240}\,\sqrt{-a^2x^2+1}\left(\left(2\left(\frac{16\,c^3}{a}+\left(25\,c^3+4\left(5\,a^2c^3x-12\,ac^3\right)x\right)x\right)x-\frac{45\,c^3}{a^2}\right)x+\frac{64\,c^3}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 3/16*c^3*arcsin(a*x)*sgn(a)/(a^2*abs(a)) + 1/240*sqrt(-a^2*x^2 + 1)*((2*(16
*c^3/a + (25*c^3 + 4*(5*a^2*c^3*x - 12*a*c^3)*x)*x)*x - 45*c^3/a^2)*x + 64*
c^3/a^3)
```

3.308 $\int e^{\tanh^{-1}(ax)}x(c-acx)^3\,dx$

Optimal. Leaf size=94

$$-\frac{1}{5}c^3x^2\left(1-a^2x^2\right)^{3/2}-\frac{c^3(14-15ax)\left(1-a^2x^2\right)^{3/2}}{30a^2}-\frac{c^3x\sqrt{1-a^2x^2}}{4a}-\frac{c^3\sin^{-1}(ax)}{4a^2}$$

```
[Out] -(c^3*x*Sqrt[1 - a^2*x^2])/(4*a) - (c^3*x^2*(1 - a^2*x^2)^(3/2))/5 - (c^3*(
14 - 15*a*x)*(1 - a^2*x^2)^(3/2))/(30*a^2) - (c^3*ArcSin[a*x])/(4*a^2)
```

Rubi [A] time = 0.128173, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6128, 1809, 780, 195, 216}

$$-\frac{1}{5}c^3x^2\left(1-a^2x^2\right)^{3/2}-\frac{c^3(14-15ax)\left(1-a^2x^2\right)^{3/2}}{30a^2}-\frac{c^3x\sqrt{1-a^2x^2}}{4a}-\frac{c^3\sin^{-1}(ax)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x*(c - a*c*x)^3,x]
```

```
[Out] -(c^3*x*Sqrt[1 - a^2*x^2])/(4*a) - (c^3*x^2*(1 - a^2*x^2)^(3/2))/5 - (c^3*(
14 - 15*a*x)*(1 - a^2*x^2)^(3/2))/(30*a^2) - (c^3*ArcSin[a*x])/(4*a^2)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x(c-acx)^3\,dx = c\int x(c-acx)^2\sqrt{1-a^2x^2}\,dx$$
$$= -\frac{1}{5}c^3x^2\left(1-a^2x^2\right)^{3/2} - \frac{c\int x\left(-7a^2c^2+10a^3c^2x\right)\sqrt{1-a^2x^2}\,dx}{5a^2}$$
$$= -\frac{1}{5}c^3x^2\left(1-a^2x^2\right)^{3/2} - \frac{c^3(14-15ax)\left(1-a^2x^2\right)^{3/2}}{30a^2} - \frac{c^3\int\sqrt{1-a^2x^2}\,dx}{2a}$$
$$= -\frac{c^3x\sqrt{1-a^2x^2}}{4a} - \frac{1}{5}c^3x^2\left(1-a^2x^2\right)^{3/2} - \frac{c^3(14-15ax)\left(1-a^2x^2\right)^{3/2}}{30a^2} - \frac{c^3\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{4a}$$
$$= -\frac{c^3x\sqrt{1-a^2x^2}}{4a} - \frac{1}{5}c^3x^2\left(1-a^2x^2\right)^{3/2} - \frac{c^3(14-15ax)\left(1-a^2x^2\right)^{3/2}}{30a^2} - \frac{c^3\sin^{-1}(ax)}{4a^2}$$

Mathematica [A] time = 0.103941, size = 75, normalized size = 0.8

$$\frac{c^3\left(\sqrt{1-a^2x^2}\left(12a^4x^4-30a^3x^3+16a^2x^2+15ax-28\right)+30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{60a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(Sqrt[1 - a^2*x^2]*(-28 + 15*a*x + 16*a^2*x^2 - 30*a^3*x^3 + 12*a^4*x^
4) + 30*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(60*a^2)
```

Maple [A] time = 0.046, size = 140, normalized size = 1.5

$$\frac{c^3a^2x^4}{5}\sqrt{-a^2x^2+1}+\frac{4\,c^3x^2}{15}\sqrt{-a^2x^2+1}-\frac{7\,c^3}{15\,a^2}\sqrt{-a^2x^2+1}-\frac{c^3ax^3}{2}\sqrt{-a^2x^2+1}+\frac{c^3x}{4\,a}\sqrt{-a^2x^2+1}-\frac{c^3}{4\,a}\arctan\left(x\sqrt{a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^3,x)
```

```
[Out] 1/5*c^3*a^2*x^4*(-a^2*x^2+1)^(1/2)+4/15*c^3*x^2*(-a^2*x^2+1)^(1/2)-7/15*c^3
/a^2*(-a^2*x^2+1)^(1/2)-1/2*c^3*a*x^3*(-a^2*x^2+1)^(1/2)+1/4*c^3*x*(-a^2*x^
2+1)^(1/2)/a-1/4*c^3/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.43816, size = 176, normalized size = 1.87

$$\frac{1}{5}\sqrt{-a^2x^2+1}a^2c^3x^4-\frac{1}{2}\sqrt{-a^2x^2+1}ac^3x^3+\frac{4}{15}\sqrt{-a^2x^2+1}c^3x^2+\frac{\sqrt{-a^2x^2+1}c^3x}{4\,a}-\frac{c^3\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{4\sqrt{a^2}a}-\frac{7\sqrt{-a^2x^2+1}}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(-a^2*x^2 + 1)*a^2*c^3*x^4 - 1/2*sqrt(-a^2*x^2 + 1)*a*c^3*x^3 + 4/1
5*sqrt(-a^2*x^2 + 1)*c^3*x^2 + 1/4*sqrt(-a^2*x^2 + 1)*c^3*x/a - 1/4*c^3*arc
sin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a) - 7/15*sqrt(-a^2*x^2 + 1)*c^3/a^2
```

Fricas [A] time = 1.62365, size = 205, normalized size = 2.18

$$\frac{30\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(12\,a^4c^3x^4-30\,a^3c^3x^3+16\,a^2c^3x^2+15\,ac^3x-28\,c^3\right)\sqrt{-a^2x^2+1}}{60\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/60*(30*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (12*a^4*c^3*x^4 - 30*
a^3*c^3*x^3 + 16*a^2*c^3*x^2 + 15*a*c^3*x - 28*c^3)*sqrt(-a^2*x^2 + 1))/a^2
```

Sympy [A] time = 11.21, size = 355, normalized size = 3.78

$$-a^4c^3\left(\begin{cases}-\frac{x^4\sqrt{-a^2x^2+1}}{5a^2}-\frac{4x^2\sqrt{-a^2x^2+1}}{15a^4}-\frac{8\sqrt{-a^2x^2+1}}{15a^6} & \text{for}\ a\neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)+2a^3c^3\left(\begin{cases}-\frac{ix^5}{4\sqrt{a^2x^2-1}}-\frac{ix^3}{8a^2\sqrt{a^2x^2-1}}+\frac{3ix}{8a^4\sqrt{a^2x^2-1}}-\frac{3i\operatorname{acosh}(ax)}{8a^5}\\ \frac{x^5}{4\sqrt{-a^2x^2+1}}+\frac{x^3}{8a^2\sqrt{-a^2x^2+1}}-\frac{3x}{8a^4\sqrt{-a^2x^2+1}}+\frac{3\operatorname{asin}(ax)}{8a^5}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(-a*c*x+c)**3,x)
```

```
[Out] -a**4*c**3*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/(5*a**2) - 4*x**2*sqrt(-a*
*2*x**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(15*a**6), Ne(a, 0)), (x**6
/6, True)) + 2*a**3*c**3*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**
3/(8*a**2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*a
cosh(a*x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x
**3/(8*a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*a
sin(a*x)/(8*a**5), True)) - 2*a*c**3*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2
*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x*
*2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) + c
**3*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
```

Giac [A] time = 1.20422, size = 109, normalized size = 1.16

$$-\frac{c^3\arcsin(ax)\operatorname{sgn}(a)}{4\,a|a|}+\frac{1}{60}\sqrt{-a^2x^2+1}\left(\left(\frac{15\,c^3}{a}+2\left(8\,c^3+3\left(2\,a^2c^3x-5\,ac^3\right)x\right)x\right)x-\frac{28\,c^3}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*c^3*arcsin(a*x)*sgn(a)/(a*abs(a)) + 1/60*sqrt(-a^2*x^2 + 1)*((15*c^3/a
 + 2*(8*c^3 + 3*(2*a^2*c^3*x - 5*a*c^3)*x)*x)*x - 28*c^3/a^2)
```

3.309 $\int e^{\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=91

$$\frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a}+\frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{5}{8}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\sin^{-1}(ax)}{8a}$$

```
[Out] (5*c^3*x*Sqrt[1 - a^2*x^2])/8 + (5*c^3*(1 - a^2*x^2)^(3/2))/(12*a) + (c^3*(
1 - a*x)*(1 - a^2*x^2)^(3/2))/(4*a) + (5*c^3*ArcSin[a*x])/(8*a)
```

Rubi [A] time = 0.057276, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6127, 671, 641, 195, 216}

$$\frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a}+\frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{5}{8}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^3,x]
```

```
[Out] (5*c^3*x*Sqrt[1 - a^2*x^2])/8 + (5*c^3*(1 - a^2*x^2)^(3/2))/(12*a) + (c^3*(
1 - a*x)*(1 - a^2*x^2)^(3/2))/(4*a) + (5*c^3*ArcSin[a*x])/(8*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}(c-acx)^3\,dx = c\int (c-acx)^2\sqrt{1-a^2x^2}\,dx$$
$$= \frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a} + \frac{1}{4}\left(5c^2\right)\int (c-acx)\sqrt{1-a^2x^2}\,dx$$
$$= \frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a} + \frac{1}{4}\left(5c^3\right)\int \sqrt{1-a^2x^2}\,dx$$
$$= \frac{5}{8}c^3x\sqrt{1-a^2x^2} + \frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a} + \frac{1}{8}\left(5c^3\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{5}{8}c^3x\sqrt{1-a^2x^2} + \frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a} + \frac{5c^3\sin^{-1}(ax)}{8a}$$

Mathematica [A] time = 0.0833492, size = 67, normalized size = 0.74

$$\frac{c^3\left(\sqrt{1-a^2x^2}\left(6a^3x^3-16a^2x^2+9ax+16\right)-30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(Sqrt[1 - a^2*x^2]*(16 + 9*a*x - 16*a^2*x^2 + 6*a^3*x^3) - 30*ArcSin[S
qrt[1 - a*x]/Sqrt[2]]))/(24*a)
```

Maple [A] time = 0.036, size = 114, normalized size = 1.3

$$\frac{c^3a^2x^3}{4}\sqrt{-a^2x^2+1} + \frac{3\,c^3x}{8}\sqrt{-a^2x^2+1} + \frac{5\,c^3}{8}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} - \frac{2\,c^3ax^2}{3}\sqrt{-a^2x^2+1} + \frac{2\,c^3}{3\,a}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3,x)
```

```
[Out] 1/4*c^3*a^2*x^3*(-a^2*x^2+1)^(1/2)+3/8*c^3*x*(-a^2*x^2+1)^(1/2)+5/8*c^3/(a^
2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-2/3*c^3*a*x^2*(-a^2*x^2+1
)^(1/2)+2/3*c^3*(-a^2*x^2+1)^(1/2)/a
```

Maxima [A] time = 1.44562, size = 140, normalized size = 1.54

$$\frac{1}{4}\sqrt{-a^2x^2+1}a^2c^3x^3 - \frac{2}{3}\sqrt{-a^2x^2+1}ac^3x^2 + \frac{3}{8}\sqrt{-a^2x^2+1}c^3x + \frac{5\,c^3\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\sqrt{a^2}} + \frac{2\sqrt{-a^2x^2+1}c^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(-a^2*x^2 + 1)*a^2*c^3*x^3 - 2/3*sqrt(-a^2*x^2 + 1)*a*c^3*x^2 + 3/8
*sqrt(-a^2*x^2 + 1)*c^3*x + 5/8*c^3*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + 2/3
```

```
*sqrt(-a^2*x^2 + 1)*c^3/a
```

Fricas [A] time = 1.6324, size = 178, normalized size = 1.96

$$-\frac{30\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(6\,a^3c^3x^3-16\,a^2c^3x^2+9\,ac^3x+16\,c^3\right)\sqrt{-a^2x^2+1}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(30*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (6*a^3*c^3*x^3 - 16*
a^2*c^3*x^2 + 9*a*c^3*x + 16*c^3)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [A] time = 6.46426, size = 136, normalized size = 1.49

$$\begin{cases} -\frac{-2c^3\sqrt{-a^2x^2+1}-2c^3\left(\begin{cases}\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3}-\sqrt{-a^2x^2+1} & \text{for}\ ax>-1\wedge ax<1\end{cases}\right)+c^3\left(\begin{cases}\frac{ax\left(-2a^2x^2+1\right)\sqrt{-a^2x^2+1}}{8}-\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{3\operatorname{asin}(ax)}{8}\end{cases}\right.}{a} \\ c^3x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**3,x)
```

```
[Out] Piecewise((-(-2*c**3*sqrt(-a**2*x**2 + 1) - 2*c**3*Piecewise(((-a**2*x**2 +
 1)**(3/2)/3 - sqrt(-a**2*x**2 + 1), (a*x > -1) & (a*x < 1))) + c**3*Piecew
ise((a*x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/8 - a*x*sqrt(-a**2*x**2 +
1)/2 + 3*asin(a*x)/8, (a*x > -1) & (a*x < 1))) - c**3*asin(a*x))/a, Ne(a, 0
)), (c**3*x, True))
```

Giac [A] time = 1.3918, size = 89, normalized size = 0.98

$$\frac{5\,c^3\arcsin(ax)\operatorname{sgn}(a)}{8\,|a|}+\frac{1}{24}\sqrt{-a^2x^2+1}\left(\frac{16\,c^3}{a}+\left(9\,c^3+2\left(3\,a^2c^3x-8\,ac^3\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 5/8*c^3*arcsin(a*x)*sgn(a)/abs(a) + 1/24*sqrt(-a^2*x^2 + 1)*(16*c^3/a + (9*
c^3 + 2*(3*a^2*c^3*x - 8*a*c^3)*x)*x)
```

3.310 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x} dx$

Optimal. Leaf size=75

$$-\frac{1}{3}c^3\left(1-a^2x^2\right)^{3/2}+c^3(1-ax)\sqrt{1-a^2x^2}-c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-c^3\sin^{-1}(ax)$$

```
[Out] c^3*(1 - a*x)*Sqrt[1 - a^2*x^2] - (c^3*(1 - a^2*x^2)^(3/2))/3 - c^3*ArcSin[
a*x] - c^3*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.173301, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1809, 815, 844, 216, 266, 63, 208}

$$-\frac{1}{3}c^3\left(1-a^2x^2\right)^{3/2}+c^3(1-ax)\sqrt{1-a^2x^2}-c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-c^3\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x,x]
```

```
[Out] c^3*(1 - a*x)*Sqrt[1 - a^2*x^2] - (c^3*(1 - a^2*x^2)^(3/2))/3 - c^3*ArcSin[
a*x] - c^3*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x}\,dx &= c\int \frac{(c-acx)^2\sqrt{1-a^2x^2}}{x}\,dx\\
&= -\frac{1}{3}c^3\left(1-a^2x^2\right)^{3/2}-\frac{c\int \frac{\left(-3a^2c^2+6a^3c^2x\right)\sqrt{1-a^2x^2}}{x}\,dx}{3a^2}\\
&= c^3(1-ax)\sqrt{1-a^2x^2}-\frac{1}{3}c^3\left(1-a^2x^2\right)^{3/2}+\frac{c\int \frac{6a^4c^2-6a^5c^2x}{x\sqrt{1-a^2x^2}}\,dx}{6a^4}\\
&= c^3(1-ax)\sqrt{1-a^2x^2}-\frac{1}{3}c^3\left(1-a^2x^2\right)^{3/2}+c^3\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx-\left(ac^3\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= c^3(1-ax)\sqrt{1-a^2x^2}-\frac{1}{3}c^3\left(1-a^2x^2\right)^{3/2}-c^3\sin^{-1}(ax)+\frac{1}{2}c^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,\right.\\
&= c^3(1-ax)\sqrt{1-a^2x^2}-\frac{1}{3}c^3\left(1-a^2x^2\right)^{3/2}-c^3\sin^{-1}(ax)-\frac{c^3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2}\right.}{a^2}\\
&= c^3(1-ax)\sqrt{1-a^2x^2}-\frac{1}{3}c^3\left(1-a^2x^2\right)^{3/2}-c^3\sin^{-1}(ax)-c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.103919, size = 135, normalized size = 1.8

$$
\frac{c^3\left(-2a^4x^4+6a^3x^3-2a^2x^2+3\sqrt{1-a^2x^2}\sin^{-1}(ax)+18\sqrt{1-a^2x^2}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-6\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\right.}{6\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x,x]
```

```
[Out] (c^3*(4 - 6*a*x - 2*a^2*x^2 + 6*a^3*x^3 - 2*a^4*x^4 + 3*Sqrt[1 - a^2*x^2]*A
rcSin[a*x] + 18*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]] - 6*Sqrt[1
- a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(6*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.036, size = 110, normalized size = 1.5

$$\frac{a^2c^3x^2}{3}\sqrt{-a^2x^2+1}+\frac{2\,c^3}{3}\sqrt{-a^2x^2+1}-c^3ax\sqrt{-a^2x^2+1}-c^3a\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-c^3\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x,x)
```

```
[Out] 1/3*c^3*a^2*x^2*(-a^2*x^2+1)^(1/2)+2/3*c^3*(-a^2*x^2+1)^(1/2)-c^3*a*x*(-a^2
*x^2+1)^(1/2)-c^3*a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-c^
3*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.43526, size = 153, normalized size = 2.04

$$\frac{1}{3}\sqrt{-a^2x^2+1}a^2c^3x^2-\sqrt{-a^2x^2+1}ac^3x-\frac{ac^3\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}-c^3\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{2}{3}\sqrt{-a^2x^2+1}c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-a^2*x^2 + 1)*a^2*c^3*x^2 - sqrt(-a^2*x^2 + 1)*a*c^3*x - a*c^3*arc
sin(a^2*x/sqrt(a^2))/sqrt(a^2) - c^3*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/ab
s(x)) + 2/3*sqrt(-a^2*x^2 + 1)*c^3
```

Fricas [A] time = 1.62816, size = 193, normalized size = 2.57

$$2\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+c^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\frac{1}{3}\left(a^2c^3x^2-3\,ac^3x+2\,c^3\right)\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x,x, algorithm="fricas")
```

```
[Out] 2*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + c^3*log((sqrt(-a^2*x^2 + 1)
- 1)/x) + 1/3*(a^2*c^3*x^2 - 3*a*c^3*x + 2*c^3)*sqrt(-a^2*x^2 + 1)
```

Sympy [C] time = 13.2081, size = 226, normalized size = 3.01

$$-a^4c^3\left(\begin{cases}-\frac{x^2\sqrt{-a^2x^2+1}}{3a^2}-\frac{2\sqrt{-a^2x^2+1}}{3a^4} & \text{for } a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)+2a^3c^3\left(\begin{cases}-\frac{ix\sqrt{a^2x^2-1}}{2a^2}-\frac{i\operatorname{acosh}(ax)}{2a^3} & \text{for } |a^2x^2|>1\\ \frac{x^3}{2\sqrt{-a^2x^2+1}}-\frac{x}{2a^2\sqrt{-a^2x^2+1}}+\frac{\operatorname{asin}(ax)}{2a^3} & \text{otherwise}\end{cases}\right)-2ac^3\left(\left\{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**3/x,x)
```

```
[Out] -a**4*c**3*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x*
*2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) + 2*a**3*c**3*Piecewise((-I*x*
sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1),
(x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x
)/(2*a**3), True)) - 2*a*c**3*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)),
a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + c**3*Piecewise
((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))
```

Giac [A] time = 1.25793, size = 128, normalized size = 1.71

$$-\frac{ac^3 \arcsin(ax)\operatorname{sgn}(a)}{|a|} - \frac{ac^3 \log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|} + \frac{1}{3}\sqrt{-a^2x^2+1}\left(2c^3 + \left(a^2c^3x - 3ac^3\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x,x, algorithm="giac")
```

```
[Out] -a*c^3*arcsin(a*x)*sgn(a)/abs(a) - a*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*
abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 1/3*sqrt(-a^2*x^2 + 1)*(2*c^3 + (a^2*c
^3*x - 3*a*c^3)*x)
```

3.311 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^2} dx$

Optimal. Leaf size=83

$$-\frac{c^3(1-a^2x^2)^{3/2}}{x}-\frac{1}{2}ac^3(ax+4)\sqrt{1-a^2x^2}+2ac^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{1}{2}ac^3\sin^{-1}(ax)$$

```
[Out] -(a*c^3*(4 + a*x)*Sqrt[1 - a^2*x^2])/2 - (c^3*(1 - a^2*x^2)^(3/2))/x - (a*c
^3*ArcSin[a*x])/2 + 2*a*c^3*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.174132, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1807, 815, 844, 216, 266, 63, 208}

$$-\frac{c^3(1-a^2x^2)^{3/2}}{x}-\frac{1}{2}ac^3(ax+4)\sqrt{1-a^2x^2}+2ac^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{1}{2}ac^3\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^2,x]
```

```
[Out] -(a*c^3*(4 + a*x)*Sqrt[1 - a^2*x^2])/2 - (c^3*(1 - a^2*x^2)^(3/2))/x - (a*c
^3*ArcSin[a*x])/2 + 2*a*c^3*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^2}\,dx = c\int \frac{(c-acx)^2\sqrt{1-a^2x^2}}{x^2}\,dx$$
$$= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{x} - c\int \frac{\left(2ac^2+a^2c^2x\right)\sqrt{1-a^2x^2}}{x}\,dx$$
$$= -\frac{1}{2}ac^3(4+ax)\sqrt{1-a^2x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{x} + \frac{c\int \frac{-4a^3c^2-a^4c^2x}{x\sqrt{1-a^2x^2}}\,dx}{2a^2}$$
$$= -\frac{1}{2}ac^3(4+ax)\sqrt{1-a^2x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{x} - \left(2ac^3\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - \frac{1}{2}\left(a^2c^3\right)\int \frac{}{\sqrt{1-}}$$
$$= -\frac{1}{2}ac^3(4+ax)\sqrt{1-a^2x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{x} - \frac{1}{2}ac^3\sin^{-1}(ax) - \left(ac^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2}}\right.$$
$$= -\frac{1}{2}ac^3(4+ax)\sqrt{1-a^2x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{x} - \frac{1}{2}ac^3\sin^{-1}(ax) + \frac{\left(2c^3\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,\right.}{a}$$
$$= -\frac{1}{2}ac^3(4+ax)\sqrt{1-a^2x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{x} - \frac{1}{2}ac^3\sin^{-1}(ax) + 2ac^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] time = 0.102182, size = 143, normalized size = 1.72

$$-\frac{c^3\left(a^4x^4-4a^3x^3-3a^2x^2+2ax\sqrt{1-a^2x^2}\sin^{-1}(ax)+2ax\sqrt{1-a^2x^2}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-4ax\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x}\right.\right.}{2x\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^2,x]
```

```
[Out] -(c^3*(2 + 4*a*x - 3*a^2*x^2 - 4*a^3*x^3 + a^4*x^4 + 2*a*x*Sqrt[1 - a^2*x^2
]*ArcSin[a*x] + 2*a*x*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]] - 4*a
*x*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(2*x*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.042, size = 113, normalized size = 1.4

$$\frac{a^2c^3x}{2}\sqrt{-a^2x^2+1}-\frac{c^3a^2}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-2c^3a\sqrt{-a^2x^2+1}-\frac{c^3}{x}\sqrt{-a^2x^2+1}+2c^3a\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^2,x)
```

```
[Out] 1/2*c^3*a^2*x*(-a^2*x^2+1)^(1/2)-1/2*c^3*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)
*x/(-a^2*x^2+1)^(1/2))-2*c^3*a*(-a^2*x^2+1)^(1/2)-c^3/x*(-a^2*x^2+1)^(1/2)+
2*c^3*a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.44037, size = 157, normalized size = 1.89

$$\frac{1}{2}\sqrt{-a^2x^2+1}a^2c^3x-\frac{a^2c^3\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}}+2ac^3\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-2\sqrt{-a^2x^2+1}ac^3-\frac{\sqrt{-a^2x^2+1}c^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*a^2*c^3*x - 1/2*a^2*c^3*arcsin(a^2*x/sqrt(a^2))/sqrt
(a^2) + 2*a*c^3*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 2*sqrt(-a^2*x
^2 + 1)*a*c^3 - sqrt(-a^2*x^2 + 1)*c^3/x
```

Fricas [A] time = 1.54244, size = 228, normalized size = 2.75

$$\frac{2ac^3x\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-4ac^3x\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-4ac^3x+\left(a^2c^3x^2-4ac^3x-2c^3\right)\sqrt{-a^2x^2+1}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^2,x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*a*c^3*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 4*a*c^3*x*log((sqrt
(-a^2*x^2 + 1) - 1)/x) - 4*a*c^3*x + (a^2*c^3*x^2 - 4*a*c^3*x - 2*c^3)*sqrt
(-a^2*x^2 + 1))/x
```

Sympy [C] time = 7.11276, size = 199, normalized size = 2.4

$$-a^4c^3\left(\begin{cases} -\frac{ix\sqrt{a^2x^2-1}}{2a^2} - \frac{i\,\mathrm{acosh}\,(ax)}{2a^3} & \text{for } |a^2x^2| > 1 \\ \frac{x^3}{2\sqrt{-a^2x^2+1}} - \frac{x}{2a^2\sqrt{-a^2x^2+1}} + \frac{\mathrm{asin}\,(ax)}{2a^3} & \text{otherwise} \end{cases}\right) + 2a^3c^3\left(\begin{cases} \frac{x^2}{2} & \text{for } a^2 = 0 \\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise} \end{cases}\right) - 2ac^3\left(\begin{cases} -\,\mathrm{acosh}\left(\frac{1}{ax}\right) \\ i\,\mathrm{asin}\left(\frac{1}{ax}\right) \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**3/x**2,x)
```

```
[Out] -a**4*c**3*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a
**3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(
-a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) + 2*a**3*c**3*Piecewise((x**2
/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) - 2*a*c**3*Piecewise(
(-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True)) + c**3*Pi
ecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 +
 1)/x, True))
```

Giac [B] time = 1.27673, size = 205, normalized size = 2.47

$$\frac{a^4c^3x}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|} - \frac{a^2c^3\arcsin(ax)\,\mathrm{sgn}(a)}{2|a|} + \frac{2a^2c^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|} - \frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)c^3}{2x|a|} + \frac{1}{2}\left(a^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a^4*c^3*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - 1/2*a^2*c^3*arcsin
(a*x)*sgn(a)/abs(a) + 2*a^2*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
2*a)/(a^2*abs(x)))/abs(a) - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3/(x*abs(
a)) + 1/2*(a^2*c^3*x - 4*a*c^3)*sqrt(-a^2*x^2 + 1)
```

3.312 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^3} dx$

Optimal. Leaf size=92

$$-\frac{c^3(1-a^2x^2)^{3/2}}{2x^2}+\frac{ac^3(ax+4)\sqrt{1-a^2x^2}}{2x}-\frac{1}{2}a^2c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+2a^2c^3\sin^{-1}(ax)$$

```
[Out] (a*c^3*(4 + a*x)*Sqrt[1 - a^2*x^2])/(2*x) - (c^3*(1 - a^2*x^2)^(3/2))/(2*x^
2) + 2*a^2*c^3*ArcSin[a*x] - (a^2*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rubi [A] time = 0.179135, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1807, 813, 844, 216, 266, 63, 208}

$$-\frac{c^3(1-a^2x^2)^{3/2}}{2x^2}+\frac{ac^3(ax+4)\sqrt{1-a^2x^2}}{2x}-\frac{1}{2}a^2c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+2a^2c^3\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^3,x]
```

```
[Out] (a*c^3*(4 + a*x)*Sqrt[1 - a^2*x^2])/(2*x) - (c^3*(1 - a^2*x^2)^(3/2))/(2*x^
2) + 2*a^2*c^3*ArcSin[a*x] - (a^2*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^3}\,dx &= c\int \frac{(c-acx)^2\sqrt{1-a^2x^2}}{x^3}\,dx\\
&= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2x^2}-\frac{1}{2}c\int \frac{\left(4ac^2-a^2c^2x\right)\sqrt{1-a^2x^2}}{x^2}\,dx\\
&= \frac{ac^3(4+ax)\sqrt{1-a^2x^2}}{2x}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2x^2}+\frac{1}{4}c\int \frac{2a^2c^2+8a^3c^2x}{x\sqrt{1-a^2x^2}}\,dx\\
&= \frac{ac^3(4+ax)\sqrt{1-a^2x^2}}{2x}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2x^2}+\frac{1}{2}\left(a^2c^3\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx+\left(2a^3c^3\right)\int \frac{1}{\sqrt{1-
\end{aligned}$$

$$\begin{aligned}
&= \frac{ac^3(4+ax)\sqrt{1-a^2x^2}}{2x}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2x^2}+2a^2c^3\sin^{-1}(ax)+\frac{1}{4}\left(a^2c^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a
\end{aligned}$$

$$\begin{aligned}
&= \frac{ac^3(4+ax)\sqrt{1-a^2x^2}}{2x}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2x^2}+2a^2c^3\sin^{-1}(ax)-\frac{1}{2}c^3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x
\end{aligned}$$

$$\begin{aligned}
&= \frac{ac^3(4+ax)\sqrt{1-a^2x^2}}{2x}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2x^2}+2a^2c^3\sin^{-1}(ax)-\frac{1}{2}a^2c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.123942, size = 155, normalized size = 1.68

$$\frac{c^3\left(-4a^4x^4-8a^3x^3+6a^2x^2+a^2x^2\sqrt{1-a^2x^2}\sin^{-1}(ax)-14a^2x^2\sqrt{1-a^2x^2}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-2a^2x^2\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{\right.}\right.}{4x^2\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^3,x]
```

```
[Out] (c^3*(-2 + 8*a*x + 6*a^2*x^2 - 8*a^3*x^3 - 4*a^4*x^4 + a^2*x^2*Sqrt[1 - a^2
*x^2]*ArcSin[a*x] - 14*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[
2]] - 2*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(4*x^2*Sqrt[
1 - a^2*x^2])
```

Maple [A] time = 0.04, size = 116, normalized size = 1.3

$$c^3a^2\sqrt{-a^2x^2+1}+2\frac{c^3a^3}{\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+2\frac{c^3a\sqrt{-a^2x^2+1}}{x}-\frac{c^3}{2x^2}\sqrt{-a^2x^2+1}-\frac{c^3a^2}{2}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^3,x)
```

```
[Out] c^3*a^2*(-a^2*x^2+1)^(1/2)+2*c^3*a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2
*x^2+1)^(1/2))+2*c^3*a/x*(-a^2*x^2+1)^(1/2)-1/2*c^3/x^2*(-a^2*x^2+1)^(1/2)-
1/2*c^3*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.43181, size = 161, normalized size = 1.75

$$\frac{2a^3c^3\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}-\frac{1}{2}a^2c^3\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\sqrt{-a^2x^2+1}a^2c^3+\frac{2\sqrt{-a^2x^2+1}ac^3}{x}-\frac{\sqrt{-a^2x^2+1}c^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^3,x, algorithm="maxima"
)
```

```
[Out] 2*a^3*c^3*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - 1/2*a^2*c^3*log(2*sqrt(-a^2*x
^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*x^2 + 1)*a^2*c^3 + 2*sqrt(-a^2*x^2 +
 1)*a*c^3/x - 1/2*sqrt(-a^2*x^2 + 1)*c^3/x^2
```

Fricas [A] time = 1.61909, size = 246, normalized size = 2.67

$$-\frac{8a^2c^3x^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-a^2c^3x^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-2a^2c^3x^2-\left(2a^2c^3x^2+4ac^3x-c^3\right)\sqrt{-a^2x^2+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^3,x, algorithm="fricas"
)
```

```
[Out] -1/2*(8*a^2*c^3*x^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - a^2*c^3*x^2*lo
g((sqrt(-a^2*x^2 + 1) - 1)/x) - 2*a^2*c^3*x^2 - (2*a^2*c^3*x^2 + 4*a*c^3*x
- c^3)*sqrt(-a^2*x^2 + 1))/x^2
```

Sympy [C] time = 5.46274, size = 228, normalized size = 2.48

$$-a^4c^3\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)+2a^3c^3\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\end{cases}\right)-2ac^3\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for } \left|a^2x^2\right|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**3/x**3,x)
```

```
[Out] -a**4*c**3*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, Tr
ue)) + 2*a**3*c**3*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0),
(sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - 2*a*c**3*Piecewise((-I*sq
rt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))
+ c**3*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x)
, 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**
2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))
```

Giac [B] time = 1.30847, size = 286, normalized size = 3.11

$$\frac{2\,a^3c^3\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{|a|}-\frac{a^3c^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}+\sqrt{-a^2x^2+1}a^2c^3+\frac{\left(a^3c^3-\frac{8\left(\sqrt{-a^2x^2+1}|a|+a\right)ac^3}{x}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}|a|+a\right)^2|a|}+\frac{8\left(\sqrt{-a\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^3,x, algorithm="giac")
```

```
[Out] 2*a^3*c^3*arcsin(a*x)*sgn(a)/abs(a) - 1/2*a^3*c^3*log(1/2*abs(-2*sqrt(-a^2*
x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*a^2*c^3 +
1/8*(a^3*c^3 - 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a*c^3/x)*a^4*x^2/((sqrt(-a
^2*x^2 + 1)*abs(a) + a)^2*abs(a)) + 1/8*(8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*
a*c^3*abs(a)/x - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3*abs(a)/(a*x^2))/a^2
```

3.313 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^4} dx$

Optimal. Leaf size=88

$$-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{ac^3(1-ax)\sqrt{1-a^2x^2}}{x^2}-a^3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-a^3c^3\sin^{-1}(ax)$$

```
[Out] (a*c^3*(1 - a*x)*Sqrt[1 - a^2*x^2])/x^2 - (c^3*(1 - a^2*x^2)^(3/2))/(3*x^3)
 - a^3*c^3*ArcSin[a*x] - a^3*c^3*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.178863, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1807, 811, 844, 216, 266, 63, 208}

$$-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{ac^3(1-ax)\sqrt{1-a^2x^2}}{x^2}-a^3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-a^3c^3\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^4,x]
```

```
[Out] (a*c^3*(1 - a*x)*Sqrt[1 - a^2*x^2])/x^2 - (c^3*(1 - a^2*x^2)^(3/2))/(3*x^3)
 - a^3*c^3*ArcSin[a*x] - a^3*c^3*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 && !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^4}\,dx &= c\int \frac{(c-acx)^2\sqrt{1-a^2x^2}}{x^4}\,dx\\
&= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{3x^3}-\frac{1}{3}c\int \frac{\left(6ac^2-3a^2c^2x\right)\sqrt{1-a^2x^2}}{x^3}\,dx\\
&= \frac{ac^3(1-ax)\sqrt{1-a^2x^2}}{x^2}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{1}{12}c\int \frac{12a^3c^2-12a^4c^2x}{x\sqrt{1-a^2x^2}}\,dx\\
&= \frac{ac^3(1-ax)\sqrt{1-a^2x^2}}{x^2}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{3x^3}+\left(a^3c^3\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx-\left(a^4c^3\right)\int \frac{1}{\sqrt{1-a^2x^2}}\\
&= \frac{ac^3(1-ax)\sqrt{1-a^2x^2}}{x^2}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{3x^3}-a^3c^3\sin^{-1}(ax)+\frac{1}{2}\left(a^3c^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2}}\right.\\
&= \frac{ac^3(1-ax)\sqrt{1-a^2x^2}}{x^2}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{3x^3}-a^3c^3\sin^{-1}(ax)-\left(ac^3\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x\right.\\
&= \frac{ac^3(1-ax)\sqrt{1-a^2x^2}}{x^2}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{3x^3}-a^3c^3\sin^{-1}(ax)-a^3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.104318, size = 156, normalized size = 1.77

$$\frac{c^3\left(4a^4x^4-6a^3x^3-2a^2x^2+3a^3x^3\sqrt{1-a^2x^2}\sin^{-1}(ax)+18a^3x^3\sqrt{1-a^2x^2}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-6a^3x^3\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{\right.\right.}{6x^3\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^4,x]
```

```
[Out] (c^3*(-2 + 6*a*x - 2*a^2*x^2 - 6*a^3*x^3 + 4*a^4*x^4 + 3*a^3*x^3*Sqrt[1 - a
^2*x^2]*ArcSin[a*x] + 18*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqr
t[2]] - 6*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(6*x^3*Sqr
t[1 - a^2*x^2])
```

Maple [A] time = 0.042, size = 119, normalized size = 1.4

$$-a^4c^3\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-c^3a^3\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{c^3a}{x^2}\sqrt{-a^2x^2+1}-\frac{c^3}{3x^3}\sqrt{-a^2x^2+1}-\frac{2c^3a^2}{3x}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^4,x)
```

```
[Out] -c^3*a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-c^3*a^3*arcta
nh(1/(-a^2*x^2+1)^(1/2))+c^3*a/x^2*(-a^2*x^2+1)^(1/2)-1/3*c^3/x^3*(-a^2*x^2
+1)^(1/2)-2/3*c^3*a^2/x*(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.44074, size = 165, normalized size = 1.88

$$-\frac{a^4c^3\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}-a^3c^3\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{2\sqrt{-a^2x^2+1}a^2c^3}{3x}+\frac{\sqrt{-a^2x^2+1}ac^3}{x^2}-\frac{\sqrt{-a^2x^2+1}c^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^4,x, algorithm="maxima"
)
```

```
[Out] -a^4*c^3*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - a^3*c^3*log(2*sqrt(-a^2*x^2 +
1)/abs(x) + 2/abs(x)) - 2/3*sqrt(-a^2*x^2 + 1)*a^2*c^3/x + sqrt(-a^2*x^2 +
1)*a*c^3/x^2 - 1/3*sqrt(-a^2*x^2 + 1)*c^3/x^3
```

Fricas [A] time = 1.65232, size = 225, normalized size = 2.56

$$\frac{6a^3c^3x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+3a^3c^3x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(2a^2c^3x^2-3ac^3x+c^3\right)\sqrt{-a^2x^2+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^4,x, algorithm="fricas"
)
```

```
[Out] 1/3*(6*a^3*c^3*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 3*a^3*c^3*x^3*l
og((sqrt(-a^2*x^2 + 1) - 1)/x) - (2*a^2*c^3*x^2 - 3*a*c^3*x + c^3)*sqrt(-a^
2*x^2 + 1))/x^3
```

Sympy [C] time = 6.57562, size = 279, normalized size = 3.17

$$-a^4c^3\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)+2a^3c^3\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)-2ac^3\left(\begin{cases}-\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\cdots}}{2x}\\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}\cdots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**3/x**4,x)
```

```
[Out] -a**4*c**3*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1
/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + 2*a**3*c**3*Piecewise((-acosh(1/(
a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True)) - 2*a*c**3*Piecewise
((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**
2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2
*a*x**3*sqrt(1 - 1/(a**2*x**2))), True)) + c**3*Piecewise((-2*I*a**2*sqrt(a
**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) > 1),
(-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), True))
```

Giac [B] time = 1.23791, size = 338, normalized size = 3.84

$$-\frac{a^4c^3\arcsin(ax)\operatorname{sgn}(a)}{|a|}-\frac{a^4c^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|}+\frac{\left(a^4c^3-\frac{6\left(\sqrt{-a^2x^2+1}|a|+a\right)a^2c^3}{x}+\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^3}{x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|}-\frac{9(\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^4,x, algorithm="giac")
```

```
[Out] -a^4*c^3*arcsin(a*x)*sgn(a)/abs(a) - a^4*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 +
 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 1/24*(a^4*c^3 - 6*(sqrt(-a^2*x^2 +
 1)*abs(a) + a)*a^2*c^3/x + 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3/x^2)*a^
6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*abs(a)) - 1/24*(9*(sqrt(-a^2*x^2 +
 1)*abs(a) + a)*a^4*c^3/x - 6*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^3/x^2
 + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^3/x^3)/(a^2*abs(a))
```

3.314 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^5}\,dx$

Optimal. Leaf size=102

$$-\frac{5a^2c^3\sqrt{1-a^2x^2}}{8x^2}+\frac{2ac^3\left(1-a^2x^2\right)^{3/2}}{3x^3}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{5}{8}a^4c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] (-5*a^2*c^3*Sqrt[1 - a^2*x^2])/(8*x^2) - (c^3*(1 - a^2*x^2)^(3/2))/(4*x^4)
+ (2*a*c^3*(1 - a^2*x^2)^(3/2))/(3*x^3) + (5*a^4*c^3*ArcTanh[Sqrt[1 - a^2*x
^2]])/8
```

Rubi [A] time = 0.163448, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 1807, 807, 266, 47, 63, 208}

$$-\frac{5a^2c^3\sqrt{1-a^2x^2}}{8x^2}+\frac{2ac^3\left(1-a^2x^2\right)^{3/2}}{3x^3}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{5}{8}a^4c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^5,x]
```

```
[Out] (-5*a^2*c^3*Sqrt[1 - a^2*x^2])/(8*x^2) - (c^3*(1 - a^2*x^2)^(3/2))/(4*x^4)
+ (2*a*c^3*(1 - a^2*x^2)^(3/2))/(3*x^3) + (5*a^4*c^3*ArcTanh[Sqrt[1 - a^2*x
^2]])/8
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^5}\,dx &= c\int \frac{(c-acx)^2\sqrt{1-a^2x^2}}{x^5}\,dx\\
&= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{1}{4}c\int \frac{\left(8ac^2-5a^2c^2x\right)\sqrt{1-a^2x^2}}{x^4}\,dx\\
&= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{2ac^3\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{1}{4}\left(5a^2c^3\right)\int \frac{\sqrt{1-a^2x^2}}{x^3}\,dx\\
&= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{2ac^3\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{1}{8}\left(5a^2c^3\right)\text{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{5a^2c^3\sqrt{1-a^2x^2}}{8x^2}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{2ac^3\left(1-a^2x^2\right)^{3/2}}{3x^3}-\frac{1}{16}\left(5a^4c^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-}\right.\\
&= -\frac{5a^2c^3\sqrt{1-a^2x^2}}{8x^2}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{2ac^3\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{1}{8}\left(5a^2c^3\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\right.\\
&= -\frac{5a^2c^3\sqrt{1-a^2x^2}}{8x^2}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{2ac^3\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{5}{8}a^4c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0322196, size = 99, normalized size = 0.97

$$
\frac{c^3\left(16a^5x^5+9a^4x^4-32a^3x^3-3a^2x^2+15a^4x^4\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+16ax-6\right)}{24x^4\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^5,x]
```

```
[Out] (c^3*(-6 + 16*a*x - 3*a^2*x^2 - 32*a^3*x^3 + 9*a^4*x^4 + 16*a^5*x^5 + 15*a^
4*x^4*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(24*x^4*Sqrt[1 - a^2*x
```

```
^2])
```

Maple [A] time = 0.046, size = 144, normalized size = 1.4

$$-c^3\left(\frac{1}{4x^4}\sqrt{-a^2x^2+1}-\frac{3a^2}{4}\left(-\frac{1}{2x^2}\sqrt{-a^2x^2+1}-\frac{a^2}{2}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)+2\frac{a^3\sqrt{-a^2x^2+1}}{x}-a^4\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^5,x)
```

```
[Out] -c^3*(1/4*(-a^2*x^2+1)^(1/2)/x^4-3/4*a^2*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a
^2*arctanh(1/(-a^2*x^2+1)^(1/2)))+2*a^3*(-a^2*x^2+1)^(1/2)/x-a^4*arctanh(1/
(-a^2*x^2+1)^(1/2))+2*a*(-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2+1)^(
1/2)/x))
```

Maxima [A] time = 1.43689, size = 165, normalized size = 1.62

$$\frac{5}{8}a^4c^3\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{2\sqrt{-a^2x^2+1}a^3c^3}{3x}-\frac{3\sqrt{-a^2x^2+1}a^2c^3}{8x^2}+\frac{2\sqrt{-a^2x^2+1}ac^3}{3x^3}-\frac{\sqrt{-a^2x^2+1}c^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^5,x, algorithm="maxima"
)
```

```
[Out] 5/8*a^4*c^3*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 2/3*sqrt(-a^2*x^2
 + 1)*a^3*c^3/x - 3/8*sqrt(-a^2*x^2 + 1)*a^2*c^3/x^2 + 2/3*sqrt(-a^2*x^2 +
1)*a*c^3/x^3 - 1/4*sqrt(-a^2*x^2 + 1)*c^3/x^4
```

Fricas [A] time = 1.56625, size = 182, normalized size = 1.78

$$-\frac{15a^4c^3x^4\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(16a^3c^3x^3+9a^2c^3x^2-16ac^3x+6c^3\right)\sqrt{-a^2x^2+1}}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^5,x, algorithm="fricas"
)
```

```
[Out] -1/24*(15*a^4*c^3*x^4*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (16*a^3*c^3*x^3 + 9
*a^2*c^3*x^2 - 16*a*c^3*x + 6*c^3)*sqrt(-a^2*x^2 + 1))/x^4
```

Sympy [C] time = 8.71955, size = 347, normalized size = 3.4

$$-a^4c^3\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)+2a^3c^3\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\ \left|a^2x^2\right|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)-2ac^3\left(\begin{cases}-\frac{2ia^2\sqrt{a^2x^2-1}}{3x}-\frac{i\sqrt{a^2x^2-1}}{3x^3} & \text{for}\ \left|a^2\right.\\ -\frac{2a^2\sqrt{-a^2x^2+1}}{3x}-\frac{\sqrt{-a^2x^2+1}}{3x^3} & \text{other}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**3/x**5,x)
```

```
[Out] -a**4*c**3*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*
x)), True)) + 2*a**3*c**3*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**
2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True)) - 2*a*c**3*Piecewise((-2*I*a**2*s
qrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) >
 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), T
rue)) + c**3*Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*a**3/(8*x*sqrt(-1 + 1/
(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 +
 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*asin(1/(a*x))/8 - 3*I*a*
*3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I
/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True))
```

Giac [B] time = 1.16439, size = 405, normalized size = 3.97

$$\frac{\left(3\,a^5c^3 - \frac{16\left(\sqrt{-a^2x^2+1}|a|+a\right)a^3c^3}{x} + \frac{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^2ac^3}{x^2} + \frac{48\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c^3}{ax^3}\right)a^8x^4}{192\left(\sqrt{-a^2x^2+1}|a|+a\right)^4|a|} + \frac{5\,a^5c^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{8\,|a|} - \frac{48\left(\sqrt{\ }\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^5*c^3 - 16*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^3*c^3/x + 24*(sqrt(
-a^2*x^2 + 1)*abs(a) + a)^2*a*c^3/x^2 + 48*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^
3*c^3/(a*x^3))*a^8*x^4/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*abs(a)) + 5/8*a^5
*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) -
 1/192*(48*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^5*c^3*abs(a)/x + 24*(sqrt(-a^2
*x^2 + 1)*abs(a) + a)^2*a^3*c^3*abs(a)/x^2 - 16*(sqrt(-a^2*x^2 + 1)*abs(a)
+ a)^3*a*c^3*abs(a)/x^3 + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^3*abs(a)/(a
*x^4))/a^4
```

3.315 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^6}\,dx$

Optimal. Leaf size=129

$$\frac{a^3c^3\sqrt{1-a^2x^2}}{4x^2}-\frac{7a^2c^3\left(1-a^2x^2\right)^{3/2}}{15x^3}+\frac{ac^3\left(1-a^2x^2\right)^{3/2}}{2x^4}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{1}{4}a^5c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] (a^3*c^3*Sqrt[1 - a^2*x^2])/(4*x^2) - (c^3*(1 - a^2*x^2)^(3/2))/(5*x^5) + (
a*c^3*(1 - a^2*x^2)^(3/2))/(2*x^4) - (7*a^2*c^3*(1 - a^2*x^2)^(3/2))/(15*x^
3) - (a^5*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/4
```

Rubi [A] time = 0.190251, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1807, 835, 807, 266, 47, 63, 208}

$$\frac{a^3c^3\sqrt{1-a^2x^2}}{4x^2}-\frac{7a^2c^3\left(1-a^2x^2\right)^{3/2}}{15x^3}+\frac{ac^3\left(1-a^2x^2\right)^{3/2}}{2x^4}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{1}{4}a^5c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^6,x]
```

```
[Out] (a^3*c^3*Sqrt[1 - a^2*x^2])/(4*x^2) - (c^3*(1 - a^2*x^2)^(3/2))/(5*x^5) + (
a*c^3*(1 - a^2*x^2)^(3/2))/(2*x^4) - (7*a^2*c^3*(1 - a^2*x^2)^(3/2))/(15*x^
3) - (a^5*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/4
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^6}\,dx = c\int \frac{(c-acx)^2\sqrt{1-a^2x^2}}{x^6}\,dx$$

$$= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{1}{5}c\int \frac{\left(10ac^2-7a^2c^2x\right)\sqrt{1-a^2x^2}}{x^5}\,dx$$

$$= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^3\left(1-a^2x^2\right)^{3/2}}{2x^4} + \frac{1}{20}c\int \frac{\left(28a^2c^2-10a^3c^2x\right)\sqrt{1-a^2x^2}}{x^4}\,dx$$

$$= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^3\left(1-a^2x^2\right)^{3/2}}{2x^4} - \frac{7a^2c^3\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{2}\left(a^3c^3\right)\int \frac{\sqrt{1-a^2x^2}}{x^3}\,dx$$

$$= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^3\left(1-a^2x^2\right)^{3/2}}{2x^4} - \frac{7a^2c^3\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{4}\left(a^3c^3\right)\mathrm{Subst}\left(\int \frac{\sqrt{1-a^2}}{x^2}\right.$$

$$= \frac{a^3c^3\sqrt{1-a^2x^2}}{4x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^3\left(1-a^2x^2\right)^{3/2}}{2x^4} - \frac{7a^2c^3\left(1-a^2x^2\right)^{3/2}}{15x^3} + \frac{1}{8}\left(a^5c^3\right)\mathrm{Su}$$

$$= \frac{a^3c^3\sqrt{1-a^2x^2}}{4x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^3\left(1-a^2x^2\right)^{3/2}}{2x^4} - \frac{7a^2c^3\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{4}\left(a^3c^3\right)\mathrm{Su}$$

$$= \frac{a^3c^3\sqrt{1-a^2x^2}}{4x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^3\left(1-a^2x^2\right)^{3/2}}{2x^4} - \frac{7a^2c^3\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{4}a^5c^3\tanh$$

Mathematica [A] time = 0.0332235, size = 107, normalized size = 0.83

$$-\frac{c^3\left(28a^6x^6-15a^5x^5-44a^4x^4+45a^3x^3+4a^2x^2+15a^5x^5\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-30ax+12\right)}{60x^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^6,x]
```

```
[Out] -(c^3*(12 - 30*a*x + 4*a^2*x^2 + 45*a^3*x^3 - 44*a^4*x^4 - 15*a^5*x^5 + 28*
a^6*x^6 + 15*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(60*x^5
*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.043, size = 190, normalized size = 1.5

$$-c^3\left(2a\left(-1/4\frac{\sqrt{-a^2x^2+1}}{x^4}+3/4a^2\left(-1/2\frac{\sqrt{-a^2x^2+1}}{x^2}-1/2a^2\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)\right)-\frac{a^4}{x}\sqrt{-a^2x^2+1}+\frac{1}{5x^5}\sqrt{-a^2x^2+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^6,x)
```

```
[Out] -c^3*(2*a*(-1/4*(-a^2*x^2+1)^(1/2)/x^4+3/4*a^2*(-1/2*(-a^2*x^2+1)^(1/2)/x^2
-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))))-a^4/x*(-a^2*x^2+1)^(1/2)+1/5/x^5*(
-a^2*x^2+1)^(1/2)-4/5*a^2*(-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2+1)
^(1/2)/x)-2*a^3*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)
^(1/2))))
```

Maxima [A] time = 1.42988, size = 196, normalized size = 1.52

$$-\frac{1}{4}a^5c^3\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{7\sqrt{-a^2x^2+1}a^4c^3}{15x}-\frac{\sqrt{-a^2x^2+1}a^3c^3}{4x^2}-\frac{4\sqrt{-a^2x^2+1}a^2c^3}{15x^3}+\frac{\sqrt{-a^2x^2+1}ac^3}{2x^4}-\frac{\sqrt{-a^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^6,x, algorithm="maxima"
)
```

```
[Out] -1/4*a^5*c^3*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 7/15*sqrt(-a^2*x
^2 + 1)*a^4*c^3/x - 1/4*sqrt(-a^2*x^2 + 1)*a^3*c^3/x^2 - 4/15*sqrt(-a^2*x^2
 + 1)*a^2*c^3/x^3 + 1/2*sqrt(-a^2*x^2 + 1)*a*c^3/x^4 - 1/5*sqrt(-a^2*x^2 +
1)*c^3/x^5
```

Fricas [A] time = 1.6267, size = 207, normalized size = 1.6

$$\frac{15a^5c^3x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(28a^4c^3x^4-15a^3c^3x^3-16a^2c^3x^2+30ac^3x-12c^3\right)\sqrt{-a^2x^2+1}}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^6,x, algorithm="fricas"
)
```

```
[Out] 1/60*(15*a^5*c^3*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (28*a^4*c^3*x^4 - 15
*a^3*c^3*x^3 - 16*a^2*c^3*x^2 + 30*a*c^3*x - 12*c^3)*sqrt(-a^2*x^2 + 1))/x^
5
```

Sympy [C] time = 10.1219, size = 476, normalized size = 3.69

$$-a^4c^3\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for } |a^2x^2|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)+2a^3c^3\left(\begin{cases}-\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} & \text{for } \frac{1}{|a^2x^2|}>1\\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)-2ac^3\left(\begin{cases}-\frac{3a^4 a\ldots}{\ldots}\\ \frac{3ia^4\operatorname{asi}\ldots}{8}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**3/x**6,x)
```

```
[Out] -a**4*c**3*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt
(-a**2*x**2 + 1)/x, True)) + 2*a**3*c**3*Piecewise((-a**2*acosh(1/(a*x))/2
- a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(
a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*
x**2))), True)) - 2*a*c**3*Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*a**3/(8*
x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - 1/(4*a*
x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*asin(1/(a*
x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1 - 1/(a*
*2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True)) + c**3*Piecewise(
(-8*a**5*sqrt(-1 + 1/(a**2*x**2))/15 - 4*a**3*sqrt(-1 + 1/(a**2*x**2))/(15*
x**2) - a*sqrt(-1 + 1/(a**2*x**2))/(5*x**4), 1/Abs(a**2*x**2) > 1), (-8*I*a
**5*sqrt(1 - 1/(a**2*x**2))/15 - 4*I*a**3*sqrt(1 - 1/(a**2*x**2))/(15*x**2)
 - I*a*sqrt(1 - 1/(a**2*x**2))/(5*x**4), True))
```

Giac [B] time = 1.1874, size = 401, normalized size = 3.11

$$\frac{\left(3\,a^6c^3-\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4c^3}{x}+\frac{25\left(\sqrt{-a^2x^2+1}|a|+a\right)^2a^2c^3}{x^2}-\frac{90\left(\sqrt{-a^2x^2+1}|a|+a\right)^4c^3}{a^2x^4}\right)a^{10}x^5}{480\left(\sqrt{-a^2x^2+1}|a|+a\right)^5|a|}-\frac{a^6c^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{4\,|a|}+\frac{90\left(\sqrt{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^6,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^6*c^3 - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^3/x + 25*(sqrt(
-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^3/x^2 - 90*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)^4*c^3/(a^2*x^4))*a^10*x^5/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*abs(a)) - 1/
4*a^6*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs
(a) + 1/480*(90*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^8*c^3/x - 25*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^3*a^4*c^3/x^3 + 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*a^
2*c^3/x^4 - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^3/x^5)/(a^4*abs(a))
```

3.316 $\int e^{\tanh^{-1}(ax)}x^3(c-acx)^4\,dx$

Optimal. Leaf size=173

$$\frac{1}{8}ac^4x^5\left(1-a^2x^2\right)^{3/2}-\frac{3}{7}c^4x^4\left(1-a^2x^2\right)^{3/2}+\frac{29c^4x^3\left(1-a^2x^2\right)^{3/2}}{48a}-\frac{19c^4x^2\left(1-a^2x^2\right)^{3/2}}{35a^2}-\frac{29c^4x\sqrt{1-a^2x^2}}{128a^3}-\frac{c^4(2432-3}{}$$

```
[Out] (-29*c^4*x*Sqrt[1 - a^2*x^2])/(128*a^3) - (19*c^4*x^2*(1 - a^2*x^2)^(3/2))/
(35*a^2) + (29*c^4*x^3*(1 - a^2*x^2)^(3/2))/(48*a) - (3*c^4*x^4*(1 - a^2*x^
2)^(3/2))/7 + (a*c^4*x^5*(1 - a^2*x^2)^(3/2))/8 - (c^4*(2432 - 3045*a*x)*(1
 - a^2*x^2)^(3/2))/(6720*a^4) - (29*c^4*ArcSin[a*x])/(128*a^4)
```

Rubi [A] time = 0.333121, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 1809, 833, 780, 195, 216}

$$\frac{1}{8}ac^4x^5\left(1-a^2x^2\right)^{3/2}-\frac{3}{7}c^4x^4\left(1-a^2x^2\right)^{3/2}+\frac{29c^4x^3\left(1-a^2x^2\right)^{3/2}}{48a}-\frac{19c^4x^2\left(1-a^2x^2\right)^{3/2}}{35a^2}-\frac{29c^4x\sqrt{1-a^2x^2}}{128a^3}-\frac{c^4(2432-3}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^3*(c - a*c*x)^4,x]
```


```
[Out] (-29*c^4*x*Sqrt[1 - a^2*x^2])/(128*a^3) - (19*c^4*x^2*(1 - a^2*x^2)^(3/2))/
(35*a^2) + (29*c^4*x^3*(1 - a^2*x^2)^(3/2))/(48*a) - (3*c^4*x^4*(1 - a^2*x^
2)^(3/2))/7 + (a*c^4*x^5*(1 - a^2*x^2)^(3/2))/8 - (c^4*(2432 - 3045*a*x)*(1
 - a^2*x^2)^(3/2))/(6720*a^4) - (29*c^4*ArcSin[a*x])/(128*a^4)
```


Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^3(c-acx)^4\,dx = c\int x^3(c-acx)^3\sqrt{1-a^2x^2}\,dx$$
$$= \frac{1}{8}ac^4x^5\left(1-a^2x^2\right)^{3/2} - \frac{c\int x^3\sqrt{1-a^2x^2}\left(-8a^2c^3+29a^3c^3x-24a^4c^3x^2\right)dx}{8a^2}$$
$$= -\frac{3}{7}c^4x^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{8}ac^4x^5\left(1-a^2x^2\right)^{3/2} + \frac{c\int x^3\left(152a^4c^3-203a^5c^3x\right)\sqrt{1-a^2x^2}\,d}{56a^4}$$
$$= \frac{29c^4x^3\left(1-a^2x^2\right)^{3/2}}{48a} - \frac{3}{7}c^4x^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{8}ac^4x^5\left(1-a^2x^2\right)^{3/2} - \frac{c\int x^2\left(609a^5c^3-9\right.}{3}$$
$$= -\frac{19c^4x^2\left(1-a^2x^2\right)^{3/2}}{35a^2} + \frac{29c^4x^3\left(1-a^2x^2\right)^{3/2}}{48a} - \frac{3}{7}c^4x^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{8}ac^4x^5\left(1-a^2x^2\right)$$
$$= -\frac{19c^4x^2\left(1-a^2x^2\right)^{3/2}}{35a^2} + \frac{29c^4x^3\left(1-a^2x^2\right)^{3/2}}{48a} - \frac{3}{7}c^4x^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{8}ac^4x^5\left(1-a^2x^2\right)$$
$$= -\frac{29c^4x\sqrt{1-a^2x^2}}{128a^3} - \frac{19c^4x^2\left(1-a^2x^2\right)^{3/2}}{35a^2} + \frac{29c^4x^3\left(1-a^2x^2\right)^{3/2}}{48a} - \frac{3}{7}c^4x^4\left(1-a^2x^2\right)^{3/2} +$$
$$= -\frac{29c^4x\sqrt{1-a^2x^2}}{128a^3} - \frac{19c^4x^2\left(1-a^2x^2\right)^{3/2}}{35a^2} + \frac{29c^4x^3\left(1-a^2x^2\right)^{3/2}}{48a} - \frac{3}{7}c^4x^4\left(1-a^2x^2\right)^{3/2} +$$

Mathematica [A] time = 0.221875, size = 99, normalized size = 0.57

$$-\frac{c^4\left(\sqrt{1-a^2x^2}\left(1680a^7x^7-5760a^6x^6+6440a^5x^5-1536a^4x^4-2030a^3x^3+2432a^2x^2-3045ax+4864\right)-6090\sin^{-}\right.}{13440a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^3*(c - a*c*x)^4,x]
```

```
[Out] -(c^4*(Sqrt[1 - a^2*x^2]*(4864 - 3045*a*x + 2432*a^2*x^2 - 2030*a^3*x^3 - 1
536*a^4*x^4 + 6440*a^5*x^5 - 5760*a^6*x^6 + 1680*a^7*x^7) - 6090*ArcSin[Sqr
t[1 - a*x]/Sqrt[2]]))/(13440*a^4)
```

Maple [A] time = 0.071, size = 209, normalized size = 1.2

$$-\frac{c^4a^3x^7}{8}\sqrt{-a^2x^2+1}-\frac{23\,c^4ax^5}{48}\sqrt{-a^2x^2+1}+\frac{29\,c^4x^3}{192\,a}\sqrt{-a^2x^2+1}+\frac{29\,c^4x}{128\,a^3}\sqrt{-a^2x^2+1}-\frac{29\,c^4}{128\,a^3}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^4,x)
```

```
[Out] -1/8*c^4*a^3*x^7*(-a^2*x^2+1)^(1/2)-23/48*c^4*a*x^5*(-a^2*x^2+1)^(1/2)+29/1
92*c^4/a*x^3*(-a^2*x^2+1)^(1/2)+29/128*c^4*x*(-a^2*x^2+1)^(1/2)/a^3-29/128*
c^4/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+3/7*c^4*a^2*x^
6*(-a^2*x^2+1)^(1/2)+4/35*c^4*x^4*(-a^2*x^2+1)^(1/2)-19/105*c^4*x^2/a^2*(-a
^2*x^2+1)^(1/2)-38/105*c^4/a^4*(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.44966, size = 269, normalized size = 1.55

$$-\frac{1}{8}\sqrt{-a^2x^2+1}a^3c^4x^7+\frac{3}{7}\sqrt{-a^2x^2+1}a^2c^4x^6-\frac{23}{48}\sqrt{-a^2x^2+1}ac^4x^5+\frac{4}{35}\sqrt{-a^2x^2+1}c^4x^4+\frac{29\sqrt{-a^2x^2+1}c^4x^3}{192\,a}-\frac{19\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] -1/8*sqrt(-a^2*x^2 + 1)*a^3*c^4*x^7 + 3/7*sqrt(-a^2*x^2 + 1)*a^2*c^4*x^6 -
23/48*sqrt(-a^2*x^2 + 1)*a*c^4*x^5 + 4/35*sqrt(-a^2*x^2 + 1)*c^4*x^4 + 29/1
92*sqrt(-a^2*x^2 + 1)*c^4*x^3/a - 19/105*sqrt(-a^2*x^2 + 1)*c^4*x^2/a^2 + 2
9/128*sqrt(-a^2*x^2 + 1)*c^4*x/a^3 - 29/128*c^4*arcsin(a^2*x/sqrt(a^2))/(sq
rt(a^2)*a^3) - 38/105*sqrt(-a^2*x^2 + 1)*c^4/a^4
```

Fricas [A] time = 1.58328, size = 302, normalized size = 1.75

$$\frac{6090\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(1680\,a^7c^4x^7-5760\,a^6c^4x^6+6440\,a^5c^4x^5-1536\,a^4c^4x^4-2030\,a^3c^4x^3+2432\,a^2c^4x^2-30\right.}{13440\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] 1/13440*(6090*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (1680*a^7*c^4*x^
7 - 5760*a^6*c^4*x^6 + 6440*a^5*c^4*x^5 - 1536*a^4*c^4*x^4 - 2030*a^3*c^4*x
^3 + 2432*a^2*c^4*x^2 - 3045*a*c^4*x + 4864*c^4)*sqrt(-a^2*x^2 + 1))/a^4
```

Sympy [A] time = 23.7803, size = 842, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3*(-a*c*x+c)**4,x)
```

```
[Out] a**5*c**4*Piecewise((-I*x**9/(8*sqrt(a**2*x**2 - 1)) - I*x**7/(48*a**2*sqrt
(a**2*x**2 - 1)) - 7*I*x**5/(192*a**4*sqrt(a**2*x**2 - 1)) - 35*I*x**3/(384
*a**6*sqrt(a**2*x**2 - 1)) + 35*I*x/(128*a**8*sqrt(a**2*x**2 - 1)) - 35*I*a
cosh(a*x)/(128*a**9), Abs(a**2*x**2) > 1), (x**9/(8*sqrt(-a**2*x**2 + 1)) +
 x**7/(48*a**2*sqrt(-a**2*x**2 + 1)) + 7*x**5/(192*a**4*sqrt(-a**2*x**2 + 1
)) + 35*x**3/(384*a**6*sqrt(-a**2*x**2 + 1)) - 35*x/(128*a**8*sqrt(-a**2*x*
*2 + 1)) + 35*asin(a*x)/(128*a**9), True)) - 3*a**4*c**4*Piecewise((-x**6*s
qrt(-a**2*x**2 + 1)/(7*a**2) - 6*x**4*sqrt(-a**2*x**2 + 1)/(35*a**4) - 8*x*
*2*sqrt(-a**2*x**2 + 1)/(35*a**6) - 16*sqrt(-a**2*x**2 + 1)/(35*a**8), Ne(a
, 0)), (x**8/8, True)) + 2*a**3*c**4*Piecewise((-I*x**7/(6*sqrt(a**2*x**2 -
 1)) - I*x**5/(24*a**2*sqrt(a**2*x**2 - 1)) - 5*I*x**3/(48*a**4*sqrt(a**2*x
**2 - 1)) + 5*I*x/(16*a**6*sqrt(a**2*x**2 - 1)) - 5*I*acosh(a*x)/(16*a**7),
 Abs(a**2*x**2) > 1), (x**7/(6*sqrt(-a**2*x**2 + 1)) + x**5/(24*a**2*sqrt(-
a**2*x**2 + 1)) + 5*x**3/(48*a**4*sqrt(-a**2*x**2 + 1)) - 5*x/(16*a**6*sqrt
(-a**2*x**2 + 1)) + 5*asin(a*x)/(16*a**7), True)) + 2*a**2*c**4*Piecewise((
-x**4*sqrt(-a**2*x**2 + 1)/(5*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)/(15*a**4)
 - 8*sqrt(-a**2*x**2 + 1)/(15*a**6), Ne(a, 0)), (x**6/6, True)) - 3*a*c**4*
Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(a**2*x**2
- 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a**5), Abs(a
**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqrt(-a**2*x*
*2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**5), True))
 + c**4*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2
+ 1)/(3*a**4), Ne(a, 0)), (x**4/4, True))
```

Giac [A] time = 1.19732, size = 158, normalized size = 0.91

$$-\frac{29\,c^4\arcsin(ax)\,\mathrm{sgn}(a)}{128\,a^3|a|}-\frac{1}{13440}\sqrt{-a^2x^2+1}\left(\left(2\left(\frac{1216\,c^4}{a^2}-\left(\frac{1015\,c^4}{a}+4\left(192\,c^4-5\left(161\,ac^4+6\left(7\,a^3c^4x-24\,a^2c^4\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -29/128*c^4*arcsin(a*x)*sgn(a)/(a^3*abs(a)) - 1/13440*sqrt(-a^2*x^2 + 1)*((
2*(1216*c^4/a^2 - (1015*c^4/a + 4*(192*c^4 - 5*(161*a*c^4 + 6*(7*a^3*c^4*x
- 24*a^2*c^4)*x)*x)*x)*x)*x - 3045*c^4/a^3)*x + 4864*c^4/a^4)
```

3.317 $\int e^{\tanh^{-1}(ax)}x^2(c-acx)^4\,dx$

Optimal. Leaf size=146

$$\frac{1}{7}ac^4x^4\left(1-a^2x^2\right)^{3/2}-\frac{1}{2}c^4x^3\left(1-a^2x^2\right)^{3/2}+\frac{5c^4x^2\left(1-a^2x^2\right)^{3/2}}{7a}+\frac{5c^4(16-21ax)\left(1-a^2x^2\right)^{3/2}}{168a^3}+\frac{5c^4x\sqrt{1-a^2x^2}}{16a^2}+\frac{5c^4\sin}{1}$$

```
[Out] (5*c^4*x*Sqrt[1 - a^2*x^2])/(16*a^2) + (5*c^4*x^2*(1 - a^2*x^2)^(3/2))/(7*a
) - (c^4*x^3*(1 - a^2*x^2)^(3/2))/2 + (a*c^4*x^4*(1 - a^2*x^2)^(3/2))/7 + (
5*c^4*(16 - 21*a*x)*(1 - a^2*x^2)^(3/2))/(168*a^3) + (5*c^4*ArcSin[a*x])/(1
6*a^3)
```

Rubi [A] time = 0.304134, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 1809, 833, 780, 195, 216}

$$\frac{1}{7}ac^4x^4\left(1-a^2x^2\right)^{3/2}-\frac{1}{2}c^4x^3\left(1-a^2x^2\right)^{3/2}+\frac{5c^4x^2\left(1-a^2x^2\right)^{3/2}}{7a}+\frac{5c^4(16-21ax)\left(1-a^2x^2\right)^{3/2}}{168a^3}+\frac{5c^4x\sqrt{1-a^2x^2}}{16a^2}+\frac{5c^4\sin}{1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2*(c - a*c*x)^4,x]
```

```
[Out] (5*c^4*x*Sqrt[1 - a^2*x^2])/(16*a^2) + (5*c^4*x^2*(1 - a^2*x^2)^(3/2))/(7*a
) - (c^4*x^3*(1 - a^2*x^2)^(3/2))/2 + (a*c^4*x^4*(1 - a^2*x^2)^(3/2))/7 + (
5*c^4*(16 - 21*a*x)*(1 - a^2*x^2)^(3/2))/(168*a^3) + (5*c^4*ArcSin[a*x])/(1
6*a^3)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^2(c-acx)^4\,dx = c\int x^2(c-acx)^3\sqrt{1-a^2x^2}\,dx$$
$$= \frac{1}{7}ac^4x^4\left(1-a^2x^2\right)^{3/2} - \frac{c\int x^2\sqrt{1-a^2x^2}\left(-7a^2c^3+25a^3c^3x-21a^4c^3x^2\right)dx}{7a^2}$$
$$= -\frac{1}{2}c^4x^3\left(1-a^2x^2\right)^{3/2} + \frac{1}{7}ac^4x^4\left(1-a^2x^2\right)^{3/2} + \frac{c\int x^2\left(105a^4c^3-150a^5c^3x\right)\sqrt{1-a^2x^2}\,d\ldots}{42a^4}$$
$$= \frac{5c^4x^2\left(1-a^2x^2\right)^{3/2}}{7a} - \frac{1}{2}c^4x^3\left(1-a^2x^2\right)^{3/2} + \frac{1}{7}ac^4x^4\left(1-a^2x^2\right)^{3/2} - \frac{c\int x\left(300a^5c^3-525\ldots\right.}{21\ldots}$$
$$= \frac{5c^4x^2\left(1-a^2x^2\right)^{3/2}}{7a} - \frac{1}{2}c^4x^3\left(1-a^2x^2\right)^{3/2} + \frac{1}{7}ac^4x^4\left(1-a^2x^2\right)^{3/2} + \frac{5c^4(16-21ax)(1-\ldots}{168a^3}$$
$$= \frac{5c^4x\sqrt{1-a^2x^2}}{16a^2} + \frac{5c^4x^2\left(1-a^2x^2\right)^{3/2}}{7a} - \frac{1}{2}c^4x^3\left(1-a^2x^2\right)^{3/2} + \frac{1}{7}ac^4x^4\left(1-a^2x^2\right)^{3/2} + \frac{5c\ldots}{}$$
$$= \frac{5c^4x\sqrt{1-a^2x^2}}{16a^2} + \frac{5c^4x^2\left(1-a^2x^2\right)^{3/2}}{7a} - \frac{1}{2}c^4x^3\left(1-a^2x^2\right)^{3/2} + \frac{1}{7}ac^4x^4\left(1-a^2x^2\right)^{3/2} + \frac{5c\ldots}{}$$

Mathematica [A] time = 0.103453, size = 91, normalized size = 0.62

$$-\frac{c^4\left(\sqrt{1-a^2x^2}\left(48a^6x^6-168a^5x^5+192a^4x^4-42a^3x^3-80a^2x^2+105ax-160\right)+210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{336a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^2*(c - a*c*x)^4,x]
```

```
[Out] -(c^4*(Sqrt[1 - a^2*x^2]*(-160 + 105*a*x - 80*a^2*x^2 - 42*a^3*x^3 + 192*a^
4*x^4 - 168*a^5*x^5 + 48*a^6*x^6) + 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(33
6*a^3)
```

Maple [A] time = 0.062, size = 186, normalized size = 1.3

$$-\frac{c^4a^3x^6}{7}\sqrt{-a^2x^2+1}-\frac{4\,c^4ax^4}{7}\sqrt{-a^2x^2+1}+\frac{5\,c^4x^2}{21\,a}\sqrt{-a^2x^2+1}+\frac{10\,c^4}{21\,a^3}\sqrt{-a^2x^2+1}+\frac{c^4a^2x^5}{2}\sqrt{-a^2x^2+1}+\frac{c^4x^3}{8}\sqrt{-a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^4,x)
```

```
[Out] -1/7*c^4*a^3*x^6*(-a^2*x^2+1)^(1/2)-4/7*c^4*a*x^4*(-a^2*x^2+1)^(1/2)+5/21*c
^4/a*x^2*(-a^2*x^2+1)^(1/2)+10/21*c^4/a^3*(-a^2*x^2+1)^(1/2)+1/2*c^4*a^2*x^
5*(-a^2*x^2+1)^(1/2)+1/8*c^4*x^3*(-a^2*x^2+1)^(1/2)-5/16*c^4*x*(-a^2*x^2+1)
^(1/2)/a^2+5/16*c^4/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2)
)
```

Maxima [A] time = 1.45246, size = 238, normalized size = 1.63

$$-\frac{1}{7}\sqrt{-a^2x^2+1}a^3c^4x^6+\frac{1}{2}\sqrt{-a^2x^2+1}a^2c^4x^5-\frac{4}{7}\sqrt{-a^2x^2+1}ac^4x^4+\frac{1}{8}\sqrt{-a^2x^2+1}c^4x^3+\frac{5\sqrt{-a^2x^2+1}c^4x^2}{21\,a}-\frac{5\sqrt{-a^2x^2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] -1/7*sqrt(-a^2*x^2 + 1)*a^3*c^4*x^6 + 1/2*sqrt(-a^2*x^2 + 1)*a^2*c^4*x^5 -
4/7*sqrt(-a^2*x^2 + 1)*a*c^4*x^4 + 1/8*sqrt(-a^2*x^2 + 1)*c^4*x^3 + 5/21*sq
rt(-a^2*x^2 + 1)*c^4*x^2/a - 5/16*sqrt(-a^2*x^2 + 1)*c^4*x/a^2 + 5/16*c^4*a
rcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2) + 10/21*sqrt(-a^2*x^2 + 1)*c^4/a^3
```

Fricas [A] time = 1.61897, size = 261, normalized size = 1.79

$$-\frac{210\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(48\,a^6c^4x^6-168\,a^5c^4x^5+192\,a^4c^4x^4-42\,a^3c^4x^3-80\,a^2c^4x^2+105\,ac^4x-160\,c^4\right)\sqrt{-a^2x^2}}{336\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] -1/336*(210*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (48*a^6*c^4*x^6 -
168*a^5*c^4*x^5 + 192*a^4*c^4*x^4 - 42*a^3*c^4*x^3 - 80*a^2*c^4*x^2 + 105*a
*c^4*x - 160*c^4)*sqrt(-a^2*x^2 + 1))/a^3
```

Sympy [C] time = 16.7034, size = 683, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(-a*c*x+c)**4,x)
```

```
[Out] a**5*c**4*Piecewise((-x**6*sqrt(-a**2*x**2 + 1)/(7*a**2) - 6*x**4*sqrt(-a**
2*x**2 + 1)/(35*a**4) - 8*x**2*sqrt(-a**2*x**2 + 1)/(35*a**6) - 16*sqrt(-a*
*2*x**2 + 1)/(35*a**8), Ne(a, 0)), (x**8/8, True)) - 3*a**4*c**4*Piecewise(
(-I*x**7/(6*sqrt(a**2*x**2 - 1)) - I*x**5/(24*a**2*sqrt(a**2*x**2 - 1)) - 5
*I*x**3/(48*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(16*a**6*sqrt(a**2*x**2 - 1))
 - 5*I*acosh(a*x)/(16*a**7), Abs(a**2*x**2) > 1), (x**7/(6*sqrt(-a**2*x**2
+ 1)) + x**5/(24*a**2*sqrt(-a**2*x**2 + 1)) + 5*x**3/(48*a**4*sqrt(-a**2*x*
*2 + 1)) - 5*x/(16*a**6*sqrt(-a**2*x**2 + 1)) + 5*asin(a*x)/(16*a**7), True
)) + 2*a**3*c**4*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/(5*a**2) - 4*x**2*sq
rt(-a**2*x**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(15*a**6), Ne(a, 0)),
 (x**6/6, True)) + 2*a**2*c**4*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) -
 I*x**3/(8*a**2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) -
 3*I*acosh(a*x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1
)) + x**3/(8*a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1))
 + 3*asin(a*x)/(8*a**5), True)) - 3*a*c**4*Piecewise((-x**2*sqrt(-a**2*x**2
 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)
) + c**4*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**
3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a
**2*x**2 + 1)) + asin(a*x)/(2*a**3), True))
```

Giac [A] time = 1.20694, size = 140, normalized size = 0.96

$$\frac{5\,c^4\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{16\,a^2|a|}-\frac{1}{336}\,\sqrt{-a^2x^2+1}\left(\left(\frac{105\,c^4}{a^2}-2\left(\frac{40\,c^4}{a}+3\left(7\,c^4-4\left(8\,ac^4+\left(2\,a^3c^4x-7\,a^2c^4\right)x\right)x\right)x\right)x-\frac{16}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 5/16*c^4*arcsin(a*x)*sgn(a)/(a^2*abs(a)) - 1/336*sqrt(-a^2*x^2 + 1)*((105*c
^4/a^2 - 2*(40*c^4/a + 3*(7*c^4 - 4*(8*a*c^4 + (2*a^3*c^4*x - 7*a^2*c^4)*x)
*x)*x)*x)*x - 160*c^4/a^3)
```

3.318 $\int e^{\tanh^{-1}(ax)}x(c-acx)^4\,dx$

Optimal. Leaf size=158

$$-\frac{c^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}}{6a^2}-\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{10a^2}-\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{40a^2}-\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{24a^2}-\frac{7c^4x\sqrt{1-a^2x^2}}{16a}-$$

```
[Out] (-7*c^4*x*Sqrt[1 - a^2*x^2])/(16*a) - (7*c^4*(1 - a^2*x^2)^(3/2))/(24*a^2)
- (7*c^4*(1 - a*x)*(1 - a^2*x^2)^(3/2))/(40*a^2) - (c^4*(1 - a*x)^2*(1 - a^
2*x^2)^(3/2))/(10*a^2) - (c^4*(1 - a*x)^3*(1 - a^2*x^2)^(3/2))/(6*a^2) - (7
*c^4*ArcSin[a*x])/(16*a^2)
```

Rubi [A] time = 0.141219, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6128, 795, 671, 641, 195, 216}

$$-\frac{c^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}}{6a^2}-\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{10a^2}-\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{40a^2}-\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{24a^2}-\frac{7c^4x\sqrt{1-a^2x^2}}{16a}-$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x*(c - a*c*x)^4,x]
```

```
[Out] (-7*c^4*x*Sqrt[1 - a^2*x^2])/(16*a) - (7*c^4*(1 - a^2*x^2)^(3/2))/(24*a^2)
- (7*c^4*(1 - a*x)*(1 - a^2*x^2)^(3/2))/(40*a^2) - (c^4*(1 - a*x)^2*(1 - a^
2*x^2)^(3/2))/(10*a^2) - (c^4*(1 - a*x)^3*(1 - a^2*x^2)^(3/2))/(6*a^2) - (7
*c^4*ArcSin[a*x])/(16*a^2)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}x(c-acx)^4\,dx &= c\int x(c-acx)^3\sqrt{1-a^2x^2}\,dx\\
&= -\frac{c^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}}{6a^2}-\frac{c\int(c-acx)^3\sqrt{1-a^2x^2}\,dx}{2a}\\
&= -\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{10a^2}-\frac{c^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}}{6a^2}-\frac{\left(7c^2\right)\int(c-acx)^2\sqrt{1-a^2x^2}\,dx}{10a}\\
&= -\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{40a^2}-\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{10a^2}-\frac{c^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}}{6a^2}-\frac{\left(7c^3\right)\cdots}{\cdots}\\
&= -\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{24a^2}-\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{40a^2}-\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{10a^2}-\frac{c^4(1-ax)^3\left(\cdots\right.}{6a\cdots}\\
&= -\frac{7c^4x\sqrt{1-a^2x^2}}{16a}-\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{24a^2}-\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{40a^2}-\frac{c^4(1-ax)^2\left(1-a^2x^2\right)\cdots}{10a^2}\\
&= -\frac{7c^4x\sqrt{1-a^2x^2}}{16a}-\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{24a^2}-\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{40a^2}-\frac{c^4(1-ax)^2\left(1-a^2x^2\right)\cdots}{10a^2}
\end{aligned}$$

Mathematica [A] time = 0.124399, size = 83, normalized size = 0.53

$$-\frac{c^4\left(\sqrt{1-a^2x^2}\left(40a^5x^5-144a^4x^4+170a^3x^3-32a^2x^2-105ax+176\right)-210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{240a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x*(c - a*c*x)^4,x]
```

```
[Out] -(c^4*(Sqrt[1 - a^2*x^2]*(176 - 105*a*x - 32*a^2*x^2 + 170*a^3*x^3 - 144*a^
4*x^4 + 40*a^5*x^5) - 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(240*a^2)
```

Maple [A] time = 0.052, size = 163, normalized size = 1.

$$-\frac{c^4a^3x^5}{6}\sqrt{-a^2x^2+1}-\frac{17\,c^4ax^3}{24}\sqrt{-a^2x^2+1}+\frac{7\,c^4x}{16\,a}\sqrt{-a^2x^2+1}-\frac{7\,c^4}{16\,a}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{3\,c^4a^2x^4}{5}\sqrt{-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^4,x)
```

```
[Out] -1/6*c^4*a^3*x^5*(-a^2*x^2+1)^(1/2)-17/24*c^4*a*x^3*(-a^2*x^2+1)^(1/2)+7/16
*c^4*x*(-a^2*x^2+1)^(1/2)/a-7/16*c^4/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a
^2*x^2+1)^(1/2))+3/5*c^4*a^2*x^4*(-a^2*x^2+1)^(1/2)+2/15*c^4*x^2*(-a^2*x^2+
1)^(1/2)-11/15*c^4/a^2*(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.43336, size = 207, normalized size = 1.31

$$-\frac{1}{6}\sqrt{-a^2x^2+1}a^3c^4x^5+\frac{3}{5}\sqrt{-a^2x^2+1}a^2c^4x^4-\frac{17}{24}\sqrt{-a^2x^2+1}ac^4x^3+\frac{2}{15}\sqrt{-a^2x^2+1}c^4x^2+\frac{7\sqrt{-a^2x^2+1}c^4x}{16a}-\frac{7c^4\,\mathrm{arc}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(-a^2*x^2 + 1)*a^3*c^4*x^5 + 3/5*sqrt(-a^2*x^2 + 1)*a^2*c^4*x^4 -
17/24*sqrt(-a^2*x^2 + 1)*a*c^4*x^3 + 2/15*sqrt(-a^2*x^2 + 1)*c^4*x^2 + 7/16
*sqrt(-a^2*x^2 + 1)*c^4*x/a - 7/16*c^4*arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a
) - 11/15*sqrt(-a^2*x^2 + 1)*c^4/a^2
```

Fricas [A] time = 1.6199, size = 236, normalized size = 1.49

$$\frac{210\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(40\,a^5c^4x^5-144\,a^4c^4x^4+170\,a^3c^4x^3-32\,a^2c^4x^2-105\,ac^4x+176\,c^4\right)\sqrt{-a^2x^2+1}}{240\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/240*(210*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (40*a^5*c^4*x^5 - 1
44*a^4*c^4*x^4 + 170*a^3*c^4*x^3 - 32*a^2*c^4*x^2 - 105*a*c^4*x + 176*c^4)*
sqrt(-a^2*x^2 + 1))/a^2
```

Sympy [A] time = 15.6389, size = 617, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(-a*c*x+c)**4,x)
```

```
[Out] a**5*c**4*Piecewise((-I*x**7/(6*sqrt(a**2*x**2 - 1)) - I*x**5/(24*a**2*sqrt
(a**2*x**2 - 1)) - 5*I*x**3/(48*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(16*a**6*
sqrt(a**2*x**2 - 1)) - 5*I*acosh(a*x)/(16*a**7), Abs(a**2*x**2) > 1), (x**7
/(6*sqrt(-a**2*x**2 + 1)) + x**5/(24*a**2*sqrt(-a**2*x**2 + 1)) + 5*x**3/(4
8*a**4*sqrt(-a**2*x**2 + 1)) - 5*x/(16*a**6*sqrt(-a**2*x**2 + 1)) + 5*asin(
a*x)/(16*a**7), True)) - 3*a**4*c**4*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/
(5*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(
15*a**6), Ne(a, 0)), (x**6/6, True)) + 2*a**3*c**4*Piecewise((-I*x**5/(4*sq
```

```
rt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sq
rt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4
*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*s
qrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**5), True)) + 2*a**2*c**4*Piecewise
((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne
(a, 0)), (x**4/4, True)) - 3*a*c**4*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*
a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**
2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) + c*
*4*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
```

Giac [A] time = 1.22086, size = 127, normalized size = 0.8

$$-\frac{7\,c^4\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{16\,a|a|}-\frac{1}{240}\,\sqrt{-a^2x^2+1}\left(\frac{176\,c^4}{a^2}-\left(\frac{105\,c^4}{a}+2\left(16\,c^4-\left(85\,ac^4+4\left(5\,a^3c^4x-18\,a^2c^4\right)x\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -7/16*c^4*arcsin(a*x)*sgn(a)/(a*abs(a)) - 1/240*sqrt(-a^2*x^2 + 1)*(176*c^4
/a^2 - (105*c^4/a + 2*(16*c^4 - (85*a*c^4 + 4*(5*a^3*c^4*x - 18*a^2*c^4)*x)
*x)*x)*x)
```

3.319 $\int e^{\tanh^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=123

$$\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7}{8}c^4x\sqrt{1-a^2x^2}+\frac{7c^4\sin^{-1}(ax)}{8a}$$

```
[Out] (7*c^4*x*Sqrt[1 - a^2*x^2])/8 + (7*c^4*(1 - a^2*x^2)^(3/2))/(12*a) + (7*c^4
*(1 - a*x)*(1 - a^2*x^2)^(3/2))/(20*a) + (c^4*(1 - a*x)^2*(1 - a^2*x^2)^(3/
2))/(5*a) + (7*c^4*ArcSin[a*x])/(8*a)
```

Rubi [A] time = 0.0812557, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6127, 671, 641, 195, 216}

$$\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7}{8}c^4x\sqrt{1-a^2x^2}+\frac{7c^4\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^4,x]
```

```
[Out] (7*c^4*x*Sqrt[1 - a^2*x^2])/8 + (7*c^4*(1 - a^2*x^2)^(3/2))/(12*a) + (7*c^4
*(1 - a*x)*(1 - a^2*x^2)^(3/2))/(20*a) + (c^4*(1 - a*x)^2*(1 - a^2*x^2)^(3/
2))/(5*a) + (7*c^4*ArcSin[a*x])/(8*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}(c-acx)^4\,dx &= c\int (c-acx)^3\sqrt{1-a^2x^2}\,dx \\
&= \frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{5}\left(7c^2\right)\int (c-acx)^2\sqrt{1-a^2x^2}\,dx \\
&= \frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{4}\left(7c^3\right)\int (c-acx)\sqrt{1-a^2x^2}\,dx \\
&= \frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{4}\left(7c^4\right)\int \sqrt{1-
\\
&= \frac{7}{8}c^4x\sqrt{1-a^2x^2}+\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}
\\
&= \frac{7}{8}c^4x\sqrt{1-a^2x^2}+\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}
\end{aligned}$$

Mathematica [A] time = 0.0951718, size = 75, normalized size = 0.61

$$-\frac{c^4\left(\sqrt{1-a^2x^2}\left(24a^4x^4-90a^3x^3+112a^2x^2-15ax-136\right)+210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^4,x]
```

```
[Out] -(c^4*(Sqrt[1 - a^2*x^2]*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3 + 24*a^4
*x^4) + 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(120*a)
```

Maple [A] time = 0.043, size = 137, normalized size = 1.1

$$-\frac{c^4x^4a^3}{5}\sqrt{-a^2x^2+1}-\frac{14\,c^4ax^2}{15}\sqrt{-a^2x^2+1}+\frac{17\,c^4}{15\,a}\sqrt{-a^2x^2+1}+\frac{3\,a^2c^4x^3}{4}\sqrt{-a^2x^2+1}+\frac{c^4x}{8}\sqrt{-a^2x^2+1}+\frac{7\,c^4}{8}\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4,x)
```

```
[Out] -1/5*c^4*a^3*x^4*(-a^2*x^2+1)^(1/2)-14/15*c^4*a*x^2*(-a^2*x^2+1)^(1/2)+17/1
5*c^4*(-a^2*x^2+1)^(1/2)/a+3/4*c^4*a^2*x^3*(-a^2*x^2+1)^(1/2)+1/8*c^4*x*(-a
^2*x^2+1)^(1/2)+7/8*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2)
)
```

Maxima [A] time = 1.44511, size = 171, normalized size = 1.39

$$-\frac{1}{5}\sqrt{-a^2x^2+1}a^3c^4x^4+\frac{3}{4}\sqrt{-a^2x^2+1}a^2c^4x^3-\frac{14}{15}\sqrt{-a^2x^2+1}ac^4x^2+\frac{1}{8}\sqrt{-a^2x^2+1}c^4x+\frac{7\,c^4\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\sqrt{a^2}}+\frac{17\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-a^2*x^2 + 1)*a^3*c^4*x^4 + 3/4*sqrt(-a^2*x^2 + 1)*a^2*c^4*x^3 -
14/15*sqrt(-a^2*x^2 + 1)*a*c^4*x^2 + 1/8*sqrt(-a^2*x^2 + 1)*c^4*x + 7/8*c^4
*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + 17/15*sqrt(-a^2*x^2 + 1)*c^4/a
```

Fricas [A] time = 1.58102, size = 209, normalized size = 1.7

$$-\frac{210\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(24\,a^4c^4x^4-90\,a^3c^4x^3+112\,a^2c^4x^2-15\,ac^4x-136\,c^4\right)\sqrt{-a^2x^2+1}}{120\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/120*(210*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (24*a^4*c^4*x^4 -
90*a^3*c^4*x^3 + 112*a^2*c^4*x^2 - 15*a*c^4*x - 136*c^4)*sqrt(-a^2*x^2 + 1)
)/a
```

Sympy [A] time = 8.73666, size = 226, normalized size = 1.84

$$\begin{cases}\frac{3c^4\sqrt{-a^2x^2+1}+2c^4\left(\begin{cases}-\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\operatorname{asin}(ax)}{2} & \text{for}\: ax>-1\wedge ax<1\end{cases}\right)+2c^4\left(\begin{cases}\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3}-\sqrt{-a^2x^2+1} & \text{for}\: ax>-1\wedge ax<1\end{cases}\right)- \cdots}{\cdots} \\ c^4x\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4,x)
```

```
[Out] Piecewise(((3*c**4*sqrt(-a**2*x**2 + 1) + 2*c**4*Piecewise((-a*x*sqrt(-a**2
*x**2 + 1)/2 + asin(a*x)/2, (a*x > -1) & (a*x < 1))) + 2*c**4*Piecewise(((-
a**2*x**2 + 1)**(3/2)/3 - sqrt(-a**2*x**2 + 1), (a*x > -1) & (a*x < 1))) -
3*c**4*Piecewise((a*x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/8 - a*x*sqrt(
-a**2*x**2 + 1)/2 + 3*asin(a*x)/8, (a*x > -1) & (a*x < 1))) + c**4*Piecewis
e((-(-a**2*x**2 + 1)**(5/2)/5 + 2*(-a**2*x**2 + 1)**(3/2)/3 - sqrt(-a**2*x*
*2 + 1), (a*x > -1) & (a*x < 1))) + c**4*asin(a*x))/a, Ne(a, 0)), (c**4*x,
True))
```

Giac [A] time = 1.25185, size = 105, normalized size = 0.85

$$\frac{7\,c^4\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{8\,|a|}+\frac{1}{120}\,\sqrt{-a^2x^2+1}\left(\frac{136\,c^4}{a}+\left(15\,c^4-2\left(56\,ac^4+3\left(4\,a^3c^4x-15\,a^2c^4\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 7/8*c^4*arcsin(a*x)*sgn(a)/abs(a) + 1/120*sqrt(-a^2*x^2 + 1)*(136*c^4/a + (
15*c^4 - 2*(56*a*c^4 + 3*(4*a^3*c^4*x - 15*a^2*c^4)*x)*x)*x)
```

3.320 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x}\,dx$

Optimal. Leaf size=101

$$\frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2}-c^4\left(1-a^2x^2\right)^{3/2}+\frac{1}{8}c^4(8-13ax)\sqrt{1-a^2x^2}-c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{13}{8}c^4\sin^{-1}(ax)$$

```
[Out] (c^4*(8 - 13*a*x)*Sqrt[1 - a^2*x^2])/8 - c^4*(1 - a^2*x^2)^(3/2) + (a*c^4*x
*(1 - a^2*x^2)^(3/2))/4 - (13*c^4*ArcSin[a*x])/8 - c^4*ArcTanh[Sqrt[1 - a^2
*x^2]]
```

Rubi [A] time = 0.246412, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1809, 815, 844, 216, 266, 63, 208}

$$\frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2}-c^4\left(1-a^2x^2\right)^{3/2}+\frac{1}{8}c^4(8-13ax)\sqrt{1-a^2x^2}-c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{13}{8}c^4\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x,x]
```

```
[Out] (c^4*(8 - 13*a*x)*Sqrt[1 - a^2*x^2])/8 - c^4*(1 - a^2*x^2)^(3/2) + (a*c^4*x
*(1 - a^2*x^2)^(3/2))/4 - (13*c^4*ArcSin[a*x])/8 - c^4*ArcTanh[Sqrt[1 - a^2
*x^2]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x}\,dx = c\int \frac{(c-acx)^3\sqrt{1-a^2x^2}}{x}\,dx$$

$$= \frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2} - \frac{c\int \frac{\sqrt{1-a^2x^2}\left(-4a^2c^3+13a^3c^3x-12a^4c^3x^2\right)}{x}\,dx}{4a^2}$$

$$= -c^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2} + \frac{c\int \frac{\left(12a^4c^3-39a^5c^3x\right)\sqrt{1-a^2x^2}}{x}\,dx}{12a^4}$$

$$= \frac{1}{8}c^4(8-13ax)\sqrt{1-a^2x^2} - c^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2} - \frac{c\int \frac{-24a^6c^3+39a^7c^3x}{x\sqrt{1-a^2x^2}}\,dx}{24a^6}$$

$$= \frac{1}{8}c^4(8-13ax)\sqrt{1-a^2x^2} - c^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2} + c^4\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - \frac{1}{8}$$

$$= \frac{1}{8}c^4(8-13ax)\sqrt{1-a^2x^2} - c^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2} - \frac{13}{8}c^4\sin^{-1}(ax) + \frac{1}{2}c^4\,\mathrm{Sub}$$

$$= \frac{1}{8}c^4(8-13ax)\sqrt{1-a^2x^2} - c^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2} - \frac{13}{8}c^4\sin^{-1}(ax) - \frac{c^4\,\mathrm{Subs}}{}$$

$$= \frac{1}{8}c^4(8-13ax)\sqrt{1-a^2x^2} - c^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2} - \frac{13}{8}c^4\sin^{-1}(ax) - c^4\tanh$$

Mathematica [A] time = 0.0961237, size = 142, normalized size = 1.41

$$\frac{c^4\left(2a^5x^5-8a^4x^4+9a^3x^3+8a^2x^2+4\sqrt{1-a^2x^2}\sin^{-1}(ax)+34\sqrt{1-a^2x^2}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-8\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a\right.\right.}{8\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x,x]
```

```
[Out] (c^4*(-11*a*x + 8*a^2*x^2 + 9*a^3*x^3 - 8*a^4*x^4 + 2*a^5*x^5 + 4*Sqrt[1 -
a^2*x^2]*ArcSin[a*x] + 34*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]] -
 8*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(8*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.04, size = 115, normalized size = 1.1

$$-\frac{c^4a^3x^3}{4}\sqrt{-a^2x^2+1}-\frac{11\,c^4ax}{8}\sqrt{-a^2x^2+1}-\frac{13\,c^4a}{8}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+c^4a^2x^2\sqrt{-a^2x^2+1}-c^4\text{Artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x,x)
```

```
[Out] -1/4*c^4*a^3*x^3*(-a^2*x^2+1)^(1/2)-11/8*c^4*a*x*(-a^2*x^2+1)^(1/2)-13/8*c^
4*a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+c^4*a^2*x^2*(-a^2*
x^2+1)^(1/2)-c^4*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.44072, size = 159, normalized size = 1.57

$$-\frac{1}{4}\sqrt{-a^2x^2+1}a^3c^4x^3+\sqrt{-a^2x^2+1}a^2c^4x^2-\frac{11}{8}\sqrt{-a^2x^2+1}ac^4x-\frac{13\,ac^4\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\sqrt{a^2}}-c^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-a^2*x^2 + 1)*a^3*c^4*x^3 + sqrt(-a^2*x^2 + 1)*a^2*c^4*x^2 - 11/8
*sqrt(-a^2*x^2 + 1)*a*c^4*x - 13/8*a*c^4*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2)
- c^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A] time = 1.66374, size = 212, normalized size = 2.1

$$\frac{13}{4}c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+c^4\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\frac{1}{8}\left(2\,a^3c^4x^3-8\,a^2c^4x^2+11\,ac^4x\right)\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x,x, algorithm="fricas")
```

```
[Out] 13/4*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + c^4*log((sqrt(-a^2*x^2 +
1) - 1)/x) - 1/8*(2*a^3*c^4*x^3 - 8*a^2*c^4*x^2 + 11*a*c^4*x)*sqrt(-a^2*x^2
```

```
+ 1)
```

Sympy [C] time = 20.6904, size = 420, normalized size = 4.16

$$a^5c^4\left(\begin{cases}-\frac{ix^5}{4\sqrt{a^2x^2-1}}-\frac{ix^3}{8a^2\sqrt{a^2x^2-1}}+\frac{3ix}{8a^4\sqrt{a^2x^2-1}}-\frac{3i\,\mathrm{acosh}\,(ax)}{8a^5} & \text{for } |a^2x^2|>1\\ \frac{x^5}{4\sqrt{-a^2x^2+1}}+\frac{x^3}{8a^2\sqrt{-a^2x^2+1}}-\frac{3x}{8a^4\sqrt{-a^2x^2+1}}+\frac{3\,\mathrm{asin}\,(ax)}{8a^5} & \text{otherwise}\end{cases}\right)-3a^4c^4\left(\begin{cases}-\frac{x^2\sqrt{-a^2x^2+1}}{3a^2}-\frac{2\sqrt{-a^2x^2+1}}{3a^4} & \text{for } a\neq 0\\ \frac{x^4}{4} & \text{otherwis}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4/x,x)
```

```
[Out] a**5*c**4*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(
a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a*
*5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqr
t(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**
5), True)) - 3*a**4*c**4*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2
*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) + 2*a**3*c**4*Pi
ecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**
2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 +
1)) + asin(a*x)/(2*a**3), True)) + 2*a**2*c**4*Piecewise((x**2/2, Eq(a**2,
0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) - 3*a*c**4*Piecewise((sqrt(a**(-2)
)*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2
< 0)) + c**4*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(
a*x)), True))
```

Giac [A] time = 1.30434, size = 135, normalized size = 1.34

$$-\frac{13\,ac^4\arcsin(ax)\,\mathrm{sgn}(a)}{8\,|a|}-\frac{ac^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\frac{1}{8}\left(11\,ac^4+2\left(a^3c^4x-4\,a^2c^4\right)x\right)\sqrt{-a^2x^2+1}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x,x, algorithm="giac")
```

```
[Out] -13/8*a*c^4*arcsin(a*x)*sgn(a)/abs(a) - a*c^4*log(1/2*abs(-2*sqrt(-a^2*x^2
+ 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/8*(11*a*c^4 + 2*(a^3*c^4*x - 4*
a^2*c^4)*x)*sqrt(-a^2*x^2 + 1)*x
```

3.321 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^2}\,dx$

Optimal. Leaf size=106

$$\frac{1}{3}ac^4\left(1-a^2x^2\right)^{3/2}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{x}-\frac{1}{2}ac^4(6-ax)\sqrt{1-a^2x^2}+3ac^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{1}{2}ac^4\sin^{-1}(ax)$$

```
[Out] -(a*c^4*(6 - a*x)*Sqrt[1 - a^2*x^2])/2 + (a*c^4*(1 - a^2*x^2)^(3/2))/3 - (c
^4*(1 - a^2*x^2)^(3/2))/x + (a*c^4*ArcSin[a*x])/2 + 3*a*c^4*ArcTanh[Sqrt[1
- a^2*x^2]]
```

Rubi [A] time = 0.24186, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {6128, 1807, 1809, 815, 844, 216, 266, 63, 208}

$$\frac{1}{3}ac^4\left(1-a^2x^2\right)^{3/2}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{x}-\frac{1}{2}ac^4(6-ax)\sqrt{1-a^2x^2}+3ac^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{1}{2}ac^4\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^2,x]
```

```
[Out] -(a*c^4*(6 - a*x)*Sqrt[1 - a^2*x^2])/2 + (a*c^4*(1 - a^2*x^2)^(3/2))/3 - (c
^4*(1 - a^2*x^2)^(3/2))/x + (a*c^4*ArcSin[a*x])/2 + 3*a*c^4*ArcTanh[Sqrt[1
- a^2*x^2]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^2}\,dx = c\int \frac{(c-acx)^3\sqrt{1-a^2x^2}}{x^2}\,dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{x} - c\int \frac{\sqrt{1-a^2x^2}\left(3ac^3-a^2c^3x+a^3c^3x^2\right)}{x}\,dx$$
$$= \frac{1}{3}ac^4\left(1-a^2x^2\right)^{3/2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{c\int \frac{\left(-9a^3c^3+3a^4c^3x\right)\sqrt{1-a^2x^2}}{x}\,dx}{3a^2}$$
$$= -\frac{1}{2}ac^4(6-ax)\sqrt{1-a^2x^2} + \frac{1}{3}ac^4\left(1-a^2x^2\right)^{3/2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{x} - \frac{c\int \frac{18a^5c^3-3a^6c^3x}{x\sqrt{1-a^2x^2}}\,dx}{6a^4}$$
$$= -\frac{1}{2}ac^4(6-ax)\sqrt{1-a^2x^2} + \frac{1}{3}ac^4\left(1-a^2x^2\right)^{3/2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{x} - \left(3ac^4\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,d$$
$$= -\frac{1}{2}ac^4(6-ax)\sqrt{1-a^2x^2} + \frac{1}{3}ac^4\left(1-a^2x^2\right)^{3/2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{1}{2}ac^4\sin^{-1}(ax) - \frac{1}{2}(3a$$
$$= -\frac{1}{2}ac^4(6-ax)\sqrt{1-a^2x^2} + \frac{1}{3}ac^4\left(1-a^2x^2\right)^{3/2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{1}{2}ac^4\sin^{-1}(ax) + \frac{\left(3c^4\right.}{}$$
$$= -\frac{1}{2}ac^4(6-ax)\sqrt{1-a^2x^2} + \frac{1}{3}ac^4\left(1-a^2x^2\right)^{3/2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{1}{2}ac^4\sin^{-1}(ax) + 3ac^4$$

Mathematica [A] time = 0.127345, size = 152, normalized size = 1.43

$$-\frac{c^4\left(-2a^5x^5+9a^4x^4-14a^3x^3-15a^2x^2+9ax\sqrt{1-a^2x^2}\sin^{-1}(ax)+24ax\sqrt{1-a^2x^2}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-18ax\sqrt{1-a^2x^2}\,\mathrm{ta}\right.}{6x\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^2,x]
```

```
[Out] -(c^4*(6 + 16*a*x - 15*a^2*x^2 - 14*a^3*x^3 + 9*a^4*x^4 - 2*a^5*x^5 + 9*a*x
*Sqrt[1 - a^2*x^2]*ArcSin[a*x] + 24*a*x*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a
*x]/Sqrt[2]] - 18*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(6*x*S
qrt[1 - a^2*x^2])
```

Maple [A] time = 0.046, size = 136, normalized size = 1.3

$$-\frac{c^4a^3x^2}{3}\sqrt{-a^2x^2+1} - \frac{8\,c^4a}{3}\sqrt{-a^2x^2+1} + \frac{3\,c^4a^2x}{2}\sqrt{-a^2x^2+1} + \frac{c^4a^2}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} - \frac{c^4}{x}\sqrt{-a^2x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^2,x)
```

```
[Out] -1/3*c^4*a^3*x^2*(-a^2*x^2+1)^(1/2)-8/3*c^4*a*(-a^2*x^2+1)^(1/2)+3/2*c^4*a^
2*x*(-a^2*x^2+1)^(1/2)+1/2*c^4*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x
^2+1)^(1/2))-c^4/x*(-a^2*x^2+1)^(1/2)+3*c^4*a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.44635, size = 188, normalized size = 1.77

$$-\frac{1}{3}\sqrt{-a^2x^2+1}a^3c^4x^2+\frac{3}{2}\sqrt{-a^2x^2+1}a^2c^4x+\frac{a^2c^4\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}}+3\,ac^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{8}{3}\sqrt{-a^2x^2+1}ac^4-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^2,x, algorithm="maxima"
)
```

```
[Out] -1/3*sqrt(-a^2*x^2 + 1)*a^3*c^4*x^2 + 3/2*sqrt(-a^2*x^2 + 1)*a^2*c^4*x + 1/
2*a^2*c^4*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + 3*a*c^4*log(2*sqrt(-a^2*x^2 +
 1)/abs(x) + 2/abs(x)) - 8/3*sqrt(-a^2*x^2 + 1)*a*c^4 - sqrt(-a^2*x^2 + 1)*
c^4/x
```

Fricas [A] time = 1.58546, size = 258, normalized size = 2.43

$$-\frac{6\,ac^4x\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+18\,ac^4x\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+16\,ac^4x+\left(2\,a^3c^4x^3-9\,a^2c^4x^2+16\,ac^4x+6\,c^4\right)\sqrt{-a^2x^2+1}}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^2,x, algorithm="fricas"
)
```

```
[Out] -1/6*(6*a*c^4*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 18*a*c^4*x*log((sq
rt(-a^2*x^2 + 1) - 1)/x) + 16*a*c^4*x + (2*a^3*c^4*x^3 - 9*a^2*c^4*x^2 + 16
*a*c^4*x + 6*c^4)*sqrt(-a^2*x^2 + 1))/x
```

Sympy [C] time = 7.41335, size = 306, normalized size = 2.89

$$a^5c^4\left(\begin{cases}-\frac{x^2\sqrt{-a^2x^2+1}}{3a^2}-\frac{2\sqrt{-a^2x^2+1}}{3a^4} & \text{for } a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)-3a^4c^4\left(\begin{cases}-\frac{ix\sqrt{a^2x^2-1}}{2a^2}-\frac{i\operatorname{acosh}(ax)}{2a^3} & \text{for } \left|a^2x^2\right|>1\\ \frac{x^3}{2\sqrt{-a^2x^2+1}}-\frac{x}{2a^2\sqrt{-a^2x^2+1}}+\frac{\operatorname{asin}(ax)}{2a^3} & \text{otherwise}\end{cases}\right)+2a^3c^4\left(\begin{cases}\ldots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4/x**2,x)
```

```
[Out] a**5*c**4*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**
2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) - 3*a**4*c**4*Piecewise((-I*x*s
qrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1), (
x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)
/(2*a**3), True)) + 2*a**3*c**4*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a*
*2*x**2 + 1)/a**2, True)) + 2*a**2*c**4*Piecewise((sqrt(a**(-2))*asin(x*sqr
t(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - 3*a*
c**4*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), T
rue)) + c**4*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sq
rt(-a**2*x**2 + 1)/x, True))
```

Giac [A] time = 1.28515, size = 221, normalized size = 2.08

$$\frac{a^4c^4x}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|}+\frac{a^2c^4\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{2\,|a|}+\frac{3\,a^2c^4\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)c^4}{2\,x|a|}-\frac{1}{6}\left(16\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a^4*c^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) + 1/2*a^2*c^4*arcsin
(a*x)*sgn(a)/abs(a) + 3*a^2*c^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
2*a)/(a^2*abs(x)))/abs(a) - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/(x*abs(
a)) - 1/6*(16*a*c^4 + (2*a^3*c^4*x - 9*a^2*c^4)*x)*sqrt(-a^2*x^2 + 1)
```

3.322 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^3}\,dx$

Optimal. Leaf size=116

$$\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{x}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2}+\frac{5}{2}a^2c^4(ax+1)\sqrt{1-a^2x^2}-\frac{5}{2}a^2c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{5}{2}a^2c^4\sin^{-1}(ax)$$

```
[Out] (5*a^2*c^4*(1 + a*x)*Sqrt[1 - a^2*x^2])/2 - (c^4*(1 - a^2*x^2)^(3/2))/(2*x^
2) + (3*a*c^4*(1 - a^2*x^2)^(3/2))/x + (5*a^2*c^4*ArcSin[a*x])/2 - (5*a^2*c
^4*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rubi [A] time = 0.246947, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1807, 815, 844, 216, 266, 63, 208}

$$\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{x}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2}+\frac{5}{2}a^2c^4(ax+1)\sqrt{1-a^2x^2}-\frac{5}{2}a^2c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{5}{2}a^2c^4\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^3,x]
```

```
[Out] (5*a^2*c^4*(1 + a*x)*Sqrt[1 - a^2*x^2])/2 - (c^4*(1 - a^2*x^2)^(3/2))/(2*x^
2) + (3*a*c^4*(1 - a^2*x^2)^(3/2))/x + (5*a^2*c^4*ArcSin[a*x])/2 - (5*a^2*c
^4*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^3}\,dx = c\int \frac{(c-acx)^3\sqrt{1-a^2x^2}}{x^3}\,dx$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2} - \frac{1}{2}c\int \frac{\sqrt{1-a^2x^2}\left(6ac^3-5a^2c^3x+2a^3c^3x^2\right)}{x^2}\,dx$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{1}{2}c\int \frac{\left(5a^2c^3+10a^3c^3x\right)\sqrt{1-a^2x^2}}{x}\,dx$$

$$= \frac{5}{2}a^2c^4(1+ax)\sqrt{1-a^2x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{x} - \frac{c\int \frac{-10a^4c^3-10a^5c^3x}{x\sqrt{1-a^2x^2}}\,dx}{4a^2}$$

$$= \frac{5}{2}a^2c^4(1+ax)\sqrt{1-a^2x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{1}{2}\left(5a^2c^4\right)\int \frac{1}{x\sqrt{1-a^2x}}$$

$$= \frac{5}{2}a^2c^4(1+ax)\sqrt{1-a^2x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{5}{2}a^2c^4\sin^{-1}(ax) + \frac{1}{4}\left(5\right.$$

$$= \frac{5}{2}a^2c^4(1+ax)\sqrt{1-a^2x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{5}{2}a^2c^4\sin^{-1}(ax) - \frac{1}{2}\left(5\right.$$

$$= \frac{5}{2}a^2c^4(1+ax)\sqrt{1-a^2x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{5}{2}a^2c^4\sin^{-1}(ax) - \frac{5}{2}a^2c$$

Mathematica [A] time = 0.214591, size = 106, normalized size = 0.91

$$\frac{1}{4}c^4\left(\frac{2(ax+1)^2\left(a^3x^3-8a^2x^2+8ax-1\right)}{x^2\sqrt{1-a^2x^2}}-10a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+5a^2\sin^{-1}(ax)-10a^2\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^3,x]
```

```
[Out] (c^4*((2*(1 + a*x)^2*(-1 + 8*a*x - 8*a^2*x^2 + a^3*x^3))/(x^2*Sqrt[1 - a^2*
x^2]) + 5*a^2*ArcSin[a*x] - 10*a^2*ArcSin[Sqrt[1 - a*x]/Sqrt[2]] - 10*a^2*A
rcTanh[Sqrt[1 - a^2*x^2]]))/4
```

Maple [A] time = 0.046, size = 138, normalized size = 1.2

$$-\frac{c^4a^3x}{2}\sqrt{-a^2x^2+1}+\frac{5\,c^4a^3}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+3\,c^4a^2\sqrt{-a^2x^2+1}+3\,\frac{c^4a\sqrt{-a^2x^2+1}}{x}-\frac{5\,c^4a^2}{2}\mathrm{Artanh}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^3,x)
```

```
[Out] -1/2*c^4*a^3*x*(-a^2*x^2+1)^(1/2)+5/2*c^4*a^3/(a^2)^(1/2)*arctan((a^2)^(1/2
)*x/(-a^2*x^2+1)^(1/2))+3*c^4*a^2*(-a^2*x^2+1)^(1/2)+3*c^4*a/x*(-a^2*x^2+1)
^(1/2)-5/2*c^4*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))-1/2*c^4/x^2*(-a^2*x^2+1)^(
1/2)
```

Maxima [A] time = 1.43377, size = 190, normalized size = 1.64

$$-\frac{1}{2}\sqrt{-a^2x^2+1}a^3c^4x+\frac{5\,a^3c^4\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\,\sqrt{a^2}}-\frac{5}{2}\,a^2c^4\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+3\,\sqrt{-a^2x^2+1}a^2c^4+\frac{3\,\sqrt{-a^2x^2+1}ac^4}{x}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^3,x, algorithm="maxima"
)
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*a^3*c^4*x + 5/2*a^3*c^4*arcsin(a^2*x/sqrt(a^2))/sqr
t(a^2) - 5/2*a^2*c^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 3*sqrt(-
a^2*x^2 + 1)*a^2*c^4 + 3*sqrt(-a^2*x^2 + 1)*a*c^4/x - 1/2*sqrt(-a^2*x^2 + 1
)*c^4/x^2
```

Fricas [A] time = 1.69706, size = 269, normalized size = 2.32

$$-\frac{10\,a^2c^4x^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-5\,a^2c^4x^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-6\,a^2c^4x^2+\left(a^3c^4x^3-6\,a^2c^4x^2-6\,ac^4x+c^4\right)\sqrt{-a^2x^2+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^3,x, algorithm="fricas"
)
```

```
[Out] -1/2*(10*a^2*c^4*x^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 5*a^2*c^4*x^2
*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 6*a^2*c^4*x^2 + (a^3*c^4*x^3 - 6*a^2*c^4
*x^2 - 6*a*c^4*x + c^4)*sqrt(-a^2*x^2 + 1))/x^2
```

Sympy [C] time = 8.89793, size = 357, normalized size = 3.08

$$a^5c^4\left(\begin{cases} -\frac{ix\sqrt{a^2x^2-1}}{2a^2} - \frac{i\operatorname{acosh}(ax)}{2a^3} & \text{for } |a^2x^2| > 1 \\ \frac{x^3}{2\sqrt{-a^2x^2+1}} - \frac{x}{2a^2\sqrt{-a^2x^2+1}} + \frac{\operatorname{asin}(ax)}{2a^3} & \text{otherwise} \end{cases}\right) - 3a^4c^4\left(\begin{cases} \frac{x^2}{2} & \text{for } a^2 = 0 \\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise} \end{cases}\right) + 2a^3c^4\left(\begin{cases} \sqrt{\frac{1}{a^2}}\operatorname{asin}(x \\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4/x**3,x)
```

```
[Out] a**5*c**4*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a*
*3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-
a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) - 3*a**4*c**4*Piecewise((x**2/
2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) + 2*a**3*c**4*Piecewis
e((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt
(-a**2)), a**2 < 0)) + 2*a**2*c**4*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x
**2) > 1), (I*asin(1/(a*x)), True)) - 3*a*c**4*Piecewise((-I*sqrt(a**2*x**2
 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True)) + c**4*Piece
wise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2
*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) +
I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))
```

Giac [B] time = 1.24292, size = 302, normalized size = 2.6

$$\frac{5\,a^3c^4\arcsin(ax)\operatorname{sgn}(a)}{2\,|a|} - \frac{5\,a^3c^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|} + \frac{\left(a^3c^4 - \frac{12\left(\sqrt{-a^2x^2+1}|a|+a\right)ac^4}{x}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}|a|+a\right)^2|a|} - \frac{1}{2}\left(a^3c^4x - 6\,a^2c^4\right)\sqrt{-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^3,x, algorithm="giac")
```

```
[Out] 5/2*a^3*c^4*arcsin(a*x)*sgn(a)/abs(a) - 5/2*a^3*c^4*log(1/2*abs(-2*sqrt(-a^
2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 1/8*(a^3*c^4 - 12*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)*a*c^4/x)*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*
abs(a)) - 1/2*(a^3*c^4*x - 6*a^2*c^4)*sqrt(-a^2*x^2 + 1) + 1/8*(12*(sqrt(-a
^2*x^2 + 1)*abs(a) + a)*a*c^4*abs(a)/x - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*
c^4*abs(a)/(a*x^2))/a^2
```

3.323 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^4}\,dx$

Optimal. Leaf size=120

$$\frac{3ac^4(1-a^2x^2)^{3/2}}{2x^2}-\frac{c^4(1-a^2x^2)^{3/2}}{3x^3}-\frac{a^2c^4(6-ax)\sqrt{1-a^2x^2}}{2x}-\frac{1}{2}a^3c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-3a^3c^4\sin^{-1}(ax)$$

```
[Out] -(a^2*c^4*(6 - a*x)*Sqrt[1 - a^2*x^2])/(2*x) - (c^4*(1 - a^2*x^2)^(3/2))/(3
*x^3) + (3*a*c^4*(1 - a^2*x^2)^(3/2))/(2*x^2) - 3*a^3*c^4*ArcSin[a*x] - (a^
3*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rubi [A] time = 0.249044, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1807, 813, 844, 216, 266, 63, 208}

$$\frac{3ac^4(1-a^2x^2)^{3/2}}{2x^2}-\frac{c^4(1-a^2x^2)^{3/2}}{3x^3}-\frac{a^2c^4(6-ax)\sqrt{1-a^2x^2}}{2x}-\frac{1}{2}a^3c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-3a^3c^4\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^4,x]
```

```
[Out] -(a^2*c^4*(6 - a*x)*Sqrt[1 - a^2*x^2])/(2*x) - (c^4*(1 - a^2*x^2)^(3/2))/(3
*x^3) + (3*a*c^4*(1 - a^2*x^2)^(3/2))/(2*x^2) - 3*a^3*c^4*ArcSin[a*x] - (a^
3*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^4}\,dx &= c\int \frac{(c-acx)^3\sqrt{1-a^2x^2}}{x^4}\,dx\\
&= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3x^3}-\frac{1}{3}c\int \frac{\sqrt{1-a^2x^2}\left(9ac^3-9a^2c^3x+3a^3c^3x^2\right)}{x^3}\,dx\\
&= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{2x^2}+\frac{1}{6}c\int \frac{\left(18a^2c^3+3a^3c^3x\right)\sqrt{1-a^2x^2}}{x^2}\,dx\\
&= -\frac{a^2c^4(6-ax)\sqrt{1-a^2x^2}}{2x}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{2x^2}-\frac{1}{12}c\int \frac{-6a^3c^3+36a^4c}{x\sqrt{1-a^2x^2}}\\
&= -\frac{a^2c^4(6-ax)\sqrt{1-a^2x^2}}{2x}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{2x^2}+\frac{1}{2}\left(a^3c^4\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\\
&= -\frac{a^2c^4(6-ax)\sqrt{1-a^2x^2}}{2x}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{2x^2}-3a^3c^4\sin^{-1}(ax)+\frac{1}{4}\left(a\right.\\
&= -\frac{a^2c^4(6-ax)\sqrt{1-a^2x^2}}{2x}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{2x^2}-3a^3c^4\sin^{-1}(ax)-\frac{1}{2}\left(a\right.\\
&= -\frac{a^2c^4(6-ax)\sqrt{1-a^2x^2}}{2x}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{2x^2}-3a^3c^4\sin^{-1}(ax)-\frac{1}{2}a^3c
\end{aligned}$$

Mathematica [A] time = 0.135312, size = 164, normalized size = 1.37

$$\frac{c^4\left(12a^5x^5+32a^4x^4-30a^3x^3-28a^2x^2+3a^3x^3\sqrt{1-a^2x^2}\sin^{-1}(ax)+78a^3x^3\sqrt{1-a^2x^2}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-6a^3x^3\sqrt{1-a^2x^2}\ t\right.}{12x^3\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^4,x]
```

```
[Out] (c^4*(-4 + 18*a*x - 28*a^2*x^2 - 30*a^3*x^3 + 32*a^4*x^4 + 12*a^5*x^5 + 3*a
^3*x^3*Sqrt[1 - a^2*x^2]*ArcSin[a*x] + 78*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcSin[
Sqrt[1 - a*x]/Sqrt[2]] - 6*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x
^2]]))/(12*x^3*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.042, size = 140, normalized size = 1.2

$$-c^4a^3\sqrt{-a^2x^2+1}-3\,\frac{c^4a^4}{\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)-\frac{8\,c^4a^2}{3\,x}\sqrt{-a^2x^2+1}-\frac{c^4a^3}{2}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{3\,c^4a}{2\,x^2}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^4,x)
```

```
[Out] -c^4*a^3*(-a^2*x^2+1)^(1/2)-3*c^4*a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^
2*x^2+1)^(1/2))-8/3*c^4*a^2/x*(-a^2*x^2+1)^(1/2)-1/2*c^4*a^3*arctanh(1/(-a^
2*x^2+1)^(1/2))+3/2*c^4*a/x^2*(-a^2*x^2+1)^(1/2)-1/3*c^4/x^3*(-a^2*x^2+1)^(
1/2)
```

Maxima [A] time = 1.43947, size = 193, normalized size = 1.61

$$-\frac{3\,a^4c^4\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}-\frac{1}{2}\,a^3c^4\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\sqrt{-a^2x^2+1}a^3c^4-\frac{8\,\sqrt{-a^2x^2+1}a^2c^4}{3\,x}+\frac{3\,\sqrt{-a^2x^2+1}ac^4}{2\,x^2}-\frac{\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^4,x, algorithm="maxima"
)
```

```
[Out] -3*a^4*c^4*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - 1/2*a^3*c^4*log(2*sqrt(-a^2*
x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2*x^2 + 1)*a^3*c^4 - 8/3*sqrt(-a^2*x^
2 + 1)*a^2*c^4/x + 3/2*sqrt(-a^2*x^2 + 1)*a*c^4/x^2 - 1/3*sqrt(-a^2*x^2 + 1
)*c^4/x^3
```

Fricas [A] time = 1.64987, size = 274, normalized size = 2.28

$$\frac{36\,a^3c^4x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+3\,a^3c^4x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-6\,a^3c^4x^3-\left(6\,a^3c^4x^3+16\,a^2c^4x^2-9\,ac^4x+2\,c^4\right)\sqrt{-a^2x^2+1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^4,x, algorithm="fricas"
)
```

```
[Out] 1/6*(36*a^3*c^4*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 3*a^3*c^4*x^3*
log((sqrt(-a^2*x^2 + 1) - 1)/x) - 6*a^3*c^4*x^3 - (6*a^3*c^4*x^3 + 16*a^2*c
^4*x^2 - 9*a*c^4*x + 2*c^4)*sqrt(-a^2*x^2 + 1))/x^3
```

Sympy [C] time = 7.75275, size = 359, normalized size = 2.99

$$a^5c^4\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-3a^4c^4\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\end{cases}\right)+2a^3c^4\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for } \frac{1}{|a^2x^2|}>\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4/x**4,x)
```

```
[Out] a**5*c**4*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, Tru
e)) - 3*a**4*c**4*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (
sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + 2*a**3*c**4*Piecewise((-ac
osh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True)) + 2*a**2*c**4
*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**
2 + 1)/x, True)) - 3*a*c**4*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 +
 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a
/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True
)) + c**4*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2
 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) -
sqrt(-a**2*x**2 + 1)/(3*x**3), True))
```

Giac [B] time = 1.24267, size = 365, normalized size = 3.04

$$-\frac{3\,a^4c^4\arcsin(ax)\operatorname{sgn}(a)}{|a|}-\frac{a^4c^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{2|a|}-\sqrt{-a^2x^2+1}a^3c^4+\frac{\left(a^4c^4-\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)a^2c^4}{x}+\frac{33\left(\sqrt{-a^2x^2\cdot\right.}{x^2}\right.}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^4,x, algorithm="giac")
```

```
[Out] -3*a^4*c^4*arcsin(a*x)*sgn(a)/abs(a) - 1/2*a^4*c^4*log(1/2*abs(-2*sqrt(-a^2
*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*a^3*c^4 +
 1/24*(a^4*c^4 - 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2*c^4/x + 33*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)^2*c^4/x^2)*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^
3*abs(a)) - 1/24*(33*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^4/x - 9*(sqrt(-a
^2*x^2 + 1)*abs(a) + a)^2*a^2*c^4/x^2 + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c
^4/x^3)/(a^2*abs(a))
```

3.324 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^5}\,dx$

Optimal. Leaf size=110

$$-\frac{11a^2c^4\sqrt{1-a^2x^2}}{8x^2}+\frac{ac^4\sqrt{1-a^2x^2}}{x^3}-\frac{c^4\sqrt{1-a^2x^2}}{4x^4}+\frac{13}{8}a^4c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+a^4c^4\sin^{-1}(ax)$$

```
[Out] -(c^4*Sqrt[1 - a^2*x^2])/(4*x^4) + (a*c^4*Sqrt[1 - a^2*x^2])/x^3 - (11*a^2*
c^4*Sqrt[1 - a^2*x^2])/(8*x^2) + a^4*c^4*ArcSin[a*x] + (13*a^4*c^4*ArcTanh[
Sqrt[1 - a^2*x^2]])/8
```

Rubi [A] time = 0.247726, antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1807, 811, 844, 216, 266, 63, 208}

$$\frac{ac^4\left(1-a^2x^2\right)^{3/2}}{x^3}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{a^2c^4(13-8ax)\sqrt{1-a^2x^2}}{8x^2}+\frac{13}{8}a^4c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+a^4c^4\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^5,x]
```

```
[Out] -(a^2*c^4*(13 - 8*a*x)*Sqrt[1 - a^2*x^2])/(8*x^2) - (c^4*(1 - a^2*x^2)^(3/2
))/(4*x^4) + (a*c^4*(1 - a^2*x^2)^(3/2))/x^3 + a^4*c^4*ArcSin[a*x] + (13*a^
4*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/8
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 && !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^5}\,dx = c\int \frac{(c-acx)^3\sqrt{1-a^2x^2}}{x^5}\,dx$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{1}{4}c\int \frac{\sqrt{1-a^2x^2}\left(12ac^3-13a^2c^3x+4a^3c^3x^2\right)}{x^4}\,dx$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^4\left(1-a^2x^2\right)^{3/2}}{x^3} + \frac{1}{12}c\int \frac{\left(39a^2c^3-12a^3c^3x\right)\sqrt{1-a^2x^2}}{x^3}\,dx$$

$$= -\frac{a^2c^4(13-8ax)\sqrt{1-a^2x^2}}{8x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^4\left(1-a^2x^2\right)^{3/2}}{x^3} - \frac{1}{48}c\int \frac{78a^4c^3-48a^5}{x\sqrt{1-a^2x^2}}$$

$$= -\frac{a^2c^4(13-8ax)\sqrt{1-a^2x^2}}{8x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^4\left(1-a^2x^2\right)^{3/2}}{x^3} - \frac{1}{8}\left(13a^4c^4\right)\int \frac{1}{x\sqrt{1-}}$$

$$= -\frac{a^2c^4(13-8ax)\sqrt{1-a^2x^2}}{8x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^4\left(1-a^2x^2\right)^{3/2}}{x^3} + a^4c^4\sin^{-1}(ax) - \frac{1}{16}($$

$$= -\frac{a^2c^4(13-8ax)\sqrt{1-a^2x^2}}{8x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^4\left(1-a^2x^2\right)^{3/2}}{x^3} + a^4c^4\sin^{-1}(ax) + \frac{1}{8}(1$$

$$= -\frac{a^2c^4(13-8ax)\sqrt{1-a^2x^2}}{8x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^4\left(1-a^2x^2\right)^{3/2}}{x^3} + a^4c^4\sin^{-1}(ax) + \frac{13}{8}a^4$$

Mathematica [A] time = 0.210056, size = 125, normalized size = 1.14

$$\frac{1}{16}c^4\left(-\frac{2\left(-11a^4x^4+8a^3x^3+9a^2x^2+29a^4x^4\sqrt{1-a^2x^2}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-8ax+2\right)}{x^4\sqrt{1-a^2x^2}}+26a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-13a^4\sin^{-}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^5,x]
```

```
[Out] (c^4*(-13*a^4*ArcSin[a*x] - (2*(2 - 8*a*x + 9*a^2*x^2 + 8*a^3*x^3 - 11*a^4*
x^4 + 29*a^4*x^4*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(x^4*Sqr
t[1 - a^2*x^2]) + 26*a^4*ArcTanh[Sqrt[1 - a^2*x^2]]))/16
```

Maple [A] time = 0.046, size = 118, normalized size = 1.1

$$c^4a^5\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{c^4}{4x^4}\sqrt{-a^2x^2+1}-\frac{11a^2c^4}{8x^2}\sqrt{-a^2x^2+1}+\frac{13c^4a^4}{8}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{ac^4}{x^3}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^5,x)
```

```
[Out] c^4*a^5/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-1/4*c^4*(-a^2*
x^2+1)^(1/2)/x^4-11/8*a^2*c^4*(-a^2*x^2+1)^(1/2)/x^2+13/8*c^4*a^4*arctanh(1
/(-a^2*x^2+1)^(1/2))+a*c^4*(-a^2*x^2+1)^(1/2)/x^3
```

Maxima [A] time = 1.42836, size = 163, normalized size = 1.48

$$\frac{a^5c^4\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}+\frac{13}{8}a^4c^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{11\sqrt{-a^2x^2+1}a^2c^4}{8x^2}+\frac{\sqrt{-a^2x^2+1}ac^4}{x^3}-\frac{\sqrt{-a^2x^2+1}c^4}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^5,x, algorithm="maxima"
)
```

```
[Out] a^5*c^4*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + 13/8*a^4*c^4*log(2*sqrt(-a^2*x^
2 + 1)/abs(x) + 2/abs(x)) - 11/8*sqrt(-a^2*x^2 + 1)*a^2*c^4/x^2 + sqrt(-a^2
*x^2 + 1)*a*c^4/x^3 - 1/4*sqrt(-a^2*x^2 + 1)*c^4/x^4
```

Fricas [A] time = 1.54803, size = 234, normalized size = 2.13

$$-\frac{16a^4c^4x^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+13a^4c^4x^4\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(11a^2c^4x^2-8ac^4x+2c^4\right)\sqrt{-a^2x^2+1}}{8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^5,x, algorithm="fricas"
)
```

```
[Out] -1/8*(16*a^4*c^4*x^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 13*a^4*c^4*x^
4*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (11*a^2*c^4*x^2 - 8*a*c^4*x + 2*c^4)*sq
rt(-a^2*x^2 + 1))/x^4
```

Sympy [C] time = 11.2849, size = 505, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4/x**5,x)
```

```
[Out] a**5*c**4*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/
a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - 3*a**4*c**4*Piecewise((-acosh(1/(a
*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True)) + 2*a**3*c**4*Piecewi
se((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x
, True)) + 2*a**2*c**4*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a
**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x
*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True)) -
3*a*c**4*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2
- 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - s
qrt(-a**2*x**2 + 1)/(3*x**3), True)) + c**4*Piecewise((-3*a**4*acosh(1/(a*x
))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*
x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*
I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x*
*3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True))
```

Giac [B] time = 1.17904, size = 427, normalized size = 3.88

$$\frac{a^5c^4\arcsin(ax)\,\mathrm{sgn}(a)}{|a|}+\frac{13\,a^5c^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{8\,|a|}+\frac{\left(a^5c^4-\frac{8\left(\sqrt{-a^2x^2+1}|a|+a\right)a^3c^4}{x}+\frac{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^2ac^4}{x^2}-\frac{8\left(\sqrt{-a^2\cdots\right.}{}\cdots\right.}{64\left(\sqrt{-a^2x^2+1}|a|+a\right)^4|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^5,x, algorithm="giac")
```

```
[Out] a^5*c^4*arcsin(a*x)*sgn(a)/abs(a) + 13/8*a^5*c^4*log(1/2*abs(-2*sqrt(-a^2*x
^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 1/64*(a^5*c^4 - 8*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)*a^3*c^4/x + 24*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a*c^4/
x^2 - 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^4/(a*x^3))*a^8*x^4/((sqrt(-a^2*
x^2 + 1)*abs(a) + a)^4*abs(a)) + 1/64*(8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^
5*c^4*abs(a)/x - 24*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^3*c^4*abs(a)/x^2 +
8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a*c^4*abs(a)/x^3 - (sqrt(-a^2*x^2 + 1)*
abs(a) + a)^4*c^4*abs(a)/(a*x^4))/a^4
```

3.325 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^6}\,dx$

Optimal. Leaf size=129

$$\frac{7a^3c^4\sqrt{1-a^2x^2}}{8x^2}-\frac{17a^2c^4\left(1-a^2x^2\right)^{3/2}}{15x^3}+\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{7}{8}a^5c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] (7*a^3*c^4*Sqrt[1 - a^2*x^2])/(8*x^2) - (c^4*(1 - a^2*x^2)^(3/2))/(5*x^5) +
 (3*a*c^4*(1 - a^2*x^2)^(3/2))/(4*x^4) - (17*a^2*c^4*(1 - a^2*x^2)^(3/2))/(
15*x^3) - (7*a^5*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/8
```

Rubi [A] time = 0.245766, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 1807, 807, 266, 47, 63, 208}

$$\frac{7a^3c^4\sqrt{1-a^2x^2}}{8x^2}-\frac{17a^2c^4\left(1-a^2x^2\right)^{3/2}}{15x^3}+\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{7}{8}a^5c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^6,x]
```

```
[Out] (7*a^3*c^4*Sqrt[1 - a^2*x^2])/(8*x^2) - (c^4*(1 - a^2*x^2)^(3/2))/(5*x^5) +
 (3*a*c^4*(1 - a^2*x^2)^(3/2))/(4*x^4) - (17*a^2*c^4*(1 - a^2*x^2)^(3/2))/(
15*x^3) - (7*a^5*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/8
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^6}\,dx = c\int \frac{(c-acx)^3\sqrt{1-a^2x^2}}{x^6}\,dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{1}{5}c\int \frac{\sqrt{1-a^2x^2}\left(15ac^3-17a^2c^3x+5a^3c^3x^2\right)}{x^5}\,dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{1}{20}c\int \frac{\left(68a^2c^3-35a^3c^3x\right)\sqrt{1-a^2x^2}}{x^4}\,dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{17a^2c^4\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{4}\left(7a^3c^4\right)\int \frac{\sqrt{1-a^2x^2}}{x^3}\,d$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{17a^2c^4\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{8}\left(7a^3c^4\right)\text{Subst}\left(\int \frac{\sqrt{1-}}{x}\right.$$
$$= \frac{7a^3c^4\sqrt{1-a^2x^2}}{8x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{17a^2c^4\left(1-a^2x^2\right)^{3/2}}{15x^3} + \frac{1}{16}\left(7a^5\right.$$
$$= \frac{7a^3c^4\sqrt{1-a^2x^2}}{8x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{17a^2c^4\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{8}\left(7a^3c^4\right.$$
$$= \frac{7a^3c^4\sqrt{1-a^2x^2}}{8x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{17a^2c^4\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{7}{8}a^5c^4\,\text{ta}$$

Mathematica [A] time = 0.0355584, size = 107, normalized size = 0.83

$$-\frac{c^4\left(136a^6x^6+15a^5x^5-248a^4x^4+75a^3x^3+88a^2x^2+105a^5x^5\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-90ax+24\right)}{120x^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^6,x]
```

```
[Out] -(c^4*(24 - 90*a*x + 88*a^2*x^2 + 75*a^3*x^3 - 248*a^4*x^4 + 15*a^5*x^5 + 1
36*a^6*x^6 + 105*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(12
0*x^5*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.053, size = 207, normalized size = 1.6

$$c^4\left(-3\,a\left(-1/4\,\frac{\sqrt{-a^2x^2+1}}{x^4}+3/4\,a^2\left(-1/2\,\frac{\sqrt{-a^2x^2+1}}{x^2}-1/2\,a^2\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)\right)+3\,\frac{a^4\sqrt{-a^2x^2+1}}{x}-\frac{1}{5\,x^5}\sqrt{-a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^6,x)
```

```
[Out] c^4*(-3*a*(-1/4*(-a^2*x^2+1)^(1/2)/x^4+3/4*a^2*(-1/2*(-a^2*x^2+1)^(1/2)/x^2
-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))))+3*a^4/x*(-a^2*x^2+1)^(1/2)-1/5/x^5
*(-a^2*x^2+1)^(1/2)+14/5*a^2*(-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2
+1)^(1/2)/x)-a^5*arctanh(1/(-a^2*x^2+1)^(1/2))+2*a^3*(-1/2*(-a^2*x^2+1)^(1/
2)/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))))
```

Maxima [A] time = 1.42999, size = 196, normalized size = 1.52

$$-\frac{7}{8}\,a^5c^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{17\sqrt{-a^2x^2+1}a^4c^4}{15\,x}+\frac{\sqrt{-a^2x^2+1}a^3c^4}{8\,x^2}-\frac{14\sqrt{-a^2x^2+1}a^2c^4}{15\,x^3}+\frac{3\sqrt{-a^2x^2+1}ac^4}{4\,x^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^6,x, algorithm="maxima"
)
```

```
[Out] -7/8*a^5*c^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 17/15*sqrt(-a^2*
x^2 + 1)*a^4*c^4/x + 1/8*sqrt(-a^2*x^2 + 1)*a^3*c^4/x^2 - 14/15*sqrt(-a^2*x
^2 + 1)*a^2*c^4/x^3 + 3/4*sqrt(-a^2*x^2 + 1)*a*c^4/x^4 - 1/5*sqrt(-a^2*x^2
+ 1)*c^4/x^5
```

Fricas [A] time = 1.58292, size = 212, normalized size = 1.64

$$\frac{105\,a^5c^4x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(136\,a^4c^4x^4+15\,a^3c^4x^3-112\,a^2c^4x^2+90\,ac^4x-24\,c^4\right)\sqrt{-a^2x^2+1}}{120\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^6,x, algorithm="fricas"
)
```

```
[Out] 1/120*(105*a^5*c^4*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (136*a^4*c^4*x^4 +
 15*a^3*c^4*x^3 - 112*a^2*c^4*x^2 + 90*a*c^4*x - 24*c^4)*sqrt(-a^2*x^2 + 1)
)/x^5
```

Sympy [C] time = 12.3919, size = 607, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4/x**6,x)
```

```
[Out] a**5*c**4*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x
)), True)) - 3*a**4*c**4*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2
) > 1), (-sqrt(-a**2*x**2 + 1)/x, True)) + 2*a**3*c**4*Piecewise((-a**2*aco
sh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I
*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqr
t(1 - 1/(a**2*x**2))), True)) + 2*a**2*c**4*Piecewise((-2*I*a**2*sqrt(a**2*
x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*
a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), True)) - 3
*a*c**4*Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2
*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a
**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8
*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a
*x**5*sqrt(1 - 1/(a**2*x**2))), True)) + c**4*Piecewise((-8*a**5*sqrt(-1 +
1/(a**2*x**2))/15 - 4*a**3*sqrt(-1 + 1/(a**2*x**2))/(15*x**2) - a*sqrt(-1 +
 1/(a**2*x**2))/(5*x**4), 1/Abs(a**2*x**2) > 1), (-8*I*a**5*sqrt(1 - 1/(a**
2*x**2))/15 - 4*I*a**3*sqrt(1 - 1/(a**2*x**2))/(15*x**2) - I*a*sqrt(1 - 1/(
a**2*x**2))/(5*x**4), True))
```

Giac [B] time = 1.18675, size = 478, normalized size = 3.71

$$\frac{\left(6\,a^6c^4 - \frac{45\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4c^4}{x} + \frac{130\left(\sqrt{-a^2x^2+1}|a|+a\right)^2a^2c^4}{x^2} - \frac{120\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c^4}{x^3} - \frac{420\left(\sqrt{-a^2x^2+1}|a|+a\right)^4c^4}{a^2x^4}\right)a^{10}x^5}{960\left(\sqrt{-a^2x^2+1}|a|+a\right)^5|a|} - \frac{7\,a^6c^4\log\left(\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^6,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^6*c^4 - 45*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^4/x + 130*(sqrt
(-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^4/x^2 - 120*(sqrt(-a^2*x^2 + 1)*abs(a) +
 a)^3*c^4/x^3 - 420*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^2*x^4))*a^10*x
^5/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*abs(a)) - 7/8*a^6*c^4*log(1/2*abs(-2*
sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 1/960*(420*(sqrt(-a
^2*x^2 + 1)*abs(a) + a)*a^8*c^4/x + 120*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a
^6*c^4/x^2 - 130*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a^4*c^4/x^3 + 45*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^4*a^2*c^4/x^4 - 6*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^
5*c^4/x^5)/(a^4*abs(a))
```

3.326 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^7}\,dx$

Optimal. Leaf size=156

$$-\frac{7a^4c^4\sqrt{1-a^2x^2}}{16x^2}+\frac{11a^3c^4\left(1-a^2x^2\right)^{3/2}}{15x^3}-\frac{7a^2c^4\left(1-a^2x^2\right)^{3/2}}{8x^4}+\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6}+\frac{7}{16}a^6c^4\tanh^{-1}\left(\right.$$

```
[Out] (-7*a^4*c^4*Sqrt[1 - a^2*x^2])/(16*x^2) - (c^4*(1 - a^2*x^2)^(3/2))/(6*x^6)
 + (3*a*c^4*(1 - a^2*x^2)^(3/2))/(5*x^5) - (7*a^2*c^4*(1 - a^2*x^2)^(3/2))/
(8*x^4) + (11*a^3*c^4*(1 - a^2*x^2)^(3/2))/(15*x^3) + (7*a^6*c^4*ArcTanh[Sq
rt[1 - a^2*x^2]])/16
```

Rubi [A] time = 0.263734, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1807, 835, 807, 266, 47, 63, 208}

$$-\frac{7a^4c^4\sqrt{1-a^2x^2}}{16x^2}+\frac{11a^3c^4\left(1-a^2x^2\right)^{3/2}}{15x^3}-\frac{7a^2c^4\left(1-a^2x^2\right)^{3/2}}{8x^4}+\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6}+\frac{7}{16}a^6c^4\tanh^{-1}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^7,x]
```



```
[Out] (-7*a^4*c^4*Sqrt[1 - a^2*x^2])/(16*x^2) - (c^4*(1 - a^2*x^2)^(3/2))/(6*x^6)
 + (3*a*c^4*(1 - a^2*x^2)^(3/2))/(5*x^5) - (7*a^2*c^4*(1 - a^2*x^2)^(3/2))/
(8*x^4) + (11*a^3*c^4*(1 - a^2*x^2)^(3/2))/(15*x^3) + (7*a^6*c^4*ArcTanh[Sq
rt[1 - a^2*x^2]])/16
```


Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^7}\,dx = c\int \frac{(c-acx)^3\sqrt{1-a^2x^2}}{x^7}\,dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6} - \frac{1}{6}c\int \frac{\sqrt{1-a^2x^2}\left(18ac^3-21a^2c^3x+6a^3c^3x^2\right)}{x^6}\,dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{1}{30}c\int \frac{\left(105a^2c^3-66a^3c^3x\right)\sqrt{1-a^2x^2}}{x^5}\,dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{7a^2c^4\left(1-a^2x^2\right)^{3/2}}{8x^4} - \frac{1}{120}c\int \frac{\left(264a^3c^3-105a^4c^3x\right.}{x^4}$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{7a^2c^4\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{11a^3c^4\left(1-a^2x^2\right)^{3/2}}{15x^3} + \frac{1}{8}\left(7a^4\right.$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{7a^2c^4\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{11a^3c^4\left(1-a^2x^2\right)^{3/2}}{15x^3} + \frac{1}{16}\left(7a\right.$$
$$= -\frac{7a^4c^4\sqrt{1-a^2x^2}}{16x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{7a^2c^4\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{11a^3c^4\left(1-\right.}{15x}$$
$$= -\frac{7a^4c^4\sqrt{1-a^2x^2}}{16x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{7a^2c^4\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{11a^3c^4\left(1-\right.}{15x}$$
$$= -\frac{7a^4c^4\sqrt{1-a^2x^2}}{16x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{7a^2c^4\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{11a^3c^4\left(1-\right.}{15x}$$

Mathematica [A] time = 0.0367067, size = 115, normalized size = 0.74

$$\frac{c^4\left(176a^7x^7-105a^6x^6-208a^5x^5+275a^4x^4-112a^3x^3-130a^2x^2+105a^6x^6\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+144ax-40\right.}{240x^6\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^7,x]
```

```
[Out] (c^4*(-40 + 144*a*x - 130*a^2*x^2 - 112*a^3*x^3 + 275*a^4*x^4 - 208*a^5*x^5
 - 105*a^6*x^6 + 176*a^7*x^7 + 105*a^6*x^6*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1
 - a^2*x^2]]))/(240*x^6*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.051, size = 255, normalized size = 1.6

$$c^4\left(\frac{17\,a^2}{6}\left(-\frac{1}{4\,x^4}\sqrt{-a^2x^2+1}+\frac{3\,a^2}{4}\left(-\frac{1}{2\,x^2}\sqrt{-a^2x^2+1}-\frac{a^2}{2}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)\right)-\frac{a^5}{x}\sqrt{-a^2x^2+1}-3\,a\left(-1/5\,\frac{\sqrt{-a^2x}}{x^5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^7,x)
```

```
[Out] c^4*(17/6*a^2*(-1/4*(-a^2*x^2+1)^(1/2)/x^4+3/4*a^2*(-1/2*(-a^2*x^2+1)^(1/2)
/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))))-a^5/x*(-a^2*x^2+1)^(1/2)-3*a*(
-1/5/x^5*(-a^2*x^2+1)^(1/2)+4/5*a^2*(-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-
a^2*x^2+1)^(1/2)/x))-1/6/x^6*(-a^2*x^2+1)^(1/2)-3*a^4*(-1/2*(-a^2*x^2+1)^(1
/2)/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2)))+2*a^3*(-1/3*(-a^2*x^2+1)^(1/
```

```
2)/x^3-2/3*a^2*(-a^2*x^2+1)^(1/2)/x))
```

Maxima [A] time = 1.43569, size = 227, normalized size = 1.46

$$\frac{7}{16}\,a^6c^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{11\sqrt{-a^2x^2+1}a^5c^4}{15\,x}+\frac{7\sqrt{-a^2x^2+1}a^4c^4}{16\,x^2}+\frac{2\sqrt{-a^2x^2+1}a^3c^4}{15\,x^3}-\frac{17\sqrt{-a^2x^2+1}a^2c}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^7,x, algorithm="maxima"
)
```

```
[Out] 7/16*a^6*c^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 11/15*sqrt(-a^2*
x^2 + 1)*a^5*c^4/x + 7/16*sqrt(-a^2*x^2 + 1)*a^4*c^4/x^2 + 2/15*sqrt(-a^2*x
^2 + 1)*a^3*c^4/x^3 - 17/24*sqrt(-a^2*x^2 + 1)*a^2*c^4/x^4 + 3/5*sqrt(-a^2*
x^2 + 1)*a*c^4/x^5 - 1/6*sqrt(-a^2*x^2 + 1)*c^4/x^6
```

Fricas [A] time = 1.6049, size = 239, normalized size = 1.53

$$-\frac{105\,a^6c^4x^6\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(176\,a^5c^4x^5-105\,a^4c^4x^4-32\,a^3c^4x^3+170\,a^2c^4x^2-144\,ac^4x+40\,c^4\right)\sqrt{-a^2x^2+1}}{240\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^7,x, algorithm="fricas"
)
```

```
[Out] -1/240*(105*a^6*c^4*x^6*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (176*a^5*c^4*x^5
- 105*a^4*c^4*x^4 - 32*a^3*c^4*x^3 + 170*a^2*c^4*x^2 - 144*a*c^4*x + 40*c^4
)*sqrt(-a^2*x^2 + 1))/x^6
```

Sympy [C] time = 17.6997, size = 801, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4/x**7,x)
```

```
[Out] a**5*c**4*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(
-a**2*x**2 + 1)/x, True)) - 3*a**4*c**4*Piecewise((-a**2*acosh(1/(a*x))/2 -
 a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a
*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x
**2))), True)) + 2*a**3*c**4*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x)
 - I*sqrt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2
*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), True)) + 2*a**2*c**4*Piec
ewise((-3*a**4*acosh(1/(a*x))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a
/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))),
 1/Abs(a**2*x**2) > 1), (3*I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 -
1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1
 - 1/(a**2*x**2))), True)) - 3*a*c**4*Piecewise((-8*a**5*sqrt(-1 + 1/(a**2*
```

```
x**2))/15 - 4*a**3*sqrt(-1 + 1/(a**2*x**2))/(15*x**2) - a*sqrt(-1 + 1/(a**2
*x**2))/(5*x**4), 1/Abs(a**2*x**2) > 1), (-8*I*a**5*sqrt(1 - 1/(a**2*x**2))
/15 - 4*I*a**3*sqrt(1 - 1/(a**2*x**2))/(15*x**2) - I*a*sqrt(1 - 1/(a**2*x**
2))/(5*x**4), True)) + c**4*Piecewise((-5*a**6*acosh(1/(a*x))/16 + 5*a**5/(
16*x*sqrt(-1 + 1/(a**2*x**2))) - 5*a**3/(48*x**3*sqrt(-1 + 1/(a**2*x**2)))
- a/(24*x**5*sqrt(-1 + 1/(a**2*x**2))) - 1/(6*a*x**7*sqrt(-1 + 1/(a**2*x**2
))), 1/Abs(a**2*x**2) > 1), (5*I*a**6*asin(1/(a*x))/16 - 5*I*a**5/(16*x*sqr
t(1 - 1/(a**2*x**2))) + 5*I*a**3/(48*x**3*sqrt(1 - 1/(a**2*x**2))) + I*a/(2
4*x**5*sqrt(1 - 1/(a**2*x**2))) + I/(6*a*x**7*sqrt(1 - 1/(a**2*x**2))), Tru
e))
```

Giac [B] time = 1.17995, size = 572, normalized size = 3.67

$$\frac{\left(5\,a^7c^4 - \frac{36\left(\sqrt{-a^2x^2+1}|a|+a\right)a^5c^4}{x} + \frac{105\left(\sqrt{-a^2x^2+1}|a|+a\right)^2a^3c^4}{x^2} - \frac{140\left(\sqrt{-a^2x^2+1}|a|+a\right)^3ac^4}{x^3} - \frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^4c^4}{ax^4} + \frac{600\left(\sqrt{-a^2x^2+1}|a|+a\right)^5c^4}{a^3x^5}\right)a}{1920\left(\sqrt{-a^2x^2+1}|a|+a\right)^6|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^7,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^7*c^4 - 36*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^5*c^4/x + 105*(sqr
t(-a^2*x^2 + 1)*abs(a) + a)^2*a^3*c^4/x^2 - 140*(sqrt(-a^2*x^2 + 1)*abs(a)
+ a)^3*a*c^4/x^3 - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a*x^4) + 600*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^3*x^5))*a^12*x^6/((sqrt(-a^2*x^2 +
1)*abs(a) + a)^6*abs(a)) + 7/16*a^7*c^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*a
bs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/1920*(600*(sqrt(-a^2*x^2 + 1)*abs(a)
+ a)*a^9*c^4*abs(a)/x - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^7*c^4*abs(a)
/x^2 - 140*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a^5*c^4*abs(a)/x^3 + 105*(sqrt
(-a^2*x^2 + 1)*abs(a) + a)^4*a^3*c^4*abs(a)/x^4 - 36*(sqrt(-a^2*x^2 + 1)*ab
s(a) + a)^5*a*c^4*abs(a)/x^5 + 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6*c^4*abs(
a)/(a*x^6))/a^6
```

3.327 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{c-acx}\,dx$

Optimal. Leaf size=146

$$\frac{x^3\sqrt{1-a^2x^2}}{4a^2c}+\frac{2x^2\sqrt{1-a^2x^2}}{3a^3c}+\frac{11x\sqrt{1-a^2x^2}}{8a^4c}+\frac{13\sqrt{1-a^2x^2}}{3a^5c}+\frac{(ax+1)^2}{a^5c\sqrt{1-a^2x^2}}-\frac{27\sin^{-1}(ax)}{8a^5c}$$

```
[Out] (1 + a*x)^2/(a^5*c*Sqrt[1 - a^2*x^2]) + (13*Sqrt[1 - a^2*x^2])/(3*a^5*c) +
(11*x*Sqrt[1 - a^2*x^2])/(8*a^4*c) + (2*x^2*Sqrt[1 - a^2*x^2])/(3*a^3*c) +
(x^3*Sqrt[1 - a^2*x^2])/(4*a^2*c) - (27*ArcSin[a*x])/(8*a^5*c)
```

Rubi [A] time = 0.343283, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 852, 1635, 1815, 641, 216}

$$\frac{x^3\sqrt{1-a^2x^2}}{4a^2c}+\frac{2x^2\sqrt{1-a^2x^2}}{3a^3c}+\frac{11x\sqrt{1-a^2x^2}}{8a^4c}+\frac{13\sqrt{1-a^2x^2}}{3a^5c}+\frac{(ax+1)^2}{a^5c\sqrt{1-a^2x^2}}-\frac{27\sin^{-1}(ax)}{8a^5c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a*c*x),x]
```

```
[Out] (1 + a*x)^2/(a^5*c*Sqrt[1 - a^2*x^2]) + (13*Sqrt[1 - a^2*x^2])/(3*a^5*c) +
(11*x*Sqrt[1 - a^2*x^2])/(8*a^4*c) + (2*x^2*Sqrt[1 - a^2*x^2])/(3*a^3*c) +
(x^3*Sqrt[1 - a^2*x^2])/(4*a^2*c) - (27*ArcSin[a*x])/(8*a^5*c)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
```

```
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{c-acx}\,dx = c\int \frac{x^4\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx$$

$$= \frac{\int \frac{x^4(c+acx)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$

$$= \frac{(1+ax)^2}{a^5c\sqrt{1-a^2x^2}} - \frac{\int \frac{(c+acx)\left(\frac{2}{a^4}+\frac{x}{a^3}+\frac{x^2}{a^2}+\frac{x^3}{a}\right)}{\sqrt{1-a^2x^2}}\,dx}{c^2}$$

$$= \frac{(1+ax)^2}{a^5c\sqrt{1-a^2x^2}} + \frac{x^3\sqrt{1-a^2x^2}}{4a^2c} + \frac{\int \frac{-\frac{8c}{a^2}-\frac{12cx}{a}-11cx^2-8acx^3}{\sqrt{1-a^2x^2}}\,dx}{4a^2c^2}$$

$$= \frac{(1+ax)^2}{a^5c\sqrt{1-a^2x^2}} + \frac{2x^2\sqrt{1-a^2x^2}}{3a^3c} + \frac{x^3\sqrt{1-a^2x^2}}{4a^2c} - \frac{\int \frac{24c+52acx+33a^2cx^2}{\sqrt{1-a^2x^2}}\,dx}{12a^4c^2}$$

$$= \frac{(1+ax)^2}{a^5c\sqrt{1-a^2x^2}} + \frac{11x\sqrt{1-a^2x^2}}{8a^4c} + \frac{2x^2\sqrt{1-a^2x^2}}{3a^3c} + \frac{x^3\sqrt{1-a^2x^2}}{4a^2c} + \frac{\int \frac{-81a^2c-104a^3cx}{\sqrt{1-a^2x^2}}\,dx}{24a^6c^2}$$

$$= \frac{(1+ax)^2}{a^5c\sqrt{1-a^2x^2}} + \frac{13\sqrt{1-a^2x^2}}{3a^5c} + \frac{11x\sqrt{1-a^2x^2}}{8a^4c} + \frac{2x^2\sqrt{1-a^2x^2}}{3a^3c} + \frac{x^3\sqrt{1-a^2x^2}}{4a^2c} - \frac{27\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{8a^4c}$$

$$= \frac{(1+ax)^2}{a^5c\sqrt{1-a^2x^2}} + \frac{13\sqrt{1-a^2x^2}}{3a^5c} + \frac{11x\sqrt{1-a^2x^2}}{8a^4c} + \frac{2x^2\sqrt{1-a^2x^2}}{3a^3c} + \frac{x^3\sqrt{1-a^2x^2}}{4a^2c} - \frac{27\sin^{-1}(ax)}{8a^5c}$$

Mathematica [A] time = 0.0647541, size = 81, normalized size = 0.55

$$\frac{162\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-\frac{\sqrt{ax+1}\left(6a^4x^4+10a^3x^3+17a^2x^2+47ax-128\right)}{\sqrt{1-ax}}}{24a^5c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a*c*x),x]
```

```
[Out] (-((Sqrt[1 + a*x]*(-128 + 47*a*x + 17*a^2*x^2 + 10*a^3*x^3 + 6*a^4*x^4))/Sq
rt[1 - a*x]) + 162*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])/(24*a^5*c)
```

Maple [A] time = 0.046, size = 166, normalized size = 1.1

$$\frac{x^3}{4a^2c}\sqrt{-a^2x^2+1}+\frac{11x}{8a^4c}\sqrt{-a^2x^2+1}-\frac{27}{8a^4c}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{2x^2}{3a^3c}\sqrt{-a^2x^2+1}+\frac{10}{3a^5c}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c),x)
```

```
[Out] 1/4*x^3*(-a^2*x^2+1)^(1/2)/a^2/c+11/8*x*(-a^2*x^2+1)^(1/2)/a^4/c-27/8/c/a^4
/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+2/3*x^2*(-a^2*x^2+1)^
(1/2)/a^3/c+10/3*(-a^2*x^2+1)^(1/2)/a^5/c-2/c/a^6/(x-1/a)*(-a^2*(x-1/a)^2-2
*a*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56577, size = 228, normalized size = 1.56

$$\frac{128\,ax+162\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(6\,a^4x^4+10\,a^3x^3+17\,a^2x^2+47\,ax-128\right)\sqrt{-a^2x^2+1}-128}{24\left(a^6cx-a^5c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/24*(128*a*x + 162*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (6*a
^4*x^4 + 10*a^3*x^3 + 17*a^2*x^2 + 47*a*x - 128)*sqrt(-a^2*x^2 + 1) - 128)/
(a^6*c*x - a^5*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{x^4}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int\frac{ax^5}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a*c*x+c),x)
```

```
[Out] -(Integral(x**4/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Int
egral(a*x**5/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

Giac [A] time = 1.21367, size = 154, normalized size = 1.05

$$\frac{1}{24}\sqrt{-a^2x^2+1}\left(\left(2\,x\left(\frac{3\,x}{a^2c}+\frac{8}{a^3c}\right)+\frac{33}{a^4c}\right)x+\frac{80}{a^5c}\right)-\frac{27\,\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{8\,a^4c|a|}+\frac{4}{a^4c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(-a^2*x^2 + 1)*((2*x*(3*x/(a^2*c) + 8/(a^3*c)) + 33/(a^4*c))*x + 8
0/(a^5*c)) - 27/8*arcsin(a*x)*sgn(a)/(a^4*c*abs(a)) + 4/(a^4*c*((sqrt(-a^2*
x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

3.328 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{c-acx}\,dx$

Optimal. Leaf size=114

$$\frac{x^2\sqrt{1-a^2x^2}}{3a^2c}+\frac{x\sqrt{1-a^2x^2}}{a^3c}+\frac{11\sqrt{1-a^2x^2}}{3a^4c}+\frac{(ax+1)^2}{a^4c\sqrt{1-a^2x^2}}-\frac{3\sin^{-1}(ax)}{a^4c}$$

```
[Out] (1 + a*x)^2/(a^4*c*Sqrt[1 - a^2*x^2]) + (11*Sqrt[1 - a^2*x^2])/(3*a^4*c) +
(x*Sqrt[1 - a^2*x^2])/(a^3*c) + (x^2*Sqrt[1 - a^2*x^2])/(3*a^2*c) - (3*ArcS
in[a*x])/(a^4*c)
```

Rubi [A] time = 0.285799, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 852, 1635, 1815, 641, 216}

$$\frac{x^2\sqrt{1-a^2x^2}}{3a^2c}+\frac{x\sqrt{1-a^2x^2}}{a^3c}+\frac{11\sqrt{1-a^2x^2}}{3a^4c}+\frac{(ax+1)^2}{a^4c\sqrt{1-a^2x^2}}-\frac{3\sin^{-1}(ax)}{a^4c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a*c*x),x]
```

```
[Out] (1 + a*x)^2/(a^4*c*Sqrt[1 - a^2*x^2]) + (11*Sqrt[1 - a^2*x^2])/(3*a^4*c) +
(x*Sqrt[1 - a^2*x^2])/(a^3*c) + (x^2*Sqrt[1 - a^2*x^2])/(3*a^2*c) - (3*ArcS
in[a*x])/(a^4*c)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
```

```
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^3}{c-acx}\,dx &= c\int \frac{x^3\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx \\
&= \frac{\int \frac{x^3(c+acx)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3} \\
&= \frac{(1+ax)^2}{a^4c\sqrt{1-a^2x^2}} - \frac{\int \frac{(c+acx)\left(\frac{2}{a^3}+\frac{x}{a^2}+\frac{x^2}{a}\right)}{\sqrt{1-a^2x^2}}\,dx}{c^2} \\
&= \frac{(1+ax)^2}{a^4c\sqrt{1-a^2x^2}} + \frac{x^2\sqrt{1-a^2x^2}}{3a^2c} + \frac{\int \frac{-\frac{6c}{a}-11cx-6acx^2}{\sqrt{1-a^2x^2}}\,dx}{3a^2c^2} \\
&= \frac{(1+ax)^2}{a^4c\sqrt{1-a^2x^2}} + \frac{x\sqrt{1-a^2x^2}}{a^3c} + \frac{x^2\sqrt{1-a^2x^2}}{3a^2c} - \frac{\int \frac{18ac+22a^2cx}{\sqrt{1-a^2x^2}}\,dx}{6a^4c^2} \\
&= \frac{(1+ax)^2}{a^4c\sqrt{1-a^2x^2}} + \frac{11\sqrt{1-a^2x^2}}{3a^4c} + \frac{x\sqrt{1-a^2x^2}}{a^3c} + \frac{x^2\sqrt{1-a^2x^2}}{3a^2c} - \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3c} \\
&= \frac{(1+ax)^2}{a^4c\sqrt{1-a^2x^2}} + \frac{11\sqrt{1-a^2x^2}}{3a^4c} + \frac{x\sqrt{1-a^2x^2}}{a^3c} + \frac{x^2\sqrt{1-a^2x^2}}{3a^2c} - \frac{3\sin^{-1}(ax)}{a^4c}
\end{aligned}
$$

Mathematica [A] time = 0.0509299, size = 72, normalized size = 0.63

$$
\frac{18\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-\frac{\sqrt{ax+1}\left(a^3x^3+2a^2x^2+5ax-14\right)}{\sqrt{1-ax}}}{3a^4c}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a*c*x),x]
```

```
[Out] (-((Sqrt[1 + a*x]*(-14 + 5*a*x + 2*a^2*x^2 + a^3*x^3))/Sqrt[1 - a*x]) + 18*
ArcSin[Sqrt[1 - a*x]/Sqrt[2]])/(3*a^4*c)
```

Maple [A] time = 0.042, size = 142, normalized size = 1.3

$$
\frac{x^2}{3\,a^2c}\sqrt{-a^2x^2+1}+\frac{8}{3\,a^4c}\sqrt{-a^2x^2+1}+\frac{x}{a^3c}\sqrt{-a^2x^2+1}-3\,\frac{1}{a^3c\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)-2\,\frac{1}{ca^5}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c),x)
```

```
[Out] 1/3*x^2*(-a^2*x^2+1)^(1/2)/a^2/c+8/3*(-a^2*x^2+1)^(1/2)/a^4/c+x*(-a^2*x^2+1
)^(1/2)/a^3/c-3/c/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-
2/c/a^5/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5502, size = 198, normalized size = 1.74

$$\frac{14\,ax+18\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(a^3x^3+2\,a^2x^2+5\,ax-14\right)\sqrt{-a^2x^2+1}-14}{3\left(a^5cx-a^4c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/3*(14*a*x + 18*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a^3*x^
3 + 2*a^2*x^2 + 5*a*x - 14)*sqrt(-a^2*x^2 + 1) - 14)/(a^5*c*x - a^4*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{x^3}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int\frac{ax^4}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a*c*x+c),x)
```

```
[Out] -(Integral(x**3/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Int
egral(a*x**4/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

Giac [A] time = 1.3224, size = 136, normalized size = 1.19

$$\frac{1}{3}\sqrt{-a^2x^2+1}\left(x\left(\frac{x}{a^2c}+\frac{3}{a^3c}\right)+\frac{8}{a^4c}\right)-\frac{3\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{a^3c|a|}+\frac{4}{a^3c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(-a^2*x^2 + 1)*(x*(x/(a^2*c) + 3/(a^3*c)) + 8/(a^4*c)) - 3*arcsin(a
*x)*sgn(a)/(a^3*c*abs(a)) + 4/(a^3*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*
x) - 1)*abs(a))
```

3.329 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{c-acx}\,dx$

Optimal. Leaf size=72

$$\frac{(ax+1)^2}{a^3c\sqrt{1-a^2x^2}}+\frac{(ax+6)\sqrt{1-a^2x^2}}{2a^3c}-\frac{5\sin^{-1}(ax)}{2a^3c}$$

```
[Out] (1 + a*x)^2/(a^3*c*Sqrt[1 - a^2*x^2]) + ((6 + a*x)*Sqrt[1 - a^2*x^2])/(2*a^
3*c) - (5*ArcSin[a*x])/(2*a^3*c)
```

Rubi [A] time = 0.200876, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6128, 852, 1635, 780, 216}

$$\frac{(ax+1)^2}{a^3c\sqrt{1-a^2x^2}}+\frac{(ax+6)\sqrt{1-a^2x^2}}{2a^3c}-\frac{5\sin^{-1}(ax)}{2a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a*c*x),x]
```

```
[Out] (1 + a*x)^2/(a^3*c*Sqrt[1 - a^2*x^2]) + ((6 + a*x)*Sqrt[1 - a^2*x^2])/(2*a^
3*c) - (5*ArcSin[a*x])/(2*a^3*c)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
```

```
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^2}{c-acx}\,dx &= c\int \frac{x^2\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx\\ &= \frac{\int \frac{x^2(c+acx)^2}{(1-a^2x^2)^{3/2}}\,dx}{c^3}\\ &= \frac{(1+ax)^2}{a^3c\sqrt{1-a^2x^2}}-\frac{\int \frac{\left(\frac{2}{a^2}+\frac{x}{a}\right)(c+acx)}{\sqrt{1-a^2x^2}}\,dx}{c^2}\\ &= \frac{(1+ax)^2}{a^3c\sqrt{1-a^2x^2}}+\frac{(6+ax)\sqrt{1-a^2x^2}}{2a^3c}-\frac{5\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^2c}\\ &= \frac{(1+ax)^2}{a^3c\sqrt{1-a^2x^2}}+\frac{(6+ax)\sqrt{1-a^2x^2}}{2a^3c}-\frac{5\sin^{-1}(ax)}{2a^3c}\end{aligned}$$

Mathematica [A] time = 0.0437367, size = 64, normalized size = 0.89

$$\frac{10\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-\frac{\sqrt{ax+1}\left(a^2x^2+3ax-8\right)}{\sqrt{1-ax}}}{2a^3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a*c*x),x]
```

```
[Out] (-((Sqrt[1 + a*x]*(-8 + 3*a*x + a^2*x^2))/Sqrt[1 - a*x]) + 10*ArcSin[Sqrt[1
 - a*x]/Sqrt[2]])/(2*a^3*c)
```

Maple [A] time = 0.044, size = 120, normalized size = 1.7

$$\frac{x}{2\,a^2c}\sqrt{-a^2x^2+1}-\frac{5}{2\,a^2c}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+2\,\frac{\sqrt{-a^2x^2+1}}{a^3c}-2\,\frac{1}{ca^4}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c),x)
```

```
[Out] 1/2/c*x/a^2*(-a^2*x^2+1)^(1/2)-5/2/c/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(
-a^2*x^2+1)^(1/2))+2*(-a^2*x^2+1)^(1/2)/a^3/c-2/c/a^4/(x-1/a)*(-a^2*(x-1/a)
^2-2*a*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57929, size = 178, normalized size = 2.47

$$\frac{8\,ax + 10\,(ax - 1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(a^2x^2 + 3\,ax - 8\right)\sqrt{-a^2x^2 + 1} - 8}{2\,\left(a^4cx - a^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(8*a*x + 10*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a^2*x^2
 + 3*a*x - 8)*sqrt(-a^2*x^2 + 1) - 8)/(a^4*c*x - a^3*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^2}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx + \int \frac{ax^3}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a*c*x+c),x)
```

```
[Out] -(Integral(x**2/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Int
egral(a*x**3/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

Giac [A] time = 1.18892, size = 122, normalized size = 1.69

$$\frac{1}{2}\,\sqrt{-a^2x^2 + 1}\left(\frac{x}{a^2c} + \frac{4}{a^3c}\right) - \frac{5\,\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{2\,a^2c|a|} + \frac{4}{a^2c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*(x/(a^2*c) + 4/(a^3*c)) - 5/2*arcsin(a*x)*sgn(a)/(a^
2*c*abs(a)) + 4/(a^2*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)
)
```

3.330 $\int \frac{e^{\tanh^{-1}(ax)}x}{c-acx}\,dx$

Optimal. Leaf size=64

$$\frac{\left(1-a^2x^2\right)^{3/2}}{a^2c(1-ax)^2}+\frac{2\sqrt{1-a^2x^2}}{a^2c}-\frac{2\sin^{-1}(ax)}{a^2c}$$

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a^2*c) + (1 - a^2*x^2)^(3/2)/(a^2*c*(1 - a*x)^2) - (
2*ArcSin[a*x])/(a^2*c)
```

Rubi [A] time = 0.0782248, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6128, 793, 665, 216}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{a^2c(1-ax)^2}+\frac{2\sqrt{1-a^2x^2}}{a^2c}-\frac{2\sin^{-1}(ax)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a*c*x),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a^2*c) + (1 - a^2*x^2)^(3/2)/(a^2*c*(1 - a*x)^2) - (
2*ArcSin[a*x])/(a^2*c)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x}{c-acx}\,dx = c\int \frac{x\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}}{a^2c(1-ax)^2} - \frac{2\int \frac{\sqrt{1-a^2x^2}}{c-acx}\,dx}{a}$$
$$= \frac{2\sqrt{1-a^2x^2}}{a^2c} + \frac{\left(1-a^2x^2\right)^{3/2}}{a^2c(1-ax)^2} - \frac{2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{ac}$$
$$= \frac{2\sqrt{1-a^2x^2}}{a^2c} + \frac{\left(1-a^2x^2\right)^{3/2}}{a^2c(1-ax)^2} - \frac{2\sin^{-1}(ax)}{a^2c}$$

Mathematica [A] time = 0.0350603, size = 53, normalized size = 0.83

$$\frac{\frac{\sqrt{ax+1}(3-ax)}{\sqrt{1-ax}}+4\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{a^2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a*c*x),x]
```

```
[Out] (((3 - a*x)*Sqrt[1 + a*x])/Sqrt[1 - a*x] + 4*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])
/(a^2*c)
```

Maple [A] time = 0.037, size = 98, normalized size = 1.5

$$\frac{1}{a^2c}\sqrt{-a^2x^2+1}-2\,\frac{1}{ac\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)-2\,\frac{1}{a^3c}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c),x)
```

```
[Out] (-a^2*x^2+1)^(1/2)/a^2/c-2/c/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1
)^(1/2))-2/c/a^3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50456, size = 155, normalized size = 2.42

$$\frac{3\,ax + 4\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2+1}(ax-3) - 3}{a^3cx - a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] (3*a*x + 4*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2
 + 1)*(a*x - 3) - 3)/(a^3*c*x - a^2*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx + \int \frac{ax^2}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a*c*x+c),x)
```

```
[Out] -(Integral(x/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integr
al(a*x**2/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

Giac [A] time = 1.23721, size = 105, normalized size = 1.64

$$-\frac{2\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{ac|a|} + \frac{\sqrt{-a^2x^2+1}}{a^2c} + \frac{4}{ac\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -2*arcsin(a*x)*sgn(a)/(a*c*abs(a)) + sqrt(-a^2*x^2 + 1)/(a^2*c) + 4/(a*c*((
sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

3.331 $\int \frac{e^{\tanh^{-1}(ax)}}{c-acx} dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{1-a^2x^2}}{ac(1-ax)} - \frac{\sin^{-1}(ax)}{ac}$$

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*c*(1 - a*x)) - ArcSin[a*x]/(a*c)
```

Rubi [A] time = 0.0414235, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6127, 663, 216}

$$\frac{2\sqrt{1-a^2x^2}}{ac(1-ax)} - \frac{\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*c*(1 - a*x)) - ArcSin[a*x]/(a*c)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{c-acx}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx\\
&= \frac{2\sqrt{1-a^2x^2}}{ac(1-ax)} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= \frac{2\sqrt{1-a^2x^2}}{ac(1-ax)} - \frac{\sin^{-1}(ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.0229442, size = 46, normalized size = 1.07

$$\frac{2\left(\frac{\sqrt{ax+1}}{\sqrt{1-ax}}+\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x),x]
```

```
[Out] (2*(Sqrt[1 + a*x]/Sqrt[1 - a*x] + ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*c)
```

Maple [A] time = 0.04, size = 76, normalized size = 1.8

$$-\frac{1}{c}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-2\frac{1}{a^2c}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x)
```

```
[Out] -1/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-2/c/a^2/(x-1/a)*(
-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54886, size = 136, normalized size = 3.16

$$\frac{2\left(ax+(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\sqrt{-a^2x^2+1}-1\right)}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 2*(a*x + (a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 +
 1) - 1)/(a^2*c*x - a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c),x)
```

```
[Out] -(Integral(a*x/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Inte
gral(1/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

Giac [A] time = 1.21642, size = 72, normalized size = 1.67

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{c|a|}+\frac{4}{c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(c*abs(a)) + 4/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2
*x) - 1)*abs(a))
```

3.332 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-acx)}\,dx$

Optimal. Leaf size=45

$$\frac{2(ax+1)}{c\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

```
[Out] (2*(1 + a*x))/(c*Sqrt[1 - a^2*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c
```

Rubi [A] time = 0.167767, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 852, 1805, 12, 266, 63, 208}

$$\frac{2(ax+1)}{c\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(c - a*c*x)),x]

[Out] (2*(1 + a*x))/(c*Sqrt[1 - a^2*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x(c-acx)}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x(c-acx)^2}\,dx\\
&= \frac{\int \frac{(c+acx)^2}{x\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}\\
&= \frac{2(1+ax)}{c\sqrt{1-a^2x^2}}+\frac{\int \frac{c^2}{x\sqrt{1-a^2x^2}}\,dx}{c^3}\\
&= \frac{2(1+ax)}{c\sqrt{1-a^2x^2}}+\frac{\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c}\\
&= \frac{2(1+ax)}{c\sqrt{1-a^2x^2}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2c}\\
&= \frac{2(1+ax)}{c\sqrt{1-a^2x^2}}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2c}\\
&= \frac{2(1+ax)}{c\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}
\end{aligned}
$$

Mathematica [A] time = 0.0245177, size = 55, normalized size = 1.22

$$
\frac{-\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+2ax+2}{c\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a*c*x)),x]
```

```
[Out] (2 + 2*a*x - Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(c*Sqrt[1 - a^2*
x^2])
```

Maple [A] time = 0.04, size = 61, normalized size = 1.4

$$-\frac{1}{c}\left(\operatorname{Artanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)+2\frac{1}{a}\sqrt{-a^{2}\left(x-a^{-1}\right)^{2}-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c),x)
```

```
[Out] -1/c*(arctanh(1/(-a^2*x^2+1)^(1/2))+2/a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a)
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}(acx-c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)*x), x)
```

Fricas [A] time = 1.53129, size = 124, normalized size = 2.76

$$\frac{2\,ax+(ax-1)\log\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{x}\right)-2\sqrt{-a^{2}x^{2}+1}-2}{acx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] (2*a*x + (a*x - 1)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 2*sqrt(-a^2*x^2 + 1) -
 2)/(a*c*x - c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{ax}{ax^{2}\sqrt{-a^{2}x^{2}+1}-x\sqrt{-a^{2}x^{2}+1}}\,dx+\int\frac{1}{ax^{2}\sqrt{-a^{2}x^{2}+1}-x\sqrt{-a^{2}x^{2}+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a*c*x+c),x)
```

```
[Out] -(Integral(a*x/(a*x**2*sqrt(-a**2*x**2 + 1) - x*sqrt(-a**2*x**2 + 1)), x) +
 Integral(1/(a*x**2*sqrt(-a**2*x**2 + 1) - x*sqrt(-a**2*x**2 + 1)), x))/c
```

Giac [A] time = 1.27417, size = 108, normalized size = 2.4

$$-\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{c|a|}+\frac{4a}{c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c*abs(a))
 + 4*a/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

3.333 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-acx)}\,dx$

Optimal. Leaf size=69

$$\frac{2a(ax+1)}{c\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{cx}-\frac{2a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

```
[Out] (2*a*(1 + a*x))/(c*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(c*x) - (2*a*ArcT
anh[Sqrt[1 - a^2*x^2]])/c
```

Rubi [A] time = 0.193182, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 852, 1805, 807, 266, 63, 208}

$$\frac{2a(ax+1)}{c\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{cx}-\frac{2a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a*c*x)),x]
```

```
[Out] (2*a*(1 + a*x))/(c*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(c*x) - (2*a*ArcT
anh[Sqrt[1 - a^2*x^2]])/c
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-acx)}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x^2(c-acx)^2}\,dx\\
&= \frac{\int \frac{(c+acx)^2}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}\\
&= \frac{2a(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\int \frac{-c^2-2ac^2x}{x^2\sqrt{1-a^2x^2}}\,dx}{c^3}\\
&= \frac{2a(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{cx} + \frac{(2a)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c}\\
&= \frac{2a(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{cx} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{c}\\
&= \frac{2a(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{cx} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{ac}\\
&= \frac{2a(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{cx} - \frac{2a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}
\end{aligned}$$

Mathematica [A] time = 0.0248545, size = 68, normalized size = 0.99

$$\frac{3a^2x^2 - 2ax\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) + 2ax - 1}{cx\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a*c*x)),x]
```

```
[Out] (-1 + 2*a*x + 3*a^2*x^2 - 2*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]
])/(c*x*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.042, size = 77, normalized size = 1.1

$$-\frac{1}{c}\left(\frac{1}{x}\sqrt{-a^2x^2+1}+2\,a\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+2\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c),x)
```

```
[Out] -1/c*((-a^2*x^2+1)^(1/2)/x+2*a*arctanh(1/(-a^2*x^2+1)^(1/2))+2/(x-1/a)*(-a^
2*(x-1/a)^2-2*a*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)*x^2), x)
```

Fricas [A] time = 1.61137, size = 165, normalized size = 2.39

$$\frac{2\,a^2x^2-2\,ax+2\left(a^2x^2-ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}(3\,ax-1)}{acx^2-cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] (2*a^2*x^2 - 2*a*x + 2*(a^2*x^2 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sq
rt(-a^2*x^2 + 1)*(3*a*x - 1))/(a*c*x^2 - c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax}{ax^3\sqrt{-a^2x^2+1}-x^2\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{ax^3\sqrt{-a^2x^2+1}-x^2\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a*c*x+c),x)
```

```
[Out] -(Integral(a*x/(a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x
) + Integral(1/(a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x
))/c
```

Giac [B] time = 1.25233, size = 215, normalized size = 3.12

$$-\frac{2\,a^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{c|a|}-\frac{\left(a^2-\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)}{x}\right)a^2x}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}-\frac{\sqrt{-a^2x^2+1}|a|+a}{2\,cx|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -2*a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c*abs
(a)) - 1/2*(a^2 - 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/x)*a^2*x/((sqrt(-a^2*x^
2 + 1)*abs(a) + a)*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
- 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(c*x*abs(a))
```

3.334 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-acx)} dx$

Optimal. Leaf size=100

$$\frac{2a^2(ax+1)}{c\sqrt{1-a^2x^2}}-\frac{2a\sqrt{1-a^2x^2}}{cx}-\frac{\sqrt{1-a^2x^2}}{2cx^2}-\frac{5a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}$$

```
[Out] (2*a^2*(1 + a*x))/(c*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(2*c*x^2) - (2*
a*Sqrt[1 - a^2*x^2])/(c*x) - (5*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c)
```

Rubi [A] time = 0.258057, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 1807, 807, 266, 63, 208}

$$\frac{2a^2(ax+1)}{c\sqrt{1-a^2x^2}}-\frac{2a\sqrt{1-a^2x^2}}{cx}-\frac{\sqrt{1-a^2x^2}}{2cx^2}-\frac{5a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a*c*x)),x]
```

```
[Out] (2*a^2*(1 + a*x))/(c*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(2*c*x^2) - (2*
a*Sqrt[1 - a^2*x^2])/(c*x) - (5*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-acx)}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^3(c-acx)^2}\,dx$$
$$= \frac{\int \frac{(c+acx)^2}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$
$$= \frac{2a^2(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\int \frac{-c^2-2ac^2x-2a^2c^2x^2}{x^3\sqrt{1-a^2x^2}}\,dx}{c^3}$$
$$= \frac{2a^2(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2cx^2} + \frac{\int \frac{4ac^2+5a^2c^2x}{x^2\sqrt{1-a^2x^2}}\,dx}{2c^3}$$
$$= \frac{2a^2(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2cx^2} - \frac{2a\sqrt{1-a^2x^2}}{cx} + \frac{\left(5a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{2c}$$
$$= \frac{2a^2(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2cx^2} - \frac{2a\sqrt{1-a^2x^2}}{cx} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{4c}$$
$$= \frac{2a^2(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2cx^2} - \frac{2a\sqrt{1-a^2x^2}}{cx} - \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{2c}$$
$$= \frac{2a^2(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2cx^2} - \frac{2a\sqrt{1-a^2x^2}}{cx} - \frac{5a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}$$

Mathematica [A] time = 0.0401988, size = 83, normalized size = 0.83

$$-\frac{-8a^3x^3-5a^2x^2+5a^2x^2\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+4ax+1}{2cx^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a*c*x)),x]
```

```
[Out] -(1 + 4*a*x - 5*a^2*x^2 - 8*a^3*x^3 + 5*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcTanh[S
qrt[1 - a^2*x^2]])/(2*c*x^2*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.05, size = 99, normalized size = 1.

$$-\frac{1}{c}\left(2\frac{a\sqrt{-a^2x^2+1}}{x}+\frac{5a^2}{2}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+2a\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}+\frac{1}{2x^2}\sqrt{-a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c),x)
```

```
[Out] -1/c*(2*a*(-a^2*x^2+1)^(1/2)/x+5/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))+2*a/(x
-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1/2*(-a^2*x^2+1)^(1/2)/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)*x^3), x)
```

Fricas [A] time = 1.52659, size = 200, normalized size = 2.

$$\frac{4\,a^3x^3-4\,a^2x^2+5\left(a^3x^3-a^2x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(8\,a^2x^2-3\,ax-1\right)\sqrt{-a^2x^2+1}}{2\left(acx^3-cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^3*x^3 - 4*a^2*x^2 + 5*(a^3*x^3 - a^2*x^2)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) - (8*a^2*x^2 - 3*a*x - 1)*sqrt(-a^2*x^2 + 1))/(a*c*x^3 - c*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax}{ax^4\sqrt{-a^2x^2+1}-x^3\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{ax^4\sqrt{-a^2x^2+1}-x^3\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a*c*x+c),x)
```

```
[Out] -(Integral(a*x/(a*x**4*sqrt(-a**2*x**2 + 1) - x**3*sqrt(-a**2*x**2 + 1)), x
) + Integral(1/(a*x**4*sqrt(-a**2*x**2 + 1) - x**3*sqrt(-a**2*x**2 + 1)), x
))/c
```

Giac [B] time = 1.26876, size = 302, normalized size = 3.02

$$-\frac{\left(a^3+\frac{7\left(\sqrt{-a^2x^2+1}|a|+a\right)a}{x}-\frac{40\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{ax^2}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}-\frac{5\,a^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,c|a|}-\frac{\frac{8\left(\sqrt{-a^2x^2+1}|a|+a\right)ac|a|}{x}+\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)^2 c}{ax^2}}{8\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/8*(a^3 + 7*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 40*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^2/(a*x^2))*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c*((sqrt(
```

```
-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) - 5/2*a^3*log(1/2*abs(-2*sqr
t(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c*abs(a)) - 1/8*(8*(sqrt(-a^2*
x^2 + 1)*abs(a) + a)*a*c*abs(a)/x + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c*abs
(a)/(a*x^2))/(a^2*c^2)
```

3.335 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-acx)}\,dx$

Optimal. Leaf size=125

$$\frac{2a^3(ax+1)}{c\sqrt{1-a^2x^2}}-\frac{8a^2\sqrt{1-a^2x^2}}{3cx}-\frac{a\sqrt{1-a^2x^2}}{cx^2}-\frac{\sqrt{1-a^2x^2}}{3cx^3}-\frac{3a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

```
[Out] (2*a^3*(1 + a*x))/(c*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(3*c*x^3) - (a*
Sqrt[1 - a^2*x^2])/(c*x^2) - (8*a^2*Sqrt[1 - a^2*x^2])/(3*c*x) - (3*a^3*Arc
Tanh[Sqrt[1 - a^2*x^2]])/c
```

Rubi [A] time = 0.329187, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 1807, 807, 266, 63, 208}

$$\frac{2a^3(ax+1)}{c\sqrt{1-a^2x^2}}-\frac{8a^2\sqrt{1-a^2x^2}}{3cx}-\frac{a\sqrt{1-a^2x^2}}{cx^2}-\frac{\sqrt{1-a^2x^2}}{3cx^3}-\frac{3a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*(c - a*c*x)),x]
```

```
[Out] (2*a^3*(1 + a*x))/(c*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(3*c*x^3) - (a*
Sqrt[1 - a^2*x^2])/(c*x^2) - (8*a^2*Sqrt[1 - a^2*x^2])/(3*c*x) - (3*a^3*Arc
Tanh[Sqrt[1 - a^2*x^2]])/c
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-acx)}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x^4(c-acx)^2}\,dx \\
&= \frac{\int \frac{(c+acx)^2}{x^4\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3} \\
&= \frac{2a^3(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\int \frac{-c^2-2ac^2x-2a^2c^2x^2-2a^3c^2x^3}{x^4\sqrt{1-a^2x^2}}\,dx}{c^3} \\
&= \frac{2a^3(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3cx^3} + \frac{\int \frac{6ac^2+8a^2c^2x+6a^3c^2x^2}{x^3\sqrt{1-a^2x^2}}\,dx}{3c^3} \\
&= \frac{2a^3(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3cx^3} - \frac{a\sqrt{1-a^2x^2}}{cx^2} - \frac{\int \frac{-16a^2c^2-18a^3c^2x}{x^2\sqrt{1-a^2x^2}}\,dx}{6c^3} \\
&= \frac{2a^3(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3cx^3} - \frac{a\sqrt{1-a^2x^2}}{cx^2} - \frac{8a^2\sqrt{1-a^2x^2}}{3cx} + \frac{\left(3a^3\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c} \\
&= \frac{2a^3(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3cx^3} - \frac{a\sqrt{1-a^2x^2}}{cx^2} - \frac{8a^2\sqrt{1-a^2x^2}}{3cx} + \frac{\left(3a^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2c} \\
&= \frac{2a^3(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3cx^3} - \frac{a\sqrt{1-a^2x^2}}{cx^2} - \frac{8a^2\sqrt{1-a^2x^2}}{3cx} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{c} \\
&= \frac{2a^3(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3cx^3} - \frac{a\sqrt{1-a^2x^2}}{cx^2} - \frac{8a^2\sqrt{1-a^2x^2}}{3cx} - \frac{3a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}
\end{aligned}
$$

Mathematica [A] time = 0.0410326, size = 91, normalized size = 0.73

$$
-\frac{-14a^4x^4-9a^3x^3+7a^2x^2+9a^3x^3\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+3ax+1}{3cx^3\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*(c - a*c*x)),x]
```

```
[Out] -(1 + 3*a*x + 7*a^2*x^2 - 9*a^3*x^3 - 14*a^4*x^4 + 9*a^3*x^3*Sqrt[1 - a^2*x
^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(3*c*x^3*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.046, size = 142, normalized size = 1.1

$$
-\frac{1}{c}\left(\frac{8\,a^2}{3\,x}\sqrt{-a^2x^2+1}+2\,a^3\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+2\,a^2\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}-2\,a\left(-1/2\,\frac{\sqrt{-a}}{}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c),x)
```

```
[Out] -1/c*(8/3*a^2*(-a^2*x^2+1)^(1/2)/x+2*a^3*arctanh(1/(-a^2*x^2+1)^(1/2))+2*a^
2/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2*a*(-1/2*(-a^2*x^2+1)^(1/2)/x
^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2)))+1/3*(-a^2*x^2+1)^(1/2)/x^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)*x^4), x)
```

Fricas [A] time = 1.57952, size = 217, normalized size = 1.74

$$\frac{6\,a^4x^4-6\,a^3x^3+9\left(a^4x^4-a^3x^3\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(14\,a^3x^3-5\,a^2x^2-2\,ax-1\right)\sqrt{-a^2x^2+1}}{3\left(acx^4-cx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/3*(6*a^4*x^4 - 6*a^3*x^3 + 9*(a^4*x^4 - a^3*x^3)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) - (14*a^3*x^3 - 5*a^2*x^2 - 2*a*x - 1)*sqrt(-a^2*x^2 + 1))/(a*c*x^4
 - c*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax}{ax^5\sqrt{-a^2x^2+1}-x^4\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{ax^5\sqrt{-a^2x^2+1}-x^4\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**4/(-a*c*x+c),x)
```

```
[Out] -(Integral(a*x/(a*x**5*sqrt(-a**2*x**2 + 1) - x**4*sqrt(-a**2*x**2 + 1)), x
) + Integral(1/(a*x**5*sqrt(-a**2*x**2 + 1) - x**4*sqrt(-a**2*x**2 + 1)), x
))/c
```

Giac [B] time = 1.26543, size = 382, normalized size = 3.06

$$-\frac{\left(a^4+\frac{5\left(\sqrt{-a^2x^2+1}|a|+a\right)a^2}{x}+\frac{27\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{x^2}-\frac{129\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^2x^3}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}-\frac{3\,a^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{c|a|}-\frac{\frac{33\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4}{x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/24*(a^4 + 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2/x + 27*(sqrt(-a^2*x^2 +
1)*abs(a) + a)^2/x^2 - 129*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^2*x^3))*a^6
*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/
(a^2*x) - 1)*abs(a)) - 3*a^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a
)/(a^2*abs(x)))/(c*abs(a)) - 1/24*(33*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c
^2/x + 6*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^2/x^2 + (sqrt(-a^2*x^2 + 1
)*abs(a) + a)^3*c^2/x^3)/(a^2*c^3*abs(a))
```

3.336 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-acx)^2}\,dx$

Optimal. Leaf size=159

$$-\frac{2(ax+1)^3}{a^5c^2\sqrt{1-a^2x^2}}+\frac{(ax+1)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}}-\frac{(ax+5)^2\sqrt{1-a^2x^2}}{3a^5c^2}-\frac{(ax+5)\sqrt{1-a^2x^2}}{6a^5c^2}-\frac{5\sqrt{1-a^2x^2}}{2a^5c^2}+\frac{17\sin^{-1}(ax)}{2a^5c^2}$$

```
[Out] (1 + a*x)^3/(3*a^5*c^2*(1 - a^2*x^2)^(3/2)) - (2*(1 + a*x)^3)/(a^5*c^2*Sqrt
[1 - a^2*x^2]) - (5*Sqrt[1 - a^2*x^2])/(2*a^5*c^2) - ((5 + a*x)*Sqrt[1 - a^
2*x^2])/(6*a^5*c^2) - ((5 + a*x)^2*Sqrt[1 - a^2*x^2])/(3*a^5*c^2) + (17*Arc
Sin[a*x])/(2*a^5*c^2)
```

Rubi [A] time = 0.517402, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {6128, 852, 1635, 1625, 1654, 21, 743, 641, 216}

$$-\frac{2(ax+1)^3}{a^5c^2\sqrt{1-a^2x^2}}+\frac{(ax+1)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}}-\frac{(ax+5)^2\sqrt{1-a^2x^2}}{3a^5c^2}-\frac{(ax+5)\sqrt{1-a^2x^2}}{6a^5c^2}-\frac{5\sqrt{1-a^2x^2}}{2a^5c^2}+\frac{17\sin^{-1}(ax)}{2a^5c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^3/(3*a^5*c^2*(1 - a^2*x^2)^(3/2)) - (2*(1 + a*x)^3)/(a^5*c^2*Sqrt
[1 - a^2*x^2]) - (5*Sqrt[1 - a^2*x^2])/(2*a^5*c^2) - ((5 + a*x)*Sqrt[1 - a^
2*x^2])/(6*a^5*c^2) - ((5 + a*x)^2*Sqrt[1 - a^2*x^2])/(3*a^5*c^2) + (17*Arc
Sin[a*x])/(2*a^5*c^2)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1625

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[(d + e*x)^(m + 1)*PolynomialQuotient[Pq, d + e*x, x]*(a + c*x^2)^p,
x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[PolynomialRemaind
er[Pq, d + e*x, x], 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-acx)^2}\,dx = c\int \frac{x^4\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx$$

$$= \frac{\int \frac{x^4(c+acx)^3}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^5}$$

$$= \frac{(1+ax)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{(c+acx)^2\left(\frac{3}{a^4}+\frac{3x}{a^3}+\frac{3x^2}{a^2}+\frac{3x^3}{a}\right)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^4}$$

$$= \frac{(1+ax)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{(c+acx)^3\left(\frac{3}{a^4c}+\frac{3x^2}{a^2c}\right)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^4}$$

$$= \frac{(1+ax)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^3}{a^5c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{\left(\frac{15}{a^4c}+\frac{3x}{a^3c}\right)(c+acx)^2}{\sqrt{1-a^2x^2}}\,dx}{3c^3}$$

$$= \frac{(1+ax)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^3}{a^5c^2\sqrt{1-a^2x^2}} - \frac{(5+ax)^2\sqrt{1-a^2x^2}}{3a^5c^2} - \frac{a^4\int \frac{\left(-\frac{45}{a^4}-\frac{9x}{a^3}\right)\left(\frac{15}{a^4c}+\frac{3x}{a^3c}\right)}{\sqrt{1-a^2x^2}}\,dx}{81c}$$

$$= \frac{(1+ax)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^3}{a^5c^2\sqrt{1-a^2x^2}} - \frac{(5+ax)^2\sqrt{1-a^2x^2}}{3a^5c^2} + \frac{a^4\int \frac{\left(-\frac{45}{a^4}-\frac{9x}{a^3}\right)^2}{\sqrt{1-a^2x^2}}\,dx}{243c^2}$$

$$= \frac{(1+ax)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^3}{a^5c^2\sqrt{1-a^2x^2}} - \frac{(5+ax)\sqrt{1-a^2x^2}}{6a^5c^2} - \frac{(5+ax)^2\sqrt{1-a^2x^2}}{3a^5c^2} - \frac{a^2\int \frac{-\frac{4131}{a^6}-\frac{1215x}{a^5}}{\sqrt{1-a^2x^2}}\,d}{486c^2}$$

$$= \frac{(1+ax)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^3}{a^5c^2\sqrt{1-a^2x^2}} - \frac{5\sqrt{1-a^2x^2}}{2a^5c^2} - \frac{(5+ax)\sqrt{1-a^2x^2}}{6a^5c^2} - \frac{(5+ax)^2\sqrt{1-a^2x^2}}{3a^5c^2} + \frac{17}{}$$

$$= \frac{(1+ax)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^3}{a^5c^2\sqrt{1-a^2x^2}} - \frac{5\sqrt{1-a^2x^2}}{2a^5c^2} - \frac{(5+ax)\sqrt{1-a^2x^2}}{6a^5c^2} - \frac{(5+ax)^2\sqrt{1-a^2x^2}}{3a^5c^2} + 17$$

Mathematica [A] time = 0.094149, size = 80, normalized size = 0.5

$$-\frac{\frac{\sqrt{ax+1}\left(2a^4x^4+5a^3x^3+18a^2x^2-109ax+80\right)}{(1-ax)^{3/2}}+102\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{6a^5c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a*c*x)^2,x]
```

```
[Out] -((Sqrt[1 + a*x]*(80 - 109*a*x + 18*a^2*x^2 + 5*a^3*x^3 + 2*a^4*x^4))/(1 -
a*x)^(3/2) + 102*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])/(6*a^5*c^2)
```

Maple [A] time = 0.049, size = 187, normalized size = 1.2

$$-\frac{x^2}{3c^2a^3}\sqrt{-a^2x^2+1}-\frac{17}{3a^5c^2}\sqrt{-a^2x^2+1}-\frac{3x}{2a^4c^2}\sqrt{-a^2x^2+1}+\frac{17}{2a^4c^2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{2}{3c^2a^7}\sqrt{-a^2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^2,x)
```

```
[Out] -1/3/c^2/a^3*x^2*(-a^2*x^2+1)^(1/2)-17/3*(-a^2*x^2+1)^(1/2)/a^5/c^2-3/2/c^2
/a^4*x*(-a^2*x^2+1)^(1/2)+17/2/c^2/a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a
^2*x^2+1)^(1/2))+2/3/c^2/a^7/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+2
5/3/c^2/a^6/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63265, size = 286, normalized size = 1.8

$$-\frac{80\,a^2x^2-160\,ax+102\left(a^2x^2-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^4x^4+5\,a^3x^3+18\,a^2x^2-109\,ax+80\right)\sqrt{-a^2x^2+}}{6\left(a^7c^2x^2-2\,a^6c^2x+a^5c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/6*(80*a^2*x^2 - 160*a*x + 102*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^
2 + 1) - 1)/(a*x)) + (2*a^4*x^4 + 5*a^3*x^3 + 18*a^2*x^2 - 109*a*x + 80)*sq
rt(-a^2*x^2 + 1) + 80)/(a^7*c^2*x^2 - 2*a^6*c^2*x + a^5*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^4}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^5}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(x**4/(a**2*x**2*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1)
 + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**5/(a**2*x**2*sqrt(-a**2*x**2 +
 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^4}{\sqrt{-a^2x^2+1}(acx-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*x^4/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2), x)
```

3.337 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-acx)^2}\,dx$

Optimal. Leaf size=104

$$\frac{(ax+1)^3}{3a^4c^2\left(1-a^2x^2\right)^{3/2}}-\frac{3(ax+1)^2}{a^4c^2\sqrt{1-a^2x^2}}-\frac{(ax+12)\sqrt{1-a^2x^2}}{2a^4c^2}+\frac{11\sin^{-1}(ax)}{2a^4c^2}$$

```
[Out] (1 + a*x)^3/(3*a^4*c^2*(1 - a^2*x^2)^(3/2)) - (3*(1 + a*x)^2)/(a^4*c^2*Sqrt
[1 - a^2*x^2]) - ((12 + a*x)*Sqrt[1 - a^2*x^2])/(2*a^4*c^2) + (11*ArcSin[a*
x])/(2*a^4*c^2)
```

Rubi [A] time = 0.304152, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6128, 852, 1635, 780, 216}

$$\frac{(ax+1)^3}{3a^4c^2\left(1-a^2x^2\right)^{3/2}}-\frac{3(ax+1)^2}{a^4c^2\sqrt{1-a^2x^2}}-\frac{(ax+12)\sqrt{1-a^2x^2}}{2a^4c^2}+\frac{11\sin^{-1}(ax)}{2a^4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^3/(3*a^4*c^2*(1 - a^2*x^2)^(3/2)) - (3*(1 + a*x)^2)/(a^4*c^2*Sqrt
[1 - a^2*x^2]) - ((12 + a*x)*Sqrt[1 - a^2*x^2])/(2*a^4*c^2) + (11*ArcSin[a*
x])/(2*a^4*c^2)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-acx)^2}\,dx &= c\int \frac{x^3\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx\\
&= \frac{\int \frac{x^3(c+acx)^3}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^5}\\
&= \frac{(1+ax)^3}{3a^4c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{(c+acx)^2\left(\frac{3}{a^3}+\frac{3x}{a^2}+\frac{3x^2}{a}\right)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^4}\\
&= \frac{(1+ax)^3}{3a^4c^2\left(1-a^2x^2\right)^{3/2}} - \frac{3(1+ax)^2}{a^4c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{\left(\frac{15}{a^3}+\frac{3x}{a^2}\right)(c+acx)}{\sqrt{1-a^2x^2}}\,dx}{3c^3}\\
&= \frac{(1+ax)^3}{3a^4c^2\left(1-a^2x^2\right)^{3/2}} - \frac{3(1+ax)^2}{a^4c^2\sqrt{1-a^2x^2}} - \frac{(12+ax)\sqrt{1-a^2x^2}}{2a^4c^2} + \frac{11\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^3c^2}\\
&= \frac{(1+ax)^3}{3a^4c^2\left(1-a^2x^2\right)^{3/2}} - \frac{3(1+ax)^2}{a^4c^2\sqrt{1-a^2x^2}} - \frac{(12+ax)\sqrt{1-a^2x^2}}{2a^4c^2} + \frac{11\sin^{-1}(ax)}{2a^4c^2}
\end{aligned}$$

Mathematica [A] time = 0.0834507, size = 72, normalized size = 0.69

$$-\frac{\frac{\sqrt{ax+1}\left(3a^3x^3+12a^2x^2-71ax+52\right)}{(1-ax)^{3/2}}+66\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{6a^4c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a*c*x)^2,x]
```

```
[Out] -((Sqrt[1 + a*x]*(52 - 71*a*x + 12*a^2*x^2 + 3*a^3*x^3))/(1 - a*x)^(3/2) +
66*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])/(6*a^4*c^2)
```

Maple [A] time = 0.044, size = 164, normalized size = 1.6

$$-\frac{x}{2\,c^2a^3}\sqrt{-a^2x^2+1}+\frac{11}{2\,c^2a^3}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-3\,\frac{\sqrt{-a^2x^2+1}}{a^4c^2}+\frac{2}{3\,c^2a^6}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^2,x)
```

```
[Out] -1/2/c^2/a^3*x*(-a^2*x^2+1)^(1/2)+11/2/c^2/a^3/(a^2)^(1/2)*arctan((a^2)^(1/
2)*x/(-a^2*x^2+1)^(1/2))-3/c^2/a^4*(-a^2*x^2+1)^(1/2)+2/3/c^2/a^6/(x-1/a)^2
*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+19/3/c^2/a^5/(x-1/a)*(-a^2*(x-1/a)^2-2*
a*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60952, size = 267, normalized size = 2.57

$$-\frac{52\,a^2x^2-104\,ax+66\left(a^2x^2-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3\,a^3x^3+12\,a^2x^2-71\,ax+52\right)\sqrt{-a^2x^2+1}+52}{6\left(a^6c^2x^2-2\,a^5c^2x+a^4c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/6*(52*a^2*x^2 - 104*a*x + 66*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2
 + 1) - 1)/(a*x)) + (3*a^3*x^3 + 12*a^2*x^2 - 71*a*x + 52)*sqrt(-a^2*x^2 +
1) + 52)/(a^6*c^2*x^2 - 2*a^5*c^2*x + a^4*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^4}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(x**3/(a**2*x**2*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1)
 + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**4/(a**2*x**2*sqrt(-a**2*x**2 +
 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^3}{\sqrt{-a^2x^2+1}(acx-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*x^3/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2), x)
```

3.338 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-acx)^2}\,dx$

Optimal. Leaf size=104

$$\frac{\left(1-a^2x^2\right)^{3/2}}{a^3c^2(1-ax)^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{3a^3c^2(1-ax)^3}-\frac{6\sqrt{1-a^2x^2}}{a^3c^2(1-ax)}+\frac{3\sin^{-1}(ax)}{a^3c^2}$$

```
[Out] (-6*Sqrt[1 - a^2*x^2])/(a^3*c^2*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(3*a^3*c^2
*(1 - a*x)^3) + (1 - a^2*x^2)^(3/2)/(a^3*c^2*(1 - a*x)^2) + (3*ArcSin[a*x])
/(a^3*c^2)
```

Rubi [A] time = 0.18841, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6128, 1639, 793, 663, 216}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{a^3c^2(1-ax)^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{3a^3c^2(1-ax)^3}-\frac{6\sqrt{1-a^2x^2}}{a^3c^2(1-ax)}+\frac{3\sin^{-1}(ax)}{a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a*c*x)^2,x]
```

```
[Out] (-6*Sqrt[1 - a^2*x^2])/(a^3*c^2*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(3*a^3*c^2
*(1 - a*x)^3) + (1 - a^2*x^2)^(3/2)/(a^3*c^2*(1 - a*x)^2) + (3*ArcSin[a*x])
/(a^3*c^2)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-acx)^2}\,dx &= c\int \frac{x^2\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{a^3c^2(1-ax)^2} - \frac{\int \frac{\left(2a^2c^2-3a^3c^2x\right)\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx}{a^4c}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{3a^3c^2(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{a^3c^2(1-ax)^2} - \frac{3\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx}{a^2}\\
&= -\frac{6\sqrt{1-a^2x^2}}{a^3c^2(1-ax)} + \frac{\left(1-a^2x^2\right)^{3/2}}{3a^3c^2(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{a^3c^2(1-ax)^2} + \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2c^2}\\
&= -\frac{6\sqrt{1-a^2x^2}}{a^3c^2(1-ax)} + \frac{\left(1-a^2x^2\right)^{3/2}}{3a^3c^2(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{a^3c^2(1-ax)^2} + \frac{3\sin^{-1}(ax)}{a^3c^2}
\end{aligned}$$

Mathematica [A] time = 0.0709114, size = 64, normalized size = 0.62

$$\frac{\frac{\sqrt{ax+1}\left(-3a^2x^2+19ax-14\right)}{(1-ax)^{3/2}} - 18\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{3a^3c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a*c*x)^2,x]
```

```
[Out] ((Sqrt[1 + a*x]*(-14 + 19*a*x - 3*a^2*x^2))/(1 - a*x)^(3/2) - 18*ArcSin[Sqr
t[1 - a*x]/Sqrt[2]])/(3*a^3*c^2)
```

Maple [A] time = 0.046, size = 143, normalized size = 1.4

$$-\frac{1}{c^2a^3}\sqrt{-a^2x^2+1}+3\,\frac{1}{a^2c^2\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+\frac{2}{3\,c^2a^5}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-2}+\frac{13}{3\,a^4c^2}\sqrt{-a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^2,x)
```

```
[Out] -1/c^2/a^3*(-a^2*x^2+1)^(1/2)+3/c^2/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-
a^2*x^2+1)^(1/2))+2/3/c^2/a^5/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+
```

```
13/3/c^2/a^4/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61741, size = 248, normalized size = 2.38

$$-\frac{14\,a^2x^2-28\,ax+18\left(a^2x^2-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3\,a^2x^2-19\,ax+14\right)\sqrt{-a^2x^2+1}+14}{3\left(a^5c^2x^2-2\,a^4c^2x+a^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/3*(14*a^2*x^2 - 28*a*x + 18*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2
+ 1) - 1)/(a*x)) + (3*a^2*x^2 - 19*a*x + 14)*sqrt(-a^2*x^2 + 1) + 14)/(a^5*
c^2*x^2 - 2*a^4*c^2*x + a^3*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^3}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(x**2/(a**2*x**2*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1)
 + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**3/(a**2*x**2*sqrt(-a**2*x**2 +
 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^2}{\sqrt{-a^2x^2+1}(acx-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*x^2/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2), x)
```

3.339 $\int \frac{e^{\tanh^{-1}(ax)}x}{(c-acx)^2}\,dx$

Optimal. Leaf size=74

$$\frac{\left(1-a^2x^2\right)^{3/2}}{3a^2c^2(1-ax)^3}-\frac{2\sqrt{1-a^2x^2}}{a^2c^2(1-ax)}+\frac{\sin^{-1}(ax)}{a^2c^2}$$

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a^2*c^2*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(3*a^2*c^2
*(1 - a*x)^3) + ArcSin[a*x]/(a^2*c^2)
```

Rubi [A] time = 0.0803656, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6128, 793, 663, 216}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{3a^2c^2(1-ax)^3}-\frac{2\sqrt{1-a^2x^2}}{a^2c^2(1-ax)}+\frac{\sin^{-1}(ax)}{a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a*c*x)^2,x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a^2*c^2*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(3*a^2*c^2
*(1 - a*x)^3) + ArcSin[a*x]/(a^2*c^2)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x}{(c-acx)^2}\,dx &= c\int \frac{x\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{3a^2c^2(1-ax)^3}-\frac{\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx}{a}\\
&= -\frac{2\sqrt{1-a^2x^2}}{a^2c^2(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{3a^2c^2(1-ax)^3}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{ac^2}\\
&= -\frac{2\sqrt{1-a^2x^2}}{a^2c^2(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{3a^2c^2(1-ax)^3}+\frac{\sin^{-1}(ax)}{a^2c^2}
\end{aligned}
$$

Mathematica [C] time = 0.0657945, size = 57, normalized size = 0.77

$$
-\frac{(ax+1)^{3/2}-4\sqrt{2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1-ax)\right)}{3a^2c^2(1-ax)^{3/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a*c*x)^2,x]
```

```
[Out] -((1 + a*x)^(3/2) - 4*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - a*x)
/2])/(3*a^2*c^2*(1 - a*x)^(3/2))
```

Maple [A] time = 0.042, size = 122, normalized size = 1.7

$$
\frac{1}{ac^2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{2}{3\,a^4c^2}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-2}+\frac{7}{3\,c^2a^3}\sqrt{-a^2\left(x-a^{-1}\right)^2-
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^2,x)
```

```
[Out] 1/c^2/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+2/3/c^2/a^4/(x
-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+7/3/c^2/a^3/(x-1/a)*(-a^2*(x-1/a
)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57973, size = 225, normalized size = 3.04

$$-\frac{5\,a^{2}x^{2}-10\,ax+6\left(a^{2}x^{2}-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)-\sqrt{-a^{2}x^{2}+1}(7\,ax-5)+5}{3\left(a^{4}c^{2}x^{2}-2\,a^{3}c^{2}x+a^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(5*a^2*x^2 - 10*a*x + 6*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2 +
1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*(7*a*x - 5) + 5)/(a^4*c^2*x^2 - 2*a^3*c
^2*x + a^2*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}-2ax\sqrt{-a^{2}x^{2}+1}+\sqrt{-a^{2}x^{2}+1}}\,dx+\int \frac{ax^{2}}{a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}-2ax\sqrt{-a^{2}x^{2}+1}+\sqrt{-a^{2}x^{2}+1}}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(x/(a**2*x**2*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) +
sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**2/(a**2*x**2*sqrt(-a**2*x**2 + 1)
 - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x}{\sqrt{-a^{2}x^{2}+1}(acx-c)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*x/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2), x)
```

3.340 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^2} dx$

Optimal. Leaf size=32

$$\frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3}$$

```
[Out] (1 - a^2*x^2)^(3/2)/(3*a*c^2*(1 - a*x)^3)
```

Rubi [A] time = 0.033848, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6127, 651}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^2,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(3*a*c^2*(1 - a*x)^3)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^2} dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3} dx\\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3}\end{aligned}$$

Mathematica [A] time = 0.0091477, size = 29, normalized size = 0.91

$$\frac{(ax+1)^{3/2}}{3ac^2(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^(3/2)/(3*a*c^2*(1 - a*x)^(3/2))
```

Maple [A] time = 0.03, size = 35, normalized size = 1.1

$$-\frac{(ax+1)^2}{(3\,ax-3)\,c^2a}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x)
```

```
[Out] -1/3*(a*x+1)^2/(a*x-1)/c^2/(-a^2*x^2+1)^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.60685, size = 127, normalized size = 3.97

$$\frac{a^2x^2-2\,ax+\sqrt{-a^2x^2+1}(ax+1)+1}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*x^2 - 2*a*x + sqrt(-a^2*x^2 + 1)*(a*x + 1) + 1)/(a^3*c^2*x^2 - 2*a
^2*c^2*x + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(a*x/(a**2*x**2*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1)
+ sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**2*x**2*sqrt(-a**2*x**2 + 1) -
2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2), x)
```

3.341 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-acx)^2}\,dx$

Optimal. Leaf size=74

$$\frac{4(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{5ax+3}{3c^2\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

```
[Out] (4*(1 + a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) + (3 + 5*a*x)/(3*c^2*Sqrt[1 - a^2
*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c^2
```

Rubi [A] time = 0.199597, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 823, 12, 266, 63, 208}

$$\frac{4(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{5ax+3}{3c^2\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(c - a*c*x)^2),x]
```

```
[Out] (4*(1 + a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) + (3 + 5*a*x)/(3*c^2*Sqrt[1 - a^2
*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c^2
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x(c-acx)^2}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x(c-acx)^3}\,dx$$

$$= \frac{\int \frac{(c+acx)^3}{x(1-a^2x^2)^{5/2}}\,dx}{c^5}$$

$$= \frac{4(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{-3c^3-5ac^3x}{x(1-a^2x^2)^{3/2}}\,dx}{3c^5}$$

$$= \frac{4(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{3+5ax}{3c^2\sqrt{1-a^2x^2}} - \frac{\int -\frac{3a^2c^3}{x\sqrt{1-a^2x^2}}\,dx}{3a^2c^5}$$

$$= \frac{4(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{3+5ax}{3c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c^2}$$

$$= \frac{4(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{3+5ax}{3c^2\sqrt{1-a^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{2c^2}$$

$$= \frac{4(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{3+5ax}{3c^2\sqrt{1-a^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2c^2}$$

$$= \frac{4(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{3+5ax}{3c^2\sqrt{1-a^2x^2}} - \frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

Mathematica [A] time = 0.0339987, size = 78, normalized size = 1.05

$$\frac{5a^2x^2-3(ax-1)\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-2ax-7}{3c^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a*c*x)^2),x]
```

```
[Out] (-7 - 2*a*x + 5*a^2*x^2 - 3*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a
^2*x^2]])/(3*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.04, size = 147, normalized size = 2.

$$\frac{1}{c^2}\left(-\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+2\frac{1}{a}\left(1/3\frac{1}{a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-2}-1/3\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^2,x)
```

```
[Out] 1/c^2*(-arctanh(1/(-a^2*x^2+1)^(1/2))+2/a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-
2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))-1/a/(x-1
/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2*x), x)
```

Fricas [A] time = 1.57164, size = 207, normalized size = 2.8

$$\frac{7\,a^2x^2-14\,ax+3\left(a^2x^2-2\,ax+1\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}(5\,ax-7)+7}{3\left(a^2c^2x^2-2\,ac^2x+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(7*a^2*x^2 - 14*a*x + 3*(a^2*x^2 - 2*a*x + 1)*log((sqrt(-a^2*x^2 + 1) -
 1)/x) - sqrt(-a^2*x^2 + 1)*(5*a*x - 7) + 7)/(a^2*c^2*x^2 - 2*a*c^2*x + c^2
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{a^2x^3\sqrt{-a^2x^2+1}-2ax^2\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^2x^3\sqrt{-a^2x^2+1}-2ax^2\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(a*x/(a**2*x**3*sqrt(-a**2*x**2 + 1) - 2*a*x**2*sqrt(-a**2*x**2 +
1) + x*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**2*x**3*sqrt(-a**2*x**2 +
1) - 2*a*x**2*sqrt(-a**2*x**2 + 1) + x*sqrt(-a**2*x**2 + 1)), x))/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2*x), x)
```

3.342 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-acx)^2}\,dx$

Optimal. Leaf size=99

$$\frac{4a(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{\sqrt{1-a^2x^2}}{c^2x}+\frac{a(11ax+9)}{3c^2\sqrt{1-a^2x^2}}-\frac{3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

```
[Out] (4*a*(1 + a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) + (a*(9 + 11*a*x))/(3*c^2*Sqrt[
1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(c^2*x) - (3*a*ArcTanh[Sqrt[1 - a^2*x^2]]
)/c^2
```

Rubi [A] time = 0.266828, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 852, 1805, 807, 266, 63, 208}

$$\frac{4a(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{\sqrt{1-a^2x^2}}{c^2x}+\frac{a(11ax+9)}{3c^2\sqrt{1-a^2x^2}}-\frac{3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a*c*x)^2),x]
```


```
[Out] (4*a*(1 + a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) + (a*(9 + 11*a*x))/(3*c^2*Sqrt[
1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(c^2*x) - (3*a*ArcTanh[Sqrt[1 - a^2*x^2]]
)/c^2
```


Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-acx)^2}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x^2(c-acx)^3}\,dx\\
&= \frac{\int \frac{(c+acx)^3}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx}{c^5}\\
&= \frac{4a(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{\int \frac{-3c^3-9ac^3x-8a^2c^3x^2}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^5}\\
&= \frac{4a(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{a(9+11ax)}{3c^2\sqrt{1-a^2x^2}}+\frac{\int \frac{3c^3+9ac^3x}{x^2\sqrt{1-a^2x^2}}\,dx}{3c^5}\\
&= \frac{4a(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{a(9+11ax)}{3c^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{c^2x}+\frac{(3a)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c^2}\\
&= \frac{4a(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{a(9+11ax)}{3c^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{c^2x}+\frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2c^2}\\
&= \frac{4a(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{a(9+11ax)}{3c^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{c^2x}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{ac^2}\\
&= \frac{4a(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{a(9+11ax)}{3c^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{c^2x}-\frac{3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}
\end{aligned}
$$

Mathematica [A] time = 0.0415937, size = 91, normalized size = 0.92

$$\frac{14a^3x^3 - 5a^2x^2 - 9ax(ax - 1)\sqrt{1 - a^2x^2}\tanh^{-1}\left(\sqrt{1 - a^2x^2}\right) - 16ax + 3}{3c^2x(ax - 1)\sqrt{1 - a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a*c*x)^2),x]
```

```
[Out] (3 - 16*a*x - 5*a^2*x^2 + 14*a^3*x^3 - 9*a*x*(-1 + a*x)*Sqrt[1 - a^2*x^2]*A
rcTanh[Sqrt[1 - a^2*x^2]])/(3*c^2*x*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.045, size = 118, normalized size = 1.2

$$\frac{1}{c^2}\left(-\frac{1}{x}\sqrt{-a^2x^2+1} - 3\,a\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) + \frac{2}{3\,a}\sqrt{-a^2\left(x-a^{-1}\right)^2 - 2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-2} - \frac{11}{3}\sqrt{-a^2\left(x-a^{-1}\right)^2 -}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^2,x)
```

```
[Out] 1/c^2*(-(-a^2*x^2+1)^(1/2)/x-3*a*arctanh(1/(-a^2*x^2+1)^(1/2))+2/3/a/(x-1/a
)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-11/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-
1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2*x^2), x)
```

Fricas [A] time = 1.60348, size = 252, normalized size = 2.55

$$\frac{13\,a^3x^3 - 26\,a^2x^2 + 13\,ax + 9\left(a^3x^3 - 2\,a^2x^2 + ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \left(14\,a^2x^2 - 19\,ax + 3\right)\sqrt{-a^2x^2+1}}{3\left(a^2c^2x^3 - 2\,ac^2x^2 + c^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] 1/3*(13*a^3*x^3 - 26*a^2*x^2 + 13*a*x + 9*(a^3*x^3 - 2*a^2*x^2 + a*x)*log((
sqrt(-a^2*x^2 + 1) - 1)/x) - (14*a^2*x^2 - 19*a*x + 3)*sqrt(-a^2*x^2 + 1))/
```

```
(a^2*c^2*x^3 - 2*a*c^2*x^2 + c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{a^2x^4\sqrt{-a^2x^2+1}-2ax^3\sqrt{-a^2x^2+1}+x^2\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^2x^4\sqrt{-a^2x^2+1}-2ax^3\sqrt{-a^2x^2+1}+x^2\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(a*x/(a**2*x**4*sqrt(-a**2*x**2 + 1) - 2*a*x**3*sqrt(-a**2*x**2 +
1) + x**2*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**2*x**4*sqrt(-a**2*x**2
 + 1) - 2*a*x**3*sqrt(-a**2*x**2 + 1) + x**2*sqrt(-a**2*x**2 + 1)), x))/c**
2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2*x^2), x)
```

3.343 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-acx)^2}\,dx$

Optimal. Leaf size=132

$$\frac{a^2(17ax+15)}{3c^2\sqrt{1-a^2x^2}}+\frac{4a^2(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{3a\sqrt{1-a^2x^2}}{c^2x}-\frac{\sqrt{1-a^2x^2}}{2c^2x^2}-\frac{11a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

```
[Out] (4*a^2*(1 + a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) + (a^2*(15 + 17*a*x))/(3*c^2*
Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(2*c^2*x^2) - (3*a*Sqrt[1 - a^2*x^2]
)/(c^2*x) - (11*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c^2)
```

Rubi [A] time = 0.333784, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 1807, 807, 266, 63, 208}

$$\frac{a^2(17ax+15)}{3c^2\sqrt{1-a^2x^2}}+\frac{4a^2(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{3a\sqrt{1-a^2x^2}}{c^2x}-\frac{\sqrt{1-a^2x^2}}{2c^2x^2}-\frac{11a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a*c*x)^2),x]
```


```
[Out] (4*a^2*(1 + a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) + (a^2*(15 + 17*a*x))/(3*c^2*
Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(2*c^2*x^2) - (3*a*Sqrt[1 - a^2*x^2]
)/(c^2*x) - (11*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c^2)
```


Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-acx)^2}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^3(c-acx)^3}\,dx$$

$$= \frac{\int \frac{(c+acx)^3}{x^3(1-a^2x^2)^{5/2}}\,dx}{c^5}$$

$$= \frac{4a^2(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{-3c^3-9ac^3x-12a^2c^3x^2-8a^3c^3x^3}{x^3(1-a^2x^2)^{3/2}}\,dx}{3c^5}$$

$$= \frac{4a^2(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(15+17ax)}{3c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{3c^3+9ac^3x+15a^2c^3x^2}{x^3\sqrt{1-a^2x^2}}\,dx}{3c^5}$$

$$= \frac{4a^2(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(15+17ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{\int \frac{-18ac^3-33a^2c^3x}{x^2\sqrt{1-a^2x^2}}\,dx}{6c^5}$$

$$= \frac{4a^2(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(15+17ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{3a\sqrt{1-a^2x^2}}{c^2x} + \frac{\left(11a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{2c^2}$$

$$= \frac{4a^2(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(15+17ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{3a\sqrt{1-a^2x^2}}{c^2x} + \frac{\left(11a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{4c^2}$$

$$= \frac{4a^2(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(15+17ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{3a\sqrt{1-a^2x^2}}{c^2x} - \frac{11\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{2c^2}$$

$$= \frac{4a^2(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(15+17ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{3a\sqrt{1-a^2x^2}}{c^2x} - \frac{11a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

Mathematica [A] time = 0.0462549, size = 103, normalized size = 0.78

$$\frac{52a^4x^4-19a^3x^3-59a^2x^2-33a^2x^2(ax-1)\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+15ax+3}{6c^2x^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a*c*x)^2),x]
```

```
[Out] (3 + 15*a*x - 59*a^2*x^2 - 19*a^3*x^3 + 52*a^4*x^4 - 33*a^2*x^2*(-1 + a*x)*
Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(6*c^2*x^2*(-1 + a*x)*Sqrt[1
- a^2*x^2])
```

Maple [A] time = 0.055, size = 181, normalized size = 1.4

$$\frac{1}{c^2}\left(-3\,\frac{a\sqrt{-a^2x^2+1}}{x} - \frac{11\,a^2}{2}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) + 2\,a\left(1/3\,\frac{1}{a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-2} - 1/3\sqrt{-a^2\left(\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^2,x)
```

```
[Out] 1/c^2*(-3*a*(-a^2*x^2+1)^(1/2)/x-11/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))+2*a
*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1
/a)^2-2*a*(x-1/a))^(1/2))-5*a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/
2*(-a^2*x^2+1)^(1/2)/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2*x^3), x)
```

Fricas [A] time = 1.58376, size = 285, normalized size = 2.16

$$\frac{38\,a^4x^4 - 76\,a^3x^3 + 38\,a^2x^2 + 33\left(a^4x^4 - 2\,a^3x^3 + a^2x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \left(52\,a^3x^3 - 71\,a^2x^2 + 12\,ax + 3\right)\sqrt{-a^2x^2+1}}{6\left(a^2c^2x^4 - 2\,ac^2x^3 + c^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] 1/6*(38*a^4*x^4 - 76*a^3*x^3 + 38*a^2*x^2 + 33*(a^4*x^4 - 2*a^3*x^3 + a^2*x
^2)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (52*a^3*x^3 - 71*a^2*x^2 + 12*a*x + 3
)*sqrt(-a^2*x^2 + 1))/(a^2*c^2*x^4 - 2*a*c^2*x^3 + c^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{a^2x^5\sqrt{-a^2x^2+1}-2ax^4\sqrt{-a^2x^2+1}+x^3\sqrt{-a^2x^2+1}}\,dx + \int \frac{1}{a^2x^5\sqrt{-a^2x^2+1}-2ax^4\sqrt{-a^2x^2+1}+x^3\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(a*x/(a**2*x**5*sqrt(-a**2*x**2 + 1) - 2*a*x**4*sqrt(-a**2*x**2 +
1) + x**3*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**2*x**5*sqrt(-a**2*x**2
 + 1) - 2*a*x**4*sqrt(-a**2*x**2 + 1) + x**3*sqrt(-a**2*x**2 + 1)), x))/c**
2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2*x^3), x)
```

3.344 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-acx)^2}\,dx$

Optimal. Leaf size=161

$$\frac{a^3(23ax+21)}{3c^2\sqrt{1-a^2x^2}}+\frac{4a^3(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{17a^2\sqrt{1-a^2x^2}}{3c^2x}-\frac{3a\sqrt{1-a^2x^2}}{2c^2x^2}-\frac{\sqrt{1-a^2x^2}}{3c^2x^3}-\frac{17a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

```
[Out] (4*a^3*(1 + a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) + (a^3*(21 + 23*a*x))/(3*c^2*
Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(3*c^2*x^3) - (3*a*Sqrt[1 - a^2*x^2]
)/(2*c^2*x^2) - (17*a^2*Sqrt[1 - a^2*x^2])/(3*c^2*x) - (17*a^3*ArcTanh[Sqrt
[1 - a^2*x^2]])/(2*c^2)
```

Rubi [A] time = 0.422952, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 1807, 807, 266, 63, 208}

$$\frac{a^3(23ax+21)}{3c^2\sqrt{1-a^2x^2}}+\frac{4a^3(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{17a^2\sqrt{1-a^2x^2}}{3c^2x}-\frac{3a\sqrt{1-a^2x^2}}{2c^2x^2}-\frac{\sqrt{1-a^2x^2}}{3c^2x^3}-\frac{17a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*(c - a*c*x)^2),x]
```

```
[Out] (4*a^3*(1 + a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) + (a^3*(21 + 23*a*x))/(3*c^2*
Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(3*c^2*x^3) - (3*a*Sqrt[1 - a^2*x^2]
)/(2*c^2*x^2) - (17*a^2*Sqrt[1 - a^2*x^2])/(3*c^2*x) - (17*a^3*ArcTanh[Sqrt
[1 - a^2*x^2]])/(2*c^2)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-acx)^2}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^4(c-acx)^3}\,dx$$

$$= \frac{\int \frac{(c+acx)^3}{x^4(1-a^2x^2)^{5/2}}\,dx}{c^5}$$

$$= \frac{4a^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{-3c^3-9ac^3x-12a^2c^3x^2-12a^3c^3x^3-8a^4c^3x^4}{x^4(1-a^2x^2)^{3/2}}\,dx}{3c^5}$$

$$= \frac{4a^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(21+23ax)}{3c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{3c^3+9ac^3x+15a^2c^3x^2+21a^3c^3x^3}{x^4\sqrt{1-a^2x^2}}\,dx}{3c^5}$$

$$= \frac{4a^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(21+23ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{\int \frac{-27ac^3-51a^2c^3x-63a^3c^3x^2}{x^3\sqrt{1-a^2x^2}}\,dx}{9c^5}$$

$$= \frac{4a^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(21+23ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{3a\sqrt{1-a^2x^2}}{2c^2x^2} + \frac{\int \frac{102a^2c^3+153a^3c^3x}{x^2\sqrt{1-a^2x^2}}\,dx}{18c^5}$$

$$= \frac{4a^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(21+23ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{3a\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{17a^2\sqrt{1-a^2x^2}}{3c^2x} + \frac{\left(17a^3\right)\int \frac{}{x\sqrt{1}}}{2c^2}$$

$$= \frac{4a^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(21+23ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{3a\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{17a^2\sqrt{1-a^2x^2}}{3c^2x} + \frac{\left(17a^3\right)\mathrm{Subs}}{}$$

$$= \frac{4a^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(21+23ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{3a\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{17a^2\sqrt{1-a^2x^2}}{3c^2x} - \frac{(17a)\,\mathrm{Subst}}{}$$

$$= \frac{4a^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(21+23ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{3a\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{17a^2\sqrt{1-a^2x^2}}{3c^2x} - \frac{17a^3\tanh^{-1}}{2c}$$

Mathematica [A] time = 0.053057, size = 111, normalized size = 0.69

$$\frac{80a^5x^5-29a^4x^4-91a^3x^3+23a^2x^2-51a^3x^3(ax-1)\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+7ax+2}{6c^2x^3(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*(c - a*c*x)^2),x]
```

```
[Out] (2 + 7*a*x + 23*a^2*x^2 - 91*a^3*x^3 - 29*a^4*x^4 + 80*a^5*x^5 - 51*a^3*x^3
*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(6*c^2*x^3*(-1 +
a*x)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.048, size = 226, normalized size = 1.4

$$\frac{1}{c^2}\left(-\frac{17\,a^2}{3\,x}\sqrt{-a^2x^2+1}-7\,a^3\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+2\,a^2\left(1/3\,\frac{1}{a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-2}-1/3\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c)^2,x)
```

```
[Out] 1/c^2*(-17/3*a^2*(-a^2*x^2+1)^(1/2)/x-7*a^3*arctanh(1/(-a^2*x^2+1)^(1/2))+2
*a^2*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*
(x-1/a)^2-2*a*(x-1/a))^(1/2))-7*a^2/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1
/2)+3*a*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2)))
-1/3*(-a^2*x^2+1)^(1/2)/x^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2*x^4), x)
```

Fricas [A] time = 1.62905, size = 304, normalized size = 1.89

$$\frac{50\,a^5x^5-100\,a^4x^4+50\,a^3x^3+51\left(a^5x^5-2\,a^4x^4+a^3x^3\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(80\,a^4x^4-109\,a^3x^3+18\,a^2x^2+5\,ax+2\right)\sqrt{-}}{6\left(a^2c^2x^5-2\,ac^2x^4+c^2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] 1/6*(50*a^5*x^5 - 100*a^4*x^4 + 50*a^3*x^3 + 51*(a^5*x^5 - 2*a^4*x^4 + a^3*
x^3)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (80*a^4*x^4 - 109*a^3*x^3 + 18*a^2*x
^2 + 5*a*x + 2)*sqrt(-a^2*x^2 + 1))/(a^2*c^2*x^5 - 2*a*c^2*x^4 + c^2*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{a^2x^6\sqrt{-a^2x^2+1}-2ax^5\sqrt{-a^2x^2+1}+x^4\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^2x^6\sqrt{-a^2x^2+1}-2ax^5\sqrt{-a^2x^2+1}+x^4\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**4/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(a*x/(a**2*x**6*sqrt(-a**2*x**2 + 1) - 2*a*x**5*sqrt(-a**2*x**2 +
1) + x**4*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**2*x**6*sqrt(-a**2*x**2
 + 1) - 2*a*x**5*sqrt(-a**2*x**2 + 1) + x**4*sqrt(-a**2*x**2 + 1)), x))/c**
2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2*x^4), x)
```

3.345 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-acx)^3}\,dx$

Optimal. Leaf size=135

$$\frac{(ax+1)^4}{5a^5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{19(ax+1)^3}{15a^5c^3\left(1-a^2x^2\right)^{3/2}}+\frac{6(ax+1)^2}{a^5c^3\sqrt{1-a^2x^2}}+\frac{(ax+20)\sqrt{1-a^2x^2}}{2a^5c^3}-\frac{19\sin^{-1}(ax)}{2a^5c^3}$$

```
[Out] (1 + a*x)^4/(5*a^5*c^3*(1 - a^2*x^2)^(5/2)) - (19*(1 + a*x)^3)/(15*a^5*c^3*
(1 - a^2*x^2)^(3/2)) + (6*(1 + a*x)^2)/(a^5*c^3*Sqrt[1 - a^2*x^2]) + ((20 +
 a*x)*Sqrt[1 - a^2*x^2])/(2*a^5*c^3) - (19*ArcSin[a*x])/(2*a^5*c^3)
```

Rubi [A] time = 0.415101, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6128, 852, 1635, 780, 216}

$$\frac{(ax+1)^4}{5a^5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{19(ax+1)^3}{15a^5c^3\left(1-a^2x^2\right)^{3/2}}+\frac{6(ax+1)^2}{a^5c^3\sqrt{1-a^2x^2}}+\frac{(ax+20)\sqrt{1-a^2x^2}}{2a^5c^3}-\frac{19\sin^{-1}(ax)}{2a^5c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a*c*x)^3,x]
```

```
[Out] (1 + a*x)^4/(5*a^5*c^3*(1 - a^2*x^2)^(5/2)) - (19*(1 + a*x)^3)/(15*a^5*c^3*
(1 - a^2*x^2)^(3/2)) + (6*(1 + a*x)^2)/(a^5*c^3*Sqrt[1 - a^2*x^2]) + ((20 +
 a*x)*Sqrt[1 - a^2*x^2])/(2*a^5*c^3) - (19*ArcSin[a*x])/(2*a^5*c^3)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-acx)^3}\,dx &= c\int \frac{x^4\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx\\
&= \frac{\int \frac{x^4(c+acx)^4}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^7}\\
&= \frac{(1+ax)^4}{5a^5c^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{(c+acx)^3\left(\frac{4}{a^4}+\frac{5x}{a^3}+\frac{5x^2}{a^2}+\frac{5x^3}{a}\right)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^6}\\
&= \frac{(1+ax)^4}{5a^5c^3\left(1-a^2x^2\right)^{5/2}} - \frac{19(1+ax)^3}{15a^5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{(c+acx)^2\left(\frac{45}{a^4}+\frac{30x}{a^3}+\frac{15x^2}{a^2}\right)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^5}\\
&= \frac{(1+ax)^4}{5a^5c^3\left(1-a^2x^2\right)^{5/2}} - \frac{19(1+ax)^3}{15a^5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{6(1+ax)^2}{a^5c^3\sqrt{1-a^2x^2}} - \frac{\int \frac{\left(\frac{135}{a^4}+\frac{15x}{a^3}\right)(c+acx)}{\sqrt{1-a^2x^2}}\,dx}{15c^4}\\
&= \frac{(1+ax)^4}{5a^5c^3\left(1-a^2x^2\right)^{5/2}} - \frac{19(1+ax)^3}{15a^5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{6(1+ax)^2}{a^5c^3\sqrt{1-a^2x^2}} + \frac{(20+ax)\sqrt{1-a^2x^2}}{2a^5c^3} - \frac{19\int \frac{1}{\sqrt{1-a^2x}}}{2a^4c^3}\\
&= \frac{(1+ax)^4}{5a^5c^3\left(1-a^2x^2\right)^{5/2}} - \frac{19(1+ax)^3}{15a^5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{6(1+ax)^2}{a^5c^3\sqrt{1-a^2x^2}} + \frac{(20+ax)\sqrt{1-a^2x^2}}{2a^5c^3} - \frac{19\sin^{-1}(ax}{2a^5c^3}
\end{aligned}$$

Mathematica [C] time = 0.135883, size = 122, normalized size = 0.9

$$\frac{140\sqrt{2}(ax-1)\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1-ax)\right)+\sqrt{ax+1}\left(-15a^4x^4-75a^3x^3+433a^2x^2-639ax+308\right.}{30a^5c^3(1-ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a*c*x)^3,x]
```

```
[Out] (Sqrt[1 + a*x]*(308 - 639*a*x + 433*a^2*x^2 - 75*a^3*x^3 - 15*a^4*x^4) + 36
0*(1 - a*x)^(5/2)*ArcSin[Sqrt[1 - a*x]/Sqrt[2]] + 140*Sqrt[2]*(-1 + a*x)*Hy
pergeometric2F1[-3/2, -3/2, -1/2, (1 - a*x)/2])/(30*a^5*c^3*(1 - a*x)^(5/2)
)
```

Maple [A] time = 0.052, size = 208, normalized size = 1.5

$$\frac{x}{2\,a^4c^3}\sqrt{-a^2x^2+1}-\frac{19}{2\,a^4c^3}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+4\,\frac{\sqrt{-a^2x^2+1}}{c^3a^5}-\frac{2}{5\,c^3a^8}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^3,x)
```

```
[Out] 1/2/c^3/a^4*x*(-a^2*x^2+1)^(1/2)-19/2/c^3/a^4/(a^2)^(1/2)*arctan((a^2)^(1/2
)*x/(-a^2*x^2+1)^(1/2))+4/c^3/a^5*(-a^2*x^2+1)^(1/2)-2/5/c^3/a^8/(x-1/a)^3*
(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-41/15/c^3/a^7/(x-1/a)^2*(-a^2*(x-1/a)^2-
2*a*(x-1/a))^(1/2)-199/15/c^3/a^6/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69128, size = 355, normalized size = 2.63

$$\frac{448\,a^3x^3-1344\,a^2x^2+1344\,ax+570\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(15\,a^4x^4+75\,a^3x^3-713\,a^2x^2+1\right.}{30\left(a^8c^3x^3-3\,a^7c^3x^2+3\,a^6c^3x-a^5c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/30*(448*a^3*x^3 - 1344*a^2*x^2 + 1344*a*x + 570*(a^3*x^3 - 3*a^2*x^2 + 3*
a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (15*a^4*x^4 + 75*a^3*x^3
- 713*a^2*x^2 + 1059*a*x - 448)*sqrt(-a^2*x^2 + 1) - 448)/(a^8*c^3*x^3 - 3*
a^7*c^3*x^2 + 3*a^6*c^3*x - a^5*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^4}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^5}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(x**4/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x*
*2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral
(a*x**5/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1)
+ 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [A] time = 1.24176, size = 267, normalized size = 1.98

$$\frac{1}{2}\sqrt{-a^2x^2+1}\left(\frac{x}{a^4c^3}+\frac{8}{a^5c^3}\right)-\frac{19\arcsin(ax)\operatorname{sgn}(a)}{2a^4c^3|a|}-\frac{2\left(\frac{685\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{1025\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{615\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}\right.}{15a^4c^3\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*(x/(a^4*c^3) + 8/(a^5*c^3)) - 19/2*arcsin(a*x)*sgn(a
)/(a^4*c^3*abs(a)) - 2/15*(685*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 10
25*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^4*x^2) + 615*(sqrt(-a^2*x^2 + 1)*ab
s(a) + a)^3/(a^6*x^3) - 135*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 1
64)/(a^4*c^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

3.346 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-acx)^3}\,dx$

Optimal. Leaf size=137

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^3(1-ax)^2}-\frac{14\left(1-a^2x^2\right)^{3/2}}{15a^4c^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^4c^3(1-ax)^4}+\frac{8\sqrt{1-a^2x^2}}{a^4c^3(1-ax)}-\frac{4\sin^{-1}(ax)}{a^4c^3}$$

```
[Out] (8*Sqrt[1 - a^2*x^2])/(a^4*c^3*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(5*a^4*c^3*
(1 - a*x)^4) - (14*(1 - a^2*x^2)^(3/2))/(15*a^4*c^3*(1 - a*x)^3) - (1 - a^2
*x^2)^(3/2)/(a^4*c^3*(1 - a*x)^2) - (4*ArcSin[a*x])/(a^4*c^3)
```

Rubi [A] time = 0.332826, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 1639, 1637, 659, 651, 663, 216}

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^3(1-ax)^2}-\frac{14\left(1-a^2x^2\right)^{3/2}}{15a^4c^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^4c^3(1-ax)^4}+\frac{8\sqrt{1-a^2x^2}}{a^4c^3(1-ax)}-\frac{4\sin^{-1}(ax)}{a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a*c*x)^3,x]
```

```
[Out] (8*Sqrt[1 - a^2*x^2])/(a^4*c^3*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(5*a^4*c^3*
(1 - a*x)^4) - (14*(1 - a^2*x^2)^(3/2))/(15*a^4*c^3*(1 - a*x)^3) - (1 - a^2
*x^2)^(3/2)/(a^4*c^3*(1 - a*x)^2) - (4*ArcSin[a*x])/(a^4*c^3)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-acx)^3}\,dx &= c\int \frac{x^3\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx\\
&= -\frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^3(1-ax)^2}+\frac{\int \frac{\sqrt{1-a^2x^2}\left(2a^2c^3-5a^3c^3x+4a^4c^3x^2\right)}{(c-acx)^4}\,dx}{a^5c^2}\\
&= -\frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^3(1-ax)^2}+\frac{\int \left(\frac{a^2\sqrt{1-a^2x^2}}{c(-1+ax)^4}+\frac{3a^2\sqrt{1-a^2x^2}}{c(-1+ax)^3}+\frac{4a^2\sqrt{1-a^2x^2}}{c(-1+ax)^2}\right)dx}{a^5c^2}\\
&= -\frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^3(1-ax)^2}+\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{a^3c^3}+\frac{3\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{a^3c^3}+\frac{4\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}\,dx}{a^3c^3}\\
&= \frac{8\sqrt{1-a^2x^2}}{a^4c^3(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^4c^3(1-ax)^4}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^3(1-ax)^3}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^3(1-ax)^2}-\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{5a^3c^3}-\frac{4\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3c^3}\\
&= \frac{8\sqrt{1-a^2x^2}}{a^4c^3(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^4c^3(1-ax)^4}-\frac{14\left(1-a^2x^2\right)^{3/2}}{15a^4c^3(1-ax)^3}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^3(1-ax)^2}-\frac{4\sin^{-1}(ax)}{a^4c^3}
\end{aligned}
$$

Mathematica [A] time = 0.0724086, size = 72, normalized size = 0.53

$$
\frac{\frac{\sqrt{ax+1}\left(-15a^3x^3+149a^2x^2-222ax+94\right)}{(1-ax)^{5/2}}+120\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{15a^4c^3}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a*c*x)^3,x]
```

```
[Out] ((Sqrt[1 + a*x]*(94 - 222*a*x + 149*a^2*x^2 - 15*a^3*x^3))/(1 - a*x)^(5/2)
+ 120*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])/(15*a^4*c^3)
```

Maple [A] time = 0.049, size = 186, normalized size = 1.4

$$\frac{1}{a^4c^3}\sqrt{-a^2x^2+1}-4\frac{1}{c^3a^3\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)-\frac{2}{5c^3a^7}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}-\frac{31}{15a^6c^3}\sqrt{-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^3,x)
```

```
[Out] 1/c^3/a^4*(-a^2*x^2+1)^(1/2)-4/c^3/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a
^2*x^2+1)^(1/2))-2/5/c^3/a^7/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-3
1/15/c^3/a^6/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-104/15/c^3/a^5/(x
-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68404, size = 329, normalized size = 2.4

$$\frac{94a^3x^3-282a^2x^2+282ax+120\left(a^3x^3-3a^2x^2+3ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(15a^3x^3-149a^2x^2+222ax-94\right)\sqrt{}}{15\left(a^7c^3x^3-3a^6c^3x^2+3a^5c^3x-a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/15*(94*a^3*x^3 - 282*a^2*x^2 + 282*a*x + 120*(a^3*x^3 - 3*a^2*x^2 + 3*a*x
 - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (15*a^3*x^3 - 149*a^2*x^2 +
222*a*x - 94)*sqrt(-a^2*x^2 + 1) - 94)/(a^7*c^3*x^3 - 3*a^6*c^3*x^2 + 3*a^5
*c^3*x - a^4*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{x^3}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int\frac{ax^4}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(x**3/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x*
*2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral
(a*x**4/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1)
+ 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [A] time = 1.20401, size = 251, normalized size = 1.83

$$-\frac{4\,\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{a^{3}c^{3}|a|}+\frac{\sqrt{-a^{2}x^{2}+1}}{a^{4}c^{3}}-\frac{2\left(\frac{335\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)}{a^{2}x}-\frac{505\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)^{2}}{a^{4}x^{2}}+\frac{285\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)^{3}}{a^{6}x^{3}}-\frac{60\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)^{4}}{a^{8}x^{4}}-79\right)}{15\,a^{3}c^{3}\left(\frac{\sqrt{-a^{2}x^{2}+1}|a|+a}{a^{2}x}-1\right)^{5}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -4*arcsin(a*x)*sgn(a)/(a^3*c^3*abs(a)) + sqrt(-a^2*x^2 + 1)/(a^4*c^3) - 2/1
5*(335*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 505*(sqrt(-a^2*x^2 + 1)*ab
s(a) + a)^2/(a^4*x^2) + 285*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 6
0*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 79)/(a^3*c^3*((sqrt(-a^2*x^
2 + 1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

3.347 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-acx)^3}\,dx$

Optimal. Leaf size=107

$$-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^3c^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3c^3(1-ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^3c^3(1-ax)}-\frac{\sin^{-1}(ax)}{a^3c^3}$$

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a^3*c^3*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(5*a^3*c^3*
(1 - a*x)^4) - (3*(1 - a^2*x^2)^(3/2))/(5*a^3*c^3*(1 - a*x)^3) - ArcSin[a*x
]/(a^3*c^3)
```

Rubi [A] time = 0.220088, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 1637, 659, 651, 663, 216}

$$-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^3c^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3c^3(1-ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^3c^3(1-ax)}-\frac{\sin^{-1}(ax)}{a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a*c*x)^3,x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a^3*c^3*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(5*a^3*c^3*
(1 - a*x)^4) - (3*(1 - a^2*x^2)^(3/2))/(5*a^3*c^3*(1 - a*x)^3) - ArcSin[a*x
]/(a^3*c^3)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
```

```
0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-acx)^3}\,dx &= c\int \frac{x^2\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx \\
&= c\int \left(\frac{\sqrt{1-a^2x^2}}{a^2c^4(-1+ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^2c^4(-1+ax)^3}+\frac{\sqrt{1-a^2x^2}}{a^2c^4(-1+ax)^2}\right)dx \\
&= \frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{a^2c^3}+\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}\,dx}{a^2c^3}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{a^2c^3} \\
&= \frac{2\sqrt{1-a^2x^2}}{a^3c^3(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3c^3(1-ax)^4}-\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^3c^3(1-ax)^3}-\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{5a^2c^3}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2c^3} \\
&= \frac{2\sqrt{1-a^2x^2}}{a^3c^3(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3c^3(1-ax)^4}-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^3c^3(1-ax)^3}-\frac{\sin^{-1}(ax)}{a^3c^3}
\end{aligned}$$

Mathematica [C] time = 0.0583067, size = 77, normalized size = 0.72

$$\frac{20\sqrt{2}(ax-1)\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1-ax)\right)+\sqrt{ax+1}\left(-a^2x^2+3ax+4\right)}{15a^3c^3(1-ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a*c*x)^3,x]
```

```
[Out] (Sqrt[1 + a*x]*(4 + 3*a*x - a^2*x^2) + 20*Sqrt[2]*(-1 + a*x)*Hypergeometric
2F1[-3/2, -3/2, -1/2, (1 - a*x)/2])/(15*a^3*c^3*(1 - a*x)^(5/2))
```

Maple [A] time = 0.058, size = 167, normalized size = 1.6

$$-\frac{1}{c^3a^2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{2}{5\,a^6c^3}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}-\frac{7}{5\,c^3a^5}\sqrt{-a^2\left(x-a^{-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^3,x)
```

```
[Out] -1/c^3/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-2/5/c^3/a^6
/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-7/5/c^3/a^5/(x-1/a)^2*(-a^2*(
x-1/a)^2-2*a*(x-1/a))^(1/2)-13/5/c^3/a^4/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a
))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56742, size = 300, normalized size = 2.8

$$\frac{8\,a^3x^3 - 24\,a^2x^2 + 24\,ax + 10\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - \left(13\,a^2x^2 - 19\,ax + 8\right)\sqrt{-a^2x^2+1} - 8}{5\left(a^6c^3x^3 - 3\,a^5c^3x^2 + 3\,a^4c^3x - a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/5*(8*a^3*x^3 - 24*a^2*x^2 + 24*a*x + 10*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)
*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (13*a^2*x^2 - 19*a*x + 8)*sqrt(-a
^2*x^2 + 1) - 8)/(a^6*c^3*x^3 - 3*a^5*c^3*x^2 + 3*a^4*c^3*x - a^3*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^2}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx + \int \frac{ax^3}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(x**2/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x*
*2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral
(a*x**3/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1)
+ 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [A] time = 1.18566, size = 225, normalized size = 2.1

$$-\frac{\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{a^2c^3|a|} - \frac{2\left(\frac{35\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x} - \frac{55\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2} + \frac{25\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3} - \frac{5\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4} - 8\right)}{5\,a^2c^3\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(a^2*c^3*abs(a)) - 2/5*(35*(sqrt(-a^2*x^2 + 1)*abs(a) +
 a)/(a^2*x) - 55*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^4*x^2) + 25*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a
^8*x^4) - 8)/(a^2*c^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a
))
```

3.348 $\int \frac{e^{\tanh^{-1}(ax)}x}{(c-acx)^3}\,dx$

Optimal. Leaf size=65

$$\frac{(1-a^2x^2)^{3/2}}{5a^2c^3(1-ax)^4}-\frac{4(1-a^2x^2)^{3/2}}{15a^2c^3(1-ax)^3}$$

```
[Out] (1 - a^2*x^2)^(3/2)/(5*a^2*c^3*(1 - a*x)^4) - (4*(1 - a^2*x^2)^(3/2))/(15*a
^2*c^3*(1 - a*x)^3)
```

Rubi [A] time = 0.0784652, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6128, 793, 651}

$$\frac{(1-a^2x^2)^{3/2}}{5a^2c^3(1-ax)^4}-\frac{4(1-a^2x^2)^{3/2}}{15a^2c^3(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a*c*x)^3,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(5*a^2*c^3*(1 - a*x)^4) - (4*(1 - a^2*x^2)^(3/2))/(15*a
^2*c^3*(1 - a*x)^3)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x}{(c-acx)^3}\,dx = c\int \frac{x\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}}{5a^2c^3(1-ax)^4} - \frac{4\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx}{5a}$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}}{5a^2c^3(1-ax)^4} - \frac{4\left(1-a^2x^2\right)^{3/2}}{15a^2c^3(1-ax)^3}$$

Mathematica [A] time = 0.0182955, size = 35, normalized size = 0.54

$$\frac{(ax+1)^{3/2}(4ax-1)}{15a^2c^3(1-ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a*c*x)^3,x]
```

```
[Out] ((1 + a*x)^(3/2)*(-1 + 4*a*x))/(15*a^2*c^3*(1 - a*x)^(5/2))
```

Maple [A] time = 0.034, size = 41, normalized size = 0.6

$$\frac{(4\,ax-1)\,(ax+1)^2}{15\,c^3\,(ax-1)^2\,a^2}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^3,x)
```

```
[Out] 1/15*(4*a*x-1)*(a*x+1)^2/(a*x-1)^2/c^3/(-a^2*x^2+1)^(1/2)/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56598, size = 189, normalized size = 2.91

$$-\frac{a^3x^3 - 3\,a^2x^2 + 3\,ax + \left(4\,a^2x^2 + 3\,ax - 1\right)\sqrt{-a^2x^2+1} - 1}{15\left(a^5c^3x^3 - 3\,a^4c^3x^2 + 3\,a^3c^3x - a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x + (4*a^2*x^2 + 3*a*x - 1)*sqrt(-a^2*x^2
+ 1) - 1)/(a^5*c^3*x^3 - 3*a^4*c^3*x^2 + 3*a^3*c^3*x - a^2*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^2}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(x/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2
+ 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(a*
x**2/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + 3
*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [B] time = 1.21856, size = 163, normalized size = 2.51

$$\frac{2\left(\frac{5\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}+\frac{5\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}-1\right)}{15\,ac^3\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 2/15*(5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 5*(sqrt(-a^2*x^2 + 1)*abs
(a) + a)^2/(a^4*x^2) + 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 1)/
(a*c^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

3.349 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^3} dx$

Optimal. Leaf size=65

$$\frac{(1-a^2x^2)^{3/2}}{15ac^3(1-ax)^3}+\frac{(1-a^2x^2)^{3/2}}{5ac^3(1-ax)^4}$$

```
[Out] (1 - a^2*x^2)^(3/2)/(5*a*c^3*(1 - a*x)^4) + (1 - a^2*x^2)^(3/2)/(15*a*c^3*(
1 - a*x)^3)
```

Rubi [A] time = 0.0516873, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6127, 659, 651}

$$\frac{(1-a^2x^2)^{3/2}}{15ac^3(1-ax)^3}+\frac{(1-a^2x^2)^{3/2}}{5ac^3(1-ax)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^3,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(5*a*c^3*(1 - a*x)^4) + (1 - a^2*x^2)^(3/2)/(15*a*c^3*(
1 - a*x)^3)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^3}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}}{5ac^3(1-ax)^4} + \frac{1}{5}\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}}{5ac^3(1-ax)^4} + \frac{\left(1-a^2x^2\right)^{3/2}}{15ac^3(1-ax)^3}$$

Mathematica [A] time = 0.0160793, size = 35, normalized size = 0.54

$$\frac{(4-ax)(ax+1)^{3/2}}{15ac^3(1-ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^3,x]
```

```
[Out] ((4 - a*x)*(1 + a*x)^(3/2))/(15*a*c^3*(1 - a*x)^(5/2))
```

Maple [A] time = 0.033, size = 40, normalized size = 0.6

$$-\frac{(ax-4)\,(ax+1)^2}{15\,(ax-1)^2\,c^3a}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x)
```

```
[Out] -1/15*(a*x-4)*(a*x+1)^2/(a*x-1)^2/c^3/(-a^2*x^2+1)^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57504, size = 188, normalized size = 2.89

$$\frac{4\,a^3x^3-12\,a^2x^2+12\,ax+\left(a^2x^2-3\,ax-4\right)\sqrt{-a^2x^2+1}-4}{15\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(4*a^3*x^3 - 12*a^2*x^2 + 12*a*x + (a^2*x^2 - 3*a*x - 4)*sqrt(-a^2*x^2
 + 1) - 4)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(a*x/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**
2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(
1/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + 3*a*
x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [B] time = 1.16223, size = 196, normalized size = 3.02

$$-\frac{2\left(\frac{5\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{25\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}-\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4}-4\right)}{15\,c^3\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -2/15*(5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 25*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)^2/(a^4*x^2) + 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 1
5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 4)/(c^3*((sqrt(-a^2*x^2 + 1
)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

3.350 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-acx)^3}\,dx$

Optimal. Leaf size=97

$$\frac{4ax}{5c^3\left(1-a^2x^2\right)^{3/2}}+\frac{8ax+5}{5c^3\sqrt{1-a^2x^2}}+\frac{8(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

```
[Out] (8*(1 + a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (4*a*x)/(5*c^3*(1 - a^2*x^2)^(3
/2)) + (5 + 8*a*x)/(5*c^3*Sqrt[1 - a^2*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c
^3
```

Rubi [A] time = 0.274138, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 823, 12, 266, 63, 208}

$$\frac{4ax}{5c^3\left(1-a^2x^2\right)^{3/2}}+\frac{8ax+5}{5c^3\sqrt{1-a^2x^2}}+\frac{8(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(c - a*c*x)^3),x]
```

```
[Out] (8*(1 + a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (4*a*x)/(5*c^3*(1 - a^2*x^2)^(3
/2)) + (5 + 8*a*x)/(5*c^3*Sqrt[1 - a^2*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c
^3
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x(c-acx)^3}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x(c-acx)^4}\,dx$$

$$= \frac{\int \frac{(c+acx)^4}{x(1-a^2x^2)^{7/2}}\,dx}{c^7}$$

$$= \frac{8(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{-5c^4-12ac^4x+5a^2c^4x^2}{x(1-a^2x^2)^{5/2}}\,dx}{5c^7}$$

$$= \frac{8(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4ax}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{15c^4+24ac^4x}{x(1-a^2x^2)^{3/2}}\,dx}{15c^7}$$

$$= \frac{8(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4ax}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{5+8ax}{5c^3\sqrt{1-a^2x^2}} + \frac{\int \frac{15a^2c^4}{x\sqrt{1-a^2x^2}}\,dx}{15a^2c^7}$$

$$= \frac{8(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4ax}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{5+8ax}{5c^3\sqrt{1-a^2x^2}} + \frac{\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c^3}$$

$$= \frac{8(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4ax}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{5+8ax}{5c^3\sqrt{1-a^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{2c^3}$$

$$= \frac{8(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4ax}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{5+8ax}{5c^3\sqrt{1-a^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2c^3}$$

$$= \frac{8(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4ax}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{5+8ax}{5c^3\sqrt{1-a^2x^2}} - \frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

Mathematica [C] time = 0.0861565, size = 71, normalized size = 0.73

$$\frac{3\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-a^2x^2\right)+24a^5x^5-60a^3x^3+5a^2x^2+60ax+16}{15c^3\left(1-a^2x^2\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a*c*x)^3),x]
```

```
[Out] (16 + 60*a*x + 5*a^2*x^2 - 60*a^3*x^3 + 24*a^5*x^5 + 3*Hypergeometric2F1[-5
/2, 1, -3/2, 1 - a^2*x^2])/(15*c^3*(1 - a^2*x^2)^(5/2))
```

Maple [B] time = 0.048, size = 275, normalized size = 2.8

$$-\frac{1}{c^3}\left(2\frac{1}{a^2}\left(1/5\frac{1}{a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}-2/5a\left(1/3\frac{1}{a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-2}-1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^3,x)
```

```
[Out] -1/c^3*(2/a^2*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2/5*a*(1/
3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^
2-2*a*(x-1/a))^(1/2)))+arctanh(1/(-a^2*x^2+1)^(1/2))-1/a*(1/3/a/(x-1/a)^2*(
-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^
(1/2))+1/a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^3*x), x)
```

Fricas [A] time = 1.64981, size = 284, normalized size = 2.93

$$\frac{13\,a^3x^3-39\,a^2x^2+39\,ax+5\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(8\,a^2x^2-19\,ax+13\right)\sqrt{-a^2x^2+1}-13}{5\left(a^3c^3x^3-3\,a^2c^3x^2+3\,ac^3x-c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/5*(13*a^3*x^3 - 39*a^2*x^2 + 39*a*x + 5*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)
*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (8*a^2*x^2 - 19*a*x + 13)*sqrt(-a^2*x^2
+ 1) - 13)/(a^3*c^3*x^3 - 3*a^2*c^3*x^2 + 3*a*c^3*x - c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax}{a^3x^4\sqrt{-a^2x^2+1}-3a^2x^3\sqrt{-a^2x^2+1}+3ax^2\sqrt{-a^2x^2+1}-x\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^3x^4\sqrt{-a^2x^2+1}-3a^2x^3\sqrt{-a^2x^2+1}+3ax^2\sqrt{-a^2x^2+1}-x\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(a*x/(a**3*x**4*sqrt(-a**2*x**2 + 1) - 3*a**2*x**3*sqrt(-a**2*x**
2 + 1) + 3*a*x**2*sqrt(-a**2*x**2 + 1) - x*sqrt(-a**2*x**2 + 1)), x) + Inte
gral(1/(a**3*x**4*sqrt(-a**2*x**2 + 1) - 3*a**2*x**3*sqrt(-a**2*x**2 + 1) +
 3*a*x**2*sqrt(-a**2*x**2 + 1) - x*sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [B] time = 1.22046, size = 255, normalized size = 2.63

$$-\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{c^3|a|}+\frac{2\left(13\,a-\frac{45\left(\sqrt{-a^2x^2+1}|a|+a\right)}{ax}+\frac{75\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^3x^2}-\frac{55\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^5x^3}+\frac{20\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^7x^4}\right)}{5\,c^3\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c^3*abs(a
)) + 2/5*(13*a - 45*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a*x) + 75*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^2/(a^3*x^2) - 55*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^5
*x^3) + 20*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^7*x^4))/(c^3*((sqrt(-a^2*x^
2 + 1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

3.351 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-acx)^3}\,dx$

Optimal. Leaf size=127

$$\frac{8a(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{\sqrt{1-a^2x^2}}{c^3x}+\frac{a(79ax+60)}{15c^3\sqrt{1-a^2x^2}}+\frac{4a(8ax+5)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{4a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

```
[Out] (8*a*(1 + a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (4*a*(5 + 8*a*x))/(15*c^3*(1
- a^2*x^2)^(3/2)) + (a*(60 + 79*a*x))/(15*c^3*Sqrt[1 - a^2*x^2]) - Sqrt[1 -
 a^2*x^2]/(c^3*x) - (4*a*ArcTanh[Sqrt[1 - a^2*x^2]])/c^3
```

Rubi [A] time = 0.351343, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 852, 1805, 807, 266, 63, 208}

$$\frac{8a(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{\sqrt{1-a^2x^2}}{c^3x}+\frac{a(79ax+60)}{15c^3\sqrt{1-a^2x^2}}+\frac{4a(8ax+5)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{4a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a*c*x)^3),x]
```

```
[Out] (8*a*(1 + a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (4*a*(5 + 8*a*x))/(15*c^3*(1
- a^2*x^2)^(3/2)) + (a*(60 + 79*a*x))/(15*c^3*Sqrt[1 - a^2*x^2]) - Sqrt[1 -
 a^2*x^2]/(c^3*x) - (4*a*ArcTanh[Sqrt[1 - a^2*x^2]])/c^3
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-acx)^3}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^2(c-acx)^4}\,dx$$

$$= \frac{\int \frac{(c+acx)^4}{x^2(1-a^2x^2)^{7/2}}\,dx}{c^7}$$

$$= \frac{8a(1+ax)}{5c^3(1-a^2x^2)^{5/2}} - \frac{\int \frac{-5c^4-20ac^4x-27a^2c^4x^2}{x^2(1-a^2x^2)^{5/2}}\,dx}{5c^7}$$

$$= \frac{8a(1+ax)}{5c^3(1-a^2x^2)^{5/2}} + \frac{4a(5+8ax)}{15c^3(1-a^2x^2)^{3/2}} + \frac{\int \frac{15c^4+60ac^4x+64a^2c^4x^2}{x^2(1-a^2x^2)^{3/2}}\,dx}{15c^7}$$

$$= \frac{8a(1+ax)}{5c^3(1-a^2x^2)^{5/2}} + \frac{4a(5+8ax)}{15c^3(1-a^2x^2)^{3/2}} + \frac{a(60+79ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\int \frac{-15c^4-60ac^4x}{x^2\sqrt{1-a^2x^2}}\,dx}{15c^7}$$

$$= \frac{8a(1+ax)}{5c^3(1-a^2x^2)^{5/2}} + \frac{4a(5+8ax)}{15c^3(1-a^2x^2)^{3/2}} + \frac{a(60+79ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^3x} + \frac{(4a)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c^3}$$

$$= \frac{8a(1+ax)}{5c^3(1-a^2x^2)^{5/2}} + \frac{4a(5+8ax)}{15c^3(1-a^2x^2)^{3/2}} + \frac{a(60+79ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^3x} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx\right.}{c^3}$$

$$= \frac{8a(1+ax)}{5c^3(1-a^2x^2)^{5/2}} + \frac{4a(5+8ax)}{15c^3(1-a^2x^2)^{3/2}} + \frac{a(60+79ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^3x} - \frac{4\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \right.}{ac^3}$$

$$= \frac{8a(1+ax)}{5c^3(1-a^2x^2)^{5/2}} + \frac{4a(5+8ax)}{15c^3(1-a^2x^2)^{3/2}} + \frac{a(60+79ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^3x} - \frac{4a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

Mathematica [A] time = 0.0546973, size = 101, normalized size = 0.8

$$\frac{94a^4x^4-128a^3x^3-73a^2x^2-60ax(ax-1)^2\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+134ax-15}{15c^3x(ax-1)^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a*c*x)^3),x]
```

```
[Out] (-15 + 134*a*x - 73*a^2*x^2 - 128*a^3*x^3 + 94*a^4*x^4 - 60*a*x*(-1 + a*x)^
2*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(15*c^3*x*(-1 + a*x)^2*Sqrt
[1 - a^2*x^2])
```

Maple [B] time = 0.048, size = 248, normalized size = 2.

$$-\frac{1}{c^3}\left(2\frac{1}{a}\left(1/5\frac{1}{a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}-2/5a\left(1/3\frac{1}{a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^3,x)
```

```
[Out] -1/c^3*(2/a*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2/5*a*(1/3/
a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-
2*a*(x-1/a))^(1/2)))+(-a^2*x^2+1)^(1/2)/x+4*a*arctanh(1/(-a^2*x^2+1)^(1/2))
-1/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+5/(x-1/a)*(-a^2*(x-1/a)^2
-2*a*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] -integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^3*x^2), x)
```

Fricas [A] time = 1.48822, size = 338, normalized size = 2.66

$$\frac{104\,a^4x^4-312\,a^3x^3+312\,a^2x^2-104\,ax+60\left(a^4x^4-3\,a^3x^3+3\,a^2x^2-ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(94\,a^3x^3-222\,a^2x^2+149\right.}{15\left(a^3c^3x^4-3\,a^2c^3x^3+3\,ac^3x^2-c^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/15*(104*a^4*x^4 - 312*a^3*x^3 + 312*a^2*x^2 - 104*a*x + 60*(a^4*x^4 - 3*a
^3*x^3 + 3*a^2*x^2 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (94*a^3*x^3 - 2
22*a^2*x^2 + 149*a*x - 15)*sqrt(-a^2*x^2 + 1))/(a^3*c^3*x^4 - 3*a^2*c^3*x^3
 + 3*a*c^3*x^2 - c^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax}{a^3x^5\sqrt{-a^2x^2+1}-3a^2x^4\sqrt{-a^2x^2+1}+3ax^3\sqrt{-a^2x^2+1}-x^2\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^3x^5\sqrt{-a^2x^2+1}-3a^2x^4\sqrt{-a^2x^2+1}+3ax^3\sqrt{-a^2x^2+1}-x^2\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(a*x/(a**3*x**5*sqrt(-a**2*x**2 + 1) - 3*a**2*x**4*sqrt(-a**2*x**
2 + 1) + 3*a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x) + I
ntegral(1/(a**3*x**5*sqrt(-a**2*x**2 + 1) - 3*a**2*x**4*sqrt(-a**2*x**2 + 1
) + 3*a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [B] time = 1.23041, size = 363, normalized size = 2.86

$$-\frac{4\,a^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{c^3|a|}-\frac{\sqrt{-a^2x^2+1}|a|+a}{2\,c^3x|a|}-\frac{\left(15\,a^2-\frac{491\left(\sqrt{-a^2x^2+1}|a|+a\right)}{x}+\frac{1690\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^2x^2}-\frac{2570\left(\sqrt{-a^2x^2+1}|a|\cdots\right.}{a^4x^3}\cdots\right.}{30\left(\sqrt{-a^2x^2+1}|a|+a\right)c^3\left(\frac{\sqrt{-a^2\cdots}}{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -4*a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c^3*a
bs(a)) - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(c^3*x*abs(a)) - 1/30*(15*a^2
- 491*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/x + 1690*(sqrt(-a^2*x^2 + 1)*abs(a) +
 a)^2/(a^2*x^2) - 2570*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^4*x^3) + 1815*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^6*x^4) - 555*(sqrt(-a^2*x^2 + 1)*abs(a)
 + a)^5/(a^8*x^5))*a^2*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3*((sqrt(-a^2*x
^2 + 1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

3.352 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-acx)^3} dx$

Optimal. Leaf size=162

$$\frac{a^2(164ax+135)}{15c^3\sqrt{1-a^2x^2}}+\frac{4a^2(13ax+10)}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{8a^2(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{4a\sqrt{1-a^2x^2}}{c^3x}-\frac{\sqrt{1-a^2x^2}}{2c^3x^2}-\frac{19a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^3}$$

```
[Out] (8*a^2*(1 + a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (4*a^2*(10 + 13*a*x))/(15*c
^3*(1 - a^2*x^2)^(3/2)) + (a^2*(135 + 164*a*x))/(15*c^3*Sqrt[1 - a^2*x^2])
- Sqrt[1 - a^2*x^2]/(2*c^3*x^2) - (4*a*Sqrt[1 - a^2*x^2])/(c^3*x) - (19*a^2
*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c^3)
```

Rubi [A] time = 0.430019, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 1807, 807, 266, 63, 208}

$$\frac{a^2(164ax+135)}{15c^3\sqrt{1-a^2x^2}}+\frac{4a^2(13ax+10)}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{8a^2(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{4a\sqrt{1-a^2x^2}}{c^3x}-\frac{\sqrt{1-a^2x^2}}{2c^3x^2}-\frac{19a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a*c*x)^3),x]
```

```
[Out] (8*a^2*(1 + a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (4*a^2*(10 + 13*a*x))/(15*c
^3*(1 - a^2*x^2)^(3/2)) + (a^2*(135 + 164*a*x))/(15*c^3*Sqrt[1 - a^2*x^2])
- Sqrt[1 - a^2*x^2]/(2*c^3*x^2) - (4*a*Sqrt[1 - a^2*x^2])/(c^3*x) - (19*a^2
*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c^3)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-acx)^3}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^3(c-acx)^4}\,dx$$

$$= \frac{\int \frac{(c+acx)^4}{x^3\left(1-a^2x^2\right)^{7/2}}\,dx}{c^7}$$

$$= \frac{8a^2(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{-5c^4-20ac^4x-35a^2c^4x^2-32a^3c^4x^3}{x^3\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^7}$$

$$= \frac{8a^2(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^2(10+13ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{15c^4+60ac^4x+120a^2c^4x^2+104a^3c^4x^3}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^7}$$

$$= \frac{8a^2(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^2(10+13ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(135+164ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\int \frac{-15c^4-60ac^4x-135a^2c^4x^2}{x^3\sqrt{1-a^2x^2}}\,dx}{15c^7}$$

$$= \frac{8a^2(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^2(10+13ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(135+164ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^3x^2} + \frac{\int \frac{120ac^4+285a^2c^4x}{x^2\sqrt{1-a^2x^2}}\,dx}{30c^7}$$

$$= \frac{8a^2(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^2(10+13ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(135+164ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{4a\sqrt{1-a^2x^2}}{c^3x} + \frac{\left(19a^2\right)\int -}{2c}$$

$$= \frac{8a^2(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^2(10+13ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(135+164ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{4a\sqrt{1-a^2x^2}}{c^3x} + \frac{\left(19a^2\right)\text{Sub}}{}$$

$$= \frac{8a^2(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^2(10+13ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(135+164ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{4a\sqrt{1-a^2x^2}}{c^3x} - \frac{19\,\text{Subst}}{}$$

$$= \frac{8a^2(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^2(10+13ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(135+164ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{4a\sqrt{1-a^2x^2}}{c^3x} - \frac{19a^2\tanh}{}$$

Mathematica [A] time = 0.0593456, size = 113, normalized size = 0.7

$$\frac{448a^5x^5-611a^4x^4-346a^3x^3+638a^2x^2-285a^2x^2(ax-1)^2\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-90ax-15}{30c^3x^2(ax-1)^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a*c*x)^3),x]
```

```
[Out] (-15 - 90*a*x + 638*a^2*x^2 - 346*a^3*x^3 - 611*a^4*x^4 + 448*a^5*x^5 - 285
*a^2*x^2*(-1 + a*x)^2*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(30*c^3
*x^2*(-1 + a*x)^2*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.055, size = 223, normalized size = 1.4

$$-\frac{1}{c^3}\left(\frac{2}{5a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}-\frac{29a}{5}\left(\frac{1}{3a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-2}-\frac{1}{3}\sqrt{-a^2\left(x\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^3,x)
```

```
[Out] -1/c^3*(2/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-29/5*a*(1/3/a/(x
-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*
(x-1/a))^(1/2))+4*a*(-a^2*x^2+1)^(1/2)/x+19/2*a^2*arctanh(1/(-a^2*x^2+1)^(1
/2))+9*a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1/2*(-a^2*x^2+1)^(1/2)/
x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] -integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^3*x^3), x)
```

Fricas [A] time = 1.63086, size = 375, normalized size = 2.31

$$\frac{398\,a^5x^5 - 1194\,a^4x^4 + 1194\,a^3x^3 - 398\,a^2x^2 + 285\left(a^5x^5 - 3\,a^4x^4 + 3\,a^3x^3 - a^2x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \left(448\,a^4x^4 - 105\right.}{30\left(a^3c^3x^5 - 3\,a^2c^3x^4 + 3\,ac^3x^3 - c^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/30*(398*a^5*x^5 - 1194*a^4*x^4 + 1194*a^3*x^3 - 398*a^2*x^2 + 285*(a^5*x^
5 - 3*a^4*x^4 + 3*a^3*x^3 - a^2*x^2)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (448
*a^4*x^4 - 1059*a^3*x^3 + 713*a^2*x^2 - 75*a*x - 15)*sqrt(-a^2*x^2 + 1))/(a
^3*c^3*x^5 - 3*a^2*c^3*x^4 + 3*a*c^3*x^3 - c^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax}{a^3x^6\sqrt{-a^2x^2+1}-3a^2x^5\sqrt{-a^2x^2+1}+3ax^4\sqrt{-a^2x^2+1}-x^3\sqrt{-a^2x^2+1}}\,dx + \int \frac{1}{a^3x^6\sqrt{-a^2x^2+1}-3a^2x^5\sqrt{-a^2x^2+1}+3ax^4\sqrt{-a^2x^2+1}-x^3\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(a*x/(a**3*x**6*sqrt(-a**2*x**2 + 1) - 3*a**2*x**5*sqrt(-a**2*x**
2 + 1) + 3*a*x**4*sqrt(-a**2*x**2 + 1) - x**3*sqrt(-a**2*x**2 + 1)), x) + I
ntegral(1/(a**3*x**6*sqrt(-a**2*x**2 + 1) - 3*a**2*x**5*sqrt(-a**2*x**2 + 1
```

```
) + 3*a*x**4*sqrt(-a**2*x**2 + 1) - x**3*sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [B] time = 1.26039, size = 456, normalized size = 2.81

$$-\frac{19\,a^3\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,c^3|a|}-\frac{\left(15\,a^3+\frac{165\left(\sqrt{-a^2x^2+1}|a|+a\right)a}{x}-\frac{4234\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{ax^2}+\frac{14330\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^3x^3}-\frac{20965\left(\sqrt{-a^2x^2+1}|a\right.}{a^5x^4}\right.}{120\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^3\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -19/2*a^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c^
3*abs(a)) - 1/120*(15*a^3 + 165*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 4234*
(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a*x^2) + 14330*(sqrt(-a^2*x^2 + 1)*abs(a
) + a)^3/(a^3*x^3) - 20965*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^5*x^4) + 14
385*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5/(a^7*x^5) - 4080*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^6/(a^9*x^6))*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3*((s
qrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a)) - 1/8*(16*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)*a*c^3*abs(a)/x + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3*
abs(a)/(a*x^2))/(a^2*c^6)
```

3.353 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-acx)^3} dx$

Optimal. Leaf size=187

$$\frac{a^3(93ax+80)}{5c^3\sqrt{1-a^2x^2}}+\frac{4a^3(6ax+5)}{5c^3\left(1-a^2x^2\right)^{3/2}}+\frac{8a^3(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{29a^2\sqrt{1-a^2x^2}}{3c^3x}-\frac{2a\sqrt{1-a^2x^2}}{c^3x^2}-\frac{\sqrt{1-a^2x^2}}{3c^3x^3}-\frac{18a^3\tanh^{-1}\left(\cdot\right.}{c^3}$$

```
[Out] (8*a^3*(1 + a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (4*a^3*(5 + 6*a*x))/(5*c^3*
(1 - a^2*x^2)^(3/2)) + (a^3*(80 + 93*a*x))/(5*c^3*Sqrt[1 - a^2*x^2]) - Sqrt
[1 - a^2*x^2]/(3*c^3*x^3) - (2*a*Sqrt[1 - a^2*x^2])/(c^3*x^2) - (29*a^2*Sqr
t[1 - a^2*x^2])/(3*c^3*x) - (18*a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/c^3
```

Rubi [A] time = 0.524817, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 1807, 807, 266, 63, 208}

$$\frac{a^3(93ax+80)}{5c^3\sqrt{1-a^2x^2}}+\frac{4a^3(6ax+5)}{5c^3\left(1-a^2x^2\right)^{3/2}}+\frac{8a^3(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{29a^2\sqrt{1-a^2x^2}}{3c^3x}-\frac{2a\sqrt{1-a^2x^2}}{c^3x^2}-\frac{\sqrt{1-a^2x^2}}{3c^3x^3}-\frac{18a^3\tanh^{-1}\left(\cdot\right.}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*(c - a*c*x)^3),x]
```

```
[Out] (8*a^3*(1 + a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (4*a^3*(5 + 6*a*x))/(5*c^3*
(1 - a^2*x^2)^(3/2)) + (a^3*(80 + 93*a*x))/(5*c^3*Sqrt[1 - a^2*x^2]) - Sqrt
[1 - a^2*x^2]/(3*c^3*x^3) - (2*a*Sqrt[1 - a^2*x^2])/(c^3*x^2) - (29*a^2*Sqr
t[1 - a^2*x^2])/(3*c^3*x) - (18*a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/c^3
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-acx)^3}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^4(c-acx)^4}\,dx$$

$$= \frac{\int \frac{(c+acx)^4}{x^4(1-a^2x^2)^{7/2}}\,dx}{c^7}$$

$$= \frac{8a^3(1+ax)}{5c^3(1-a^2x^2)^{5/2}} - \frac{\int \frac{-5c^4-20ac^4x-35a^2c^4x^2-40a^3c^4x^3-32a^4c^4x^4}{x^4(1-a^2x^2)^{5/2}}\,dx}{5c^7}$$

$$= \frac{8a^3(1+ax)}{5c^3(1-a^2x^2)^{5/2}} + \frac{4a^3(5+6ax)}{5c^3(1-a^2x^2)^{3/2}} + \frac{\int \frac{15c^4+60ac^4x+120a^2c^4x^2+180a^3c^4x^3+144a^4c^4x^4}{x^4(1-a^2x^2)^{3/2}}\,dx}{15c^7}$$

$$= \frac{8a^3(1+ax)}{5c^3(1-a^2x^2)^{5/2}} + \frac{4a^3(5+6ax)}{5c^3(1-a^2x^2)^{3/2}} + \frac{a^3(80+93ax)}{5c^3\sqrt{1-a^2x^2}} - \frac{\int \frac{-15c^4-60ac^4x-135a^2c^4x^2-240a^3c^4x^3}{x^4\sqrt{1-a^2x^2}}\,dx}{15c^7}$$

$$= \frac{8a^3(1+ax)}{5c^3(1-a^2x^2)^{5/2}} + \frac{4a^3(5+6ax)}{5c^3(1-a^2x^2)^{3/2}} + \frac{a^3(80+93ax)}{5c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^3x^3} + \frac{\int \frac{180ac^4+435a^2c^4x+720a^3c^4x^2}{x^3\sqrt{1-a^2x^2}}\,d}{45c^7}$$

$$= \frac{8a^3(1+ax)}{5c^3(1-a^2x^2)^{5/2}} + \frac{4a^3(5+6ax)}{5c^3(1-a^2x^2)^{3/2}} + \frac{a^3(80+93ax)}{5c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^3x^3} - \frac{2a\sqrt{1-a^2x^2}}{c^3x^2} - \frac{\int \frac{-870a^2c^4-}{x^2\sqrt{1-}}}{90}$$

$$= \frac{8a^3(1+ax)}{5c^3(1-a^2x^2)^{5/2}} + \frac{4a^3(5+6ax)}{5c^3(1-a^2x^2)^{3/2}} + \frac{a^3(80+93ax)}{5c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^3x^3} - \frac{2a\sqrt{1-a^2x^2}}{c^3x^2} - \frac{29a^2\sqrt{1-}}{3c^3x}$$

$$= \frac{8a^3(1+ax)}{5c^3(1-a^2x^2)^{5/2}} + \frac{4a^3(5+6ax)}{5c^3(1-a^2x^2)^{3/2}} + \frac{a^3(80+93ax)}{5c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^3x^3} - \frac{2a\sqrt{1-a^2x^2}}{c^3x^2} - \frac{29a^2\sqrt{1-}}{3c^3x}$$

$$= \frac{8a^3(1+ax)}{5c^3(1-a^2x^2)^{5/2}} + \frac{4a^3(5+6ax)}{5c^3(1-a^2x^2)^{3/2}} + \frac{a^3(80+93ax)}{5c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^3x^3} - \frac{2a\sqrt{1-a^2x^2}}{c^3x^2} - \frac{29a^2\sqrt{1-}}{3c^3x}$$

$$= \frac{8a^3(1+ax)}{5c^3(1-a^2x^2)^{5/2}} + \frac{4a^3(5+6ax)}{5c^3(1-a^2x^2)^{3/2}} + \frac{a^3(80+93ax)}{5c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^3x^3} - \frac{2a\sqrt{1-a^2x^2}}{c^3x^2} - \frac{29a^2\sqrt{1-}}{3c^3x}$$

Mathematica [A] time = 0.0619358, size = 121, normalized size = 0.65

$$\frac{424a^6x^6-578a^5x^5-328a^4x^4+604a^3x^3-85a^2x^2-270a^3x^3(ax-1)^2\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-20ax-5}{15c^3x^3(ax-1)^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*(c - a*c*x)^3),x]
```

```
[Out] (-5 - 20*a*x - 85*a^2*x^2 + 604*a^3*x^3 - 328*a^4*x^4 - 578*a^5*x^5 + 424*a
^6*x^6 - 270*a^3*x^3*(-1 + a*x)^2*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^
2]])/(15*c^3*x^3*(-1 + a*x)^2*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.054, size = 355, normalized size = 1.9

$$-\frac{1}{c^3}\left(2\,a\left(1/5\,\frac{1}{a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}-2/5\,a\left(1/3\,\frac{1}{a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-2}-1/\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c)^3,x)
```

```
[Out] -1/c^3*(2*a*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2/5*a*(1/3/
a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-
2*a*(x-1/a))^(1/2)))+29/3*a^2*(-a^2*x^2+1)^(1/2)/x+16*a^3*arctanh(1/(-a^2*x
^2+1)^(1/2))-7*a^2*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/
(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))+16*a^2/(x-1/a)*(-a^2*(x-1/a)^2-
2*a*(x-1/a))^(1/2)-4*a*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2
*x^2+1)^(1/2)))+1/3*(-a^2*x^2+1)^(1/2)/x^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] -integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^3*x^4), x)
```

Fricas [A] time = 1.66657, size = 389, normalized size = 2.08

$$\frac{324\,a^6x^6-972\,a^5x^5+972\,a^4x^4-324\,a^3x^3+270\left(a^6x^6-3\,a^5x^5+3\,a^4x^4-a^3x^3\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(424\,a^5x^5-1002\,a^4x\right.}{15\left(a^3c^3x^6-3\,a^2c^3x^5+3\,ac^3x^4-c^3x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/15*(324*a^6*x^6 - 972*a^5*x^5 + 972*a^4*x^4 - 324*a^3*x^3 + 270*(a^6*x^6
- 3*a^5*x^5 + 3*a^4*x^4 - a^3*x^3)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (424*a
^5*x^5 - 1002*a^4*x^4 + 674*a^3*x^3 - 70*a^2*x^2 - 15*a*x - 5)*sqrt(-a^2*x^
2 + 1))/(a^3*c^3*x^6 - 3*a^2*c^3*x^5 + 3*a*c^3*x^4 - c^3*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax}{a^3x^7\sqrt{-a^2x^2+1}-3a^2x^6\sqrt{-a^2x^2+1}+3ax^5\sqrt{-a^2x^2+1}-x^4\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^3x^7\sqrt{-a^2x^2+1}-3a^2x^6\sqrt{-a^2x^2+1}+3ax^5\sqrt{-a^2x^2+1}-x^4\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**4/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(a*x/(a**3*x**7*sqrt(-a**2*x**2 + 1) - 3*a**2*x**6*sqrt(-a**2*x**
2 + 1) + 3*a*x**5*sqrt(-a**2*x**2 + 1) - x**4*sqrt(-a**2*x**2 + 1)), x) + I
ntegral(1/(a**3*x**7*sqrt(-a**2*x**2 + 1) - 3*a**2*x**6*sqrt(-a**2*x**2 + 1
) + 3*a*x**5*sqrt(-a**2*x**2 + 1) - x**4*sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [B] time = 1.34403, size = 531, normalized size = 2.84

$$-\frac{18\,a^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{c^3|a|}-\frac{\left(5\,a^4+\frac{35\left(\sqrt{-a^2x^2+1}|a|+a\right)a^2}{x}+\frac{335\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{x^2}-\frac{7559\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^2x^3}+\frac{25195\left(\sqrt{-a^2x^2+1}|a\right.}{a^4x^4}\cdots\right.}{120\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c^3\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -18*a^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c^3*
abs(a)) - 1/120*(5*a^4 + 35*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2/x + 335*(sq
rt(-a^2*x^2 + 1)*abs(a) + a)^2/x^2 - 7559*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3
/(a^2*x^3) + 25195*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^4*x^4) - 36035*(sqr
t(-a^2*x^2 + 1)*abs(a) + a)^5/(a^6*x^5) + 24225*(sqrt(-a^2*x^2 + 1)*abs(a)
+ a)^6/(a^8*x^6) - 6585*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7/(a^10*x^7))*a^6*x
^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/
(a^2*x) - 1)^5*abs(a)) - 1/24*(117*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^6/
x + 12*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^6/x^2 + (sqrt(-a^2*x^2 + 1)*
abs(a) + a)^3*c^6/x^3)/(a^2*c^9*abs(a))
```

3.354 $\int \frac{e^{\tanh^{-1}(ax)}x^5}{(c-acx)^4}\,dx$

Optimal. Leaf size=166

$$\frac{(ax+1)^5}{7a^6c^4\left(1-a^2x^2\right)^{7/2}}-\frac{33(ax+1)^4}{35a^6c^4\left(1-a^2x^2\right)^{5/2}}+\frac{317(ax+1)^3}{105a^6c^4\left(1-a^2x^2\right)^{3/2}}-\frac{10(ax+1)^2}{a^6c^4\sqrt{1-a^2x^2}}-\frac{(ax+30)\sqrt{1-a^2x^2}}{2a^6c^4}+\frac{29\sin^{-1}(ax)}{2a^6c^4}$$

```
[Out] (1 + a*x)^5/(7*a^6*c^4*(1 - a^2*x^2)^(7/2)) - (33*(1 + a*x)^4)/(35*a^6*c^4*
(1 - a^2*x^2)^(5/2)) + (317*(1 + a*x)^3)/(105*a^6*c^4*(1 - a^2*x^2)^(3/2))
- (10*(1 + a*x)^2)/(a^6*c^4*Sqrt[1 - a^2*x^2]) - ((30 + a*x)*Sqrt[1 - a^2*x
^2])/(2*a^6*c^4) + (29*ArcSin[a*x])/(2*a^6*c^4)
```

Rubi [A] time = 0.533396, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6128, 852, 1635, 780, 216}

$$\frac{(ax+1)^5}{7a^6c^4\left(1-a^2x^2\right)^{7/2}}-\frac{33(ax+1)^4}{35a^6c^4\left(1-a^2x^2\right)^{5/2}}+\frac{317(ax+1)^3}{105a^6c^4\left(1-a^2x^2\right)^{3/2}}-\frac{10(ax+1)^2}{a^6c^4\sqrt{1-a^2x^2}}-\frac{(ax+30)\sqrt{1-a^2x^2}}{2a^6c^4}+\frac{29\sin^{-1}(ax)}{2a^6c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^5)/(c - a*c*x)^4,x]
```

```
[Out] (1 + a*x)^5/(7*a^6*c^4*(1 - a^2*x^2)^(7/2)) - (33*(1 + a*x)^4)/(35*a^6*c^4*
(1 - a^2*x^2)^(5/2)) + (317*(1 + a*x)^3)/(105*a^6*c^4*(1 - a^2*x^2)^(3/2))
- (10*(1 + a*x)^2)/(a^6*c^4*Sqrt[1 - a^2*x^2]) - ((30 + a*x)*Sqrt[1 - a^2*x
^2])/(2*a^6*c^4) + (29*ArcSin[a*x])/(2*a^6*c^4)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^5}{(c-acx)^4}\,dx = c\int \frac{x^5\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx$$

$$= \frac{\int \frac{x^5(c+acx)^5}{(1-a^2x^2)^{9/2}}\,dx}{c^9}$$

$$= \frac{(1+ax)^5}{7a^6c^4(1-a^2x^2)^{7/2}} - \frac{\int \frac{(c+acx)^4\left(\frac{5}{a^5}+\frac{7x}{a^4}+\frac{7x^2}{a^3}+\frac{7x^3}{a^2}+\frac{7x^4}{a}\right)}{(1-a^2x^2)^{7/2}}\,dx}{7c^8}$$

$$= \frac{(1+ax)^5}{7a^6c^4(1-a^2x^2)^{7/2}} - \frac{33(1+ax)^4}{35a^6c^4(1-a^2x^2)^{5/2}} + \frac{\int \frac{(c+acx)^3\left(\frac{107}{a^5}+\frac{105x}{a^4}+\frac{70x^2}{a^3}+\frac{35x^3}{a^2}\right)}{(1-a^2x^2)^{5/2}}\,dx}{35c^7}$$

$$= \frac{(1+ax)^5}{7a^6c^4(1-a^2x^2)^{7/2}} - \frac{33(1+ax)^4}{35a^6c^4(1-a^2x^2)^{5/2}} + \frac{317(1+ax)^3}{105a^6c^4(1-a^2x^2)^{3/2}} - \frac{\int \frac{(c+acx)^2\left(\frac{630}{a^5}+\frac{315x}{a^4}+\frac{105x^2}{a^3}\right)}{(1-a^2x^2)^{3/2}}\,dx}{105c^6}$$

$$= \frac{(1+ax)^5}{7a^6c^4(1-a^2x^2)^{7/2}} - \frac{33(1+ax)^4}{35a^6c^4(1-a^2x^2)^{5/2}} + \frac{317(1+ax)^3}{105a^6c^4(1-a^2x^2)^{3/2}} - \frac{10(1+ax)^2}{a^6c^4\sqrt{1-a^2x^2}} + \frac{\int \frac{\left(\frac{1470}{a^5}+\ldots\right.}{\sqrt{1-\ldots}}}{10\ldots}$$

$$= \frac{(1+ax)^5}{7a^6c^4(1-a^2x^2)^{7/2}} - \frac{33(1+ax)^4}{35a^6c^4(1-a^2x^2)^{5/2}} + \frac{317(1+ax)^3}{105a^6c^4(1-a^2x^2)^{3/2}} - \frac{10(1+ax)^2}{a^6c^4\sqrt{1-a^2x^2}} - \frac{(30+ax)\ldots}{2a\ldots}$$

$$= \frac{(1+ax)^5}{7a^6c^4(1-a^2x^2)^{7/2}} - \frac{33(1+ax)^4}{35a^6c^4(1-a^2x^2)^{5/2}} + \frac{317(1+ax)^3}{105a^6c^4(1-a^2x^2)^{3/2}} - \frac{10(1+ax)^2}{a^6c^4\sqrt{1-a^2x^2}} - \frac{(30+ax)\ldots}{2a\ldots}$$

Mathematica [A] time = 0.323606, size = 126, normalized size = 0.76

$$-\frac{(ax+1)\left(\sqrt{1-a^2x^2}\left(105a^5x^5+630a^4x^4-8404a^3x^3+18916a^2x^2-16091ax+4784\right)-945(ax-1)^4\sin^{-1}(ax)+420\ldots\right.}{210a^6c^4(ax-1)^3\left(a^2x^2-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^5)/(c - a*c*x)^4,x]
```

```
[Out] -((1 + a*x)*(Sqrt[1 - a^2*x^2]*(4784 - 16091*a*x + 18916*a^2*x^2 - 8404*a^3
*x^3 + 630*a^4*x^4 + 105*a^5*x^5) - 945*(-1 + a*x)^4*ArcSin[a*x] + 4200*(-1
```

```
+ a*x)^4*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(210*a^6*c^4*(-1 + a*x)^3*(-1 + a
^2*x^2))
```

Maple [A] time = 0.057, size = 252, normalized size = 1.5

$$-\frac{x}{2\,c^4a^5}\sqrt{-a^2x^2+1}+\frac{29}{2\,c^4a^5}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-5\,\frac{\sqrt{-a^2x^2+1}}{a^6c^4}+\frac{71}{35\,c^4a^9}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a*c*x+c)^4,x)
```

```
[Out] -1/2/c^4/a^5*x*(-a^2*x^2+1)^(1/2)+29/2/c^4/a^5/(a^2)^(1/2)*arctan((a^2)^(1/
2)*x/(-a^2*x^2+1)^(1/2))-5/c^4/a^6*(-a^2*x^2+1)^(1/2)+71/35/c^4/a^9/(x-1/a)
^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+733/105/c^4/a^8/(x-1/a)^2*(-a^2*(x-1/
a)^2-2*a*(x-1/a))^(1/2)+2417/105/c^4/a^7/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a
))^(1/2)+2/7/c^4/a^10/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73425, size = 454, normalized size = 2.73

$$-\frac{4784\,a^4x^4-19136\,a^3x^3+28704\,a^2x^2-19136\,ax+6090\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(105\right.}{210\left(a^{10}c^4x^4-4\,a^9c^4x^3+6\,a^8c^4x^2-4\,a^7c^4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] -1/210*(4784*a^4*x^4 - 19136*a^3*x^3 + 28704*a^2*x^2 - 19136*a*x + 6090*(a^
4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/
(a*x)) + (105*a^5*x^5 + 630*a^4*x^4 - 8404*a^3*x^3 + 18916*a^2*x^2 - 16091*
a*x + 4784)*sqrt(-a^2*x^2 + 1) + 4784)/(a^10*c^4*x^4 - 4*a^9*c^4*x^3 + 6*a^
8*c^4*x^2 - 4*a^7*c^4*x + a^6*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^5}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^6}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**5/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(x**5/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*x**3*sqrt(-a**2*x**
2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sq
rt(-a**2*x**2 + 1)), x) + Integral(a*x**6/(a**4*x**4*sqrt(-a**2*x**2 + 1) -
 4*a**3*x**3*sqrt(-a**2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*
x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

Giac [A] time = 1.27729, size = 340, normalized size = 2.05

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\left(\frac{x}{a^5c^4}+\frac{10}{a^6c^4}\right)+\frac{29\arcsin(ax)\,\mathrm{sgn}(a)}{2a^5c^4|a|}+\frac{2\left(\frac{11599\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{29442\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{38500\left(\sqrt{-a^2x^2+\cdots\right.}{a^6x^3}\cdots\right.}{105\,a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(x/(a^5*c^4) + 10/(a^6*c^4)) + 29/2*arcsin(a*x)*sgn
(a)/(a^5*c^4*abs(a)) + 2/105*(11599*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x)
 - 29442*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^4*x^2) + 38500*(sqrt(-a^2*x^2
 + 1)*abs(a) + a)^3/(a^6*x^3) - 26845*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^
8*x^4) + 9765*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5/(a^10*x^5) - 1470*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)^6/(a^12*x^6) - 1867)/(a^5*c^4*((sqrt(-a^2*x^2 + 1)*a
bs(a) + a)/(a^2*x) - 1)^7*abs(a))
```

3.355 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-acx)^4} dx$

Optimal. Leaf size=168

$$\frac{(1-a^2x^2)^{3/2}}{a^5c^4(1-ax)^2}+\frac{184(1-a^2x^2)^{3/2}}{105a^5c^4(1-ax)^3}-\frac{26(1-a^2x^2)^{3/2}}{35a^5c^4(1-ax)^4}+\frac{(1-a^2x^2)^{3/2}}{7a^5c^4(1-ax)^5}-\frac{10\sqrt{1-a^2x^2}}{a^5c^4(1-ax)}+\frac{5\sin^{-1}(ax)}{a^5c^4}$$

```
[Out] (-10*Sqrt[1 - a^2*x^2])/(a^5*c^4*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(7*a^5*c^
4*(1 - a*x)^5) - (26*(1 - a^2*x^2)^(3/2))/(35*a^5*c^4*(1 - a*x)^4) + (184*(
1 - a^2*x^2)^(3/2))/(105*a^5*c^4*(1 - a*x)^3) + (1 - a^2*x^2)^(3/2)/(a^5*c^
4*(1 - a*x)^2) + (5*ArcSin[a*x])/(a^5*c^4)
```

Rubi [A] time = 0.402663, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 1639, 1637, 659, 651, 663, 216}

$$\frac{(1-a^2x^2)^{3/2}}{a^5c^4(1-ax)^2}+\frac{184(1-a^2x^2)^{3/2}}{105a^5c^4(1-ax)^3}-\frac{26(1-a^2x^2)^{3/2}}{35a^5c^4(1-ax)^4}+\frac{(1-a^2x^2)^{3/2}}{7a^5c^4(1-ax)^5}-\frac{10\sqrt{1-a^2x^2}}{a^5c^4(1-ax)}+\frac{5\sin^{-1}(ax)}{a^5c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a*c*x)^4,x]
```

```
[Out] (-10*Sqrt[1 - a^2*x^2])/(a^5*c^4*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(7*a^5*c^
4*(1 - a*x)^5) - (26*(1 - a^2*x^2)^(3/2))/(35*a^5*c^4*(1 - a*x)^4) + (184*(
1 - a^2*x^2)^(3/2))/(105*a^5*c^4*(1 - a*x)^3) + (1 - a^2*x^2)^(3/2)/(a^5*c^
4*(1 - a*x)^2) + (5*ArcSin[a*x])/(a^5*c^4)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-acx)^4}\,dx = c\int \frac{x^4\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}}{a^5c^4(1-ax)^2} - \frac{\int \frac{\sqrt{1-a^2x^2}\left(2a^2c^4-7a^3c^4x+9a^4c^4x^2-5a^5c^4x^3\right)}{(c-acx)^5}\,dx}{a^6c^3}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}}{a^5c^4(1-ax)^2} - \frac{\int \left(\frac{a^2\sqrt{1-a^2x^2}}{c(-1+ax)^5} + \frac{4a^2\sqrt{1-a^2x^2}}{c(-1+ax)^4} + \frac{6a^2\sqrt{1-a^2x^2}}{c(-1+ax)^3} + \frac{5a^2\sqrt{1-a^2x^2}}{c(-1+ax)^2}\right)dx}{a^6c^3}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}}{a^5c^4(1-ax)^2} - \frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^5}\,dx}{a^4c^4} - \frac{4\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{a^4c^4} - \frac{5\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}\,dx}{a^4c^4} - \frac{6\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{a^4c^4}$$

$$= -\frac{10\sqrt{1-a^2x^2}}{a^5c^4(1-ax)} + \frac{\left(1-a^2x^2\right)^{3/2}}{7a^5c^4(1-ax)^5} - \frac{4\left(1-a^2x^2\right)^{3/2}}{5a^5c^4(1-ax)^4} + \frac{2\left(1-a^2x^2\right)^{3/2}}{a^5c^4(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{a^5c^4(1-ax)^2} + \frac{2\int \frac{\sqrt{1-a^2x^2}}{(-1+ax}}{7a^4c^4}$$

$$= -\frac{10\sqrt{1-a^2x^2}}{a^5c^4(1-ax)} + \frac{\left(1-a^2x^2\right)^{3/2}}{7a^5c^4(1-ax)^5} - \frac{26\left(1-a^2x^2\right)^{3/2}}{35a^5c^4(1-ax)^4} + \frac{26\left(1-a^2x^2\right)^{3/2}}{15a^5c^4(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{a^5c^4(1-ax)^2} + \frac{5\sin^{-1}}{a^5c}$$

$$= -\frac{10\sqrt{1-a^2x^2}}{a^5c^4(1-ax)} + \frac{\left(1-a^2x^2\right)^{3/2}}{7a^5c^4(1-ax)^5} - \frac{26\left(1-a^2x^2\right)^{3/2}}{35a^5c^4(1-ax)^4} + \frac{184\left(1-a^2x^2\right)^{3/2}}{105a^5c^4(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{a^5c^4(1-ax)^2} + \frac{5\sin^{-}}{a^5}$$

Mathematica [C] time = 0.0867586, size = 95, normalized size = 0.57

$$-\frac{\sqrt{ax+1}\left(105a^4x^4-44a^3x^3-244a^2x^2+29ax+124\right)-700\sqrt{2}(ax-1)^2\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1-ax\right.}{105a^5c^4(1-ax)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a*c*x)^4,x]
```

```
[Out] -(Sqrt[1 + a*x]*(124 + 29*a*x - 244*a^2*x^2 - 44*a^3*x^3 + 105*a^4*x^4) - 7
00*Sqrt[2]*(-1 + a*x)^2*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - a*x)/2])/(
105*a^5*c^4*(1 - a*x)^(7/2))
```

Maple [A] time = 0.056, size = 231, normalized size = 1.4

$$-\frac{1}{c^4a^5}\sqrt{-a^2x^2+1}+5\,\frac{1}{c^4a^4\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+\frac{57}{35\,c^4a^8}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}+\frac{446}{105\,c^4a^7}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^4,x)
```

```
[Out] -1/c^4/a^5*(-a^2*x^2+1)^(1/2)+5/c^4/a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-
a^2*x^2+1)^(1/2))+57/35/c^4/a^8/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2
)+446/105/c^4/a^7/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1024/105/c^4
/a^6/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+2/7/c^4/a^9/(x-1/a)^4*(-a^2
*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59119, size = 423, normalized size = 2.52

$$-\frac{824\,a^4x^4-3296\,a^3x^3+4944\,a^2x^2-3296\,ax+1050\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(105\,a^4x\right.}{105\left(a^9c^4x^4-4\,a^8c^4x^3+6\,a^7c^4x^2-4\,a^6c^4x+a^5c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] -1/105*(824*a^4*x^4 - 3296*a^3*x^3 + 4944*a^2*x^2 - 3296*a*x + 1050*(a^4*x^
4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x
)) + (105*a^4*x^4 - 1444*a^3*x^3 + 3256*a^2*x^2 - 2771*a*x + 824)*sqrt(-a^2
*x^2 + 1) + 824)/(a^9*c^4*x^4 - 4*a^8*c^4*x^3 + 6*a^7*c^4*x^2 - 4*a^6*c^4*x
 + a^5*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^4}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx + \int \frac{ax^5}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(x**4/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*x**3*sqrt(-a**2*x**
2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sq
rt(-a**2*x**2 + 1)), x) + Integral(a*x**5/(a**4*x**4*sqrt(-a**2*x**2 + 1) -
 4*a**3*x**3*sqrt(-a**2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*
x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

Giac [A] time = 1.199, size = 325, normalized size = 1.93

$$\frac{5\arcsin(ax)\,\mathrm{sgn}(a)}{a^4c^4|a|} - \frac{\sqrt{-a^2x^2+1}}{a^5c^4} + \frac{2\left(\frac{4508\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x} - \frac{11529\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2} + \frac{15050\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3} - \frac{10115\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4} + \cdots\right)}{105\,a^4c^4\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^7 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 5*arcsin(a*x)*sgn(a)/(a^4*c^4*abs(a)) - sqrt(-a^2*x^2 + 1)/(a^5*c^4) + 2/10
5*(4508*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 11529*(sqrt(-a^2*x^2 + 1)
*abs(a) + a)^2/(a^4*x^2) + 15050*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3
) - 10115*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 3570*(sqrt(-a^2*x^2
 + 1)*abs(a) + a)^5/(a^10*x^5) - 525*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a^1
2*x^6) - 719)/(a^4*c^4*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^7*abs(
a))
```

3.356 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-acx)^4} dx$

Optimal. Leaf size=138

$$\frac{86\left(1-a^2x^2\right)^{3/2}}{105a^4c^4(1-ax)^3}-\frac{19\left(1-a^2x^2\right)^{3/2}}{35a^4c^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^4c^4(1-ax)^5}-\frac{2\sqrt{1-a^2x^2}}{a^4c^4(1-ax)}+\frac{\sin^{-1}(ax)}{a^4c^4}$$

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a^4*c^4*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(7*a^4*c^4
*(1 - a*x)^5) - (19*(1 - a^2*x^2)^(3/2))/(35*a^4*c^4*(1 - a*x)^4) + (86*(1
- a^2*x^2)^(3/2))/(105*a^4*c^4*(1 - a*x)^3) + ArcSin[a*x]/(a^4*c^4)
```

Rubi [A] time = 0.271601, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 1637, 659, 651, 663, 216}

$$\frac{86\left(1-a^2x^2\right)^{3/2}}{105a^4c^4(1-ax)^3}-\frac{19\left(1-a^2x^2\right)^{3/2}}{35a^4c^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^4c^4(1-ax)^5}-\frac{2\sqrt{1-a^2x^2}}{a^4c^4(1-ax)}+\frac{\sin^{-1}(ax)}{a^4c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a*c*x)^4,x]
```


```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a^4*c^4*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(7*a^4*c^4
*(1 - a*x)^5) - (19*(1 - a^2*x^2)^(3/2))/(35*a^4*c^4*(1 - a*x)^4) + (86*(1
- a^2*x^2)^(3/2))/(105*a^4*c^4*(1 - a*x)^3) + ArcSin[a*x]/(a^4*c^4)
```


Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
```

```
0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-acx)^4}\,dx &= c\int \frac{x^3\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx \\
&= c\int \left(-\frac{\sqrt{1-a^2x^2}}{a^3c^5(-1+ax)^5}-\frac{3\sqrt{1-a^2x^2}}{a^3c^5(-1+ax)^4}-\frac{3\sqrt{1-a^2x^2}}{a^3c^5(-1+ax)^3}-\frac{\sqrt{1-a^2x^2}}{a^3c^5(-1+ax)^2}\right)dx \\
&= -\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^5}\,dx}{a^3c^4}-\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}\,dx}{a^3c^4}-\frac{3\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{a^3c^4}-\frac{3\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{a^3c^4} \\
&= -\frac{2\sqrt{1-a^2x^2}}{a^4c^4(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^4c^4(1-ax)^5}-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^4c^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^4(1-ax)^3}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{7a^3c^4}+\frac{3\int \frac{\sqrt{1-a^2x}}{(-1+ax}}{5a^3c^4} \\
&= -\frac{2\sqrt{1-a^2x^2}}{a^4c^4(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^4c^4(1-ax)^5}-\frac{19\left(1-a^2x^2\right)^{3/2}}{35a^4c^4(1-ax)^4}+\frac{4\left(1-a^2x^2\right)^{3/2}}{5a^4c^4(1-ax)^3}+\frac{\sin^{-1}(ax)}{a^4c^4}-\frac{2\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,d}{35a^3c^4} \\
&= -\frac{2\sqrt{1-a^2x^2}}{a^4c^4(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^4c^4(1-ax)^5}-\frac{19\left(1-a^2x^2\right)^{3/2}}{35a^4c^4(1-ax)^4}+\frac{86\left(1-a^2x^2\right)^{3/2}}{105a^4c^4(1-ax)^3}+\frac{\sin^{-1}(ax)}{a^4c^4}
\end{aligned}$$

Mathematica [A] time = 0.233386, size = 94, normalized size = 0.68

$$\frac{\sqrt{ax+1}\left(\sqrt{1-a^2x^2}\left(296a^3x^3-659a^2x^2+559ax-166\right)+105(ax-1)^4\sin^{-1}(ax)\right)}{105a^4c^4(1-ax)^{7/2}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a*c*x)^4,x]
```

```
[Out] (Sqrt[1 + a*x]*(Sqrt[1 - a^2*x^2]*(-166 + 559*a*x - 659*a^2*x^2 + 296*a^3*x
^3) + 105*(-1 + a*x)^4*ArcSin[a*x]))/(105*a^4*c^4*(1 - a*x)^(7/2)*Sqrt[1 -
a^2*x^2])
```

Maple [A] time = 0.065, size = 210, normalized size = 1.5

$$\frac{1}{c^4a^3}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{43}{35\,c^4a^7}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}+\frac{229}{105\,a^6c^4}\sqrt{-a^2\left(x-a^{-1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^4,x)
```

```
[Out] 1/c^4/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+43/35/c^4/a^
7/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+229/105/c^4/a^6/(x-1/a)^2*(-
a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+296/105/c^4/a^5/(x-1/a)*(-a^2*(x-1/a)^2-2*
a*(x-1/a))^(1/2)+2/7/c^4/a^8/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64324, size = 394, normalized size = 2.86

$$-\frac{166\,a^4x^4-664\,a^3x^3+996\,a^2x^2-664\,ax+210\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(296\,a^3x^3-65\right.}{105\left(a^8c^4x^4-4\,a^7c^4x^3+6\,a^6c^4x^2-4\,a^5c^4x+a^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] -1/105*(166*a^4*x^4 - 664*a^3*x^3 + 996*a^2*x^2 - 664*a*x + 210*(a^4*x^4 -
4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) -
 (296*a^3*x^3 - 659*a^2*x^2 + 559*a*x - 166)*sqrt(-a^2*x^2 + 1) + 166)/(a^8
*c^4*x^4 - 4*a^7*c^4*x^3 + 6*a^6*c^4*x^2 - 4*a^5*c^4*x + a^4*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^4}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(x**3/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*x**3*sqrt(-a**2*x**
2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sq
rt(-a**2*x**2 + 1)), x) + Integral(a*x**4/(a**4*x**4*sqrt(-a**2*x**2 + 1) -
 4*a**3*x**3*sqrt(-a**2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*
x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

Giac [A] time = 1.22032, size = 297, normalized size = 2.15

$$\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{a^3c^4|a|}+\frac{2\left(\frac{1057\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{2751\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{3640\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}-\frac{2170\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4}+\cdots\right.}{105\,a^3c^4\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/(a^3*c^4*abs(a)) + 2/105*(1057*(sqrt(-a^2*x^2 + 1)*abs(a
) + a)/(a^2*x) - 2751*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^4*x^2) + 3640*(s
qrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 2170*(sqrt(-a^2*x^2 + 1)*abs(a)
 + a)^4/(a^8*x^4) + 735*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5/(a^10*x^5) - 105*
(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a^12*x^6) - 166)/(a^3*c^4*((sqrt(-a^2*x^
2 + 1)*abs(a) + a)/(a^2*x) - 1)^7*abs(a))
```

3.357 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-acx)^4}\,dx$

Optimal. Leaf size=97

$$\frac{23\left(1-a^2x^2\right)^{3/2}}{105a^3c^4(1-ax)^3}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3c^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^3c^4(1-ax)^5}$$

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a^3*c^4*(1 - a*x)^5) - (12*(1 - a^2*x^2)^(3/2))/(35*
a^3*c^4*(1 - a*x)^4) + (23*(1 - a^2*x^2)^(3/2))/(105*a^3*c^4*(1 - a*x)^3)
```

Rubi [A] time = 0.208041, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6128, 1639, 793, 659, 651}

$$\frac{23\left(1-a^2x^2\right)^{3/2}}{105a^3c^4(1-ax)^3}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3c^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^3c^4(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a*c*x)^4,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a^3*c^4*(1 - a*x)^5) - (12*(1 - a^2*x^2)^(3/2))/(35*
a^3*c^4*(1 - a*x)^4) + (23*(1 - a^2*x^2)^(3/2))/(105*a^3*c^4*(1 - a*x)^3)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-acx)^4}\,dx &= c\int \frac{x^2\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx\\ &= -\frac{\left(1-a^2x^2\right)^{3/2}}{a^3c^4(1-ax)^4}+\frac{\int \frac{\left(4a^2c^2-3a^3c^2x\right)\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx}{a^4c}\\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3c^4(1-ax)^5}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3c^4(1-ax)^4}+\frac{23\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx}{7a^2}\\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3c^4(1-ax)^5}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3c^4(1-ax)^4}+\frac{23\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx}{35a^2c}\\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3c^4(1-ax)^5}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3c^4(1-ax)^4}+\frac{23\left(1-a^2x^2\right)^{3/2}}{105a^3c^4(1-ax)^3}\end{aligned}$$

Mathematica [A] time = 0.0263635, size = 43, normalized size = 0.44

$$-\frac{(ax+1)^{3/2}\left(-23a^2x^2+10ax-2\right)}{105a^3c^4(1-ax)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a*c*x)^4,x]
```

```
[Out] -((1 + a*x)^(3/2)*(-2 + 10*a*x - 23*a^2*x^2))/(105*a^3*c^4*(1 - a*x)^(7/2))
```

Maple [A] time = 0.034, size = 49, normalized size = 0.5

$$-\frac{\left(23\,a^2x^2-10\,ax+2\right)(ax+1)^2}{105\,c^4\,(ax-1)^3\,a^3}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^4,x)
```

```
[Out] -1/105*(23*a^2*x^2-10*a*x+2)*(a*x+1)^2/(a*x-1)^3/c^4/(-a^2*x^2+1)^(1/2)/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79434, size = 250, normalized size = 2.58

$$\frac{2\,a^{4}x^{4}-8\,a^{3}x^{3}+12\,a^{2}x^{2}-8\,ax+\left(23\,a^{3}x^{3}+13\,a^{2}x^{2}-8\,ax+2\right)\sqrt{-a^{2}x^{2}+1}+2}{105\left(a^{7}c^{4}x^{4}-4\,a^{6}c^{4}x^{3}+6\,a^{5}c^{4}x^{2}-4\,a^{4}c^{4}x+a^{3}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] 1/105*(2*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 8*a*x + (23*a^3*x^3 + 13*a^2*x^
2 - 8*a*x + 2)*sqrt(-a^2*x^2 + 1) + 2)/(a^7*c^4*x^4 - 4*a^6*c^4*x^3 + 6*a^5
*c^4*x^2 - 4*a^4*c^4*x + a^3*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^{2}}{a^{4}x^{4}\sqrt{-a^{2}x^{2}+1}-4a^{3}x^{3}\sqrt{-a^{2}x^{2}+1}+6a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}-4ax\sqrt{-a^{2}x^{2}+1}+\sqrt{-a^{2}x^{2}+1}}\,dx+\int \frac{ax^{3}}{a^{4}x^{4}\sqrt{-a^{2}x^{2}+1}-4a^{3}x^{3}\sqrt{-a^{2}x^{2}+1}+6a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}-4ax\sqrt{-a^{2}x^{2}+1}}}{c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(x**2/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*x**3*sqrt(-a**2*x**
2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sq
rt(-a**2*x**2 + 1)), x) + Integral(a*x**3/(a**4*x**4*sqrt(-a**2*x**2 + 1) -
 4*a**3*x**3*sqrt(-a**2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*
x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

Giac [A] time = 1.29527, size = 200, normalized size = 2.06

$$-\frac{4\left(\frac{7\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)}{a^{2}x}-\frac{21\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)^{2}}{a^{4}x^{2}}-\frac{35\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)^{3}}{a^{6}x^{3}}-\frac{70\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)^{4}}{a^{8}x^{4}}-1\right)}{105\,a^{2}c^{4}\left(\frac{\sqrt{-a^{2}x^{2}+1}|a|+a}{a^{2}x}-1\right)^{7}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -4/105*(7*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 21*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^2/(a^4*x^2) - 35*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) -
70*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 1)/(a^2*c^4*((sqrt(-a^2*x^
2 + 1)*abs(a) + a)/(a^2*x) - 1)^7*abs(a))
```

3.358 $\int \frac{e^{\tanh^{-1}(ax)}x}{(c-acx)^4}\,dx$

Optimal. Leaf size=97

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{21a^2c^4(1-ax)^3}-\frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^5}$$

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a^2*c^4*(1 - a*x)^5) - (1 - a^2*x^2)^(3/2)/(7*a^2*c^
4*(1 - a*x)^4) - (1 - a^2*x^2)^(3/2)/(21*a^2*c^4*(1 - a*x)^3)
```

Rubi [A] time = 0.100774, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6128, 793, 659, 651}

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{21a^2c^4(1-ax)^3}-\frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a*c*x)^4,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a^2*c^4*(1 - a*x)^5) - (1 - a^2*x^2)^(3/2)/(7*a^2*c^
4*(1 - a*x)^4) - (1 - a^2*x^2)^(3/2)/(21*a^2*c^4*(1 - a*x)^3)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
```

```
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x}{(c-acx)^4}\,dx &= c\int \frac{x\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^5}-\frac{5\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx}{7a}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^5}-\frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^4}-\frac{\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx}{7ac}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^5}-\frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^4}-\frac{\left(1-a^2x^2\right)^{3/2}}{21a^2c^4(1-ax)^3}
\end{aligned}$$

Mathematica [A] time = 0.0192582, size = 42, normalized size = 0.43

$$-\frac{(ax+1)^{3/2}\left(a^2x^2-5ax+1\right)}{21a^2c^4(1-ax)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a*c*x)^4,x]
```

```
[Out] -((1 + a*x)^(3/2)*(1 - 5*a*x + a^2*x^2))/(21*a^2*c^4*(1 - a*x)^(7/2))
```

Maple [A] time = 0.038, size = 48, normalized size = 0.5

$$\frac{\left(a^2x^2-5\,ax+1\right)(ax+1)^2}{21\,c^4\,(ax-1)^3\,a^2}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^4,x)
```

```
[Out] 1/21*(a^2*x^2-5*a*x+1)*(a*x+1)^2/(a*x-1)^3/c^4/(-a^2*x^2+1)^(1/2)/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88299, size = 240, normalized size = 2.47

$$-\frac{a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+\left(a^{3}x^{3}-4\,a^{2}x^{2}-4\,ax+1\right)\sqrt{-a^{2}x^{2}+1}+1}{21\,\left(a^{6}c^{4}x^{4}-4\,a^{5}c^{4}x^{3}+6\,a^{4}c^{4}x^{2}-4\,a^{3}c^{4}x+a^{2}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/21*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + (a^3*x^3 - 4*a^2*x^2 - 4*a
*x + 1)*sqrt(-a^2*x^2 + 1) + 1)/(a^6*c^4*x^4 - 4*a^5*c^4*x^3 + 6*a^4*c^4*x^
2 - 4*a^3*c^4*x + a^2*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^{4}x^{4}\sqrt{-a^{2}x^{2}+1}-4a^{3}x^{3}\sqrt{-a^{2}x^{2}+1}+6a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}-4ax\sqrt{-a^{2}x^{2}+1}+\sqrt{-a^{2}x^{2}+1}}\,dx+\int \frac{ax^{2}}{a^{4}x^{4}\sqrt{-a^{2}x^{2}+1}-4a^{3}x^{3}\sqrt{-a^{2}x^{2}+1}+6a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}-4ax\sqrt{-a^{2}x^{2}+1}+\sqrt{-a^{2}x^{2}+1}}\,dx}{c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(x/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*x**3*sqrt(-a**2*x**2 +
 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sqrt(
-a**2*x**2 + 1)), x) + Integral(a*x**2/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*
a**3*x**3*sqrt(-a**2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*s
qrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

Giac [A] time = 1.19962, size = 200, normalized size = 2.06

$$\frac{2\left(\frac{7\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)}{a^{2}x}+\frac{28\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)^{3}}{a^{6}x^{3}}-\frac{7\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)^{4}}{a^{8}x^{4}}+\frac{21\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)^{5}}{a^{10}x^{5}}-1\right)}{21\,ac^{4}\left(\frac{\sqrt{-a^{2}x^{2}+1}|a|+a}{a^{2}x}-1\right)^{7}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 2/21*(7*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 28*(sqrt(-a^2*x^2 + 1)*ab
s(a) + a)^3/(a^6*x^3) - 7*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 21*
(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5/(a^10*x^5) - 1)/(a*c^4*((sqrt(-a^2*x^2 +
1)*abs(a) + a)/(a^2*x) - 1)^7*abs(a))
```

3.359 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=97

$$\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^4(1-ax)^3}+\frac{2\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}$$

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a*c^4*(1 - a*x)^5) + (2*(1 - a^2*x^2)^(3/2))/(35*a*c
^4*(1 - a*x)^4) + (2*(1 - a^2*x^2)^(3/2))/(105*a*c^4*(1 - a*x)^3)
```

Rubi [A] time = 0.070342, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6127, 659, 651}

$$\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^4(1-ax)^3}+\frac{2\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^4,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a*c^4*(1 - a*x)^5) + (2*(1 - a^2*x^2)^(3/2))/(35*a*c
^4*(1 - a*x)^4) + (2*(1 - a^2*x^2)^(3/2))/(105*a*c^4*(1 - a*x)^3)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^4}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}+\frac{2}{7}\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx}{35c}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^4(1-ax)^3}
\end{aligned}
$$

Mathematica [A] time = 0.0197159, size = 43, normalized size = 0.44

$$
-\frac{(ax+1)^{3/2}\left(-2a^2x^2+10ax-23\right)}{105ac^4(1-ax)^{7/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^4,x]
```

```
[Out] -((1 + a*x)^(3/2)*(-23 + 10*a*x - 2*a^2*x^2))/(105*a*c^4*(1 - a*x)^(7/2))
```

Maple [A] time = 0.035, size = 49, normalized size = 0.5

$$
-\frac{\left(2\,a^2x^2-10\,ax+23\right)(ax+1)^2}{105\,(ax-1)^3\,c^4a}\frac{1}{\sqrt{-a^2x^2+1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x)
```

```
[Out] -1/105*(2*a^2*x^2-10*a*x+23)*(a*x+1)^2/(a*x-1)^3/c^4/(-a^2*x^2+1)^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88557, size = 254, normalized size = 2.62

$$
\frac{23\,a^4x^4-92\,a^3x^3+138\,a^2x^2-92\,ax+\left(2\,a^3x^3-8\,a^2x^2+13\,ax+23\right)\sqrt{-a^2x^2+1}+23}{105\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(23*a^4*x^4 - 92*a^3*x^3 + 138*a^2*x^2 - 92*a*x + (2*a^3*x^3 - 8*a^2*
x^2 + 13*a*x + 23)*sqrt(-a^2*x^2 + 1) + 23)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 +
6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx + \int \frac{1}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(a*x/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*x**3*sqrt(-a**2*x**2
 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sqr
t(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**
3*x**3*sqrt(-a**2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt
(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

Giac [B] time = 1.29471, size = 269, normalized size = 2.77

$$-\frac{2\left(\frac{56\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x} - \frac{273\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2} + \frac{350\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3} - \frac{455\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4} + \frac{210\left(\sqrt{-a^2x^2+1}|a|+a\right)^5}{a^{10}x^5} - \frac{105\left(\sqrt{-a^2x^2+1}|a|+a\right)^6}{a^{12}x^6} - 23\right)}{105\,c^4\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)^7 |a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -2/105*(56*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 273*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)^2/(a^4*x^2) + 350*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3)
 - 455*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 210*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)^5/(a^10*x^5) - 105*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a^12*x^
6) - 23)/(c^4*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^7*abs(a))
```

3.360 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-acx)^4}\,dx$

Optimal. Leaf size=128

$$-\frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{166ax+105}{105c^4\sqrt{1-a^2x^2}}+\frac{83ax+35}{105c^4\left(1-a^2x^2\right)^{3/2}}+\frac{16(ax+1)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^4}$$

```
[Out] (16*(1 + a*x))/(7*c^4*(1 - a^2*x^2)^(7/2)) - (4*(7 - 3*a*x))/(35*c^4*(1 - a
^2*x^2)^(5/2)) + (35 + 83*a*x)/(105*c^4*(1 - a^2*x^2)^(3/2)) + (105 + 166*a
*x)/(105*c^4*Sqrt[1 - a^2*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c^4
```

Rubi [A] time = 0.31061, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 823, 12, 266, 63, 208}

$$-\frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{166ax+105}{105c^4\sqrt{1-a^2x^2}}+\frac{83ax+35}{105c^4\left(1-a^2x^2\right)^{3/2}}+\frac{16(ax+1)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(c - a*c*x)^4),x]
```

```
[Out] (16*(1 + a*x))/(7*c^4*(1 - a^2*x^2)^(7/2)) - (4*(7 - 3*a*x))/(35*c^4*(1 - a
^2*x^2)^(5/2)) + (35 + 83*a*x)/(105*c^4*(1 - a^2*x^2)^(3/2)) + (105 + 166*a
*x)/(105*c^4*Sqrt[1 - a^2*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c^4
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x(c-acx)^4}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x(c-acx)^5}\,dx$$
$$= \frac{\int \frac{(c+acx)^5}{x(1-a^2x^2)^{9/2}}\,dx}{c^9}$$
$$= \frac{16(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{\int \frac{-7c^5-19ac^5x+35a^2c^5x^2+7a^3c^5x^3}{x(1-a^2x^2)^{7/2}}\,dx}{7c^9}$$
$$= \frac{16(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{\int \frac{35c^5+83ac^5x}{x(1-a^2x^2)^{5/2}}\,dx}{35c^9}$$
$$= \frac{16(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{35+83ax}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{105a^2c^5+166a^3c^5x}{x(1-a^2x^2)^{3/2}}\,dx}{105a^2c^9}$$
$$= \frac{16(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{35+83ax}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{105+166ax}{105c^4\sqrt{1-a^2x^2}} + \frac{\int \frac{105a^4c^5}{x\sqrt{1-a^2x^2}}\,dx}{105a^4c^9}$$
$$= \frac{16(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{35+83ax}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{105+166ax}{105c^4\sqrt{1-a^2x^2}} + \frac{\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c^4}$$
$$= \frac{16(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{35+83ax}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{105+166ax}{105c^4\sqrt{1-a^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,d\right.}{2c^4}$$
$$= \frac{16(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{35+83ax}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{105+166ax}{105c^4\sqrt{1-a^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,\right.}{a^2c^4}$$
$$= \frac{16(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{35+83ax}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{105+166ax}{105c^4\sqrt{1-a^2x^2}} - \frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right.}{c^4}$$

Mathematica [C] time = 0.171819, size = 79, normalized size = 0.62

$$\frac{15\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},1-a^2x^2\right)-166a^7x^7+581a^5x^5-700a^3x^3+105a^2x^2+525ax+120}{105c^4\left(1-a^2x^2\right)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a*c*x)^4),x]
```

```
[Out] (120 + 525*a*x + 105*a^2*x^2 - 700*a^3*x^3 + 581*a^5*x^5 - 166*a^7*x^7 + 15
*Hypergeometric2F1[-7/2, 1, -5/2, 1 - a^2*x^2])/(105*c^4*(1 - a^2*x^2)^(7/2
))
```

Maple [B] time = 0.047, size = 451, normalized size = 3.5

$$\frac{1}{c^4}\left(-\frac{1}{a^2}\left(\frac{1}{5a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}-\frac{2a}{5}\left(\frac{1}{3a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-2}-\frac{1}{3}\sqrt{-a\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^4,x)
```

```
[Out] 1/c^4*(-1/a^2*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2/5*a*(1/
3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^
2-2*a*(x-1/a))^(1/2)))+2/a^3*(1/7/a/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^
(1/2)-3/7*a*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2/5*a*(1/3/
a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-
2*a*(x-1/a))^(1/2))))-arctanh(1/(-a^2*x^2+1)^(1/2))+1/a*(1/3/a/(x-1/a)^2*(-
a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(
1/2))-1/a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,(acx-c)^4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^4*x), x)
```

Fricas [A] time = 1.77956, size = 379, normalized size = 2.96

$$\frac{296\,a^4x^4 - 1184\,a^3x^3 + 1776\,a^2x^2 - 1184\,ax + 105\left(a^4x^4 - 4\,a^3x^3 + 6\,a^2x^2 - 4\,ax + 1\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \left(166\,a^3x^3 - 5\right.}{105\left(a^4c^4x^4 - 4\,a^3c^4x^3 + 6\,a^2c^4x^2 - 4\,ac^4x + c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(296*a^4*x^4 - 1184*a^3*x^3 + 1776*a^2*x^2 - 1184*a*x + 105*(a^4*x^4
- 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (166
*a^3*x^3 - 559*a^2*x^2 + 659*a*x - 296)*sqrt(-a^2*x^2 + 1) + 296)/(a^4*c^4*
x^4 - 4*a^3*c^4*x^3 + 6*a^2*c^4*x^2 - 4*a*c^4*x + c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{a^4x^5\sqrt{-a^2x^2+1}-4a^3x^4\sqrt{-a^2x^2+1}+6a^2x^3\sqrt{-a^2x^2+1}-4ax^2\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx + \int \frac{1}{a^4x^5\sqrt{-a^2x^2+1}-4a^3x^4\sqrt{-a^2x^2+1}+6a^2x^3\sqrt{-a^2x^2+1}-4ax^2\sqrt{-}}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(a*x/(a**4*x**5*sqrt(-a**2*x**2 + 1) - 4*a**3*x**4*sqrt(-a**2*x**2
 + 1) + 6*a**2*x**3*sqrt(-a**2*x**2 + 1) - 4*a*x**2*sqrt(-a**2*x**2 + 1) +
x*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**5*sqrt(-a**2*x**2 + 1) -
```

```
4*a**3*x**4*sqrt(-a**2*x**2 + 1) + 6*a**2*x**3*sqrt(-a**2*x**2 + 1) - 4*a*x
**2*sqrt(-a**2*x**2 + 1) + x*sqrt(-a**2*x**2 + 1)), x))/c**4
```

Giac [B] time = 1.34777, size = 328, normalized size = 2.56

$$-\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{c^4|a|}+\frac{2\left(296a-\frac{1547\left(\sqrt{-a^2x^2+1}|a|+a\right)}{ax}+\frac{4011\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^3x^2}-\frac{5600\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^5x^3}+\frac{4760\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^7x^4}\right.}{105c^4\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c^4*abs(a
)) + 2/105*(296*a - 1547*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a*x) + 4011*(sqrt
(-a^2*x^2 + 1)*abs(a) + a)^2/(a^3*x^2) - 5600*(sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3/(a^5*x^3) + 4760*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^7*x^4) - 2205*(s
qrt(-a^2*x^2 + 1)*abs(a) + a)^5/(a^9*x^5) + 525*(sqrt(-a^2*x^2 + 1)*abs(a)
+ a)^6/(a^11*x^6))/(c^4*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^7*abs
(a))
```

3.361 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-acx)^4}\,dx$

Optimal. Leaf size=155

$$\frac{16a(ax+1)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{\sqrt{1-a^2x^2}}{c^4x}+\frac{a(719ax+525)}{105c^4\sqrt{1-a^2x^2}}+\frac{a(307ax+175)}{105c^4\left(1-a^2x^2\right)^{3/2}}+\frac{4a(17ax+7)}{35c^4\left(1-a^2x^2\right)^{5/2}}-\frac{5a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^4}$$

```
[Out] (16*a*(1 + a*x))/(7*c^4*(1 - a^2*x^2)^(7/2)) + (4*a*(7 + 17*a*x))/(35*c^4*(
1 - a^2*x^2)^(5/2)) + (a*(175 + 307*a*x))/(105*c^4*(1 - a^2*x^2)^(3/2)) + (
a*(525 + 719*a*x))/(105*c^4*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(c^4*x)
- (5*a*ArcTanh[Sqrt[1 - a^2*x^2]])/c^4
```

Rubi [A] time = 0.449472, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 852, 1805, 807, 266, 63, 208}

$$\frac{16a(ax+1)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{\sqrt{1-a^2x^2}}{c^4x}+\frac{a(719ax+525)}{105c^4\sqrt{1-a^2x^2}}+\frac{a(307ax+175)}{105c^4\left(1-a^2x^2\right)^{3/2}}+\frac{4a(17ax+7)}{35c^4\left(1-a^2x^2\right)^{5/2}}-\frac{5a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a*c*x)^4),x]
```

```
[Out] (16*a*(1 + a*x))/(7*c^4*(1 - a^2*x^2)^(7/2)) + (4*a*(7 + 17*a*x))/(35*c^4*(
1 - a^2*x^2)^(5/2)) + (a*(175 + 307*a*x))/(105*c^4*(1 - a^2*x^2)^(3/2)) + (
a*(525 + 719*a*x))/(105*c^4*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(c^4*x)
- (5*a*ArcTanh[Sqrt[1 - a^2*x^2]])/c^4
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-acx)^4}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^2(c-acx)^5}\,dx$$

$$= \frac{\int \frac{(c+acx)^5}{x^2\left(1-a^2x^2\right)^{9/2}}\,dx}{c^9}$$

$$= \frac{16a(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{\int \frac{-7c^5-35ac^5x-61a^2c^5x^2+7a^3c^5x^3}{x^2\left(1-a^2x^2\right)^{7/2}}\,dx}{7c^9}$$

$$= \frac{16a(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a(7+17ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{\int \frac{35c^5+175ac^5x+272a^2c^5x^2}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx}{35c^9}$$

$$= \frac{16a(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a(7+17ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a(175+307ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{-105c^5-525ac^5x-614a^2c^5x^2}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{105c^9}$$

$$= \frac{16a(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a(7+17ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a(175+307ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a(525+719ax)}{105c^4\sqrt{1-a^2x^2}} + \frac{\int \frac{105c^5+525ac^5x}{x^2\sqrt{1-a^2x^2}}}{105c^9}$$

$$= \frac{16a(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a(7+17ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a(175+307ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a(525+719ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^4x} + \frac{(5}{}$$

$$= \frac{16a(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a(7+17ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a(175+307ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a(525+719ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^4x} + \frac{(5}{}$$

$$= \frac{16a(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a(7+17ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a(175+307ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a(525+719ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^4x} - \frac{5}{}$$

$$= \frac{16a(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a(7+17ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a(175+307ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a(525+719ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^4x} - \frac{5}{}$$

Mathematica [A] time = 0.0610804, size = 109, normalized size = 0.7

$$\frac{824a^5x^5 - 1947a^4x^4 + 485a^3x^3 + 1812a^2x^2 - 525ax(ax-1)^3\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - 1339ax + 105}{105c^4x(ax-1)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a*c*x)^4),x]
```

```
[Out] (105 - 1339*a*x + 1812*a^2*x^2 + 485*a^3*x^3 - 1947*a^4*x^4 + 824*a^5*x^5 -
 525*a*x*(-1 + a*x)^3*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(105*c^
4*x*(-1 + a*x)^3*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.053, size = 423, normalized size = 2.7

$$\frac{1}{c^4}\left(-3\,\frac{1}{a}\left(1/5\,\frac{1}{a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3} - 2/5\,a\left(1/3\,\frac{1}{a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^4,x)
```

```
[Out] 1/c^4*(-3/a*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2/5*a*(1/3/
a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-
2*a*(x-1/a))^(1/2)))+4/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-19/
3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-(-a^2*x^2+1)^(1/2)/x+2/a^2*(1/
7/a/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-3/7*a*(1/5/a/(x-1/a)^3*(-a
^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2/5*a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*
(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))))-5*a*arctan
h(1/(-a^2*x^2+1)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)^4x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^4*x^2), x)
```

Fricas [A] time = 1.9712, size = 429, normalized size = 2.77

$$\frac{1024\,a^5x^5 - 4096\,a^4x^4 + 6144\,a^3x^3 - 4096\,a^2x^2 + 1024\,ax + 525\left(a^5x^5 - 4\,a^4x^4 + 6\,a^3x^3 - 4\,a^2x^2 + ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) -}{105\left(a^4c^4x^5 - 4\,a^3c^4x^4 + 6\,a^2c^4x^3 - 4\,ac^4x^2 + c^4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] 1/105*(1024*a^5*x^5 - 4096*a^4*x^4 + 6144*a^3*x^3 - 4096*a^2*x^2 + 1024*a*x
 + 525*(a^5*x^5 - 4*a^4*x^4 + 6*a^3*x^3 - 4*a^2*x^2 + a*x)*log((sqrt(-a^2*x
^2 + 1) - 1)/x) - (824*a^4*x^4 - 2771*a^3*x^3 + 3256*a^2*x^2 - 1444*a*x + 1
05)*sqrt(-a^2*x^2 + 1))/(a^4*c^4*x^5 - 4*a^3*c^4*x^4 + 6*a^2*c^4*x^3 - 4*a*
c^4*x^2 + c^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{a^4x^6\sqrt{-a^2x^2+1} - 4a^3x^5\sqrt{-a^2x^2+1} + 6a^2x^4\sqrt{-a^2x^2+1} - 4ax^3\sqrt{-a^2x^2+1} + x^2\sqrt{-a^2x^2+1}}\,dx + \int \frac{1}{a^4x^6\sqrt{-a^2x^2+1} - 4a^3x^5\sqrt{-a^2x^2+1} + 6a^2x^4\sqrt{-a^2x^2+1} - 4ax^3\sqrt{-a^2}}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(a*x/(a**4*x**6*sqrt(-a**2*x**2 + 1) - 4*a**3*x**5*sqrt(-a**2*x**2
 + 1) + 6*a**2*x**4*sqrt(-a**2*x**2 + 1) - 4*a*x**3*sqrt(-a**2*x**2 + 1) +
x**2*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**6*sqrt(-a**2*x**2 + 1)
 - 4*a**3*x**5*sqrt(-a**2*x**2 + 1) + 6*a**2*x**4*sqrt(-a**2*x**2 + 1) - 4*
a*x**3*sqrt(-a**2*x**2 + 1) + x**2*sqrt(-a**2*x**2 + 1)), x))/c**4
```

Giac [B] time = 1.26586, size = 436, normalized size = 2.81

$$-\frac{5\,a^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{c^4|a|}-\frac{\sqrt{-a^2x^2+1}|a|+a}{2\,c^4x|a|}-\frac{\left(105\,a^2-\frac{4831\left(\sqrt{-a^2x^2+1}|a|+a\right)}{x}+\frac{24997\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^2x^2}-\frac{61131\left(\sqrt{-a^2x^2\cdots\right.}{a^4x^3}\cdots\right.}{210\left(\sqrt{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -5*a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c^4*a
bs(a)) - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(c^4*x*abs(a)) - 1/210*(105*a^
2 - 4831*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/x + 24997*(sqrt(-a^2*x^2 + 1)*abs(
a) + a)^2/(a^2*x^2) - 61131*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^4*x^3) + 8
2915*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^6*x^4) - 66325*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)^5/(a^8*x^5) + 29295*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a^10*x
^6) - 5985*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7/(a^12*x^7))*a^2*x/((sqrt(-a^2*
x^2 + 1)*abs(a) + a)*c^4*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^7*ab
s(a))
```

3.362 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-acx)^4}\,dx$

Optimal. Leaf size=192

$$\frac{a^2(1867ax+1470)}{105c^4\sqrt{1-a^2x^2}}+\frac{a^2(671ax+455)}{105c^4\left(1-a^2x^2\right)^{3/2}}+\frac{4a^2(31ax+21)}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{16a^2(ax+1)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{5a\sqrt{1-a^2x^2}}{c^4x}-\frac{\sqrt{1-a^2x^2}}{2c^4x^2}-\frac{29a^2\,\mathrm{ta}}{}$$

```
[Out] (16*a^2*(1 + a*x))/(7*c^4*(1 - a^2*x^2)^(7/2)) + (4*a^2*(21 + 31*a*x))/(35*
c^4*(1 - a^2*x^2)^(5/2)) + (a^2*(455 + 671*a*x))/(105*c^4*(1 - a^2*x^2)^(3/
2)) + (a^2*(1470 + 1867*a*x))/(105*c^4*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^
2]/(2*c^4*x^2) - (5*a*Sqrt[1 - a^2*x^2])/(c^4*x) - (29*a^2*ArcTanh[Sqrt[1 -
 a^2*x^2]])/(2*c^4)
```

Rubi [A] time = 0.523759, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 1807, 807, 266, 63, 208}

$$\frac{a^2(1867ax+1470)}{105c^4\sqrt{1-a^2x^2}}+\frac{a^2(671ax+455)}{105c^4\left(1-a^2x^2\right)^{3/2}}+\frac{4a^2(31ax+21)}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{16a^2(ax+1)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{5a\sqrt{1-a^2x^2}}{c^4x}-\frac{\sqrt{1-a^2x^2}}{2c^4x^2}-\frac{29a^2\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a*c*x)^4),x]
```


```
[Out] (16*a^2*(1 + a*x))/(7*c^4*(1 - a^2*x^2)^(7/2)) + (4*a^2*(21 + 31*a*x))/(35*
c^4*(1 - a^2*x^2)^(5/2)) + (a^2*(455 + 671*a*x))/(105*c^4*(1 - a^2*x^2)^(3/
2)) + (a^2*(1470 + 1867*a*x))/(105*c^4*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^
2]/(2*c^4*x^2) - (5*a*Sqrt[1 - a^2*x^2])/(c^4*x) - (29*a^2*ArcTanh[Sqrt[1 -
 a^2*x^2]])/(2*c^4)
```


Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-acx)^4}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^3(c-acx)^5}\,dx$$

$$= \frac{\int \frac{(c+acx)^5}{x^3\left(1-a^2x^2\right)^{9/2}}\,dx}{c^9}$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{\int \frac{-7c^5-35ac^5x-77a^2c^5x^2-89a^3c^5x^3}{x^3\left(1-a^2x^2\right)^{7/2}}\,dx}{7c^9}$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a^2(21+31ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{\int \frac{35c^5+175ac^5x+420a^2c^5x^2+496a^3c^5x^3}{x^3\left(1-a^2x^2\right)^{5/2}}\,dx}{35c^9}$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a^2(21+31ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2(455+671ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{-105c^5-525ac^5x-1365a^2c^5x^2-1342a^3c^5x^3}{x^3\left(1-a^2x^2\right)^{3/2}}\,d}{105c^9}$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a^2(21+31ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2(455+671ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(1470+1867ax)}{105c^4\sqrt{1-a^2x^2}} + \frac{\int \frac{105c^5+525ac^5x+}{x^3\sqrt{1-a}}}{105}$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a^2(21+31ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2(455+671ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(1470+1867ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^4x^2} - \int$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a^2(21+31ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2(455+671ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(1470+1867ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^4x^2} - 5$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a^2(21+31ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2(455+671ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(1470+1867ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^4x^2} - 5$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a^2(21+31ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2(455+671ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(1470+1867ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^4x^2} - 5$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a^2(21+31ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2(455+671ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(1470+1867ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^4x^2} - 5$$

Mathematica [A] time = 0.0654024, size = 121, normalized size = 0.63

$$\frac{4784a^6x^6-11307a^5x^5+2825a^4x^4+10512a^3x^3-7774a^2x^2-3045a^2x^2(ax-1)^3\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+735ax}{210c^4x^2(ax-1)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a*c*x)^4),x]
```

```
[Out] (105 + 735*a*x - 7774*a^2*x^2 + 10512*a^3*x^3 + 2825*a^4*x^4 - 11307*a^5*x^
5 + 4784*a^6*x^6 - 3045*a^2*x^2*(-1 + a*x)^3*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt
[1 - a^2*x^2]])/(210*c^4*x^2*(-1 + a*x)^3*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.052, size = 397, normalized size = 2.1

$$\frac{1}{c^4}\left(-\frac{1}{a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}+11a\left(1/3\frac{1}{a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-2}-1/3\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^4,x)
```

```
[Out] 1/c^4*(-1/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+11*a*(1/3/a/(x-1/a
)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1
/a))^(1/2))-5*a*(-a^2*x^2+1)^(1/2)/x+2/a*(1/7/a/(x-1/a)^4*(-a^2*(x-1/a)^2-2
*a*(x-1/a))^(1/2)-3/7*a*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
-2/5*a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^
2*(x-1/a)^2-2*a*(x-1/a))^(1/2))))-29/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))-14
*a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/2*(-a^2*x^2+1)^(1/2)/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}(acx-c)^4x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^4*x^3), x)
```

Fricas [A] time = 1.97885, size = 471, normalized size = 2.45

$$\frac{4834\,a^6x^6-19336\,a^5x^5+29004\,a^4x^4-19336\,a^3x^3+4834\,a^2x^2+3045\left(a^6x^6-4\,a^5x^5+6\,a^4x^4-4\,a^3x^3+a^2x^2\right)\log\left(\right.}{210\left(a^4c^4x^6-4\,a^3c^4x^5+6\,a^2c^4x^4-4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] 1/210*(4834*a^6*x^6 - 19336*a^5*x^5 + 29004*a^4*x^4 - 19336*a^3*x^3 + 4834*
a^2*x^2 + 3045*(a^6*x^6 - 4*a^5*x^5 + 6*a^4*x^4 - 4*a^3*x^3 + a^2*x^2)*log(
(sqrt(-a^2*x^2 + 1) - 1)/x) - (4784*a^5*x^5 - 16091*a^4*x^4 + 18916*a^3*x^3
 - 8404*a^2*x^2 + 630*a*x + 105)*sqrt(-a^2*x^2 + 1))/(a^4*c^4*x^6 - 4*a^3*c
^4*x^5 + 6*a^2*c^4*x^4 - 4*a*c^4*x^3 + c^4*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{a^4x^7\sqrt{-a^2x^2+1}-4a^3x^6\sqrt{-a^2x^2+1}+6a^2x^5\sqrt{-a^2x^2+1}-4ax^4\sqrt{-a^2x^2+1}+x^3\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^4x^7\sqrt{-a^2x^2+1}-4a^3x^6\sqrt{-a^2x^2+1}+6a^2x^5\sqrt{-a^2x^2+1}-4ax^4\sqrt{\right.}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(a*x/(a**4*x**7*sqrt(-a**2*x**2 + 1) - 4*a**3*x**6*sqrt(-a**2*x**2
 + 1) + 6*a**2*x**5*sqrt(-a**2*x**2 + 1) - 4*a*x**4*sqrt(-a**2*x**2 + 1) +
x**3*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**7*sqrt(-a**2*x**2 + 1)
 - 4*a**3*x**6*sqrt(-a**2*x**2 + 1) + 6*a**2*x**5*sqrt(-a**2*x**2 + 1) - 4*
a*x**4*sqrt(-a**2*x**2 + 1) + x**3*sqrt(-a**2*x**2 + 1)), x))/c**4
```

Giac [B] time = 1.22932, size = 529, normalized size = 2.76

$$-\frac{29\,a^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,c^4|a|}-\frac{\left(105\,a^3+\frac{1365\left(\sqrt{-a^2x^2+1}|a|+a\right)a}{x}-\frac{51167\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{ax^2}+\frac{260729\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^3x^3}-\frac{621537\left(\sqrt{-a^2\ldots\right.}{a^5x\ldots}\right.}{840\left(\sqrt{-a^2x^2+\ldots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -29/2*a^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c^
4*abs(a)) - 1/840*(105*a^3 + 1365*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 511
67*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a*x^2) + 260729*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)^3/(a^3*x^3) - 621537*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^5*x^4)
 + 826175*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5/(a^7*x^5) - 642005*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^6/(a^9*x^6) + 274995*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7/
(a^11*x^7) - 52500*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^8/(a^13*x^8))*a^4*x^2/((
sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^4*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*
x) - 1)^7*abs(a)) - 1/8*(20*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a*c^4*abs(a)/x
+ (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^4*abs(a)/(a*x^2))/(a^2*c^8)
```

3.363 $\int e^{\tanh^{-1}(x)}x(1+x)\,dx$

Optimal. Leaf size=61

$$-\frac{1}{3}\sqrt{1-x}(x+1)^{5/2}-\frac{1}{3}\sqrt{1-x}(x+1)^{3/2}-\sqrt{1-x}\sqrt{x+1}+\sin^{-1}(x)$$

```
[Out] -(Sqrt[1 - x]*Sqrt[1 + x]) - (Sqrt[1 - x]*(1 + x)^(3/2))/3 - (Sqrt[1 - x]*(
1 + x)^(5/2))/3 + ArcSin[x]
```

Rubi [A] time = 0.0319597, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6129, 80, 50, 41, 216}

$$-\frac{1}{3}\sqrt{1-x}(x+1)^{5/2}-\frac{1}{3}\sqrt{1-x}(x+1)^{3/2}-\sqrt{1-x}\sqrt{x+1}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*x*(1 + x),x]
```

```
[Out] -(Sqrt[1 - x]*Sqrt[1 + x]) - (Sqrt[1 - x]*(1 + x)^(3/2))/3 - (Sqrt[1 - x]*(
1 + x)^(5/2))/3 + ArcSin[x]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(x)}x(1+x)\,dx &= \int \frac{x(1+x)^{3/2}}{\sqrt{1-x}}\,dx\\
&= -\frac{1}{3}\sqrt{1-x}(1+x)^{5/2}+\frac{2}{3}\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx\\
&= -\frac{1}{3}\sqrt{1-x}(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}(1+x)^{5/2}+\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx\\
&= -\sqrt{1-x}\sqrt{1+x}-\frac{1}{3}\sqrt{1-x}(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}(1+x)^{5/2}+\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx\\
&= -\sqrt{1-x}\sqrt{1+x}-\frac{1}{3}\sqrt{1-x}(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}(1+x)^{5/2}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\sqrt{1-x}\sqrt{1+x}-\frac{1}{3}\sqrt{1-x}(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}(1+x)^{5/2}+\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0296669, size = 42, normalized size = 0.69

$$-\frac{1}{3}\sqrt{1-x^2}\left(x^2+3x+5\right)-2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*x*(1 + x),x]
```

```
[Out] -(Sqrt[1 - x^2]*(5 + 3*x + x^2))/3 - 2*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

Maple [A] time = 0.036, size = 41, normalized size = 0.7

$$-\frac{x^2}{3}\sqrt{-x^2+1}-\frac{5}{3}\sqrt{-x^2+1}-x\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/(-x^2+1)^(1/2)*x,x)
```

```
[Out] -1/3*x^2*(-x^2+1)^(1/2)-5/3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2)+arcsin(x)
```

Maxima [A] time = 1.42634, size = 54, normalized size = 0.89

$$-\frac{1}{3}\sqrt{-x^2+1}x^2-\sqrt{-x^2+1}x-\frac{5}{3}\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^2 + 1)*x^2 - sqrt(-x^2 + 1)*x - 5/3*sqrt(-x^2 + 1) + arcsin(x)
```

Fricas [A] time = 1.7552, size = 97, normalized size = 1.59

$$-\frac{1}{3}\left(x^2 + 3x + 5\right)\sqrt{-x^2 + 1} - 2\arctan\left(\frac{\sqrt{-x^2 + 1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] -1/3*(x^2 + 3*x + 5)*sqrt(-x^2 + 1) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.420643, size = 37, normalized size = 0.61

$$-\frac{x^2\sqrt{1 - x^2}}{3} - x\sqrt{1 - x^2} - \frac{5\sqrt{1 - x^2}}{3} + \operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/(-x**2+1)**(1/2)*x,x)
```

```
[Out] -x**2*sqrt(1 - x**2)/3 - x*sqrt(1 - x**2) - 5*sqrt(1 - x**2)/3 + asin(x)
```

Giac [A] time = 1.15787, size = 28, normalized size = 0.46

$$-\frac{1}{3}\left((x + 3)x + 5\right)\sqrt{-x^2 + 1} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2)*x,x, algorithm="giac")
```

```
[Out] -1/3*((x + 3)*x + 5)*sqrt(-x^2 + 1) + arcsin(x)
```

3.364 $\int e^{\tanh^{-1}(x)}(1+x)\,dx$

Optimal. Leaf size=47

$$-\frac{1}{2}\sqrt{1-x}(x+1)^{3/2}-\frac{3}{2}\sqrt{1-x}\sqrt{x+1}+\frac{3}{2}\sin^{-1}(x)$$

```
[Out] (-3*Sqrt[1 - x]*Sqrt[1 + x])/2 - (Sqrt[1 - x]*(1 + x)^(3/2))/2 + (3*ArcSin[
x])/2
```

Rubi [A] time = 0.0174384, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6129, 50, 41, 216}

$$-\frac{1}{2}\sqrt{1-x}(x+1)^{3/2}-\frac{3}{2}\sqrt{1-x}\sqrt{x+1}+\frac{3}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*(1 + x),x]
```

```
[Out] (-3*Sqrt[1 - x]*Sqrt[1 + x])/2 - (Sqrt[1 - x]*(1 + x)^(3/2))/2 + (3*ArcSin[
x])/2
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(x)}(1+x)\,dx = \int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx$$
$$= -\frac{1}{2}\sqrt{1-x}(1+x)^{3/2} + \frac{3}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx$$
$$= -\frac{3}{2}\sqrt{1-x}\sqrt{1+x} - \frac{1}{2}\sqrt{1-x}(1+x)^{3/2} + \frac{3}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx$$
$$= -\frac{3}{2}\sqrt{1-x}\sqrt{1+x} - \frac{1}{2}\sqrt{1-x}(1+x)^{3/2} + \frac{3}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{3}{2}\sqrt{1-x}\sqrt{1+x} - \frac{1}{2}\sqrt{1-x}(1+x)^{3/2} + \frac{3}{2}\sin^{-1}(x)$$

Mathematica [A] time = 0.0119762, size = 37, normalized size = 0.79

$$-\frac{1}{2}\sqrt{1-x^2}(x+4) - 3\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*(1 + x),x]
```

```
[Out] -((4 + x)*Sqrt[1 - x^2])/2 - 3*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

Maple [A] time = 0.034, size = 29, normalized size = 0.6

$$-\frac{x}{2}\sqrt{-x^2+1} + \frac{3\arcsin(x)}{2} - 2\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/(-x^2+1)^(1/2),x)
```

```
[Out] -1/2*x*(-x^2+1)^(1/2)+3/2*arcsin(x)-2*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.43312, size = 38, normalized size = 0.81

$$-\frac{1}{2}\sqrt{-x^2+1}x - 2\sqrt{-x^2+1} + \frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x - 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

Fricas [A] time = 1.79934, size = 86, normalized size = 1.83

$$-\frac{1}{2}\sqrt{-x^2+1}(x+4) - 3\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*(x + 4) - 3*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.236391, size = 27, normalized size = 0.57

$$-\frac{x\sqrt{1-x^2}}{2} - 2\sqrt{1-x^2} + \frac{3\,\mathrm{asin}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -x*sqrt(1 - x**2)/2 - 2*sqrt(1 - x**2) + 3*asin(x)/2
```

Giac [A] time = 1.12413, size = 26, normalized size = 0.55

$$-\frac{1}{2}\sqrt{-x^2+1}(x+4) + \frac{3}{2}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*(x + 4) + 3/2*arcsin(x)
```

3.365 $\int e^{\tanh^{-1}(x)}x(1+x)^2\,dx$

Optimal. Leaf size=87

$$-\frac{1}{4}\sqrt{1-x}(x+1)^{7/2}-\frac{1}{4}\sqrt{1-x}(x+1)^{5/2}-\frac{5}{8}\sqrt{1-x}(x+1)^{3/2}-\frac{15}{8}\sqrt{1-x}\sqrt{x+1}+\frac{15}{8}\sin^{-1}(x)$$

```
[Out] (-15*Sqrt[1 - x]*Sqrt[1 + x])/8 - (5*Sqrt[1 - x]*(1 + x)^(3/2))/8 - (Sqrt[1
 - x]*(1 + x)^(5/2))/4 - (Sqrt[1 - x]*(1 + x)^(7/2))/4 + (15*ArcSin[x])/8
```

Rubi [A] time = 0.0482521, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6129, 80, 50, 41, 216}

$$-\frac{1}{4}\sqrt{1-x}(x+1)^{7/2}-\frac{1}{4}\sqrt{1-x}(x+1)^{5/2}-\frac{5}{8}\sqrt{1-x}(x+1)^{3/2}-\frac{15}{8}\sqrt{1-x}\sqrt{x+1}+\frac{15}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*x*(1 + x)^2,x]
```

```
[Out] (-15*Sqrt[1 - x]*Sqrt[1 + x])/8 - (5*Sqrt[1 - x]*(1 + x)^(3/2))/8 - (Sqrt[1
 - x]*(1 + x)^(5/2))/4 - (Sqrt[1 - x]*(1 + x)^(7/2))/4 + (15*ArcSin[x])/8
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(x)}x(1+x)^2\,dx &= \int \frac{x(1+x)^{5/2}}{\sqrt{1-x}}\,dx \\
&= -\frac{1}{4}\sqrt{1-x}(1+x)^{7/2}+\frac{3}{4}\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx \\
&= -\frac{1}{4}\sqrt{1-x}(1+x)^{5/2}-\frac{1}{4}\sqrt{1-x}(1+x)^{7/2}+\frac{5}{4}\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx \\
&= -\frac{5}{8}\sqrt{1-x}(1+x)^{3/2}-\frac{1}{4}\sqrt{1-x}(1+x)^{5/2}-\frac{1}{4}\sqrt{1-x}(1+x)^{7/2}+\frac{15}{8}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx \\
&= -\frac{15}{8}\sqrt{1-x}\sqrt{1+x}-\frac{5}{8}\sqrt{1-x}(1+x)^{3/2}-\frac{1}{4}\sqrt{1-x}(1+x)^{5/2}-\frac{1}{4}\sqrt{1-x}(1+x)^{7/2}+\frac{15}{8}\int \frac{}{\sqrt{1-}} \\
&= -\frac{15}{8}\sqrt{1-x}\sqrt{1+x}-\frac{5}{8}\sqrt{1-x}(1+x)^{3/2}-\frac{1}{4}\sqrt{1-x}(1+x)^{5/2}-\frac{1}{4}\sqrt{1-x}(1+x)^{7/2}+\frac{15}{8}\int \frac{1}{\sqrt{1-}} \\
&= -\frac{15}{8}\sqrt{1-x}\sqrt{1+x}-\frac{5}{8}\sqrt{1-x}(1+x)^{3/2}-\frac{1}{4}\sqrt{1-x}(1+x)^{5/2}-\frac{1}{4}\sqrt{1-x}(1+x)^{7/2}+\frac{15}{8}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0310331, size = 51, normalized size = 0.59

$$\frac{1}{8}\left(-\sqrt{1-x^2}\left(2x^3+8x^2+15x+24\right)-30\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*x*(1 + x)^2,x]
```

```
[Out] (-(Sqrt[1 - x^2]*(24 + 15*x + 8*x^2 + 2*x^3)) - 30*ArcSin[Sqrt[1 - x]/Sqrt[
2]])/8
```

Maple [A] time = 0.035, size = 57, normalized size = 0.7

$$-\frac{x^3}{4}\sqrt{-x^2+1}-\frac{15\,x}{8}\sqrt{-x^2+1}+\frac{15\arcsin(x)}{8}-x^2\sqrt{-x^2+1}-3\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^3/(-x^2+1)^(1/2)*x,x)
```

```
[Out] -1/4*x^3*(-x^2+1)^(1/2)-15/8*x*(-x^2+1)^(1/2)+15/8*arcsin(x)-x^2*(-x^2+1)^(
1/2)-3*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.43023, size = 76, normalized size = 0.87

$$-\frac{1}{4}\sqrt{-x^2+1}x^3-\sqrt{-x^2+1}x^2-\frac{15}{8}\sqrt{-x^2+1}x-3\sqrt{-x^2+1}+\frac{15}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-x^2+1)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-x^2 + 1)*x^3 - sqrt(-x^2 + 1)*x^2 - 15/8*sqrt(-x^2 + 1)*x - 3*sq
rt(-x^2 + 1) + 15/8*arcsin(x)
```

Fricas [A] time = 1.86501, size = 117, normalized size = 1.34

$$-\frac{1}{8}\left(2x^3+8x^2+15x+24\right)\sqrt{-x^2+1}-\frac{15}{4}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-x^2+1)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] -1/8*(2*x^3 + 8*x^2 + 15*x + 24)*sqrt(-x^2 + 1) - 15/4*arctan((sqrt(-x^2 +
1) - 1)/x)
```

Sympy [A] time = 0.865044, size = 54, normalized size = 0.62

$$-\frac{x^3\sqrt{1-x^2}}{4}-x^2\sqrt{1-x^2}-\frac{15x\sqrt{1-x^2}}{8}-3\sqrt{1-x^2}+\frac{15\operatorname{asin}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**3/(-x**2+1)**(1/2)*x,x)
```

```
[Out] -x**3*sqrt(1 - x**2)/4 - x**2*sqrt(1 - x**2) - 15*x*sqrt(1 - x**2)/8 - 3*sq
rt(1 - x**2) + 15*asin(x)/8
```

Giac [A] time = 1.28611, size = 38, normalized size = 0.44

$$-\frac{1}{8}\left((2(x+4)x+15)x+24\right)\sqrt{-x^2+1}+\frac{15}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-x^2+1)^(1/2)*x,x, algorithm="giac")
```

```
[Out] -1/8*((2*(x + 4)*x + 15)*x + 24)*sqrt(-x^2 + 1) + 15/8*arcsin(x)
```

3.366 $\int e^{\tanh^{-1}(x)}(1+x)^2\,dx$

Optimal. Leaf size=67

$$-\frac{1}{3}\sqrt{1-x}(x+1)^{5/2}-\frac{5}{6}\sqrt{1-x}(x+1)^{3/2}-\frac{5}{2}\sqrt{1-x}\sqrt{x+1}+\frac{5}{2}\sin^{-1}(x)$$

```
[Out] (-5*Sqrt[1 - x]*Sqrt[1 + x])/2 - (5*Sqrt[1 - x]*(1 + x)^(3/2))/6 - (Sqrt[1
- x]*(1 + x)^(5/2))/3 + (5*ArcSin[x])/2
```

Rubi [A] time = 0.0290799, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6129, 50, 41, 216}

$$-\frac{1}{3}\sqrt{1-x}(x+1)^{5/2}-\frac{5}{6}\sqrt{1-x}(x+1)^{3/2}-\frac{5}{2}\sqrt{1-x}\sqrt{x+1}+\frac{5}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*(1 + x)^2,x]
```

```
[Out] (-5*Sqrt[1 - x]*Sqrt[1 + x])/2 - (5*Sqrt[1 - x]*(1 + x)^(3/2))/6 - (Sqrt[1
- x]*(1 + x)^(5/2))/3 + (5*ArcSin[x])/2
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(x)}(1+x)^2\,dx &= \int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx \\
&= -\frac{1}{3}\sqrt{1-x}(1+x)^{5/2}+\frac{5}{3}\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx \\
&= -\frac{5}{6}\sqrt{1-x}(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}(1+x)^{5/2}+\frac{5}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx \\
&= -\frac{5}{2}\sqrt{1-x}\sqrt{1+x}-\frac{5}{6}\sqrt{1-x}(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}(1+x)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx \\
&= -\frac{5}{2}\sqrt{1-x}\sqrt{1+x}-\frac{5}{6}\sqrt{1-x}(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}(1+x)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{5}{2}\sqrt{1-x}\sqrt{1+x}-\frac{5}{6}\sqrt{1-x}(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}(1+x)^{5/2}+\frac{5}{2}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0216032, size = 44, normalized size = 0.66

$$-\frac{1}{6}\sqrt{1-x^2}\left(2x^2+9x+22\right)-5\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*(1 + x)^2,x]
```

```
[Out] -(Sqrt[1 - x^2]*(22 + 9*x + 2*x^2))/6 - 5*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

Maple [A] time = 0.045, size = 43, normalized size = 0.6

$$-\frac{x^2}{3}\sqrt{-x^2+1}-\frac{11}{3}\sqrt{-x^2+1}-\frac{3x}{2}\sqrt{-x^2+1}+\frac{5\arcsin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^3/(-x^2+1)^(1/2),x)
```

```
[Out] -1/3*x^2*(-x^2+1)^(1/2)-11/3*(-x^2+1)^(1/2)-3/2*x*(-x^2+1)^(1/2)+5/2*arcsin
(x)
```

Maxima [A] time = 1.44777, size = 57, normalized size = 0.85

$$-\frac{1}{3}\sqrt{-x^2+1}x^2-\frac{3}{2}\sqrt{-x^2+1}x-\frac{11}{3}\sqrt{-x^2+1}+\frac{5}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^2 + 1)*x^2 - 3/2*sqrt(-x^2 + 1)*x - 11/3*sqrt(-x^2 + 1) + 5/2*
arcsin(x)
```

Fricas [A] time = 1.73717, size = 101, normalized size = 1.51

$$-\frac{1}{6}\left(2\,x^2+9\,x+22\right)\sqrt{-x^2+1}-5\,\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*x^2 + 9*x + 22)*sqrt(-x^2 + 1) - 5*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.418913, size = 44, normalized size = 0.66

$$-\frac{x^2\sqrt{1-x^2}}{3}-\frac{3x\sqrt{1-x^2}}{2}-\frac{11\sqrt{1-x^2}}{3}+\frac{5\,\mathrm{asin}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**3/(-x**2+1)**(1/2),x)
```

```
[Out] -x**2*sqrt(1 - x**2)/3 - 3*x*sqrt(1 - x**2)/2 - 11*sqrt(1 - x**2)/3 + 5*asi
n(x)/2
```

Giac [A] time = 1.17571, size = 34, normalized size = 0.51

$$-\frac{1}{6}\,((2\,x+9)x+22)\sqrt{-x^2+1}+\frac{5}{2}\,\arcsin\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*((2*x + 9)*x + 22)*sqrt(-x^2 + 1) + 5/2*arcsin(x)
```

3.367 $\int \frac{e^{\tanh^{-1}(x)}x}{1+x}\,dx$

Optimal. Leaf size=18

$$-\sqrt{1-x}\sqrt{x+1}$$

```
[Out] -(Sqrt[1 - x]*Sqrt[1 + x])
```

Rubi [A] time = 0.0340615, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6129, 74}

$$-\sqrt{1-x}\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[x]*x)/(1 + x),x]

[Out] -(Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(x)}x}{1+x}\,dx &= \int \frac{x}{\sqrt{1-x}\sqrt{1+x}}\,dx\\ &= -\sqrt{1-x}\sqrt{1+x}\end{aligned}$$

Mathematica [A] time = 0.0052176, size = 13, normalized size = 0.72

$$-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[x]*x)/(1 + x),x]

[Out] -Sqrt[1 - x^2]
```

Maple [A] time = 0.027, size = 17, normalized size = 0.9

$$(1+x)(-1+x)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)*x,x)
```

```
[Out] (1+x)*(-1+x)/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.937585, size = 15, normalized size = 0.83

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)
```

Fricas [C] time = 1.75239, size = 23, normalized size = 1.28

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 1)
```

Sympy [A] time = 0.139766, size = 8, normalized size = 0.44

$$-\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(1 - x**2)
```

Giac [A] time = 1.14251, size = 15, normalized size = 0.83

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)
```

3.368 $\int \frac{e^{\tanh^{-1}(x)}}{1+x} dx$

Optimal. Leaf size=2

$$\sin^{-1}(x)$$

```
[Out] ArcSin[x]
```

Rubi [A] time = 0.0194153, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6129, 41, 216}

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]/(1 + x),x]

[Out] ArcSin[x]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(x)}}{1+x} dx &= \int \frac{1}{\sqrt{1-x}\sqrt{1+x}} dx\\ &= \int \frac{1}{\sqrt{1-x^2}} dx\\ &= \sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0030071, size = 2, normalized size = 1.

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]/(1 + x),x]
```

```
[Out] ArcSin[x]
```

Maple [A] time = 0.033, size = 3, normalized size = 1.5

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2),x)
```

```
[Out] arcsin(x)
```

Maxima [A] time = 1.42471, size = 3, normalized size = 1.5

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x)
```

Fricas [B] time = 1.68103, size = 47, normalized size = 23.5

$$-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.135614, size = 2, normalized size = 1.

$$\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2),x)
```

```
[Out] asin(x)
```

Giac [A] time = 1.13114, size = 3, normalized size = 1.5

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(x)
```

3.369 $\int \frac{e^{\tanh^{-1}(x)}x}{(1+x)^2}\,dx$

Optimal. Leaf size=20

$$\frac{\sqrt{1-x}}{\sqrt{x+1}}+\sin^{-1}(x)$$

```
[Out] Sqrt[1 - x]/Sqrt[1 + x] + ArcSin[x]
```

Rubi [A] time = 0.0335342, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6129, 78, 41, 216}

$$\frac{\sqrt{1-x}}{\sqrt{x+1}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[x]*x)/(1 + x)^2,x]
```

```
[Out] Sqrt[1 - x]/Sqrt[1 + x] + ArcSin[x]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(x)}x}{(1+x)^2}\,dx &= \int \frac{x}{\sqrt{1-x}(1+x)^{3/2}}\,dx \\
&= \frac{\sqrt{1-x}}{\sqrt{1+x}} + \int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx \\
&= \frac{\sqrt{1-x}}{\sqrt{1+x}} + \int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{\sqrt{1-x}}{\sqrt{1+x}} + \sin^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.034002, size = 20, normalized size = 1.

$$
\frac{\sqrt{1-x}}{\sqrt{x+1}} + \sin^{-1}(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[x]*x)/(1 + x)^2,x]
```

```
[Out] Sqrt[1 - x]/Sqrt[1 + x] + ArcSin[x]
```

Maple [A] time = 0.032, size = 24, normalized size = 1.2

$$
\arcsin(x) + \frac{1}{1+x}\sqrt{-(1+x)^2 + 2x + 2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(-x^2+1)^(1/2)*x,x)
```

```
[Out] arcsin(x)+1/(1+x)*(-(1+x)^2+2*x+2)^(1/2)
```

Maxima [A] time = 1.42316, size = 24, normalized size = 1.2

$$
\frac{\sqrt{-x^2+1}}{x+1} + \arcsin(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^2+1)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 1)/(x + 1) + arcsin(x)
```

Fricas [B] time = 1.87501, size = 105, normalized size = 5.25

$$
-\frac{2(x+1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - x - \sqrt{-x^2+1} - 1}{x+1}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^2+1)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] -(2*(x + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - x - sqrt(-x^2 + 1) - 1)/(x + 1
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(x-1)(x+1)}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x**2+1)**(1/2)*x,x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)*(x + 1))*(x + 1)), x)
```

Giac [A] time = 1.23876, size = 32, normalized size = 1.6

$$-\frac{2}{\frac{\sqrt{-x^2+1}-1}{x}-1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^2+1)^(1/2)*x,x, algorithm="giac")
```

```
[Out] -2/((sqrt(-x^2 + 1) - 1)/x - 1) + arcsin(x)
```

3.370 $\int \frac{e^{\tanh^{-1}(x)}}{(1+x)^2} dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

```
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

Rubi [A] time = 0.0181478, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6129, 37}

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]/(1 + x)^2,x]

[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{\tanh^{-1}(x)}}{(1+x)^2} dx &= \int \frac{1}{\sqrt{1-x}(1+x)^{3/2}} dx \\ &= -\frac{\sqrt{1-x}}{\sqrt{1+x}} \end{aligned}$$

Mathematica [A] time = 0.0039239, size = 18, normalized size = 1.

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]/(1 + x)^2,x]
```

```
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

Maple [A] time = 0.029, size = 14, normalized size = 0.8

$$(-1+x)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(-x^2+1)^(1/2),x)
```

```
[Out] (-1+x)/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.41851, size = 22, normalized size = 1.22

$$-\frac{\sqrt{-x^2+1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)/(x + 1)
```

Fricas [A] time = 1.73147, size = 47, normalized size = 2.61

$$-\frac{x+\sqrt{-x^2+1}+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(x + sqrt(-x^2 + 1) + 1)/(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*(x + 1)), x)
```

Giac [A] time = 1.27452, size = 28, normalized size = 1.56

$$\frac{2}{\frac{\sqrt{-x^2+1}-1}{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/((sqrt(-x^2 + 1) - 1)/x - 1)
```

3.371 $\int e^{\tanh^{-1}(x)}x(1+x)^{3/2}\,dx$

Optimal. Leaf size=49

$$\frac{2}{7}(1-x)^{7/2}-2(1-x)^{5/2}+\frac{16}{3}(1-x)^{3/2}-8\sqrt{1-x}$$

```
[Out] -8*Sqrt[1 - x] + (16*(1 - x)^(3/2))/3 - 2*(1 - x)^(5/2) + (2*(1 - x)^(7/2))
/7
```

Rubi [A] time = 0.0479878, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6129, 77}

$$\frac{2}{7}(1-x)^{7/2}-2(1-x)^{5/2}+\frac{16}{3}(1-x)^{3/2}-8\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*x*(1 + x)^(3/2),x]
```

```
[Out] -8*Sqrt[1 - x] + (16*(1 - x)^(3/2))/3 - 2*(1 - x)^(5/2) + (2*(1 - x)^(7/2))
/7
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(x)}x(1+x)^{3/2}\,dx &= \int \frac{x(1+x)^2}{\sqrt{1-x}}\,dx\\ &= \int \left(\frac{4}{\sqrt{1-x}}-8\sqrt{1-x}+5(1-x)^{3/2}-(1-x)^{5/2}\right)dx\\ &= -8\sqrt{1-x}+\frac{16}{3}(1-x)^{3/2}-2(1-x)^{5/2}+\frac{2}{7}(1-x)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0111833, size = 28, normalized size = 0.57

$$-\frac{2}{21}\sqrt{1-x}\left(3x^3+12x^2+23x+46\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]*x*(1 + x)^(3/2),x]
```

```
[Out] (-2*Sqrt[1 - x]*(46 + 23*x + 12*x^2 + 3*x^3))/21
```

Maple [A] time = 0.03, size = 35, normalized size = 0.7

$$\frac{(-2+2x)\left(3x^3+12x^2+23x+46\right)}{21}\sqrt{1+x}\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(-x^2+1)^(1/2)*x,x)
```

```
[Out] 2/21*(-1+x)*(3*x^3+12*x^2+23*x+46)*(1+x)^(1/2)/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.94956, size = 39, normalized size = 0.8

$$\frac{2\left(3x^4+9x^3+11x^2+23x-46\right)}{21\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] 2/21*(3*x^4 + 9*x^3 + 11*x^2 + 23*x - 46)/sqrt(-x + 1)
```

Fricas [A] time = 1.80191, size = 85, normalized size = 1.73

$$-\frac{2\left(3x^3+12x^2+23x+46\right)\sqrt{-x^2+1}}{21\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] -2/21*(3*x^3 + 12*x^2 + 23*x + 46)*sqrt(-x^2 + 1)/sqrt(x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(-x**2+1)**(1/2)*x,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28964, size = 70, normalized size = 1.43

$$-\frac{2}{7}(x-1)^3\sqrt{-x+1}-2(x-1)^2\sqrt{-x+1}+\frac{16}{3}(-x+1)^{\frac{3}{2}}+\frac{64}{21}\sqrt{2}-8\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*x,x, algorithm="giac")
```

```
[Out] -2/7*(x - 1)^3*sqrt(-x + 1) - 2*(x - 1)^2*sqrt(-x + 1) + 16/3*(-x + 1)^(3/2
) + 64/21*sqrt(2) - 8*sqrt(-x + 1)
```

3.372 $\int e^{\tanh^{-1}(x)}(1+x)^{3/2}\,dx$

Optimal. Leaf size=38

$$-\frac{2}{5}(1-x)^{5/2}+\frac{8}{3}(1-x)^{3/2}-8\sqrt{1-x}$$

```
[Out] -8*Sqrt[1 - x] + (8*(1 - x)^(3/2))/3 - (2*(1 - x)^(5/2))/5
```

Rubi [A] time = 0.0288797, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 43}

$$-\frac{2}{5}(1-x)^{5/2}+\frac{8}{3}(1-x)^{3/2}-8\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*(1 + x)^(3/2),x]
```

```
[Out] -8*Sqrt[1 - x] + (8*(1 - x)^(3/2))/3 - (2*(1 - x)^(5/2))/5
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(x)}(1+x)^{3/2}\,dx&=\int\frac{(1+x)^2}{\sqrt{1-x}}\,dx\\&=\int\left(\frac{4}{\sqrt{1-x}}-4\sqrt{1-x}+(1-x)^{3/2}\right)dx\\&=-8\sqrt{1-x}+\frac{8}{3}(1-x)^{3/2}-\frac{2}{5}(1-x)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0089038, size = 23, normalized size = 0.61

$$-\frac{2}{15}\sqrt{1-x}\left(3x^2+14x+43\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]*(1 + x)^(3/2),x]
```

```
[Out] (-2*Sqrt[1 - x]*(43 + 14*x + 3*x^2))/15
```

Maple [A] time = 0.03, size = 30, normalized size = 0.8

$$\frac{(-2+2x)(3x^2+14x+43)}{15}\sqrt{1+x}\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 2/15*(-1+x)*(3*x^2+14*x+43)*(1+x)^(1/2)/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.953294, size = 32, normalized size = 0.84

$$\frac{2(3x^3+11x^2+29x-43)}{15\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*x^3 + 11*x^2 + 29*x - 43)/sqrt(-x + 1)
```

Fricas [A] time = 1.67183, size = 73, normalized size = 1.92

$$-\frac{2(3x^2+14x+43)\sqrt{-x^2+1}}{15\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*x^2 + 14*x + 43)*sqrt(-x^2 + 1)/sqrt(x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1949, size = 51, normalized size = 1.34

$$-\frac{2}{5}(x-1)^2\sqrt{-x+1}+\frac{8}{3}(-x+1)^{\frac{3}{2}}+\frac{64}{15}\sqrt{2}-8\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/5*(x - 1)^2*sqrt(-x + 1) + 8/3*(-x + 1)^(3/2) + 64/15*sqrt(2) - 8*sqrt(-
x + 1)
```

3.373 $\int e^{\tanh^{-1}(x)}(1-x)^{3/2}x\,dx$

Optimal. Leaf size=34

$$-\frac{2}{7}(x+1)^{7/2}+\frac{6}{5}(x+1)^{5/2}-\frac{4}{3}(x+1)^{3/2}$$

```
[Out] (-4*(1 + x)^(3/2))/3 + (6*(1 + x)^(5/2))/5 - (2*(1 + x)^(7/2))/7
```

Rubi [A] time = 0.0620389, antiderivative size = 47, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6128, 795, 627, 43}

$$-\frac{2}{7}\sqrt{1-x}\left(1-x^2\right)^{3/2}+\frac{2}{35}(x+1)^{5/2}-\frac{4}{21}(x+1)^{3/2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcTanh[x]*(1 - x)^(3/2)*x,x]
```

```
[Out] (-4*(1 + x)^(3/2))/21 + (2*(1 + x)^(5/2))/35 - (2*Sqrt[1 - x]*(1 - x^2)^(3/
2))/7
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(x)}(1-x)^{3/2}x\,dx &= \int \sqrt{1-x}x\sqrt{1-x^2}\,dx \\
&= -\frac{2}{7}\sqrt{1-x}\left(1-x^2\right)^{3/2} - \frac{1}{7}\int \sqrt{1-x}\sqrt{1-x^2}\,dx \\
&= -\frac{2}{7}\sqrt{1-x}\left(1-x^2\right)^{3/2} - \frac{1}{7}\int (1-x)\sqrt{1+x}\,dx \\
&= -\frac{2}{7}\sqrt{1-x}\left(1-x^2\right)^{3/2} - \frac{1}{7}\int \left(2\sqrt{1+x}-(1+x)^{3/2}\right)dx \\
&= -\frac{4}{21}(1+x)^{3/2} + \frac{2}{35}(1+x)^{5/2} - \frac{2}{7}\sqrt{1-x}\left(1-x^2\right)^{3/2}
\end{aligned}
$$

Mathematica [A] time = 0.0098989, size = 21, normalized size = 0.62

$$
-\frac{2}{105}(x+1)^{3/2}\left(15x^2-33x+22\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]*(1 - x)^(3/2)*x,x]
```

```
[Out] (-2*(1 + x)^(3/2)*(22 - 33*x + 15*x^2))/105
```

Maple [A] time = 0.028, size = 34, normalized size = 1.

$$
-\frac{2\left(15x^2-33x+22\right)(1+x)^2}{105}\sqrt{1-x}\frac{1}{\sqrt{-x^2+1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x,x)
```

```
[Out] -2/105*(1+x)^2*(15*x^2-33*x+22)*(1-x)^(1/2)/(-x^2+1)^(1/2)
```

Maxima [B] time = 0.971683, size = 65, normalized size = 1.91

$$
-\frac{2\left(15x^4-24x^3+13x^2-52x-104\right)}{105\sqrt{x+1}} - \frac{2\left(x^3-2x^2+3x+6\right)}{5\sqrt{x+1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] -2/105*(15*x^4 - 24*x^3 + 13*x^2 - 52*x - 104)/sqrt(x + 1) - 2/5*(x^3 - 2*x
^2 + 3*x + 6)/sqrt(x + 1)
```

Fricas [A] time = 1.72861, size = 99, normalized size = 2.91

$$
\frac{2\left(15x^3-18x^2-11x+22\right)\sqrt{-x^2+1}\sqrt{-x+1}}{105(x-1)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] 2/105*(15*x^3 - 18*x^2 - 11*x + 22)*sqrt(-x^2 + 1)*sqrt(-x + 1)/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(1-x\right)^{\frac{3}{2}}\left(x+1\right)}{\sqrt{-\left(x-1\right)\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*(1-x)**(3/2)*x,x)
```

```
[Out] Integral(x*(1 - x)**(3/2)*(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [A] time = 1.2457, size = 36, normalized size = 1.06

$$-\frac{2}{7}\left(x+1\right)^{\frac{7}{2}}+\frac{6}{5}\left(x+1\right)^{\frac{5}{2}}-\frac{4}{3}\left(x+1\right)^{\frac{3}{2}}+\frac{16}{105}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x,x, algorithm="giac")
```

```
[Out] -2/7*(x + 1)^(7/2) + 6/5*(x + 1)^(5/2) - 4/3*(x + 1)^(3/2) + 16/105*sqrt(2)
```

3.374 $\int e^{\tanh^{-1}(x)}(1-x)^{3/2}\,dx$

Optimal. Leaf size=23

$$\frac{4}{3}(x+1)^{3/2}-\frac{2}{5}(x+1)^{5/2}$$

```
[Out] (4*(1 + x)^(3/2))/3 - (2*(1 + x)^(5/2))/5
```

Rubi [A] time = 0.033876, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6127, 627, 43}

$$\frac{4}{3}(x+1)^{3/2}-\frac{2}{5}(x+1)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*(1 - x)^(3/2),x]

[Out] (4*(1 + x)^(3/2))/3 - (2*(1 + x)^(5/2))/5
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(x)}(1-x)^{3/2}\,dx &= \int \sqrt{1-x}\sqrt{1-x^2}\,dx\\ &= \int (1-x)\sqrt{1+x}\,dx\\ &= \int \left(2\sqrt{1+x}-(1+x)^{3/2}\right)dx\\ &= \frac{4}{3}(1+x)^{3/2}-\frac{2}{5}(1+x)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0069197, size = 16, normalized size = 0.7

$$-\frac{2}{15}(x+1)^{3/2}(3x-7)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]*(1 - x)^(3/2),x]
```

```
[Out] (-2*(1 + x)^(3/2)*(-7 + 3*x))/15
```

Maple [A] time = 0.03, size = 29, normalized size = 1.3

$$-\frac{2\,(3x-7)\,(1+x)^{2}}{15}\sqrt{1-x}\frac{1}{\sqrt{-x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2),x)
```

```
[Out] -2/15*(1+x)^2*(3*x-7)*(1-x)^(1/2)/(-x^2+1)^(1/2)
```

Maxima [B] time = 0.973383, size = 49, normalized size = 2.13

$$-\frac{2\left(x^{3}-2\,x^{2}+3\,x+6\right)}{5\,\sqrt{x+1}}-\frac{2\left(x^{2}-4\,x-5\right)}{3\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(x^3 - 2*x^2 + 3*x + 6)/sqrt(x + 1) - 2/3*(x^2 - 4*x - 5)/sqrt(x + 1)
```

Fricas [B] time = 1.80654, size = 81, normalized size = 3.52

$$\frac{2\left(3\,x^{2}-4\,x-7\right)\sqrt{-x^{2}+1}\sqrt{-x+1}}{15\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 - 4*x - 7)*sqrt(-x^2 + 1)*sqrt(-x + 1)/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(1-x)^{\frac{3}{2}}\,(x+1)}{\sqrt{-(x-1)\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*(1-x)**(3/2),x)
```

```
[Out] Integral((1 - x)**(3/2)*(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [A] time = 1.20362, size = 27, normalized size = 1.17

$$-\frac{2}{5}(x+1)^{\frac{5}{2}}+\frac{4}{3}(x+1)^{\frac{3}{2}}-\frac{16}{15}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/5*(x + 1)^(5/2) + 4/3*(x + 1)^(3/2) - 16/15*sqrt(2)
```

3.375 $\int e^{\tanh^{-1}(x)} x\sqrt{1+x}\,dx$

Optimal. Leaf size=36

$$-\frac{2}{5}(1-x)^{5/2}+2(1-x)^{3/2}-4\sqrt{1-x}$$

```
[Out] -4*Sqrt[1 - x] + 2*(1 - x)^(3/2) - (2*(1 - x)^(5/2))/5
```

Rubi [A] time = 0.0415186, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6129, 77}

$$-\frac{2}{5}(1-x)^{5/2}+2(1-x)^{3/2}-4\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*x*Sqrt[1 + x],x]
```

```
[Out] -4*Sqrt[1 - x] + 2*(1 - x)^(3/2) - (2*(1 - x)^(5/2))/5
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(x)} x\sqrt{1+x}\,dx &= \int \frac{x(1+x)}{\sqrt{1-x}}\,dx\\ &= \int \left(\frac{2}{\sqrt{1-x}}-3\sqrt{1-x}+(1-x)^{3/2}\right)dx\\ &= -4\sqrt{1-x}+2(1-x)^{3/2}-\frac{2}{5}(1-x)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0083818, size = 21, normalized size = 0.58

$$-\frac{2}{5}\sqrt{1-x}\left(x^2+3x+6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]*x*Sqrt[1 + x],x]

[Out] (-2*Sqrt[1 - x]*(6 + 3*x + x^2))/5
```

Maple [A] time = 0.036, size = 28, normalized size = 0.8

$$\frac{(-2+2x)\left(x^2+3x+6\right)}{5}\sqrt{1+x}\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(-x^2+1)^(1/2)*x,x)

[Out] 2/5*(-1+x)*(x^2+3*x+6)*(1+x)^(1/2)/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.94335, size = 30, normalized size = 0.83

$$\frac{2\left(x^3+2x^2+3x-6\right)}{5\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*x,x, algorithm="maxima")

[Out] 2/5*(x^3 + 2*x^2 + 3*x - 6)/sqrt(-x + 1)
```

Fricas [A] time = 1.7774, size = 66, normalized size = 1.83

$$-\frac{2\left(x^2+3x+6\right)\sqrt{-x^2+1}}{5\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*x,x, algorithm="fricas")

[Out] -2/5*(x^2 + 3*x + 6)*sqrt(-x^2 + 1)/sqrt(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x(x+1)^{\frac{3}{2}}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(-x**2+1)**(1/2)*x,x)

[Out] Integral(x*(x + 1)**(3/2)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [A] time = 1.28932, size = 51, normalized size = 1.42

$$-\frac{2}{5}(x-1)^2\sqrt{-x+1}+2(-x+1)^{\frac{3}{2}}+\frac{8}{5}\sqrt{2}-4\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*x,x, algorithm="giac")
```

```
[Out] -2/5*(x - 1)^2*sqrt(-x + 1) + 2*(-x + 1)^(3/2) + 8/5*sqrt(2) - 4*sqrt(-x +
1)
```

3.376 $\int e^{\tanh^{-1}(x)}\sqrt{1+x}\,dx$

Optimal. Leaf size=25

$$\frac{2}{3}(1-x)^{3/2}-4\sqrt{1-x}$$

```
[Out] -4*Sqrt[1 - x] + (2*(1 - x)^(3/2))/3
```

Rubi [A] time = 0.0234036, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 43}

$$\frac{2}{3}(1-x)^{3/2}-4\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*Sqrt[1 + x],x]
```

```
[Out] -4*Sqrt[1 - x] + (2*(1 - x)^(3/2))/3
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(x)}\sqrt{1+x}\,dx &= \int \frac{1+x}{\sqrt{1-x}}\,dx\\ &= \int \left(\frac{2}{\sqrt{1-x}}-\sqrt{1-x}\right)dx\\ &= -4\sqrt{1-x}+\frac{2}{3}(1-x)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.005727, size = 16, normalized size = 0.64

$$-\frac{2}{3}\sqrt{1-x}(x+5)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]*Sqrt[1 + x],x]
```

```
[Out] (-2*Sqrt[1 - x]*(5 + x))/3
```

Maple [A] time = 0.029, size = 23, normalized size = 0.9

$$\frac{(-2+2x)(x+5)}{3}\sqrt{1+x}\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 2/3*(-1+x)*(x+5)*(1+x)^(1/2)/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.947353, size = 23, normalized size = 0.92

$$\frac{2\left(x^2+4x-5\right)}{3\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x^2 + 4*x - 5)/sqrt(-x + 1)
```

Fricas [A] time = 1.70803, size = 55, normalized size = 2.2

$$-\frac{2\sqrt{-x^2+1}(x+5)}{3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-x^2 + 1)*(x + 5)/sqrt(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(x+1)^{\frac{3}{2}}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral((x + 1)**(3/2)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [A] time = 1.23354, size = 32, normalized size = 1.28

$$\frac{2}{3}(-x+1)^{\frac{3}{2}}+\frac{8}{3}\sqrt{2}-4\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(-x + 1)^(3/2) + 8/3*sqrt(2) - 4*sqrt(-x + 1)
```

3.377 $\int e^{\tanh^{-1}(x)}\sqrt{1-x}x\,dx$

Optimal. Leaf size=23

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

```
[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

Rubi [A] time = 0.0393961, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6128, 26, 43}

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*Sqrt[1 - x]*x,x]
```

```
[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(x)}\sqrt{1-x}x\,dx &= \int \frac{x\sqrt{1-x^2}}{\sqrt{1-x}}\,dx\\ &= \int x\sqrt{1+x}\,dx\\ &= \int \left(-\sqrt{1+x}+(1+x)^{3/2}\right)dx\\ &= -\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0069671, size = 16, normalized size = 0.7

$$\frac{2}{15}(x+1)^{3/2}(3x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]*Sqrt[1 - x]*x,x]
```

```
[Out] (2*(1 + x)^(3/2)*(-2 + 3*x))/15
```

Maple [A] time = 0.031, size = 29, normalized size = 1.3

$$\frac{2\,(1+x)^2\,(3\,x-2)}{15}\sqrt{1-x}\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x,x)
```

```
[Out] 2/15*(1+x)^2*(3*x-2)*(1-x)^(1/2)/(-x^2+1)^(1/2)
```

Maxima [B] time = 0.984432, size = 51, normalized size = 2.22

$$\frac{2\left(3\,x^3-x^2+4\,x+8\right)}{15\,\sqrt{x+1}}+\frac{2\left(x^2-x-2\right)}{3\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] 2/15*(3*x^3 - x^2 + 4*x + 8)/sqrt(x + 1) + 2/3*(x^2 - x - 2)/sqrt(x + 1)
```

Fricas [B] time = 1.86095, size = 80, normalized size = 3.48

$$-\frac{2\left(3\,x^2+x-2\right)\sqrt{-x^2+1}\sqrt{-x+1}}{15\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] -2/15*(3*x^2 + x - 2)*sqrt(-x^2 + 1)*sqrt(-x + 1)/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{1-x}\,(x+1)}{\sqrt{-(x-1)\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*(1-x)**(1/2)*x,x)
```

```
[Out] Integral(x*sqrt(1 - x)*(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [A] time = 1.18087, size = 27, normalized size = 1.17

$$\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{4}{15}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x,x, algorithm="giac")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2) - 4/15*sqrt(2)
```

3.378 $\int e^{\tanh^{-1}(x)}\sqrt{1-x}\,dx$

Optimal. Leaf size=11

$$\frac{2}{3}(x+1)^{3/2}$$

```
[Out] (2*(1 + x)^(3/2))/3
```

Rubi [A] time = 0.0200117, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6127, 26, 32}

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*Sqrt[1 - x],x]

[Out] (2*(1 + x)^(3/2))/3
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int e^{\tanh^{-1}(x)}\sqrt{1-x}\,dx &= \int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx \\ &= \int \sqrt{1+x}\,dx \\ &= \frac{2}{3}(1+x)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.0040061, size = 11, normalized size = 1.

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]*Sqrt[1 - x],x]

[Out] (2*(1 + x)^(3/2))/3
```

Maple [B] time = 0.029, size = 24, normalized size = 2.2

$$\frac{2\left(1+x\right)^{2}}{3}\sqrt{1-x}\frac{1}{\sqrt{-x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2),x)

[Out] 2/3*(1+x)^2*(1-x)^(1/2)/(-x^2+1)^(1/2)
```

Maxima [B] time = 0.980432, size = 31, normalized size = 2.82

$$\frac{2\left(x^{2}-x-2\right)}{3\sqrt{x+1}}+2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2),x, algorithm="maxima")

[Out] 2/3*(x^2 - x - 2)/sqrt(x + 1) + 2*sqrt(x + 1)
```

Fricas [B] time = 1.87658, size = 68, normalized size = 6.18

$$-\frac{2\sqrt{-x^{2}+1}(x+1)\sqrt{-x+1}}{3\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2),x, algorithm="fricas")

[Out] -2/3*sqrt(-x^2 + 1)*(x + 1)*sqrt(-x + 1)/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{1-x}\left(x+1\right)}{\sqrt{-\left(x-1\right)\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*(1-x)**(1/2),x)

[Out] Integral(sqrt(1 - x)*(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [A] time = 1.19436, size = 18, normalized size = 1.64

$$\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{4}{3}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2) - 4/3*sqrt(2)
```

3.379 $\int \frac{e^{\tanh^{-1}(x)}x}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=25

$$\frac{2}{3}(1-x)^{3/2}-2\sqrt{1-x}$$

```
[Out] -2*Sqrt[1 - x] + (2*(1 - x)^(3/2))/3
```

Rubi [A] time = 0.0390745, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6129, 43}

$$\frac{2}{3}(1-x)^{3/2}-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[x]*x)/Sqrt[1 + x],x]
```

```
[Out] -2*Sqrt[1 - x] + (2*(1 - x)^(3/2))/3
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(x)}x}{\sqrt{1+x}}\,dx &= \int \frac{x}{\sqrt{1-x}}\,dx\\ &= \int \left(\frac{1}{\sqrt{1-x}}-\sqrt{1-x}\right)dx\\ &= -2\sqrt{1-x}+\frac{2}{3}(1-x)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0066284, size = 16, normalized size = 0.64

$$-\frac{2}{3}\sqrt{1-x}(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[x]*x)/Sqrt[1 + x],x]
```

```
[Out] (-2*Sqrt[1 - x]*(2 + x))/3
```

Maple [A] time = 0.032, size = 23, normalized size = 0.9

$$\frac{(-2+2x)(x+2)}{3}\sqrt{1+x}\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(-x^2+1)^(1/2)*x,x)
```

```
[Out] 2/3*(-1+x)*(x+2)*(1+x)^(1/2)/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.961726, size = 20, normalized size = 0.8

$$\frac{2\left(x^2+x-2\right)}{3\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] 2/3*(x^2 + x - 2)/sqrt(-x + 1)
```

Fricas [A] time = 2.05257, size = 55, normalized size = 2.2

$$-\frac{2\sqrt{-x^2+1}(x+2)}{3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-x^2 + 1)*(x + 2)/sqrt(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{x+1}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(-x**2+1)**(1/2)*x,x)
```

```
[Out] Integral(x*sqrt(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [A] time = 1.15428, size = 32, normalized size = 1.28

$$\frac{2}{3}(-x+1)^{\frac{3}{2}} + \frac{2}{3}\sqrt{2} - 2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*x,x, algorithm="giac")
```

```
[Out] 2/3*(-x + 1)^(3/2) + 2/3*sqrt(2) - 2*sqrt(-x + 1)
```

3.380 $\int \frac{e^{\tanh^{-1}(x)}}{\sqrt{1+x}} dx$

Optimal. Leaf size=11

$$-2\sqrt{1-x}$$

```
[Out] -2*Sqrt[1 - x]
```

Rubi [A] time = 0.0192077, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 32}

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]/Sqrt[1 + x],x]

[Out] -2*Sqrt[1 - x]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(x)}}{\sqrt{1+x}} dx &= \int \frac{1}{\sqrt{1-x}} dx \\ &= -2\sqrt{1-x}\end{aligned}$$

Mathematica [A] time = 0.003727, size = 11, normalized size = 1.

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]/Sqrt[1 + x],x]

[Out] -2*Sqrt[1 - x]
```

Maple [B] time = 0.029, size = 20, normalized size = 1.8

$$2\frac{(-1+x)\sqrt{1+x}}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 2*(-1+x)*(1+x)^(1/2)/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.969603, size = 16, normalized size = 1.45

$$\frac{2(x-1)}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(x - 1)/sqrt(-x + 1)
```

Fricas [C] time = 1.84967, size = 42, normalized size = 3.82

$$-\frac{2\sqrt{-x^2+1}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-x^2 + 1)/sqrt(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+1}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [A] time = 1.15086, size = 20, normalized size = 1.82

$$2\sqrt{2}-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")

[Out] 2*sqrt(2) - 2*sqrt(-x + 1)
```

3.381 $\int \frac{e^{\tanh^{-1}(x)}x}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=42

$$-\frac{2}{3}(x+1)^{3/2}-2\sqrt{x+1}+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)$$

```
[Out] -2*Sqrt[1 + x] - (2*(1 + x)^(3/2))/3 + 2*Sqrt[2]*ArcTanh[Sqrt[1 + x]/Sqrt[2
]]
```

Rubi [A] time = 0.0463722, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6129, 80, 50, 63, 206}

$$-\frac{2}{3}(x+1)^{3/2}-2\sqrt{x+1}+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[x]*x)/Sqrt[1 - x],x]
```

```
[Out] -2*Sqrt[1 + x] - (2*(1 + x)^(3/2))/3 + 2*Sqrt[2]*ArcTanh[Sqrt[1 + x]/Sqrt[2
]]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(x)}x}{\sqrt{1-x}}\,dx &= \int \frac{x\sqrt{1+x}}{1-x}\,dx\\ &= -\frac{2}{3}(1+x)^{3/2}+\int \frac{\sqrt{1+x}}{1-x}\,dx\\ &= -2\sqrt{1+x}-\frac{2}{3}(1+x)^{3/2}+2\int \frac{1}{(1-x)\sqrt{1+x}}\,dx\\ &= -2\sqrt{1+x}-\frac{2}{3}(1+x)^{3/2}+4\,\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+x}\right)\\ &= -2\sqrt{1+x}-\frac{2}{3}(1+x)^{3/2}+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0201375, size = 36, normalized size = 0.86

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)-\frac{2}{3}\sqrt{x+1}(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[x]*x)/Sqrt[1 - x],x]
```

```
[Out] (-2*Sqrt[1 + x]*(4 + x))/3 + 2*Sqrt[2]*ArcTanh[Sqrt[1 + x]/Sqrt[2]]
```

Maple [A] time = 0.042, size = 61, normalized size = 1.5

$$-\frac{2}{3x-3}\sqrt{-x^2+1}\sqrt{1-x}\left(3\,\mathrm{Artanh}\left(1/2\sqrt{1+x}\sqrt{2}\right)\sqrt{2}-\sqrt{1+x}x-4\sqrt{1+x}\right)\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*x/(1-x)^(1/2),x)
```

```
[Out] -2/3*(-x^2+1)^(1/2)*(1-x)^(1/2)*(3*arctanh(1/2*(1+x)^(1/2)*2^(1/2))*2^(1/2)
-(1+x)^(1/2)*x-4*(1+x)^(1/2))/(-1+x)/(1+x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)x}{\sqrt{-x^2+1}\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*x/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)*x/(sqrt(-x^2 + 1)*sqrt(-x + 1)), x)
```

Fricas [B] time = 1.81759, size = 204, normalized size = 4.86

$$\frac{3\sqrt{2}(x-1)\log\left(-\frac{x^2-2\sqrt{2}\sqrt{-x^2+1}\sqrt{-x+1}+2x-3}{x^2-2x+1}\right)+2\sqrt{-x^2+1}(x+4)\sqrt{-x+1}}{3(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*x/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*sqrt(2)*(x - 1)*log(-(x^2 - 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(-x + 1) +
2*x - 3)/(x^2 - 2*x + 1)) + 2*sqrt(-x^2 + 1)*(x + 4)*sqrt(-x + 1))/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x(x+1)}{\sqrt{-(x-1)(x+1)}\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*x/(1-x)**(1/2),x)
```

```
[Out] Integral(x*(x + 1)/(sqrt(-(x - 1)*(x + 1))*sqrt(1 - x)), x)
```

Giac [A] time = 1.23088, size = 59, normalized size = 1.4

$$-\frac{2}{3}(x+1)^{\frac{3}{2}}-\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{x+1}}{\sqrt{2}+\sqrt{x+1}}\right)-2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*x/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(x + 1)^(3/2) - sqrt(2)*log((sqrt(2) - sqrt(x + 1))/(sqrt(2) + sqrt(x
+ 1))) - 2*sqrt(x + 1)
```

3.382 $\int \frac{e^{\tanh^{-1}(x)}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=31

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)-2\sqrt{x+1}$$

```
[Out] -2*Sqrt[1 + x] + 2*Sqrt[2]*ArcTanh[Sqrt[1 + x]/Sqrt[2]]
```

Rubi [A] time = 0.0336543, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6127, 627, 50, 63, 206}

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)-2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]/Sqrt[1 - x],x]
```

```
[Out] -2*Sqrt[1 + x] + 2*Sqrt[2]*ArcTanh[Sqrt[1 + x]/Sqrt[2]]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(x)}}{\sqrt{1-x}}\,dx &= \int \frac{\sqrt{1-x^2}}{(1-x)^{3/2}}\,dx\\ &= \int \frac{\sqrt{1+x}}{1-x}\,dx\\ &= -2\sqrt{1+x}+2\int \frac{1}{(1-x)\sqrt{1+x}}\,dx\\ &= -2\sqrt{1+x}+4\,\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+x}\right)\\ &= -2\sqrt{1+x}+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0054061, size = 31, normalized size = 1.

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)-2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]/Sqrt[1 - x],x]
```

```
[Out] -2*Sqrt[1 + x] + 2*Sqrt[2]*ArcTanh[Sqrt[1 + x]/Sqrt[2]]
```

Maple [B] time = 0.037, size = 52, normalized size = 1.7

$$-2\,\frac{\sqrt{-x^2+1}\sqrt{1-x}\left(\mathrm{Artanh}\left(1/2\,\sqrt{1+x}\sqrt{2}\right)\sqrt{2}-\sqrt{1+x}\right)}{(-1+x)\,\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)/(1-x)^(1/2),x)
```

```
[Out] -2*(-x^2+1)^(1/2)*(1-x)^(1/2)*(arctanh(1/2*(1+x)^(1/2)*2^(1/2))*2^(1/2)-(1+
x)^(1/2))/(-1+x)/(1+x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{-x^2+1}\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(-x^2 + 1)*sqrt(-x + 1)), x)
```

Fricas [B] time = 1.82272, size = 185, normalized size = 5.97

$$\frac{\sqrt{2}(x-1)\log\left(-\frac{x^2-2\sqrt{2}\sqrt{-x^2+1}\sqrt{-x+1}+2x-3}{x^2-2x+1}\right)+2\sqrt{-x^2+1}\sqrt{-x+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(x - 1)*log(-(x^2 - 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(-x + 1) + 2*x -
3)/(x^2 - 2*x + 1)) + 2*sqrt(-x^2 + 1)*sqrt(-x + 1))/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{-(x-1)(x+1)}\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)/(1-x)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt(-(x - 1)*(x + 1))*sqrt(1 - x)), x)
```

Giac [A] time = 1.20945, size = 50, normalized size = 1.61

$$-\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{x+1}}{\sqrt{2}+\sqrt{x+1}}\right)-2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*log((sqrt(2) - sqrt(x + 1))/(sqrt(2) + sqrt(x + 1))) - 2*sqrt(x +
1)
```

3.383 $\int \frac{e^{\tanh^{-1}(x)}x}{(1+x)^{3/2}}\,dx$

Optimal. Leaf size=34

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)-2\sqrt{1-x}$$

```
[Out] -2*Sqrt[1 - x] + Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]]
```

Rubi [A] time = 0.045287, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6129, 80, 63, 206}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[x]*x)/(1 + x)^(3/2),x]

[Out] -2*Sqrt[1 - x] + Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(x)}x}{(1+x)^{3/2}}\,dx &= \int \frac{x}{\sqrt{1-x}(1+x)}\,dx \\
&= -2\sqrt{1-x} - \int \frac{1}{\sqrt{1-x}(1+x)}\,dx \\
&= -2\sqrt{1-x} + 2\,\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1-x}\right) \\
&= -2\sqrt{1-x} + \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0068915, size = 34, normalized size = 1.

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) - 2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[x]*x)/(1 + x)^(3/2),x]

[Out] -2*Sqrt[1 - x] + Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]]
```

Maple [A] time = 0.065, size = 50, normalized size = 1.5

$$\sqrt{-x^2+1}\left(\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1-x}\right)\sqrt{2} - 2\sqrt{1-x}\right)\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(1/2)/(-x^2+1)^(1/2)*x,x)

[Out] 1/(1+x)^(1/2)*(-x^2+1)^(1/2)/(1-x)^(1/2)*(arctanh(1/2*(1-x)^(1/2)*2^(1/2))*
2^(1/2)-2*(1-x)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^2+1}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2)*x,x, algorithm="maxima")

[Out] integrate(x/(sqrt(-x^2 + 1)*sqrt(x + 1)), x)
```

Fricas [B] time = 1.73524, size = 188, normalized size = 5.53

$$\frac{\sqrt{2}(x+1)\log\left(-\frac{x^2-2\sqrt{2}\sqrt{-x^2+1}\sqrt{x+1}-2x-3}{x^2+2x+1}\right) - 4\sqrt{-x^2+1}\sqrt{x+1}}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*(x + 1)*log(-(x^2 - 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(x + 1) - 2*x
 - 3)/(x^2 + 2*x + 1)) - 4*sqrt(-x^2 + 1)*sqrt(x + 1))/(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(x-1)(x+1)}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2)/(-x**2+1)**(1/2)*x,x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)*(x + 1))*sqrt(x + 1)), x)
```

Giac [A] time = 1.25002, size = 58, normalized size = 1.71

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{2}+\sqrt{-x+1}}\right)-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2)*x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log((sqrt(2) - sqrt(-x + 1))/(sqrt(2) + sqrt(-x + 1))) - 2*sqr
t(-x + 1)
```

3.384 $\int \frac{e^{\tanh^{-1}(x)}}{(1+x)^{3/2}}\,dx$

Optimal. Leaf size=23

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]])
```

Rubi [A] time = 0.028107, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6129, 63, 206}

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]/(1 + x)^(3/2),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(x)}}{(1+x)^{3/2}}\,dx &= \int \frac{1}{\sqrt{1-x}(1+x)}\,dx\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1-x}\right)\right)\\
&= -\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0043876, size = 23, normalized size = 1.

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]/(1 + x)^(3/2),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]])
```

Maple [B] time = 0.062, size = 40, normalized size = 1.7

$$-\sqrt{2}\sqrt{-x^2+1}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1-x}\right)\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] -1/(1+x)^(1/2)*(-x^2+1)^(1/2)/(1-x)^(1/2)*2^(1/2)*arctanh(1/2*(1-x)^(1/2)*2
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(x + 1)), x)
```

Fricas [B] time = 1.78585, size = 122, normalized size = 5.3

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{x^2+2\sqrt{2}\sqrt{-x^2+1}\sqrt{x+1}-2x-3}{x^2+2x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(x^2 + 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(x + 1) - 2*x - 3)/(x^
2 + 2*x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(x + 1)), x)
```

Giac [B] time = 1.20302, size = 50, normalized size = 2.17

$$-\frac{1}{2}\sqrt{2}\log\left(\sqrt{2}+\sqrt{-x+1}\right)+\frac{1}{2}\sqrt{2}\log\left(\sqrt{2}-\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(sqrt(2) + sqrt(-x + 1)) + 1/2*sqrt(2)*log(sqrt(2) - sqrt(-
x + 1))
```

3.385 $\int \frac{e^{\tanh^{-1}(x)}x}{(1-x)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{(x+1)^{3/2}}{2(1-x)}+\frac{5\sqrt{x+1}}{2}-\frac{5\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] (5*Sqrt[1 + x])/2 + (1 + x)^(3/2)/(2*(1 - x)) - (5*ArcTanh[Sqrt[1 + x]/Sqrt
[2]])/Sqrt[2]
```

Rubi [A] time = 0.0545211, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6129, 78, 50, 63, 206}

$$\frac{(x+1)^{3/2}}{2(1-x)}+\frac{5\sqrt{x+1}}{2}-\frac{5\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[x]*x)/(1 - x)^(3/2),x]
```

```
[Out] (5*Sqrt[1 + x])/2 + (1 + x)^(3/2)/(2*(1 - x)) - (5*ArcTanh[Sqrt[1 + x]/Sqrt
[2]])/Sqrt[2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(x)}x}{(1-x)^{3/2}}\,dx &= \int \frac{x\sqrt{1+x}}{(1-x)^2}\,dx\\ &= \frac{(1+x)^{3/2}}{2(1-x)} - \frac{5}{4}\int \frac{\sqrt{1+x}}{1-x}\,dx\\ &= \frac{5\sqrt{1+x}}{2} + \frac{(1+x)^{3/2}}{2(1-x)} - \frac{5}{2}\int \frac{1}{(1-x)\sqrt{1+x}}\,dx\\ &= \frac{5\sqrt{1+x}}{2} + \frac{(1+x)^{3/2}}{2(1-x)} - 5\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+x}\right)\\ &= \frac{5\sqrt{1+x}}{2} + \frac{(1+x)^{3/2}}{2(1-x)} - \frac{5\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0203339, size = 40, normalized size = 0.78

$$\frac{\sqrt{x+1}(2x-3)}{x-1} - \frac{5\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[x]*x)/(1 - x)^(3/2),x]
```

```
[Out] (Sqrt[1 + x]*(-3 + 2*x))/(-1 + x) - (5*ArcTanh[Sqrt[1 + x]/Sqrt[2]])/Sqrt[2
]
```

Maple [B] time = 0.045, size = 78, normalized size = 1.5

$$\frac{1}{2\,(-1+x)^2}\sqrt{-x^2+1}\sqrt{1-x}\left(5\,\sqrt{2}\text{Artanh}\left(1/2\,\sqrt{1+x}\sqrt{2}\right)x - 5\,\text{Artanh}\left(1/2\,\sqrt{1+x}\sqrt{2}\right)\sqrt{2} - 4\,\sqrt{1+x}x + 6\,\sqrt{1+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*x/(1-x)^(3/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(1-x)^(1/2)*(5*2^(1/2)*arctanh(1/2*(1+x)^(1/2)*2^(1/2))*
x-5*arctanh(1/2*(1+x)^(1/2)*2^(1/2))*2^(1/2)-4*(1+x)^(1/2)*x+6*(1+x)^(1/2))
/(-1+x)^2/(1+x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)x}{\sqrt{-x^2+1}(-x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*x/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)*x/(sqrt(-x^2 + 1)*(-x + 1)^(3/2)), x)
```

Fricas [B] time = 1.7736, size = 228, normalized size = 4.47

$$\frac{5\sqrt{2}\left(x^2-2x+1\right)\log\left(-\frac{x^2+2\sqrt{2}\sqrt{-x^2+1}\sqrt{-x+1}+2x-3}{x^2-2x+1}\right)-4\sqrt{-x^2+1}(2x-3)\sqrt{-x+1}}{4\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*x/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(5*sqrt(2)*(x^2 - 2*x + 1)*log(-(x^2 + 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(-x
 + 1) + 2*x - 3)/(x^2 - 2*x + 1)) - 4*sqrt(-x^2 + 1)*(2*x - 3)*sqrt(-x + 1)
)/(x^2 - 2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,(x+1)}{\sqrt{-\,(x-1)\,(x+1)}\,(1-x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*x/(1-x)**(3/2),x)
```

```
[Out] Integral(x*(x + 1)/(sqrt(-(x - 1)*(x + 1))*(1 - x)**(3/2)), x)
```

Giac [A] time = 1.26011, size = 66, normalized size = 1.29

$$\frac{5}{4}\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{x+1}}{\sqrt{2}+\sqrt{x+1}}\right)+2\sqrt{x+1}-\frac{\sqrt{x+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*x/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] 5/4*sqrt(2)*log((sqrt(2) - sqrt(x + 1))/(sqrt(2) + sqrt(x + 1))) + 2*sqrt(x
 + 1) - sqrt(x + 1)/(x - 1)
```

3.386 $\int \frac{e^{\tanh^{-1}(x)}}{(1-x)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{x+1}}{1-x}-\frac{\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] Sqrt[1 + x]/(1 - x) - ArcTanh[Sqrt[1 + x]/Sqrt[2]]/Sqrt[2]
```

Rubi [A] time = 0.0370145, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6127, 627, 47, 63, 206}

$$\frac{\sqrt{x+1}}{1-x}-\frac{\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]/(1 - x)^(3/2),x]
```

```
[Out] Sqrt[1 + x]/(1 - x) - ArcTanh[Sqrt[1 + x]/Sqrt[2]]/Sqrt[2]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(x)}}{(1-x)^{3/2}}\,dx &= \int \frac{\sqrt{1-x^2}}{(1-x)^{5/2}}\,dx \\
&= \int \frac{\sqrt{1+x}}{(1-x)^2}\,dx \\
&= \frac{\sqrt{1+x}}{1-x} - \frac{1}{2}\int \frac{1}{(1-x)\sqrt{1+x}}\,dx \\
&= \frac{\sqrt{1+x}}{1-x} - \text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+x}\right) \\
&= \frac{\sqrt{1+x}}{1-x} - \frac{\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0233544, size = 36, normalized size = 0.97

$$-\frac{\sqrt{x+1}}{x-1} - \frac{\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]/(1 - x)^(3/2),x]
```

```
[Out] -(Sqrt[1 + x]/(-1 + x)) - ArcTanh[Sqrt[1 + x]/Sqrt[2]]/Sqrt[2]
```

Maple [B] time = 0.043, size = 69, normalized size = 1.9

$$\frac{1}{2\,(-1+x)^2}\sqrt{-x^2+1}\sqrt{1-x}\left(\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+x}\right)x - \text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+x}\right)\sqrt{2} + 2\sqrt{1+x}\right)\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)/(1-x)^(3/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(1-x)^(1/2)*(2^(1/2)*arctanh(1/2*(1+x)^(1/2)*2^(1/2))*x-
arctanh(1/2*(1+x)^(1/2)*2^(1/2))*2^(1/2)+2*(1+x)^(1/2))/(-1+x)^2/(1+x)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{-x^2+1}(-x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(-x^2 + 1)*(-x + 1)^(3/2)), x)
```

Fricas [B] time = 1.9207, size = 212, normalized size = 5.73

$$\frac{\sqrt{2}\left(x^2-2\,x+1\right)\log\left(-\frac{x^2+2\,\sqrt{2}\sqrt{-x^2+1}\sqrt{-x+1}+2\,x-3}{x^2-2\,x+1}\right)+4\,\sqrt{-x^2+1}\sqrt{-x+1}}{4\left(x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(x^2 - 2*x + 1)*log(-(x^2 + 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(-x +
 1) + 2*x - 3)/(x^2 - 2*x + 1)) + 4*sqrt(-x^2 + 1)*sqrt(-x + 1))/(x^2 - 2*x
 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{-\left(x-1\right)\left(x+1\right)}\left(1-x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)/(1-x)**(3/2),x)
```

```
[Out] Integral((x + 1)/(sqrt(-(x - 1)*(x + 1))*(1 - x)**(3/2)), x)
```

Giac [A] time = 1.18524, size = 57, normalized size = 1.54

$$\frac{1}{4}\,\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{x+1}}{\sqrt{2}+\sqrt{x+1}}\right)-\frac{\sqrt{x+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log((sqrt(2) - sqrt(x + 1))/(sqrt(2) + sqrt(x + 1))) - sqrt(x +
 1)/(x - 1)
```

3.387 $\int e^{\tanh^{-1}(ax)} x^m \sqrt{c-acx}\,dx$

Optimal. Leaf size=64

$$\frac{2c(ax+1)\sqrt{1-a^2x^2}x^m(-ax)^{-m}\text{Hypergeometric2F1}\left(\frac{3}{2},-m,\frac{5}{2},ax+1\right)}{3a\sqrt{c-acx}}$$

```
[Out] (2*c*x^m*(1 + a*x)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[3/2, -m, 5/2, 1 + a*
x])/(3*a*(-(a*x))^m*Sqrt[c - a*c*x])
```

Rubi [A] time = 0.124062, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {6128, 892, 67, 65}

$$\frac{2c(ax+1)\sqrt{1-a^2x^2}x^m(-ax)^{-m}\,{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};ax+1\right)}{3a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*Sqrt[c - a*c*x],x]
```

```
[Out] (2*c*x^m*(1 + a*x)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[3/2, -m, 5/2, 1 + a*
x])/(3*a*(-(a*x))^m*Sqrt[c - a*c*x])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/
e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^m\sqrt{c-acx}\,dx = c\int \frac{x^m\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx$$
$$= \frac{\left(c\sqrt{1-a^2x^2}\right)\int x^m\sqrt{\frac{1}{c}+\frac{ax}{c}}\,dx}{\sqrt{\frac{1}{c}+\frac{ax}{c}}\sqrt{c-acx}}$$
$$= \frac{\left(cx^m(-ax)^{-m}\sqrt{1-a^2x^2}\right)\int(-ax)^m\sqrt{\frac{1}{c}+\frac{ax}{c}}\,dx}{\sqrt{\frac{1}{c}+\frac{ax}{c}}\sqrt{c-acx}}$$
$$= \frac{2cx^m(-ax)^{-m}(1+ax)\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};1+ax\right)}{3a\sqrt{c-acx}}$$

Mathematica [A] time = 0.01767, size = 46, normalized size = 0.72

$$\frac{x^{m+1}\sqrt{c-acx}\text{Hypergeometric2F1}\left(-\frac{1}{2},m+1,m+2,-ax\right)}{(m+1)\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*Sqrt[c - a*c*x],x]
```

```
[Out] (x^(1 + m)*Sqrt[c - a*c*x]*Hypergeometric2F1[-1/2, 1 + m, 2 + m, -(a*x)])/(
(1 + m)*Sqrt[1 - a*x])
```

Maple [F] time = 0.368, size = 0, normalized size = 0.

$$\int (ax+1)\,x^m\sqrt{-acx+c}\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a*c*x+c)^(1/2),x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a*c*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}x^m}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*x^m/(a*x - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{-c\,(ax-1)}\,(ax+1)}{\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(a*x - 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

3.388 $\int e^{\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=107

$$\frac{2c^2x^2\left(1-a^2x^2\right)^{3/2}}{7a(c-acx)^{3/2}}-\frac{8c^2\left(1-a^2x^2\right)^{3/2}}{105a^3(c-acx)^{3/2}}+\frac{8c\left(1-a^2x^2\right)^{3/2}}{35a^3\sqrt{c-acx}}$$

```
[Out] (-8*c^2*(1 - a^2*x^2)^(3/2))/(105*a^3*(c - a*c*x)^(3/2)) + (2*c^2*x^2*(1 -
a^2*x^2)^(3/2))/(7*a*(c - a*c*x)^(3/2)) + (8*c*(1 - a^2*x^2)^(3/2))/(35*a^3
*Sqrt[c - a*c*x])
```

Rubi [A] time = 0.158381, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {6128, 871, 795, 649}

$$\frac{2c^2x^2\left(1-a^2x^2\right)^{3/2}}{7a(c-acx)^{3/2}}-\frac{8c^2\left(1-a^2x^2\right)^{3/2}}{105a^3(c-acx)^{3/2}}+\frac{8c\left(1-a^2x^2\right)^{3/2}}{35a^3\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (-8*c^2*(1 - a^2*x^2)^(3/2))/(105*a^3*(c - a*c*x)^(3/2)) + (2*c^2*x^2*(1 -
a^2*x^2)^(3/2))/(7*a*(c - a*c*x)^(3/2)) + (8*c*(1 - a^2*x^2)^(3/2))/(35*a^3
*Sqrt[c - a*c*x])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 871

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p
+ 1))/(c*(m - n - 1)), x] - Dist[(n*(e*f + d*g))/(e*(m - n - 1)), Int[(d +
e*x)^m*(f + g*x)^(n - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&
 EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Integ
erQ[n])
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
```

```
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx &= c\int \frac{x^2\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx\\
&= \frac{2c^2x^2\left(1-a^2x^2\right)^{3/2}}{7a(c-acx)^{3/2}} - \frac{(4c)\int \frac{x\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx}{7a}\\
&= \frac{2c^2x^2\left(1-a^2x^2\right)^{3/2}}{7a(c-acx)^{3/2}} + \frac{8c\left(1-a^2x^2\right)^{3/2}}{35a^3\sqrt{c-acx}} - \frac{(4c)\int \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx}{35a^2}\\
&= -\frac{8c^2\left(1-a^2x^2\right)^{3/2}}{105a^3(c-acx)^{3/2}} + \frac{2c^2x^2\left(1-a^2x^2\right)^{3/2}}{7a(c-acx)^{3/2}} + \frac{8c\left(1-a^2x^2\right)^{3/2}}{35a^3\sqrt{c-acx}}
\end{aligned}$$

Mathematica [A] time = 0.0282366, size = 51, normalized size = 0.48

$$\frac{2(ax+1)^{3/2}\left(15a^2x^2-12ax+8\right)\sqrt{c-acx}}{105a^3\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + a*x)^(3/2)*Sqrt[c - a*c*x]*(8 - 12*a*x + 15*a^2*x^2))/(105*a^3*Sqrt
[1 - a*x])
```

Maple [A] time = 0.039, size = 48, normalized size = 0.5

$$\frac{2\,(ax+1)^2\left(15\,a^2x^2-12\,ax+8\right)}{105\,a^3}\sqrt{-acx+c}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/105*(a*x+1)^2*(15*a^2*x^2-12*a*x+8)*(-a*c*x+c)^(1/2)/a^3/(-a^2*x^2+1)^(1/
2)
```

Maxima [A] time = 1.03429, size = 143, normalized size = 1.34

$$\frac{2\left(5\,a^4\sqrt{c}x^4-a^3\sqrt{c}x^3+2\,a^2\sqrt{c}x^2-8\,a\sqrt{c}x-16\,\sqrt{c}\right)}{35\,\sqrt{ax+1}a^3}+\frac{2\left(3\,a^3\sqrt{c}x^3-a^2\sqrt{c}x^2+4\,a\sqrt{c}x+8\,\sqrt{c}\right)}{15\,\sqrt{ax+1}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] 2/35*(5*a^4*sqrt(c)*x^4 - a^3*sqrt(c)*x^3 + 2*a^2*sqrt(c)*x^2 - 8*a*sqrt(c)
*x - 16*sqrt(c))/(sqrt(a*x + 1)*a^3) + 2/15*(3*a^3*sqrt(c)*x^3 - a^2*sqrt(c
)*x^2 + 4*a*sqrt(c)*x + 8*sqrt(c))/(sqrt(a*x + 1)*a^3)
```

Fricas [A] time = 1.80732, size = 128, normalized size = 1.2

$$-\frac{2\left(15\,a^3x^3+3\,a^2x^2-4\,ax+8\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{105\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -2/105*(15*a^3*x^3 + 3*a^2*x^2 - 4*a*x + 8)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x
+ c)/(a^4*x - a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-c\,(ax-1)}\,(ax+1)}{\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [A] time = 1.37634, size = 88, normalized size = 0.82

$$-\frac{2\,c\left(\frac{22\,\sqrt{2}\sqrt{c}}{a^2}-\frac{15\,(acx+c)^{\frac{7}{2}}-42\,(acx+c)^{\frac{5}{2}}c+35\,(acx+c)^{\frac{3}{2}}c^2}{a^2c^3}\right)}{105\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] -2/105*c*(22*sqrt(2)*sqrt(c)/a^2 - (15*(a*c*x + c)^(7/2) - 42*(a*c*x + c)^(
5/2)*c + 35*(a*c*x + c)^(3/2)*c^2/(a^2*c^3))/(a*abs(c))
```

3.389 $\int e^{\tanh^{-1}(ax)} x\sqrt{c - acx}\,dx$

Optimal. Leaf size=69

$$\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{15a^2(c-acx)^{3/2}}-\frac{2c\left(1-a^2x^2\right)^{3/2}}{5a^2\sqrt{c-acx}}$$

```
[Out] (2*c^2*(1 - a^2*x^2)^(3/2))/(15*a^2*(c - a*c*x)^(3/2)) - (2*c*(1 - a^2*x^2)
^(3/2))/(5*a^2*Sqrt[c - a*c*x])
```

Rubi [A] time = 0.0897471, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6128, 795, 649}

$$\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{15a^2(c-acx)^{3/2}}-\frac{2c\left(1-a^2x^2\right)^{3/2}}{5a^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x*Sqrt[c - a*c*x],x]
```


```
[Out] (2*c^2*(1 - a^2*x^2)^(3/2))/(15*a^2*(c - a*c*x)^(3/2)) - (2*c*(1 - a^2*x^2)
^(3/2))/(5*a^2*Sqrt[c - a*c*x])
```


Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx = c\int \frac{x\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx$$
$$= -\frac{2c\left(1-a^2x^2\right)^{3/2}}{5a^2\sqrt{c-acx}} + \frac{c\int \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}dx}{5a}$$
$$= \frac{2c^2\left(1-a^2x^2\right)^{3/2}}{15a^2(c-acx)^{3/2}} - \frac{2c\left(1-a^2x^2\right)^{3/2}}{5a^2\sqrt{c-acx}}$$

Mathematica [A] time = 0.0227393, size = 43, normalized size = 0.62

$$\frac{2(ax+1)^{3/2}(3ax-2)\sqrt{c-acx}}{15a^2\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + a*x)^(3/2)*(-2 + 3*a*x)*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - a*x])
```

Maple [A] time = 0.033, size = 40, normalized size = 0.6

$$\frac{2\,(ax+1)^2\,(3\,ax-2)}{15\,a^2}\sqrt{-acx+c}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/15*(a*x+1)^2*(3*a*x-2)*(-a*c*x+c)^(1/2)/a^2/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.03025, size = 112, normalized size = 1.62

$$\frac{2\left(3\,a^3\sqrt{c}x^3 - a^2\sqrt{c}x^2 + 4\,a\sqrt{c}x + 8\,\sqrt{c}\right)}{15\,\sqrt{ax+1}a^2} + \frac{2\left(a^2\sqrt{c}x^2 - a\sqrt{c}x - 2\,\sqrt{c}\right)}{3\,\sqrt{ax+1}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2/15*(3*a^3*sqrt(c)*x^3 - a^2*sqrt(c)*x^2 + 4*a*sqrt(c)*x + 8*sqrt(c))/(sqr
t(a*x + 1)*a^2) + 2/3*(a^2*sqrt(c)*x^2 - a*sqrt(c)*x - 2*sqrt(c))/(sqrt(a*x
 + 1)*a^2)
```

Fricas [A] time = 1.83253, size = 107, normalized size = 1.55

$$-\frac{2\left(3\,a^2x^2 + ax - 2\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{15\left(a^3x - a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] -2/15*(3*a^2*x^2 + a*x - 2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^3*x - a^
2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\,(ax-1)}\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [A] time = 1.36497, size = 70, normalized size = 1.01

$$-\frac{2\,c\left(\frac{2\,\sqrt{2}\sqrt{c}}{a}-\frac{3\,(acx+c)^{\frac{5}{2}}-5\,(acx+c)^{\frac{3}{2}}c}{ac^2}\right)}{15\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] -2/15*c*(2*sqrt(2)*sqrt(c)/a - (3*(a*c*x + c)^(5/2) - 5*(a*c*x + c)^(3/2)*c
)/(a*c^2))/(a*abs(c))
```

3.390 $\int e^{\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=35

$$\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}$$

```
[Out] (2*c^2*(1 - a^2*x^2)^(3/2))/(3*a*(c - a*c*x)^(3/2))
```

Rubi [A] time = 0.0451377, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6127, 649}

$$\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - a*c*x],x]
```

```
[Out] (2*c^2*(1 - a^2*x^2)^(3/2))/(3*a*(c - a*c*x)^(3/2))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}\sqrt{c-acx}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx\\ &= \frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.012753, size = 37, normalized size = 1.06

$$\frac{2(ax+1)^{3/2}\sqrt{c-acx}}{3a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + a*x)^(3/2)*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - a*x])
```

Maple [A] time = 0.03, size = 34, normalized size = 1.

$$\frac{2 (ax+1)^2}{3a}\sqrt{-acx+c}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/3*(a*x+1)^2*(-a*c*x+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.01662, size = 78, normalized size = 2.23

$$\frac{2\left(a^2\sqrt{c}x^2 - a\sqrt{c}x - 2\sqrt{c}\right)}{3\sqrt{ax+1}a} + \frac{2\left(a\sqrt{c}x + \sqrt{c}\right)}{\sqrt{ax+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 2/3*(a^2*sqrt(c)*x^2 - a*sqrt(c)*x - 2*sqrt(c))/(sqrt(a*x + 1)*a) + 2*(a*sq
rt(c)*x + sqrt(c))/(sqrt(a*x + 1)*a)
```

Fricas [A] time = 1.78012, size = 86, normalized size = 2.46

$$-\frac{2\sqrt{-a^2x^2+1}\sqrt{-acx+c}(ax+1)}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/3*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x + 1)/(a^2*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c(ax-1)}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [A] time = 1.35827, size = 43, normalized size = 1.23

$$-\frac{2\left(2\sqrt{2}\sqrt{c}-\frac{(acx+c)^{\frac{3}{2}}}{c}\right)c}{3\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(2*sqrt(2)*sqrt(c) - (a*c*x + c)^(3/2)/c)*c/(a*abs(c))
```

3.391 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=68

$$\frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] (2*c*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x] - 2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1
 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rubi [A] time = 0.151622, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {6128, 865, 875, 208}

$$\frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*c*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x] - 2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1
 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g
*(m - n - 1)), x] - Dist[(c*m*(e*f + d*g))/(e^2*g*(m - n - 1)), Int[(d + e*
x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f,
 g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Int
egerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}\,dx\\ &= \frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}}+\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx\\ &= \frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}}+\left(2a^2c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\ &= \frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\end{aligned}$$

Mathematica [A] time = 0.0195518, size = 46, normalized size = 0.68

$$\frac{\sqrt{c-acx}\left(2\sqrt{ax+1}-2\tanh^{-1}\left(\sqrt{ax+1}\right)\right)}{\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (Sqrt[c - a*c*x]*(2*Sqrt[1 + a*x] - 2*ArcTanh[Sqrt[1 + a*x]]))/Sqrt[1 - a*x
]
```

Maple [A] time = 0.097, size = 71, normalized size = 1.

$$2\,\frac{\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}}{(ax-1)\sqrt{c\,(ax+1)}}\left(\sqrt{c}\text{Artanh}\left(\frac{\sqrt{c\,(ax+1)}}{\sqrt{c}}\right)-\sqrt{c\,(ax+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)/x,x)
```

```
[Out] 2*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(c^(1/2)*arctanh((c*(a*x+1))^(1/2)/
c^(1/2))-(c*(a*x+1))^(1/2))/(a*x-1)/(c*(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{c}\int \frac{1}{\sqrt{ax+1}x}\,dx+\frac{2\left(a\sqrt{c}x+\sqrt{c}\right)}{\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxim
a")
```

```
[Out] sqrt(c)*integrate(1/(sqrt(a*x + 1)*x), x) + 2*(a*sqrt(c)*x + sqrt(c))/sqrt(
a*x + 1)
```

Fricas [A] time = 1.89636, size = 417, normalized size = 6.13

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{ax-1}, -\frac{2\left((ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{-c}}{a^2cx^2-c}\right)\right.}{ax-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="frica
s")
```

```
[Out] [((a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*
c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) - 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c
))/(a*x - 1), -2*((a*x - 1)*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x
+ c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a*x
- 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c(ax-1)}\,(ax+1)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [A] time = 1.26251, size = 108, normalized size = 1.59

$$\frac{2c^2\left(\frac{\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)+\sqrt{2}\sqrt{-c}}{\sqrt{-c}\sqrt{c}}+\frac{\sqrt{acx+c}}{c}\right)}{|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac"
)
```

```
[Out] 2*c^2*(arctan(sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) - (sqrt(c)*arctan(sqrt(2)*
sqrt(c)/sqrt(-c)) + sqrt(2)*sqrt(-c))/(sqrt(-c)*sqrt(c)) + sqrt(a*c*x + c)/
c)/abs(c)
```

3.392 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}-a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] -((c*Sqrt[1 - a^2*x^2])/(x*Sqrt[c - a*c*x])) - a*Sqrt[c]*ArcTanh[(Sqrt[c]*S
qrt[1 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rubi [A] time = 0.156802, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {6128, 863, 875, 208}

$$-\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}-a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -((c*Sqrt[1 - a^2*x^2])/(x*Sqrt[c - a*c*x])) - a*Sqrt[c]*ArcTanh[(Sqrt[c]*S
qrt[1 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 863

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g*
(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x^2\sqrt{c-acx}}\,dx\\
&= -\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}+\frac{1}{2}a\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}+\left(a^3c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\
&= -\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}-a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0242757, size = 57, normalized size = 0.79

$$-\frac{\sqrt{c-acx}\left(ax+ax\sqrt{ax+1}\tanh^{-1}\left(\sqrt{ax+1}\right)+1\right)}{x\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -((Sqrt[c - a*c*x]*(1 + a*x + a*x*Sqrt[1 + a*x]*ArcTanh[Sqrt[1 + a*x]]))/(x
*Sqrt[1 - a^2*x^2]))
```

Maple [A] time = 0.1, size = 78, normalized size = 1.1

$$\frac{1}{(ax-1)x}\sqrt{-a^2x^2+1}\sqrt{-c(ax-1)}\left(\text{Artanh}\left(\sqrt{c(ax+1)}\frac{1}{\sqrt{c}}\right)xac+\sqrt{c(ax+1)}\sqrt{c}\right)\frac{1}{\sqrt{c(ax+1)}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)/x^2,x)
```

```
[Out] (-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(arctanh((c*(a*x+1))^(1/2)/c^(1/2))*x
*a*c+(c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)/(c*(a*x+1))^(1/2)/x/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}(ax+1)}{\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Fricas [A] time = 1.93985, size = 441, normalized size = 6.12

$$\left[\frac{\left(a^2x^2-ax\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{2\left(ax^2-x\right)},-\frac{\left(a^2x^2-ax\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2x^2+1}\cdots}{a^2\cdots}\right)\cdots}{ax\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] [1/2*((a^2*x^2 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1
)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(
-a*c*x + c))/(a*x^2 - x), -((a^2*x^2 - a*x)*sqrt(-c)*arctan(sqrt(-a^2*x^2 +
 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt(-a^2*x^2 + 1)*sqrt(-a
*c*x + c))/(a*x^2 - x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\left(ax-1\right)}\left(ax+1\right)}{x^2\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [A] time = 1.28845, size = 136, normalized size = 1.89

$$\frac{\left(a^2c\left(\frac{\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{acx+c}}{acx}\right)-\frac{a^2c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)-\sqrt{2}a^2\sqrt{-c}c}{\sqrt{-c}\sqrt{c}}\right)c}{a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="gia
c")
```

```
[Out] (a^2*c*(arctan(sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(a*c*x + c)/(a*c*x)
) - (a^2*c^(3/2)*arctan(sqrt(2)*sqrt(c)/sqrt(-c)) - sqrt(2)*a^2*sqrt(-c)*c)
/(sqrt(-c)*sqrt(c)))*c/(a*abs(c))
```

3.393 $\int e^{2\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=101

$$-\frac{2(c-acx)^{9/2}}{9a^4c^4}+\frac{10(c-acx)^{7/2}}{7a^4c^3}-\frac{18(c-acx)^{5/2}}{5a^4c^2}+\frac{14(c-acx)^{3/2}}{3a^4c}-\frac{4\sqrt{c-acx}}{a^4}$$

```
[Out] (-4*Sqrt[c - a*c*x])/a^4 + (14*(c - a*c*x)^(3/2))/(3*a^4*c) - (18*(c - a*c*
x)^(5/2))/(5*a^4*c^2) + (10*(c - a*c*x)^(7/2))/(7*a^4*c^3) - (2*(c - a*c*x)
^(9/2))/(9*a^4*c^4)
```

Rubi [A] time = 0.132296, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6130, 21, 77}

$$-\frac{2(c-acx)^{9/2}}{9a^4c^4}+\frac{10(c-acx)^{7/2}}{7a^4c^3}-\frac{18(c-acx)^{5/2}}{5a^4c^2}+\frac{14(c-acx)^{3/2}}{3a^4c}-\frac{4\sqrt{c-acx}}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a^4 + (14*(c - a*c*x)^(3/2))/(3*a^4*c) - (18*(c - a*c*
x)^(5/2))/(5*a^4*c^2) + (10*(c - a*c*x)^(7/2))/(7*a^4*c^3) - (2*(c - a*c*x)
^(9/2))/(9*a^4*c^4)
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\int\frac{x^3(1+ax)\sqrt{c-acx}}{1-ax}\,dx$$
$$=c\int\frac{x^3(1+ax)}{\sqrt{c-acx}}\,dx$$
$$=c\int\left(\frac{2}{a^3\sqrt{c-acx}}-\frac{7\sqrt{c-acx}}{a^3c}+\frac{9(c-acx)^{3/2}}{a^3c^2}-\frac{5(c-acx)^{5/2}}{a^3c^3}+\frac{(c-acx)^{7/2}}{a^3c^4}\right)dx$$
$$=-\frac{4\sqrt{c-acx}}{a^4}+\frac{14(c-acx)^{3/2}}{3a^4c}-\frac{18(c-acx)^{5/2}}{5a^4c^2}+\frac{10(c-acx)^{7/2}}{7a^4c^3}-\frac{2(c-acx)^{9/2}}{9a^4c^4}$$

Mathematica [A] time = 0.0660835, size = 48, normalized size = 0.48

$$-\frac{2\left(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272\right)\sqrt{c-acx}}{315a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(272 + 136*a*x + 102*a^2*x^2 + 85*a^3*x^3 + 35*a^4*x^4)
)/(315*a^4)
```

Maple [A] time = 0.035, size = 45, normalized size = 0.5

$$-\frac{70\,x^4a^4+170\,x^3a^3+204\,a^2x^2+272\,ax+544}{315\,a^4}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/315*(-a*c*x+c)^(1/2)*(35*a^4*x^4+85*a^3*x^3+102*a^2*x^2+136*a*x+272)/a^4
```

Maxima [A] time = 0.96522, size = 100, normalized size = 0.99

$$-\frac{2\left(35\,(-acx+c)^{\frac{9}{2}}-225\,(-acx+c)^{\frac{7}{2}}c+567\,(-acx+c)^{\frac{5}{2}}c^2-735\,(-acx+c)^{\frac{3}{2}}c^3+630\,\sqrt{-acx+c}c^4\right)}{315\,a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -2/315*(35*(-a*c*x + c)^(9/2) - 225*(-a*c*x + c)^(7/2)*c + 567*(-a*c*x + c)
^(5/2)*c^2 - 735*(-a*c*x + c)^(3/2)*c^3 + 630*sqrt(-a*c*x + c)*c^4)/(a^4*c^
4)
```

Fricas [A] time = 1.76348, size = 115, normalized size = 1.14

$$-\frac{2\left(35\,a^4x^4+85\,a^3x^3+102\,a^2x^2+136\,ax+272\right)\sqrt{-acx+c}}{315\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/315*(35*a^4*x^4 + 85*a^3*x^3 + 102*a^2*x^2 + 136*a*x + 272)*sqrt(-a*c*x
+ c)/a^4
```

Sympy [A] time = 10.9475, size = 83, normalized size = 0.82

$$\frac{2\left(-2c^4\sqrt{-acx+c}+\frac{7c^3(-acx+c)^{\frac{3}{2}}}{3}-\frac{9c^2(-acx+c)^{\frac{5}{2}}}{5}+\frac{5c(-acx+c)^{\frac{7}{2}}}{7}-\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(-a*c*x+c)**(1/2),x)
```

```
[Out] 2*(-2*c**4*sqrt(-a*c*x + c) + 7*c**3*(-a*c*x + c)**(3/2)/3 - 9*c**2*(-a*c*x
 + c)**(5/2)/5 + 5*c*(-a*c*x + c)**(7/2)/7 - (-a*c*x + c)**(9/2)/9)/(a**4*c
**4)
```

Giac [A] time = 1.23064, size = 140, normalized size = 1.39

$$-\frac{2\left(35\,(acx-c)^4\sqrt{-acx+c}+225\,(acx-c)^3\sqrt{-acx+c}c+567\,(acx-c)^2\sqrt{-acx+c}c^2-735\,(-acx+c)^{\frac{3}{2}}c^3+630\,\sqrt{-acx}\right.}{315\,a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c) + 225*(a*c*x - c)^3*sqrt(-a*c*x +
 c)*c + 567*(a*c*x - c)^2*sqrt(-a*c*x + c)*c^2 - 735*(-a*c*x + c)^(3/2)*c^3
 + 630*sqrt(-a*c*x + c)*c^4)/(a^4*c^4)
```

3.394 $\int e^{2\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=80

$$\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{8(c-acx)^{5/2}}{5a^3c^2}+\frac{10(c-acx)^{3/2}}{3a^3c}-\frac{4\sqrt{c-acx}}{a^3}$$

```
[Out] (-4*Sqrt[c - a*c*x])/a^3 + (10*(c - a*c*x)^(3/2))/(3*a^3*c) - (8*(c - a*c*x
)^(5/2))/(5*a^3*c^2) + (2*(c - a*c*x)^(7/2))/(7*a^3*c^3)
```

Rubi [A] time = 0.162893, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6130, 21, 77}

$$\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{8(c-acx)^{5/2}}{5a^3c^2}+\frac{10(c-acx)^{3/2}}{3a^3c}-\frac{4\sqrt{c-acx}}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^2*Sqrt[c - a*c*x],x]
```


```
[Out] (-4*Sqrt[c - a*c*x])/a^3 + (10*(c - a*c*x)^(3/2))/(3*a^3*c) - (8*(c - a*c*x
)^(5/2))/(5*a^3*c^2) + (2*(c - a*c*x)^(7/2))/(7*a^3*c^3)
```


Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \int \frac{x^2(1+ax)\sqrt{c-acx}}{1-ax}\,dx$$
$$= c\int \frac{x^2(1+ax)}{\sqrt{c-acx}}\,dx$$
$$= c\int \left(\frac{2}{a^2\sqrt{c-acx}} - \frac{5\sqrt{c-acx}}{a^2c} + \frac{4(c-acx)^{3/2}}{a^2c^2} - \frac{(c-acx)^{5/2}}{a^2c^3}\right)dx$$
$$= -\frac{4\sqrt{c-acx}}{a^3} + \frac{10(c-acx)^{3/2}}{3a^3c} - \frac{8(c-acx)^{5/2}}{5a^3c^2} + \frac{2(c-acx)^{7/2}}{7a^3c^3}$$

Mathematica [A] time = 0.052234, size = 40, normalized size = 0.5

$$-\frac{2\left(15a^3x^3+39a^2x^2+52ax+104\right)\sqrt{c-acx}}{105a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(104 + 52*a*x + 39*a^2*x^2 + 15*a^3*x^3))/(105*a^3)
```

Maple [A] time = 0.031, size = 37, normalized size = 0.5

$$-\frac{30\,x^3a^3+78\,a^2x^2+104\,ax+208}{105\,a^3}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/105*(-a*c*x+c)^(1/2)*(15*a^3*x^3+39*a^2*x^2+52*a*x+104)/a^3
```

Maxima [A] time = 0.963379, size = 81, normalized size = 1.01

$$\frac{2\left(15\,(-acx+c)^{\frac{7}{2}}-84\,(-acx+c)^{\frac{5}{2}}c+175\,(-acx+c)^{\frac{3}{2}}c^2-210\,\sqrt{-acx+c}c^3\right)}{105\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 2/105*(15*(-a*c*x + c)^(7/2) - 84*(-a*c*x + c)^(5/2)*c + 175*(-a*c*x + c)^(
3/2)*c^2 - 210*sqrt(-a*c*x + c)*c^3)/(a^3*c^3)
```

Fricas [A] time = 1.80303, size = 95, normalized size = 1.19

$$-\frac{2\left(15\,a^3x^3+39\,a^2x^2+52\,ax+104\right)\sqrt{-acx+c}}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/105*(15*a^3*x^3 + 39*a^2*x^2 + 52*a*x + 104)*sqrt(-a*c*x + c)/a^3
```

Sympy [A] time = 8.01316, size = 68, normalized size = 0.85

$$-\frac{2\left(2c^3\sqrt{-acx+c}-\frac{5c^2(-acx+c)^{\frac{3}{2}}}{3}+\frac{4c(-acx+c)^{\frac{5}{2}}}{5}-\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] -2*(2*c**3*sqrt(-a*c*x + c) - 5*c**2*(-a*c*x + c)**(3/2)/3 + 4*c*(-a*c*x +
c)**(5/2)/5 - (-a*c*x + c)**(7/2)/7)/(a**3*c**3)
```

Giac [A] time = 1.2239, size = 108, normalized size = 1.35

$$-\frac{2\left(15\,(acx-c)^3\sqrt{-acx+c}+84\,(acx-c)^2\sqrt{-acx+c}c-175\,(-acx+c)^{\frac{3}{2}}c^2+210\,\sqrt{-acx+c}c^3\right)}{105\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/105*(15*(a*c*x - c)^3*sqrt(-a*c*x + c) + 84*(a*c*x - c)^2*sqrt(-a*c*x +
c)*c - 175*(-a*c*x + c)^(3/2)*c^2 + 210*sqrt(-a*c*x + c)*c^3)/(a^3*c^3)
```

3.395 $\int e^{2\tanh^{-1}(ax)} x\sqrt{c-acx}\,dx$

Optimal. Leaf size=57

$$-\frac{2(c-acx)^{5/2}}{5a^2c^2}+\frac{2(c-acx)^{3/2}}{a^2c}-\frac{4\sqrt{c-acx}}{a^2}$$

```
[Out] (-4*Sqrt[c - a*c*x])/a^2 + (2*(c - a*c*x)^(3/2))/(a^2*c) - (2*(c - a*c*x)^(
5/2))/(5*a^2*c^2)
```

Rubi [A] time = 0.0857842, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6130, 21, 77}

$$-\frac{2(c-acx)^{5/2}}{5a^2c^2}+\frac{2(c-acx)^{3/2}}{a^2c}-\frac{4\sqrt{c-acx}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a^2 + (2*(c - a*c*x)^(3/2))/(a^2*c) - (2*(c - a*c*x)^(
5/2))/(5*a^2*c^2)
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx=\int \frac{x(1+ax)\sqrt{c-acx}}{1-ax}\,dx$$
$$=c\int \frac{x(1+ax)}{\sqrt{c-acx}}\,dx$$
$$=c\int \left(\frac{2}{a\sqrt{c-acx}}-\frac{3\sqrt{c-acx}}{ac}+\frac{(c-acx)^{3/2}}{ac^2}\right)dx$$
$$=-\frac{4\sqrt{c-acx}}{a^2}+\frac{2(c-acx)^{3/2}}{a^2c}-\frac{2(c-acx)^{5/2}}{5a^2c^2}$$

Mathematica [A] time = 0.0404044, size = 31, normalized size = 0.54

$$-\frac{2\left(a^2x^2+3ax+6\right)\sqrt{c-acx}}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(6 + 3*a*x + a^2*x^2))/(5*a^2)
```

Maple [A] time = 0.031, size = 28, normalized size = 0.5

$$-\frac{2\,a^2x^2+6\,ax+12}{5\,a^2}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/5*(-a*c*x+c)^(1/2)*(a^2*x^2+3*a*x+6)/a^2
```

Maxima [A] time = 0.94846, size = 59, normalized size = 1.04

$$-\frac{2\left((-acx+c)^{\frac{5}{2}}-5\,(-acx+c)^{\frac{3}{2}}c+10\,\sqrt{-acx+c}c^2\right)}{5\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*((-a*c*x + c)^(5/2) - 5*(-a*c*x + c)^(3/2)*c + 10*sqrt(-a*c*x + c)*c^2
)/(a^2*c^2)
```

Fricas [A] time = 1.90873, size = 66, normalized size = 1.16

$$-\frac{2\left(a^2x^2+3\,ax+6\right)\sqrt{-acx+c}}{5\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(a^2*x^2 + 3*a*x + 6)*sqrt(-a*c*x + c)/a^2
```

Sympy [A] time = 6.45034, size = 48, normalized size = 0.84

$$\frac{2\left(-2c^2\sqrt{-acx+c}+c\left(-acx+c\right)^{\frac{3}{2}}-\frac{\left(-acx+c\right)^{\frac{5}{2}}}{5}\right)}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(-a*c*x+c)**(1/2),x)
```

```
[Out] 2*(-2*c**2*sqrt(-a*c*x + c) + c*(-a*c*x + c)**(3/2) - (-a*c*x + c)**(5/2)/5
)/(a**2*c**2)
```

Giac [A] time = 1.22635, size = 74, normalized size = 1.3

$$-\frac{2\left(\left(acx-c\right)^2\sqrt{-acx+c}-5\left(-acx+c\right)^{\frac{3}{2}}c+10\sqrt{-acx+c}c^2\right)}{5a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/5*((a*c*x - c)^2*sqrt(-a*c*x + c) - 5*(-a*c*x + c)^(3/2)*c + 10*sqrt(-a*
c*x + c)*c^2)/(a^2*c^2)
```

3.396 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=38

$$\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}$$

```
[Out] (-4*Sqrt[c - a*c*x])/a + (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Rubi [A] time = 0.0480732, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6130, 21, 43}

$$\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x],x]

[Out] (-4*Sqrt[c - a*c*x])/a + (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx &= \int \frac{(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\
&= c\int \frac{1+ax}{\sqrt{c-acx}}\,dx\\
&= c\int \left(\frac{2}{\sqrt{c-acx}}-\frac{\sqrt{c-acx}}{c}\right)dx\\
&= -\frac{4\sqrt{c-acx}}{a}+\frac{2(c-acx)^{3/2}}{3ac}
\end{aligned}$$

Mathematica [A] time = 0.023291, size = 23, normalized size = 0.61

$$-\frac{2(ax+5)\sqrt{c-acx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (-2*(5 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Maple [A] time = 0.033, size = 20, normalized size = 0.5

$$-\frac{2\,ax+10}{3\,a}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/3*(-a*c*x+c)^(1/2)*(a*x+5)/a
```

Maxima [A] time = 0.95218, size = 41, normalized size = 1.08

$$\frac{2\left((-acx+c)^{\frac{3}{2}}-6\,\sqrt{-acx+c}c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

Fricas [A] time = 1.81769, size = 47, normalized size = 1.24

$$-\frac{2\,\sqrt{-acx+c}(ax+5)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-a*c*x + c)*(a*x + 5)/a
```

Sympy [A] time = 4.18732, size = 31, normalized size = 0.82

$$-\frac{2\left(2c\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(1/2),x)
```

```
[Out] -2*(2*c*sqrt(-a*c*x + c) - (-a*c*x + c)**(3/2)/3)/(a*c)
```

Giac [A] time = 1.18508, size = 41, normalized size = 1.08

$$\frac{2\left((-acx+c)^{\frac{3}{2}}-6\sqrt{-acx+c}c\right)}{3ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

3.397 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=39

$$-2\sqrt{c-acx}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] -2*Sqrt[c - a*c*x] - 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rubi [A] time = 0.10492, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6130, 21, 80, 63, 208}

$$-2\sqrt{c-acx}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x,x]

[Out] -2*Sqrt[c - a*c*x] - 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx &= \int \frac{(1+ax)\sqrt{c-acx}}{x(1-ax)}\,dx \\
&= c\int \frac{1+ax}{x\sqrt{c-acx}}\,dx \\
&= -2\sqrt{c-acx}+c\int \frac{1}{x\sqrt{c-acx}}\,dx \\
&= -2\sqrt{c-acx}-\frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right)}{a} \\
&= -2\sqrt{c-acx}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.030375, size = 39, normalized size = 1.

$$
-2\sqrt{c-acx}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] -2*Sqrt[c - a*c*x] - 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Maple [A] time = 0.035, size = 32, normalized size = 0.8

$$
-2\,\text{Artanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}-2\sqrt{-acx+c}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x,x)
```

```
[Out] -2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-2*(-a*c*x+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86101, size = 207, normalized size = 5.31

$$\left[\sqrt{c}\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)-2\sqrt{-acx+c}, 2\sqrt{-c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-2\sqrt{-acx+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(c)*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) - 2*sqrt(-a*c*x
+ c), 2*sqrt(-c)*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*sqrt(-a*c*x + c)]
```

Sympy [A] time = 6.20633, size = 39, normalized size = 1.

$$\frac{2c\operatorname{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(1/2)/x,x)
```

```
[Out] 2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 2*sqrt(-a*c*x + c)
```

Giac [A] time = 1.27942, size = 49, normalized size = 1.26

$$\frac{2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 2*sqrt(-a*c*x + c)
```

3.398 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=43

$$-\frac{\sqrt{c-acx}}{x}-3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] -(Sqrt[c - a*c*x]/x) - 3*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rubi [A] time = 0.106619, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6130, 21, 78, 63, 208}

$$-\frac{\sqrt{c-acx}}{x}-3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -(Sqrt[c - a*c*x]/x) - 3*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx &= \int \frac{(1+ax)\sqrt{c-acx}}{x^2(1-ax)}\,dx \\
&= c\int \frac{1+ax}{x^2\sqrt{c-acx}}\,dx \\
&= -\frac{\sqrt{c-acx}}{x} + \frac{1}{2}(3ac)\int \frac{1}{x\sqrt{c-acx}}\,dx \\
&= -\frac{\sqrt{c-acx}}{x} - 3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right) \\
&= -\frac{\sqrt{c-acx}}{x} - 3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0279141, size = 43, normalized size = 1.

$$-\frac{\sqrt{c-acx}}{x} - 3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -(Sqrt[c - a*c*x]/x) - 3*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Maple [A] time = 0.04, size = 45, normalized size = 1.1

$$2\,ac\left(-1/2\,\frac{\sqrt{-acx+c}}{acx} - 3/2\,\frac{1}{\sqrt{c}}\text{Artanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^2,x)
```

```
[Out] 2*a*c*(-1/2*(-a*c*x+c)^(1/2)/a/c/x-3/2/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(
1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98908, size = 234, normalized size = 5.44

$$\left[\frac{3\,a\sqrt{c}x\log\left(\frac{acx+2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)-2\,\sqrt{-acx+c}}{2\,x},\ \frac{3\,a\sqrt{-c}x\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-\sqrt{-acx+c}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fricas"
)
```

```
[Out] [1/2*(3*a*sqrt(c)*x*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) - 2*s
qrt(-a*c*x + c))/x, (3*a*sqrt(-c)*x*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - s
qrt(-a*c*x + c))/x]
```

Sympy [B] time = 12.9676, size = 119, normalized size = 2.77

$$\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(-c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}-\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}+\frac{2ac\operatorname{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{-acx+c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(1/2)/x**2,x)
```

```
[Out] a*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 - a*c**2
*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 + 2*a*c*atan(sq
rt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)/x
```

Giac [A] time = 1.23356, size = 54, normalized size = 1.26

$$\frac{3\,ac\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{-acx+c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 3*a*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)/x
```

3.399 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=68

$$-\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-\frac{\sqrt{c-acx}}{2x^2}-\frac{7a\sqrt{c-acx}}{4x}$$

```
[Out] -Sqrt[c - a*c*x]/(2*x^2) - (7*a*Sqrt[c - a*c*x])/(4*x) - (7*a^2*Sqrt[c]*Arc
Tanh[Sqrt[c - a*c*x]/Sqrt[c]])/4
```

Rubi [A] time = 0.115961, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 21, 78, 51, 63, 208}

$$-\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-\frac{\sqrt{c-acx}}{2x^2}-\frac{7a\sqrt{c-acx}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] -Sqrt[c - a*c*x]/(2*x^2) - (7*a*Sqrt[c - a*c*x])/(4*x) - (7*a^2*Sqrt[c]*Arc
Tanh[Sqrt[c - a*c*x]/Sqrt[c]])/4
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx &= \int \frac{(1+ax)\sqrt{c-acx}}{x^3(1-ax)}\,dx\\
&= c\int \frac{1+ax}{x^3\sqrt{c-acx}}\,dx\\
&= -\frac{\sqrt{c-acx}}{2x^2}+\frac{1}{4}(7ac)\int \frac{1}{x^2\sqrt{c-acx}}\,dx\\
&= -\frac{\sqrt{c-acx}}{2x^2}-\frac{7a\sqrt{c-acx}}{4x}+\frac{1}{8}\left(7a^2c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx\\
&= -\frac{\sqrt{c-acx}}{2x^2}-\frac{7a\sqrt{c-acx}}{4x}-\frac{1}{4}(7a)\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right)\\
&= -\frac{\sqrt{c-acx}}{2x^2}-\frac{7a\sqrt{c-acx}}{4x}-\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0451259, size = 55, normalized size = 0.81

$$-\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-\frac{(7ax+2)\sqrt{c-acx}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] -((2 + 7*a*x)*Sqrt[c - a*c*x])/(4*x^2) - (7*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*
c*x]/Sqrt[c]])/4
```

Maple [A] time = 0.043, size = 65, normalized size = 1.

$$-2\,a^2c^2\left(\frac{1}{a^2c^2x^2}\left(-\frac{7\,(-acx+c)^{3/2}}{8\,c}+\frac{9\,\sqrt{-acx+c}}{8}\right)+\frac{7}{8\,c^{3/2}}\mathrm{Artanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^3,x)
```

```
[Out] -2*a^2*c^2*((-7/8/c*(-a*c*x+c)^(3/2)+9/8*(-a*c*x+c)^(1/2))/a^2/c^2/x^2+7/8/
c^(3/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94544, size = 288, normalized size = 4.24

$$\left[\frac{7\,a^2\sqrt{c}x^2\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)-2\sqrt{-acx+c}(7\,ax+2)}{8\,x^2},\frac{7\,a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-\sqrt{-acx+c}(7\,ax+2)}{4\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="fricas"
)
```

```
[Out] [1/8*(7*a^2*sqrt(c)*x^2*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) -
 2*sqrt(-a*c*x + c)*(7*a*x + 2))/x^2, 1/4*(7*a^2*sqrt(-c)*x^2*arctan(sqrt(-
a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(7*a*x + 2))/x^2]
```

Sympy [B] time = 23.728, size = 270, normalized size = 3.97

$$-\frac{10a^2c^4\sqrt{-acx+c}}{16ac^4x-8c^4+8c^2(-acx+c)^2}+\frac{6a^2c^3(-acx+c)^{\frac{3}{2}}}{16ac^4x-8c^4+8c^2(-acx+c)^2}+\frac{3a^2c^3\sqrt{\frac{1}{c^5}}\log\left(-c^3\sqrt{\frac{1}{c^5}}+\sqrt{-acx+c}\right)}{8}-\frac{3a^2c^3\sqrt{\frac{1}{c^5}}\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(1/2)/x**3,x)
```

```
[Out] -10*a**2*c**4*sqrt(-a*c*x + c)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)*
*2) + 6*a**2*c**3*(-a*c*x + c)**(3/2)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*
x + c)**2) + 3*a**2*c**3*sqrt(c**(-5))*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*
x + c))/8 - 3*a**2*c**3*sqrt(c**(-5))*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x
+ c))/8 + a**2*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c
))/2 - a**2*c**2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2
 - a*sqrt(-a*c*x + c)/x
```

Giac [A] time = 1.17247, size = 97, normalized size = 1.43

$$\frac{7\,a^2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}}+\frac{7\,(-acx+c)^{\frac{3}{2}}a^2c-9\sqrt{-acx+c}a^2c^2}{4\,a^2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 7/4*a^2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/4*(7*(-a*c*x + c)^
(3/2)*a^2*c - 9*sqrt(-a*c*x + c)*a^2*c^2)/(a^2*c^2*x^2)
```

3.400 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=89

$$-\frac{11a^2\sqrt{c-acx}}{8x}-\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-\frac{11a\sqrt{c-acx}}{12x^2}-\frac{\sqrt{c-acx}}{3x^3}$$

```
[Out] -Sqrt[c - a*c*x]/(3*x^3) - (11*a*Sqrt[c - a*c*x])/(12*x^2) - (11*a^2*Sqrt[c
 - a*c*x])/(8*x) - (11*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/8
```

Rubi [A] time = 0.128482, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 21, 78, 51, 63, 208}

$$-\frac{11a^2\sqrt{c-acx}}{8x}-\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-\frac{11a\sqrt{c-acx}}{12x^2}-\frac{\sqrt{c-acx}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^4,x]
```



```
[Out] -Sqrt[c - a*c*x]/(3*x^3) - (11*a*Sqrt[c - a*c*x])/(12*x^2) - (11*a^2*Sqrt[c
 - a*c*x])/(8*x) - (11*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/8
```


Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx &= \int \frac{(1+ax)\sqrt{c-acx}}{x^4(1-ax)}\,dx \\
&= c\int \frac{1+ax}{x^4\sqrt{c-acx}}\,dx \\
&= -\frac{\sqrt{c-acx}}{3x^3}+\frac{1}{6}(11ac)\int \frac{1}{x^3\sqrt{c-acx}}\,dx \\
&= -\frac{\sqrt{c-acx}}{3x^3}-\frac{11a\sqrt{c-acx}}{12x^2}+\frac{1}{8}\left(11a^2c\right)\int \frac{1}{x^2\sqrt{c-acx}}\,dx \\
&= -\frac{\sqrt{c-acx}}{3x^3}-\frac{11a\sqrt{c-acx}}{12x^2}-\frac{11a^2\sqrt{c-acx}}{8x}+\frac{1}{16}\left(11a^3c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx \\
&= -\frac{\sqrt{c-acx}}{3x^3}-\frac{11a\sqrt{c-acx}}{12x^2}-\frac{11a^2\sqrt{c-acx}}{8x}-\frac{1}{8}\left(11a^2\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right) \\
&= -\frac{\sqrt{c-acx}}{3x^3}-\frac{11a\sqrt{c-acx}}{12x^2}-\frac{11a^2\sqrt{c-acx}}{8x}-\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0563852, size = 63, normalized size = 0.71

$$
-\frac{\left(33a^2x^2+22ax+8\right)\sqrt{c-acx}}{24x^3}-\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] -(Sqrt[c - a*c*x]*(8 + 22*a*x + 33*a^2*x^2))/(24*x^3) - (11*a^3*Sqrt[c]*Arc
Tanh[Sqrt[c - a*c*x]/Sqrt[c]])/8
```

Maple [A] time = 0.043, size = 80, normalized size = 0.9

$$
2\,a^3c^3\left(-\frac{1}{a^3c^3x^3}\left(\frac{11\,(-acx+c)^{5/2}}{16\,c^2}-\frac{11\,(-acx+c)^{3/2}}{6\,c}+\frac{21\,\sqrt{-acx+c}}{16}\right)-\frac{11}{16\,c^{5/2}}\operatorname{Artanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^4,x)
```

```
[Out] 2*a^3*c^3*(-(11/16/c^2*(-a*c*x+c)^(5/2)-11/6/c*(-a*c*x+c)^(3/2)+21/16*(-a*c
*x+c)^(1/2))/a^3/c^3/x^3-11/16/c^(5/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84949, size = 331, normalized size = 3.72

$$\left[\frac{33\,a^3\sqrt{c}x^3\log\left(\frac{acx+2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)-2\left(33\,a^2x^2+22\,ax+8\right)\sqrt{-acx+c}}{48\,x^3},\frac{33\,a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-\left(33\,a^2x^2+2\right.}{24\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="fricas"
)
```

```
[Out] [1/48*(33*a^3*sqrt(c)*x^3*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x)
 - 2*(33*a^2*x^2 + 22*a*x + 8)*sqrt(-a*c*x + c))/x^3, 1/24*(33*a^3*sqrt(-c)
*x^3*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (33*a^2*x^2 + 22*a*x + 8)*sqrt(-
a*c*x + c))/x^3]
```

Sympy [B] time = 42.3672, size = 439, normalized size = 4.93

$$\frac{66a^3c^6\sqrt{-acx+c}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}-\frac{80a^3c^5\left(-acx+c\right)^{\frac{3}{2}}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}+\frac{}{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(1/2)/x**4,x)
```

```
[Out] 66*a**3*c**6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x +
 c)**2 + 48*c**3*(-a*c*x + c)**3) - 80*a**3*c**5*(-a*c*x + c)**(3/2)/(-144*
a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) +
30*a**3*c**4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*
x + c)**2 + 48*c**3*(-a*c*x + c)**3) - 10*a**3*c**4*sqrt(-a*c*x + c)/(16*a*
c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) + 5*a**3*c**4*sqrt(c**(-7))*log(-
c**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 - 5*a**3*c**4*sqrt(c**(-7))*log(c
**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 + 6*a**3*c**3*(-a*c*x + c)**(3/2)/
(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) + 3*a**3*c**3*sqrt(c**(-5))
*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8 - 3*a**3*c**3*sqrt(c**(-5))*
log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8
```

Giac [A] time = 1.21523, size = 134, normalized size = 1.51

$$\frac{11\,a^3c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}}-\frac{33\,(acx-c)^2\sqrt{-acx+c}a^3c-88\,(-acx+c)^{\frac{3}{2}}a^3c^2+63\,\sqrt{-acx+c}a^3c^3}{24\,a^3c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 11/8*a^3*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 1/24*(33*(a*c*x - c
)^2*sqrt(-a*c*x + c)*a^3*c - 88*(-a*c*x + c)^(3/2)*a^3*c^2 + 63*sqrt(-a*c*x
 + c)*a^3*c^3)/(a^3*c^3*x^3)
```

3.401 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=110

$$-\frac{25a^2\sqrt{c-acx}}{32x^2}-\frac{75a^3\sqrt{c-acx}}{64x}-\frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-\frac{5a\sqrt{c-acx}}{8x^3}-\frac{\sqrt{c-acx}}{4x^4}$$

```
[Out] -Sqrt[c - a*c*x]/(4*x^4) - (5*a*Sqrt[c - a*c*x])/(8*x^3) - (25*a^2*Sqrt[c -
 a*c*x])/(32*x^2) - (75*a^3*Sqrt[c - a*c*x])/(64*x) - (75*a^4*Sqrt[c]*ArcTa
nh[Sqrt[c - a*c*x]/Sqrt[c]])/64
```

Rubi [A] time = 0.144642, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 21, 78, 51, 63, 208}

$$-\frac{25a^2\sqrt{c-acx}}{32x^2}-\frac{75a^3\sqrt{c-acx}}{64x}-\frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-\frac{5a\sqrt{c-acx}}{8x^3}-\frac{\sqrt{c-acx}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] -Sqrt[c - a*c*x]/(4*x^4) - (5*a*Sqrt[c - a*c*x])/(8*x^3) - (25*a^2*Sqrt[c -
 a*c*x])/(32*x^2) - (75*a^3*Sqrt[c - a*c*x])/(64*x) - (75*a^4*Sqrt[c]*ArcTa
nh[Sqrt[c - a*c*x]/Sqrt[c]])/64
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx &= \int \frac{(1+ax)\sqrt{c-acx}}{x^5(1-ax)}\,dx \\
&= c\int \frac{1+ax}{x^5\sqrt{c-acx}}\,dx \\
&= -\frac{\sqrt{c-acx}}{4x^4}+\frac{1}{8}(15ac)\int \frac{1}{x^4\sqrt{c-acx}}\,dx \\
&= -\frac{\sqrt{c-acx}}{4x^4}-\frac{5a\sqrt{c-acx}}{8x^3}+\frac{1}{16}\left(25a^2c\right)\int \frac{1}{x^3\sqrt{c-acx}}\,dx \\
&= -\frac{\sqrt{c-acx}}{4x^4}-\frac{5a\sqrt{c-acx}}{8x^3}-\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{1}{64}\left(75a^3c\right)\int \frac{1}{x^2\sqrt{c-acx}}\,dx \\
&= -\frac{\sqrt{c-acx}}{4x^4}-\frac{5a\sqrt{c-acx}}{8x^3}-\frac{25a^2\sqrt{c-acx}}{32x^2}-\frac{75a^3\sqrt{c-acx}}{64x}+\frac{1}{128}\left(75a^4c\right)\int \frac{1}{x\sqrt{c-acx}} \\
&= -\frac{\sqrt{c-acx}}{4x^4}-\frac{5a\sqrt{c-acx}}{8x^3}-\frac{25a^2\sqrt{c-acx}}{32x^2}-\frac{75a^3\sqrt{c-acx}}{64x}-\frac{1}{64}\left(75a^3\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-} \right. \\
&= -\frac{\sqrt{c-acx}}{4x^4}-\frac{5a\sqrt{c-acx}}{8x^3}-\frac{25a^2\sqrt{c-acx}}{32x^2}-\frac{75a^3\sqrt{c-acx}}{64x}-\frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right.
\end{aligned}$$

Mathematica [A] time = 0.0672636, size = 71, normalized size = 0.65

$$-\frac{\left(75a^3x^3+50a^2x^2+40ax+16\right)\sqrt{c-acx}}{64x^4}-\frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] -(Sqrt[c - a*c*x]*(16 + 40*a*x + 50*a^2*x^2 + 75*a^3*x^3))/(64*x^4) - (75*a
^4*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64
```

Maple [A] time = 0.043, size = 93, normalized size = 0.9

$$-2a^4c^4\left(\frac{1}{a^4c^4x^4}\left(-\frac{75(-acx+c)^{7/2}}{128c^3}+\frac{275(-acx+c)^{5/2}}{128c^2}-\frac{365(-acx+c)^{3/2}}{128c}+\frac{181\sqrt{-acx+c}}{128}\right)+\frac{75}{128c^{7/2}}\text{Artanh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^5,x)
```

```
[Out] -2*a^4*c^4*((-75/128/c^3*(-a*c*x+c)^(7/2)+275/128/c^2*(-a*c*x+c)^(5/2)-365/
128/c*(-a*c*x+c)^(3/2)+181/128*(-a*c*x+c)^(1/2))/a^4/c^4/x^4+75/128/c^(7/2)
*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94722, size = 370, normalized size = 3.36

$$\left[\frac{75\,a^4\sqrt{c}x^4\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)-2\left(75\,a^3x^3+50\,a^2x^2+40\,ax+16\right)\sqrt{-acx+c}}{128\,x^4},\ \frac{75\,a^4\sqrt{-c}x^4\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="fricas"
)
```

```
[Out] [1/128*(75*a^4*sqrt(c)*x^4*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x
) - 2*(75*a^3*x^3 + 50*a^2*x^2 + 40*a*x + 16)*sqrt(-a*c*x + c))/x^4, 1/64*(
75*a^4*sqrt(-c)*x^4*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (75*a^3*x^3 + 50*
a^2*x^2 + 40*a*x + 16)*sqrt(-a*c*x + c))/x^4]
```

Sympy [B] time = 102.727, size = 639, normalized size = 5.81

$$-\frac{558a^4c^8\sqrt{-acx+c}}{1536ac^8x-1152c^8+2304c^6\left(-acx+c\right)^2-1536c^5\left(-acx+c\right)^3+384c^4\left(-acx+c\right)^4}+\frac{102}{1536ac^8x-1152c^8+2304c^6\left(-ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(1/2)/x**5,x)
```

```
[Out] -558*a**4*c**8*sqrt(-a*c*x + c)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*
c*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) + 1022*
a**4*c**7*(-a*c*x + c)**(3/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*
x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) - 770*a**
4*c**6*(-a*c*x + c)**(5/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*x +
 c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) + 66*a**4*c*
```

```
*6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 4
8*c**3*(-a*c*x + c)**3) + 210*a**4*c**5*(-a*c*x + c)**(7/2)/(1536*a*c**8*x
- 1152*c**8 + 2304*c**6*(-a*c*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c
**4*(-a*c*x + c)**4) - 80*a**4*c**5*(-a*c*x + c)**(3/2)/(-144*a*c**6*x + 96
*c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) + 35*a**4*c**5*
sqrt(c**(-9))*log(-c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 - 35*a**4*c**
5*sqrt(c**(-9))*log(c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 + 30*a**4*c*
*4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2
+ 48*c**3*(-a*c*x + c)**3) + 5*a**4*c**4*sqrt(c**(-7))*log(-c**4*sqrt(c**(-
7)) + sqrt(-a*c*x + c))/16 - 5*a**4*c**4*sqrt(c**(-7))*log(c**4*sqrt(c**(-7
)) + sqrt(-a*c*x + c))/16
```

Giac [A] time = 1.22184, size = 170, normalized size = 1.55

$$\frac{75\,a^4c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{64\,\sqrt{-c}} - \frac{75\,(acx-c)^3\sqrt{-acx+c}a^4c + 275\,(acx-c)^2\sqrt{-acx+c}a^4c^2 - 365\,(-acx+c)^{\frac{3}{2}}a^4c^3 + 181\,\sqrt{-}}{64\,a^4c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 75/64*a^4*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 1/64*(75*(a*c*x -
c)^3*sqrt(-a*c*x + c)*a^4*c + 275*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c^2 -
365*(-a*c*x + c)^(3/2)*a^4*c^3 + 181*sqrt(-a*c*x + c)*a^4*c^4)/(a^4*c^4*x^4
)
```

3.402 $\int e^{3\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=249

$$-\frac{2(ax+1)^{9/2}(c-acx)^{3/2}}{9a^4c(1-ax)^{3/2}}+\frac{2(ax+1)^{7/2}(c-acx)^{3/2}}{7a^4c(1-ax)^{3/2}}-\frac{2(ax+1)^{5/2}(c-acx)^{3/2}}{5a^4c(1-ax)^{3/2}}-\frac{2(ax+1)^{3/2}(c-acx)^{3/2}}{3a^4c(1-ax)^{3/2}}-\frac{4\sqrt{ax+1}(c-a}{a^4c(1-ax)^{}}$$

```
[Out] (-4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(a^4*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)
^(3/2)*(c - a*c*x)^(3/2))/(3*a^4*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)^(5/2)*(c
 - a*c*x)^(3/2))/(5*a^4*c*(1 - a*x)^(3/2)) + (2*(1 + a*x)^(7/2)*(c - a*c*x)
^(3/2))/(7*a^4*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)^(9/2)*(c - a*c*x)^(3/2))/(
9*a^4*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*(c - a*c*x)^(3/2)*ArcTanh[Sqrt[1 + a*
x]/Sqrt[2]])/(a^4*c*(1 - a*x)^(3/2))
```

Rubi [A] time = 0.17964, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 23, 88, 50, 63, 208}

$$-\frac{2(ax+1)^{9/2}(c-acx)^{3/2}}{9a^4c(1-ax)^{3/2}}+\frac{2(ax+1)^{7/2}(c-acx)^{3/2}}{7a^4c(1-ax)^{3/2}}-\frac{2(ax+1)^{5/2}(c-acx)^{3/2}}{5a^4c(1-ax)^{3/2}}-\frac{2(ax+1)^{3/2}(c-acx)^{3/2}}{3a^4c(1-ax)^{3/2}}-\frac{4\sqrt{ax+1}(c-a}{a^4c(1-ax)^{}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^3*Sqrt[c - a*c*x],x]
```


```
[Out] (-4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(a^4*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)
^(3/2)*(c - a*c*x)^(3/2))/(3*a^4*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)^(5/2)*(c
 - a*c*x)^(3/2))/(5*a^4*c*(1 - a*x)^(3/2)) + (2*(1 + a*x)^(7/2)*(c - a*c*x)
^(3/2))/(7*a^4*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)^(9/2)*(c - a*c*x)^(3/2))/(
9*a^4*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*(c - a*c*x)^(3/2)*ArcTanh[Sqrt[1 + a*
x]/Sqrt[2]])/(a^4*c*(1 - a*x)^(3/2))
```


Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx &= \int \frac{x^3(1+ax)^{3/2}\sqrt{c-acx}}{(1-ax)^{3/2}}\,dx \\
&= \frac{(c-acx)^{3/2}\int \frac{x^3(1+ax)^{3/2}}{c-acx}\,dx}{(1-ax)^{3/2}} \\
&= \frac{(c-acx)^{3/2}\int \left(-\frac{(1+ax)^{3/2}}{a^3c}+\frac{(1+ax)^{5/2}}{a^3c}-\frac{(1+ax)^{7/2}}{a^3c}+\frac{(1+ax)^{3/2}}{a^3(c-acx)}\right)dx}{(1-ax)^{3/2}} \\
&= -\frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^4c(1-ax)^{3/2}}+\frac{2(1+ax)^{7/2}(c-acx)^{3/2}}{7a^4c(1-ax)^{3/2}}-\frac{2(1+ax)^{9/2}(c-acx)^{3/2}}{9a^4c(1-ax)^{3/2}}+\frac{(c-ac}{a} \\
&= -\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^4c(1-ax)^{3/2}}-\frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^4c(1-ax)^{3/2}}+\frac{2(1+ax)^{7/2}(c-acx)^{3/2}}{7a^4c(1-ax)^{3/2}}-\frac{2(1+a}{9a} \\
&= -\frac{4\sqrt{1+ax}(c-acx)^{3/2}}{a^4c(1-ax)^{3/2}}-\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^4c(1-ax)^{3/2}}-\frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^4c(1-ax)^{3/2}}+\frac{2(1+ax)}{7a^4c} \\
&= -\frac{4\sqrt{1+ax}(c-acx)^{3/2}}{a^4c(1-ax)^{3/2}}-\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^4c(1-ax)^{3/2}}-\frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^4c(1-ax)^{3/2}}+\frac{2(1+ax)}{7a^4c} \\
&= -\frac{4\sqrt{1+ax}(c-acx)^{3/2}}{a^4c(1-ax)^{3/2}}-\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^4c(1-ax)^{3/2}}-\frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^4c(1-ax)^{3/2}}+\frac{2(1+ax)}{7a^4c}
\end{aligned}
$$

Mathematica [A] time = 0.0801823, size = 92, normalized size = 0.37

$$
-\frac{2\sqrt{c-acx}\left(\sqrt{ax+1}\left(35a^4x^4+95a^3x^3+138a^2x^2+236ax+788\right)-630\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{315a^4\sqrt{1-ax}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(Sqrt[1 + a*x]*(788 + 236*a*x + 138*a^2*x^2 + 95*a^3*x^
3 + 35*a^4*x^4) - 630*Sqrt[2]*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]]))/(315*a^4*Sqr
t[1 - a*x])
```

Maple [A] time = 0.099, size = 146, normalized size = 0.6

$$-\frac{2}{(315\,ax-315)\,a^4}\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}\left(-35\,x^4a^4\sqrt{c\,(ax+1)}-95\,x^3a^3\sqrt{c\,(ax+1)}-138\,x^2a^2\sqrt{c\,(ax+1)}+630\,\sqrt{c}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/315*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(-35*x^4*a^4*(c*(a*x+1))^(1/2)
-95*x^3*a^3*(c*(a*x+1))^(1/2)-138*x^2*a^2*(c*(a*x+1))^(1/2)+630*c^(1/2)*2^(
1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-236*x*a*(c*(a*x+1))^(1/
2)-788*(c*(a*x+1))^(1/2))/(a*x-1)/(c*(a*x+1))^(1/2)/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}(ax+1)^3x^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3*x^3/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 1.94997, size = 659, normalized size = 2.65

$$\left[\frac{2\left(315\,\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+\left(35\,a^4x^4+95\,a^3x^3+138\,a^2x^2+236\,ax+788\right)\sqrt{-a^2x^2}\right.}{315\left(a^5x-a^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [2/315*(315*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)
*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1))
+ (35*a^4*x^4 + 95*a^3*x^3 + 138*a^2*x^2 + 236*a*x + 788)*sqrt(-a^2*x^2 + 1
)*sqrt(-a*c*x + c))/(a^5*x - a^4), 2/315*(630*sqrt(2)*(a*x - 1)*sqrt(-c)*ar
ctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c))
+ (35*a^4*x^4 + 95*a^3*x^3 + 138*a^2*x^2 + 236*a*x + 788)*sqrt(-a^2*x^2 + 1
)*sqrt(-a*c*x + c))/(a^5*x - a^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-c\,(ax-1)}\,(ax+1)^3}{(-\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**3*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2)
, x)
```

Giac [A] time = 1.36205, size = 201, normalized size = 0.81

$$\frac{4\sqrt{2}\left(315\,c^2\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)+646\,\sqrt{-c}c^{\frac{3}{2}}\right)}{315\,a^4\sqrt{-c}|c|}-\frac{2\left(\frac{630\,\sqrt{2}c^5\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\,\sqrt{-c}}\right)}{\sqrt{-c}}+35\,(acx+c)^{\frac{9}{2}}-45\,(acx+c)^{\frac{7}{2}}c+63\,(acx+c)^{\frac{5}{2}}$$

$$315\,a^4c^3|c|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] 4/315*sqrt(2)*(315*c^2*arctan(sqrt(c)/sqrt(-c)) + 646*sqrt(-c)*c^(3/2))/(a^
4*sqrt(-c)*abs(c)) - 2/315*(630*sqrt(2)*c^5*arctan(1/2*sqrt(2)*sqrt(a*c*x +
 c)/sqrt(-c))/sqrt(-c) + 35*(a*c*x + c)^(9/2) - 45*(a*c*x + c)^(7/2)*c + 63
*(a*c*x + c)^(5/2)*c^2 + 105*(a*c*x + c)^(3/2)*c^3 + 630*sqrt(a*c*x + c)*c^
4)/(a^4*c^3*abs(c))
```

3.403 $\int e^{3\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=169

$$-\frac{2(ax+1)^{7/2}(c-acx)^{3/2}}{7a^3c(1-ax)^{3/2}}-\frac{2(ax+1)^{3/2}(c-acx)^{3/2}}{3a^3c(1-ax)^{3/2}}-\frac{4\sqrt{ax+1}(c-acx)^{3/2}}{a^3c(1-ax)^{3/2}}+\frac{4\sqrt{2}(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{a^3c(1-ax)^{3/2}}$$

```
[Out] (-4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(a^3*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)
^(3/2)*(c - a*c*x)^(3/2))/(3*a^3*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)^(7/2)*(c
 - a*c*x)^(3/2))/(7*a^3*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*(c - a*c*x)^(3/2)*A
rcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(a^3*c*(1 - a*x)^(3/2))
```

Rubi [A] time = 0.159788, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 23, 88, 50, 63, 208}

$$-\frac{2(ax+1)^{7/2}(c-acx)^{3/2}}{7a^3c(1-ax)^{3/2}}-\frac{2(ax+1)^{3/2}(c-acx)^{3/2}}{3a^3c(1-ax)^{3/2}}-\frac{4\sqrt{ax+1}(c-acx)^{3/2}}{a^3c(1-ax)^{3/2}}+\frac{4\sqrt{2}(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{a^3c(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^2*Sqrt[c - a*c*x],x]
```


```
[Out] (-4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(a^3*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)
^(3/2)*(c - a*c*x)^(3/2))/(3*a^3*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)^(7/2)*(c
 - a*c*x)^(3/2))/(7*a^3*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*(c - a*c*x)^(3/2)*A
rcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(a^3*c*(1 - a*x)^(3/2))
```


Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\int \frac{x^2(1+ax)^{3/2}\sqrt{c-acx}}{(1-ax)^{3/2}}\,dx$$

$$=\frac{(c-acx)^{3/2}\int \frac{x^2(1+ax)^{3/2}}{c-acx}\,dx}{(1-ax)^{3/2}}$$

$$=\frac{(c-acx)^{3/2}\int \left(-\frac{(1+ax)^{5/2}}{a^2c}+\frac{(1+ax)^{3/2}}{a^2(c-acx)}\right)dx}{(1-ax)^{3/2}}$$

$$=-\frac{2(1+ax)^{7/2}(c-acx)^{3/2}}{7a^3c(1-ax)^{3/2}}+\frac{(c-acx)^{3/2}\int \frac{(1+ax)^{3/2}}{c-acx}\,dx}{a^2(1-ax)^{3/2}}$$

$$=-\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^3c(1-ax)^{3/2}}-\frac{2(1+ax)^{7/2}(c-acx)^{3/2}}{7a^3c(1-ax)^{3/2}}+\frac{\left(2(c-acx)^{3/2}\right)\int \frac{\sqrt{1+ax}}{c-acx}\,dx}{a^2(1-ax)^{3/2}}$$

$$=-\frac{4\sqrt{1+ax}(c-acx)^{3/2}}{a^3c(1-ax)^{3/2}}-\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^3c(1-ax)^{3/2}}-\frac{2(1+ax)^{7/2}(c-acx)^{3/2}}{7a^3c(1-ax)^{3/2}}+\frac{\left(4(c-ac\right.}{}$$

$$=-\frac{4\sqrt{1+ax}(c-acx)^{3/2}}{a^3c(1-ax)^{3/2}}-\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^3c(1-ax)^{3/2}}-\frac{2(1+ax)^{7/2}(c-acx)^{3/2}}{7a^3c(1-ax)^{3/2}}+\frac{\left(8(c-ac\right.}{}$$

$$=-\frac{4\sqrt{1+ax}(c-acx)^{3/2}}{a^3c(1-ax)^{3/2}}-\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^3c(1-ax)^{3/2}}-\frac{2(1+ax)^{7/2}(c-acx)^{3/2}}{7a^3c(1-ax)^{3/2}}+\frac{4\sqrt{2}(c-}{}$$

Mathematica [A] time = 0.0637418, size = 84, normalized size = 0.5

$$-\frac{2\sqrt{c-acx}\left(\sqrt{ax+1}\left(3a^3x^3+9a^2x^2+16ax+52\right)-42\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{21a^3\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(Sqrt[1 + a*x]*(52 + 16*a*x + 9*a^2*x^2 + 3*a^3*x^3) -
42*Sqrt[2]*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]]))/(21*a^3*Sqrt[1 - a*x])
```

Maple [A] time = 0.13, size = 129, normalized size = 0.8

$$-\frac{2}{\left(21\,ax-21\right)a^{3}}\sqrt{-a^{2}x^{2}+1}\sqrt{-c\left(ax-1\right)}\left(-3\,x^{3}a^{3}\sqrt{c\left(ax+1\right)}-9\,x^{2}a^{2}\sqrt{c\left(ax+1\right)}+42\,\sqrt{c}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\left(ax+1\right)}\sqrt{2}}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/21*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(-3*x^3*a^3*(c*(a*x+1))^(1/2)-9
*x^2*a^2*(c*(a*x+1))^(1/2)+42*c^(1/2)*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)
*2^(1/2)/c^(1/2))-16*x*a*(c*(a*x+1))^(1/2)-52*(c*(a*x+1))^(1/2))/(a*x-1)/(c
*(a*x+1))^(1/2)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}(ax+1)^{3}x^{2}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3*x^2/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 1.84437, size = 605, normalized size = 3.58

$$\left[\frac{2\left(21\,\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{a^{2}cx^{2}+2\,acx-2\,\sqrt{2}\sqrt{-a^{2}x^{2}+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^{2}x^{2}-2\,ax+1}\right)+\left(3\,a^{3}x^{3}+9\,a^{2}x^{2}+16\,ax+52\right)\sqrt{-a^{2}x^{2}+1}\sqrt{-acx+c}\right)}{21\left(a^{4}x-a^{3}\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [2/21*(21*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*s
qrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) +
(3*a^3*x^3 + 9*a^2*x^2 + 16*a*x + 52)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/
(a^4*x - a^3), 2/21*(42*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2
*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (3*a^3*x^3 + 9*a^2*x
^2 + 16*a*x + 52)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^4*x - a^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}\sqrt{-c\left(ax-1\right)}\left(ax+1\right)^{3}}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2)
, x)
```

Giac [A] time = 1.36461, size = 169, normalized size = 1.

$$\frac{4\sqrt{2}\left(21\,c^2\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)+40\,\sqrt{-c}c^{\frac{3}{2}}\right)}{21\,a^3\sqrt{-c}|c|}-\frac{2\left(\frac{42\,\sqrt{2}c^4\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\,\sqrt{-c}}\right)}{\sqrt{-c}}+3\,(acx+c)^{\frac{7}{2}}+7\,(acx+c)^{\frac{3}{2}}c^2+42\,\sqrt{acx+c}c^3\right)}{21\,a^3c^2|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] 4/21*sqrt(2)*(21*c^2*arctan(sqrt(c)/sqrt(-c)) + 40*sqrt(-c)*c^(3/2))/(a^3*s
qrt(-c)*abs(c)) - 2/21*(42*sqrt(2)*c^4*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/s
qrt(-c))/sqrt(-c) + 3*(a*c*x + c)^(7/2) + 7*(a*c*x + c)^(3/2)*c^2 + 42*sqrt
(a*c*x + c)*c^3)/(a^3*c^2*abs(c))
```

3.404 $\int e^{3\tanh^{-1}(ax)} x\sqrt{c-acx}\,dx$

Optimal. Leaf size=169

$$-\frac{2(ax+1)^{5/2}(c-acx)^{3/2}}{5a^2c(1-ax)^{3/2}}-\frac{2(ax+1)^{3/2}(c-acx)^{3/2}}{3a^2c(1-ax)^{3/2}}-\frac{4\sqrt{ax+1}(c-acx)^{3/2}}{a^2c(1-ax)^{3/2}}+\frac{4\sqrt{2}(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{a^2c(1-ax)^{3/2}}$$

```
[Out] (-4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(a^2*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)
^(3/2)*(c - a*c*x)^(3/2))/(3*a^2*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)^(5/2)*(c
 - a*c*x)^(3/2))/(5*a^2*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*(c - a*c*x)^(3/2)*A
rcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(a^2*c*(1 - a*x)^(3/2))
```

Rubi [A] time = 0.114421, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6130, 23, 80, 50, 63, 208}

$$-\frac{2(ax+1)^{5/2}(c-acx)^{3/2}}{5a^2c(1-ax)^{3/2}}-\frac{2(ax+1)^{3/2}(c-acx)^{3/2}}{3a^2c(1-ax)^{3/2}}-\frac{4\sqrt{ax+1}(c-acx)^{3/2}}{a^2c(1-ax)^{3/2}}+\frac{4\sqrt{2}(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{a^2c(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(a^2*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)
^(3/2)*(c - a*c*x)^(3/2))/(3*a^2*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)^(5/2)*(c
 - a*c*x)^(3/2))/(5*a^2*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*(c - a*c*x)^(3/2)*A
rcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(a^2*c*(1 - a*x)^(3/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx &= \int \frac{x(1+ax)^{3/2}\sqrt{c-acx}}{(1-ax)^{3/2}}\,dx\\
&= \frac{(c-acx)^{3/2}\int \frac{x(1+ax)^{3/2}}{c-acx}\,dx}{(1-ax)^{3/2}}\\
&= -\frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^2c(1-ax)^{3/2}}+\frac{(c-acx)^{3/2}\int \frac{(1+ax)^{3/2}}{c-acx}\,dx}{a(1-ax)^{3/2}}\\
&= -\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^2c(1-ax)^{3/2}}-\frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^2c(1-ax)^{3/2}}+\frac{\left(2(c-acx)^{3/2}\right)\int \frac{\sqrt{1+ax}}{c-acx}\,dx}{a(1-ax)^{3/2}}\\
&= -\frac{4\sqrt{1+ax}(c-acx)^{3/2}}{a^2c(1-ax)^{3/2}}-\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^2c(1-ax)^{3/2}}-\frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^2c(1-ax)^{3/2}}+\frac{\left(4(c-acx\right.}{}\\
&= -\frac{4\sqrt{1+ax}(c-acx)^{3/2}}{a^2c(1-ax)^{3/2}}-\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^2c(1-ax)^{3/2}}-\frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^2c(1-ax)^{3/2}}+\frac{\left(8(c-acx\right.}{}\\
&= -\frac{4\sqrt{1+ax}(c-acx)^{3/2}}{a^2c(1-ax)^{3/2}}-\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^2c(1-ax)^{3/2}}-\frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^2c(1-ax)^{3/2}}+\frac{4\sqrt{2}(c-a}{a}
\end{aligned}
$$

Mathematica [A] time = 0.04188, size = 76, normalized size = 0.45

$$
-\frac{2\sqrt{c-acx}\left(\sqrt{ax+1}\left(3a^2x^2+11ax+38\right)-30\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{15a^2\sqrt{1-ax}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(Sqrt[1 + a*x]*(38 + 11*a*x + 3*a^2*x^2) - 30*Sqrt[2]*A
rcTanh[Sqrt[1 + a*x]/Sqrt[2]]))/(15*a^2*Sqrt[1 - a*x])
```

Maple [A] time = 0.118, size = 112, normalized size = 0.7

$$
-\frac{2}{(15ax-15)a^2}\sqrt{-a^2x^2+1}\sqrt{-c(ax-1)}\left(-3x^2a^2\sqrt{c(ax+1)}+30\sqrt{c}\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{c(ax+1)}\sqrt{2}}{\sqrt{c}}\right)-11xa\sqrt{c(ax\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/15*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(-3*x^2*a^2*(c*(a*x+1))^(1/2)+3
0*c^(1/2)*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-11*x*a*(c*
(a*x+1))^(1/2)-38*(c*(a*x+1))^(1/2))/(a*x-1)/(c*(a*x+1))^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}(ax+1)^3 x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3*x/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 1.89258, size = 572, normalized size = 3.38

$$\left[\frac{2\left(15\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+\left(3\,a^2x^2+11\,ax+38\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}\right)}{15\left(a^3x-a^2\right)}, \frac{2\left(30\sqrt{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [2/15*(15*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*s
qrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) +
(3*a^2*x^2 + 11*a*x + 38)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^3*x - a^2
), 2/15*(30*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sq
rt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (3*a^2*x^2 + 11*a*x + 38)*sqrt(-
a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^3*x - a^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\,(ax-1)}\,(ax+1)^3}{(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x
)
```

Giac [A] time = 1.27842, size = 166, normalized size = 0.98

$$\frac{4\sqrt{2}\left(15\,c^2\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)+26\,\sqrt{-c}c^{\frac{3}{2}}\right)}{15\,a^2\sqrt{-c}|c|}-\frac{2\left(\frac{30\,\sqrt{2}c^3\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\,\sqrt{-c}}\right)}{\sqrt{-c}}+3\,(acx+c)^{\frac{5}{2}}+5\,(acx+c)^{\frac{3}{2}}c+30\,\sqrt{acx+c}c^2\right)}{15\,a^2c|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] 4/15*sqrt(2)*(15*c^2*arctan(sqrt(c)/sqrt(-c)) + 26*sqrt(-c)*c^(3/2))/(a^2*s
qrt(-c)*abs(c)) - 2/15*(30*sqrt(2)*c^3*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/s
qrt(-c))/sqrt(-c) + 3*(a*c*x + c)^(5/2) + 5*(a*c*x + c)^(3/2)*c + 30*sqrt(a
*c*x + c)*c^2)/(a^2*c*abs(c))
```

3.405 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=119

$$-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}-\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a}$$

```
[Out] (-4*c*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x]) - (2*c^2*(1 - a^2*x^2)^(3/2))/
(3*a*(c - a*c*x)^(3/2)) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*
x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/a
```

Rubi [A] time = 0.114404, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6127, 665, 661, 208}

$$-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}-\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*c*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x]) - (2*c^2*(1 - a^2*x^2)^(3/2))/
(3*a*(c - a*c*x)^(3/2)) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*
x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/a
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\sqrt{c-acx}\,dx &= c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{5/2}}\,dx\\
&= -\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}+\left(2c^2\right)\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{3/2}}\,dx\\
&= -\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}+(4c)\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}-\left(8ac^2\right)\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\
&= -\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0316262, size = 67, normalized size = 0.56

$$
-\frac{2\sqrt{c-acx}\left(\sqrt{ax+1}(ax+7)-6\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{3a\sqrt{1-ax}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(Sqrt[1 + a*x]*(7 + a*x) - 6*Sqrt[2]*ArcTanh[Sqrt[1 + a
*x]/Sqrt[2]]))/(3*a*Sqrt[1 - a*x])
```

Maple [A] time = 0.108, size = 95, normalized size = 0.8

$$
-\frac{2}{(3ax-3)a}\sqrt{-a^2x^2+1}\sqrt{-c(ax-1)}\left(6\sqrt{c}\sqrt{2}\text{Artanh}\left(1/2\frac{\sqrt{c(ax+1)}\sqrt{2}}{\sqrt{c}}\right)-xa\sqrt{c(ax+1)}-7\sqrt{c(ax+1)}\right)\frac{1}{\sqrt{c(ax+1)}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/3*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(6*c^(1/2)*2^(1/2)*arctanh(1/2*(
c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-x*a*(c*(a*x+1))^(1/2)-7*(c*(a*x+1))^(1/2)
)/(a*x-1)/(c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{-acx+c}(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 1.78455, size = 518, normalized size = 4.35

$$\left[\frac{2\left(3\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+\sqrt{-a^2x^2+1}\sqrt{-acx+c}(ax+7)\right)}{3\left(a^2x-a\right)}, \frac{2\left(6\sqrt{2}(ax-1)\sqrt{-c}\,\mathrm{arc}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] [2/3*(3*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqr
t(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + sq
rt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x + 7))/(a^2*x - a), 2/3*(6*sqrt(2)*(a
*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c
)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x + 7))/(a^2*x
- a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)}\,(ax+1)^3}{\left(-(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A] time = 1.18129, size = 142, normalized size = 1.19

$$-\frac{2\left(\frac{6\sqrt{2}c^2\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+(acx+c)^{\frac{3}{2}}+6\sqrt{acx+c}\,c\right)}{3\,a|c|}+\frac{4\sqrt{2}\left(3\,c^2\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)+4\sqrt{-c}c^{\frac{3}{2}}\right)}{3\,a\sqrt{-c}|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] -2/3*(6*sqrt(2)*c^2*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) +
 (a*c*x + c)^(3/2) + 6*sqrt(a*c*x + c)*c)/(a*abs(c)) + 4/3*sqrt(2)*(3*c^2*a
rctan(sqrt(c)/sqrt(-c)) + 4*sqrt(-c)*c^(3/2))/(a*sqrt(-c)*abs(c))
```

3.406 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=119

$$-\frac{2\sqrt{ax+1}(c-acx)^{3/2}}{c(1-ax)^{3/2}}-\frac{2(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{c(1-ax)^{3/2}}+\frac{4\sqrt{2}(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}$$

```
[Out] (-2*Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(c*(1 - a*x)^(3/2)) - (2*(c - a*c*x)^(
3/2)*ArcTanh[Sqrt[1 + a*x]])/(c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*(c - a*c*x)^(
3/2)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(c*(1 - a*x)^(3/2))
```

Rubi [A] time = 0.136005, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 23, 84, 156, 63, 208}

$$-\frac{2\sqrt{ax+1}(c-acx)^{3/2}}{c(1-ax)^{3/2}}-\frac{2(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{c(1-ax)^{3/2}}+\frac{4\sqrt{2}(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (-2*Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(c*(1 - a*x)^(3/2)) - (2*(c - a*c*x)^(
3/2)*ArcTanh[Sqrt[1 + a*x]])/(c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*(c - a*c*x)^(
3/2)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(c*(1 - a*x)^(3/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx &= \int \frac{(1+ax)^{3/2}\sqrt{c-acx}}{x(1-ax)^{3/2}}\,dx\\
&= \frac{(c-acx)^{3/2}\int \frac{(1+ax)^{3/2}}{x(c-acx)}\,dx}{(1-ax)^{3/2}}\\
&= -\frac{2\sqrt{1+ax}(c-acx)^{3/2}}{c(1-ax)^{3/2}} - \frac{(c-acx)^{3/2}\int \frac{-ac-3a^2cx}{x\sqrt{1+ax}(c-acx)}\,dx}{ac(1-ax)^{3/2}}\\
&= -\frac{2\sqrt{1+ax}(c-acx)^{3/2}}{c(1-ax)^{3/2}} + \frac{\left(4a(c-acx)^{3/2}\right)\int \frac{1}{\sqrt{1+ax}(c-acx)}\,dx}{(1-ax)^{3/2}} + \frac{(c-acx)^{3/2}\int \frac{1}{x\sqrt{1+ax}}\,dx}{c(1-ax)^{3/2}}\\
&= -\frac{2\sqrt{1+ax}(c-acx)^{3/2}}{c(1-ax)^{3/2}} + \frac{\left(8(c-acx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{2c-cx^2}\,dx, x, \sqrt{1+ax}\right)}{(1-ax)^{3/2}} + \frac{\left(2(c-acx)^{3/2}\right)\text{S}}{}\\
&= -\frac{2\sqrt{1+ax}(c-acx)^{3/2}}{c(1-ax)^{3/2}} - \frac{2(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{1+ax}\right)}{c(1-ax)^{3/2}} + \frac{4\sqrt{2}(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+ax}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0322904, size = 66, normalized size = 0.55

$$-\frac{2\sqrt{c-acx}\left(\sqrt{ax+1}+\tanh^{-1}\left(\sqrt{ax+1}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(Sqrt[1 + a*x] + ArcTanh[Sqrt[1 + a*x]] - 2*Sqrt[2]*Arc
Tanh[Sqrt[1 + a*x]/Sqrt[2]]))/Sqrt[1 - a*x]
```

Maple [A] time = 0.104, size = 98, normalized size = 0.8

$$-2\,\frac{\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}}{(ax-1)\,\sqrt{c\,(ax+1)}}\left(2\,\sqrt{c}\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)-\sqrt{c}\text{Artanh}\left(\frac{\sqrt{c\,(ax+1)}}{\sqrt{c}}\right)-\sqrt{c\,(ax+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x,x)
```

```
[Out] -2*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(2*c^(1/2)*2^(1/2)*arctanh(1/2*(c*
(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-c^(1/2)*arctanh((c*(a*x+1))^(1/2)/c^(1/2))-
(c*(a*x+1))^(1/2))/(a*x-1)/(c*(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx + c}(ax + 1)^3}{\left(-a^2x^2 + 1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Fricas [A] time = 1.94456, size = 753, normalized size = 6.33

$$\left[\frac{2\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+(ax-1)\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\,\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2\,c}{ax^2-x}\right)+2\,\sqrt{}}{ax-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] [(2*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a
^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + (a*x -
 1)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)
*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a*x
- 1), 2*(2*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqr
t(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) - (a*x - 1)*sqrt(-c)*arctan(sqrt(-a
^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 + 1)
*sqrt(-a*c*x + c))/(a*x - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(ax-1\right)}\left(ax+1\right)^3}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1))**(3/2)),
 x)
```

Giac [A] time = 1.23092, size = 188, normalized size = 1.58

$$-2\,c\left(\frac{2\,\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\,\sqrt{-c}}\right)}{\sqrt{-c}|c|}-\frac{c\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}|c|}+\frac{\sqrt{acx+c}}{|c|}\right)-\frac{\sqrt{2}\left(\sqrt{2}c^2\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)-4\,c^2\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)-2\,\sqrt{-c}\right.}{\sqrt{-c}|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] -2*c*(2*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*ab
s(c)) - c*arctan(sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*abs(c)) + sqrt(a*c*x +
 c)/abs(c)) - sqrt(2)*(sqrt(2)*c^2*arctan(sqrt(2)*sqrt(c)/sqrt(-c)) - 4*c^2
*arctan(sqrt(c)/sqrt(-c)) - 2*sqrt(-c)*c^(3/2))/(sqrt(-c)*abs(c))
```

3.407 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=124

$$-\frac{\sqrt{ax+1}(c-acx)^{3/2}}{cx(1-ax)^{3/2}}-\frac{5a(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{c(1-ax)^{3/2}}+\frac{4\sqrt{2}a(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}$$

```
[Out] -((Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(c*x*(1 - a*x)^(3/2))) - (5*a*(c - a*c*
x)^(3/2)*ArcTanh[Sqrt[1 + a*x]])/(c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*a*(c - a*
c*x)^(3/2)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(c*(1 - a*x)^(3/2))
```

Rubi [A] time = 0.135035, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 23, 98, 156, 63, 208}

$$-\frac{\sqrt{ax+1}(c-acx)^{3/2}}{cx(1-ax)^{3/2}}-\frac{5a(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{c(1-ax)^{3/2}}+\frac{4\sqrt{2}a(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -((Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(c*x*(1 - a*x)^(3/2))) - (5*a*(c - a*c*
x)^(3/2)*ArcTanh[Sqrt[1 + a*x]])/(c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*a*(c - a*
c*x)^(3/2)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(c*(1 - a*x)^(3/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx &= \int \frac{(1+ax)^{3/2}\sqrt{c-acx}}{x^2(1-ax)^{3/2}}\,dx \\
&= \frac{(c-acx)^{3/2}\int \frac{(1+ax)^{3/2}}{x^2(c-acx)}\,dx}{(1-ax)^{3/2}} \\
&= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{cx(1-ax)^{3/2}} - \frac{(c-acx)^{3/2}\int \frac{-\frac{5ac}{2}-\frac{3}{2}a^2cx}{x\sqrt{1+ax}(c-acx)}\,dx}{c(1-ax)^{3/2}} \\
&= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{cx(1-ax)^{3/2}} + \frac{\left(4a^2(c-acx)^{3/2}\right)\int \frac{1}{\sqrt{1+ax}(c-acx)}\,dx}{(1-ax)^{3/2}} + \frac{\left(5a(c-acx)^{3/2}\right)\int \frac{1}{x\sqrt{1+ax}}\,dx}{2c(1-ax)^{3/2}} \\
&= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{cx(1-ax)^{3/2}} + \frac{\left(8a(c-acx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{2c-cx^2}\,dx, x, \sqrt{1+ax}\right)}{(1-ax)^{3/2}} + \frac{\left(5(c-acx)^{3/2}\right)\text{S}}{} \\
&= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{cx(1-ax)^{3/2}} - \frac{5a(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{1+ax}\right)}{c(1-ax)^{3/2}} + \frac{4\sqrt{2}a(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+ax}}{\sqrt{2}}\right.}{c(1-ax)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0358617, size = 75, normalized size = 0.6

$$-\frac{\sqrt{c-acx}\left(\sqrt{ax+1}+5ax\tanh^{-1}\left(\sqrt{ax+1}\right)-4\sqrt{2}ax\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -((Sqrt[c - a*c*x]*(Sqrt[1 + a*x] + 5*a*x*ArcTanh[Sqrt[1 + a*x]] - 4*Sqrt[2
]*a*x*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]]))/(x*Sqrt[1 - a*x]))
```

Maple [A] time = 0.124, size = 105, normalized size = 0.9

$$\frac{1}{(ax-1)x}\left(-4\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{c(ax+1)}\sqrt{2}}{\sqrt{c}}\right)xac+5\,\text{Artanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)xac+\sqrt{c(ax+1)}\sqrt{c}\right)\sqrt{-a^2x^2+1}\sqrt{-c(ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^2,x)
```

```
[Out] (-4*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c+5*arctanh(
(c*(a*x+1))^(1/2)/c^(1/2))*x*a*c+(c*(a*x+1))^(1/2)*c^(1/2))*(-a^2*x^2+1)^(1
/2)*(-c*(a*x-1))^(1/2)/(a*x-1)/(c*(a*x+1))^(1/2)/c^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

Fricas [A] time = 1.98907, size = 799, normalized size = 6.44

$$\left[\frac{4\sqrt{2}\left(a^2x^2-ax\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+5\left(a^2x^2-ax\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\,\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2}{ax^2-x}\right.}{2\left(ax^2-x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="f
ricas")
```

```
[Out] [1/2*(4*sqrt(2)*(a^2*x^2 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(
2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)
) + 5*(a^2*x^2 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1
)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(
-a*c*x + c))/(a*x^2 - x), (4*sqrt(2)*(a^2*x^2 - a*x)*sqrt(-c)*arctan(sqrt(2
)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) - 5*(a^2*x^
2 - a*x)*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*
c*x^2 - c)) + sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a*x^2 - x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)}\,(ax+1)^3}{x^2\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)**3/(x**2*(-(a*x - 1)*(a*x + 1))**(3/2
)), x)
```

Giac [A] time = 1.28635, size = 211, normalized size = 1.7

$$-ac^2\left(\frac{4\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}|c|} - \frac{5\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}|c|} + \frac{\sqrt{acx+c}}{acx|c|}\right) - \frac{\sqrt{2}\left(5\sqrt{2}ac^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right) - 8ac^{\frac{5}{2}}\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right) - 2\right.}{2\sqrt{-c}\sqrt{c}|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="g
iac")
```

```
[Out] -a*c^2*(4*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*ab
s(c)) - 5*arctan(sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*abs(c)) + sqrt(a*c*x +
 c)/(a*c*x*abs(c))) - 1/2*sqrt(2)*(5*sqrt(2)*a*c^(5/2)*arctan(sqrt(2)*sqrt(
c)/sqrt(-c)) - 8*a*c^(5/2)*arctan(sqrt(c)/sqrt(-c)) - 2*a*sqrt(-c)*c^2)/(sq
rt(-c)*sqrt(c)*abs(c))
```

3.408 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=173

$$-\frac{23a^2(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{4c(1-ax)^{3/2}}+\frac{4\sqrt{2}a^2(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}-\frac{\sqrt{ax+1}(c-acx)^{3/2}}{2cx^2(1-ax)^{3/2}}-\frac{9a\sqrt{ax+1}(c-acx}{4cx(1-ax)^{3/2}}$$

```
[Out] -(Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(2*c*x^2*(1 - a*x)^(3/2)) - (9*a*Sqrt[1
+ a*x]*(c - a*c*x)^(3/2))/(4*c*x*(1 - a*x)^(3/2)) - (23*a^2*(c - a*c*x)^(3/
2)*ArcTanh[Sqrt[1 + a*x]])/(4*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*a^2*(c - a*c*
x)^(3/2)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(c*(1 - a*x)^(3/2))
```

Rubi [A] time = 0.149217, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6130, 23, 98, 151, 156, 63, 208}

$$-\frac{23a^2(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{4c(1-ax)^{3/2}}+\frac{4\sqrt{2}a^2(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}-\frac{\sqrt{ax+1}(c-acx)^{3/2}}{2cx^2(1-ax)^{3/2}}-\frac{9a\sqrt{ax+1}(c-acx}{4cx(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] -(Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(2*c*x^2*(1 - a*x)^(3/2)) - (9*a*Sqrt[1
+ a*x]*(c - a*c*x)^(3/2))/(4*c*x*(1 - a*x)^(3/2)) - (23*a^2*(c - a*c*x)^(3/
2)*ArcTanh[Sqrt[1 + a*x]])/(4*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*a^2*(c - a*c*
x)^(3/2)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(c*(1 - a*x)^(3/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx &= \int \frac{(1+ax)^{3/2}\sqrt{c-acx}}{x^3(1-ax)^{3/2}}\,dx \\
&= \frac{(c-acx)^{3/2}\int \frac{(1+ax)^{3/2}}{x^3(c-acx)}\,dx}{(1-ax)^{3/2}} \\
&= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{2cx^2(1-ax)^{3/2}} - \frac{(c-acx)^{3/2}\int \frac{-\frac{9ac}{2}-\frac{7}{2}a^2cx}{x^2\sqrt{1+ax}(c-acx)}\,dx}{2c(1-ax)^{3/2}} \\
&= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{2cx^2(1-ax)^{3/2}} - \frac{9a\sqrt{1+ax}(c-acx)^{3/2}}{4cx(1-ax)^{3/2}} + \frac{(c-acx)^{3/2}\int \frac{\frac{23a^2c^2}{4}+\frac{9}{4}a^3c^2x}{x\sqrt{1+ax}(c-acx)}\,dx}{2c^2(1-ax)^{3/2}} \\
&= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{2cx^2(1-ax)^{3/2}} - \frac{9a\sqrt{1+ax}(c-acx)^{3/2}}{4cx(1-ax)^{3/2}} + \frac{\left(4a^3(c-acx)^{3/2}\right)\int \frac{1}{\sqrt{1+ax}(c-acx)}\,dx}{(1-ax)^{3/2}} + \frac{\left(23a^2\right.}{} \\
&= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{2cx^2(1-ax)^{3/2}} - \frac{9a\sqrt{1+ax}(c-acx)^{3/2}}{4cx(1-ax)^{3/2}} + \frac{\left(8a^2(c-acx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{2c-cx^2}\,dx, x, \sqrt{1}\right.}{(1-ax)^{3/2}} \\
&= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{2cx^2(1-ax)^{3/2}} - \frac{9a\sqrt{1+ax}(c-acx)^{3/2}}{4cx(1-ax)^{3/2}} - \frac{23a^2(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{1+ax}\right)}{4c(1-ax)^{3/2}} + \frac{4\sqrt{2}}{}
\end{aligned}$$

Mathematica [A] time = 0.0536777, size = 92, normalized size = 0.53

$$-\frac{\sqrt{c-acx}\left(23a^2x^2\tanh^{-1}\left(\sqrt{ax+1}\right)-16\sqrt{2}a^2x^2\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)+\sqrt{ax+1}(9ax+2)\right)}{4x^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] -(Sqrt[c - a*c*x]*(Sqrt[1 + a*x]*(2 + 9*a*x) + 23*a^2*x^2*ArcTanh[Sqrt[1 +
a*x]] - 16*Sqrt[2]*a^2*x^2*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]]))/(4*x^2*Sqrt[1 -
 a*x])
```

Maple [A] time = 0.11, size = 131, normalized size = 0.8

$$-\frac{1}{(4\,ax-4)\,x^{2}}\sqrt{-a^{2}x^{2}+1}\sqrt{-c\,(ax-1)}\left(16\,\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^{2}a^{2}c-23\,c\mathrm{Artanh}\left(\frac{\sqrt{c\,(ax+1)}}{\sqrt{c}}\right)x^{2}a^{2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^3,x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(16*2^(1/2)*arctanh(1/2*(c*(a*x+
1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-23*c*arctanh((c*(a*x+1))^(1/2)/c^(1/2)
)*x^2*a^2-9*x*a*(c*(a*x+1))^(1/2)*c^(1/2)-2*(c*(a*x+1))^(1/2)*c^(1/2))/c^(1
/2)/(a*x-1)/(c*(a*x+1))^(1/2)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}(ax+1)^{3}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Fricas [A] time = 1.99908, size = 869, normalized size = 5.02

$$\left[\frac{16\,\sqrt{2}\left(a^{3}x^{3}-a^{2}x^{2}\right)\sqrt{c}\log\left(-\frac{a^{2}cx^{2}+2\,acx-2\,\sqrt{2}\sqrt{-a^{2}x^{2}+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^{2}x^{2}-2\,ax+1}\right)+23\left(a^{3}x^{3}-a^{2}x^{2}\right)\sqrt{c}\log\left(-\frac{a^{2}cx^{2}+acx+2\,\sqrt{-a^{2}x^{2}+1}\sqrt{-ac}}{ax^{2}-x}\right.}{8\left(ax^{3}-x^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="f
ricas")
```

```
[Out] [1/8*(16*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*
sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x
 + 1)) + 23*(a^3*x^3 - a^2*x^2)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a
^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2
```

```
+ 1)*sqrt(-a*c*x + c)*(9*a*x + 2))/(a*x^3 - x^2), 1/4*(16*sqrt(2)*(a^3*x^3
- a^2*x^2)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt
(-c)/(a^2*c*x^2 - c)) - 23*(a^3*x^3 - a^2*x^2)*sqrt(-c)*arctan(sqrt(-a^2*x^
2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 + 1)*sqrt
(-a*c*x + c)*(9*a*x + 2))/(a*x^3 - x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c(ax-1)}\,(ax+1)^3}{x^3\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)**3/(x**3*(-(a*x - 1)*(a*x + 1))**(3/2
)), x)
```

Giac [A] time = 1.3226, size = 244, normalized size = 1.41

$$-\frac{1}{4}a^2c^3\left(\frac{16\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}c|c|}-\frac{23\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c|c|}+\frac{9(acx+c)^{\frac{3}{2}}-7\sqrt{acx+c}c}{a^2c^3x^2|c|}\right)-\frac{\sqrt{2}\left(23\sqrt{2}a^2c^2\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="g
iac")
```

```
[Out] -1/4*a^2*c^3*(16*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(sqrt
(-c)*c*abs(c)) - 23*arctan(sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c*abs(c)) +
(9*(a*c*x + c)^(3/2) - 7*sqrt(a*c*x + c)*c)/(a^2*c^3*x^2*abs(c))) - 1/8*sqr
t(2)*(23*sqrt(2)*a^2*c^2*arctan(sqrt(2)*sqrt(c)/sqrt(-c)) - 32*a^2*c^2*arct
an(sqrt(c)/sqrt(-c)) - 22*a^2*sqrt(-c)*c^(3/2))/(sqrt(-c)*abs(c))
```

3.409 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=216

$$-\frac{19a^2\sqrt{ax+1}(c-acx)^{3/2}}{8cx(1-ax)^{3/2}}-\frac{45a^3(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{8c(1-ax)^{3/2}}+\frac{4\sqrt{2}a^3(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}-\frac{13a\sqrt{ax+1}(c}{12cx^2(1-}$$

```
[Out] -(Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(3*c*x^3*(1 - a*x)^(3/2)) - (13*a*Sqrt[1
 + a*x]*(c - a*c*x)^(3/2))/(12*c*x^2*(1 - a*x)^(3/2)) - (19*a^2*Sqrt[1 + a*
x]*(c - a*c*x)^(3/2))/(8*c*x*(1 - a*x)^(3/2)) - (45*a^3*(c - a*c*x)^(3/2)*A
rcTanh[Sqrt[1 + a*x]])/(8*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*a^3*(c - a*c*x)^(
3/2)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(c*(1 - a*x)^(3/2))
```

Rubi [A] time = 0.166241, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6130, 23, 98, 151, 156, 63, 208}

$$-\frac{19a^2\sqrt{ax+1}(c-acx)^{3/2}}{8cx(1-ax)^{3/2}}-\frac{45a^3(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{8c(1-ax)^{3/2}}+\frac{4\sqrt{2}a^3(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}-\frac{13a\sqrt{ax+1}(c}{12cx^2(1-}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] -(Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(3*c*x^3*(1 - a*x)^(3/2)) - (13*a*Sqrt[1
 + a*x]*(c - a*c*x)^(3/2))/(12*c*x^2*(1 - a*x)^(3/2)) - (19*a^2*Sqrt[1 + a*
x]*(c - a*c*x)^(3/2))/(8*c*x*(1 - a*x)^(3/2)) - (45*a^3*(c - a*c*x)^(3/2)*A
rcTanh[Sqrt[1 + a*x]])/(8*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*a^3*(c - a*c*x)^(
3/2)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(c*(1 - a*x)^(3/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{(1+ax)^{3/2}\sqrt{c-acx}}{x^4(1-ax)^{3/2}}\,dx$$

$$= \frac{(c-acx)^{3/2}\int \frac{(1+ax)^{3/2}}{x^4(c-acx)}\,dx}{(1-ax)^{3/2}}$$

$$= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{3cx^3(1-ax)^{3/2}} - \frac{(c-acx)^{3/2}\int \frac{-\frac{13ac}{2}-\frac{11}{2}a^2cx}{x^3\sqrt{1+ax}(c-acx)}\,dx}{3c(1-ax)^{3/2}}$$

$$= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{3cx^3(1-ax)^{3/2}} - \frac{13a\sqrt{1+ax}(c-acx)^{3/2}}{12cx^2(1-ax)^{3/2}} + \frac{(c-acx)^{3/2}\int \frac{\frac{57a^2c^2}{4}+\frac{39}{4}a^3c^2x}{x^2\sqrt{1+ax}(c-acx)}\,dx}{6c^2(1-ax)^{3/2}}$$

$$= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{3cx^3(1-ax)^{3/2}} - \frac{13a\sqrt{1+ax}(c-acx)^{3/2}}{12cx^2(1-ax)^{3/2}} - \frac{19a^2\sqrt{1+ax}(c-acx)^{3/2}}{8cx(1-ax)^{3/2}} - \frac{(c-acx)^{3/2}\int}{6c^3(1}$$

$$= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{3cx^3(1-ax)^{3/2}} - \frac{13a\sqrt{1+ax}(c-acx)^{3/2}}{12cx^2(1-ax)^{3/2}} - \frac{19a^2\sqrt{1+ax}(c-acx)^{3/2}}{8cx(1-ax)^{3/2}} + \frac{(4a^4(c-acx)^3}{(1}$$

$$= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{3cx^3(1-ax)^{3/2}} - \frac{13a\sqrt{1+ax}(c-acx)^{3/2}}{12cx^2(1-ax)^{3/2}} - \frac{19a^2\sqrt{1+ax}(c-acx)^{3/2}}{8cx(1-ax)^{3/2}} + \frac{(8a^3(c-acx)^3}{}$$

$$= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{3cx^3(1-ax)^{3/2}} - \frac{13a\sqrt{1+ax}(c-acx)^{3/2}}{12cx^2(1-ax)^{3/2}} - \frac{19a^2\sqrt{1+ax}(c-acx)^{3/2}}{8cx(1-ax)^{3/2}} - \frac{45a^3(c-acx)^3}{8c(}$$

Mathematica [A] time = 0.0644, size = 100, normalized size = 0.46

$$-\frac{\sqrt{c-acx}\left(\sqrt{ax+1}\left(57a^2x^2+26ax+8\right)+135a^3x^3\tanh^{-1}\left(\sqrt{ax+1}\right)-96\sqrt{2}a^3x^3\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{24x^3\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] -(Sqrt[c - a*c*x]*(Sqrt[1 + a*x]*(8 + 26*a*x + 57*a^2*x^2) + 135*a^3*x^3*Ar
cTanh[Sqrt[1 + a*x]] - 96*Sqrt[2]*a^3*x^3*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]]))/
(24*x^3*Sqrt[1 - a*x])
```

Maple [A] time = 0.112, size = 151, normalized size = 0.7

$$-\frac{1}{(24\,ax-24)\,x^3}\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}\left(96\,\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^3a^3c-135\,c\text{Artanh}\left(\frac{\sqrt{c\,(ax+1)}}{\sqrt{c}}\right)x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^4,x)
```

```
[Out] -1/24*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(96*2^(1/2)*arctanh(1/2*(c*(a*x
+1))^(1/2)*2^(1/2)/c^(1/2))*x^3*a^3*c-135*c*arctanh((c*(a*x+1))^(1/2)/c^(1/
2))*x^3*a^3-57*x^2*a^2*(c*(a*x+1))^(1/2)*c^(1/2)-26*x*a*(c*(a*x+1))^(1/2)*c
^(1/2)-8*(c*(a*x+1))^(1/2)*c^(1/2))/c^(1/2)/(a*x-1)/(c*(a*x+1))^(1/2)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x^4), x)
```

Fricas [A] time = 2.09233, size = 913, normalized size = 4.23

$$\left[\frac{96\sqrt{2}\left(a^4x^4-a^3x^3\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+135\left(a^4x^4-a^3x^3\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\,\sqrt{-a^2x^2+1}\sqrt{-c}}{ax^2-x}\right.}{48\left(ax^4-x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="f
ricas")
```

```
[Out] [1/48*(96*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2
*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*
x + 1)) + 135*(a^4*x^4 - a^3*x^3)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(
-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*(57*a^2*x^2
+ 26*a*x + 8)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a*x^4 - x^3), 1/24*(96*
sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt
(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) - 135*(a^4*x^4 - a^3*x^3)*sqrt(-c)*a
rctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (57*a
^2*x^2 + 26*a*x + 8)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a*x^4 - x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)}\,(ax+1)^3}{x^4\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)**3/(x**4*(-(a*x - 1)*(a*x + 1))**(3/2
)), x)
```

Giac [A] time = 1.37695, size = 262, normalized size = 1.21

$$-\frac{1}{24}\,a^3c^4\left(\frac{96\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\,\sqrt{-c}}\right)}{\sqrt{-c}c^2|c|}-\frac{135\,\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2|c|}+\frac{57\,(acx+c)^{\frac{5}{2}}-88\,(acx+c)^{\frac{3}{2}}c+39\,\sqrt{acx+c}c^2}{a^3c^5x^3|c|}\right)-\frac{\sqrt{2}(135}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="g
iac")
```

```
[Out] -1/24*a^3*c^4*(96*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(sqr
t(-c)*c^2*abs(c)) - 135*arctan(sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c^2*abs(
c)) + (57*(a*c*x + c)^(5/2) - 88*(a*c*x + c)^(3/2)*c + 39*sqrt(a*c*x + c)*c
^2)/(a^3*c^5*x^3*abs(c))) - 1/48*sqrt(2)*(135*sqrt(2)*a^3*c^2*arctan(sqrt(2
)*sqrt(c)/sqrt(-c)) - 192*a^3*c^2*arctan(sqrt(c)/sqrt(-c)) - 182*a^3*sqrt(-
c)*c^(3/2))/(sqrt(-c)*abs(c))
```

3.410 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=259

$$-\frac{107a^2\sqrt{ax+1}(c-acx)^{3/2}}{96cx^2(1-ax)^{3/2}}-\frac{149a^3\sqrt{ax+1}(c-acx)^{3/2}}{64cx(1-ax)^{3/2}}-\frac{363a^4(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{64c(1-ax)^{3/2}}+\frac{4\sqrt{2}a^4(c-acx)^{3/2}\text{ ta}}{c(1-ax)}$$

```
[Out] -(Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(4*c*x^4*(1 - a*x)^(3/2)) - (17*a*Sqrt[1
 + a*x]*(c - a*c*x)^(3/2))/(24*c*x^3*(1 - a*x)^(3/2)) - (107*a^2*Sqrt[1 + a
*x]*(c - a*c*x)^(3/2))/(96*c*x^2*(1 - a*x)^(3/2)) - (149*a^3*Sqrt[1 + a*x]*
(c - a*c*x)^(3/2))/(64*c*x*(1 - a*x)^(3/2)) - (363*a^4*(c - a*c*x)^(3/2)*Ar
cTanh[Sqrt[1 + a*x]])/(64*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*a^4*(c - a*c*x)^(
3/2)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(c*(1 - a*x)^(3/2))
```

Rubi [A] time = 0.191846, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6130, 23, 98, 151, 156, 63, 208}

$$-\frac{107a^2\sqrt{ax+1}(c-acx)^{3/2}}{96cx^2(1-ax)^{3/2}}-\frac{149a^3\sqrt{ax+1}(c-acx)^{3/2}}{64cx(1-ax)^{3/2}}-\frac{363a^4(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{64c(1-ax)^{3/2}}+\frac{4\sqrt{2}a^4(c-acx)^{3/2}\text{ ta}}{c(1-ax)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^5,x]
```


```
[Out] -(Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(4*c*x^4*(1 - a*x)^(3/2)) - (17*a*Sqrt[1
 + a*x]*(c - a*c*x)^(3/2))/(24*c*x^3*(1 - a*x)^(3/2)) - (107*a^2*Sqrt[1 + a
*x]*(c - a*c*x)^(3/2))/(96*c*x^2*(1 - a*x)^(3/2)) - (149*a^3*Sqrt[1 + a*x]*
(c - a*c*x)^(3/2))/(64*c*x*(1 - a*x)^(3/2)) - (363*a^4*(c - a*c*x)^(3/2)*Ar
cTanh[Sqrt[1 + a*x]])/(64*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*a^4*(c - a*c*x)^(
3/2)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(c*(1 - a*x)^(3/2))
```


Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{(1+ax)^{3/2}\sqrt{c-acx}}{x^5(1-ax)^{3/2}}\,dx$$
$$= \frac{(c-acx)^{3/2}\int \frac{(1+ax)^{3/2}}{x^5(c-acx)}\,dx}{(1-ax)^{3/2}}$$
$$= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{4cx^4(1-ax)^{3/2}} - \frac{(c-acx)^{3/2}\int \frac{-\frac{17ac}{2}-\frac{15}{2}a^2cx}{x^4\sqrt{1+ax}(c-acx)}\,dx}{4c(1-ax)^{3/2}}$$
$$= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{4cx^4(1-ax)^{3/2}} - \frac{17a\sqrt{1+ax}(c-acx)^{3/2}}{24cx^3(1-ax)^{3/2}} + \frac{(c-acx)^{3/2}\int \frac{\frac{107a^2c^2}{4}+\frac{85}{4}a^3c^2x}{x^3\sqrt{1+ax}(c-acx)}\,dx}{12c^2(1-ax)^{3/2}}$$
$$= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{4cx^4(1-ax)^{3/2}} - \frac{17a\sqrt{1+ax}(c-acx)^{3/2}}{24cx^3(1-ax)^{3/2}} - \frac{107a^2\sqrt{1+ax}(c-acx)^{3/2}}{96cx^2(1-ax)^{3/2}} - \frac{(c-acx)^{3}}{2}$$
$$= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{4cx^4(1-ax)^{3/2}} - \frac{17a\sqrt{1+ax}(c-acx)^{3/2}}{24cx^3(1-ax)^{3/2}} - \frac{107a^2\sqrt{1+ax}(c-acx)^{3/2}}{96cx^2(1-ax)^{3/2}} - \frac{149a^3\sqrt{1}}{64cx}$$
$$= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{4cx^4(1-ax)^{3/2}} - \frac{17a\sqrt{1+ax}(c-acx)^{3/2}}{24cx^3(1-ax)^{3/2}} - \frac{107a^2\sqrt{1+ax}(c-acx)^{3/2}}{96cx^2(1-ax)^{3/2}} - \frac{149a^3\sqrt{1}}{64cx}$$
$$= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{4cx^4(1-ax)^{3/2}} - \frac{17a\sqrt{1+ax}(c-acx)^{3/2}}{24cx^3(1-ax)^{3/2}} - \frac{107a^2\sqrt{1+ax}(c-acx)^{3/2}}{96cx^2(1-ax)^{3/2}} - \frac{149a^3\sqrt{1}}{64cx}$$
$$= -\frac{\sqrt{1+ax}(c-acx)^{3/2}}{4cx^4(1-ax)^{3/2}} - \frac{17a\sqrt{1+ax}(c-acx)^{3/2}}{24cx^3(1-ax)^{3/2}} - \frac{107a^2\sqrt{1+ax}(c-acx)^{3/2}}{96cx^2(1-ax)^{3/2}} - \frac{149a^3\sqrt{1}}{64cx}$$

Mathematica [A] time = 0.0747908, size = 108, normalized size = 0.42

$$-\frac{\sqrt{c-acx}\left(\sqrt{ax+1}\left(447a^3x^3+214a^2x^2+136ax+48\right)+1089a^4x^4\tanh^{-1}\left(\sqrt{ax+1}\right)-768\sqrt{2}a^4x^4\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{192x^4\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] -(Sqrt[c - a*c*x]*(Sqrt[1 + a*x]*(48 + 136*a*x + 214*a^2*x^2 + 447*a^3*x^3)
 + 1089*a^4*x^4*ArcTanh[Sqrt[1 + a*x]] - 768*Sqrt[2]*a^4*x^4*ArcTanh[Sqrt[1
 + a*x]/Sqrt[2]]))/(192*x^4*Sqrt[1 - a*x])
```

Maple [A] time = 0.109, size = 171, normalized size = 0.7

$$-\frac{1}{(192\,ax-192)\,x^4}\sqrt{-a^2x^2+1}\sqrt{-c\,(ax-1)}\left(768\,\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)ca^4x^4-1089\,c\mathrm{Artanh}\left(\frac{\sqrt{c\,(ax+1)}}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^5,x)
```

```
[Out] -1/192*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(768*2^(1/2)*arctanh(1/2*(c*(a
*x+1))^(1/2)*2^(1/2)/c^(1/2))*c*a^4*x^4-1089*c*arctanh((c*(a*x+1))^(1/2)/c^
```

```
(1/2))*x^4*a^4-447*x^3*a^3*(c*(a*x+1))^(1/2)*c^(1/2)-214*x^2*a^2*(c*(a*x+1)
)^(1/2)*c^(1/2)-136*x*a*(c*(a*x+1))^(1/2)*c^(1/2)-48*(c*(a*x+1))^(1/2)*c^(1
/2))/c^(1/2)/(a*x-1)/(c*(a*x+1))^(1/2)/x^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x^5), x)
```

Fricas [A] time = 2.13876, size = 967, normalized size = 3.73

$$\left[\frac{768\sqrt{2}\left(a^5x^5-a^4x^4\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+1089\left(a^5x^5-a^4x^4\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\,\sqrt{-a^2x^2+1}\sqrt{-ac}}{ax^2-x}\right.}{384\left(ax^5-x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="f
ricas")
```

```
[Out] [1/384*(768*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x -
 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*
a*x + 1)) + 1089*(a^5*x^5 - a^4*x^4)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sq
rt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*(447*a^3*
x^3 + 214*a^2*x^2 + 136*a*x + 48)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a*x
^5 - x^4), 1/192*(768*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(-c)*arctan(sqrt(2)*s
qrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) - 1089*(a^5*x^
5 - a^4*x^4)*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(
a^2*c*x^2 - c)) + (447*a^3*x^3 + 214*a^2*x^2 + 136*a*x + 48)*sqrt(-a^2*x^2
+ 1)*sqrt(-a*c*x + c))/(a*x^5 - x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41393, size = 279, normalized size = 1.08

$$-\frac{1}{192}a^4c^5\left(\frac{768\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}c^3|c|}-\frac{1089\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^3|c|}+\frac{447\,(acx+c)^{\frac{7}{2}}-1127\,(acx+c)^{\frac{5}{2}}c+1049\,(acx+c)^{\frac{3}{2}}}{a^4c^7x^4|c|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="g
iac")
```

```
[Out] -1/192*a^4*c^5*(768*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(s
qrt(-c)*c^3*abs(c)) - 1089*arctan(sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c^3*a
bs(c)) + (447*(a*c*x + c)^(7/2) - 1127*(a*c*x + c)^(5/2)*c + 1049*(a*c*x +
c)^(3/2)*c^2 - 321*sqrt(a*c*x + c)*c^3)/(a^4*c^7*x^4*abs(c))) - 1/384*sqrt(
2)*(1089*sqrt(2)*a^4*c^2*arctan(sqrt(2)*sqrt(c)/sqrt(-c)) - 1536*a^4*c^2*ar
ctan(sqrt(c)/sqrt(-c)) - 1690*a^4*sqrt(-c)*c^(3/2))/(sqrt(-c)*abs(c))
```

3.411 $\int e^{-\tanh^{-1}(ax)} x^m \sqrt{c-acx}\, dx$

Optimal. Leaf size=114

$$\frac{2(4m+5)(ax+1)x^m\sqrt{c-acx}(-ax)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},ax+1\right)}{a(2m+3)\sqrt{1-a^2x^2}}-\frac{2c\sqrt{1-a^2x^2}x^{m+1}}{(2m+3)\sqrt{c-acx}}$$

```
[Out] (-2*c*x^(1 + m)*Sqrt[1 - a^2*x^2])/((3 + 2*m)*Sqrt[c - a*c*x]) + (2*(5 + 4*
m)*x^m*(1 + a*x)*Sqrt[c - a*c*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + a*x])/
(a*(3 + 2*m)*(-(a*x))^m*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.184865, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6128, 881, 892, 67, 65}

$$\frac{2(4m+5)(ax+1)x^m\sqrt{c-acx}(-ax)^{-m}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};ax+1\right)}{a(2m+3)\sqrt{1-a^2x^2}}-\frac{2c\sqrt{1-a^2x^2}x^{m+1}}{(2m+3)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a*c*x])/E^ArcTanh[a*x],x]
```

```
[Out] (-2*c*x^(1 + m)*Sqrt[1 - a^2*x^2])/((3 + 2*m)*Sqrt[c - a*c*x]) + (2*(5 + 4*
m)*x^m*(1 + a*x)*Sqrt[c - a*c*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + a*x])/
(a*(3 + 2*m)*(-(a*x))^m*Sqrt[1 - a^2*x^2])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/
e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)} x^m \sqrt{c-acx}\,dx &= \frac{\int \frac{x^m (c-acx)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{2cx^{1+m}\sqrt{1-a^2x^2}}{(3+2m)\sqrt{c-acx}} + \frac{(5+4m)\int \frac{x^m\sqrt{c-acx}}{\sqrt{1-a^2x^2}}\,dx}{3+2m}\\
&= -\frac{2cx^{1+m}\sqrt{1-a^2x^2}}{(3+2m)\sqrt{c-acx}} + \frac{\left((5+4m)\sqrt{\frac{1}{c}+\frac{ax}{c}}\sqrt{c-acx}\right)\int \frac{x^m}{\sqrt{\frac{1}{c}+\frac{ax}{c}}}\,dx}{(3+2m)\sqrt{1-a^2x^2}}\\
&= -\frac{2cx^{1+m}\sqrt{1-a^2x^2}}{(3+2m)\sqrt{c-acx}} + \frac{\left((5+4m)x^m(-ax)^{-m}\sqrt{\frac{1}{c}+\frac{ax}{c}}\sqrt{c-acx}\right)\int \frac{(-ax)^m}{\sqrt{\frac{1}{c}+\frac{ax}{c}}}\,dx}{(3+2m)\sqrt{1-a^2x^2}}\\
&= -\frac{2cx^{1+m}\sqrt{1-a^2x^2}}{(3+2m)\sqrt{c-acx}} + \frac{2(5+4m)x^m(-ax)^{-m}(1+ax)\sqrt{c-acx}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+ax\right)}{a(3+2m)\sqrt{1-a^2x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0412233, size = 77, normalized size = 0.68

$$
-\frac{c\sqrt{1-ax}x^{m+1}\left(2(m+1)\sqrt{ax+1}-(4m+5)\text{Hypergeometric2F1}\left(\frac{1}{2},m+1,m+2,-ax\right)\right)}{(m+1)(2m+3)\sqrt{c-acx}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^m*Sqrt[c - a*c*x])/E^ArcTanh[a*x],x]
```

```
[Out] -((c*x^(1 + m)*Sqrt[1 - a*x]*(2*(1 + m)*Sqrt[1 + a*x] - (5 + 4*m)*Hypergeom
etric2F1[1/2, 1 + m, 2 + m, -(a*x)]))/((1 + m)*(3 + 2*m)*Sqrt[c - a*c*x]))
```

Maple [F] time = 0.497, size = 0, normalized size = 0.

$$
\int \frac{x^m}{ax+1}\sqrt{-acx+c}\sqrt{-a^2x^2+1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*x^m/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}x^m}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*x^m/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{-c\left(ax-1\right)}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*x^m/(a*x + 1), x)
```

3.412 $\int e^{-\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=135

$$-\frac{2cx^3\sqrt{1-a^2x^2}}{7\sqrt{c-acx}}+\frac{26cx^2\sqrt{1-a^2x^2}}{35a\sqrt{c-acx}}+\frac{104\sqrt{1-a^2x^2}\sqrt{c-acx}}{105a^3}+\frac{104c\sqrt{1-a^2x^2}}{105a^3\sqrt{c-acx}}$$

```
[Out] (104*c*Sqrt[1 - a^2*x^2])/(105*a^3*Sqrt[c - a*c*x]) + (26*c*x^2*Sqrt[1 - a^
2*x^2])/(35*a*Sqrt[c - a*c*x]) - (2*c*x^3*Sqrt[1 - a^2*x^2])/(7*Sqrt[c - a*
c*x]) + (104*Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2])/(105*a^3)
```

Rubi [A] time = 0.21296, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6128, 881, 871, 795, 649}

$$-\frac{2cx^3\sqrt{1-a^2x^2}}{7\sqrt{c-acx}}+\frac{26cx^2\sqrt{1-a^2x^2}}{35a\sqrt{c-acx}}+\frac{104\sqrt{1-a^2x^2}\sqrt{c-acx}}{105a^3}+\frac{104c\sqrt{1-a^2x^2}}{105a^3\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^ArcTanh[a*x],x]
```

```
[Out] (104*c*Sqrt[1 - a^2*x^2])/(105*a^3*Sqrt[c - a*c*x]) + (26*c*x^2*Sqrt[1 - a^
2*x^2])/(35*a*Sqrt[c - a*c*x]) - (2*c*x^3*Sqrt[1 - a^2*x^2])/(7*Sqrt[c - a*
c*x]) + (104*Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2])/(105*a^3)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 871

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p
+ 1))/(c*(m - n - 1)), x] - Dist[(n*(e*f + d*g))/(e*(m - n - 1)), Int[(d +
e*x)^m*(f + g*x)^(n - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&
 EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Integ
erQ[n])
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx &= \frac{\int \frac{x^2(c-acx)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{2cx^3\sqrt{1-a^2x^2}}{7\sqrt{c-acx}}+\frac{13}{7}\int \frac{x^2\sqrt{c-acx}}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{26cx^2\sqrt{1-a^2x^2}}{35a\sqrt{c-acx}}-\frac{2cx^3\sqrt{1-a^2x^2}}{7\sqrt{c-acx}}-\frac{52\int \frac{x\sqrt{c-acx}}{\sqrt{1-a^2x^2}}\,dx}{35a}\\
&= \frac{26cx^2\sqrt{1-a^2x^2}}{35a\sqrt{c-acx}}-\frac{2cx^3\sqrt{1-a^2x^2}}{7\sqrt{c-acx}}+\frac{104\sqrt{c-acx}\sqrt{1-a^2x^2}}{105a^3}+\frac{52\int \frac{\sqrt{c-acx}}{\sqrt{1-a^2x^2}}\,dx}{105a^2}\\
&= \frac{104c\sqrt{1-a^2x^2}}{105a^3\sqrt{c-acx}}+\frac{26cx^2\sqrt{1-a^2x^2}}{35a\sqrt{c-acx}}-\frac{2cx^3\sqrt{1-a^2x^2}}{7\sqrt{c-acx}}+\frac{104\sqrt{c-acx}\sqrt{1-a^2x^2}}{105a^3}
\end{aligned}$$

Mathematica [A] time = 0.036173, size = 55, normalized size = 0.41

$$-\frac{2c\sqrt{1-a^2x^2}\left(15a^3x^3-39a^2x^2+52ax-104\right)}{105a^3\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^ArcTanh[a*x],x]
```

```
[Out] (-2*c*Sqrt[1 - a^2*x^2]*(-104 + 52*a*x - 39*a^2*x^2 + 15*a^3*x^3))/(105*a^3
*Sqrt[c - a*c*x])
```

Maple [A] time = 0.032, size = 56, normalized size = 0.4

$$\frac{30\,x^3a^3-78\,a^2x^2+104\,ax-208}{(105\,ax-105)\,a^3}\sqrt{-a^2x^2+1}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2/105*(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)*(15*a^3*x^3-39*a^2*x^2+52*a*x-104
)/(a*x-1)/a^3
```

Maxima [A] time = 1.02985, size = 84, normalized size = 0.62

$$-\frac{2\left(15\,a^{3}\sqrt{c}x^{3}-39\,a^{2}\sqrt{c}x^{2}+52\,a\sqrt{c}x-104\,\sqrt{c}\right)\sqrt{ax+1}(ax-1)}{105\left(a^{4}x-a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] -2/105*(15*a^3*sqrt(c)*x^3 - 39*a^2*sqrt(c)*x^2 + 52*a*sqrt(c)*x - 104*sqrt
(c))*sqrt(a*x + 1)*(a*x - 1)/(a^4*x - a^3)
```

Fricas [A] time = 1.74107, size = 132, normalized size = 0.98

$$\frac{2\left(15\,a^{3}x^{3}-39\,a^{2}x^{2}+52\,ax-104\right)\sqrt{-a^{2}x^{2}+1}\sqrt{-acx+c}}{105\left(a^{4}x-a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/105*(15*a^3*x^3 - 39*a^2*x^2 + 52*a*x - 104)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c
*x + c)/(a^4*x - a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}\sqrt{-c\,(ax-1)}\sqrt{-\,(ax-1)\,(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [A] time = 1.18971, size = 100, normalized size = 0.74

$$-\frac{2\left(\frac{76\,\sqrt{2}c^{\frac{3}{2}}}{a^{3}}+\frac{15\,(acx+c)^{\frac{7}{2}}-84\,(acx+c)^{\frac{5}{2}}c+175\,(acx+c)^{\frac{3}{2}}c^{2}-210\,\sqrt{acx+c}c^{3}}{a^{3}c^{2}}\right)|c|}{105\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] -2/105*(76*sqrt(2)*c^(3/2)/a^3 + (15*(a*c*x + c)^(7/2) - 84*(a*c*x + c)^(5/
2)*c + 175*(a*c*x + c)^(3/2)*c^2 - 210*sqrt(a*c*x + c)*c^3)/(a^3*c^2))*abs(
c)/c^2
```

3.413 $\int e^{-\tanh^{-1}(ax)} x\sqrt{c-acx}\, dx$

Optimal. Leaf size=101

$$-\frac{2\sqrt{1-a^2x^2}(c-acx)^{3/2}}{5a^2c}-\frac{2\sqrt{1-a^2x^2}\sqrt{c-acx}}{5a^2}-\frac{8c\sqrt{1-a^2x^2}}{5a^2\sqrt{c-acx}}$$

```
[Out] (-8*c*Sqrt[1 - a^2*x^2])/(5*a^2*Sqrt[c - a*c*x]) - (2*Sqrt[c - a*c*x]*Sqrt[
1 - a^2*x^2])/(5*a^2) - (2*(c - a*c*x)^(3/2)*Sqrt[1 - a^2*x^2])/(5*a^2*c)
```

Rubi [A] time = 0.121145, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {6128, 795, 657, 649}

$$-\frac{2\sqrt{1-a^2x^2}(c-acx)^{3/2}}{5a^2c}-\frac{2\sqrt{1-a^2x^2}\sqrt{c-acx}}{5a^2}-\frac{8c\sqrt{1-a^2x^2}}{5a^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a*c*x])/E^ArcTanh[a*x],x]
```

```
[Out] (-8*c*Sqrt[1 - a^2*x^2])/(5*a^2*Sqrt[c - a*c*x]) - (2*Sqrt[c - a*c*x]*Sqrt[
1 - a^2*x^2])/(5*a^2) - (2*(c - a*c*x)^(3/2)*Sqrt[1 - a^2*x^2])/(5*a^2*c)
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{\int \frac{x(c-acx)^{3/2}}{\sqrt{1-a^2x^2}}dx}{c}$$
$$= -\frac{2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{5a^2c} - \frac{3\int \frac{(c-acx)^{3/2}}{\sqrt{1-a^2x^2}}dx}{5ac}$$
$$= -\frac{2\sqrt{c-acx}\sqrt{1-a^2x^2}}{5a^2} - \frac{2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{5a^2c} - \frac{4\int \frac{\sqrt{c-acx}}{\sqrt{1-a^2x^2}}dx}{5a}$$
$$= -\frac{8c\sqrt{1-a^2x^2}}{5a^2\sqrt{c-acx}} - \frac{2\sqrt{c-acx}\sqrt{1-a^2x^2}}{5a^2} - \frac{2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{5a^2c}$$

Mathematica [A] time = 0.0287383, size = 46, normalized size = 0.46

$$-\frac{2c\sqrt{1-a^2x^2}\left(a^2x^2-3ax+6\right)}{5a^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^ArcTanh[a*x],x]
```

```
[Out] (-2*c*Sqrt[1 - a^2*x^2]*(6 - 3*a*x + a^2*x^2))/(5*a^2*Sqrt[c - a*c*x])
```

Maple [A] time = 0.032, size = 47, normalized size = 0.5

$$\frac{2\,a^2x^2-6\,ax+12}{(5\,ax-5)\,a^2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2/5*(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)*(a^2*x^2-3*a*x+6)/(a*x-1)/a^2
```

Maxima [A] time = 1.02028, size = 68, normalized size = 0.67

$$-\frac{2\left(a^2\sqrt{c}x^2-3\,a\sqrt{c}x+6\,\sqrt{c}\right)\sqrt{ax+1}(ax-1)}{5\left(a^3x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2/5*(a^2*sqrt(c)*x^2 - 3*a*sqrt(c)*x + 6*sqrt(c))*sqrt(a*x + 1)*(a*x - 1)/
(a^3*x - a^2)
```

Fricas [A] time = 1.84683, size = 104, normalized size = 1.03

$$\frac{2\left(a^2x^2-3\,ax+6\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{5\left(a^3x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 2/5*(a^2*x^2 - 3*a*x + 6)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^3*x - a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\sqrt{-c\,(ax-1)}\sqrt{-(ax-1)\,(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [A] time = 1.19132, size = 81, normalized size = 0.8

$$\frac{2\left(\frac{4\sqrt{2}c^{\frac{5}{2}}}{a}-\frac{(acx+c)^{\frac{5}{2}}-5\,(acx+c)^{\frac{3}{2}}c+10\,\sqrt{acx+c}c^2}{a}\right)|c|}{5\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] 2/5*(4*sqrt(2)*c^(5/2)/a - ((a*c*x + c)^(5/2) - 5*(a*c*x + c)^(3/2)*c + 10*
sqrt(a*c*x + c)*c^2)/a)*abs(c)/(a*c^3)
```

3.414 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=66

$$\frac{8c\sqrt{1-a^2x^2}}{3a\sqrt{c-acx}}+\frac{2\sqrt{1-a^2x^2}\sqrt{c-acx}}{3a}$$

```
[Out] (8*c*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[c - a*c*x]) + (2*Sqrt[c - a*c*x]*Sqrt[1 -
 a^2*x^2])/(3*a)
```

Rubi [A] time = 0.0652532, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6127, 657, 649}

$$\frac{8c\sqrt{1-a^2x^2}}{3a\sqrt{c-acx}}+\frac{2\sqrt{1-a^2x^2}\sqrt{c-acx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^ArcTanh[a*x],x]
```

```
[Out] (8*c*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[c - a*c*x]) + (2*Sqrt[c - a*c*x]*Sqrt[1 -
 a^2*x^2])/(3*a)
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}\sqrt{c-acx}\,dx&=\frac{\int\frac{(c-acx)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{c}\\&=\frac{2\sqrt{c-acx}\sqrt{1-a^2x^2}}{3a}+\frac{4}{3}\int\frac{\sqrt{c-acx}}{\sqrt{1-a^2x^2}}\,dx\\&=\frac{8c\sqrt{1-a^2x^2}}{3a\sqrt{c-acx}}+\frac{2\sqrt{c-acx}\sqrt{1-a^2x^2}}{3a}\end{aligned}$$

Mathematica [A] time = 0.0189387, size = 38, normalized size = 0.58

$$-\frac{2c(ax-5)\sqrt{1-a^2x^2}}{3a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^ArcTanh[a*x],x]
```

```
[Out] (-2*c*(-5 + a*x)*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[c - a*c*x])
```

Maple [A] time = 0.03, size = 39, normalized size = 0.6

$$\frac{2\,ax-10}{(3\,ax-3)\,a}\sqrt{-a^2x^2+1}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2/3*(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)*(a*x-5)/(a*x-1)/a
```

Maxima [A] time = 0.999403, size = 50, normalized size = 0.76

$$-\frac{2\left(a\sqrt{c}x-5\sqrt{c}\right)\sqrt{ax+1}(ax-1)}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -2/3*(a*sqrt(c)*x - 5*sqrt(c))*sqrt(a*x + 1)*(a*x - 1)/(a^2*x - a)
```

Fricas [A] time = 1.79553, size = 85, normalized size = 1.29

$$\frac{2\sqrt{-a^2x^2+1}\sqrt{-acx+c}(ax-5)}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/3*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x - 5)/(a^2*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\,(ax-1)}\sqrt{-\,(ax-1)\,(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [A] time = 1.19726, size = 58, normalized size = 0.88

$$-\frac{2\left(\frac{4\sqrt{2}c^{\frac{3}{2}}}{a}+\frac{(acx+c)^{\frac{3}{2}}-6\sqrt{acx+c}c}{a}\right)|c|}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(4*sqrt(2)*c^(3/2)/a + ((a*c*x + c)^(3/2) - 6*sqrt(a*c*x + c)*c)/a)*ab
s(c)/c^2
```

3.415 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=68

$$-\frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] (-2*c*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x] - 2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[
1 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rubi [A] time = 0.158214, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6128, 881, 875, 208}

$$-\frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^ArcTanh[a*x]*x),x]
```

```
[Out] (-2*c*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x] - 2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[
1 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{\int \frac{(c-acx)^{3/2}}{x\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= -\frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}} + \int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}} + \left(2a^2c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$
$$= -\frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Mathematica [A] time = 0.0198962, size = 44, normalized size = 0.65

$$-\frac{2c\sqrt{1-ax}\left(\sqrt{ax+1}+\tanh^{-1}\left(\sqrt{ax+1}\right)\right)}{\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^ArcTanh[a*x]*x),x]
```

```
[Out] (-2*c*Sqrt[1 - a*x]*(Sqrt[1 + a*x] + ArcTanh[Sqrt[1 + a*x]]))/Sqrt[c - a*c*
x]
```

Maple [A] time = 0.097, size = 69, normalized size = 1.

$$2\,\frac{\sqrt{-c\,(ax-1)}\sqrt{-a^2x^2+1}}{(ax-1)\,\sqrt{c\,(ax+1)}}\left(\sqrt{c}\text{Artanh}\left(\frac{\sqrt{c\,(ax+1)}}{\sqrt{c}}\right)+\sqrt{c\,(ax+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] 2*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(c^(1/2)*arctanh((c*(a*x+1))^(1/2)/
c^(1/2))+(c*(a*x+1))^(1/2))/(a*x-1)/(c*(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/((a*x + 1)*x), x)
```

Fricas [A] time = 1.90054, size = 417, normalized size = 6.13

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{ax-1}, -\frac{2\left((ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{-c}}{a^2cx^2-c}\right)\ldots\right.}{ax-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="frica
s")
```

```
[Out] [((a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*
c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c
))/(a*x - 1), -2*((a*x - 1)*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x
+ c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a*x
- 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c(ax-1)}\sqrt{-(ax-1)(ax+1)}}{x(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(x*(a*x + 1)), x)
```

Giac [A] time = 1.21521, size = 109, normalized size = 1.6

$$\frac{2\left(\left(\frac{c\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\sqrt{acx+c}\right)c-\frac{c^2\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)-\sqrt{2}\sqrt{-c}c^{\frac{3}{2}}}{\sqrt{-c}}\right)|c|}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac"
)
```

```
[Out] 2*((c*arctan(sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(a*c*x + c))*c - (c^2
*arctan(sqrt(2)*sqrt(c)/sqrt(-c)) - sqrt(2)*sqrt(-c)*c^(3/2))/sqrt(-c))*abs
(c)/c^2
```

3.416 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=72

$$3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)-\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}$$

```
[Out] -((c*Sqrt[1 - a^2*x^2])/(x*Sqrt[c - a*c*x])) + 3*a*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rubi [A] time = 0.160621, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6128, 879, 875, 208}

$$3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)-\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] -((c*Sqrt[1 - a^2*x^2])/(x*Sqrt[c - a*c*x])) + 3*a*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\int \frac{(c-acx)^{3/2}}{x^2\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= -\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}} - \frac{1}{2}(3a)\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}} - \left(3a^3c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$
$$= -\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}} + 3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Mathematica [A] time = 0.0303993, size = 52, normalized size = 0.72

$$\frac{\sqrt{1-ax}\left(3ac\tanh^{-1}\left(\sqrt{ax+1}\right)-\frac{c\sqrt{ax+1}}{x}\right)}{\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] (Sqrt[1 - a*x]*(-((c*Sqrt[1 + a*x])/x) + 3*a*c*ArcTanh[Sqrt[1 + a*x]]))/Sqr
t[c - a*c*x]
```

Maple [A] time = 0.1, size = 79, normalized size = 1.1

$$\frac{1}{(ax-1)\,x}\sqrt{-c\,(ax-1)}\sqrt{-a^2x^2+1}\left(-3\,\mathrm{Artanh}\left(\frac{\sqrt{c\,(ax+1)}}{\sqrt{c}}\right)xac+\sqrt{c\,(ax+1)}\sqrt{c}\right)\frac{1}{\sqrt{c\,(ax+1)}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] (-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(-3*arctanh((c*(a*x+1))^(1/2)/c^(1/2)
)*x*a*c+(c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)/(c*(a*x+1))^(1/2)/x/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/((a*x + 1)*x^2), x)
```

Fricas [A] time = 1.86449, size = 446, normalized size = 6.19

$$\left[\frac{3\left(a^2x^2-ax\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{2\left(ax^2-x\right)}, \frac{3\left(a^2x^2-ax\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2x^2}}{}\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] [1/2*(3*(a^2*x^2 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a^2*x^2 +
 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqr
t(-a*c*x + c))/(a*x^2 - x), (3*(a^2*x^2 - a*x)*sqrt(-c)*arctan(sqrt(-a^2*x^
2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 + 1)*sqrt
(-a*c*x + c))/(a*x^2 - x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(ax-1\right)}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{x^2\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(x**2*(a*x + 1)), x)
```

Giac [A] time = 1.2062, size = 131, normalized size = 1.82

$$-\frac{\left(ac^2\left(\frac{3\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\sqrt{acx+c}}{acx}\right)-\frac{3ac^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)+\sqrt{2}a\sqrt{-c}c^2}{\sqrt{-c}\sqrt{c}}\right)|c|}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="gia
c")
```

```
[Out] -(a*c^2*(3*arctan(sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) + sqrt(a*c*x + c)/(a*c
*x)) - (3*a*c^(5/2)*arctan(sqrt(2)*sqrt(c)/sqrt(-c)) + sqrt(2)*a*sqrt(-c)*c
^2)/(sqrt(-c)*sqrt(c)))*abs(c)/c^2
```

3.417 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=112

$$\frac{7ac\sqrt{1-a^2x^2}}{4x\sqrt{c-acx}}-\frac{c\sqrt{1-a^2x^2}}{2x^2\sqrt{c-acx}}-\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(2*x^2*Sqrt[c - a*c*x]) + (7*a*c*Sqrt[1 - a^2*x^2])/
(4*x*Sqrt[c - a*c*x]) - (7*a^2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/
Sqrt[c - a*c*x]])/4
```

Rubi [A] time = 0.195998, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6128, 879, 873, 875, 208}

$$\frac{7ac\sqrt{1-a^2x^2}}{4x\sqrt{c-acx}}-\frac{c\sqrt{1-a^2x^2}}{2x^2\sqrt{c-acx}}-\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^ArcTanh[a*x]*x^3),x]
```


```
[Out] -(c*Sqrt[1 - a^2*x^2])/(2*x^2*Sqrt[c - a*c*x]) + (7*a*c*Sqrt[1 - a^2*x^2])/
(4*x*Sqrt[c - a*c*x]) - (7*a^2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/
Sqrt[c - a*c*x]])/4
```


Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 873

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*
x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g)), x] - Dist[(e*(m - n - 2))/((n + 1)
*(e*f + d*g)), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2
, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx &= \frac{\int \frac{(c-acx)^{3/2}}{x^3\sqrt{1-a^2x^2}}\,dx}{c}\\ &= -\frac{c\sqrt{1-a^2x^2}}{2x^2\sqrt{c-acx}}-\frac{1}{4}(7a)\int \frac{\sqrt{c-acx}}{x^2\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{c\sqrt{1-a^2x^2}}{2x^2\sqrt{c-acx}}+\frac{7ac\sqrt{1-a^2x^2}}{4x\sqrt{c-acx}}+\frac{1}{8}\left(7a^2\right)\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{c\sqrt{1-a^2x^2}}{2x^2\sqrt{c-acx}}+\frac{7ac\sqrt{1-a^2x^2}}{4x\sqrt{c-acx}}+\frac{1}{4}\left(7a^4c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\ &= -\frac{c\sqrt{1-a^2x^2}}{2x^2\sqrt{c-acx}}+\frac{7ac\sqrt{1-a^2x^2}}{4x\sqrt{c-acx}}-\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\end{aligned}$$

Mathematica [A] time = 0.032561, size = 64, normalized size = 0.57

$$-\frac{c\sqrt{1-ax}\left(7a^2x^2\tanh^{-1}\left(\sqrt{ax+1}\right)+(2-7ax)\sqrt{ax+1}\right)}{4x^2\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^ArcTanh[a*x]*x^3),x]
```

```
[Out] -(c*Sqrt[1 - a*x]*((2 - 7*a*x)*Sqrt[1 + a*x] + 7*a^2*x^2*ArcTanh[Sqrt[1 + a
*x]]))/(4*x^2*Sqrt[c - a*c*x])
```

Maple [A] time = 0.102, size = 101, normalized size = 0.9

$$\frac{1}{(4\,ax-4)\,x^2}\sqrt{-c\,(ax-1)}\sqrt{-a^2x^2+1}\left(7\,c\text{Artanh}\left(\frac{\sqrt{c\,(ax+1)}}{\sqrt{c}}\right)x^2a^2-7\,xa\sqrt{c\,(ax+1)}\sqrt{c}+2\,\sqrt{c\,(ax+1)}\sqrt{c}\right)\frac{1}{\sqrt{c\,(ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x)
```

```
[Out] 1/4*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(7*c*arctanh((c*(a*x+1))^(1/2)/c^
(1/2))*x^2*a^2-7*x*a*(c*(a*x+1))^(1/2)*c^(1/2)+2*(c*(a*x+1))^(1/2)*c^(1/2))
/c^(1/2)/(a*x-1)/(c*(a*x+1))^(1/2)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{(ax+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/((a*x + 1)*x^3), x)
```

Fricas [A] time = 1.92857, size = 501, normalized size = 4.47

$$\left[\frac{7\left(a^3x^3-a^2x^2\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}(7ax-2)}{8\left(ax^3-x^2\right)},-\frac{7\left(a^3x^3-a^2x^2\right)\sqrt{-c}\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="fri
cas")
```

```
[Out] [1/8*(7*(a^3*x^3 - a^2*x^2)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x
^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) - 2*sqrt(-a^2*x^2 + 1)
*sqrt(-a*c*x + c)*(7*a*x - 2))/(a*x^3 - x^2), -1/4*(7*(a^3*x^3 - a^2*x^2)*s
qrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)
) + sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(7*a*x - 2))/(a*x^3 - x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)}\sqrt{-(ax-1)\,(ax+1)}}{x^3\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(x**3*(a*x + 1)), x)
```

Giac [A] time = 1.31209, size = 159, normalized size = 1.42

$$\frac{\left(a^2c^3\left(\frac{7\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}+\frac{7(acx+c)^{\frac{3}{2}}-9\sqrt{acx+c}c}{a^2c^3x^2}\right)-\frac{7a^2c^2\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)+5\sqrt{2}a^2\sqrt{-c}c^{\frac{3}{2}}}{\sqrt{-c}}\right)|c|}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="gia
c")
```

```
[Out] 1/4*(a^2*c^3*(7*arctan(sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c) + (7*(a*c*x +
 c)^(3/2) - 9*sqrt(a*c*x + c)*c)/(a^2*c^3*x^2)) - (7*a^2*c^2*arctan(sqrt(2)
*sqrt(c)/sqrt(-c)) + 5*sqrt(2)*a^2*sqrt(-c)*c^(3/2))/sqrt(-c))*abs(c)/c^2
```

3.418 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=148

$$-\frac{11a^2c\sqrt{1-a^2x^2}}{8x\sqrt{c-acx}}+\frac{11ac\sqrt{1-a^2x^2}}{12x^2\sqrt{c-acx}}-\frac{c\sqrt{1-a^2x^2}}{3x^3\sqrt{c-acx}}+\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(3*x^3*Sqrt[c - a*c*x]) + (11*a*c*Sqrt[1 - a^2*x^2])
/(12*x^2*Sqrt[c - a*c*x]) - (11*a^2*c*Sqrt[1 - a^2*x^2])/(8*x*Sqrt[c - a*c*
x]) + (11*a^3*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]])
/8
```

Rubi [A] time = 0.236645, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6128, 879, 873, 875, 208}

$$-\frac{11a^2c\sqrt{1-a^2x^2}}{8x\sqrt{c-acx}}+\frac{11ac\sqrt{1-a^2x^2}}{12x^2\sqrt{c-acx}}-\frac{c\sqrt{1-a^2x^2}}{3x^3\sqrt{c-acx}}+\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^ArcTanh[a*x]*x^4),x]
```

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(3*x^3*Sqrt[c - a*c*x]) + (11*a*c*Sqrt[1 - a^2*x^2])
/(12*x^2*Sqrt[c - a*c*x]) - (11*a^2*c*Sqrt[1 - a^2*x^2])/(8*x*Sqrt[c - a*c*
x]) + (11*a^3*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]])
/8
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 873

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*
x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g)), x] - Dist[(e*(m - n - 2))/((n + 1)
*(e*f + d*g)), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2
, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx &= \frac{\int \frac{(c-acx)^{3/2}}{x^4\sqrt{1-a^2x^2}}\,dx}{c}\\ &= -\frac{c\sqrt{1-a^2x^2}}{3x^3\sqrt{c-acx}}-\frac{1}{6}(11a)\int \frac{\sqrt{c-acx}}{x^3\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{c\sqrt{1-a^2x^2}}{3x^3\sqrt{c-acx}}+\frac{11ac\sqrt{1-a^2x^2}}{12x^2\sqrt{c-acx}}+\frac{1}{8}\left(11a^2\right)\int \frac{\sqrt{c-acx}}{x^2\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{c\sqrt{1-a^2x^2}}{3x^3\sqrt{c-acx}}+\frac{11ac\sqrt{1-a^2x^2}}{12x^2\sqrt{c-acx}}-\frac{11a^2c\sqrt{1-a^2x^2}}{8x\sqrt{c-acx}}-\frac{1}{16}\left(11a^3\right)\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{c\sqrt{1-a^2x^2}}{3x^3\sqrt{c-acx}}+\frac{11ac\sqrt{1-a^2x^2}}{12x^2\sqrt{c-acx}}-\frac{11a^2c\sqrt{1-a^2x^2}}{8x\sqrt{c-acx}}-\frac{1}{8}\left(11a^5c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x}\right.\\ &= -\frac{c\sqrt{1-a^2x^2}}{3x^3\sqrt{c-acx}}+\frac{11ac\sqrt{1-a^2x^2}}{12x^2\sqrt{c-acx}}-\frac{11a^2c\sqrt{1-a^2x^2}}{8x\sqrt{c-acx}}+\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\end{aligned}$$

Mathematica [C] time = 0.0223248, size = 56, normalized size = 0.38

$$\frac{c\sqrt{1-a^2x^2}\left(11a^3x^3\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},ax+1\right)-1\right)}{3x^3\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^ArcTanh[a*x]*x^4),x]
```

```
[Out] (c*Sqrt[1 - a^2*x^2]*(-1 + 11*a^3*x^3*Hypergeometric2F1[1/2, 3, 3/2, 1 + a*
x]))/(3*x^3*Sqrt[c - a*c*x])
```

Maple [A] time = 0.105, size = 121, normalized size = 0.8

$$-\frac{1}{(24\,ax-24)\,x^3}\sqrt{-c\,(ax-1)}\sqrt{-a^2x^2+1}\left(33\,c\text{Artanh}\left(\frac{\sqrt{c\,(ax+1)}}{\sqrt{c}}\right)x^3a^3-33\,x^2a^2\sqrt{c\,(ax+1)}\sqrt{c}+22\,xa\sqrt{c\,(ax+1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x)
```

```
[Out] -1/24*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(33*c*arctanh((c*(a*x+1))^(1/2)
/c^(1/2))*x^3*a^3-33*x^2*a^2*(c*(a*x+1))^(1/2)*c^(1/2)+22*x*a*(c*(a*x+1))^(
```

```
1/2)*c^(1/2)-8*(c*(a*x+1))^(1/2)*c^(1/2))/c^(1/2)/(a*x-1)/(c*(a*x+1))^(1/2)
/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{(ax+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/((a*x + 1)*x^4), x)
```

Fricas [A] time = 1.96078, size = 543, normalized size = 3.67

$$\left[\frac{33\left(a^4x^4-a^3x^3\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)+2\left(33a^2x^2-22ax+8\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{48\left(ax^4-x^3\right)}, \frac{33\left(a^4x^4-a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="fri
cas")
```

```
[Out] [1/48*(33*(a^4*x^4 - a^3*x^3)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a^2
*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*(33*a^2*x^2 - 22
*a*x + 8)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a*x^4 - x^3), 1/24*(33*(a^4
*x^4 - a^3*x^3)*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c
)/(a^2*c*x^2 - c)) + (33*a^2*x^2 - 22*a*x + 8)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c
*x + c))/(a*x^4 - x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c(ax-1)}\sqrt{-(ax-1)(ax+1)}}{x^4(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(x**4*(a*x + 1)), x)
```

Giac [A] time = 1.31783, size = 177, normalized size = 1.2

$$-\frac{\left(a^3c^4\left(\frac{33\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2}+\frac{33(acx+c)^{\frac{5}{2}}-88(acx+c)^{\frac{3}{2}}c+63\sqrt{acx+c}c^2}{a^3c^5x^3}\right)-\frac{33a^3c^2\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)+19\sqrt{2}a^3\sqrt{-c}c^{\frac{3}{2}}}{\sqrt{-c}}\right)|c|}{24c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="gia
c")
```

```
[Out] -1/24*(a^3*c^4*(33*arctan(sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c^2) + (33*(a
*c*x + c)^(5/2) - 88*(a*c*x + c)^(3/2)*c + 63*sqrt(a*c*x + c)*c^2)/(a^3*c^5
*x^3)) - (33*a^3*c^2*arctan(sqrt(2)*sqrt(c)/sqrt(-c)) + 19*sqrt(2)*a^3*sqrt
(-c)*c^(3/2))/sqrt(-c))*abs(c)/c^2
```

3.419 $\int e^{-2\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=139

$$-\frac{2(c-acx)^{9/2}}{9a^4c^4}+\frac{2(c-acx)^{7/2}}{7a^4c^3}-\frac{2(c-acx)^{5/2}}{5a^4c^2}-\frac{2(c-acx)^{3/2}}{3a^4c}-\frac{4\sqrt{c-acx}}{a^4}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

```
[Out] (-4*Sqrt[c - a*c*x])/a^4 - (2*(c - a*c*x)^(3/2))/(3*a^4*c) - (2*(c - a*c*x)
^(5/2))/(5*a^4*c^2) + (2*(c - a*c*x)^(7/2))/(7*a^4*c^3) - (2*(c - a*c*x)^(9
/2))/(9*a^4*c^4) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt
[c])])/a^4
```

Rubi [A] time = 0.172503, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 21, 88, 50, 63, 206}

$$-\frac{2(c-acx)^{9/2}}{9a^4c^4}+\frac{2(c-acx)^{7/2}}{7a^4c^3}-\frac{2(c-acx)^{5/2}}{5a^4c^2}-\frac{2(c-acx)^{3/2}}{3a^4c}-\frac{4\sqrt{c-acx}}{a^4}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a*c*x])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a^4 - (2*(c - a*c*x)^(3/2))/(3*a^4*c) - (2*(c - a*c*x)
^(5/2))/(5*a^4*c^2) + (2*(c - a*c*x)^(7/2))/(7*a^4*c^3) - (2*(c - a*c*x)^(9
/2))/(9*a^4*c^4) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt
[c])])/a^4
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx &= \int \frac{x^3(1-ax)\sqrt{c-acx}}{1+ax}\,dx \\
&= \frac{\int \frac{x^3(c-acx)^{3/2}}{1+ax}\,dx}{c} \\
&= \frac{\int \left(\frac{(c-acx)^{3/2}}{a^3}-\frac{(c-acx)^{3/2}}{a^3(1+ax)}-\frac{(c-acx)^{5/2}}{a^3c}+\frac{(c-acx)^{7/2}}{a^3c^2}\right)dx}{c} \\
&= -\frac{2(c-acx)^{5/2}}{5a^4c^2}+\frac{2(c-acx)^{7/2}}{7a^4c^3}-\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{a^3c} \\
&= -\frac{2(c-acx)^{3/2}}{3a^4c}-\frac{2(c-acx)^{5/2}}{5a^4c^2}+\frac{2(c-acx)^{7/2}}{7a^4c^3}-\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{2\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{a^3} \\
&= -\frac{4\sqrt{c-acx}}{a^4}-\frac{2(c-acx)^{3/2}}{3a^4c}-\frac{2(c-acx)^{5/2}}{5a^4c^2}+\frac{2(c-acx)^{7/2}}{7a^4c^3}-\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{(4c)\int \frac{}{(}}{} \\
&= -\frac{4\sqrt{c-acx}}{a^4}-\frac{2(c-acx)^{3/2}}{3a^4c}-\frac{2(c-acx)^{5/2}}{5a^4c^2}+\frac{2(c-acx)^{7/2}}{7a^4c^3}-\frac{2(c-acx)^{9/2}}{9a^4c^4}+\frac{8\,\mathrm{Subst}}{} \\
&= -\frac{4\sqrt{c-acx}}{a^4}-\frac{2(c-acx)^{3/2}}{3a^4c}-\frac{2(c-acx)^{5/2}}{5a^4c^2}+\frac{2(c-acx)^{7/2}}{7a^4c^3}-\frac{2(c-acx)^{9/2}}{9a^4c^4}+\frac{4\sqrt{2}\sqrt{c}}{}
\end{aligned}$$

Mathematica [A] time = 0.121177, size = 87, normalized size = 0.63

$$\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}-\frac{2\left(35a^4x^4-95a^3x^3+138a^2x^2-236ax+788\right)\sqrt{c-acx}}{315a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a*c*x])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(788 - 236*a*x + 138*a^2*x^2 - 95*a^3*x^3 + 35*a^4*x^4)
)/(315*a^4) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
)/a^4
```

Maple [A] time = 0.036, size = 101, normalized size = 0.7

$$-2\,\frac{1}{c^4a^4}\left(1/9\,(-acx+c)^{9/2}-1/7\,c\,(-acx+c)^{7/2}+1/5\,c^2\,(-acx+c)^{5/2}+1/3\,c^3\,(-acx+c)^{3/2}+2\,\sqrt{-acx+c}c^4-2\,c^{9/2}\sqrt{2}\mathrm{Ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -2/c^4/a^4*(1/9*(-a*c*x+c)^(9/2)-1/7*c*(-a*c*x+c)^(7/2)+1/5*c^2*(-a*c*x+c)^
(5/2)+1/3*c^3*(-a*c*x+c)^(3/2)+2*(-a*c*x+c)^(1/2)*c^4-2*c^(9/2)*2^(1/2)*arc
tanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97642, size = 454, normalized size = 3.27

$$\left[\frac{2\left(315\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-\left(35a^4x^4-95a^3x^3+138a^2x^2-236ax+788\right)\sqrt{-acx+c}\right)}{315a^4},-\frac{2\left(630\sqrt{2}\sqrt{-c}\,\mathrm{ar}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] [2/315*(315*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) - (35*a^4*x^4 - 95*a^3*x^3 + 138*a^2*x^2 - 236*a*x + 788
)*sqrt(-a*c*x + c))/a^4, -2/315*(630*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sq
rt(-a*c*x + c)*sqrt(-c)/c) + (35*a^4*x^4 - 95*a^3*x^3 + 138*a^2*x^2 - 236*a
*x + 788)*sqrt(-a*c*x + c))/a^4]
```

Sympy [A] time = 13.1902, size = 126, normalized size = 0.91

$$\frac{2\left(-\frac{2\sqrt{2}c^5\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-2c^4\sqrt{-acx+c}-\frac{c^3(-acx+c)^{\frac{3}{2}}}{3}-\frac{c^2(-acx+c)^{\frac{5}{2}}}{5}+\frac{c(-acx+c)^{\frac{7}{2}}}{7}-\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] 2*(-2*sqrt(2)*c**5*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) - 2
*c**4*sqrt(-a*c*x + c) - c**3*(-a*c*x + c)**(3/2)/3 - c**2*(-a*c*x + c)**(5
/2)/5 + c*(-a*c*x + c)**(7/2)/7 - (-a*c*x + c)**(9/2)/9)/(a**4*c**4)
```

Giac [A] time = 1.29246, size = 215, normalized size = 1.55

$$-\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^4\sqrt{-c}}-\frac{2\left(35\,(acx-c)^4\sqrt{-acx+c}a^{32}c^{32}+45\,(acx-c)^3\sqrt{-acx+c}a^{32}c^{33}+63\,(acx-c)^2\sqrt{-acx}\right.}{315\,a^{36}c^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^4*sqrt(-c)) -
 2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c)*a^32*c^32 + 45*(a*c*x - c)^3*sqrt
(-a*c*x + c)*a^32*c^33 + 63*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^32*c^34 + 105*
(-a*c*x + c)^(3/2)*a^32*c^35 + 630*sqrt(-a*c*x + c)*a^32*c^36)/(a^36*c^36)
```

3.420 $\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=97

$$\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{2(c-acx)^{3/2}}{3a^3c}+\frac{4\sqrt{c-acx}}{a^3}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

```
[Out] (4*Sqrt[c - a*c*x])/a^3 + (2*(c - a*c*x)^(3/2))/(3*a^3*c) + (2*(c - a*c*x)^
(7/2))/(7*a^3*c^3) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/a^3
```

Rubi [A] time = 0.157777, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 21, 88, 50, 63, 206}

$$\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{2(c-acx)^{3/2}}{3a^3c}+\frac{4\sqrt{c-acx}}{a^3}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^3 + (2*(c - a*c*x)^(3/2))/(3*a^3*c) + (2*(c - a*c*x)^
(7/2))/(7*a^3*c^3) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/a^3
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx &= \int \frac{x^2(1-ax)\sqrt{c-acx}}{1+ax}\,dx\\
&= \frac{\int \frac{x^2(c-acx)^{3/2}}{1+ax}\,dx}{c}\\
&= \frac{\int \left(\frac{(c-acx)^{3/2}}{a^2(1+ax)}-\frac{(c-acx)^{5/2}}{a^2c}\right)dx}{c}\\
&= \frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{a^2c}\\
&= \frac{2(c-acx)^{3/2}}{3a^3c}+\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{2\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{a^2}\\
&= \frac{4\sqrt{c-acx}}{a^3}+\frac{2(c-acx)^{3/2}}{3a^3c}+\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{a^2}\\
&= \frac{4\sqrt{c-acx}}{a^3}+\frac{2(c-acx)^{3/2}}{3a^3c}+\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{8\,\mathrm{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{a^3}\\
&= \frac{4\sqrt{c-acx}}{a^3}+\frac{2(c-acx)^{3/2}}{3a^3c}+\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.0838122, size = 78, normalized size = 0.8

$$\frac{2\left(-3a^3x^3+9a^2x^2-16ax+52\right)\sqrt{c-acx}-84\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{21a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(52 - 16*a*x + 9*a^2*x^2 - 3*a^3*x^3) - 84*Sqrt[2]*Sqrt[
c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(21*a^3)
```

Maple [A] time = 0.042, size = 75, normalized size = 0.8

$$2\,\frac{1}{c^3a^3}\left(1/7\,(-acx+c)^{7/2}+1/3\,(-acx+c)^{3/2}c^2+2\,\sqrt{-acx+c}c^3-2\,c^{7/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 2/c^3/a^3*(1/7*(-a*c*x+c)^(7/2)+1/3*(-a*c*x+c)^(3/2)*c^2+2*(-a*c*x+c)^(1/2)
*c^3-2*c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90403, size = 398, normalized size = 4.1

$$\left[\frac{2\left(21\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-\left(3a^3x^3-9a^2x^2+16ax-52\right)\sqrt{-acx+c}\right)}{21a^3}, \frac{2\left(42\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] [2/21*(21*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) - (3*a^3*x^3 - 9*a^2*x^2 + 16*a*x - 52)*sqrt(-a*c*x + c))/
a^3, 2/21*(42*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)
/c) - (3*a^3*x^3 - 9*a^2*x^2 + 16*a*x - 52)*sqrt(-a*c*x + c))/a^3]
```

Sympy [A] time = 10.483, size = 97, normalized size = 1.

$$-\frac{2\left(-\frac{2\sqrt{2}c^4\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-2c^3\sqrt{-acx+c}-\frac{c^2(-acx+c)^{\frac{3}{2}}}{3}-\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -2*(-2*sqrt(2)*c**4*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) -
2*c**3*sqrt(-a*c*x + c) - c**2*(-a*c*x + c)**(3/2)/3 - (-a*c*x + c)**(7/2)/
7)/(a**3*c**3)
```

Giac [A] time = 1.19883, size = 142, normalized size = 1.46

$$\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^3\sqrt{-c}}-\frac{2\left(3\,(acx-c)^3\sqrt{-acx+c}a^{18}c^{18}-7\,(-acx+c)^{\frac{3}{2}}a^{18}c^{20}-42\,\sqrt{-acx+c}a^{18}c^{21}\right)}{21\,a^{21}c^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^3*sqrt(-c)) -
2/21*(3*(a*c*x - c)^3*sqrt(-a*c*x + c)*a^18*c^18 - 7*(-a*c*x + c)^(3/2)*a^1
8*c^20 - 42*sqrt(-a*c*x + c)*a^18*c^21)/(a^21*c^21)
```

3.421 $\int e^{-2\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=97

$$-\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{2(c-acx)^{3/2}}{3a^2c}-\frac{4\sqrt{c-acx}}{a^2}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

```
[Out] (-4*Sqrt[c - a*c*x])/a^2 - (2*(c - a*c*x)^(3/2))/(3*a^2*c) - (2*(c - a*c*x)
^(5/2))/(5*a^2*c^2) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*S
qrt[c])])/a^2
```

Rubi [A] time = 0.106804, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6130, 21, 80, 50, 63, 206}

$$-\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{2(c-acx)^{3/2}}{3a^2c}-\frac{4\sqrt{c-acx}}{a^2}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a*c*x])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a^2 - (2*(c - a*c*x)^(3/2))/(3*a^2*c) - (2*(c - a*c*x)
^(5/2))/(5*a^2*c^2) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*S
qrt[c])])/a^2
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx &= \int \frac{x(1-ax)\sqrt{c-acx}}{1+ax}\,dx\\
&= \frac{\int \frac{x(c-acx)^{3/2}}{1+ax}\,dx}{c}\\
&= -\frac{2(c-acx)^{5/2}}{5a^2c^2} - \frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{ac}\\
&= -\frac{2(c-acx)^{3/2}}{3a^2c} - \frac{2(c-acx)^{5/2}}{5a^2c^2} - \frac{2\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{a}\\
&= -\frac{4\sqrt{c-acx}}{a^2} - \frac{2(c-acx)^{3/2}}{3a^2c} - \frac{2(c-acx)^{5/2}}{5a^2c^2} - \frac{(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{a}\\
&= -\frac{4\sqrt{c-acx}}{a^2} - \frac{2(c-acx)^{3/2}}{3a^2c} - \frac{2(c-acx)^{5/2}}{5a^2c^2} + \frac{8\,\mathrm{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{a^2}\\
&= -\frac{4\sqrt{c-acx}}{a^2} - \frac{2(c-acx)^{3/2}}{3a^2c} - \frac{2(c-acx)^{5/2}}{5a^2c^2} + \frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.0755803, size = 70, normalized size = 0.72

$$\frac{60\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right) - 2\left(3a^2x^2 - 11ax + 38\right)\sqrt{c-acx}}{15a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(38 - 11*a*x + 3*a^2*x^2) + 60*Sqrt[2]*Sqrt[c]*ArcTanh[
Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(15*a^2)
```

Maple [A] time = 0.035, size = 73, normalized size = 0.8

$$-2\,\frac{1}{a^2c^2}\left(1/5\,(-acx+c)^{5/2} + 1/3\,c\,(-acx+c)^{3/2} + 2\,\sqrt{-acx+c}c^2 - 2\,c^{5/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -2/c^2/a^2*(1/5*(-a*c*x+c)^(5/2)+1/3*c*(-a*c*x+c)^(3/2)+2*(-a*c*x+c)^(1/2)*
c^2-2*c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72137, size = 367, normalized size = 3.78

$$\left[\frac{2\left(15\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-\left(3a^2x^2-11ax+38\right)\sqrt{-acx+c}\right)}{15a^2}, -\frac{2\left(30\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)+\left(3a^2\right.\right.}{15a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [2/15*(15*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) - (3*a^2*x^2 - 11*a*x + 38)*sqrt(-a*c*x + c))/a^2, -2/15*(
30*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) + (3*a^
2*x^2 - 11*a*x + 38)*sqrt(-a*c*x + c))/a^2]
```

Sympy [A] time = 8.68225, size = 94, normalized size = 0.97

$$\frac{2\left(-\frac{2\sqrt{2}c^3\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-2c^2\sqrt{-acx+c}-\frac{c(-acx+c)^{\frac{3}{2}}}{3}-\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] 2*(-2*sqrt(2)*c**3*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) - 2
*c**2*sqrt(-a*c*x + c) - c*(-a*c*x + c)**(3/2)/3 - (-a*c*x + c)**(5/2)/5)/(
a**2*c**2)
```

Giac [A] time = 1.29087, size = 142, normalized size = 1.46

$$-\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}}-\frac{2\left(3(acx-c)^2\sqrt{-acx+c}a^8c^8+5(-acx+c)^{\frac{3}{2}}a^8c^9+30\sqrt{-acx+c}a^8c^{10}\right)}{15a^{10}c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^2*sqrt(-c)) -
 2/15*(3*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^8*c^8 + 5*(-a*c*x + c)^(3/2)*a^8*
c^9 + 30*sqrt(-a*c*x + c)*a^8*c^10)/(a^10*c^10)
```

3.422 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=76

$$\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{c-acx}}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] (4*Sqrt[c - a*c*x])/a + (2*(c - a*c*x)^(3/2))/(3*a*c) - (4*Sqrt[2]*Sqrt[c]*
ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.0662254, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6130, 21, 50, 63, 206}

$$\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{c-acx}}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a + (2*(c - a*c*x)^(3/2))/(3*a*c) - (4*Sqrt[2]*Sqrt[c]*
ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx &= \int \frac{(1-ax)\sqrt{c-acx}}{1+ax}\,dx \\
&= \frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{c} \\
&= \frac{2(c-acx)^{3/2}}{3ac} + 2\int \frac{\sqrt{c-acx}}{1+ax}\,dx \\
&= \frac{4\sqrt{c-acx}}{a} + \frac{2(c-acx)^{3/2}}{3ac} + (4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx \\
&= \frac{4\sqrt{c-acx}}{a} + \frac{2(c-acx)^{3/2}}{3ac} - \frac{8\,\mathrm{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{a} \\
&= \frac{4\sqrt{c-acx}}{a} + \frac{2(c-acx)^{3/2}}{3ac} - \frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0385392, size = 61, normalized size = 0.8

$$
-\frac{2(ax-7)\sqrt{c-acx}+12\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{3a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(2*(-7 + a*x)*Sqrt[c - a*c*x] + 12*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]
/(Sqrt[2]*Sqrt[c])])/(3*a)
```

Maple [A] time = 0.034, size = 59, normalized size = 0.8

$$
2\frac{1}{ac}\left(1/3\,(-acx+c)^{3/2}+2\,c\sqrt{-acx+c}-2\,c^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 2/c/a*(1/3*(-a*c*x+c)^(3/2)+2*c*(-a*c*x+c)^(1/2)-2*c^(3/2)*2^(1/2)*arctanh(
1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60835, size = 312, normalized size = 4.11

$$\left[\frac{2\left(3\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-\sqrt{-acx+c}(ax-7)\right)}{3a},\ \frac{2\left(6\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-\sqrt{-acx+c}(ax-7)\right)}{3a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3
*c)/(a*x + 1)) - sqrt(-a*c*x + c)*(a*x - 7))/a, 2/3*(6*sqrt(2)*sqrt(-c)*arc
tan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(a*x - 7))/
a]
```

Sympy [A] time = 6.07183, size = 75, normalized size = 0.99

$$-\frac{2\left(-\frac{2\sqrt{2}c^{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-2c\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -2*(-2*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) -
2*c*sqrt(-a*c*x + c) - (-a*c*x + c)**(3/2)/3)/(a*c)
```

Giac [A] time = 1.2975, size = 104, normalized size = 1.37

$$\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{2\left((-acx+c)^{\frac{3}{2}}a^{2}c^{2}+6\sqrt{-acx+c}a^{2}c^{3}\right)}{3a^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) + 2/
3*((-a*c*x + c)^(3/2)*a^2*c^2 + 6*sqrt(-a*c*x + c)*a^2*c^3)/(a^3*c^3)
```

3.423 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=74

$$-2\sqrt{c-acx}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] -2*Sqrt[c - a*c*x] - 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] + 4*Sqrt[2]
*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.137556, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6130, 21, 84, 156, 63, 208, 206}

$$-2\sqrt{c-acx}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] -2*Sqrt[c - a*c*x] - 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] + 4*Sqrt[2]
*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx &= \int \frac{(1-ax)\sqrt{c-acx}}{x(1+ax)}\,dx\\
&= \frac{\int \frac{(c-acx)^{3/2}}{x(1+ax)}\,dx}{c}\\
&= -2\sqrt{c-acx} + \frac{\int \frac{ac^2-3a^2c^2x}{x(1+ax)\sqrt{c-acx}}\,dx}{ac}\\
&= -2\sqrt{c-acx} + c\int \frac{1}{x\sqrt{c-acx}}\,dx - (4ac)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx\\
&= -2\sqrt{c-acx} + 8\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right) - \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right)}{a}\\
&= -2\sqrt{c-acx} - 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0260978, size = 74, normalized size = 1.

$$-2\sqrt{c-acx} - 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] -2*Sqrt[c - a*c*x] - 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] + 4*Sqrt[2]
*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Maple [A] time = 0.041, size = 58, normalized size = 0.8

$$-2\,\text{Artanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c} + 4\,\text{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\sqrt{2}\sqrt{c} - 2\,\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x)
```

```
[Out] -2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*arctanh(1/2*(-a*c*x+c)^(1/2)
*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)-2*(-a*c*x+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59106, size = 417, normalized size = 5.64

$$\left[2\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{c}\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)-2\sqrt{-acx+c}, -4\sqrt{2}\sqrt{-c}\,\mathrm{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(
a*x + 1)) + sqrt(c)*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) - 2*s
qrt(-a*c*x + c), -4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sq
rt(-c)/c) + 2*sqrt(-c)*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*sqrt(-a*c*x
+ c)]
```

Sympy [A] time = 7.31216, size = 80, normalized size = 1.08

$$\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{4\sqrt{2}c\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x,x)
```

```
[Out] 2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 4*sqrt(2)*c*atan(sqrt(2)*sqr
t(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) - 2*sqrt(-a*c*x + c)
```

Giac [A] time = 1.23271, size = 90, normalized size = 1.22

$$-\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*c*a
rctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 2*sqrt(-a*c*x + c)
```

3.424 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{c-acx}}{x}+5a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] -(Sqrt[c - a*c*x]/x) + 5*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] - 4*Sqr
t[2]*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.139717, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6130, 21, 98, 156, 63, 208, 206}

$$-\frac{\sqrt{c-acx}}{x}+5a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] -(Sqrt[c - a*c*x]/x) + 5*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] - 4*Sqr
t[2]*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx &= \int \frac{(1-ax)\sqrt{c-acx}}{x^2(1+ax)}\,dx \\
&= \frac{\int \frac{(c-acx)^{3/2}}{x^2(1+ax)}\,dx}{c} \\
&= -\frac{\sqrt{c-acx}}{x} - \frac{\int \frac{\frac{5ac^2}{2}-\frac{3}{2}a^2c^2x}{x(1+ax)\sqrt{c-acx}}\,dx}{c} \\
&= -\frac{\sqrt{c-acx}}{x} - \frac{1}{2}(5ac)\int \frac{1}{x\sqrt{c-acx}}\,dx + \left(4a^2c\right)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx \\
&= -\frac{\sqrt{c-acx}}{x} + 5\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right) - (8a)\,\mathrm{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right) \\
&= -\frac{\sqrt{c-acx}}{x} + 5a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0415392, size = 79, normalized size = 1.

$$
-\frac{\sqrt{c-acx}}{x} + 5a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] -(Sqrt[c - a*c*x]/x) + 5*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] - 4*Sqr
t[2]*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Maple [A] time = 0.043, size = 71, normalized size = 0.9

$$
2\,ac\left(-2\,\frac{\sqrt{2}}{\sqrt{c}}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right) - 1/2\,\frac{\sqrt{-acx+c}}{acx} + 5/2\,\frac{1}{\sqrt{c}}\mathrm{Artanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x)
```

```
[Out] 2*a*c*(-2*2^(1/2)/c^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))-1/2
*(-a*c*x+c)^(1/2)/a/c/x+5/2/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66565, size = 454, normalized size = 5.75

$$\left[\frac{4\sqrt{2}a\sqrt{c}x\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+5a\sqrt{c}x\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)-2\sqrt{-acx+c}}{2x}, \frac{4\sqrt{2}a\sqrt{-c}x\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="fricas"
)
```

```
[Out] [1/2*(4*sqrt(2)*a*sqrt(c)*x*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) + 5*a*sqrt(c)*x*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c)
- 2*c)/x) - 2*sqrt(-a*c*x + c))/x, (4*sqrt(2)*a*sqrt(-c)*x*arctan(1/2*sqrt(
2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 5*a*sqrt(-c)*x*arctan(sqrt(-a*c*x + c)*sq
rt(-c)/c) - sqrt(-a*c*x + c))/x]
```

Sympy [B] time = 11.3228, size = 162, normalized size = 2.05

$$\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(-c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}-\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}-\frac{6ac\operatorname{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{4\sqrt{2}ac\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-ac}}{2\sqrt{-c}}\right.}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**2,x)
```

```
[Out] a*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 - a*c**2
*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 - 6*a*c*atan(sq
rt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 4*sqrt(2)*a*c*atan(sqrt(2)*sqrt(-a*c*x
+ c)/(2*sqrt(-c)))/sqrt(-c) - sqrt(-a*c*x + c)/x
```

Giac [A] time = 1.20888, size = 97, normalized size = 1.23

$$\frac{4\sqrt{2}ac\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{5\,ac\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{-acx+c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 5*a*
c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)/x
```

3.425 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=106

$$-\frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)-\frac{\sqrt{c-acx}}{2x^2}+\frac{9a\sqrt{c-acx}}{4x}$$

```
[Out] -Sqrt[c - a*c*x]/(2*x^2) + (9*a*Sqrt[c - a*c*x])/(4*x) - (23*a^2*Sqrt[c]*Ar
cTanh[Sqrt[c - a*c*x]/Sqrt[c]])/4 + 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c -
a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.158577, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6130, 21, 98, 151, 156, 63, 208, 206}

$$-\frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)-\frac{\sqrt{c-acx}}{2x^2}+\frac{9a\sqrt{c-acx}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] -Sqrt[c - a*c*x]/(2*x^2) + (9*a*Sqrt[c - a*c*x])/(4*x) - (23*a^2*Sqrt[c]*Ar
cTanh[Sqrt[c - a*c*x]/Sqrt[c]])/4 + 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c -
a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{(1-ax)\sqrt{c-acx}}{x^3(1+ax)}\,dx$$

$$= \frac{\int \frac{(c-acx)^{3/2}}{x^3(1+ax)}\,dx}{c}$$

$$= -\frac{\sqrt{c-acx}}{2x^2} - \frac{\int \frac{\frac{9ac^2}{2}-\frac{7}{2}a^2c^2x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{2c}$$

$$= -\frac{\sqrt{c-acx}}{2x^2} + \frac{9a\sqrt{c-acx}}{4x} + \frac{\int \frac{\frac{23a^2c^3}{4}-\frac{9}{4}a^3c^3x}{x(1+ax)\sqrt{c-acx}}\,dx}{2c^2}$$

$$= -\frac{\sqrt{c-acx}}{2x^2} + \frac{9a\sqrt{c-acx}}{4x} + \frac{1}{8}\left(23a^2c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx - \left(4a^3c\right)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx$$

$$= -\frac{\sqrt{c-acx}}{2x^2} + \frac{9a\sqrt{c-acx}}{4x} - \frac{1}{4}(23a)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right) + \left(8a^2\right)\text{Subst}\left(\int \right.$$

$$= -\frac{\sqrt{c-acx}}{2x^2} + \frac{9a\sqrt{c-acx}}{4x} - \frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] time = 0.0702709, size = 93, normalized size = 0.88

$$-\frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{(9ax-2)\sqrt{c-acx}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] ((-2 + 9*a*x)*Sqrt[c - a*c*x])/(4*x^2) - (23*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a
*c*x]/Sqrt[c]])/4 + 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*
Sqrt[c])]
```

Maple [A] time = 0.043, size = 95, normalized size = 0.9

$$-2a^2c^2\left(-2\frac{\sqrt{2}}{c^{3/2}}\mathrm{Artanh}\left(1/2\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)-\frac{1}{c}\left(\frac{1}{a^2c^2x^2}\left(-\frac{9(-acx+c)^{3/2}}{8}+\frac{7c\sqrt{-acx+c}}{8}\right)-\frac{23}{8\sqrt{c}}\mathrm{Artanh}\left(\frac{\sqrt{-acx}}{\sqrt{c}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x)
```

```
[Out] -2*a^2*c^2*(-2/c^(3/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)
)-1/c*((-9/8*(-a*c*x+c)^(3/2)+7/8*c*(-a*c*x+c)^(1/2))/a^2/c^2/x^2-23/8/c^(1
/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68449, size = 525, normalized size = 4.95

$$\left[\frac{16\sqrt{2}a^2\sqrt{c}x^2\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+23a^2\sqrt{c}x^2\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\sqrt{-acx+c}(9ax-2)}{8x^2},-\frac{16\sqrt{2}a^2\sqrt{-c}}{\quad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="fricas"
)
```

```
[Out] [1/8*(16*sqrt(2)*a^2*sqrt(c)*x^2*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sq
rt(c) - 3*c)/(a*x + 1)) + 23*a^2*sqrt(c)*x^2*log((a*c*x + 2*sqrt(-a*c*x + c
)*sqrt(c) - 2*c)/x) + 2*sqrt(-a*c*x + c)*(9*a*x - 2))/x^2, -1/4*(16*sqrt(2)
```

```
*a^2*sqrt(-c)*x^2*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 23*a^2*
sqrt(-c)*x^2*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(9*a*x
- 2))/x^2]
```

Sympy [B] time = 28.2064, size = 352, normalized size = 3.32

$$-\frac{10a^2c^4\sqrt{-acx+c}}{16ac^4x-8c^4+8c^2\left(-acx+c\right)^2}+\frac{6a^2c^3\left(-acx+c\right)^{\frac{3}{2}}}{16ac^4x-8c^4+8c^2\left(-acx+c\right)^2}+\frac{3a^2c^3\sqrt{\frac{1}{c^5}}\log\left(-c^3\sqrt{\frac{1}{c^5}}+\sqrt{-acx+c}\right)}{8}-\frac{3a^2c^3\sqrt{\frac{1}{c^5}}\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**3,x)
```

```
[Out] -10*a**2*c**4*sqrt(-a*c*x + c)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)*
*2) + 6*a**2*c**3*(-a*c*x + c)**(3/2)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*
x + c)**2) + 3*a**2*c**3*sqrt(c**(-5))*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*
x + c))/8 - 3*a**2*c**3*sqrt(c**(-5))*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x
+ c))/8 - 3*a**2*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x +
 c))/2 + 3*a**2*c**2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c
))/2 + 8*a**2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 4*sqrt(2)*a**2*c
*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 3*a*sqrt(-a*c*x + c
)/x
```

Giac [A] time = 1.20463, size = 143, normalized size = 1.35

$$-\frac{4\sqrt{2}a^2c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{23\,a^2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}}-\frac{9\left(-acx+c\right)^{\frac{3}{2}}a^2c-7\sqrt{-acx+c}a^2c^2}{4\,a^2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a^2*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2
3/4*a^2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 1/4*(9*(-a*c*x + c)^
(3/2)*a^2*c - 7*sqrt(-a*c*x + c)*a^2*c^2)/(a^2*c^2*x^2)
```

3.426 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=127

$$-\frac{19a^2\sqrt{c-acx}}{8x}+\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{13a\sqrt{c-acx}}{12x^2}-\frac{\sqrt{c-acx}}{3x^3}$$

```
[Out] -Sqrt[c - a*c*x]/(3*x^3) + (13*a*Sqrt[c - a*c*x])/(12*x^2) - (19*a^2*Sqrt[c
 - a*c*x])/(8*x) + (45*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/8 - 4*
Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.181513, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6130, 21, 98, 151, 156, 63, 208, 206}

$$-\frac{19a^2\sqrt{c-acx}}{8x}+\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{13a\sqrt{c-acx}}{12x^2}-\frac{\sqrt{c-acx}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] -Sqrt[c - a*c*x]/(3*x^3) + (13*a*Sqrt[c - a*c*x])/(12*x^2) - (19*a^2*Sqrt[c
 - a*c*x])/(8*x) + (45*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/8 - 4*
Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{(1-ax)\sqrt{c-acx}}{x^4(1+ax)}\,dx$$

$$= \frac{\int \frac{(c-acx)^{3/2}}{x^4(1+ax)}\,dx}{c}$$

$$= -\frac{\sqrt{c-acx}}{3x^3} - \frac{\int \frac{\frac{13ac^2}{2}-\frac{11}{2}a^2c^2x}{x^3(1+ax)\sqrt{c-acx}}\,dx}{3c}$$

$$= -\frac{\sqrt{c-acx}}{3x^3} + \frac{13a\sqrt{c-acx}}{12x^2} + \frac{\int \frac{\frac{57a^2c^3}{4}-\frac{39}{4}a^3c^3x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{6c^2}$$

$$= -\frac{\sqrt{c-acx}}{3x^3} + \frac{13a\sqrt{c-acx}}{12x^2} - \frac{19a^2\sqrt{c-acx}}{8x} - \frac{\int \frac{\frac{135a^3c^4}{8}-\frac{57}{8}a^4c^4x}{x(1+ax)\sqrt{c-acx}}\,dx}{6c^3}$$

$$= -\frac{\sqrt{c-acx}}{3x^3} + \frac{13a\sqrt{c-acx}}{12x^2} - \frac{19a^2\sqrt{c-acx}}{8x} - \frac{1}{16}\left(45a^3c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx + \left(4a^4c\right)\int \frac{}{(1+}$$

$$= -\frac{\sqrt{c-acx}}{3x^3} + \frac{13a\sqrt{c-acx}}{12x^2} - \frac{19a^2\sqrt{c-acx}}{8x} + \frac{1}{8}\left(45a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right)$$

$$= -\frac{\sqrt{c-acx}}{3x^3} + \frac{13a\sqrt{c-acx}}{12x^2} - \frac{19a^2\sqrt{c-acx}}{8x} + \frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^3\sqrt{c}\tan$$

Mathematica [A] time = 0.0924259, size = 101, normalized size = 0.8

$$\frac{\left(-57a^2x^2+26ax-8\right)\sqrt{c-acx}}{24x^3}+\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - a*c*x]*(-8 + 26*a*x - 57*a^2*x^2))/(24*x^3) + (45*a^3*Sqrt[c]*Arc
Tanh[Sqrt[c - a*c*x]/Sqrt[c]])/8 - 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a
*c*x]/(Sqrt[2]*Sqrt[c])]
```

Maple [A] time = 0.045, size = 110, normalized size = 0.9

$$2\,a^3c^3\left(-2\,\frac{\sqrt{2}}{c^{5/2}}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)-\frac{1}{c^2}\left(-\frac{1}{a^3c^3x^3}\left(-\frac{19\,(-acx+c)^{5/2}}{16}+\frac{11\,c\,(-acx+c)^{3/2}}{6}-\frac{13\,\sqrt{-acx+c}c^2}{16}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x)
```

```
[Out] 2*a^3*c^3*(-2/c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))
-1/c^2*(-(-19/16*(-a*c*x+c)^(5/2)+11/6*c*(-a*c*x+c)^(3/2)-13/16*(-a*c*x+c)^
(1/2)*c^2)/a^3/c^3/x^3-45/16/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70569, size = 567, normalized size = 4.46

$$\left[\frac{96\,\sqrt{2}a^3\sqrt{c}x^3\log\left(\frac{acx+2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+135\,a^3\sqrt{c}x^3\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)-2\left(57\,a^2x^2-26\,ax+8\right)\sqrt{-acx+c}\quad 9}{48\,x^3},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="fricas"
)
```

```
[Out] [1/48*(96*sqrt(2)*a^3*sqrt(c)*x^3*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*s
qrt(c) - 3*c)/(a*x + 1)) + 135*a^3*sqrt(c)*x^3*log((a*c*x - 2*sqrt(-a*c*x +
```

```
c)*sqrt(c) - 2*c)/x) - 2*(57*a^2*x^2 - 26*a*x + 8)*sqrt(-a*c*x + c))/x^3,
1/24*(96*sqrt(2)*a^3*sqrt(-c)*x^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(
-c)/c) - 135*a^3*sqrt(-c)*x^3*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (57*a^2
*x^2 - 26*a*x + 8)*sqrt(-a*c*x + c))/x^3]
```

Sympy [B] time = 131.152, size = 614, normalized size = 4.83

$$\frac{66a^3c^6\sqrt{-acx+c}}{-144ac^6x+96c^6-144c^4(-acx+c)^2+48c^3(-acx+c)^3}-\frac{80a^3c^5(-acx+c)^{\frac{3}{2}}}{-144ac^6x+96c^6-144c^4(-acx+c)^2+48c^3(-acx+c)^3}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**4,x)
```

```
[Out] 66*a**3*c**6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x +
 c)**2 + 48*c**3*(-a*c*x + c)**3) - 80*a**3*c**5*(-a*c*x + c)**(3/2)/(-144*
a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) +
30*a**3*c**4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*
x + c)**2 + 48*c**3*(-a*c*x + c)**3) + 30*a**3*c**4*sqrt(-a*c*x + c)/(16*a*
c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) + 5*a**3*c**4*sqrt(c**(-7))*log(-
c**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 - 5*a**3*c**4*sqrt(c**(-7))*log(c
**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 - 18*a**3*c**3*(-a*c*x + c)**(3/2)
/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) - 9*a**3*c**3*sqrt(c**(-5)
)*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8 + 9*a**3*c**3*sqrt(c**(-5))
*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8 + 2*a**3*c**2*sqrt(c**(-3))*l
og(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c)) - 2*a**3*c**2*sqrt(c**(-3))*log(
c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c)) - 8*a**3*c*atan(sqrt(-a*c*x + c)/sqr
t(-c))/sqrt(-c) + 4*sqrt(2)*a**3*c*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c
)))/sqrt(-c) - 4*a**2*sqrt(-a*c*x + c)/x
```

Giac [A] time = 1.16098, size = 180, normalized size = 1.42

$$\frac{4\sqrt{2}a^3c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{45a^3c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{8\sqrt{-c}}-\frac{57(acx-c)^2\sqrt{-acx+c}a^3c-88(-acx+c)^{\frac{3}{2}}a^3c^2+39\sqrt{-acx+\ldots}}{24a^3c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^3*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 45
/8*a^3*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 1/24*(57*(a*c*x - c)^
2*sqrt(-a*c*x + c)*a^3*c - 88*(-a*c*x + c)^(3/2)*a^3*c^2 + 39*sqrt(-a*c*x +
 c)*a^3*c^3)/(a^3*c^3*x^3)
```

3.427 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

<u>**Optimal**</u>. Leaf size=148

$$-\frac{107a^2\sqrt{c-acx}}{96x^2}+\frac{149a^3\sqrt{c-acx}}{64x}-\frac{363}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{17a\sqrt{c-acx}}{24x^3}-$$

```
[Out] -Sqrt[c - a*c*x]/(4*x^4) + (17*a*Sqrt[c - a*c*x])/(24*x^3) - (107*a^2*Sqrt[
c - a*c*x])/(96*x^2) + (149*a^3*Sqrt[c - a*c*x])/(64*x) - (363*a^4*Sqrt[c]*
ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64 + 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c
 - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

<u>**Rubi**</u> **[A]** time = 0.208883, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6130, 21, 98, 151, 156, 63, 208, 206}

$$-\frac{107a^2\sqrt{c-acx}}{96x^2}+\frac{149a^3\sqrt{c-acx}}{64x}-\frac{363}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{17a\sqrt{c-acx}}{24x^3}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x^5),x]
```

```
[Out] -Sqrt[c - a*c*x]/(4*x^4) + (17*a*Sqrt[c - a*c*x])/(24*x^3) - (107*a^2*Sqrt[
c - a*c*x])/(96*x^2) + (149*a^3*Sqrt[c - a*c*x])/(64*x) - (363*a^4*Sqrt[c]*
ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64 + 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c
 - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

<u>Rule 6130</u>

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

<u>Rule 21</u>

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

<u>Rule 98</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

<u>Rule 151</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{(1-ax)\sqrt{c-acx}}{x^5(1+ax)}\,dx$$
$$= \frac{\int \frac{(c-acx)^{3/2}}{x^5(1+ax)}\,dx}{c}$$
$$= -\frac{\sqrt{c-acx}}{4x^4} - \frac{\int \frac{\frac{17ac^2}{2}-\frac{15}{2}a^2c^2x}{x^4(1+ax)\sqrt{c-acx}}\,dx}{4c}$$
$$= -\frac{\sqrt{c-acx}}{4x^4} + \frac{17a\sqrt{c-acx}}{24x^3} + \frac{\int \frac{\frac{107a^2c^3}{4}-\frac{85}{4}a^3c^3x}{x^3(1+ax)\sqrt{c-acx}}\,dx}{12c^2}$$
$$= -\frac{\sqrt{c-acx}}{4x^4} + \frac{17a\sqrt{c-acx}}{24x^3} - \frac{107a^2\sqrt{c-acx}}{96x^2} - \frac{\int \frac{\frac{447a^3c^4}{8}-\frac{321}{8}a^4c^4x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{24c^3}$$
$$= -\frac{\sqrt{c-acx}}{4x^4} + \frac{17a\sqrt{c-acx}}{24x^3} - \frac{107a^2\sqrt{c-acx}}{96x^2} + \frac{149a^3\sqrt{c-acx}}{64x} + \frac{\int \frac{\frac{1089a^4c^5}{16}-\frac{447}{16}a^5c^5x}{x(1+ax)\sqrt{c-acx}}\,dx}{24c^4}$$
$$= -\frac{\sqrt{c-acx}}{4x^4} + \frac{17a\sqrt{c-acx}}{24x^3} - \frac{107a^2\sqrt{c-acx}}{96x^2} + \frac{149a^3\sqrt{c-acx}}{64x} + \frac{1}{128}\left(363a^4c\right)\int \frac{}{x\sqrt{c}}$$
$$= -\frac{\sqrt{c-acx}}{4x^4} + \frac{17a\sqrt{c-acx}}{24x^3} - \frac{107a^2\sqrt{c-acx}}{96x^2} + \frac{149a^3\sqrt{c-acx}}{64x} - \frac{1}{64}\left(363a^3\right)\text{Subst}\left(\int\right.$$
$$= -\frac{\sqrt{c-acx}}{4x^4} + \frac{17a\sqrt{c-acx}}{24x^3} - \frac{107a^2\sqrt{c-acx}}{96x^2} + \frac{149a^3\sqrt{c-acx}}{64x} - \frac{363}{64}a^4\sqrt{c}\tanh^{-1}\left(\sqrt{c}\right.$$

Mathematica [A] time = 0.104788, size = 109, normalized size = 0.74

$$\frac{\left(447a^3x^3-214a^2x^2+136ax-48\right)\sqrt{c-acx}}{192x^4} - \frac{363}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a*c*x]*(-48 + 136*a*x - 214*a^2*x^2 + 447*a^3*x^3))/(192*x^4) - (
363*a^4*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64 + 4*Sqrt[2]*a^4*Sqrt[c
]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Maple [A] time = 0.047, size = 123, normalized size = 0.8

$$-2\,a^4c^4\left(-2\,\frac{\sqrt{2}}{c^{7/2}}\text{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right) - \frac{1}{c^3}\left(\frac{1}{a^4c^4x^4}\left(-\frac{149\,(-acx+c)^{7/2}}{128} + \frac{1127\,c\,(-acx+c)^{5/2}}{384} - \frac{1049\,(-acx+}{384}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x)
```

```
[Out] -2*a^4*c^4*(-2/c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)
)-1/c^3*((-149/128*(-a*c*x+c)^(7/2)+1127/384*c*(-a*c*x+c)^(5/2)-1049/384*(-
a*c*x+c)^(3/2)*c^2+107/128*(-a*c*x+c)^(1/2)*c^3)/a^4/c^4/x^4-363/128/c^(1/2
)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6854, size = 622, normalized size = 4.2

$$\left[\frac{768\sqrt{2}a^4\sqrt{c}x^4\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+1089a^4\sqrt{c}x^4\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\left(447a^3x^3-214a^2x^2+136ax-48\right.}{384x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="fricas"
)
```

```
[Out] [1/384*(768*sqrt(2)*a^4*sqrt(c)*x^4*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)
*sqrt(c) - 3*c)/(a*x + 1)) + 1089*a^4*sqrt(c)*x^4*log((a*c*x + 2*sqrt(-a*c*
x + c)*sqrt(c) - 2*c)/x) + 2*(447*a^3*x^3 - 214*a^2*x^2 + 136*a*x - 48)*sqr
t(-a*c*x + c))/x^4, -1/192*(768*sqrt(2)*a^4*sqrt(-c)*x^4*arctan(1/2*sqrt(2)
*sqrt(-a*c*x + c)*sqrt(-c)/c) - 1089*a^4*sqrt(-c)*x^4*arctan(sqrt(-a*c*x +
c)*sqrt(-c)/c) - (447*a^3*x^3 - 214*a^2*x^2 + 136*a*x - 48)*sqrt(-a*c*x + c
))/x^4]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16699, size = 216, normalized size = 1.46

$$-\frac{4\sqrt{2}a^4c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{363a^4c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{64\sqrt{-c}}+\frac{447(acx-c)^3\sqrt{-acx+c}a^4c+1127(acx-c)^2\sqrt{-acx+c}a^4c}{192a^4c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a^4*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 3
63/64*a^4*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/192*(447*(a*c*x
- c)^3*sqrt(-a*c*x + c)*a^4*c + 1127*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c^2
 - 1049*(-a*c*x + c)^(3/2)*a^4*c^3 + 321*sqrt(-a*c*x + c)*a^4*c^4)/(a^4*c^4
*x^4)
```

3.428 $\int e^{-3\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=235

$$\frac{2c^2(1-ax)^{3/2}(ax+1)^{9/2}}{9a^4(c-acx)^{3/2}}-\frac{2c^2(1-ax)^{3/2}(ax+1)^{7/2}}{a^4(c-acx)^{3/2}}+\frac{38c^2(1-ax)^{3/2}(ax+1)^{5/2}}{5a^4(c-acx)^{3/2}}-\frac{50c^2(1-ax)^{3/2}(ax+1)^{3/2}}{3a^4(c-acx)^{3/2}}+\frac{32c^2(1-}{a^4(c}$$

```
[Out] (8*c^2*(1 - a*x)^(3/2))/(a^4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (32*c^2*(1
- a*x)^(3/2)*Sqrt[1 + a*x])/(a^4*(c - a*c*x)^(3/2)) - (50*c^2*(1 - a*x)^(3/
2)*(1 + a*x)^(3/2))/(3*a^4*(c - a*c*x)^(3/2)) + (38*c^2*(1 - a*x)^(3/2)*(1
+ a*x)^(5/2))/(5*a^4*(c - a*c*x)^(3/2)) - (2*c^2*(1 - a*x)^(3/2)*(1 + a*x)^
(7/2))/(a^4*(c - a*c*x)^(3/2)) + (2*c^2*(1 - a*x)^(3/2)*(1 + a*x)^(9/2))/(9
*a^4*(c - a*c*x)^(3/2))
```

Rubi [A] time = 0.153605, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6130, 23, 88}

$$\frac{2c^2(1-ax)^{3/2}(ax+1)^{9/2}}{9a^4(c-acx)^{3/2}}-\frac{2c^2(1-ax)^{3/2}(ax+1)^{7/2}}{a^4(c-acx)^{3/2}}+\frac{38c^2(1-ax)^{3/2}(ax+1)^{5/2}}{5a^4(c-acx)^{3/2}}-\frac{50c^2(1-ax)^{3/2}(ax+1)^{3/2}}{3a^4(c-acx)^{3/2}}+\frac{32c^2(1-}{a^4(c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a*c*x])/E^(3*ArcTanh[a*x]),x]
```


```
[Out] (8*c^2*(1 - a*x)^(3/2))/(a^4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (32*c^2*(1
- a*x)^(3/2)*Sqrt[1 + a*x])/(a^4*(c - a*c*x)^(3/2)) - (50*c^2*(1 - a*x)^(3/
2)*(1 + a*x)^(3/2))/(3*a^4*(c - a*c*x)^(3/2)) + (38*c^2*(1 - a*x)^(3/2)*(1
+ a*x)^(5/2))/(5*a^4*(c - a*c*x)^(3/2)) - (2*c^2*(1 - a*x)^(3/2)*(1 + a*x)^
(7/2))/(a^4*(c - a*c*x)^(3/2)) + (2*c^2*(1 - a*x)^(3/2)*(1 + a*x)^(9/2))/(9
*a^4*(c - a*c*x)^(3/2))
```


Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\int\frac{x^3(1-ax)^{3/2}\sqrt{c-acx}}{(1+ax)^{3/2}}\,dx$$
$$=\frac{(1-ax)^{3/2}\int\frac{x^3(c-acx)^2}{(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}$$
$$=\frac{(1-ax)^{3/2}\int\left(-\frac{4c^2}{a^3(1+ax)^{3/2}}+\frac{16c^2}{a^3\sqrt{1+ax}}-\frac{25c^2\sqrt{1+ax}}{a^3}+\frac{19c^2(1+ax)^{3/2}}{a^3}-\frac{7c^2(1+ax)^{5/2}}{a^3}+\frac{c^2(1+ax)^{7/2}}{a^3}\right.}{(c-acx)^{3/2}}$$
$$=\frac{8c^2(1-ax)^{3/2}}{a^4\sqrt{1+ax}(c-acx)^{3/2}}+\frac{32c^2(1-ax)^{3/2}\sqrt{1+ax}}{a^4(c-acx)^{3/2}}-\frac{50c^2(1-ax)^{3/2}(1+ax)^{3/2}}{3a^4(c-acx)^{3/2}}+\frac{38c^2(}{5}$$

Mathematica [A] time = 0.048824, size = 76, normalized size = 0.32

$$\frac{2c\sqrt{1-ax}\left(5a^5x^5-20a^4x^4+41a^3x^3-82a^2x^2+328ax+656\right)}{45a^4\sqrt{ax+1}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a*c*x])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (2*c*Sqrt[1 - a*x]*(656 + 328*a*x - 82*a^2*x^2 + 41*a^3*x^3 - 20*a^4*x^4 +
5*a^5*x^5))/(45*a^4*Sqrt[1 + a*x]*Sqrt[c - a*c*x])
```

Maple [A] time = 0.033, size = 79, normalized size = 0.3

$$\frac{10\,x^5a^5-40\,x^4a^4+82\,x^3a^3-164\,a^2x^2+656\,ax+1312}{45\,(ax+1)^2\,(ax-1)^2\,a^4}\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 2/45*(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)*(5*a^5*x^5-20*a^4*x^4+41*a^3*x^3-8
2*a^2*x^2+328*a*x+656)/(a*x+1)^2/(a*x-1)^2/a^4
```

Maxima [A] time = 1.04195, size = 116, normalized size = 0.49

$$\frac{2\left(5\,a^5\sqrt{c}x^5-20\,a^4\sqrt{c}x^4+41\,a^3\sqrt{c}x^3-82\,a^2\sqrt{c}x^2+328\,a\sqrt{c}x+656\,\sqrt{c}\right)\sqrt{ax+1}(ax-1)}{45\left(a^6x^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="m
axima")
```

```
[Out] 2/45*(5*a^5*sqrt(c)*x^5 - 20*a^4*sqrt(c)*x^4 + 41*a^3*sqrt(c)*x^3 - 82*a^2*
sqrt(c)*x^2 + 328*a*sqrt(c)*x + 656*sqrt(c))*sqrt(a*x + 1)*(a*x - 1)/(a^6*x
^2 - a^4)
```

Fricas [A] time = 1.53572, size = 170, normalized size = 0.72

$$-\frac{2\left(5\,a^5x^5-20\,a^4x^4+41\,a^3x^3-82\,a^2x^2+328\,ax+656\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{45\left(a^6x^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="f
ricas")
```

```
[Out] -2/45*(5*a^5*x^5 - 20*a^4*x^4 + 41*a^3*x^3 - 82*a^2*x^2 + 328*a*x + 656)*sq
rt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^6*x^2 - a^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\sqrt{-c\,(ax-1)}\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3
, x)
```

Giac [A] time = 1.28954, size = 146, normalized size = 0.62

$$-\frac{928\sqrt{2}|c|}{45\,a^4\sqrt{c}}+\frac{2\left(5\,(acx+c)^{\frac{9}{2}}|c|-45\,(acx+c)^{\frac{7}{2}}c|c|+171\,(acx+c)^{\frac{5}{2}}c^2|c|-375\,(acx+c)^{\frac{3}{2}}c^3|c|+720\,\sqrt{acx+c}c^4|c|+\frac{180\,c^5|c|}{\sqrt{acx+c}}\right)}{45\,a^4c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="g
iac")
```

```
[Out] -928/45*sqrt(2)*abs(c)/(a^4*sqrt(c)) + 2/45*(5*(a*c*x + c)^(9/2)*abs(c) - 4
5*(a*c*x + c)^(7/2)*c*abs(c) + 171*(a*c*x + c)^(5/2)*c^2*abs(c) - 375*(a*c*
x + c)^(3/2)*c^3*abs(c) + 720*sqrt(a*c*x + c)*c^4*abs(c) + 180*c^5*abs(c)/s
qrt(a*c*x + c))/(a^4*c^5)
```

3.429 $\int e^{-3\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=197

$$\frac{2c^2(1-ax)^{3/2}(ax+1)^{7/2}}{7a^3(c-acx)^{3/2}}-\frac{12c^2(1-ax)^{3/2}(ax+1)^{5/2}}{5a^3(c-acx)^{3/2}}+\frac{26c^2(1-ax)^{3/2}(ax+1)^{3/2}}{3a^3(c-acx)^{3/2}}-\frac{24c^2(1-ax)^{3/2}\sqrt{ax+1}}{a^3(c-acx)^{3/2}}-\frac{8c^2}{a^3\sqrt{ax}}$$

```
[Out] (-8*c^2*(1 - a*x)^(3/2))/(a^3*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (24*c^2*(1
 - a*x)^(3/2)*Sqrt[1 + a*x])/(a^3*(c - a*c*x)^(3/2)) + (26*c^2*(1 - a*x)^(3
/2)*(1 + a*x)^(3/2))/(3*a^3*(c - a*c*x)^(3/2)) - (12*c^2*(1 - a*x)^(3/2)*(1
 + a*x)^(5/2))/(5*a^3*(c - a*c*x)^(3/2)) + (2*c^2*(1 - a*x)^(3/2)*(1 + a*x)
^(7/2))/(7*a^3*(c - a*c*x)^(3/2))
```

Rubi [A] time = 0.149603, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6130, 23, 88}

$$\frac{2c^2(1-ax)^{3/2}(ax+1)^{7/2}}{7a^3(c-acx)^{3/2}}-\frac{12c^2(1-ax)^{3/2}(ax+1)^{5/2}}{5a^3(c-acx)^{3/2}}+\frac{26c^2(1-ax)^{3/2}(ax+1)^{3/2}}{3a^3(c-acx)^{3/2}}-\frac{24c^2(1-ax)^{3/2}\sqrt{ax+1}}{a^3(c-acx)^{3/2}}-\frac{8c^2}{a^3\sqrt{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^(3*ArcTanh[a*x]),x]
```


```
[Out] (-8*c^2*(1 - a*x)^(3/2))/(a^3*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (24*c^2*(1
 - a*x)^(3/2)*Sqrt[1 + a*x])/(a^3*(c - a*c*x)^(3/2)) + (26*c^2*(1 - a*x)^(3
/2)*(1 + a*x)^(3/2))/(3*a^3*(c - a*c*x)^(3/2)) - (12*c^2*(1 - a*x)^(3/2)*(1
 + a*x)^(5/2))/(5*a^3*(c - a*c*x)^(3/2)) + (2*c^2*(1 - a*x)^(3/2)*(1 + a*x)
^(7/2))/(7*a^3*(c - a*c*x)^(3/2))
```


Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \int \frac{x^2(1-ax)^{3/2}\sqrt{c-acx}}{(1+ax)^{3/2}}\,dx$$
$$= \frac{(1-ax)^{3/2}\int \frac{x^2(c-acx)^2}{(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}$$
$$= \frac{(1-ax)^{3/2}\int \left(\frac{4c^2}{a^2(1+ax)^{3/2}} - \frac{12c^2}{a^2\sqrt{1+ax}} + \frac{13c^2\sqrt{1+ax}}{a^2} - \frac{6c^2(1+ax)^{3/2}}{a^2} + \frac{c^2(1+ax)^{5/2}}{a^2}\right)dx}{(c-acx)^{3/2}}$$
$$= -\frac{8c^2(1-ax)^{3/2}}{a^3\sqrt{1+ax}(c-acx)^{3/2}} - \frac{24c^2(1-ax)^{3/2}\sqrt{1+ax}}{a^3(c-acx)^{3/2}} + \frac{26c^2(1-ax)^{3/2}(1+ax)^{3/2}}{3a^3(c-acx)^{3/2}} - \frac{12c^2(1-}{5a}$$

Mathematica [A] time = 0.0425967, size = 68, normalized size = 0.35

$$\frac{2c\sqrt{1-ax}\left(15a^4x^4-66a^3x^3+167a^2x^2-668ax-1336\right)}{105a^3\sqrt{ax+1}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (2*c*Sqrt[1 - a*x]*(-1336 - 668*a*x + 167*a^2*x^2 - 66*a^3*x^3 + 15*a^4*x^4
))/(105*a^3*Sqrt[1 + a*x]*Sqrt[c - a*c*x])
```

Maple [A] time = 0.032, size = 71, normalized size = 0.4

$$\frac{30\,x^4a^4-132\,x^3a^3+334\,a^2x^2-1336\,ax-2672}{105\,(ax+1)^2\,(ax-1)^2\,a^3}\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 2/105*(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)*(15*a^4*x^4-66*a^3*x^3+167*a^2*x^
2-668*a*x-1336)/(a*x+1)^2/(a*x-1)^2/a^3
```

Maxima [A] time = 1.03741, size = 101, normalized size = 0.51

$$\frac{2\left(15\,a^4\sqrt{c}x^4-66\,a^3\sqrt{c}x^3+167\,a^2\sqrt{c}x^2-668\,a\sqrt{c}x-1336\,\sqrt{c}\right)\sqrt{ax+1}(ax-1)}{105\left(a^5x^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="m
axima")
```

```
[Out] 2/105*(15*a^4*sqrt(c)*x^4 - 66*a^3*sqrt(c)*x^3 + 167*a^2*sqrt(c)*x^2 - 668*
a*sqrt(c)*x - 1336*sqrt(c))*sqrt(a*x + 1)*(a*x - 1)/(a^5*x^2 - a^3)
```

Fricas [A] time = 1.53482, size = 158, normalized size = 0.8

$$-\frac{2\left(15\,a^4x^4-66\,a^3x^3+167\,a^2x^2-668\,ax-1336\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{105\left(a^5x^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="f
ricas")
```

```
[Out] -2/105*(15*a^4*x^4 - 66*a^3*x^3 + 167*a^2*x^2 - 668*a*x - 1336)*sqrt(-a^2*x
^2 + 1)*sqrt(-a*c*x + c)/(a^5*x^2 - a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\sqrt{-c\,(ax-1)}\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3
, x)
```

Giac [A] time = 1.32868, size = 126, normalized size = 0.64

$$\frac{1888\,\sqrt{2}|c|}{105\,a^3\sqrt{c}}+\frac{2\left(15\,(acx+c)^{\frac{7}{2}}|c|-126\,(acx+c)^{\frac{5}{2}}c|c|+455\,(acx+c)^{\frac{3}{2}}c^2|c|-1260\,\sqrt{acx+c}c^3|c|-\frac{420\,c^4|c|}{\sqrt{acx+c}}\right)}{105\,a^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="g
iac")
```

```
[Out] 1888/105*sqrt(2)*abs(c)/(a^3*sqrt(c)) + 2/105*(15*(a*c*x + c)^(7/2)*abs(c)
- 126*(a*c*x + c)^(5/2)*c*abs(c) + 455*(a*c*x + c)^(3/2)*c^2*abs(c) - 1260*
sqrt(a*c*x + c)*c^3*abs(c) - 420*c^4*abs(c)/sqrt(a*c*x + c))/(a^3*c^4)
```

3.430 $\int e^{-3\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=157

$$\frac{2c^2(1-ax)^{3/2}(ax+1)^{5/2}}{5a^2(c-acx)^{3/2}}-\frac{10c^2(1-ax)^{3/2}(ax+1)^{3/2}}{3a^2(c-acx)^{3/2}}+\frac{16c^2(1-ax)^{3/2}\sqrt{ax+1}}{a^2(c-acx)^{3/2}}+\frac{8c^2(1-ax)^{3/2}}{a^2\sqrt{ax+1}(c-acx)^{3/2}}$$

```
[Out] (8*c^2*(1 - a*x)^(3/2))/(a^2*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (16*c^2*(1
- a*x)^(3/2)*Sqrt[1 + a*x])/(a^2*(c - a*c*x)^(3/2)) - (10*c^2*(1 - a*x)^(3/
2)*(1 + a*x)^(3/2))/(3*a^2*(c - a*c*x)^(3/2)) + (2*c^2*(1 - a*x)^(3/2)*(1 +
 a*x)^(5/2))/(5*a^2*(c - a*c*x)^(3/2))
```

Rubi [A] time = 0.105492, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6130, 23, 77}

$$\frac{2c^2(1-ax)^{3/2}(ax+1)^{5/2}}{5a^2(c-acx)^{3/2}}-\frac{10c^2(1-ax)^{3/2}(ax+1)^{3/2}}{3a^2(c-acx)^{3/2}}+\frac{16c^2(1-ax)^{3/2}\sqrt{ax+1}}{a^2(c-acx)^{3/2}}+\frac{8c^2(1-ax)^{3/2}}{a^2\sqrt{ax+1}(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a*c*x])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (8*c^2*(1 - a*x)^(3/2))/(a^2*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (16*c^2*(1
- a*x)^(3/2)*Sqrt[1 + a*x])/(a^2*(c - a*c*x)^(3/2)) - (10*c^2*(1 - a*x)^(3/
2)*(1 + a*x)^(3/2))/(3*a^2*(c - a*c*x)^(3/2)) + (2*c^2*(1 - a*x)^(3/2)*(1 +
 a*x)^(5/2))/(5*a^2*(c - a*c*x)^(3/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx = \int \frac{x(1-ax)^{3/2}\sqrt{c-acx}}{(1+ax)^{3/2}}\,dx$$
$$= \frac{(1-ax)^{3/2}\int \frac{x(c-acx)^2}{(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}$$
$$= \frac{(1-ax)^{3/2}\int \left(-\frac{4c^2}{a(1+ax)^{3/2}}+\frac{8c^2}{a\sqrt{1+ax}}-\frac{5c^2\sqrt{1+ax}}{a}+\frac{c^2(1+ax)^{3/2}}{a}\right)dx}{(c-acx)^{3/2}}$$
$$= \frac{8c^2(1-ax)^{3/2}}{a^2\sqrt{1+ax}(c-acx)^{3/2}}+\frac{16c^2(1-ax)^{3/2}\sqrt{1+ax}}{a^2(c-acx)^{3/2}}-\frac{10c^2(1-ax)^{3/2}(1+ax)^{3/2}}{3a^2(c-acx)^{3/2}}+\frac{2c^2(1-}{5a}$$

Mathematica [A] time = 0.0331603, size = 60, normalized size = 0.38

$$\frac{2c\sqrt{1-ax}\left(3a^3x^3-16a^2x^2+79ax+158\right)}{15a^2\sqrt{ax+1}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (2*c*Sqrt[1 - a*x]*(158 + 79*a*x - 16*a^2*x^2 + 3*a^3*x^3))/(15*a^2*Sqrt[1
+ a*x]*Sqrt[c - a*c*x])
```

Maple [A] time = 0.033, size = 63, normalized size = 0.4

$$\frac{6\,x^3a^3-32\,a^2x^2+158\,ax+316}{15\,(ax+1)^2\,(ax-1)^2\,a^2}\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 2/15*(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)*(3*a^3*x^3-16*a^2*x^2+79*a*x+158)/
(a*x+1)^2/(a*x-1)^2/a^2
```

Maxima [A] time = 1.02841, size = 86, normalized size = 0.55

$$\frac{2\left(3\,a^3\sqrt{c}x^3-16\,a^2\sqrt{c}x^2+79\,a\sqrt{c}x+158\,\sqrt{c}\right)\sqrt{ax+1}(ax-1)}{15\left(a^4x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="max
ima")
```

```
[Out] 2/15*(3*a^3*sqrt(c)*x^3 - 16*a^2*sqrt(c)*x^2 + 79*a*sqrt(c)*x + 158*sqrt(c)
)*sqrt(a*x + 1)*(a*x - 1)/(a^4*x^2 - a^2)
```

Fricas [A] time = 1.60425, size = 134, normalized size = 0.85

$$-\frac{2\left(3\,a^3x^3-16\,a^2x^2+79\,ax+158\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{15\left(a^4x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fri
cas")
```

```
[Out] -2/15*(3*a^3*x^3 - 16*a^2*x^2 + 79*a*x + 158)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*
x + c)/(a^4*x^2 - a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\,(ax-1)}\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x
)
```

Giac [A] time = 1.26133, size = 105, normalized size = 0.67

$$-\frac{224\,\sqrt{2}|c|}{15\,a^2\sqrt{c}}+\frac{2\left(3\,(acx+c)^{\frac{5}{2}}|c|-25\,(acx+c)^{\frac{3}{2}}c|c|+120\,\sqrt{acx+c}c^2|c|+\frac{60\,c^3|c|}{\sqrt{acx+c}}\right)}{15\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gia
c")
```

```
[Out] -224/15*sqrt(2)*abs(c)/(a^2*sqrt(c)) + 2/15*(3*(a*c*x + c)^(5/2)*abs(c) - 2
5*(a*c*x + c)^(3/2)*c*abs(c) + 120*sqrt(a*c*x + c)*c^2*abs(c) + 60*c^3*abs(
c)/sqrt(a*c*x + c))/(a^2*c^3)
```

3.431 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=103

$$\frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}}+\frac{16(c-acx)^{3/2}}{3ac\sqrt{1-a^2x^2}}-\frac{64\sqrt{c-acx}}{3a\sqrt{1-a^2x^2}}$$

```
[Out] (-64*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - a^2*x^2]) + (16*(c - a*c*x)^(3/2))/(3*a
*c*Sqrt[1 - a^2*x^2]) + (2*(c - a*c*x)^(5/2))/(3*a*c^2*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0961596, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6127, 657, 649}

$$\frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}}+\frac{16(c-acx)^{3/2}}{3ac\sqrt{1-a^2x^2}}-\frac{64\sqrt{c-acx}}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-64*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - a^2*x^2]) + (16*(c - a*c*x)^(3/2))/(3*a
*c*Sqrt[1 - a^2*x^2]) + (2*(c - a*c*x)^(5/2))/(3*a*c^2*Sqrt[1 - a^2*x^2])
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\int \frac{(c-acx)^{7/2}}{(1-a^2x^2)^{3/2}}dx}{c^3}$$
$$= \frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}} + \frac{8\int \frac{(c-acx)^{5/2}}{(1-a^2x^2)^{3/2}}dx}{3c^2}$$
$$= \frac{16(c-acx)^{3/2}}{3ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}} + \frac{32\int \frac{(c-acx)^{3/2}}{(1-a^2x^2)^{3/2}}dx}{3c}$$
$$= -\frac{64\sqrt{c-acx}}{3a\sqrt{1-a^2x^2}} + \frac{16(c-acx)^{3/2}}{3ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.025651, size = 51, normalized size = 0.5

$$\frac{2c\sqrt{1-ax}\left(a^2x^2-10ax-23\right)}{3a\sqrt{ax+1}\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (2*c*Sqrt[1 - a*x]*(-23 - 10*a*x + a^2*x^2))/(3*a*Sqrt[1 + a*x]*Sqrt[c - a*
c*x])
```

Maple [A] time = 0.033, size = 54, normalized size = 0.5

$$\frac{2\,a^2x^2-20\,ax-46}{3\,(ax+1)^2\,(ax-1)^2\,a}\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 2/3*(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)*(a^2*x^2-10*a*x-23)/(a*x+1)^2/(a*x-
1)^2/a
```

Maxima [A] time = 1.02229, size = 68, normalized size = 0.66

$$\frac{2\left(a^2\sqrt{c}x^2-10\,a\sqrt{c}x-23\,\sqrt{c}\right)\sqrt{ax+1}(ax-1)}{3\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/3*(a^2*sqrt(c)*x^2 - 10*a*sqrt(c)*x - 23*sqrt(c))*sqrt(a*x + 1)*(a*x - 1)
/(a^3*x^2 - a)
```

Fricas [A] time = 1.51483, size = 108, normalized size = 1.05

$$-\frac{2\left(a^{2}x^{2}-10\,ax-23\right)\sqrt{-a^{2}x^{2}+1}\sqrt{-acx+c}}{3\left(a^{3}x^{2}-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="frica
s")
```

```
[Out] -2/3*(a^2*x^2 - 10*a*x - 23)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^3*x^2 -
 a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(ax-1\right)}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)
```

Giac [A] time = 1.25759, size = 77, normalized size = 0.75

$$\frac{32\,\sqrt{2}|c|}{3\,a\sqrt{c}}+\frac{2\left(\left(acx+c\right)^{\frac{3}{2}}-12\,\sqrt{acx+c}c-\frac{12\,c^{2}}{\sqrt{acx+c}}\right)|c|}{3\,ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac"
)
```

```
[Out] 32/3*sqrt(2)*abs(c)/(a*sqrt(c)) + 2/3*((a*c*x + c)^(3/2) - 12*sqrt(a*c*x +
c)*c - 12*c^2/sqrt(a*c*x + c))*abs(c)/(a*c^2)
```

3.432 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=107

$$\frac{2c^2(1-ax)^{3/2}\sqrt{ax+1}}{(c-acx)^{3/2}}+\frac{8c^2(1-ax)^{3/2}}{\sqrt{ax+1}(c-acx)^{3/2}}-\frac{2c^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{(c-acx)^{3/2}}$$

```
[Out] (8*c^2*(1 - a*x)^(3/2))/(Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (2*c^2*(1 - a*x
)^(3/2)*Sqrt[1 + a*x])/(c - a*c*x)^(3/2) - (2*c^2*(1 - a*x)^(3/2)*ArcTanh[S
qrt[1 + a*x]])/(c - a*c*x)^(3/2)
```

Rubi [A] time = 0.128615, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6130, 23, 87, 63, 208}

$$\frac{2c^2(1-ax)^{3/2}\sqrt{ax+1}}{(c-acx)^{3/2}}+\frac{8c^2(1-ax)^{3/2}}{\sqrt{ax+1}(c-acx)^{3/2}}-\frac{2c^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] (8*c^2*(1 - a*x)^(3/2))/(Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (2*c^2*(1 - a*x
)^(3/2)*Sqrt[1 + a*x])/(c - a*c*x)^(3/2) - (2*c^2*(1 - a*x)^(3/2)*ArcTanh[S
qrt[1 + a*x]])/(c - a*c*x)^(3/2)
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx &= \int \frac{(1-ax)^{3/2}\sqrt{c-acx}}{x(1+ax)^{3/2}}\,dx\\
&= \frac{(1-ax)^{3/2}\int \frac{(c-acx)^2}{x(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}\\
&= \frac{(1-ax)^{3/2}\int \left(-\frac{4ac^2}{(1+ax)^{3/2}}+\frac{ac^2}{\sqrt{1+ax}}+\frac{c^2}{x\sqrt{1+ax}}\right)dx}{(c-acx)^{3/2}}\\
&= \frac{8c^2(1-ax)^{3/2}}{\sqrt{1+ax}(c-acx)^{3/2}}+\frac{2c^2(1-ax)^{3/2}\sqrt{1+ax}}{(c-acx)^{3/2}}+\frac{\left(c^2(1-ax)^{3/2}\right)\int \frac{1}{x\sqrt{1+ax}}\,dx}{(c-acx)^{3/2}}\\
&= \frac{8c^2(1-ax)^{3/2}}{\sqrt{1+ax}(c-acx)^{3/2}}+\frac{2c^2(1-ax)^{3/2}\sqrt{1+ax}}{(c-acx)^{3/2}}+\frac{\left(2c^2(1-ax)^{3/2}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{a}+\frac{x^2}{a}}\,dx,x,\sqrt{1+ax}\right)}{a(c-acx)^{3/2}}\\
&= \frac{8c^2(1-ax)^{3/2}}{\sqrt{1+ax}(c-acx)^{3/2}}+\frac{2c^2(1-ax)^{3/2}\sqrt{1+ax}}{(c-acx)^{3/2}}-\frac{2c^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{1+ax}\right)}{(c-acx)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0347293, size = 51, normalized size = 0.48

$$\frac{2c\sqrt{1-ax}\left(\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},ax+1\right)+ax+4\right)}{\sqrt{ax+1}\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] (2*c*Sqrt[1 - a*x]*(4 + a*x + Hypergeometric2F1[-1/2, 1, 1/2, 1 + a*x]))/(S
qrt[1 + a*x]*Sqrt[c - a*c*x])
```

Maple [A] time = 0.108, size = 78, normalized size = 0.7

$$2\,\frac{\sqrt{-c\,(ax-1)}\sqrt{-a^2x^2+1}}{(ax-1)\,(ax+1)\,c}\left(\sqrt{c}\text{Artanh}\left(\frac{\sqrt{c\,(ax+1)}}{\sqrt{c}}\right)\sqrt{c\,(ax+1)}-acx-5\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x)
```

```
[Out] 2*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(c^(1/2)*arctanh((c*(a*x+1))^(1/2)/
c^(1/2))*(c*(a*x+1))^(1/2)-a*c*x-5*c)/(a*x-1)/c/(a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}}{(ax+1)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(-a*c*x + c)/((a*x + 1)^3*x), x)
```

Fricas [A] time = 1.60018, size = 466, normalized size = 4.36

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}(ax+5)}{a^2x^2-1}, -\frac{2\left(\left(a^2x^2-1\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\cdots}\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="fri
cas")
```

```
[Out] [((a^2*x^2 - 1)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1)*sqrt
(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) - 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x
 + c)*(a*x + 5))/(a^2*x^2 - 1), -2*((a^2*x^2 - 1)*sqrt(-c)*arctan(sqrt(-a^2
*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 + 1)*s
qrt(-a*c*x + c)*(a*x + 5))/(a^2*x^2 - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)}\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}{x\,(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(x*(a*x + 1)**3),
 x)
```

Giac [A] time = 1.22053, size = 127, normalized size = 1.19

$$2\left(\frac{\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{4}{\sqrt{acx+c}}+\frac{\sqrt{acx+c}}{c}\right)|c|-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}|c|\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)+6\sqrt{-c}|c|\right)}{\sqrt{-c}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="gia
c")
```

```
[Out] 2*(arctan(sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) + 4/sqrt(a*c*x + c) + sqrt(a*c
*x + c)/c)*abs(c) - sqrt(2)*(sqrt(2)*sqrt(c)*abs(c)*arctan(sqrt(2)*sqrt(c)/
sqrt(-c)) + 6*sqrt(-c)*abs(c))/(sqrt(-c)*sqrt(c))
```

3.433 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=112

$$-\frac{9ac^2(1-ax)^{3/2}}{\sqrt{ax+1}(c-acx)^{3/2}}-\frac{c^2(1-ax)^{3/2}}{x\sqrt{ax+1}(c-acx)^{3/2}}+\frac{7ac^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{(c-acx)^{3/2}}$$

```
[Out] (-9*a*c^2*(1 - a*x)^(3/2))/(Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (c^2*(1 - a*
x)^(3/2))/(x*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (7*a*c^2*(1 - a*x)^(3/2)*Ar
cTanh[Sqrt[1 + a*x]])/(c - a*c*x)^(3/2)
```

Rubi [A] time = 0.118861, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 23, 89, 78, 63, 208}

$$-\frac{9ac^2(1-ax)^{3/2}}{\sqrt{ax+1}(c-acx)^{3/2}}-\frac{c^2(1-ax)^{3/2}}{x\sqrt{ax+1}(c-acx)^{3/2}}+\frac{7ac^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] (-9*a*c^2*(1 - a*x)^(3/2))/(Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (c^2*(1 - a*
x)^(3/2))/(x*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (7*a*c^2*(1 - a*x)^(3/2)*Ar
cTanh[Sqrt[1 + a*x]])/(c - a*c*x)^(3/2)
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx &= \int \frac{(1-ax)^{3/2}\sqrt{c-acx}}{x^2(1+ax)^{3/2}}\,dx\\
&= \frac{(1-ax)^{3/2}\int \frac{(c-acx)^2}{x^2(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}\\
&= -\frac{c^2(1-ax)^{3/2}}{x\sqrt{1+ax}(c-acx)^{3/2}} + \frac{(1-ax)^{3/2}\int \frac{-\frac{7ac^2}{2}+a^2c^2x}{x(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}\\
&= -\frac{9ac^2(1-ax)^{3/2}}{\sqrt{1+ax}(c-acx)^{3/2}} - \frac{c^2(1-ax)^{3/2}}{x\sqrt{1+ax}(c-acx)^{3/2}} - \frac{\left(7ac^2(1-ax)^{3/2}\right)\int \frac{1}{x\sqrt{1+ax}}\,dx}{2(c-acx)^{3/2}}\\
&= -\frac{9ac^2(1-ax)^{3/2}}{\sqrt{1+ax}(c-acx)^{3/2}} - \frac{c^2(1-ax)^{3/2}}{x\sqrt{1+ax}(c-acx)^{3/2}} - \frac{\left(7c^2(1-ax)^{3/2}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{a}+\frac{x^2}{a}}\,dx, x, \sqrt{1\right.}{(c-acx)^{3/2}}\\
&= -\frac{9ac^2(1-ax)^{3/2}}{\sqrt{1+ax}(c-acx)^{3/2}} - \frac{c^2(1-ax)^{3/2}}{x\sqrt{1+ax}(c-acx)^{3/2}} + \frac{7ac^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{1+ax}\right)}{(c-acx)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0292104, size = 64, normalized size = 0.57

$$
\frac{c\sqrt{1-ax}\left(-9ax+7ax\sqrt{ax+1}\tanh^{-1}\left(\sqrt{ax+1}\right)-1\right)}{x\sqrt{ax+1}\sqrt{c-acx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] (c*Sqrt[1 - a*x]*(-1 - 9*a*x + 7*a*x*Sqrt[1 + a*x]*ArcTanh[Sqrt[1 + a*x]]))
/(x*Sqrt[1 + a*x]*Sqrt[c - a*c*x])
```

Maple [A] time = 0.128, size = 82, normalized size = 0.7

$$
\frac{1}{(ax+1)(ax-1)x}\left(-7\,\text{Artanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)xa\sqrt{c(ax+1)}+9\,xa\sqrt{c}+\sqrt{c}\right)\sqrt{-c(ax-1)}\sqrt{-a^2x^2+1}\frac{1}{\sqrt{c}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x)
```

```
[Out] (-7*arctanh((c*(a*x+1))^(1/2)/c^(1/2))*x*a*(c*(a*x+1))^(1/2)+9*x*a*c^(1/2)+
c^(1/2))*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/c^(1/2)/(a*x+1)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}}{\left(ax+1\right)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(-a*c*x + c)/((a*x + 1)^3*x^2), x)
```

Fricas [A] time = 1.68149, size = 483, normalized size = 4.31

$$\left[\frac{7\left(a^3x^3-ax\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}(9ax+1)}{2\left(a^2x^3-x\right)}, \frac{7\left(a^3x^3-ax\right)\sqrt{-c}\,\mathrm{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="f
ricas")
```

```
[Out] [1/2*(7*(a^3*x^3 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a^2*x^2 +
 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqr
t(-a*c*x + c)*(9*a*x + 1))/(a^2*x^3 - x), (7*(a^3*x^3 - a*x)*sqrt(-c)*arcta
n(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2
*x^2 + 1)*sqrt(-a*c*x + c)*(9*a*x + 1))/(a^2*x^3 - x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(ax-1\right)}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{x^2\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(x**2*(a*x + 1)**
3), x)
```

Giac [A] time = 1.23867, size = 144, normalized size = 1.29

$$-a\left(\frac{7\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{9\,acx+c}{(acx+c)^{\frac{3}{2}}-\sqrt{acx+c}c}\right)|c|+\frac{\sqrt{2}\left(7\sqrt{2}a\sqrt{c}|c|\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)+10\,a\sqrt{-c}|c|\right)}{2\sqrt{-c}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="g
iac")
```

```
[Out] -a*(7*arctan(sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) + (9*a*c*x + c)/((a*c*x + c
)^(3/2) - sqrt(a*c*x + c)*c))*abs(c) + 1/2*sqrt(2)*(7*sqrt(2)*a*sqrt(c)*abs
(c)*arctan(sqrt(2)*sqrt(c)/sqrt(-c)) + 10*a*sqrt(-c)*abs(c))/(sqrt(-c)*sqrt
(c))
```

3.434 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=163

$$\frac{47a^2c^2(1-ax)^{3/2}}{4\sqrt{ax+1}(c-acx)^{3/2}}-\frac{47a^2c^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{4(c-acx)^{3/2}}-\frac{c^2(1-ax)^{3/2}}{2x^2\sqrt{ax+1}(c-acx)^{3/2}}+\frac{13ac^2(1-ax)^{3/2}}{4x\sqrt{ax+1}(c-acx)^{3/2}}$$

```
[Out] (47*a^2*c^2*(1 - a*x)^(3/2))/(4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (c^2*(1
- a*x)^(3/2))/(2*x^2*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (13*a*c^2*(1 - a*x)
^(3/2))/(4*x*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (47*a^2*c^2*(1 - a*x)^(3/2)
*ArcTanh[Sqrt[1 + a*x]])/(4*(c - a*c*x)^(3/2))
```

Rubi [A] time = 0.129531, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6130, 23, 89, 78, 51, 63, 208}

$$\frac{47a^2c^2(1-ax)^{3/2}}{4\sqrt{ax+1}(c-acx)^{3/2}}-\frac{47a^2c^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{4(c-acx)^{3/2}}-\frac{c^2(1-ax)^{3/2}}{2x^2\sqrt{ax+1}(c-acx)^{3/2}}+\frac{13ac^2(1-ax)^{3/2}}{4x\sqrt{ax+1}(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] (47*a^2*c^2*(1 - a*x)^(3/2))/(4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (c^2*(1
- a*x)^(3/2))/(2*x^2*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (13*a*c^2*(1 - a*x)
^(3/2))/(4*x*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (47*a^2*c^2*(1 - a*x)^(3/2)
*ArcTanh[Sqrt[1 + a*x]])/(4*(c - a*c*x)^(3/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx &= \int \frac{(1-ax)^{3/2}\sqrt{c-acx}}{x^3(1+ax)^{3/2}}\,dx \\
&= \frac{(1-ax)^{3/2}\int \frac{(c-acx)^2}{x^3(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}} \\
&= -\frac{c^2(1-ax)^{3/2}}{2x^2\sqrt{1+ax}(c-acx)^{3/2}} + \frac{(1-ax)^{3/2}\int \frac{-\frac{13ac^2}{2}+2a^2c^2x}{x^2(1+ax)^{3/2}}\,dx}{2(c-acx)^{3/2}} \\
&= -\frac{c^2(1-ax)^{3/2}}{2x^2\sqrt{1+ax}(c-acx)^{3/2}} + \frac{13ac^2(1-ax)^{3/2}}{4x\sqrt{1+ax}(c-acx)^{3/2}} + \frac{\left(47a^2c^2(1-ax)^{3/2}\right)\int \frac{1}{x(1+ax)^{3/2}}\,dx}{8(c-acx)^{3/2}} \\
&= \frac{47a^2c^2(1-ax)^{3/2}}{4\sqrt{1+ax}(c-acx)^{3/2}} - \frac{c^2(1-ax)^{3/2}}{2x^2\sqrt{1+ax}(c-acx)^{3/2}} + \frac{13ac^2(1-ax)^{3/2}}{4x\sqrt{1+ax}(c-acx)^{3/2}} + \frac{\left(47a^2c^2(1-ax\right.}{8(c-}\\
&= \frac{47a^2c^2(1-ax)^{3/2}}{4\sqrt{1+ax}(c-acx)^{3/2}} - \frac{c^2(1-ax)^{3/2}}{2x^2\sqrt{1+ax}(c-acx)^{3/2}} + \frac{13ac^2(1-ax)^{3/2}}{4x\sqrt{1+ax}(c-acx)^{3/2}} + \frac{\left(47ac^2(1-ax)\right.}{} \\
&= \frac{47a^2c^2(1-ax)^{3/2}}{4\sqrt{1+ax}(c-acx)^{3/2}} - \frac{c^2(1-ax)^{3/2}}{2x^2\sqrt{1+ax}(c-acx)^{3/2}} + \frac{13ac^2(1-ax)^{3/2}}{4x\sqrt{1+ax}(c-acx)^{3/2}} - \frac{47a^2c^2(1-ax)}{4(c}
\end{aligned}$$

Mathematica [C] time = 0.0270787, size = 65, normalized size = 0.4

$$\frac{c\sqrt{1-ax}\left(47a^2x^2\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},ax+1\right)+13ax-2\right)}{4x^2\sqrt{ax+1}\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] (c*Sqrt[1 - a*x]*(-2 + 13*a*x + 47*a^2*x^2*Hypergeometric2F1[-1/2, 1, 1/2,
1 + a*x]))/(4*x^2*Sqrt[1 + a*x]*Sqrt[c - a*c*x])
```

Maple [A] time = 0.113, size = 100, normalized size = 0.6

$$\frac{1}{(4ax-4)(ax+1)x^2}\sqrt{-c(ax-1)}\sqrt{-a^2x^2+1}\left(47\,\mathrm{Artanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)x^2a^2\sqrt{c(ax+1)}-47\,x^2a^2\sqrt{c}-13\,xa\sqrt{c}+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x)
```

```
[Out] 1/4*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(47*arctanh((c*(a*x+1))^(1/2)/c^(
1/2))*x^2*a^2*(c*(a*x+1))^(1/2)-47*x^2*a^2*c^(1/2)-13*x*a*c^(1/2)+2*c^(1/2)
)/c^(1/2)/(a*x-1)/(a*x+1)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}}{(ax+1)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="m
axima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(-a*c*x + c)/((a*x + 1)^3*x^3), x)
```

Fricas [A] time = 1.84219, size = 547, normalized size = 3.36

$$\left[\frac{47\left(a^4x^4-a^2x^2\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)-2\left(47\,a^2x^2+13\,ax-2\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{8\left(a^2x^4-x^2\right)},-\frac{47\left(a^4x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="f
ricas")
```

```
[Out] [1/8*(47*(a^4*x^4 - a^2*x^2)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*
x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) - 2*(47*a^2*x^2 + 13*
a*x - 2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^2*x^4 - x^2), -1/4*(47*(a^
4*x^4 - a^2*x^2)*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-
c)/(a^2*c*x^2 - c)) + (47*a^2*x^2 + 13*a*x - 2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*
c*x + c))/(a^2*x^4 - x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)}\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}{x^3\,(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(x**3*(a*x + 1)**
3), x)
```

Giac [A] time = 1.23052, size = 178, normalized size = 1.09

$$\frac{1}{4}\,a^2c\left(\frac{47\,\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}+\frac{32}{\sqrt{acx+c}c}+\frac{15\,(acx+c)^{\frac{3}{2}}-17\,\sqrt{acx+c}c}{a^2c^3x^2}\right)|c|-\frac{\sqrt{2}\left(47\,\sqrt{2}a^2\sqrt{c}|c|\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)+58\,a^2\sqrt{-c}|c|\right)}{8\,\sqrt{-c}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="g
iac")
```

```
[Out] 1/4*a^2*c*(47*arctan(sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c) + 32/(sqrt(a*c*
x + c)*c) + (15*(a*c*x + c)^(3/2) - 17*sqrt(a*c*x + c)*c)/(a^2*c^3*x^2))*ab
s(c) - 1/8*sqrt(2)*(47*sqrt(2)*a^2*sqrt(c)*abs(c)*arctan(sqrt(2)*sqrt(c)/sq
rt(-c)) + 58*a^2*sqrt(-c)*abs(c))/(sqrt(-c)*sqrt(c))
```

3.435 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=206

$$-\frac{119a^3c^2(1-ax)^{3/2}}{8\sqrt{ax+1}(c-acx)^{3/2}}-\frac{119a^2c^2(1-ax)^{3/2}}{24x\sqrt{ax+1}(c-acx)^{3/2}}+\frac{119a^3c^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{8(c-acx)^{3/2}}+\frac{19ac^2(1-ax)^{3/2}}{12x^2\sqrt{ax+1}(c-acx)^{3/2}}-$$

```
[Out] (-119*a^3*c^2*(1 - a*x)^(3/2))/(8*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (c^2*(
1 - a*x)^(3/2))/(3*x^3*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (19*a*c^2*(1 - a*
x)^(3/2))/(12*x^2*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (119*a^2*c^2*(1 - a*x)
^(3/2))/(24*x*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (119*a^3*c^2*(1 - a*x)^(3/
2)*ArcTanh[Sqrt[1 + a*x]])/(8*(c - a*c*x)^(3/2))
```

Rubi [A] time = 0.138261, antiderivative size = 209, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6130, 23, 89, 78, 51, 63, 208}

$$-\frac{119a^2c^2(1-ax)^{3/2}\sqrt{ax+1}}{8x(c-acx)^{3/2}}+\frac{119a^2c^2(1-ax)^{3/2}}{12x\sqrt{ax+1}(c-acx)^{3/2}}+\frac{119a^3c^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{8(c-acx)^{3/2}}+\frac{19ac^2(1-ax)^{3/2}}{12x^2\sqrt{ax+1}(c-acx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] -(c^2*(1 - a*x)^(3/2))/(3*x^3*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (19*a*c^2*
(1 - a*x)^(3/2))/(12*x^2*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (119*a^2*c^2*(1
 - a*x)^(3/2))/(12*x*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (119*a^2*c^2*(1 - a
*x)^(3/2)*Sqrt[1 + a*x])/(8*x*(c - a*c*x)^(3/2)) + (119*a^3*c^2*(1 - a*x)^(
3/2)*ArcTanh[Sqrt[1 + a*x]])/(8*(c - a*c*x)^(3/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{(1-ax)^{3/2}\sqrt{c-acx}}{x^4(1+ax)^{3/2}}\,dx$$

$$= \frac{(1-ax)^{3/2}\int \frac{(c-acx)^2}{x^4(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}$$

$$= -\frac{c^2(1-ax)^{3/2}}{3x^3\sqrt{1+ax}(c-acx)^{3/2}} + \frac{(1-ax)^{3/2}\int \frac{-\frac{19ac^2}{2}+3a^2c^2x}{x^3(1+ax)^{3/2}}\,dx}{3(c-acx)^{3/2}}$$

$$= -\frac{c^2(1-ax)^{3/2}}{3x^3\sqrt{1+ax}(c-acx)^{3/2}} + \frac{19ac^2(1-ax)^{3/2}}{12x^2\sqrt{1+ax}(c-acx)^{3/2}} + \frac{\left(119a^2c^2(1-ax)^{3/2}\right)\int \frac{1}{x^2(1+ax)^{3/2}}\,dx}{24(c-acx)^{3/2}}$$

$$= -\frac{c^2(1-ax)^{3/2}}{3x^3\sqrt{1+ax}(c-acx)^{3/2}} + \frac{19ac^2(1-ax)^{3/2}}{12x^2\sqrt{1+ax}(c-acx)^{3/2}} + \frac{119a^2c^2(1-ax)^{3/2}}{12x\sqrt{1+ax}(c-acx)^{3/2}} + \frac{\left(119a^2c^2\right.}{}$$

$$= -\frac{c^2(1-ax)^{3/2}}{3x^3\sqrt{1+ax}(c-acx)^{3/2}} + \frac{19ac^2(1-ax)^{3/2}}{12x^2\sqrt{1+ax}(c-acx)^{3/2}} + \frac{119a^2c^2(1-ax)^{3/2}}{12x\sqrt{1+ax}(c-acx)^{3/2}} - \frac{119a^2c^2(}{8x}$$

$$= -\frac{c^2(1-ax)^{3/2}}{3x^3\sqrt{1+ax}(c-acx)^{3/2}} + \frac{19ac^2(1-ax)^{3/2}}{12x^2\sqrt{1+ax}(c-acx)^{3/2}} + \frac{119a^2c^2(1-ax)^{3/2}}{12x\sqrt{1+ax}(c-acx)^{3/2}} - \frac{119a^2c^2(}{8x}$$

$$= -\frac{c^2(1-ax)^{3/2}}{3x^3\sqrt{1+ax}(c-acx)^{3/2}} + \frac{19ac^2(1-ax)^{3/2}}{12x^2\sqrt{1+ax}(c-acx)^{3/2}} + \frac{119a^2c^2(1-ax)^{3/2}}{12x\sqrt{1+ax}(c-acx)^{3/2}} - \frac{119a^2c^2(}{8x}$$

Mathematica [C] time = 0.0282059, size = 65, normalized size = 0.32

$$-\frac{c\sqrt{1-ax}\left(119a^3x^3\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},ax+1\right)-19ax+4\right)}{12x^3\sqrt{ax+1}\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] -(c*Sqrt[1 - a*x]*(4 - 19*a*x + 119*a^3*x^3*Hypergeometric2F1[-1/2, 2, 1/2,
 1 + a*x]))/(12*x^3*Sqrt[1 + a*x]*Sqrt[c - a*c*x])
```

Maple [A] time = 0.112, size = 111, normalized size = 0.5

$$-\frac{1}{(24\,ax-24)\,(ax+1)\,x^3}\sqrt{-c\,(ax-1)}\sqrt{-a^2x^2+1}\left(357\,\text{Artanh}\left(\frac{\sqrt{c\,(ax+1)}}{\sqrt{c}}\right)x^3a^3\sqrt{c\,(ax+1)}-357\,x^3a^3\sqrt{c}-119\,x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x)
```

```
[Out] -1/24*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(357*arctanh((c*(a*x+1))^(1/2)/
c^(1/2))*x^3*a^3*(c*(a*x+1))^(1/2)-357*x^3*a^3*c^(1/2)-119*x^2*a^2*c^(1/2)+
38*x*a*c^(1/2)-8*c^(1/2))/c^(1/2)/(a*x-1)/(a*x+1)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}}{\left(ax+1\right)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="m
axima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(-a*c*x + c)/((a*x + 1)^3*x^4), x)
```

Fricas [A] time = 1.94263, size = 591, normalized size = 2.87

$$\left[\frac{357\left(a^5x^5-a^3x^3\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)+2\left(357\,a^3x^3+119\,a^2x^2-38\,ax+8\right)\sqrt{-a^2x^2+1}\sqrt{-acx}}{48\left(a^2x^5-x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="f
ricas")
```

```
[Out] [1/48*(357*(a^5*x^5 - a^3*x^3)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a^
2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*(357*a^3*x^3 +
119*a^2*x^2 - 38*a*x + 8)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^2*x^5 - x
^3), 1/24*(357*(a^5*x^5 - a^3*x^3)*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(
-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (357*a^3*x^3 + 119*a^2*x^2 - 38*a*x
 + 8)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^2*x^5 - x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25029, size = 198, normalized size = 0.96

$$-\frac{1}{24}a^3c^2\left(\frac{357\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2}+\frac{192}{\sqrt{acx+c}c^2}+\frac{165(acx+c)^{\frac{5}{2}}-376(acx+c)^{\frac{3}{2}}c+219\sqrt{acx+c}c^2}{a^3c^5x^3}\right)|c|+\frac{\sqrt{2}\left(357\sqrt{2}a^3\sqrt{\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="g
iac")
```

```
[Out] -1/24*a^3*c^2*(357*arctan(sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c^2) + 192/(s
qrt(a*c*x + c)*c^2) + (165*(a*c*x + c)^(5/2) - 376*(a*c*x + c)^(3/2)*c + 21
9*sqrt(a*c*x + c)*c^2)/(a^3*c^5*x^3))*abs(c) + 1/48*sqrt(2)*(357*sqrt(2)*a^
3*sqrt(c)*abs(c)*arctan(sqrt(2)*sqrt(c)/sqrt(-c)) + 446*a^3*sqrt(-c)*abs(c)
)/(sqrt(-c)*sqrt(c))
```

3.436 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=249

$$-\frac{223a^2c^2(1-ax)^{3/2}}{96x^2\sqrt{ax+1}(c-acx)^{3/2}}+\frac{1115a^4c^2(1-ax)^{3/2}}{64\sqrt{ax+1}(c-acx)^{3/2}}+\frac{1115a^3c^2(1-ax)^{3/2}}{192x\sqrt{ax+1}(c-acx)^{3/2}}-\frac{1115a^4c^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{64(c-acx)^{3/2}}$$

```
[Out] (1115*a^4*c^2*(1 - a*x)^(3/2))/(64*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (c^2*
(1 - a*x)^(3/2))/(4*x^4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (25*a*c^2*(1 - a
*x)^(3/2))/(24*x^3*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (223*a^2*c^2*(1 - a*x
)^(3/2))/(96*x^2*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (1115*a^3*c^2*(1 - a*x)
^(3/2))/(192*x*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (1115*a^4*c^2*(1 - a*x)^(
3/2)*ArcTanh[Sqrt[1 + a*x]])/(64*(c - a*c*x)^(3/2))
```

Rubi [A] time = 0.154656, antiderivative size = 252, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6130, 23, 89, 78, 51, 63, 208}

$$-\frac{1115a^2c^2(1-ax)^{3/2}\sqrt{ax+1}}{96x^2(c-acx)^{3/2}}+\frac{223a^2c^2(1-ax)^{3/2}}{24x^2\sqrt{ax+1}(c-acx)^{3/2}}+\frac{1115a^3c^2(1-ax)^{3/2}\sqrt{ax+1}}{64x(c-acx)^{3/2}}-\frac{1115a^4c^2(1-ax)^{3/2}\tanh^{-1}}{64(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] -(c^2*(1 - a*x)^(3/2))/(4*x^4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (25*a*c^2*
(1 - a*x)^(3/2))/(24*x^3*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (223*a^2*c^2*(1
 - a*x)^(3/2))/(24*x^2*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (1115*a^2*c^2*(1
- a*x)^(3/2)*Sqrt[1 + a*x])/(96*x^2*(c - a*c*x)^(3/2)) + (1115*a^3*c^2*(1 -
 a*x)^(3/2)*Sqrt[1 + a*x])/(64*x*(c - a*c*x)^(3/2)) - (1115*a^4*c^2*(1 - a*
x)^(3/2)*ArcTanh[Sqrt[1 + a*x]])/(64*(c - a*c*x)^(3/2))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{(1-ax)^{3/2}\sqrt{c-acx}}{x^5(1+ax)^{3/2}}\,dx$$

$$= \frac{(1-ax)^{3/2}\int \frac{(c-acx)^2}{x^5(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}$$

$$= -\frac{c^2(1-ax)^{3/2}}{4x^4\sqrt{1+ax}(c-acx)^{3/2}} + \frac{(1-ax)^{3/2}\int \frac{-\frac{25ac^2}{2}+4a^2c^2x}{x^4(1+ax)^{3/2}}\,dx}{4(c-acx)^{3/2}}$$

$$= -\frac{c^2(1-ax)^{3/2}}{4x^4\sqrt{1+ax}(c-acx)^{3/2}} + \frac{25ac^2(1-ax)^{3/2}}{24x^3\sqrt{1+ax}(c-acx)^{3/2}} + \frac{\left(223a^2c^2(1-ax)^{3/2}\right)\int \frac{1}{x^3(1+ax)^{3/2}}}{48(c-acx)^{3/2}}$$

$$= -\frac{c^2(1-ax)^{3/2}}{4x^4\sqrt{1+ax}(c-acx)^{3/2}} + \frac{25ac^2(1-ax)^{3/2}}{24x^3\sqrt{1+ax}(c-acx)^{3/2}} + \frac{223a^2c^2(1-ax)^{3/2}}{24x^2\sqrt{1+ax}(c-acx)^{3/2}} + \frac{(111}{}$$

$$= -\frac{c^2(1-ax)^{3/2}}{4x^4\sqrt{1+ax}(c-acx)^{3/2}} + \frac{25ac^2(1-ax)^{3/2}}{24x^3\sqrt{1+ax}(c-acx)^{3/2}} + \frac{223a^2c^2(1-ax)^{3/2}}{24x^2\sqrt{1+ax}(c-acx)^{3/2}} - \frac{1115}{}$$

$$= -\frac{c^2(1-ax)^{3/2}}{4x^4\sqrt{1+ax}(c-acx)^{3/2}} + \frac{25ac^2(1-ax)^{3/2}}{24x^3\sqrt{1+ax}(c-acx)^{3/2}} + \frac{223a^2c^2(1-ax)^{3/2}}{24x^2\sqrt{1+ax}(c-acx)^{3/2}} - \frac{1115}{}$$

$$= -\frac{c^2(1-ax)^{3/2}}{4x^4\sqrt{1+ax}(c-acx)^{3/2}} + \frac{25ac^2(1-ax)^{3/2}}{24x^3\sqrt{1+ax}(c-acx)^{3/2}} + \frac{223a^2c^2(1-ax)^{3/2}}{24x^2\sqrt{1+ax}(c-acx)^{3/2}} - \frac{1115}{}$$

$$= -\frac{c^2(1-ax)^{3/2}}{4x^4\sqrt{1+ax}(c-acx)^{3/2}} + \frac{25ac^2(1-ax)^{3/2}}{24x^3\sqrt{1+ax}(c-acx)^{3/2}} + \frac{223a^2c^2(1-ax)^{3/2}}{24x^2\sqrt{1+ax}(c-acx)^{3/2}} - \frac{1115}{}$$

Mathematica [C] time = 0.0290418, size = 65, normalized size = 0.26

$$\frac{c\sqrt{1-ax}\left(223a^4x^4\text{Hypergeometric2F1}\left(-\frac{1}{2},3,\frac{1}{2},ax+1\right)+25ax-6\right)}{24x^4\sqrt{ax+1}\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] (c*Sqrt[1 - a*x]*(-6 + 25*a*x + 223*a^4*x^4*Hypergeometric2F1[-1/2, 3, 1/2,
 1 + a*x]))/(24*x^4*Sqrt[1 + a*x]*Sqrt[c - a*c*x])
```

Maple [A] time = 0.125, size = 122, normalized size = 0.5

$$\frac{1}{(192\,ax-192)\,(ax+1)\,x^4}\sqrt{-c\,(ax-1)}\sqrt{-a^2x^2+1}\left(3345\,\text{Artanh}\left(\frac{\sqrt{c\,(ax+1)}}{\sqrt{c}}\right)x^4a^4\sqrt{c\,(ax+1)}-3345\,x^4a^4\sqrt{c}-111\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x)
```

```
[Out] 1/192*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(3345*arctanh((c*(a*x+1))^(1/2)
/c^(1/2))*x^4*a^4*(c*(a*x+1))^(1/2)-3345*x^4*a^4*c^(1/2)-1115*x^3*a^3*c^(1/
2)+446*x^2*a^2*c^(1/2)-200*x*a*c^(1/2)+48*c^(1/2))/c^(1/2)/(a*x-1)/(a*x+1)/
```

```
x^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}}{\left(ax+1\right)^3x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="m
axima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(-a*c*x + c)/((a*x + 1)^3*x^5), x)
```

Fricas [A] time = 1.92656, size = 647, normalized size = 2.6

$$\left[\frac{3345\left(a^6x^6-a^4x^4\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)-2\left(3345\,a^4x^4+1115\,a^3x^3-446\,a^2x^2+200\,ax-48\right)\sqrt{-a^2}}{384\left(a^2x^6-x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="f
ricas")
```

```
[Out] [1/384*(3345*(a^6*x^6 - a^4*x^4)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-
a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) - 2*(3345*a^4*x^4
 + 1115*a^3*x^3 - 446*a^2*x^2 + 200*a*x - 48)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*
x + c))/(a^2*x^6 - x^4), -1/192*(3345*(a^6*x^6 - a^4*x^4)*sqrt(-c)*arctan(s
qrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (3345*a^4*x^
4 + 1115*a^3*x^3 - 446*a^2*x^2 + 200*a*x - 48)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c
*x + c))/(a^2*x^6 - x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32463, size = 216, normalized size = 0.87

$$\frac{1}{192}a^4c^3\left(\frac{3345\ \arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^3}+\frac{1536}{\sqrt{acx+c}c^3}+\frac{1809\left(acx+c\right)^{\frac{7}{2}}-6121\left(acx+c\right)^{\frac{5}{2}}c+7063\left(acx+c\right)^{\frac{3}{2}}c^2-2799\sqrt{acx+}}{a^4c^7x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="g
iac")
```

```
[Out] 1/192*a^4*c^3*(3345*arctan(sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c^3) + 1536/
(sqrt(a*c*x + c)*c^3) + (1809*(a*c*x + c)^(7/2) - 6121*(a*c*x + c)^(5/2)*c
+ 7063*(a*c*x + c)^(3/2)*c^2 - 2799*sqrt(a*c*x + c)*c^3)/(a^4*c^7*x^4))*abs
(c) - 1/384*sqrt(2)*(3345*sqrt(2)*a^4*sqrt(c)*abs(c)*arctan(sqrt(2)*sqrt(c)
/sqrt(-c)) + 4166*a^4*sqrt(-c)*abs(c))/(sqrt(-c)*sqrt(c))
```

3.437 $\int e^{-2p\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=61

$$-\frac{2^{-p}(1-ax)^p(c-acx)^{p+1}\text{Hypergeometric2F1}\left(p,2p+1,2(p+1),\frac{1}{2}(1-ax)\right)}{ac(2p+1)}$$

```
[Out] -(((1 - a*x)^p*(c - a*c*x)^(1 + p)*Hypergeometric2F1[p, 1 + 2*p, 2*(1 + p),
 (1 - a*x)/2])/(2^p*a*c*(1 + 2*p)))
```

Rubi [A] time = 0.0587585, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6130, 23, 69}

$$-\frac{2^{-p}(1-ax)^p(c-acx)^{p+1}\,{}_2F_1\left(p,2p+1;2(p+1);\frac{1}{2}(1-ax)\right)}{ac(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^p/E^(2*p*ArcTanh[a*x]),x]
```

```
[Out] -(((1 - a*x)^p*(c - a*c*x)^(1 + p)*Hypergeometric2F1[p, 1 + 2*p, 2*(1 + p),
 (1 - a*x)/2])/(2^p*a*c*(1 + 2*p)))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int e^{-2p\tanh^{-1}(ax)}(c-acx)^p\,dx &= \int(1-ax)^p(1+ax)^{-p}(c-acx)^p\,dx\\
&= \left((1-ax)^p(c-acx)^{-p}\right)\int(1+ax)^{-p}(c-acx)^{2p}\,dx\\
&= -\frac{2^{-p}(1-ax)^p(c-acx)^{1+p}\,{}_2F_1\left(p,1+2p;2(1+p);\frac{1}{2}(1-ax)\right)}{ac(1+2p)}
\end{aligned}$$

Mathematica [A] time = 0.0258814, size = 56, normalized size = 0.92

$$-\frac{2^{-p}(1-ax)^{p+1}(c-acx)^p\text{Hypergeometric2F1}\left(p,2p+1,2p+2,\frac{1}{2}-\frac{ax}{2}\right)}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^p/E^(2*p*ArcTanh[a*x]),x]
```

```
[Out] -(((1 - a*x)^(1 + p)*(c - a*c*x)^p*Hypergeometric2F1[p, 1 + 2*p, 2 + 2*p, 1
/2 - (a*x)/2])/(2^p*(a + 2*a*p)))
```

Maple [F] time = 0.343, size = 0, normalized size = 0.

$$\int \frac{(-acx+c)^p}{e^{2p\operatorname{Artanh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p/exp(2*p*arctanh(a*x)),x)
```

```
[Out] int((-a*c*x+c)^p/exp(2*p*arctanh(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-acx+c)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/exp(2*p*arctanh(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^p/((a*x + 1)/(a*x - 1))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-acx+c)^p}{\left(\frac{ax+1}{ax-1}\right)^p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/exp(2*p*arctanh(a*x)),x, algorithm="fricas")
```

```
[Out] integral((-a*c*x + c)^p/((a*x + 1)/(a*x - 1))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**p/exp(2*p*atanh(a*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-acx + c)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/exp(2*p*arctanh(a*x)),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^p/((a*x + 1)/(a*x - 1))^p, x)
```

3.438 $\int e^{2p\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=37

$$\frac{(1-ax)^{-p}(ax+1)^{p+1}(c-acx)^p}{a(p+1)}$$

```
[Out] ((1 + a*x)^(1 + p)*(c - a*c*x)^p)/(a*(1 + p)*(1 - a*x)^p)
```

Rubi [A] time = 0.0346184, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6130, 23, 32}

$$\frac{(1-ax)^{-p}(ax+1)^{p+1}(c-acx)^p}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*p*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] ((1 + a*x)^(1 + p)*(c - a*c*x)^p)/(a*(1 + p)*(1 - a*x)^p)
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int e^{2p\tanh^{-1}(ax)}(c-acx)^p\,dx &= \int (1-ax)^{-p}(1+ax)^p(c-acx)^p\,dx\\
&= \left((1-ax)^{-p}(c-acx)^p\right)\int (1+ax)^p\,dx\\
&= \frac{(1-ax)^{-p}(1+ax)^{1+p}(c-acx)^p}{a(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.0236089, size = 32, normalized size = 0.86

$$\frac{(ax+1)(c-acx)^p e^{2p\tanh^{-1}(ax)}}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*p*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (E^(2*p*ArcTanh[a*x])*(1 + a*x)*(c - a*c*x)^p)/(a*(1 + p))
```

Maple [A] time = 0.027, size = 32, normalized size = 0.9

$$\frac{(ax+1)\,e^{2p\mathrm{Artanh}(ax)}\,(-acx+c)^{p}}{a\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arctanh(a*x))*(-a*c*x+c)^p,x)
```

```
[Out] (a*x+1)/a/(1+p)*exp(2*p*arctanh(a*x))*(-a*c*x+c)^p
```

Maxima [A] time = 0.988049, size = 41, normalized size = 1.11

$$\frac{\left(a\,(-c)^{p}\,x+(-c)^{p}\right)(ax+1)^{p}}{a\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] (a*(-c)^p*x + (-c)^p)*(a*x + 1)^p/(a*(p + 1))
```

Fricas [A] time = 1.90134, size = 81, normalized size = 2.19

$$\frac{(ax+1)(-acx+c)^{p}\left(\frac{ax+1}{ax-1}\right)^{p}}{ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] (a*x + 1)*(-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^p/(a*p + a)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*atanh(a*x))*(-a*c*x+c)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx + c)^p \left(\frac{ax + 1}{ax - 1}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^p, x)
```

3.439 $\int e^{n\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=82

$$-\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}(c-acx)^{p+1}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2}+p+1,-\frac{n}{2}+p+2,\frac{1}{2}(1-ax)\right)}{ac(-n+2p+2)}$$

```
[Out] -((2^(1 + n/2)*(c - a*c*x)^(1 + p)*Hypergeometric2F1[-n/2, 1 - n/2 + p, 2 -
 n/2 + p, (1 - a*x)/2])/(a*c*(2 - n + 2*p)*(1 - a*x)^(n/2)))
```

Rubi [A] time = 0.0623685, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6130, 23, 69}

$$-\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}(c-acx)^{p+1}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2}+p+1;-\frac{n}{2}+p+2;\frac{1}{2}(1-ax)\right)}{ac(-n+2p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] -((2^(1 + n/2)*(c - a*c*x)^(1 + p)*Hypergeometric2F1[-n/2, 1 - n/2 + p, 2 -
 n/2 + p, (1 - a*x)/2])/(a*c*(2 - n + 2*p)*(1 - a*x)^(n/2)))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int e^{n\tanh^{-1}(ax)}(c-acx)^p\,dx &= \int (1-ax)^{-n/2}(1+ax)^{n/2}(c-acx)^p\,dx\\
&= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{-\frac{n}{2}+p}\,dx\\
&= -\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}(c-acx)^{1+p}\,{}_2F_1\left(-\frac{n}{2},1-\frac{n}{2}+p;2-\frac{n}{2}+p;\frac{1}{2}(1-ax)\right)}{ac(2-n+2p)}
\end{aligned}$$

Mathematica [A] time = 0.0240423, size = 77, normalized size = 0.94

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{1-\frac{n}{2}}(c-acx)^p\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2}+p+1,-\frac{n}{2}+p+2,\frac{1}{2}-\frac{ax}{2}\right)}{a(n-2(p+1))}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (2^(1 + n/2)*(1 - a*x)^(1 - n/2)*(c - a*c*x)^p*Hypergeometric2F1[-n/2, 1 -
n/2 + p, 2 - n/2 + p, 1/2 - (a*x)/2])/(a*(n - 2*(1 + p)))
```

Maple [F] time = 0.261, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}(-acx+c)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a*c*x+c)^p,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a*c*x+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx+c)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-acx+c)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] integral((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-c(ax-1))^p e^{n\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a*c*x+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1))**p*exp(n*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx+c)^p \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.440 $\int e^{n\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=68

$$-\frac{c^3 2^{\frac{n}{2}+1}(1-ax)^{4-\frac{n}{2}}\text{Hypergeometric2F1}\left(4-\frac{n}{2},-\frac{n}{2},5-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a(8-n)}$$

```
[Out] -((2^(1 + n/2)*c^3*(1 - a*x)^(4 - n/2)*Hypergeometric2F1[4 - n/2, -n/2, 5 -
 n/2, (1 - a*x)/2])/(a*(8 - n)))
```

Rubi [A] time = 0.0469197, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 69}

$$-\frac{c^3 2^{\frac{n}{2}+1}(1-ax)^{4-\frac{n}{2}}\,{}_2F_1\left(4-\frac{n}{2},-\frac{n}{2};5-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(8-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a*c*x)^3,x]
```

```
[Out] -((2^(1 + n/2)*c^3*(1 - a*x)^(4 - n/2)*Hypergeometric2F1[4 - n/2, -n/2, 5 -
 n/2, (1 - a*x)/2])/(a*(8 - n)))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}(c-acx)^3\,dx &= c^3\int (1-ax)^{3-\frac{n}{2}}(1+ax)^{n/2}\,dx\\ &= -\frac{2^{1+\frac{n}{2}}c^3(1-ax)^{4-\frac{n}{2}}\,{}_2F_1\left(4-\frac{n}{2},-\frac{n}{2};5-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(8-n)}\end{aligned}$$

Mathematica [A] time = 0.0196658, size = 65, normalized size = 0.96

$$\frac{c^3 2^{\frac{n}{2}+1}(1-ax)^{4-\frac{n}{2}}\text{Hypergeometric2F1}\left(4-\frac{n}{2},-\frac{n}{2},5-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a(n-8)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (2^(1 + n/2)*c^3*(1 - a*x)^(4 - n/2)*Hypergeometric2F1[4 - n/2, -n/2, 5 - n
/2, (1 - a*x)/2])/(a*(-8 + n))
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int e^{n \operatorname{Artanh}(ax)} (-acx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a*c*x+c)^3,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a*c*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int (acx - c)^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate((a*c*x - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\left(a^3c^3x^3 - 3a^2c^3x^2 + 3ac^3x - c^3\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^3*c^3*x^3 - 3*a^2*c^3*x^2 + 3*a*c^3*x - c^3)*((a*x + 1)/(a*x -
 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^3\left(\int 3axe^{n \operatorname{atanh}(ax)}\, dx + \int -3a^2x^2e^{n \operatorname{atanh}(ax)}\, dx + \int a^3x^3e^{n \operatorname{atanh}(ax)}\, dx + \int -e^{n \operatorname{atanh}(ax)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a*c*x+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a*x*exp(n*atanh(a*x)), x) + Integral(-3*a**2*x**2*exp(n*a
tanh(a*x)), x) + Integral(a**3*x**3*exp(n*atanh(a*x)), x) + Integral(-exp(n
*atanh(a*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -(acx-c)^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a*c*x - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.441 $\int e^{n\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=68

$$-\frac{c^2 2^{\frac{n}{2}+1}(1-ax)^{3-\frac{n}{2}}\text{Hypergeometric2F1}\left(3-\frac{n}{2},-\frac{n}{2},4-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a(6-n)}$$

```
[Out] -((2^(1 + n/2)*c^2*(1 - a*x)^(3 - n/2)*Hypergeometric2F1[3 - n/2, -n/2, 4 -
 n/2, (1 - a*x)/2])/(a*(6 - n)))
```

Rubi [A] time = 0.0489045, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 69}

$$-\frac{c^2 2^{\frac{n}{2}+1}(1-ax)^{3-\frac{n}{2}}\,{}_2F_1\left(3-\frac{n}{2},-\frac{n}{2};4-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(6-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a*c*x)^2,x]
```

```
[Out] -((2^(1 + n/2)*c^2*(1 - a*x)^(3 - n/2)*Hypergeometric2F1[3 - n/2, -n/2, 4 -
 n/2, (1 - a*x)/2])/(a*(6 - n)))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}(c-acx)^2\,dx &= c^2\int (1-ax)^{2-\frac{n}{2}}(1+ax)^{n/2}\,dx\\ &= -\frac{2^{1+\frac{n}{2}}c^2(1-ax)^{3-\frac{n}{2}}\,{}_2F_1\left(3-\frac{n}{2},-\frac{n}{2};4-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(6-n)}\end{aligned}$$

Mathematica [A] time = 0.0172727, size = 65, normalized size = 0.96

$$\frac{c^2 2^{\frac{n}{2}+1}(1-ax)^{3-\frac{n}{2}}\text{Hypergeometric2F1}\left(3-\frac{n}{2},-\frac{n}{2},4-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a(n-6)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (2^(1 + n/2)*c^2*(1 - a*x)^(3 - n/2)*Hypergeometric2F1[3 - n/2, -n/2, 4 - n
/2, (1 - a*x)/2])/(a*(-6 + n))
```

Maple [F] time = 0.159, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{Artanh}(ax)} (-acx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a*c*x+c)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a*c*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (acx - c)^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*c*x - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2c^2x^2 - 2\,ac^2x + c^2\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int -2axe^{n\,\mathrm{atanh}\,(ax)}\,dx + \int a^2x^2e^{n\,\mathrm{atanh}\,(ax)}\,dx + \int e^{n\,\mathrm{atanh}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a*c*x+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a*x*exp(n*atanh(a*x)), x) + Integral(a**2*x**2*exp(n*atan
h(a*x)), x) + Integral(exp(n*atanh(a*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (acx-c)^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*c*x - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.442 $\int e^{n\tanh^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=66

$$-\frac{c2^{\frac{n}{2}+1}(1-ax)^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(2-\frac{n}{2},-\frac{n}{2},3-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a(4-n)}$$

```
[Out] -((2^(1 + n/2)*c*(1 - a*x)^(2 - n/2)*Hypergeometric2F1[2 - n/2, -n/2, 3 - n
/2, (1 - a*x)/2])/(a*(4 - n)))
```

Rubi [A] time = 0.0376154, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6129, 69}

$$-\frac{c2^{\frac{n}{2}+1}(1-ax)^{2-\frac{n}{2}}\,{}_2F_1\left(2-\frac{n}{2},-\frac{n}{2};3-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(4-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a*c*x),x]
```

```
[Out] -((2^(1 + n/2)*c*(1 - a*x)^(2 - n/2)*Hypergeometric2F1[2 - n/2, -n/2, 3 - n
/2, (1 - a*x)/2])/(a*(4 - n)))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}(c-acx)\,dx &= c\int (1-ax)^{1-\frac{n}{2}}(1+ax)^{n/2}\,dx\\ &= -\frac{2^{1+\frac{n}{2}}c(1-ax)^{2-\frac{n}{2}}\,{}_2F_1\left(2-\frac{n}{2},-\frac{n}{2};3-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(4-n)}\end{aligned}$$

Mathematica [A] time = 0.0166896, size = 63, normalized size = 0.95

$$\frac{c2^{\frac{n}{2}+1}(1-ax)^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(2-\frac{n}{2},-\frac{n}{2},3-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a(n-4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a*c*x),x]
```

```
[Out] (2^(1 + n/2)*c*(1 - a*x)^(2 - n/2)*Hypergeometric2F1[2 - n/2, -n/2, 3 - n/2
, (1 - a*x)/2])/(a*(-4 + n))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int e^{n\operatorname{Artanh}(ax)} (-acx + c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a*c*x+c),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a*c*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int (acx - c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -integrate((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-(acx - c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-(a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int axe^{n\operatorname{atanh}(ax)}\, dx + \int -e^{n\operatorname{atanh}(ax)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a*c*x+c),x)
```

```
[Out] -c*(Integral(a*x*exp(n*atanh(a*x)), x) + Integral(-exp(n*atanh(a*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -(acx - c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-(a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.443 $\int \frac{e^{n \tanh^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=59

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{acn}$$

```
[Out] (2^(1 + n/2)*Hypergeometric2F1[-n/2, -n/2, 1 - n/2, (1 - a*x)/2])/(a*c*n*(1
 - a*x)^(n/2))
```

Rubi [A] time = 0.0430239, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 69}

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a*c*x),x]
```

```
[Out] (2^(1 + n/2)*Hypergeometric2F1[-n/2, -n/2, 1 - n/2, (1 - a*x)/2])/(a*c*n*(1
 - a*x)^(n/2))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n \tanh^{-1}(ax)}}{c-acx}\,dx &= \frac{\int (1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c}\\ &= \frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{acn}\end{aligned}$$

Mathematica [A] time = 0.0101542, size = 59, normalized size = 1.

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{acn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a*c*x),x]
```

```
[Out] (2^(1 + n/2)*Hypergeometric2F1[-n/2, -n/2, 1 - n/2, (1 - a*x)/2])/(a*c*n*(1
 - a*x)^(n/2))
```

Maple [F] time = 0.204, size = 0, normalized size = 0.

$$\int \frac{e^{n\operatorname{Artanh}(ax)}}{-acx + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a*c*x+c),x)
```

```
[Out] int(exp(n*arctanh(a*x))/(-a*c*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx - c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx - c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{e^{n\operatorname{atanh}(ax)}}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a*c*x+c),x)
```

```
[Out] -Integral(exp(n*atanh(a*x))/(a*x - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

3.444 $\int \frac{e^{n \tanh^{-1}(ax)}}{(c-acx)^2} dx$

Optimal. Leaf size=39

$$\frac{(1-ax)^{-\frac{n}{2}-1}(ax+1)^{\frac{n+2}{2}}}{ac^2(n+2)}$$

```
[Out] ((1 - a*x)^(-1 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^2*(2 + n))
```

Rubi [A] time = 0.0389181, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 37}

$$\frac{(1-ax)^{-\frac{n}{2}-1}(ax+1)^{\frac{n+2}{2}}}{ac^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a*c*x)^2,x]
```

```
[Out] ((1 - a*x)^(-1 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^2*(2 + n))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{n \tanh^{-1}(ax)}}{(c-acx)^2} dx &= \frac{\int (1-ax)^{-2-\frac{n}{2}}(1+ax)^{n/2} dx}{c^2} \\ &= \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^2(2+n)} \end{aligned}$$

Mathematica [A] time = 0.0191819, size = 39, normalized size = 1.

$$\frac{(1-ax)^{-\frac{n}{2}-1}(ax+1)^{\frac{n}{2}+1}}{ac^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a*c*x)^2,x]
```

```
[Out] ((1 - a*x)^(-1 - n/2)*(1 + a*x)^(1 + n/2))/(a*c^2*(2 + n))
```

Maple [A] time = 0.033, size = 33, normalized size = 0.9

$$-\frac{e^{n \operatorname{Artanh}(ax)} (ax+1)}{(ax-1)\, c^2 (2+n)\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a*c*x+c)^2,x)
```

```
[Out] -exp(n*arctanh(a*x))*(a*x+1)/(a*x-1)/c^2/(2+n)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^2, x)
```

Fricas [A] time = 1.92778, size = 120, normalized size = 3.08

$$\frac{(ax+1)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^2n + 2\,ac^2 - \left(a^2c^2n + 2\,a^2c^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] (a*x + 1)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n + 2*a*c^2 - (a^2*c^2*n + 2
*a^2*c^2)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a*c*x+c)**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.11933, size = 43, normalized size = 1.1

$$-\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n+1}}{ac^2(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -((a*x + 1)/(a*x - 1))^(1/2*n + 1)/(a*c^2*(n + 2))
```

3.445 $\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=84

$$\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-1}}{ac^3\left(n^2+6n+8\right)}+\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-2}}{ac^3(n+4)}$$

```
[Out] ((1 - a*x)^(-2 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^3*(4 + n)) + ((1 - a*x)^(
-1 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^3*(8 + 6*n + n^2))
```

Rubi [A] time = 0.0558024, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 45, 37}

$$\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-1}}{ac^3\left(n^2+6n+8\right)}+\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-2}}{ac^3(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a*c*x)^3,x]
```


```
[Out] ((1 - a*x)^(-2 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^3*(4 + n)) + ((1 - a*x)^(
-1 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^3*(8 + 6*n + n^2))
```


Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\int (1-ax)^{-3-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c^3}$$
$$= \frac{(1-ax)^{-2-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^3(4+n)} + \frac{\int (1-ax)^{-2-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c^3(4+n)}$$
$$= \frac{(1-ax)^{-2-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^3(4+n)} + \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^3(2+n)(4+n)}$$

Mathematica [A] time = 0.0311324, size = 51, normalized size = 0.61

$$\frac{(1-ax)^{-\frac{n}{2}-2}(-ax+n+3)(ax+1)^{\frac{n}{2}+1}}{ac^3(n+2)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a*c*x)^3,x]
```

```
[Out] ((1 - a*x)^(-2 - n/2)*(3 + n - a*x)*(1 + a*x)^(1 + n/2))/(a*c^3*(2 + n)*(4
+ n))
```

Maple [A] time = 0.032, size = 46, normalized size = 0.6

$$-\frac{e^{n\operatorname{Artanh}(ax)}(ax-n-3)(ax+1)}{(ax-1)^2c^3\left(n^2+6n+8\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a*c*x+c)^3,x)
```

```
[Out] -exp(n*arctanh(a*x))*(a*x-n-3)*(a*x+1)/(a*x-1)^2/c^3/(n^2+6*n+8)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^3, x)
```

Fricas [A] time = 2.25484, size = 259, normalized size = 3.08

$$-\frac{\left(a^2x^2-(an+2a)x-n-3\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^3n^2+6ac^3n+8ac^3+\left(a^3c^3n^2+6a^3c^3n+8a^3c^3\right)x^2-2\left(a^2c^3n^2+6a^2c^3n+8a^2c^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(a^2*x^2 - (a*n + 2*a)*x - n - 3)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^2
 + 6*a*c^3*n + 8*a*c^3 + (a^3*c^3*n^2 + 6*a^3*c^3*n + 8*a^3*c^3)*x^2 - 2*(a
^2*c^3*n^2 + 6*a^2*c^3*n + 8*a^2*c^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a*c*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^3, x)
```

3.446 $\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=135

$$\frac{2(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-1}}{ac^4(n+6)\left(n^2+6n+8\right)}+\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-3}}{ac^4(n+6)}+\frac{2(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-2}}{ac^4(n+4)(n+6)}$$

```
[Out] ((1 - a*x)^(-3 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^4*(6 + n)) + (2*(1 - a*x)
^(-2 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^4*(4 + n)*(6 + n)) + (2*(1 - a*x)^(
-1 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^4*(6 + n)*(8 + 6*n + n^2))
```

Rubi [A] time = 0.0807233, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 45, 37}

$$\frac{2(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-1}}{ac^4(n+6)\left(n^2+6n+8\right)}+\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-3}}{ac^4(n+6)}+\frac{2(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-2}}{ac^4(n+4)(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a*c*x)^4,x]
```

```
[Out] ((1 - a*x)^(-3 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^4*(6 + n)) + (2*(1 - a*x)
^(-2 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^4*(4 + n)*(6 + n)) + (2*(1 - a*x)^(
-1 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^4*(6 + n)*(8 + 6*n + n^2))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int (1-ax)^{-4-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c^4}$$
$$= \frac{(1-ax)^{-3-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^4(6+n)} + \frac{2\int (1-ax)^{-3-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c^4(6+n)}$$
$$= \frac{(1-ax)^{-3-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^4(6+n)} + \frac{2(1-ax)^{-2-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^4(4+n)(6+n)} + \frac{2\int (1-ax)^{-2-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c^4(4+n)(6+n)}$$
$$= \frac{(1-ax)^{-3-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^4(6+n)} + \frac{2(1-ax)^{-2-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^4(4+n)(6+n)} + \frac{2(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^4(2+n)(4+n)(6+n)}$$

Mathematica [A] time = 0.0438406, size = 74, normalized size = 0.55

$$\frac{(1-ax)^{-\frac{n}{2}-3}(ax+1)^{\frac{n}{2}+1}\left(2a^2x^2-2anx-8ax+n^2+8n+14\right)}{ac^4(n+2)(n+4)(n+6)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a*c*x)^4,x]
```

```
[Out] ((1 - a*x)^(-3 - n/2)*(1 + a*x)^(1 + n/2)*(14 + 8*n + n^2 - 8*a*x - 2*a*n*x
 + 2*a^2*x^2))/(a*c^4*(2 + n)*(4 + n)*(6 + n))
```

Maple [A] time = 0.04, size = 68, normalized size = 0.5

$$-\frac{\left(2\,a^2x^2-2\,anx-8\,ax+n^2+8\,n+14\right)(ax+1)\,e^{n\,\mathrm{Artanh}(ax)}}{(ax-1)^3\,c^4a\left(n^2+8\,n+12\right)(4+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a*c*x+c)^4,x)
```

```
[Out] -(a*x+1)*(2*a^2*x^2-2*a*n*x-8*a*x+n^2+8*n+14)*exp(n*arctanh(a*x))/(a*x-1)^3
/c^4/a/(n^2+8*n+12)/(4+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^4, x)
```

Fricas [A] time = 2.26367, size = 483, normalized size = 3.58

$$\frac{\left(2\,a^{3}x^{3}-2\left(a^{2}n+3\,a^{2}\right)x^{2}+n^{2}+\left(an^{2}+6\,an+6\,a\right)x+8\,n+14\right)}{ac^{4}n^{3}+12\,ac^{4}n^{2}+44\,ac^{4}n+48\,ac^{4}-\left(a^{4}c^{4}n^{3}+12\,a^{4}c^{4}n^{2}+44\,a^{4}c^{4}n+48\,a^{4}c^{4}\right)x^{3}+3\left(a^{3}c^{4}n^{3}+12\,a^{3}c^{4}n^{2}+44\,a^{3}c^{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] (2*a^3*x^3 - 2*(a^2*n + 3*a^2)*x^2 + n^2 + (a*n^2 + 6*a*n + 6*a)*x + 8*n +
14)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^4*n^3 + 12*a*c^4*n^2 + 44*a*c^4*n +
48*a*c^4 - (a^4*c^4*n^3 + 12*a^4*c^4*n^2 + 44*a^4*c^4*n + 48*a^4*c^4)*x^3 +
 3*(a^3*c^4*n^3 + 12*a^3*c^4*n^2 + 44*a^3*c^4*n + 48*a^3*c^4)*x^2 - 3*(a^2*
c^4*n^3 + 12*a^2*c^4*n^2 + 44*a^2*c^4*n + 48*a^2*c^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a*c*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(acx-c)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^4, x)
```

3.447 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=60

$$\frac{x(1-ax)^{-p}F_1\left(1-p;\frac{1}{2}-p,-\frac{1}{2};2-p;ax,-ax\right)\left(c-\frac{c}{ax}\right)^p}{1-p}$$

```
[Out] ((c - c/(a*x))^p*x*AppellF1[1 - p, 1/2 - p, -1/2, 2 - p, a*x, -(a*x)])/((1
- p)*(1 - a*x)^p)
```

Rubi [A] time = 0.101507, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6134, 6129, 133}

$$\frac{x(1-ax)^{-p}F_1\left(1-p;\frac{1}{2}-p,-\frac{1}{2};2-p;ax,-ax\right)\left(c-\frac{c}{ax}\right)^p}{1-p}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x))^p,x]
```

```
[Out] ((c - c/(a*x))^p*x*AppellF1[1 - p, 1/2 - p, -1/2, 2 - p, a*x, -(a*x)])/((1
- p)*(1 - a*x)^p)
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx &= \left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int e^{\tanh^{-1}(ax)}x^{-p}(1-ax)^p\,dx\\
&= \left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int x^{-p}(1-ax)^{-\frac{1}{2}+p}\sqrt{1+ax}\,dx\\
&= \frac{\left(c-\frac{c}{ax}\right)^p x(1-ax)^{-p}F_1\left(1-p;\frac{1}{2}-p,-\frac{1}{2};2-p;ax,-ax\right)}{1-p}
\end{aligned}$$

Mathematica [F] time = 0.802792, size = 0, normalized size = 0.

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^p,x]
```

```
[Out] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^p, x]
```

Maple [F] time = 0.369, size = 0, normalized size = 0.

$$\int (ax+1)\left(c-\frac{c}{ax}\right)^p\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^p,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^p/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(\frac{acx-c}{ax}\right)^p}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^p,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*((a*c*x - c)/(a*x))^p/(a*x - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^p(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^p/sqrt(-a^2*x^2 + 1), x)
```

3.448 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=125

$$\frac{3c^4\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3}-\frac{c^4(6-ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{3c^4\sin^{-1}(ax)}{a}$$

```
[Out] -(c^4*(6 - a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^4*(1 - a^2*x^2)^(3/2))/(3
*a^4*x^3) + (3*c^4*(1 - a^2*x^2)^(3/2))/(2*a^3*x^2) - (3*c^4*ArcSin[a*x])/a
 - (c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rubi [A] time = 0.255857, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6131, 6128, 1807, 813, 844, 216, 266, 63, 208}

$$\frac{3c^4\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3}-\frac{c^4(6-ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{3c^4\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x))^4,x]
```

```
[Out] -(c^4*(6 - a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^4*(1 - a^2*x^2)^(3/2))/(3
*a^4*x^3) + (3*c^4*(1 - a^2*x^2)^(3/2))/(2*a^3*x^2) - (3*c^4*ArcSin[a*x])/a
 - (c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
```

```
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\int \frac{e^{\tanh^{-1}(ax)}(1-ax)^4}{x^4}dx}{a^4}$$
$$= \frac{c^4\int \frac{(1-ax)^3\sqrt{1-a^2x^2}}{x^4}dx}{a^4}$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} - \frac{c^4\int \frac{\sqrt{1-a^2x^2}\left(9a-9a^2x+3a^3x^2\right)}{x^3}dx}{3a^4}$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^4\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} + \frac{c^4\int \frac{\left(18a^2+3a^3x\right)\sqrt{1-a^2x^2}}{x^2}dx}{6a^4}$$
$$= -\frac{c^4(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^4\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{c^4\int \frac{-6a^3+36a^4x}{x\sqrt{1-a^2x^2}}dx}{12a^4}$$
$$= -\frac{c^4(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^4\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \left(3c^4\right)\int \frac{1}{\sqrt{1-a^2x^2}}dx + \cdots$$
$$= -\frac{c^4(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^4\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{3c^4\sin^{-1}(ax)}{a} + \frac{c^4\,\mathrm{Subst}\cdots}{\cdots}$$
$$= -\frac{c^4(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^4\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{3c^4\sin^{-1}(ax)}{a} - \frac{c^4\,\mathrm{Subst}\cdots}{\cdots}$$
$$= -\frac{c^4(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^4\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{3c^4\sin^{-1}(ax)}{a} - \frac{c^4\tanh^{-1}\cdots}{\cdots}$$

Mathematica [A] time = 0.241563, size = 86, normalized size = 0.69

$$\frac{c^4\left(-\frac{\sqrt{1-a^2x^2}\left(6a^3x^3+16a^2x^2-9ax+2\right)}{a^3x^3}-3\log\left(\sqrt{1-a^2x^2}+1\right)+3\log(ax)-18\sin^{-1}(ax)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^4,x]
```

```
[Out] (c^4*(-((Sqrt[1 - a^2*x^2]*(2 - 9*a*x + 16*a^2*x^2 + 6*a^3*x^3))/(a^3*x^3))
 - 18*ArcSin[a*x] + 3*Log[a*x] - 3*Log[1 + Sqrt[1 - a^2*x^2]]))/(6*a)
```

Maple [A] time = 0.044, size = 142, normalized size = 1.1

$$-\frac{c^4}{a}\sqrt{-a^2x^2+1}-3\frac{c^4}{\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)-\frac{8c^4}{3a^2x}\sqrt{-a^2x^2+1}-\frac{c^4}{2a}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{3c^4}{2x^2a^3}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^4,x)
```

```
[Out] -c^4*(-a^2*x^2+1)^(1/2)/a-3*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))-8/3*c^4*(-a^2*x^2+1)^(1/2)/a^2/x-1/2*c^4/a*arctanh(1/(-a^2*x^2+1)
^(1/2))+3/2*c^4*(-a^2*x^2+1)^(1/2)/x^2/a^3-1/3*c^4*(-a^2*x^2+1)^(1/2)/a^4/x
^3
```

Maxima [A] time = 1.46655, size = 269, normalized size = 2.15

$$-\frac{3\,c^4\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}-\frac{2\,c^4\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)}{a}-\frac{\sqrt{-a^2x^2+1}c^4}{a}+\frac{3\left(a^2\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{\sqrt{-a^2x^2+1}}{x^2}\right)c^4}{2\,a^3}-\frac{2\,\sqrt{-a^2x^2}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -3*c^4*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - 2*c^4*log(2*sqrt(-a^2*x^2 + 1)/a
bs(x) + 2/abs(x))/a - sqrt(-a^2*x^2 + 1)*c^4/a + 3/2*(a^2*log(2*sqrt(-a^2*x
^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*x^2 + 1)/x^2)*c^4/a^3 - 2*sqrt(-a^2*
x^2 + 1)*c^4/(a^2*x) - 1/3*(2*sqrt(-a^2*x^2 + 1)*a^2/x + sqrt(-a^2*x^2 + 1)
/x^3)*c^4/a^4
```

Fricas [A] time = 2.17275, size = 282, normalized size = 2.26

$$\frac{36\,a^3c^4x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+3\,a^3c^4x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-6\,a^3c^4x^3-\left(6\,a^3c^4x^3+16\,a^2c^4x^2-9\,ac^4x+2\,c^4\right)\sqrt{-a^2x^2+1}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(36*a^3*c^4*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 3*a^3*c^4*x^3*
log((sqrt(-a^2*x^2 + 1) - 1)/x) - 6*a^3*c^4*x^3 - (6*a^3*c^4*x^3 + 16*a^2*c
^4*x^2 - 9*a*c^4*x + 2*c^4)*sqrt(-a^2*x^2 + 1))/(a^4*x^3)
```

Sympy [A] time = 11.2067, size = 357, normalized size = 2.86

$$ac^4\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-3c^4\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)+\frac{2c^4\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}+\frac{2c^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**4,x)
```

```
[Out] a*c**4*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
 - 3*c**4*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/
a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + 2*c**4*Piecewise((-acosh(1/(a*x)),
 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a + 2*c**4*Piecewise((-I*s
qrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))
/a**2 - 3*c**4*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2
))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1
- 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3 + c**
```

```
4*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3
*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a*
*2*x**2 + 1)/(3*x**3), True))/a**4
```

Giac [B] time = 1.17919, size = 355, normalized size = 2.84

$$\frac{\left(c^4 - \frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)c^4}{a^2x} + \frac{33\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^4}{a^4x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|} - \frac{3\,c^4\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{|a|} - \frac{c^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|} - \frac{\sqrt{-a^2x^2+1}\cdots}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] 1/24*(c^4 - 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/(a^2*x) + 33*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^2*c^4/(a^4*x^2))*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*abs(a)) - 3*c^4*arcsin(a*x)*sgn(a)/abs(a) - 1/2*c^4*log(1/2*abs(-2*sqr
t(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c^4
/a - 1/24*(33*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/x - 9*(sqrt(-a^2*x^2 + 1)
*abs(a) + a)^2*c^4/(a^2*x^2) + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^4/(a^4*x
^3))/(a^2*abs(a))
```

3.449 $\int e^{\tanh^{-1}(ax)} \left(c - \frac{c}{ax}\right)^3 dx$

Optimal. Leaf size=97

$$\frac{c^3 \left(1 - a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{c^3(ax+4)\sqrt{1-a^2x^2}}{2a^2x} + \frac{c^3 \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a} - \frac{2c^3 \sin^{-1}(ax)}{a}$$

```
[Out] -(c^3*(4 + a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) + (c^3*(1 - a^2*x^2)^(3/2))/(2
*a^3*x^2) - (2*c^3*ArcSin[a*x])/a + (c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rubi [A] time = 0.191305, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6131, 6128, 1807, 813, 844, 216, 266, 63, 208}

$$\frac{c^3 \left(1 - a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{c^3(ax+4)\sqrt{1-a^2x^2}}{2a^2x} + \frac{c^3 \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a} - \frac{2c^3 \sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x))^3,x]
```

```
[Out] -(c^3*(4 + a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) + (c^3*(1 - a^2*x^2)^(3/2))/(2
*a^3*x^2) - (2*c^3*ArcSin[a*x])/a + (c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
```

```
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3dx = -\frac{c^3\int\frac{e^{\tanh^{-1}(ax)}(1-ax)^3}{x^3}dx}{a^3}$$
$$= -\frac{c^3\int\frac{(1-ax)^2\sqrt{1-a^2x^2}}{x^3}dx}{a^3}$$
$$= \frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}+\frac{c^3\int\frac{\left(4a-a^2x\right)\sqrt{1-a^2x^2}}{x^2}dx}{2a^3}$$
$$= -\frac{c^3(4+ax)\sqrt{1-a^2x^2}}{2a^2x}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{c^3\int\frac{2a^2+8a^3x}{x\sqrt{1-a^2x^2}}dx}{4a^3}$$
$$= -\frac{c^3(4+ax)\sqrt{1-a^2x^2}}{2a^2x}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\left(2c^3\right)\int\frac{1}{\sqrt{1-a^2x^2}}dx-\frac{c^3\int\frac{1}{x\sqrt{1-a^2x^2}}dx}{2a}$$
$$= -\frac{c^3(4+ax)\sqrt{1-a^2x^2}}{2a^2x}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{2c^3\sin^{-1}(ax)}{a}-\frac{c^3\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-a^2x}}dx,x,x^2\right)}{4a}$$
$$= -\frac{c^3(4+ax)\sqrt{1-a^2x^2}}{2a^2x}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{2c^3\sin^{-1}(ax)}{a}+\frac{c^3\,\text{Subst}\left(\int\frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}dx,x,\sqrt{1-a^2x^2}\right)}{2a^3}$$
$$= -\frac{c^3(4+ax)\sqrt{1-a^2x^2}}{2a^2x}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{2c^3\sin^{-1}(ax)}{a}+\frac{c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}$$

Mathematica [A] time = 0.185543, size = 75, normalized size = 0.77

$$\frac{c^3\left(\frac{\sqrt{1-a^2x^2}\left(-2a^2x^2-4ax+1\right)}{a^2x^2}+\log\left(\sqrt{1-a^2x^2}+1\right)-\log(ax)-4\sin^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^3,x]
```

```
[Out] (c^3*(((1 - 4*a*x - 2*a^2*x^2)*Sqrt[1 - a^2*x^2])/(a^2*x^2) - 4*ArcSin[a*x]
 - Log[a*x] + Log[1 + Sqrt[1 - a^2*x^2]]))/(2*a)
```

Maple [A] time = 0.04, size = 119, normalized size = 1.2

$$-\frac{c^3}{a}\sqrt{-a^2x^2+1}-2\frac{c^3}{\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)-2\frac{c^3\sqrt{-a^2x^2+1}}{a^2x}+\frac{c^3}{2x^2a^3}\sqrt{-a^2x^2+1}+\frac{c^3}{2a}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^3,x)
```

```
[Out] -c^3*(-a^2*x^2+1)^(1/2)/a-2*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))-2*c^3*(-a^2*x^2+1)^(1/2)/a^2/x+1/2*c^3*(-a^2*x^2+1)^(1/2)/x^2/a^3
+1/2*c^3/a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.46918, size = 162, normalized size = 1.67

$$-\frac{2\,c^3\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}-\frac{\sqrt{-a^2x^2+1}c^3}{a}+\frac{\left(a^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{\sqrt{-a^2x^2+1}}{x^2}\right)c^3}{2\,a^3}-\frac{2\sqrt{-a^2x^2+1}c^3}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -2*c^3*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - sqrt(-a^2*x^2 + 1)*c^3/a + 1/2*(
a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*x^2 + 1)/x^2)*c
^3/a^3 - 2*sqrt(-a^2*x^2 + 1)*c^3/(a^2*x)
```

Fricas [A] time = 2.25605, size = 252, normalized size = 2.6

$$\frac{8\,a^2c^3x^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-a^2c^3x^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-2\,a^2c^3x^2-\left(2\,a^2c^3x^2+4\,ac^3x-c^3\right)\sqrt{-a^2x^2+1}}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(8*a^2*c^3*x^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - a^2*c^3*x^2*log
((sqrt(-a^2*x^2 + 1) - 1)/x) - 2*a^2*c^3*x^2 - (2*a^2*c^3*x^2 + 4*a*c^3*x -
 c^3)*sqrt(-a^2*x^2 + 1))/(a^3*x^2)
```

Sympy [A] time = 6.93348, size = 228, normalized size = 2.35

$$ac^3\left(\begin{cases}\frac{x^2}{2} & \text{for}\: a^2=0\\-\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-2c^3\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for}\: a^2>0\\\sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\: a^2<0\end{cases}\right)+\frac{2c^3\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\: \left|a^2x^2\right|>1\\-\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)}{a^2}-\frac{c^3 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**3,x)
```

```
[Out] a*c**3*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
 - 2*c**3*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/
a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + 2*c**3*Piecewise((-I*sqrt(a**2*x**
2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a**2 - c**3
*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Ab
s(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2
))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3
```

Giac [B] time = 1.20748, size = 279, normalized size = 2.88

$$-\frac{\left(c^{3}-\frac{8\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)c^{3}}{a^{2}x}\right)a^{4}x^{2}}{8\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)^{2}|a|}-\frac{2\,c^{3}\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{|a|}+\frac{c^{3}\log\left(\frac{\left|-2\,\sqrt{-a^{2}x^{2}+1}|a|-2\,a\right|}{2\,a^{2}|x|}\right)}{2\,|a|}-\frac{\sqrt{-a^{2}x^{2}+1}c^{3}}{a}-\frac{\frac{8\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)c^{3}|}{a^{2}x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -1/8*(c^3 - 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3/(a^2*x))*a^4*x^2/((sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*abs(a)) - 2*c^3*arcsin(a*x)*sgn(a)/abs(a) + 1/2*
c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) -
sqrt(-a^2*x^2 + 1)*c^3/a - 1/8*(8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3*abs(a
)/(a^2*x) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3*abs(a)/(a^4*x^2))/a^2
```

3.450 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=65

$$-\frac{c^2(ax+1)\sqrt{1-a^2x^2}}{a^2x}+\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{c^2\sin^{-1}(ax)}{a}$$

```
[Out] -((c^2*(1 + a*x)*Sqrt[1 - a^2*x^2])/(a^2*x)) - (c^2*ArcSin[a*x])/a + (c^2*A
rcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rubi [A] time = 0.122547, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6131, 6128, 813, 844, 216, 266, 63, 208}

$$-\frac{c^2(ax+1)\sqrt{1-a^2x^2}}{a^2x}+\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{c^2\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x))^2,x]
```

```
[Out] -((c^2*(1 + a*x)*Sqrt[1 - a^2*x^2])/(a^2*x)) - (c^2*ArcSin[a*x])/a + (c^2*A
rcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx &= \frac{c^2\int \frac{e^{\tanh^{-1}(ax)}(1-ax)^2}{x^2}\,dx}{a^2}\\
&= \frac{c^2\int \frac{(1-ax)\sqrt{1-a^2x^2}}{x^2}\,dx}{a^2}\\
&= -\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\int \frac{2a+2a^2x}{x\sqrt{1-a^2x^2}}\,dx}{2a^2}\\
&= -\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{a^2x}-c^2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx-\frac{c^2\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{a}\\
&= -\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\sin^{-1}(ax)}{a}-\frac{c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2a}\\
&= -\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\sin^{-1}(ax)}{a}+\frac{c^2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^3}\\
&= -\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\sin^{-1}(ax)}{a}+\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.100301, size = 78, normalized size = 1.2

$$
-\frac{c^2\left(ax\sqrt{1-a^2x^2}+\sqrt{1-a^2x^2}-ax\log\left(\sqrt{1-a^2x^2}+1\right)+ax\log(ax)+ax\sin^{-1}(ax)\right)}{a^2x}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^2,x]
```

```
[Out] -((c^2*(Sqrt[1 - a^2*x^2] + a*x*Sqrt[1 - a^2*x^2] + a*x*ArcSin[a*x] + a*x*L
og[a*x] - a*x*Log[1 + Sqrt[1 - a^2*x^2]]))/(a^2*x))
```

Maple [A] time = 0.039, size = 95, normalized size = 1.5

$$-\frac{c^2}{a}\sqrt{-a^2x^2+1}-c^2\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{c^2}{a^2x}\sqrt{-a^2x^2+1}+\frac{c^2}{a}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^2,x)
```

```
[Out] -c^2*(-a^2*x^2+1)^(1/2)/a-c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)
^(1/2))-c^2*(-a^2*x^2+1)^(1/2)/a^2/x+c^2/a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.44266, size = 132, normalized size = 2.03

$$-\frac{c^2\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}+\frac{c^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)}{a}-\frac{\sqrt{-a^2x^2+1}c^2}{a}-\frac{\sqrt{-a^2x^2+1}c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -c^2*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + c^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x
) + 2/abs(x))/a - sqrt(-a^2*x^2 + 1)*c^2/a - sqrt(-a^2*x^2 + 1)*c^2/(a^2*x)
```

Fricas [A] time = 2.2136, size = 201, normalized size = 3.09

$$\frac{2\,ac^2x\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-ac^2x\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-ac^2x-\left(ac^2x+c^2\right)\sqrt{-a^2x^2+1}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] (2*a*c^2*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - a*c^2*x*log((sqrt(-a^2*
x^2 + 1) - 1)/x) - a*c^2*x - (a*c^2*x + c^2)*sqrt(-a^2*x^2 + 1))/(a^2*x)
```

Sympy [A] time = 6.37038, size = 151, normalized size = 2.32

$$ac^2\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-c^2\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)-\frac{c^2\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}+\frac{c^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**2,x)
```

```
[Out] a*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
 - c**2*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a*
*2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - c**2*Piecewise((-acosh(1/(a*x)), 1/A
bs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a + c**2*Piecewise((-I*sqrt(a*
*2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a**2
```

Giac [B] time = 1.19354, size = 188, normalized size = 2.89

$$\frac{a^2c^2x}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|}-\frac{c^2\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{|a|}+\frac{c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|}-\frac{\sqrt{-a^2x^2+1}c^2}{a}-\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)c^2}{2a^2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*c^2*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - c^2*arcsin(a*x)*sg
n(a)/abs(a) + c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(
x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c^2/a - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)*c^2/(a^2*x*abs(a))
```

3.451 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=41

$$\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{c\sqrt{1-a^2x^2}}{a}$$

```
[Out] -((c*Sqrt[1 - a^2*x^2])/a) + (c*ArcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rubi [A] time = 0.0677943, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6131, 6128, 266, 50, 63, 208}

$$\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{c\sqrt{1-a^2x^2}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x)),x]
```

```
[Out] -((c*Sqrt[1 - a^2*x^2])/a) + (c*ArcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= -\frac{c\int\frac{e^{\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}\\
&= -\frac{c\int\frac{\sqrt{1-a^2x^2}}{x}\,dx}{a}\\
&= -\frac{c\,\mathrm{Subst}\left(\int\frac{\sqrt{1-a^2x}}{x}\,dx,x,x^2\right)}{2a}\\
&= -\frac{c\sqrt{1-a^2x^2}}{a}-\frac{c\,\mathrm{Subst}\left(\int\frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2a}\\
&= -\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\,\mathrm{Subst}\left(\int\frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^3}\\
&= -\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0396717, size = 42, normalized size = 1.02

$$
-\frac{c\left(\sqrt{1-a^2x^2}-\log\left(\sqrt{1-a^2x^2}+1\right)+\log(x)\right)}{a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x)),x]
```

```
[Out] -((c*(Sqrt[1 - a^2*x^2] + Log[x] - Log[1 + Sqrt[1 - a^2*x^2]]))/a)
```

Maple [A] time = 0.036, size = 34, normalized size = 0.8

$$
\frac{c}{a}\left(-\sqrt{-a^2x^2+1}+\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x),x)
```

```
[Out] c/a*(-(-a^2*x^2+1)^(1/2)+arctanh(1/(-a^2*x^2+1)^(1/2)))
```

Maxima [A] time = 0.947166, size = 68, normalized size = 1.66

$$\frac{c\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)}{a}-\frac{\sqrt{-a^2x^2+1}c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] c*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))/a - sqrt(-a^2*x^2 + 1)*c/a
```

Fricas [A] time = 2.10142, size = 85, normalized size = 2.07

$$-\frac{c\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] -(c*log((sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1)*c)/a
```

Sympy [A] time = 16.261, size = 61, normalized size = 1.49

$$\begin{cases}\frac{-c\sqrt{-a^2x^2+1}+\frac{c\left(-\log\left(-1+\frac{1}{\sqrt{-a^2x^2+1}}\right)+\log\left(1+\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)}{2}}{a} & \text{for } a\neq 0\\ cx+\tilde{\infty}c\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x),x)
```

```
[Out] Piecewise(((-c*sqrt(-a**2*x**2 + 1) + c*(-log(-1 + 1/sqrt(-a**2*x**2 + 1))
+ log(1 + 1/sqrt(-a**2*x**2 + 1)))/2)/a, Ne(a, 0)), (c*x + zoo*c*log(x), Tr
ue))
```

Giac [A] time = 1.12334, size = 74, normalized size = 1.8

$$-\frac{c\left(2\sqrt{-a^2x^2+1}-\log\left(\sqrt{-a^2x^2+1}+1\right)+\log\left(-\sqrt{-a^2x^2+1}+1\right)\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x),x, algorithm="giac")
```

```
[Out] -1/2*c*(2*sqrt(-a^2*x^2 + 1) - log(sqrt(-a^2*x^2 + 1) + 1) + log(-sqrt(-a^2
*x^2 + 1) + 1))/a
```

3.452 $\int \frac{e^{\tanh^{-1}(ax)}}{c-\frac{c}{ax}} dx$

Optimal. Leaf size=65

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{ac(1-ax)^2}-\frac{2\sqrt{1-a^2x^2}}{ac}+\frac{2\sin^{-1}(ax)}{ac}$$

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a*c) - (1 - a^2*x^2)^(3/2)/(a*c*(1 - a*x)^2) + (2*A
rcSin[a*x])/(a*c)
```

Rubi [A] time = 0.0956761, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6131, 6128, 793, 665, 216}

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{ac(1-ax)^2}-\frac{2\sqrt{1-a^2x^2}}{ac}+\frac{2\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a*x)),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a*c) - (1 - a^2*x^2)^(3/2)/(a*c*(1 - a*x)^2) + (2*A
rcSin[a*x])/(a*c)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
```

```
|| EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx &= -\frac{a\int \frac{e^{\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}\\
&= -\frac{a\int \frac{x\sqrt{1-a^2x^2}}{(1-ax)^2}\,dx}{c}\\
&= -\frac{\left(1-a^2x^2\right)^{3/2}}{ac(1-ax)^2}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx}{c}\\
&= -\frac{2\sqrt{1-a^2x^2}}{ac}-\frac{\left(1-a^2x^2\right)^{3/2}}{ac(1-ax)^2}+\frac{2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{2\sqrt{1-a^2x^2}}{ac}-\frac{\left(1-a^2x^2\right)^{3/2}}{ac(1-ax)^2}+\frac{2\sin^{-1}(ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.0382358, size = 52, normalized size = 0.8

$$\frac{\frac{(ax-3)\sqrt{ax+1}}{\sqrt{1-ax}}-4\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a*x)),x]
```

```
[Out] (((-3 + a*x)*Sqrt[1 + a*x])/Sqrt[1 - a*x] - 4*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]
)/(a*c)
```

Maple [A] time = 0.043, size = 96, normalized size = 1.5

$$-\frac{1}{ac}\sqrt{-a^2x^2+1}+2\frac{1}{c\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+2\frac{1}{a^2c}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a/c+2/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(
1/2))+2/a^2/c/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))), x)
```

Fricas [A] time = 2.09259, size = 154, normalized size = 2.37

$$-\frac{3\,ax + 4\,(ax - 1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2+1}(ax - 3) - 3}{a^2cx - ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] -(3*a*x + 4*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x - 3) - 3)/(a^2*c*x - a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\left(\int \frac{x}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx + \int \frac{ax^2}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a/x),x)
```

```
[Out] a*(Integral(x/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integ
ral(a*x**2/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

Giac [A] time = 1.17706, size = 99, normalized size = 1.52

$$\frac{2\,\arcsin(ax)\,\mathrm{sgn}(a)}{c|a|} - \frac{\sqrt{-a^2x^2+1}}{ac} - \frac{4}{c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] 2*arcsin(a*x)*sgn(a)/(c*abs(a)) - sqrt(-a^2*x^2 + 1)/(a*c) - 4/(c*((sqrt(-a
^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

3.453 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal. Leaf size=104

$$\frac{\left(1-a^2x^2\right)^{3/2}}{ac^2(1-ax)^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3}-\frac{6\sqrt{1-a^2x^2}}{ac^2(1-ax)}+\frac{3\sin^{-1}(ax)}{ac^2}$$

```
[Out] (-6*Sqrt[1 - a^2*x^2])/(a*c^2*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(3*a*c^2*(1
- a*x)^3) + (1 - a^2*x^2)^(3/2)/(a*c^2*(1 - a*x)^2) + (3*ArcSin[a*x])/(a*c^
2)
```

Rubi [A] time = 0.185402, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6131, 6128, 1639, 793, 663, 216}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{ac^2(1-ax)^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3}-\frac{6\sqrt{1-a^2x^2}}{ac^2(1-ax)}+\frac{3\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a*x))^2,x]
```

```
[Out] (-6*Sqrt[1 - a^2*x^2])/(a*c^2*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(3*a*c^2*(1
- a*x)^3) + (1 - a^2*x^2)^(3/2)/(a*c^2*(1 - a*x)^2) + (3*ArcSin[a*x])/(a*c^
2)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
```

```
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx &= \frac{a^2\int \frac{e^{\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}\\
&= \frac{a^2\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^3}\,dx}{c^2}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{ac^2(1-ax)^2}-\frac{\int \frac{\left(2a^2-3a^3x\right)\sqrt{1-a^2x^2}}{(1-ax)^3}\,dx}{a^2c^2}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{ac^2(1-ax)^2}-\frac{3\int \frac{\sqrt{1-a^2x^2}}{(1-ax)^2}\,dx}{c^2}\\
&= -\frac{6\sqrt{1-a^2x^2}}{ac^2(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{ac^2(1-ax)^2}+\frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^2}\\
&= -\frac{6\sqrt{1-a^2x^2}}{ac^2(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{ac^2(1-ax)^2}+\frac{3\sin^{-1}(ax)}{ac^2}
\end{aligned}$$

Mathematica [A] time = 0.133426, size = 53, normalized size = 0.51

$$\frac{\frac{\sqrt{1-a^2x^2}\left(-3a^2x^2+19ax-14\right)}{(ax-1)^2}+9\sin^{-1}(ax)}{3ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a*x))^2,x]
```

```
[Out] (((-14 + 19*a*x - 3*a^2*x^2)*Sqrt[1 - a^2*x^2])/(-1 + a*x)^2 + 9*ArcSin[a*x
])/(3*a*c^2)
```

Maple [A] time = 0.047, size = 140, normalized size = 1.4

$$-\frac{1}{ac^2}\sqrt{-a^2x^2+1}+3\,\frac{1}{c^2\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+\frac{2}{3\,a^3c^2}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-2}+\frac{13}{3\,a^2c^2}\sqrt{-a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^2,x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a/c^2+3/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))+2/3/a^3/c^2/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+13/3/a^2
/c^2/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^2), x)
```

Fricas [A] time = 2.11676, size = 246, normalized size = 2.37

$$-\frac{14\,a^2x^2-28\,ax+18\left(a^2x^2-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3\,a^2x^2-19\,ax+14\right)\sqrt{-a^2x^2+1}+14}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(14*a^2*x^2 - 28*a*x + 18*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2
+ 1) - 1)/(a*x)) + (3*a^2*x^2 - 19*a*x + 14)*sqrt(-a^2*x^2 + 1) + 14)/(a^3*
c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\left(\int\frac{x^2}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int\frac{ax^3}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*(Integral(x**2/(a**2*x**2*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2
 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**3/(a**2*x**2*sqrt(-a**2*x
```

```
**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

Giac [A] time = 1.18989, size = 171, normalized size = 1.64

$$\frac{3 \arcsin(ax)\operatorname{sgn}(a)}{c^2|a|} - \frac{\sqrt{-a^2x^2+1}}{ac^2} + \frac{2\left(\frac{24\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x} - \frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2} - 11\right)}{3\,c^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] 3*arcsin(a*x)*sgn(a)/(c^2*abs(a)) - sqrt(-a^2*x^2 + 1)/(a*c^2) + 2/3*(24*(s
qrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2
/(a^4*x^2) - 11)/(c^2*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^3*abs(a
))
```

3.454 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx$

Optimal. Leaf size=136

$$\frac{(1-a^2x^2)^{3/2}}{ac^3(1-ax)^2}+\frac{14\left(1-a^2x^2\right)^{3/2}}{15ac^3(1-ax)^3}-\frac{(1-a^2x^2)^{3/2}}{5ac^3(1-ax)^4}-\frac{8\sqrt{1-a^2x^2}}{ac^3(1-ax)}+\frac{4\sin^{-1}(ax)}{ac^3}$$

```
[Out] (-8*Sqrt[1 - a^2*x^2])/(a*c^3*(1 - a*x)) - (1 - a^2*x^2)^(3/2)/(5*a*c^3*(1
 - a*x)^4) + (14*(1 - a^2*x^2)^(3/2))/(15*a*c^3*(1 - a*x)^3) + (1 - a^2*x^2)
^(3/2)/(a*c^3*(1 - a*x)^2) + (4*ArcSin[a*x])/(a*c^3)
```

Rubi [A] time = 0.281984, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6131, 6128, 1639, 1637, 659, 651, 663, 216}

$$\frac{(1-a^2x^2)^{3/2}}{ac^3(1-ax)^2}+\frac{14\left(1-a^2x^2\right)^{3/2}}{15ac^3(1-ax)^3}-\frac{(1-a^2x^2)^{3/2}}{5ac^3(1-ax)^4}-\frac{8\sqrt{1-a^2x^2}}{ac^3(1-ax)}+\frac{4\sin^{-1}(ax)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a*x))^3,x]
```

```
[Out] (-8*Sqrt[1 - a^2*x^2])/(a*c^3*(1 - a*x)) - (1 - a^2*x^2)^(3/2)/(5*a*c^3*(1
 - a*x)^4) + (14*(1 - a^2*x^2)^(3/2))/(15*a*c^3*(1 - a*x)^3) + (1 - a^2*x^2)
^(3/2)/(a*c^3*(1 - a*x)^2) + (4*ArcSin[a*x])/(a*c^3)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
```

```
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx &= -\frac{a^3\int \frac{e^{\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}\\
&= -\frac{a^3\int \frac{x^3\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx}{c^3}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{ac^3(1-ax)^2}-\frac{\int \frac{\sqrt{1-a^2x^2}\left(2a^2-5a^3x+4a^4x^2\right)}{(1-ax)^4}\,dx}{a^2c^3}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{ac^3(1-ax)^2}-\frac{\int \left(\frac{a^2\sqrt{1-a^2x^2}}{(-1+ax)^4}+\frac{3a^2\sqrt{1-a^2x^2}}{(-1+ax)^3}+\frac{4a^2\sqrt{1-a^2x^2}}{(-1+ax)^2}\right)dx}{a^2c^3}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{ac^3(1-ax)^2}-\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{c^3}-\frac{3\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{c^3}-\frac{4\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}\,dx}{c^3}\\
&= -\frac{8\sqrt{1-a^2x^2}}{ac^3(1-ax)}-\frac{\left(1-a^2x^2\right)^{3/2}}{5ac^3(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{ac^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{ac^3(1-ax)^2}+\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{5c^3}+\frac{4\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^3}\\
&= -\frac{8\sqrt{1-a^2x^2}}{ac^3(1-ax)}-\frac{\left(1-a^2x^2\right)^{3/2}}{5ac^3(1-ax)^4}+\frac{14\left(1-a^2x^2\right)^{3/2}}{15ac^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{ac^3(1-ax)^2}+\frac{4\sin^{-1}(ax)}{ac^3}
\end{aligned}$$

Mathematica [A] time = 0.146847, size = 61, normalized size = 0.45

$$\frac{\frac{\sqrt{1-a^2x^2}(-15a^3x^3+149a^2x^2-222ax+94)}{(ax-1)^3}+60\sin^{-1}(ax)}{15ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a*x))^3,x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(94 - 222*a*x + 149*a^2*x^2 - 15*a^3*x^3))/(-1 + a*x)^3
 + 60*ArcSin[a*x])/(15*a*c^3)
```

Maple [A] time = 0.046, size = 184, normalized size = 1.4

$$-\frac{1}{ac^3}\sqrt{-a^2x^2+1}+4\frac{1}{c^3\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+\frac{2}{5a^4c^3}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}+\frac{31}{15a^3c^3}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^3,x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a/c^3+4/c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))+2/5/a^4/c^3/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+31/15/a^
3/c^3/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+104/15/a^2/c^3/(x-1/a)*(
-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^3), x)
```

Fricas [A] time = 2.2657, size = 328, normalized size = 2.41

$$-\frac{94\,a^3x^3-282\,a^2x^2+282\,ax+120\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(15\,a^3x^3-149\,a^2x^2+222\,ax-9\right.}{15\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(94*a^3*x^3 - 282*a^2*x^2 + 282*a*x + 120*(a^3*x^3 - 3*a^2*x^2 + 3*a*
x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (15*a^3*x^3 - 149*a^2*x^2 +
 222*a*x - 94)*sqrt(-a^2*x^2 + 1) - 94)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^
```

```
2*c^3*x - a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3\left(\int \frac{x^3}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^4}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*(Integral(x**3/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**
2*x**2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Inte
gral(a*x**4/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 +
 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [A] time = 1.17342, size = 244, normalized size = 1.79

$$\frac{4\arcsin(ax)\,\mathrm{sgn}(a)}{c^3|a|}-\frac{\sqrt{-a^2x^2+1}}{ac^3}+\frac{2\left(\frac{335\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{505\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{285\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}-\frac{60\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4}-\cdots\right.}{15\,c^3\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] 4*arcsin(a*x)*sgn(a)/(c^3*abs(a)) - sqrt(-a^2*x^2 + 1)/(a*c^3) + 2/15*(335*
(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 505*(sqrt(-a^2*x^2 + 1)*abs(a) +
a)^2/(a^4*x^2) + 285*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 60*(sqrt
(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 79)/(c^3*((sqrt(-a^2*x^2 + 1)*abs(
a) + a)/(a^2*x) - 1)^5*abs(a))
```

3.455 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

Optimal. Leaf size=168

$$\frac{(1-a^2x^2)^{3/2}}{ac^4(1-ax)^2}+\frac{184(1-a^2x^2)^{3/2}}{105ac^4(1-ax)^3}-\frac{26(1-a^2x^2)^{3/2}}{35ac^4(1-ax)^4}+\frac{(1-a^2x^2)^{3/2}}{7ac^4(1-ax)^5}-\frac{10\sqrt{1-a^2x^2}}{ac^4(1-ax)}+\frac{5\sin^{-1}(ax)}{ac^4}$$

```
[Out] (-10*Sqrt[1 - a^2*x^2])/(a*c^4*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(7*a*c^4*(1
 - a*x)^5) - (26*(1 - a^2*x^2)^(3/2))/(35*a*c^4*(1 - a*x)^4) + (184*(1 - a^
2*x^2)^(3/2))/(105*a*c^4*(1 - a*x)^3) + (1 - a^2*x^2)^(3/2)/(a*c^4*(1 - a*x
)^2) + (5*ArcSin[a*x])/(a*c^4)
```

Rubi [A] time = 0.351977, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6131, 6128, 1639, 1637, 659, 651, 663, 216}

$$\frac{(1-a^2x^2)^{3/2}}{ac^4(1-ax)^2}+\frac{184(1-a^2x^2)^{3/2}}{105ac^4(1-ax)^3}-\frac{26(1-a^2x^2)^{3/2}}{35ac^4(1-ax)^4}+\frac{(1-a^2x^2)^{3/2}}{7ac^4(1-ax)^5}-\frac{10\sqrt{1-a^2x^2}}{ac^4(1-ax)}+\frac{5\sin^{-1}(ax)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a*x))^4,x]
```

```
[Out] (-10*Sqrt[1 - a^2*x^2])/(a*c^4*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(7*a*c^4*(1
 - a*x)^5) - (26*(1 - a^2*x^2)^(3/2))/(35*a*c^4*(1 - a*x)^4) + (184*(1 - a^
2*x^2)^(3/2))/(105*a*c^4*(1 - a*x)^3) + (1 - a^2*x^2)^(3/2)/(a*c^4*(1 - a*x
)^2) + (5*ArcSin[a*x])/(a*c^4)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\int\frac{e^{\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}$$
$$= \frac{a^4\int\frac{x^4\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx}{c^4}$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}}{ac^4(1-ax)^2} - \frac{\int\frac{\sqrt{1-a^2x^2}\left(2a^2-7a^3x+9a^4x^2-5a^5x^3\right)}{(1-ax)^5}\,dx}{a^2c^4}$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}}{ac^4(1-ax)^2} - \frac{\int\left(\frac{a^2\sqrt{1-a^2x^2}}{(-1+ax)^5}+\frac{4a^2\sqrt{1-a^2x^2}}{(-1+ax)^4}+\frac{6a^2\sqrt{1-a^2x^2}}{(-1+ax)^3}+\frac{5a^2\sqrt{1-a^2x^2}}{(-1+ax)^2}\right)dx}{a^2c^4}$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}}{ac^4(1-ax)^2} - \frac{\int\frac{\sqrt{1-a^2x^2}}{(-1+ax)^5}\,dx}{c^4} - \frac{4\int\frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{c^4} - \frac{5\int\frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}\,dx}{c^4} - \frac{6\int\frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{c^4}$$
$$= -\frac{10\sqrt{1-a^2x^2}}{ac^4(1-ax)} + \frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5} - \frac{4\left(1-a^2x^2\right)^{3/2}}{5ac^4(1-ax)^4} + \frac{2\left(1-a^2x^2\right)^{3/2}}{ac^4(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{ac^4(1-ax)^2} + \frac{2\int\frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{7c^4}$$
$$= -\frac{10\sqrt{1-a^2x^2}}{ac^4(1-ax)} + \frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5} - \frac{26\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4} + \frac{26\left(1-a^2x^2\right)^{3/2}}{15ac^4(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{ac^4(1-ax)^2} + \frac{5\sin^{-1}(ax)}{ac^4}$$
$$= -\frac{10\sqrt{1-a^2x^2}}{ac^4(1-ax)} + \frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5} - \frac{26\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4} + \frac{184\left(1-a^2x^2\right)^{3/2}}{105ac^4(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{ac^4(1-ax)^2} + \frac{5\sin^{-1}(ax)}{ac^4}$$

Mathematica [A] time = 0.178585, size = 69, normalized size = 0.41

$$\frac{\frac{\sqrt{1-a^2x^2}\left(-105a^4x^4+1444a^3x^3-3256a^2x^2+2771ax-824\right)}{(ax-1)^4}+525\sin^{-1}(ax)}{105ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a*x))^4,x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(-824 + 2771*a*x - 3256*a^2*x^2 + 1444*a^3*x^3 - 105*a^
4*x^4))/(-1 + a*x)^4 + 525*ArcSin[a*x])/(105*a*c^4)
```

Maple [A] time = 0.05, size = 228, normalized size = 1.4

$$-\frac{1}{ac^4}\sqrt{-a^2x^2+1}+5\frac{1}{c^4\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+\frac{57}{35\,a^4c^4}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}+\frac{446}{105\,a^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^4,x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a/c^4+5/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))+57/35/a^4/c^4/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+446/10
5/a^3/c^4/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1024/105/a^2/c^4/(x-
1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+2/7/a^5/c^4/(x-1/a)^4*(-a^2*(x-1/a)
^2-2*a*(x-1/a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^4), x)
```

Fricas [A] time = 2.25498, size = 420, normalized size = 2.5

$$-\frac{824\,a^4x^4-3296\,a^3x^3+4944\,a^2x^2-3296\,ax+1050\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(105\,a^4x\right.}{105\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] -1/105*(824*a^4*x^4 - 3296*a^3*x^3 + 4944*a^2*x^2 - 3296*a*x + 1050*(a^4*x^
4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x
)) + (105*a^4*x^4 - 1444*a^3*x^3 + 3256*a^2*x^2 - 2771*a*x + 824)*sqrt(-a^2
*x^2 + 1) + 824)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x
 + a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^4\left(\int \frac{x^4}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^5}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2}\right.}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*(Integral(x**4/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*x**3*sqrt(-a**
2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1)
 + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**5/(a**4*x**4*sqrt(-a**2*x**2 +
 1) - 4*a**3*x**3*sqrt(-a**2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) -
 4*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

Giac [A] time = 1.16328, size = 317, normalized size = 1.89

$$\frac{5\arcsin(ax)\,\mathrm{sgn}(a)}{c^4|a|}-\frac{\sqrt{-a^2x^2+1}}{ac^4}+\frac{2\left(\frac{4508\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{11529\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{15050\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}-\frac{10115\left(\sqrt{-a^2x^2+1}\right.}{a^8x^4}\right.}{105\,c^4\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] 5*arcsin(a*x)*sgn(a)/(c^4*abs(a)) - sqrt(-a^2*x^2 + 1)/(a*c^4) + 2/105*(450
8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 11529*(sqrt(-a^2*x^2 + 1)*abs(a
) + a)^2/(a^4*x^2) + 15050*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 10
115*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 3570*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^5/(a^10*x^5) - 525*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a^12*x^6)
 - 719)/(c^4*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^7*abs(a))
```

3.456 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=59

$$-\frac{(2-p)\left(c-\frac{c}{ax}\right)^p \text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{1}{ax}\right)}{ap}-x\left(c-\frac{c}{ax}\right)^p$$

```
[Out] -((c - c/(a*x))^p*x) - ((2 - p)*(c - c/(a*x))^p*Hypergeometric2F1[1, p, 1 +
 p, 1 - 1/(a*x)])/(a*p)
```

Rubi [A] time = 0.0803001, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6133, 25, 514, 375, 78, 65}

$$-\frac{(2-p)\left(c-\frac{c}{ax}\right)^p {}_2F_1\left(1,p;p+1;1-\frac{1}{ax}\right)}{ap}-x\left(c-\frac{c}{ax}\right)^p$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] -((c - c/(a*x))^p*x) - ((2 - p)*(c - c/(a*x))^p*Hypergeometric2F1[1, p, 1 +
 p, 1 - 1/(a*x)])/(a*p)
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx &= \int \frac{\left(c-\frac{c}{ax}\right)^p(1+ax)}{1-ax}\,dx\\
&= -\frac{c\int \frac{\left(c-\frac{c}{ax}\right)^{-1+p}(1+ax)}{x}\,dx}{a}\\
&= -\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{-1+p}\,dx}{a}\\
&= \frac{c\,\mathrm{Subst}\left(\int \frac{(a+x)\left(c-\frac{cx}{a}\right)^{-1+p}}{x^2}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\left(c-\frac{c}{ax}\right)^p x+\frac{(c(2-p))\,\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{-1+p}}{x}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\left(c-\frac{c}{ax}\right)^p x-\frac{(2-p)\left(c-\frac{c}{ax}\right)^p {}_2F_1\left(1,p;1+p;1-\frac{1}{ax}\right)}{ap}
\end{aligned}$$

Mathematica [A] time = 0.0275809, size = 46, normalized size = 0.78

$$\frac{\left(c-\frac{c}{ax}\right)^p\left((p-2)\text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{1}{ax}\right)-apx\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] ((c - c/(a*x))^p*(-(a*p*x) + (-2 + p)*Hypergeometric2F1[1, p, 1 + p, 1 - 1/
(a*x)]))/(a*p)
```

Maple [F] time = 0.335, size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2}{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^p,x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{ax}\right)^p}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^p,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(c - c/(a*x))^p/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{(ax+1)\left(\frac{acx-c}{ax}\right)^p}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^p,x, algorithm="fricas")
```

```
[Out] integral(-(a*x + 1)*((a*c*x - c)/(a*x))^p/(a*x - 1), x)
```

Sympy [C] time = 7.34323, size = 274, normalized size = 4.64

$$-a\left(\begin{cases} \frac{0^p x}{a}+\frac{0^p\log(ax-1)}{a^2}-\frac{a^{-p}c^p p x^2 x^{-p} e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}1-p,2-p\\3-p\end{matrix}\middle|ax\right)}{\Gamma(3-p)\Gamma(p+1)} & \text{for } |ax|>1\\ \frac{0^p x}{a}+\frac{0^p\log(-ax+1)}{a^2}-\frac{a^{-p}c^p p x^2 x^{-p} e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}1-p,2-p\\3-p\end{matrix}\middle|ax\right)}{\Gamma(3-p)\Gamma(p+1)} & \text{otherwise}\end{cases}\right)-\left(\begin{cases} \frac{0^p\log(ax-1)}{a}-\frac{a^{-p}c^p p x x^{-p} e^{i\pi p}\Gamma(p)\Gamma(1-p)\,{}_2F_1 \cdots}{\Gamma(2-p)\Gamma(p+\cdots} \\ \frac{0^p\log(-ax+1)}{a}-\frac{a^{-p}c^p p x x^{-p} e^{i\pi p}\Gamma(p)\Gamma(1-p)\,{}_2 \cdots}{\Gamma(2-p)\Gamma(p \cdots} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**p,x)
```

```
[Out] -a*Piecewise((0**p*x/a + 0**p*log(a*x - 1)/a**2 - a**(-p)*c**p*p*x**2*x**(-
p)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 - p,), a*x)/(
gamma(3 - p)*gamma(p + 1)), Abs(a*x) > 1), (0**p*x/a + 0**p*log(-a*x + 1)/a
**2 - a**(-p)*c**p*p*x**2*x**(-p)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((
1 - p, 2 - p), (3 - p,), a*x)/(gamma(3 - p)*gamma(p + 1)), True)) - Piecewi
se((0**p*log(a*x - 1)/a - a**(-p)*c**p*p*x*x**(-p)*exp(I*pi*p)*gamma(p)*gam
ma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a*x)/(gamma(2 - p)*gamma(p + 1)),
 Abs(a*x) > 1), (0**p*log(-a*x + 1)/a - a**(-p)*c**p*p*x*x**(-p)*exp(I*pi*p
)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a*x)/(gamma(2 - p)*
gamma(p + 1)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)^2\left(c-\frac{c}{ax}\right)^p}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^p,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2*(c - c/(a*x))^p/(a^2*x^2 - 1), x)
```

3.457 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

Optimal. Leaf size=62

$$\frac{c^5}{a^3x^2}-\frac{c^5}{a^4x^3}+\frac{c^5}{4a^5x^4}+\frac{2c^5}{a^2x}+\frac{3c^5\log(x)}{a}+c^5(-x)$$

```
[Out] c^5/(4*a^5*x^4) - c^5/(a^4*x^3) + c^5/(a^3*x^2) + (2*c^5)/(a^2*x) - c^5*x +
 (3*c^5*Log[x])/a
```

Rubi [A] time = 0.113199, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 75}

$$\frac{c^5}{a^3x^2}-\frac{c^5}{a^4x^3}+\frac{c^5}{4a^5x^4}+\frac{2c^5}{a^2x}+\frac{3c^5\log(x)}{a}+c^5(-x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] c^5/(4*a^5*x^4) - c^5/(a^4*x^3) + c^5/(a^3*x^2) + (2*c^5)/(a^2*x) - c^5*x +
 (3*c^5*Log[x])/a
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx = -\frac{c^5\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)^5}{x^5}dx}{a^5}$$
$$= -\frac{c^5\int \frac{(1-ax)^4(1+ax)}{x^5}dx}{a^5}$$
$$= -\frac{c^5\int \left(a^5+\frac{1}{x^5}-\frac{3a}{x^4}+\frac{2a^2}{x^3}+\frac{2a^3}{x^2}-\frac{3a^4}{x}\right)dx}{a^5}$$
$$= \frac{c^5}{4a^5x^4}-\frac{c^5}{a^4x^3}+\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}-c^5x+\frac{3c^5\log(x)}{a}$$

Mathematica [A] time = 0.216819, size = 64, normalized size = 1.03

$$\frac{c^5}{a^3x^2}-\frac{c^5}{a^4x^3}+\frac{c^5}{4a^5x^4}+\frac{2c^5}{a^2x}+\frac{3c^5\log(ax)}{a}+c^5(-x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] c^5/(4*a^5*x^4) - c^5/(a^4*x^3) + c^5/(a^3*x^2) + (2*c^5)/(a^2*x) - c^5*x +
 (3*c^5*Log[a*x])/a
```

Maple [A] time = 0.036, size = 61, normalized size = 1.

$$\frac{c^5}{4\,a^5x^4}-\frac{c^5}{a^4x^3}+\frac{c^5}{x^2a^3}+2\,\frac{c^5}{a^2x}-c^5x+3\,\frac{c^5\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^5,x)
```

```
[Out] 1/4*c^5/a^5/x^4-c^5/a^4/x^3+c^5/x^2/a^3+2*c^5/a^2/x-c^5*x+3*c^5*ln(x)/a
```

Maxima [A] time = 0.947721, size = 78, normalized size = 1.26

$$-c^5x+\frac{3\,c^5\log(x)}{a}+\frac{8\,a^3c^5x^3+4\,a^2c^5x^2-4\,ac^5x+c^5}{4\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^5,x, algorithm="maxima")
```

```
[Out] -c^5*x + 3*c^5*log(x)/a + 1/4*(8*a^3*c^5*x^3 + 4*a^2*c^5*x^2 - 4*a*c^5*x +
c^5)/(a^5*x^4)
```

Fricas [A] time = 2.07021, size = 143, normalized size = 2.31

$$-\frac{4\,a^5c^5x^5-12\,a^4c^5x^4\log(x)-8\,a^3c^5x^3-4\,a^2c^5x^2+4\,ac^5x-c^5}{4\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^5*c^5*x^5 - 12*a^4*c^5*x^4*log(x) - 8*a^3*c^5*x^3 - 4*a^2*c^5*x^2
 + 4*a*c^5*x - c^5)/(a^5*x^4)
```

Sympy [A] time = 0.461193, size = 63, normalized size = 1.02

$$\frac{-a^5c^5x + 3a^4c^5\log(x) + \frac{8a^3c^5x^3+4a^2c^5x^2-4ac^5x+c^5}{4x^4}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**5,x)
```

```
[Out] (-a**5*c**5*x + 3*a**4*c**5*log(x) + (8*a**3*c**5*x**3 + 4*a**2*c**5*x**2 -
 4*a*c**5*x + c**5)/(4*x**4))/a**5
```

Giac [A] time = 1.19038, size = 80, normalized size = 1.29

$$-c^5x + \frac{3\,c^5\log(|x|)}{a} + \frac{8\,a^3c^5x^3 + 4\,a^2c^5x^2 - 4\,ac^5x + c^5}{4\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] -c^5*x + 3*c^5*log(abs(x))/a + 1/4*(8*a^3*c^5*x^3 + 4*a^2*c^5*x^2 - 4*a*c^5
*x + c^5)/(a^5*x^4)
```

3.458 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=40

$$\frac{c^4}{a^3x^2}-\frac{c^4}{3a^4x^3}+\frac{2c^4\log(x)}{a}+c^4(-x)$$

```
[Out] -c^4/(3*a^4*x^3) + c^4/(a^3*x^2) - c^4*x + (2*c^4*Log[x])/a
```

Rubi [A] time = 0.102118, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 75}

$$\frac{c^4}{a^3x^2}-\frac{c^4}{3a^4x^3}+\frac{2c^4\log(x)}{a}+c^4(-x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] -c^4/(3*a^4*x^3) + c^4/(a^3*x^2) - c^4*x + (2*c^4*Log[x])/a
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx &= \frac{c^4\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)^4}{x^4}\,dx}{a^4}\\
&= \frac{c^4\int \frac{(1-ax)^3(1+ax)}{x^4}\,dx}{a^4}\\
&= \frac{c^4\int \left(-a^4+\frac{1}{x^4}-\frac{2a}{x^3}+\frac{2a^3}{x}\right)dx}{a^4}\\
&= -\frac{c^4}{3a^4x^3}+\frac{c^4}{a^3x^2}-c^4x+\frac{2c^4\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.148076, size = 42, normalized size = 1.05

$$\frac{c^4}{a^3x^2}-\frac{c^4}{3a^4x^3}+\frac{2c^4\log(ax)}{a}+c^4(-x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] -c^4/(3*a^4*x^3) + c^4/(a^3*x^2) - c^4*x + (2*c^4*Log[a*x])/a
```

Maple [A] time = 0.034, size = 39, normalized size = 1.

$$-\frac{c^4}{3\,a^4x^3}+\frac{c^4}{x^2a^3}-c^4x+2\,\frac{c^4\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^4,x)
```

```
[Out] -1/3*c^4/a^4/x^3+c^4/x^2/a^3-c^4*x+2*c^4*ln(x)/a
```

Maxima [A] time = 0.949206, size = 51, normalized size = 1.27

$$-c^4x+\frac{2\,c^4\log(x)}{a}+\frac{3\,ac^4x-c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -c^4*x + 2*c^4*log(x)/a + 1/3*(3*a*c^4*x - c^4)/(a^4*x^3)
```

Fricas [A] time = 2.06078, size = 99, normalized size = 2.48

$$-\frac{3\,a^4c^4x^4-6\,a^3c^4x^3\log(x)-3\,ac^4x+c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^4*c^4*x^4 - 6*a^3*c^4*x^3*log(x) - 3*a*c^4*x + c^4)/(a^4*x^3)
```

Sympy [A] time = 0.362913, size = 39, normalized size = 0.98

$$\frac{-a^4c^4x+2a^3c^4\log(x)+\frac{3ac^4x-c^4}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**4,x)
```

```
[Out] (-a**4*c**4*x + 2*a**3*c**4*log(x) + (3*a*c**4*x - c**4)/(3*x**3))/a**4
```

Giac [A] time = 1.20082, size = 53, normalized size = 1.32

$$-c^4x + \frac{2\,c^4\log\left(|x|\right)}{a} + \frac{3\,ac^4x - c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -c^4*x + 2*c^4*log(abs(x))/a + 1/3*(3*a*c^4*x - c^4)/(a^4*x^3)
```

3.459 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=40

$$\frac{c^3}{2a^3x^2}-\frac{c^3}{a^2x}+\frac{c^3\log(x)}{a}+c^3(-x)$$

```
[Out] c^3/(2*a^3*x^2) - c^3/(a^2*x) - c^3*x + (c^3*Log[x])/a
```

Rubi [A] time = 0.0984824, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 75}

$$\frac{c^3}{2a^3x^2}-\frac{c^3}{a^2x}+\frac{c^3\log(x)}{a}+c^3(-x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] c^3/(2*a^3*x^2) - c^3/(a^2*x) - c^3*x + (c^3*Log[x])/a
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx &= -\frac{c^3\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)^3}{x^3}\,dx}{a^3}\\
&= -\frac{c^3\int \frac{(1-ax)^2(1+ax)}{x^3}\,dx}{a^3}\\
&= -\frac{c^3\int \left(a^3+\frac{1}{x^3}-\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{a^3}\\
&= \frac{c^3}{2a^3x^2}-\frac{c^3}{a^2x}-c^3x+\frac{c^3\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.120661, size = 42, normalized size = 1.05

$$\frac{c^3}{2a^3x^2} - \frac{c^3}{a^2x} + \frac{c^3\log(ax)}{a} + c^3(-x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] c^3/(2*a^3*x^2) - c^3/(a^2*x) - c^3*x + (c^3*Log[a*x])/a
```

Maple [A] time = 0.037, size = 39, normalized size = 1.

$$\frac{c^3}{2\,x^2a^3} - \frac{c^3}{a^2x} - c^3x + \frac{c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^3,x)
```

```
[Out] 1/2*c^3/x^2/a^3-c^3/a^2/x-c^3*x+c^3*ln(x)/a
```

Maxima [A] time = 0.948866, size = 50, normalized size = 1.25

$$-c^3x + \frac{c^3\log(x)}{a} - \frac{2\,ac^3x - c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -c^3*x + c^3*log(x)/a - 1/2*(2*a*c^3*x - c^3)/(a^3*x^2)
```

Fricas [A] time = 2.01761, size = 99, normalized size = 2.48

$$-\frac{2\,a^3c^3x^3 - 2\,a^2c^3x^2\log(x) + 2\,ac^3x - c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*c^3*x^3 - 2*a^2*c^3*x^2*log(x) + 2*a*c^3*x - c^3)/(a^3*x^2)
```

Sympy [A] time = 0.336779, size = 37, normalized size = 0.92

$$\frac{-a^3c^3x + a^2c^3\log(x) - \frac{2ac^3x-c^3}{2x^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**3,x)
```

```
[Out] (-a**3*c**3*x + a**2*c**3*log(x) - (2*a*c**3*x - c**3)/(2*x**2))/a**3
```

Giac [A] time = 1.138, size = 51, normalized size = 1.27

$$-c^3x + \frac{c^3 \log(|x|)}{a} - \frac{2\,ac^3x - c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -c^3*x + c^3*log(abs(x))/a - 1/2*(2*a*c^3*x - c^3)/(a^3*x^2)
```

3.460 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=18

$$c^2(-x)-\frac{c^2}{a^2x}$$

```
[Out] -(c^2/(a^2*x)) - c^2*x
```

Rubi [A] time = 0.089764, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6131, 6129, 73, 14}

$$c^2(-x)-\frac{c^2}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] -(c^2/(a^2*x)) - c^2*x
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \frac{c^2\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^2}{x^2}dx}{a^2}$$
$$= \frac{c^2\int\frac{(1-ax)(1+ax)}{x^2}dx}{a^2}$$
$$= \frac{c^2\int\frac{1-a^2x^2}{x^2}dx}{a^2}$$
$$= \frac{c^2\int\left(-a^2+\frac{1}{x^2}\right)dx}{a^2}$$
$$= -\frac{c^2}{a^2x}-c^2x$$

Mathematica [A] time = 0.0870168, size = 18, normalized size = 1.

$$c^2(-x)-\frac{c^2}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] -(c^2/(a^2*x)) - c^2*x
```

Maple [A] time = 0.033, size = 20, normalized size = 1.1

$$\frac{c^2}{a^2}\left(-a^2x-x^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^2,x)
```

```
[Out] c^2/a^2*(-a^2*x-1/x)
```

Maxima [A] time = 0.955711, size = 24, normalized size = 1.33

$$-c^2x-\frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -c^2*x - c^2/(a^2*x)
```

Fricas [A] time = 1.93103, size = 41, normalized size = 2.28

$$-\frac{a^2c^2x^2+c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*c^2*x^2 + c^2)/(a^2*x)
```

Sympy [A] time = 0.261791, size = 17, normalized size = 0.94

$$\frac{-a^2c^2x - \frac{c^2}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**2,x)
```

```
[Out] (-a**2*c**2*x - c**2/x)/a**2
```

Giac [A] time = 1.12906, size = 24, normalized size = 1.33

$$-c^2x - \frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -c^2*x - c^2/(a^2*x)
```

3.461 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=13

$$-\frac{c\log(x)}{a}-cx$$

[Out] -(c*x) - (c*Log[x])/a

Rubi [A] time = 0.0541777, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6131, 6129, 43}

$$-\frac{c\log(x)}{a}-cx$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x)),x]

[Out] -(c*x) - (c*Log[x])/a

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= -\frac{c\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}\\
&= -\frac{c\int\frac{1+ax}{x}\,dx}{a}\\
&= -\frac{c\int\left(a+\frac{1}{x}\right)dx}{a}\\
&= -cx-\frac{c\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.034086, size = 13, normalized size = 1.

$$-\frac{c\log(x)}{a} - cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x)),x]
```

```
[Out] -(c*x) - (c*Log[x])/a
```

Maple [A] time = 0.029, size = 14, normalized size = 1.1

$$-cx - \frac{c\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x),x)
```

```
[Out] -c*x-c*ln(x)/a
```

Maxima [A] time = 0.936903, size = 18, normalized size = 1.38

$$-cx - \frac{c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -c*x - c*log(x)/a
```

Fricas [A] time = 2.00883, size = 31, normalized size = 2.38

$$-\frac{acx + c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] -(a*c*x + c*log(x))/a
```

Sympy [A] time = 0.095783, size = 12, normalized size = 0.92

$$\frac{-acx - c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x),x)
```

```
[Out] (-a*c*x - c*log(x))/a
```

Giac [A] time = 1.13038, size = 19, normalized size = 1.46

$$-cx - \frac{c \log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x),x, algorithm="giac")
```

```
[Out] -c*x - c*log(abs(x))/a
```

3.462 $\int \frac{e^{2\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=38

$$-\frac{2}{ac(1-ax)}-\frac{3\log(1-ax)}{ac}-\frac{x}{c}$$

```
[Out] -(x/c) - 2/(a*c*(1 - a*x)) - (3*Log[1 - a*x])/(a*c)
```

Rubi [A] time = 0.0878648, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 77}

$$-\frac{2}{ac(1-ax)}-\frac{3\log(1-ax)}{ac}-\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a*x)),x]
```

```
[Out] -(x/c) - 2/(a*c*(1 - a*x)) - (3*Log[1 - a*x])/(a*c)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{a\int \frac{e^{2\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}$$
$$= -\frac{a\int \frac{x(1+ax)}{(1-ax)^2}\,dx}{c}$$
$$= -\frac{a\int \left(\frac{1}{a}+\frac{2}{a(-1+ax)^2}+\frac{3}{a(-1+ax)}\right)dx}{c}$$
$$= -\frac{x}{c}-\frac{2}{ac(1-ax)}-\frac{3\log(1-ax)}{ac}$$

Mathematica [A] time = 0.0287888, size = 31, normalized size = 0.82

$$-\frac{ax+\frac{2}{1-ax}+3\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a*x)),x]
```

```
[Out] -((a*x + 2/(1 - a*x) + 3*Log[1 - a*x])/(a*c))
```

Maple [A] time = 0.038, size = 37, normalized size = 1.

$$-\frac{x}{c}+2\,\frac{1}{ac\,(ax-1)}-3\,\frac{\ln(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x),x)
```

```
[Out] -x/c+2/a/c/(a*x-1)-3/a/c*ln(a*x-1)
```

Maxima [A] time = 0.948848, size = 49, normalized size = 1.29

$$-\frac{x}{c}+\frac{2}{a^2cx-ac}-\frac{3\,\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -x/c + 2/(a^2*c*x - a*c) - 3*log(a*x - 1)/(a*c)
```

Fricas [A] time = 2.03134, size = 88, normalized size = 2.32

$$-\frac{a^2x^2-ax+3\,(ax-1)\log(ax-1)-2}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] -(a^2*x^2 - a*x + 3*(a*x - 1)*log(a*x - 1) - 2)/(a^2*c*x - a*c)
```

Sympy [A] time = 0.338379, size = 26, normalized size = 0.68

$$\frac{2}{a^2cx - ac} - \frac{x}{c} - \frac{3\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a/x),x)
```

```
[Out] 2/(a**2*c*x - a*c) - x/c - 3*log(a*x - 1)/(a*c)
```

Giac [A] time = 1.17017, size = 50, normalized size = 1.32

$$-\frac{x}{c} - \frac{3\log(|ax-1|)}{ac} + \frac{2}{(ax-1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] -x/c - 3*log(abs(a*x - 1))/(a*c) + 2/((a*x - 1)*a*c)
```

3.463 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=53

$$-\frac{5}{ac^2(1-ax)}+\frac{1}{ac^2(1-ax)^2}-\frac{4\log(1-ax)}{ac^2}-\frac{x}{c^2}$$

```
[Out] -(x/c^2) + 1/(a*c^2*(1 - a*x)^2) - 5/(a*c^2*(1 - a*x)) - (4*Log[1 - a*x])/(
a*c^2)
```

Rubi [A] time = 0.113906, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 77}

$$-\frac{5}{ac^2(1-ax)}+\frac{1}{ac^2(1-ax)^2}-\frac{4\log(1-ax)}{ac^2}-\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] -(x/c^2) + 1/(a*c^2*(1 - a*x)^2) - 5/(a*c^2*(1 - a*x)) - (4*Log[1 - a*x])/(
a*c^2)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int\frac{e^{2\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}$$
$$= \frac{a^2\int\frac{x^2(1+ax)}{(1-ax)^3}\,dx}{c^2}$$
$$= \frac{a^2\int\left(-\frac{1}{a^2}-\frac{2}{a^2(-1+ax)^3}-\frac{5}{a^2(-1+ax)^2}-\frac{4}{a^2(-1+ax)}\right)dx}{c^2}$$
$$= -\frac{x}{c^2}+\frac{1}{ac^2(1-ax)^2}-\frac{5}{ac^2(1-ax)}-\frac{4\log(1-ax)}{ac^2}$$

Mathematica [A] time = 0.0897144, size = 51, normalized size = 0.96

$$\frac{5}{ac^2(ax-1)}+\frac{1}{ac^2(ax-1)^2}-\frac{4\log(1-ax)}{ac^2}-\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] -(x/c^2) + 1/(a*c^2*(-1 + a*x)^2) + 5/(a*c^2*(-1 + a*x)) - (4*Log[1 - a*x])
/(a*c^2)
```

Maple [A] time = 0.037, size = 51, normalized size = 1.

$$-\frac{x}{c^2}+\frac{1}{ac^2\left(ax-1\right)^2}+5\,\frac{1}{ac^2\left(ax-1\right)}-4\,\frac{\ln\left(ax-1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^2,x)
```

```
[Out] -x/c^2+1/a/c^2/(a*x-1)^2+5/a/c^2/(a*x-1)-4/c^2/a*ln(a*x-1)
```

Maxima [A] time = 0.945463, size = 74, normalized size = 1.4

$$\frac{5\,ax-4}{a^3c^2x^2-2\,a^2c^2x+ac^2}-\frac{x}{c^2}-\frac{4\,\log\left(ax-1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] (5*a*x - 4)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) - x/c^2 - 4*log(a*x - 1)/(a
*c^2)
```

Fricas [A] time = 2.17622, size = 150, normalized size = 2.83

$$-\frac{a^3x^3-2\,a^2x^2-4\,ax+4\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+4}{a^3c^2x^2-2\,a^2c^2x+ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] -(a^3*x^3 - 2*a^2*x^2 - 4*a*x + 4*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 4)/(
a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [A] time = 0.426509, size = 49, normalized size = 0.92

$$\frac{5ax-4}{a^3c^2x^2-2a^2c^2x+ac^2}-\frac{x}{c^2}-\frac{4\log(ax-1)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a/x)**2,x)
```

```
[Out] (5*a*x - 4)/(a**3*c**2*x**2 - 2*a**2*c**2*x + a*c**2) - x/c**2 - 4*log(a*x
- 1)/(a*c**2)
```

Giac [A] time = 1.15494, size = 57, normalized size = 1.08

$$-\frac{x}{c^2}-\frac{4\log(|ax-1|)}{ac^2}+\frac{5ax-4}{(ax-1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -x/c^2 - 4*log(abs(a*x - 1))/(a*c^2) + (5*a*x - 4)/((a*x - 1)^2*a*c^2)
```

3.464 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=74

$$-\frac{9}{ac^3(1-ax)}+\frac{7}{2ac^3(1-ax)^2}-\frac{2}{3ac^3(1-ax)^3}-\frac{5\log(1-ax)}{ac^3}-\frac{x}{c^3}$$

```
[Out] -(x/c^3) - 2/(3*a*c^3*(1 - a*x)^3) + 7/(2*a*c^3*(1 - a*x)^2) - 9/(a*c^3*(1
- a*x)) - (5*Log[1 - a*x])/(a*c^3)
```

Rubi [A] time = 0.133356, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 77}

$$-\frac{9}{ac^3(1-ax)}+\frac{7}{2ac^3(1-ax)^2}-\frac{2}{3ac^3(1-ax)^3}-\frac{5\log(1-ax)}{ac^3}-\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] -(x/c^3) - 2/(3*a*c^3*(1 - a*x)^3) + 7/(2*a*c^3*(1 - a*x)^2) - 9/(a*c^3*(1
- a*x)) - (5*Log[1 - a*x])/(a*c^3)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{a^3\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}$$
$$= -\frac{a^3\int \frac{x^3(1+ax)}{(1-ax)^4}\,dx}{c^3}$$
$$= -\frac{a^3\int \left(\frac{1}{a^3}+\frac{2}{a^3(-1+ax)^4}+\frac{7}{a^3(-1+ax)^3}+\frac{9}{a^3(-1+ax)^2}+\frac{5}{a^3(-1+ax)}\right)dx}{c^3}$$
$$= -\frac{x}{c^3}-\frac{2}{3ac^3(1-ax)^3}+\frac{7}{2ac^3(1-ax)^2}-\frac{9}{ac^3(1-ax)}-\frac{5\log(1-ax)}{ac^3}$$

Mathematica [A] time = 0.122096, size = 63, normalized size = 0.85

$$\frac{-6a^4x^4+18a^3x^3+36a^2x^2-81ax-30(ax-1)^3\log(1-ax)+37}{6ac^3(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] (37 - 81*a*x + 36*a^2*x^2 + 18*a^3*x^3 - 6*a^4*x^4 - 30*(-1 + a*x)^3*Log[1
- a*x])/(6*a*c^3*(-1 + a*x)^3)
```

Maple [A] time = 0.035, size = 67, normalized size = 0.9

$$-\frac{x}{c^3}+\frac{2}{3\,ac^3\,(ax-1)^3}+\frac{7}{2\,ac^3\,(ax-1)^2}+9\,\frac{1}{ac^3\,(ax-1)}-5\,\frac{\ln(ax-1)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^3,x)
```

```
[Out] -x/c^3+2/3/a/c^3/(a*x-1)^3+7/2/a/c^3/(a*x-1)^2+9/a/c^3/(a*x-1)-5/a/c^3*ln(a
*x-1)
```

Maxima [A] time = 0.972135, size = 103, normalized size = 1.39

$$\frac{54\,a^2x^2-87\,ax+37}{6\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}-\frac{x}{c^3}-\frac{5\,\log(ax-1)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(54*a^2*x^2 - 87*a*x + 37)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x -
 a*c^3) - x/c^3 - 5*log(a*x - 1)/(a*c^3)
```

Fricas [A] time = 2.17975, size = 219, normalized size = 2.96

$$-\frac{6\,a^4x^4-18\,a^3x^3-36\,a^2x^2+81\,ax+30\,\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(ax-1\right)-37}{6\,\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^4*x^4 - 18*a^3*x^3 - 36*a^2*x^2 + 81*a*x + 30*(a^3*x^3 - 3*a^2*x^
2 + 3*a*x - 1)*log(a*x - 1) - 37)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*
x - a*c^3)
```

Sympy [A] time = 0.551466, size = 73, normalized size = 0.99

$$\frac{54a^2x^2-87ax+37}{6a^4c^3x^3-18a^3c^3x^2+18a^2c^3x-6ac^3}-\frac{x}{c^3}-\frac{5\log\left(ax-1\right)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a/x)**3,x)
```

```
[Out] (54*a**2*x**2 - 87*a*x + 37)/(6*a**4*c**3*x**3 - 18*a**3*c**3*x**2 + 18*a**
2*c**3*x - 6*a*c**3) - x/c**3 - 5*log(a*x - 1)/(a*c**3)
```

Giac [A] time = 1.16687, size = 69, normalized size = 0.93

$$-\frac{x}{c^3}-\frac{5\,\log\left(|ax-1|\right)}{ac^3}+\frac{54\,a^2x^2-87\,ax+37}{6\,(ax-1)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -x/c^3 - 5*log(abs(a*x - 1))/(a*c^3) + 1/6*(54*a^2*x^2 - 87*a*x + 37)/((a*x
 - 1)^3*a*c^3)
```

3.465 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

Optimal. Leaf size=88

$$-\frac{14}{ac^4(1-ax)}+\frac{8}{ac^4(1-ax)^2}-\frac{3}{ac^4(1-ax)^3}+\frac{1}{2ac^4(1-ax)^4}-\frac{6\log(1-ax)}{ac^4}-\frac{x}{c^4}$$

```
[Out] -(x/c^4) + 1/(2*a*c^4*(1 - a*x)^4) - 3/(a*c^4*(1 - a*x)^3) + 8/(a*c^4*(1 - a*x)^2) - 14/(a*c^4*(1 - a*x)) - (6*Log[1 - a*x])/(a*c^4)
```

Rubi [A] time = 0.138521, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 77}

$$-\frac{14}{ac^4(1-ax)}+\frac{8}{ac^4(1-ax)^2}-\frac{3}{ac^4(1-ax)^3}+\frac{1}{2ac^4(1-ax)^4}-\frac{6\log(1-ax)}{ac^4}-\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] -(x/c^4) + 1/(2*a*c^4*(1 - a*x)^4) - 3/(a*c^4*(1 - a*x)^3) + 8/(a*c^4*(1 - a*x)^2) - 14/(a*c^4*(1 - a*x)) - (6*Log[1 - a*x])/(a*c^4)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\int \frac{e^{2\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}$$
$$= \frac{a^4\int \frac{x^4(1+ax)}{(1-ax)^5}\,dx}{c^4}$$
$$= \frac{a^4\int \left(-\frac{1}{a^4}-\frac{2}{a^4(-1+ax)^5}-\frac{9}{a^4(-1+ax)^4}-\frac{16}{a^4(-1+ax)^3}-\frac{14}{a^4(-1+ax)^2}-\frac{6}{a^4(-1+ax)}\right)dx}{c^4}$$
$$= -\frac{x}{c^4}+\frac{1}{2ac^4(1-ax)^4}-\frac{3}{ac^4(1-ax)^3}+\frac{8}{ac^4(1-ax)^2}-\frac{14}{ac^4(1-ax)}-\frac{6\log(1-ax)}{ac^4}$$

Mathematica [A] time = 0.147132, size = 71, normalized size = 0.81

$$\frac{-2a^5x^5+8a^4x^4+16a^3x^3-60a^2x^2+56ax-12(ax-1)^4\log(1-ax)-17}{2ac^4(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (-17 + 56*a*x - 60*a^2*x^2 + 16*a^3*x^3 + 8*a^4*x^4 - 2*a^5*x^5 - 12*(-1 + a*x)^4*Log[1 - a*x])/(2*a*c^4*(-1 + a*x)^4)
```

Maple [A] time = 0.035, size = 82, normalized size = 0.9

$$-\frac{x}{c^4}+3\frac{1}{ac^4(ax-1)^3}+8\frac{1}{ac^4(ax-1)^2}+14\frac{1}{ac^4(ax-1)}-6\frac{\ln(ax-1)}{ac^4}+\frac{1}{2ac^4(ax-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^4,x)
```

```
[Out] -x/c^4+3/a/c^4/(a*x-1)^3+8/c^4/a/(a*x-1)^2+14/c^4/a/(a*x-1)-6/c^4/a*ln(a*x-1)+1/2/a/c^4/(a*x-1)^4
```

Maxima [A] time = 0.95665, size = 127, normalized size = 1.44

$$\frac{28a^3x^3-68a^2x^2+58ax-17}{2\left(a^5c^4x^4-4a^4c^4x^3+6a^3c^4x^2-4a^2c^4x+ac^4\right)}-\frac{x}{c^4}-\frac{6\log(ax-1)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/2*(28*a^3*x^3 - 68*a^2*x^2 + 58*a*x - 17)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4) - x/c^4 - 6*log(a*x - 1)/(a*c^4)
```

Fricas [A] time = 2.18946, size = 273, normalized size = 3.1

$$-\frac{2\,a^{5}x^{5}-8\,a^{4}x^{4}-16\,a^{3}x^{3}+60\,a^{2}x^{2}-56\,ax+12\left(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1\right)\log\left(ax-1\right)+17}{2\left(a^{5}c^{4}x^{4}-4\,a^{4}c^{4}x^{3}+6\,a^{3}c^{4}x^{2}-4\,a^{2}c^{4}x+ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^5*x^5 - 8*a^4*x^4 - 16*a^3*x^3 + 60*a^2*x^2 - 56*a*x + 12*(a^4*x^
4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 17)/(a^5*c^4*x^4 - 4*
a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [A] time = 0.651929, size = 94, normalized size = 1.07

$$\frac{28a^{3}x^{3}-68a^{2}x^{2}+58ax-17}{2a^{5}c^{4}x^{4}-8a^{4}c^{4}x^{3}+12a^{3}c^{4}x^{2}-8a^{2}c^{4}x+2ac^{4}}-\frac{x}{c^{4}}-\frac{6\log\left(ax-1\right)}{ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a/x)**4,x)
```

```
[Out] (28*a**3*x**3 - 68*a**2*x**2 + 58*a*x - 17)/(2*a**5*c**4*x**4 - 8*a**4*c**4
*x**3 + 12*a**3*c**4*x**2 - 8*a**2*c**4*x + 2*a*c**4) - x/c**4 - 6*log(a*x
- 1)/(a*c**4)
```

Giac [A] time = 1.13769, size = 80, normalized size = 0.91

$$-\frac{x}{c^{4}}-\frac{6\,\log\left(|ax-1|\right)}{ac^{4}}+\frac{28\,a^{3}x^{3}-68\,a^{2}x^{2}+58\,ax-17}{2\,(ax-1)^{4}ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -x/c^4 - 6*log(abs(a*x - 1))/(a*c^4) + 1/2*(28*a^3*x^3 - 68*a^2*x^2 + 58*a*
x - 17)/((a*x - 1)^4*a*c^4)
```

3.466 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$

Optimal. Leaf size=103

$$-\frac{c^4(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^4(3ax+2)\sqrt{1-a^2x^2}}{2a^2x}-\frac{3c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}+\frac{c^4\sin^{-1}(ax)}{a}$$

```
[Out] (c^4*(2 + 3*a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^4*(2 - 3*a*x)*(1 - a^2*x
^2)^(3/2))/(6*a^4*x^3) + (c^4*ArcSin[a*x])/a - (3*c^4*ArcTanh[Sqrt[1 - a^2*
x^2]])/(2*a)
```

Rubi [A] time = 0.167968, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6131, 6128, 811, 813, 844, 216, 266, 63, 208}

$$-\frac{c^4(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^4(3ax+2)\sqrt{1-a^2x^2}}{2a^2x}-\frac{3c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}+\frac{c^4\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] (c^4*(2 + 3*a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^4*(2 - 3*a*x)*(1 - a^2*x
^2)^(3/2))/(6*a^4*x^3) + (c^4*ArcSin[a*x])/a - (3*c^4*ArcTanh[Sqrt[1 - a^2*
x^2]])/(2*a)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
```

```
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\int \frac{e^{3\tanh^{-1}(ax)}(1-ax)^4}{x^4}dx}{a^4}$$
$$= \frac{c^4\int \frac{(1-ax)(1-a^2x^2)^{3/2}}{x^4}dx}{a^4}$$
$$= -\frac{c^4(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} - \frac{c^4\int \frac{(4a^2-6a^3x)\sqrt{1-a^2x^2}}{x^2}dx}{4a^4}$$
$$= \frac{c^4(2+3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^4(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + \frac{c^4\int \frac{12a^3+8a^4x}{x\sqrt{1-a^2x^2}}dx}{8a^4}$$
$$= \frac{c^4(2+3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^4(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + c^4\int \frac{1}{\sqrt{1-a^2x^2}}dx + \frac{(3c^4)\int \frac{1}{x\sqrt{1-a}}$$
$$= \frac{c^4(2+3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^4(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + \frac{c^4\sin^{-1}(ax)}{a} + \frac{(3c^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1}}\right.}{4a}$$
$$= \frac{c^4(2+3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^4(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + \frac{c^4\sin^{-1}(ax)}{a} - \frac{(3c^4)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-}\right.}{2a}$$
$$= \frac{c^4(2+3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^4(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + \frac{c^4\sin^{-1}(ax)}{a} - \frac{3c^4\tanh^{-1}\left(\sqrt{1-a}\right.}{2a}$$

Mathematica [A] time = 0.243841, size = 85, normalized size = 0.83

$$\frac{c^4\left(\frac{\sqrt{1-a^2x^2}(6a^3x^3+8a^2x^2+3ax-2)}{a^3x^3} - 9\log\left(\sqrt{1-a^2x^2}+1\right) + 9\log(ax) + 6\sin^{-1}(ax)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] (c^4*((Sqrt[1 - a^2*x^2]*(-2 + 3*a*x + 8*a^2*x^2 + 6*a^3*x^3))/(a^3*x^3) +
6*ArcSin[a*x] + 9*Log[a*x] - 9*Log[1 + Sqrt[1 - a^2*x^2]]))/(6*a)
```

Maple [A] time = 0.052, size = 180, normalized size = 1.8

$$-c^4ax^2\frac{1}{\sqrt{-a^2x^2+1}} + \frac{c^4}{2a}\frac{1}{\sqrt{-a^2x^2+1}} - \frac{4c^4x}{3}\frac{1}{\sqrt{-a^2x^2+1}} + c^4\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} + \frac{5c^4}{3a^2x}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^4,x)
```

```
[Out] -c^4*a*x^2/(-a^2*x^2+1)^(1/2)+1/2*c^4/a/(-a^2*x^2+1)^(1/2)-4/3*c^4*x/(-a^2*
x^2+1)^(1/2)+c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+5/3*c
^4/a^2/x/(-a^2*x^2+1)^(1/2)-3/2*c^4/a*arctanh(1/(-a^2*x^2+1)^(1/2))+1/2*c^4
/a^3/x^2/(-a^2*x^2+1)^(1/2)-1/3*c^4/a^4/x^3/(-a^2*x^2+1)^(1/2)
```

Maxima [B] time = 1.47497, size = 487, normalized size = 4.73

$$-a^3c^4\left(\frac{x^2}{\sqrt{-a^2x^2+1}a^2}-\frac{2}{\sqrt{-a^2x^2+1}a^4}\right)-a^2c^4\left(\frac{x}{\sqrt{-a^2x^2+1}a^2}-\frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2}\right)+\frac{3c^4x}{\sqrt{-a^2x^2+1}}+\frac{3c^4\left(\frac{1}{\sqrt{-a^2x^2+1}}-\log\left(\frac{2\sqrt{}}{}\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -a^3*c^4*(x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4)) - a^2*
c^4*(x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2))
+ 3*c^4*x/sqrt(-a^2*x^2 + 1) + 3*c^4*(1/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-a^
2*x^2 + 1)/abs(x) + 2/abs(x)))/a - 3*(2*a^2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(
-a^2*x^2 + 1)*x))*c^4/a^2 - 3*c^4/(sqrt(-a^2*x^2 + 1)*a) + 1/2*(3*a^2*log(2
*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 3*a^2/sqrt(-a^2*x^2 + 1) + 1/(sqrt
(-a^2*x^2 + 1)*x^2))*c^4/a^3 + 1/3*(8*a^4*x/sqrt(-a^2*x^2 + 1) - 4*a^2/(sqr
t(-a^2*x^2 + 1)*x) - 1/(sqrt(-a^2*x^2 + 1)*x^3))*c^4/a^4
```

Fricas [A] time = 2.1517, size = 282, normalized size = 2.74

$$-\frac{12\,a^3c^4x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-9\,a^3c^4x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-6\,a^3c^4x^3-\left(6\,a^3c^4x^3+8\,a^2c^4x^2+3\,ac^4x-2\,c^4\right)\sqrt{-a^2x^2+1}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(12*a^3*c^4*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 9*a^3*c^4*x^3
*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 6*a^3*c^4*x^3 - (6*a^3*c^4*x^3 + 8*a^2*c
^4*x^2 + 3*a*c^4*x - 2*c^4)*sqrt(-a^2*x^2 + 1))/(a^4*x^3)
```

Sympy [A] time = 13.3834, size = 354, normalized size = 3.44

$$-ac^4\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)+c^4\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\end{cases}\right)+\frac{2c^4\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for } \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}-\frac{2c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**4,x)
```

```
[Out] -a*c**4*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)
) + c**4*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a
**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + 2*c**4*Piecewise((-acosh(1/(a*x)),
1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a - 2*c**4*Piecewise((-I*sq
rt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/
a**2 - c**4*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/
```

```
(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1
/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3 + c**4*P
iecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x*
*3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*
x**2 + 1)/(3*x**3), True))/a**4
```

Giac [B] time = 1.19015, size = 354, normalized size = 3.44

$$\frac{\left(c^4 - \frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)c^4}{a^2x} - \frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^4}{a^4x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|} + \frac{c^4\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{|a|} - \frac{3\,c^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|} + \frac{\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] 1/24*(c^4 - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/(a^2*x) - 15*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^2*c^4/(a^4*x^2))*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*abs(a)) + c^4*arcsin(a*x)*sgn(a)/abs(a) - 3/2*c^4*log(1/2*abs(-2*sqrt(
-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c^4/a
 + 1/24*(15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/x + 3*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)^2*c^4/(a^2*x^2) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^4/(a^4*x^3
))/(a^2*abs(a))
```

3.467 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=77

$$\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}+\frac{3c^3\sqrt{1-a^2x^2}}{2a}-\frac{3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}$$

```
[Out] (3*c^3*Sqrt[1 - a^2*x^2])/(2*a) + (c^3*(1 - a^2*x^2)^(3/2))/(2*a^3*x^2) - (
3*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rubi [A] time = 0.121125, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6131, 6128, 266, 47, 50, 63, 208}

$$\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}+\frac{3c^3\sqrt{1-a^2x^2}}{2a}-\frac{3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] (3*c^3*Sqrt[1 - a^2*x^2])/(2*a) + (c^3*(1 - a^2*x^2)^(3/2))/(2*a^3*x^2) - (
3*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx &= -\frac{c^3\int \frac{e^{3\tanh^{-1}(ax)}(1-ax)^3}{x^3}\,dx}{a^3}\\
&= -\frac{c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{x^3}\,dx}{a^3}\\
&= -\frac{c^3\,\text{Subst}\left(\int \frac{\left(1-a^2x\right)^{3/2}}{x^2}\,dx,x,x^2\right)}{2a^3}\\
&= \frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}+\frac{\left(3c^3\right)\text{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x}\,dx,x,x^2\right)}{4a}\\
&= \frac{3c^3\sqrt{1-a^2x^2}}{2a}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}+\frac{\left(3c^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{4a}\\
&= \frac{3c^3\sqrt{1-a^2x^2}}{2a}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{\left(3c^3\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{2a^3}\\
&= \frac{3c^3\sqrt{1-a^2x^2}}{2a}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.129802, size = 77, normalized size = 1.

$$
\frac{\sqrt{1-a^2x^2}\left(\frac{c^3}{2a^2x^2}+c^3\right)}{a}-\frac{3c^3\log\left(\sqrt{1-a^2x^2}+1\right)}{2a}+\frac{3c^3\log(ax)}{2a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] ((c^3 + c^3/(2*a^2*x^2))*Sqrt[1 - a^2*x^2])/a + (3*c^3*Log[a*x])/(2*a) - (3
*c^3*Log[1 + Sqrt[1 - a^2*x^2]])/(2*a)
```

Maple [A] time = 0.043, size = 118, normalized size = 1.5

$$\frac{c^3}{a^3}\left(a^6\left(-\frac{x^2}{a^2}\frac{1}{\sqrt{-a^2x^2+1}}+2\frac{1}{a^4\sqrt{-a^2x^2+1}}\right)-3\frac{a^2}{\sqrt{-a^2x^2+1}}+\frac{3a^2}{2}\left(\frac{1}{\sqrt{-a^2x^2+1}}-\operatorname{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)+\frac{1}{2x^2}\frac{1}{\sqrt{-a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^3,x)
```

```
[Out] c^3/a^3*(a^6*(-x^2/a^2/(-a^2*x^2+1)^(1/2)+2/a^4/(-a^2*x^2+1)^(1/2))-3*a^2/(
-a^2*x^2+1)^(1/2)+3/2*a^2*(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2
)))+1/2/x^2/(-a^2*x^2+1)^(1/2))
```

Maxima [B] time = 1.44512, size = 254, normalized size = 3.3

$$-a^3c^3\left(\frac{x^2}{\sqrt{-a^2x^2+1}a^2}-\frac{2}{\sqrt{-a^2x^2+1}a^4}\right)+\frac{3c^3\left(\frac{1}{\sqrt{-a^2x^2+1}}-\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)\right)}{a}-\frac{3c^3}{\sqrt{-a^2x^2+1}a}+\frac{\left(3a^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -a^3*c^3*(x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4)) + 3*c^
3*(1/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)))/a -
3*c^3/(sqrt(-a^2*x^2 + 1)*a) + 1/2*(3*a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) +
 2/abs(x)) - 3*a^2/sqrt(-a^2*x^2 + 1) + 1/(sqrt(-a^2*x^2 + 1)*x^2))*c^3/a^3
```

Fricas [A] time = 2.08806, size = 165, normalized size = 2.14

$$\frac{3a^2c^3x^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+2a^2c^3x^2+\left(2a^2c^3x^2+c^3\right)\sqrt{-a^2x^2+1}}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^2*c^3*x^2*log((sqrt(-a^2*x^2 + 1) - 1)/x) + 2*a^2*c^3*x^2 + (2*a^2
*c^3*x^2 + c^3)*sqrt(-a^2*x^2 + 1))/(a^3*x^2)
```

Sympy [A] time = 21.5522, size = 104, normalized size = 1.35

$$\frac{2c^3\sqrt{-a^2x^2+1}+\frac{3c^3\log\left(-1+\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}-\frac{3c^3\log\left(1+\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{c^3}{2\left(1+\frac{1}{\sqrt{-a^2x^2+1}}\right)}+\frac{c^3}{2\left(-1+\frac{1}{\sqrt{-a^2x^2+1}}\right)}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**3,x)
```

```
[Out] (2*c**3*sqrt(-a**2*x**2 + 1) + 3*c**3*log(-1 + 1/sqrt(-a**2*x**2 + 1))/2 -
3*c**3*log(1 + 1/sqrt(-a**2*x**2 + 1))/2 + c**3/(2*(1 + 1/sqrt(-a**2*x**2 +
 1))) + c**3/(2*(-1 + 1/sqrt(-a**2*x**2 + 1))))/(2*a)
```

Giac [A] time = 1.16145, size = 107, normalized size = 1.39

$$\frac{c^3\left(4\sqrt{-a^2x^2+1}+\frac{2\sqrt{-a^2x^2+1}}{a^2x^2}-3\log\left(\sqrt{-a^2x^2+1}+1\right)+3\log\left(-\sqrt{-a^2x^2+1}+1\right)\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] 1/4*c^3*(4*sqrt(-a^2*x^2 + 1) + 2*sqrt(-a^2*x^2 + 1)/(a^2*x^2) - 3*log(sqrt
(-a^2*x^2 + 1) + 1) + 3*log(-sqrt(-a^2*x^2 + 1) + 1))/a
```

3.468 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$

Optimal. Leaf size=67

$$-\frac{c^2(1-ax)\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{c^2\sin^{-1}(ax)}{a}$$

```
[Out] -((c^2*(1 - a*x)*Sqrt[1 - a^2*x^2])/(a^2*x)) - (c^2*ArcSin[a*x])/a - (c^2*A
rcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rubi [A] time = 0.158355, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6131, 6128, 850, 813, 844, 216, 266, 63, 208}

$$-\frac{c^2(1-ax)\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{c^2\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] -((c^2*(1 - a*x)*Sqrt[1 - a^2*x^2])/(a^2*x)) - (c^2*ArcSin[a*x])/a - (c^2*A
rcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
```

```
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx &= \frac{c^2\int \frac{e^{3\tanh^{-1}(ax)}(1-ax)^2}{x^2}dx}{a^2}\\
&= \frac{c^2\int \frac{\left(1-a^2x^2\right)^{3/2}}{x^2(1-ax)}dx}{a^2}\\
&= \frac{c^2\int \frac{(1+ax)\sqrt{1-a^2x^2}}{x^2}dx}{a^2}\\
&= -\frac{c^2(1-ax)\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\int \frac{-2a+2a^2x}{x\sqrt{1-a^2x^2}}dx}{2a^2}\\
&= -\frac{c^2(1-ax)\sqrt{1-a^2x^2}}{a^2x}-c^2\int \frac{1}{\sqrt{1-a^2x^2}}dx+\frac{c^2\int \frac{1}{x\sqrt{1-a^2x^2}}dx}{a}\\
&= -\frac{c^2(1-ax)\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\sin^{-1}(ax)}{a}+\frac{c^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}dx,x,x^2\right)}{2a}\\
&= -\frac{c^2(1-ax)\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\sin^{-1}(ax)}{a}-\frac{c^2\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}dx,x,\sqrt{1-a^2x^2}\right)}{a^3}\\
&= -\frac{c^2(1-ax)\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\sin^{-1}(ax)}{a}-\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.110829, size = 82, normalized size = 1.22

$$
\frac{\sqrt{1-a^2x^2}\left(c^2-\frac{c^2}{ax}\right)}{a}-\frac{c^2\log\left(\sqrt{1-a^2x^2}+1\right)}{a}+\frac{c^2\log(ax)}{a}-\frac{c^2\sin^{-1}(ax)}{a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] ((c^2 - c^2/(a*x))*Sqrt[1 - a^2*x^2])/a - (c^2*ArcSin[a*x])/a + (c^2*Log[a*
x])/a - (c^2*Log[1 + Sqrt[1 - a^2*x^2]])/a
```

Maple [B] time = 0.044, size = 133, normalized size = 2.

$$
-ac^2x^2\frac{1}{\sqrt{-a^2x^2+1}}+\frac{c^2}{a}\frac{1}{\sqrt{-a^2x^2+1}}+xc^2\frac{1}{\sqrt{-a^2x^2+1}}-c^2\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{c^2}{a^2x}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{c^2}{a}\mathrm{Art}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^2,x)
```

```
[Out] -c^2*a*x^2/(-a^2*x^2+1)^(1/2)+c^2/a/(-a^2*x^2+1)^(1/2)+c^2*x/(-a^2*x^2+1)^(
1/2)-c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-c^2/a^2/x/(-a
^2*x^2+1)^(1/2)-c^2/a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [B] time = 1.45598, size = 298, normalized size = 4.45

$$-a^3c^2\left(\frac{x^2}{\sqrt{-a^2x^2+1}a^2}-\frac{2}{\sqrt{-a^2x^2+1}a^4}\right)+a^2c^2\left(\frac{x}{\sqrt{-a^2x^2+1}a^2}-\frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2}\right)-\frac{2c^2x}{\sqrt{-a^2x^2+1}}+\frac{c^2\left(\frac{1}{\sqrt{-a^2x^2+1}}-\log\left(\frac{2}{\cdots}\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -a^3*c^2*(x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4)) + a^2*
c^2*(x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2))
- 2*c^2*x/sqrt(-a^2*x^2 + 1) + c^2*(1/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-a^2*
x^2 + 1)/abs(x) + 2/abs(x)))/a + (2*a^2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2
*x^2 + 1)*x))*c^2/a^2 - 2*c^2/(sqrt(-a^2*x^2 + 1)*a)
```

Fricas [A] time = 2.11682, size = 201, normalized size = 3.

$$\frac{2\,ac^2x\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+ac^2x\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+ac^2x+\left(ac^2x-c^2\right)\sqrt{-a^2x^2+1}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] (2*a*c^2*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + a*c^2*x*log((sqrt(-a^2*
x^2 + 1) - 1)/x) + a*c^2*x + (a*c^2*x - c^2)*sqrt(-a^2*x^2 + 1))/(a^2*x)
```

Sympy [A] time = 9.19275, size = 151, normalized size = 2.25

$$-ac^2\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-c^2\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)+\frac{c^2\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**2,x)
```

```
[Out] -a*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)
) - c**2*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a
**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + c**2*Piecewise((-acosh(1/(a*x)), 1/
Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a + c**2*Piecewise((-I*sqrt(a
**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a**2
```

Giac [B] time = 1.19415, size = 188, normalized size = 2.81

$$\frac{a^2c^2x}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|}-\frac{c^2\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{|a|}-\frac{c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|}+\frac{\sqrt{-a^2x^2+1}c^2}{a}-\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)c\cdots}{2a^2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*c^2*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - c^2*arcsin(a*x)*sg
n(a)/abs(a) - c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(
x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c^2/a - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)*c^2/(a^2*x*abs(a))
```

3.469 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=50

$$\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{2c\sin^{-1}(ax)}{a}$$

```
[Out] (c*Sqrt[1 - a^2*x^2])/a - (2*c*ArcSin[a*x])/a + (c*ArcTanh[Sqrt[1 - a^2*x^2
]])/a
```

Rubi [A] time = 0.163831, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6131, 6128, 852, 1809, 844, 216, 266, 63, 208}

$$\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{2c\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a*x)),x]
```


```
[Out] (c*Sqrt[1 - a^2*x^2])/a - (2*c*ArcSin[a*x])/a + (c*ArcTanh[Sqrt[1 - a^2*x^2
]])/a
```


Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= -\frac{c\int \frac{e^{3\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}\\
&= -\frac{c\int \frac{\left(1-a^2x^2\right)^{3/2}}{x(1-ax)^2}\,dx}{a}\\
&= -\frac{c\int \frac{(1+ax)^2}{x\sqrt{1-a^2x^2}}\,dx}{a}\\
&= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\int \frac{-a^2-2a^3x}{x\sqrt{1-a^2x^2}}\,dx}{a^3}\\
&= \frac{c\sqrt{1-a^2x^2}}{a}-(2c)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx-\frac{c\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{a}\\
&= \frac{c\sqrt{1-a^2x^2}}{a}-\frac{2c\sin^{-1}(ax)}{a}-\frac{c\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2a}\\
&= \frac{c\sqrt{1-a^2x^2}}{a}-\frac{2c\sin^{-1}(ax)}{a}+\frac{c\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^3}\\
&= \frac{c\sqrt{1-a^2x^2}}{a}-\frac{2c\sin^{-1}(ax)}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.053473, size = 47, normalized size = 0.94

$$\frac{c\left(\sqrt{1-a^2x^2}+\log\left(\sqrt{1-a^2x^2}+1\right)-2\sin^{-1}(ax)-\log(x)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a*x)),x]
```

```
[Out] (c*(Sqrt[1 - a^2*x^2] - 2*ArcSin[a*x] - Log[x] + Log[1 + Sqrt[1 - a^2*x^2]]
))/a
```

Maple [A] time = 0.041, size = 84, normalized size = 1.7

$$-acx^2\frac{1}{\sqrt{-a^2x^2+1}}+\frac{c}{a}\frac{1}{\sqrt{-a^2x^2+1}}-2\frac{c}{\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+\frac{c}{a}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x),x)
```

```
[Out] -c*a*x^2/(-a^2*x^2+1)^(1/2)+c/a/(-a^2*x^2+1)^(1/2)-2*c/(a^2)^(1/2)*arctan((
a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+c/a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [B] time = 1.48107, size = 205, normalized size = 4.1

$$-a^3c\left(\frac{x^2}{\sqrt{-a^2x^2+1}a^2}-\frac{2}{\sqrt{-a^2x^2+1}a^4}\right)+2\,a^2c\left(\frac{x}{\sqrt{-a^2x^2+1}a^2}-\frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2}\right)-\frac{2\,cx}{\sqrt{-a^2x^2+1}}-\frac{c\left(\frac{1}{\sqrt{-a^2x^2+1}}-\log\left(\frac{2\sqrt{}}{}\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -a^3*c*(x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4)) + 2*a^2*
c*(x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2)) -
2*c*x/sqrt(-a^2*x^2 + 1) - c*(1/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-a^2*x^2 +
1)/abs(x) + 2/abs(x)))/a
```

Fricas [A] time = 2.11897, size = 144, normalized size = 2.88

$$\frac{4\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-c\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (4*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - c*log((sqrt(-a^2*x^2 + 1) - 1
)/x) + sqrt(-a^2*x^2 + 1)*c)/a
```

Sympy [A] time = 7.76892, size = 104, normalized size = 2.08

$$-ac\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2 = 0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right) - 2c\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2 > 0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2 < 0\end{cases}\right) - \frac{c\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for } \frac{1}{|a^2x^2|} > 1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x),x)
```

```
[Out] -a*c*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) -
 2*c*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)
*asinh(x*sqrt(-a**2)), a**2 < 0)) - c*Piecewise((-acosh(1/(a*x)), 1/Abs(a**
2*x**2) > 1), (I*asin(1/(a*x)), True))/a
```

Giac [A] time = 1.19219, size = 92, normalized size = 1.84

$$-\frac{2\,c\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{|a|} + \frac{c\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|} + \frac{\sqrt{-a^2x^2+1}c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x),x, algorithm="giac")
```

```
[Out] -2*c*arcsin(a*x)*sgn(a)/abs(a) + c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a)
 - 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c/a
```

3.470 $\int \frac{e^{3\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=99

$$-\frac{\left(1-a^2x^2\right)^{5/2}}{3ac(1-ax)^4}+\frac{8\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^2}+\frac{4\sqrt{1-a^2x^2}}{ac}-\frac{4\sin^{-1}(ax)}{ac}$$

```
[Out] (4*Sqrt[1 - a^2*x^2])/(a*c) + (8*(1 - a^2*x^2)^(3/2))/(3*a*c*(1 - a*x)^2) -
 (1 - a^2*x^2)^(5/2)/(3*a*c*(1 - a*x)^4) - (4*ArcSin[a*x])/(a*c)
```

Rubi [A] time = 0.120271, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6131, 6128, 793, 663, 665, 216}

$$-\frac{\left(1-a^2x^2\right)^{5/2}}{3ac(1-ax)^4}+\frac{8\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^2}+\frac{4\sqrt{1-a^2x^2}}{ac}-\frac{4\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a*x)),x]
```

```
[Out] (4*Sqrt[1 - a^2*x^2])/(a*c) + (8*(1 - a^2*x^2)^(3/2))/(3*a*c*(1 - a*x)^2) -
 (1 - a^2*x^2)^(5/2)/(3*a*c*(1 - a*x)^4) - (4*ArcSin[a*x])/(a*c)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
```

```
2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx &= -\frac{a\int \frac{e^{3\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}\\
&= -\frac{a\int \frac{x\left(1-a^2x^2\right)^{3/2}}{(1-ax)^4}\,dx}{c}\\
&= -\frac{\left(1-a^2x^2\right)^{5/2}}{3ac(1-ax)^4}+\frac{4\int \frac{\left(1-a^2x^2\right)^{3/2}}{(1-ax)^3}\,dx}{3c}\\
&= \frac{8\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^2}-\frac{\left(1-a^2x^2\right)^{5/2}}{3ac(1-ax)^4}-\frac{4\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx}{c}\\
&= \frac{4\sqrt{1-a^2x^2}}{ac}+\frac{8\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^2}-\frac{\left(1-a^2x^2\right)^{5/2}}{3ac(1-ax)^4}-\frac{4\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= \frac{4\sqrt{1-a^2x^2}}{ac}+\frac{8\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^2}-\frac{\left(1-a^2x^2\right)^{5/2}}{3ac(1-ax)^4}-\frac{4\sin^{-1}(ax)}{ac}
\end{aligned}
$$

Mathematica [C] time = 0.0338071, size = 62, normalized size = 0.63

$$
-\frac{16\sqrt{2}(ax-1)\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2},\frac{1}{2}(1-ax)\right)+(ax+1)^{5/2}}{3ac(1-ax)^{3/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a*x)),x]
```

```
[Out] -((1 + a*x)^(5/2) + 16*Sqrt[2]*(-1 + a*x)*Hypergeometric2F1[-3/2, -1/2, 1/2
, (1 - a*x)/2])/(3*a*c*(1 - a*x)^(3/2))
```

Maple [A] time = 0.046, size = 168, normalized size = 1.7

$$
-\frac{ax^2}{c}\frac{1}{\sqrt{-a^2x^2+1}}+9\frac{1}{ac\sqrt{-a^2x^2+1}}+12\frac{x}{c\sqrt{-a^2x^2+1}}-4\frac{1}{c\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+\frac{8}{3a^2c}\left(x-a^{-1}\right)^{-1}\frac{}{\sqrt{-a^2\left(x-a\right.}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x),x)
```

```
[Out] -a/c*x^2/(-a^2*x^2+1)^(1/2)+9/a/c/(-a^2*x^2+1)^(1/2)+12/c*x/(-a^2*x^2+1)^(1
/2)-4/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+8/3/a^2/c/(x-1
/a)/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-16/3/c/(-a^2*(x-1/a)^2-2*a*(x-1/a))^
(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))), x)
```

Fricas [A] time = 2.17749, size = 236, normalized size = 2.38

$$\frac{19\,a^2x^2-38\,ax+24\left(a^2x^2-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3\,a^2x^2-26\,ax+19\right)\sqrt{-a^2x^2+1}+19}{3\left(a^3cx^2-2\,a^2cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] 1/3*(19*a^2*x^2 - 38*a*x + 24*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2 +
 1) - 1)/(a*x)) + (3*a^2*x^2 - 26*a*x + 19)*sqrt(-a^2*x^2 + 1) + 19)/(a^3*c
*x^2 - 2*a^2*c*x + a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\left(\int \frac{x}{-a^3x^3\sqrt{-a^2x^2+1}+a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{3ax^2}{-a^3x^3\sqrt{-a^2x^2+1}+a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{}{-a^3x^3}\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a/x),x)
```

```
[Out] a*(Integral(x/(-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2
+ 1) + a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(3*a*
x**2/(-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1) + a*
x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(3*a**2*x**3/(
-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1) + a*x*sqrt
(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**4/(-a**3*x*
*3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1) + a*x*sqrt(-a**2*x
**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

Giac [A] time = 1.16348, size = 170, normalized size = 1.72

$$-\frac{4\,\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{c|a|}+\frac{\sqrt{-a^2x^2+1}}{ac}-\frac{8\left(\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}-4\right)}{3\,c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] -4*arcsin(a*x)*sgn(a)/(c*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c) - 8/3*(9*(sqrt(
-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^
4*x^2) - 4)/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^3*abs(a))
```

3.471 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal. Leaf size=128

$$\frac{(ax+1)^5}{5ac^2\left(1-a^2x^2\right)^{5/2}}-\frac{2(ax+1)^4}{3ac^2\left(1-a^2x^2\right)^{3/2}}+\frac{10(ax+1)^2}{3ac^2\sqrt{1-a^2x^2}}+\frac{5\sqrt{1-a^2x^2}}{ac^2}-\frac{5\sin^{-1}(ax)}{ac^2}$$

```
[Out] (1 + a*x)^5/(5*a*c^2*(1 - a^2*x^2)^(5/2)) - (2*(1 + a*x)^4)/(3*a*c^2*(1 - a
^2*x^2)^(3/2)) + (10*(1 + a*x)^2)/(3*a*c^2*Sqrt[1 - a^2*x^2]) + (5*Sqrt[1 -
 a^2*x^2])/(a*c^2) - (5*ArcSin[a*x])/(a*c^2)
```

Rubi [A] time = 0.257292, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6131, 6128, 852, 1635, 21, 669, 641, 216}

$$\frac{(ax+1)^5}{5ac^2\left(1-a^2x^2\right)^{5/2}}-\frac{2(ax+1)^4}{3ac^2\left(1-a^2x^2\right)^{3/2}}+\frac{10(ax+1)^2}{3ac^2\sqrt{1-a^2x^2}}+\frac{5\sqrt{1-a^2x^2}}{ac^2}-\frac{5\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] (1 + a*x)^5/(5*a*c^2*(1 - a^2*x^2)^(5/2)) - (2*(1 + a*x)^4)/(3*a*c^2*(1 - a
^2*x^2)^(3/2)) + (10*(1 + a*x)^2)/(3*a*c^2*Sqrt[1 - a^2*x^2]) + (5*Sqrt[1 -
 a^2*x^2])/(a*c^2) - (5*ArcSin[a*x])/(a*c^2)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
```

```
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx &= \frac{a^2\int \frac{e^{3\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2} \\
&= \frac{a^2\int \frac{x^2(1-a^2x^2)^{3/2}}{(1-ax)^5}\,dx}{c^2} \\
&= \frac{a^2\int \frac{x^2(1+ax)^5}{(1-a^2x^2)^{7/2}}\,dx}{c^2} \\
&= \frac{(1+ax)^5}{5ac^2(1-a^2x^2)^{5/2}} - \frac{a^2\int \frac{\left(\frac{5}{a^2}+\frac{5x}{a}\right)(1+ax)^4}{(1-a^2x^2)^{5/2}}\,dx}{5c^2} \\
&= \frac{(1+ax)^5}{5ac^2(1-a^2x^2)^{5/2}} - \frac{\int \frac{(1+ax)^5}{(1-a^2x^2)^{5/2}}\,dx}{c^2} \\
&= \frac{(1+ax)^5}{5ac^2(1-a^2x^2)^{5/2}} - \frac{2(1+ax)^4}{3ac^2(1-a^2x^2)^{3/2}} + \frac{5\int \frac{(1+ax)^3}{(1-a^2x^2)^{3/2}}\,dx}{3c^2} \\
&= \frac{(1+ax)^5}{5ac^2(1-a^2x^2)^{5/2}} - \frac{2(1+ax)^4}{3ac^2(1-a^2x^2)^{3/2}} + \frac{10(1+ax)^2}{3ac^2\sqrt{1-a^2x^2}} - \frac{5\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx}{c^2} \\
&= \frac{(1+ax)^5}{5ac^2(1-a^2x^2)^{5/2}} - \frac{2(1+ax)^4}{3ac^2(1-a^2x^2)^{3/2}} + \frac{10(1+ax)^2}{3ac^2\sqrt{1-a^2x^2}} + \frac{5\sqrt{1-a^2x^2}}{ac^2} - \frac{5\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^2} \\
&= \frac{(1+ax)^5}{5ac^2(1-a^2x^2)^{5/2}} - \frac{2(1+ax)^4}{3ac^2(1-a^2x^2)^{3/2}} + \frac{10(1+ax)^2}{3ac^2\sqrt{1-a^2x^2}} + \frac{5\sqrt{1-a^2x^2}}{ac^2} - \frac{5\sin^{-1}(ax)}{ac^2}
\end{aligned}
$$

Mathematica [A] time = 0.123158, size = 61, normalized size = 0.48

$$
\frac{\frac{\sqrt{1-a^2x^2}(15a^3x^3-188a^2x^2+279ax-118)}{(ax-1)^3} - 75\sin^{-1}(ax)}{15ac^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(-118 + 279*a*x - 188*a^2*x^2 + 15*a^3*x^3))/(-1 + a*x)
^3 - 75*ArcSin[a*x])/(15*a*c^2)
```

Maple [A] time = 0.051, size = 212, normalized size = 1.7

$$
-\frac{ax^2}{c^2}\frac{1}{\sqrt{-a^2x^2+1}} + 14\frac{1}{ac^2\sqrt{-a^2x^2+1}} + 25\frac{x}{c^2\sqrt{-a^2x^2+1}} - 5\frac{1}{c^2\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right) + \frac{8}{5a^3c^2}\left(x-a^{-1}\right)^{-2}\frac{}{\sqrt{-}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^2,x)
```

[Out] -a/c^2*x^2/(-a^2*x^2+1)^(1/2)+14/a/c^2/(-a^2*x^2+1)^(1/2)+25*x/c^2/(-a^2*x^2+1)^(1/2)-5/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+8/5/a^3/c^2/(x-1/a)^2/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+116/15/a^2/c^2/(x-1/a)/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-232/15/c^2/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x

---

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^2,x, algorithm="maxima")

[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^2), x)

Fricas [A] time = 2.20754, size = 331, normalized size = 2.59

$$\frac{118\,a^3x^3-354\,a^2x^2+354\,ax+150\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(15\,a^3x^3-188\,a^2x^2+279\,ax-118\right)}{15\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^2,x, algorithm="fricas")

[Out] 1/15*(118*a^3*x^3 - 354*a^2*x^2 + 354*a*x + 150*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (15*a^3*x^3 - 188*a^2*x^2 + 279*a*x - 118)*sqrt(-a^2*x^2 + 1) - 118)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\left(\int \frac{x^2}{-a^4x^4\sqrt{-a^2x^2+1}+2a^3x^3\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{3ax^3}{-a^4x^4\sqrt{-a^2x^2+1}+2a^3x^3\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{}{-a^4}\right.}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a/x)**2,x)

[Out] a**2*(Integral(x**2/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(3*a*x**3/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(3*a**2*x**4/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**5/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2

Giac [A] time = 1.22684, size = 243, normalized size = 1.9

$$-\frac{5\arcsin(ax)\,\mathrm{sgn}(a)}{c^2|a|}+\frac{\sqrt{-a^2x^2+1}}{ac^2}-\frac{2\left(\frac{440\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{670\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{360\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}-\frac{75\left(\sqrt{-a^2x^2+1}|a|+\right.}{a^8x^4}\right.}{15\,c^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -5*arcsin(a*x)*sgn(a)/(c^2*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c^2) - 2/15*(440
*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 670*(sqrt(-a^2*x^2 + 1)*abs(a) +
 a)^2/(a^4*x^2) + 360*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 75*(sqr
t(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 103)/(c^2*((sqrt(-a^2*x^2 + 1)*ab
s(a) + a)/(a^2*x) - 1)^5*abs(a))
```

3.472 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=155

$$-\frac{(ax+1)^6}{7ac^3\left(1-a^2x^2\right)^{7/2}}+\frac{4(ax+1)^5}{7ac^3\left(1-a^2x^2\right)^{5/2}}-\frac{(ax+1)^4}{ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{4(ax+1)^2}{ac^3\sqrt{1-a^2x^2}}+\frac{6\sqrt{1-a^2x^2}}{ac^3}-\frac{6\sin^{-1}(ax)}{ac^3}$$

```
[Out] -(1 + a*x)^6/(7*a*c^3*(1 - a^2*x^2)^(7/2)) + (4*(1 + a*x)^5)/(7*a*c^3*(1 -
a^2*x^2)^(5/2)) - (1 + a*x)^4/(a*c^3*(1 - a^2*x^2)^(3/2)) + (4*(1 + a*x)^2)
/(a*c^3*Sqrt[1 - a^2*x^2]) + (6*Sqrt[1 - a^2*x^2])/(a*c^3) - (6*ArcSin[a*x]
)/(a*c^3)
```

Rubi [A] time = 0.334214, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6131, 6128, 852, 1635, 789, 669, 641, 216}

$$-\frac{(ax+1)^6}{7ac^3\left(1-a^2x^2\right)^{7/2}}+\frac{4(ax+1)^5}{7ac^3\left(1-a^2x^2\right)^{5/2}}-\frac{(ax+1)^4}{ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{4(ax+1)^2}{ac^3\sqrt{1-a^2x^2}}+\frac{6\sqrt{1-a^2x^2}}{ac^3}-\frac{6\sin^{-1}(ax)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] -(1 + a*x)^6/(7*a*c^3*(1 - a^2*x^2)^(7/2)) + (4*(1 + a*x)^5)/(7*a*c^3*(1 -
a^2*x^2)^(5/2)) - (1 + a*x)^4/(a*c^3*(1 - a^2*x^2)^(3/2)) + (4*(1 + a*x)^2)
/(a*c^3*Sqrt[1 - a^2*x^2]) + (6*Sqrt[1 - a^2*x^2])/(a*c^3) - (6*ArcSin[a*x]
)/(a*c^3)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
```

```
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx = -\frac{a^3\int \frac{e^{3\tanh^{-1}(ax)}x^3}{(1-ax)^3}dx}{c^3}$$

$$= -\frac{a^3\int \frac{x^3(1-a^2x^2)^{3/2}}{(1-ax)^6}dx}{c^3}$$

$$= -\frac{a^3\int \frac{x^3(1+ax)^6}{(1-a^2x^2)^{9/2}}dx}{c^3}$$

$$= -\frac{(1+ax)^6}{7ac^3(1-a^2x^2)^{7/2}} + \frac{a^3\int \frac{(1+ax)^5\left(\frac{6}{a^3}+\frac{7x}{a^2}+\frac{7x^2}{a}\right)}{(1-a^2x^2)^{7/2}}dx}{7c^3}$$

$$= -\frac{(1+ax)^6}{7ac^3(1-a^2x^2)^{7/2}} + \frac{4(1+ax)^5}{7ac^3(1-a^2x^2)^{5/2}} - \frac{a^3\int \frac{\left(\frac{70}{a^3}+\frac{35x}{a^2}\right)(1+ax)^4}{(1-a^2x^2)^{5/2}}dx}{35c^3}$$

$$= -\frac{(1+ax)^6}{7ac^3(1-a^2x^2)^{7/2}} + \frac{4(1+ax)^5}{7ac^3(1-a^2x^2)^{5/2}} - \frac{(1+ax)^4}{ac^3(1-a^2x^2)^{3/2}} + \frac{2\int \frac{(1+ax)^3}{(1-a^2x^2)^{3/2}}dx}{c^3}$$

$$= -\frac{(1+ax)^6}{7ac^3(1-a^2x^2)^{7/2}} + \frac{4(1+ax)^5}{7ac^3(1-a^2x^2)^{5/2}} - \frac{(1+ax)^4}{ac^3(1-a^2x^2)^{3/2}} + \frac{4(1+ax)^2}{ac^3\sqrt{1-a^2x^2}} - \frac{6\int \frac{1+ax}{\sqrt{1-a^2x^2}}dx}{c^3}$$

$$= -\frac{(1+ax)^6}{7ac^3(1-a^2x^2)^{7/2}} + \frac{4(1+ax)^5}{7ac^3(1-a^2x^2)^{5/2}} - \frac{(1+ax)^4}{ac^3(1-a^2x^2)^{3/2}} + \frac{4(1+ax)^2}{ac^3\sqrt{1-a^2x^2}} + \frac{6\sqrt{1-a^2x^2}}{ac^3} - \frac{6\int \frac{}{\sqrt{1}}}{c}$$

$$= -\frac{(1+ax)^6}{7ac^3(1-a^2x^2)^{7/2}} + \frac{4(1+ax)^5}{7ac^3(1-a^2x^2)^{5/2}} - \frac{(1+ax)^4}{ac^3(1-a^2x^2)^{3/2}} + \frac{4(1+ax)^2}{ac^3\sqrt{1-a^2x^2}} + \frac{6\sqrt{1-a^2x^2}}{ac^3} - \frac{6\sin^{-1}}{ac^3}$$

Mathematica [A] time = 0.148317, size = 69, normalized size = 0.45

$$\frac{\frac{\sqrt{1-a^2x^2}(7a^4x^4-116a^3x^3+261a^2x^2-222ax+66)}{(ax-1)^4} - 42\sin^{-1}(ax)}{7ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(66 - 222*a*x + 261*a^2*x^2 - 116*a^3*x^3 + 7*a^4*x^4))
/(-1 + a*x)^4 - 42*ArcSin[a*x])/(7*a*c^3)
```

Maple [A] time = 0.052, size = 256, normalized size = 1.7

$$-\frac{ax^2}{c^3}\frac{1}{\sqrt{-a^2x^2+1}} + 20\frac{1}{ac^3\sqrt{-a^2x^2+1}} + 44\frac{x}{c^3\sqrt{-a^2x^2+1}} - 6\frac{1}{c^3\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right) + \frac{8}{7a^4c^3}\left(x-a^{-1}\right)^{-3}\frac{}{\sqrt{-a^2(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^3,x)
```

```
[Out] -a/c^3*x^2/(-a^2*x^2+1)^(1/2)+20/a/c^3/(-a^2*x^2+1)^(1/2)+44*x/c^3/(-a^2*x^
2+1)^(1/2)-6/c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+8/7/a
^4/c^3/(x-1/a)^3/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+44/7/a^3/c^3/(x-1/a)^2/
(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+110/7/a^2/c^3/(x-1/a)/(-a^2*(x-1/a)^2-2*
a*(x-1/a))^(1/2)-220/7/c^3/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^3), x)
```

Fricas [A] time = 2.20865, size = 398, normalized size = 2.57

$$\frac{66\,a^4x^4-264\,a^3x^3+396\,a^2x^2-264\,ax+84\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(7\,a^4x^4-116\,a\right.}{7\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/7*(66*a^4*x^4 - 264*a^3*x^3 + 396*a^2*x^2 - 264*a*x + 84*(a^4*x^4 - 4*a^3
*x^3 + 6*a^2*x^2 - 4*a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (7*a
^4*x^4 - 116*a^3*x^3 + 261*a^2*x^2 - 222*a*x + 66)*sqrt(-a^2*x^2 + 1) + 66)
/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3\left(\int\frac{x^3}{-a^5x^5\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-2a^3x^3\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int\frac{}{-a^5x^5\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-2a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*(Integral(x**3/(-a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a*
*2*x**2 + 1) - 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x*
*2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral
(3*a*x**4/(-a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 +
1) - 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) +
3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(3*a**2*x*
*5/(-a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*
a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + 3*a*x*s
qrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**6/(-a**5
```

```
*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**3*x**3
*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + 3*a*x*sqrt(-a**2
*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [A] time = 1.22897, size = 316, normalized size = 2.04

$$-\frac{6\arcsin(ax)\operatorname{sgn}(a)}{c^3|a|}+\frac{\sqrt{-a^2x^2+1}}{ac^3}-\frac{2\left(\frac{371\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{952\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{1246\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}-\frac{819\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^8x^4}\right.}{7c^3\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -6*arcsin(a*x)*sgn(a)/(c^3*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c^3) - 2/7*(371*
(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 952*(sqrt(-a^2*x^2 + 1)*abs(a) +
a)^2/(a^4*x^2) + 1246*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 819*(sq
rt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 287*(sqrt(-a^2*x^2 + 1)*abs(a) +
 a)^5/(a^10*x^5) - 42*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a^12*x^6) - 59)/(c
^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^7*abs(a))
```

3.473 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

Optimal. Leaf size=190

$$\frac{(ax+1)^7}{9ac^4\left(1-a^2x^2\right)^{9/2}}-\frac{34(ax+1)^6}{63ac^4\left(1-a^2x^2\right)^{7/2}}+\frac{344(ax+1)^5}{315ac^4\left(1-a^2x^2\right)^{5/2}}-\frac{4(ax+1)^4}{3ac^4\left(1-a^2x^2\right)^{3/2}}+\frac{14(ax+1)^2}{3ac^4\sqrt{1-a^2x^2}}+\frac{7\sqrt{1-a^2x^2}}{ac^4}-$$

```
[Out] (1 + a*x)^7/(9*a*c^4*(1 - a^2*x^2)^(9/2)) - (34*(1 + a*x)^6)/(63*a*c^4*(1 -
 a^2*x^2)^(7/2)) + (344*(1 + a*x)^5)/(315*a*c^4*(1 - a^2*x^2)^(5/2)) - (4*(
1 + a*x)^4)/(3*a*c^4*(1 - a^2*x^2)^(3/2)) + (14*(1 + a*x)^2)/(3*a*c^4*Sqrt[
1 - a^2*x^2]) + (7*Sqrt[1 - a^2*x^2])/(a*c^4) - (7*ArcSin[a*x])/(a*c^4)
```

Rubi [A] time = 0.443257, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6131, 6128, 852, 1635, 789, 669, 641, 216}

$$\frac{(ax+1)^7}{9ac^4\left(1-a^2x^2\right)^{9/2}}-\frac{34(ax+1)^6}{63ac^4\left(1-a^2x^2\right)^{7/2}}+\frac{344(ax+1)^5}{315ac^4\left(1-a^2x^2\right)^{5/2}}-\frac{4(ax+1)^4}{3ac^4\left(1-a^2x^2\right)^{3/2}}+\frac{14(ax+1)^2}{3ac^4\sqrt{1-a^2x^2}}+\frac{7\sqrt{1-a^2x^2}}{ac^4}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (1 + a*x)^7/(9*a*c^4*(1 - a^2*x^2)^(9/2)) - (34*(1 + a*x)^6)/(63*a*c^4*(1 -
 a^2*x^2)^(7/2)) + (344*(1 + a*x)^5)/(315*a*c^4*(1 - a^2*x^2)^(5/2)) - (4*(
1 + a*x)^4)/(3*a*c^4*(1 - a^2*x^2)^(3/2)) + (14*(1 + a*x)^2)/(3*a*c^4*Sqrt[
1 - a^2*x^2]) + (7*Sqrt[1 - a^2*x^2])/(a*c^4) - (7*ArcSin[a*x])/(a*c^4)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
```

```
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}dx = \frac{a^4\int \frac{e^{3\tanh^{-1}(ax)}x^4}{(1-ax)^4}dx}{c^4}$$

$$= \frac{a^4\int \frac{x^4(1-a^2x^2)^{3/2}}{(1-ax)^7}dx}{c^4}$$

$$= \frac{a^4\int \frac{x^4(1+ax)^7}{(1-a^2x^2)^{11/2}}dx}{c^4}$$

$$= \frac{(1+ax)^7}{9ac^4(1-a^2x^2)^{9/2}} - \frac{a^4\int \frac{(1+ax)^6\left(\frac{7}{a^4}+\frac{9x}{a^3}+\frac{9x^2}{a^2}+\frac{9x^3}{a}\right)}{(1-a^2x^2)^{9/2}}dx}{9c^4}$$

$$= \frac{(1+ax)^7}{9ac^4(1-a^2x^2)^{9/2}} - \frac{34(1+ax)^6}{63ac^4(1-a^2x^2)^{7/2}} + \frac{a^4\int \frac{(1+ax)^5\left(\frac{155}{a^4}+\frac{126x}{a^3}+\frac{63x^2}{a^2}\right)}{(1-a^2x^2)^{7/2}}dx}{63c^4}$$

$$= \frac{(1+ax)^7}{9ac^4(1-a^2x^2)^{9/2}} - \frac{34(1+ax)^6}{63ac^4(1-a^2x^2)^{7/2}} + \frac{344(1+ax)^5}{315ac^4(1-a^2x^2)^{5/2}} - \frac{a^4\int \frac{\left(\frac{945}{a^4}+\frac{315x}{a^3}\right)(1+ax)^4}{(1-a^2x^2)^{5/2}}dx}{315c^4}$$

$$= \frac{(1+ax)^7}{9ac^4(1-a^2x^2)^{9/2}} - \frac{34(1+ax)^6}{63ac^4(1-a^2x^2)^{7/2}} + \frac{344(1+ax)^5}{315ac^4(1-a^2x^2)^{5/2}} - \frac{4(1+ax)^4}{3ac^4(1-a^2x^2)^{3/2}} + \frac{7\int \frac{(1+ax)}{(1-a^2x^2}}{3c^4}$$

$$= \frac{(1+ax)^7}{9ac^4(1-a^2x^2)^{9/2}} - \frac{34(1+ax)^6}{63ac^4(1-a^2x^2)^{7/2}} + \frac{344(1+ax)^5}{315ac^4(1-a^2x^2)^{5/2}} - \frac{4(1+ax)^4}{3ac^4(1-a^2x^2)^{3/2}} + \frac{14(1+ax}{3ac^4\sqrt{1-}}$$

$$= \frac{(1+ax)^7}{9ac^4(1-a^2x^2)^{9/2}} - \frac{34(1+ax)^6}{63ac^4(1-a^2x^2)^{7/2}} + \frac{344(1+ax)^5}{315ac^4(1-a^2x^2)^{5/2}} - \frac{4(1+ax)^4}{3ac^4(1-a^2x^2)^{3/2}} + \frac{14(1+ax}{3ac^4\sqrt{1-}}$$

$$= \frac{(1+ax)^7}{9ac^4(1-a^2x^2)^{9/2}} - \frac{34(1+ax)^6}{63ac^4(1-a^2x^2)^{7/2}} + \frac{344(1+ax)^5}{315ac^4(1-a^2x^2)^{5/2}} - \frac{4(1+ax)^4}{3ac^4(1-a^2x^2)^{3/2}} + \frac{14(1+ax}{3ac^4\sqrt{1-}}$$

Mathematica [A] time = 0.182409, size = 77, normalized size = 0.41

$$\frac{\frac{\sqrt{1-a^2x^2}(315a^5x^5-6539a^4x^4+19780a^3x^3-25347a^2x^2+15115ax-3464)}{(ax-1)^5} - 2205\sin^{-1}(ax)}{315ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(-3464 + 15115*a*x - 25347*a^2*x^2 + 19780*a^3*x^3 - 65
39*a^4*x^4 + 315*a^5*x^5))/(-1 + a*x)^5 - 2205*ArcSin[a*x])/(315*a*c^4)
```

Maple [A] time = 0.059, size = 300, normalized size = 1.6

$$-\frac{ax^2}{c^4}\frac{1}{\sqrt{-a^2x^2+1}} + 27\frac{1}{ac^4\sqrt{-a^2x^2+1}} + 70\frac{x}{c^4\sqrt{-a^2x^2+1}} - 7\frac{1}{c^4\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right) + \frac{8}{9a^5c^4}\left(x-a^{-1}\right)^{-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^4,x)
```

```
[Out] -a/c^4*x^2/(-a^2*x^2+1)^(1/2)+27/a/c^4/(-a^2*x^2+1)^(1/2)+70*x/c^4/(-a^2*x^
2+1)^(1/2)-7/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+8/9/a
^5/c^4/(x-1/a)^4/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+356/63/a^4/c^4/(x-1/a)^
3/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+5002/315/a^3/c^4/(x-1/a)^2/(-a^2*(x-1/
a)^2-2*a*(x-1/a))^(1/2)+8543/315/a^2/c^4/(x-1/a)/(-a^2*(x-1/a)^2-2*a*(x-1/a
))^(1/2)-17086/315/c^4/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^4), x)
```

Fricas [A] time = 2.26225, size = 516, normalized size = 2.72

$$\frac{3464\,a^5x^5-17320\,a^4x^4+34640\,a^3x^3-34640\,a^2x^2+17320\,ax+4410\left(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+5\,ax-1\right)\mathrm{arc}}{315\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(3464*a^5*x^5 - 17320*a^4*x^4 + 34640*a^3*x^3 - 34640*a^2*x^2 + 17320
*a*x + 4410*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*arc
tan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (315*a^5*x^5 - 6539*a^4*x^4 + 19780*a
^3*x^3 - 25347*a^2*x^2 + 15115*a*x - 3464)*sqrt(-a^2*x^2 + 1) - 3464)/(a^6*
c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a
*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int \frac{x^4}{-a^6x^6\sqrt{-a^2x^2+1}+4a^5x^5\sqrt{-a^2x^2+1}-5a^4x^4\sqrt{-a^2x^2+1}+5a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{}{-a^6x^6\sqrt{-a^2x^2+1}+4a^5x^5\sqrt{-a^2x^2+1}-5a^4x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*(Integral(x**4/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 4*a**5*x**5*sqrt(-a*
*2*x**2 + 1) - 5*a**4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**2*x**2*sqrt(-a**2*x*
```

```
*2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral
(3*a*x**5/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 4*a**5*x**5*sqrt(-a**2*x**2 +
1) - 5*a**4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**2*x**2*sqrt(-a**2*x**2 + 1) -
4*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(3*a**2*x*
*6/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 4*a**5*x**5*sqrt(-a**2*x**2 + 1) - 5*
a**4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*s
qrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**7/(-a**6
*x**6*sqrt(-a**2*x**2 + 1) + 4*a**5*x**5*sqrt(-a**2*x**2 + 1) - 5*a**4*x**4
*sqrt(-a**2*x**2 + 1) + 5*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2
*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

Giac [A] time = 1.31472, size = 389, normalized size = 2.05

$$-\frac{7\arcsin(ax)\operatorname{sgn}(a)}{c^4|a|}+\frac{\sqrt{-a^2x^2+1}}{ac^4}-\frac{2\left(\frac{26136\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{93834\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{188706\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}-\frac{229194\left(\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -7*arcsin(a*x)*sgn(a)/(c^4*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c^4) - 2/315*(26
136*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 93834*(sqrt(-a^2*x^2 + 1)*abs
(a) + a)^2/(a^4*x^2) + 188706*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) -
 229194*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 167580*(sqrt(-a^2*x^2
 + 1)*abs(a) + a)^5/(a^10*x^5) - 75810*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a
^12*x^6) + 19530*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7/(a^14*x^7) - 2205*(sqrt(
-a^2*x^2 + 1)*abs(a) + a)^8/(a^16*x^8) - 3149)/(c^4*((sqrt(-a^2*x^2 + 1)*ab
s(a) + a)/(a^2*x) - 1)^9*abs(a))
```

3.474 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=93

$$\frac{(4-p)\left(c-\frac{c}{ax}\right)^p \text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{1}{ax}\right)}{ap}-\frac{c(5-p)\left(c-\frac{c}{ax}\right)^{p-1}}{a(1-p)}+cx\left(c-\frac{c}{ax}\right)^{p-1}$$

```
[Out] -((c*(5 - p)*(c - c/(a*x))^(-1 + p))/(a*(1 - p))) + c*(c - c/(a*x))^(-1 + p
)*x + ((4 - p)*(c - c/(a*x))^p*Hypergeometric2F1[1, p, 1 + p, 1 - 1/(a*x)])
/(a*p)
```

Rubi [A] time = 0.114792, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6133, 25, 514, 375, 89, 79, 65}

$$\frac{(4-p)\left(c-\frac{c}{ax}\right)^p {}_2F_1\left(1,p;p+1;1-\frac{1}{ax}\right)}{ap}-\frac{c(5-p)\left(c-\frac{c}{ax}\right)^{p-1}}{a(1-p)}+cx\left(c-\frac{c}{ax}\right)^{p-1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] -((c*(5 - p)*(c - c/(a*x))^(-1 + p))/(a*(1 - p))) + c*(c - c/(a*x))^(-1 + p
)*x + ((4 - p)*(c - c/(a*x))^p*Hypergeometric2F1[1, p, 1 + p, 1 - 1/(a*x)])
/(a*p)
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx &= \int \frac{\left(c-\frac{c}{ax}\right)^p(1+ax)^2}{(1-ax)^2}\,dx\\
&= \frac{c^2\int \frac{\left(c-\frac{c}{ax}\right)^{-2+p}(1+ax)^2}{x^2}\,dx}{a^2}\\
&= \frac{c^2\int \left(a+\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{-2+p}\,dx}{a^2}\\
&= -\frac{c^2\operatorname{Subst}\left(\int \frac{(a+x)^2\left(c-\frac{cx}{a}\right)^{-2+p}}{x^2}\,dx,x,\frac{1}{x}\right)}{a^2}\\
&= c\left(c-\frac{c}{ax}\right)^{-1+p}x-\frac{c\operatorname{Subst}\left(\int \frac{(ac(4-p)+cx)\left(c-\frac{cx}{a}\right)^{-2+p}}{x}\,dx,x,\frac{1}{x}\right)}{a^2}\\
&= -\frac{c(5-p)\left(c-\frac{c}{ax}\right)^{-1+p}}{a(1-p)}+c\left(c-\frac{c}{ax}\right)^{-1+p}x-\frac{(c(4-p))\operatorname{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{-1+p}}{x}\,dx,x,\frac{1}{x}\right)}{a}\\
&= -\frac{c(5-p)\left(c-\frac{c}{ax}\right)^{-1+p}}{a(1-p)}+c\left(c-\frac{c}{ax}\right)^{-1+p}x+\frac{(4-p)\left(c-\frac{c}{ax}\right)^p{}_2F_1\left(1,p;1+p;1-\frac{1}{ax}\right)}{ap}
\end{aligned}
$$

Mathematica [A] time = 0.0443018, size = 81, normalized size = 0.87

$$
\frac{\left(c-\frac{c}{ax}\right)^p\left(apx(p(ax-1)-ax+5)-\left(p^2-5p+4\right)(ax-1)\text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{1}{ax}\right)\right)}{a(p-1)p(ax-1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] ((c - c/(a*x))^p*(a*p*x*(5 - a*x + p*(-1 + a*x)) - (4 - 5*p + p^2)*(-1 + a*
x)*Hypergeometric2F1[1, p, 1 + p, 1 - 1/(a*x)]))/(a*(-1 + p)*p*(-1 + a*x))
```

Maple [F] time = 0.353, size = 0, normalized size = 0.

$$\int \frac{(ax+1)^4}{(-a^2x^2+1)^2}\left(c-\frac{c}{ax}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^p,x)
```

```
[Out] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^4\left(c-\frac{c}{ax}\right)^p}{(a^2x^2-1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^4*(c - c/(a*x))^p/(a^2*x^2 - 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(a^2x^2+2\,ax+1)\left(\frac{acx-c}{ax}\right)^p}{a^2x^2-2\,ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*((a*c*x - c)/(a*x))^p/(a^2*x^2 - 2*a*x + 1),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^p (ax+1)^2}{(ax-1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a/x)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p*(a*x + 1)**2/(a*x - 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^4\left(c-\frac{c}{ax}\right)^p}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^4*(c - c/(a*x))^p/(a^2*x^2 - 1)^2, x)
```

3.475 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

Optimal. Leaf size=64

$$-\frac{c^5}{a^3x^2}-\frac{c^5}{3a^4x^3}+\frac{c^5}{4a^5x^4}+\frac{2c^5}{a^2x}-\frac{c^5\log(x)}{a}+c^5x$$

```
[Out] c^5/(4*a^5*x^4) - c^5/(3*a^4*x^3) - c^5/(a^3*x^2) + (2*c^5)/(a^2*x) + c^5*x
 - (c^5*Log[x])/a
```

Rubi [A] time = 0.11063, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$-\frac{c^5}{a^3x^2}-\frac{c^5}{3a^4x^3}+\frac{c^5}{4a^5x^4}+\frac{2c^5}{a^2x}-\frac{c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] c^5/(4*a^5*x^4) - c^5/(3*a^4*x^3) - c^5/(a^3*x^2) + (2*c^5)/(a^2*x) + c^5*x
 - (c^5*Log[x])/a
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx = -\frac{c^5\int \frac{e^{4\tanh^{-1}(ax)}(1-ax)^5}{x^5}dx}{a^5}$$
$$= -\frac{c^5\int \frac{(1-ax)^3(1+ax)^2}{x^5}dx}{a^5}$$
$$= -\frac{c^5\int \left(-a^5+\frac{1}{x^5}-\frac{a}{x^4}-\frac{2a^2}{x^3}+\frac{2a^3}{x^2}+\frac{a^4}{x}\right)dx}{a^5}$$
$$= \frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}+c^5x-\frac{c^5\log(x)}{a}$$

Mathematica [A] time = 0.276819, size = 66, normalized size = 1.03

$$-\frac{c^5}{a^3x^2}-\frac{c^5}{3a^4x^3}+\frac{c^5}{4a^5x^4}+\frac{2c^5}{a^2x}-\frac{c^5\log(ax)}{a}+c^5x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] c^5/(4*a^5*x^4) - c^5/(3*a^4*x^3) - c^5/(a^3*x^2) + (2*c^5)/(a^2*x) + c^5*x
 - (c^5*Log[a*x])/a
```

Maple [A] time = 0.036, size = 61, normalized size = 1.

$$\frac{c^5}{4\,a^5x^4}-\frac{c^5}{3\,a^4x^3}-\frac{c^5}{x^2a^3}+2\,\frac{c^5}{a^2x}+c^5x-\frac{c^5\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^5,x)
```

```
[Out] 1/4*c^5/a^5/x^4-1/3*c^5/a^4/x^3-c^5/x^2/a^3+2*c^5/a^2/x+c^5*x-c^5*ln(x)/a
```

Maxima [A] time = 0.955641, size = 80, normalized size = 1.25

$$c^5x-\frac{c^5\log(x)}{a}+\frac{24\,a^3c^5x^3-12\,a^2c^5x^2-4\,ac^5x+3\,c^5}{12\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^5,x, algorithm="maxima")
```

```
[Out] c^5*x - c^5*log(x)/a + 1/12*(24*a^3*c^5*x^3 - 12*a^2*c^5*x^2 - 4*a*c^5*x +
3*c^5)/(a^5*x^4)
```

Fricas [A] time = 1.9712, size = 150, normalized size = 2.34

$$\frac{12\,a^5c^5x^5-12\,a^4c^5x^4\log(x)+24\,a^3c^5x^3-12\,a^2c^5x^2-4\,ac^5x+3\,c^5}{12\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^5*c^5*x^5 - 12*a^4*c^5*x^4*log(x) + 24*a^3*c^5*x^3 - 12*a^2*c^5*
x^2 - 4*a*c^5*x + 3*c^5)/(a^5*x^4)
```

Sympy [A] time = 0.512202, size = 63, normalized size = 0.98

$$\frac{a^5c^5x - a^4c^5\log(x) + \frac{24a^3c^5x^3 - 12a^2c^5x^2 - 4ac^5x + 3c^5}{12x^4}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a/x)**5,x)
```

```
[Out] (a**5*c**5*x - a**4*c**5*log(x) + (24*a**3*c**5*x**3 - 12*a**2*c**5*x**2 -
4*a*c**5*x + 3*c**5)/(12*x**4))/a**5
```

Giac [A] time = 1.1245, size = 81, normalized size = 1.27

$$c^5x - \frac{c^5\log(|x|)}{a} + \frac{24\,a^3c^5x^3 - 12\,a^2c^5x^2 - 4\,ac^5x + 3\,c^5}{12\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] c^5*x - c^5*log(abs(x))/a + 1/12*(24*a^3*c^5*x^3 - 12*a^2*c^5*x^2 - 4*a*c^5
*x + 3*c^5)/(a^5*x^4)
```

3.476 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=30

$$-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

```
[Out] -c^4/(3*a^4*x^3) + (2*c^4)/(a^2*x) + c^4*x
```

Rubi [A] time = 0.0975286, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6131, 6129, 73, 270}

$$-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] -c^4/(3*a^4*x^3) + (2*c^4)/(a^2*x) + c^4*x
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\int \frac{e^{4\tanh^{-1}(ax)}(1-ax)^4}{x^4}dx}{a^4}$$
$$= \frac{c^4\int \frac{(1-ax)^2(1+ax)^2}{x^4}dx}{a^4}$$
$$= \frac{c^4\int \frac{(1-a^2x^2)^2}{x^4}dx}{a^4}$$
$$= \frac{c^4\int \left(a^4+\frac{1}{x^4}-\frac{2a^2}{x^2}\right)dx}{a^4}$$
$$= -\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

Mathematica [A] time = 0.182125, size = 30, normalized size = 1.

$$-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] -c^4/(3*a^4*x^3) + (2*c^4)/(a^2*x) + c^4*x
```

Maple [A] time = 0.033, size = 27, normalized size = 0.9

$$\frac{c^4}{a^4}\left(xa^4+2\frac{a^2}{x}-\frac{1}{3x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^4,x)
```

```
[Out] c^4/a^4*(x*a^4+2*a^2/x-1/3/x^3)
```

Maxima [A] time = 0.943456, size = 42, normalized size = 1.4

$$c^4x+\frac{6a^2c^4x^2-c^4}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] c^4*x + 1/3*(6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

Fricas [A] time = 1.90811, size = 72, normalized size = 2.4

$$\frac{3a^4c^4x^4+6a^2c^4x^2-c^4}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^4*x^4 + 6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

Sympy [A] time = 0.370765, size = 31, normalized size = 1.03

$$\frac{a^4c^4x + \frac{6a^2c^4x^2-c^4}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a/x)**4,x)
```

```
[Out] (a**4*c**4*x + (6*a**2*c**4*x**2 - c**4)/(3*x**3))/a**4
```

Giac [A] time = 1.13233, size = 42, normalized size = 1.4

$$c^4x + \frac{6\,a^2c^4x^2 - c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] c^4*x + 1/3*(6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

3.477 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=38

$$\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+\frac{c^3\log(x)}{a}+c^3x$$

```
[Out] c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x + (c^3*Log[x])/a
```

Rubi [A] time = 0.101055, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 75}

$$\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+\frac{c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x + (c^3*Log[x])/a
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx &= -\frac{c^3\int\frac{e^{4\tanh^{-1}(ax)}(1-ax)^3}{x^3}\,dx}{a^3}\\
&= -\frac{c^3\int\frac{(1-ax)(1+ax)^2}{x^3}\,dx}{a^3}\\
&= -\frac{c^3\int\left(-a^3+\frac{1}{x^3}+\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{a^3}\\
&= \frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{c^3\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.134335, size = 40, normalized size = 1.05

$$\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+\frac{c^3\log(ax)}{a}+c^3x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x + (c^3*Log[a*x])/a
```

Maple [A] time = 0.031, size = 37, normalized size = 1.

$$\frac{c^3}{2\,x^2a^3}+\frac{c^3}{a^2x}+c^3x+\frac{c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^3,x)
```

```
[Out] 1/2*c^3/x^2/a^3+c^3/a^2/x+c^3*x+c^3*ln(x)/a
```

Maxima [A] time = 0.939259, size = 46, normalized size = 1.21

$$c^3x+\frac{c^3\log(x)}{a}+\frac{2\,ac^3x+c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] c^3*x + c^3*log(x)/a + 1/2*(2*a*c^3*x + c^3)/(a^3*x^2)
```

Fricas [A] time = 1.96404, size = 97, normalized size = 2.55

$$\frac{2\,a^3c^3x^3+2\,a^2c^3x^2\log(x)+2\,ac^3x+c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*c^3*x^3 + 2*a^2*c^3*x^2*log(x) + 2*a*c^3*x + c^3)/(a^3*x^2)
```

Sympy [A] time = 0.386818, size = 37, normalized size = 0.97

$$\frac{a^3c^3x+a^2c^3\log(x)+\frac{2ac^3x+c^3}{2x^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a/x)**3,x)
```

```
[Out] (a**3*c**3*x + a**2*c**3*log(x) + (2*a*c**3*x + c**3)/(2*x**2))/a**3
```

Giac [A] time = 1.17048, size = 47, normalized size = 1.24

$$c^3x + \frac{c^3\log(|x|)}{a} + \frac{2\,ac^3x + c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] c^3*x + c^3*log(abs(x))/a + 1/2*(2*a*c^3*x + c^3)/(a^3*x^2)
```

3.478 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$

Optimal. Leaf size=27

$$-\frac{c^2}{a^2x}+\frac{2c^2\log(x)}{a}+c^2x$$

```
[Out] -(c^2/(a^2*x)) + c^2*x + (2*c^2*Log[x])/a
```

Rubi [A] time = 0.0961591, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 43}

$$-\frac{c^2}{a^2x}+\frac{2c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] -(c^2/(a^2*x)) + c^2*x + (2*c^2*Log[x])/a
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx &= \frac{c^2\int \frac{e^{4\tanh^{-1}(ax)}(1-ax)^2}{x^2}\,dx}{a^2}\\
&= \frac{c^2\int \frac{(1+ax)^2}{x^2}\,dx}{a^2}\\
&= \frac{c^2\int \left(a^2+\frac{1}{x^2}+\frac{2a}{x}\right)dx}{a^2}\\
&= -\frac{c^2}{a^2x}+c^2x+\frac{2c^2\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0936694, size = 29, normalized size = 1.07

$$-\frac{c^2}{a^2x}+\frac{2c^2\log(ax)}{a}+c^2x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] -(c^2/(a^2*x)) + c^2*x + (2*c^2*Log[a*x])/a
```

Maple [A] time = 0.034, size = 28, normalized size = 1.

$$-\frac{c^2}{a^2x}+xc^2+2\frac{c^2\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^2,x)
```

```
[Out] -c^2/a^2/x+x*c^2+2*c^2*ln(x)/a
```

Maxima [A] time = 0.95113, size = 36, normalized size = 1.33

$$c^2x+\frac{2c^2\log(x)}{a}-\frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] c^2*x + 2*c^2*log(x)/a - c^2/(a^2*x)
```

Fricas [A] time = 2.02426, size = 65, normalized size = 2.41

$$\frac{a^2c^2x^2+2\,ac^2x\log(x)-c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] (a^2*c^2*x^2 + 2*a*c^2*x*log(x) - c^2)/(a^2*x)
```

Sympy [A] time = 0.328515, size = 26, normalized size = 0.96

$$\frac{a^2c^2x+2ac^2\log(x)-\frac{c^2}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a/x)**2,x)
```

```
[Out] (a**2*c**2*x + 2*a*c**2*log(x) - c**2/x)/a**2
```

Giac [A] time = 1.17434, size = 38, normalized size = 1.41

$$c^2x + \frac{2\,c^2\log\left(|x|\right)}{a} - \frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] c^2*x + 2*c^2*log(abs(x))/a - c^2/(a^2*x)
```

3.479 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=25

$$-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}+cx$$

```
[Out] c*x - (c*Log[x])/a + (4*c*Log[1 - a*x])/a
```

Rubi [A] time = 0.0654436, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6131, 6129, 72}

$$-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*x - (c*Log[x])/a + (4*c*Log[1 - a*x])/a
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx &= -\frac{c\int\frac{e^{4\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}\\
&= -\frac{c\int\frac{(1+ax)^2}{x(1-ax)}\,dx}{a}\\
&= -\frac{c\int\left(-a+\frac{1}{x}-\frac{4a}{-1+ax}\right)\,dx}{a}\\
&= cx-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0376014, size = 25, normalized size = 1.

$$-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*x - (c*Log[x])/a + (4*c*Log[1 - a*x])/a
```

Maple [A] time = 0.035, size = 25, normalized size = 1.

$$cx-\frac{c\ln(x)}{a}+4\frac{c\ln(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x),x)
```

```
[Out] c*x-c*ln(x)/a+4*c/a*ln(a*x-1)
```

Maxima [A] time = 0.958637, size = 32, normalized size = 1.28

$$cx+\frac{4c\log(ax-1)}{a}-\frac{c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] c*x + 4*c*log(a*x - 1)/a - c*log(x)/a
```

Fricas [A] time = 2.05495, size = 55, normalized size = 2.2

$$\frac{acx+4c\log(ax-1)-c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (a*c*x + 4*c*log(a*x - 1) - c*log(x))/a
```

Sympy [A] time = 0.420343, size = 17, normalized size = 0.68

$$cx+\frac{c\left(-\log(x)+4\log\left(x-\frac{1}{a}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a/x),x)
```

```
[Out] c*x + c*(-log(x) + 4*log(x - 1/a))/a
```

Giac [A] time = 1.11784, size = 35, normalized size = 1.4

$$cx + \frac{4\,c\log\left(|ax - 1|\right)}{a} - \frac{c\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x),x, algorithm="giac")
```

```
[Out] c*x + 4*c*log(abs(a*x - 1))/a - c*log(abs(x))/a
```

3.480 $\int \frac{e^{4\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=53

$$\frac{8}{ac(1-ax)}-\frac{2}{ac(1-ax)^2}+\frac{5\log(1-ax)}{ac}+\frac{x}{c}$$

```
[Out] x/c - 2/(a*c*(1 - a*x)^2) + 8/(a*c*(1 - a*x)) + (5*Log[1 - a*x])/(a*c)
```

Rubi [A] time = 0.101552, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 77}

$$\frac{8}{ac(1-ax)}-\frac{2}{ac(1-ax)^2}+\frac{5\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - c/(a*x)),x]
```

```
[Out] x/c - 2/(a*c*(1 - a*x)^2) + 8/(a*c*(1 - a*x)) + (5*Log[1 - a*x])/(a*c)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{a\int \frac{e^{4\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}$$
$$= -\frac{a\int \frac{x(1+ax)^2}{(1-ax)^3}\,dx}{c}$$
$$= -\frac{a\int \left(-\frac{1}{a}-\frac{4}{a(-1+ax)^3}-\frac{8}{a(-1+ax)^2}-\frac{5}{a(-1+ax)}\right)dx}{c}$$
$$= \frac{x}{c}-\frac{2}{ac(1-ax)^2}+\frac{8}{ac(1-ax)}+\frac{5\log(1-ax)}{ac}$$

Mathematica [A] time = 0.0287099, size = 51, normalized size = 0.96

$$-\frac{a\left(-\frac{8}{a^2(1-ax)}+\frac{2}{a^2(1-ax)^2}-\frac{5\log(1-ax)}{a^2}-\frac{x}{a}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - c/(a*x)),x]
```

```
[Out] -((a*(-(x/a) + 2/(a^2*(1 - a*x)^2) - 8/(a^2*(1 - a*x)) - (5*Log[1 - a*x])/a
^2))/c)
```

Maple [A] time = 0.036, size = 51, normalized size = 1.

$$\frac{x}{c}-2\,\frac{1}{ac\,(ax-1)^2}-8\,\frac{1}{ac\,(ax-1)}+5\,\frac{\ln(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x),x)
```

```
[Out] x/c-2/a/c/(a*x-1)^2-8/a/c/(a*x-1)+5/a/c*ln(a*x-1)
```

Maxima [A] time = 0.952897, size = 66, normalized size = 1.25

$$-\frac{2\,(4\,ax-3)}{a^3cx^2-2\,a^2cx+ac}+\frac{x}{c}+\frac{5\,\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*(4*a*x - 3)/(a^3*c*x^2 - 2*a^2*c*x + a*c) + x/c + 5*log(a*x - 1)/(a*c)
```

Fricas [A] time = 1.96432, size = 140, normalized size = 2.64

$$\frac{a^3x^3-2\,a^2x^2-7\,ax+5\left(a^2x^2-2\,ax+1\right)\log(ax-1)+6}{a^3cx^2-2\,a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - 2*a^2*x^2 - 7*a*x + 5*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 6)/(a
^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [A] time = 0.453774, size = 41, normalized size = 0.77

$$-\frac{8ax-6}{a^3cx^2-2a^2cx+ac}+\frac{x}{c}+\frac{5\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(c-c/a/x),x)
```

```
[Out] -(8*a*x - 6)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + x/c + 5*log(a*x - 1)/(a*c)
```

Giac [A] time = 1.2103, size = 57, normalized size = 1.08

$$\frac{x}{c}+\frac{5\log(|ax-1|)}{ac}-\frac{2(4ax-3)}{(ax-1)^2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x),x, algorithm="giac")
```

```
[Out] x/c + 5*log(abs(a*x - 1))/(a*c) - 2*(4*a*x - 3)/((a*x - 1)^2*a*c)
```

3.481 $\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=71

$$\frac{13}{ac^2(1-ax)}-\frac{6}{ac^2(1-ax)^2}+\frac{4}{3ac^2(1-ax)^3}+\frac{6\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

```
[Out] x/c^2 + 4/(3*a*c^2*(1 - a*x)^3) - 6/(a*c^2*(1 - a*x)^2) + 13/(a*c^2*(1 - a*
x)) + (6*Log[1 - a*x])/(a*c^2)
```

Rubi [A] time = 0.126946, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$\frac{13}{ac^2(1-ax)}-\frac{6}{ac^2(1-ax)^2}+\frac{4}{3ac^2(1-ax)^3}+\frac{6\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] x/c^2 + 4/(3*a*c^2*(1 - a*x)^3) - 6/(a*c^2*(1 - a*x)^2) + 13/(a*c^2*(1 - a*
x)) + (6*Log[1 - a*x])/(a*c^2)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{a^2\int \frac{e^{4\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}$$
$$=\frac{a^2\int \frac{x^2(1+ax)^2}{(1-ax)^4}\,dx}{c^2}$$
$$=\frac{a^2\int \left(\frac{1}{a^2}+\frac{4}{a^2(-1+ax)^4}+\frac{12}{a^2(-1+ax)^3}+\frac{13}{a^2(-1+ax)^2}+\frac{6}{a^2(-1+ax)}\right)dx}{c^2}$$
$$=\frac{x}{c^2}+\frac{4}{3ac^2(1-ax)^3}-\frac{6}{ac^2(1-ax)^2}+\frac{13}{ac^2(1-ax)}+\frac{6\log(1-ax)}{ac^2}$$

Mathematica [A] time = 0.105242, size = 63, normalized size = 0.89

$$\frac{3a^4x^4-9a^3x^3-30a^2x^2+57ax+18(ax-1)^3\log(1-ax)-25}{3ac^2(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] (-25 + 57*a*x - 30*a^2*x^2 - 9*a^3*x^3 + 3*a^4*x^4 + 18*(-1 + a*x)^3*Log[1
- a*x])/(3*a*c^2*(-1 + a*x)^3)
```

Maple [A] time = 0.034, size = 66, normalized size = 0.9

$$\frac{x}{c^2}-\frac{4}{3\,ac^2\,(ax-1)^3}-6\,\frac{1}{ac^2\,(ax-1)^2}-13\,\frac{1}{ac^2\,(ax-1)}+6\,\frac{\ln(ax-1)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^2,x)
```

```
[Out] x/c^2-4/3/a/c^2/(a*x-1)^3-6/a/c^2/(a*x-1)^2-13/a/c^2/(a*x-1)+6/c^2/a*ln(a*x
-1)
```

Maxima [A] time = 0.959683, size = 101, normalized size = 1.42

$$-\frac{39\,a^2x^2-60\,ax+25}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}+\frac{x}{c^2}+\frac{6\,\log(ax-1)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(39*a^2*x^2 - 60*a*x + 25)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2) + x/c^2 + 6*log(a*x - 1)/(a*c^2)
```

Fricas [A] time = 1.95806, size = 216, normalized size = 3.04

$$\frac{3\,a^4x^4 - 9\,a^3x^3 - 30\,a^2x^2 + 57\,ax + 18\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\log\left(ax - 1\right) - 25}{3\left(a^4c^2x^3 - 3\,a^3c^2x^2 + 3\,a^2c^2x - ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*x^4 - 9*a^3*x^3 - 30*a^2*x^2 + 57*a*x + 18*(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)*log(a*x - 1) - 25)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2)
```

Sympy [A] time = 0.57543, size = 73, normalized size = 1.03

$$-\frac{39a^2x^2 - 60ax + 25}{3a^4c^2x^3 - 9a^3c^2x^2 + 9a^2c^2x - 3ac^2} + \frac{x}{c^2} + \frac{6\log\left(ax - 1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(c-c/a/x)**2,x)
```

```
[Out] -(39*a**2*x**2 - 60*a*x + 25)/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2
*c**2*x - 3*a*c**2) + x/c**2 + 6*log(a*x - 1)/(a*c**2)
```

Giac [A] time = 1.14921, size = 68, normalized size = 0.96

$$\frac{x}{c^2} + \frac{6\,\log\left(|ax - 1|\right)}{ac^2} - \frac{39\,a^2x^2 - 60\,ax + 25}{3\left(ax - 1\right)^3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] x/c^2 + 6*log(abs(a*x - 1))/(a*c^2) - 1/3*(39*a^2*x^2 - 60*a*x + 25)/((a*x
- 1)^3*a*c^2)
```

3.482 $\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=89

$$\frac{19}{ac^3(1-ax)}-\frac{25}{2ac^3(1-ax)^2}+\frac{16}{3ac^3(1-ax)^3}-\frac{1}{ac^3(1-ax)^4}+\frac{7\log(1-ax)}{ac^3}+\frac{x}{c^3}$$

```
[Out] x/c^3 - 1/(a*c^3*(1 - a*x)^4) + 16/(3*a*c^3*(1 - a*x)^3) - 25/(2*a*c^3*(1 -
 a*x)^2) + 19/(a*c^3*(1 - a*x)) + (7*Log[1 - a*x])/(a*c^3)
```

Rubi [A] time = 0.145035, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$\frac{19}{ac^3(1-ax)}-\frac{25}{2ac^3(1-ax)^2}+\frac{16}{3ac^3(1-ax)^3}-\frac{1}{ac^3(1-ax)^4}+\frac{7\log(1-ax)}{ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] x/c^3 - 1/(a*c^3*(1 - a*x)^4) + 16/(3*a*c^3*(1 - a*x)^3) - 25/(2*a*c^3*(1 -
 a*x)^2) + 19/(a*c^3*(1 - a*x)) + (7*Log[1 - a*x])/(a*c^3)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{a^3\int \frac{e^{4\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}$$
$$= -\frac{a^3\int \frac{x^3(1+ax)^2}{(1-ax)^5}\,dx}{c^3}$$
$$= -\frac{a^3\int \left(-\frac{1}{a^3}-\frac{4}{a^3(-1+ax)^5}-\frac{16}{a^3(-1+ax)^4}-\frac{25}{a^3(-1+ax)^3}-\frac{19}{a^3(-1+ax)^2}-\frac{7}{a^3(-1+ax)}\right)dx}{c^3}$$
$$= \frac{x}{c^3}-\frac{1}{ac^3(1-ax)^4}+\frac{16}{3ac^3(1-ax)^3}-\frac{25}{2ac^3(1-ax)^2}+\frac{19}{ac^3(1-ax)}+\frac{7\log(1-ax)}{ac^3}$$

Mathematica [A] time = 0.137104, size = 71, normalized size = 0.8

$$\frac{6a^5x^5-24a^4x^4-78a^3x^3+243a^2x^2-218ax+42(ax-1)^4\log(1-ax)+65}{6ac^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] (65 - 218*a*x + 243*a^2*x^2 - 78*a^3*x^3 - 24*a^4*x^4 + 6*a^5*x^5 + 42*(-1
+ a*x)^4*Log[1 - a*x])/(6*a*c^3*(-1 + a*x)^4)
```

Maple [A] time = 0.037, size = 81, normalized size = 0.9

$$\frac{x}{c^3}-\frac{16}{3\,ac^3\,(ax-1)^3}-\frac{25}{2\,ac^3\,(ax-1)^2}-19\,\frac{1}{ac^3\,(ax-1)}+7\,\frac{\ln(ax-1)}{ac^3}-\frac{1}{ac^3\,(ax-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^3,x)
```

```
[Out] x/c^3-16/3/a/c^3/(a*x-1)^3-25/2/a/c^3/(a*x-1)^2-19/a/c^3/(a*x-1)+7/a/c^3*ln
(a*x-1)-1/a/c^3/(a*x-1)^4
```

Maxima [A] time = 0.975467, size = 126, normalized size = 1.42

$$-\frac{114\,a^3x^3-267\,a^2x^2+224\,ax-65}{6\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}+\frac{x}{c^3}+\frac{7\,\log(ax-1)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(114*a^3*x^3 - 267*a^2*x^2 + 224*a*x - 65)/(a^5*c^3*x^4 - 4*a^4*c^3*x^
3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + x/c^3 + 7*log(a*x - 1)/(a*c^3)
```

Fricas [A] time = 1.98953, size = 275, normalized size = 3.09

$$\frac{6\,a^5x^5 - 24\,a^4x^4 - 78\,a^3x^3 + 243\,a^2x^2 - 218\,ax + 42\left(a^4x^4 - 4\,a^3x^3 + 6\,a^2x^2 - 4\,ax + 1\right)\log\left(ax - 1\right) + 65}{6\left(a^5c^3x^4 - 4\,a^4c^3x^3 + 6\,a^3c^3x^2 - 4\,a^2c^3x + ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^5*x^5 - 24*a^4*x^4 - 78*a^3*x^3 + 243*a^2*x^2 - 218*a*x + 42*(a^4*
x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 65)/(a^5*c^3*x^4 -
4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [A] time = 0.704141, size = 94, normalized size = 1.06

$$-\frac{114a^3x^3 - 267a^2x^2 + 224ax - 65}{6a^5c^3x^4 - 24a^4c^3x^3 + 36a^3c^3x^2 - 24a^2c^3x + 6ac^3} + \frac{x}{c^3} + \frac{7\log\left(ax - 1\right)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(c-c/a/x)**3,x)
```

```
[Out] -(114*a**3*x**3 - 267*a**2*x**2 + 224*a*x - 65)/(6*a**5*c**3*x**4 - 24*a**4
*c**3*x**3 + 36*a**3*c**3*x**2 - 24*a**2*c**3*x + 6*a*c**3) + x/c**3 + 7*lo
g(a*x - 1)/(a*c**3)
```

Giac [A] time = 1.16917, size = 78, normalized size = 0.88

$$\frac{x}{c^3} + \frac{7\,\log\left(|ax - 1|\right)}{ac^3} - \frac{114\,a^3x^3 - 267\,a^2x^2 + 224\,ax - 65}{6\left(ax - 1\right)^4ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] x/c^3 + 7*log(abs(a*x - 1))/(a*c^3) - 1/6*(114*a^3*x^3 - 267*a^2*x^2 + 224*
a*x - 65)/((a*x - 1)^4*a*c^3)
```

3.483 $\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=105

$$\frac{26}{ac^4(1-ax)}-\frac{22}{ac^4(1-ax)^2}+\frac{41}{3ac^4(1-ax)^3}-\frac{5}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}+\frac{8\log(1-ax)}{ac^4}+\frac{x}{c^4}$$

```
[Out] x/c^4 + 4/(5*a*c^4*(1 - a*x)^5) - 5/(a*c^4*(1 - a*x)^4) + 41/(3*a*c^4*(1 -
a*x)^3) - 22/(a*c^4*(1 - a*x)^2) + 26/(a*c^4*(1 - a*x)) + (8*Log[1 - a*x])/
(a*c^4)
```

Rubi [A] time = 0.148823, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$\frac{26}{ac^4(1-ax)}-\frac{22}{ac^4(1-ax)^2}+\frac{41}{3ac^4(1-ax)^3}-\frac{5}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}+\frac{8\log(1-ax)}{ac^4}+\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] x/c^4 + 4/(5*a*c^4*(1 - a*x)^5) - 5/(a*c^4*(1 - a*x)^4) + 41/(3*a*c^4*(1 -
a*x)^3) - 22/(a*c^4*(1 - a*x)^2) + 26/(a*c^4*(1 - a*x)) + (8*Log[1 - a*x])/
(a*c^4)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\int \frac{e^{4\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}$$
$$= \frac{a^4\int \frac{x^4(1+ax)^2}{(1-ax)^6}\,dx}{c^4}$$
$$= \frac{a^4\int \left(\frac{1}{a^4}+\frac{4}{a^4(-1+ax)^6}+\frac{20}{a^4(-1+ax)^5}+\frac{41}{a^4(-1+ax)^4}+\frac{44}{a^4(-1+ax)^3}+\frac{26}{a^4(-1+ax)^2}+\frac{8}{a^4(-1+ax)}\right)dx}{c^4}$$
$$= \frac{x}{c^4}+\frac{4}{5ac^4(1-ax)^5}-\frac{5}{ac^4(1-ax)^4}+\frac{41}{3ac^4(1-ax)^3}-\frac{22}{ac^4(1-ax)^2}+\frac{26}{ac^4(1-ax)}+\frac{8\log(1-ax)}{ac^4}$$

Mathematica [A] time = 0.16034, size = 79, normalized size = 0.75

$$\frac{15a^6x^6-75a^5x^5-240a^4x^4+1080a^3x^3-1480a^2x^2+890ax+120(ax-1)^5\log(1-ax)-202}{15ac^4(ax-1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (-202 + 890*a*x - 1480*a^2*x^2 + 1080*a^3*x^3 - 240*a^4*x^4 - 75*a^5*x^5 +
15*a^6*x^6 + 120*(-1 + a*x)^5*Log[1 - a*x])/(15*a*c^4*(-1 + a*x)^5)
```

Maple [A] time = 0.052, size = 96, normalized size = 0.9

$$\frac{x}{c^4}-\frac{41}{3\,ac^4\,(ax-1)^3}-22\,\frac{1}{ac^4\,(ax-1)^2}-26\,\frac{1}{ac^4\,(ax-1)}-\frac{4}{5\,ac^4\,(ax-1)^5}+8\,\frac{\ln(ax-1)}{ac^4}-5\,\frac{1}{ac^4\,(ax-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^4,x)
```

```
[Out] x/c^4-41/3/a/c^4/(a*x-1)^3-22/c^4/a/(a*x-1)^2-26/c^4/a/(a*x-1)-4/5/a/c^4/(a
*x-1)^5+8/c^4/a*ln(a*x-1)-5/a/c^4/(a*x-1)^4
```

Maxima [A] time = 0.972854, size = 153, normalized size = 1.46

$$-\frac{390\,a^4x^4-1230\,a^3x^3+1555\,a^2x^2-905\,ax+202}{15\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}+\frac{x}{c^4}+\frac{8\,\log(ax-1)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/15*(390*a^4*x^4 - 1230*a^3*x^3 + 1555*a^2*x^2 - 905*a*x + 202)/(a^6*c^4*
x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4
) + x/c^4 + 8*log(a*x - 1)/(a*c^4)
```

Fricas [A] time = 2.01305, size = 347, normalized size = 3.3

$$\frac{15\,a^6x^6-75\,a^5x^5-240\,a^4x^4+1080\,a^3x^3-1480\,a^2x^2+890\,ax+120\left(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+5\,ax-1\right)\log\left(a\right.}{15\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^6*x^6 - 75*a^5*x^5 - 240*a^4*x^4 + 1080*a^3*x^3 - 1480*a^2*x^2 +
 890*a*x + 120*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*
log(a*x - 1) - 202)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*
c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [A] time = 0.890723, size = 114, normalized size = 1.09

$$-\frac{390a^4x^4-1230a^3x^3+1555a^2x^2-905ax+202}{15a^6c^4x^5-75a^5c^4x^4+150a^4c^4x^3-150a^3c^4x^2+75a^2c^4x-15ac^4}+\frac{x}{c^4}+\frac{8\log\left(ax-1\right)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(c-c/a/x)**4,x)
```

```
[Out] -(390*a**4*x**4 - 1230*a**3*x**3 + 1555*a**2*x**2 - 905*a*x + 202)/(15*a**6
*c**4*x**5 - 75*a**5*c**4*x**4 + 150*a**4*c**4*x**3 - 150*a**3*c**4*x**2 +
75*a**2*c**4*x - 15*a*c**4) + x/c**4 + 8*log(a*x - 1)/(a*c**4)
```

Giac [A] time = 1.16825, size = 89, normalized size = 0.85

$$\frac{x}{c^4}+\frac{8\,\log\left(|ax-1|\right)}{ac^4}-\frac{390\,a^4x^4-1230\,a^3x^3+1555\,a^2x^2-905\,ax+202}{15\left(ax-1\right)^5ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] x/c^4 + 8*log(abs(a*x - 1))/(a*c^4) - 1/15*(390*a^4*x^4 - 1230*a^3*x^3 + 15
55*a^2*x^2 - 905*a*x + 202)/((a*x - 1)^5*a*c^4)
```

3.484 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=60

$$\frac{x(1-ax)^{-p}F_1\left(1-p;-p-\frac{1}{2},\frac{1}{2};2-p;ax,-ax\right)\left(c-\frac{c}{ax}\right)^p}{1-p}$$

```
[Out] ((c - c/(a*x))^p*x*AppellF1[1 - p, -1/2 - p, 1/2, 2 - p, a*x, -(a*x)])/((1
- p)*(1 - a*x)^p)
```

Rubi [A] time = 0.0952823, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6134, 6129, 133}

$$\frac{x(1-ax)^{-p}F_1\left(1-p;-p-\frac{1}{2},\frac{1}{2};2-p;ax,-ax\right)\left(c-\frac{c}{ax}\right)^p}{1-p}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^p/E^ArcTanh[a*x],x]
```

```
[Out] ((c - c/(a*x))^p*x*AppellF1[1 - p, -1/2 - p, 1/2, 2 - p, a*x, -(a*x)])/((1
- p)*(1 - a*x)^p)
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx = \left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int e^{-\tanh^{-1}(ax)}x^{-p}(1-ax)^p dx$$
$$= \left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int \frac{x^{-p}(1-ax)^{\frac{1}{2}+p}}{\sqrt{1+ax}}dx$$
$$= \frac{\left(c-\frac{c}{ax}\right)^p x(1-ax)^{-p}F_1\left(1-p;-\frac{1}{2}-p,\frac{1}{2};2-p;ax,-ax\right)}{1-p}$$

Mathematica [F] time = 1.04295, size = 0, normalized size = 0.

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c - c/(a*x))^p/E^ArcTanh[a*x],x]
```

```
[Out] Integrate[(c - c/(a*x))^p/E^ArcTanh[a*x], x]
```

Maple [F] time = 0.332, size = 0, normalized size = 0.

$$\int \frac{1}{ax+1}\left(c-\frac{c}{ax}\right)^p\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int((c-c/a/x)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^p}{ax+1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^p/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(\frac{acx-c}{ax}\right)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*((a*c*x - c)/(a*x))^p/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{p}\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^{2}x^{2}+1}\left(c-\frac{c}{ax}\right)^{p}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^p/(a*x + 1), x)
```

3.485 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=140

$$\frac{c^4\sqrt{1-a^2x^2}}{a}-\frac{32c^4\sqrt{1-a^2x^2}}{3a^2x}+\frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3}+\frac{25c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}+\frac{5c^4\sin^{-1}(ax)}{a}$$

```
[Out] (c^4*Sqrt[1 - a^2*x^2])/a - (c^4*Sqrt[1 - a^2*x^2])/(3*a^4*x^3) + (5*c^4*Sq
rt[1 - a^2*x^2])/(2*a^3*x^2) - (32*c^4*Sqrt[1 - a^2*x^2])/(3*a^2*x) + (5*c^
4*ArcSin[a*x])/a + (25*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rubi [A] time = 0.419824, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6131, 6128, 1807, 1809, 844, 216, 266, 63, 208}

$$\frac{c^4\sqrt{1-a^2x^2}}{a}-\frac{32c^4\sqrt{1-a^2x^2}}{3a^2x}+\frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3}+\frac{25c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}+\frac{5c^4\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^4/E^ArcTanh[a*x],x]
```

```
[Out] (c^4*Sqrt[1 - a^2*x^2])/a - (c^4*Sqrt[1 - a^2*x^2])/(3*a^4*x^3) + (5*c^4*Sq
rt[1 - a^2*x^2])/(2*a^3*x^2) - (32*c^4*Sqrt[1 - a^2*x^2])/(3*a^2*x) + (5*c^
4*ArcSin[a*x])/a + (25*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
```

```
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\int \frac{e^{-\tanh^{-1}(ax)}(1-ax)^4}{x^4}dx}{a^4}$$
$$= \frac{c^4\int \frac{(1-ax)^5}{x^4\sqrt{1-a^2x^2}}dx}{a^4}$$
$$= -\frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3} - \frac{c^4\int \frac{15a-32a^2x+30a^3x^2-15a^4x^3+3a^5x^4}{x^3\sqrt{1-a^2x^2}}dx}{3a^4}$$
$$= -\frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3} + \frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2} + \frac{c^4\int \frac{64a^2-75a^3x+30a^4x^2-6a^5x^3}{x^2\sqrt{1-a^2x^2}}dx}{6a^4}$$
$$= -\frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3} + \frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{32c^4\sqrt{1-a^2x^2}}{3a^2x} - \frac{c^4\int \frac{75a^3-30a^4x+6a^5x^2}{x\sqrt{1-a^2x^2}}dx}{6a^4}$$
$$= \frac{c^4\sqrt{1-a^2x^2}}{a} - \frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3} + \frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{32c^4\sqrt{1-a^2x^2}}{3a^2x} + \frac{c^4\int \frac{-75a^5+30a^6x}{x\sqrt{1-a^2x^2}}dx}{6a^6}$$
$$= \frac{c^4\sqrt{1-a^2x^2}}{a} - \frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3} + \frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{32c^4\sqrt{1-a^2x^2}}{3a^2x} + \left(5c^4\right)\int \frac{1}{\sqrt{1-a^2x^2}}dx - $$
$$= \frac{c^4\sqrt{1-a^2x^2}}{a} - \frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3} + \frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{32c^4\sqrt{1-a^2x^2}}{3a^2x} + \frac{5c^4\sin^{-1}(ax)}{a} - \frac{\left(25c^4\right)\mathrm{S}}{}$$
$$= \frac{c^4\sqrt{1-a^2x^2}}{a} - \frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3} + \frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{32c^4\sqrt{1-a^2x^2}}{3a^2x} + \frac{5c^4\sin^{-1}(ax)}{a} + \frac{\left(25c^4\right)\mathrm{S}}{}$$
$$= \frac{c^4\sqrt{1-a^2x^2}}{a} - \frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3} + \frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{32c^4\sqrt{1-a^2x^2}}{3a^2x} + \frac{5c^4\sin^{-1}(ax)}{a} + \frac{25c^4\tan}{}$$

Mathematica [A] time = 0.239494, size = 85, normalized size = 0.61

$$\frac{c^4\left(\frac{\sqrt{1-a^2x^2}\left(6a^3x^3-64a^2x^2+15ax-2\right)}{a^3x^3}+75\log\left(\sqrt{1-a^2x^2}+1\right)-75\log(ax)+30\sin^{-1}(ax)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^4/E^ArcTanh[a*x],x]
```

```
[Out] (c^4*((Sqrt[1 - a^2*x^2]*(-2 + 15*a*x - 64*a^2*x^2 + 6*a^3*x^3))/(a^3*x^3)
+ 30*ArcSin[a*x] - 75*Log[a*x] + 75*Log[1 + Sqrt[1 - a^2*x^2]]))/(6*a)
```

Maple [A] time = 0.052, size = 232, normalized size = 1.7

$$-11\,\frac{c^4\left(-a^2x^2+1\right)^{3/2}}{a^2x}-11\,c^4x\sqrt{-a^2x^2+1}-11\,\frac{c^4}{\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+\frac{25\,c^4}{2\,a}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{25\,c^4}{2\,a}\sqrt{-a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -11*c^4/a^2/x*(-a^2*x^2+1)^(3/2)-11*c^4*x*(-a^2*x^2+1)^(1/2)-11*c^4/(a^2)^(
1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+25/2*c^4/a*arctanh(1/(-a^2*x^
```

```
2+1)^(1/2))-25/2*c^4*(-a^2*x^2+1)^(1/2)/a+16*c^4/a*(-a^2*(x+1/a)^2+2*a*(x+1
/a))^(1/2)+16*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1
/a))^(1/2))+5/2*c^4*(-a^2*x^2+1)^(3/2)/x^2/a^3-1/3*c^4*(-a^2*x^2+1)^(3/2)/a
^4/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13532, size = 286, normalized size = 2.04

$$-\frac{60\,a^3c^4x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+75\,a^3c^4x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-6\,a^3c^4x^3-\left(6\,a^3c^4x^3-64\,a^2c^4x^2+15\,ac^4x-2\,c^4\right)\sqrt{-a}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(60*a^3*c^4*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 75*a^3*c^4*x^
3*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 6*a^3*c^4*x^3 - (6*a^3*c^4*x^3 - 64*a^2
*c^4*x^2 + 15*a*c^4*x - 2*c^4)*sqrt(-a^2*x^2 + 1))/(a^4*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^4\left(\int\frac{\sqrt{-a^2x^2+1}}{ax^5+x^4}\,dx+\int-\frac{4ax\sqrt{-a^2x^2+1}}{ax^5+x^4}\,dx+\int\frac{6a^2x^2\sqrt{-a^2x^2+1}}{ax^5+x^4}\,dx+\int-\frac{4a^3x^3\sqrt{-a^2x^2+1}}{ax^5+x^4}\,dx+\int\frac{a^4x^4\sqrt{-a^2x^2+1}}{ax^5+x^4}\,dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**4/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c**4*(Integral(sqrt(-a**2*x**2 + 1)/(a*x**5 + x**4), x) + Integral(-4*a*x*s
qrt(-a**2*x**2 + 1)/(a*x**5 + x**4), x) + Integral(6*a**2*x**2*sqrt(-a**2*x
**2 + 1)/(a*x**5 + x**4), x) + Integral(-4*a**3*x**3*sqrt(-a**2*x**2 + 1)/(
a*x**5 + x**4), x) + Integral(a**4*x**4*sqrt(-a**2*x**2 + 1)/(a*x**5 + x**4
), x))/a**4
```

Giac [B] time = 1.18686, size = 354, normalized size = 2.53

$$\frac{\left(c^4-\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)c^4}{a^2x}+\frac{129\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^4}{a^4x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|}+\frac{5\,c^4\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{|a|}+\frac{25\,c^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}+\frac{\sqrt{-a^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(c^4 - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/(a^2*x) + 129*(sqrt(-a^2
*x^2 + 1)*abs(a) + a)^2*c^4/(a^4*x^2))*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a)
+ a)^3*abs(a)) + 5*c^4*arcsin(a*x)*sgn(a)/abs(a) + 25/2*c^4*log(1/2*abs(-2*
sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*
c^4/a - 1/24*(129*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/x - 15*(sqrt(-a^2*x^2
 + 1)*abs(a) + a)^2*c^4/(a^2*x^2) + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^4/(
a^4*x^3))/(a^2*abs(a))
```

3.486 $\int e^{-\tanh^{-1}(ax)} \left(c - \frac{c}{ax}\right)^3 dx$

Optimal. Leaf size=111

$$\frac{c^3\sqrt{1-a^2x^2}}{a} - \frac{4c^3\sqrt{1-a^2x^2}}{a^2x} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} + \frac{13c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a} + \frac{4c^3\sin^{-1}(ax)}{a}$$

```
[Out] (c^3*Sqrt[1 - a^2*x^2])/a + (c^3*Sqrt[1 - a^2*x^2])/(2*a^3*x^2) - (4*c^3*Sq
rt[1 - a^2*x^2])/(a^2*x) + (4*c^3*ArcSin[a*x])/a + (13*c^3*ArcTanh[Sqrt[1 -
 a^2*x^2]])/(2*a)
```

Rubi [A] time = 0.31196, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6131, 6128, 1807, 1809, 844, 216, 266, 63, 208}

$$\frac{c^3\sqrt{1-a^2x^2}}{a} - \frac{4c^3\sqrt{1-a^2x^2}}{a^2x} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} + \frac{13c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a} + \frac{4c^3\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^3/E^ArcTanh[a*x],x]
```

```
[Out] (c^3*Sqrt[1 - a^2*x^2])/a + (c^3*Sqrt[1 - a^2*x^2])/(2*a^3*x^2) - (4*c^3*Sq
rt[1 - a^2*x^2])/(a^2*x) + (4*c^3*ArcSin[a*x])/a + (13*c^3*ArcTanh[Sqrt[1 -
 a^2*x^2]])/(2*a)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
```

```
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{c^3\int \frac{e^{-\tanh^{-1}(ax)}(1-ax)^3}{x^3}dx}{a^3}$$
$$= -\frac{c^3\int \frac{(1-ax)^4}{x^3\sqrt{1-a^2x^2}}dx}{a^3}$$
$$= \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}+\frac{c^3\int \frac{8a-13a^2x+8a^3x^2-2a^4x^3}{x^2\sqrt{1-a^2x^2}}dx}{2a^3}$$
$$= \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{4c^3\sqrt{1-a^2x^2}}{a^2x}-\frac{c^3\int \frac{13a^2-8a^3x+2a^4x^2}{x\sqrt{1-a^2x^2}}dx}{2a^3}$$
$$= \frac{c^3\sqrt{1-a^2x^2}}{a}+\frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{4c^3\sqrt{1-a^2x^2}}{a^2x}+\frac{c^3\int \frac{-13a^4+8a^5x}{x\sqrt{1-a^2x^2}}dx}{2a^5}$$
$$= \frac{c^3\sqrt{1-a^2x^2}}{a}+\frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{4c^3\sqrt{1-a^2x^2}}{a^2x}+\left(4c^3\right)\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{\left(13c^3\right)\int \frac{1}{x\sqrt{1-a^2}}$$
$$= \frac{c^3\sqrt{1-a^2x^2}}{a}+\frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{4c^3\sqrt{1-a^2x^2}}{a^2x}+\frac{4c^3\sin^{-1}(ax)}{a}-\frac{\left(13c^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2}}\right.}{4a}$$
$$= \frac{c^3\sqrt{1-a^2x^2}}{a}+\frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{4c^3\sqrt{1-a^2x^2}}{a^2x}+\frac{4c^3\sin^{-1}(ax)}{a}+\frac{\left(13c^3\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\right.}{2a^3}$$
$$= \frac{c^3\sqrt{1-a^2x^2}}{a}+\frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{4c^3\sqrt{1-a^2x^2}}{a^2x}+\frac{4c^3\sin^{-1}(ax)}{a}+\frac{13c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right.}{2a}$$

Mathematica [A] time = 0.181021, size = 77, normalized size = 0.69

$$\frac{c^3\left(\frac{\sqrt{1-a^2x^2}\left(2a^2x^2-8ax+1\right)}{a^2x^2}+13\log\left(\sqrt{1-a^2x^2}+1\right)-13\log(ax)+8\sin^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^3/E^ArcTanh[a*x],x]
```

```
[Out] (c^3*((Sqrt[1 - a^2*x^2]*(1 - 8*a*x + 2*a^2*x^2))/(a^2*x^2) + 8*ArcSin[a*x]
 - 13*Log[a*x] + 13*Log[1 + Sqrt[1 - a^2*x^2]]))/(2*a)
```

Maple [B] time = 0.053, size = 209, normalized size = 1.9

$$-4\frac{c^3\left(-a^2x^2+1\right)^{3/2}}{a^2x}-4c^3x\sqrt{-a^2x^2+1}-4\frac{c^3}{\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+\frac{13c^3}{2a}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{13c^3}{2a}\sqrt{-a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -4*c^3/a^2/x*(-a^2*x^2+1)^(3/2)-4*c^3*x*(-a^2*x^2+1)^(1/2)-4*c^3/(a^2)^(1/2
)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+13/2*c^3/a*arctanh(1/(-a^2*x^2+1
)^(1/2))-13/2*c^3*(-a^2*x^2+1)^(1/2)/a+8*c^3/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))
^(1/2)+8*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^
```

```
(1/2))+1/2*c^3*(-a^2*x^2+1)^(3/2)/x^2/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12384, size = 259, normalized size = 2.33

$$-\frac{16\,a^{2}c^{3}x^{2}\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)+13\,a^{2}c^{3}x^{2}\log\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{x}\right)-2\,a^{2}c^{3}x^{2}-\left(2\,a^{2}c^{3}x^{2}-8\,ac^{3}x+c^{3}\right)\sqrt{-a^{2}x^{2}+1}}{2\,a^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(16*a^2*c^3*x^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 13*a^2*c^3*x^
2*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 2*a^2*c^3*x^2 - (2*a^2*c^3*x^2 - 8*a*c^
3*x + c^3)*sqrt(-a^2*x^2 + 1))/(a^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^{3}\left(\int -\frac{\sqrt{-a^{2}x^{2}+1}}{ax^{4}+x^{3}}\,dx+\int \frac{3ax\sqrt{-a^{2}x^{2}+1}}{ax^{4}+x^{3}}\,dx+\int -\frac{3a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}}{ax^{4}+x^{3}}\,dx+\int \frac{a^{3}x^{3}\sqrt{-a^{2}x^{2}+1}}{ax^{4}+x^{3}}\,dx\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**3/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c**3*(Integral(-sqrt(-a**2*x**2 + 1)/(a*x**4 + x**3), x) + Integral(3*a*x*s
qrt(-a**2*x**2 + 1)/(a*x**4 + x**3), x) + Integral(-3*a**2*x**2*sqrt(-a**2*
x**2 + 1)/(a*x**4 + x**3), x) + Integral(a**3*x**3*sqrt(-a**2*x**2 + 1)/(a*
x**4 + x**3), x))/a**3
```

Giac [B] time = 1.20003, size = 278, normalized size = 2.5

$$-\frac{\left(c^{3}-\frac{16\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)c^{3}}{a^{2}x}\right)a^{4}x^{2}}{8\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)^{2}|a|}+\frac{4\,c^{3}\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{|a|}+\frac{13\,c^{3}\log\left(\frac{\left|-2\,\sqrt{-a^{2}x^{2}+1}|a|-2\,a\right|}{2\,a^{2}|x|}\right)}{2\,|a|}+\frac{\sqrt{-a^{2}x^{2}+1}c^{3}}{a}-\frac{16\left(\sqrt{-a^{2}x^{2}+1}|a|\right.}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(c^3 - 16*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3/(a^2*x))*a^4*x^2/((sqrt(
-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)) + 4*c^3*arcsin(a*x)*sgn(a)/abs(a) + 13/
2*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a)
+ sqrt(-a^2*x^2 + 1)*c^3/a - 1/8*(16*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3*ab
s(a)/(a^2*x) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3*abs(a)/(a^4*x^2))/a^2
```

3.487 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=82

$$\frac{c^2\sqrt{1-a^2x^2}}{a}-\frac{c^2\sqrt{1-a^2x^2}}{a^2x}+\frac{3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}+\frac{3c^2\sin^{-1}(ax)}{a}$$

```
[Out] (c^2*Sqrt[1 - a^2*x^2])/a - (c^2*Sqrt[1 - a^2*x^2])/(a^2*x) + (3*c^2*ArcSin
[a*x])/a + (3*c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rubi [A] time = 0.237808, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6131, 6128, 1807, 1809, 844, 216, 266, 63, 208}

$$\frac{c^2\sqrt{1-a^2x^2}}{a}-\frac{c^2\sqrt{1-a^2x^2}}{a^2x}+\frac{3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}+\frac{3c^2\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^2/E^ArcTanh[a*x],x]
```

```
[Out] (c^2*Sqrt[1 - a^2*x^2])/a - (c^2*Sqrt[1 - a^2*x^2])/(a^2*x) + (3*c^2*ArcSin
[a*x])/a + (3*c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
```

```
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \frac{c^2\int\frac{e^{-\tanh^{-1}(ax)}(1-ax)^2}{x^2}dx}{a^2}$$
$$= \frac{c^2\int\frac{(1-ax)^3}{x^2\sqrt{1-a^2x^2}}dx}{a^2}$$
$$= -\frac{c^2\sqrt{1-a^2x^2}}{a^2x} - \frac{c^2\int\frac{3a-3a^2x+a^3x^2}{x\sqrt{1-a^2x^2}}dx}{a^2}$$
$$= \frac{c^2\sqrt{1-a^2x^2}}{a} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} + \frac{c^2\int\frac{-3a^3+3a^4x}{x\sqrt{1-a^2x^2}}dx}{a^4}$$
$$= \frac{c^2\sqrt{1-a^2x^2}}{a} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} + (3c^2)\int\frac{1}{\sqrt{1-a^2x^2}}dx - \frac{(3c^2)\int\frac{1}{x\sqrt{1-a^2x^2}}dx}{a}$$
$$= \frac{c^2\sqrt{1-a^2x^2}}{a} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} + \frac{3c^2\sin^{-1}(ax)}{a} - \frac{(3c^2)\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-a^2x}}dx, x, x^2\right)}{2a}$$
$$= \frac{c^2\sqrt{1-a^2x^2}}{a} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} + \frac{3c^2\sin^{-1}(ax)}{a} + \frac{(3c^2)\,\text{Subst}\left(\int\frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}dx, x, \sqrt{1-a^2x^2}\right)}{a^3}$$
$$= \frac{c^2\sqrt{1-a^2x^2}}{a} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} + \frac{3c^2\sin^{-1}(ax)}{a} + \frac{3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}$$

Mathematica [A] time = 0.116056, size = 83, normalized size = 1.01

$$\frac{\sqrt{1-a^2x^2}\left(c^2-\frac{c^2}{ax}\right)}{a} + \frac{3c^2\log\left(\sqrt{1-a^2x^2}+1\right)}{a} - \frac{3c^2\log(ax)}{a} + \frac{3c^2\sin^{-1}(ax)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^2/E^ArcTanh[a*x],x]
```

```
[Out] ((c^2 - c^2/(a*x))*Sqrt[1 - a^2*x^2])/a + (3*c^2*ArcSin[a*x])/a - (3*c^2*Lo
g[a*x])/a + (3*c^2*Log[1 + Sqrt[1 - a^2*x^2]])/a
```

Maple [B] time = 0.044, size = 186, normalized size = 2.3

$$-\frac{c^2}{a^2x}\left(-a^2x^2+1\right)^{\frac{3}{2}} - c^2x\sqrt{-a^2x^2+1} - c^2\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} + 3\frac{c^2\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{a} - 3\frac{c^2\sqrt{-a^2x^2+1}}{a} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -c^2/a^2/x*(-a^2*x^2+1)^(3/2)-c^2*x*(-a^2*x^2+1)^(1/2)-c^2/(a^2)^(1/2)*arct
an((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+3*c^2/a*arctanh(1/(-a^2*x^2+1)^(1/2))-
3*c^2*(-a^2*x^2+1)^(1/2)/a+4*c^2/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+4*c^2
/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.18273, size = 205, normalized size = 2.5

$$-\frac{6\,ac^{2}x\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)+3\,ac^{2}x\log\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{x}\right)-ac^{2}x-\left(ac^{2}x-c^{2}\right)\sqrt{-a^{2}x^{2}+1}}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(6*a*c^2*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 3*a*c^2*x*log((sqrt(-a
^2*x^2 + 1) - 1)/x) - a*c^2*x - (a*c^2*x - c^2)*sqrt(-a^2*x^2 + 1))/(a^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^{2}\left(\int\frac{\sqrt{-a^{2}x^{2}+1}}{ax^{3}+x^{2}}\,dx+\int-\frac{2ax\sqrt{-a^{2}x^{2}+1}}{ax^{3}+x^{2}}\,dx+\int\frac{a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}}{ax^{3}+x^{2}}\,dx\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**2/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c**2*(Integral(sqrt(-a**2*x**2 + 1)/(a*x**3 + x**2), x) + Integral(-2*a*x*s
qrt(-a**2*x**2 + 1)/(a*x**3 + x**2), x) + Integral(a**2*x**2*sqrt(-a**2*x**
2 + 1)/(a*x**3 + x**2), x))/a**2
```

Giac [A] time = 1.14011, size = 188, normalized size = 2.29

$$\frac{a^{2}c^{2}x}{2\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)|a|}+\frac{3\,c^{2}\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{|a|}+\frac{3\,c^{2}\log\left(\frac{\left|-2\,\sqrt{-a^{2}x^{2}+1}|a|-2\,a\right|}{2\,a^{2}|x|}\right)}{|a|}+\frac{\sqrt{-a^{2}x^{2}+1}c^{2}}{a}-\frac{\left(\sqrt{-a^{2}x^{2}+1}|a|+\right.}{2\,a^{2}x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*a^2*c^2*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) + 3*c^2*arcsin(a*x)*
sgn(a)/abs(a) + 3*c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*
abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c^2/a - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a)
 + a)*c^2/(a^2*x*abs(a))
```

3.488 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=50

$$\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}+\frac{2c\sin^{-1}(ax)}{a}$$

```
[Out] (c*Sqrt[1 - a^2*x^2])/a + (2*c*ArcSin[a*x])/a + (c*ArcTanh[Sqrt[1 - a^2*x^2
]])/a
```

Rubi [A] time = 0.151384, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6131, 6128, 1809, 844, 216, 266, 63, 208}

$$\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}+\frac{2c\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))/E^ArcTanh[a*x],x]
```

```
[Out] (c*Sqrt[1 - a^2*x^2])/a + (2*c*ArcSin[a*x])/a + (c*ArcTanh[Sqrt[1 - a^2*x^2
]])/a
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= -\frac{c\int\frac{e^{-\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}\\
&= -\frac{c\int\frac{(1-ax)^2}{x\sqrt{1-a^2x^2}}\,dx}{a}\\
&= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\int\frac{-a^2+2a^3x}{x\sqrt{1-a^2x^2}}\,dx}{a^3}\\
&= \frac{c\sqrt{1-a^2x^2}}{a}+(2c)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx-\frac{c\int\frac{1}{x\sqrt{1-a^2x^2}}\,dx}{a}\\
&= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{2c\sin^{-1}(ax)}{a}-\frac{c\,\mathrm{Subst}\left(\int\frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2a}\\
&= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{2c\sin^{-1}(ax)}{a}+\frac{c\,\mathrm{Subst}\left(\int\frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^3}\\
&= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{2c\sin^{-1}(ax)}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0528544, size = 47, normalized size = 0.94

$$\frac{c\left(\sqrt{1-a^2x^2}+\log\left(\sqrt{1-a^2x^2}+1\right)+2\sin^{-1}(ax)-\log(x)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))/E^ArcTanh[a*x],x]
```

```
[Out] (c*(Sqrt[1 - a^2*x^2] + 2*ArcSin[a*x] - Log[x] + Log[1 + Sqrt[1 - a^2*x^2]]
))/a
```

Maple [B] time = 0.045, size = 106, normalized size = 2.1

$$-\frac{c}{a}\sqrt{-a^2x^2+1}+\frac{c}{a}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+2\,\frac{c\sqrt{-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)}}{a}+2\,\frac{c}{\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -c*(-a^2*x^2+1)^(1/2)/a+c/a*arctanh(1/(-a^2*x^2+1)^(1/2))+2*c/a*(-a^2*(x+1/
a)^2+2*a*(x+1/a))^(1/2)+2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^
2+2*a*(x+1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12852, size = 146, normalized size = 2.92

$$-\frac{4\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+c\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(4*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + c*log((sqrt(-a^2*x^2 + 1) -
1)/x) - sqrt(-a^2*x^2 + 1)*c)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c\left(\int -\frac{\sqrt{-a^2x^2+1}}{ax^2+x}\,dx+\int \frac{ax\sqrt{-a^2x^2+1}}{ax^2+x}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c*(Integral(-sqrt(-a**2*x**2 + 1)/(a*x**2 + x), x) + Integral(a*x*sqrt(-a**
2*x**2 + 1)/(a*x**2 + x), x))/a
```

Giac [A] time = 1.1359, size = 92, normalized size = 1.84

$$\frac{2\,c\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{|a|}+\frac{c\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\frac{\sqrt{-a^2x^2+1}c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2*c*arcsin(a*x)*sgn(a)/abs(a) + c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a)
- 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c/a
```

3.489 $\int \frac{e^{-\tanh^{-1}(ax)}}{c-\frac{c}{ax}} dx$

Optimal. Leaf size=21

$$\frac{\sqrt{1-a^2x^2}}{ac}$$

```
[Out] Sqrt[1 - a^2*x^2]/(a*c)
```

Rubi [A] time = 0.0738308, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6128, 261}

$$\frac{\sqrt{1-a^2x^2}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a*x))),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a*c)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{c-\frac{c}{ax}} dx &= -\frac{a\int \frac{e^{-\tanh^{-1}(ax)}x}{1-ax} dx}{c}\\
&= -\frac{a\int \frac{x}{\sqrt{1-a^2x^2}} dx}{c}\\
&= \frac{\sqrt{1-a^2x^2}}{ac}
\end{aligned}$$

Mathematica [A] time = 0.0127588, size = 21, normalized size = 1.

$$\frac{\sqrt{1-a^2x^2}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a*x))),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a*c)
```

Maple [A] time = 0.032, size = 20, normalized size = 1.

$$\frac{1}{ac}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x),x)
```

```
[Out] (-a^2*x^2+1)^(1/2)/a/c
```

Maxima [A] time = 0.974711, size = 30, normalized size = 1.43

$$\frac{\sqrt{ax+1}\sqrt{-ax+1}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] sqrt(a*x + 1)*sqrt(-a*x + 1)/(a*c)
```

Fricas [A] time = 1.9748, size = 35, normalized size = 1.67

$$\frac{\sqrt{-a^2x^2+1}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] sqrt(-a^2*x^2 + 1)/(a*c)
```

Sympy [A] time = 8.53791, size = 53, normalized size = 2.52

$$a\left(\begin{cases} \tilde{\infty}\left(\begin{cases} \frac{x^2}{2} & \text{for}\ a^2=0 \\ -\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3a^2} & \text{otherwise} \end{cases}\right) & \text{for}\ c=0 \\ -\frac{x^2}{2c} & \text{for}\ a^2=0 \\ \frac{\sqrt{-a^2x^2+1}}{a^2c} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a/x),x)
```

```
[Out] a*Piecewise((zoo*Piecewise((x**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2)
/(3*a**2), True)), Eq(c, 0)), (-x**2/(2*c), Eq(a**2, 0)), (sqrt(-a**2*x**2
+ 1)/(a**2*c), True))
```

Giac [A] time = 1.13544, size = 26, normalized size = 1.24

$$\frac{\sqrt{-a^2x^2+1}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] sqrt(-a^2*x^2 + 1)/(a*c)
```

3.490 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal. Leaf size=63

$$\frac{\sqrt{1-a^2x^2}}{ac^2(1-ax)}+\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{\sin^{-1}(ax)}{ac^2}$$

```
[Out] Sqrt[1 - a^2*x^2]/(a*c^2) + Sqrt[1 - a^2*x^2]/(a*c^2*(1 - a*x)) - ArcSin[a*
x]/(a*c^2)
```

Rubi [A] time = 0.179248, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6131, 6128, 1639, 12, 793, 216}

$$\frac{\sqrt{1-a^2x^2}}{ac^2(1-ax)}+\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a*x))^2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a*c^2) + Sqrt[1 - a^2*x^2]/(a*c^2*(1 - a*x)) - ArcSin[a*
x]/(a*c^2)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx &= \frac{a^2\int \frac{e^{-\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}\\ &= \frac{a^2\int \frac{x^2}{(1-ax)\sqrt{1-a^2x^2}}\,dx}{c^2}\\ &= \frac{\sqrt{1-a^2x^2}}{ac^2}+\frac{\int \frac{a^3x}{(1-ax)\sqrt{1-a^2x^2}}\,dx}{a^2c^2}\\ &= \frac{\sqrt{1-a^2x^2}}{ac^2}+\frac{a\int \frac{x}{(1-ax)\sqrt{1-a^2x^2}}\,dx}{c^2}\\ &= \frac{\sqrt{1-a^2x^2}}{ac^2}+\frac{\sqrt{1-a^2x^2}}{ac^2(1-ax)}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^2}\\ &= \frac{\sqrt{1-a^2x^2}}{ac^2}+\frac{\sqrt{1-a^2x^2}}{ac^2(1-ax)}-\frac{\sin^{-1}(ax)}{ac^2}\end{aligned}$$

Mathematica [A] time = 0.0949117, size = 47, normalized size = 0.75

$$\frac{\sqrt{1-a^2x^2}\left(\frac{1}{c^2}-\frac{1}{c^2(ax-1)}\right)}{a}-\frac{\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a*x))^2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(c^(-2) - 1/(c^2*(-1 + a*x))))/a - ArcSin[a*x]/(a*c^2)
```

Maple [B] time = 0.046, size = 198, normalized size = 3.1

$$\frac{1}{2a^3c^2}\left(-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)\right)^{\frac{3}{2}}\left(x-a^{-1}\right)^{-2}+\frac{5}{4ac^2}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}-\frac{5}{4c^2}\arctan\left(x\sqrt{a^2}\frac{}{\sqrt{-a^2(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^2,x)
```

```
[Out] 1/2/a^3/c^2/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+5/4/a/c^2*(-a^2*(x
-1/a)^2-2*a*(x-1/a))^(1/2)-5/4/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(
x-1/a)^2-2*a*(x-1/a))^(1/2))+1/4/a/c^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1
/4/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a*x))^2), x)
```

Fricas [A] time = 2.10926, size = 158, normalized size = 2.51

$$\frac{2\,ax+2\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}(ax-2)-2}{a^2c^2x-ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] (2*a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2
 + 1)*(a*x - 2) - 2)/(a^2*c^2*x - a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\int \frac{x^2\sqrt{-a^2x^2+1}}{a^3x^3-a^2x^2-ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2*sqrt(-a**2*x**2 + 1)/(a**3*x**3 - a**2*x**2 - a*x + 1),
x)/c**2
```

Giac [A] time = 1.14473, size = 97, normalized size = 1.54

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{c^2|a|}+\frac{\sqrt{-a^2x^2+1}}{ac^2}+\frac{2}{c^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(c^2*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c^2) + 2/(c^2*((sq
rt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

3.491 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx$

Optimal. Leaf size=94

$$-\frac{(ax+1)^2}{3ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{8(ax+1)}{3ac^3\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac^3}-\frac{2\sin^{-1}(ax)}{ac^3}$$

```
[Out] -(1 + a*x)^2/(3*a*c^3*(1 - a^2*x^2)^(3/2)) + (8*(1 + a*x))/(3*a*c^3*Sqrt[1
- a^2*x^2]) + Sqrt[1 - a^2*x^2]/(a*c^3) - (2*ArcSin[a*x])/(a*c^3)
```

Rubi [A] time = 0.259897, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6131, 6128, 852, 1635, 641, 216}

$$-\frac{(ax+1)^2}{3ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{8(ax+1)}{3ac^3\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac^3}-\frac{2\sin^{-1}(ax)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a*x))^3),x]
```

```
[Out] -(1 + a*x)^2/(3*a*c^3*(1 - a^2*x^2)^(3/2)) + (8*(1 + a*x))/(3*a*c^3*Sqrt[1
- a^2*x^2]) + Sqrt[1 - a^2*x^2]/(a*c^3) - (2*ArcSin[a*x])/(a*c^3)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
```

```
& GtQ[m, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx &= -\frac{a^3\int \frac{e^{-\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}\\
&= -\frac{a^3\int \frac{x^3}{(1-ax)^2\sqrt{1-a^2x^2}}\,dx}{c^3}\\
&= -\frac{a^3\int \frac{x^3(1+ax)^2}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^3}\\
&= -\frac{(1+ax)^2}{3ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{a^3\int \frac{(1+ax)\left(\frac{2}{a^3}+\frac{3x}{a^2}+\frac{3x^2}{a}\right)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^3}\\
&= -\frac{(1+ax)^2}{3ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{8(1+ax)}{3ac^3\sqrt{1-a^2x^2}}-\frac{a^3\int \frac{\frac{6}{a^3}+\frac{3x}{a^2}}{\sqrt{1-a^2x^2}}\,dx}{3c^3}\\
&= -\frac{(1+ax)^2}{3ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{8(1+ax)}{3ac^3\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac^3}-\frac{2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^3}\\
&= -\frac{(1+ax)^2}{3ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{8(1+ax)}{3ac^3\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac^3}-\frac{2\sin^{-1}(ax)}{ac^3}
\end{aligned}$$

Mathematica [A] time = 0.125562, size = 53, normalized size = 0.56

$$\frac{\frac{\sqrt{1-a^2x^2}\left(3a^2x^2-14ax+10\right)}{(ax-1)^2}-6\sin^{-1}(ax)}{3ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a*x))^3),x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(10 - 14*a*x + 3*a^2*x^2))/(-1 + a*x)^2 - 6*ArcSin[a*x]
)/(3*a*c^3)
```

Maple [B] time = 0.049, size = 242, normalized size = 2.6

$$\frac{1}{6 a^4 c^3}\left(-a^2 \left(x-a^{-1}\right)^2-2 a \left(x-a^{-1}\right)\right)^{\frac{3}{2}} \left(x-a^{-1}\right)^{-3}+\frac{5}{4 a^3 c^3}\left(-a^2 \left(x-a^{-1}\right)^2-2 a \left(x-a^{-1}\right)\right)^{\frac{3}{2}} \left(x-a^{-1}\right)^{-2}+\frac{17}{8 a c^3}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^3,x)
```

```
[Out] 1/6/a^4/c^3/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+5/4/a^3/c^3/(x-1/a
)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+17/8/a/c^3*(-a^2*(x-1/a)^2-2*a*(x-1/
a))^(1/2)-17/8/c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-
1/a))^(1/2))+1/8/a/c^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1/8/c^3/(a^2)^(1/
2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2 x^2+1}}{(ax+1)\left(c-\frac{c}{ax}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a*x))^3), x)
```

Fricas [A] time = 2.18325, size = 244, normalized size = 2.6

$$\frac{10 a^2 x^2-20 ax+12 \left(a^2 x^2-2 ax+1\right) \arctan\left(\frac{\sqrt{-a^2 x^2+1}-1}{ax}\right)+\left(3 a^2 x^2-14 ax+10\right)\sqrt{-a^2 x^2+1}+10}{3 \left(a^3 c^3 x^2-2 a^2 c^3 x+a c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(10*a^2*x^2 - 20*a*x + 12*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2 +
 1) - 1)/(a*x)) + (3*a^2*x^2 - 14*a*x + 10)*sqrt(-a^2*x^2 + 1) + 10)/(a^3*c
^3*x^2 - 2*a^2*c^3*x + a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3 \int \frac{x^3 \sqrt{-a^2 x^2+1}}{a^4 x^4-2 a^3 x^3+2 ax-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*Integral(x**3*sqrt(-a**2*x**2 + 1)/(a**4*x**4 - 2*a**3*x**3 + 2*a*x -
1), x)/c**3
```

Giac [A] time = 1.21157, size = 170, normalized size = 1.81

$$-\frac{2\arcsin(ax)\operatorname{sgn}(a)}{c^3|a|}+\frac{\sqrt{-a^2x^2+1}}{ac^3}-\frac{2\left(\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{6\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}-7\right)}{3\,c^3\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -2*arcsin(a*x)*sgn(a)/(c^3*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c^3) - 2/3*(15*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 6*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^
2/(a^4*x^2) - 7)/(c^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^3*abs(a
))
```

3.492 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

Optimal. Leaf size=125

$$\frac{(ax+1)^3}{5ac^4\left(1-a^2x^2\right)^{5/2}}-\frac{6(ax+1)^2}{5ac^4\left(1-a^2x^2\right)^{3/2}}+\frac{24(ax+1)}{5ac^4\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac^4}-\frac{3\sin^{-1}(ax)}{ac^4}$$

```
[Out] (1 + a*x)^3/(5*a*c^4*(1 - a^2*x^2)^(5/2)) - (6*(1 + a*x)^2)/(5*a*c^4*(1 - a
^2*x^2)^(3/2)) + (24*(1 + a*x))/(5*a*c^4*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*
x^2]/(a*c^4) - (3*ArcSin[a*x])/(a*c^4)
```

Rubi [A] time = 0.353745, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6131, 6128, 852, 1635, 641, 216}

$$\frac{(ax+1)^3}{5ac^4\left(1-a^2x^2\right)^{5/2}}-\frac{6(ax+1)^2}{5ac^4\left(1-a^2x^2\right)^{3/2}}+\frac{24(ax+1)}{5ac^4\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac^4}-\frac{3\sin^{-1}(ax)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a*x))^4),x]
```

```
[Out] (1 + a*x)^3/(5*a*c^4*(1 - a^2*x^2)^(5/2)) - (6*(1 + a*x)^2)/(5*a*c^4*(1 - a
^2*x^2)^(3/2)) + (24*(1 + a*x))/(5*a*c^4*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*
x^2]/(a*c^4) - (3*ArcSin[a*x])/(a*c^4)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
```

```
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx &= \frac{a^4\int \frac{e^{-\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}\\
&= \frac{a^4\int \frac{x^4}{(1-ax)^3\sqrt{1-a^2x^2}}\,dx}{c^4}\\
&= \frac{a^4\int \frac{x^4(1+ax)^3}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^4}\\
&= \frac{(1+ax)^3}{5ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{a^4\int \frac{(1+ax)^2\left(\frac{3}{a^4}+\frac{5x}{a^3}+\frac{5x^2}{a^2}+\frac{5x^3}{a}\right)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^4}\\
&= \frac{(1+ax)^3}{5ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{6(1+ax)^2}{5ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{a^4\int \frac{(1+ax)\left(\frac{27}{a^4}+\frac{30x}{a^3}+\frac{15x^2}{a^2}\right)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^4}\\
&= \frac{(1+ax)^3}{5ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{6(1+ax)^2}{5ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{24(1+ax)}{5ac^4\sqrt{1-a^2x^2}} - \frac{a^4\int \frac{\frac{45}{a^4}+\frac{15x}{a^3}}{\sqrt{1-a^2x^2}}\,dx}{15c^4}\\
&= \frac{(1+ax)^3}{5ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{6(1+ax)^2}{5ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{24(1+ax)}{5ac^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}}{ac^4} - \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^4}\\
&= \frac{(1+ax)^3}{5ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{6(1+ax)^2}{5ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{24(1+ax)}{5ac^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}}{ac^4} - \frac{3\sin^{-1}(ax)}{ac^4}
\end{aligned}
$$

Mathematica [A] time = 0.158374, size = 61, normalized size = 0.49

$$
\frac{\frac{\sqrt{1-a^2x^2}\left(5a^3x^3-39a^2x^2+57ax-24\right)}{(ax-1)^3}-15\sin^{-1}(ax)}{5ac^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a*x))^4),x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(-24 + 57*a*x - 39*a^2*x^2 + 5*a^3*x^3))/(-1 + a*x)^3 -
 15*ArcSin[a*x])/(5*a*c^4)
```

Maple [B] time = 0.053, size = 286, normalized size = 2.3

$$\frac{1}{10a^5c^4}\left(-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)\right)^{\frac{3}{2}}\left(x-a^{-1}\right)^{-4}+\frac{11}{20a^4c^4}\left(-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)\right)^{\frac{3}{2}}\left(x-a^{-1}\right)^{-3}+\frac{17}{8a^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^4,x)
```

```
[Out] 1/10/a^5/c^4/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+11/20/a^4/c^4/(x-
1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+17/8/a^3/c^4/(x-1/a)^2*(-a^2*(x-1
/a)^2-2*a*(x-1/a))^(3/2)+49/16/a/c^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-49/
16/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))
+1/16/a/c^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1/16/c^4/(a^2)^(1/2)*arctan(
(a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{ax}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a*x))^4), x)
```

Fricas [A] time = 2.10625, size = 317, normalized size = 2.54

$$\frac{24a^3x^3-72a^2x^2+72ax+30\left(a^3x^3-3a^2x^2+3ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(5a^3x^3-39a^2x^2+57ax-24\right)\sqrt{-a^2}}{5\left(a^4c^4x^3-3a^3c^4x^2+3a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(24*a^3*x^3 - 72*a^2*x^2 + 72*a*x + 30*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1
)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (5*a^3*x^3 - 39*a^2*x^2 + 57*a*x
 - 24)*sqrt(-a^2*x^2 + 1) - 24)/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x
- a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^4\int\frac{x^4\sqrt{-a^2x^2+1}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*Integral(x**4*sqrt(-a**2*x**2 + 1)/(a**5*x**5 - 3*a**4*x**4 + 2*a**3*x
**3 + 2*a**2*x**2 - 3*a*x + 1), x)/c**4
```

Giac [A] time = 1.18799, size = 243, normalized size = 1.94

$$-\frac{3\arcsin(ax)\operatorname{sgn}(a)}{c^4|a|}+\frac{\sqrt{-a^2x^2+1}}{ac^4}-\frac{2\left(\frac{80\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{120\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{70\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}-\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4}-\right.}{5c^4\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -3*arcsin(a*x)*sgn(a)/(c^4*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c^4) - 2/5*(80*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 120*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)^2/(a^4*x^2) + 70*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 15*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 19)/(c^4*((sqrt(-a^2*x^2 + 1)*abs(a)
 + a)/(a^2*x) - 1)^5*abs(a))
```

3.493 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=114

$$-\frac{\left(c-\frac{c}{ax}\right)^{p+2}\text{Hypergeometric2F1}\left(1,p+2,p+3,\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(p+2)}+\frac{\left(c-\frac{c}{ax}\right)^{p+2}\text{Hypergeometric2F1}\left(1,p+2,p+3,1-\frac{1}{ax}\right)}{ac^2}$$

```
[Out] -(((c - c/(a*x))^(2 + p)*x)/c^2) - ((c - c/(a*x))^(2 + p)*Hypergeometric2F1
[1, 2 + p, 3 + p, (a - x^(-1))/(2*a)])/(2*a*c^2*(2 + p)) + ((c - c/(a*x))^(
2 + p)*Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a*x)])/(a*c^2)
```

Rubi [A] time = 0.130092, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6133, 25, 514, 375, 103, 156, 65, 68}

$$-\frac{\left(c-\frac{c}{ax}\right)^{p+2}{}_2F_1\left(1,p+2;p+3;\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(p+2)}+\frac{\left(c-\frac{c}{ax}\right)^{p+2}{}_2F_1\left(1,p+2;p+3;1-\frac{1}{ax}\right)}{ac^2}-\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^p/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(((c - c/(a*x))^(2 + p)*x)/c^2) - ((c - c/(a*x))^(2 + p)*Hypergeometric2F1
[1, 2 + p, 3 + p, (a - x^(-1))/(2*a)])/(2*a*c^2*(2 + p)) + ((c - c/(a*x))^(
2 + p)*Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a*x)])/(a*c^2)
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx &= \int \frac{\left(c-\frac{c}{ax}\right)^p(1-ax)}{1+ax}\,dx\\
&= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{1+p}x}{1+ax}\,dx}{c}\\
&= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{1+p}}{a+\frac{1}{x}}\,dx}{c}\\
&= \frac{a\,\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}}{x^2(a+x)}\,dx,x,\frac{1}{x}\right)}{c}\\
&= -\frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}-\frac{\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}\left(c(2+p)+\frac{c(1+p)x}{a}\right)}{x(a+x)}\,dx,x,\frac{1}{x}\right)}{c^2}\\
&= -\frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}+\frac{\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}}{a+x}\,dx,x,\frac{1}{x}\right)}{ac}-\frac{(2+p)\,\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}}{x}\,dx,x,\frac{1}{x}\right)}{ac}\\
&= -\frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}-\frac{\left(c-\frac{c}{ax}\right)^{2+p}{}_2F_1\left(1,2+p;3+p;\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(2+p)}+\frac{\left(c-\frac{c}{ax}\right)^{2+p}{}_2F_1\left(1,2+p;3+p;\right.}{ac^2}
\end{aligned}
$$

Mathematica [A] time = 0.0479664, size = 87, normalized size = 0.76

$$-\frac{(ax-1)^2\left(c-\frac{c}{ax}\right)^p\left(\text{Hypergeometric2F1}\left(1,p+2,p+3,\frac{a-\frac{1}{x}}{2a}\right)+2(p+2)\left(ax-\text{Hypergeometric2F1}\left(1,p+2,p+\dots\right.\right.\right.}{2a^3(p+2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^p/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -((c - c/(a*x))^p*(-1 + a*x)^2*(Hypergeometric2F1[1, 2 + p, 3 + p, (a - x^(
-1))/(2*a)] + 2*(2 + p)*(a*x - Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a*
x)])))/(2*a^3*(2 + p)*x^2)
```

Maple [F] time = 0.233, size = 0, normalized size = 0.

$$\int\frac{-a^2x^2+1}{(ax+1)^2}\left(c-\frac{c}{ax}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^p/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] int((c-c/a/x)^p/(a*x+1)^2*(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(a^2x^2-1)\left(c-\frac{c}{ax}\right)^p}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a*x))^p/(a*x + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ax-1)\left(\frac{acx-c}{ax}\right)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-(a*x - 1)*((a*c*x - c)/(a*x))^p/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\left(c-\frac{c}{ax}\right)^{p}}{ax+1}\,dx-\int \frac{ax\left(c-\frac{c}{ax}\right)^{p}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**p/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-(c - c/(a*x))**p/(a*x + 1), x) - Integral(a*x*(c - c/(a*x))**p/(
a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(a^{2}x^{2}-1\right)\left(c-\frac{c}{ax}\right)^{p}}{(ax+1)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*(c - c/(a*x))^p/(a*x + 1)^2, x)
```

3.494 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=66

$$\frac{3c^4}{a^3x^2}-\frac{c^4}{3a^4x^3}-\frac{16c^4}{a^2x}-\frac{26c^4\log(x)}{a}+\frac{32c^4\log(ax+1)}{a}+c^4(-x)$$

```
[Out] -c^4/(3*a^4*x^3) + (3*c^4)/(a^3*x^2) - (16*c^4)/(a^2*x) - c^4*x - (26*c^4*L
og[x])/a + (32*c^4*Log[1 + a*x])/a
```

Rubi [A] time = 0.117514, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$\frac{3c^4}{a^3x^2}-\frac{c^4}{3a^4x^3}-\frac{16c^4}{a^2x}-\frac{26c^4\log(x)}{a}+\frac{32c^4\log(ax+1)}{a}+c^4(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^4/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -c^4/(3*a^4*x^3) + (3*c^4)/(a^3*x^2) - (16*c^4)/(a^2*x) - c^4*x - (26*c^4*L
og[x])/a + (32*c^4*Log[1 + a*x])/a
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)^4}{x^4}dx}{a^4}$$
$$= \frac{c^4\int \frac{(1-ax)^5}{x^4(1+ax)}dx}{a^4}$$
$$= \frac{c^4\int \left(-a^4+\frac{1}{x^4}-\frac{6a}{x^3}+\frac{16a^2}{x^2}-\frac{26a^3}{x}+\frac{32a^4}{1+ax}\right)dx}{a^4}$$
$$= -\frac{c^4}{3a^4x^3}+\frac{3c^4}{a^3x^2}-\frac{16c^4}{a^2x}-c^4x-\frac{26c^4\log(x)}{a}+\frac{32c^4\log(1+ax)}{a}$$

Mathematica [A] time = 0.164862, size = 68, normalized size = 1.03

$$\frac{3c^4}{a^3x^2}-\frac{c^4}{3a^4x^3}-\frac{16c^4}{a^2x}-\frac{26c^4\log(ax)}{a}+\frac{32c^4\log(ax+1)}{a}+c^4(-x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^4/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -c^4/(3*a^4*x^3) + (3*c^4)/(a^3*x^2) - (16*c^4)/(a^2*x) - c^4*x - (26*c^4*L
og[a*x])/a + (32*c^4*Log[1 + a*x])/a
```

Maple [A] time = 0.039, size = 65, normalized size = 1.

$$-\frac{c^4}{3\,a^4x^3}+3\,\frac{c^4}{x^2a^3}-16\,\frac{c^4}{a^2x}-c^4x-26\,\frac{c^4\ln(x)}{a}+32\,\frac{c^4\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^4/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/3*c^4/a^4/x^3+3*c^4/x^2/a^3-16*c^4/a^2/x-c^4*x-26*c^4*ln(x)/a+32*c^4*ln(
a*x+1)/a
```

Maxima [A] time = 0.957922, size = 82, normalized size = 1.24

$$-c^4x+\frac{32\,c^4\log(ax+1)}{a}-\frac{26\,c^4\log(x)}{a}-\frac{48\,a^2c^4x^2-9\,ac^4x+c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -c^4*x + 32*c^4*log(a*x + 1)/a - 26*c^4*log(x)/a - 1/3*(48*a^2*c^4*x^2 - 9*
a*c^4*x + c^4)/(a^4*x^3)
```

Fricas [A] time = 2.17741, size = 163, normalized size = 2.47

$$-\frac{3\,a^4c^4x^4-96\,a^3c^4x^3\log(ax+1)+78\,a^3c^4x^3\log(x)+48\,a^2c^4x^2-9\,ac^4x+c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^4*c^4*x^4 - 96*a^3*c^4*x^3*log(a*x + 1) + 78*a^3*c^4*x^3*log(x) +
 48*a^2*c^4*x^2 - 9*a*c^4*x + c^4)/(a^4*x^3)
```

Sympy [A] time = 0.726072, size = 58, normalized size = 0.88

$$-c^4x-\frac{2c^4\left(13\log(x)-16\log\left(x+\frac{1}{a}\right)\right)}{a}-\frac{48a^2c^4x^2-9ac^4x+c^4}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**4/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -c**4*x - 2*c**4*(13*log(x) - 16*log(x + 1/a))/a - (48*a**2*c**4*x**2 - 9*a
*c**4*x + c**4)/(3*a**4*x**3)
```

Giac [A] time = 1.13984, size = 151, normalized size = 2.29

$$-\frac{6\,c^4\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}-\frac{26\,c^4\log\left(\left|-\frac{1}{ax+1}+1\right|\right)}{a}+\frac{\left(3\,c^4+\frac{49\,c^4}{ax+1}-\frac{117\,c^4}{(ax+1)^2}+\frac{66\,c^4}{(ax+1)^3}\right)(ax+1)}{3\,a\left(\frac{1}{ax+1}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -6*c^4*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a - 26*c^4*log(abs(-1/(a*x +
1) + 1))/a + 1/3*(3*c^4 + 49*c^4/(a*x + 1) - 117*c^4/(a*x + 1)^2 + 66*c^4/(
a*x + 1)^3)*(a*x + 1)/(a*(1/(a*x + 1) - 1)^3)
```

3.495 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$

Optimal. Leaf size=55

$$\frac{c^3}{2a^3x^2}-\frac{5c^3}{a^2x}-\frac{11c^3\log(x)}{a}+\frac{16c^3\log(ax+1)}{a}+c^3(-x)$$

```
[Out] c^3/(2*a^3*x^2) - (5*c^3)/(a^2*x) - c^3*x - (11*c^3*Log[x])/a + (16*c^3*Log
[1 + a*x])/a
```

Rubi [A] time = 0.116294, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$\frac{c^3}{2a^3x^2}-\frac{5c^3}{a^2x}-\frac{11c^3\log(x)}{a}+\frac{16c^3\log(ax+1)}{a}+c^3(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^3/E^(2*ArcTanh[a*x]),x]
```


```
[Out] c^3/(2*a^3*x^2) - (5*c^3)/(a^2*x) - c^3*x - (11*c^3*Log[x])/a + (16*c^3*Log
[1 + a*x])/a
```


Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{c^3\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)^3}{x^3}dx}{a^3}$$
$$= -\frac{c^3\int \frac{(1-ax)^4}{x^3(1+ax)}dx}{a^3}$$
$$= -\frac{c^3\int \left(a^3+\frac{1}{x^3}-\frac{5a}{x^2}+\frac{11a^2}{x}-\frac{16a^3}{1+ax}\right)dx}{a^3}$$
$$= \frac{c^3}{2a^3x^2}-\frac{5c^3}{a^2x}-c^3x-\frac{11c^3\log(x)}{a}+\frac{16c^3\log(1+ax)}{a}$$

Mathematica [A] time = 0.124412, size = 57, normalized size = 1.04

$$\frac{c^3}{2a^3x^2}-\frac{5c^3}{a^2x}-\frac{11c^3\log(ax)}{a}+\frac{16c^3\log(ax+1)}{a}+c^3(-x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] c^3/(2*a^3*x^2) - (5*c^3)/(a^2*x) - c^3*x - (11*c^3*Log[a*x])/a + (16*c^3*L
og[1 + a*x])/a
```

Maple [A] time = 0.037, size = 54, normalized size = 1.

$$\frac{c^3}{2x^2a^3}-5\frac{c^3}{a^2x}-c^3x-11\frac{c^3\ln(x)}{a}+16\frac{c^3\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^3/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/2*c^3/x^2/a^3-5*c^3/a^2/x-c^3*x-11*c^3*ln(x)/a+16*c^3*ln(a*x+1)/a
```

Maxima [A] time = 0.966151, size = 70, normalized size = 1.27

$$-c^3x+\frac{16c^3\log(ax+1)}{a}-\frac{11c^3\log(x)}{a}-\frac{10ac^3x-c^3}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -c^3*x + 16*c^3*log(a*x + 1)/a - 11*c^3*log(x)/a - 1/2*(10*a*c^3*x - c^3)/(
a^3*x^2)
```

Fricas [A] time = 2.22177, size = 142, normalized size = 2.58

$$-\frac{2a^3c^3x^3-32a^2c^3x^2\log(ax+1)+22a^2c^3x^2\log(x)+10ac^3x-c^3}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*c^3*x^3 - 32*a^2*c^3*x^2*log(a*x + 1) + 22*a^2*c^3*x^2*log(x) +
 10*a*c^3*x - c^3)/(a^3*x^2)
```

Sympy [A] time = 0.629576, size = 44, normalized size = 0.8

$$-c^3x - \frac{c^3\left(11\log\left(x\right) - 16\log\left(x + \frac{1}{a}\right)\right)}{a} - \frac{10ac^3x - c^3}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**3/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -c**3*x - c**3*(11*log(x) - 16*log(x + 1/a))/a - (10*a*c**3*x - c**3)/(2*a*
*3*x**2)
```

Giac [A] time = 1.16233, size = 135, normalized size = 2.45

$$-\frac{5\,c^3\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a} - \frac{11\,c^3\log\left(\left|-\frac{1}{ax+1} + 1\right|\right)}{a} - \frac{\left(2\,c^3 + \frac{7\,c^3}{ax+1} - \frac{10\,c^3}{(ax+1)^2}\right)(ax+1)}{2\,a\left(\frac{1}{ax+1} - 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -5*c^3*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a - 11*c^3*log(abs(-1/(a*x +
1) + 1))/a - 1/2*(2*c^3 + 7*c^3/(a*x + 1) - 10*c^3/(a*x + 1)^2)*(a*x + 1)/(
a*(1/(a*x + 1) - 1)^2)
```

3.496 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=42

$$-\frac{c^2}{a^2x}-\frac{4c^2\log(x)}{a}+\frac{8c^2\log(ax+1)}{a}+c^2(-x)$$

```
[Out] -(c^2/(a^2*x)) - c^2*x - (4*c^2*Log[x])/a + (8*c^2*Log[1 + a*x])/a
```

Rubi [A] time = 0.110625, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$-\frac{c^2}{a^2x}-\frac{4c^2\log(x)}{a}+\frac{8c^2\log(ax+1)}{a}+c^2(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^2/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c^2/(a^2*x)) - c^2*x - (4*c^2*Log[x])/a + (8*c^2*Log[1 + a*x])/a
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx &= \frac{c^2\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)^2}{x^2}\,dx}{a^2}\\
&= \frac{c^2\int \frac{(1-ax)^3}{x^2(1+ax)}\,dx}{a^2}\\
&= \frac{c^2\int \left(-a^2+\frac{1}{x^2}-\frac{4a}{x}+\frac{8a^2}{1+ax}\right)dx}{a^2}\\
&= -\frac{c^2}{a^2x}-c^2x-\frac{4c^2\log(x)}{a}+\frac{8c^2\log(1+ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0974576, size = 44, normalized size = 1.05

$$-\frac{c^2}{a^2x}-\frac{4c^2\log(ax)}{a}+\frac{8c^2\log(ax+1)}{a}+c^2(-x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^2/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c^2/(a^2*x)) - c^2*x - (4*c^2*Log[a*x])/a + (8*c^2*Log[1 + a*x])/a
```

Maple [A] time = 0.036, size = 43, normalized size = 1.

$$-\frac{c^2}{a^2x}-xc^2-4\,\frac{c^2\ln(x)}{a}+8\,\frac{c^2\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^2/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -c^2/a^2/x-x*c^2-4*c^2*ln(x)/a+8*c^2*ln(a*x+1)/a
```

Maxima [A] time = 0.959153, size = 57, normalized size = 1.36

$$-c^2x+\frac{8\,c^2\log(ax+1)}{a}-\frac{4\,c^2\log(x)}{a}-\frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -c^2*x + 8*c^2*log(a*x + 1)/a - 4*c^2*log(x)/a - c^2/(a^2*x)
```

Fricas [A] time = 2.24187, size = 100, normalized size = 2.38

$$-\frac{a^2c^2x^2-8\,ac^2x\log(ax+1)+4\,ac^2x\log(x)+c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -(a^2*c^2*x^2 - 8*a*c^2*x*log(a*x + 1) + 4*a*c^2*x*log(x) + c^2)/(a^2*x)
```

Sympy [A] time = 0.522319, size = 32, normalized size = 0.76

$$-c^2x-\frac{4c^2\left(\log(x)-2\log\left(x+\frac{1}{a}\right)\right)}{a}-\frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**2/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -c**2*x - 4*c**2*(log(x) - 2*log(x + 1/a))/a - c**2/(a**2*x)
```

Giac [A] time = 1.20323, size = 97, normalized size = 2.31

$$-\frac{4\,c^2\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}-\frac{4\,c^2\log\left(\left|-\frac{1}{ax+1}+1\right|\right)}{a}+\frac{(ax+1)c^2}{a\left(\frac{1}{ax+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -4*c^2*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a - 4*c^2*log(abs(-1/(a*x + 1
) + 1))/a + (a*x + 1)*c^2/(a*(1/(a*x + 1) - 1))
```

3.497 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=25

$$-\frac{c\log(x)}{a}+\frac{4c\log(ax+1)}{a}-cx$$

```
[Out] -(c*x) - (c*Log[x])/a + (4*c*Log[1 + a*x])/a
```

Rubi [A] time = 0.0656118, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6131, 6129, 72}

$$-\frac{c\log(x)}{a}+\frac{4c\log(ax+1)}{a}-cx$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c*x) - (c*Log[x])/a + (4*c*Log[1 + a*x])/a
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= -\frac{c\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}\\ &= -\frac{c\int\frac{(1-ax)^2}{x(1+ax)}\,dx}{a}\\ &= -\frac{c\int\left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{a}\\ &= -cx-\frac{c\log(x)}{a}+\frac{4c\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0364962, size = 25, normalized size = 1.

$$-\frac{c\log(x)}{a}+\frac{4c\log(ax+1)}{a}-cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c*x) - (c*Log[x])/a + (4*c*Log[1 + a*x])/a
```

Maple [A] time = 0.034, size = 26, normalized size = 1.

$$-cx-\frac{c\ln(x)}{a}+4\,\frac{c\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -c*x-c*ln(x)/a+4*c*ln(a*x+1)/a
```

Maxima [A] time = 0.960578, size = 34, normalized size = 1.36

$$-cx+\frac{4\,c\log(ax+1)}{a}-\frac{c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -c*x + 4*c*log(a*x + 1)/a - c*log(x)/a
```

Fricas [A] time = 2.29126, size = 57, normalized size = 2.28

$$-\frac{acx-4\,c\log(ax+1)+c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -(a*c*x - 4*c*log(a*x + 1) + c*log(x))/a
```

Sympy [A] time = 0.426521, size = 19, normalized size = 0.76

$$-cx-\frac{c\left(\log(x)-4\log\left(x+\frac{1}{a}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -c*x - c*(log(x) - 4*log(x + 1/a))/a
```

Giac [B] time = 1.18673, size = 76, normalized size = 3.04

$$-\frac{(ax+1)c}{a} - \frac{3\,c\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a} - \frac{c\log\left(\left|-\frac{1}{ax+1}+1\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -(a*x + 1)*c/a - 3*c*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a - c*log(abs(-
1/(a*x + 1) + 1))/a
```

3.498 $\int \frac{e^{-2\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=20

$$\frac{\log(ax+1)}{ac}-\frac{x}{c}$$

```
[Out] -(x/c) + Log[1 + a*x]/(a*c)
```

Rubi [A] time = 0.0875015, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 43}

$$\frac{\log(ax+1)}{ac}-\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))),x]
```

```
[Out] -(x/c) + Log[1 + a*x]/(a*c)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx &= -\frac{a\int \frac{e^{-2\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}\\
&= -\frac{a\int \frac{x}{1+ax}\,dx}{c}\\
&= -\frac{a\int \left(\frac{1}{a}-\frac{1}{a(1+ax)}\right)dx}{c}\\
&= -\frac{x}{c}+\frac{\log(1+ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.0152997, size = 18, normalized size = 0.9

$$\frac{\log(ax+1)-ax}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))),x]
```

```
[Out] (-(a*x) + Log[1 + a*x])/(a*c)
```

Maple [A] time = 0.028, size = 21, normalized size = 1.1

$$-\frac{x}{c}+\frac{\ln(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x),x)
```

```
[Out] -x/c+ln(a*x+1)/a/c
```

Maxima [A] time = 0.940878, size = 27, normalized size = 1.35

$$-\frac{x}{c}+\frac{\log(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -x/c + log(a*x + 1)/(a*c)
```

Fricas [A] time = 2.20964, size = 39, normalized size = 1.95

$$-\frac{ax-\log(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] -(a*x - log(a*x + 1))/(a*c)
```

Sympy [A] time = 0.30708, size = 19, normalized size = 0.95

$$-a\left(\frac{x}{ac}-\frac{\log(ax+1)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x),x)
```

```
[Out] -a*(x/(a*c) - log(a*x + 1)/(a**2*c))
```

Giac [B] time = 1.25834, size = 55, normalized size = 2.75

$$-\frac{ax+1}{ac}-\frac{\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] -(a*x + 1)/(a*c) - log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/(a*c)
```

3.499 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal. Leaf size=18

$$\frac{\tanh^{-1}(ax)}{ac^2} - \frac{x}{c^2}$$

```
[Out] -(x/c^2) + ArcTanh[a*x]/(a*c^2)
```

Rubi [A] time = 0.109727, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6131, 6129, 72, 207}

$$\frac{\tanh^{-1}(ax)}{ac^2} - \frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] -(x/c^2) + ArcTanh[a*x]/(a*c^2)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{e^{-2\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}$$
$$= \frac{a^2\int \frac{x^2}{(1-ax)(1+ax)}\,dx}{c^2}$$
$$= \frac{a^2\int \left(-\frac{1}{a^2}-\frac{1}{a^2(-1+a^2x^2)}\right)dx}{c^2}$$
$$= -\frac{x}{c^2}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{c^2}$$
$$= -\frac{x}{c^2}+\frac{\tanh^{-1}(ax)}{ac^2}$$

Mathematica [B] time = 0.0728216, size = 40, normalized size = 2.22

$$-\frac{\log(1-ax)}{2ac^2}+\frac{\log(ax+1)}{2ac^2}-\frac{x}{c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] -(x/c^2) - Log[1 - a*x]/(2*a*c^2) + Log[1 + a*x]/(2*a*c^2)
```

Maple [A] time = 0.036, size = 36, normalized size = 2.

$$-\frac{x}{c^2}+\frac{\ln(ax+1)}{2\,ac^2}-\frac{\ln(ax-1)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^2,x)
```

```
[Out] -x/c^2+1/2*ln(a*x+1)/a/c^2-1/2/c^2/a*ln(a*x-1)
```

Maxima [A] time = 0.964698, size = 47, normalized size = 2.61

$$-\frac{x}{c^2}+\frac{\log(ax+1)}{2\,ac^2}-\frac{\log(ax-1)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -x/c^2 + 1/2*log(a*x + 1)/(a*c^2) - 1/2*log(a*x - 1)/(a*c^2)
```

Fricas [A] time = 2.20241, size = 70, normalized size = 3.89

$$-\frac{2\,ax-\log(ax+1)+\log(ax-1)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x - log(a*x + 1) + log(a*x - 1))/(a*c^2)
```

Sympy [B] time = 0.353467, size = 36, normalized size = 2.

$$-a^2\left(\frac{x}{a^2c^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{2}-\frac{\log\left(x+\frac{1}{a}\right)}{2}}{a^3c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x)**2,x)
```

```
[Out] -a**2*(x/(a**2*c**2) + (log(x - 1/a)/2 - log(x + 1/a)/2)/(a**3*c**2))
```

Giac [A] time = 1.21909, size = 47, normalized size = 2.61

$$-\frac{ax+1}{ac^2}-\frac{\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -(a*x + 1)/(a*c^2) - 1/2*log(abs(-2/(a*x + 1) + 1))/(a*c^2)
```

3.500 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=58

$$-\frac{1}{2ac^3(1-ax)}-\frac{5\log(1-ax)}{4ac^3}+\frac{\log(ax+1)}{4ac^3}-\frac{x}{c^3}$$

```
[Out] -(x/c^3) - 1/(2*a*c^3*(1 - a*x)) - (5*Log[1 - a*x])/(4*a*c^3) + Log[1 + a*x
]/(4*a*c^3)
```

Rubi [A] time = 0.124837, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$-\frac{1}{2ac^3(1-ax)}-\frac{5\log(1-ax)}{4ac^3}+\frac{\log(ax+1)}{4ac^3}-\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] -(x/c^3) - 1/(2*a*c^3*(1 - a*x)) - (5*Log[1 - a*x])/(4*a*c^3) + Log[1 + a*x
]/(4*a*c^3)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{a^3\int \frac{e^{-2\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}$$
$$= -\frac{a^3\int \frac{x^3}{(1-ax)^2(1+ax)}\,dx}{c^3}$$
$$= -\frac{a^3\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{c^3}$$
$$= -\frac{x}{c^3}-\frac{1}{2ac^3(1-ax)}-\frac{5\log(1-ax)}{4ac^3}+\frac{\log(1+ax)}{4ac^3}$$

Mathematica [A] time = 0.107489, size = 57, normalized size = 0.98

$$\frac{1}{2ac^3(ax-1)}-\frac{5\log(1-ax)}{4ac^3}+\frac{\log(ax+1)}{4ac^3}-\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] -(x/c^3) + 1/(2*a*c^3*(-1 + a*x)) - (5*Log[1 - a*x])/(4*a*c^3) + Log[1 + a*
x]/(4*a*c^3)
```

Maple [A] time = 0.038, size = 51, normalized size = 0.9

$$-\frac{x}{c^3}+\frac{\ln(ax+1)}{4\,ac^3}+\frac{1}{2\,ac^3\,(ax-1)}-\frac{5\,\ln(ax-1)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^3,x)
```

```
[Out] -x/c^3+1/4*ln(a*x+1)/a/c^3+1/2/a/c^3/(a*x-1)-5/4/a/c^3*ln(a*x-1)
```

Maxima [A] time = 0.956659, size = 73, normalized size = 1.26

$$\frac{1}{2\left(a^2c^3x-ac^3\right)}-\frac{x}{c^3}+\frac{\log(ax+1)}{4\,ac^3}-\frac{5\,\log(ax-1)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] 1/2/(a^2*c^3*x - a*c^3) - x/c^3 + 1/4*log(a*x + 1)/(a*c^3) - 5/4*log(a*x -
1)/(a*c^3)
```

Fricas [A] time = 2.21859, size = 138, normalized size = 2.38

$$-\frac{4\,a^2x^2-4\,ax-(ax-1)\log(ax+1)+5\,(ax-1)\log(ax-1)-2}{4\left(a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
 - 2)/(a^2*c^3*x - a*c^3)
```

Sympy [A] time = 0.556586, size = 58, normalized size = 1.

$$-a^{3}\left(-\frac{1}{2a^{5}c^{3}x-2a^{4}c^{3}}+\frac{x}{a^{3}c^{3}}+\frac{\frac{5\log\left(x-\frac{1}{a}\right)}{4}-\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^{4}c^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x)**3,x)
```

```
[Out] -a**3*(-1/(2*a**5*c**3*x - 2*a**4*c**3) + x/(a**3*c**3) + (5*log(x - 1/a)/4
 - log(x + 1/a)/4)/(a**4*c**3))
```

Giac [A] time = 1.23157, size = 115, normalized size = 1.98

$$-\frac{(ax+1)\left(\frac{9}{ax+1}-4\right)}{4\,ac^{3}\left(\frac{2}{ax+1}-1\right)}+\frac{\log\left(\frac{|ax+1|}{(ax+1)^{2}|a|}\right)}{ac^{3}}-\frac{5\,\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{4\,ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a*x + 1)*(9/(a*x + 1) - 4)/(a*c^3*(2/(a*x + 1) - 1)) + log(abs(a*x +
1)/((a*x + 1)^2*abs(a)))/(a*c^3) - 5/4*log(abs(-2/(a*x + 1) + 1))/(a*c^3)
```

3.501 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=76

$$-\frac{7}{4ac^4(1-ax)}+\frac{1}{4ac^4(1-ax)^2}-\frac{17\log(1-ax)}{8ac^4}+\frac{\log(ax+1)}{8ac^4}-\frac{x}{c^4}$$

```
[Out] -(x/c^4) + 1/(4*a*c^4*(1 - a*x)^2) - 7/(4*a*c^4*(1 - a*x)) - (17*Log[1 - a*
x])/(8*a*c^4) + Log[1 + a*x]/(8*a*c^4)
```

Rubi [A] time = 0.132721, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$-\frac{7}{4ac^4(1-ax)}+\frac{1}{4ac^4(1-ax)^2}-\frac{17\log(1-ax)}{8ac^4}+\frac{\log(ax+1)}{8ac^4}-\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^4),x]
```


```
[Out] -(x/c^4) + 1/(4*a*c^4*(1 - a*x)^2) - 7/(4*a*c^4*(1 - a*x)) - (17*Log[1 - a*
x])/(8*a*c^4) + Log[1 + a*x]/(8*a*c^4)
```


Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\int \frac{e^{-2\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}$$
$$= \frac{a^4\int \frac{x^4}{(1-ax)^3(1+ax)}\,dx}{c^4}$$
$$= \frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{2a^4(-1+ax)^3}-\frac{7}{4a^4(-1+ax)^2}-\frac{17}{8a^4(-1+ax)}+\frac{1}{8a^4(1+ax)}\right)dx}{c^4}$$
$$= -\frac{x}{c^4}+\frac{1}{4ac^4(1-ax)^2}-\frac{7}{4ac^4(1-ax)}-\frac{17\log(1-ax)}{8ac^4}+\frac{\log(1+ax)}{8ac^4}$$

Mathematica [A] time = 0.138221, size = 69, normalized size = 0.91

$$\frac{-8a^3x^3+16a^2x^2+6ax-17(ax-1)^2\log(1-ax)+(ax-1)^2\log(ax+1)-12}{8ac^4(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] (-12 + 6*a*x + 16*a^2*x^2 - 8*a^3*x^3 - 17*(-1 + a*x)^2*Log[1 - a*x] + (-1
+ a*x)^2*Log[1 + a*x])/(8*a*c^4*(-1 + a*x)^2)
```

Maple [A] time = 0.037, size = 66, normalized size = 0.9

$$-\frac{x}{c^4}+\frac{\ln(ax+1)}{8ac^4}+\frac{1}{4ac^4(ax-1)^2}+\frac{7}{4ac^4(ax-1)}-\frac{17\ln(ax-1)}{8ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^4,x)
```

```
[Out] -x/c^4+1/8*ln(a*x+1)/a/c^4+1/4/c^4/a/(a*x-1)^2+7/4/c^4/a/(a*x-1)-17/8/c^4/a
*ln(a*x-1)
```

Maxima [A] time = 0.948014, size = 95, normalized size = 1.25

$$\frac{7ax-6}{4\left(a^3c^4x^2-2a^2c^4x+ac^4\right)}-\frac{x}{c^4}+\frac{\log(ax+1)}{8ac^4}-\frac{17\log(ax-1)}{8ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/4*(7*a*x - 6)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4) - x/c^4 + 1/8*log(a*x +
 1)/(a*c^4) - 17/8*log(a*x - 1)/(a*c^4)
```

Fricas [A] time = 2.28046, size = 212, normalized size = 2.79

$$-\frac{8\,a^3x^3-16\,a^2x^2-6\,ax-\left(a^2x^2-2\,ax+1\right)\log\left(ax+1\right)+17\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+12}{8\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
 17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a
*c^4)
```

Sympy [A] time = 0.675496, size = 75, normalized size = 0.99

$$-a^4\left(-\frac{7ax-6}{4a^7c^4x^2-8a^6c^4x+4a^5c^4}+\frac{x}{a^4c^4}+\frac{\frac{17\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x)**4,x)
```

```
[Out] -a**4*(-(7*a*x - 6)/(4*a**7*c**4*x**2 - 8*a**6*c**4*x + 4*a**5*c**4) + x/(a
**4*c**4) + (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**4))
```

Giac [A] time = 1.16696, size = 128, normalized size = 1.68

$$\frac{2\,\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{ac^4}-\frac{17\,\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{8\,ac^4}+\frac{(ax+1)\left(\frac{77}{ax+1}-\frac{88}{(ax+1)^2}-16\right)}{16\,ac^4\left(\frac{2}{ax+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] 2*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/(a*c^4) - 17/8*log(abs(-2/(a*x + 1
) + 1))/(a*c^4) + 1/16*(a*x + 1)*(77/(a*x + 1) - 88/(a*x + 1)^2 - 16)/(a*c^
4*(2/(a*x + 1) - 1)^2)
```

3.502 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=140

$$-\frac{c^3\sqrt{1-a^2x^2}}{a}-\frac{6c^3\sqrt{1-a^2x^2}}{a^2x}+\frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}}+\frac{33c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{6c^3\sin^{-1}(ax)}{a}$$

```
[Out] (-32*c^3*(1 - a*x))/(a*Sqrt[1 - a^2*x^2]) - (c^3*Sqrt[1 - a^2*x^2])/a + (c^
3*Sqrt[1 - a^2*x^2])/(2*a^3*x^2) - (6*c^3*Sqrt[1 - a^2*x^2])/(a^2*x) - (6*c
^3*ArcSin[a*x])/a + (33*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rubi [A] time = 0.367923, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6131, 6128, 1805, 1807, 1809, 844, 216, 266, 63, 208}

$$-\frac{c^3\sqrt{1-a^2x^2}}{a}-\frac{6c^3\sqrt{1-a^2x^2}}{a^2x}+\frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}}+\frac{33c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{6c^3\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^3/E^(3*ArcTanh[a*x]),x]
```


```
[Out] (-32*c^3*(1 - a*x))/(a*Sqrt[1 - a^2*x^2]) - (c^3*Sqrt[1 - a^2*x^2])/a + (c^
3*Sqrt[1 - a^2*x^2])/(2*a^3*x^2) - (6*c^3*Sqrt[1 - a^2*x^2])/(a^2*x) - (6*c
^3*ArcSin[a*x])/a + (33*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```


Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
```

```
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{c^3\int \frac{e^{-3\tanh^{-1}(ax)}(1-ax)^3}{x^3}dx}{a^3}$$

$$= -\frac{c^3\int \frac{(1-ax)^6}{x^3(1-a^2x^2)^{3/2}}dx}{a^3}$$

$$= -\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}} + \frac{c^3\int \frac{-1+6ax-16a^2x^2-6a^3x^3+a^4x^4}{x^3\sqrt{1-a^2x^2}}dx}{a^3}$$

$$= -\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{c^3\int \frac{-12a+33a^2x+12a^3x^2-2a^4x^3}{x^2\sqrt{1-a^2x^2}}dx}{2a^3}$$

$$= -\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{6c^3\sqrt{1-a^2x^2}}{a^2x} + \frac{c^3\int \frac{-33a^2-12a^3x+2a^4x^2}{x\sqrt{1-a^2x^2}}dx}{2a^3}$$

$$= -\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^3\sqrt{1-a^2x^2}}{a} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{6c^3\sqrt{1-a^2x^2}}{a^2x} - \frac{c^3\int \frac{33a^4+12a^5x}{x\sqrt{1-a^2x^2}}dx}{2a^5}$$

$$= -\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^3\sqrt{1-a^2x^2}}{a} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{6c^3\sqrt{1-a^2x^2}}{a^2x} - \left(6c^3\right)\int \frac{1}{\sqrt{1-a^2x^2}}dx$$

$$= -\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^3\sqrt{1-a^2x^2}}{a} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{6c^3\sqrt{1-a^2x^2}}{a^2x} - \frac{6c^3\sin^{-1}(ax)}{a} - \frac{(33c^3}{}$$

$$= -\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^3\sqrt{1-a^2x^2}}{a} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{6c^3\sqrt{1-a^2x^2}}{a^2x} - \frac{6c^3\sin^{-1}(ax)}{a} + \frac{(33c^3}{}$$

$$= -\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^3\sqrt{1-a^2x^2}}{a} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{6c^3\sqrt{1-a^2x^2}}{a^2x} - \frac{6c^3\sin^{-1}(ax)}{a} + \frac{33c^3}{}$$

Mathematica [A] time = 0.281584, size = 93, normalized size = 0.66

$$\frac{c^3\left(-\frac{\sqrt{1-a^2x^2}(2a^3x^3+78a^2x^2+11ax-1)}{a^2x^2(ax+1)} + 33\log\left(\sqrt{1-a^2x^2}+1\right) - 33\log(ax) - 12\sin^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^3*(-((Sqrt[1 - a^2*x^2]*(-1 + 11*a*x + 78*a^2*x^2 + 2*a^3*x^3))/(a^2*x^2
*(1 + a*x))) - 12*ArcSin[a*x] - 33*Log[a*x] + 33*Log[1 + Sqrt[1 - a^2*x^2]]
))/(2*a)
```

Maple [B] time = 0.063, size = 352, normalized size = 2.5

$$-8\frac{c^3\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)\right)^{5/2}}{a^4\left(x+a^{-1}\right)^3} - 4\frac{c^3\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)\right)^{5/2}}{a^3\left(x+a^{-1}\right)^2} + 2\frac{c^3\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -8*c^3/a^4/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-4*c^3/a^3/(x+1/a)^2
*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)+2*c^3/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3
/2)+3*c^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x+3*c^3/(a^2)^(1/2)*arctan((a^
2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-6*c^3/a^2/x*(-a^2*x^2+1)^(5/
2)-6*c^3*x*(-a^2*x^2+1)^(3/2)-9*c^3*x*(-a^2*x^2+1)^(1/2)-9*c^3/(a^2)^(1/2)*
arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-11/2*c^3*(-a^2*x^2+1)^(3/2)/a-33/2
*c^3*(-a^2*x^2+1)^(1/2)/a+33/2*c^3/a*arctanh(1/(-a^2*x^2+1)^(1/2))+1/2*c^3/
a^3/x^2*(-a^2*x^2+1)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^3}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^3/(a*x + 1)^3, x)
```

Fricas [A] time = 2.51443, size = 365, normalized size = 2.61

$$-\frac{66\,a^3c^3x^3+66\,a^2c^3x^2-24\left(a^3c^3x^3+a^2c^3x^2\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+33\left(a^3c^3x^3+a^2c^3x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(2\,a^3c^3x^3\right.}{2\left(a^4x^3+a^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(66*a^3*c^3*x^3 + 66*a^2*c^3*x^2 - 24*(a^3*c^3*x^3 + a^2*c^3*x^2)*arct
an((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 33*(a^3*c^3*x^3 + a^2*c^3*x^2)*log((sq
rt(-a^2*x^2 + 1) - 1)/x) + (2*a^3*c^3*x^3 + 78*a^2*c^3*x^2 + 11*a*c^3*x - c
^3)*sqrt(-a^2*x^2 + 1))/(a^4*x^3 + a^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^3\left(\int -\frac{\sqrt{-a^2x^2+1}}{a^3x^6+3a^2x^5+3ax^4+x^3}\,dx+\int \frac{3ax\sqrt{-a^2x^2+1}}{a^3x^6+3a^2x^5+3ax^4+x^3}\,dx+\int -\frac{2a^2x^2\sqrt{-a^2x^2+1}}{a^3x^6+3a^2x^5+3ax^4+x^3}\,dx+\int -\frac{2a^3x^3\sqrt{-a^2x^2+1}}{a^3x^6+3a^2x^5+3ax^4+x^3}\,dx+\int \frac{3a^4x^4\sqrt{-a^2}}{a^3x^6+3a^2x^5+}\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**3/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] c**3*(Integral(-sqrt(-a**2*x**2 + 1)/(a**3*x**6 + 3*a**2*x**5 + 3*a*x**4 +
x**3), x) + Integral(3*a*x*sqrt(-a**2*x**2 + 1)/(a**3*x**6 + 3*a**2*x**5 +
3*a*x**4 + x**3), x) + Integral(-2*a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**3*x**
```

```
6 + 3*a**2*x**5 + 3*a*x**4 + x**3), x) + Integral(-2*a**3*x**3*sqrt(-a**2*x
**2 + 1)/(a**3*x**6 + 3*a**2*x**5 + 3*a*x**4 + x**3), x) + Integral(3*a**4*
x**4*sqrt(-a**2*x**2 + 1)/(a**3*x**6 + 3*a**2*x**5 + 3*a*x**4 + x**3), x) +
 Integral(-a**5*x**5*sqrt(-a**2*x**2 + 1)/(a**3*x**6 + 3*a**2*x**5 + 3*a*x*
*4 + x**3), x))/a**3
```

Giac [B] time = 1.17329, size = 358, normalized size = 2.56

$$-\frac{6\,c^3 \arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{|a|}-\frac{\left(c^3-\frac{23\left(\sqrt{-a^2x^2+1}|a|+a\right)c^3}{a^2x}-\frac{536\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^3}{a^4x^2}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}|a|+a\right)^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}+\frac{33\,c^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{\sqrt{-a^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -6*c^3*arcsin(a*x)*sgn(a)/abs(a) - 1/8*(c^3 - 23*(sqrt(-a^2*x^2 + 1)*abs(a)
 + a)*c^3/(a^2*x) - 536*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3/(a^4*x^2))*a^
4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(
a^2*x) + 1)*abs(a)) + 33/2*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2
*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c^3/a - 1/8*(24*(sqrt(-a^2*x^
2 + 1)*abs(a) + a)*c^3*abs(a)/(a^2*x) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c
^3*abs(a)/(a^4*x^2))/a^2
```

3.503 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=111

$$-\frac{c^2\sqrt{1-a^2x^2}}{a}-\frac{c^2\sqrt{1-a^2x^2}}{a^2x}-\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}}+\frac{5c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{5c^2\sin^{-1}(ax)}{a}$$

```
[Out] (-16*c^2*(1 - a*x))/(a*Sqrt[1 - a^2*x^2]) - (c^2*Sqrt[1 - a^2*x^2])/a - (c^
2*Sqrt[1 - a^2*x^2])/(a^2*x) - (5*c^2*ArcSin[a*x])/a + (5*c^2*ArcTanh[Sqrt[
1 - a^2*x^2]])/a
```

Rubi [A] time = 0.295462, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6131, 6128, 1805, 1807, 1809, 844, 216, 266, 63, 208}

$$-\frac{c^2\sqrt{1-a^2x^2}}{a}-\frac{c^2\sqrt{1-a^2x^2}}{a^2x}-\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}}+\frac{5c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{5c^2\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^2/E^(3*ArcTanh[a*x]),x]
```


```
[Out] (-16*c^2*(1 - a*x))/(a*Sqrt[1 - a^2*x^2]) - (c^2*Sqrt[1 - a^2*x^2])/a - (c^
2*Sqrt[1 - a^2*x^2])/(a^2*x) - (5*c^2*ArcSin[a*x])/a + (5*c^2*ArcTanh[Sqrt[
1 - a^2*x^2]])/a
```


Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
```

```
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \frac{c^2\int \frac{e^{-3\tanh^{-1}(ax)}(1-ax)^2}{x^2}dx}{a^2}$$

$$= \frac{c^2\int \frac{(1-ax)^5}{x^2(1-a^2x^2)^{3/2}}dx}{a^2}$$

$$= -\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^2\int \frac{-1+5ax+5a^2x^2-a^3x^3}{x^2\sqrt{1-a^2x^2}}dx}{a^2}$$

$$= -\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} + \frac{c^2\int \frac{-5a-5a^2x+a^3x^2}{x\sqrt{1-a^2x^2}}dx}{a^2}$$

$$= -\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^2\sqrt{1-a^2x^2}}{a} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} - \frac{c^2\int \frac{5a^3+5a^4x}{x\sqrt{1-a^2x^2}}dx}{a^4}$$

$$= -\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^2\sqrt{1-a^2x^2}}{a} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} - (5c^2)\int \frac{1}{\sqrt{1-a^2x^2}}dx - \frac{(5c^2)\int \frac{1}{x\sqrt{1-a^2x^2}}}{a}$$

$$= -\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^2\sqrt{1-a^2x^2}}{a} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} - \frac{5c^2\sin^{-1}(ax)}{a} - \frac{(5c^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}dx\right.}{2a}$$

$$= -\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^2\sqrt{1-a^2x^2}}{a} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} - \frac{5c^2\sin^{-1}(ax)}{a} + \frac{(5c^2)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}dx,\right.}{a^3}$$

$$= -\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^2\sqrt{1-a^2x^2}}{a} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} - \frac{5c^2\sin^{-1}(ax)}{a} + \frac{5c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}$$

Mathematica [A] time = 0.216785, size = 81, normalized size = 0.73

$$\frac{c^2\left(-\frac{\sqrt{1-a^2x^2}(a^2x^2+18ax+1)}{ax(ax+1)} + 5\log\left(\sqrt{1-a^2x^2}+1\right) - 5\log(ax) - 5\sin^{-1}(ax)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^2*(-((Sqrt[1 - a^2*x^2]*(1 + 18*a*x + a^2*x^2))/(a*x*(1 + a*x))) - 5*Arc
Sin[a*x] - 5*Log[a*x] + 5*Log[1 + Sqrt[1 - a^2*x^2]]))/a
```

Maple [B] time = 0.052, size = 329, normalized size = 3.

$$-4\frac{c^2\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)\right)^{5/2}}{a^4\left(x+a^{-1}\right)^3} - 4\frac{c^2\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)\right)^{5/2}}{a^3\left(x+a^{-1}\right)^2} - \frac{7c^2}{3a}\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -4*c^2/a^4/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-4*c^2/a^3/(x+1/a)^2
*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-7/3*c^2/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^
```

```
(3/2)-7/2*c^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-7/2*c^2/(a^2)^(1/2)*arct
an((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-c^2/a^2/x*(-a^2*x^2+1)
^(5/2)-c^2*x*(-a^2*x^2+1)^(3/2)-3/2*c^2*x*(-a^2*x^2+1)^(1/2)-3/2*c^2/(a^2)^
(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-5/3*c^2*(-a^2*x^2+1)^(3/2)/a
-5*c^2*(-a^2*x^2+1)^(1/2)/a+5*c^2/a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^2}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^2/(a*x + 1)^3, x)
```

Fricas [A] time = 2.47556, size = 313, normalized size = 2.82

$$-\frac{17\,a^2c^2x^2+17\,ac^2x-10\left(a^2c^2x^2+ac^2x\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+5\left(a^2c^2x^2+ac^2x\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(a^2c^2x^2+18\,a\right.}{a^3x^2+a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(17*a^2*c^2*x^2 + 17*a*c^2*x - 10*(a^2*c^2*x^2 + a*c^2*x)*arctan((sqrt(-a^
2*x^2 + 1) - 1)/(a*x)) + 5*(a^2*c^2*x^2 + a*c^2*x)*log((sqrt(-a^2*x^2 + 1)
 - 1)/x) + (a^2*c^2*x^2 + 18*a*c^2*x + c^2)*sqrt(-a^2*x^2 + 1))/(a^3*x^2 + a
^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2\left(\int \frac{\sqrt{-a^2x^2+1}}{a^3x^5+3a^2x^4+3ax^3+x^2}\,dx+\int -\frac{2ax\sqrt{-a^2x^2+1}}{a^3x^5+3a^2x^4+3ax^3+x^2}\,dx+\int \frac{2a^3x^3\sqrt{-a^2x^2+1}}{a^3x^5+3a^2x^4+3ax^3+x^2}\,dx+\int -\frac{a^4x^4\sqrt{-a^2x^2+1}}{a^3x^5+3a^2x^4+3ax^3+x^2}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**2/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] c**2*(Integral(sqrt(-a**2*x**2 + 1)/(a**3*x**5 + 3*a**2*x**4 + 3*a*x**3 + x
**2), x) + Integral(-2*a*x*sqrt(-a**2*x**2 + 1)/(a**3*x**5 + 3*a**2*x**4 +
3*a*x**3 + x**2), x) + Integral(2*a**3*x**3*sqrt(-a**2*x**2 + 1)/(a**3*x**5
 + 3*a**2*x**4 + 3*a*x**3 + x**2), x) + Integral(-a**4*x**4*sqrt(-a**2*x**2
 + 1)/(a**3*x**5 + 3*a**2*x**4 + 3*a*x**3 + x**2), x))/a**2
```

Giac [A] time = 1.20234, size = 266, normalized size = 2.4

$$-\frac{5\,c^2\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{|a|}+\frac{5\,c^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\frac{\sqrt{-a^2x^2+1}c^2}{a}+\frac{\left(c^2+\frac{65\left(\sqrt{-a^2x^2+1}|a|+a\right)c^2}{a^2x}\right)a^2x}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -5*c^2*arcsin(a*x)*sgn(a)/abs(a) + 5*c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*
abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c^2/a + 1/2*(c^2 +
65*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/(a^2*x))*a^2*x/((sqrt(-a^2*x^2 + 1)*
abs(a) + a)*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a)) - 1/2*(sq
rt(-a^2*x^2 + 1)*abs(a) + a)*c^2/(a^2*x*abs(a))
```

3.504 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=77

$$-\frac{c\sqrt{1-a^2x^2}}{a}-\frac{8c(1-ax)}{a\sqrt{1-a^2x^2}}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{4c\sin^{-1}(ax)}{a}$$

```
[Out] (-8*c*(1 - a*x))/(a*Sqrt[1 - a^2*x^2]) - (c*Sqrt[1 - a^2*x^2])/a - (4*c*Arc
Sin[a*x])/a + (c*ArcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rubi [A] time = 0.205672, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6131, 6128, 1805, 1809, 844, 216, 266, 63, 208}

$$-\frac{c\sqrt{1-a^2x^2}}{a}-\frac{8c(1-ax)}{a\sqrt{1-a^2x^2}}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{4c\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-8*c*(1 - a*x))/(a*Sqrt[1 - a^2*x^2]) - (c*Sqrt[1 - a^2*x^2])/a - (4*c*Arc
Sin[a*x])/a + (c*ArcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
```

```
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = -\frac{c\int\frac{e^{-3\tanh^{-1}(ax)}(1-ax)}{x}dx}{a}$$
$$= -\frac{c\int\frac{(1-ax)^4}{x(1-a^2x^2)^{3/2}}dx}{a}$$
$$= -\frac{8c(1-ax)}{a\sqrt{1-a^2x^2}}+\frac{c\int\frac{-1-4ax+a^2x^2}{x\sqrt{1-a^2x^2}}dx}{a}$$
$$= -\frac{8c(1-ax)}{a\sqrt{1-a^2x^2}}-\frac{c\sqrt{1-a^2x^2}}{a}-\frac{c\int\frac{a^2+4a^3x}{x\sqrt{1-a^2x^2}}dx}{a^3}$$
$$= -\frac{8c(1-ax)}{a\sqrt{1-a^2x^2}}-\frac{c\sqrt{1-a^2x^2}}{a}-(4c)\int\frac{1}{\sqrt{1-a^2x^2}}dx-\frac{c\int\frac{1}{x\sqrt{1-a^2x^2}}dx}{a}$$
$$= -\frac{8c(1-ax)}{a\sqrt{1-a^2x^2}}-\frac{c\sqrt{1-a^2x^2}}{a}-\frac{4c\sin^{-1}(ax)}{a}-\frac{c\,\mathrm{Subst}\left(\int\frac{1}{x\sqrt{1-a^2x}}dx,x,x^2\right)}{2a}$$
$$= -\frac{8c(1-ax)}{a\sqrt{1-a^2x^2}}-\frac{c\sqrt{1-a^2x^2}}{a}-\frac{4c\sin^{-1}(ax)}{a}+\frac{c\,\mathrm{Subst}\left(\int\frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}dx,x,\sqrt{1-a^2x^2}\right)}{a^3}$$
$$= -\frac{8c(1-ax)}{a\sqrt{1-a^2x^2}}-\frac{c\sqrt{1-a^2x^2}}{a}-\frac{4c\sin^{-1}(ax)}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}$$

Mathematica [A] time = 0.136717, size = 61, normalized size = 0.79

$$\frac{c\left(-\frac{\sqrt{1-a^2x^2}(ax+9)}{ax+1}+\log\left(\sqrt{1-a^2x^2}+1\right)-4\sin^{-1}(ax)-\log(x)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c*(-(((9 + a*x)*Sqrt[1 - a^2*x^2])/(1 + a*x)) - 4*ArcSin[a*x] - Log[x] + L
og[1 + Sqrt[1 - a^2*x^2]]))/a
```

Maple [B] time = 0.052, size = 223, normalized size = 2.9

$$-2\,\frac{c\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right)^{5/2}}{a^4\left(x+a^{-1}\right)^3}-3\,\frac{c\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right)^{5/2}}{a^3\left(x+a^{-1}\right)^2}-\frac{8\,c}{3\,a}\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -2*c/a^4/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-3*c/a^3/(x+1/a)^2*(-a
^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-8/3*c/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)-
4*c*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-4*c/(a^2)^(1/2)*arctan((a^2)^(1/2)
*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-1/3*c*(-a^2*x^2+1)^(3/2)/a-c*(-a^2*x
```

```
^2+1)^(1/2)/a+c/a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-a^2x^2+1)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))/(a*x + 1)^3, x)
```

Fricas [A] time = 2.50258, size = 224, normalized size = 2.91

$$-\frac{9\,acx-8\,(acx+c)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+(acx+c)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}(acx+9\,c)+9\,c}{a^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(9*a*c*x - 8*(a*c*x + c)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a*c*x +
 c)*log((sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1)*(a*c*x + 9*c) + 9*
c)/(a^2*x + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c\left(\int -\frac{\sqrt{-a^2x^2+1}}{a^3x^4+3a^2x^3+3ax^2+x}\,dx+\int \frac{ax\sqrt{-a^2x^2+1}}{a^3x^4+3a^2x^3+3ax^2+x}\,dx+\int \frac{a^2x^2\sqrt{-a^2x^2+1}}{a^3x^4+3a^2x^3+3ax^2+x}\,dx+\int -\frac{a^3x^3\sqrt{-a^2x^2+1}}{a^3x^4+3a^2x^3+3ax^2+x}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] c*(Integral(-sqrt(-a**2*x**2 + 1)/(a**3*x**4 + 3*a**2*x**3 + 3*a*x**2 + x),
 x) + Integral(a*x*sqrt(-a**2*x**2 + 1)/(a**3*x**4 + 3*a**2*x**3 + 3*a*x**2
 + x), x) + Integral(a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**3*x**4 + 3*a**2*x**
3 + 3*a*x**2 + x), x) + Integral(-a**3*x**3*sqrt(-a**2*x**2 + 1)/(a**3*x**4
 + 3*a**2*x**3 + 3*a*x**2 + x), x))/a
```

Giac [A] time = 1.33726, size = 140, normalized size = 1.82

$$-\frac{4\,c\arcsin(ax)\,\mathrm{sgn}(a)}{|a|}+\frac{c\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\frac{\sqrt{-a^2x^2+1}c}{a}+\frac{16\,c}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -4*c*arcsin(a*x)*sgn(a)/abs(a) + c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a)
 - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c/a + 16*c/(((sqrt(-a^2*x
^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

3.505 $\int \frac{e^{-3\tanh^{-1}(ax)}}{c-\frac{c}{ax}} dx$

Optimal. Leaf size=65

$$-\frac{(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-a^2x^2}}{ac}-\frac{2\sin^{-1}(ax)}{ac}$$

```
[Out] -((1 - a*x)^2/(a*c*Sqrt[1 - a^2*x^2])) - (2*Sqrt[1 - a^2*x^2])/(a*c) - (2*A
rcSin[a*x])/(a*c)
```

Rubi [A] time = 0.104094, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6131, 6128, 789, 641, 216}

$$-\frac{(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-a^2x^2}}{ac}-\frac{2\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))),x]
```

```
[Out] -((1 - a*x)^2/(a*c*Sqrt[1 - a^2*x^2])) - (2*Sqrt[1 - a^2*x^2])/(a*c) - (2*A
rcSin[a*x])/(a*c)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx &= -\frac{a\int \frac{e^{-3\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}\\
&= -\frac{a\int \frac{x(1-ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c}\\
&= -\frac{(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{2\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-a^2x^2}}{ac}-\frac{2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-a^2x^2}}{ac}-\frac{2\sin^{-1}(ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.0329864, size = 67, normalized size = 1.03

$$\frac{a^2x^2+4\sqrt{1-a^2x^2}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)+2ax-3}{ac\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))),x]
```

```
[Out] (-3 + 2*a*x + a^2*x^2 + 4*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])/
(a*c*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.052, size = 292, normalized size = 4.5

$$-\frac{1}{2a^4c\left(x+a^{-1}\right)^3}\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)\right)^{\frac{5}{2}}-\frac{5}{4a^3c\left(x+a^{-1}\right)^2}\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)\right)^{\frac{5}{2}}-\frac{31}{24ac}\left(-a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x),x)
```

```
[Out] -1/2/a^4/c/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-5/4/a^3/c/(x+1/a)^2
*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-31/24/c/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^
(3/2)-31/16/c*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-31/16/c/(a^2)^(1/2)*arct
an((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))+1/24/a/c*(-a^2*(x-1/a)
^2-2*a*(x-1/a))^(3/2)-1/16/c*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x-1/16/c/(a
^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))), x)
```

Fricas [A] time = 2.21634, size = 154, normalized size = 2.37

$$-\frac{3\,ax-4\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}(ax+3)+3}{a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] -(3*a*x - 4*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x + 3) + 3)/(a^2*c*x + a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\left(\int \frac{x\sqrt{-a^2x^2+1}}{a^4x^4+2a^3x^3-2ax-1}\,dx+\int -\frac{a^2x^3\sqrt{-a^2x^2+1}}{a^4x^4+2a^3x^3-2ax-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x),x)
```

```
[Out] a*(Integral(x*sqrt(-a**2*x**2 + 1)/(a**4*x**4 + 2*a**3*x**3 - 2*a*x - 1), x
) + Integral(-a**2*x**3*sqrt(-a**2*x**2 + 1)/(a**4*x**4 + 2*a**3*x**3 - 2*a
*x - 1), x))/c
```

Giac [A] time = 1.24643, size = 99, normalized size = 1.52

$$-\frac{2\,\arcsin(ax)\,\mathrm{sgn}(a)}{c|a|}-\frac{\sqrt{-a^2x^2+1}}{ac}+\frac{4}{c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] -2*arcsin(a*x)*sgn(a)/(c*abs(a)) - sqrt(-a^2*x^2 + 1)/(a*c) + 4/(c*((sqrt(-
a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

3.506 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=63

$$-\frac{1-ax}{ac^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{\sin^{-1}(ax)}{ac^2}$$

```
[Out] -((1 - a*x)/(a*c^2*Sqrt[1 - a^2*x^2])) - Sqrt[1 - a^2*x^2]/(a*c^2) - ArcSin
[a*x]/(a*c^2)
```

Rubi [A] time = 0.124952, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6131, 6128, 797, 641, 216, 637}

$$-\frac{1-ax}{ac^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] -((1 - a*x)/(a*c^2*Sqrt[1 - a^2*x^2])) - Sqrt[1 - a^2*x^2]/(a*c^2) - ArcSin
[a*x]/(a*c^2)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx &= \frac{a^2\int \frac{e^{-3\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}\\
&= \frac{a^2\int \frac{x^2(1-ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^2}\\
&= \frac{\int \frac{1-ax}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^2}-\frac{\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx}{c^2}\\
&= -\frac{1-ax}{ac^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^2}\\
&= -\frac{1-ax}{ac^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{\sin^{-1}(ax)}{ac^2}
\end{aligned}
$$

Mathematica [A] time = 0.134099, size = 46, normalized size = 0.73

$$
-\frac{\sqrt{1-a^2x^2}(ax+2)+(ax+1)\sin^{-1}(ax)}{ac^2(ax+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^2),x]

[Out] -(((2 + a*x)*Sqrt[1 - a^2*x^2] + (1 + a*x)*ArcSin[a*x])/(a*c^2*(1 + a*x)))
```

Maple [B] time = 0.053, size = 336, normalized size = 5.3

$$
-\frac{1}{4a^4c^2\left(x+a^{-1}\right)^3}\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)\right)^{\frac{5}{2}}-\frac{3}{4a^3c^2\left(x+a^{-1}\right)^2}\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)\right)^{\frac{5}{2}}-\frac{37}{48ac^2}\left(-a^2\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^2,x)

[Out] -1/4/a^4/c^2/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-3/4/a^3/c^2/(x+1/
a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-37/48/a/c^2*(-a^2*(x+1/a)^2+2*a*(x+
1/a))^(3/2)-37/32/c^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-37/32/c^2/(a^2)^
(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-1/8/a^3/c^2/
(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)-5/48/a/c^2*(-a^2*(x-1/a)^2-2*a
*(x-1/a))^(3/2)+5/32/c^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x+5/32/c^2/(a^2
)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^2), x)
```

Fricas [A] time = 2.33273, size = 159, normalized size = 2.52

$$-\frac{2\,ax-2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}(ax+2)+2}{a^2c^2x+ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x + 2) + 2)/(a^2*c^2*x + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\left(\int \frac{x^2\sqrt{-a^2x^2+1}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\,dx+\int -\frac{a^2x^4\sqrt{-a^2x^2+1}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*(Integral(x**2*sqrt(-a**2*x**2 + 1)/(a**5*x**5 + a**4*x**4 - 2*a**3*x*
*3 - 2*a**2*x**2 + a*x + 1), x) + Integral(-a**2*x**4*sqrt(-a**2*x**2 + 1)/
(a**5*x**5 + a**4*x**4 - 2*a**3*x**3 - 2*a**2*x**2 + a*x + 1), x))/c**2
```

Giac [A] time = 1.17544, size = 99, normalized size = 1.57

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{c^2|a|}-\frac{\sqrt{-a^2x^2+1}}{ac^2}+\frac{2}{c^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(c^2*abs(a)) - sqrt(-a^2*x^2 + 1)/(a*c^2) + 2/(c^2*((sq
rt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

3.507 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=45

$$-\frac{\sqrt{1-a^2x^2}}{ac^3}-\frac{1}{ac^3\sqrt{1-a^2x^2}}$$

```
[Out] -(1/(a*c^3*Sqrt[1 - a^2*x^2])) - Sqrt[1 - a^2*x^2]/(a*c^3)
```

Rubi [A] time = 0.118959, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6131, 6128, 266, 43}

$$-\frac{\sqrt{1-a^2x^2}}{ac^3}-\frac{1}{ac^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] -(1/(a*c^3*Sqrt[1 - a^2*x^2])) - Sqrt[1 - a^2*x^2]/(a*c^3)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{a^3\int \frac{e^{-3\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}$$
$$= -\frac{a^3\int \frac{x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$
$$= -\frac{a^3\,\text{Subst}\left(\int \frac{x}{\left(1-a^2x\right)^{3/2}}\,dx, x, x^2\right)}{2c^3}$$
$$= -\frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{a^2\left(1-a^2x\right)^{3/2}}-\frac{1}{a^2\sqrt{1-a^2x}}\right)dx, x, x^2\right)}{2c^3}$$
$$= -\frac{1}{ac^3\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac^3}$$

Mathematica [A] time = 0.131466, size = 30, normalized size = 0.67

$$\frac{a^2x^2-2}{ac^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] (-2 + a^2*x^2)/(a*c^3*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.034, size = 43, normalized size = 1.

$$\frac{a^2x^2-2}{a\left(ax-1\right)^2c^3\left(ax+1\right)^2}\left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^3,x)
```

```
[Out] 1/a*(-a^2*x^2+1)^(3/2)*(a^2*x^2-2)/(a*x-1)^2/c^3/(a*x+1)^2
```

Maxima [A] time = 0.98244, size = 61, normalized size = 1.36

$$-\frac{\left(a^2x^2-2\right)\sqrt{ax+1}\sqrt{-ax+1}}{a^3c^3x^2-ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -(a^2*x^2 - 2)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^3*c^3*x^2 - a*c^3)
```

Fricas [A] time = 2.19505, size = 101, normalized size = 2.24

$$-\frac{2\,a^2x^2+\left(a^2x^2-2\right)\sqrt{-a^2x^2+1}-2}{a^3c^3x^2-ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] -(2*a^2*x^2 + (a^2*x^2 - 2)*sqrt(-a^2*x^2 + 1) - 2)/(a^3*c^3*x^2 - a*c^3)
```

Sympy [A] time = 23.5224, size = 34, normalized size = 0.76

$$-\frac{2\left(\frac{\sqrt{-a^2x^2+1}}{2c^3}+\frac{1}{2c^3\sqrt{-a^2x^2+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x)**3,x)
```

```
[Out] -2*(sqrt(-a**2*x**2 + 1)/(2*c**3) + 1/(2*c**3*sqrt(-a**2*x**2 + 1)))/a
```

Giac [A] time = 1.17183, size = 45, normalized size = 1.

$$-\frac{\sqrt{-a^2x^2+1}+\frac{1}{\sqrt{-a^2x^2+1}}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -(sqrt(-a^2*x^2 + 1) + 1/sqrt(-a^2*x^2 + 1))/(a*c^3)
```

3.508 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=96

$$\frac{a^2x^3(ax+1)}{3c^4\left(1-a^2x^2\right)^{3/2}}-\frac{x(4ax+3)}{3c^4\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3ac^4}+\frac{\sin^{-1}(ax)}{ac^4}$$

```
[Out] (a^2*x^3*(1 + a*x))/(3*c^4*(1 - a^2*x^2)^(3/2)) - (x*(3 + 4*a*x))/(3*c^4*Sq
rt[1 - a^2*x^2]) - (8*Sqrt[1 - a^2*x^2])/(3*a*c^4) + ArcSin[a*x]/(a*c^4)
```

Rubi [A] time = 0.164805, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6131, 6128, 850, 819, 641, 216}

$$\frac{a^2x^3(ax+1)}{3c^4\left(1-a^2x^2\right)^{3/2}}-\frac{x(4ax+3)}{3c^4\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3ac^4}+\frac{\sin^{-1}(ax)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] (a^2*x^3*(1 + a*x))/(3*c^4*(1 - a^2*x^2)^(3/2)) - (x*(3 + 4*a*x))/(3*c^4*Sq
rt[1 - a^2*x^2]) - (8*Sqrt[1 - a^2*x^2])/(3*a*c^4) + ArcSin[a*x]/(a*c^4)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
```

```
!ILtQ[m + 2*p + 3, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx &= \frac{a^4\int \frac{e^{-3\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}\\
&= \frac{a^4\int \frac{x^4}{(1-ax)\left(1-a^2x^2\right)^{3/2}}\,dx}{c^4}\\
&= \frac{a^4\int \frac{x^4(1+ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^4}\\
&= \frac{a^2x^3(1+ax)}{3c^4\left(1-a^2x^2\right)^{3/2}}-\frac{a^2\int \frac{x^2(3+4ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^4}\\
&= \frac{a^2x^3(1+ax)}{3c^4\left(1-a^2x^2\right)^{3/2}}-\frac{x(3+4ax)}{3c^4\sqrt{1-a^2x^2}}+\frac{\int \frac{3+8ax}{\sqrt{1-a^2x^2}}\,dx}{3c^4}\\
&= \frac{a^2x^3(1+ax)}{3c^4\left(1-a^2x^2\right)^{3/2}}-\frac{x(3+4ax)}{3c^4\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3ac^4}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^4}\\
&= \frac{a^2x^3(1+ax)}{3c^4\left(1-a^2x^2\right)^{3/2}}-\frac{x(3+4ax)}{3c^4\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3ac^4}+\frac{\sin^{-1}(ax)}{ac^4}
\end{aligned}
$$

Mathematica [A] time = 0.174464, size = 68, normalized size = 0.71

$$
\frac{\frac{\sqrt{1-a^2x^2}\left(-3a^3x^3+7a^2x^2+5ax-8\right)}{(ax-1)^2(ax+1)}+3\sin^{-1}(ax)}{3ac^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(-8 + 5*a*x + 7*a^2*x^2 - 3*a^3*x^3))/((-1 + a*x)^2*(1
+ a*x)) + 3*ArcSin[a*x])/(3*a*c^4)
```

Maple [B] time = 0.064, size = 424, normalized size = 4.4

$$-\frac{43}{48\,a^3c^4}\left(-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)\right)^{\frac{5}{2}}\left(x-a^{-1}\right)^{-2}+\frac{87\,x}{64\,c^4}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}+\frac{87}{64\,c^4}\arctan\left(x\sqrt{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^4,x)
```

```
[Out] -43/48/a^3/c^4/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)+87/64/c^4*(-a^2
*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x+87/64/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/
(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))+1/24/a^5/c^4/(x-1/a)^4*(-a^2*(x-1/a)^2-
2*a*(x-1/a))^(5/2)+17/48/a^4/c^4/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/
2)-1/16/a^4/c^4/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-1/4/a^3/c^4/(x
+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-23/64/c^4*(-a^2*(x+1/a)^2+2*a*(x
+1/a))^(1/2)*x-23/64/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2
*a*(x+1/a))^(1/2))-29/32/a/c^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)-23/96/a/c
^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^4), x)
```

Fricas [A] time = 2.47331, size = 297, normalized size = 3.09

$$-\frac{8\,a^3x^3-8\,a^2x^2-8\,ax+6\left(a^3x^3-a^2x^2-ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3\,a^3x^3-7\,a^2x^2-5\,ax+8\right)\sqrt{-a^2x^2+1}+8}{3\left(a^4c^4x^3-a^3c^4x^2-a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(8*a^3*x^3 - 8*a^2*x^2 - 8*a*x + 6*(a^3*x^3 - a^2*x^2 - a*x + 1)*arcta
n((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (3*a^3*x^3 - 7*a^2*x^2 - 5*a*x + 8)*sqr
t(-a^2*x^2 + 1) + 8)/(a^4*c^4*x^3 - a^3*c^4*x^2 - a^2*c^4*x + a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^4\left(\int \frac{x^4\sqrt{-a^2x^2+1}}{a^7x^7-a^6x^6-3a^5x^5+3a^4x^4+3a^3x^3-3a^2x^2-ax+1}\,dx+\int -\frac{a^2x^6\sqrt{-a^2x^2+1}}{a^7x^7-a^6x^6-3a^5x^5+3a^4x^4+3a^3x^3-3a^2x^2-ax+1}\,dx\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*(Integral(x**4*sqrt(-a**2*x**2 + 1)/(a**7*x**7 - a**6*x**6 - 3*a**5*x*
*5 + 3*a**4*x**4 + 3*a**3*x**3 - 3*a**2*x**2 - a*x + 1), x) + Integral(-a**
2*x**6*sqrt(-a**2*x**2 + 1)/(a**7*x**7 - a**6*x**6 - 3*a**5*x**5 + 3*a**4*x
**4 + 3*a**3*x**3 - 3*a**2*x**2 - a*x + 1), x))/c**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3\left(c-\frac{c}{ax}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^4), x)
```

3.509 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx$

Optimal. Leaf size=125

$$-\frac{(ax+1)^2}{5ac^5\left(1-a^2x^2\right)^{5/2}}+\frac{22(ax+1)}{15ac^5\left(1-a^2x^2\right)^{3/2}}-\frac{\sqrt{1-a^2x^2}}{ac^5}-\frac{2(23ax+30)}{15ac^5\sqrt{1-a^2x^2}}+\frac{2\sin^{-1}(ax)}{ac^5}$$

```
[Out] -(1 + a*x)^2/(5*a*c^5*(1 - a^2*x^2)^(5/2)) + (22*(1 + a*x))/(15*a*c^5*(1 -
a^2*x^2)^(3/2)) - (2*(30 + 23*a*x))/(15*a*c^5*Sqrt[1 - a^2*x^2]) - Sqrt[1 -
 a^2*x^2]/(a*c^5) + (2*ArcSin[a*x])/(a*c^5)
```

Rubi [A] time = 0.317702, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6131, 6128, 852, 1635, 1814, 641, 216}

$$-\frac{(ax+1)^2}{5ac^5\left(1-a^2x^2\right)^{5/2}}+\frac{22(ax+1)}{15ac^5\left(1-a^2x^2\right)^{3/2}}-\frac{\sqrt{1-a^2x^2}}{ac^5}-\frac{2(23ax+30)}{15ac^5\sqrt{1-a^2x^2}}+\frac{2\sin^{-1}(ax)}{ac^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^5),x]
```

```
[Out] -(1 + a*x)^2/(5*a*c^5*(1 - a^2*x^2)^(5/2)) + (22*(1 + a*x))/(15*a*c^5*(1 -
a^2*x^2)^(3/2)) - (2*(30 + 23*a*x))/(15*a*c^5*Sqrt[1 - a^2*x^2]) - Sqrt[1 -
 a^2*x^2]/(a*c^5) + (2*ArcSin[a*x])/(a*c^5)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
```

```
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx &= -\frac{a^5\int \frac{e^{-3\tanh^{-1}(ax)}x^5}{(1-ax)^5}\,dx}{c^5}\\
&= -\frac{a^5\int \frac{x^5}{(1-ax)^2\left(1-a^2x^2\right)^{3/2}}\,dx}{c^5}\\
&= -\frac{a^5\int \frac{x^5(1+ax)^2}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^5}\\
&= -\frac{(1+ax)^2}{5ac^5\left(1-a^2x^2\right)^{5/2}}+\frac{a^5\int \frac{(1+ax)\left(\frac{2}{a^5}+\frac{5x}{a^4}+\frac{5x^2}{a^3}+\frac{5x^3}{a^2}+\frac{5x^4}{a}\right)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^5}\\
&= -\frac{(1+ax)^2}{5ac^5\left(1-a^2x^2\right)^{5/2}}+\frac{22(1+ax)}{15ac^5\left(1-a^2x^2\right)^{3/2}}-\frac{a^5\int \frac{\frac{16}{a^5}+\frac{45x}{a^4}+\frac{30x^2}{a^3}+\frac{15x^3}{a^2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^5}\\
&= -\frac{(1+ax)^2}{5ac^5\left(1-a^2x^2\right)^{5/2}}+\frac{22(1+ax)}{15ac^5\left(1-a^2x^2\right)^{3/2}}-\frac{2(30+23ax)}{15ac^5\sqrt{1-a^2x^2}}+\frac{a^5\int \frac{\frac{30}{a^5}+\frac{15x}{a^4}}{\sqrt{1-a^2x^2}}\,dx}{15c^5}\\
&= -\frac{(1+ax)^2}{5ac^5\left(1-a^2x^2\right)^{5/2}}+\frac{22(1+ax)}{15ac^5\left(1-a^2x^2\right)^{3/2}}-\frac{2(30+23ax)}{15ac^5\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac^5}+\frac{2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^5}\\
&= -\frac{(1+ax)^2}{5ac^5\left(1-a^2x^2\right)^{5/2}}+\frac{22(1+ax)}{15ac^5\left(1-a^2x^2\right)^{3/2}}-\frac{2(30+23ax)}{15ac^5\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac^5}+\frac{2\sin^{-1}(ax)}{ac^5}
\end{aligned}
$$

Mathematica [A] time = 0.215928, size = 76, normalized size = 0.61

$$\frac{\frac{\sqrt{1-a^2x^2}(-15a^4x^4+76a^3x^3-32a^2x^2-82ax+56)}{(ax-1)^3(ax+1)}+30\sin^{-1}(ax)}{15ac^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^5),x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(56 - 82*a*x - 32*a^2*x^2 + 76*a^3*x^3 - 15*a^4*x^4))/(
(-1 + a*x)^3*(1 + a*x)) + 30*ArcSin[a*x])/(15*a*c^5)
```

Maple [B] time = 0.086, size = 468, normalized size = 3.7

$$-\frac{187}{128\,ac^5}\left(-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)\right)^{\frac{3}{2}}+\frac{7}{48\,a^5c^5}\left(-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)\right)^{\frac{5}{2}}\left(x-a^{-1}\right)^{-4}+\frac{31}{48\,a^4c^5}\left(-a^2\left(x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^5,x)
```

```
[Out] -187/128/a/c^5*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+7/48/a^5/c^5/(x-1/a)^4*(-
a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)+31/48/a^4/c^5/(x-1/a)^3*(-a^2*(x-1/a)^2-2*
a*(x-1/a))^(5/2)-139/96/a^3/c^5/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2
)+561/256/c^5*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x+561/256/c^5/(a^2)^(1/2)*
arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))-1/32/a^4/c^5/(x+1/
a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-9/64/a^3/c^5/(x+1/a)^2*(-a^2*(x+1/a
)^2+2*a*(x+1/a))^(5/2)-49/256/c^5*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-49/2
56/c^5/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
+1/40/a^6/c^5/(x-1/a)^5*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)-49/384/a/c^5*(-a
^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^5,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^5), x)
```

Fricas [A] time = 2.44363, size = 342, normalized size = 2.74

$$-\frac{56\,a^4x^4-112\,a^3x^3+112\,ax+60\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(15\,a^4x^4-76\,a^3x^3+32\,a^2x^2+82\right.}{15\left(a^5c^5x^4-2\,a^4c^5x^3+2\,a^2c^5x-ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^5,x, algorithm="fricas")
```

```
[Out] -1/15*(56*a^4*x^4 - 112*a^3*x^3 + 112*a*x + 60*(a^4*x^4 - 2*a^3*x^3 + 2*a*x
 - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (15*a^4*x^4 - 76*a^3*x^3 + 3
2*a^2*x^2 + 82*a*x - 56)*sqrt(-a^2*x^2 + 1) - 56)/(a^5*c^5*x^4 - 2*a^4*c^5*
x^3 + 2*a^2*c^5*x - a*c^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^5\left(\int\frac{x^5\sqrt{-a^2x^2+1}}{a^8x^8-2a^7x^7-2a^6x^6+6a^5x^5-6a^3x^3+2a^2x^2+2ax-1}\,dx+\int-\frac{a^2x^7\sqrt{-a^2x^2+1}}{a^8x^8-2a^7x^7-2a^6x^6+6a^5x^5-6a^3x^3+2a^2x^2+2ax-1}\,dx\right)}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x)**5,x)
```

```
[Out] a**5*(Integral(x**5*sqrt(-a**2*x**2 + 1)/(a**8*x**8 - 2*a**7*x**7 - 2*a**6*
x**6 + 6*a**5*x**5 - 6*a**3*x**3 + 2*a**2*x**2 + 2*a*x - 1), x) + Integral(
-a**2*x**7*sqrt(-a**2*x**2 + 1)/(a**8*x**8 - 2*a**7*x**7 - 2*a**6*x**6 + 6*
a**5*x**5 - 6*a**3*x**3 + 2*a**2*x**2 + 2*a*x - 1), x))/c**5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^5), x)
```

3.510 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=225

$$-\frac{10a^2x^3\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{9/2}}{21(1-ax)^{5/2}}-\frac{a^3x^4(54-227ax)\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{9/2}}{105(1-ax)^{9/2}}-\frac{7a^{7/2}x^{9/2}\left(c-\frac{c}{ax}\right)^{9/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{9/2}}+\frac{2ax^2\sqrt{ax}}{5(1}$$

```
[Out] -(a^3*(c - c/(a*x))^(9/2)*x^4*(54 - 227*a*x)*Sqrt[1 + a*x])/(105*(1 - a*x)^
(9/2)) - (10*a^2*(c - c/(a*x))^(9/2)*x^3*Sqrt[1 + a*x])/(21*(1 - a*x)^(5/2)
) + (2*a*(c - c/(a*x))^(9/2)*x^2*Sqrt[1 + a*x])/(5*(1 - a*x)^(3/2)) - (2*(c
 - c/(a*x))^(9/2)*x*Sqrt[1 + a*x])/(7*Sqrt[1 - a*x]) - (7*a^(7/2)*(c - c/(a
*x))^(9/2)*x^(9/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(9/2)
```

Rubi [A] time = 0.208767, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6134, 6129, 97, 150, 143, 54, 215}

$$-\frac{10a^2x^3\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{9/2}}{21(1-ax)^{5/2}}-\frac{a^3x^4(54-227ax)\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{9/2}}{105(1-ax)^{9/2}}-\frac{7a^{7/2}x^{9/2}\left(c-\frac{c}{ax}\right)^{9/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{9/2}}+\frac{2ax^2\sqrt{ax}}{5(1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x))^(9/2),x]
```


```
[Out] -(a^3*(c - c/(a*x))^(9/2)*x^4*(54 - 227*a*x)*Sqrt[1 + a*x])/(105*(1 - a*x)^
(9/2)) - (10*a^2*(c - c/(a*x))^(9/2)*x^3*Sqrt[1 + a*x])/(21*(1 - a*x)^(5/2)
) + (2*a*(c - c/(a*x))^(9/2)*x^2*Sqrt[1 + a*x])/(5*(1 - a*x)^(3/2)) - (2*(c
 - c/(a*x))^(9/2)*x*Sqrt[1 + a*x])/(7*Sqrt[1 - a*x]) - (7*a^(7/2)*(c - c/(a
*x))^(9/2)*x^(9/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(9/2)
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int \frac{e^{\tanh^{-1}(ax)}(1-ax)^{9/2}}{x^{9/2}}dx}{(1-ax)^{9/2}}$$

$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int \frac{(1-ax)^4\sqrt{1+ax}}{x^{9/2}}dx}{(1-ax)^{9/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+ax}}{7\sqrt{1-ax}} + \frac{\left(2\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int \frac{(1-ax)^3\left(-\frac{7a}{2}-\frac{9a^2x}{2}\right)}{x^{7/2}\sqrt{1+ax}}dx}{7(1-ax)^{9/2}}$$

$$= \frac{2a\left(c-\frac{c}{ax}\right)^{9/2}x^2\sqrt{1+ax}}{5(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+ax}}{7\sqrt{1-ax}} + \frac{\left(4\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int \frac{(1-ax)^2\left(\frac{25a^2}{4}+\frac{59}{}\right.}{x^{5/2}\sqrt{1+ax}}}{35(1-ax)^{9/2}}$$

$$= -\frac{10a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3\sqrt{1+ax}}{21(1-ax)^{5/2}} + \frac{2a\left(c-\frac{c}{ax}\right)^{9/2}x^2\sqrt{1+ax}}{5(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+ax}}{7\sqrt{1-ax}} + \frac{(8(}{}$$

$$= -\frac{a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4(54-227ax)\sqrt{1+ax}}{105(1-ax)^{9/2}} - \frac{10a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3\sqrt{1+ax}}{21(1-ax)^{5/2}} + \frac{2a\left(c-\frac{c}{ax}\right)^{9/2}x^2}{5(1-ax)^{}}$$

$$= -\frac{a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4(54-227ax)\sqrt{1+ax}}{105(1-ax)^{9/2}} - \frac{10a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3\sqrt{1+ax}}{21(1-ax)^{5/2}} + \frac{2a\left(c-\frac{c}{ax}\right)^{9/2}x^2}{5(1-ax)^{}}$$

$$= -\frac{a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4(54-227ax)\sqrt{1+ax}}{105(1-ax)^{9/2}} - \frac{10a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3\sqrt{1+ax}}{21(1-ax)^{5/2}} + \frac{2a\left(c-\frac{c}{ax}\right)^{9/2}x^2}{5(1-ax)^{}}$$

Mathematica [C] time = 0.0789492, size = 101, normalized size = 0.45

$$\frac{c^4\sqrt{c-\frac{c}{ax}}\left(245a^2x^2\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},-ax\right)+\sqrt{ax+1}\left(105a^4x^4-688a^3x^3-601a^2x^2+162ax-30\right)\right.}{105a^4x^3\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x]*(-30 + 162*a*x - 601*a^2*x^2 - 688*a^
3*x^3 + 105*a^4*x^4) + 245*a^2*x^2*Hypergeometric2F1[-3/2, -3/2, -1/2, -(a*
x)]))/(105*a^4*x^3*Sqrt[1 - a*x])
```

Maple [A] time = 0.18, size = 172, normalized size = 0.8

$$-\frac{c^4}{210\,x^3\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(210\,a^{9/2}\sqrt{-(ax+1)\,x}x^4+735\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)x^4a^4+584\,a^{7/}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(9/2),x)
```

```
[Out] -1/210*(c*(a*x-1)/a/x)^(1/2)/x^3*c^4/a^(9/2)*(-a^2*x^2+1)^(1/2)*(210*a^(9/2
)*(-(a*x+1)*x)^(1/2)*x^4+735*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2
))*x^4*a^4+584*a^(7/2)*x^3*(-(a*x+1)*x)^(1/2)-712*a^(5/2)*x^2*(-(a*x+1)*x)^
```

```
(1/2)+324*a^(3/2)*x*(-(a*x+1)*x)^(1/2)-60*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-
1)/(-(a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(9/2)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [A] time = 3.0308, size = 810, normalized size = 3.6

$$\left[\frac{735\left(a^4c^4x^4-a^3c^4x^3\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(105a^4c^4x^4+292a^3c^4x^3-356a^2c^4x^2+\right.}{420\left(a^5x^4-a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(735*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*
x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))
 - c)/(a*x - 1)) - 4*(105*a^4*c^4*x^4 + 292*a^3*c^4*x^3 - 356*a^2*c^4*x^2 +
 162*a*c^4*x - 30*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4
 - a^4*x^3), 1/210*(735*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*arctan(2*sqrt(-
a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c))
 - 2*(105*a^4*c^4*x^4 + 292*a^3*c^4*x^3 - 356*a^2*c^4*x^2 + 162*a*c^4*x - 3
0*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(9/2)/sqrt(-a^2*x^2 + 1), x)
```

3.511 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=181

$$-\frac{a^2x^3\sqrt{ax+1}(31ax+18)\left(c-\frac{c}{ax}\right)^{7/2}}{15(1-ax)^{7/2}}+\frac{5a^{5/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{7/2}}+\frac{2ax^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{7/2}}{3(1-ax)^{3/2}}-\frac{2x\sqrt{ax+1}\left(c-\right.}{5\sqrt{1-ax}}$$

```
[Out] (2*a*(c - c/(a*x))^(7/2)*x^2*Sqrt[1 + a*x])/(3*(1 - a*x)^(3/2)) - (2*(c - c
/(a*x))^(7/2)*x*Sqrt[1 + a*x])/(5*Sqrt[1 - a*x]) - (a^2*(c - c/(a*x))^(7/2)
*x^3*Sqrt[1 + a*x]*(18 + 31*a*x))/(15*(1 - a*x)^(7/2)) + (5*a^(5/2)*(c - c/
(a*x))^(7/2)*x^(7/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(7/2)
```

Rubi [A] time = 0.165372, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6134, 6129, 97, 150, 143, 54, 215}

$$-\frac{a^2x^3\sqrt{ax+1}(31ax+18)\left(c-\frac{c}{ax}\right)^{7/2}}{15(1-ax)^{7/2}}+\frac{5a^{5/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{7/2}}+\frac{2ax^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{7/2}}{3(1-ax)^{3/2}}-\frac{2x\sqrt{ax+1}\left(c-\right.}{5\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x))^(7/2),x]
```

```
[Out] (2*a*(c - c/(a*x))^(7/2)*x^2*Sqrt[1 + a*x])/(3*(1 - a*x)^(3/2)) - (2*(c - c
/(a*x))^(7/2)*x*Sqrt[1 + a*x])/(5*Sqrt[1 - a*x]) - (a^2*(c - c/(a*x))^(7/2)
*x^3*Sqrt[1 + a*x]*(18 + 31*a*x))/(15*(1 - a*x)^(7/2)) + (5*a^(5/2)*(c - c/
(a*x))^(7/2)*x^(7/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(7/2)
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{e^{\tanh^{-1}(ax)}(1-ax)^{7/2}}{x^{7/2}}dx}{(1-ax)^{7/2}}$$

$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{(1-ax)^3\sqrt{1+ax}}{x^{7/2}}dx}{(1-ax)^{7/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}}+\frac{\left(2\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{(1-ax)^2\left(-\frac{5a}{2}-\frac{7a^2x}{2}\right)}{x^{5/2}\sqrt{1+ax}}dx}{5(1-ax)^{7/2}}$$

$$= \frac{2a\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}}{3(1-ax)^{3/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}}+\frac{\left(4\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{(1-ax)\left(\frac{9a^2}{4}+\frac{31a^3}{4}\right.}{x^{3/2}\sqrt{1+ax}}}{15(1-ax)^{7/2}}$$

$$= \frac{2a\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}}{3(1-ax)^{3/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}}-\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3\sqrt{1+ax}(18+31ax}{15(1-ax)^{7/2}}$$

$$= \frac{2a\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}}{3(1-ax)^{3/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}}-\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3\sqrt{1+ax}(18+31ax}{15(1-ax)^{7/2}}$$

$$= \frac{2a\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}}{3(1-ax)^{3/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}}-\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3\sqrt{1+ax}(18+31ax}{15(1-ax)^{7/2}}$$

Mathematica [A] time = 0.074095, size = 95, normalized size = 0.52

$$\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{ax+1}\left(15a^3x^3+56a^2x^2-28ax+6\right)-75a^{5/2}x^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{15a^3x^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x]*(6 - 28*a*x + 56*a^2*x^2 + 15*a^3*x^3
) - 75*a^(5/2)*x^(5/2)*ArcSinh[Sqrt[a]*Sqrt[x]]))/(15*a^3*x^2*Sqrt[1 - a*x]
)
```

Maple [A] time = 0.145, size = 154, normalized size = 0.9

$$-\frac{c^3}{30\,x^2\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(30\,a^{7/2}x^3\sqrt{-(ax+1)\,x}+75\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)x^3a^3+112\,a^{5/2}x^2\sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(7/2),x)
```

```
[Out] -1/30*(c*(a*x-1)/a/x)^(1/2)/x^2*c^3/a^(7/2)*(-a^2*x^2+1)^(1/2)*(30*a^(7/2)*
x^3*(-(a*x+1)*x)^(1/2)+75*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*
x^3*a^3+112*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2)-56*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+
12*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-(a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(7/2)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [A] time = 2.92512, size = 745, normalized size = 4.12

$$\left[\frac{75\left(a^3c^3x^3-a^2c^3x^2\right)\sqrt{-c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(15\,a^3c^3x^3+56\,a^2c^3x^2-28\,ac^3x+6\,c^3\right)\sqrt{}}{60\left(a^4x^3-a^3x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*(75*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) -
 c)/(a*x - 1)) - 4*(15*a^3*c^3*x^3 + 56*a^2*c^3*x^2 - 28*a*c^3*x + 6*c^3)*s
qrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(75*(a
^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*s
```

```
qrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(15*a^3*c^3*x^3 + 56*
a^2*c^3*x^2 - 28*a*c^3*x + 6*c^3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)
))/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(7/2)/sqrt(-a^2*x^2 + 1), x)
```

3.512 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=171

$$-\frac{3a^2x^3\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}{(1-ax)^{5/2}}-\frac{3a^{3/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{5/2}}+\frac{4ax^2(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{(1-ax)^{5/2}}-\frac{2x(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3(1-ax)^{5/2}}$$

```
[Out] (-3*a^2*(c - c/(a*x))^(5/2)*x^3*Sqrt[1 + a*x])/(1 - a*x)^(5/2) - (2*(c - c/
(a*x))^(5/2)*x*(1 + a*x)^(3/2))/(3*(1 - a*x)^(5/2)) + (4*a*(c - c/(a*x))^(5
/2)*x^2*(1 + a*x)^(3/2))/(1 - a*x)^(5/2) - (3*a^(3/2)*(c - c/(a*x))^(5/2)*x
^(5/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(5/2)
```

Rubi [A] time = 0.157882, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6134, 6129, 89, 78, 50, 54, 215}

$$-\frac{3a^2x^3\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}{(1-ax)^{5/2}}-\frac{3a^{3/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{5/2}}+\frac{4ax^2(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{(1-ax)^{5/2}}-\frac{2x(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3(1-ax)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x))^(5/2),x]
```

```
[Out] (-3*a^2*(c - c/(a*x))^(5/2)*x^3*Sqrt[1 + a*x])/(1 - a*x)^(5/2) - (2*(c - c/
(a*x))^(5/2)*x*(1 + a*x)^(3/2))/(3*(1 - a*x)^(5/2)) + (4*a*(c - c/(a*x))^(5
/2)*x^2*(1 + a*x)^(3/2))/(1 - a*x)^(5/2) - (3*a^(3/2)*(c - c/(a*x))^(5/2)*x
^(5/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(5/2)
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] && !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx &= \frac{\left(\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{e^{\tanh^{-1}(ax)}(1-ax)^{5/2}}{x^{5/2}}dx}{(1-ax)^{5/2}}\\
&= \frac{\left(\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{(1-ax)^2\sqrt{1+ax}}{x^{5/2}}dx}{(1-ax)^{5/2}}\\
&= -\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x(1+ax)^{3/2}}{3(1-ax)^{5/2}}+\frac{\left(2\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{\sqrt{1+ax}\left(-3a+\frac{3a^2x}{2}\right)}{x^{3/2}}dx}{3(1-ax)^{5/2}}\\
&= -\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x(1+ax)^{3/2}}{3(1-ax)^{5/2}}+\frac{4a\left(c-\frac{c}{ax}\right)^{5/2}x^2(1+ax)^{3/2}}{(1-ax)^{5/2}}-\frac{\left(3a^2\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{\sqrt{1+ax}}{\sqrt{x}}\cdots}{(1-ax)^{5/2}}\\
&= -\frac{3a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3\sqrt{1+ax}}{(1-ax)^{5/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x(1+ax)^{3/2}}{3(1-ax)^{5/2}}+\frac{4a\left(c-\frac{c}{ax}\right)^{5/2}x^2(1+ax)^{3/2}}{(1-ax)^{5/2}}-\cdots\\
&= -\frac{3a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3\sqrt{1+ax}}{(1-ax)^{5/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x(1+ax)^{3/2}}{3(1-ax)^{5/2}}+\frac{4a\left(c-\frac{c}{ax}\right)^{5/2}x^2(1+ax)^{3/2}}{(1-ax)^{5/2}}-\cdots\\
&= -\frac{3a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3\sqrt{1+ax}}{(1-ax)^{5/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x(1+ax)^{3/2}}{3(1-ax)^{5/2}}+\frac{4a\left(c-\frac{c}{ax}\right)^{5/2}x^2(1+ax)^{3/2}}{(1-ax)^{5/2}}-\cdots
\end{aligned}$$

Mathematica [A] time = 0.0639127, size = 87, normalized size = 0.51

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{ax+1}\left(3a^2x^2+10ax-2\right)-9a^{3/2}x^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{3a^2x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x]*(-2 + 10*a*x + 3*a^2*x^2) - 9*a^(3/2)
*x^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]]))/(3*a^2*x*Sqrt[1 - a*x])
```

Maple [A] time = 0.144, size = 136, normalized size = 0.8

$$-\frac{c^2}{6\,(ax-1)\,x}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(6\,a^{5/2}x^2\sqrt{-(ax+1)\,x}+9\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)x^2a^2+20\,a^{3/2}x\sqrt{-(ax+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(5/2),x)
```

```
[Out] -1/6*(c*(a*x-1)/a/x)^(1/2)/x*c^2/a^(5/2)*(-a^2*x^2+1)^(1/2)*(6*a^(5/2)*x^2*
(-(a*x+1)*x)^(1/2)+9*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^2*a
^2+20*a^(3/2)*x*(-(a*x+1)*x)^(1/2)-4*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-
(a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(5/2)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [A] time = 2.77078, size = 676, normalized size = 3.95

$$\left[\frac{9\left(a^2c^2x^2-ac^2x\right)\sqrt{-c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(3\,a^2c^2x^2+10\,ac^2x-2\,c^2\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{12\left(a^3x^2-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(9*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(
2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) - 4*(3*a^2*c^2*x^2 + 10*a*c^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1)*sqrt(
(a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x), 1/6*(9*(a^2*c^2*x^2 - a*c^2*x)*sqrt(
c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c
*x^2 - a*c*x - c)) - 2*(3*a^2*c^2*x^2 + 10*a*c^2*x - 2*c^2)*sqrt(-a^2*x^2 +
```

```
1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(5/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(5/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(5/2)/sqrt(-a^2*x^2 + 1), x)
```

3.513 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=128

$$-\frac{2x\left(1-a^2x^2\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^3}+\frac{ax^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}+\frac{\sqrt{a}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{3/2}}$$

```
[Out] (a*(c - c/(a*x))^(3/2)*x^2*Sqrt[1 + a*x])/(1 - a*x)^(3/2) - (2*(c - c/(a*x)
)^(3/2)*x*(1 - a^2*x^2)^(3/2))/(1 - a*x)^3 + (Sqrt[a]*(c - c/(a*x))^(3/2)*x
^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(3/2)
```

Rubi [A] time = 0.192049, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6134, 6128, 879, 848, 50, 54, 215}

$$-\frac{2x\left(1-a^2x^2\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^3}+\frac{ax^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}+\frac{\sqrt{a}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x))^(3/2),x]
```

```
[Out] (a*(c - c/(a*x))^(3/2)*x^2*Sqrt[1 + a*x])/(1 - a*x)^(3/2) - (2*(c - c/(a*x)
)^(3/2)*x*(1 - a^2*x^2)^(3/2))/(1 - a*x)^3 + (Sqrt[a]*(c - c/(a*x))^(3/2)*x
^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(3/2)
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
```

```
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx &= \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{e^{\tanh^{-1}(ax)}(1-ax)^{3/2}}{x^{3/2}}\,dx}{(1-ax)^{3/2}}\\
&= \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{\sqrt{1-ax}\sqrt{1-a^2x^2}}{x^{3/2}}\,dx}{(1-ax)^{3/2}}\\
&= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\left(1-a^2x^2\right)^{3/2}}{(1-ax)^3}+\frac{\left(a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx}{(1-ax)^{3/2}}\\
&= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\left(1-a^2x^2\right)^{3/2}}{(1-ax)^3}+\frac{\left(a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{\sqrt{1+ax}}{\sqrt{x}}\,dx}{(1-ax)^{3/2}}\\
&= \frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\left(1-a^2x^2\right)^{3/2}}{(1-ax)^3}+\frac{\left(a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{1}{\sqrt{x}\sqrt{1+ax}}\,d}{2(1-ax)^{3/2}}\\
&= \frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\left(1-a^2x^2\right)^{3/2}}{(1-ax)^3}+\frac{\left(a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\text{Subst}\left(\int\frac{}{\sqrt{}}\right.}{(1-ax)^{3/2}}\\
&= \frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\left(1-a^2x^2\right)^{3/2}}{(1-ax)^3}+\frac{\sqrt{a}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\sinh^{-1}\left(\sqrt{a}\right.}{(1-ax)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0501064, size = 70, normalized size = 0.55

$$\frac{c\sqrt{c-\frac{c}{ax}}\left(\sqrt{ax+1}(ax+2)-\sqrt{a}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x]*(2 + a*x) - Sqrt[a]*Sqrt[x]*ArcSinh[Sqr
t[a]*Sqrt[x]]))/(a*Sqrt[1 - a*x])
```

Maple [A] time = 0.138, size = 108, normalized size = 0.8

$$-\frac{c}{2ax-2}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(2a^{3/2}x\sqrt{-(ax+1)x}+\arctan\left(\frac{2ax+1}{2}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)x}}\right)xa+4\sqrt{a}\sqrt{-(ax+1)x}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(3/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)*(-a^2*x^2+1)^(1/2)*(2*a^(3/2)*x*(-(a*x
+1)*x)^(1/2)+arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a+4*a^(1/2)
*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-(a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(3/2)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [A] time = 2.67263, size = 559, normalized size = 4.37

$$\left[\frac{(acx-c)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4(2a^2x^2+ax)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\sqrt{-a^2x^2+1}(acx+2c)\sqrt{\frac{acx-c}{ax}}}{4(a^2x-a)},\ \frac{(acx-c)\sqrt{c}\arctan\left(\frac{2\cdots}{\cdots}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*c*x - c)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x
)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) - 4*s
qrt(-a^2*x^2 + 1)*(a*c*x + 2*c)*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(
(a*c*x - c)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c
)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*sqrt(-a^2*x^2 + 1)*(a*c*x + 2*c)*sq
rt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(3/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(3/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(3/2)/sqrt(-a^2*x^2 + 1), x)
```

3.514 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}-\frac{c\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -((c*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - c/(a*x)])) + (Sqrt[c - c/(a*x)]*Sqrt[x]
*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rubi [A] time = 0.144027, antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6134, 6128, 848, 50, 54, 215}

$$\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/Sqrt[1 - a*x] + (Sqrt[c - c/(a*x)]*Sqrt
[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}}+\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}}\\
&= \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}}+\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{\sqrt{1-ax}}\\
&= \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}}+\frac{\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}
\end{aligned}$$

Mathematica [A] time = 0.0377752, size = 69, normalized size = 0.82

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}+\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{\sqrt{a}\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]
*Sqrt[x]]))/(Sqrt[a]*Sqrt[1 - a*x])
```

Maple [A] time = 0.128, size = 89, normalized size = 1.1

$$\frac{x}{2\,ax-2}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(-2\sqrt{a}\sqrt{-(ax+1)\,x}+\arctan\left(\frac{2\,ax+1}{2}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)\,x}}\right)\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(-2*a^(1/2)*(-(a*x+1)*x)^(1/
2)+arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))/(a*x-1)/(-(a*x+1)*x)^(
1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/sqrt(-a^2*x^2 + 1), x)
```

Fricas [A] time = 2.66618, size = 529, normalized size = 6.3

$$\left[-\frac{4\sqrt{-a^2x^2+1}ax\sqrt{\frac{acx-c}{ax}}-(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)}{4\left(a^2x-a\right)},-\frac{2\sqrt{-a^2x^2+1}ax\sqrt{\frac{acx-c}{ax}}+(ax\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) - (a*x - 1)*sqrt(-c
)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt
(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)))/(a^2*x - a), -1/2*(2*sqrt(-a^
2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) + (a*x - 1)*sqrt(c)*arctan(2*sqrt(-a
^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)))
/(a^2*x - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/sqrt(-a^2*x^2 + 1), x)
```

3.515 $\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=157

$$-\frac{3\sqrt{1-ax}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}+\frac{2\sqrt{2}\sqrt{1-ax}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{1-ax}\sqrt{ax+1}}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -((Sqrt[1 - a*x]*Sqrt[1 + a*x])/(a*Sqrt[c - c/(a*x)])) - (3*Sqrt[1 - a*x]*A
rcSinh[Sqrt[a]*Sqrt[x]])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x]) + (2*Sqrt[2]*S
qrt[1 - a*x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a^(3/2)*Sqr
t[c - c/(a*x)]*Sqrt[x])
```

Rubi [A] time = 0.157716, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6134, 6129, 101, 157, 54, 215, 93, 206}

$$-\frac{3\sqrt{1-ax}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}+\frac{2\sqrt{2}\sqrt{1-ax}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{1-ax}\sqrt{ax+1}}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/Sqrt[c - c/(a*x)],x]
```


```
[Out] -((Sqrt[1 - a*x]*Sqrt[1 + a*x])/(a*Sqrt[c - c/(a*x)])) - (3*Sqrt[1 - a*x]*A
rcSinh[Sqrt[a]*Sqrt[x]])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x]) + (2*Sqrt[2]*S
qrt[1 - a*x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a^(3/2)*Sqr
t[c - c/(a*x)]*Sqrt[x])
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-ax}\int \frac{e^{\tanh^{-1}(ax)}\sqrt{x}}{\sqrt{1-ax}}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= \frac{\sqrt{1-ax}\int \frac{\sqrt{x}\sqrt{1+ax}}{1-ax}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} + \frac{\sqrt{1-ax}\int \frac{\frac{1}{2}+\frac{3ax}{2}}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} - \frac{\left(3\sqrt{1-ax}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a\sqrt{c-\frac{c}{ax}}\sqrt{x}} + \frac{\left(2\sqrt{1-ax}\right)\int \frac{1}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} - \frac{\left(3\sqrt{1-ax}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}} + \frac{\left(4\sqrt{1-ax}\right)\mathrm{Subst}\left(\int \frac{1}{1-2ax^2}\,dx,x,\ldots\right.}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} - \frac{3\sqrt{1-ax}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{c-\frac{c}{ax}}\sqrt{x}} + \frac{2\sqrt{2}\sqrt{1-ax}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right)}{a^{3/2}\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

Mathematica [A] time = 0.0644654, size = 105, normalized size = 0.67

$$-\frac{\sqrt{1-ax}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}+3\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/Sqrt[c - c/(a*x)],x]
```

```
[Out] -((Sqrt[1 - a*x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x] + 3*ArcSinh[Sqrt[a]*Sqrt[x]
] - 2*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]))/(a^(3/2)*S
qrt[c - c/(a*x)]*Sqrt[x]))
```

Maple [A] time = 0.157, size = 168, normalized size = 1.1

$$-\frac{x\sqrt{2}}{4c(ax-1)}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(2\sqrt{-(ax+1)x}a^{3/2}\sqrt{2}\sqrt{-a^{-1}}-3\arctan\left(1/2\frac{2ax+1}{\sqrt{a}\sqrt{-(ax+1)x}}\right)a\sqrt{2}\sqrt{-a^{-1}}+4\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(1/2),x)
```

```
[Out] -1/4*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(2*(-(a*x+1)*x)^(1/2)*a^(3/
2)*2^(1/2)*(-1/a)^(1/2)-3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*
a*2^(1/2)*(-1/a)^(1/2)+4*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*
a*x-1)/(a*x-1))*a^(1/2))*2^(1/2)/a^(3/2)/c/(a*x-1)/(-(a*x+1)*x)^(1/2)/(-1/a
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))), x)
```

3.516 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=198

$$\frac{5(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7(1-ax)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{2}a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{2\sqrt{ax+1}(1-ax)^{3/2}}{a^2x\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\sqrt{ax+1}\sqrt{1-ax}}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] (Sqrt[1 - a*x]*Sqrt[1 + a*x])/(a*(c - c/(a*x))^(3/2)) + (2*(1 - a*x)^(3/2)*
Sqrt[1 + a*x])/(a^2*(c - c/(a*x))^(3/2)*x) + (5*(1 - a*x)^(3/2)*ArcSinh[Sqr
t[a]*Sqrt[x]])/(a^(5/2)*(c - c/(a*x))^(3/2)*x^(3/2)) - (7*(1 - a*x)^(3/2)*A
rcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(Sqrt[2]*a^(5/2)*(c - c/(a
*x))^(3/2)*x^(3/2))
```

Rubi [A] time = 0.186758, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6134, 6129, 97, 154, 157, 54, 215, 93, 206}

$$\frac{5(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7(1-ax)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{2}a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{2\sqrt{ax+1}(1-ax)^{3/2}}{a^2x\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\sqrt{ax+1}\sqrt{1-ax}}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a*x))^(3/2),x]
```

```
[Out] (Sqrt[1 - a*x]*Sqrt[1 + a*x])/(a*(c - c/(a*x))^(3/2)) + (2*(1 - a*x)^(3/2)*
Sqrt[1 + a*x])/(a^2*(c - c/(a*x))^(3/2)*x) + (5*(1 - a*x)^(3/2)*ArcSinh[Sqr
t[a]*Sqrt[x]])/(a^(5/2)*(c - c/(a*x))^(3/2)*x^(3/2)) - (7*(1 - a*x)^(3/2)*A
rcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(Sqrt[2]*a^(5/2)*(c - c/(a
*x))^(3/2)*x^(3/2))
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{(1-ax)^{3/2}\int \frac{e^{\tanh^{-1}(ax)}x^{3/2}}{(1-ax)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{(1-ax)^{3/2}\int \frac{x^{3/2}\sqrt{1+ax}}{(1-ax)^2}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{(1-ax)^{3/2}\int \frac{\sqrt{x}\left(\frac{3}{2}+2ax\right)}{(1-ax)\sqrt{1+ax}}\,dx}{a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{2(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{(1-ax)^{3/2}\int \frac{-a-\frac{5a^2x}{2}}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a^3\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{2(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{\left(5(1-ax)^{3/2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}} - \frac{\left(7(1-ax)^{3/2}\right)\int \frac{1}{\sqrt{x}(1-ax)\sqrt{1+a}}}{2a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{2(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{\left(5(1-ax)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}} - \frac{\left(7(1-ax)^{3/2}\right)}{a}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{2(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{5(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}} - \frac{7(1-ax)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right)}{\sqrt{2}a^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

Mathematica [A] time = 0.149247, size = 126, normalized size = 0.64

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{ax+1}(ax-2)+10(ax-1)\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-7\sqrt{2}(ax-1)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{2a^{3/2}c\sqrt{x}\sqrt{1-ax}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a*x))^(3/2),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*(-2 + a*x)*Sqrt[1 + a*x] + 10*(-1 + a*x)*ArcSinh[Sqrt[a]
*Sqrt[x]] - 7*Sqrt[2]*(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 +
 a*x]])/(2*a^(3/2)*c*Sqrt[c - c/(a*x)]*Sqrt[x]*Sqrt[1 - a*x])
```

Maple [A] time = 0.152, size = 276, normalized size = 1.4

$$-\frac{x\sqrt{2}}{4c^2(ax-1)^2}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(2\sqrt{-(ax+1)x}a^{5/2}\sqrt{2}\sqrt{-a^{-1}}x-5a^2\arctan\left(1/2\frac{2ax+1}{\sqrt{a}\sqrt{-(ax+1)x}}\right)\sqrt{2}\sqrt{-a^{-1}}x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(3/2),x)
```

```
[Out] -1/4*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)/c^2*(-a^2*x^2+1)^(1/2)*(2*(-(a*x+1)*x)
^(1/2)*a^(5/2)*2^(1/2)*(-1/a)^(1/2)*x-5*a^2*arctan(1/2/a^(1/2)*(2*a*x+1)/(-
(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)*x-4*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/
2)*(-1/a)^(1/2)+7*a^(3/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3
```

```
*a*x-1)/(a*x-1))*x+5*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(
1/2)*(-1/a)^(1/2)-7*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1
)/(a*x-1))*a^(1/2))*2^(1/2)/(a*x-1)^2/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(3/2)), x)
```

Fricas [A] time = 3.13346, size = 1146, normalized size = 5.79

$$\left[-\frac{7\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx+4\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+10\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\,\mathrm{lo}}{8\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(7*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*
c*x^2 - 13*a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*
sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 10*(a^2*x
^2 - 2*a*x + 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x)*
sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(a^
2*x^2 - 2*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2
*a^2*c^2*x + a*c^2), 1/4*(7*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*arctan(2*
sqrt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2
 - 2*a*c*x - c)) - 10*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2
+ 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 4*(a^
2*x^2 - 2*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2
*a^2*c^2*x + a*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(3/2)*sqrt(-(a*x - 1)*(a*x + 1))),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(3/2)), x)
```

3.517 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=249

$$-\frac{23\sqrt{ax+1}(1-ax)^{5/2}}{8a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{7(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{79(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{8\sqrt{2}a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11\sqrt{ax+1}(1-ax)^{3/2}}{8a^2x\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\sqrt{a\cdots}}{2a\cdots}$$

```
[Out] (Sqrt[1 - a*x]*Sqrt[1 + a*x])/(2*a*(c - c/(a*x))^(5/2)) - (11*(1 - a*x)^(3/
2)*Sqrt[1 + a*x])/(8*a^2*(c - c/(a*x))^(5/2)*x) - (23*(1 - a*x)^(5/2)*Sqrt[
1 + a*x])/(8*a^3*(c - c/(a*x))^(5/2)*x^2) - (7*(1 - a*x)^(5/2)*ArcSinh[Sqrt
[a]*Sqrt[x]])/(a^(7/2)*(c - c/(a*x))^(5/2)*x^(5/2)) + (79*(1 - a*x)^(5/2)*A
rcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(8*Sqrt[2]*a^(7/2)*(c - c/
(a*x))^(5/2)*x^(5/2))
```

Rubi [A] time = 0.202928, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6134, 6129, 97, 149, 154, 157, 54, 215, 93, 206}

$$-\frac{23\sqrt{ax+1}(1-ax)^{5/2}}{8a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{7(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{79(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{8\sqrt{2}a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11\sqrt{ax+1}(1-ax)^{3/2}}{8a^2x\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\sqrt{a\cdots}}{2a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a*x))^(5/2),x]
```

```
[Out] (Sqrt[1 - a*x]*Sqrt[1 + a*x])/(2*a*(c - c/(a*x))^(5/2)) - (11*(1 - a*x)^(3/
2)*Sqrt[1 + a*x])/(8*a^2*(c - c/(a*x))^(5/2)*x) - (23*(1 - a*x)^(5/2)*Sqrt[
1 + a*x])/(8*a^3*(c - c/(a*x))^(5/2)*x^2) - (7*(1 - a*x)^(5/2)*ArcSinh[Sqrt
[a]*Sqrt[x]])/(a^(7/2)*(c - c/(a*x))^(5/2)*x^(5/2)) + (79*(1 - a*x)^(5/2)*A
rcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(8*Sqrt[2]*a^(7/2)*(c - c/
(a*x))^(5/2)*x^(5/2))
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{(1-ax)^{5/2}\int \frac{e^{\tanh^{-1}(ax)}x^{5/2}}{(1-ax)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{5/2}\int \frac{x^{5/2}\sqrt{1+ax}}{(1-ax)^3}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{2a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{(1-ax)^{5/2}\int \frac{x^{3/2}\left(\frac{5}{2}+3ax\right)}{(1-ax)^2\sqrt{1+ax}}\,dx}{2a\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{2a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{5/2}x} - \frac{(1-ax)^{5/2}\int \frac{\sqrt{x}\left(-\frac{33a}{4}-\frac{23a^2x}{2}\right)}{(1-ax)\sqrt{1+ax}}\,dx}{4a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{2a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{5/2}x} - \frac{23(1-ax)^{5/2}\sqrt{1+ax}}{8a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1-ax)^{5/2}\int \frac{\frac{23a^2}{4}+14a^3x}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{4a^5\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{2a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{5/2}x} - \frac{23(1-ax)^{5/2}\sqrt{1+ax}}{8a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} - \frac{\left(7(1-ax)^{5/2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}} +$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{2a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{5/2}x} - \frac{23(1-ax)^{5/2}\sqrt{1+ax}}{8a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} - \frac{\left(7(1-ax)^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\right.}{a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{2a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{5/2}x} - \frac{23(1-ax)^{5/2}\sqrt{1+ax}}{8a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} - \frac{7(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}} +$$

Mathematica [A] time = 0.170979, size = 139, normalized size = 0.56

$$\frac{-2\sqrt{a}\sqrt{x}\sqrt{ax+1}\left(8a^2x^2-35ax+23\right)-112(ax-1)^2\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+79\sqrt{2}(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{16a^{3/2}c^2\sqrt{x}(1-ax)^{3/2}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a*x))^(5/2),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*(23 - 35*a*x + 8*a^2*x^2) - 112*(-1 + a*x
)^2*ArcSinh[Sqrt[a]*Sqrt[x]] + 79*Sqrt[2]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqr
t[a]*Sqrt[x])/Sqrt[1 + a*x]])/(16*a^(3/2)*c^2*Sqrt[c - c/(a*x)]*Sqrt[x]*(1
- a*x)^(3/2))
```

Maple [B] time = 0.148, size = 390, normalized size = 1.6

$$-\frac{x\sqrt{2}}{32\,c^3\,(ax-1)^3}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(16\,\sqrt{-(ax+1)\,x}a^{7/2}\sqrt{2}\sqrt{-a^{-1}}x^2-70\,\sqrt{-(ax+1)\,x}a^{5/2}\sqrt{2}\sqrt{-a^{-1}}x-56\,a^3\,\text{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(5/2),x)
```

```
[Out] -1/32*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(16*(-(a*x+1)*x)^(1/2)*a^(
7/2)*2^(1/2)*(-1/a)^(1/2)*x^2-70*(-(a*x+1)*x)^(1/2)*a^(5/2)*2^(1/2)*(-1/a)^
(1/2)*x-56*a^3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1
/a)^(1/2)*x^2+79*a^(5/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*
a*x-1)/(a*x-1))*x^2+46*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2)+112*
a^2*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)*x
-158*a^(3/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-
1))*x-56*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)^
(1/2)+79*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*
a^(1/2))*2^(1/2)/a^(3/2)/c^3/(a*x-1)^3/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(5/2)), x)
```

Fricas [A] time = 3.00449, size = 1305, normalized size = 5.24

$$\left[-\frac{79\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{-c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx+4\sqrt{2}\left(3a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+112\left(a^3x^3-3a^2x^2+\right.}{64\left(a^4c^3x^3-3a^3c^3x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(79*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(17*a^3*
c*x^3 - 3*a^2*c*x^2 - 13*a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2
+ 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1
)) + 112*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a
*c*x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*
x)) - c)/(a*x - 1)) + 8*(8*a^3*x^3 - 35*a^2*x^2 + 23*a*x)*sqrt(-a^2*x^2 + 1
)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c
^3), 1/32*(79*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*arctan(2*sq
rt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
 2*a*c*x - c)) - 112*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*arctan(2*sqr
t(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
c)) - 4*(8*a^3*x^3 - 35*a^2*x^2 + 23*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x -
c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(5/2)*sqrt(-(a*x - 1)*(a*x + 1))),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(5/2)), x)
```

3.518 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx$

Optimal. Leaf size=145

$$-\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-x\left(c-\frac{c}{ax}\right)^{9/2}$$

```
[Out] (-5*c^4*Sqrt[c - c/(a*x)])/a - (5*c^3*(c - c/(a*x))^(3/2))/(3*a) - (c^2*(c
- c/(a*x))^(5/2))/a - (5*c*(c - c/(a*x))^(7/2))/(7*a) - (c - c/(a*x))^(9/2)
*x + (5*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rubi [A] time = 0.185047, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 78, 50, 63, 208}

$$-\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-x\left(c-\frac{c}{ax}\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] (-5*c^4*Sqrt[c - c/(a*x)])/a - (5*c^3*(c - c/(a*x))^(3/2))/(3*a) - (c^2*(c
- c/(a*x))^(5/2))/a - (5*c*(c - c/(a*x))^(7/2))/(7*a) - (c - c/(a*x))^(9/2)
*x + (5*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{9/2}(1+ax)}{1-ax}dx$$
$$=-\frac{c\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}{x}dx}{a}$$
$$=-\frac{c\int\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}dx}{a}$$
$$=\frac{c\,\mathrm{Subst}\left(\int\frac{(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}{x^2}dx,x,\frac{1}{x}\right)}{a}$$
$$=-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{(5c)\,\mathrm{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{7/2}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{(5c^2)\,\mathrm{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=-\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{(5c^3)\,\mathrm{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=-\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{(5c^4)\,\mathrm{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x}\right.}{2a}$$
$$=-\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{(5c^5)\,\mathrm{Su}}{}$$
$$=-\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x+(5c^4)\,\mathrm{Su}$$
$$=-\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x+\frac{5c^{9/2}\,\mathrm{ta}}{}$$

Mathematica [A] time = 0.140204, size = 91, normalized size = 0.63

$$\frac{5c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{c^4\left(21a^4x^4+92a^3x^3+4a^2x^2-18ax+6\right)\sqrt{c-\frac{c}{ax}}}{21a^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] -(c^4*Sqrt[c - c/(a*x)]*(6 - 18*a*x + 4*a^2*x^2 + 92*a^3*x^3 + 21*a^4*x^4))
/(21*a^4*x^3) + (5*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Maple [A] time = 0.131, size = 163, normalized size = 1.1

$$\frac{c^4}{42\,x^4}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-210\,a^{9/2}\sqrt{ax^2-x}x^5+105\,\ln\left(1/2\,\frac{2\,\sqrt{ax^2-x}\sqrt{a}+2\,ax-1}{\sqrt{a}}\right)x^5a^4+168\,a^{7/2}\left(ax^2-x\right)^{3/2}x^3-16\,a^{5/2}\right($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(9/2),x)
```

```
[Out] 1/42*(c*(a*x-1)/a/x)^(1/2)/x^4*c^4*(-210*a^(9/2)*(a*x^2-x)^(1/2)*x^5+105*ln
(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^5*a^4+168*a^(7/2)*(a*x^
2-x)^(3/2)*x^3-16*a^(5/2)*(a*x^2-x)^(3/2)*x^2-24*a^(3/2)*(a*x^2-x)^(3/2)*x+
12*(a*x^2-x)^(3/2)*a^(1/2))/((a*x-1)*x)^(1/2)/a^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(c - c/(a*x))^(9/2)/(a^2*x^2 - 1), x)
```

Fricas [A] time = 2.11413, size = 517, normalized size = 3.57

$$\left[\frac{105\,a^3c^{\frac{9}{2}}x^3\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(21\,a^4c^4x^4+92\,a^3c^4x^3+4\,a^2c^4x^2-18\,ac^4x+6\,c^4\right)\sqrt{\frac{acx-c}{ax}}}{42\,a^4x^3},-\frac{105\,a^3\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/42*(105*a^3*c^(9/2)*x^3*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) + c) - 2*(21*a^4*c^4*x^4 + 92*a^3*c^4*x^3 + 4*a^2*c^4*x^2 - 18*a*c^4*x
 + 6*c^4)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3), -1/21*(105*a^3*sqrt(-c)*c^4*x
^3*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (21*a^4*c^4*x^4 + 92*a^3*c^
4*x^3 + 4*a^2*c^4*x^2 - 18*a*c^4*x + 6*c^4)*sqrt((a*c*x - c)/(a*x)))/(a^4*x
^3)]
```

Sympy [C] time = 30.9838, size = 2210, normalized size = 15.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(9/2),x)
```

```
[Out] -c**4*Piecewise((sqrt(a)*sqrt(c)*x**(3/2)/sqrt(a*x - 1) - sqrt(c)*acosh(sqr
t(a)*sqrt(x))/a - sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(a*x - 1)), Abs(a*x) > 1), (
I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)*sqrt(-a*x + 1)/sqrt(a
), True)) - 4*c**5*atan(sqrt(c - c/(a*x))/sqrt(-c))/(a*sqrt(-c)) - 4*c**4*s
```

```
qrt(c - c/(a*x))/a - 2*c**4*Piecewise((-4*a**(11/2)*sqrt(c)*x**(7/2)/(15*a*
*(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*a**(9/2)*sqrt(c)*x**(5/2)/(15*a
**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*a**5*sqrt(c)*x**3*sqrt(a*x - 1
)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) - 2*a**4*sqrt(c)*x**2*sqrt(
a*x - 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) - 8*a**3*sqrt(c)*x*s
qrt(a*x - 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 6*a**2*sqrt(c)
*sqrt(a*x - 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)), Abs(a*x) > 1)
, (-4*a**(11/2)*sqrt(c)*x**(7/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/
2)) + 4*a**(9/2)*sqrt(c)*x**(5/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5
/2)) + 4*I*a**5*sqrt(c)*x**3*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(
5/2)*x**(5/2)) - 2*I*a**4*sqrt(c)*x**2*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2)
 - 15*a**(5/2)*x**(5/2)) - 8*I*a**3*sqrt(c)*x*sqrt(-a*x + 1)/(15*a**(7/2)*x
**(7/2) - 15*a**(5/2)*x**(5/2)) + 6*I*a**2*sqrt(c)*sqrt(-a*x + 1)/(15*a**(7
/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)), True))/a**3 + c**4*Piecewise((-16*a**
(19/2)*sqrt(c)*x**(13/2)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2)
 + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) + 48*a**(17/2)*sqrt(c)*x*
*(11/2)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x
**(9/2) - 105*a**(7/2)*x**(7/2)) - 48*a**(15/2)*sqrt(c)*x**(9/2)/(105*a**(1
3/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(
7/2)*x**(7/2)) + 16*a**(13/2)*sqrt(c)*x**(7/2)/(105*a**(13/2)*x**(13/2) - 3
15*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) + 1
6*a**9*sqrt(c)*x**6*sqrt(a*x - 1)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*
x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) - 40*a**8*sqrt(c
)*x**5*sqrt(a*x - 1)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 3
15*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) + 30*a**7*sqrt(c)*x**4*sqrt(a
*x - 1)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x
**(9/2) - 105*a**(7/2)*x**(7/2)) - 40*a**6*sqrt(c)*x**3*sqrt(a*x - 1)/(105*
a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105
*a**(7/2)*x**(7/2)) + 100*a**5*sqrt(c)*x**2*sqrt(a*x - 1)/(105*a**(13/2)*x*
*(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x*
*(7/2)) - 96*a**4*sqrt(c)*x*sqrt(a*x - 1)/(105*a**(13/2)*x**(13/2) - 315*a*
*(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) + 30*a**
3*sqrt(c)*sqrt(a*x - 1)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2)
+ 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)), Abs(a*x) > 1), (-16*a**(1
9/2)*sqrt(c)*x**(13/2)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) +
 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) + 48*a**(17/2)*sqrt(c)*x**(
11/2)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**
(9/2) - 105*a**(7/2)*x**(7/2)) - 48*a**(15/2)*sqrt(c)*x**(9/2)/(105*a**(13/
2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/
2)*x**(7/2)) + 16*a**(13/2)*sqrt(c)*x**(7/2)/(105*a**(13/2)*x**(13/2) - 315
*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) + 16*
I*a**9*sqrt(c)*x**6*sqrt(-a*x + 1)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)
*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) - 40*I*a**8*sqr
t(c)*x**5*sqrt(-a*x + 1)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2)
 + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) + 30*I*a**7*sqrt(c)*x**4*
sqrt(-a*x + 1)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**
(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) - 40*I*a**6*sqrt(c)*x**3*sqrt(-a*x
+ 1)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(
9/2) - 105*a**(7/2)*x**(7/2)) + 100*I*a**5*sqrt(c)*x**2*sqrt(-a*x + 1)/(105
*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 10
5*a**(7/2)*x**(7/2)) - 96*I*a**4*sqrt(c)*x*sqrt(-a*x + 1)/(105*a**(13/2)*x*
*(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x*
*(7/2)) + 30*I*a**3*sqrt(c)*sqrt(-a*x + 1)/(105*a**(13/2)*x**(13/2) - 315*a
**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)), True))
/a**4
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.519 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=120

$$-\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-x\left(c-\frac{c}{ax}\right)^{7/2}$$

```
[Out] (-3*c^3*Sqrt[c - c/(a*x)])/a - (c^2*(c - c/(a*x))^(3/2))/a - (3*c*(c - c/(a
*x))^(5/2))/(5*a) - (c - c/(a*x))^(7/2)*x + (3*c^(7/2)*ArcTanh[Sqrt[c - c/(
a*x)]/Sqrt[c]])/a
```

Rubi [A] time = 0.160838, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 78, 50, 63, 208}

$$-\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-x\left(c-\frac{c}{ax}\right)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (-3*c^3*Sqrt[c - c/(a*x)])/a - (c^2*(c - c/(a*x))^(3/2))/a - (3*c*(c - c/(a
*x))^(5/2))/(5*a) - (c - c/(a*x))^(7/2)*x + (3*c^(7/2)*ArcTanh[Sqrt[c - c/(
a*x)]/Sqrt[c]])/a
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}{1-ax}dx$$
$$=-\frac{c\int\frac{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}{x}dx}{a}$$
$$=-\frac{c\int\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}dx}{a}$$
$$=\frac{c\,\text{Subst}\left(\int\frac{(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}{x^2}dx,x,\frac{1}{x}\right)}{a}$$
$$=-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{(3c)\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{\left(3c^2\right)\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=-\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{\left(3c^3\right)\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=-\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{\left(3c^4\right)\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}dx\right.}{2a}$$
$$=-\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x+\left(3c^3\right)\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}d\right.$$
$$=-\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.0861841, size = 83, normalized size = 0.69

$$\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{c^3\left(5a^3x^3+8a^2x^2+4ax-2\right)\sqrt{c-\frac{c}{ax}}}{5a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] -(c^3*Sqrt[c - c/(a*x)]*(-2 + 4*a*x + 8*a^2*x^2 + 5*a^3*x^3))/(5*a^3*x^2) +
 (3*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Maple [A] time = 0.138, size = 144, normalized size = 1.2

$$\frac{c^3}{10\,x^3}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-30\,\sqrt{ax^2-x}a^{7/2}x^4+20\,a^{5/2}\left(ax^2-x\right)^{3/2}x^2+15\,\ln\left(1/2\,\frac{2\,\sqrt{ax^2-x}\sqrt{a}+2\,ax-1}{\sqrt{a}}\right)x^4a^3+4\,a^{3/2}\left(ax^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(7/2),x)
```

```
[Out] 1/10*(c*(a*x-1)/a/x)^(1/2)/x^3*c^3*(-30*(a*x^2-x)^(1/2)*a^(7/2)*x^4+20*a^(5
/2)*(a*x^2-x)^(3/2)*x^2+15*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/
2))*x^4*a^3+4*a^(3/2)*(a*x^2-x)^(3/2)*x-4*(a*x^2-x)^(3/2)*a^(1/2))/((a*x-1)
*x)^(1/2)/a^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(c - c/(a*x))^(7/2)/(a^2*x^2 - 1), x)
```

Fricas [A] time = 2.17002, size = 462, normalized size = 3.85

$$\left[\frac{15\,a^2c^{\frac{7}{2}}x^2\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(5\,a^3c^3x^3+8\,a^2c^3x^2+4\,ac^3x-2\,c^3\right)\sqrt{\frac{acx-c}{ax}}}{10\,a^3x^2},-\frac{15\,a^2\sqrt{-c}c^3x^2\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/10*(15*a^2*c^(7/2)*x^2*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) + c) - 2*(5*a^3*c^3*x^3 + 8*a^2*c^3*x^2 + 4*a*c^3*x - 2*c^3)*sqrt((a*c*
x - c)/(a*x)))/(a^3*x^2), -1/5*(15*a^2*sqrt(-c)*c^3*x^2*arctan(sqrt(-c)*sqr
t((a*c*x - c)/(a*x))/c) + (5*a^3*c^3*x^3 + 8*a^2*c^3*x^2 + 4*a*c^3*x - 2*c^
3)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2)]
```

Sympy [C] time = 23.4733, size = 729, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(7/2),x)
```

```
[Out] -c**3*Piecewise((sqrt(a)*sqrt(c)*x**(3/2)/sqrt(a*x - 1) - sqrt(c)*acosh(sqr
t(a)*sqrt(x))/a - sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(a*x - 1)), Abs(a*x) > 1), (
I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)*sqrt(-a*x + 1)/sqrt(a
), True)) - 2*c**4*atan(sqrt(c - c/(a*x))/sqrt(-c))/(a*sqrt(-c)) - 2*c**3*s
qrt(c - c/(a*x))/a + c**3*Piecewise((0, Eq(c, 0)), (2*a*(c - c/(a*x))**(3/2
)/(3*c), True))/a**2 - c**3*Piecewise((-4*a**(11/2)*sqrt(c)*x**(7/2)/(15*a*
*(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*a**(9/2)*sqrt(c)*x**(5/2)/(15*a
```

```
**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*a**5*sqrt(c)*x**3*sqrt(a*x - 1
)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) - 2*a**4*sqrt(c)*x**2*sqrt(
a*x - 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) - 8*a**3*sqrt(c)*x*s
qrt(a*x - 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 6*a**2*sqrt(c)
*sqrt(a*x - 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)), Abs(a*x) > 1)
, (-4*a**(11/2)*sqrt(c)*x**(7/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/
2)) + 4*a**(9/2)*sqrt(c)*x**(5/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5
/2)) + 4*I*a**5*sqrt(c)*x**3*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(
5/2)*x**(5/2)) - 2*I*a**4*sqrt(c)*x**2*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2)
 - 15*a**(5/2)*x**(5/2)) - 8*I*a**3*sqrt(c)*x*sqrt(-a*x + 1)/(15*a**(7/2)*x
**(7/2) - 15*a**(5/2)*x**(5/2)) + 6*I*a**2*sqrt(c)*sqrt(-a*x + 1)/(15*a**(7
/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)), True))/a**3
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.520 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=96

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-x\left(c-\frac{c}{ax}\right)^{5/2}$$

```
[Out] -((c^2*Sqrt[c - c/(a*x)])/a) - (c*(c - c/(a*x))^(3/2))/(3*a) - (c - c/(a*x)
)^(5/2)*x + (c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rubi [A] time = 0.142908, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 78, 50, 63, 208}

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-x\left(c-\frac{c}{ax}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] -((c^2*Sqrt[c - c/(a*x)])/a) - (c*(c - c/(a*x))^(3/2))/(3*a) - (c - c/(a*x)
)^(5/2)*x + (c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}{1-ax}dx$$
$$=-\frac{c\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}{x}dx}{a}$$
$$=-\frac{c\int\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}dx}{a}$$
$$=\frac{c\,\text{Subst}\left(\int\frac{(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}{x^2}dx,x,\frac{1}{x}\right)}{a}$$
$$=-\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{c\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{c^2\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=-\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{c^3\,\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{2a}$$
$$=-\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\left(c-\frac{c}{ax}\right)^{5/2}x+c^2\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}dx,x,\sqrt{c-\frac{c}{ax}}\right)$$
$$=-\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.0622871, size = 75, normalized size = 0.78

$$\frac{c^2\left(-3a^2x^2+2ax-2\right)\sqrt{c-\frac{c}{ax}}+3ac^{5/2}x\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{3a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(-2 + 2*a*x - 3*a^2*x^2) + 3*a*c^(5/2)*x*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]])/(3*a^2*x)
```

Maple [A] time = 0.127, size = 108, normalized size = 1.1

$$\frac{c^2}{6x^2}\sqrt{\frac{c(ax-1)}{ax}}\left(-6\sqrt{ax^2-x}a^{5/2}x^3+3\ln\left(1/2\,\frac{2\sqrt{ax^2-x}\sqrt{a}+2ax-1}{\sqrt{a}}\right)x^3a^2+4\left(ax^2-x\right)^{3/2}\sqrt{a}\right)\frac{1}{\sqrt{(ax-1)x}}a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(5/2),x)
```

```
[Out] 1/6*(c*(a*x-1)/a/x)^(1/2)/x^2*c^2*(-6*(a*x^2-x)^(1/2)*a^(5/2)*x^3+3*ln(1/2*
(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^3*a^2+4*(a*x^2-x)^(3/2)*a^(1
```

```
/2))/((a*x-1)*x)^(1/2)/a^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(c - c/(a*x))^(5/2)/(a^2*x^2 - 1), x)
```

Fricas [A] time = 2.13375, size = 398, normalized size = 4.15

$$\left[\frac{3\,ac^{\frac{5}{2}}x\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(3\,a^2c^2x^2-2\,ac^2x+2\,c^2\right)\sqrt{\frac{acx-c}{ax}}}{6\,a^2x},-\frac{3\,a\sqrt{-c}c^2x\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)+\left(3\,a^2c^2\right.}{3\,a^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*c^(5/2)*x*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) +
c) - 2*(3*a^2*c^2*x^2 - 2*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*x)
, -1/3*(3*a*sqrt(-c)*c^2*x*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (3*
a^2*c^2*x^2 - 2*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*x)]
```

Sympy [C] time = 7.31768, size = 143, normalized size = 1.49

$$-c^2\left(\begin{cases}\frac{\sqrt{a}\sqrt{c}x^{\frac{3}{2}}}{\sqrt{ax-1}}-\frac{\sqrt{c}\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a}-\frac{\sqrt{c}\sqrt{x}}{\sqrt{a}\sqrt{ax-1}} & \text{for } |ax|>1\\ \frac{i\sqrt{c}\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{i\sqrt{c}\sqrt{x}\sqrt{-ax+1}}{\sqrt{a}} & \text{otherwise}\end{cases}\right)+\frac{c^2\left(\begin{cases}0 & \text{for } c=0\\ \frac{2a\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(5/2),x)
```

```
[Out] -c**2*Piecewise((sqrt(a)*sqrt(c)*x**(3/2)/sqrt(a*x - 1) - sqrt(c)*acosh(sqr
t(a)*sqrt(x))/a - sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(a*x - 1)), Abs(a*x) > 1), (
I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)*sqrt(-a*x + 1)/sqrt(a
), True)) + c**2*Piecewise((0, Eq(c, 0)), (2*a*(c - c/(a*x))**(3/2)/(3*c),
True))/a**2
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.521 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=71

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{c\sqrt{c-\frac{c}{ax}}}{a}-x\left(c-\frac{c}{ax}\right)^{3/2}$$

```
[Out] (c*Sqrt[c - c/(a*x)])/a - (c - c/(a*x))^(3/2)*x - (c^(3/2)*ArcTanh[Sqrt[c -
 c/(a*x)]/Sqrt[c]])/a
```

Rubi [A] time = 0.120584, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 78, 50, 63, 208}

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{c\sqrt{c-\frac{c}{ax}}}{a}-x\left(c-\frac{c}{ax}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)])/a - (c - c/(a*x))^(3/2)*x - (c^(3/2)*ArcTanh[Sqrt[c -
 c/(a*x)]/Sqrt[c]])/a
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx &= \int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}{1-ax}\,dx\\
&= -\frac{c\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x}\,dx}{a}\\
&= -\frac{c\int \left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}\,dx}{a}\\
&= \frac{c\,\text{Subst}\left(\int \frac{(a+x)\sqrt{c-\frac{cx}{a}}}{x^2}\,dx,x,\frac{1}{x}\right)}{a}\\
&= -\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{c\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= \frac{c\sqrt{c-\frac{c}{ax}}}{a}-\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= \frac{c\sqrt{c-\frac{c}{ax}}}{a}-\left(c-\frac{c}{ax}\right)^{3/2}x-c\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)\\
&= \frac{c\sqrt{c-\frac{c}{ax}}}{a}-\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0470816, size = 57, normalized size = 0.8

$$\frac{c(2-ax)\sqrt{c-\frac{c}{ax}}-c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(2 - a*x) - c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
)/a
```

Maple [A] time = 0.121, size = 103, normalized size = 1.5

$$-\frac{c}{2x}\sqrt{\frac{c(ax-1)}{ax}}\left(-2\sqrt{ax^2-x}a^{3/2}x^2+4\left(ax^2-x\right)^{3/2}\sqrt{a}+\ln\left(\frac{1}{2}\left(2\sqrt{ax^2-x}\sqrt{a}+2ax-1\right)\frac{1}{\sqrt{a}}\right)x^2a\right)\frac{1}{\sqrt{(ax-1)x}}a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(3/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)/x*c*(-2*(a*x^2-x)^(1/2)*a^(3/2)*x^2+4*(a*x^2-x)^
(3/2)*a^(1/2)+ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a)/((
a*x-1)*x)^(1/2)/a^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(ax+1)^2\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(c - c/(a*x))^(3/2)/(a^2*x^2 - 1), x)
```

Fricas [A] time = 2.15055, size = 297, normalized size = 4.18

$$\left[\frac{c^{\frac{3}{2}}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-2\,(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{2\,a},\ \frac{\sqrt{-c}c\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(c^(3/2)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 2
*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a, (sqrt(-c)*c*arctan(sqrt(-c)*sqrt
```

```
((a*c*x - c)/(a*x))/c) - (a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a]
```

Sympy [C] time = 21.4053, size = 163, normalized size = 2.3

$$-c\left(\begin{cases}\frac{\sqrt{a}\sqrt{c}x^{\frac{3}{2}}}{\sqrt{ax-1}}-\frac{\sqrt{c}\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a}-\frac{\sqrt{c}\sqrt{x}}{\sqrt{a}\sqrt{ax-1}} & \text{for } |ax|>1\\ \frac{i\sqrt{c}\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{i\sqrt{c}\sqrt{x}\sqrt{-ax+1}}{\sqrt{a}} & \text{otherwise}\end{cases}\right)+\frac{2c^2\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{2c\sqrt{c-\frac{c}{ax}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(3/2),x)
```

```
[Out] -c*Piecewise((sqrt(a)*sqrt(c)*x**(3/2)/sqrt(a*x - 1) - sqrt(c)*acosh(sqrt(a
)*sqrt(x))/a - sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(a*x - 1)), Abs(a*x) > 1), (I*s
qrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)*sqrt(-a*x + 1)/sqrt(a),
True)) + 2*c**2*atan(sqrt(c - c/(a*x))/sqrt(-c))/(a*sqrt(-c)) + 2*c*sqrt(c
- c/(a*x))/a
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.522 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=51

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] -(Sqrt[c - c/(a*x)]*x) - (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rubi [A] time = 0.101261, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6133, 25, 514, 375, 78, 63, 208}

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*x) - (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= \int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{1-ax}\,dx\\
&= -\frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x}\,dx}{a}\\
&= -\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}\\
&= \frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= -\sqrt{c-\frac{c}{ax}}x+\frac{(3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= -\sqrt{c-\frac{c}{ax}}x-3\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)\\
&= -\sqrt{c-\frac{c}{ax}}x-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0301668, size = 51, normalized size = 1.

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*x) - (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Maple [B] time = 0.131, size = 120, normalized size = 2.4

$$\frac{x}{2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2\sqrt{ax^2-x}\sqrt{a}-4\sqrt{(ax-1)\,x}\sqrt{a}-\ln\left(\frac{1}{2}\left(2\sqrt{ax^2-x}\sqrt{a}+2\,ax-1\right)\frac{1}{\sqrt{a}}\right)-2\,\ln\left(1/2\,\frac{2\sqrt{(ax-1)\,x}\sqrt{a}+2}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*(a*x^2-x)^(1/2)*a^(1/2)-4*((a*x-1)*x)^(1/2)*
a^(1/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))-2*ln(1/2*(2*((a
*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))/(a^2*x^2 - 1), x)
```

Fricas [A] time = 2.19437, size = 275, normalized size = 5.39

$$\left[-\frac{2\,ax\sqrt{\frac{acx-c}{ax}}-3\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\,-\frac{ax\sqrt{\frac{acx-c}{ax}}-3\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)
*x*sqrt((a*c*x - c)/(a*x)) + c))/a, -(a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(
-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(1/2),x)
```

```
[Out] -Integral(sqrt(c - c/(a*x))/(a*x - 1), x) - Integral(a*x*sqrt(c - c/(a*x))/
(a*x - 1), x)
```

Giac [B] time = 1.20874, size = 132, normalized size = 2.59

$$-\frac{3\sqrt{c}\log\left(|a|\sqrt{|c|}\right)\operatorname{sgn}(x)}{2a}+\frac{3\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)|a|+a\sqrt{c}\right|\right)}{2a\operatorname{sgn}(x)}-\frac{\sqrt{a^2cx^2-acx}|a|}{a^2\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(c)*log(abs(a)*sqrt(abs(c)))*sgn(x)/a + 3/2*sqrt(c)*log(abs(-2*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*abs(a) + a*sqrt(c)))/(a*sgn(x)) - sq
rt(a^2*c*x^2 - a*c*x)*abs(a)/(a^2*sgn(x))
```

3.523 $\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=71

$$-\frac{x}{\sqrt{c-\frac{c}{ax}}}+\frac{5}{a\sqrt{c-\frac{c}{ax}}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] 5/(a*Sqrt[c - c/(a*x)]) - x/Sqrt[c - c/(a*x)] - (5*ArcTanh[Sqrt[c - c/(a*x)
]/Sqrt[c]])/(a*Sqrt[c])
```

Rubi [A] time = 0.162234, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 78, 51, 63, 208}

$$-\frac{x}{\sqrt{c-\frac{c}{ax}}}+\frac{5}{a\sqrt{c-\frac{c}{ax}}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] 5/(a*Sqrt[c - c/(a*x)]) - x/Sqrt[c - c/(a*x)] - (5*ArcTanh[Sqrt[c - c/(a*x)
]/Sqrt[c]])/(a*Sqrt[c])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}(1-ax)}\,dx$$
$$= -\frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{3/2}x}\,dx}{a}$$
$$= -\frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx}{a}$$
$$= \frac{c\,\mathrm{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= -\frac{x}{\sqrt{c-\frac{c}{ax}}} + \frac{(5c)\,\mathrm{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \frac{5}{a\sqrt{c-\frac{c}{ax}}} - \frac{x}{\sqrt{c-\frac{c}{ax}}} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \frac{5}{a\sqrt{c-\frac{c}{ax}}} - \frac{x}{\sqrt{c-\frac{c}{ax}}} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c}$$
$$= \frac{5}{a\sqrt{c-\frac{c}{ax}}} - \frac{x}{\sqrt{c-\frac{c}{ax}}} - \frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

Mathematica [C] time = 0.0263909, size = 44, normalized size = 0.62

$$\frac{5\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\frac{1}{ax}\right)-ax}{a\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (-(a*x) + 5*Hypergeometric2F1[-1/2, 1, 1/2, 1 - 1/(a*x)])/(a*Sqrt[c - c/(a*
x)])
```

Maple [B] time = 0.144, size = 194, normalized size = 2.7

$$-\frac{x}{2\,c\,(ax-1)^2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(10\,a^{5/2}\sqrt{(ax-1)\,x}x^2+5\,\ln\left(1/2\,\frac{2\,\sqrt{(ax-1)\,x}\sqrt{a}+2\,ax-1}{\sqrt{a}}\right)x^2a^2-8\,a^{3/2}\,((ax-1)\,x)^{3/2}-20\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(1/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(10*a^(5/2)*((a*x-1)*x)^(1/2)*x^2+5*ln(1/2*(2*
((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a^2-8*a^(3/2)*((a*x-1)*x)^(
3/2)-20*a^(3/2)*((a*x-1)*x)^(1/2)*x-10*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+
2*a*x-1)/a^(1/2))*x*a+10*((a*x-1)*x)^(1/2)*a^(1/2)+5*ln(1/2*(2*((a*x-1)*x)^
(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/c/(a*x-1)^2/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2}{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*sqrt(c - c/(a*x))), x)
```

Fricas [A] time = 2.10614, size = 375, normalized size = 5.28

$$\left[\frac{5\,(ax-1)\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(a^2x^2-5\,ax\right)\sqrt{\frac{acx-c}{ax}}}{2\left(a^2cx-ac\right)},\ \frac{5\,(ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-\left(a^2x^2-\right.}{a^2cx-ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(5*(a*x - 1)*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) + c) - 2*(a^2*x^2 - 5*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c), (5
*(a*x - 1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (a^2*x^2 -
 5*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{ax\sqrt{c-\frac{c}{ax}}-\sqrt{c-\frac{c}{ax}}}\,dx-\int \frac{1}{ax\sqrt{c-\frac{c}{ax}}-\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a/x)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x*sqrt(c - c/(a*x)) - sqrt(c - c/(a*x))), x) - Integral(1/
(a*x*sqrt(c - c/(a*x)) - sqrt(c - c/(a*x))), x)
```

Giac [B] time = 1.25152, size = 166, normalized size = 2.34

$$ac\left(\frac{5\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c}+\frac{4c-\frac{5(acx-c)}{ax}}{\left(c\sqrt{\frac{acx-c}{ax}}-\frac{(acx-c)\sqrt{\frac{acx-c}{ax}}}{ax}\right)a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] a*c*(5*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c) + (4*c - 5
*(a*c*x - c)/(a*x))/((c*sqrt((a*c*x - c)/(a*x)) - (a*c*x - c)*sqrt((a*c*x -
 c)/(a*x))/(a*x))*a^2*c))
```

3.524 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}-\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{7}{ac\sqrt{c-\frac{c}{ax}}}+\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] 7/(3*a*(c - c/(a*x))^(3/2)) + 7/(a*c*Sqrt[c - c/(a*x)]) - x/(c - c/(a*x))^(
3/2) - (7*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(3/2))
```

Rubi [A] time = 0.150278, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 78, 51, 63, 208}

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}-\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{7}{ac\sqrt{c-\frac{c}{ax}}}+\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a*x))^(3/2),x]
```


```
[Out] 7/(3*a*(c - c/(a*x))^(3/2)) + 7/(a*c*Sqrt[c - c/(a*x)]) - x/(c - c/(a*x))^(
3/2) - (7*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(3/2))
```


Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{3/2}(1-ax)}\,dx$$
$$= -\frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{5/2}x}\,dx}{a}$$
$$= -\frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx}{a}$$
$$= \frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{a}$$
$$= -\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{(7c)\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{2a}$$
$$= \frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2a}$$
$$= \frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7}{ac\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2ac}$$
$$= \frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7}{ac\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)}{c^2}$$
$$= \frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7}{ac\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}$$

Mathematica [C] time = 0.0278301, size = 55, normalized size = 0.57

$$\frac{x\left(7\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\frac{1}{ax}\right)-3ax\right)}{3c(ax-1)\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] (x*(-3*a*x + 7*Hypergeometric2F1[-3/2, 1, -1/2, 1 - 1/(a*x)]))/(3*c*Sqrt[c
 - c/(a*x)]*(-1 + a*x))
```

Maple [B] time = 0.128, size = 260, normalized size = 2.7

$$-\frac{x}{6c^2(ax-1)^3}\sqrt{\frac{c(ax-1)}{ax}}\left(42a^{7/2}\sqrt{(ax-1)x}x^3+21\ln\left(1/2\,\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{\sqrt{a}}\right)x^3a^3-36a^{5/2}((ax-1)x)^{3/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(3/2),x)
```

```
[Out] -1/6*(c*(a*x-1)/a/x)^(1/2)*x*(42*a^(7/2)*((a*x-1)*x)^(1/2)*x^3+21*ln(1/2*(2
*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^3*a^3-36*a^(5/2)*((a*x-1)*x)
^(3/2)*x-126*a^(5/2)*((a*x-1)*x)^(1/2)*x^2-63*ln(1/2*(2*((a*x-1)*x)^(1/2)*a
^(1/2)+2*a*x-1)/a^(1/2))*x^2*a^2+28*a^(3/2)*((a*x-1)*x)^(3/2)+126*a^(3/2)*(
(a*x-1)*x)^(1/2)*x+63*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))
*x*a-42*((a*x-1)*x)^(1/2)*a^(1/2)-21*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*
a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/c^2/(a*x-1)^3/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2}{(a^2x^2-1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a*x))^(3/2)), x)
```

Fricas [A] time = 1.89251, size = 513, normalized size = 5.34

$$\left[\frac{21\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(3\,a^3x^3-28\,a^2x^2+21\,ax\right)\sqrt{\frac{acx-c}{ax}}}{6\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)},\frac{21\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(21*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a
*c*x - c)/(a*x)) + c) - 2*(3*a^3*x^3 - 28*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c
)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), 1/3*(21*(a^2*x^2 - 2*a*x + 1
)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (3*a^3*x^3 - 28*a^2
*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{acx\sqrt{c-\frac{c}{ax}}-2c\sqrt{c-\frac{c}{ax}}+\frac{c\sqrt{c-\frac{c}{ax}}}{ax}}\,dx-\int \frac{1}{acx\sqrt{c-\frac{c}{ax}}-2c\sqrt{c-\frac{c}{ax}}+\frac{c\sqrt{c-\frac{c}{ax}}}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a/x)**(3/2),x)
```

```
[Out] -Integral(a*x/(a*c*x*sqrt(c - c/(a*x)) - 2*c*sqrt(c - c/(a*x)) + c*sqrt(c -
 c/(a*x))/(a*x)), x) - Integral(1/(a*c*x*sqrt(c - c/(a*x)) - 2*c*sqrt(c - c
/(a*x)) + c*sqrt(c - c/(a*x))/(a*x)), x)
```

Giac [A] time = 1.28289, size = 193, normalized size = 2.01

$$\frac{1}{3}\,ac\left(\frac{2\left(2\,c+\frac{9\,(acx-c)}{ax}\right)x}{(acx-c)ac^2\sqrt{\frac{acx-c}{ax}}}+\frac{21\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^2}-\frac{3\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*a*c*(2*(2*c + 9*(a*c*x - c)/(a*x))*x/((a*c*x - c)*a*c^2*sqrt((a*c*x - c
)/(a*x))) + 21*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^2)
- 3*sqrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c^2))
```

3.525 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=119

$$\frac{9}{ac^2\sqrt{c-\frac{c}{ax}}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}-\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] 9/(5*a*(c - c/(a*x))^(5/2)) + 3/(a*c*(c - c/(a*x))^(3/2)) + 9/(a*c^2*Sqrt[c
 - c/(a*x)]) - x/(c - c/(a*x))^(5/2) - (9*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]
])/(a*c^(5/2))
```

Rubi [A] time = 0.159088, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 78, 51, 63, 208}

$$\frac{9}{ac^2\sqrt{c-\frac{c}{ax}}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}-\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] 9/(5*a*(c - c/(a*x))^(5/2)) + 3/(a*c*(c - c/(a*x))^(3/2)) + 9/(a*c^2*Sqrt[c
 - c/(a*x)]) - x/(c - c/(a*x))^(5/2) - (9*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]
])/(a*c^(5/2))
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
```

```
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{5/2}(1-ax)}\,dx$$
$$= -\frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{7/2}x}\,dx}{a}$$
$$= -\frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx}{a}$$
$$= \frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= -\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{(9c)\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{9\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{9\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2ac}$$
$$= \frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{9\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$
$$= \frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^3}$$
$$= \frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}$$

Mathematica [C] time = 0.0298084, size = 59, normalized size = 0.5

$$\frac{9\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-\frac{1}{ax}\right)}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] -(x/(c - c/(a*x))^(5/2)) + (9*Hypergeometric2F1[-5/2, 1, -3/2, 1 - 1/(a*x)]
)/(5*a*(c - c/(a*x))^(5/2))
```

Maple [B] time = 0.129, size = 328, normalized size = 2.8

$$-\frac{x}{10 c^3 (ax-1)^4}\sqrt{\frac{c(ax-1)}{ax}}\left(90 a^{9/2}\sqrt{(ax-1)x}x^4+45\ln\left(1/2\,\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{\sqrt{a}}\right)x^4a^4-80a^{7/2}((ax-1)x)^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(5/2),x)
```

```
[Out] -1/10*(c*(a*x-1)/a/x)^(1/2)*x*(90*a^(9/2)*((a*x-1)*x)^(1/2)*x^4+45*ln(1/2*(
2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^4*a^4-80*a^(7/2)*((a*x-1)*x
)^(3/2)*x^2-360*a^(7/2)*((a*x-1)*x)^(1/2)*x^3-180*ln(1/2*(2*((a*x-1)*x)^(1/
2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^3*a^3+132*a^(5/2)*((a*x-1)*x)^(3/2)*x+540*a^
(5/2)*((a*x-1)*x)^(1/2)*x^2+270*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1
)/a^(1/2))*x^2*a^2-60*a^(3/2)*((a*x-1)*x)^(3/2)-360*a^(3/2)*((a*x-1)*x)^(1/
2)*x-180*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x*a+90*((a*x
-1)*x)^(1/2)*a^(1/2)+45*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2
)))/((a*x-1)*x)^(1/2)/c^3/(a*x-1)^4/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2}{(a^2x^2-1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a*x))^(5/2)), x)
```

Fricas [A] time = 1.83678, size = 628, normalized size = 5.28

$$\left[\frac{45\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-2acx+2a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(5a^4x^4-69a^3x^3+105a^2x^2-45ax\right)\sqrt{\frac{acx-c}{ax}}\ 4}{10\left(a^4c^3x^3-3a^3c^3x^2+3a^2c^3x-ac^3\right)},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/10*(45*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-2*a*c*x + 2*a*sqrt
(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 2*(5*a^4*x^4 - 69*a^3*x^3 + 105*a^2*x^
2 - 45*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c
^3*x - a*c^3), 1/5*(45*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(sq
rt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (5*a^4*x^4 - 69*a^3*x^3 + 105*a^2*x^2 -
 45*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*
x - a*c^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2878, size = 220, normalized size = 1.85

$$\frac{1}{5}\,ac\left(\frac{2\left(2\,c^2+\frac{5\,(acx-c)c}{ax}+\frac{20\,(acx-c)^2}{a^2x^2}\right)x^2}{(acx-c)^2c^3\sqrt{\frac{acx-c}{ax}}}+\frac{45\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^3}-\frac{5\,\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/5*a*c*(2*(2*c^2 + 5*(a*c*x - c)*c/(a*x) + 20*(a*c*x - c)^2/(a^2*x^2))*x^2
/((a*c*x - c)^2*c^3*sqrt((a*c*x - c)/(a*x))) + 45*arctan(sqrt((a*c*x - c)/(
a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^3) - 5*sqrt((a*c*x - c)/(a*x))/(a^2*(c - (a
*c*x - c)/(a*x))*c^3))
```

3.526 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=146

$$\frac{11}{ac^3\sqrt{c-\frac{c}{ax}}}+\frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{11\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}-\frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}}$$

```
[Out] 11/(7*a*(c - c/(a*x))^(7/2)) + 11/(5*a*c*(c - c/(a*x))^(5/2)) + 11/(3*a*c^2
*(c - c/(a*x))^(3/2)) + 11/(a*c^3*Sqrt[c - c/(a*x)]) - x/(c - c/(a*x))^(7/2
) - (11*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(7/2))
```

Rubi [A] time = 0.189725, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 78, 51, 63, 208}

$$\frac{11}{ac^3\sqrt{c-\frac{c}{ax}}}+\frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{11\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}-\frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a*x))^(7/2),x]
```

```
[Out] 11/(7*a*(c - c/(a*x))^(7/2)) + 11/(5*a*c*(c - c/(a*x))^(5/2)) + 11/(3*a*c^2
*(c - c/(a*x))^(3/2)) + 11/(a*c^3*Sqrt[c - c/(a*x)]) - x/(c - c/(a*x))^(7/2
) - (11*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(7/2))
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
```

```
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{7/2}(1-ax)}\,dx$$

$$= -\frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{9/2}x}\,dx}{a}$$

$$= -\frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx}{a}$$

$$= \frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{(11c)\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= \frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= \frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11\,\text{Subst}\left(\int \frac{1}{x\sqrt{c \ldots}} \ldots\right)}{2ac^3}$$

$$= \frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\,\text{Subst}\left(\int \frac{1}{a-\ldots} \ldots\right)}{c \ldots}$$

$$= \frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\tanh^{-1}\left(\frac{\sqrt{c-\ldots}}{\sqrt{c}}\right)}{ac^{7/2}}$$

Mathematica [C] time = 0.0395115, size = 46, normalized size = 0.32

$$\frac{\frac{11\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},1-\frac{1}{ax}\right)}{a} - 7x}{7\left(c-\frac{c}{ax}\right)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a*x))^(7/2),x]
```

```
[Out] (-7*x + (11*Hypergeometric2F1[-7/2, 1, -5/2, 1 - 1/(a*x)])/a)/(7*(c - c/(a*
x))^(7/2))
```

Maple [B] time = 0.139, size = 396, normalized size = 2.7

$$-\frac{x}{210\,c^4\,(ax-1)^5}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2310\,a^{11/2}\sqrt{(ax-1)\,x}x^5+1155\,\ln\left(1/2\,\frac{2\,\sqrt{(ax-1)\,x}\sqrt{a}+2\,ax-1}{\sqrt{a}}\right)x^5a^5-2100\,a^{9/2}\,((ax-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(7/2),x)
```

```
[Out] -1/210*(c*(a*x-1)/a/x)^(1/2)*x*(2310*a^(11/2)*((a*x-1)*x)^(1/2)*x^5+1155*ln
(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^5*a^5-2100*a^(9/2)*((
a*x-1)*x)^(3/2)*x^3-11550*a^(9/2)*((a*x-1)*x)^(1/2)*x^4-5775*ln(1/2*(2*((a*
x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^4*a^4+5368*a^(7/2)*((a*x-1)*x)^(3
/2)*x^2+23100*a^(7/2)*((a*x-1)*x)^(1/2)*x^3+11550*ln(1/2*(2*((a*x-1)*x)^(1/
2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^3*a^3-4928*a^(5/2)*((a*x-1)*x)^(3/2)*x-23100
*a^(5/2)*((a*x-1)*x)^(1/2)*x^2-11550*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*
a*x-1)/a^(1/2))*x^2*a^2+1540*a^(3/2)*((a*x-1)*x)^(3/2)+11550*a^(3/2)*((a*x-
1)*x)^(1/2)*x+5775*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x*
a-2310*((a*x-1)*x)^(1/2)*a^(1/2)-1155*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2
*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/c^4/(a*x-1)^5/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(ax+1)^2}{\left(a^2x^2-1\right)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a*x))^(7/2)), x)
```

Fricas [A] time = 1.77269, size = 772, normalized size = 5.29

$$\left[\frac{1155\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(105\,a^5x^5-1936\,a^4x^4+4466\,a^3x^3-38\right.}{210\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/210*(1155*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*log(-2*a
*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 2*(105*a^5*x^5 - 1936*a
^4*x^4 + 4466*a^3*x^3 - 3850*a^2*x^2 + 1155*a*x)*sqrt((a*c*x - c)/(a*x)))/(
a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4), 1/105*(
1155*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*arctan(sqrt(-c)
*sqrt((a*c*x - c)/(a*x))/c) - (105*a^5*x^5 - 1936*a^4*x^4 + 4466*a^3*x^3 -
3850*a^2*x^2 + 1155*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^5*c^4*x^4 - 4*a^4*c^4*
```

```
x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a/x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31346, size = 250, normalized size = 1.71

$$\frac{1}{105}\,ac\left(\frac{2\left(30\,c^3+\frac{63\,(acx-c)c^2}{ax}+\frac{140\,(acx-c)^2c}{a^2x^2}+\frac{525\,(acx-c)^3}{a^3x^3}\right)ax^3}{(acx-c)^3c^4\sqrt{\frac{acx-c}{ax}}}+\frac{1155\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^4}-\frac{105\,\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] 1/105*a*c*(2*(30*c^3 + 63*(a*c*x - c)*c^2/(a*x) + 140*(a*c*x - c)^2*c/(a^2*
x^2) + 525*(a*c*x - c)^3/(a^3*x^3))*a*x^3/((a*c*x - c)^3*c^4*sqrt((a*c*x -
c)/(a*x))) + 1155*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^
4) - 105*sqrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c^4))
```

3.527 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=223

$$\frac{3a^2x^3(6-17ax)(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35(1-ax)^{9/2}}-\frac{3a^3x^4\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{9/2}}{(1-ax)^{9/2}}+\frac{3a^{7/2}x^{9/2}\left(c-\frac{c}{ax}\right)^{9/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{9/2}}+\frac{6ax^2(ax+1)^{3}}{35(1-}$$

```
[Out] (-3*a^3*(c - c/(a*x))^(9/2)*x^4*Sqrt[1 + a*x])/(1 - a*x)^(9/2) + (3*a^2*(c
- c/(a*x))^(9/2)*x^3*(6 - 17*a*x)*(1 + a*x)^(3/2))/(35*(1 - a*x)^(9/2)) + (
6*a*(c - c/(a*x))^(9/2)*x^2*(1 + a*x)^(3/2))/(35*(1 - a*x)^(5/2)) - (2*(c -
 c/(a*x))^(9/2)*x*(1 + a*x)^(3/2))/(7*(1 - a*x)^(3/2)) + (3*a^(7/2)*(c - c/
(a*x))^(9/2)*x^(9/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(9/2)
```

Rubi [A] time = 0.183927, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6134, 6129, 97, 150, 143, 47, 54, 215}

$$\frac{3a^2x^3(6-17ax)(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35(1-ax)^{9/2}}-\frac{3a^3x^4\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{9/2}}{(1-ax)^{9/2}}+\frac{3a^{7/2}x^{9/2}\left(c-\frac{c}{ax}\right)^{9/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{9/2}}+\frac{6ax^2(ax+1)^{3}}{35(1-}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a*x))^(9/2),x]
```


```
[Out] (-3*a^3*(c - c/(a*x))^(9/2)*x^4*Sqrt[1 + a*x])/(1 - a*x)^(9/2) + (3*a^2*(c
- c/(a*x))^(9/2)*x^3*(6 - 17*a*x)*(1 + a*x)^(3/2))/(35*(1 - a*x)^(9/2)) + (
6*a*(c - c/(a*x))^(9/2)*x^2*(1 + a*x)^(3/2))/(35*(1 - a*x)^(5/2)) - (2*(c -
 c/(a*x))^(9/2)*x*(1 + a*x)^(3/2))/(7*(1 - a*x)^(3/2)) + (3*a^(7/2)*(c - c/
(a*x))^(9/2)*x^(9/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(9/2)
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{e^{3\tanh^{-1}(ax)}(1-ax)^{9/2}}{x^{9/2}}dx}{(1-ax)^{9/2}}$$
$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{(1-ax)^3(1+ax)^{3/2}}{x^{9/2}}dx}{(1-ax)^{9/2}}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{9/2}x(1+ax)^{3/2}}{7(1-ax)^{3/2}} + \frac{\left(2\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{(1-ax)^2\sqrt{1+ax}\left(-\frac{3a}{2}-\frac{9a^2x}{2}\right)}{x^{7/2}}dx}{7(1-ax)^{9/2}}$$
$$= \frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^2(1+ax)^{3/2}}{35(1-ax)^{5/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x(1+ax)^{3/2}}{7(1-ax)^{3/2}} + \frac{\left(4\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{(1-ax)\sqrt{1+ax}}{x}}{35(1-ax)^{9/2}}$$
$$= \frac{3a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3(6-17ax)(1+ax)^{3/2}}{35(1-ax)^{9/2}} + \frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^2(1+ax)^{3/2}}{35(1-ax)^{5/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x(1+}{7(1-ax)^{3/2}}$$
$$= -\frac{3a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4\sqrt{1+ax}}{(1-ax)^{9/2}} + \frac{3a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3(6-17ax)(1+ax)^{3/2}}{35(1-ax)^{9/2}} + \frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^2(1}{35(1-ax)^5}$$
$$= -\frac{3a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4\sqrt{1+ax}}{(1-ax)^{9/2}} + \frac{3a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3(6-17ax)(1+ax)^{3/2}}{35(1-ax)^{9/2}} + \frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^2(1}{35(1-ax)^5}$$
$$= -\frac{3a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4\sqrt{1+ax}}{(1-ax)^{9/2}} + \frac{3a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3(6-17ax)(1+ax)^{3/2}}{35(1-ax)^{9/2}} + \frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^2(1}{35(1-ax)^5}$$

Mathematica [C] time = 0.0637342, size = 85, normalized size = 0.38

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(35a^2x^2\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},-ax\right)+\left(35a^2x^2-46ax+10\right)(ax+1)^{5/2}\right)}{35a^4x^3\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] -(c^4*Sqrt[c - c/(a*x)]*((1 + a*x)^(5/2)*(10 - 46*a*x + 35*a^2*x^2) + 35*a^
2*x^2*Hypergeometric2F1[-3/2, -3/2, -1/2, -(a*x)]))/(35*a^4*x^3*Sqrt[1 - a*
x])
```

Maple [A] time = 0.137, size = 172, normalized size = 0.8

$$\frac{c^4}{70\,x^3\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(70\,a^{9/2}\sqrt{-(ax+1)\,x}x^4+105\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)x^4a^4+328\,a^{7/2}x^3\sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(9/2),x)
```

```
[Out] 1/70*(c*(a*x-1)/a/x)^(1/2)/x^3*c^4/a^(9/2)*(-a^2*x^2+1)^(1/2)*(70*a^(9/2)*(
-(a*x+1)*x)^(1/2)*x^4+105*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*
x^4*a^4+328*a^(7/2)*x^3*(-(a*x+1)*x)^(1/2)-24*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2
)-52*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+20*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-
```

```
(a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(9/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a*x))^(9/2)/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 2.15417, size = 802, normalized size = 3.6

$$\left[\frac{105\left(a^4c^4x^4-a^3c^4x^3\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(35a^4c^4x^4+164a^3c^4x^3-12a^2c^4x^2-\right.}{140\left(a^5x^4-a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(9/2),x, algorithm="fricas
")
```

```
[Out] [1/140*(105*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*
x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))
 - c)/(a*x - 1)) + 4*(35*a^4*c^4*x^4 + 164*a^3*c^4*x^3 - 12*a^2*c^4*x^2 - 2
6*a*c^4*x + 10*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 -
a^4*x^3), -1/70*(105*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*arctan(2*sqrt(-a^2
*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) -
2*(35*a^4*c^4*x^4 + 164*a^3*c^4*x^3 - 12*a^2*c^4*x^2 - 26*a*c^4*x + 10*c^4)
*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a*x))^(9/2)/(-a^2*x^2 + 1)^(3/2), x)
```

3.528 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$

Optimal. Leaf size=217

$$-\frac{a^3x^4\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{7/2}}{(1-ax)^{7/2}}+\frac{2a^2x^3(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3(1-ax)^{7/2}}-\frac{a^{5/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{7/2}}+\frac{4ax^2(ax+1)^{5/2}\left(c-\frac{c}{a}\right.}{3(1-ax)^{7/2}}$$

```
[Out] -((a^3*(c - c/(a*x))^(7/2)*x^4*Sqrt[1 + a*x])/(1 - a*x)^(7/2)) + (2*a^2*(c
- c/(a*x))^(7/2)*x^3*(1 + a*x)^(3/2))/(3*(1 - a*x)^(7/2)) - (2*(c - c/(a*x)
)^(7/2)*x*(1 + a*x)^(5/2))/(5*(1 - a*x)^(7/2)) + (4*a*(c - c/(a*x))^(7/2)*x
^2*(1 + a*x)^(5/2))/(3*(1 - a*x)^(7/2)) - (a^(5/2)*(c - c/(a*x))^(7/2)*x^(7
/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(7/2)
```

Rubi [A] time = 0.169269, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6134, 6129, 89, 78, 47, 50, 54, 215}

$$-\frac{a^3x^4\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{7/2}}{(1-ax)^{7/2}}+\frac{2a^2x^3(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3(1-ax)^{7/2}}-\frac{a^{5/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{7/2}}+\frac{4ax^2(ax+1)^{5/2}\left(c-\frac{c}{a}\right.}{3(1-ax)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a*x))^(7/2),x]
```


```
[Out] -((a^3*(c - c/(a*x))^(7/2)*x^4*Sqrt[1 + a*x])/(1 - a*x)^(7/2)) + (2*a^2*(c
- c/(a*x))^(7/2)*x^3*(1 + a*x)^(3/2))/(3*(1 - a*x)^(7/2)) - (2*(c - c/(a*x)
)^(7/2)*x*(1 + a*x)^(5/2))/(5*(1 - a*x)^(7/2)) + (4*a*(c - c/(a*x))^(7/2)*x
^2*(1 + a*x)^(5/2))/(3*(1 - a*x)^(7/2)) - (a^(5/2)*(c - c/(a*x))^(7/2)*x^(7
/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(7/2)
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{e^{3\tanh^{-1}(ax)}(1-ax)^{7/2}}{x^{7/2}}dx}{(1-ax)^{7/2}}$$
$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{(1-ax)^2(1+ax)^{3/2}}{x^{7/2}}dx}{(1-ax)^{7/2}}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x(1+ax)^{5/2}}{5(1-ax)^{7/2}} + \frac{\left(2\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{(1+ax)^{3/2}\left(-5a+\frac{5a^2x}{2}\right)}{x^{5/2}}dx}{5(1-ax)^{7/2}}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x(1+ax)^{5/2}}{5(1-ax)^{7/2}} + \frac{4a\left(c-\frac{c}{ax}\right)^{7/2}x^2(1+ax)^{5/2}}{3(1-ax)^{7/2}} - \frac{\left(a^2\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{(1+ax)}{x^{3/2}}}{3(1-ax)^{7/2}}$$
$$= \frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3(1+ax)^{3/2}}{3(1-ax)^{7/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x(1+ax)^{5/2}}{5(1-ax)^{7/2}} + \frac{4a\left(c-\frac{c}{ax}\right)^{7/2}x^2(1+ax)^{5/2}}{3(1-ax)^{7/2}}$$
$$= -\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}x^4\sqrt{1+ax}}{(1-ax)^{7/2}} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3(1+ax)^{3/2}}{3(1-ax)^{7/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x(1+ax)^{5/2}}{5(1-ax)^{7/2}} +$$
$$= -\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}x^4\sqrt{1+ax}}{(1-ax)^{7/2}} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3(1+ax)^{3/2}}{3(1-ax)^{7/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x(1+ax)^{5/2}}{5(1-ax)^{7/2}} +$$
$$= -\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}x^4\sqrt{1+ax}}{(1-ax)^{7/2}} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3(1+ax)^{3/2}}{3(1-ax)^{7/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x(1+ax)^{5/2}}{5(1-ax)^{7/2}} +$$

Mathematica [C] time = 0.0525422, size = 77, normalized size = 0.35

$$-\frac{2c^3\sqrt{c-\frac{c}{ax}}\left(5a^2x^2\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2},-ax\right)+(10ax-3)(ax+1)^{5/2}\right)}{15a^3x^2\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (-2*c^3*Sqrt[c - c/(a*x)]*((1 + a*x)^(5/2)*(-3 + 10*a*x) + 5*a^2*x^2*Hyperg
eometric2F1[-3/2, -1/2, 1/2, -(a*x)]))/(15*a^3*x^2*Sqrt[1 - a*x])
```

Maple [A] time = 0.141, size = 154, normalized size = 0.7

$$\frac{c^3}{30x^2(ax-1)}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(30a^{7/2}x^3\sqrt{-(ax+1)x}+15\arctan\left(1/2\frac{2ax+1}{\sqrt{a}\sqrt{-(ax+1)x}}\right)x^3a^3+88a^{5/2}x^2\sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(7/2),x)
```

```
[Out] 1/30*(c*(a*x-1)/a/x)^(1/2)/x^2*c^3/a^(7/2)*(-a^2*x^2+1)^(1/2)*(30*a^(7/2)*x
^3*(-(a*x+1)*x)^(1/2)+15*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x
^3*a^3+88*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2)+16*a^(3/2)*x*(-(a*x+1)*x)^(1/2)-12
*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-(a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(7/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a*x))^(7/2)/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 2.07634, size = 744, normalized size = 3.43

$$\left[\frac{15\left(a^3c^3x^3-a^2c^3x^2\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(15a^3c^3x^3+44a^2c^3x^2+8ac^3x-6c^3\right)\sqrt{-}}{60\left(a^4x^3-a^3x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(7/2),x, algorithm="fricas
")
```

```
[Out] [1/60*(15*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x
+ 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) -
 c)/(a*x - 1)) + 4*(15*a^3*c^3*x^3 + 44*a^2*c^3*x^2 + 8*a*c^3*x - 6*c^3)*sq
rt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2), -1/30*(15*(a
^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*s
qrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(15*a^3*c^3*x^3 + 44*
a^2*c^3*x^2 + 8*a*c^3*x - 6*c^3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x))
)/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a*x))^(7/2)/(-a^2*x^2 + 1)^(3/2), x)
```

3.529 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=176

$$-\frac{a^2x^3\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}{(1-ax)^{5/2}}-\frac{2x\left(1-a^2x^2\right)^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3(1-ax)^5}-\frac{a^{3/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{5/2}}+\frac{2ax^2(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3(1-ax)^{5/2}}$$

```
[Out] -((a^2*(c - c/(a*x))^(5/2)*x^3*Sqrt[1 + a*x])/(1 - a*x)^(5/2)) + (2*a*(c -
c/(a*x))^(5/2)*x^2*(1 + a*x)^(3/2))/(3*(1 - a*x)^(5/2)) - (2*(c - c/(a*x))^
(5/2)*x*(1 - a^2*x^2)^(5/2))/(3*(1 - a*x)^5) - (a^(3/2)*(c - c/(a*x))^(5/2)
*x^(5/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(5/2)
```

Rubi [A] time = 0.205523, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6134, 6128, 879, 848, 47, 50, 54, 215}

$$-\frac{a^2x^3\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}{(1-ax)^{5/2}}-\frac{2x\left(1-a^2x^2\right)^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3(1-ax)^5}-\frac{a^{3/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{5/2}}+\frac{2ax^2(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3(1-ax)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] -((a^2*(c - c/(a*x))^(5/2)*x^3*Sqrt[1 + a*x])/(1 - a*x)^(5/2)) + (2*a*(c -
c/(a*x))^(5/2)*x^2*(1 + a*x)^(3/2))/(3*(1 - a*x)^(5/2)) - (2*(c - c/(a*x))^
(5/2)*x*(1 - a^2*x^2)^(5/2))/(3*(1 - a*x)^5) - (a^(3/2)*(c - c/(a*x))^(5/2)
*x^(5/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(5/2)
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{e^{3\tanh^{-1}(ax)}(1-ax)^{5/2}}{x^{5/2}}dx}{(1-ax)^{5/2}}$$

$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{(1-a^2x^2)^{3/2}}{x^{5/2}\sqrt{1-ax}}dx}{(1-ax)^{5/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\left(1-a^2x^2\right)^{5/2}}{3(1-ax)^5} - \frac{\left(a\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{(1-a^2x^2)^{3/2}}{x^{3/2}(1-ax)^{3/2}}dx}{3(1-ax)^{5/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\left(1-a^2x^2\right)^{5/2}}{3(1-ax)^5} - \frac{\left(a\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{(1+ax)^{3/2}}{x^{3/2}}dx}{3(1-ax)^{5/2}}$$

$$= \frac{2a\left(c-\frac{c}{ax}\right)^{5/2}x^2(1+ax)^{3/2}}{3(1-ax)^{5/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\left(1-a^2x^2\right)^{5/2}}{3(1-ax)^5} - \frac{\left(a^2\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{\sqrt{1+ax}}{\sqrt{x}}d}{(1-ax)^{5/2}}$$

$$= -\frac{a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3\sqrt{1+ax}}{(1-ax)^{5/2}} + \frac{2a\left(c-\frac{c}{ax}\right)^{5/2}x^2(1+ax)^{3/2}}{3(1-ax)^{5/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\left(1-a^2x^2\right)^{5/2}}{3(1-ax)^5} - $$

$$= -\frac{a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3\sqrt{1+ax}}{(1-ax)^{5/2}} + \frac{2a\left(c-\frac{c}{ax}\right)^{5/2}x^2(1+ax)^{3/2}}{3(1-ax)^{5/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\left(1-a^2x^2\right)^{5/2}}{3(1-ax)^5} - $$

$$= -\frac{a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3\sqrt{1+ax}}{(1-ax)^{5/2}} + \frac{2a\left(c-\frac{c}{ax}\right)^{5/2}x^2(1+ax)^{3/2}}{3(1-ax)^{5/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\left(1-a^2x^2\right)^{5/2}}{3(1-ax)^5} - $$

Mathematica [C] time = 0.0397677, size = 66, normalized size = 0.38

$$-\frac{2c^2\sqrt{c-\frac{c}{ax}}\left((ax+1)^{5/2}-ax\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2},-ax\right)\right)}{3a^2x\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] (-2*c^2*Sqrt[c - c/(a*x)]*((1 + a*x)^(5/2) - a*x*Hypergeometric2F1[-3/2, -1
/2, 1/2, -(a*x)]))/(3*a^2*x*Sqrt[1 - a*x])
```

Maple [A] time = 0.142, size = 136, normalized size = 0.8

$$\frac{c^2}{6\,(ax-1)\,x}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(6\,a^{5/2}x^2\sqrt{-(ax+1)\,x}+4\,a^{3/2}x\sqrt{-(ax+1)\,x}-3\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)x^2a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(5/2),x)
```

```
[Out] 1/6*(c*(a*x-1)/a/x)^(1/2)*c^2*(-a^2*x^2+1)^(1/2)*(6*a^(5/2)*x^2*(-(a*x+1)*x
)^(1/2)+4*a^(3/2)*x*(-(a*x+1)*x)^(1/2)-3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*
x+1)*x)^(1/2))*x^2*a^2+4*a^(1/2)*(-(a*x+1)*x)^(1/2))/x/a^(5/2)/(a*x-1)/(-(a
*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a*x))^(5/2)/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 2.15303, size = 674, normalized size = 3.83

$$\left[\frac{3\left(a^2c^2x^2-ac^2x\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3a^2c^2x^2+2ac^2x+2c^2\right)\sqrt{-a^2x^2+1}\sqrt{\frac{ac}{a}}}{12\left(a^3x^2-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(5/2),x, algorithm="fricas
")
```

```
[Out] [1/12*(3*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(
2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(3*a^2*c^2*x^2 + 2*a*c^2*x + 2*c^2)*sqrt(-a^2*x^2 + 1)*sqrt((
a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x), 1/6*(3*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c
)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*
x^2 - a*c*x - c)) + 2*(3*a^2*c^2*x^2 + 2*a*c^2*x + 2*c^2)*sqrt(-a^2*x^2 + 1
)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(5/2),x, algorithm="giac")

[Out] integrate((a*x + 1)^3*(c - c/(a*x))^(5/2)/(-a^2*x^2 + 1)^(3/2), x)
```

3.530 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$\frac{3ax^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}+\frac{3\sqrt{a}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{3/2}}-\frac{2x(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}$$

```
[Out] (3*a*(c - c/(a*x))^(3/2)*x^2*Sqrt[1 + a*x])/(1 - a*x)^(3/2) - (2*(c - c/(a*
x))^(3/2)*x*(1 + a*x)^(3/2))/(1 - a*x)^(3/2) + (3*Sqrt[a]*(c - c/(a*x))^(3/
2)*x^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(3/2)
```

Rubi [A] time = 0.172567, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6134, 6128, 848, 47, 50, 54, 215}

$$\frac{3ax^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}+\frac{3\sqrt{a}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{3/2}}-\frac{2x(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (3*a*(c - c/(a*x))^(3/2)*x^2*Sqrt[1 + a*x])/(1 - a*x)^(3/2) - (2*(c - c/(a*
x))^(3/2)*x*(1 + a*x)^(3/2))/(1 - a*x)^(3/2) + (3*Sqrt[a]*(c - c/(a*x))^(3/
2)*x^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(3/2)
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] && !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx &= \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{3\tanh^{-1}(ax)}(1-ax)^{3/2}}{x^{3/2}}\,dx}{(1-ax)^{3/2}}\\
&= \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{\left(1-a^2x^2\right)^{3/2}}{x^{3/2}(1-ax)^{3/2}}\,dx}{(1-ax)^{3/2}}\\
&= \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{(1+ax)^{3/2}}{x^{3/2}}\,dx}{(1-ax)^{3/2}}\\
&= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x(1+ax)^{3/2}}{(1-ax)^{3/2}}+\frac{\left(3a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx}{(1-ax)^{3/2}}\\
&= \frac{3a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x(1+ax)^{3/2}}{(1-ax)^{3/2}}+\frac{\left(3a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2(1-ax)^{3/2}}\\
&= \frac{3a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x(1+ax)^{3/2}}{(1-ax)^{3/2}}+\frac{\left(3a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\cdots}}\right.}{(1-ax)^{3/2}}\\
&= \frac{3a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x(1+ax)^{3/2}}{(1-ax)^{3/2}}+\frac{3\sqrt{a}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\cdots\right.}{(1-ax)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0247577, size = 42, normalized size = 0.33

$$-\frac{2x\left(c-\frac{c}{ax}\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2},-ax\right)}{(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a*x))^(3/2),x]
```

[Out] (-2*(c - c/(a*x))^(3/2)*x*Hypergeometric2F1[-3/2, -1/2, 1/2, -(a*x)])/(1 - a*x)^(3/2)

Maple [A] time = 0.135, size = 109, normalized size = 0.9

$$-\frac{c}{2\,ax-2}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(-2\,a^{3/2}x\sqrt{-(ax+1)\,x}+3\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)xa+4\,\sqrt{a}\sqrt{-(ax+1)\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(3/2),x)

[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)*(-a^2*x^2+1)^(1/2)*(-2*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a+4*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-(a*x+1)*x)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(3/2),x, algorithm="maxima")

[Out] integrate((a*x + 1)^3*(c - c/(a*x))^(3/2)/(-a^2*x^2 + 1)^(3/2), x)

Fricas [A] time = 2.12708, size = 564, normalized size = 4.44

$$\left[\frac{3\,(acx-c)\sqrt{-c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,\sqrt{-a^2x^2+1}(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)},\ \frac{3\,(acx-c)\sqrt{c}\,\mathrm{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(3/2),x, algorithm="fricas")

[Out] [1/4*(3*(a*c*x - c)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*sqrt(-a^2*x^2 + 1)*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(3*(a*c*x - c)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*sqrt(-a^2*x^2 + 1)*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(3/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(3/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3
/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a*x))^(3/2)/(-a^2*x^2 + 1)^(3/2), x)
```

3.531 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=155

$$-\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{5\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{a}\sqrt{1-ax}}$$

```
[Out] -((Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/Sqrt[1 - a*x]) - (5*Sqrt[c - c/(a*x)]
*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x]) + (4*Sqrt[2]*Sqr
t[c - c/(a*x)]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(S
qrt[a]*Sqrt[1 - a*x])
```

Rubi [A] time = 0.15448, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6134, 6129, 102, 157, 54, 215, 93, 206}

$$-\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{5\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
```


```
[Out] -((Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/Sqrt[1 - a*x]) - (5*Sqrt[c - c/(a*x)]
*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x]) + (4*Sqrt[2]*Sqr
t[c - c/(a*x)]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(S
qrt[a]*Sqrt[1 - a*x])
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{e^{3\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{(1+ax)^{3/2}}{\sqrt{x}(1-ax)}\,dx}{\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{-\frac{3a}{2}-\frac{5a^2x}{2}}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{\left(5\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}}+\frac{\left(4\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{1}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{\left(5\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{\sqrt{1-ax}}+\frac{\left(8\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\operatorname{Subst}\left(\int\right.}{\sqrt{1-}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{5\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right)}{\sqrt{a}\sqrt{1-ax}}
\end{aligned}
$$

Mathematica [A] time = 0.0742003, size = 105, normalized size = 0.68

$$
-\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}+5\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)\right)}{\sqrt{a}\sqrt{1-ax}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x] + 5*ArcSinh[Sqr
t[a]*Sqrt[x]] - 4*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])
)/(Sqrt[a]*Sqrt[1 - a*x]))
```

Maple [A] time = 0.137, size = 165, normalized size = 1.1

$$\frac{x\sqrt{2}}{4\,ax-4}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(2\,\sqrt{-(ax+1)\,x}a^{3/2}\sqrt{2}\sqrt{-a^{-1}}-5\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)a\sqrt{2}\sqrt{-a^{-1}}+8\,\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/4*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(2*(-(a*x+1)*x)^(1/2)*a^(3/2
)*2^(1/2)*(-1/a)^(1/2)-5*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a
*2^(1/2)*(-1/a)^(1/2)+8*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a
*x-1)/(a*x-1))*a^(1/2))*2^(1/2)/(a*x-1)/(-(a*x+1)*x)^(1/2)/a^(3/2)/(-1/a)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 2.57575, size = 975, normalized size = 6.29

$$\left[\frac{4\sqrt{-a^2x^2+1}ax\sqrt{\frac{acx-c}{ax}}+4\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx+4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+5\,(ax-1)\sqrt{}}{4\left(a^2x-a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) + 4*sqrt(2)*(a*x - 1
)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x + 4*sqrt(2)*(3*a^2*x
```

```
^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3
 - 3*a^2*x^2 + 3*a*x - 1)) + 5*(a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c
*x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)
) - c)/(a*x - 1)))/(a^2*x - a), 1/2*(2*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x -
 c)/(a*x)) - 4*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1
)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + 5*(a*x
 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x
))/(2*a^2*c*x^2 - a*c*x - c)))/(a^2*x - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2)
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/(-a^2*x^2 + 1)^(3/2), x)
```

3.532 $\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=195

$$\frac{7\sqrt{1-ax}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{2}\sqrt{1-ax}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}+\frac{(ax+1)^{3/2}}{a\sqrt{1-ax}\sqrt{c-\frac{c}{ax}}}+\frac{2\sqrt{1-ax}\sqrt{ax+1}}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] (2*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(a*Sqrt[c - c/(a*x)]) + (1 + a*x)^(3/2)/(a*
Sqrt[c - c/(a*x)]*Sqrt[1 - a*x]) + (7*Sqrt[1 - a*x]*ArcSinh[Sqrt[a]*Sqrt[x]
])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x]) - (5*Sqrt[2]*Sqrt[1 - a*x]*ArcTanh[(
Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x]
)
```

Rubi [A] time = 0.170096, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6134, 6129, 97, 154, 157, 54, 215, 93, 206}

$$\frac{7\sqrt{1-ax}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{2}\sqrt{1-ax}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}+\frac{(ax+1)^{3/2}}{a\sqrt{1-ax}\sqrt{c-\frac{c}{ax}}}+\frac{2\sqrt{1-ax}\sqrt{ax+1}}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/Sqrt[c - c/(a*x)],x]
```


```
[Out] (2*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(a*Sqrt[c - c/(a*x)]) + (1 + a*x)^(3/2)/(a*
Sqrt[c - c/(a*x)]*Sqrt[1 - a*x]) + (7*Sqrt[1 - a*x]*ArcSinh[Sqrt[a]*Sqrt[x]
])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x]) - (5*Sqrt[2]*Sqrt[1 - a*x]*ArcTanh[(
Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x]
)
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
```

```
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx = \frac{\sqrt{1-ax}\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{x}}{\sqrt{1-ax}}dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= \frac{\sqrt{1-ax}\int \frac{\sqrt{x}(1+ax)^{3/2}}{(1-ax)^2}dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= \frac{(1+ax)^{3/2}}{a\sqrt{c-\frac{c}{ax}}\sqrt{1-ax}} - \frac{\sqrt{1-ax}\int \frac{\sqrt{1+ax}\left(\frac{1}{2}+2ax\right)}{\sqrt{x}(1-ax)}dx}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= \frac{2\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} + \frac{(1+ax)^{3/2}}{a\sqrt{c-\frac{c}{ax}}\sqrt{1-ax}} + \frac{\sqrt{1-ax}\int \frac{-\frac{3a}{2}-\frac{7a^2x}{2}}{\sqrt{x}(1-ax)\sqrt{1+ax}}dx}{a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= \frac{2\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} + \frac{(1+ax)^{3/2}}{a\sqrt{c-\frac{c}{ax}}\sqrt{1-ax}} + \frac{\left(7\sqrt{1-ax}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}dx}{2a\sqrt{c-\frac{c}{ax}}\sqrt{x}} - \frac{\left(5\sqrt{1-ax}\right)\int \frac{1}{\sqrt{x}(1-ax)\sqrt{1+ax}}}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= \frac{2\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} + \frac{(1+ax)^{3/2}}{a\sqrt{c-\frac{c}{ax}}\sqrt{1-ax}} + \frac{\left(7\sqrt{1-ax}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}dx, x, \sqrt{x}\right)}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}} - \frac{\left(10\sqrt{1-ax}\right)\text{S}}{}$$

$$= \frac{2\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} + \frac{(1+ax)^{3/2}}{a\sqrt{c-\frac{c}{ax}}\sqrt{1-ax}} + \frac{7\sqrt{1-ax}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{c-\frac{c}{ax}}\sqrt{x}} - \frac{5\sqrt{2}\sqrt{1-ax}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}}{\sqrt{1+a}}\right.}{a^{3/2}\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

Mathematica [A] time = 0.0990348, size = 120, normalized size = 0.62

$$\frac{\sqrt{a}\sqrt{x}\sqrt{ax+1}(3-ax)+(7-7ax)\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+5\sqrt{2}(ax-1)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{3/2}\sqrt{x}\sqrt{1-ax}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[a]*Sqrt[x]*(3 - a*x)*Sqrt[1 + a*x] + (7 - 7*a*x)*ArcSinh[Sqrt[a]*Sqrt
[x]] + 5*Sqrt[2]*(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]
])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x]*Sqrt[1 - a*x])
```

Maple [A] time = 0.146, size = 276, normalized size = 1.4

$$\frac{x\sqrt{2}}{4c(ax-1)^2}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(2\sqrt{-(ax+1)x}a^{5/2}\sqrt{2}\sqrt{-a^{-1}}x-7a^2\arctan\left(1/2\frac{2ax+1}{\sqrt{a}\sqrt{-(ax+1)x}}\right)\sqrt{2}\sqrt{-a^{-1}}x\cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(1/2),x)
```

```
[Out] 1/4*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)/c*(-a^2*x^2+1)^(1/2)*(2*(-(a*x+1)*x)^(1
/2)*a^(5/2)*2^(1/2)*(-1/a)^(1/2)*x-7*a^2*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*
x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)*x-6*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*
```

```
(-1/a)^(1/2)+10*a^(3/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a
*x-1)/(a*x-1))*x+7*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/
2)*(-1/a)^(1/2)-10*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)
/(a*x-1))*a^(1/2))*2^(1/2)/(a*x-1)^2/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))), x)
```

Fricas [A] time = 2.63296, size = 1131, normalized size = 5.8

$$\left[\frac{5\sqrt{2}\left(a^2cx^2-2\,acx+c\right)\sqrt{-\frac{1}{c}}\log\left(-\frac{17\,a^3x^3-3\,a^2x^2+4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-\frac{1}{c}}\sqrt{\frac{acx-c}{ax}}-13\,ax-1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)-7\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\log\left(-\right.}{4\left(a^3cx^2-2\,a^2cx+ac\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(1/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(5*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*sqrt(-1/c)*log(-(17*a^3*x^3 - 3*a
^2*x^2 + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-1/c)*sqrt((a*
c*x - c)/(a*x)) - 13*a*x - 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - 7*(a^2*x
^2 - 2*a*x + 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)*
sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(a^
2*x^2 - 3*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c*x^2 - 2*a
^2*c*x + a*c), 1/2*(7*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2
+ 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(a^
2*x^2 - 3*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)) - 5*sqrt(2)*(a^2*
c*x^2 - 2*a*c*x + c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x -
c)/(a*x))/((3*a^2*x^2 - 2*a*x - 1)*sqrt(c)))/sqrt(c))/(a^3*c*x^2 - 2*a^2*c*
x + a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral((a*x + 1)**3/(sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{(-a^2x^2+1)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))), x)
```

3.533 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=249

$$-\frac{9(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{51(1-ax)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{4\sqrt{2}a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{21\sqrt{ax+1}(1-ax)^{3/2}}{8a^2x\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{9(ax+1)^{3/2}\sqrt{1-ax}}{8a^2x\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{(a\cdots}{2a\sqrt{1-\cdots}}$$

```
[Out] (-21*(1 - a*x)^(3/2)*Sqrt[1 + a*x])/(8*a^2*(c - c/(a*x))^(3/2)*x) + (1 + a*
x)^(3/2)/(2*a*(c - c/(a*x))^(3/2)*Sqrt[1 - a*x]) - (9*Sqrt[1 - a*x]*(1 + a*
x)^(3/2))/(8*a^2*(c - c/(a*x))^(3/2)*x) - (9*(1 - a*x)^(3/2)*ArcSinh[Sqrt[a
]*Sqrt[x]])/(a^(5/2)*(c - c/(a*x))^(3/2)*x^(3/2)) + (51*(1 - a*x)^(3/2)*Arc
Tanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(4*Sqrt[2]*a^(5/2)*(c - c/(a
*x))^(3/2)*x^(3/2))
```

Rubi [A] time = 0.205266, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6134, 6129, 97, 149, 154, 157, 54, 215, 93, 206}

$$-\frac{9(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{51(1-ax)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{4\sqrt{2}a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{21\sqrt{ax+1}(1-ax)^{3/2}}{8a^2x\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{9(ax+1)^{3/2}\sqrt{1-ax}}{8a^2x\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{(a\cdots}{2a\sqrt{1-\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a*x))^(3/2),x]
```


```
[Out] (-21*(1 - a*x)^(3/2)*Sqrt[1 + a*x])/(8*a^2*(c - c/(a*x))^(3/2)*x) + (1 + a*
x)^(3/2)/(2*a*(c - c/(a*x))^(3/2)*Sqrt[1 - a*x]) - (9*Sqrt[1 - a*x]*(1 + a*
x)^(3/2))/(8*a^2*(c - c/(a*x))^(3/2)*x) - (9*(1 - a*x)^(3/2)*ArcSinh[Sqrt[a
]*Sqrt[x]])/(a^(5/2)*(c - c/(a*x))^(3/2)*x^(3/2)) + (51*(1 - a*x)^(3/2)*Arc
Tanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(4*Sqrt[2]*a^(5/2)*(c - c/(a
*x))^(3/2)*x^(3/2))
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{(1-ax)^{3/2}\int \frac{e^{3\tanh^{-1}(ax)}x^{3/2}}{(1-ax)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{(1-ax)^{3/2}\int \frac{x^{3/2}(1+ax)^{3/2}}{(1-ax)^3}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{(1+ax)^{3/2}}{2a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1-ax}} - \frac{(1-ax)^{3/2}\int \frac{\sqrt{x}\sqrt{1+ax}\left(\frac{3}{2}+3ax\right)}{(1-ax)^2}\,dx}{2a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{(1+ax)^{3/2}}{2a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1-ax}} - \frac{9\sqrt{1-ax}(1+ax)^{3/2}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{(1-ax)^{3/2}\int \frac{\sqrt{1+ax}\left(-\frac{9a}{4}-\frac{21a^2x}{2}\right)}{\sqrt{x}(1-ax)}\,dx}{4a^3\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{21(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{(1+ax)^{3/2}}{2a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1-ax}} - \frac{9\sqrt{1-ax}(1+ax)^{3/2}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{(1-ax)^{3/2}\int \frac{\frac{15a^2}{2}+18a^3x}{\sqrt{x}(1-ax)\sqrt{1+ax}}}{4a^4\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{21(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{(1+ax)^{3/2}}{2a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1-ax}} - \frac{9\sqrt{1-ax}(1+ax)^{3/2}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{\left(9(1-ax)^{3/2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{21(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{(1+ax)^{3/2}}{2a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1-ax}} - \frac{9\sqrt{1-ax}(1+ax)^{3/2}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{\left(9(1-ax)^{3/2}\right)\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x}$$

$$= -\frac{21(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{(1+ax)^{3/2}}{2a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1-ax}} - \frac{9\sqrt{1-ax}(1+ax)^{3/2}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{9(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

Mathematica [A] time = 0.140049, size = 139, normalized size = 0.56

$$-\frac{-2\sqrt{a}\sqrt{x}\sqrt{ax+1}\left(4a^2x^2-23ax+15\right)-72(ax-1)^2\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+51\sqrt{2}(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{8a^{3/2}c\sqrt{x}(1-ax)^{3/2}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] -(-2*Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*(15 - 23*a*x + 4*a^2*x^2) - 72*(-1 + a*x
)^2*ArcSinh[Sqrt[a]*Sqrt[x]] + 51*Sqrt[2]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqr
t[a]*Sqrt[x])/Sqrt[1 + a*x]])/(8*a^(3/2)*c*Sqrt[c - c/(a*x)]*Sqrt[x]*(1 - a
*x)^(3/2))
```

Maple [B] time = 0.148, size = 390, normalized size = 1.6

$$\frac{x\sqrt{2}}{16\,c^2\,(ax-1)^3}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(8\,\sqrt{-(ax+1)\,x}a^{7/2}\sqrt{2}\sqrt{-a^{-1}}x^2-36\,a^3\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)\sqrt{2}\sqrt{-a^{-1}}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(3/2),x)
```

```
[Out] 1/16*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)/c^2*(-a^2*x^2+1)^(1/2)*(8*(-(a*x+1)*x)
^(1/2)*a^(7/2)*2^(1/2)*(-1/a)^(1/2)*x^2-36*a^3*arctan(1/2/a^(1/2)*(2*a*x+1)
/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)*x^2-46*(-(a*x+1)*x)^(1/2)*a^(5/2)
*2^(1/2)*(-1/a)^(1/2)*x+51*a^(5/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^
(1/2)*a-3*a*x-1)/(a*x-1))*x^2+72*a^2*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)
*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)*x+30*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1
/a)^(1/2)-102*a^(3/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x
-1)/(a*x-1))*x-36*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2
)*(-1/a)^(1/2)+51*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/
(a*x-1))*a^(1/2))*2^(1/2)/(a*x-1)^3/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(3/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(3/2)), x)
```

3.534 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=293

$$\frac{103\sqrt{ax+1}(1-ax)^{5/2}}{32a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{43(ax+1)^{3/2}(1-ax)^{3/2}}{32a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{249(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{16\sqrt{2}a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}-$$

```
[Out] (103*(1 - a*x)^(5/2)*Sqrt[1 + a*x])/(32*a^3*(c - c/(a*x))^(5/2)*x^2) + (1 +
 a*x)^(3/2)/(3*a*(c - c/(a*x))^(5/2)*Sqrt[1 - a*x]) - (13*Sqrt[1 - a*x]*(1
+ a*x)^(3/2))/(24*a^2*(c - c/(a*x))^(5/2)*x) + (43*(1 - a*x)^(3/2)*(1 + a*x
)^(3/2))/(32*a^3*(c - c/(a*x))^(5/2)*x^2) + (11*(1 - a*x)^(5/2)*ArcSinh[Sqr
t[a]*Sqrt[x]])/(a^(7/2)*(c - c/(a*x))^(5/2)*x^(5/2)) - (249*(1 - a*x)^(5/2)
*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(16*Sqrt[2]*a^(7/2)*(c -
 c/(a*x))^(5/2)*x^(5/2))
```

Rubi [A] time = 0.231221, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6134, 6129, 97, 149, 154, 157, 54, 215, 93, 206}

$$\frac{103\sqrt{ax+1}(1-ax)^{5/2}}{32a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{43(ax+1)^{3/2}(1-ax)^{3/2}}{32a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{249(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{16\sqrt{2}a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a*x))^(5/2),x]
```


```
[Out] (103*(1 - a*x)^(5/2)*Sqrt[1 + a*x])/(32*a^3*(c - c/(a*x))^(5/2)*x^2) + (1 +
 a*x)^(3/2)/(3*a*(c - c/(a*x))^(5/2)*Sqrt[1 - a*x]) - (13*Sqrt[1 - a*x]*(1
+ a*x)^(3/2))/(24*a^2*(c - c/(a*x))^(5/2)*x) + (43*(1 - a*x)^(3/2)*(1 + a*x
)^(3/2))/(32*a^3*(c - c/(a*x))^(5/2)*x^2) + (11*(1 - a*x)^(5/2)*ArcSinh[Sqr
t[a]*Sqrt[x]])/(a^(7/2)*(c - c/(a*x))^(5/2)*x^(5/2)) - (249*(1 - a*x)^(5/2)
*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(16*Sqrt[2]*a^(7/2)*(c -
 c/(a*x))^(5/2)*x^(5/2))
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
```

```
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{(1-ax)^{5/2}\int \frac{e^{3\tanh^{-1}(ax)}x^{5/2}}{(1-ax)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{5/2}\int \frac{x^{5/2}(1+ax)^{3/2}}{(1-ax)^4}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1+ax)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{5/2}\sqrt{1-ax}} - \frac{(1-ax)^{5/2}\int \frac{x^{3/2}\sqrt{1+ax}\left(\frac{5}{2}+4ax\right)}{(1-ax)^3}\,dx}{3a\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1+ax)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{5/2}\sqrt{1-ax}} - \frac{13\sqrt{1-ax}(1+ax)^{3/2}}{24a^2\left(c-\frac{c}{ax}\right)^{5/2}x} - \frac{(1-ax)^{5/2}\int \frac{\sqrt{x}\sqrt{1+ax}\left(-\frac{39a}{4}-\frac{45a^2x}{2}\right)}{(1-ax)^2}\,dx}{12a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1+ax)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{5/2}\sqrt{1-ax}} - \frac{13\sqrt{1-ax}(1+ax)^{3/2}}{24a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{43(1-ax)^{3/2}(1+ax)^{3/2}}{32a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} - \frac{(1-ax)^{5/2}\int \frac{\sqrt{1+ax}\left(\frac{129}{8}\cdots\right.}{\sqrt{x}(1\cdots}}{24a^5\left(c-\frac{c}{ax}\right)^{5/\cdots}}$$

$$= \frac{103(1-ax)^{5/2}\sqrt{1+ax}}{32a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1+ax)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{5/2}\sqrt{1-ax}} - \frac{13\sqrt{1-ax}(1+ax)^{3/2}}{24a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{43(1-ax)^{3/2}(1+ax)^{3/\cdots}}{32a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2}$$

$$= \frac{103(1-ax)^{5/2}\sqrt{1+ax}}{32a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1+ax)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{5/2}\sqrt{1-ax}} - \frac{13\sqrt{1-ax}(1+ax)^{3/2}}{24a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{43(1-ax)^{3/2}(1+ax)^{3/\cdots}}{32a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2}$$

$$= \frac{103(1-ax)^{5/2}\sqrt{1+ax}}{32a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1+ax)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{5/2}\sqrt{1-ax}} - \frac{13\sqrt{1-ax}(1+ax)^{3/2}}{24a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{43(1-ax)^{3/2}(1+ax)^{3/\cdots}}{32a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2}$$

$$= \frac{103(1-ax)^{5/2}\sqrt{1+ax}}{32a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1+ax)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{5/2}\sqrt{1-ax}} - \frac{13\sqrt{1-ax}(1+ax)^{3/2}}{24a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{43(1-ax)^{3/2}(1+ax)^{3/\cdots}}{32a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2}$$

Mathematica [A] time = 0.160495, size = 147, normalized size = 0.5

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{ax+1}\left(-48a^3x^3+415a^2x^2-554ax+219\right)-1056(ax-1)^3\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+747\sqrt{2}(ax-1)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cdots}{\sqrt{ax+\cdots}}\right.}{96a^{3/2}c^2\sqrt{x}(1-ax)^{5/2}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*(219 - 554*a*x + 415*a^2*x^2 - 48*a^3*x^3)
 - 1056*(-1 + a*x)^3*ArcSinh[Sqrt[a]*Sqrt[x]] + 747*Sqrt[2]*(-1 + a*x)^3*Ar
cTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(96*a^(3/2)*c^2*Sqrt[c - c/
(a*x)]*Sqrt[x]*(1 - a*x)^(5/2))
```

Maple [B] time = 0.177, size = 504, normalized size = 1.7

$$\frac{x\sqrt{2}}{192\,c^3\,(ax-1)^4}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(96\,a^{9/2}\sqrt{2}\sqrt{-a^{-1}}\sqrt{-(ax+1)\,x}x^3-830\,\sqrt{-(ax+1)\,x}a^{7/2}\sqrt{2}\sqrt{-a^{-1}}x^2+747\,a\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(5/2),x)
```

```
[Out] 1/192*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(96*a^(9/2)*2^(1/2)*(-1/a)
^(1/2)*(-(a*x+1)*x)^(1/2)*x^3-830*(-(a*x+1)*x)^(1/2)*a^(7/2)*2^(1/2)*(-1/a)
^(1/2)*x^2+747*a^(7/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*
x-1)/(a*x-1))*x^3-528*a^4*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*
2^(1/2)*(-1/a)^(1/2)*x^3+1108*(-(a*x+1)*x)^(1/2)*a^(5/2)*2^(1/2)*(-1/a)^(1/
2)*x-2241*a^(5/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/
(a*x-1))*x^2+1584*a^3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1
/2)*(-1/a)^(1/2)*x^2-438*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2)-15
84*a^2*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2
)*x+2241*a^(3/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(
a*x-1))*x+528*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-
1/a)^(1/2)-747*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*
x-1))*a^(1/2))*2^(1/2)/a^(3/2)/c^3/(a*x-1)^4/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{(-a^2x^2+1)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(5/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{(-a^2x^2+1)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(5/2)), x)
```

3.535 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=225

$$\frac{a^3x^4\sqrt{ax+1}(521ax+2718)\left(c-\frac{c}{ax}\right)^{9/2}}{105(1-ax)^{9/2}}-\frac{94a^2x^3\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{9/2}}{21(1-ax)^{5/2}}+\frac{11a^{7/2}x^{9/2}\left(c-\frac{c}{ax}\right)^{9/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{9/2}}+\frac{6ax^2\sqrt{ax\cdots}}{5(1\cdots}$$

```
[Out] (-94*a^2*(c - c/(a*x))^(9/2)*x^3*Sqrt[1 + a*x])/(21*(1 - a*x)^(5/2)) + (6*a
*(c - c/(a*x))^(9/2)*x^2*Sqrt[1 + a*x])/(5*(1 - a*x)^(3/2)) - (2*(c - c/(a*
x))^(9/2)*x*Sqrt[1 + a*x])/(7*Sqrt[1 - a*x]) + (a^3*(c - c/(a*x))^(9/2)*x^4
*Sqrt[1 + a*x]*(2718 + 521*a*x))/(105*(1 - a*x)^(9/2)) + (11*a^(7/2)*(c - c
/(a*x))^(9/2)*x^(9/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(9/2)
```

Rubi [A] time = 0.18765, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6134, 6129, 98, 150, 143, 54, 215}

$$\frac{a^3x^4\sqrt{ax+1}(521ax+2718)\left(c-\frac{c}{ax}\right)^{9/2}}{105(1-ax)^{9/2}}-\frac{94a^2x^3\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{9/2}}{21(1-ax)^{5/2}}+\frac{11a^{7/2}x^{9/2}\left(c-\frac{c}{ax}\right)^{9/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{9/2}}+\frac{6ax^2\sqrt{ax\cdots}}{5(1\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(9/2)/E^ArcTanh[a*x],x]
```



```
[Out] (-94*a^2*(c - c/(a*x))^(9/2)*x^3*Sqrt[1 + a*x])/(21*(1 - a*x)^(5/2)) + (6*a
*(c - c/(a*x))^(9/2)*x^2*Sqrt[1 + a*x])/(5*(1 - a*x)^(3/2)) - (2*(c - c/(a*
x))^(9/2)*x*Sqrt[1 + a*x])/(7*Sqrt[1 - a*x]) + (a^3*(c - c/(a*x))^(9/2)*x^4
*Sqrt[1 + a*x]*(2718 + 521*a*x))/(105*(1 - a*x)^(9/2)) + (11*a^(7/2)*(c - c
/(a*x))^(9/2)*x^(9/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(9/2)
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{e^{-\tanh^{-1}(ax)}(1-ax)^{9/2}}{x^{9/2}}dx}{(1-ax)^{9/2}}$$

$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{(1-ax)^5}{x^{9/2}\sqrt{1+ax}}dx}{(1-ax)^{9/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+ax}}{7\sqrt{1-ax}} - \frac{\left(2\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{(1-ax)^3\left(\frac{21a}{2}-\frac{5a^2x}{2}\right)}{x^{7/2}\sqrt{1+ax}}dx}{7(1-ax)^{9/2}}$$

$$= \frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^2\sqrt{1+ax}}{5(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+ax}}{7\sqrt{1-ax}} - \frac{\left(4\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{(1-ax)^2\left(-\frac{235a^2}{4}-17\cdots\right.}{x^{5/2}\sqrt{1+ax}}\cdots}{35(1-ax)^{9/2}}$$

$$= -\frac{94a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3\sqrt{1+ax}}{21(1-ax)^{5/2}} + \frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^2\sqrt{1+ax}}{5(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+ax}}{7\sqrt{1-ax}} - \frac{\left(8\left(c\cdots\right.\right.}{\cdots}$$

$$= -\frac{94a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3\sqrt{1+ax}}{21(1-ax)^{5/2}} + \frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^2\sqrt{1+ax}}{5(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+ax}}{7\sqrt{1-ax}} + \frac{a^3\left(c\cdots\right.}{\cdots}$$

$$= -\frac{94a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3\sqrt{1+ax}}{21(1-ax)^{5/2}} + \frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^2\sqrt{1+ax}}{5(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+ax}}{7\sqrt{1-ax}} + \frac{a^3\left(c\cdots\right.}{\cdots}$$

$$= -\frac{94a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3\sqrt{1+ax}}{21(1-ax)^{5/2}} + \frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^2\sqrt{1+ax}}{5(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+ax}}{7\sqrt{1-ax}} + \frac{a^3\left(c\cdots\right.}{\cdots}$$

Mathematica [A] time = 2.63019, size = 108, normalized size = 0.48

$$\frac{c^4\sqrt{c-\frac{c}{ax}}\left(\sqrt{-ax(ax+1)}\left(105a^4x^4-4156a^3x^3+1028a^2x^2-246ax+30\right)-1155a^4x^4\sin^{-1}\left(\sqrt{-ax}\right)\right)}{105a^3x^2(-ax)^{3/2}\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(9/2)/E^ArcTanh[a*x],x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(Sqrt[-(a*x*(1 + a*x))]*(30 - 246*a*x + 1028*a^2*x^2
 - 4156*a^3*x^3 + 105*a^4*x^4) - 1155*a^4*x^4*ArcSin[Sqrt[-(a*x)]]))/(105*a
^3*x^2*(-(a*x))^(3/2)*Sqrt[1 - a*x])
```

Maple [A] time = 0.145, size = 172, normalized size = 0.8

$$\frac{c^4}{210\,x^3\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(210\,a^{9/2}\sqrt{-(ax+1)\,x}x^4+1155\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)x^4a^4-8312\,a^{7/2}x^3\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/210*(c*(a*x-1)/a/x)^(1/2)/x^3*c^4/a^(9/2)*(-a^2*x^2+1)^(1/2)*(210*a^(9/2)
*(-(a*x+1)*x)^(1/2)*x^4+1155*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2
))*x^4*a^4-8312*a^(7/2)*x^3*(-(a*x+1)*x)^(1/2)+2056*a^(5/2)*x^2*(-(a*x+1)*x
```

```
)^(1/2)-492*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+60*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*
x-1)/(-(a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(9/2)/(a*x + 1), x)
```

Fricas [A] time = 2.35472, size = 819, normalized size = 3.64

$$\left[\frac{1155\left(a^4c^4x^4-a^3c^4x^3\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(105a^4c^4x^4-4156a^3c^4x^3+1028a^2\right.}{420\left(a^5x^4-a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/420*(1155*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c
*x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)
) - c)/(a*x - 1)) + 4*(105*a^4*c^4*x^4 - 4156*a^3*c^4*x^3 + 1028*a^2*c^4*x^
2 - 246*a*c^4*x + 30*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*
x^4 - a^4*x^3), -1/210*(1155*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*arctan(2*s
qrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x
- c)) - 2*(105*a^4*c^4*x^4 - 4156*a^3*c^4*x^3 + 1028*a^2*c^4*x^2 - 246*a*c^
4*x + 30*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^
3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(9/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(9/2)/(a*x + 1), x)
```

3.536 $\int e^{-\tanh^{-1}(ax)} \left(c - \frac{c}{ax}\right)^{7/2} dx$

Optimal. Leaf size=179

$$-\frac{a^2x^3\sqrt{ax+1}(7ax+66)\left(c-\frac{c}{ax}\right)^{7/2}}{5(1-ax)^{7/2}}-\frac{9a^{5/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{7/2}}+\frac{2ax^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{7/2}}{(1-ax)^{3/2}}-\frac{2x\sqrt{ax+1}\,(c}{5\sqrt{1-}}$$

```
[Out] (2*a*(c - c/(a*x))^(7/2)*x^2*Sqrt[1 + a*x])/(1 - a*x)^(3/2) - (2*(c - c/(a*
x))^(7/2)*x*Sqrt[1 + a*x])/(5*Sqrt[1 - a*x]) - (a^2*(c - c/(a*x))^(7/2)*x^3
*Sqrt[1 + a*x]*(66 + 7*a*x))/(5*(1 - a*x)^(7/2)) - (9*a^(5/2)*(c - c/(a*x))
^(7/2)*x^(7/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(7/2)
```

Rubi [A] time = 0.166558, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6134, 6129, 98, 150, 143, 54, 215}

$$-\frac{a^2x^3\sqrt{ax+1}(7ax+66)\left(c-\frac{c}{ax}\right)^{7/2}}{5(1-ax)^{7/2}}-\frac{9a^{5/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{7/2}}+\frac{2ax^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{7/2}}{(1-ax)^{3/2}}-\frac{2x\sqrt{ax+1}\,(c}{5\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(7/2)/E^ArcTanh[a*x],x]
```


```
[Out] (2*a*(c - c/(a*x))^(7/2)*x^2*Sqrt[1 + a*x])/(1 - a*x)^(3/2) - (2*(c - c/(a*
x))^(7/2)*x*Sqrt[1 + a*x])/(5*Sqrt[1 - a*x]) - (a^2*(c - c/(a*x))^(7/2)*x^3
*Sqrt[1 + a*x]*(66 + 7*a*x))/(5*(1 - a*x)^(7/2)) - (9*a^(5/2)*(c - c/(a*x))
^(7/2)*x^(7/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(7/2)
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{e^{-\tanh^{-1}(ax)}(1-ax)^{7/2}}{x^{7/2}}\,dx}{(1-ax)^{7/2}}$$

$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{(1-ax)^4}{x^{7/2}\sqrt{1+ax}}\,dx}{(1-ax)^{7/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}} - \frac{\left(2\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{(1-ax)^2\left(\frac{15a}{2}-\frac{3a^2x}{2}\right)}{x^{5/2}\sqrt{1+ax}}\,dx}{5(1-ax)^{7/2}}$$

$$= \frac{2a\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}} - \frac{\left(4\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{(1-ax)\left(-\frac{99a^2}{4}-\frac{21a^3}{4}\right.}{x^{3/2}\sqrt{1+ax}}}{15(1-ax)^{7/2}}$$

$$= \frac{2a\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3\sqrt{1+ax}(66+7ax)}{5(1-ax)^{7/2}} -$$

$$= \frac{2a\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3\sqrt{1+ax}(66+7ax)}{5(1-ax)^{7/2}} -$$

$$= \frac{2a\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3\sqrt{1+ax}(66+7ax)}{5(1-ax)^{7/2}} -$$

Mathematica [A] time = 0.0891843, size = 95, normalized size = 0.53

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{ax+1}\left(5a^3x^3-92a^2x^2+16ax-2\right)-45a^{5/2}x^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{5a^3x^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^ArcTanh[a*x],x]
```

```
[Out] -(c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x]*(-2 + 16*a*x - 92*a^2*x^2 + 5*a^3*x^
3) - 45*a^(5/2)*x^(5/2)*ArcSinh[Sqrt[a]*Sqrt[x]]))/(5*a^3*x^2*Sqrt[1 - a*x]
)
```

Maple [A] time = 0.164, size = 154, normalized size = 0.9

$$\frac{c^3}{10\,x^2\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(10\,a^{7/2}x^3\sqrt{-(ax+1)\,x}+45\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)x^3a^3-184\,a^{5/2}x^2\sqrt{\cdot}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/10*(c*(a*x-1)/a/x)^(1/2)/x^2*c^3/a^(7/2)*(-a^2*x^2+1)^(1/2)*(10*a^(7/2)*x
^3*(-(a*x+1)*x)^(1/2)+45*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x
^3*a^3-184*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2)+32*a^(3/2)*x*(-(a*x+1)*x)^(1/2)-4
*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-(a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(7/2)/(a*x + 1), x)
```

Fricas [A] time = 2.52005, size = 744, normalized size = 4.16

$$\left[\frac{45\left(a^3c^3x^3-a^2c^3x^2\right)\sqrt{-c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(5\,a^3c^3x^3-92\,a^2c^3x^2+16\,ac^3x-2\,c^3\right)}{20\left(a^4x^3-a^3x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/20*(45*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x
+ 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) -
 c)/(a*x - 1)) + 4*(5*a^3*c^3*x^3 - 92*a^2*c^3*x^2 + 16*a*c^3*x - 2*c^3)*sq
rt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2), -1/10*(45*(a
^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*s
qrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(5*a^3*c^3*x^3 - 92*a
^2*c^3*x^2 + 16*a*c^3*x - 2*c^3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x))
)/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(7/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(7/2)/(a*x + 1), x)
```

3.537 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=137

$$\frac{7a^{3/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{5/2}}+\frac{ax^2(18-ax)\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3(1-ax)^{5/2}}-\frac{2x\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-ax}}$$

```
[Out] (-2*(c - c/(a*x))^(5/2)*x*Sqrt[1 + a*x])/(3*Sqrt[1 - a*x]) + (a*(c - c/(a*x
))^(5/2)*x^2*(18 - a*x)*Sqrt[1 + a*x])/(3*(1 - a*x)^(5/2)) + (7*a^(3/2)*(c
- c/(a*x))^(5/2)*x^(5/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(5/2)
```

Rubi [A] time = 0.154269, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6134, 6129, 98, 143, 54, 215}

$$\frac{7a^{3/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{5/2}}+\frac{ax^2(18-ax)\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3(1-ax)^{5/2}}-\frac{2x\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(5/2)/E^ArcTanh[a*x],x]
```


```
[Out] (-2*(c - c/(a*x))^(5/2)*x*Sqrt[1 + a*x])/(3*Sqrt[1 - a*x]) + (a*(c - c/(a*x
))^(5/2)*x^2*(18 - a*x)*Sqrt[1 + a*x])/(3*(1 - a*x)^(5/2)) + (7*a^(3/2)*(c
- c/(a*x))^(5/2)*x^(5/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(5/2)
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
```

```
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{\left(\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{e^{-\tanh^{-1}(ax)}(1-ax)^{5/2}}{x^{5/2}}dx}{(1-ax)^{5/2}}$$
$$=\frac{\left(\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{(1-ax)^3}{x^{5/2}\sqrt{1+ax}}dx}{(1-ax)^{5/2}}$$
$$=-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}}{3\sqrt{1-ax}}-\frac{\left(2\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{(1-ax)\left(\frac{9a}{2}-\frac{a^2x}{2}\right)}{x^{3/2}\sqrt{1+ax}}dx}{3(1-ax)^{5/2}}$$
$$=-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}}{3\sqrt{1-ax}}+\frac{a\left(c-\frac{c}{ax}\right)^{5/2}x^2(18-ax)\sqrt{1+ax}}{3(1-ax)^{5/2}}+\frac{\left(7a^2\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{}{\sqrt{x}\sqrt{}}}{2(1-ax)^{5/2}}$$
$$=-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}}{3\sqrt{1-ax}}+\frac{a\left(c-\frac{c}{ax}\right)^{5/2}x^2(18-ax)\sqrt{1+ax}}{3(1-ax)^{5/2}}+\frac{\left(7a^2\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\text{Subs}}{(1-ax)}$$
$$=-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}}{3\sqrt{1-ax}}+\frac{a\left(c-\frac{c}{ax}\right)^{5/2}x^2(18-ax)\sqrt{1+ax}}{3(1-ax)^{5/2}}+\frac{7a^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\sinh^{-1}}{(1-ax)^{5/2}}$$

Mathematica [A] time = 0.0613968, size = 87, normalized size = 0.64

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{ax+1}\left(3a^2x^2-22ax+2\right)-21a^{3/2}x^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{3a^2x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^ArcTanh[a*x],x]
```

```
[Out] -(c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x]*(2 - 22*a*x + 3*a^2*x^2) - 21*a^(3/2
)*x^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]]))/(3*a^2*x*Sqrt[1 - a*x])
```

Maple [A] time = 0.159, size = 136, normalized size = 1.

$$\frac{c^2}{6\,(ax-1)\,x}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(6\,a^{5/2}x^2\sqrt{-(ax+1)\,x}+21\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)x^2a^2-44\,a^{3/2}x\sqrt{-(ax+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/6*(c*(a*x-1)/a/x)^(1/2)/x*c^2/a^(5/2)*(-a^2*x^2+1)^(1/2)*(6*a^(5/2)*x^2*(
-(a*x+1)*x)^(1/2)+21*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^2*a
^2-44*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+4*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-
(a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(5/2)/(a*x + 1), x)
```

Fricas [A] time = 2.46867, size = 680, normalized size = 4.96

$$\left[\frac{21\left(a^2c^2x^2-ac^2x\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3a^2c^2x^2-22ac^2x+2c^2\right)\sqrt{-a^2x^2+1}\sqrt{\cdots}}{12\left(a^3x^2-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(21*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*
(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/
(a*x - 1)) + 4*(3*a^2*c^2*x^2 - 22*a*c^2*x + 2*c^2)*sqrt(-a^2*x^2 + 1)*sqrt
((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x), -1/6*(21*(a^2*c^2*x^2 - a*c^2*x)*sq
rt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^
2*c*x^2 - a*c*x - c)) - 2*(3*a^2*c^2*x^2 - 22*a*c^2*x + 2*c^2)*sqrt(-a^2*x^
2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(5/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(5/2)/(a*x + 1), x)
```

3.538 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=126

$$\frac{ax^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}-\frac{5\sqrt{a}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{3/2}}-\frac{2x\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}$$

```
[Out] (-2*(c - c/(a*x))^(3/2)*x*Sqrt[1 + a*x])/(1 - a*x)^(3/2) + (a*(c - c/(a*x))
^(3/2)*x^2*Sqrt[1 + a*x])/(1 - a*x)^(3/2) - (5*Sqrt[a]*(c - c/(a*x))^(3/2)*
x^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(3/2)
```

Rubi [A] time = 0.151152, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6134, 6129, 89, 80, 54, 215}

$$\frac{ax^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}-\frac{5\sqrt{a}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{3/2}}-\frac{2x\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(3/2)/E^ArcTanh[a*x],x]
```

```
[Out] (-2*(c - c/(a*x))^(3/2)*x*Sqrt[1 + a*x])/(1 - a*x)^(3/2) + (a*(c - c/(a*x))
^(3/2)*x^2*Sqrt[1 + a*x])/(1 - a*x)^(3/2) - (5*Sqrt[a]*(c - c/(a*x))^(3/2)*
x^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(3/2)
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx &= \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{e^{-\tanh^{-1}(ax)}(1-ax)^{3/2}}{x^{3/2}}dx}{(1-ax)^{3/2}}\\
&= \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{(1-ax)^2}{x^{3/2}\sqrt{1+ax}}dx}{(1-ax)^{3/2}}\\
&= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\sqrt{1+ax}}{(1-ax)^{3/2}}+\frac{\left(2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{-a+\frac{a^2x}{2}}{\sqrt{x}\sqrt{1+ax}}dx}{(1-ax)^{3/2}}\\
&= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\sqrt{1+ax}}{(1-ax)^{3/2}}+\frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}}-\frac{\left(5a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{1}{\sqrt{x}\sqrt{1+ax}}dx}{2(1-ax)^{3/2}}\\
&= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\sqrt{1+ax}}{(1-ax)^{3/2}}+\frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}}-\frac{\left(5a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{1+ax^{\cdots}}}\right.}{(1-ax)^{3/2}}\\
&= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\sqrt{1+ax}}{(1-ax)^{3/2}}+\frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}}-\frac{5\sqrt{a}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0462589, size = 71, normalized size = 0.56

$$
-\frac{c\sqrt{c-\frac{c}{ax}}\left((ax-2)\sqrt{ax+1}-5\sqrt{a}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{a\sqrt{1-ax}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^ArcTanh[a*x],x]
```

```
[Out] -((c*Sqrt[c - c/(a*x)]*((-2 + a*x)*Sqrt[1 + a*x] - 5*Sqrt[a]*Sqrt[x]*ArcSin
h[Sqrt[a]*Sqrt[x]]))/(a*Sqrt[1 - a*x]))
```

Maple [A] time = 0.14, size = 109, normalized size = 0.9

$$
\frac{c}{2\,ax-2}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(2\,a^{3/2}x\sqrt{-(ax+1)\,x}+5\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)xa-4\,\sqrt{a}\sqrt{-(ax+1)\,x}\right)a^{-\frac{3}{2}}\frac{\cdots}{\sqrt{\cdots}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)*(-a^2*x^2+1)^(1/2)*(2*a^(3/2)*x*(-(a*x+
1)*x)^(1/2)+5*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a-4*a^(1/2
)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-(a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(3/2)/(a*x + 1), x)
```

Fricas [A] time = 2.14547, size = 566, normalized size = 4.49

$$\left[\frac{5\,(acx-c)\sqrt{-c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^2x^2+ax)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,\sqrt{-a^2x^2+1}(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)}, -\frac{5\,(acx-c)\sqrt{c}\,\mathrm{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(5*(a*c*x - c)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a
*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4
*sqrt(-a^2*x^2 + 1)*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a), -1/
2*(5*(a*c*x - c)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*
x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*sqrt(-a^2*x^2 + 1)*(a*c*x - 2*
c)*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(3/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(3/2)*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(3/2)/(a*x + 1), x)
```

3.539 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=90

$$\frac{3\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{1-ax}$$

```
[Out] -((Sqrt[c - c/(a*x)]*x*Sqrt[1 - a^2*x^2])/(1 - a*x)) + (3*Sqrt[c - c/(a*x)]
*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rubi [A] time = 0.164407, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6134, 6128, 881, 848, 54, 215}

$$\frac{3\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{1-ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^ArcTanh[a*x],x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x*Sqrt[1 - a^2*x^2])/(1 - a*x)) + (3*Sqrt[c - c/(a*x)]
*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
```

```
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}} \\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^{3/2}}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}} \\
&= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1-a^2x^2}}{1-ax}+\frac{\left(3\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-ax}}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx}{2\sqrt{1-ax}} \\
&= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1-a^2x^2}}{1-ax}+\frac{\left(3\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}} \\
&= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1-a^2x^2}}{1-ax}+\frac{\left(3\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{\sqrt{1-ax}} \\
&= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1-a^2x^2}}{1-ax}+\frac{3\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}
\end{aligned}
$$

Mathematica [A] time = 0.0484186, size = 67, normalized size = 0.74

$$
\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\frac{3\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}-\sqrt{x}\sqrt{ax+1}\right)}{\sqrt{1-ax}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(-(Sqrt[x]*Sqrt[1 + a*x]) + (3*ArcSinh[Sqrt[a]*S
qrt[x]])/Sqrt[a]))/Sqrt[1 - a*x]
```

Maple [A] time = 0.142, size = 91, normalized size = 1.

$$
\frac{x}{2\,ax-2}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(2\,\sqrt{a}\sqrt{-(ax+1)\,x}+3\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)\,x}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(2*a^(1/2)*(-(a*x+1)*x)^(1/2
)+3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))/(a*x-1)/(-(a*x+1)*x)^
(1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

Fricas [A] time = 2.13557, size = 532, normalized size = 5.91

$$\left[\frac{4\sqrt{-a^2x^2+1}ax\sqrt{\frac{acx-c}{ax}}+3(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)}{4\left(a^2x-a\right)}, \frac{2\sqrt{-a^2x^2+1}ax\sqrt{\frac{acx-c}{ax}}-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) + 3*(a*x - 1)*sqrt(-
c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqr
t(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)))/(a^2*x - a), 1/2*(2*sqrt(-a^
2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) - 3*(a*x - 1)*sqrt(c)*arctan(2*sqrt(
-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)
))/(a^2*x - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

3.540 $\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}} dx$

Optimal. Leaf size=88

$$\frac{\sqrt{1-ax}\sqrt{ax+1}}{a\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{1-ax}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}$$

```
[Out] (Sqrt[1 - a*x]*Sqrt[1 + a*x])/(a*Sqrt[c - c/(a*x)]) - (Sqrt[1 - a*x]*ArcSin
h[Sqrt[a]*Sqrt[x]])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x])
```

Rubi [A] time = 0.145846, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6134, 6128, 848, 50, 54, 215}

$$\frac{\sqrt{1-ax}\sqrt{ax+1}}{a\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{1-ax}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)]),x]
```


```
[Out] (Sqrt[1 - a*x]*Sqrt[1 + a*x])/(a*Sqrt[c - c/(a*x)]) - (Sqrt[1 - a*x]*ArcSin
h[Sqrt[a]*Sqrt[x]])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x])
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] && !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx &= \frac{\sqrt{1-ax}\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{x}}{\sqrt{1-ax}}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}\\
&= \frac{\sqrt{1-ax}\int \frac{\sqrt{x}\sqrt{1-ax}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}\\
&= \frac{\sqrt{1-ax}\int \frac{\sqrt{x}}{\sqrt{1+ax}}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}\\
&= \frac{\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} - \frac{\sqrt{1-ax}\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a\sqrt{c-\frac{c}{ax}}\sqrt{x}}\\
&= \frac{\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} - \frac{\sqrt{1-ax}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}}\\
&= \frac{\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} - \frac{\sqrt{1-ax}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{c-\frac{c}{ax}}\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.0368407, size = 71, normalized size = 0.81

$$\frac{\sqrt{1-ax}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}-\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[1 - a*x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x] - ArcSinh[Sqrt[a]*Sqrt[x]]))/
(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x])
```

Maple [A] time = 0.125, size = 92, normalized size = 1.1

$$\frac{x}{2c(ax-1)}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(2\sqrt{a}\sqrt{-(ax+1)x}+\arctan\left(\frac{2ax+1}{2}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)x}}\right)\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x/a^(1/2)/c*(-a^2*x^2+1)^(1/2)*(2*a^(1/2)*(-(a*x+
1)*x)^(1/2)+arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))/(a*x-1)/(-(a*
x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*sqrt(c - c/(a*x))), x)
```

Fricas [A] time = 2.08492, size = 537, normalized size = 6.1

$$\left[\frac{4\sqrt{-a^2x^2+1}ax\sqrt{\frac{acx-c}{ax}}-(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4(2a^2x^2+ax)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)}{4(a^2cx-ac)}, \frac{2\sqrt{-a^2x^2+1}ax\sqrt{\frac{acx-c}{ax}}-(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(1/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) - (a*x - 1)*sqrt(-c)
*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(
-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)))/(a^2*c*x - a*c), 1/2*(2*sqrt(-
a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) - (a*x - 1)*sqrt(c)*arctan(2*sqrt(
-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)
))/(a^2*c*x - a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*sqrt(c - c/(a*x))), x)
```

3.541 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=159

$$-\frac{(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\sqrt{2}(1-ax)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\sqrt{ax+1}(1-ax)^{3/2}}{a^2x\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] -(((1 - a*x)^(3/2)*Sqrt[1 + a*x])/(a^2*(c - c/(a*x))^(3/2)*x)) - ((1 - a*x)
^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(a^(5/2)*(c - c/(a*x))^(3/2)*x^(3/2)) + (S
qrt[2]*(1 - a*x)^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a
^(5/2)*(c - c/(a*x))^(3/2)*x^(3/2))
```

Rubi [A] time = 0.179565, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6134, 6129, 102, 21, 105, 54, 215, 93, 206}

$$-\frac{(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\sqrt{2}(1-ax)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\sqrt{ax+1}(1-ax)^{3/2}}{a^2x\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a*x))^(3/2)),x]
```

```
[Out] -(((1 - a*x)^(3/2)*Sqrt[1 + a*x])/(a^2*(c - c/(a*x))^(3/2)*x)) - ((1 - a*x)
^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(a^(5/2)*(c - c/(a*x))^(3/2)*x^(3/2)) + (S
qrt[2]*(1 - a*x)^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a
^(5/2)*(c - c/(a*x))^(3/2)*x^(3/2))
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{(1-ax)^{3/2}\int \frac{e^{-\tanh^{-1}(ax)}x^{3/2}}{(1-ax)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$
$$= \frac{(1-ax)^{3/2}\int \frac{x^{3/2}}{(1-ax)\sqrt{1+ax}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$
$$= -\frac{(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{(1-ax)^{3/2}\int \frac{-\frac{1}{2}-\frac{ax}{2}}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$
$$= -\frac{(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{(1-ax)^{3/2}\int \frac{\sqrt{1+ax}}{\sqrt{x}(1-ax)}\,dx}{2a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$
$$= -\frac{(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{(1-ax)^{3/2}\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}} + \frac{(1-ax)^{3/2}\int \frac{1}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$
$$= -\frac{(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{(1-ax)^{3/2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}} + \frac{\left(2(1-ax)^{3/2}\right)\text{Subst}\left(\int \frac{1}{1-2ax^2}\,dx\right.}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$
$$= -\frac{(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}} + \frac{\sqrt{2}(1-ax)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right)}{a^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

Mathematica [A] time = 0.058452, size = 105, normalized size = 0.66

$$\frac{\sqrt{1-ax}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}+\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)\right)}{a^{3/2}c\sqrt{x}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (Sqrt[1 - a*x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]] -
Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]))/(a^(3/2)*c*Sqrt[
c - c/(a*x)]*Sqrt[x])
```

Maple [A] time = 0.145, size = 168, normalized size = 1.1

$$\frac{x\sqrt{2}}{4\,c^2\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(2\,\sqrt{-(ax+1)\,x}a^{3/2}\sqrt{2}\sqrt{-a^{-1}}-\arctan\left(\frac{2\,ax+1}{2}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)\,x}}\right)a\sqrt{2}\sqrt{-a^{-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(3/2),x)
```

```
[Out] 1/4*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(2*(-(a*x+1)*x)^(1/2)*a^(3/2
)*2^(1/2)*(-1/a)^(1/2)-arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2
^(1/2)*(-1/a)^(1/2)+2*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x
-1)/(a*x-1))*a^(1/2))*2^(1/2)/a^(3/2)/c^2/(a*x-1)/(-(a*x+1)*x)^(1/2)/(-1/a)
```

```
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a*x))^(3/2)), x)
```

Fricas [A] time = 2.45272, size = 986, normalized size = 6.2

$$\left[\frac{4\sqrt{-a^2x^2+1}ax\sqrt{\frac{acx-c}{ax}}+\sqrt{2}(acx-c)\sqrt{-\frac{1}{c}}\log\left(-\frac{17a^3x^3-3a^2x^2+4\sqrt{2}(3a^2x^2+ax)\sqrt{-a^2x^2+1}\sqrt{-\frac{1}{c}}\sqrt{\frac{acx-c}{ax}}-13ax-1}{a^3x^3-3a^2x^2+3ax-1}\right)-(ax-1)\sqrt{-c}\log}{4\left(a^2c^2x-ac^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) + sqrt(2)*(a*c*x - c
)*sqrt(-1/c)*log(-(17*a^3*x^3 - 3*a^2*x^2 + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqr
t(-a^2*x^2 + 1)*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x)) - 13*a*x - 1)/(a^3*x^3 -
 3*a^2*x^2 + 3*a*x - 1)) - (a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x +
 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)))/(a^2*c^2*x - a*c^2), 1/2*(2*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c
*x - c)/(a*x)) + (a*x - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*
sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - sqrt(2)*(a*c*x - c)*ar
ctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x))/((3*a^2*x^2 -
 2*a*x - 1)*sqrt(c)))/sqrt(c))/(a^2*c^2*x - a*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(-1 + 1/(a*x)))**(3/2)*(a*x + 1)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a*x))^(3/2)), x)
```

3.542 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=208

$$\frac{3\sqrt{ax+1}(1-ax)^{5/2}}{2a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{3(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{9(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{2\sqrt{2}a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\sqrt{ax+1}(1-ax)^{3/2}}{2a^2x\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] ((1 - a*x)^(3/2)*Sqrt[1 + a*x])/(2*a^2*(c - c/(a*x))^(5/2)*x) + (3*(1 - a*x
)^(5/2)*Sqrt[1 + a*x])/(2*a^3*(c - c/(a*x))^(5/2)*x^2) + (3*(1 - a*x)^(5/2)
*ArcSinh[Sqrt[a]*Sqrt[x]])/(a^(7/2)*(c - c/(a*x))^(5/2)*x^(5/2)) - (9*(1 -
a*x)^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(2*Sqrt[2]*a^(
7/2)*(c - c/(a*x))^(5/2)*x^(5/2))
```

Rubi [A] time = 0.189436, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6134, 6129, 98, 154, 157, 54, 215, 93, 206}

$$\frac{3\sqrt{ax+1}(1-ax)^{5/2}}{2a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{3(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{9(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{2\sqrt{2}a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\sqrt{ax+1}(1-ax)^{3/2}}{2a^2x\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a*x))^(5/2)),x]
```

```
[Out] ((1 - a*x)^(3/2)*Sqrt[1 + a*x])/(2*a^2*(c - c/(a*x))^(5/2)*x) + (3*(1 - a*x
)^(5/2)*Sqrt[1 + a*x])/(2*a^3*(c - c/(a*x))^(5/2)*x^2) + (3*(1 - a*x)^(5/2)
*ArcSinh[Sqrt[a]*Sqrt[x]])/(a^(7/2)*(c - c/(a*x))^(5/2)*x^(5/2)) - (9*(1 -
a*x)^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(2*Sqrt[2]*a^(
7/2)*(c - c/(a*x))^(5/2)*x^(5/2))
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx = \frac{(1-ax)^{5/2}\int \frac{e^{-\tanh^{-1}(ax)}x^{5/2}}{(1-ax)^{5/2}}dx}{\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{5/2}\int \frac{x^{5/2}}{(1-ax)^2\sqrt{1+ax}}dx}{\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}x} - \frac{(1-ax)^{5/2}\int \frac{\sqrt{x}\left(\frac{3}{2}+3ax\right)}{(1-ax)\sqrt{1+ax}}dx}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{3(1-ax)^{5/2}\sqrt{1+ax}}{2a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1-ax)^{5/2}\int \frac{-\frac{3a}{2}-3a^2x}{\sqrt{x}(1-ax)\sqrt{1+ax}}dx}{2a^4\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{3(1-ax)^{5/2}\sqrt{1+ax}}{2a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{\left(3(1-ax)^{5/2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}dx}{2a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}} - \frac{\left(9(1-ax)^{5/2}\right)\int \frac{1}{\sqrt{x}(1-ax)\sqrt{}}}{4a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{3(1-ax)^{5/2}\sqrt{1+ax}}{2a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{\left(3(1-ax)^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}dx, x, \sqrt{x}\right)}{a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}} - \frac{\left(9(1-ax)^{5}\right.}{}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{3(1-ax)^{5/2}\sqrt{1+ax}}{2a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{3(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}} - \frac{9(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{1-}}\right.}{2\sqrt{2}a^{7/2}\left(c-\frac{c}{ax}\right)^{5/2}x^{5}}$$

Mathematica [A] time = 0.167171, size = 127, normalized size = 0.61

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{ax+1}(3-2ax)-12(ax-1)\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+9\sqrt{2}(ax-1)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{4a^{3/2}c^2\sqrt{x}\sqrt{1-ax}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*(3 - 2*a*x)*Sqrt[1 + a*x] - 12*(-1 + a*x)*ArcSinh[Sqrt[a
]*Sqrt[x]] + 9*Sqrt[2]*(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1
+ a*x]])/(4*a^(3/2)*c^2*Sqrt[c - c/(a*x)]*Sqrt[x]*Sqrt[1 - a*x])
```

Maple [A] time = 0.152, size = 276, normalized size = 1.3

$$\frac{x\sqrt{2}}{8c^3(ax-1)^2}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(4\sqrt{-(ax+1)x}a^{5/2}\sqrt{2}\sqrt{-a^{-1}}x-6\sqrt{-(ax+1)x}a^{3/2}\sqrt{2}\sqrt{-a^{-1}}-6a^2\arctan\left(1/2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(5/2),x)
```

```
[Out] 1/8*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(4*(-(a*x+1)*x)^(1/2)*a^(5/2
)*2^(1/2)*(-1/a)^(1/2)*x-6*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2)-
6*a^2*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)
*x+9*a^(3/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-
```

```
1))*x+6*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)^(
1/2)-9*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*a^
(1/2))*2^(1/2)/a^(3/2)/c^3/(a*x-1)^2/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a*x))^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(5/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(-1 + 1/(a*x)))**(5/2)*(a*x + 1)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a*x))^(5/2)), x)
```

3.543 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=252

$$-\frac{35\sqrt{ax+1}(1-ax)^{7/2}}{16a^4x^3\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{15\sqrt{ax+1}(1-ax)^{5/2}}{16a^3x^2\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{5(1-ax)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{9/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{115(1-ax)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{16\sqrt{2}a^{9/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{\sqrt{a}\ldots}{4\ldots}$$

```
[Out] ((1 - a*x)^(3/2)*Sqrt[1 + a*x])/(4*a^2*(c - c/(a*x))^(7/2)*x) - (15*(1 - a*
x)^(5/2)*Sqrt[1 + a*x])/(16*a^3*(c - c/(a*x))^(7/2)*x^2) - (35*(1 - a*x)^(7
/2)*Sqrt[1 + a*x])/(16*a^4*(c - c/(a*x))^(7/2)*x^3) - (5*(1 - a*x)^(7/2)*Ar
cSinh[Sqrt[a]*Sqrt[x]])/(a^(9/2)*(c - c/(a*x))^(7/2)*x^(7/2)) + (115*(1 - a
*x)^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(16*Sqrt[2]*a^(
9/2)*(c - c/(a*x))^(7/2)*x^(7/2))
```

Rubi [A] time = 0.204563, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6134, 6129, 98, 149, 154, 157, 54, 215, 93, 206}

$$-\frac{35\sqrt{ax+1}(1-ax)^{7/2}}{16a^4x^3\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{15\sqrt{ax+1}(1-ax)^{5/2}}{16a^3x^2\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{5(1-ax)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{9/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{115(1-ax)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{16\sqrt{2}a^{9/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{\sqrt{a}\ldots}{4\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a*x))^(7/2)),x]
```

```
[Out] ((1 - a*x)^(3/2)*Sqrt[1 + a*x])/(4*a^2*(c - c/(a*x))^(7/2)*x) - (15*(1 - a*
x)^(5/2)*Sqrt[1 + a*x])/(16*a^3*(c - c/(a*x))^(7/2)*x^2) - (35*(1 - a*x)^(7
/2)*Sqrt[1 + a*x])/(16*a^4*(c - c/(a*x))^(7/2)*x^3) - (5*(1 - a*x)^(7/2)*Ar
cSinh[Sqrt[a]*Sqrt[x]])/(a^(9/2)*(c - c/(a*x))^(7/2)*x^(7/2)) + (115*(1 - a
*x)^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(16*Sqrt[2]*a^(
9/2)*(c - c/(a*x))^(7/2)*x^(7/2))
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
```

```
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{(1-ax)^{7/2}\int \frac{e^{-\tanh^{-1}(ax)}x^{7/2}}{(1-ax)^{7/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{7/2}\int \frac{x^{7/2}}{(1-ax)^3\sqrt{1+ax}}\,dx}{\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{4a^2\left(c-\frac{c}{ax}\right)^{7/2}x} - \frac{(1-ax)^{7/2}\int \frac{x^{3/2}\left(\frac{5}{2}+5ax\right)}{(1-ax)^2\sqrt{1+ax}}\,dx}{4a^2\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{4a^2\left(c-\frac{c}{ax}\right)^{7/2}x} - \frac{15(1-ax)^{5/2}\sqrt{1+ax}}{16a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2} - \frac{(1-ax)^{7/2}\int \frac{\sqrt{x}\left(-\frac{45a}{4}-\frac{35a^2x}{2}\right)}{(1-ax)\sqrt{1+ax}}\,dx}{8a^4\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{4a^2\left(c-\frac{c}{ax}\right)^{7/2}x} - \frac{15(1-ax)^{5/2}\sqrt{1+ax}}{16a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2} - \frac{35(1-ax)^{7/2}\sqrt{1+ax}}{16a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} + \frac{(1-ax)^{7/2}\int \frac{\frac{35a^2}{4}+20a^3x}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,d}{8a^6\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{4a^2\left(c-\frac{c}{ax}\right)^{7/2}x} - \frac{15(1-ax)^{5/2}\sqrt{1+ax}}{16a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2} - \frac{35(1-ax)^{7/2}\sqrt{1+ax}}{16a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} - \frac{\left(5(1-ax)^{7/2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a^4\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{4a^2\left(c-\frac{c}{ax}\right)^{7/2}x} - \frac{15(1-ax)^{5/2}\sqrt{1+ax}}{16a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2} - \frac{35(1-ax)^{7/2}\sqrt{1+ax}}{16a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} - \frac{\left(5(1-ax)^{7/2}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{a^4\left(c-\frac{c}{ax}\right)^{7/2}x^{7}}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{4a^2\left(c-\frac{c}{ax}\right)^{7/2}x} - \frac{15(1-ax)^{5/2}\sqrt{1+ax}}{16a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2} - \frac{35(1-ax)^{7/2}\sqrt{1+ax}}{16a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} - \frac{5(1-ax)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{9/2}\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

Mathematica [A] time = 0.216278, size = 139, normalized size = 0.55

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{ax+1}\left(16a^2x^2-55ax+35\right)+160(ax-1)^2\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-115\sqrt{2}(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{32a^{3/2}c^3\sqrt{x}(1-ax)^{3/2}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*(35 - 55*a*x + 16*a^2*x^2) + 160*(-1 + a*x
)^2*ArcSinh[Sqrt[a]*Sqrt[x]] - 115*Sqrt[2]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sq
rt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(32*a^(3/2)*c^3*Sqrt[c - c/(a*x)]*Sqrt[x]*(1
 - a*x)^(3/2))
```

Maple [A] time = 0.151, size = 390, normalized size = 1.6

$$\frac{x\sqrt{2}}{64\,c^4\,(ax-1)^3}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(32\,\sqrt{-(ax+1)\,x}a^{7/2}\sqrt{2}\sqrt{-a^{-1}}x^2-110\,\sqrt{-(ax+1)\,x}a^{5/2}\sqrt{2}\sqrt{-a^{-1}}x-80\,a^3\,\text{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(7/2),x)
```

```
[Out] 1/64*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(32*(-(a*x+1)*x)^(1/2)*a^(7
/2)*2^(1/2)*(-1/a)^(1/2)*x^2-110*(-(a*x+1)*x)^(1/2)*a^(5/2)*2^(1/2)*(-1/a)^
(1/2)*x-80*a^3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1
/a)^(1/2)*x^2+115*a^(5/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3
*a*x-1)/(a*x-1))*x^2+70*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2)+160
*a^2*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)*
x-230*a^(3/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x
-1))*x-80*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)
^(1/2)+115*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1)
)*a^(1/2))*2^(1/2)/a^(3/2)/c^4/(a*x-1)^3/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(7/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a*x))^(7/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(7/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a/x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a*x))^(7/2)), x)
```

3.544 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=189

$$\frac{51c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{19c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\frac{13c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{64\sqrt{2}c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-$$

```
[Out] (51*c^4*Sqrt[c - c/(a*x)])/a + (19*c^3*(c - c/(a*x))^(3/2))/(3*a) + (3*c^2*
(c - c/(a*x))^(5/2))/(5*a) - (5*c*(c - c/(a*x))^(7/2))/(7*a) - (c - c/(a*x)
)^(9/2)*x + (13*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a - (64*Sqrt[2]
*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.26489, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6133, 25, 514, 375, 98, 154, 156, 63, 208}

$$\frac{51c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{19c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\frac{13c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{64\sqrt{2}c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(9/2)/E^(2*ArcTanh[a*x]),x]
```


```
[Out] (51*c^4*Sqrt[c - c/(a*x)])/a + (19*c^3*(c - c/(a*x))^(3/2))/(3*a) + (3*c^2*
(c - c/(a*x))^(5/2))/(5*a) - (5*c*(c - c/(a*x))^(7/2))/(7*a) - (c - c/(a*x)
)^(9/2)*x + (13*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a - (64*Sqrt[2]
*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```


Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
```

```
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{9/2}(1-ax)}{1+ax}dx$$

$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{11/2}x}{1+ax}dx}{c}$$

$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{11/2}}{a+\frac{1}{x}}dx}{c}$$

$$=\frac{a\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{11/2}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{7/2}\left(\frac{13c^2}{2}+\frac{5c^2x}{2a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{2\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{5/2}\left(\frac{91c^3}{4}-\frac{21c^3x}{4a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{7c}$$

$$=\frac{3c^2\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{4\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}\left(\frac{455c^4}{8}-\frac{665c^4x}{8a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{35c}$$

$$=\frac{19c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{8\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}\left(\frac{136\cdots}{1\cdots}\right.}{x(\cdots}\right.}{105\cdots}$$

$$=\frac{51c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{19c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{16\,\text{S}\cdots}{\cdots}$$

$$=\frac{51c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{19c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{(13c\cdots}{\cdots}$$

$$=\frac{51c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{19c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x+(13c\cdots$$

$$=\frac{51c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{19c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x+\frac{13c^{9\cdots}}{\cdots}$$

<u>Mathematica</u> [A] time = 0.323213, size = 133, normalized size = 0.7

$$\frac{c^4\left(-105a^4x^4+6428a^3x^3-1196a^2x^2+258ax-30\right)\sqrt{c-\frac{c}{ax}}}{105a^4x^3}+\frac{13c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{64\sqrt{2}c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(9/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(-30 + 258*a*x - 1196*a^2*x^2 + 6428*a^3*x^3 - 105*a
^4*x^4))/(105*a^4*x^3) + (13*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

```
- (64*Sqrt[2]*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Maple [A] time = 0.155, size = 305, normalized size = 1.6

$$-\frac{c^4}{210\,x^4}\sqrt{\frac{c\,(ax-1)}{ax}}\left(6720\,a^{9/2}\sqrt{a^{-1}}\sqrt{(ax-1)\,x}x^5-17430\,a^{9/2}\sqrt{a^{-1}}\sqrt{ax^2-x}x^5-6720\,a^{7/2}\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax-1)}}{ax+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(9/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/210*(c*(a*x-1)/a/x)^(1/2)*c^4*(6720*a^(9/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2
)*x^5-17430*a^(9/2)*(1/a)^(1/2)*(a*x^2-x)^(1/2)*x^5-6720*a^(7/2)*2^(1/2)*ln
((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^5+10920*a^(
7/2)*(1/a)^(1/2)*(a*x^2-x)^(3/2)*x^3-1936*a^(5/2)*(a*x^2-x)^(3/2)*x^2*(1/a)
^(1/2)+8715*(1/a)^(1/2)*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))
*x^5*a^4-10080*(1/a)^(1/2)*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(
1/2))*x^5*a^4+456*a^(3/2)*(a*x^2-x)^(3/2)*x*(1/a)^(1/2)-60*(a*x^2-x)^(3/2)*
a^(1/2)*(1/a)^(1/2))/x^4/a^(9/2)/((a*x-1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a*x))^(9/2)/(a*x + 1)^2, x)
```

Fricas [A] time = 1.73634, size = 811, normalized size = 4.29

$$\left[\frac{6720\sqrt{2}a^3c^{\frac{9}{2}}x^3\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+1365\,a^3c^{\frac{9}{2}}x^3\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(105\,a^4c^4x^4-6428\,a^3\right.}{210\,a^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/210*(6720*sqrt(2)*a^3*c^(9/2)*x^3*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
 - c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + 1365*a^3*c^(9/2)*x^3*log(-2*a*c*x
- 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 2*(105*a^4*c^4*x^4 - 6428*a^
3*c^4*x^3 + 1196*a^2*c^4*x^2 - 258*a*c^4*x + 30*c^4)*sqrt((a*c*x - c)/(a*x)
))/(a^4*x^3), 1/105*(6720*sqrt(2)*a^3*sqrt(-c)*c^4*x^3*arctan(1/2*sqrt(2)*s
qrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 1365*a^3*sqrt(-c)*c^4*x^3*arctan(sqrt(
-c)*sqrt((a*c*x - c)/(a*x))/c) - (105*a^4*c^4*x^4 - 6428*a^3*c^4*x^3 + 1196
```

```
*a^2*c^4*x^2 - 258*a*c^4*x + 30*c^4)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(9/2)/(a*x+1)**2*(-a**2*x**2+1),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.545 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=164

$$\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{32\sqrt{2}c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-x\left(c-\frac{c}{ax}\right)^{7/2}$$

```
[Out] (21*c^3*Sqrt[c - c/(a*x)])/a + (5*c^2*(c - c/(a*x))^(3/2))/(3*a) - (3*c*(c
- c/(a*x))^(5/2))/(5*a) - (c - c/(a*x))^(7/2)*x + (11*c^(7/2)*ArcTanh[Sqrt[
c - c/(a*x)]/Sqrt[c]])/a - (32*Sqrt[2]*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/(S
qrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.226423, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6133, 25, 514, 375, 98, 154, 156, 63, 208}

$$\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{32\sqrt{2}c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-x\left(c-\frac{c}{ax}\right)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(7/2)/E^(2*ArcTanh[a*x]),x]
```


```
[Out] (21*c^3*Sqrt[c - c/(a*x)])/a + (5*c^2*(c - c/(a*x))^(3/2))/(3*a) - (3*c*(c
- c/(a*x))^(5/2))/(5*a) - (c - c/(a*x))^(7/2)*x + (11*c^(7/2)*ArcTanh[Sqrt[
c - c/(a*x)]/Sqrt[c]])/a - (32*Sqrt[2]*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/(S
qrt[2]*Sqrt[c])])/a
```


Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
```

```
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}(1-ax)}{1+ax}dx$$
$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{9/2}x}{1+ax}dx}{c}$$
$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{9/2}}{a+\frac{1}{x}}dx}{c}$$
$$=\frac{a\,\mathrm{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{9/2}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c}$$
$$=-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{\mathrm{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{5/2}\left(\frac{11c^2}{2}+\frac{3c^2x}{2a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{c}$$
$$=-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{2\,\mathrm{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}\left(\frac{55c^3}{4}-\frac{25c^3x}{4a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{5c}$$
$$=\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{4\,\mathrm{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}\left(\frac{165c^4}{8}-\frac{315c^4x}{8a}\right)}{x(a+x)}dx,x,\right.}{15c}$$
$$=\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{8\,\mathrm{Subst}\left(\int\frac{\frac{165c^5}{16}-\frac{795}{1}}{x(a+x)\sqrt{c}}\right.}{15c}$$
$$=\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{(11c^4)\,\mathrm{Subst}\left(\int\frac{}{x\sqrt{}}\right.}{2a}$$
$$=\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x+(11c^3)\,\mathrm{Subst}\left(\int\frac{}{a}\right.$$
$$=\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{a}}}{\sqrt{c}}\right.}{a}$$

Mathematica [A] time = 0.17077, size = 125, normalized size = 0.76

$$\frac{c^3\left(-15a^3x^3+376a^2x^2-52ax+6\right)\sqrt{c-\frac{c}{ax}}}{15a^3x^2}+\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{32\sqrt{2}c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(6 - 52*a*x + 376*a^2*x^2 - 15*a^3*x^3))/(15*a^3*x^2
) + (11*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a - (32*Sqrt[2]*c^(7/2)
*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Maple [B] time = 0.128, size = 281, normalized size = 1.7

$$-\frac{c^3}{30\,x^3}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-1110\,\sqrt{a x^2-x}\,a^{7/2}\sqrt{a^{-1}}x^4+480\,a^{7/2}\sqrt{a^{-1}}\sqrt{(ax-1)\,x}x^4+660\,a^{5/2}\left(a x^2-x\right)^{3/2}x^2\sqrt{a^{-1}}+555\,\ln\left|1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/30*(c*(a*x-1)/a/x)^(1/2)/x^3*c^3/a^(7/2)*(-1110*(a*x^2-x)^(1/2)*a^(7/2)*
(1/a)^(1/2)*x^4+480*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^4+660*a^(5/2)*(
a*x^2-x)^(3/2)*x^2*(1/a)^(1/2)+555*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-
1)/a^(1/2))*(1/a)^(1/2)*x^4*a^3-480*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/
2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^4-720*ln(1/2*(2*((a*x-1)*x)^(1/2
)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^4*a^3-92*a^(3/2)*(a*x^2-x)^(3/2)*
x*(1/a)^(1/2)+12*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/(1/
a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\left(ax+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a*x))^(7/2)/(a*x + 1)^2, x)
```

Fricas [A] time = 1.97427, size = 741, normalized size = 4.52

$$\left[\frac{480\,\sqrt{2}a^2c^{\frac{7}{2}}x^2\log\left(\frac{2\,\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+165\,a^2c^{\frac{7}{2}}x^2\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(15\,a^3c^3x^3-376\,a^2c^3x^2+5\right.}{30\,a^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/30*(480*sqrt(2)*a^2*c^(7/2)*x^2*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
 c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + 165*a^2*c^(7/2)*x^2*log(-2*a*c*x - 2
*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 2*(15*a^3*c^3*x^3 - 376*a^2*c^3
*x^2 + 52*a*c^3*x - 6*c^3)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2), 1/15*(480*sq
rt(2)*a^2*sqrt(-c)*c^3*x^2*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*
x))/c) - 165*a^2*sqrt(-c)*c^3*x^2*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c
) - (15*a^3*c^3*x^3 - 376*a^2*c^3*x^2 + 52*a*c^3*x - 6*c^3)*sqrt((a*c*x - c
)/(a*x)))/(a^3*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{4c^3\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx-\int \frac{6c^3\sqrt{c-\frac{c}{ax}}}{a^2x^2+ax}\,dx-\int -\frac{4c^3\sqrt{c-\frac{c}{ax}}}{a^3x^3+a^2x^2}\,dx-\int \frac{c^3\sqrt{c-\frac{c}{ax}}}{a^4x^4+a^3x^3}\,dx-\int \frac{ac^3x\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(7/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-4*c**3*sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(6*c**3*sqrt(c
- c/(a*x))/(a**2*x**2 + a*x), x) - Integral(-4*c**3*sqrt(c - c/(a*x))/(a**3
*x**3 + a**2*x**2), x) - Integral(c**3*sqrt(c - c/(a*x))/(a**4*x**4 + a**3*
x**3), x) - Integral(a*c**3*x*sqrt(c - c/(a*x))/(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.546 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx$

Optimal. Leaf size=139

$$\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{9c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{16\sqrt{2}c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-x\left(c-\frac{c}{ax}\right)^{5/2}$$

```
[Out] (7*c^2*Sqrt[c - c/(a*x)])/a - (c*(c - c/(a*x))^(3/2))/(3*a) - (c - c/(a*x))
^(5/2)*x + (9*c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a - (16*Sqrt[2]*c
^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.198139, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6133, 25, 514, 375, 98, 154, 156, 63, 208}

$$\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{9c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{16\sqrt{2}c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-x\left(c-\frac{c}{ax}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(5/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (7*c^2*Sqrt[c - c/(a*x)])/a - (c*(c - c/(a*x))^(3/2))/(3*a) - (c - c/(a*x))
^(5/2)*x + (9*c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a - (16*Sqrt[2]*c
^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{5/2}(1-ax)}{1+ax}dx$$
$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}x}{1+ax}dx}{c}$$
$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}}{a+\frac{1}{x}}dx}{c}$$
$$=\frac{a\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{7/2}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c}$$
$$=-\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}\left(\frac{9c^2}{2}+\frac{c^2x}{2a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{c}$$
$$=-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{2\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}\left(\frac{27c^3}{4}-\frac{21c^3x}{4a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{3c}$$
$$=\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{4\,\text{Subst}\left(\int\frac{\frac{27c^4}{8}-\frac{69c^4x}{8a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{3c}$$
$$=\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{(9c^3)\,\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{2a}+\frac{(16c^3)\,\text{S}$$
$$=\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\left(c-\frac{c}{ax}\right)^{5/2}x+(9c^2)\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}dx,x,\sqrt{c-\frac{c}{ax}}\right)-$$
$$=\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{9c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{16\sqrt{2}c^{5/2}\tanh^{-1}\Big(}{a}$$

Mathematica [A] time = 0.100779, size = 116, normalized size = 0.83

$$\frac{c^2\left(-3a^2x^2+26ax-2\right)\sqrt{c-\frac{c}{ax}}+27ac^{5/2}x\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-48\sqrt{2}ac^{5/2}x\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{3a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(-2 + 26*a*x - 3*a^2*x^2) + 27*a*c^(5/2)*x*ArcTanh[S
qrt[c - c/(a*x)]/Sqrt[c]] - 48*Sqrt[2]*a*c^(5/2)*x*ArcTanh[Sqrt[c - c/(a*x)
]/(Sqrt[2]*Sqrt[c])])/(3*a^2*x)
```

Maple [B] time = 0.136, size = 257, normalized size = 1.9

$$-\frac{c^2}{6x^2}\sqrt{\frac{c(ax-1)}{ax}}\left(-90\sqrt{ax^2-x}a^{5/2}\sqrt{a^{-1}}x^3+48a^{5/2}\sqrt{a^{-1}}\sqrt{(ax-1)x}x^3+48a^{3/2}\left(ax^2-x\right)^{3/2}x\sqrt{a^{-1}}+45\ln\left(1/2\frac{2\sqrt{ax^2}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/6*(c*(a*x-1)/a/x)^(1/2)/x^2*c^2/a^(5/2)*(-90*(a*x^2-x)^(1/2)*a^(5/2)*(1/
a)^(1/2)*x^3+48*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^3+48*a^(3/2)*(a*x^2
-x)^(3/2)*x*(1/a)^(1/2)+45*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/
2))*(1/a)^(1/2)*x^3*a^2-48*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-
1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^3-72*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+
2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^3*a^2-4*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2)
)/((a*x-1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a*x))^(5/2)/(a*x + 1)^2, x)
```

Fricas [A] time = 1.90505, size = 655, normalized size = 4.71

$$\left[\frac{48\sqrt{2}ac^{\frac{5}{2}}x\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+27\,ac^{\frac{5}{2}}x\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(3\,a^2c^2x^2-26\,ac^2x+2\,c^2\right)\sqrt{\frac{ac}{}}}{6\,a^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/6*(48*sqrt(2)*a*c^(5/2)*x*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) - 3*a*c*x + c)/(a*x + 1)) + 27*a*c^(5/2)*x*log(-2*a*c*x - 2*a*sqrt(c)*
x*sqrt((a*c*x - c)/(a*x)) + c) - 2*(3*a^2*c^2*x^2 - 26*a*c^2*x + 2*c^2)*sqr
t((a*c*x - c)/(a*x)))/(a^2*x), 1/3*(48*sqrt(2)*a*sqrt(-c)*c^2*x*arctan(1/2*
sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 27*a*sqrt(-c)*c^2*x*arctan(sq
rt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (3*a^2*c^2*x^2 - 26*a*c^2*x + 2*c^2)*sq
rt((a*c*x - c)/(a*x)))/(a^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{3c^2\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx-\int \frac{3c^2\sqrt{c-\frac{c}{ax}}}{a^2x^2+ax}\,dx-\int -\frac{c^2\sqrt{c-\frac{c}{ax}}}{a^3x^3+a^2x^2}\,dx-\int \frac{ac^2x\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(5/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-3*c**2*sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(3*c**2*sqrt(c
- c/(a*x))/(a**2*x**2 + a*x), x) - Integral(-c**2*sqrt(c - c/(a*x))/(a**3*x
**3 + a**2*x**2), x) - Integral(a*c**2*x*sqrt(c - c/(a*x))/(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.547 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=113

$$\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{8\sqrt{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}+\frac{c\sqrt{c-\frac{c}{ax}}}{a}-x\left(c-\frac{c}{ax}\right)^{3/2}$$

```
[Out] (c*Sqrt[c - c/(a*x)])/a - (c - c/(a*x))^(3/2)*x + (7*c^(3/2)*ArcTanh[Sqrt[c
 - c/(a*x)]/Sqrt[c]])/a - (8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqr
t[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.212502, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6133, 25, 514, 375, 98, 154, 156, 63, 208}

$$\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{8\sqrt{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}+\frac{c\sqrt{c-\frac{c}{ax}}}{a}-x\left(c-\frac{c}{ax}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(3/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)])/a - (c - c/(a*x))^(3/2)*x + (7*c^(3/2)*ArcTanh[Sqrt[c
 - c/(a*x)]/Sqrt[c]])/a - (8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqr
t[2]*Sqrt[c])])/a
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}(1-ax)}{1+ax}dx$$
$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{5/2}x}{1+ax}dx}{c}$$
$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{5/2}}{a+\frac{1}{x}}dx}{c}$$
$$=\frac{a\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c}$$
$$=-\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}\left(\frac{7c^2}{2}-\frac{c^2x}{2a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{c}$$
$$=\frac{c\sqrt{c-\frac{c}{ax}}}{a}-\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{2\,\text{Subst}\left(\int\frac{\frac{7c^3}{4}-\frac{9c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{c}$$
$$=\frac{c\sqrt{c-\frac{c}{ax}}}{a}-\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{\left(7c^2\right)\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{2a}+\frac{\left(8c^2\right)\text{Subst}\left(\int\frac{1}{(a+x)\sqrt{c\ldots}}\right.}{a}$$
$$=\frac{c\sqrt{c-\frac{c}{ax}}}{a}-\left(c-\frac{c}{ax}\right)^{3/2}x+(7c)\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}dx,x,\sqrt{c-\frac{c}{ax}}\right)-(16c)\,\text{Subst}\left(\int\right.$$
$$=\frac{c\sqrt{c-\frac{c}{ax}}}{a}-\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{8\sqrt{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.0805365, size = 96, normalized size = 0.85

$$\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-8\sqrt{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)+c(2-ax)\sqrt{c-\frac{c}{ax}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(2 - a*x) + 7*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c
]] - 8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Maple [B] time = 0.124, size = 229, normalized size = 2.

$$-\frac{c}{2x}\sqrt{\frac{c(ax-1)}{ax}}\left(-10\sqrt{ax^2-x}a^{3/2}\sqrt{a^{-1}}x^2+8a^{3/2}\sqrt{a^{-1}}\sqrt{(ax-1)x}x^2+4\left(ax^2-x\right)^{3/2}\sqrt{a}\sqrt{a^{-1}}+5\ln\left(1/2\frac{2\sqrt{ax^2-\ldots}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)/x*c/a^(3/2)*(-10*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(
1/2)*x^2+8*a^(3/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^2+4*(a*x^2-x)^(3/2)*a^(1
/2)*(1/a)^(1/2)+5*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)
^(1/2)*x^2*a-8*a^(1/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*
a-3*a*x+1)/(a*x+1))*x^2-12*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(
1/2))*(1/a)^(1/2)*x^2*a)/((a*x-1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(a^2x^2-1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a*x))^(3/2)/(a*x + 1)^2, x)
```

Fricas [A] time = 1.9695, size = 539, normalized size = 4.77

$$\left[\frac{8\sqrt{2}c^{\frac{3}{2}}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+7\,c^{\frac{3}{2}}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-2\,(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{2\,a},\ \frac{8\sqrt{2}\sqrt{-c}c\arctan\left(\frac{\sqrt{2}\ldots}{\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/2*(8*sqrt(2)*c^(3/2)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))
 - 3*a*c*x + c)/(a*x + 1)) + 7*c^(3/2)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*
c*x - c)/(a*x)) + c) - 2*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a, (8*sqrt(
2)*sqrt(-c)*c*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 7*sq
rt(-c)*c*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (a*c*x - 2*c)*sqrt((a
*c*x - c)/(a*x)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{2c\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx-\int \frac{c\sqrt{c-\frac{c}{ax}}}{a^2x^2+ax}\,dx-\int \frac{acx\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(3/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-2*c*sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(c*sqrt(c - c/(a*x
))/(a**2*x**2 + a*x), x) - Integral(a*c*x*sqrt(c - c/(a*x))/(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.548 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=93

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)+\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -(Sqrt[c - c/(a*x)]*x) + (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a -
 (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.162651, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 98, 156, 63, 208}

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)+\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*x) + (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a -
 (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{1+ax}\,dx$$
$$=-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{1+ax}\,dx}{c}$$
$$=-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}\,dx}{c}$$
$$=\frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2(a+x)}\,dx,x,\frac{1}{x}\right)}{c}$$
$$=-\sqrt{c-\frac{c}{ax}}x-\frac{\text{Subst}\left(\int \frac{\frac{5c^2}{2}-\frac{3c^2x}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{c}$$
$$=-\sqrt{c-\frac{c}{ax}}x-\frac{(5c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}+\frac{(4c)\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$
$$=-\sqrt{c-\frac{c}{ax}}x+5\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)-8\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)$$
$$=-\sqrt{c-\frac{c}{ax}}x+\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.0493812, size = 93, normalized size = 1.

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)+\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*x) + (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a -
 (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Maple [B] time = 0.124, size = 190, normalized size = 2.

$$\frac{x}{2}\sqrt{\frac{c(ax-1)}{ax}}\left(2\sqrt{ax^2-x}a^{3/2}\sqrt{a^{-1}}-4\sqrt{(ax-1)x}a^{3/2}\sqrt{a^{-1}}-\ln\left(\frac{1}{2}\left(2\sqrt{ax^2-x}\sqrt{a}+2ax-1\right)\frac{1}{\sqrt{a}}\right)a\sqrt{a^{-1}}+4\sqrt{2}\ln\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(1/2)-4*((a*x-
1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/
a^(1/2))*a*(1/a)^(1/2)+4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2
```

```
)*a-3*a*x+1)/(a*x+1))*a^(1/2)+6*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1
)/a^(1/2))*a*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/a^(3/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))/(a*x + 1)^2, x)
```

Fricas [A] time = 1.9688, size = 509, normalized size = 5.47

$$\left[-\frac{2\,ax\sqrt{\frac{acx-c}{ax}}-4\,\sqrt{2}\sqrt{c}\log\left(\frac{2\,\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)-5\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},-\frac{ax\sqrt{\frac{acx-c}{ax}}-4\,\sqrt{2}\sqrt{-c}\,\mathrm{a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) - 4*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*s
qrt(c)*x*sqrt((a*c*x - c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) - 5*sqrt(c)*log(
-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, -(a*x*sqrt((a*c*x
 - c)/(a*x)) - 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x -
 c)/(a*x))/c) + 5*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(a*x*sqrt(c - c/(a*x))
/(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.549 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=96

$$-\frac{x\sqrt{c-\frac{c}{ax}}}{c}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] -((Sqrt[c - c/(a*x)]*x)/c) + (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*Sqrt
[c]) - (2*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*Sqrt[c])
```

Rubi [A] time = 0.161138, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 99, 156, 63, 208}

$$-\frac{x\sqrt{c-\frac{c}{ax}}}{c}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x)/c) + (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*Sqrt
[c]) - (2*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*Sqrt[c])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx = \int \frac{1-ax}{\sqrt{c-\frac{c}{ax}}(1+ax)}dx$$
$$= -\frac{a\int \frac{\sqrt{c-\frac{c}{ax}}x}{1+ax}dx}{c}$$
$$= -\frac{a\int \frac{\sqrt{c-\frac{c}{ax}}}{a+\frac{1}{x}}dx}{c}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2(a+x)}dx, x, \frac{1}{x}\right)}{c}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}x}{c} + \frac{\text{Subst}\left(\int \frac{-\frac{3c}{2}+\frac{cx}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}dx, x, \frac{1}{x}\right)}{c}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}x}{c} - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}dx, x, \frac{1}{x}\right)}{2a} + \frac{2\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}dx, x, \frac{1}{x}\right)}{a}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}x}{c} + \frac{3\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c} - \frac{4\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}x}{c} + \frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

Mathematica [A] time = 0.0474643, size = 96, normalized size = 1.

$$-\frac{x\sqrt{c-\frac{c}{ax}}}{c} + \frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x)/c) + (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*Sqrt
[c]) - (2*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*Sqrt[c])
```

Maple [A] time = 0.126, size = 136, normalized size = 1.4

$$-\frac{x}{2c}\sqrt{\frac{c(ax-1)}{ax}}\left(2\sqrt{(ax-1)x}a^{3/2}\sqrt{a^{-1}} - 3\ln\left(1/2\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{\sqrt{a}}\right)a\sqrt{a^{-1}} - 2\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax-1}}{ax+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(1/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-3*ln(
1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)-2*2^(1/2)*
```

```
ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))/((
a*x-1)*x)^(1/2)/c/a^(3/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^2x^2-1}{(ax+1)^2\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*sqrt(c - c/(a*x))), x)
```

Fricas [A] time = 1.78072, size = 535, normalized size = 5.57

$$\left[-\frac{2\,ax\sqrt{\frac{acx-c}{ax}}-2\,\sqrt{2}\sqrt{c}\log\left(\frac{\frac{2\,\sqrt{2}ax\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}-3\,ax+1}{ax+1}\right)-3\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,ac},\ \frac{2\,\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}ax\sqrt{-\frac{1}{c}}\sqrt{\cdots}}{ax-1}\right.\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) - 2*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*x
*sqrt((a*c*x - c)/(a*x))/sqrt(c) - 3*a*x + 1)/(a*x + 1)) - 3*sqrt(c)*log(-2
*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/(a*c), (2*sqrt(2)*c*sq
rt(-1/c)*arctan(sqrt(2)*a*x*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(a*x - 1)) -
 a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/
(a*x))/c))/(a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{ax\sqrt{c-\frac{c}{ax}}+\sqrt{c-\frac{c}{ax}}}\,dx-\int -\frac{1}{ax\sqrt{c-\frac{c}{ax}}+\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x*sqrt(c - c/(a*x)) + sqrt(c - c/(a*x))), x) - Integral(-1
/(a*x*sqrt(c - c/(a*x)) + sqrt(c - c/(a*x))), x)
```

Giac [B] time = 1.32843, size = 227, normalized size = 2.36

$$ac\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}c}-\frac{3\arctan\left(\frac{\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c}-\frac{\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}}{a^2\left(c+\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}\right)c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] a*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-(c - 2*c/(a*x + 1))/(1/(a*x + 1) -
1))/sqrt(-c))/(a^2*sqrt(-c)*c) - 3*arctan(sqrt(-(c - 2*c/(a*x + 1))/(1/(a*x
 + 1) - 1))/sqrt(-c))/(a^2*sqrt(-c)*c) - sqrt(-(c - 2*c/(a*x + 1))/(1/(a*x
+ 1) - 1))/(a^2*(c + (c - 2*c/(a*x + 1))/(1/(a*x + 1) - 1))*c))
```

3.550 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{x\sqrt{c-\frac{c}{ax}}}{c^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

```
[Out] -((Sqrt[c - c/(a*x)]*x)/c^2) + ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*c^(3/2
)) - (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Rubi [A] time = 0.163008, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6133, 25, 514, 375, 103, 21, 83, 63, 208}

$$-\frac{x\sqrt{c-\frac{c}{ax}}}{c^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x)/c^2) + ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*c^(3/2
)) - (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}\,dx$$

$$= -\frac{a\int \frac{x}{\sqrt{c-\frac{c}{ax}}(1+ax)}\,dx}{c}$$

$$= -\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}\,dx}{c}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} - \frac{\text{Subst}\left(\int \frac{\frac{c}{2}-\frac{cx}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} - \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x(a+x)}\,dx, x, \frac{1}{x}\right)}{2c^2}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac} + \frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} + \frac{\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^2} - \frac{2\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^2}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Mathematica [A] time = 0.05085, size = 95, normalized size = 1.

$$-\frac{x\sqrt{c-\frac{c}{ax}}}{c^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x)/c^2) + ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*c^(3/2
)) - (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Maple [A] time = 0.148, size = 134, normalized size = 1.4

$$\frac{x}{2c^2}\sqrt{\frac{c(ax-1)}{ax}}\left(-2\sqrt{(ax-1)x}a^{3/2}\sqrt{a^{-1}} + \ln\left(\frac{1}{2}\left(2\sqrt{(ax-1)x}\sqrt{a}+2ax-1\right)\frac{1}{\sqrt{a}}\right)a\sqrt{a^{-1}} + \sqrt{2}\ln\left(\frac{1}{ax+1}\left(2\sqrt{2}\sqrt{a^{-1}}\sqrt{\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(3/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(-2*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/
2)+ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)+2^(1
/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2)
)/((a*x-1)*x)^(1/2)/c^2/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^2x^2-1}{(ax+1)^2\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a*x))^(3/2)), x)
```

Fricas [A] time = 1.90191, size = 529, normalized size = 5.57

$$\left[-\frac{2\,ax\sqrt{\frac{acx-c}{ax}}-\sqrt{2}\sqrt{c}\log\left(\frac{\frac{2\sqrt{2}ax\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}-3\,ax+1}{ax+1}\right)-\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,ac^2}, \frac{\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}ax\sqrt{-\frac{1}{c}}\sqrt{\frac{acx-}{ax}}}{ax-1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) - sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*x*s
qrt((a*c*x - c)/(a*x))/sqrt(c) - 3*a*x + 1)/(a*x + 1)) - sqrt(c)*log(-2*a*c
*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/(a*c^2), (sqrt(2)*c*sqrt(-
1/c)*arctan(sqrt(2)*a*x*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(a*x - 1)) - a*x
*sqrt((a*c*x - c)/(a*x)) - sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))
/c))/(a*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{acx\sqrt{c-\frac{c}{ax}}-\frac{c\sqrt{c-\frac{c}{ax}}}{ax}}\,dx-\int -\frac{1}{acx\sqrt{c-\frac{c}{ax}}-\frac{c\sqrt{c-\frac{c}{ax}}}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x)**(3/2),x)
```

```
[Out] -Integral(a*x/(a*c*x*sqrt(c - c/(a*x)) - c*sqrt(c - c/(a*x))/(a*x)), x) - I
ntegral(-1/(a*c*x*sqrt(c - c/(a*x)) - c*sqrt(c - c/(a*x))/(a*x)), x)
```

Giac [B] time = 1.43671, size = 225, normalized size = 2.37

$$ac\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}c^2}-\frac{\arctan\left(\frac{\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^2}-\frac{\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}}{a^2\left(c+\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}\right)c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] a*c*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-(c - 2*c/(a*x + 1))/(1/(a*x + 1) - 1)
)/sqrt(-c))/(a^2*sqrt(-c)*c^2) - arctan(sqrt(-(c - 2*c/(a*x + 1))/(1/(a*x +
 1) - 1))/sqrt(-c))/(a^2*sqrt(-c)*c^2) - sqrt(-(c - 2*c/(a*x + 1))/(1/(a*x
+ 1) - 1))/(a^2*(c + (c - 2*c/(a*x + 1))/(1/(a*x + 1) - 1))*c^2))
```

3.551 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}+\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

```
[Out] 2/(a*c^2*Sqrt[c - c/(a*x)]) - x/(c^2*Sqrt[c - c/(a*x)]) - ArcTanh[Sqrt[c -
c/(a*x)]/Sqrt[c]]/(a*c^(5/2)) - ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])
]/(Sqrt[2]*a*c^(5/2))
```

Rubi [A] time = 0.190284, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6133, 25, 514, 375, 103, 152, 156, 63, 208}

$$-\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}+\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] 2/(a*c^2*Sqrt[c - c/(a*x)]) - x/(c^2*Sqrt[c - c/(a*x)]) - ArcTanh[Sqrt[c -
c/(a*x)]/Sqrt[c]]/(a*c^(5/2)) - ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])
]/(Sqrt[2]*a*c^(5/2))
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
```

```
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}\,dx$$

$$= -\frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}\,dx}{c}$$

$$= -\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}\,dx}{c}$$

$$= \frac{a\,\mathrm{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{x}{c^2\sqrt{c-\frac{c}{ax}}} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{c}{2}-\frac{3cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} + \frac{\mathrm{Subst}\left(\int \frac{\frac{c^2}{2}+\frac{c^2x}{a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= \frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} + \frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2} + \frac{\mathrm{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= \frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} - \frac{\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^3} - \frac{\mathrm{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^3}$$

$$= \frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

Mathematica [C] time = 0.0517435, size = 67, normalized size = 0.56

$$\frac{\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a-\frac{1}{x}}{2a}\right)+\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\frac{1}{ax}\right)-ax}{ac^2\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (-(a*x) + Hypergeometric2F1[-1/2, 1, 1/2, (a - x^(-1))/(2*a)] + Hypergeomet
ric2F1[-1/2, 1, 1/2, 1 - 1/(a*x)])/(a*c^2*Sqrt[c - c/(a*x)])
```

Maple [B] time = 0.128, size = 370, normalized size = 3.1

$$-\frac{x}{4c^3(ax-1)^2}\sqrt{\frac{c(ax-1)}{ax}}\left(8a^{7/2}\sqrt{a^{-1}}\sqrt{(ax-1)x}x^2+2\ln\left(1/2\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{\sqrt{a}}\right)\sqrt{a^{-1}}x^2a^3-a^{\frac{5}{2}}\sqrt{2}\ln\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(5/2),x)
```

```
[Out] -1/4*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(8*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/
2)*x^2+2*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*
x^2*a^3-a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x
+1)/(a*x+1))*x^2-4*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(3/2)-16*a^(5/2)*(1/a)^(
1/2)*((a*x-1)*x)^(1/2)*x-4*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(
1/2))*(1/a)^(1/2)*x*a^2+2*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1
)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x+8*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+2*
ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)-2^(1/2)
*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))/(
(a*x-1)*x)^(1/2)/c^3/(a*x-1)^2/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^2x^2-1}{(ax+1)^2\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a*x))^(5/2)), x)
```

Fricas [A] time = 1.94808, size = 649, normalized size = 5.45

$$\left[\frac{\sqrt{2}(ax-1)\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+2\,(ax-1)\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-4\left(a^2x^2-2\,ax\right)\sqrt{\frac{acx-c}{ax}}}{4\left(a^2c^3x-ac^3\right)}, \frac{\sqrt{2}(\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*(a*x - 1)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)
/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + 2*(a*x - 1)*sqrt(c)*log(-2*a*c*x + 2*a*
sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 4*(a^2*x^2 - 2*a*x)*sqrt((a*c*x -
c)/(a*x)))/(a^2*c^3*x - a*c^3), 1/2*(sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(1/2*
sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 2*(a*x - 1)*sqrt(-c)*arctan(s
qrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(a^2*x^2 - 2*a*x)*sqrt((a*c*x - c)/(
a*x)))/(a^2*c^3*x - a*c^3)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x)**(5/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 1.77423, size = 301, normalized size = 2.53

$$\frac{1}{2}\,ac\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}c^3}+\frac{2\arctan\left(\frac{\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^3}+\frac{2\left(c+\frac{2\left(c-\frac{2c}{ax+1}\right)}{\frac{1}{ax+1}-1}\right)}{\left(c\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}+\frac{\left(c-\frac{2c}{ax+1}\right)\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}}{\frac{1}{ax+1}-1}\right)a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*a*c*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-(c - 2*c/(a*x + 1))/(1/(a*x + 1)
- 1))/sqrt(-c))/(a^2*sqrt(-c)*c^3) + 2*arctan(sqrt(-(c - 2*c/(a*x + 1))/(1/
(a*x + 1) - 1))/sqrt(-c))/(a^2*sqrt(-c)*c^3) + 2*(c + 2*(c - 2*c/(a*x + 1))
/(1/(a*x + 1) - 1))/((c*sqrt(-(c - 2*c/(a*x + 1))/(1/(a*x + 1) - 1)) + (c -
 2*c/(a*x + 1))*sqrt(-(c - 2*c/(a*x + 1))/(1/(a*x + 1) - 1))/(1/(a*x + 1) -
 1))*a^2*c^3))
```

3.552 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}}+\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

```
[Out] 4/(3*a*c^2*(c - c/(a*x))^(3/2)) + 7/(2*a*c^3*Sqrt[c - c/(a*x)]) - x/(c^2*(c
 - c/(a*x))^(3/2)) - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(7/2)) - A
rcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]/(2*Sqrt[2]*a*c^(7/2))
```

Rubi [A] time = 0.227748, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6133, 25, 514, 375, 103, 152, 156, 63, 208}

$$-\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}}+\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] 4/(3*a*c^2*(c - c/(a*x))^(3/2)) + 7/(2*a*c^3*Sqrt[c - c/(a*x)]) - x/(c^2*(c
 - c/(a*x))^(3/2)) - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(7/2)) - A
rcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]/(2*Sqrt[2]*a*c^(7/2))
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
```

```
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}\,dx$$
$$= -\frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}\,dx}{c}$$
$$= -\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}\,dx}{c}$$
$$= \frac{a\,\mathrm{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= -\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{3c}{2}-\frac{5cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$
$$= \frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\mathrm{Subst}\left(\int \frac{\frac{9c^2}{2}+\frac{6c^2x}{a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^4}$$
$$= \frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{9c^3}{2}-\frac{21c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{3c^6}$$
$$= \frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\mathrm{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{4ac^3} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-}}\right.}{2ac^3}$$
$$= \frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\mathrm{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{2c^4} - \frac{3\,\mathrm{Subst}\left(\int \frac{}{a-}\right.}{}$$
$$= \frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

Mathematica [C] time = 0.0563473, size = 77, normalized size = 0.52

$$\frac{x\left(\mathrm{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a-\frac{1}{x}}{2a}\right)+3\mathrm{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\frac{1}{ax}\right)-3ax\right)}{3c^3(ax-1)\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (x*(-3*a*x + Hypergeometric2F1[-3/2, 1, -1/2, (a - x^(-1))/(2*a)] + 3*Hyper
geometric2F1[-3/2, 1, -1/2, 1 - 1/(a*x)]))/(3*c^3*Sqrt[c - c/(a*x)]*(-1 + a
*x))
```

Maple [B] time = 0.133, size = 497, normalized size = 3.4

$$-\frac{x}{24c^4(ax-1)^3}\sqrt{\frac{c(ax-1)}{ax}}\left(84a^{9/2}\sqrt{a^{-1}}\sqrt{(ax-1)x}x^3+36\ln\left(1/2\,\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{\sqrt{a}}\right)\sqrt{a^{-1}}x^3a^4-3a^{7/2}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(7/2),x)
```

```
[Out] -1/24*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(84*a^(9/2)*(1/a)^(1/2)*((a*x-1)*x)^(
1/2)*x^3+36*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/
2)*x^3*a^4-3*a^(7/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-
3*a*x+1)/(a*x+1))*x^3-60*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(3/2)*x-252*a^(7/2
)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^2-108*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)
+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^2*a^3+9*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a
)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^2+52*a^(5/2)*(1/a)^(1/2)*((
a*x-1)*x)^(3/2)+252*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x+108*ln(1/2*(2*(
(a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x*a^2-9*a^(3/2)*2^(1
/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x-84*((
a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-36*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+
2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)+3*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1
)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))/((a*x-1)*x)^(1/2)/c^4/(a*x-1)^3/(1/
a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{a^2x^2-1}{(ax+1)^2\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a*x))^(7/2)), x)
```

Fricas [A] time = 1.99972, size = 811, normalized size = 5.48

$$\left[\frac{3\sqrt{2}\left(a^2x^2-2ax+1\right)\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3acx+c}{ax+1}\right)+36\left(a^2x^2-2ax+1\right)\sqrt{c}\log\left(-2acx+2a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-4(6}{24\left(a^3c^4x^2-2a^2c^4x+ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*s
qrt((a*c*x - c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + 36*(a^2*x^2 - 2*a*x + 1)
*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 4*(6*a
^3*x^3 - 29*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2
*c^4*x + a*c^4), 1/12*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(1/2*
sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 36*(a^2*x^2 - 2*a*x + 1)*sqrt
```

```
(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(6*a^3*x^3 - 29*a^2*x^2
 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x)**(7/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 2.30167, size = 346, normalized size = 2.34

$$\frac{1}{12}\,ac\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}c^4}-\frac{2\left(2c-\frac{15\left(c-\frac{2c}{ax+1}\right)}{\frac{1}{ax+1}-1}\right)\left(\frac{1}{ax+1}-1\right)}{a^2\left(c-\frac{2c}{ax+1}\right)c^4\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}}+\frac{36\arctan\left(\frac{\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^4}-\frac{12\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}}{a^2\left(c+\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}\right)c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] 1/12*a*c*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-(c - 2*c/(a*x + 1))/(1/(a*x +
1) - 1))/sqrt(-c))/(a^2*sqrt(-c)*c^4) - 2*(2*c - 15*(c - 2*c/(a*x + 1))/(1/
(a*x + 1) - 1))*(1/(a*x + 1) - 1)/(a^2*(c - 2*c/(a*x + 1))*c^4*sqrt(-(c - 2
*c/(a*x + 1))/(1/(a*x + 1) - 1))) + 36*arctan(sqrt(-(c - 2*c/(a*x + 1))/(1/
(a*x + 1) - 1))/sqrt(-c))/(a^2*sqrt(-c)*c^4) - 12*sqrt(-(c - 2*c/(a*x + 1))
/(1/(a*x + 1) - 1))/(a^2*(c + (c - 2*c/(a*x + 1))/(1/(a*x + 1) - 1))*c^4))
```

3.553 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}+\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

```
[Out] 6/(5*a*c^2*(c - c/(a*x))^(5/2)) + 11/(6*a*c^3*(c - c/(a*x))^(3/2)) + 21/(4*
a*c^4*Sqrt[c - c/(a*x)]) - x/(c^2*(c - c/(a*x))^(5/2)) - (5*ArcTanh[Sqrt[c
- c/(a*x)]/Sqrt[c]])/(a*c^(9/2)) - ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[
c])]/(4*Sqrt[2]*a*c^(9/2))
```

Rubi [A] time = 0.256622, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6133, 25, 514, 375, 103, 152, 156, 63, 208}

$$-\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}+\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^(9/2)),x]
```


```
[Out] 6/(5*a*c^2*(c - c/(a*x))^(5/2)) + 11/(6*a*c^3*(c - c/(a*x))^(3/2)) + 21/(4*
a*c^4*Sqrt[c - c/(a*x)]) - x/(c^2*(c - c/(a*x))^(5/2)) - (5*ArcTanh[Sqrt[c
- c/(a*x)]/Sqrt[c]])/(a*c^(9/2)) - ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[
c])]/(4*Sqrt[2]*a*c^(9/2))
```


Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx = \int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{9/2}(1+ax)}\,dx$$

$$= -\frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}\,dx}{c}$$

$$= -\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}\,dx}{c}$$

$$= \frac{a\,\mathrm{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{5c}{2}-\frac{7cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\mathrm{Subst}\left(\int \frac{\frac{25c^2}{2}+\frac{15c^2x}{a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^4}$$

$$= \frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{75c^3}{2}-\frac{165c^3x}{4a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^6}$$

$$= \frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\mathrm{Subst}\left(\int \frac{\frac{75c^4}{2}+\frac{315c^4x}{8a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x,\right.}{15c^8}$$

$$= \frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\mathrm{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x,\right.}{8ac^4}$$

$$= \frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\mathrm{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\right.}{4c^5}$$

$$= \frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}} - \frac{\tanh^{-}}{4\sqrt{}}$$

Mathematica [C] time = 0.0576646, size = 82, normalized size = 0.47

$$-\frac{ax^2\left(-\mathrm{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{a-\frac{1}{x}}{2a}\right)-5\mathrm{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-\frac{1}{ax}\right)+5ax\right)}{5c^4(ax-1)^2\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^(9/2)),x]
```

```
[Out] -(a*x^2*(5*a*x - Hypergeometric2F1[-5/2, 1, -3/2, (a - x^(-1))/(2*a)] - 5*H
ypergeometric2F1[-5/2, 1, -3/2, 1 - 1/(a*x)]))/(5*c^4*Sqrt[c - c/(a*x)]*(-1
```

```
+ a*x)^2)
```

Maple [B] time = 0.161, size = 626, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(9/2),x)
```

```
[Out] 1/240*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(-1260*((a*x-1)*x)^(1/2)*a^(11/2)*(1/
a)^(1/2)*x^4+1020*((a*x-1)*x)^(3/2)*a^(9/2)*(1/a)^(1/2)*x^2-600*ln(1/2*(2*(
(a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^4*a^5+15*a^(9/2)*2
^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^4+
5040*a^(9/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^3-1792*a^(7/2)*(1/a)^(1/2)*((a
*x-1)*x)^(3/2)*x+2400*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))
*(1/a)^(1/2)*x^3*a^4-60*a^(7/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*
x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^3-7560*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)
*x^2+820*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(3/2)-3600*ln(1/2*(2*((a*x-1)*x)^(
1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^2*a^3+90*a^(5/2)*2^(1/2)*ln((2
*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^2+5040*a^(5/2)
*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x+2400*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2
*a*x-1)/a^(1/2))*(1/a)^(1/2)*x*a^2-60*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(
1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x-1260*((a*x-1)*x)^(1/2)*a^(3/2)
*(1/a)^(1/2)-600*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1
/a)^(1/2)+15*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)
/(a*x+1))*a^(1/2))/((a*x-1)*x)^(1/2)/c^5/(a*x-1)^4/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^2x^2-1}{(ax+1)^2\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(9/2),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a*x))^(9/2)), x)
```

Fricas [A] time = 1.98411, size = 973, normalized size = 5.62

$$\left[\frac{15\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{cx}\sqrt{\frac{acx-c}{ax}}-3acx+c}{ax+1}\right)+600\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-2acx+2a\sqrt{cx}\right.}{240\left(a^4c^5x^3-3a^3c^5x^2+3a^2c^5x-ac^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log((2*sqrt(2)
*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + 600*(a^3*x
^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*
x - c)/(a*x)) + c) - 4*(60*a^4*x^4 - 497*a^3*x^3 + 740*a^2*x^2 - 315*a*x)*s
qrt((a*c*x - c)/(a*x)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
, 1/120*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(1/2*s
qrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 600*(a^3*x^3 - 3*a^2*x^2 + 3*a
*x - 1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(60*a^4*x^4
 - 497*a^3*x^3 + 740*a^2*x^2 - 315*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^5*x
^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x)**(9/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 3.87115, size = 389, normalized size = 2.25

$$\frac{1}{120}ac\left(\frac{15\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}c^5}+\frac{2\left(12c^2-\frac{50\left(c-\frac{2c}{ax+1}\right)c}{\frac{1}{ax+1}-1}+\frac{255\left(c-\frac{2c}{ax+1}\right)^2}{\left(\frac{1}{ax+1}-1\right)^2}\right)\left(\frac{1}{ax+1}-1\right)^2}{a^2\left(c-\frac{2c}{ax+1}\right)^2c^5\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}}+\frac{600\arctan\left(\frac{\sqrt{-\frac{c-\frac{2c}{ax+1}}{\frac{1}{ax+1}-1}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^5}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/120*a*c*(15*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-(c - 2*c/(a*x + 1))/(1/(a*x
+ 1) - 1))/sqrt(-c))/(a^2*sqrt(-c)*c^5) + 2*(12*c^2 - 50*(c - 2*c/(a*x + 1)
)*c/(1/(a*x + 1) - 1) + 255*(c - 2*c/(a*x + 1))^2/(1/(a*x + 1) - 1)^2)*(1/(
a*x + 1) - 1)^2/(a^2*(c - 2*c/(a*x + 1))^2*c^5*sqrt(-(c - 2*c/(a*x + 1))/(1
/(a*x + 1) - 1))) + 600*arctan(sqrt(-(c - 2*c/(a*x + 1))/(1/(a*x + 1) - 1))
/sqrt(-c))/(a^2*sqrt(-c)*c^5) - 120*sqrt(-(c - 2*c/(a*x + 1))/(1/(a*x + 1)
- 1))/(a^2*(c + (c - 2*c/(a*x + 1))/(1/(a*x + 1) - 1))*c^5))
```

3.554 $\int e^{-3\tanh^{-1}(ax)} \left(c-\frac{c}{ax}\right)^{9/2} dx$

Optimal. Leaf size=267

$$\frac{5a^4x^5(587-109ax)\left(c-\frac{c}{ax}\right)^{9/2}}{7(1-ax)^{9/2}\sqrt{ax+1}}+\frac{70a^3x^4\left(c-\frac{c}{ax}\right)^{9/2}}{(1-ax)^{5/2}\sqrt{ax+1}}-\frac{50a^2x^3\left(c-\frac{c}{ax}\right)^{9/2}}{7(1-ax)^{3/2}\sqrt{ax+1}}-\frac{15a^{7/2}x^{9/2}\left(c-\frac{c}{ax}\right)^{9/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{9/2}}+\frac{10\cdots}{7\sqrt{\cdots}}$$

```
[Out] (5*a^4*(c - c/(a*x))^(9/2)*x^5*(587 - 109*a*x))/(7*(1 - a*x)^(9/2)*Sqrt[1 +
 a*x]) + (70*a^3*(c - c/(a*x))^(9/2)*x^4)/((1 - a*x)^(5/2)*Sqrt[1 + a*x]) -
 (50*a^2*(c - c/(a*x))^(9/2)*x^3)/(7*(1 - a*x)^(3/2)*Sqrt[1 + a*x]) + (10*a
*(c - c/(a*x))^(9/2)*x^2)/(7*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (2*(c - c/(a*x)
)^(9/2)*x*Sqrt[1 - a*x])/(7*Sqrt[1 + a*x]) - (15*a^(7/2)*(c - c/(a*x))^(9/2
)*x^(9/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(9/2)
```

Rubi [A] time = 0.242904, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6134, 6129, 98, 150, 143, 54, 215}

$$\frac{5a^4x^5(587-109ax)\left(c-\frac{c}{ax}\right)^{9/2}}{7(1-ax)^{9/2}\sqrt{ax+1}}+\frac{70a^3x^4\left(c-\frac{c}{ax}\right)^{9/2}}{(1-ax)^{5/2}\sqrt{ax+1}}-\frac{50a^2x^3\left(c-\frac{c}{ax}\right)^{9/2}}{7(1-ax)^{3/2}\sqrt{ax+1}}-\frac{15a^{7/2}x^{9/2}\left(c-\frac{c}{ax}\right)^{9/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{9/2}}+\frac{10\cdots}{7\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(9/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (5*a^4*(c - c/(a*x))^(9/2)*x^5*(587 - 109*a*x))/(7*(1 - a*x)^(9/2)*Sqrt[1 +
 a*x]) + (70*a^3*(c - c/(a*x))^(9/2)*x^4)/((1 - a*x)^(5/2)*Sqrt[1 + a*x]) -
 (50*a^2*(c - c/(a*x))^(9/2)*x^3)/(7*(1 - a*x)^(3/2)*Sqrt[1 + a*x]) + (10*a
*(c - c/(a*x))^(9/2)*x^2)/(7*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (2*(c - c/(a*x)
)^(9/2)*x*Sqrt[1 - a*x])/(7*Sqrt[1 + a*x]) - (15*a^(7/2)*(c - c/(a*x))^(9/2
)*x^(9/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(9/2)
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{e^{-3\tanh^{-1}(ax)}(1-ax)^{9/2}}{x^{9/2}}dx}{(1-ax)^{9/2}}$$
$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{(1-ax)^6}{x^{9/2}(1+ax)^{3/2}}dx}{(1-ax)^{9/2}}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1-ax}}{7\sqrt{1+ax}} - \frac{\left(2\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{(1-ax)^4\left(\frac{25a}{2}-\frac{5a^2x}{2}\right)}{x^{7/2}(1+ax)^{3/2}}dx}{7(1-ax)^{9/2}}$$
$$= \frac{10a\left(c-\frac{c}{ax}\right)^{9/2}x^2}{7\sqrt{1-ax}\sqrt{1+ax}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1-ax}}{7\sqrt{1+ax}} - \frac{\left(4\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{(1-ax)^3\left(-\frac{375a^2}{4}-\frac{25a^3x}{4}\right)}{x^{5/2}(1+ax)^{3/2}}dx}{35(1-ax)^{9/2}}$$
$$= -\frac{50a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3}{7(1-ax)^{3/2}\sqrt{1+ax}} + \frac{10a\left(c-\frac{c}{ax}\right)^{9/2}x^2}{7\sqrt{1-ax}\sqrt{1+ax}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1-ax}}{7\sqrt{1+ax}} - \frac{\left(8\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right.}{105}$$
$$= \frac{70a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4}{(1-ax)^{5/2}\sqrt{1+ax}} - \frac{50a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3}{7(1-ax)^{3/2}\sqrt{1+ax}} + \frac{10a\left(c-\frac{c}{ax}\right)^{9/2}x^2}{7\sqrt{1-ax}\sqrt{1+ax}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1-ax}}{7\sqrt{1+ax}}$$
$$= \frac{5a^4\left(c-\frac{c}{ax}\right)^{9/2}x^5(587-109ax)}{7(1-ax)^{9/2}\sqrt{1+ax}} + \frac{70a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4}{(1-ax)^{5/2}\sqrt{1+ax}} - \frac{50a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3}{7(1-ax)^{3/2}\sqrt{1+ax}} + \frac{10a\left(c-\right.}{7\sqrt{1-ax}}$$
$$= \frac{5a^4\left(c-\frac{c}{ax}\right)^{9/2}x^5(587-109ax)}{7(1-ax)^{9/2}\sqrt{1+ax}} + \frac{70a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4}{(1-ax)^{5/2}\sqrt{1+ax}} - \frac{50a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3}{7(1-ax)^{3/2}\sqrt{1+ax}} + \frac{10a\left(c-\right.}{7\sqrt{1-ax}}$$
$$= \frac{5a^4\left(c-\frac{c}{ax}\right)^{9/2}x^5(587-109ax)}{7(1-ax)^{9/2}\sqrt{1+ax}} + \frac{70a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4}{(1-ax)^{5/2}\sqrt{1+ax}} - \frac{50a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3}{7(1-ax)^{3/2}\sqrt{1+ax}} + \frac{10a\left(c-\right.}{7\sqrt{1-ax}}$$

Mathematica [C] time = 10.0585, size = 234, normalized size = 0.88

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(-\frac{7168(-ax(ax+1))^{5/2}(ax-1)^4\text{HypergeometricPFQ}\left(\left\{-\frac{3}{2},2,2,2,\frac{5}{2}\right\},\left\{1,1,1,\frac{7}{2}\right\},-ax\right)}{(ax+1)^{3/2}} + \sqrt{-ax(ax+1)}\left(70000a^8x^8-214760a^7x^7+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(9/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(c^4*Sqrt[c - c/(a*x)]*(Sqrt[-(a*x*(1 + a*x))]*(3091 - 12955*a*x + 34100*a
^2*x^2 - 59750*a^3*x^3 - 375805*a^4*x^4 + 84329*a^5*x^5 + 165830*a^6*x^6 -
214760*a^7*x^7 + 70000*a^8*x^8) + 105*(-27 + 81*a*x - 54*a^2*x^2 - 54*a^3*x
^3 + 209*a^4*x^4 + 101*a^5*x^5)*ArcSin[Sqrt[-(a*x)]] - (7168*(-1 + a*x)^4*(
-(a*x*(1 + a*x)))^(5/2)*HypergeometricPFQ[{-3/2, 2, 2, 2, 5/2}, {1, 1, 1, 7
/2}, -(a*x)])/(1 + a*x)^(3/2)))/(896*a^4*x^3*Sqrt[-(a*x*(1 + a*x))]*Sqrt[1
- a^2*x^2])
```

Maple [A] time = 0.158, size = 227, normalized size = 0.9

$$-\frac{c^4}{14x^3(ax+1)(ax-1)}\sqrt{\frac{c(ax-1)}{ax}}\left(14\sqrt{-(ax+1)x}a^{11/2}x^5+105\arctan\left(1/2\frac{2ax+1}{\sqrt{a}\sqrt{-(ax+1)x}}\right)x^5a^5+3510a^{9/2}\sqrt{-(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/14*(c*(a*x-1)/a/x)^(1/2)/x^3*c^4/a^(9/2)*(14*(-(a*x+1)*x)^(1/2)*a^(11/2)
*x^5+105*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^5*a^5+3510*a^(9
/2)*(-(a*x+1)*x)^(1/2)*x^4+105*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1
/2))*x^4*a^4+1440*a^(7/2)*x^3*(-(a*x+1)*x)^(1/2)-220*a^(5/2)*x^2*(-(a*x+1)*
x)^(1/2)+40*a^(3/2)*x*(-(a*x+1)*x)^(1/2)-4*a^(1/2)*(-(a*x+1)*x)^(1/2))*(-a^
2*x^2+1)^(1/2)/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a x}\right)^{\frac{9}{2}}}{(a x+1)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(9/2)/(a*x + 1)^3, x)
```

Fricas [A] time = 2.19902, size = 848, normalized size = 3.18

$$\left[\frac{105\left(a^{5} c^{4} x^{5}-a^{3} c^{4} x^{3}\right)\sqrt{-c}\log\left(-\frac{8\,a^{3} c x^{3}-7\,a c x-4\left(2\,a^{2} x^{2}+a x\right)\sqrt{-a^{2} x^{2}+1}\sqrt{-c}\sqrt{\frac{a c x-c}{a x}}-c}{a x-1}\right)-4\left(7\,a^{5} c^{4} x^{5}+1755\,a^{4} c^{4} x^{4}+720\,a^{3} c^{4} x^{3}\right.}{28\left(a^{6} x^{5}-a^{4} x^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/28*(105*(a^5*c^4*x^5 - a^3*c^4*x^3)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))
- c)/(a*x - 1)) - 4*(7*a^5*c^4*x^5 + 1755*a^4*c^4*x^4 + 720*a^3*c^4*x^3 - 1
10*a^2*c^4*x^2 + 20*a*c^4*x - 2*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a
*x)))/(a^6*x^5 - a^4*x^3), 1/14*(105*(a^5*c^4*x^5 - a^3*c^4*x^3)*sqrt(c)*ar
ctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
- a*c*x - c)) - 2*(7*a^5*c^4*x^5 + 1755*a^4*c^4*x^4 + 720*a^3*c^4*x^3 - 110
*a^2*c^4*x^2 + 20*a*c^4*x - 2*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x
)))/(a^6*x^5 - a^4*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(9/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(9/2)/(a*x + 1)^3, x)
```

3.555 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=225

$$-\frac{a^3x^4(2525-427ax)\left(c-\frac{c}{ax}\right)^{7/2}}{15(1-ax)^{7/2}\sqrt{ax+1}}-\frac{398a^2x^3\left(c-\frac{c}{ax}\right)^{7/2}}{15(1-ax)^{3/2}\sqrt{ax+1}}+\frac{13a^{5/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{7/2}}+\frac{38ax^2\left(c-\frac{c}{ax}\right)^{7/2}}{15\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -(a^3*(c - c/(a*x))^(7/2)*x^4*(2525 - 427*a*x))/(15*(1 - a*x)^(7/2)*Sqrt[1
+ a*x]) - (398*a^2*(c - c/(a*x))^(7/2)*x^3)/(15*(1 - a*x)^(3/2)*Sqrt[1 + a*
x]) + (38*a*(c - c/(a*x))^(7/2)*x^2)/(15*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (2*
(c - c/(a*x))^(7/2)*x*Sqrt[1 - a*x])/(5*Sqrt[1 + a*x]) + (13*a^(5/2)*(c - c
/(a*x))^(7/2)*x^(7/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(7/2)
```

Rubi [A] time = 0.206663, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6134, 6129, 98, 150, 143, 54, 215}

$$-\frac{a^3x^4(2525-427ax)\left(c-\frac{c}{ax}\right)^{7/2}}{15(1-ax)^{7/2}\sqrt{ax+1}}-\frac{398a^2x^3\left(c-\frac{c}{ax}\right)^{7/2}}{15(1-ax)^{3/2}\sqrt{ax+1}}+\frac{13a^{5/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{7/2}}+\frac{38ax^2\left(c-\frac{c}{ax}\right)^{7/2}}{15\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(7/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(a^3*(c - c/(a*x))^(7/2)*x^4*(2525 - 427*a*x))/(15*(1 - a*x)^(7/2)*Sqrt[1
+ a*x]) - (398*a^2*(c - c/(a*x))^(7/2)*x^3)/(15*(1 - a*x)^(3/2)*Sqrt[1 + a*
x]) + (38*a*(c - c/(a*x))^(7/2)*x^2)/(15*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (2*
(c - c/(a*x))^(7/2)*x*Sqrt[1 - a*x])/(5*Sqrt[1 + a*x]) + (13*a^(5/2)*(c - c
/(a*x))^(7/2)*x^(7/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(7/2)
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{e^{-3\tanh^{-1}(ax)}(1-ax)^{7/2}}{x^{7/2}}dx}{(1-ax)^{7/2}}$$

$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{(1-ax)^5}{x^{7/2}(1+ax)^{3/2}}dx}{(1-ax)^{7/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1-ax}}{5\sqrt{1+ax}} - \frac{\left(2\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{(1-ax)^3\left(\frac{19a}{2}-\frac{3a^2x}{2}\right)}{x^{5/2}(1+ax)^{3/2}}dx}{5(1-ax)^{7/2}}$$

$$= \frac{38a\left(c-\frac{c}{ax}\right)^{7/2}x^2}{15\sqrt{1-ax}\sqrt{1+ax}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1-ax}}{5\sqrt{1+ax}} - \frac{\left(4\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{(1-ax)^2\left(-\frac{199a^2}{4}-\frac{29a^3}{4}\right.}{x^{3/2}(1+ax)^{3/2}}}{15(1-ax)^{7/2}}$$

$$= -\frac{398a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3}{15(1-ax)^{3/2}\sqrt{1+ax}} + \frac{38a\left(c-\frac{c}{ax}\right)^{7/2}x^2}{15\sqrt{1-ax}\sqrt{1+ax}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1-ax}}{5\sqrt{1+ax}} - \frac{\left(8\left(c-\frac{c}{ax}\right)^{7/}\right.}{}$$

$$= -\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}x^4(2525-427ax)}{15(1-ax)^{7/2}\sqrt{1+ax}} - \frac{398a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3}{15(1-ax)^{3/2}\sqrt{1+ax}} + \frac{38a\left(c-\frac{c}{ax}\right)^{7/2}x^2}{15\sqrt{1-ax}\sqrt{1+ax}} - \frac{2(}{}$$

$$= -\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}x^4(2525-427ax)}{15(1-ax)^{7/2}\sqrt{1+ax}} - \frac{398a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3}{15(1-ax)^{3/2}\sqrt{1+ax}} + \frac{38a\left(c-\frac{c}{ax}\right)^{7/2}x^2}{15\sqrt{1-ax}\sqrt{1+ax}} - \frac{2(}{}$$

$$= -\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}x^4(2525-427ax)}{15(1-ax)^{7/2}\sqrt{1+ax}} - \frac{398a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3}{15(1-ax)^{3/2}\sqrt{1+ax}} + \frac{38a\left(c-\frac{c}{ax}\right)^{7/2}x^2}{15\sqrt{1-ax}\sqrt{1+ax}} - \frac{2(}{}$$

Mathematica [C] time = 3.54371, size = 200, normalized size = 0.89

$$-\frac{c^3x\sqrt{c-\frac{c}{ax}}\left(1040a^4x^4(ax-1)^3\sqrt{ax+1}\sqrt{-ax(ax+1)}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{9}{2},\frac{11}{2},-ax\right)-3\sqrt{-ax(ax+1)}\left(520a^6\right.\right.}{720(-ax)^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(c^3*Sqrt[c - c/(a*x)]*x*(-3*Sqrt[-(a*x*(1 + a*x))]*(-6921 + 19192*a*x - 2
1508*a^2*x^2 - 28706*a^3*x^3 + 6325*a^4*x^4 - 2470*a^5*x^5 + 520*a^6*x^6) +
 585*(-35 + 70*a*x - 86*a^3*x^3 + 19*a^4*x^4)*ArcSin[Sqrt[-(a*x)]] + 1040*a
^4*x^4*(-1 + a*x)^3*Sqrt[1 + a*x]*Sqrt[-(a*x*(1 + a*x))]*Hypergeometric2F1[
3/2, 9/2, 11/2, -(a*x)]))/(720*(-(a*x))^(7/2)*Sqrt[1 + a*x]*Sqrt[1 - a^2*x^
2])
```

Maple [A] time = 0.153, size = 209, normalized size = 0.9

$$-\frac{c^3}{30x^2(ax+1)(ax-1)}\sqrt{\frac{c(ax-1)}{ax}}\left(30a^{9/2}\sqrt{-(ax+1)x}x^4+195\arctan\left(1/2\frac{2ax+1}{\sqrt{a}\sqrt{-(ax+1)x}}\right)x^4a^4+3182a^{7/2}x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/30*(c*(a*x-1)/a/x)^(1/2)/x^2*c^3/a^(7/2)*(30*a^(9/2)*(-(a*x+1)*x)^(1/2)*
x^4+195*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^4*a^4+3182*a^(7/
2)*x^3*(-(a*x+1)*x)^(1/2)+195*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/
2))*x^3*a^3+1096*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2)-124*a^(3/2)*x*(-(a*x+1)*x)^
(1/2)+12*a^(1/2)*(-(a*x+1)*x)^(1/2))*(-a^2*x^2+1)^(1/2)/(a*x+1)/(-(a*x+1)*x
)^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(7/2)/(a*x + 1)^3, x)
```

Fricas [A] time = 2.21649, size = 802, normalized size = 3.56

$$\left[\frac{195\left(a^4c^3x^4-a^2c^3x^2\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(15a^4c^3x^4+1591a^3c^3x^3+548a^2c^3x^2-\right.}{60\left(a^5x^4-a^3x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/60*(195*(a^4*c^3*x^4 - a^2*c^3*x^2)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))
- c)/(a*x - 1)) - 4*(15*a^4*c^3*x^4 + 1591*a^3*c^3*x^3 + 548*a^2*c^3*x^2 -
62*a*c^3*x + 6*c^3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 -
a^3*x^2), 1/30*(195*(a^4*c^3*x^4 - a^2*c^3*x^2)*sqrt(c)*arctan(2*sqrt(-a^2*
x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2
*(15*a^4*c^3*x^4 + 1591*a^3*c^3*x^3 + 548*a^2*c^3*x^2 - 62*a*c^3*x + 6*c^3)
*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^3*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(7/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(7/2)/(a*x + 1)^3, x)
```

3.556 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx$

Optimal. Leaf size=181

$$\frac{a^2x^3(191-25ax)\left(c-\frac{c}{ax}\right)^{5/2}}{3(1-ax)^{5/2}\sqrt{ax+1}}-\frac{11a^{3/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{5/2}}+\frac{26ax^2\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-ax}\sqrt{ax+1}}-\frac{2x\sqrt{1-ax}\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{ax+1}}$$

```
[Out] (a^2*(c - c/(a*x))^(5/2)*x^3*(191 - 25*a*x))/(3*(1 - a*x)^(5/2)*Sqrt[1 + a*
x]) + (26*a*(c - c/(a*x))^(5/2)*x^2)/(3*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (2*(
c - c/(a*x))^(5/2)*x*Sqrt[1 - a*x])/(3*Sqrt[1 + a*x]) - (11*a^(3/2)*(c - c/
(a*x))^(5/2)*x^(5/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(5/2)
```

Rubi [A] time = 0.189898, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6134, 6129, 98, 150, 143, 54, 215}

$$\frac{a^2x^3(191-25ax)\left(c-\frac{c}{ax}\right)^{5/2}}{3(1-ax)^{5/2}\sqrt{ax+1}}-\frac{11a^{3/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{5/2}}+\frac{26ax^2\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-ax}\sqrt{ax+1}}-\frac{2x\sqrt{1-ax}\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(5/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (a^2*(c - c/(a*x))^(5/2)*x^3*(191 - 25*a*x))/(3*(1 - a*x)^(5/2)*Sqrt[1 + a*
x]) + (26*a*(c - c/(a*x))^(5/2)*x^2)/(3*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (2*(
c - c/(a*x))^(5/2)*x*Sqrt[1 - a*x])/(3*Sqrt[1 + a*x]) - (11*a^(3/2)*(c - c/
(a*x))^(5/2)*x^(5/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(5/2)
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{\left(\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{e^{-3\tanh^{-1}(ax)}(1-ax)^{5/2}}{x^{5/2}}\,dx}{(1-ax)^{5/2}}$$

$$=\frac{\left(\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{(1-ax)^4}{x^{5/2}(1+ax)^{3/2}}\,dx}{(1-ax)^{5/2}}$$

$$=-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1-ax}}{3\sqrt{1+ax}}-\frac{\left(2\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{(1-ax)^2\left(\frac{13a}{2}-\frac{a^2x}{2}\right)}{x^{3/2}(1+ax)^{3/2}}\,dx}{3(1-ax)^{5/2}}$$

$$=\frac{26a\left(c-\frac{c}{ax}\right)^{5/2}x^2}{3\sqrt{1-ax}\sqrt{1+ax}}-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1-ax}}{3\sqrt{1+ax}}-\frac{\left(4\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{(1-ax)\left(-\frac{79a^2}{4}-\frac{25a^3x}{4}\right)}{\sqrt{x}(1+ax)^{3/2}}\,d}{3(1-ax)^{5/2}}$$

$$=\frac{a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3(191-25ax)}{3(1-ax)^{5/2}\sqrt{1+ax}}+\frac{26a\left(c-\frac{c}{ax}\right)^{5/2}x^2}{3\sqrt{1-ax}\sqrt{1+ax}}-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1-ax}}{3\sqrt{1+ax}}-\frac{\left(11a^2\left(\right.\right.}{}$$

$$=\frac{a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3(191-25ax)}{3(1-ax)^{5/2}\sqrt{1+ax}}+\frac{26a\left(c-\frac{c}{ax}\right)^{5/2}x^2}{3\sqrt{1-ax}\sqrt{1+ax}}-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1-ax}}{3\sqrt{1+ax}}-\frac{\left(11a^2\left(\right.\right.}{}$$

$$=\frac{a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3(191-25ax)}{3(1-ax)^{5/2}\sqrt{1+ax}}+\frac{26a\left(c-\frac{c}{ax}\right)^{5/2}x^2}{3\sqrt{1-ax}\sqrt{1+ax}}-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1-ax}}{3\sqrt{1+ax}}-\frac{11a^{3/2}\left(\right.}{}$$

Mathematica [A] time = 0.077821, size = 97, normalized size = 0.54

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\left(3a^3x^3+133a^2x^2-33a^{3/2}x^{3/2}\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+32ax-2\right)}{3a^2x\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(-2 + 32*a*x + 133*a^2*x^2 + 3*a^3*x^3 - 33*a^(3/2)*
x^(3/2)*Sqrt[1 + a*x]*ArcSinh[Sqrt[a]*Sqrt[x]]))/(3*a^2*x*Sqrt[1 - a^2*x^2]
)
```

Maple [A] time = 0.157, size = 191, normalized size = 1.1

$$-\frac{c^2}{6\,(ax+1)\,(ax-1)\,x}\sqrt{\frac{c\,(ax-1)}{ax}}\left(6\,a^{7/2}x^3\sqrt{-(ax+1)\,x}+33\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)x^3a^3+266\,a^{5/2}x^2\sqrt{-(ax+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/6*(c*(a*x-1)/a/x)^(1/2)/x*c^2/a^(5/2)*(6*a^(7/2)*x^3*(-(a*x+1)*x)^(1/2)+
33*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^3*a^3+266*a^(5/2)*x^2
*(-(a*x+1)*x)^(1/2)+33*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^2
*a^2+64*a^(3/2)*x*(-(a*x+1)*x)^(1/2)-4*a^(1/2)*(-(a*x+1)*x)^(1/2))*(-a^2*x^
2+1)^(1/2)/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(5/2)/(a*x + 1)^3, x)
```

Fricas [A] time = 2.17677, size = 728, normalized size = 4.02

$$\left[\frac{33\left(a^3c^2x^3-ac^2x\right)\sqrt{-c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(3\,a^3c^2x^3+133\,a^2c^2x^2+32\,ac^2x-2\,c^2\right)\sqrt{-c}}{12\left(a^4x^3-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/12*(33*(a^3*c^2*x^3 - a*c^2*x)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*
(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/
(a*x - 1)) - 4*(3*a^3*c^2*x^3 + 133*a^2*c^2*x^2 + 32*a*c^2*x - 2*c^2)*sqrt(
-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3 - a^2*x), 1/6*(33*(a^3*c^2*
x^3 - a*c^2*x)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x
- c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(3*a^3*c^2*x^3 + 133*a^2*c^2*x^2
 + 32*a*c^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3
 - a^2*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(5/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(5/2)/(a*x + 1)^3, x)
```

3.557 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=133

$$-\frac{ax^2(23-ax)\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}\sqrt{ax+1}}+\frac{9\sqrt{a}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{3/2}}-\frac{2x\sqrt{1-ax}\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{ax+1}}$$

```
[Out] (-2*(c - c/(a*x))^(3/2)*x*Sqrt[1 - a*x])/Sqrt[1 + a*x] - (a*(c - c/(a*x))^(
3/2)*x^2*(23 - a*x))/((1 - a*x)^(3/2)*Sqrt[1 + a*x]) + (9*Sqrt[a]*(c - c/(a
*x))^(3/2)*x^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(3/2)
```

Rubi [A] time = 0.175303, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6134, 6129, 98, 143, 54, 215}

$$-\frac{ax^2(23-ax)\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}\sqrt{ax+1}}+\frac{9\sqrt{a}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{3/2}}-\frac{2x\sqrt{1-ax}\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(3/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-2*(c - c/(a*x))^(3/2)*x*Sqrt[1 - a*x])/Sqrt[1 + a*x] - (a*(c - c/(a*x))^(
3/2)*x^2*(23 - a*x))/((1 - a*x)^(3/2)*Sqrt[1 + a*x]) + (9*Sqrt[a]*(c - c/(a
*x))^(3/2)*x^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(3/2)
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
```

```
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{-3\tanh^{-1}(ax)}(1-ax)^{3/2}}{x^{3/2}}\,dx}{(1-ax)^{3/2}}$$

$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{(1-ax)^3}{x^{3/2}(1+ax)^{3/2}}\,dx}{(1-ax)^{3/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\left(2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{(1-ax)\left(\frac{7a}{2}+\frac{a^2x}{2}\right)}{\sqrt{x}(1+ax)^{3/2}}\,dx}{(1-ax)^{3/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2(23-ax)}{(1-ax)^{3/2}\sqrt{1+ax}} + \frac{\left(9a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}}{2(1-ax)^{3/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2(23-ax)}{(1-ax)^{3/2}\sqrt{1+ax}} + \frac{\left(9a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\text{Subst}\left(\int\right.}{(1-ax)^{3/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2(23-ax)}{(1-ax)^{3/2}\sqrt{1+ax}} + \frac{9\sqrt{a}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\sinh^{-1}\left(\sqrt{a}\right.}{(1-ax)^{3/2}}$$

Mathematica [A] time = 0.0512834, size = 80, normalized size = 0.6

$$\frac{c\sqrt{c-\frac{c}{ax}}\left(a^2x^2+19ax-9\sqrt{a}\sqrt{x}\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+2\right)}{a\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(2 + 19*a*x + a^2*x^2 - 9*Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x
]*ArcSinh[Sqrt[a]*Sqrt[x]]))/(a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.157, size = 164, normalized size = 1.2

$$-\frac{c}{(2\,ax+2)\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2\,a^{5/2}x^2\sqrt{-(ax+1)\,x}+9\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)x^2a^2+38\,a^{3/2}x\sqrt{-(ax+1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)*(2*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2)+9*ar
ctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^2*a^2+38*a^(3/2)*x*(-(a*x+
1)*x)^(1/2)+9*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a+4*a^(1/2
)*(-(a*x+1)*x)^(1/2))*(-a^2*x^2+1)^(1/2)/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(3/2)/(a*x + 1)^3, x)
```

Fricas [A] time = 2.12749, size = 621, normalized size = 4.67

$$\left[\frac{9\left(a^2cx^2-c\right)\sqrt{-c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(a^2cx^2+19\,acx+2\,c\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{4\left(a^3x^2-a\right)},\ \frac{9\left(a^2cx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(9*(a^2*c*x^2 - c)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2
 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1))
 - 4*(a^2*c*x^2 + 19*a*c*x + 2*c)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)
))/(a^3*x^2 - a), 1/2*(9*(a^2*c*x^2 - c)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1
)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(a^2*c
*x^2 + 19*a*c*x + 2*c)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2
 - a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(3/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(3/2)/(a*x + 1)^3, x)
```

3.558 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=123

$$\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{8x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{7\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

```
[Out] (8*Sqrt[c - c/(a*x)]*x)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a*x)]*
x*Sqrt[1 + a*x])/Sqrt[1 - a*x] - (7*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[
a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rubi [A] time = 0.158976, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6134, 6129, 89, 80, 54, 215}

$$\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{8x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{7\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (8*Sqrt[c - c/(a*x)]*x)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a*x)]*
x*Sqrt[1 + a*x])/Sqrt[1 - a*x] - (7*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[
a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^2}{\sqrt{x}(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{8\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\frac{3a^2}{2}-\frac{a^3x}{2}}{\sqrt{x}\sqrt{1+ax}}\,dx}{a^2\sqrt{1-ax}}\\
&= \frac{8\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{\left(7\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}}\\
&= \frac{8\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{\left(7\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{\sqrt{1-ax}}\\
&= \frac{8\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{7\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}
\end{aligned}
$$

Mathematica [A] time = 0.0509674, size = 80, normalized size = 0.65

$$
\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}(ax+9)-7\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{\sqrt{a}\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*(9 + a*x) - 7*Sqrt[1 + a*x]*Arc
Sinh[Sqrt[a]*Sqrt[x]]))/(Sqrt[a]*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.139, size = 140, normalized size = 1.1

$$
-\frac{x}{(2\,ax+2)\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2\,a^{3/2}x\sqrt{-(ax+1)\,x}+7\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)xa+18\,\sqrt{a}\sqrt{-(ax+1)\,x}+7
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+7*arctan(1/2/a
^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a+18*a^(1/2)*(-(a*x+1)*x)^(1/2)+7*ar
ctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))*(-a^2*x^2+1)^(1/2)/a^(1/2)/
(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/(a*x + 1)^3, x)
```

Fricas [A] time = 2.25012, size = 586, normalized size = 4.76

$$\left[\frac{7\left(a^2x^2-1\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(a^2x^2+9ax\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{4\left(a^3x^2-a\right)}, \frac{7\left(a^2x^2-1\right)\sqrt{c}\,a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(7*(a^2*x^2 - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 +
 a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) -
 4*(a^2*x^2 + 9*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 -
 a), 1/2*(7*(a^2*x^2 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*s
qrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(a^2*x^2 + 9*a*x)*sqr
t(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-a^2x^2+1)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/(a*x + 1)^3, x)
```

3.559 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=127

$$-\frac{x(1-ax)}{\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}+\frac{5\sqrt{1-ax}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{1-ax}}{a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}$$

```
[Out] (-5*Sqrt[1 - a*x])/(a*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x]) - (x*(1 - a*x))/(Sqr
t[c - c/(a*x)]*Sqrt[1 - a^2*x^2]) + (5*Sqrt[1 - a*x]*ArcSinh[Sqrt[a]*Sqrt[x
]])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x])
```

Rubi [A] time = 0.203986, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6134, 6128, 881, 848, 47, 54, 215}

$$-\frac{x(1-ax)}{\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}+\frac{5\sqrt{1-ax}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{1-ax}}{a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (-5*Sqrt[1 - a*x])/(a*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x]) - (x*(1 - a*x))/(Sqr
t[c - c/(a*x)]*Sqrt[1 - a^2*x^2]) + (5*Sqrt[1 - a*x]*ArcSinh[Sqrt[a]*Sqrt[x
]])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-ax}\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{x}}{\sqrt{1-ax}}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= \frac{\sqrt{1-ax}\int \frac{\sqrt{x}(1-ax)^{5/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{x(1-ax)}{\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}} + \frac{\left(5\sqrt{1-ax}\right)\int \frac{\sqrt{x}(1-ax)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{2\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{x(1-ax)}{\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}} + \frac{\left(5\sqrt{1-ax}\right)\int \frac{\sqrt{x}}{(1+ax)^{3/2}}\,dx}{2\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{5\sqrt{1-ax}}{a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}} - \frac{x(1-ax)}{\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}} + \frac{\left(5\sqrt{1-ax}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{5\sqrt{1-ax}}{a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}} - \frac{x(1-ax)}{\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}} + \frac{\left(5\sqrt{1-ax}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{5\sqrt{1-ax}}{a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}} - \frac{x(1-ax)}{\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}} + \frac{5\sqrt{1-ax}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

Mathematica [A] time = 0.0602446, size = 86, normalized size = 0.68

$$\frac{\sqrt{1-ax}\left(5\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-\sqrt{a}\sqrt{x}(ax+5)\right)}{a^{3/2}\sqrt{x}\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[1 - a*x]*(-(Sqrt[a]*Sqrt[x]*(5 + a*x)) + 5*Sqrt[1 + a*x]*ArcSinh[Sqrt
[a]*Sqrt[x]]))/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x]*Sqrt[1 + a*x])
```

Maple [A] time = 0.142, size = 143, normalized size = 1.1

$$-\frac{x}{2\,(ax-1)\,(ax+1)\,c}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2\,a^{3/2}x\sqrt{-(ax+1)\,x}+5\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)xa+10\,\sqrt{a}\sqrt{-(ax+1)\,x}+5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(1/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x/a^(1/2)/c*(2*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+5*ar
ctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a+10*a^(1/2)*(-(a*x+1)*x)^
(1/2)+5*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))*(-a^2*x^2+1)^(1/2
)/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*sqrt(c - c/(a*x))), x)
```

Fricas [A] time = 2.30044, size = 598, normalized size = 4.71

$$\left[-\frac{5\left(a^2x^2-1\right)\sqrt{-c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+5\,ax\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{4\left(a^3cx^2-ac\right)},\frac{5\left(a^2x^2-1\right)\sqrt{c}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(1/2),x, algorithm="fric
as")
```

```
[Out] [-1/4*(5*(a^2*x^2 - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2
+ a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1))
+ 4*(a^2*x^2 + 5*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c*x^
2 - a*c), 1/2*(5*(a^2*x^2 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c
)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(a^2*x^2 + 5*a*x
)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c*x^2 - a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**
3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*sqrt(c - c/(a*x))), x)
```

3.560 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{3(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3\sqrt{ax+1}(1-ax)^{3/2}}{a^2x\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2(1-ax)^{3/2}}{a\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] (-2*(1 - a*x)^(3/2))/(a*(c - c/(a*x))^(3/2)*Sqrt[1 + a*x]) + (3*(1 - a*x)^(
3/2)*Sqrt[1 + a*x])/(a^2*(c - c/(a*x))^(3/2)*x) - (3*(1 - a*x)^(3/2)*ArcSin
h[Sqrt[a]*Sqrt[x]])/(a^(5/2)*(c - c/(a*x))^(3/2)*x^(3/2))
```

Rubi [A] time = 0.193546, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6134, 6128, 848, 47, 50, 54, 215}

$$-\frac{3(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3\sqrt{ax+1}(1-ax)^{3/2}}{a^2x\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2(1-ax)^{3/2}}{a\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (-2*(1 - a*x)^(3/2))/(a*(c - c/(a*x))^(3/2)*Sqrt[1 + a*x]) + (3*(1 - a*x)^(
3/2)*Sqrt[1 + a*x])/(a^2*(c - c/(a*x))^(3/2)*x) - (3*(1 - a*x)^(3/2)*ArcSin
h[Sqrt[a]*Sqrt[x]])/(a^(5/2)*(c - c/(a*x))^(3/2)*x^(3/2))
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx &= \frac{(1-ax)^{3/2}\int \frac{e^{-3\tanh^{-1}(ax)}x^{3/2}}{(1-ax)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}\\
&= \frac{(1-ax)^{3/2}\int \frac{x^{3/2}(1-ax)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}\\
&= \frac{(1-ax)^{3/2}\int \frac{x^{3/2}}{(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}\\
&= -\frac{2(1-ax)^{3/2}}{a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1+ax}}+\frac{\left(3(1-ax)^{3/2}\right)\int \frac{\sqrt{x}}{\sqrt{1+ax}}\,dx}{a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}\\
&= -\frac{2(1-ax)^{3/2}}{a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1+ax}}+\frac{3(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x}-\frac{\left(3(1-ax)^{3/2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}\\
&= -\frac{2(1-ax)^{3/2}}{a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1+ax}}+\frac{3(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x}-\frac{\left(3(1-ax)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}\\
&= -\frac{2(1-ax)^{3/2}}{a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1+ax}}+\frac{3(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x}-\frac{3(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0612196, size = 44, normalized size = 0.34

$$\frac{2x(1-ax)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{2},\frac{7}{2},-ax\right)}{5\left(c-\frac{c}{ax}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (2*x*(1 - a*x)^(3/2)*Hypergeometric2F1[3/2, 5/2, 7/2, -(a*x)])/(5*(c - c/(a
*x))^(3/2))
```

Maple [A] time = 0.137, size = 143, normalized size = 1.1

$$-\frac{x}{2\,c^2\,(ax+1)\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2\,a^{3/2}x\sqrt{-(ax+1)\,x}+3\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)xa+6\,\sqrt{a}\sqrt{-(ax+1)\,x}+3\,\mathrm{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(3/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x/a^(1/2)/c^2*(2*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+3*
arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a+6*a^(1/2)*(-(a*x+1)*x)
^(1/2)+3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))*(-a^2*x^2+1)^(1/
2)/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^(3/2)), x)
```

Fricas [A] time = 2.13781, size = 609, normalized size = 4.65

$$\left[-\frac{3\left(a^2x^2-1\right)\sqrt{-c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+3\,ax\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{4\left(a^3c^2x^2-ac^2\right)},\ \frac{3\left(a^2x^2-1\right)\sqrt{c}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(3/2),x, algorithm="fric
as")
```

```
[Out] [-1/4*(3*(a^2*x^2 - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2
+ a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1))
+ 4*(a^2*x^2 + 3*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*
x^2 - a*c^2), 1/2*(3*(a^2*x^2 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sq
```

```
rt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(a^2*x^2 + 3
*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - a*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^(3/2)), x)
```

3.561 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=199

$$-\frac{2\sqrt{ax+1}(1-ax)^{5/2}}{a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{2}a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{(1-ax)^{5/2}}{a^2x\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] (1 - a*x)^(5/2)/(a^2*(c - c/(a*x))^(5/2)*x*Sqrt[1 + a*x]) - (2*(1 - a*x)^(5
/2)*Sqrt[1 + a*x])/(a^3*(c - c/(a*x))^(5/2)*x^2) + ((1 - a*x)^(5/2)*ArcSinh
[Sqrt[a]*Sqrt[x]])/(a^(7/2)*(c - c/(a*x))^(5/2)*x^(5/2)) + ((1 - a*x)^(5/2)
*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(Sqrt[2]*a^(7/2)*(c - c/
(a*x))^(5/2)*x^(5/2))
```

Rubi [A] time = 0.205231, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6134, 6129, 98, 154, 157, 54, 215, 93, 206}

$$-\frac{2\sqrt{ax+1}(1-ax)^{5/2}}{a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{2}a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{(1-ax)^{5/2}}{a^2x\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (1 - a*x)^(5/2)/(a^2*(c - c/(a*x))^(5/2)*x*Sqrt[1 + a*x]) - (2*(1 - a*x)^(5
/2)*Sqrt[1 + a*x])/(a^3*(c - c/(a*x))^(5/2)*x^2) + ((1 - a*x)^(5/2)*ArcSinh
[Sqrt[a]*Sqrt[x]])/(a^(7/2)*(c - c/(a*x))^(5/2)*x^(5/2)) + ((1 - a*x)^(5/2)
*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(Sqrt[2]*a^(7/2)*(c - c/
(a*x))^(5/2)*x^(5/2))
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{(1-ax)^{5/2}\int \frac{e^{-3\tanh^{-1}(ax)}x^{5/2}}{(1-ax)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{5/2}\int \frac{x^{5/2}}{(1-ax)(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{5/2}}{a^2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}} - \frac{(1-ax)^{5/2}\int \frac{\sqrt{x}\left(\frac{3}{2}-2ax\right)}{(1-ax)\sqrt{1+ax}}\,dx}{a^2\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{5/2}}{a^2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}} - \frac{2(1-ax)^{5/2}\sqrt{1+ax}}{a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1-ax)^{5/2}\int \frac{a-\frac{a^2x}{2}}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a^4\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{5/2}}{a^2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}} - \frac{2(1-ax)^{5/2}\sqrt{1+ax}}{a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1-ax)^{5/2}\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}} + \frac{(1-ax)^{5/2}\int \frac{1}{\sqrt{x}(1-ax)\sqrt{1}}}{2a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/}}$$

$$= \frac{(1-ax)^{5/2}}{a^2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}} - \frac{2(1-ax)^{5/2}\sqrt{1+ax}}{a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1-ax)^{5/2}\,\text{Subst}\left(\int \frac{1}{1-2ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{1+ax}}\right)}{a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}} + \frac{(1-ax}{}$$

$$= \frac{(1-ax)^{5/2}}{a^2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}} - \frac{2(1-ax)^{5/2}\sqrt{1+ax}}{a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}} + \frac{(1-ax)^{5/2}\tanh^{-1}\left(\right.}{\sqrt{2}a^{7/2}\left(c-\frac{c}{ax}\right)^{5/2}}$$

Mathematica [C] time = 0.153795, size = 162, normalized size = 0.81

$$\frac{\sqrt{1-ax}\left(5\left(2\sqrt{a}\sqrt{x}-4\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+\sqrt{2ax+2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)\right)-4a^{5/2}x^{5/2}\sqrt{ax+1}\text{Hypergeometric2F1}}{10a^{3/2}c^2\sqrt{x}\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (Sqrt[1 - a*x]*(5*(2*Sqrt[a]*Sqrt[x] - 4*Sqrt[1 + a*x]*ArcSinh[Sqrt[a]*Sqrt
[x]] + Sqrt[2 + 2*a*x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]) -
4*a^(5/2)*x^(5/2)*Sqrt[1 + a*x]*Hypergeometric2F1[3/2, 5/2, 7/2, -(a*x)]))/
(10*a^(3/2)*c^2*Sqrt[c - c/(a*x)]*Sqrt[x]*Sqrt[1 + a*x])
```

Maple [A] time = 0.147, size = 279, normalized size = 1.4

$$-\frac{x\sqrt{2}}{4\,c^3\,(ax+1)\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2\,\sqrt{-(ax+1)\,x}a^{5/2}\sqrt{2}\sqrt{-a^{-1}}x+a^2\arctan\left(\frac{2\,ax+1}{2}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)\,x}}\right)\sqrt{2}\sqrt{-a^{-1}}x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(5/2),x)
```

```
[Out] -1/4*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)/c^3*2^(1/2)*(2*(-(a*x+1)*x)^(1/2)*a^(5
/2)*2^(1/2)*(-1/a)^(1/2)*x+a^2*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1
/2))*2^(1/2)*(-1/a)^(1/2)*x+4*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/
```

```
2)+arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)^(1/2)+
a^(3/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*x
+ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*a^(1/2))
*(-a^2*x^2+1)^(1/2)/(-1/a)^(1/2)/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^(5/2)), x)
```

Fricas [A] time = 2.65381, size = 1057, normalized size = 5.31

$$\left[-\frac{\sqrt{2}\left(a^2x^2-1\right)\sqrt{-c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx+4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\left(a^2x^2-1\right)\sqrt{-c}\log\left(-\frac{8\,a^3cx^3-7\,acx+\cdots}{\cdots}\right)\cdots}{8\left(a^3c^3x^2-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(5/2),x, algorithm="fric
as")
```

```
[Out] [-1/8*(sqrt(2)*(a^2*x^2 - 1)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13
*a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*
x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 2*(a^2*x^2 - 1)*sqr
t(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*
sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(a^2*x^2 + 2*a*x)*sqrt
(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^3*x^2 - a*c^3), 1/4*(sqrt(2)
*(a^2*x^2 - 1)*sqrt(c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt
((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + 2*(a^2*x^2 - 1)*sqrt(c)*
arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^
2 - a*c*x - c)) - 4*(a^2*x^2 + 2*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(
a*x)))/(a^3*c^3*x^2 - a*c^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^(5/2)), x)
```

3.562 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=251

$$\frac{7\sqrt{ax+1}(1-ax)^{7/2}}{4a^4x^3\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{(1-ax)^{7/2}}{4a^3x^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{(1-ax)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{9/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{11(1-ax)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{4\sqrt{2}a^{9/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{}{2a^2}$$

```
[Out] (1 - a*x)^(5/2)/(2*a^2*(c - c/(a*x))^(7/2)*x*Sqrt[1 + a*x]) - (1 - a*x)^(7/
2)/(4*a^3*(c - c/(a*x))^(7/2)*x^2*Sqrt[1 + a*x]) + (7*(1 - a*x)^(7/2)*Sqrt[
1 + a*x])/(4*a^4*(c - c/(a*x))^(7/2)*x^3) + ((1 - a*x)^(7/2)*ArcSinh[Sqrt[a
]*Sqrt[x]])/(a^(9/2)*(c - c/(a*x))^(7/2)*x^(7/2)) - (11*(1 - a*x)^(7/2)*Arc
Tanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(4*Sqrt[2]*a^(9/2)*(c - c/(a
*x))^(7/2)*x^(7/2))
```

Rubi [A] time = 0.221361, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6134, 6129, 98, 150, 154, 157, 54, 215, 93, 206}

$$\frac{7\sqrt{ax+1}(1-ax)^{7/2}}{4a^4x^3\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{(1-ax)^{7/2}}{4a^3x^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{(1-ax)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{9/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{11(1-ax)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{4\sqrt{2}a^{9/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (1 - a*x)^(5/2)/(2*a^2*(c - c/(a*x))^(7/2)*x*Sqrt[1 + a*x]) - (1 - a*x)^(7/
2)/(4*a^3*(c - c/(a*x))^(7/2)*x^2*Sqrt[1 + a*x]) + (7*(1 - a*x)^(7/2)*Sqrt[
1 + a*x])/(4*a^4*(c - c/(a*x))^(7/2)*x^3) + ((1 - a*x)^(7/2)*ArcSinh[Sqrt[a
]*Sqrt[x]])/(a^(9/2)*(c - c/(a*x))^(7/2)*x^(7/2)) - (11*(1 - a*x)^(7/2)*Arc
Tanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(4*Sqrt[2]*a^(9/2)*(c - c/(a
*x))^(7/2)*x^(7/2))
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
```

```
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{(1-ax)^{7/2}\int \frac{e^{-3\tanh^{-1}(ax)}x^{7/2}}{(1-ax)^{7/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{7/2}\int \frac{x^{7/2}}{(1-ax)^2(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{5/2}}{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}} - \frac{(1-ax)^{7/2}\int \frac{x^{3/2}\left(\frac{5}{2}+3ax\right)}{(1-ax)(1+ax)^{3/2}}\,dx}{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{5/2}}{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}} - \frac{(1-ax)^{7/2}}{4a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}} - \frac{(1-ax)^{7/2}\int \frac{\sqrt{x}\left(-\frac{3a}{4}+\frac{7a^2x}{2}\right)}{(1-ax)\sqrt{1+ax}}\,dx}{2a^4\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{5/2}}{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}} - \frac{(1-ax)^{7/2}}{4a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}} + \frac{7(1-ax)^{7/2}\sqrt{1+ax}}{4a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} + \frac{(1-ax)^{7/2}\int \frac{-\cdots}{\sqrt{x}(1\cdots}}{2a^6\left(c-\frac{c}{ax}\right)\cdots}$$

$$= \frac{(1-ax)^{5/2}}{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}} - \frac{(1-ax)^{7/2}}{4a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}} + \frac{7(1-ax)^{7/2}\sqrt{1+ax}}{4a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} + \frac{(1-ax)^{7/2}\int \frac{1}{\sqrt{x}\sqrt{\cdots}}}{2a^4\left(c-\frac{c}{ax}\right)^{7/2}\cdots}$$

$$= \frac{(1-ax)^{5/2}}{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}} - \frac{(1-ax)^{7/2}}{4a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}} + \frac{7(1-ax)^{7/2}\sqrt{1+ax}}{4a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} + \frac{(1-ax)^{7/2}\,\text{Subst}\cdots}{a^4\left(c-\cdots\right.}$$

$$= \frac{(1-ax)^{5/2}}{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}} - \frac{(1-ax)^{7/2}}{4a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}} + \frac{7(1-ax)^{7/2}\sqrt{1+ax}}{4a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} + \frac{(1-ax)^{7/2}\sinh^{-1}\cdots}{a^{9/2}\left(c-\frac{c}{ax}\right)^{7/\cdots}}$$

Mathematica [C] time = 0.599073, size = 234, normalized size = 0.93

$$\frac{-40a^{7/2}x^{7/2}(ax-1)\sqrt{ax+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{7}{2},\frac{9}{2},-ax\right)-56a^{5/2}x^{5/2}(ax-1)\sqrt{ax+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2}\cdots\right.}{560a^{3/2}c\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (35*(Sqrt[a]*Sqrt[x]*(-25 + 2*a*x + 13*a^2*x^2 + 2*a^3*x^3) + 19*(-1 + a*x)
*Sqrt[1 + a*x]*ArcSinh[Sqrt[a]*Sqrt[x]] - 22*(-1 + a*x)*Sqrt[2 + 2*a*x]*Arc
Tanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]) - 56*a^(5/2)*x^(5/2)*(-1 + a
*x)*Sqrt[1 + a*x]*Hypergeometric2F1[3/2, 5/2, 7/2, -(a*x)] - 40*a^(7/2)*x^(
7/2)*(-1 + a*x)*Sqrt[1 + a*x]*Hypergeometric2F1[3/2, 7/2, 9/2, -(a*x)])/(56
0*a^(3/2)*c^3*Sqrt[c - c/(a*x)]*Sqrt[x]*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.157, size = 315, normalized size = 1.3

$$-\frac{x\sqrt{2}}{16\,c^4\,(ax+1)\,(ax-1)^2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(8\sqrt{-(ax+1)\,x}\,a^{7/2}\sqrt{2}\sqrt{-a^{-1}}x^2+2\sqrt{-(ax+1)\,x}\,a^{5/2}\sqrt{2}\sqrt{-a^{-1}}x-4\,a^3\arctan\left(\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(7/2),x)
```

```
[Out] -1/16*(c*(a*x-1)/a/x)^(1/2)*x*2^(1/2)*(8*(-(a*x+1)*x)^(1/2)*a^(7/2)*2^(1/2)
*(-1/a)^(1/2)*x^2+2*(-(a*x+1)*x)^(1/2)*a^(5/2)*2^(1/2)*(-1/a)^(1/2)*x-4*a^3
*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)*x^2+
11*a^(5/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1)
)*x^2-14*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2)+4*arctan(1/2/a^(1/
2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)^(1/2)-11*ln((2*2^(1/2)*(-
1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*a^(1/2))*(-a^2*x^2+1)^(1/
2)/a^(3/2)/c^4/(a*x+1)/(-1/a)^(1/2)/(-(a*x+1)*x)^(1/2)/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^(7/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(7/2),x, algorithm="fric
as")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^(7/2)), x)
```

3.563 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)x^3}\,dx$

Optimal. Leaf size=105

$$-\frac{4a^2(4ax+3)}{3c\sqrt{1-a^2x^2}}-\frac{8a^2(ax+1)}{3c\left(1-a^2x^2\right)^{3/2}}+\frac{a\sqrt{1-a^2x^2}}{cx}+\frac{4a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

```
[Out] (-8*a^2*(1 + a*x))/(3*c*(1 - a^2*x^2)^(3/2)) - (4*a^2*(3 + 4*a*x))/(3*c*Sqr
t[1 - a^2*x^2]) + (a*Sqrt[1 - a^2*x^2])/(c*x) + (4*a^2*ArcTanh[Sqrt[1 - a^2
*x^2]])/c
```

Rubi [A] time = 0.321872, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {6131, 6128, 852, 1805, 807, 266, 63, 208}

$$-\frac{4a^2(4ax+3)}{3c\sqrt{1-a^2x^2}}-\frac{8a^2(ax+1)}{3c\left(1-a^2x^2\right)^{3/2}}+\frac{a\sqrt{1-a^2x^2}}{cx}+\frac{4a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/((c - c/(a*x))*x^3),x]
```

```
[Out] (-8*a^2*(1 + a*x))/(3*c*(1 - a^2*x^2)^(3/2)) - (4*a^2*(3 + 4*a*x))/(3*c*Sqr
t[1 - a^2*x^2]) + (a*Sqrt[1 - a^2*x^2])/(c*x) + (4*a^2*ArcTanh[Sqrt[1 - a^2
*x^2]])/c
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
```

```
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)x^3}\,dx = -\frac{a\int \frac{e^{3\tanh^{-1}(ax)}}{x^2(1-ax)}\,dx}{c}$$

$$= -\frac{a\int \frac{\left(1-a^2x^2\right)^{3/2}}{x^2(1-ax)^4}\,dx}{c}$$

$$= -\frac{a\int \frac{(1+ax)^4}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx}{c}$$

$$= -\frac{8a^2(1+ax)}{3c\left(1-a^2x^2\right)^{3/2}} + \frac{a\int \frac{-3-12ax-13a^2x^2}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{3c}$$

$$= -\frac{8a^2(1+ax)}{3c\left(1-a^2x^2\right)^{3/2}} - \frac{4a^2(3+4ax)}{3c\sqrt{1-a^2x^2}} - \frac{a\int \frac{3+12ax}{x^2\sqrt{1-a^2x^2}}\,dx}{3c}$$

$$= -\frac{8a^2(1+ax)}{3c\left(1-a^2x^2\right)^{3/2}} - \frac{4a^2(3+4ax)}{3c\sqrt{1-a^2x^2}} + \frac{a\sqrt{1-a^2x^2}}{cx} - \frac{\left(4a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c}$$

$$= -\frac{8a^2(1+ax)}{3c\left(1-a^2x^2\right)^{3/2}} - \frac{4a^2(3+4ax)}{3c\sqrt{1-a^2x^2}} + \frac{a\sqrt{1-a^2x^2}}{cx} - \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{c}$$

$$= -\frac{8a^2(1+ax)}{3c\left(1-a^2x^2\right)^{3/2}} - \frac{4a^2(3+4ax)}{3c\sqrt{1-a^2x^2}} + \frac{a\sqrt{1-a^2x^2}}{cx} + \frac{4\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{c}$$

$$= -\frac{8a^2(1+ax)}{3c\left(1-a^2x^2\right)^{3/2}} - \frac{4a^2(3+4ax)}{3c\sqrt{1-a^2x^2}} + \frac{a\sqrt{1-a^2x^2}}{cx} + \frac{4a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Mathematica [A] time = 0.0439308, size = 92, normalized size = 0.88

$$\frac{a\left(-19a^3x^3+7a^2x^2+12ax(ax-1)\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+23ax-3\right)}{3cx(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/((c - c/(a*x))*x^3),x]
```

```
[Out] (a*(-3 + 23*a*x + 7*a^2*x^2 - 19*a^3*x^3 + 12*a*x*(-1 + a*x)*Sqrt[1 - a^2*x
^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(3*c*x*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.049, size = 165, normalized size = 1.6

$$\frac{a}{c}\left(-a^2x\frac{1}{\sqrt{-a^2x^2+1}}+\frac{1}{x}\frac{1}{\sqrt{-a^2x^2+1}}-4a\left(\frac{1}{\sqrt{-a^2x^2+1}}-\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)+8a\left(1/3\frac{1}{a}\left(x-a^{-1}\right)^{-1}\frac{}{\sqrt{-a^2\left(x-a\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)/x^3,x)
```

```
[Out] a/c*(-a^2*x/(-a^2*x^2+1)^(1/2)+1/x/(-a^2*x^2+1)^(1/2)-4*a*(1/(-a^2*x^2+1)^(
1/2)-arctanh(1/(-a^2*x^2+1)^(1/2)))+8*a*(1/3/a/(x-1/a)/(-a^2*(x-1/a)^2-2*a*
(x-1/a))^(1/2)+1/3/a*(-2*(x-1/a)*a^2-2*a)/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2
)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{(-a^2x^2+1)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))*x^3), x)
```

Fricas [A] time = 1.89639, size = 258, normalized size = 2.46

$$-\frac{20\,a^4x^3-40\,a^3x^2+20\,a^2x+12\left(a^4x^3-2\,a^3x^2+a^2x\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(19\,a^3x^2-26\,a^2x+3\,a\right)\sqrt{-a^2x^2+1}}{3\left(a^2cx^3-2\,acx^2+cx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/3*(20*a^4*x^3 - 40*a^3*x^2 + 20*a^2*x + 12*(a^4*x^3 - 2*a^3*x^2 + a^2*x)
*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (19*a^3*x^2 - 26*a^2*x + 3*a)*sqrt(-a^2*
x^2 + 1))/(a^2*c*x^3 - 2*a*c*x^2 + c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\left(\int \frac{3ax}{-a^3x^5\sqrt{-a^2x^2+1}+a^2x^4\sqrt{-a^2x^2+1}+ax^3\sqrt{-a^2x^2+1}-x^2\sqrt{-a^2x^2+1}}\,dx+\int \frac{3a^2x^2}{-a^3x^5\sqrt{-a^2x^2+1}+a^2x^4\sqrt{-a^2x^2+1}+ax^3\sqrt{-a^2x^2+1}-x^2\sqrt{-a^2x^2+1}}\,dx+\int\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a/x)/x**3,x)
```

```
[Out] a*(Integral(3*a*x/(-a**3*x**5*sqrt(-a**2*x**2 + 1) + a**2*x**4*sqrt(-a**2*x
**2 + 1) + a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x) + I
ntegral(3*a**2*x**2/(-a**3*x**5*sqrt(-a**2*x**2 + 1) + a**2*x**4*sqrt(-a**2
*x**2 + 1) + a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x) +
 Integral(a**3*x**3/(-a**3*x**5*sqrt(-a**2*x**2 + 1) + a**2*x**4*sqrt(-a**2
*x**2 + 1) + a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x) +
 Integral(1/(-a**3*x**5*sqrt(-a**2*x**2 + 1) + a**2*x**4*sqrt(-a**2*x**2 +
1) + a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x))/c
```

Giac [B] time = 1.23646, size = 293, normalized size = 2.79

$$\frac{4\,a^{3}\log\left(\frac{\left|-2\,\sqrt{-a^{2}x^{2}+1}|a|-2\,a\right|}{2\,a^{2}|x|}\right)}{c|a|}+\frac{\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)a}{2\,cx|a|}+\frac{\left(3\,a^{3}-\frac{89\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)a}{x}+\frac{153\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)^{2}}{ax^{2}}-\frac{99\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)^{3}}{a^{3}x^{3}}\right)a^{2}x}{6\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)c\left(\frac{\sqrt{-a^{2}x^{2}+1}|a|+a}{a^{2}x}-1\right)^{3}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)/x^3,x, algorithm="giac")
```

```
[Out] 4*a^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c*abs(
a)) + 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/(c*x*abs(a)) + 1/6*(3*a^3 - 89*
(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x + 153*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2
/(a*x^2) - 99*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^3*x^3))*a^2*x/((sqrt(-a^
2*x^2 + 1)*abs(a) + a)*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^3*ab
s(a))
```

3.564 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx$

Optimal. Leaf size=57

$$\frac{2x^{m+1}\sqrt{c-\frac{c}{ax}}\text{Hypergeometric2F1}\left(-\frac{1}{2},m+\frac{1}{2},m+\frac{3}{2},-ax\right)}{(2m+1)\sqrt{1-ax}}$$

```
[Out] (2*Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[-1/2, 1/2 + m, 3/2 + m, -(
a*x)])/((1 + 2*m)*Sqrt[1 - a*x])
```

Rubi [A] time = 0.229812, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6134, 6128, 848, 64}

$$\frac{2x^{m+1}\sqrt{c-\frac{c}{ax}}\,{}_2F_1\left(-\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};-ax\right)}{(2m+1)\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)]*x^m,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[-1/2, 1/2 + m, 3/2 + m, -(
a*x)])/((1 + 2*m)*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] && !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{\tanh^{-1}(ax)}x^{-\frac{1}{2}+m}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{-\frac{1}{2}+m}\sqrt{1-a^2x^2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int x^{-\frac{1}{2}+m}\sqrt{1+ax}\,dx}{\sqrt{1-ax}}$$
$$= \frac{2\sqrt{c-\frac{c}{ax}}x^{1+m}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}+m;\frac{3}{2}+m;-ax\right)}{(1+2m)\sqrt{1-ax}}$$

Mathematica [A] time = 0.0268785, size = 56, normalized size = 0.98

$$\frac{x^{m+1}\sqrt{c-\frac{c}{ax}}\text{Hypergeometric2F1}\left(-\frac{1}{2},m+\frac{1}{2},m+\frac{3}{2},-ax\right)}{\left(m+\frac{1}{2}\right)\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)]*x^m,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[-1/2, 1/2 + m, 3/2 + m, -(a*
x)])/((1/2 + m)*Sqrt[1 - a*x])
```

Maple [F] time = 0.296, size = 0, normalized size = 0.

$$\int (ax+1)\,x^m\sqrt{c-\frac{c}{ax}}\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a/x)^(1/2),x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a/x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x^m/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}x^m\sqrt{\frac{acx-c}{ax}}}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^m*sqrt((a*c*x - c)/(a*x))/(a*x - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^m\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x^m/sqrt(-a^2*x^2 + 1), x)
```

3.565 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal. Leaf size=179

$$-\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-ax}}+\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{8a^{5/2}\sqrt{1-ax}}+\frac{x^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}}+\frac{x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{12a\sqrt{1-ax}}$$

```
[Out] -(Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(8*a^2*Sqrt[1 - a*x]) + (Sqrt[c - c/(a
*x)]*x^2*Sqrt[1 + a*x])/(12*a*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)]*x^3*Sqrt[
1 + a*x])/(3*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sq
rt[x]])/(8*a^(5/2)*Sqrt[1 - a*x])
```

Rubi [A] time = 0.250841, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6134, 6128, 848, 50, 54, 215}

$$-\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-ax}}+\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{8a^{5/2}\sqrt{1-ax}}+\frac{x^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}}+\frac{x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{12a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(8*a^2*Sqrt[1 - a*x]) + (Sqrt[c - c/(a
*x)]*x^2*Sqrt[1 + a*x])/(12*a*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)]*x^3*Sqrt[
1 + a*x])/(3*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sq
rt[x]])/(8*a^(5/2)*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{\tanh^{-1}(ax)}x^{3/2}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}\sqrt{1-a^2x^2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int x^{3/2}\sqrt{1+ax}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1+ax}}{3\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}}{\sqrt{1+ax}}\,dx}{6\sqrt{1-ax}}$$
$$= \frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1+ax}}{3\sqrt{1-ax}} - \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}}{\sqrt{1+ax}}\,dx}{8a\sqrt{1-ax}}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1+ax}}{3\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1-}}}{16a^2\sqrt{1-ax}}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1+ax}}{3\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\,(}{8a^2\sqrt{1}}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1+ax}}{3\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}(}{8a^{5/2}\sqrt{1-ax}}$$

Mathematica [A] time = 0.0625353, size = 88, normalized size = 0.49

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}\left(8a^2x^2+2ax-3\right)+3\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{24a^{5/2}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*(-3 + 2*a*x + 8*a
^2*x^2) + 3*ArcSinh[Sqrt[a]*Sqrt[x]]))/(24*a^(5/2)*Sqrt[1 - a*x])
```

Maple [A] time = 0.133, size = 125, normalized size = 0.7

$$-\frac{x}{48\,ax-48}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(16\,a^{5/2}x^2\sqrt{-(ax+1)\,x}+4\,a^{3/2}x\sqrt{-(ax+1)\,x}-6\,\sqrt{a}\sqrt{-(ax+1)\,x}-3\arctan\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(c-c/a/x)^(1/2),x)
```

```
[Out] -1/48*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(16*a^(5/2)*x^2*(-(a*x+1)*
x)^(1/2)+4*a^(3/2)*x*(-(a*x+1)*x)^(1/2)-6*a^(1/2)*(-(a*x+1)*x)^(1/2)-3*arct
an(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))/a^(5/2)/(a*x-1)/(-(a*x+1)*x)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x^2/sqrt(-a^2*x^2 + 1), x)
```

Fricas [A] time = 2.02362, size = 617, normalized size = 3.45

$$\left[\frac{3\,(ax-1)\sqrt{-c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(8\,a^3x^3+2\,a^2x^2-3\,ax\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{96\left(a^4x-a^3\right)},-\frac{3\,(ax-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/96*(3*(a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*
x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) - 4*
(8*a^3*x^3 + 2*a^2*x^2 - 3*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))
/(a^4*x - a^3), -1/48*(3*(a*x - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sq
rt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^3*x^3 +
 2*a^2*x^2 - 3*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^
3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x^2/sqrt(-a^2*x^2 + 1), x)
```

3.566 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal. Leaf size=135

$$-\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}+\frac{x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-ax}}+\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-ax}}$$

```
[Out] (Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(4*a*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)
]*x^2*Sqrt[1 + a*x])/(2*Sqrt[1 - a*x]) - (Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh
[Sqrt[a]*Sqrt[x]])/(4*a^(3/2)*Sqrt[1 - a*x])
```

Rubi [A] time = 0.188586, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6134, 6128, 848, 50, 54, 215}

$$-\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}+\frac{x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-ax}}+\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)]*x,x]
```


```
[Out] (Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(4*a*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)
]*x^2*Sqrt[1 + a*x])/(2*Sqrt[1 - a*x]) - (Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh
[Sqrt[a]*Sqrt[x]])/(4*a^(3/2)*Sqrt[1 - a*x])
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{\tanh^{-1}(ax)}\sqrt{x}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \sqrt{x}\sqrt{1+ax}\,dx}{\sqrt{1-ax}}\\
&= \frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{2\sqrt{1-ax}}+\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}}{\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}}\\
&= \frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{4a\sqrt{1-ax}}+\frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{2\sqrt{1-ax}}-\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{8a\sqrt{1-ax}}\\
&= \frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{4a\sqrt{1-ax}}+\frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{2\sqrt{1-ax}}-\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{4a\sqrt{1-ax}}\\
&= \frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{4a\sqrt{1-ax}}+\frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{2\sqrt{1-ax}}-\frac{\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}
\end{aligned}
$$

Mathematica [A] time = 0.050679, size = 80, normalized size = 0.59

$$
\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}(2ax+1)-\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{4a^{3/2}\sqrt{1-ax}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*(1 + 2*a*x) - Arc
Sinh[Sqrt[a]*Sqrt[x]]))/(4*a^(3/2)*Sqrt[1 - a*x])
```

Maple [A] time = 0.128, size = 105, normalized size = 0.8

$$
-\frac{x}{8ax-8}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(4a^{3/2}x\sqrt{-(ax+1)x}+2\sqrt{a}\sqrt{-(ax+1)x}+\arctan\left(\frac{2ax+1}{2}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)x}}\right)\right)a
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(c-c/a/x)^(1/2),x)
```

```
[Out] -1/8*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)/a^(3/2)*(4*a^(3/2)*x*(-(a*x
+1)*x)^(1/2)+2*a^(1/2)*(-(a*x+1)*x)^(1/2)+arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a
*x+1)*x)^(1/2)))/(a*x-1)/(-(a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}x}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x/sqrt(-a^2*x^2 + 1), x)
```

Fricas [A] time = 2.16785, size = 571, normalized size = 4.23

$$\left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{16\left(a^3x-a^2\right)}, \frac{(ax-1)\sqrt{c}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="fricas
")
```

```
[Out] [1/16*((a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x)
*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) - 4*(2
*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2),
1/8*((a*x - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x
- c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2
+ 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}x}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x/sqrt(-a^2*x^2 + 1), x)
```

3.567 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=85

$$\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

```
[Out] (Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/Sqrt[1 - a*x] + (Sqrt[c - c/(a*x)]*Sqrt
[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rubi [A] time = 0.141955, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6134, 6128, 848, 50, 54, 215}

$$\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/Sqrt[1 - a*x] + (Sqrt[c - c/(a*x)]*Sqrt
[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{\sqrt{1-ax}}+\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}}\\
&= \frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{\sqrt{1-ax}}+\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{\sqrt{1-ax}}\\
&= \frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{\sqrt{1-ax}}+\frac{\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}
\end{aligned}$$

Mathematica [A] time = 0.0396635, size = 69, normalized size = 0.81

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}+\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]
*Sqrt[x]]))/(Sqrt[a]*Sqrt[1 - a*x])
```

Maple [A] time = 0.13, size = 89, normalized size = 1.1

$$\frac{x}{2\,ax-2}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(-2\,\sqrt{a}\sqrt{-(ax+1)\,x}+\arctan\left(\frac{2\,ax+1}{2}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)\,x}}\right)\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(-2*a^(1/2)*(-(a*x+1)*x)^(1/
2)+arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))/(a*x-1)/(-(a*x+1)*x)^(
1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/sqrt(-a^2*x^2 + 1), x)
```

Fricas [A] time = 2.18683, size = 529, normalized size = 6.22

$$\left[-\frac{4\sqrt{-a^2x^2+1}ax\sqrt{\frac{acx-c}{ax}}-(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4(2a^2x^2+ax)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)}{4(a^2x-a)}, -\frac{2\sqrt{-a^2x^2+1}ax\sqrt{\frac{acx-c}{ax}}+(a\ldots}{2\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) - (a*x - 1)*sqrt(-c
)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt
(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)))/(a^2*x - a), -1/2*(2*sqrt(-a^
2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) + (a*x - 1)*sqrt(c)*arctan(2*sqrt(-a
^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)))
/(a^2*x - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/sqrt(-a^2*x^2 + 1), x)
```

3.568 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=86

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}-\frac{2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}$$

```
[Out] (-2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/Sqrt[1 - a*x] + (2*Sqrt[a]*Sqrt[c - c/
(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/Sqrt[1 - a*x]
```

Rubi [A] time = 0.248922, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6134, 6128, 848, 47, 54, 215}

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}-\frac{2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/Sqrt[1 - a*x] + (2*Sqrt[a]*Sqrt[c - c/
(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/Sqrt[1 - a*x]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{3/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-a^2x^2}}{x^{3/2}\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{3/2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}}+\frac{\left(a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}}+\frac{\left(2a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}}+\frac{2\sqrt{a}\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}
\end{aligned}$$

Mathematica [A] time = 0.0366927, size = 61, normalized size = 0.71

$$-\frac{2\sqrt{c-\frac{c}{ax}}\left(\sqrt{ax+1}-\sqrt{a}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x] - Sqrt[a]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt
[x]]))/Sqrt[1 - a*x]
```

Maple [A] time = 0.134, size = 90, normalized size = 1.1

$$\frac{1}{ax-1}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(\arctan\left(\frac{2\,ax+1}{2}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)\,x}}\right)xa+2\sqrt{a}\sqrt{-(ax+1)\,x}\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x,x)
```

```
[Out] (c*(a*x-1)/a/x)^(1/2)*(-a^2*x^2+1)^(1/2)*(arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a
*x+1)*x)^(1/2))*x*a+2*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-(a*x+1)*x)^(1/2
)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [A] time = 2.27025, size = 506, normalized size = 5.88

$$\left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{2(ax-1)}, -\frac{(ax-1)\sqrt{c}\arctan\left(\frac{2\sqrt{-a^2x^2+1}a\sqrt{c}\cdots}{2a^2cx^2-ac\cdots}\right)\cdots}{ax-\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas
")
```

```
[Out] [1/2*((a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)*
sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*sqr
t(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a*x - 1), -((a*x - 1)*sqrt(c)*arc
tan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 -
 a*c*x - c)) - 2*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x*sqrt(-(a*x - 1)*(a*x + 1))),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x), x)
```

3.569 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=41

$$-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}}$$

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2))/(3*x*Sqrt[1 - a*x])
```

Rubi [A] time = 0.226816, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6134, 6128, 848, 37}

$$-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2))/(3*x*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{5/2}}dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-a^2x^2}}{x^{5/2}\sqrt{1-ax}}dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{5/2}}dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{3x\sqrt{1-ax}}$$

Mathematica [A] time = 0.0212255, size = 41, normalized size = 1.

$$-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2))/(3*x*Sqrt[1 - a*x])
```

Maple [A] time = 0.083, size = 40, normalized size = 1.

$$-\frac{2\,(ax+1)^2}{3\,x}\sqrt{\frac{c\,(ax-1)}{ax}}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^2,x)
```

```
[Out] -2/3*(a*x+1)^2/x/(-a^2*x^2+1)^(1/2)*(c*(a*x-1)/a/x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Fricas [A] time = 1.90373, size = 95, normalized size = 2.32

$$\frac{2\sqrt{-a^2x^2+1}(ax+1)\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fric
as")
```

```
[Out] 2/3*sqrt(-a^2*x^2 + 1)*(a*x + 1)*sqrt((a*c*x - c)/(a*x))/(a*x^2 - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

3.570 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=84

$$\frac{4a(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{15x\sqrt{1-ax}}-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}}$$

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2))/(5*x^2*Sqrt[1 - a*x]) + (4*a*Sqrt[c
- c/(a*x)]*(1 + a*x)^(3/2))/(15*x*Sqrt[1 - a*x])
```

Rubi [A] time = 0.21596, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6134, 6128, 848, 45, 37}

$$\frac{4a(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{15x\sqrt{1-ax}}-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2))/(5*x^2*Sqrt[1 - a*x]) + (4*a*Sqrt[c
- c/(a*x)]*(1 + a*x)^(3/2))/(15*x*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{7/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-a^2x^2}}{x^{7/2}\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{7/2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{5x^2\sqrt{1-ax}} - \frac{\left(2a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{5/2}}\,dx}{5\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{5x^2\sqrt{1-ax}} + \frac{4a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{15x\sqrt{1-ax}}
\end{aligned}$$

Mathematica [A] time = 0.0275659, size = 47, normalized size = 0.56

$$\frac{2(ax+1)^{3/2}(2ax-3)\sqrt{c-\frac{c}{ax}}}{15x^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2)*(-3 + 2*a*x))/(15*x^2*Sqrt[1 - a*x])
```

Maple [A] time = 0.08, size = 46, normalized size = 0.6

$$\frac{2\,(ax+1)^2\,(2\,ax-3)}{15\,x^2}\sqrt{\frac{c\,(ax-1)}{ax}}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^3,x)
```

```
[Out] 2/15*(a*x+1)^2*(2*a*x-3)*(c*(a*x-1)/a/x)^(1/2)/x^2/(-a^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [A] time = 1.84116, size = 116, normalized size = 1.38

$$-\frac{2\left(2\,a^2x^2-ax-3\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{15\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="fric
as")
```

```
[Out] -2/15*(2*a^2*x^2 - a*x - 3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x))/(a*x
^3 - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

3.571 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=128

$$-\frac{16a^2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{105x\sqrt{1-ax}}+\frac{8a(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}}-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}}$$

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2))/(7*x^3*Sqrt[1 - a*x]) + (8*a*Sqrt[c
 - c/(a*x)]*(1 + a*x)^(3/2))/(35*x^2*Sqrt[1 - a*x]) - (16*a^2*Sqrt[c - c/(a*
x)]*(1 + a*x)^(3/2))/(105*x*Sqrt[1 - a*x])
```

Rubi [A] time = 0.230905, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6134, 6128, 848, 45, 37}

$$-\frac{16a^2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{105x\sqrt{1-ax}}+\frac{8a(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}}-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2))/(7*x^3*Sqrt[1 - a*x]) + (8*a*Sqrt[c
 - c/(a*x)]*(1 + a*x)^(3/2))/(35*x^2*Sqrt[1 - a*x]) - (16*a^2*Sqrt[c - c/(a*
x)]*(1 + a*x)^(3/2))/(105*x*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
```

```
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{9/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-a^2x^2}}{x^{9/2}\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{9/2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{7x^3\sqrt{1-ax}} - \frac{\left(4a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{7/2}}\,dx}{7\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{7x^3\sqrt{1-ax}} + \frac{8a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{35x^2\sqrt{1-ax}} + \frac{\left(8a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{5/2}}\,dx}{35\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{7x^3\sqrt{1-ax}} + \frac{8a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{35x^2\sqrt{1-ax}} - \frac{16a^2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{105x\sqrt{1-ax}}
\end{aligned}$$

Mathematica [A] time = 0.0284834, size = 55, normalized size = 0.43

$$-\frac{2(ax+1)^{3/2}\left(8a^2x^2-12ax+15\right)\sqrt{c-\frac{c}{ax}}}{105x^3\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2)*(15 - 12*a*x + 8*a^2*x^2))/(105*x^3*S
qrt[1 - a*x])
```

Maple [A] time = 0.094, size = 54, normalized size = 0.4

$$-\frac{2\,(ax+1)^2\left(8\,a^2x^2-12\,ax+15\right)}{105\,x^3}\sqrt{\frac{c\,(ax-1)}{ax}}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^4,x)
```

```
[Out] -2/105*(a*x+1)^2*(8*a^2*x^2-12*a*x+15)*(c*(a*x-1)/a/x)^(1/2)/x^3/(-a^2*x^2+
1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x^4), x)
```

Fricas [A] time = 1.89214, size = 136, normalized size = 1.06

$$\frac{2\left(8a^3x^3-4a^2x^2+3ax+15\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{105\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="fric
as")
```

```
[Out] 2/105*(8*a^3*x^3 - 4*a^2*x^2 + 3*a*x + 15)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x -
 c)/(a*x))/(a*x^4 - x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{x^4\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**4*sqrt(-(a*x - 1)*(a*x + 1))
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x^4), x)
```

3.572 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=172

$$-\frac{16a^2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{105x^2\sqrt{1-ax}}+\frac{32a^3(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{315x\sqrt{1-ax}}+\frac{4a(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{21x^3\sqrt{1-ax}}-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{9x^4\sqrt{1-ax}}$$

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2))/(9*x^4*Sqrt[1 - a*x]) + (4*a*Sqrt[c
- c/(a*x)]*(1 + a*x)^(3/2))/(21*x^3*Sqrt[1 - a*x]) - (16*a^2*Sqrt[c - c/(a*
x)]*(1 + a*x)^(3/2))/(105*x^2*Sqrt[1 - a*x]) + (32*a^3*Sqrt[c - c/(a*x)]*(1
 + a*x)^(3/2))/(315*x*Sqrt[1 - a*x])
```

Rubi [A] time = 0.232787, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6134, 6128, 848, 45, 37}

$$-\frac{16a^2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{105x^2\sqrt{1-ax}}+\frac{32a^3(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{315x\sqrt{1-ax}}+\frac{4a(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{21x^3\sqrt{1-ax}}-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{9x^4\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x^5,x]
```


```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2))/(9*x^4*Sqrt[1 - a*x]) + (4*a*Sqrt[c
- c/(a*x)]*(1 + a*x)^(3/2))/(21*x^3*Sqrt[1 - a*x]) - (16*a^2*Sqrt[c - c/(a*
x)]*(1 + a*x)^(3/2))/(105*x^2*Sqrt[1 - a*x]) + (32*a^3*Sqrt[c - c/(a*x)]*(1
 + a*x)^(3/2))/(315*x*Sqrt[1 - a*x])
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
```

```
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{11/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-a^2x^2}}{x^{11/2}\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{11/2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{9x^4\sqrt{1-ax}} - \frac{\left(2a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{9/2}}\,dx}{3\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{9x^4\sqrt{1-ax}} + \frac{4a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{21x^3\sqrt{1-ax}} + \frac{\left(8a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{7/2}}\,dx}{21\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{9x^4\sqrt{1-ax}} + \frac{4a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{21x^3\sqrt{1-ax}} - \frac{16a^2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{105x^2\sqrt{1-ax}} - \frac{\left(16a^3\sqrt{c-\frac{c}{ax}}\sqrt{\right.}}{105\sqrt{1}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{9x^4\sqrt{1-ax}} + \frac{4a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{21x^3\sqrt{1-ax}} - \frac{16a^2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{105x^2\sqrt{1-ax}} + \frac{32a^3\sqrt{c-\frac{c}{ax}}(1}{315x\sqrt{1-}}
\end{aligned}$$

Mathematica [A] time = 0.0324035, size = 63, normalized size = 0.37

$$\frac{2(ax+1)^{3/2}\left(16a^3x^3-24a^2x^2+30ax-35\right)\sqrt{c-\frac{c}{ax}}}{315x^4\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2)*(-35 + 30*a*x - 24*a^2*x^2 + 16*a^3*x^
3))/(315*x^4*Sqrt[1 - a*x])
```

Maple [A] time = 0.089, size = 62, normalized size = 0.4

$$\frac{2\,(ax+1)^2\left(16\,x^3a^3-24\,a^2x^2+30\,ax-35\right)}{315\,x^4}\sqrt{\frac{c\,(ax-1)}{ax}}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^5,x)
```

```
[Out] 2/315*(a*x+1)^2*(16*a^3*x^3-24*a^2*x^2+30*a*x-35)*(c*(a*x-1)/a/x)^(1/2)/x^4
/(-a^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x^5), x)
```

Fricas [A] time = 2.00962, size = 155, normalized size = 0.9

$$-\frac{2\left(16\,a^4x^4-8\,a^3x^3+6\,a^2x^2-5\,ax-35\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{315\left(ax^5-x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="fric
as")
```

```
[Out] -2/315*(16*a^4*x^4 - 8*a^3*x^3 + 6*a^2*x^2 - 5*a*x - 35)*sqrt(-a^2*x^2 + 1)
*sqrt((a*c*x - c)/(a*x))/(a*x^5 - x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^5\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**5*sqrt(-(a*x - 1)*(a*x + 1))
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x^5), x)
```

3.573 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

Optimal. Leaf size=130

$$-\frac{25x^2\sqrt{c-\frac{c}{ax}}}{32a^2}-\frac{75x\sqrt{c-\frac{c}{ax}}}{64a^3}-\frac{75\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}-\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}-\frac{5x^3\sqrt{c-\frac{c}{ax}}}{8a}$$

```
[Out] (-75*Sqrt[c - c/(a*x)]*x)/(64*a^3) - (25*Sqrt[c - c/(a*x)]*x^2)/(32*a^2) -
(5*Sqrt[c - c/(a*x)]*x^3)/(8*a) - (Sqrt[c - c/(a*x)]*x^4)/4 - (75*Sqrt[c]*A
rcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(64*a^4)
```

Rubi [A] time = 0.245019, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6133, 25, 514, 446, 78, 51, 63, 208}

$$-\frac{25x^2\sqrt{c-\frac{c}{ax}}}{32a^2}-\frac{75x\sqrt{c-\frac{c}{ax}}}{64a^3}-\frac{75\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}-\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}-\frac{5x^3\sqrt{c-\frac{c}{ax}}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] (-75*Sqrt[c - c/(a*x)]*x)/(64*a^3) - (25*Sqrt[c - c/(a*x)]*x^2)/(32*a^2) -
(5*Sqrt[c - c/(a*x)]*x^3)/(8*a) - (Sqrt[c - c/(a*x)]*x^4)/4 - (75*Sqrt[c]*A
rcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(64*a^4)
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x^3(1+ax)}{1-ax}\,dx$$
$$=-\frac{c\int\frac{x^2(1+ax)}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$
$$=-\frac{c\int\frac{\left(a+\frac{1}{x}\right)x^3}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$
$$=\frac{c\,\text{Subst}\left(\int\frac{a+x}{x^5\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$
$$=-\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4+\frac{(15c)\,\text{Subst}\left(\int\frac{1}{x^4\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{8a}$$
$$=-\frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a}-\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4+\frac{(25c)\,\text{Subst}\left(\int\frac{1}{x^3\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{16a^2}$$
$$=-\frac{25\sqrt{c-\frac{c}{ax}}x^2}{32a^2}-\frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a}-\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4+\frac{(75c)\,\text{Subst}\left(\int\frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{64a^3}$$
$$=-\frac{75\sqrt{c-\frac{c}{ax}}x}{64a^3}-\frac{25\sqrt{c-\frac{c}{ax}}x^2}{32a^2}-\frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a}-\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4+\frac{(75c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,\right.}{128a^4}$$
$$=-\frac{75\sqrt{c-\frac{c}{ax}}x}{64a^3}-\frac{25\sqrt{c-\frac{c}{ax}}x^2}{32a^2}-\frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a}-\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4-\frac{75\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{\right.}{64a^3}$$
$$=-\frac{75\sqrt{c-\frac{c}{ax}}x}{64a^3}-\frac{25\sqrt{c-\frac{c}{ax}}x^2}{32a^2}-\frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a}-\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4-\frac{75\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}$$

Mathematica [C] time = 0.033258, size = 50, normalized size = 0.38

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(15\text{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},1-\frac{1}{ax}\right)+a^4x^4\right)}{4a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*(a^4*x^4 + 15*Hypergeometric2F1[1/2, 4, 3/2, 1 - 1/(a*x
)]))/(4*a^4)
```

Maple [A] time = 0.131, size = 172, normalized size = 1.3

$$-\frac{x}{128}\sqrt{\frac{c(ax-1)}{ax}}\left(32x\left(ax^2-x\right)^{3/2}a^{7/2}+112\left(ax^2-x\right)^{3/2}a^{5/2}+212\sqrt{ax^2-x}a^{5/2}x-106\sqrt{ax^2-x}a^{3/2}+256a^{3/2}\sqrt{(ax-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(c-c/a/x)^(1/2),x)
```

```
[Out] -1/128*(c*(a*x-1)/a/x)^(1/2)*x*(32*x*(a*x^2-x)^(3/2)*a^(7/2)+112*(a*x^2-x)^
(3/2)*a^(5/2)+212*(a*x^2-x)^(1/2)*a^(5/2)*x-106*(a*x^2-x)^(1/2)*a^(3/2)+256
*a^(3/2)*((a*x-1)*x)^(1/2)+128*a*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-
1)/a^(1/2))-53*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a)/((a*x
-1)*x)^(1/2)/a^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}x^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))*x^3/(a^2*x^2 - 1), x)
```

Fricas [A] time = 1.86403, size = 412, normalized size = 3.17

$$\left[-\frac{2\left(16\,a^4x^4+40\,a^3x^3+50\,a^2x^2+75\,ax\right)\sqrt{\frac{acx-c}{ax}}-75\,\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{128\,a^4},-\frac{\left(16\,a^4x^4+40\,a^3x^3+50\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(2*(16*a^4*x^4 + 40*a^3*x^3 + 50*a^2*x^2 + 75*a*x)*sqrt((a*c*x - c)
/(a*x)) - 75*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) +
 c))/a^4, -1/64*((16*a^4*x^4 + 40*a^3*x^3 + 50*a^2*x^2 + 75*a*x)*sqrt((a*c*
x - c)/(a*x)) - 75*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^4
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx-\int \frac{ax^4\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(c-c/a/x)**(1/2),x)
```

```
[Out] -Integral(x**3*sqrt(c - c/(a*x))/(a*x - 1), x) - Integral(a*x**4*sqrt(c - c
/(a*x))/(a*x - 1), x)
```

Giac [A] time = 1.33258, size = 193, normalized size = 1.48

$$-\frac{1}{64}\sqrt{a^2cx^2-acx}\left(2\left(4\,x\left(\frac{2\,x|a|}{a^2\mathrm{sgn}\,(x)}+\frac{5\,|a|}{a^3\mathrm{sgn}\,(x)}\right)+\frac{25\,|a|}{a^4\mathrm{sgn}\,(x)}\right)x+\frac{75\,|a|}{a^5\mathrm{sgn}\,(x)}\right)-\frac{75\,\sqrt{c}\log\left(|a|\sqrt{|c|}\right)\mathrm{sgn}\,(x)}{128\,a^4}+\frac{75\,\sqrt{c}\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/64*sqrt(a^2*c*x^2 - a*c*x)*(2*(4*x*(2*x*abs(a)/(a^2*sgn(x)) + 5*abs(a)/(
a^3*sgn(x))) + 25*abs(a)/(a^4*sgn(x)))*x + 75*abs(a)/(a^5*sgn(x))) - 75/128
*sqrt(c)*log(abs(a)*sqrt(abs(c)))*sgn(x)/a^4 + 75/128*sqrt(c)*log(abs(-2*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*abs(a) + a*sqrt(c)))/(a^4*sgn(x))
```

3.574 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal. Leaf size=105

$$-\frac{11x\sqrt{c-\frac{c}{ax}}}{8a^2}-\frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}-\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}-\frac{11x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

```
[Out] (-11*Sqrt[c - c/(a*x)]*x)/(8*a^2) - (11*Sqrt[c - c/(a*x)]*x^2)/(12*a) - (Sq
rt[c - c/(a*x)]*x^3)/3 - (11*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(8
*a^3)
```

Rubi [A] time = 0.217048, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6133, 25, 514, 446, 78, 51, 63, 208}

$$-\frac{11x\sqrt{c-\frac{c}{ax}}}{8a^2}-\frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}-\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}-\frac{11x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (-11*Sqrt[c - c/(a*x)]*x)/(8*a^2) - (11*Sqrt[c - c/(a*x)]*x^2)/(12*a) - (Sq
rt[c - c/(a*x)]*x^3)/3 - (11*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(8
*a^3)
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx &= \int \frac{\sqrt{c-\frac{c}{ax}}x^2(1+ax)}{1-ax}\,dx \\
&= -\frac{c\int \frac{x(1+ax)}{\sqrt{c-\frac{c}{ax}}}\,dx}{a} \\
&= -\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\sqrt{c-\frac{c}{ax}}}\,dx}{a} \\
&= \frac{c\,\text{Subst}\left(\int \frac{a+x}{x^4\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a} \\
&= -\frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3+\frac{(11c)\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{6a} \\
&= -\frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a}-\frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3+\frac{(11c)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{8a^2} \\
&= -\frac{11\sqrt{c-\frac{c}{ax}}x}{8a^2}-\frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a}-\frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3+\frac{(11c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{16a^3} \\
&= -\frac{11\sqrt{c-\frac{c}{ax}}x}{8a^2}-\frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a}-\frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3-\frac{11\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)}{8a^2} \\
&= -\frac{11\sqrt{c-\frac{c}{ax}}x}{8a^2}-\frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a}-\frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3-\frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}
\end{aligned}
$$

Mathematica [C] time = 0.0348243, size = 50, normalized size = 0.48

$$
-\frac{\sqrt{c-\frac{c}{ax}}\left(11\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\frac{1}{ax}\right)+a^3x^3\right)}{3a^3}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*(a^3*x^3 + 11*Hypergeometric2F1[1/2, 3, 3/2, 1 - 1/(a*x
)]))/(3*a^3)
```

Maple [A] time = 0.128, size = 155, normalized size = 1.5

$$
-\frac{x}{48}\sqrt{\frac{c(ax-1)}{ax}}\left(16\left(ax^2-x\right)^{3/2}a^{5/2}+60\sqrt{ax^2-x}a^{5/2}x-30\sqrt{ax^2-x}a^{3/2}+96a^{3/2}\sqrt{(ax-1)x}+48a\ln\left(1/2\,\frac{2\sqrt{(ax
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(c-c/a/x)^(1/2),x)
```

```
[Out] -1/48*(c*(a*x-1)/a/x)^(1/2)*x*(16*(a*x^2-x)^(3/2)*a^(5/2)+60*(a*x^2-x)^(1/2
)*a^(5/2)*x-30*(a*x^2-x)^(1/2)*a^(3/2)+96*a^(3/2)*((a*x-1)*x)^(1/2)+48*a*ln
(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))-15*ln(1/2*(2*(a*x^2-x)^
(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a)/((a*x-1)*x)^(1/2)/a^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}x^2}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))*x^2/(a^2*x^2 - 1), x)
```

Fricas [A] time = 1.883, size = 373, normalized size = 3.55

$$\left[-\frac{2\left(8\,a^3x^3+22\,a^2x^2+33\,ax\right)\sqrt{\frac{acx-c}{ax}}-33\,\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{48\,a^3},-\frac{\left(8\,a^3x^3+22\,a^2x^2+33\,ax\right)\sqrt{\frac{acx-c}{ax}}-33}{24\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(2*(8*a^3*x^3 + 22*a^2*x^2 + 33*a*x)*sqrt((a*c*x - c)/(a*x)) - 33*sq
rt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^3, -1/24
*((8*a^3*x^3 + 22*a^2*x^2 + 33*a*x)*sqrt((a*c*x - c)/(a*x)) - 33*sqrt(-c)*a
rctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx-\int \frac{ax^3\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(c-c/a/x)**(1/2),x)
```

```
[Out] -Integral(x**2*sqrt(c - c/(a*x))/(a*x - 1), x) - Integral(a*x**3*sqrt(c - c
/(a*x))/(a*x - 1), x)
```

Giac [A] time = 1.28809, size = 173, normalized size = 1.65

$$-\frac{1}{24}\sqrt{a^2cx^2-acx}\left(2\,x\left(\frac{4\,x|a|}{a^2\mathrm{sgn}\,(x)}+\frac{11\,|a|}{a^3\mathrm{sgn}\,(x)}\right)+\frac{33\,|a|}{a^4\mathrm{sgn}\,(x)}\right)-\frac{11\,\sqrt{c}\log\left(|a|\sqrt{|c|}\right)\mathrm{sgn}\,(x)}{16\,a^3}+\frac{11\,\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2cx}-\sqrt{a^2}\right.\right.\right.}{16\,a^3\mathrm{sgn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(a^2*c*x^2 - a*c*x)*(2*x*(4*x*abs(a)/(a^2*sgn(x)) + 11*abs(a)/(a^
3*sgn(x))) + 33*abs(a)/(a^4*sgn(x))) - 11/16*sqrt(c)*log(abs(a)*sqrt(abs(c)
))*sgn(x)/a^3 + 11/16*sqrt(c)*log(abs(-2*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
a*c*x))*abs(a) + a*sqrt(c)))/(a^3*sgn(x))
```

3.575 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal. Leaf size=80

$$-\frac{7\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}-\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}-\frac{7x\sqrt{c-\frac{c}{ax}}}{4a}$$

```
[Out] (-7*Sqrt[c - c/(a*x)]*x)/(4*a) - (Sqrt[c - c/(a*x)]*x^2)/2 - (7*Sqrt[c]*Arc
Tanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(4*a^2)
```

Rubi [A] time = 0.142979, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {6133, 25, 434, 446, 78, 51, 63, 208}

$$-\frac{7\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}-\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}-\frac{7x\sqrt{c-\frac{c}{ax}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (-7*Sqrt[c - c/(a*x)]*x)/(4*a) - (Sqrt[c - c/(a*x)]*x^2)/2 - (7*Sqrt[c]*Arc
Tanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(4*a^2)
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x(1+ax)}{1-ax}\,dx$$
$$=-\frac{c\int\frac{1+ax}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$
$$=-\frac{c\int\frac{\left(a+\frac{1}{x}\right)x}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$
$$=\frac{c\,\text{Subst}\left(\int\frac{a+x}{x^3\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$
$$=-\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2+\frac{(7c)\,\text{Subst}\left(\int\frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{4a}$$
$$=-\frac{7\sqrt{c-\frac{c}{ax}}x}{4a}-\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2+\frac{(7c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{8a^2}$$
$$=-\frac{7\sqrt{c-\frac{c}{ax}}x}{4a}-\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2-\frac{7\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)}{4a}$$
$$=-\frac{7\sqrt{c-\frac{c}{ax}}x}{4a}-\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2-\frac{7\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}$$

Mathematica [A] time = 0.0710427, size = 77, normalized size = 0.96

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(ax\sqrt{1-\frac{1}{ax}}(2ax+7)+7\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\right)\right)}{4a^2\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*(a*Sqrt[1 - 1/(a*x)]*x*(7 + 2*a*x) + 7*ArcTanh[Sqrt[1 -
 1/(a*x)]]))/(4*a^2*Sqrt[1 - 1/(a*x)])
```

Maple [B] time = 0.118, size = 139, normalized size = 1.7

$$-\frac{x}{8}\sqrt{\frac{c(ax-1)}{ax}}\left(4\sqrt{ax^2-x}a^{5/2}x-2\sqrt{ax^2-x}a^{3/2}+16a^{3/2}\sqrt{(ax-1)x}+8a\ln\left(1/2\,\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{\sqrt{a}}\right)-\ln\left(\frac{1}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(c-c/a/x)^(1/2),x)
```

```
[Out] -1/8*(c*(a*x-1)/a/x)^(1/2)*x*(4*(a*x^2-x)^(1/2)*a^(5/2)*x-2*(a*x^2-x)^(1/2)
*a^(3/2)+16*a^(3/2)*((a*x-1)*x)^(1/2)+8*a*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/
```

```
2)+2*a*x-1)/a^(1/2))-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a)
/((a*x-1)*x)^(1/2)/a^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}x}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))*x/(a^2*x^2 - 1), x)
```

Fricas [A] time = 1.92521, size = 329, normalized size = 4.11

$$\left[-\frac{2\left(2a^2x^2+7ax\right)\sqrt{\frac{acx-c}{ax}}-7\sqrt{c}\log\left(-2acx+2a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{8a^2}, -\frac{\left(2a^2x^2+7ax\right)\sqrt{\frac{acx-c}{ax}}-7\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{4a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(2*a^2*x^2 + 7*a*x)*sqrt((a*c*x - c)/(a*x)) - 7*sqrt(c)*log(-2*a*c
*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^2, -1/4*((2*a^2*x^2 + 7*
a*x)*sqrt((a*c*x - c)/(a*x)) - 7*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/
(a*x))/c))/a^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx-\int \frac{ax^2\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(c-c/a/x)**(1/2),x)
```

```
[Out] -Integral(x*sqrt(c - c/(a*x))/(a*x - 1), x) - Integral(a*x**2*sqrt(c - c/(a
*x))/(a*x - 1), x)
```

Giac [A] time = 1.29714, size = 153, normalized size = 1.91

$$-\frac{1}{4}\sqrt{a^2cx^2-acx}\left(\frac{2x|a|}{a^2\mathrm{sgn}(x)}+\frac{7|a|}{a^3\mathrm{sgn}(x)}\right)-\frac{7\sqrt{c}\log\left(|a|\sqrt{|c|}\right)\mathrm{sgn}(x)}{8a^2}+\frac{7\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)|a|+a\right.\right.}{8a^2\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(a^2*c*x^2 - a*c*x)*(2*x*abs(a)/(a^2*sgn(x)) + 7*abs(a)/(a^3*sgn(x
))) - 7/8*sqrt(c)*log(abs(a)*sqrt(abs(c)))*sgn(x)/a^2 + 7/8*sqrt(c)*log(abs
(-2*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*abs(a) + a*sqrt(c)))/(a^2*sgn
(x))
```

3.576 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=51

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] -(Sqrt[c - c/(a*x)]*x) - (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rubi [A] time = 0.0962397, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6133, 25, 514, 375, 78, 63, 208}

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*x) - (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= \int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{1-ax}\,dx \\
&= -\frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x}\,dx}{a} \\
&= -\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}\,dx}{a} \\
&= \frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= -\sqrt{c-\frac{c}{ax}}x+\frac{(3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a} \\
&= -\sqrt{c-\frac{c}{ax}}x-3\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) \\
&= -\sqrt{c-\frac{c}{ax}}x-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0387472, size = 51, normalized size = 1.

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*x) - (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Maple [B] time = 0.152, size = 118, normalized size = 2.3

$$-\frac{x}{2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(4\sqrt{(ax-1)\,x}\sqrt{a}-2\sqrt{a x^2-x}\sqrt{a}+\ln\left(\frac{1}{2}\left(2\sqrt{a x^2-x}\sqrt{a}+2\,ax-1\right)\frac{1}{\sqrt{a}}\right)+2\ln\left(1/2\,\frac{2\sqrt{(ax-1)\,x}\sqrt{a}}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(4*((a*x-1)*x)^(1/2)*a^(1/2)-2*(a*x^2-x)^(1/2)
*a^(1/2)+ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))+2*ln(1/2*(2*((
a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))/(a^2*x^2 - 1), x)
```

Fricas [A] time = 1.77913, size = 275, normalized size = 5.39

$$\left[-\frac{2\,ax\sqrt{\frac{acx-c}{ax}}-3\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\ -\frac{ax\sqrt{\frac{acx-c}{ax}}-3\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)
*x*sqrt((a*c*x - c)/(a*x)) + c))/a, -(a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(
-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(1/2),x)
```

```
[Out] -Integral(sqrt(c - c/(a*x))/(a*x - 1), x) - Integral(a*x*sqrt(c - c/(a*x))/
(a*x - 1), x)
```

Giac [B] time = 1.2503, size = 132, normalized size = 2.59

$$-\frac{3\sqrt{c}\log\left(|a|\sqrt{|c|}\right)\operatorname{sgn}(x)}{2a}+\frac{3\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)|a|+a\sqrt{c}\right|\right)}{2a\operatorname{sgn}(x)}-\frac{\sqrt{a^2cx^2-acx}|a|}{a^2\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(c)*log(abs(a)*sqrt(abs(c)))*sgn(x)/a + 3/2*sqrt(c)*log(abs(-2*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*abs(a) + a*sqrt(c)))/(a*sgn(x)) - sq
rt(a^2*c*x^2 - a*c*x)*abs(a)/(a^2*sgn(x))
```

3.577 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=47

$$-2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

```
[Out] -2*Sqrt[c - c/(a*x)] - 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
```

Rubi [A] time = 0.190577, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6133, 25, 514, 446, 80, 63, 208}

$$-2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] -2*Sqrt[c - c/(a*x)] - 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx &= \int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x(1-ax)}\,dx\\
&= -\frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x^2}\,dx}{a}\\
&= -\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x}\,dx}{a}\\
&= \frac{c\,\text{Subst}\left(\int \frac{a+x}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -2\sqrt{c-\frac{c}{ax}} + c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= -2\sqrt{c-\frac{c}{ax}} - (2a)\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)\\
&= -2\sqrt{c-\frac{c}{ax}} - 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0317308, size = 47, normalized size = 1.

$$-2\sqrt{c-\frac{c}{ax}} - 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] -2*Sqrt[c - c/(a*x)] - 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
```

Maple [B] time = 0.135, size = 98, normalized size = 2.1

$$-\frac{1}{x}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2\sqrt{(ax-1)\,x}a^{3/2}x^2-2\left(ax^2-x\right)^{3/2}\sqrt{a}+\ln\left(\frac{1}{2}\left(2\sqrt{(ax-1)\,x}\sqrt{a}+2\,ax-1\right)\frac{1}{\sqrt{a}}\right)x^2a\right)\frac{1}{\sqrt{(ax-1)\,x}}\frac{1}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x,x)
```

```
[Out] -(c*(a*x-1)/a/x)^(1/2)/x*(2*((a*x-1)*x)^(1/2)*a^(3/2)*x^2-2*(a*x^2-x)^(3/2)
*a^(1/2)+ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a)/((a*x
-1)*x)^(1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}}{\left(a^2x^2-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))/((a^2*x^2 - 1)*x), x)
```

Fricas [A] time = 1.78576, size = 246, normalized size = 5.23

$$\left[\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-2\sqrt{\frac{acx-c}{ax}},2\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-2\sqrt{\frac{acx-c}{ax}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 2*sqrt
((a*c*x - c)/(a*x)), 2*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c)
- 2*sqrt((a*c*x - c)/(a*x))]
```

Sympy [A] time = 13.2401, size = 39, normalized size = 0.83

$$\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{\sqrt{-c}}-2\sqrt{c-\frac{c}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(1/2)/x,x)
```

```
[Out] 2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sqrt(-c) - 2*sqrt(c - c/(a*x))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.578 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=42

$$\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a\sqrt{c-\frac{c}{ax}}$$

```
[Out] -4*a*Sqrt[c - c/(a*x)] + (2*a*(c - c/(a*x))^(3/2))/(3*c)
```

Rubi [A] time = 0.215732, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6133, 25, 514, 444, 43}

$$\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] -4*a*Sqrt[c - c/(a*x)] + (2*a*(c - c/(a*x))^(3/2))/(3*c)
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x^2(1-ax)}\,dx$$
$$= -\frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x^3}\,dx}{a}$$
$$= -\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^2}\,dx}{a}$$
$$= \frac{c\,\text{Subst}\left(\int \frac{a+x}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= \frac{c\,\text{Subst}\left(\int \left(\frac{2a}{\sqrt{c-\frac{cx}{a}}} - \frac{a\sqrt{c-\frac{cx}{a}}}{c}\right)dx, x, \frac{1}{x}\right)}{a}$$
$$= -4a\sqrt{c-\frac{c}{ax}} + \frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}$$

Mathematica [A] time = 0.0336075, size = 28, normalized size = 0.67

$$-\frac{2(5ax+1)\sqrt{c-\frac{c}{ax}}}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + 5*a*x))/(3*x)
```

Maple [A] time = 0.082, size = 27, normalized size = 0.6

$$-\frac{10\,ax+2}{3\,x}\sqrt{\frac{c\,(ax-1)}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^2,x)
```

```
[Out] -2/3*(c*(a*x-1)/a/x)^(1/2)*(5*a*x+1)/x
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}}{(a^2x^2-1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))/((a^2*x^2 - 1)*x^2), x)
```

Fricas [A] time = 1.75613, size = 59, normalized size = 1.4

$$-\frac{2\,(5\,ax+1)\sqrt{\frac{acx-c}{ax}}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -2/3*(5*a*x + 1)*sqrt((a*c*x - c)/(a*x))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c-\frac{c}{ax}}}{ax^3-x^2}\,dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax^3-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(1/2)/x**2,x)
```

```
[Out] -Integral(sqrt(c - c/(a*x))/(a*x**3 - x**2), x) - Integral(a*x*sqrt(c - c/(
a*x))/(a*x**3 - x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.579 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=69

$$-\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c}-4a^2\sqrt{c-\frac{c}{ax}}$$

```
[Out] -4*a^2*Sqrt[c - c/(a*x)] + (2*a^2*(c - c/(a*x))^(3/2))/c - (2*a^2*(c - c/(a
*x))^(5/2))/(5*c^2)
```

Rubi [A] time = 0.221703, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6133, 25, 514, 446, 77}

$$-\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c}-4a^2\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] -4*a^2*Sqrt[c - c/(a*x)] + (2*a^2*(c - c/(a*x))^(3/2))/c - (2*a^2*(c - c/(a
*x))^(5/2))/(5*c^2)
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx &= \int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x^3(1-ax)}\,dx\\
&= -\frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x^4}\,dx}{a}\\
&= -\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^3}\,dx}{a}\\
&= \frac{c\,\text{Subst}\left(\int \frac{x(a+x)}{\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \frac{c\,\text{Subst}\left(\int \left(\frac{2a^2}{\sqrt{c-\frac{cx}{a}}}-\frac{3a^2\sqrt{c-\frac{cx}{a}}}{c}+\frac{a^2\left(c-\frac{cx}{a}\right)^{3/2}}{c^2}\right)dx,x,\frac{1}{x}\right)}{a}\\
&= -4a^2\sqrt{c-\frac{c}{ax}}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c}-\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}
\end{aligned}$$

Mathematica [A] time = 0.0413682, size = 36, normalized size = 0.52

$$-\frac{2\left(6a^2x^2+3ax+1\right)\sqrt{c-\frac{c}{ax}}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + 3*a*x + 6*a^2*x^2))/(5*x^2)
```

Maple [A] time = 0.086, size = 35, normalized size = 0.5

$$-\frac{12\,a^2x^2+6\,ax+2}{5\,x^2}\sqrt{\frac{c\,(ax-1)}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^3,x)
```

```
[Out] -2/5*(c*(a*x-1)/a/x)^(1/2)*(6*a^2*x^2+3*a*x+1)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}}{(a^2x^2-1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))/((a^2*x^2 - 1)*x^3), x)
```

Fricas [A] time = 1.82501, size = 78, normalized size = 1.13

$$-\frac{2\left(6a^2x^2+3ax+1\right)\sqrt{\frac{acx-c}{ax}}}{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -2/5*(6*a^2*x^2 + 3*a*x + 1)*sqrt((a*c*x - c)/(a*x))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c-\frac{c}{ax}}}{ax^4-x^3}\,dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax^4-x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(1/2)/x**3,x)
```

```
[Out] -Integral(sqrt(c - c/(a*x))/(a*x**4 - x**3), x) - Integral(a*x*sqrt(c - c/(
a*x))/(a*x**4 - x**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.580 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=96

$$\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^3\sqrt{c-\frac{c}{ax}}$$

```
[Out] -4*a^3*Sqrt[c - c/(a*x)] + (10*a^3*(c - c/(a*x))^(3/2))/(3*c) - (8*a^3*(c -
 c/(a*x))^(5/2))/(5*c^2) + (2*a^3*(c - c/(a*x))^(7/2))/(7*c^3)
```

Rubi [A] time = 0.224457, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6133, 25, 514, 446, 77}

$$\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^3\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] -4*a^3*Sqrt[c - c/(a*x)] + (10*a^3*(c - c/(a*x))^(3/2))/(3*c) - (8*a^3*(c -
 c/(a*x))^(5/2))/(5*c^2) + (2*a^3*(c - c/(a*x))^(7/2))/(7*c^3)
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx &= \int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x^4(1-ax)}\,dx \\
&= -\frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x^5}\,dx}{a} \\
&= -\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^4}\,dx}{a} \\
&= \frac{c\,\text{Subst}\left(\int \frac{x^2(a+x)}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= \frac{c\,\text{Subst}\left(\int \left(\frac{2a^3}{\sqrt{c-\frac{cx}{a}}} - \frac{5a^3\sqrt{c-\frac{cx}{a}}}{c} + \frac{4a^3\left(c-\frac{cx}{a}\right)^{3/2}}{c^2} - \frac{a^3\left(c-\frac{cx}{a}\right)^{5/2}}{c^3}\right)dx, x, \frac{1}{x}\right)}{a} \\
&= -4a^3\sqrt{c-\frac{c}{ax}} + \frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}
\end{aligned}$$

Mathematica [A] time = 0.0491571, size = 44, normalized size = 0.46

$$-\frac{2\left(104a^3x^3+52a^2x^2+39ax+15\right)\sqrt{c-\frac{c}{ax}}}{105x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(15 + 39*a*x + 52*a^2*x^2 + 104*a^3*x^3))/(105*x^3)
```

Maple [A] time = 0.087, size = 43, normalized size = 0.5

$$-\frac{208\,x^3a^3+104\,a^2x^2+78\,ax+30}{105\,x^3}\sqrt{\frac{c\,(ax-1)}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^4,x)
```

```
[Out] -2/105*(c*(a*x-1)/a/x)^(1/2)*(104*a^3*x^3+52*a^2*x^2+39*a*x+15)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}}{(a^2x^2-1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))/((a^2*x^2 - 1)*x^4), x)
```

Fricas [A] time = 1.79331, size = 104, normalized size = 1.08

$$-\frac{2\left(104\,a^3x^3+52\,a^2x^2+39\,ax+15\right)\sqrt{\frac{acx-c}{ax}}}{105\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -2/105*(104*a^3*x^3 + 52*a^2*x^2 + 39*a*x + 15)*sqrt((a*c*x - c)/(a*x))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c-\frac{c}{ax}}}{ax^5-x^4}\,dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax^5-x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(1/2)/x**4,x)
```

```
[Out] -Integral(sqrt(c - c/(a*x))/(a*x**5 - x**4), x) - Integral(a*x*sqrt(c - c/(
a*x))/(a*x**5 - x**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.581 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=121

$$-\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}+\frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^4\sqrt{c-\frac{c}{ax}}$$

```
[Out] -4*a^4*Sqrt[c - c/(a*x)] + (14*a^4*(c - c/(a*x))^(3/2))/(3*c) - (18*a^4*(c
- c/(a*x))^(5/2))/(5*c^2) + (10*a^4*(c - c/(a*x))^(7/2))/(7*c^3) - (2*a^4*(
c - c/(a*x))^(9/2))/(9*c^4)
```

Rubi [A] time = 0.238246, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6133, 25, 514, 446, 77}

$$-\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}+\frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^4\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] -4*a^4*Sqrt[c - c/(a*x)] + (14*a^4*(c - c/(a*x))^(3/2))/(3*c) - (18*a^4*(c
- c/(a*x))^(5/2))/(5*c^2) + (10*a^4*(c - c/(a*x))^(7/2))/(7*c^3) - (2*a^4*(
c - c/(a*x))^(9/2))/(9*c^4)
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx &= \int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x^5(1-ax)}\,dx \\
&= -\frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x^6}\,dx}{a} \\
&= -\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^5}\,dx}{a} \\
&= \frac{c\,\mathrm{Subst}\left(\int \frac{x^3(a+x)}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= \frac{c\,\mathrm{Subst}\left(\int \left(\frac{2a^4}{\sqrt{c-\frac{cx}{a}}} - \frac{7a^4\sqrt{c-\frac{cx}{a}}}{c} + \frac{9a^4\left(c-\frac{cx}{a}\right)^{3/2}}{c^2} - \frac{5a^4\left(c-\frac{cx}{a}\right)^{5/2}}{c^3} + \frac{a^4\left(c-\frac{cx}{a}\right)^{7/2}}{c^4}\right)dx, x, \frac{1}{x}\right)}{a} \\
&= -4a^4\sqrt{c-\frac{c}{ax}} + \frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}
\end{aligned}
$$

Mathematica [A] time = 0.0589913, size = 52, normalized size = 0.43

$$
-\frac{2\left(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35\right)\sqrt{c-\frac{c}{ax}}}{315x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(35 + 85*a*x + 102*a^2*x^2 + 136*a^3*x^3 + 272*a^4*x^
4))/(315*x^4)
```

Maple [A] time = 0.089, size = 51, normalized size = 0.4

$$
-\frac{544\,x^4a^4+272\,x^3a^3+204\,a^2x^2+170\,ax+70}{315\,x^4}\sqrt{\frac{c\,(ax-1)}{ax}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^5,x)
```

```
[Out] -2/315*(c*(a*x-1)/a/x)^(1/2)*(272*a^4*x^4+136*a^3*x^3+102*a^2*x^2+85*a*x+35
)/x^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}}{(a^2x^2-1)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))/((a^2*x^2 - 1)*x^5), x)
```

Fricas [A] time = 1.79809, size = 124, normalized size = 1.02

$$-\frac{2\left(272\,a^4x^4+136\,a^3x^3+102\,a^2x^2+85\,ax+35\right)\sqrt{\frac{acx-c}{ax}}}{315\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -2/315*(272*a^4*x^4 + 136*a^3*x^3 + 102*a^2*x^2 + 85*a*x + 35)*sqrt((a*c*x
- c)/(a*x))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c-\frac{c}{ax}}}{ax^6-x^5}\,dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax^6-x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] -Integral(sqrt(c - c/(a*x))/(a*x**6 - x**5), x) - Integral(a*x*sqrt(c - c/(
a*x))/(a*x**6 - x**5), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.582 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

Optimal. Leaf size=292

$$-\frac{11x^2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{24a^2\sqrt{1-ax}}-\frac{21x(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{32a^3\sqrt{1-ax}}-\frac{107x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{64a^3\sqrt{1-ax}}-\frac{363\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{64a^{7/2}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{\cdots}}{\cdots}$$

```
[Out] (-107*Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(64*a^3*Sqrt[1 - a*x]) - (21*Sqrt[
c - c/(a*x)]*x*(1 + a*x)^(3/2))/(32*a^3*Sqrt[1 - a*x]) - (11*Sqrt[c - c/(a*
x)]*x^2*(1 + a*x)^(3/2))/(24*a^2*Sqrt[1 - a*x]) - (Sqrt[c - c/(a*x)]*x^3*(1
 + a*x)^(3/2))/(4*a*Sqrt[1 - a*x]) - (363*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh
[Sqrt[a]*Sqrt[x]])/(64*a^(7/2)*Sqrt[1 - a*x]) + (4*Sqrt[2]*Sqrt[c - c/(a*x)
]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a^(7/2)*Sqrt[1
 - a*x])
```

Rubi [A] time = 0.313426, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6134, 6129, 101, 154, 157, 54, 215, 93, 206}

$$-\frac{11x^2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{24a^2\sqrt{1-ax}}-\frac{21x(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{32a^3\sqrt{1-ax}}-\frac{107x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{64a^3\sqrt{1-ax}}-\frac{363\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{64a^{7/2}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] (-107*Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(64*a^3*Sqrt[1 - a*x]) - (21*Sqrt[
c - c/(a*x)]*x*(1 + a*x)^(3/2))/(32*a^3*Sqrt[1 - a*x]) - (11*Sqrt[c - c/(a*
x)]*x^2*(1 + a*x)^(3/2))/(24*a^2*Sqrt[1 - a*x]) - (Sqrt[c - c/(a*x)]*x^3*(1
 + a*x)^(3/2))/(4*a*Sqrt[1 - a*x]) - (363*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh
[Sqrt[a]*Sqrt[x]])/(64*a^(7/2)*Sqrt[1 - a*x]) + (4*Sqrt[2]*Sqrt[c - c/(a*x)
]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a^(7/2)*Sqrt[1
 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
```

```
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{3\tanh^{-1}(ax)}x^{5/2}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{5/2}(1+ax)^{3/2}}{1-ax}dx}{\sqrt{1-ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}x^3(1+ax)^{3/2}}{4a\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}\sqrt{1+ax}\left(\frac{5}{2}+\frac{11ax}{2}\right)}{1-ax}dx}{4a\sqrt{1-ax}}$$

$$= -\frac{11\sqrt{c-\frac{c}{ax}}x^2(1+ax)^{3/2}}{24a^2\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}x^3(1+ax)^{3/2}}{4a\sqrt{1-ax}} - \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}\sqrt{1+ax}\left(-\frac{33a}{4}-\frac{63a^2x}{4}\right)}{1-ax}}{12a^3\sqrt{1-ax}}$$

$$= -\frac{21\sqrt{c-\frac{c}{ax}}x(1+ax)^{3/2}}{32a^3\sqrt{1-ax}} - \frac{11\sqrt{c-\frac{c}{ax}}x^2(1+ax)^{3/2}}{24a^2\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}x^3(1+ax)^{3/2}}{4a\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\right.}{}$$

$$= -\frac{107\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{64a^3\sqrt{1-ax}} - \frac{21\sqrt{c-\frac{c}{ax}}x(1+ax)^{3/2}}{32a^3\sqrt{1-ax}} - \frac{11\sqrt{c-\frac{c}{ax}}x^2(1+ax)^{3/2}}{24a^2\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{}}$$

$$= -\frac{107\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{64a^3\sqrt{1-ax}} - \frac{21\sqrt{c-\frac{c}{ax}}x(1+ax)^{3/2}}{32a^3\sqrt{1-ax}} - \frac{11\sqrt{c-\frac{c}{ax}}x^2(1+ax)^{3/2}}{24a^2\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{}}$$

$$= -\frac{107\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{64a^3\sqrt{1-ax}} - \frac{21\sqrt{c-\frac{c}{ax}}x(1+ax)^{3/2}}{32a^3\sqrt{1-ax}} - \frac{11\sqrt{c-\frac{c}{ax}}x^2(1+ax)^{3/2}}{24a^2\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{}}$$

$$= -\frac{107\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{64a^3\sqrt{1-ax}} - \frac{21\sqrt{c-\frac{c}{ax}}x(1+ax)^{3/2}}{32a^3\sqrt{1-ax}} - \frac{11\sqrt{c-\frac{c}{ax}}x^2(1+ax)^{3/2}}{24a^2\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{}}$$

Mathematica [A] time = 0.0990608, size = 130, normalized size = 0.45

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}x\sqrt{ax+1}\left(48a^3x^3+136a^2x^2+214ax+447\right)+1089\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-768\sqrt{2}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)\right)}{192a^{7/2}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*(Sqrt[a]*x*Sqrt[1 + a*x]*(447 + 214*a*x + 136*a^2*x^2 +
 48*a^3*x^3) + 1089*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]] - 768*Sqrt[2]*Sqrt[x]*
ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]))/(192*a^(7/2)*Sqrt[1 - a*
x])
```

Maple [A] time = 0.149, size = 247, normalized size = 0.9

$$\frac{x\sqrt{2}}{768\,ax-768}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(96\,a^{9/2}\sqrt{2}\sqrt{-a^{-1}}\sqrt{-(ax+1)\,x}x^3+272\,\sqrt{-(ax+1)\,x}a^{7/2}\sqrt{2}\sqrt{-a^{-1}}x^2+428\,\sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/768*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(96*a^(9/2)*2^(1/2)*(-1/a)
^(1/2)*(-(a*x+1)*x)^(1/2)*x^3+272*(-(a*x+1)*x)^(1/2)*a^(7/2)*2^(1/2)*(-1/a)
^(1/2)*x^2+428*(-(a*x+1)*x)^(1/2)*a^(5/2)*2^(1/2)*(-1/a)^(1/2)*x+894*(-(a*x
+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2)-1089*arctan(1/2/a^(1/2)*(2*a*x+1)
/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)^(1/2)+1536*ln((2*2^(1/2)*(-1/a)^(1/2)
*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*a^(1/2))*2^(1/2)/a^(9/2)/(a*x-1)/(-
(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}x^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))*x^3/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="fr
icas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**3*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**
(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}x^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))*x^3/(-a^2*x^2 + 1)^(3/2), x)
```

3.583 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal. Leaf size=248

$$-\frac{3x(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-ax}}-\frac{13x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-ax}}-\frac{45\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{8a^{5/2}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{5/2}\sqrt{1-ax}}-\frac{x^2}{}$$

```
[Out] (-13*Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(8*a^2*Sqrt[1 - a*x]) - (3*Sqrt[c -
 c/(a*x)]*x*(1 + a*x)^(3/2))/(4*a^2*Sqrt[1 - a*x]) - (Sqrt[c - c/(a*x)]*x^2
*(1 + a*x)^(3/2))/(3*a*Sqrt[1 - a*x]) - (45*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSi
nh[Sqrt[a]*Sqrt[x]])/(8*a^(5/2)*Sqrt[1 - a*x]) + (4*Sqrt[2]*Sqrt[c - c/(a*x
)]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a^(5/2)*Sqrt[
1 - a*x])
```

Rubi [A] time = 0.292, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6134, 6129, 101, 154, 157, 54, 215, 93, 206}

$$-\frac{3x(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-ax}}-\frac{13x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-ax}}-\frac{45\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{8a^{5/2}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{5/2}\sqrt{1-ax}}-\frac{x^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (-13*Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(8*a^2*Sqrt[1 - a*x]) - (3*Sqrt[c -
 c/(a*x)]*x*(1 + a*x)^(3/2))/(4*a^2*Sqrt[1 - a*x]) - (Sqrt[c - c/(a*x)]*x^2
*(1 + a*x)^(3/2))/(3*a*Sqrt[1 - a*x]) - (45*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSi
nh[Sqrt[a]*Sqrt[x]])/(8*a^(5/2)*Sqrt[1 - a*x]) + (4*Sqrt[2]*Sqrt[c - c/(a*x
)]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a^(5/2)*Sqrt[
1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
```

```
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{3\tanh^{-1}(ax)}x^{3/2}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$

$$=\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{x^{3/2}(1+ax)^{3/2}}{1-ax}dx}{\sqrt{1-ax}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}x^2(1+ax)^{3/2}}{3a\sqrt{1-ax}}+\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{\sqrt{x}\sqrt{1+ax}\left(\frac{3}{2}+\frac{9ax}{2}\right)}{1-ax}dx}{3a\sqrt{1-ax}}$$

$$=-\frac{3\sqrt{c-\frac{c}{ax}}x(1+ax)^{3/2}}{4a^2\sqrt{1-ax}}-\frac{\sqrt{c-\frac{c}{ax}}x^2(1+ax)^{3/2}}{3a\sqrt{1-ax}}-\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{\sqrt{1+ax}\left(-\frac{9a}{4}-\frac{39a^2x}{4}\right)}{\sqrt{x}(1-ax)}dx}{6a^3\sqrt{1-ax}}$$

$$=-\frac{13\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{8a^2\sqrt{1-ax}}-\frac{3\sqrt{c-\frac{c}{ax}}x(1+ax)^{3/2}}{4a^2\sqrt{1-ax}}-\frac{\sqrt{c-\frac{c}{ax}}x^2(1+ax)^{3/2}}{3a\sqrt{1-ax}}+\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int}{6a^4\sqrt{1}}$$

$$=-\frac{13\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{8a^2\sqrt{1-ax}}-\frac{3\sqrt{c-\frac{c}{ax}}x(1+ax)^{3/2}}{4a^2\sqrt{1-ax}}-\frac{\sqrt{c-\frac{c}{ax}}x^2(1+ax)^{3/2}}{3a\sqrt{1-ax}}-\frac{\left(45\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)}{16a^2\sqrt{1}}$$

$$=-\frac{13\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{8a^2\sqrt{1-ax}}-\frac{3\sqrt{c-\frac{c}{ax}}x(1+ax)^{3/2}}{4a^2\sqrt{1-ax}}-\frac{\sqrt{c-\frac{c}{ax}}x^2(1+ax)^{3/2}}{3a\sqrt{1-ax}}-\frac{\left(45\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)}{8}$$

$$=-\frac{13\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{8a^2\sqrt{1-ax}}-\frac{3\sqrt{c-\frac{c}{ax}}x(1+ax)^{3/2}}{4a^2\sqrt{1-ax}}-\frac{\sqrt{c-\frac{c}{ax}}x^2(1+ax)^{3/2}}{3a\sqrt{1-ax}}-\frac{45\sqrt{c-\frac{c}{ax}}\sqrt{x}\sin}{8a^{5/2}\sqrt{1}}$$

Mathematica [A] time = 0.0831088, size = 122, normalized size = 0.49

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}x\sqrt{ax+1}\left(8a^2x^2+26ax+57\right)+135\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-96\sqrt{2}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)\right)}{24a^{5/2}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*(Sqrt[a]*x*Sqrt[1 + a*x]*(57 + 26*a*x + 8*a^2*x^2) + 13
5*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]] - 96*Sqrt[2]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sq
rt[a]*Sqrt[x])/Sqrt[1 + a*x]]))/(24*a^(5/2)*Sqrt[1 - a*x])
```

Maple [A] time = 0.138, size = 219, normalized size = 0.9

$$\frac{x\sqrt{2}}{96\,ax-96}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(16\,\sqrt{-(ax+1)\,x}a^{7/2}\sqrt{2}\sqrt{-a^{-1}}x^2+52\,\sqrt{-(ax+1)\,x}a^{5/2}\sqrt{2}\sqrt{-a^{-1}}x+114\,\sqrt{-(ax+1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/96*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(16*(-(a*x+1)*x)^(1/2)*a^(7
/2)*2^(1/2)*(-1/a)^(1/2)*x^2+52*(-(a*x+1)*x)^(1/2)*a^(5/2)*2^(1/2)*(-1/a)^(
1/2)*x+114*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2)-135*arctan(1/2/a
^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)^(1/2)+192*ln((2*2^(1/
```

```
2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*a^(1/2))*2^(1/2)/a^(
7/2)/(a*x-1)/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}x^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))*x^2/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 2.69645, size = 1073, normalized size = 4.33

$$\left[\frac{96\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx+4\sqrt{2}(3a^2x^2+ax)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+135(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4}{}\right.}{96(a^4x-a^3)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="fr
icas")
```

```
[Out] [1/96*(96*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*
a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x
 - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 135*(a*x - 1)*sqrt(-
c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqr
t(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(8*a^3*x^3 + 26*a^2*x^2 +
 57*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), -1/48*(
96*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*
x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - 135*(a*x - 1)*sqrt
(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*
c*x^2 - a*c*x - c)) - 2*(8*a^3*x^3 + 26*a^2*x^2 + 57*a*x)*sqrt(-a^2*x^2 + 1
)*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**
(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))*x^2/(-a^2*x^2 + 1)^(3/2), x)
```

3.584 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal. Leaf size=204

$$-\frac{23\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{3/2}\sqrt{1-ax}}-\frac{x(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{2a\sqrt{1-ax}}-\frac{7x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-ax}}$$

```
[Out] (-7*Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(4*a*Sqrt[1 - a*x]) - (Sqrt[c - c/(a
*x)]*x*(1 + a*x)^(3/2))/(2*a*Sqrt[1 - a*x]) - (23*Sqrt[c - c/(a*x)]*Sqrt[x]
*ArcSinh[Sqrt[a]*Sqrt[x]])/(4*a^(3/2)*Sqrt[1 - a*x]) + (4*Sqrt[2]*Sqrt[c -
c/(a*x)]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a^(3/2)
*Sqrt[1 - a*x])
```

Rubi [A] time = 0.227905, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {6134, 6129, 101, 154, 157, 54, 215, 93, 206}

$$-\frac{23\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{3/2}\sqrt{1-ax}}-\frac{x(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{2a\sqrt{1-ax}}-\frac{7x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (-7*Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(4*a*Sqrt[1 - a*x]) - (Sqrt[c - c/(a
*x)]*x*(1 + a*x)^(3/2))/(2*a*Sqrt[1 - a*x]) - (23*Sqrt[c - c/(a*x)]*Sqrt[x]
*ArcSinh[Sqrt[a]*Sqrt[x]])/(4*a^(3/2)*Sqrt[1 - a*x]) + (4*Sqrt[2]*Sqrt[c -
c/(a*x)]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a^(3/2)
*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] && !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{3\tanh^{-1}(ax)}\sqrt{x}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}(1+ax)^{3/2}}{1-ax}dx}{\sqrt{1-ax}}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}x(1+ax)^{3/2}}{2a\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}\left(\frac{1}{2}+\frac{7ax}{2}\right)}{\sqrt{x}(1-ax)}dx}{2a\sqrt{1-ax}}$$
$$= -\frac{7\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{4a\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}x(1+ax)^{3/2}}{2a\sqrt{1-ax}} - \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-\frac{9a}{4}-\frac{23a^2x}{4}}{\sqrt{x}(1-ax)\sqrt{1+ax}}dx}{2a^2\sqrt{1-ax}}$$
$$= -\frac{7\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{4a\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}x(1+ax)^{3/2}}{2a\sqrt{1-ax}} - \frac{\left(23\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}dx}{8a\sqrt{1-ax}} + \frac{\left(4\sqrt{c-\right.}{}$$
$$= -\frac{7\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{4a\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}x(1+ax)^{3/2}}{2a\sqrt{1-ax}} - \frac{\left(23\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}dx, x, \right.}{4a\sqrt{1-ax}}$$
$$= -\frac{7\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{4a\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}x(1+ax)^{3/2}}{2a\sqrt{1-ax}} - \frac{23\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}} + \frac{4\sqrt{2}\sqrt{c}}{}$$

Mathematica [A] time = 0.072024, size = 114, normalized size = 0.56

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}x\sqrt{ax+1}(2ax+9)+23\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-16\sqrt{2}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)\right)}{4a^{3/2}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*(Sqrt[a]*x*Sqrt[1 + a*x]*(9 + 2*a*x) + 23*Sqrt[x]*ArcSi
nh[Sqrt[a]*Sqrt[x]] - 16*Sqrt[2]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/
Sqrt[1 + a*x]]))/(4*a^(3/2)*Sqrt[1 - a*x])
```

Maple [A] time = 0.138, size = 191, normalized size = 0.9

$$\frac{x\sqrt{2}}{16\,ax-16}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(4\,\sqrt{-(ax+1)\,xa}^{5/2}\sqrt{2}\sqrt{-a^{-1}}x+18\,\sqrt{-(ax+1)\,xa}^{3/2}\sqrt{2}\sqrt{-a^{-1}}-23\,\arctan\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/16*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(4*(-(a*x+1)*x)^(1/2)*a^(5/
2)*2^(1/2)*(-1/a)^(1/2)*x+18*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2
)-23*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)^(1/2
)+32*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*a^(1
/2))*2^(1/2)/a^(5/2)/(a*x-1)/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))*x/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="fric
as")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/
2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))*x/(-a^2*x^2 + 1)^(3/2), x)
```

3.585 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=155

$$-\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{5\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{a}\sqrt{1-ax}}$$

```
[Out] -((Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/Sqrt[1 - a*x]) - (5*Sqrt[c - c/(a*x)]
*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x]) + (4*Sqrt[2]*Sqr
t[c - c/(a*x)]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(S
qrt[a]*Sqrt[1 - a*x])
```

Rubi [A] time = 0.163309, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6134, 6129, 102, 157, 54, 215, 93, 206}

$$-\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{5\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/Sqrt[1 - a*x]) - (5*Sqrt[c - c/(a*x)]
*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x]) + (4*Sqrt[2]*Sqr
t[c - c/(a*x)]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(S
qrt[a]*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{e^{3\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{(1+ax)^{3/2}}{\sqrt{x}(1-ax)}\,dx}{\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{-\frac{3a}{2}-\frac{5a^2x}{2}}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{\left(5\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}}+\frac{\left(4\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{1}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{\left(5\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{\sqrt{1-ax}}+\frac{\left(8\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int\right.}{\sqrt{1-}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{5\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right)}{\sqrt{a}\sqrt{1-ax}}
\end{aligned}$$

Mathematica [A] time = 0.0646731, size = 105, normalized size = 0.68

$$-\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}+5\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x] + 5*ArcSinh[Sqr
t[a]*Sqrt[x]] - 4*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])
)/(Sqrt[a]*Sqrt[1 - a*x]))
```

Maple [A] time = 0.142, size = 165, normalized size = 1.1

$$\frac{x\sqrt{2}}{4\,ax-4}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(2\,\sqrt{-(ax+1)\,x}a^{3/2}\sqrt{2}\sqrt{-a^{-1}}-5\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)a\sqrt{2}\sqrt{-a^{-1}}+8\,\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/4*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(2*(-(a*x+1)*x)^(1/2)*a^(3/2
)*2^(1/2)*(-1/a)^(1/2)-5*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a
*2^(1/2)*(-1/a)^(1/2)+8*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a
*x-1)/(a*x-1))*a^(1/2))*2^(1/2)/(a*x-1)/(-(a*x+1)*x)^(1/2)/a^(3/2)/(-1/a)^(
1/2)
```

Maximma [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 2.99902, size = 975, normalized size = 6.29

$$\left[\frac{4\sqrt{-a^2x^2+1}ax\sqrt{\frac{acx-c}{ax}}+4\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx+4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+5\,(ax-1)\sqrt{}}{4\left(a^2x-a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) + 4*sqrt(2)*(a*x - 1
)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x + 4*sqrt(2)*(3*a^2*x
```

```
^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3
 - 3*a^2*x^2 + 3*a*x - 1)) + 5*(a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c
*x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)
) - c)/(a*x - 1)))/(a^2*x - a), 1/2*(2*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x -
 c)/(a*x)) - 4*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1
)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + 5*(a*x
 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x
))/(2*a^2*c*x^2 - a*c*x - c)))/(a^2*x - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2)
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/(-a^2*x^2 + 1)^(3/2), x)
```

3.586 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=154

$$-\frac{2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{2\sqrt{a}\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{a}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}$$

```
[Out] (-2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/Sqrt[1 - a*x] - (2*Sqrt[a]*Sqrt[c - c/
(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/Sqrt[1 - a*x] + (4*Sqrt[2]*Sqrt[a]
*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]
)/Sqrt[1 - a*x]
```

Rubi [A] time = 0.251498, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6134, 6129, 98, 157, 54, 215, 93, 206}

$$-\frac{2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{2\sqrt{a}\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{a}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x,x]
```


```
[Out] (-2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/Sqrt[1 - a*x] - (2*Sqrt[a]*Sqrt[c - c/
(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/Sqrt[1 - a*x] + (4*Sqrt[2]*Sqrt[a]
*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]
)/Sqrt[1 - a*x]
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{3/2}}\,dx}{\sqrt{1-ax}}$$

$$=\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1+ax)^{3/2}}{x^{3/2}(1-ax)}\,dx}{\sqrt{1-ax}}$$

$$=-\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-\frac{3a}{2}-\frac{a^2x}{2}}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{\sqrt{1-ax}}$$

$$=-\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{\left(a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}}+\frac{\left(4a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{\sqrt{1-ax}}$$

$$=-\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{\left(2a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{\sqrt{1-ax}}+\frac{\left(8a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}}{\sqrt{1-}}$$

$$=-\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{2\sqrt{a}\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{a}\sqrt{c-\frac{c}{ax}}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right.}{\sqrt{1-ax}}$$

Mathematica [A] time = 0.0468401, size = 103, normalized size = 0.67

$$-\frac{2\sqrt{c-\frac{c}{ax}}\left(\sqrt{ax+1}+\sqrt{a}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-2\sqrt{2}\sqrt{a}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x] + Sqrt[a]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt
[x]] - 2*Sqrt[2]*Sqrt[a]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 +
 a*x]]))/Sqrt[1 - a*x]
```

Maple [A] time = 0.15, size = 166, normalized size = 1.1

$$\frac{\sqrt{2}}{2\,ax-2}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(-\arctan\left(\frac{2\,ax+1}{2}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)\,x}}\right)\sqrt{2}\sqrt{-a^{-1}}xa+2\,\sqrt{-(ax+1)\,x}\sqrt{a}\sqrt{2}\sqrt{-a^{-1}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x,x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*(-a^2*x^2+1)^(1/2)*(-arctan(1/2/a^(1/2)*(2*a*x+1)
/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)*x*a+2*(-(a*x+1)*x)^(1/2)*a^(1/2)*
2^(1/2)*(-1/a)^(1/2)+4*a^(1/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2
)*a-3*a*x-1)/(a*x-1))*x)*2^(1/2)/(a*x-1)/(-(a*x+1)*x)^(1/2)/a^(1/2)/(-1/a)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Fricas [A] time = 2.95535, size = 949, normalized size = 6.16

$$\left[\frac{2\,\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx+4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+(ax-1)\sqrt{-c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^2x^2\right.}{}\right.}{2\,(ax-1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="fric
as")
```

```
[Out] [1/2*(2*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*
c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x -
```

```
c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + (a*x - 1)*sqrt(-c)*log
(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*
sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x
- c)/(a*x)))/(a*x - 1), -(2*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(2*sqrt(2)*sqrt
(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x -
 c)) - (a*x - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*
x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x
 - c)/(a*x)))/(a*x - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{x\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1))**(3
/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/((-a^2*x^2 + 1)^(3/2)*x), x)
```

3.587 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=147

$$\frac{4\sqrt{2}a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}-\frac{4a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}}$$

```
[Out] (-4*a*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/Sqrt[1 - a*x] - (2*Sqrt[c - c/(a*x)]
*(1 + a*x)^(3/2))/(3*x*Sqrt[1 - a*x]) + (4*Sqrt[2]*a^(3/2)*Sqrt[c - c/(a*x)
]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/Sqrt[1 - a*x]
```

Rubi [A] time = 0.245573, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6134, 6129, 94, 93, 206}

$$\frac{4\sqrt{2}a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}-\frac{4a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```


```
[Out] (-4*a*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/Sqrt[1 - a*x] - (2*Sqrt[c - c/(a*x)]
*(1 + a*x)^(3/2))/(3*x*Sqrt[1 - a*x]) + (4*Sqrt[2]*a^(3/2)*Sqrt[c - c/(a*x)
]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/Sqrt[1 - a*x]
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{5/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1+ax)^{3/2}}{x^{5/2}(1-ax)}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{3x\sqrt{1-ax}}+\frac{\left(2a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{3/2}(1-ax)}\,dx}{\sqrt{1-ax}}\\
&= -\frac{4a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{3x\sqrt{1-ax}}+\frac{\left(4a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{4a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{3x\sqrt{1-ax}}+\frac{\left(8a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-2ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{1+ax}}\right)}{\sqrt{1-ax}}\\
&= -\frac{4a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{3x\sqrt{1-ax}}+\frac{4\sqrt{2}a^{3/2}\sqrt{c-\frac{c}{ax}}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right)}{\sqrt{1-ax}}
\end{aligned}$$

Mathematica [A] time = 0.0538605, size = 93, normalized size = 0.63

$$\frac{2\sqrt{c-\frac{c}{ax}}\left(6\sqrt{2}a^{3/2}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)-\sqrt{ax+1}(7ax+1)\right)}{3x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(-(Sqrt[1 + a*x]*(1 + 7*a*x)) + 6*Sqrt[2]*a^(3/2)*x^(3
/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]))/(3*x*Sqrt[1 - a*x])
```

Maple [A] time = 0.154, size = 150, normalized size = 1.

$$\frac{\sqrt{2}}{3\,(ax-1)\,x}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(7\,x\sqrt{-(ax+1)\,xa}\sqrt{2}\sqrt{-a^{-1}}+6\,a\ln\left(\frac{1}{ax-1}\left(2\,\sqrt{2}\sqrt{-a^{-1}}\sqrt{-(ax+1)\,xa}-3\,ax-1\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^2,x)
```

```
[Out] 1/3*(c*(a*x-1)/a/x)^(1/2)/x*(-a^2*x^2+1)^(1/2)*(7*x*(-(a*x+1)*x)^(1/2)*a*2^
(1/2)*(-1/a)^(1/2)+6*a*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*
```

```
x-1)/(a*x-1))*x^2+(-(a*x+1)*x)^(1/2)*2^(1/2)*(-1/a)^(1/2))*2^(1/2)/(a*x-1)/
(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

Fricas [A] time = 2.59657, size = 668, normalized size = 4.54

$$\left[\frac{3\sqrt{2}\left(a^2x^2-ax\right)\sqrt{-c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx+4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\,\sqrt{-a^2x^2+1}(7\,ax+1)\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fr
icas")
```

```
[Out] [1/3*(3*sqrt(2)*(a^2*x^2 - a*x)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 -
 13*a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a
*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 2*sqrt(-a^2*x^2
+ 1)*(7*a*x + 1)*sqrt((a*c*x - c)/(a*x)))/(a*x^2 - x), -2/3*(3*sqrt(2)*(a^2
*x^2 - a*x)*sqrt(c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a
*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - sqrt(-a^2*x^2 + 1)*(7*a*x +
 1)*sqrt((a*c*x - c)/(a*x)))/(a*x^2 - x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{x^2\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(x**2*(-(a*x - 1)*(a*x + 1))*
*(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3 \sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

3.588 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=191

$$-\frac{4a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{4\sqrt{2}a^{5/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}-\frac{2(ax+1)^{5/2}\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}}-\frac{2a(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}}$$

```
[Out] (-4*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/Sqrt[1 - a*x] - (2*a*Sqrt[c - c/(a
*x)]*(1 + a*x)^(3/2))/(3*x*Sqrt[1 - a*x]) - (2*Sqrt[c - c/(a*x)]*(1 + a*x)^
(5/2))/(5*x^2*Sqrt[1 - a*x]) + (4*Sqrt[2]*a^(5/2)*Sqrt[c - c/(a*x)]*Sqrt[x]
*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/Sqrt[1 - a*x]
```

Rubi [A] time = 0.279413, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6134, 6129, 96, 94, 93, 206}

$$-\frac{4a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{4\sqrt{2}a^{5/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}-\frac{2(ax+1)^{5/2}\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}}-\frac{2a(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (-4*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/Sqrt[1 - a*x] - (2*a*Sqrt[c - c/(a
*x)]*(1 + a*x)^(3/2))/(3*x*Sqrt[1 - a*x]) - (2*Sqrt[c - c/(a*x)]*(1 + a*x)^
(5/2))/(5*x^2*Sqrt[1 - a*x]) + (4*Sqrt[2]*a^(5/2)*Sqrt[c - c/(a*x)]*Sqrt[x]
*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/Sqrt[1 - a*x]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{7/2}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1+ax)^{3/2}}{x^{7/2}(1-ax)}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{5/2}}{5x^2\sqrt{1-ax}} + \frac{\left(a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1+ax)^{3/2}}{x^{5/2}(1-ax)}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{2a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{3x\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{5/2}}{5x^2\sqrt{1-ax}} + \frac{\left(2a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{3/2}(1-ax)}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{4a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{2a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{3x\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{5/2}}{5x^2\sqrt{1-ax}} + \frac{\left(4a^3\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int -}{\sqrt{1-a}}$$

$$= -\frac{4a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{2a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{3x\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{5/2}}{5x^2\sqrt{1-ax}} + \frac{\left(8a^3\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\mathrm{Su}}{}$$

$$= -\frac{4a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{2a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{3x\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{5/2}}{5x^2\sqrt{1-ax}} + \frac{4\sqrt{2}a^{5/2}\sqrt{c-\frac{c}{ax}}\sqrt{x}\,\mathrm{t}}{\sqrt{1-}}$$

Mathematica [A] time = 0.0599593, size = 101, normalized size = 0.53

$$\frac{2\sqrt{c-\frac{c}{ax}}\left(30\sqrt{2}a^{5/2}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)-\sqrt{ax+1}\left(38a^2x^2+11ax+3\right)\right)}{15x^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(-(Sqrt[1 + a*x]*(3 + 11*a*x + 38*a^2*x^2)) + 30*Sqrt[
2]*a^(5/2)*x^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]))/(15*x
```

```
^2*Sqrt[1 - a*x])
```

Maple [A] time = 0.145, size = 181, normalized size = 1.

$$\frac{\sqrt{2}}{15\,x^{2}\left(ax-1\right)}\sqrt{\frac{c\left(ax-1\right)}{ax}}\sqrt{-a^{2}x^{2}+1}\left(38\,a^{2}\sqrt{2}\sqrt{-a^{-1}}x^{2}\sqrt{-\left(ax+1\right)x}+30\,a^{2}\ln\left(\frac{1}{ax-1}\left(2\,\sqrt{2}\sqrt{-a^{-1}}\sqrt{-\left(ax+1\right)x}a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^3,x)
```

```
[Out] 1/15*(c*(a*x-1)/a/x)^(1/2)/x^2*(-a^2*x^2+1)^(1/2)*(38*a^2*2^(1/2)*(-1/a)^(1
/2)*x^2*(-(a*x+1)*x)^(1/2)+30*a^2*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(
1/2)*a-3*a*x-1)/(a*x-1))*x^3+11*x*(-(a*x+1)*x)^(1/2)*a*2^(1/2)*(-1/a)^(1/2)
+3*(-(a*x+1)*x)^(1/2)*2^(1/2)*(-1/a)^(1/2))*2^(1/2)/(a*x-1)/(-(a*x+1)*x)^(1
/2)/(-1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax+1\right)^{3}\sqrt{c-\frac{c}{ax}}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Fricas [A] time = 2.57177, size = 728, normalized size = 3.81

$$\left[\frac{15\,\sqrt{2}\left(a^{3}x^{3}-a^{2}x^{2}\right)\sqrt{-c}\log\left(-\frac{17\,a^{3}cx^{3}-3\,a^{2}cx^{2}-13\,acx+4\,\sqrt{2}\left(3\,a^{2}x^{2}+ax\right)\sqrt{-a^{2}x^{2}+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1}\right)+2\left(38\,a^{2}x^{2}+11\,ax+3\right)\sqrt{-a^{2}x^{2}}}{15\left(ax^{3}-x^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="fr
icas")
```

```
[Out] [1/15*(15*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c
*x^2 - 13*a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*s
qrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 2*(38*a^2*
x^2 + 11*a*x + 3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a*x^3 - x^2)
, -2/15*(15*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(c)*arctan(2*sqrt(2)*sqrt(-a^2*
x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) -
 (38*a^2*x^2 + 11*a*x + 3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a*x
^3 - x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{x^3\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(x**3*(-(a*x - 1)*(a*x + 1))*
*(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

3.589 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=237

$$-\frac{104a^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{21\sqrt{1-ax}}-\frac{32a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{21x\sqrt{1-ax}}+\frac{4\sqrt{2}a^{7/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}-\frac{6a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{7x^2\sqrt{1-ax}}-\frac{2\sqrt{a}}{7}$$

```
[Out] (-104*a^3*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(21*Sqrt[1 - a*x]) - (2*Sqrt[c -
 c/(a*x)]*Sqrt[1 + a*x])/(7*x^3*Sqrt[1 - a*x]) - (6*a*Sqrt[c - c/(a*x)]*Sqr
t[1 + a*x])/(7*x^2*Sqrt[1 - a*x]) - (32*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x]
)/(21*x*Sqrt[1 - a*x]) + (4*Sqrt[2]*a^(7/2)*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcTa
nh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/Sqrt[1 - a*x]
```

Rubi [A] time = 0.277296, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6134, 6129, 98, 152, 12, 93, 206}

$$-\frac{104a^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{21\sqrt{1-ax}}-\frac{32a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{21x\sqrt{1-ax}}+\frac{4\sqrt{2}a^{7/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}-\frac{6a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{7x^2\sqrt{1-ax}}-\frac{2\sqrt{a}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (-104*a^3*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(21*Sqrt[1 - a*x]) - (2*Sqrt[c -
 c/(a*x)]*Sqrt[1 + a*x])/(7*x^3*Sqrt[1 - a*x]) - (6*a*Sqrt[c - c/(a*x)]*Sqr
t[1 + a*x])/(7*x^2*Sqrt[1 - a*x]) - (32*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x]
)/(21*x*Sqrt[1 - a*x]) + (4*Sqrt[2]*a^(7/2)*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcTa
nh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/Sqrt[1 - a*x]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{9/2}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1+ax)^{3/2}}{x^{9/2}(1-ax)}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^3\sqrt{1-ax}} - \frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-\frac{15a}{2}-\frac{13a^2x}{2}}{x^{7/2}(1-ax)\sqrt{1+ax}}\,dx}{7\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^3\sqrt{1-ax}} - \frac{6a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^2\sqrt{1-ax}} + \frac{\left(4\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{20a^2+15a^3x}{x^{5/2}(1-ax)\sqrt{1+ax}}\,dx}{35\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^3\sqrt{1-ax}} - \frac{6a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^2\sqrt{1-ax}} - \frac{32a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{21x\sqrt{1-ax}} - \frac{\left(8\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-}{x^{3/2}}}{105\sqrt{1-}}$$
$$= -\frac{104a^3\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{21\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^3\sqrt{1-ax}} - \frac{6a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^2\sqrt{1-ax}} - \frac{32a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+a}}{21x\sqrt{1-ax}}$$
$$= -\frac{104a^3\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{21\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^3\sqrt{1-ax}} - \frac{6a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^2\sqrt{1-ax}} - \frac{32a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+a}}{21x\sqrt{1-ax}}$$
$$= -\frac{104a^3\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{21\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^3\sqrt{1-ax}} - \frac{6a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^2\sqrt{1-ax}} - \frac{32a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+a}}{21x\sqrt{1-ax}}$$
$$= -\frac{104a^3\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{21\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^3\sqrt{1-ax}} - \frac{6a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^2\sqrt{1-ax}} - \frac{32a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+a}}{21x\sqrt{1-ax}}$$

Mathematica [A] time = 0.0709776, size = 109, normalized size = 0.46

$$\frac{2\sqrt{c-\frac{c}{ax}}\left(42\sqrt{2}a^{7/2}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)-\sqrt{ax+1}\left(52a^3x^3+16a^2x^2+9ax+3\right)\right)}{21x^3\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(-(Sqrt[1 + a*x]*(3 + 9*a*x + 16*a^2*x^2 + 52*a^3*x^3)
) + 42*Sqrt[2]*a^(7/2)*x^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a
*x]]))/(21*x^3*Sqrt[1 - a*x])
```

Maple [A] time = 0.145, size = 209, normalized size = 0.9

$$\frac{\sqrt{2}}{21\,x^3\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(52\,a^3\sqrt{2}\sqrt{-a^{-1}}x^3\sqrt{-(ax+1)\,x}+42\,a^3\ln\left(\frac{1}{ax-1}\left(2\,\sqrt{2}\sqrt{-a^{-1}}\sqrt{-(ax+1)\,x}a-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^4,x)
```

```
[Out] 1/21*(c*(a*x-1)/a/x)^(1/2)/x^3*(-a^2*x^2+1)^(1/2)*(52*a^3*2^(1/2)*(-1/a)^(1
/2)*x^3*(-(a*x+1)*x)^(1/2)+42*a^3*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(
```

```
1/2)*a-3*a*x-1)/(a*x-1))*x^4+16*a^2*2^(1/2)*(-1/a)^(1/2)*x^2*(-(a*x+1)*x)^(
1/2)+9*x*(-(a*x+1)*x)^(1/2)*a*2^(1/2)*(-1/a)^(1/2)+3*(-(a*x+1)*x)^(1/2)*2^(
1/2)*(-1/a)^(1/2))*2^(1/2)/(a*x-1)/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/((-a^2*x^2 + 1)^(3/2)*x^4), x)
```

Fricas [A] time = 2.51777, size = 760, normalized size = 3.21

$$\left[\frac{21\sqrt{2}\left(a^4x^4-a^3x^3\right)\sqrt{-c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx+4\sqrt{2}\left(3a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+2\left(52a^3x^3+16a^2x^2+9ax+3\right)\sqrt{}}{21\left(ax^4-x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="fr
icas")
```

```
[Out] [1/21*(21*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c
*x^2 - 13*a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*s
qrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 2*(52*a^3*
x^3 + 16*a^2*x^2 + 9*a*x + 3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(
a*x^4 - x^3), -2/21*(21*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(c)*arctan(2*sqrt(2
)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a
*c*x - c)) - (52*a^3*x^3 + 16*a^2*x^2 + 9*a*x + 3)*sqrt(-a^2*x^2 + 1)*sqrt(
(a*c*x - c)/(a*x)))/(a*x^4 - x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{x^4\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(x**4*(-(a*x - 1)*(a*x + 1))*
*(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/((-a^2*x^2 + 1)^(3/2)*x^4), x)
```

3.590 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=281

$$-\frac{92a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105x^2\sqrt{1-ax}}-\frac{1576a^4\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{315\sqrt{1-ax}}-\frac{472a^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{315x\sqrt{1-ax}}+\frac{4\sqrt{2}a^{9/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}-38$$

```
[Out] (-1576*a^4*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(315*Sqrt[1 - a*x]) - (2*Sqrt[c
 - c/(a*x)]*Sqrt[1 + a*x])/(9*x^4*Sqrt[1 - a*x]) - (38*a*Sqrt[c - c/(a*x)]*
Sqrt[1 + a*x])/(63*x^3*Sqrt[1 - a*x]) - (92*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 +
a*x])/(105*x^2*Sqrt[1 - a*x]) - (472*a^3*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(
315*x*Sqrt[1 - a*x]) + (4*Sqrt[2]*a^(9/2)*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcTanh
[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/Sqrt[1 - a*x]
```

Rubi [A] time = 0.306778, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6134, 6129, 98, 152, 12, 93, 206}

$$-\frac{92a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105x^2\sqrt{1-ax}}-\frac{1576a^4\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{315\sqrt{1-ax}}-\frac{472a^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{315x\sqrt{1-ax}}+\frac{4\sqrt{2}a^{9/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}-38$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```


```
[Out] (-1576*a^4*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(315*Sqrt[1 - a*x]) - (2*Sqrt[c
 - c/(a*x)]*Sqrt[1 + a*x])/(9*x^4*Sqrt[1 - a*x]) - (38*a*Sqrt[c - c/(a*x)]*
Sqrt[1 + a*x])/(63*x^3*Sqrt[1 - a*x]) - (92*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 +
a*x])/(105*x^2*Sqrt[1 - a*x]) - (472*a^3*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(
315*x*Sqrt[1 - a*x]) + (4*Sqrt[2]*a^(9/2)*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcTanh
[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/Sqrt[1 - a*x]
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{11/2}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1+ax)^{3/2}}{x^{11/2}(1-ax)}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{9x^4\sqrt{1-ax}} - \frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-\frac{19a}{2}-\frac{17a^2x}{2}}{x^{9/2}(1-ax)\sqrt{1+ax}}\,dx}{9\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{9x^4\sqrt{1-ax}} - \frac{38a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{63x^3\sqrt{1-ax}} + \frac{\left(4\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\frac{69a^2}{2}+\frac{57a^3x}{2}}{x^{7/2}(1-ax)\sqrt{1+ax}}\,dx}{63\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{9x^4\sqrt{1-ax}} - \frac{38a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{63x^3\sqrt{1-ax}} - \frac{92a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{105x^2\sqrt{1-ax}} - \frac{\left(8\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-\frac{177}{2}}{x^{5/2}(1-}}{315\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{9x^4\sqrt{1-ax}} - \frac{38a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{63x^3\sqrt{1-ax}} - \frac{92a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{105x^2\sqrt{1-ax}} - \frac{472a^3\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{315x\sqrt{1-ax}}$$
$$= -\frac{1576a^4\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{315\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{9x^4\sqrt{1-ax}} - \frac{38a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{63x^3\sqrt{1-ax}} - \frac{92a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+a}}{105x^2\sqrt{1-ax}}$$
$$= -\frac{1576a^4\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{315\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{9x^4\sqrt{1-ax}} - \frac{38a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{63x^3\sqrt{1-ax}} - \frac{92a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+a}}{105x^2\sqrt{1-ax}}$$
$$= -\frac{1576a^4\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{315\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{9x^4\sqrt{1-ax}} - \frac{38a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{63x^3\sqrt{1-ax}} - \frac{92a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+a}}{105x^2\sqrt{1-ax}}$$
$$= -\frac{1576a^4\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{315\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{9x^4\sqrt{1-ax}} - \frac{38a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{63x^3\sqrt{1-ax}} - \frac{92a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+a}}{105x^2\sqrt{1-ax}}$$

Mathematica [A] time = 0.0778593, size = 117, normalized size = 0.42

$$\frac{2\sqrt{c-\frac{c}{ax}}\left(630\sqrt{2}a^{9/2}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)-\sqrt{ax+1}\left(788a^4x^4+236a^3x^3+138a^2x^2+95ax+35\right)\right)}{315x^4\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(-(Sqrt[1 + a*x]*(35 + 95*a*x + 138*a^2*x^2 + 236*a^3*
x^3 + 788*a^4*x^4)) + 630*Sqrt[2]*a^(9/2)*x^(9/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*
Sqrt[x])/Sqrt[1 + a*x]]))/(315*x^4*Sqrt[1 - a*x])
```

Maple [A] time = 0.168, size = 237, normalized size = 0.8

$$\frac{\sqrt{2}}{315\,x^4\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(788\,a^4\sqrt{2}\sqrt{-a^{-1}}x^4\sqrt{-(ax+1)\,x}+630\,a^4\ln\left(\frac{1}{ax-1}\left(2\sqrt{2}\sqrt{-a^{-1}}\sqrt{-(ax+1)\,x}a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^5,x)
```

```
[Out] 1/315*(c*(a*x-1)/a/x)^(1/2)/x^4*(-a^2*x^2+1)^(1/2)*(788*a^4*2^(1/2)*(-1/a)^
(1/2)*x^4*(-(a*x+1)*x)^(1/2)+630*a^4*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x
)^(1/2)*a-3*a*x-1)/(a*x-1))*x^5+236*a^3*2^(1/2)*(-1/a)^(1/2)*x^3*(-(a*x+1)*
x)^(1/2)+138*a^2*2^(1/2)*(-1/a)^(1/2)*x^2*(-(a*x+1)*x)^(1/2)+95*x*(-(a*x+1)
*x)^(1/2)*a*2^(1/2)*(-1/a)^(1/2)+35*(-(a*x+1)*x)^(1/2)*2^(1/2)*(-1/a)^(1/2)
)*2^(1/2)/(a*x-1)/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/((-a^2*x^2 + 1)^(3/2)*x^5), x)
```

Fricas [A] time = 2.66007, size = 814, normalized size = 2.9

$$\left[\frac{315\sqrt{2}\left(a^5x^5-a^4x^4\right)\sqrt{-c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx+4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\left(788\,a^4x^4+236\,a^3x^3+138\right.}{315\left(ax^5-x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="fr
icas")
```

```
[Out] [1/315*(315*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2
*c*x^2 - 13*a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)
*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 2*(788*a
^4*x^4 + 236*a^3*x^3 + 138*a^2*x^2 + 95*a*x + 35)*sqrt(-a^2*x^2 + 1)*sqrt((
a*c*x - c)/(a*x)))/(a*x^5 - x^4), -2/315*(315*sqrt(2)*(a^5*x^5 - a^4*x^4)*s
qrt(c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - (788*a^4*x^4 + 236*a^3*x^3 + 138*a^2*x^2
 + 95*a*x + 35)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a*x^5 - x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{x^5\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(x**5*(-(a*x - 1)*(a*x + 1))*
*(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/((-a^2*x^2 + 1)^(3/2)*x^5), x)
```

3.591 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx$

Optimal. Leaf size=116

$$\frac{(4m+3)x^{m+1}\sqrt{c-\frac{c}{ax}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},m+\frac{1}{2},m+\frac{3}{2},-ax\right)}{(m+1)(2m+1)\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\,x^{m+1}\sqrt{c-\frac{c}{ax}}}{(m+1)(1-ax)}$$

```
[Out] -((Sqrt[c - c/(a*x)]*x^(1 + m)*Sqrt[1 - a^2*x^2])/((1 + m)*(1 - a*x))) + ((
3 + 4*m)*Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[1/2, 1/2 + m, 3/2 +
m, -(a*x)])/((1 + m)*(1 + 2*m)*Sqrt[1 - a*x])
```

Rubi [A] time = 0.293454, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6134, 6128, 881, 848, 64}

$$\frac{(4m+3)x^{m+1}\sqrt{c-\frac{c}{ax}}\,{}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};-ax\right)}{(m+1)(2m+1)\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\,x^{m+1}\sqrt{c-\frac{c}{ax}}}{(m+1)(1-ax)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^m)/E^ArcTanh[a*x],x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x^(1 + m)*Sqrt[1 - a^2*x^2])/((1 + m)*(1 - a*x))) + ((
3 + 4*m)*Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[1/2, 1/2 + m, 3/2 +
m, -(a*x)])/((1 + m)*(1 + 2*m)*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] && !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] && !IntegerQ[p] && EqQ[m + p - 1, 0] && !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
```

```
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{-\tanh^{-1}(ax)}x^{-\frac{1}{2}+m}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{-\frac{1}{2}+m}(1-ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x^{1+m}\sqrt{1-a^2x^2}}{(1+m)(1-ax)}+\frac{\left((3+4m)\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{-\frac{1}{2}+m}\sqrt{1-ax}}{\sqrt{1-a^2x^2}}\,dx}{2(1+m)\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x^{1+m}\sqrt{1-a^2x^2}}{(1+m)(1-ax)}+\frac{\left((3+4m)\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{-\frac{1}{2}+m}}{\sqrt{1+ax}}\,dx}{2(1+m)\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x^{1+m}\sqrt{1-a^2x^2}}{(1+m)(1-ax)}+\frac{(3+4m)\sqrt{c-\frac{c}{ax}}x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+m;\frac{3}{2}+m;-ax\right)}{(1+m)(1+2m)\sqrt{1-ax}}
\end{aligned}$$

Mathematica [A] time = 0.0561967, size = 87, normalized size = 0.75

$$-\frac{2x^{m+1}\sqrt{c-\frac{c}{ax}}\left(a(4m+3)x\text{Hypergeometric2F1}\left(\frac{1}{2},m+\frac{3}{2},m+\frac{5}{2},-ax\right)-(2m+3)\sqrt{ax+1}\right)}{\left(4m^2+8m+3\right)\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^m)/E^ArcTanh[a*x],x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*x^(1 + m)*(-((3 + 2*m)*Sqrt[1 + a*x]) + a*(3 + 4*m)*x
*Hypergeometric2F1[1/2, 3/2 + m, 5/2 + m, -(a*x)]))/((3 + 8*m + 4*m^2)*Sqrt
[1 - a*x])
```

Maple [F] time = 0.381, size = 0, normalized size = 0.

$$\int \frac{x^m}{ax+1}\sqrt{c-\frac{c}{ax}}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))*x^m/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}x^m\sqrt{\frac{acx-c}{ax}}}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m*sqrt((a*c*x - c)/(a*x))/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{-c\left(-1+\frac{1}{ax}\right)}\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))*x^m/(a*x + 1), x)
```

3.592 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal. Leaf size=182

$$-\frac{x^3\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{3(1-ax)}-\frac{11x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-ax}}+\frac{11\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{8a^{5/2}\sqrt{1-ax}}+\frac{11x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{12a\sqrt{1-ax}}$$

```
[Out] (-11*Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(8*a^2*Sqrt[1 - a*x]) + (11*Sqrt[c
- c/(a*x)]*x^2*Sqrt[1 + a*x])/(12*a*Sqrt[1 - a*x]) - (Sqrt[c - c/(a*x)]*x^3
*Sqrt[1 - a^2*x^2])/(3*(1 - a*x)) + (11*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[S
qrt[a]*Sqrt[x]])/(8*a^(5/2)*Sqrt[1 - a*x])
```

Rubi [A] time = 0.307973, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6134, 6128, 881, 848, 50, 54, 215}

$$-\frac{x^3\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{3(1-ax)}-\frac{11x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-ax}}+\frac{11\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{8a^{5/2}\sqrt{1-ax}}+\frac{11x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{12a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^ArcTanh[a*x],x]
```

```
[Out] (-11*Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(8*a^2*Sqrt[1 - a*x]) + (11*Sqrt[c
- c/(a*x)]*x^2*Sqrt[1 + a*x])/(12*a*Sqrt[1 - a*x]) - (Sqrt[c - c/(a*x)]*x^3
*Sqrt[1 - a^2*x^2])/(3*(1 - a*x)) + (11*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[S
qrt[a]*Sqrt[x]])/(8*a^(5/2)*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{-\tanh^{-1}(ax)}x^{3/2}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}(1-ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1-a^2x^2}}{3(1-ax)}+\frac{\left(11\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}\sqrt{1-ax}}{\sqrt{1-a^2x^2}}\,dx}{6\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1-a^2x^2}}{3(1-ax)}+\frac{\left(11\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}}{\sqrt{1+ax}}\,dx}{6\sqrt{1-ax}}\\
&= \frac{11\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{12a\sqrt{1-ax}}-\frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1-a^2x^2}}{3(1-ax)}-\frac{\left(11\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}}{\sqrt{1+ax}}\,dx}{8a\sqrt{1-ax}}\\
&= -\frac{11\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{8a^2\sqrt{1-ax}}+\frac{11\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{12a\sqrt{1-ax}}-\frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1-a^2x^2}}{3(1-ax)}+\frac{\left(11\sqrt{c-\frac{c}{ax}}\sqrt{\right.}}{16a^2}\\
&= -\frac{11\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{8a^2\sqrt{1-ax}}+\frac{11\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{12a\sqrt{1-ax}}-\frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1-a^2x^2}}{3(1-ax)}+\frac{\left(11\sqrt{c-\frac{c}{ax}}\sqrt{\right.}}{}\\
&= -\frac{11\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{8a^2\sqrt{1-ax}}+\frac{11\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{12a\sqrt{1-ax}}-\frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1-a^2x^2}}{3(1-ax)}+\frac{11\sqrt{c-\frac{c}{ax}}\sqrt{x}}{8a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0844775, size = 88, normalized size = 0.48

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}\left(-8a^2x^2+22ax-33\right)+33\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{24a^{5/2}\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*(-33 + 22*a*x - 8
*a^2*x^2) + 33*ArcSinh[Sqrt[a]*Sqrt[x]]))/(24*a^(5/2)*Sqrt[1 - a*x])
```

Maple [A] time = 0.132, size = 125, normalized size = 0.7

$$\frac{x}{48\,ax-48}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(16\,a^{5/2}x^2\sqrt{-(ax+1)\,x}-44\,a^{3/2}x\sqrt{-(ax+1)\,x}+66\,\sqrt{a}\sqrt{-(ax+1)\,x}+33\,\arctan\left(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/48*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)/a^(5/2)*(16*a^(5/2)*x^2*(-(
a*x+1)*x)^(1/2)-44*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+66*a^(1/2)*(-(a*x+1)*x)^(1/
2)+33*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))/(a*x-1)/(-(a*x+1)*x
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}\,x^2}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))*x^2/(a*x + 1), x)
```

Fricas [A] time = 3.23785, size = 625, normalized size = 3.43

$$\left[\frac{33\,(ax-1)\sqrt{-c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(8\,a^3x^3-22\,a^2x^2+33\,ax\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{96\left(a^4x-a^3\right)},-\frac{33\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/96*(33*(a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a
*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4
*(8*a^3*x^3 - 22*a^2*x^2 + 33*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x
)))/(a^4*x - a^3), -1/48*(33*(a*x - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*
a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(8*a^3*x
```

```
^3 - 22*a^2*x^2 + 33*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*
x - a^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}x^2}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))*x^2/(a*x + 1), x)
```

3.593 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal. Leaf size=138

$$-\frac{x^2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{2(1-ax)}-\frac{7\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}+\frac{7x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-ax}}$$

```
[Out] (7*Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(4*a*Sqrt[1 - a*x]) - (Sqrt[c - c/(a*
x)]*x^2*Sqrt[1 - a^2*x^2])/(2*(1 - a*x)) - (7*Sqrt[c - c/(a*x)]*Sqrt[x]*Arc
Sinh[Sqrt[a]*Sqrt[x]])/(4*a^(3/2)*Sqrt[1 - a*x])
```

Rubi [A] time = 0.230064, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {6134, 6128, 881, 848, 50, 54, 215}

$$-\frac{x^2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{2(1-ax)}-\frac{7\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}+\frac{7x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^ArcTanh[a*x],x]
```

```
[Out] (7*Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(4*a*Sqrt[1 - a*x]) - (Sqrt[c - c/(a*
x)]*x^2*Sqrt[1 - a^2*x^2])/(2*(1 - a*x)) - (7*Sqrt[c - c/(a*x)]*Sqrt[x]*Arc
Sinh[Sqrt[a]*Sqrt[x]])/(4*a^(3/2)*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
```

```
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{-\tanh^{-1}(ax)}\sqrt{x}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}(1-ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1-a^2x^2}}{2(1-ax)}+\frac{\left(7\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}\sqrt{1-ax}}{\sqrt{1-a^2x^2}}\,dx}{4\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1-a^2x^2}}{2(1-ax)}+\frac{\left(7\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}}{\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}}\\
&= \frac{7\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{4a\sqrt{1-ax}}-\frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1-a^2x^2}}{2(1-ax)}-\frac{\left(7\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{8a\sqrt{1-ax}}\\
&= \frac{7\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{4a\sqrt{1-ax}}-\frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1-a^2x^2}}{2(1-ax)}-\frac{\left(7\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{4a\sqrt{1-ax}}\\
&= \frac{7\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{4a\sqrt{1-ax}}-\frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1-a^2x^2}}{2(1-ax)}-\frac{7\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}
\end{aligned}
$$

Mathematica [A] time = 0.0508709, size = 80, normalized size = 0.58

$$
-\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}(2ax-7)+7\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{4a^{3/2}\sqrt{1-ax}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^ArcTanh[a*x],x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*(-7 + 2*a*x) + 7
*ArcSinh[Sqrt[a]*Sqrt[x]]))/(4*a^(3/2)*Sqrt[1 - a*x])
```

Maple [A] time = 0.123, size = 107, normalized size = 0.8

$$\frac{x}{8ax-8}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(4a^{3/2}x\sqrt{-(ax+1)x}-14\sqrt{a}\sqrt{-(ax+1)x}-7\arctan\left(1/2\frac{2ax+1}{\sqrt{a}\sqrt{-(ax+1)x}}\right)\right)a^{-\frac{3}{2}}\frac{}{\sqrt{-(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/8*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(4*a^(3/2)*x*(-(a*x+1)*x)^(1
/2)-14*a^(1/2)*(-(a*x+1)*x)^(1/2)-7*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*
x)^(1/2)))/a^(3/2)/(a*x-1)/(-(a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))*x/(a*x + 1), x)
```

Fricas [A] time = 3.00311, size = 582, normalized size = 4.22

$$\left[\frac{7(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4(2a^2x^2+ax)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4(2a^2x^2-7ax)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{16(a^3x-a^2)},\frac{7(ax-1)\sqrt{c}\,\mathrm{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] [1/16*(7*(a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*
x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*
(2*a^2*x^2 - 7*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^
2), 1/8*(7*(a*x - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((
a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(2*a^2*x^2 - 7*a*x)*sqrt(-
a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}\,x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))*x/(a*x + 1), x)
```

3.594 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=90

$$\frac{3\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{1-ax}$$

```
[Out] -((Sqrt[c - c/(a*x)]*x*Sqrt[1 - a^2*x^2])/(1 - a*x)) + (3*Sqrt[c - c/(a*x)]
*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rubi [A] time = 0.176204, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6134, 6128, 881, 848, 54, 215}

$$\frac{3\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{1-ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^ArcTanh[a*x],x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x*Sqrt[1 - a^2*x^2])/(1 - a*x)) + (3*Sqrt[c - c/(a*x)]
*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
```

```
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{e^{-\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{(1-ax)^{3/2}}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}}{1-ax}+\frac{\left(3\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{\sqrt{1-ax}}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx}{2\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}}{1-ax}+\frac{\left(3\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}}{1-ax}+\frac{\left(3\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}}{1-ax}+\frac{3\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}
\end{aligned}
$$

Mathematica [A] time = 0.0494856, size = 67, normalized size = 0.74

$$
\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\frac{3\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}-\sqrt{x}\sqrt{ax+1}\right)}{\sqrt{1-ax}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(-(Sqrt[x]*Sqrt[1 + a*x]) + (3*ArcSinh[Sqrt[a]*S
qrt[x]])/Sqrt[a]))/Sqrt[1 - a*x]
```

Maple [A] time = 0.135, size = 91, normalized size = 1.

$$
\frac{x}{2\,ax-2}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}\left(2\,\sqrt{a}\sqrt{-(ax+1)\,x}+3\,\arctan\left(1/2\,\frac{2\,ax+1}{\sqrt{a}\sqrt{-(ax+1)\,x}}\right)\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)\,x}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(2*a^(1/2)*(-(a*x+1)*x)^(1/2
)+3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))/(a*x-1)/(-(a*x+1)*x)^
(1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

Fricas [A] time = 2.90617, size = 532, normalized size = 5.91

$$\left[\frac{4\sqrt{-a^2x^2+1}ax\sqrt{\frac{acx-c}{ax}}+3(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4(2a^2x^2+ax)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)}{4(a^2x-a)}, \frac{2\sqrt{-a^2x^2+1}ax\sqrt{\frac{acx-c}{ax}}-3(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) + 3*(a*x - 1)*sqrt(-
c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqr
t(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)))/(a^2*x - a), 1/2*(2*sqrt(-a^
2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) - 3*(a*x - 1)*sqrt(c)*arctan(2*sqrt(
-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)
))/(a^2*x - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

3.595 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=89

$$-\frac{2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{1-ax}-\frac{2\sqrt{a}\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}$$

```
[Out] (-2*Sqrt[c - c/(a*x)]*Sqrt[1 - a^2*x^2])/(1 - a*x) - (2*Sqrt[a]*Sqrt[c - c/
(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/Sqrt[1 - a*x]
```

Rubi [A] time = 0.266104, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6134, 6128, 879, 848, 54, 215}

$$-\frac{2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{1-ax}-\frac{2\sqrt{a}\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcTanh[a*x]*x),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*Sqrt[1 - a^2*x^2])/(1 - a*x) - (2*Sqrt[a]*Sqrt[c - c/
(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/Sqrt[1 - a*x]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
```

```
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{3/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^{3/2}}{x^{3/2}\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{1-ax}-\frac{\left(a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-ax}}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{1-ax}-\frac{\left(a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{1-ax}-\frac{\left(2a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{1-ax}-\frac{2\sqrt{a}\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}
\end{aligned}$$

Mathematica [A] time = 0.0519567, size = 60, normalized size = 0.67

$$-\frac{2\sqrt{c-\frac{c}{ax}}\left(\sqrt{ax+1}+\sqrt{a}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcTanh[a*x]*x),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x] + Sqrt[a]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt
[x]]))/Sqrt[1 - a*x]
```

Maple [A] time = 0.137, size = 91, normalized size = 1.

$$\frac{1}{ax-1}\sqrt{\frac{c(ax-1)}{ax}}\left(-\arctan\left(\frac{2ax+1}{2}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)x}}\right)xa+2\sqrt{a}\sqrt{-(ax+1)x}\right)\sqrt{-a^2x^2+1}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] (c*(a*x-1)/a/x)^(1/2)*(-arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*
a+2*a^(1/2)*(-(a*x+1)*x)^(1/2))*(-a^2*x^2+1)^(1/2)/(a*x-1)/(-(a*x+1)*x)^(1/
2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/((a*x + 1)*x), x)
```

Fricas [A] time = 2.72097, size = 505, normalized size = 5.67

$$\left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{2(ax-1)}, \frac{(ax-1)\sqrt{c}\arctan\left(\frac{2\sqrt{-a^2x^2+1}a\sqrt{c}x\ldots}{2a^2cx^2-acx-\ldots}\right)\ldots}{ax-\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="fricas
")
```

```
[Out] [1/2*((a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x)*
sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*sqr
t(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a*x - 1), ((a*x - 1)*sqrt(c)*arct
an(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 -
a*c*x - c)) + 2*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\sqrt{-(ax-1)(ax+1)}}{x(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(x*(a*x + 1)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/((a*x + 1)*x), x)
```

3.596 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=84

$$\frac{10a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}}-\frac{2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{3x(1-ax)}$$

```
[Out] (10*a*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(3*Sqrt[1 - a*x]) - (2*Sqrt[c - c/(a
*x)]*Sqrt[1 - a^2*x^2])/(3*x*(1 - a*x))
```

Rubi [A] time = 0.259863, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6134, 6128, 879, 848, 37}

$$\frac{10a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}}-\frac{2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{3x(1-ax)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] (10*a*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(3*Sqrt[1 - a*x]) - (2*Sqrt[c - c/(a
*x)]*Sqrt[1 - a^2*x^2])/(3*x*(1 - a*x))
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
```

```
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{5/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^{3/2}}{x^{5/2}\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{3x(1-ax)} - \frac{\left(5a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-ax}}{x^{3/2}\sqrt{1-a^2x^2}}\,dx}{3\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{3x(1-ax)} - \frac{\left(5a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{x^{3/2}\sqrt{1+ax}}\,dx}{3\sqrt{1-ax}}\\
&= \frac{10a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{3\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{3x(1-ax)}
\end{aligned}$$

Mathematica [A] time = 0.0306416, size = 47, normalized size = 0.56

$$\frac{2\sqrt{ax+1}(5ax-1)\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x]*(-1 + 5*a*x))/(3*x*Sqrt[1 - a*x])
```

Maple [A] time = 0.093, size = 46, normalized size = 0.6

$$-\frac{10\,ax-2}{(3\,ax-3)\,x}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] -2/3*(5*a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^2), x)
```

Fricas [A] time = 2.33845, size = 99, normalized size = 1.18

$$-\frac{2\sqrt{-a^2x^2+1}(5\,ax-1)\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fric
as")
```

```
[Out] -2/3*sqrt(-a^2*x^2 + 1)*(5*a*x - 1)*sqrt((a*c*x - c)/(a*x))/(a*x^2 - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\sqrt{-(ax-1)(ax+1)}}{x^2(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(x**2*(a*x + 1)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^2), x)
```

3.597 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=128

$$-\frac{2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{5x^2(1-ax)}-\frac{12a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-ax}}+\frac{6a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{5x\sqrt{1-ax}}$$

```
[Out] (-12*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(5*Sqrt[1 - a*x]) + (6*a*Sqrt[c -
 c/(a*x)]*Sqrt[1 + a*x])/(5*x*Sqrt[1 - a*x]) - (2*Sqrt[c - c/(a*x)]*Sqrt[1
- a^2*x^2])/(5*x^2*(1 - a*x))
```

Rubi [A] time = 0.267113, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6134, 6128, 879, 848, 45, 37}

$$-\frac{2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{5x^2(1-ax)}-\frac{12a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-ax}}+\frac{6a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{5x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcTanh[a*x]*x^3),x]
```


```
[Out] (-12*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(5*Sqrt[1 - a*x]) + (6*a*Sqrt[c -
 c/(a*x)]*Sqrt[1 + a*x])/(5*x*Sqrt[1 - a*x]) - (2*Sqrt[c - c/(a*x)]*Sqrt[1
- a^2*x^2])/(5*x^2*(1 - a*x))
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
```

```
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{7/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^{3/2}}{x^{7/2}\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{5x^2(1-ax)}-\frac{\left(9a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-ax}}{x^{5/2}\sqrt{1-a^2x^2}}\,dx}{5\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{5x^2(1-ax)}-\frac{\left(9a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{x^{5/2}\sqrt{1+ax}}\,dx}{5\sqrt{1-ax}}\\
&= \frac{6a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{5x\sqrt{1-ax}}-\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{5x^2(1-ax)}+\frac{\left(6a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{x^{3/2}\sqrt{1+ax}}\,dx}{5\sqrt{1-ax}}\\
&= -\frac{12a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{5\sqrt{1-ax}}+\frac{6a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{5x\sqrt{1-ax}}-\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{5x^2(1-ax)}
\end{aligned}$$

Mathematica [A] time = 0.0304812, size = 55, normalized size = 0.43

$$-\frac{2\sqrt{ax+1}\left(6a^2x^2-3ax+1\right)\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcTanh[a*x]*x^3),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x]*(1 - 3*a*x + 6*a^2*x^2))/(5*x^2*Sqrt[1
- a*x])
```

Maple [A] time = 0.089, size = 54, normalized size = 0.4

$$\frac{12\,a^{2}x^{2}-6\,ax+2}{5\,x^{2}\left(ax-1\right)}\sqrt{\frac{c\left(ax-1\right)}{ax}}\sqrt{-a^{2}x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x)
```

```
[Out] 2/5*(6*a^2*x^2-3*a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2/(a*x-1
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^{2}x^{2}+1}\sqrt{c-\frac{c}{ax}}}{(ax+1)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^3), x)
```

Fricas [A] time = 2.43893, size = 116, normalized size = 0.91

$$\frac{2\left(6\,a^{2}x^{2}-3\,ax+1\right)\sqrt{-a^{2}x^{2}+1}\sqrt{\frac{acx-c}{ax}}}{5\left(ax^{3}-x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="fric
as")
```

```
[Out] 2/5*(6*a^2*x^2 - 3*a*x + 1)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x))/(a*x
^3 - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{x^{3}\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(x**3*(a*x + 1)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}{(ax+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^3), x)
```

3.598 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=172

$$-\frac{2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{7x^3(1-ax)}+\frac{208a^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105\sqrt{1-ax}}-\frac{104a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105x\sqrt{1-ax}}+\frac{26a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}}$$

```
[Out] (208*a^3*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(105*Sqrt[1 - a*x]) + (26*a*Sqrt[
c - c/(a*x)]*Sqrt[1 + a*x])/(35*x^2*Sqrt[1 - a*x]) - (104*a^2*Sqrt[c - c/(a
*x)]*Sqrt[1 + a*x])/(105*x*Sqrt[1 - a*x]) - (2*Sqrt[c - c/(a*x)]*Sqrt[1 - a
^2*x^2])/(7*x^3*(1 - a*x))
```

Rubi [A] time = 0.263483, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6134, 6128, 879, 848, 45, 37}

$$-\frac{2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{7x^3(1-ax)}+\frac{208a^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105\sqrt{1-ax}}-\frac{104a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105x\sqrt{1-ax}}+\frac{26a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcTanh[a*x]*x^4),x]
```

```
[Out] (208*a^3*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(105*Sqrt[1 - a*x]) + (26*a*Sqrt[
c - c/(a*x)]*Sqrt[1 + a*x])/(35*x^2*Sqrt[1 - a*x]) - (104*a^2*Sqrt[c - c/(a
*x)]*Sqrt[1 + a*x])/(105*x*Sqrt[1 - a*x]) - (2*Sqrt[c - c/(a*x)]*Sqrt[1 - a
^2*x^2])/(7*x^3*(1 - a*x))
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{9/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^{3/2}}{x^{9/2}\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{7x^3(1-ax)}-\frac{\left(13a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-ax}}{x^{7/2}\sqrt{1-a^2x^2}}\,dx}{7\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{7x^3(1-ax)}-\frac{\left(13a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{x^{7/2}\sqrt{1+ax}}\,dx}{7\sqrt{1-ax}}\\
&= \frac{26a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{35x^2\sqrt{1-ax}}-\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{7x^3(1-ax)}+\frac{\left(52a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{x^{5/2}\sqrt{1+ax}}\,dx}{35\sqrt{1-ax}}\\
&= \frac{26a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{35x^2\sqrt{1-ax}}-\frac{104a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{105x\sqrt{1-ax}}-\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{7x^3(1-ax)}-\frac{\left(104a^3\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int}{105\sqrt{1-a}}\\
&= \frac{208a^3\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{105\sqrt{1-ax}}+\frac{26a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{35x^2\sqrt{1-ax}}-\frac{104a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{105x\sqrt{1-ax}}-\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x}}{7x^3(1-ax)}
\end{aligned}
$$

Mathematica [A] time = 0.033091, size = 63, normalized size = 0.37

$$
\frac{2\sqrt{ax+1}\left(104a^3x^3-52a^2x^2+39ax-15\right)\sqrt{c-\frac{c}{ax}}}{105x^3\sqrt{1-ax}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcTanh[a*x]*x^4),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x]*(-15 + 39*a*x - 52*a^2*x^2 + 104*a^3*x^3
))/(105*x^3*Sqrt[1 - a*x])
```

Maple [A] time = 0.084, size = 62, normalized size = 0.4

$$-\frac{208\,x^3a^3-104\,a^2x^2+78\,ax-30}{105\,x^3\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x)
```

```
[Out] -2/105*(104*a^3*x^3-52*a^2*x^2+39*a*x-15)*(c*(a*x-1)/a/x)^(1/2)*(-a^2*x^2+1
)^(1/2)/x^3/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}{(ax+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^4), x)
```

Fricas [A] time = 2.5101, size = 143, normalized size = 0.83

$$-\frac{2\left(104\,a^3x^3-52\,a^2x^2+39\,ax-15\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{105\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="fric
as")
```

```
[Out] -2/105*(104*a^3*x^3 - 52*a^2*x^2 + 39*a*x - 15)*sqrt(-a^2*x^2 + 1)*sqrt((a*
c*x - c)/(a*x))/(a*x^4 - x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\sqrt{-(ax-1)(ax+1)}}{x^4\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(x**4*(a*x + 1)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}{(ax+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^4), x)
```

3.599 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

Optimal. Leaf size=172

$$-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{96a^2}+\frac{149x\sqrt{c-\frac{c}{ax}}}{64a^3}-\frac{363\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^4}-\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}+\frac{17x^3\sqrt{c-\frac{c}{ax}}}{24a}$$

```
[Out] (149*Sqrt[c - c/(a*x)]*x)/(64*a^3) - (107*Sqrt[c - c/(a*x)]*x^2)/(96*a^2) +
 (17*Sqrt[c - c/(a*x)]*x^3)/(24*a) - (Sqrt[c - c/(a*x)]*x^4)/4 - (363*Sqrt[
c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(64*a^4) + (4*Sqrt[2]*Sqrt[c]*ArcTan
h[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^4
```

Rubi [A] time = 0.352306, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 446, 98, 151, 156, 63, 208}

$$-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{96a^2}+\frac{149x\sqrt{c-\frac{c}{ax}}}{64a^3}-\frac{363\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^4}-\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}+\frac{17x^3\sqrt{c-\frac{c}{ax}}}{24a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^3)/E^(2*ArcTanh[a*x]),x]
```


```
[Out] (149*Sqrt[c - c/(a*x)]*x)/(64*a^3) - (107*Sqrt[c - c/(a*x)]*x^2)/(96*a^2) +
 (17*Sqrt[c - c/(a*x)]*x^3)/(24*a) - (Sqrt[c - c/(a*x)]*x^4)/4 - (363*Sqrt[
c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(64*a^4) + (4*Sqrt[2]*Sqrt[c]*ArcTan
h[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^4
```


Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x^3(1-ax)}{1+ax}\,dx$$

$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}x^4}{1+ax}\,dx}{c}$$

$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{a+\frac{1}{x}}\,dx}{c}$$

$$=\frac{a\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^5(a+x)}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4-\frac{\text{Subst}\left(\int\frac{\frac{17c^2}{2}-\frac{15c^2x}{2a}}{x^4(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{4c}$$

$$=\frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a}-\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4+\frac{\text{Subst}\left(\int\frac{\frac{107c^3}{4}-\frac{85c^3x}{4a}}{x^3(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{12ac^2}$$

$$=-\frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2}+\frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a}-\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4-\frac{\text{Subst}\left(\int\frac{\frac{447c^4}{8}-\frac{321c^4x}{8a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{24a^2c^3}$$

$$=\frac{149\sqrt{c-\frac{c}{ax}}x}{64a^3}-\frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2}+\frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a}-\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4+\frac{\text{Subst}\left(\int\frac{\frac{1089c^5}{16}-\frac{447c^5}{16a}\cdots}{x(a+x)\sqrt{c-\cdots}}\cdots\right)}{24a^3c^4}$$

$$=\frac{149\sqrt{c-\frac{c}{ax}}x}{64a^3}-\frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2}+\frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a}-\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4+\frac{(363c)\,\text{Subst}\left(\int\frac{\cdots}{x\sqrt{\cdots}}\cdots\right)}{128a^4}$$

$$=\frac{149\sqrt{c-\frac{c}{ax}}x}{64a^3}-\frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2}+\frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a}-\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4-\frac{363\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}\cdots\right)}{64a^3}$$

$$=\frac{149\sqrt{c-\frac{c}{ax}}x}{64a^3}-\frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2}+\frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a}-\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4-\frac{363\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\cdots}}{\sqrt{c}}\cdots\right)}{64a^4}$$

Mathematica [A] time = 0.136369, size = 116, normalized size = 0.67

$$\frac{ax\left(-48a^3x^3+136a^2x^2-214ax+447\right)\sqrt{c-\frac{c}{ax}}-1089\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+768\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{192a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^3)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(447 - 214*a*x + 136*a^2*x^2 - 48*a^3*x^3) - 1089*Sq
rt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] + 768*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt
[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(192*a^4)
```

Maple [A] time = 0.117, size = 259, normalized size = 1.5

$$-\frac{x}{384}\sqrt{\frac{c(ax-1)}{ax}}\left(96x\left(ax^2-x\right)^{3/2}a^{9/2}\sqrt{a^{-1}}-176\left(ax^2-x\right)^{3/2}a^{7/2}\sqrt{a^{-1}}+252\sqrt{ax^2-x}a^{7/2}\sqrt{a^{-1}}x-126\sqrt{ax^2-x}a^{5/2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/384*(c*(a*x-1)/a/x)^(1/2)*x*(96*x*(a*x^2-x)^(3/2)*a^(9/2)*(1/a)^(1/2)-17
6*(a*x^2-x)^(3/2)*a^(7/2)*(1/a)^(1/2)+252*(a*x^2-x)^(1/2)*a^(7/2)*(1/a)^(1/
2)*x-126*(a*x^2-x)^(1/2)*a^(5/2)*(1/a)^(1/2)-768*a^(5/2)*(1/a)^(1/2)*((a*x-
1)*x)^(1/2)+768*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)
*a-3*a*x+1)/(a*x+1))+1152*a^2*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/
a^(1/2))*(1/a)^(1/2)-63*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))
*(1/a)^(1/2)*a^2)/((a*x-1)*x)^(1/2)/a^(11/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}x^3}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))*x^3/(a*x + 1)^2, x)
```

Fricas [A] time = 2.54578, size = 666, normalized size = 3.87

$$\left[\frac{768\sqrt{2}\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3acx-c}{ax+1}\right)-2\left(48a^4x^4-136a^3x^3+214a^2x^2-447ax\right)\sqrt{\frac{acx-c}{ax}}+1089\sqrt{c}\log\left(-2acx+2a\right.}{384a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/384*(768*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) + 3*a*c*x - c)/(a*x + 1)) - 2*(48*a^4*x^4 - 136*a^3*x^3 + 214*a^2*x^2
- 447*a*x)*sqrt((a*c*x - c)/(a*x)) + 1089*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c
)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^4, -1/192*(768*sqrt(2)*sqrt(-c)*arctan(
1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (48*a^4*x^4 - 136*a^3*x^3
 + 214*a^2*x^2 - 447*a*x)*sqrt((a*c*x - c)/(a*x)) - 1089*sqrt(-c)*arctan(sq
rt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x^3\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx-\int\frac{ax^4\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x**3*sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(a*x**4*sqrt(c -
c/(a*x))/(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.600 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal. Leaf size=147

$$-\frac{19x\sqrt{c-\frac{c}{ax}}}{8a^2}+\frac{45\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^3}-\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}+\frac{13x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

```
[Out] (-19*Sqrt[c - c/(a*x)]*x)/(8*a^2) + (13*Sqrt[c - c/(a*x)]*x^2)/(12*a) - (Sq
rt[c - c/(a*x)]*x^3)/3 + (45*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(8
*a^3) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^
3
```

Rubi [A] time = 0.314401, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 446, 98, 151, 156, 63, 208}

$$-\frac{19x\sqrt{c-\frac{c}{ax}}}{8a^2}+\frac{45\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^3}-\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}+\frac{13x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^(2*ArcTanh[a*x]),x]
```


```
[Out] (-19*Sqrt[c - c/(a*x)]*x)/(8*a^2) + (13*Sqrt[c - c/(a*x)]*x^2)/(12*a) - (Sq
rt[c - c/(a*x)]*x^3)/3 + (45*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(8
*a^3) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^
3
```


Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}x^2(1-ax)}{1+ax}\,dx$$
$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{1+ax}\,dx}{c}$$
$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{a+\frac{1}{x}}\,dx}{c}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^4(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= -\frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 - \frac{\text{Subst}\left(\int \frac{\frac{13c^2}{2}-\frac{11c^2x}{2a}}{x^3(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{3c}$$
$$= \frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a} - \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 + \frac{\text{Subst}\left(\int \frac{\frac{57c^3}{4}-\frac{39c^3x}{4a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{6ac^2}$$
$$= -\frac{19\sqrt{c-\frac{c}{ax}}x}{8a^2} + \frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a} - \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 - \frac{\text{Subst}\left(\int \frac{\frac{135c^4}{8}-\frac{57c^4x}{8a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{6a^2c^3}$$
$$= -\frac{19\sqrt{c-\frac{c}{ax}}x}{8a^2} + \frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a} - \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 - \frac{(45c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{16a^3} + \frac{(4c)\,\text{S}}{}$$
$$= -\frac{19\sqrt{c-\frac{c}{ax}}x}{8a^2} + \frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a} - \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 + \frac{45\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{8a^2} - \frac{8\,\text{S}}{}$$
$$= -\frac{19\sqrt{c-\frac{c}{ax}}x}{8a^2} + \frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a} - \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 + \frac{45\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3} - \frac{4\sqrt{2}\sqrt{c}\tanh^{-1}}{a^3}$$

Mathematica [A] time = 0.0994495, size = 108, normalized size = 0.73

$$\frac{ax\left(-8a^2x^2+26ax-57\right)\sqrt{c-\frac{c}{ax}}+135\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-96\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(-57 + 26*a*x - 8*a^2*x^2) + 135*Sqrt[c]*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]] - 96*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqr
t[2]*Sqrt[c])])/(24*a^3)
```

Maple [A] time = 0.118, size = 237, normalized size = 1.6

$$-\frac{x}{48}\sqrt{\frac{c(ax-1)}{ax}}\left(16\left(ax^2-x\right)^{3/2}a^{7/2}\sqrt{a^{-1}}-36\sqrt{ax^2-x}a^{7/2}\sqrt{a^{-1}}x+18\sqrt{ax^2-x}a^{5/2}\sqrt{a^{-1}}+96a^{5/2}\sqrt{a^{-1}}\sqrt{(ax-1)x}-9\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/48*(c*(a*x-1)/a/x)^(1/2)*x*(16*(a*x^2-x)^(3/2)*a^(7/2)*(1/a)^(1/2)-36*(a
*x^2-x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x+18*(a*x^2-x)^(1/2)*a^(5/2)*(1/a)^(1/2)+
96*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)-96*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(
1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))-144*a^2*ln(1/2*(2*((a*x-1)
*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)+9*ln(1/2*(2*(a*x^2-x)^(1/2)
*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^2)/((a*x-1)*x)^(1/2)/a^(9/2)/(1/a)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}\,x^2}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))*x^2/(a*x + 1)^2, x)
```

Fricas [A] time = 2.68336, size = 609, normalized size = 4.14

$$\left[\frac{96\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)-2\left(8\,a^3x^3-26\,a^2x^2+57\,ax\right)\sqrt{\frac{acx-c}{ax}}+135\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{48\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/48*(96*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)
) - 3*a*c*x + c)/(a*x + 1)) - 2*(8*a^3*x^3 - 26*a^2*x^2 + 57*a*x)*sqrt((a*c
*x - c)/(a*x)) + 135*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/
(a*x)) + c))/a^3, 1/24*(96*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqr
t((a*c*x - c)/(a*x))/c) - (8*a^3*x^3 - 26*a^2*x^2 + 57*a*x)*sqrt((a*c*x - c
)/(a*x)) - 135*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x^2\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx-\int \frac{ax^3\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x**2*sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(a*x**3*sqrt(c -
c/(a*x))/(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.601 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=122

$$-\frac{23\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^2}-\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}+\frac{9x\sqrt{c-\frac{c}{ax}}}{4a}$$

```
[Out] (9*Sqrt[c - c/(a*x)]*x)/(4*a) - (Sqrt[c - c/(a*x)]*x^2)/2 - (23*Sqrt[c]*Arc
Tanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(4*a^2) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[
c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^2
```

Rubi [A] time = 0.240319, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {6133, 25, 514, 446, 98, 151, 156, 63, 208}

$$-\frac{23\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^2}-\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}+\frac{9x\sqrt{c-\frac{c}{ax}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (9*Sqrt[c - c/(a*x)]*x)/(4*a) - (Sqrt[c - c/(a*x)]*x^2)/2 - (23*Sqrt[c]*Arc
Tanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(4*a^2) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[
c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^2
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x(1-ax)}{1+ax}dx$$
$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{1+ax}dx}{c}$$
$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{a+\frac{1}{x}}dx}{c}$$
$$=\frac{a\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^3(a+x)}dx,x,\frac{1}{x}\right)}{c}$$
$$=-\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2-\frac{\text{Subst}\left(\int\frac{\frac{9c^2}{2}-\frac{7c^2x}{2a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{2c}$$
$$=\frac{9\sqrt{c-\frac{c}{ax}}x}{4a}-\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2+\frac{\text{Subst}\left(\int\frac{\frac{23c^3}{4}-\frac{9c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{2ac^2}$$
$$=\frac{9\sqrt{c-\frac{c}{ax}}x}{4a}-\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2+\frac{(23c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{8a^2}-\frac{(4c)\,\text{Subst}\left(\int\frac{1}{(a+x)\sqrt{c-}}\right.}{a^2}$$
$$=\frac{9\sqrt{c-\frac{c}{ax}}x}{4a}-\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2-\frac{23\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}dx,x,\sqrt{c-\frac{c}{ax}}\right)}{4a}+\frac{8\,\text{Subst}\left(\int\frac{1}{2a-\frac{ax^2}{c}}d\right.}{a}$$
$$=\frac{9\sqrt{c-\frac{c}{ax}}x}{4a}-\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2-\frac{23\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

Mathematica [A] time = 0.0788897, size = 100, normalized size = 0.82

$$\frac{ax(9-2ax)\sqrt{c-\frac{c}{ax}}-23\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+16\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(9 - 2*a*x) - 23*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/S
qrt[c]] + 16*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/
(4*a^2)
```

Maple [B] time = 0.115, size = 215, normalized size = 1.8

$$\frac{x}{8}\sqrt{\frac{c(ax-1)}{ax}}\left(-4\sqrt{ax^2-x}a^{7/2}\sqrt{a^{-1}}x+16a^{5/2}\sqrt{a^{-1}}\sqrt{(ax-1)x}+2\sqrt{ax^2-x}a^{5/2}\sqrt{a^{-1}}-16a^{3/2}\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax}}{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/8*(c*(a*x-1)/a/x)^(1/2)*x*(-4*(a*x^2-x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x+16*a^
(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)+2*(a*x^2-x)^(1/2)*a^(5/2)*(1/a)^(1/2)-1
6*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a
*x+1))-24*a^2*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(
1/2)+ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^2)/(
(a*x-1)*x)^(1/2)/a^(7/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}x}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))*x/(a*x + 1)^2, x)
```

Fricas [A] time = 2.52284, size = 568, normalized size = 4.66

$$\left[\frac{16\sqrt{2}\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-2\left(2\,a^2x^2-9\,ax\right)\sqrt{\frac{acx-c}{ax}}+23\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{8\,a^2},-\frac{16\sqrt{2}\sqrt{-c}\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/8*(16*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)
) + 3*a*c*x - c)/(a*x + 1)) - 2*(2*a^2*x^2 - 9*a*x)*sqrt((a*c*x - c)/(a*x))
 + 23*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^
2, -1/4*(16*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(
a*x))/c) + (2*a^2*x^2 - 9*a*x)*sqrt((a*c*x - c)/(a*x)) - 23*sqrt(-c)*arctan
(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx-\int \frac{ax^2\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x*sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(a*x**2*sqrt(c - c/(
a*x))/(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.602 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=93

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)+\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -(Sqrt[c - c/(a*x)]*x) + (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a -
 (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.157853, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 98, 156, 63, 208}

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)+\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*x) + (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a -
 (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{1+ax}\,dx$$
$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{1+ax}dx}{c}$$
$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}dx}{c}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2(a+x)}dx, x, \frac{1}{x}\right)}{c}$$
$$= -\sqrt{c-\frac{c}{ax}}x - \frac{\text{Subst}\left(\int \frac{\frac{5c^2}{2}-\frac{3c^2x}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}dx, x, \frac{1}{x}\right)}{c}$$
$$= -\sqrt{c-\frac{c}{ax}}x - \frac{(5c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}dx, x, \frac{1}{x}\right)}{2a} + \frac{(4c)\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}dx, x, \frac{1}{x}\right)}{a}$$
$$= -\sqrt{c-\frac{c}{ax}}x + 5\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}dx, x, \sqrt{c-\frac{c}{ax}}\right) - 8\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}dx, x, \sqrt{c-\frac{c}{ax}}\right.$$
$$= -\sqrt{c-\frac{c}{ax}}x + \frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a} - \frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.0449782, size = 93, normalized size = 1.

$$x\left(-\sqrt{c-\frac{c}{ax}}\right) + \frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a} - \frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*x) + (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a -
 (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Maple [B] time = 0.122, size = 190, normalized size = 2.

$$\frac{x}{2}\sqrt{\frac{c(ax-1)}{ax}}\left(2\sqrt{ax^2-x}a^{3/2}\sqrt{a^{-1}} - 4\sqrt{(ax-1)x}a^{3/2}\sqrt{a^{-1}} - \ln\left(\frac{1}{2}\left(2\sqrt{ax^2-x}\sqrt{a}+2ax-1\right)\frac{1}{\sqrt{a}}\right)a\sqrt{a^{-1}} + 4\sqrt{2}\ln\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(1/2)-4*((a*x-
1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/
a^(1/2))*a*(1/a)^(1/2)+4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2
```

```
)*a-3*a*x+1)/(a*x+1))*a^(1/2)+6*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1
)/a^(1/2))*a*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/a^(3/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(a^2x^2-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))/(a*x + 1)^2, x)
```

Fricas [A] time = 2.56418, size = 509, normalized size = 5.47

$$\left[-\frac{2\,ax\sqrt{\frac{acx-c}{ax}}-4\,\sqrt{2}\sqrt{c}\log\left(\frac{2\,\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)-5\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},-\frac{ax\sqrt{\frac{acx-c}{ax}}-4\,\sqrt{2}\sqrt{-c}\,\mathrm{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) - 4*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*s
qrt(c)*x*sqrt((a*c*x - c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) - 5*sqrt(c)*log(
-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, -(a*x*sqrt((a*c*x
 - c)/(a*x)) - 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x -
 c)/(a*x))/c) + 5*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(a*x*sqrt(c - c/(a*x))
/(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.603 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=86

$$-2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] -2*Sqrt[c - c/(a*x)] - 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] + 4*Sqr
t[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.237423, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6133, 25, 434, 446, 84, 156, 63, 208}

$$-2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] -2*Sqrt[c - c/(a*x)] - 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] + 4*Sqr
t[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx &= \int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x(1+ax)}\,dx\\
&= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{1+ax}\,dx}{c}\\
&= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x}\,dx}{c}\\
&= \frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x(a+x)}\,dx,x,\frac{1}{x}\right)}{c}\\
&= -2\sqrt{c-\frac{c}{ax}}+\frac{a\,\text{Subst}\left(\int \frac{c^2-\frac{3c^2x}{a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{c}\\
&= -2\sqrt{c-\frac{c}{ax}}+c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)-(4c)\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\\
&= -2\sqrt{c-\frac{c}{ax}}-(2a)\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)+(8a)\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\right.\right.\\
&= -2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.034305, size = 86, normalized size = 1.

$$-2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] -2*Sqrt[c - c/(a*x)] - 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] + 4*Sqr
t[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Maple [B] time = 0.126, size = 227, normalized size = 2.6

$$\frac{1}{x}\sqrt{\frac{c(ax-1)}{ax}}\left(2\sqrt{(ax-1)x}x^2a^{3/2}\sqrt{a^{-1}}-4\sqrt{ax^2-x}a^{3/2}\sqrt{a^{-1}}x^2+2\left(ax^2-x\right)^{3/2}\sqrt{a}\sqrt{a^{-1}}+2\ln\left(1/2\frac{2\sqrt{ax^2-x}\sqrt{a}+}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x)
```

```
[Out] (c*(a*x-1)/a/x)^(1/2)/x*(2*((a*x-1)*x)^(1/2)*x^2*a^(3/2)*(1/a)^(1/2)-4*(a*x
^2-x)^(1/2)*a^(3/2)*(1/a)^(1/2)*x^2+2*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2)+2
*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^2*a-2*a^
(1/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1
))*x^2-3*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*
x^2*a)/((a*x-1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}}{(ax+1)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))/((a*x + 1)^2*x), x)
```

Fricas [A] time = 2.65709, size = 482, normalized size = 5.6

$$\left[2\sqrt{2}\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-2\sqrt{\frac{acx-c}{ax}},-4\sqrt{2}\sqrt{-c}\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*
a*c*x - c)/(a*x + 1)) + sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x -
c)/(a*x)) + c) - 2*sqrt((a*c*x - c)/(a*x)), -4*sqrt(2)*sqrt(-c)*arctan(1/2*
sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 2*sqrt(-c)*arctan(sqrt(-c)*sq
rt((a*c*x - c)/(a*x))/c) - 2*sqrt((a*c*x - c)/(a*x))]
```

Sympy [A] time = 21.5545, size = 80, normalized size = 0.93

$$\frac{2c\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{4\sqrt{2}c\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c-\frac{c}{ax}}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-2\sqrt{c-\frac{c}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x,x)
```

```
[Out] 2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sqrt(-c) - 4*sqrt(2)*c*atan(sqrt(2)*sq
rt(c - c/(a*x))/(2*sqrt(-c)))/sqrt(-c) - 2*sqrt(c - c/(a*x))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.604 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=82

$$\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a\sqrt{c-\frac{c}{ax}}-4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 4*a*Sqrt[c - c/(a*x)] + (2*a*(c - c/(a*x))^(3/2))/(3*c) - 4*Sqrt[2]*a*Sqrt[
c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.235805, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6133, 25, 514, 444, 50, 63, 208}

$$\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a\sqrt{c-\frac{c}{ax}}-4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] 4*a*Sqrt[c - c/(a*x)] + (2*a*(c - c/(a*x))^(3/2))/(3*c) - 4*Sqrt[2]*a*Sqrt[
c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx &= \int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^2(1+ax)}\,dx\\
&= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x(1+ax)}\,dx}{c}\\
&= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^2}\,dx}{c}\\
&= \frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx,x,\frac{1}{x}\right)}{c}\\
&= \frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+(2a)\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx,x,\frac{1}{x}\right)\\
&= 4a\sqrt{c-\frac{c}{ax}}+\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+(4ac)\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\\
&= 4a\sqrt{c-\frac{c}{ax}}+\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-\left(8a^2\right)\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)\\
&= 4a\sqrt{c-\frac{c}{ax}}+\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0623136, size = 69, normalized size = 0.84

$$
\frac{2(7ax-1)\sqrt{c-\frac{c}{ax}}}{3x}-4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(-1 + 7*a*x))/(3*x) - 4*Sqrt[2]*a*Sqrt[c]*ArcTanh[Sqrt
[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Maple [B] time = 0.125, size = 254, normalized size = 3.1

$$-\frac{1}{3x^2}\sqrt{\frac{c(ax-1)}{ax}}\left(-18\sqrt{ax^2-x}a^{5/2}\sqrt{a^{-1}}x^3+6a^{5/2}\sqrt{a^{-1}}\sqrt{(ax-1)x}x^3+12a^{3/2}\left(ax^2-x\right)^{3/2}x\sqrt{a^{-1}}+9\ln\left(1/2\frac{2\sqrt{ax}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x)
```

```
[Out] -1/3*(c*(a*x-1)/a/x)^(1/2)/x^2*(-18*(a*x^2-x)^(1/2)*a^(5/2)*(1/a)^(1/2)*x^3
+6*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^3+12*a^(3/2)*(a*x^2-x)^(3/2)*x*(
1/a)^(1/2)+9*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2
)*x^3*a^2-6*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3
*a*x+1)/(a*x+1))*x^3-9*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)
)*(1/a)^(1/2)*x^3*a^2-2*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1
/2)/a^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}}{(ax+1)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))/((a*x + 1)^2*x^2), x)
```

Fricas [A] time = 2.4569, size = 371, normalized size = 4.52

$$\left[\frac{2\left(3\sqrt{2}a\sqrt{c}x\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3acx+c}{ax+1}\right)+(7ax-1)\sqrt{\frac{acx-c}{ax}}\right)}{3x}, \frac{2\left(6\sqrt{2}a\sqrt{-c}x\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2c}\right)+(7ax-1)\sqrt{\frac{acx-c}{ax}}\right.}{3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*a*sqrt(c)*x*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) - 3*a*c*x + c)/(a*x + 1)) + (7*a*x - 1)*sqrt((a*c*x - c)/(a*x)))/x, 2/3
*(6*sqrt(2)*a*sqrt(-c)*x*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)
)/c) + (7*a*x - 1)*sqrt((a*c*x - c)/(a*x)))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int-\frac{\sqrt{c-\frac{c}{ax}}}{ax^3+x^2}\,dx-\int\frac{ax\sqrt{c-\frac{c}{ax}}}{ax^3+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**2,x)
```

```
[Out] -Integral(-sqrt(c - c/(a*x))/(a*x**3 + x**2), x) - Integral(a*x*sqrt(c - c/
(a*x))/(a*x**3 + x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.605 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=113

$$-\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^2\sqrt{c-\frac{c}{ax}}+4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] -4*a^2*Sqrt[c - c/(a*x)] - (2*a^2*(c - c/(a*x))^(3/2))/(3*c) - (2*a^2*(c -
c/(a*x))^(5/2))/(5*c^2) + 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.253156, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6133, 25, 514, 446, 80, 50, 63, 208}

$$-\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^2\sqrt{c-\frac{c}{ax}}+4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] -4*a^2*Sqrt[c - c/(a*x)] - (2*a^2*(c - c/(a*x))^(3/2))/(3*c) - (2*a^2*(c -
c/(a*x))^(5/2))/(5*c^2) + 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^3(1+ax)}\,dx$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^2(1+ax)}\,dx}{c}$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^3}\,dx}{c}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{a^2\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \left(2a^2\right)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx, x, \frac{1}{x}\right)$$

$$= -4a^2\sqrt{c-\frac{c}{ax}} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \left(4a^2c\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,\right.$$

$$= -4a^2\sqrt{c-\frac{c}{ax}} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \left(8a^3\right)\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\right.$$

$$= -4a^2\sqrt{c-\frac{c}{ax}} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + 4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] time = 0.075404, size = 79, normalized size = 0.7

$$4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right) - \frac{2\left(38a^2x^2-11ax+3\right)\sqrt{c-\frac{c}{ax}}}{15x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(3 - 11*a*x + 38*a^2*x^2))/(15*x^2) + 4*Sqrt[2]*a^2*S
qrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Maple [B] time = 0.128, size = 278, normalized size = 2.5

$$\frac{1}{15\,x^3}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-90\sqrt{ax^2-x}a^{7/2}\sqrt{a^{-1}}x^4 + 30\,a^{7/2}\sqrt{a^{-1}}\sqrt{(ax-1)\,x}x^4 + 60\,a^{5/2}\left(ax^2-x\right)^{3/2}x^2\sqrt{a^{-1}} + 45\,\ln\left(1/2\,\frac{2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x)
```

```
[Out] 1/15*(c*(a*x-1)/a/x)^(1/2)/x^3*(-90*(a*x^2-x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x^4
+30*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^4+60*a^(5/2)*(a*x^2-x)^(3/2)*x^
2*(1/a)^(1/2)+45*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^
(1/2)*x^4*a^3-30*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2
)*a-3*a*x+1)/(a*x+1))*x^4-45*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a
^(1/2))*(1/a)^(1/2)*x^4*a^3-16*a^(3/2)*(a*x^2-x)^(3/2)*x*(1/a)^(1/2)+6*(a*x
^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}}{(ax+1)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))/((a*x + 1)^2*x^3), x)
```

Fricas [A] time = 2.36566, size = 433, normalized size = 3.83

$$\left[\frac{2\left(15\sqrt{2}a^2\sqrt{c}x^2\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-\left(38\,a^2x^2-11\,ax+3\right)\sqrt{\frac{acx-c}{ax}}\right)}{15\,x^2},-\frac{2\left(30\sqrt{2}a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)\cdots\right.}{15\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="fricas")
```

```
[Out] [2/15*(15*sqrt(2)*a^2*sqrt(c)*x^2*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
 c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) - (38*a^2*x^2 - 11*a*x + 3)*sqrt((a*c*
x - c)/(a*x)))/x^2, -2/15*(30*sqrt(2)*a^2*sqrt(-c)*x^2*arctan(1/2*sqrt(2)*s
qrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (38*a^2*x^2 - 11*a*x + 3)*sqrt((a*c*x
- c)/(a*x)))/x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{c-\frac{c}{ax}}}{ax^4+x^3}\,dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax^4+x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**3,x)
```

```
[Out] -Integral(-sqrt(c - c/(a*x))/(a*x**4 + x**3), x) - Integral(a*x*sqrt(c - c/
(a*x))/(a*x**4 + x**3), x)
```

Giac [B] time = 2.17577, size = 375, normalized size = 3.32

$$-\frac{4\sqrt{2}a^3c\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}|a|\right)}{2a\sqrt{-c}}\right)}{\sqrt{-c}|a|\operatorname{sgn}(x)}-\frac{2\left(60\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^4a^5c-45\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^3a^4\right.}{15\left(\sqrt{a^2c}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a^3*c*arctan(1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*
x))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) - 2/15*(60*(
sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a^5*c - 45*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - a*c*x))^3*a^4*c^(3/2)*abs(a) + 35*(sqrt(a^2*c)*x - sqrt(a^2*c*
x^2 - a*c*x))^2*a^5*c^2 - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^4*
c^(5/2)*abs(a) + 3*a^5*c^3)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*a^
2*abs(a)*sgn(x))
```

3.606 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=113

$$\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^3\sqrt{c-\frac{c}{ax}}-4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 4*a^3*Sqrt[c - c/(a*x)] + (2*a^3*(c - c/(a*x))^(3/2))/(3*c) + (2*a^3*(c - c
/(a*x))^(7/2))/(7*c^3) - 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(S
qrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.281289, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6133, 25, 514, 446, 88, 50, 63, 208}

$$\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^3\sqrt{c-\frac{c}{ax}}-4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] 4*a^3*Sqrt[c - c/(a*x)] + (2*a^3*(c - c/(a*x))^(3/2))/(3*c) + (2*a^3*(c - c
/(a*x))^(7/2))/(7*c^3) - 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(S
qrt[2]*Sqrt[c])]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^4(1+ax)}\,dx$$
$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^3(1+ax)}\,dx}{c}$$
$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^4}\,dx}{c}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{x^2\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{a^2\left(c-\frac{cx}{a}\right)^{3/2}}{a+x} - \frac{a\left(c-\frac{cx}{a}\right)^{5/2}}{c}\right)dx, x, \frac{1}{x}\right)}{c}$$
$$= \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{a^3\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \left(2a^3\right)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx, x, \frac{1}{x}\right)$$
$$= 4a^3\sqrt{c-\frac{c}{ax}} + \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \left(4a^3c\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$
$$= 4a^3\sqrt{c-\frac{c}{ax}} + \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \left(8a^4\right)\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$
$$= 4a^3\sqrt{c-\frac{c}{ax}} + \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - 4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] time = 0.125415, size = 87, normalized size = 0.77

$$\frac{2\left(52a^3x^3-16a^2x^2+9ax-3\right)\sqrt{c-\frac{c}{ax}}}{21x^3} - 4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(-3 + 9*a*x - 16*a^2*x^2 + 52*a^3*x^3))/(21*x^3) - 4*S
qrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Maple [B] time = 0.127, size = 302, normalized size = 2.7

$$-\frac{1}{21x^4}\sqrt{\frac{c(ax-1)}{ax}}\left(-126\sqrt{ax^2-x}a^{9/2}\sqrt{a^{-1}}x^5 + 42a^{9/2}\sqrt{a^{-1}}\sqrt{(ax-1)x}x^5 + 84\left(ax^2-x\right)^{3/2}a^{7/2}\sqrt{a^{-1}}x^3 + 63\ln\left(1/2\,\frac{2\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x)
```

```
[Out] -1/21*(c*(a*x-1)/a/x)^(1/2)/x^4*(-126*(a*x^2-x)^(1/2)*a^(9/2)*(1/a)^(1/2)*x
^5+42*a^(9/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^5+84*(a*x^2-x)^(3/2)*a^(7/2)*
(1/a)^(1/2)*x^3+63*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a
)^(1/2)*x^5*a^4-42*a^(7/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1
/2)*a-3*a*x+1)/(a*x+1))*x^5-63*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)
/a^(1/2))*(1/a)^(1/2)*x^5*a^4-20*a^(5/2)*(a*x^2-x)^(3/2)*x^2*(1/a)^(1/2)+12
*a^(3/2)*(a*x^2-x)^(3/2)*x*(1/a)^(1/2)-6*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2
))/((a*x-1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(a^2x^2-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))/((a*x + 1)^2*x^4), x)
```

Fricas [A] time = 2.14058, size = 463, normalized size = 4.1

$$\left[\frac{2\left(21\sqrt{2}a^3\sqrt{c}x^3\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3acx+c}{ax+1}\right)+\left(52a^3x^3-16a^2x^2+9ax-3\right)\sqrt{\frac{acx-c}{ax}}\right)}{21x^3},\ \frac{2\left(42\sqrt{2}a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{2}\sqrt{-}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="fricas")
```

```
[Out] [2/21*(21*sqrt(2)*a^3*sqrt(c)*x^3*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + (52*a^3*x^3 - 16*a^2*x^2 + 9*a*x - 3)
*sqrt((a*c*x - c)/(a*x)))/x^3, 2/21*(42*sqrt(2)*a^3*sqrt(-c)*x^3*arctan(1/2
*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (52*a^3*x^3 - 16*a^2*x^2 + 9
*a*x - 3)*sqrt((a*c*x - c)/(a*x)))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{c-\frac{c}{ax}}}{ax^5+x^4}\,dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax^5+x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**4,x)
```

```
[Out] -Integral(-sqrt(c - c/(a*x))/(a*x**5 + x**4), x) - Integral(a*x*sqrt(c - c/
(a*x))/(a*x**5 + x**4), x)
```

Giac [B] time = 2.43882, size = 481, normalized size = 4.26

$$\frac{4\sqrt{2}a^4c\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}|a|\right)}{2a\sqrt{-c}}\right)}{\sqrt{-c}|a|\mathrm{sgn}\left(x\right)}+\frac{2\left(84\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^6a^7c-84\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^5a^6c^{\frac{3}{2}}|a|\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^4*c*arctan(1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x
))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) + 2/21*(84*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^6*a^7*c - 84*(sqrt(a^2*c)*x - sqrt(
a^2*c*x^2 - a*c*x))^5*a^6*c^(3/2)*abs(a) + 112*(sqrt(a^2*c)*x - sqrt(a^2*c*
x^2 - a*c*x))^4*a^7*c^2 - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*a
^6*c^(5/2)*abs(a) + 63*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^7*c^3
- 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^6*c^(7/2)*abs(a) + 3*a^7*c
^4)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^7*a^3*abs(a)*sgn(x))
```

3.607 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=163

$$-\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^4\sqrt{c-\frac{c}{ax}}+4\sqrt{2}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] -4*a^4*Sqrt[c - c/(a*x)] - (2*a^4*(c - c/(a*x))^(3/2))/(3*c) - (2*a^4*(c -
c/(a*x))^(5/2))/(5*c^2) + (2*a^4*(c - c/(a*x))^(7/2))/(7*c^3) - (2*a^4*(c -
 c/(a*x))^(9/2))/(9*c^4) + 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/
(Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.302679, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6133, 25, 514, 446, 88, 50, 63, 208}

$$-\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^4\sqrt{c-\frac{c}{ax}}+4\sqrt{2}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x^5),x]
```

```
[Out] -4*a^4*Sqrt[c - c/(a*x)] - (2*a^4*(c - c/(a*x))^(3/2))/(3*c) - (2*a^4*(c -
c/(a*x))^(5/2))/(5*c^2) + (2*a^4*(c - c/(a*x))^(7/2))/(7*c^3) - (2*a^4*(c -
 c/(a*x))^(9/2))/(9*c^4) + 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/
(Sqrt[2]*Sqrt[c])]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^5(1+ax)}\,dx$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^4(1+ax)}dx}{c}$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^5}dx}{c}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^3\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{a\,\text{Subst}\left(\int \left(a^2\left(c-\frac{cx}{a}\right)^{3/2} - \frac{a^3\left(c-\frac{cx}{a}\right)^{3/2}}{a+x} - \frac{a^2\left(c-\frac{cx}{a}\right)^{5/2}}{c} + \frac{a^2\left(c-\frac{cx}{a}\right)^{7/2}}{c^2}\right)dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} - \frac{a^4\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} - \left(2a^4\right)\text{Subst}\left(\int\right.$$

$$= -4a^4\sqrt{c-\frac{c}{ax}} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} - (4$$

$$= -4a^4\sqrt{c-\frac{c}{ax}} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} + (8$$

$$= -4a^4\sqrt{c-\frac{c}{ax}} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} + 4$$

Mathematica [A] time = 0.128243, size = 95, normalized size = 0.58

$$4\sqrt{2}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right) - \frac{2\left(788a^4x^4 - 236a^3x^3 + 138a^2x^2 - 95ax + 35\right)\sqrt{c-\frac{c}{ax}}}{315x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x^5),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(35 - 95*a*x + 138*a^2*x^2 - 236*a^3*x^3 + 788*a^4*x^
4))/(315*x^4) + 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sq
rt[c])]
```

Maple [B] time = 0.131, size = 326, normalized size = 2.

$$\frac{1}{315\,x^5}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-1890\sqrt{ax^2-x}a^{11/2}\sqrt{a^{-1}}x^6 + 630\,a^{11/2}\sqrt{a^{-1}}\sqrt{(ax-1)\,x}x^6 + 1260\left(ax^2-x\right)^{3/2}a^{9/2}\sqrt{a^{-1}}x^4 + 945\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x)
```

```
[Out] 1/315*(c*(a*x-1)/a/x)^(1/2)/x^5*(-1890*(a*x^2-x)^(1/2)*a^(11/2)*(1/a)^(1/2)
*x^6+630*a^(11/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^6+1260*(a*x^2-x)^(3/2)*a^
(9/2)*(1/a)^(1/2)*x^4+945*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2
))*(1/a)^(1/2)*x^6*a^5-630*a^(9/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-
1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^6-945*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)
+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^6*a^5-316*(a*x^2-x)^(3/2)*a^(7/2)*(1/a)^(1
/2)*x^3+156*a^(5/2)*(a*x^2-x)^(3/2)*x^2*(1/a)^(1/2)-120*a^(3/2)*(a*x^2-x)^(
3/2)*x*(1/a)^(1/2)+70*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/2
)/a^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}}{(ax+1)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))/((a*x + 1)^2*x^5), x)
```

Fricas [A] time = 2.30128, size = 520, normalized size = 3.19

$$\left[\frac{2\left(315\sqrt{2}a^4\sqrt{c}x^4\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-\left(788\,a^4x^4-236\,a^3x^3+138\,a^2x^2-95\,ax+35\right)\sqrt{\frac{acx-c}{ax}}\right)}{315\,x^4}, -\frac{2\left(630\sqrt{2}a^4\sqrt{\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="fricas")
```

```
[Out] [2/315*(315*sqrt(2)*a^4*sqrt(c)*x^4*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
 - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) - (788*a^4*x^4 - 236*a^3*x^3 + 138*a
^2*x^2 - 95*a*x + 35)*sqrt((a*c*x - c)/(a*x)))/x^4, -2/315*(630*sqrt(2)*a^4
*sqrt(-c)*x^4*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (788
*a^4*x^4 - 236*a^3*x^3 + 138*a^2*x^2 - 95*a*x + 35)*sqrt((a*c*x - c)/(a*x))
)/x^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{c-\frac{c}{ax}}}{ax^6+x^5}\,dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax^6+x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**5,x)
```

```
[Out] -Integral(-sqrt(c - c/(a*x))/(a*x**6 + x**5), x) - Integral(a*x*sqrt(c - c/
(a*x))/(a*x**6 + x**5), x)
```

Giac [B] time = 2.88813, size = 586, normalized size = 3.6

$$-\frac{4\sqrt{2}a^5c\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}|a|\right)}{2a\sqrt{-c}}\right)}{\sqrt{-c}|a|\mathrm{sgn}(x)}-\frac{2\left(1260\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^8a^9c-1260\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a^5*c*arctan(1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*
x))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) - 2/315*(126
0*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^8*a^9*c - 1260*(sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - a*c*x))^7*a^8*c^(3/2)*abs(a) + 2100*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - a*c*x))^6*a^9*c^2 - 3150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c
*x))^5*a^8*c^(5/2)*abs(a) + 3528*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^
4*a^9*c^3 - 2625*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*a^8*c^(7/2)*ab
s(a) + 1215*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^9*c^4 - 315*(sqrt
(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^8*c^(9/2)*abs(a) + 35*a^9*c^5)/((sqr
t(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^9*a^4*abs(a)*sgn(x))
```

3.608 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

Optimal. Leaf size=262

$$\frac{1115x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{96a^2\sqrt{1-ax}}-\frac{1115x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{64a^3\sqrt{1-ax}}+\frac{1115\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{64a^{7/2}\sqrt{1-ax}}+\frac{x^4\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4\sqrt{1-ax}}+\frac{8x^4\sqrt{c-}}{\sqrt{1-ax}\sqrt{ax}}$$

```
[Out] (8*Sqrt[c - c/(a*x)]*x^4)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (1115*Sqrt[c - c/
(a*x)]*x*Sqrt[1 + a*x])/(64*a^3*Sqrt[1 - a*x]) + (1115*Sqrt[c - c/(a*x)]*x^
2*Sqrt[1 + a*x])/(96*a^2*Sqrt[1 - a*x]) - (223*Sqrt[c - c/(a*x)]*x^3*Sqrt[1
 + a*x])/(24*a*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)]*x^4*Sqrt[1 + a*x])/(4*Sq
rt[1 - a*x]) + (1115*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(6
4*a^(7/2)*Sqrt[1 - a*x])
```

Rubi [A] time = 0.267233, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6134, 6129, 89, 80, 50, 54, 215}

$$\frac{1115x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{96a^2\sqrt{1-ax}}-\frac{1115x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{64a^3\sqrt{1-ax}}+\frac{1115\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{64a^{7/2}\sqrt{1-ax}}+\frac{x^4\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4\sqrt{1-ax}}+\frac{8x^4\sqrt{c-}}{\sqrt{1-ax}\sqrt{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^3)/E^(3*ArcTanh[a*x]),x]
```


```
[Out] (8*Sqrt[c - c/(a*x)]*x^4)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (1115*Sqrt[c - c/
(a*x)]*x*Sqrt[1 + a*x])/(64*a^3*Sqrt[1 - a*x]) + (1115*Sqrt[c - c/(a*x)]*x^
2*Sqrt[1 + a*x])/(96*a^2*Sqrt[1 - a*x]) - (223*Sqrt[c - c/(a*x)]*x^3*Sqrt[1
 + a*x])/(24*a*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)]*x^4*Sqrt[1 + a*x])/(4*Sq
rt[1 - a*x]) + (1115*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(6
4*a^(7/2)*Sqrt[1 - a*x])
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
 1]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{-3\tanh^{-1}(ax)}x^{5/2}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{5/2}(1-ax)^2}{(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{8\sqrt{c-\frac{c}{ax}}x^4}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{5/2}\left(\frac{27a^2}{2}-\frac{a^3x}{2}\right)}{\sqrt{1+ax}}\,dx}{a^2\sqrt{1-ax}}$$
$$= \frac{8\sqrt{c-\frac{c}{ax}}x^4}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^4\sqrt{1+ax}}{4\sqrt{1-ax}} - \frac{\left(223\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{5/2}}{\sqrt{1+ax}}\,dx}{8\sqrt{1-ax}}$$
$$= \frac{8\sqrt{c-\frac{c}{ax}}x^4}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{223\sqrt{c-\frac{c}{ax}}x^3\sqrt{1+ax}}{24a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^4\sqrt{1+ax}}{4\sqrt{1-ax}} + \frac{\left(1115\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int}{48a\sqrt{1-ax}}$$
$$= \frac{8\sqrt{c-\frac{c}{ax}}x^4}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{1115\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{96a^2\sqrt{1-ax}} - \frac{223\sqrt{c-\frac{c}{ax}}x^3\sqrt{1+ax}}{24a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^4\sqrt{1+}}{4\sqrt{1-ax}}$$
$$= \frac{8\sqrt{c-\frac{c}{ax}}x^4}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{1115\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{64a^3\sqrt{1-ax}} + \frac{1115\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{96a^2\sqrt{1-ax}} - \frac{223\sqrt{c-\frac{c}{ax}}x^3\sqrt{1}}{24a\sqrt{1-ax}}$$
$$= \frac{8\sqrt{c-\frac{c}{ax}}x^4}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{1115\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{64a^3\sqrt{1-ax}} + \frac{1115\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{96a^2\sqrt{1-ax}} - \frac{223\sqrt{c-\frac{c}{ax}}x^3\sqrt{1}}{24a\sqrt{1-ax}}$$
$$= \frac{8\sqrt{c-\frac{c}{ax}}x^4}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{1115\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{64a^3\sqrt{1-ax}} + \frac{1115\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{96a^2\sqrt{1-ax}} - \frac{223\sqrt{c-\frac{c}{ax}}x^3\sqrt{1}}{24a\sqrt{1-ax}}$$

Mathematica [A] time = 0.0617415, size = 108, normalized size = 0.41

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\left(48a^4x^4-200a^3x^3+446a^2x^2-1115ax-3345\right)+3345\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{192a^{7/2}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^3)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*(-3345 - 1115*a*x + 446*a^2*x^2
 - 200*a^3*x^3 + 48*a^4*x^4) + 3345*Sqrt[1 + a*x]*ArcSinh[Sqrt[a]*Sqrt[x]])
)/(192*a^(7/2)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.149, size = 194, normalized size = 0.7

$$-\frac{x}{(384\,ax+384)\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\left(96\,a^{9/2}\sqrt{-(ax+1)\,x}x^4-400\,a^{7/2}x^3\sqrt{-(ax+1)\,x}+892\,a^{5/2}x^2\sqrt{-(ax+1)\,x}-2230\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/384*(c*(a*x-1)/a/x)^(1/2)*x*(96*a^(9/2)*(-(a*x+1)*x)^(1/2)*x^4-400*a^(7/
2)*x^3*(-(a*x+1)*x)^(1/2)+892*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2)-2230*a^(3/2)*x
```

```
*(-(a*x+1)*x)^(1/2)-3345*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x
*a-6690*a^(1/2)*(-(a*x+1)*x)^(1/2)-3345*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x
+1)*x)^(1/2)))*(-a^2*x^2+1)^(1/2)/a^(7/2)/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}\,x^3}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))*x^3/(a*x + 1)^3, x)
```

Fricas [A] time = 2.52605, size = 738, normalized size = 2.82

$$\left[\frac{3345\left(a^2x^2-1\right)\sqrt{-c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(48\,a^5x^5-200\,a^4x^4+446\,a^3x^3-1115\,a^2x^2\right.}{768\left(a^6x^2-a^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fr
icas")
```

```
[Out] [1/768*(3345*(a^2*x^2 - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*
x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
1)) - 4*(48*a^5*x^5 - 200*a^4*x^4 + 446*a^3*x^3 - 1115*a^2*x^2 - 3345*a*x)*
sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^6*x^2 - a^4), -1/384*(3345*(
a^2*x^2 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x -
c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(48*a^5*x^5 - 200*a^4*x^4 + 446*a^
3*x^3 - 1115*a^2*x^2 - 3345*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x))
)/(a^6*x^2 - a^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}\,x^3}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))*x^3/(a*x + 1)^3, x)
```

3.609 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal. Leaf size=218

$$\frac{119x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-ax}}-\frac{119\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{8a^{5/2}\sqrt{1-ax}}+\frac{x^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}}+\frac{8x^3\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{119x^2\sqrt{ax+1}\sqrt{c\cdots}}{12a\sqrt{1-ax}}$$

```
[Out] (8*Sqrt[c - c/(a*x)]*x^3)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (119*Sqrt[c - c/(
a*x)]*x*Sqrt[1 + a*x])/(8*a^2*Sqrt[1 - a*x]) - (119*Sqrt[c - c/(a*x)]*x^2*S
qrt[1 + a*x])/(12*a*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)]*x^3*Sqrt[1 + a*x])/
(3*Sqrt[1 - a*x]) - (119*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]]
)/(8*a^(5/2)*Sqrt[1 - a*x])
```

Rubi [A] time = 0.250208, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6134, 6129, 89, 80, 50, 54, 215}

$$\frac{119x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-ax}}-\frac{119\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{8a^{5/2}\sqrt{1-ax}}+\frac{x^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}}+\frac{8x^3\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{119x^2\sqrt{ax+1}\sqrt{c\cdots}}{12a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (8*Sqrt[c - c/(a*x)]*x^3)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (119*Sqrt[c - c/(
a*x)]*x*Sqrt[1 + a*x])/(8*a^2*Sqrt[1 - a*x]) - (119*Sqrt[c - c/(a*x)]*x^2*S
qrt[1 + a*x])/(12*a*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)]*x^3*Sqrt[1 + a*x])/
(3*Sqrt[1 - a*x]) - (119*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]]
)/(8*a^(5/2)*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{-3\tanh^{-1}(ax)}x^{3/2}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}(1-ax)^2}{(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}x^3}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}\left(\frac{19a^2}{2}-\frac{a^3x}{2}\right)}{\sqrt{1+ax}}\,dx}{a^2\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}x^3}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1+ax}}{3\sqrt{1-ax}} - \frac{\left(119\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}}{\sqrt{1+ax}}\,dx}{6\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}x^3}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{119\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1+ax}}{3\sqrt{1-ax}} + \frac{\left(119\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{}{\sqrt{1}}}{8a\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}x^3}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{119\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} - \frac{119\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1+ax}}{3\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}x^3}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{119\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} - \frac{119\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1+ax}}{3\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}x^3}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{119\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} - \frac{119\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1+ax}}{3\sqrt{1-ax}}$$

Mathematica [A] time = 0.0587841, size = 100, normalized size = 0.46

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\left(8a^3x^3-38a^2x^2+119ax+357\right)-357\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{24a^{5/2}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*(357 + 119*a*x - 38*a^2*x^2 + 8
*a^3*x^3) - 357*Sqrt[1 + a*x]*ArcSinh[Sqrt[a]*Sqrt[x]]))/(24*a^(5/2)*Sqrt[1
 - a^2*x^2])
```

Maple [A] time = 0.138, size = 176, normalized size = 0.8

$$-\frac{x}{(48\,ax+48)\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\left(16\,a^{7/2}x^3\sqrt{-(ax+1)\,x}-76\,a^{5/2}x^2\sqrt{-(ax+1)\,x}+238\,a^{3/2}x\sqrt{-(ax+1)\,x}+357\,\mathrm{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/48*(c*(a*x-1)/a/x)^(1/2)*x*(16*a^(7/2)*x^3*(-(a*x+1)*x)^(1/2)-76*a^(5/2)
*x^2*(-(a*x+1)*x)^(1/2)+238*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+357*arctan(1/2/a^(
1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a+714*a^(1/2)*(-(a*x+1)*x)^(1/2)+357*a
rctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))/a^(5/2)*(-a^2*x^2+1)^(1/2)
/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}\,x^2}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))*x^2/(a*x + 1)^3, x)
```

Fricas [A] time = 2.52247, size = 683, normalized size = 3.13

$$\left[\frac{357\left(a^2x^2-1\right)\sqrt{-c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(8\,a^4x^4-38\,a^3x^3+119\,a^2x^2+357\,ax\right)\sqrt{-a^2x^2}}{96\left(a^5x^2-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fr
icas")
```

```
[Out] [1/96*(357*(a^2*x^2 - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^
2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)
) - 4*(8*a^4*x^4 - 38*a^3*x^3 + 119*a^2*x^2 + 357*a*x)*sqrt(-a^2*x^2 + 1)*s
qrt((a*c*x - c)/(a*x)))/(a^5*x^2 - a^3), 1/48*(357*(a^2*x^2 - 1)*sqrt(c)*ar
ctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
- a*c*x - c)) - 2*(8*a^4*x^4 - 38*a^3*x^3 + 119*a^2*x^2 + 357*a*x)*sqrt(-a^
2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x^2 - a^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}x^2}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))*x^2/(a*x + 1)^3, x)
```

3.610 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal. Leaf size=174

$$\frac{47\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}+\frac{x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-ax}}+\frac{8x^2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{47x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-ax}}$$

```
[Out] (8*Sqrt[c - c/(a*x)]*x^2)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (47*Sqrt[c - c/(a
*x)]*x*Sqrt[1 + a*x])/(4*a*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)]*x^2*Sqrt[1 +
 a*x])/(2*Sqrt[1 - a*x]) + (47*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sq
rt[x]])/(4*a^(3/2)*Sqrt[1 - a*x])
```

Rubi [A] time = 0.198755, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {6134, 6129, 89, 80, 50, 54, 215}

$$\frac{47\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}+\frac{x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-ax}}+\frac{8x^2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{47x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^(3*ArcTanh[a*x]),x]
```


```
[Out] (8*Sqrt[c - c/(a*x)]*x^2)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (47*Sqrt[c - c/(a
*x)]*x*Sqrt[1 + a*x])/(4*a*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)]*x^2*Sqrt[1 +
 a*x])/(2*Sqrt[1 - a*x]) + (47*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sq
rt[x]])/(4*a^(3/2)*Sqrt[1 - a*x])
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{-3\tanh^{-1}(ax)}\sqrt{x}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}(1-ax)^2}{(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{8\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}\left(\frac{11a^2}{2}-\frac{a^3x}{2}\right)}{\sqrt{1+ax}}\,dx}{a^2\sqrt{1-ax}}\\
&= \frac{8\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{2\sqrt{1-ax}}-\frac{\left(47\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}}{\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}}\\
&= \frac{8\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{47\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{4a\sqrt{1-ax}}+\frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{2\sqrt{1-ax}}+\frac{\left(47\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+}}
{8a\sqrt{1-ax}}\\
&= \frac{8\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{47\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{4a\sqrt{1-ax}}+\frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{2\sqrt{1-ax}}+\frac{\left(47\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\right.}{4a\sqrt{1-}}\\
&= \frac{8\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{47\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{4a\sqrt{1-ax}}+\frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{2\sqrt{1-ax}}+\frac{47\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{\right.}}{4a^{3/2}\sqrt{1-ax}}
\end{aligned}
$$

Mathematica [A] time = 0.0563745, size = 92, normalized size = 0.53

$$
\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\left(2a^2x^2-13ax-47\right)+47\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{4a^{3/2}\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*(-47 - 13*a*x + 2*a^2*x^2) + 47
*Sqrt[1 + a*x]*ArcSinh[Sqrt[a]*Sqrt[x]]))/(4*a^(3/2)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.151, size = 158, normalized size = 0.9

$$-\frac{x}{(8ax+8)(ax-1)}\sqrt{\frac{c(ax-1)}{ax}}\left(4a^{5/2}x^2\sqrt{-(ax+1)x}-26a^{3/2}x\sqrt{-(ax+1)x}-47\arctan\left(1/2\frac{2ax+1}{\sqrt{a}\sqrt{-(ax+1)x}}\right)xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/8*(c*(a*x-1)/a/x)^(1/2)*x*(4*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2)-26*a^(3/2)*x
*(-(a*x+1)*x)^(1/2)-47*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a
-94*a^(1/2)*(-(a*x+1)*x)^(1/2)-47*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)
^(1/2)))*(-a^2*x^2+1)^(1/2)/a^(3/2)/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}x}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))*x/(a*x + 1)^3, x)
```

Fricas [A] time = 2.54116, size = 640, normalized size = 3.68

$$\left[\frac{47\left(a^2x^2-1\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(2a^3x^3-13a^2x^2-47ax\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{16\left(a^4x^2-a^2\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fric
as")
```

```
[Out] [1/16*(47*(a^2*x^2 - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2
 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1))
 - 4*(2*a^3*x^3 - 13*a^2*x^2 - 47*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/
(a*x)))/(a^4*x^2 - a^2), -1/8*(47*(a^2*x^2 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*
x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2
*(2*a^3*x^3 - 13*a^2*x^2 - 47*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x
```

```
)))/(a^4*x^2 - a^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)*
*3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}\,x}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))*x/(a*x + 1)^3, x)
```

3.611 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=123

$$\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{8x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{7\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

```
[Out] (8*Sqrt[c - c/(a*x)]*x)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a*x)]*
x*Sqrt[1 + a*x])/Sqrt[1 - a*x] - (7*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[
a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rubi [A] time = 0.144515, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6134, 6129, 89, 80, 54, 215}

$$\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{8x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{7\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (8*Sqrt[c - c/(a*x)]*x)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a*x)]*
x*Sqrt[1 + a*x])/Sqrt[1 - a*x] - (7*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[
a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^2}{\sqrt{x}(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{8\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\frac{3a^2}{2}-\frac{a^3x}{2}}{\sqrt{x}\sqrt{1+ax}}\,dx}{a^2\sqrt{1-ax}}\\
&= \frac{8\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{\left(7\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}}\\
&= \frac{8\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{\left(7\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{\sqrt{1-ax}}\\
&= \frac{8\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{7\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}
\end{aligned}$$

Mathematica [A] time = 0.0490833, size = 80, normalized size = 0.65

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}(ax+9)-7\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{\sqrt{a}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*(9 + a*x) - 7*Sqrt[1 + a*x]*Arc
Sinh[Sqrt[a]*Sqrt[x]]))/(Sqrt[a]*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.145, size = 140, normalized size = 1.1

$$-\frac{x}{(2ax+2)(ax-1)}\sqrt{\frac{c(ax-1)}{ax}}\left(2a^{3/2}x\sqrt{-(ax+1)x}+7\arctan\left(1/2\frac{2ax+1}{\sqrt{a}\sqrt{-(ax+1)x}}\right)xa+18\sqrt{a}\sqrt{-(ax+1)x}+7\text{ a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+7*arctan(1/2/a
^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a+18*a^(1/2)*(-(a*x+1)*x)^(1/2)+7*ar
ctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))*(-a^2*x^2+1)^(1/2)/a^(1/2)/
(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/(a*x + 1)^3, x)
```

Fricas [A] time = 2.43076, size = 586, normalized size = 4.76

$$\left[\frac{7\left(a^2x^2-1\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(a^2x^2+9ax\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{4\left(a^3x^2-a\right)},\ \frac{7\left(a^2x^2-1\right)\sqrt{\ldots}}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(7*(a^2*x^2 - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 +
 a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) -
 4*(a^2*x^2 + 9*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 -
 a), 1/2*(7*(a^2*x^2 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*s
qrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(a^2*x^2 + 9*a*x)*sqr
t(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/(a*x + 1)^3, x)
```

3.612 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=124

$$-\frac{10ax\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{2\sqrt{a}\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}$$

```
[Out] (-2*Sqrt[c - c/(a*x)])/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (10*a*Sqrt[c - c/(a*
x)]*x)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (2*Sqrt[a]*Sqrt[c - c/(a*x)]*Sqrt[x]
*ArcSinh[Sqrt[a]*Sqrt[x]])/Sqrt[1 - a*x]
```

Rubi [A] time = 0.236607, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6134, 6129, 89, 78, 54, 215}

$$-\frac{10ax\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{2\sqrt{a}\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)])/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (10*a*Sqrt[c - c/(a*
x)]*x)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (2*Sqrt[a]*Sqrt[c - c/(a*x)]*Sqrt[x]
*ArcSinh[Sqrt[a]*Sqrt[x]])/Sqrt[1 - a*x]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{3/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^2}{x^{3/2}(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-2a+\frac{a^2x}{2}}{\sqrt{x}(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{10a\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{10a\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(2a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{10a\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{2\sqrt{a}\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}
\end{aligned}$$

Mathematica [A] time = 0.0512749, size = 70, normalized size = 0.56

$$-\frac{2\sqrt{c-\frac{c}{ax}}\left(5ax-\sqrt{a}\sqrt{x}\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+1\right)}{\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + 5*a*x - Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*ArcSinh[Sq
rt[a]*Sqrt[x]]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.152, size = 142, normalized size = 1.2

$$\frac{1}{(ax-1)(ax+1)}\sqrt{\frac{c(ax-1)}{ax}}\left(\arctan\left(\frac{2ax+1}{2}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-(ax+1)x}}\right)x^2a^2+10a^{3/2}x\sqrt{-(ax+1)x}+\arctan\left(\frac{2ax+1}{2}\frac{1}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x)
```

```
[Out] (c*(a*x-1)/a/x)^(1/2)*(arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^2
*a^2+10*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)
*x)^(1/2))*x*a+2*a^(1/2)*(-(a*x+1)*x)^(1/2))*(-a^2*x^2+1)^(1/2)/(a*x+1)/a^(
1/2)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{(ax+1)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/((a*x + 1)^3*x), x)
```

Fricas [A] time = 2.45656, size = 560, normalized size = 4.52

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{-c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\sqrt{-a^2x^2+1}(5\,ax+1)\sqrt{\frac{acx-c}{ax}}}{2\left(a^2x^2-1\right)}, -\frac{\left(a^2x^2-1\right)\sqrt{c}\,\mathrm{arc}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="fric
as")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a
*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4
*sqrt(-a^2*x^2 + 1)*(5*a*x + 1)*sqrt((a*c*x - c)/(a*x)))/(a^2*x^2 - 1), -((
a^2*x^2 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x -
c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*sqrt(-a^2*x^2 + 1)*(5*a*x + 1)*sqr
t((a*c*x - c)/(a*x)))/(a^2*x^2 - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}{x(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2)/(x*(a*x + 1)
**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{(ax+1)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="giac
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/((a*x + 1)^3*x), x)
```

3.613 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=123

$$\frac{46a^2x\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}\sqrt{ax+1}}+\frac{20a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}\sqrt{ax+1}}-\frac{2\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (20*a*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (2*Sqrt[c - c/(a
*x)])/(3*x*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (46*a^2*Sqrt[c - c/(a*x)]*x)/(3*S
qrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.223131, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6134, 6129, 89, 78, 37}

$$\frac{46a^2x\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}\sqrt{ax+1}}+\frac{20a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}\sqrt{ax+1}}-\frac{2\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x^2),x]
```


```
[Out] (20*a*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (2*Sqrt[c - c/(a
*x)])/(3*x*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (46*a^2*Sqrt[c - c/(a*x)]*x)/(3*S
qrt[1 - a*x]*Sqrt[1 + a*x])
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{5/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^2}{x^{5/2}(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-5a+\frac{3a^2x}{2}}{x^{3/2}(1+ax)^{3/2}}\,dx}{3\sqrt{1-ax}}\\
&= \frac{20a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}\sqrt{1+ax}}-\frac{2\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(23a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}(1+ax)^{3/2}}\,dx}{3\sqrt{1-ax}}\\
&= \frac{20a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}\sqrt{1+ax}}-\frac{2\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}\sqrt{1+ax}}+\frac{46a^2\sqrt{c-\frac{c}{ax}}\,x}{3\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.0296321, size = 50, normalized size = 0.41

$$\frac{2\left(23a^2x^2+10ax-1\right)\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(-1 + 10*a*x + 23*a^2*x^2))/(3*x*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.087, size = 61, normalized size = 0.5

$$\frac{46\,a^2x^2+20\,ax-2}{3\,(ax+1)^2\,x\,(ax-1)^2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x)
```

```
[Out] 2/3*(23*a^2*x^2+10*a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(-a^2*x^2+1)^(3/2)/(a*x+1)^
2/x/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{\left(ax+1\right)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/((a*x + 1)^3*x^2), x)
```

Fricas [A] time = 2.12165, size = 120, normalized size = 0.98

$$-\frac{2\left(23\,a^2x^2+10\,ax-1\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{3\left(a^2x^3-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="fr
icas")
```

```
[Out] -2/3*(23*a^2*x^2 + 10*a*x - 1)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x))/(
a^2*x^3 - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{x^2\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2)/(x**2*(a*x +
 1)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{\left(ax+1\right)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/((a*x + 1)^3*x^2), x)
```

3.614 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=166

$$-\frac{316a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{15\sqrt{1-ax}}+\frac{158a^2\sqrt{c-\frac{c}{ax}}}{15\sqrt{1-ax}\sqrt{ax+1}}-\frac{2\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}\sqrt{ax+1}}+\frac{32a\sqrt{c-\frac{c}{ax}}}{15x\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (158*a^2*Sqrt[c - c/(a*x)])/(15*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (2*Sqrt[c -
c/(a*x)])/(5*x^2*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (32*a*Sqrt[c - c/(a*x)])/(1
5*x*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (316*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x]
)/(15*Sqrt[1 - a*x])
```

Rubi [A] time = 0.233736, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6134, 6129, 89, 78, 45, 37}

$$-\frac{316a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{15\sqrt{1-ax}}+\frac{158a^2\sqrt{c-\frac{c}{ax}}}{15\sqrt{1-ax}\sqrt{ax+1}}-\frac{2\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}\sqrt{ax+1}}+\frac{32a\sqrt{c-\frac{c}{ax}}}{15x\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x^3),x]
```


```
[Out] (158*a^2*Sqrt[c - c/(a*x)])/(15*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (2*Sqrt[c -
c/(a*x)])/(5*x^2*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (32*a*Sqrt[c - c/(a*x)])/(1
5*x*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (316*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x]
)/(15*Sqrt[1 - a*x])
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{7/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^2}{x^{7/2}(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-8a+\frac{5a^2x}{2}}{x^{5/2}(1+ax)^{3/2}}\,dx}{5\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}\sqrt{1+ax}}+\frac{32a\sqrt{c-\frac{c}{ax}}}{15x\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(79a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{x^{3/2}(1+ax)^{3/2}}\,dx}{15\sqrt{1-ax}}\\
&= \frac{158a^2\sqrt{c-\frac{c}{ax}}}{15\sqrt{1-ax}\sqrt{1+ax}}-\frac{2\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}\sqrt{1+ax}}+\frac{32a\sqrt{c-\frac{c}{ax}}}{15x\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(158a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\cdots}{15\sqrt{1-\cdots}}\\
&= \frac{158a^2\sqrt{c-\frac{c}{ax}}}{15\sqrt{1-ax}\sqrt{1+ax}}-\frac{2\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}\sqrt{1+ax}}+\frac{32a\sqrt{c-\frac{c}{ax}}}{15x\sqrt{1-ax}\sqrt{1+ax}}-\frac{316a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+\cdots}}{15\sqrt{1-ax}}
\end{aligned}
$$

Mathematica [A] time = 0.0308127, size = 58, normalized size = 0.35

$$
-\frac{2\left(158a^3x^3+79a^2x^2-16ax+3\right)\sqrt{c-\frac{c}{ax}}}{15x^2\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(3 - 16*a*x + 79*a^2*x^2 + 158*a^3*x^3))/(15*x^2*Sqrt
[1 - a^2*x^2])
```

Maple [A] time = 0.086, size = 69, normalized size = 0.4

$$-\frac{316\,x^3a^3+158\,a^2x^2-32\,ax+6}{15\,(ax+1)^2\,x^2\,(ax-1)^2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x)
```

```
[Out] -2/15*(158*a^3*x^3+79*a^2*x^2-16*a*x+3)*(c*(a*x-1)/a/x)^(1/2)*(-a^2*x^2+1)^
(3/2)/(a*x+1)^2/x^2/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{(ax+1)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/((a*x + 1)^3*x^3), x)
```

Fricas [A] time = 2.1239, size = 142, normalized size = 0.86

$$\frac{2\left(158\,a^3x^3+79\,a^2x^2-16\,ax+3\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{15\left(a^2x^4-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="fr
icas")
```

```
[Out] 2/15*(158*a^3*x^3 + 79*a^2*x^2 - 16*a*x + 3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x
 - c)/(a*x))/(a^2*x^4 - x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{\left(ax+1\right)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/((a*x + 1)^3*x^3), x)
```

3.615 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=213

$$\frac{2672a^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105\sqrt{1-ax}}-\frac{1336a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105x\sqrt{1-ax}}+\frac{334a^2\sqrt{c-\frac{c}{ax}}}{35x\sqrt{1-ax}\sqrt{ax+1}}+\frac{44a\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}\sqrt{ax+1}}-\frac{2\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (-2*Sqrt[c - c/(a*x)])/(7*x^3*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (44*a*Sqrt[c -
 c/(a*x)])/(35*x^2*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (334*a^2*Sqrt[c - c/(a*x)
])/(35*x*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (2672*a^3*Sqrt[c - c/(a*x)]*Sqrt[1
+ a*x])/(105*Sqrt[1 - a*x]) - (1336*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(1
05*x*Sqrt[1 - a*x])
```

Rubi [A] time = 0.244291, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6134, 6129, 89, 78, 45, 37}

$$\frac{2672a^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105\sqrt{1-ax}}-\frac{1336a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105x\sqrt{1-ax}}+\frac{334a^2\sqrt{c-\frac{c}{ax}}}{35x\sqrt{1-ax}\sqrt{ax+1}}+\frac{44a\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}\sqrt{ax+1}}-\frac{2\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x^4),x]
```


```
[Out] (-2*Sqrt[c - c/(a*x)])/(7*x^3*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (44*a*Sqrt[c -
 c/(a*x)])/(35*x^2*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (334*a^2*Sqrt[c - c/(a*x)
])/(35*x*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (2672*a^3*Sqrt[c - c/(a*x)]*Sqrt[1
+ a*x])/(105*Sqrt[1 - a*x]) - (1336*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(1
05*x*Sqrt[1 - a*x])
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
1]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{9/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^2}{x^{9/2}(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-11a+\frac{7a^2x}{2}}{x^{7/2}(1+ax)^{3/2}}\,dx}{7\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}\sqrt{1+ax}}+\frac{44a\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(167a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{x^{5/2}(1+ax)^{3/2}}\,dx}{35\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}\sqrt{1+ax}}+\frac{44a\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}\sqrt{1+ax}}+\frac{334a^2\sqrt{c-\frac{c}{ax}}}{35x\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(668a^2\sqrt{c-\frac{c}{ax}}\right.}{35\sqrt{}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}\sqrt{1+ax}}+\frac{44a\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}\sqrt{1+ax}}+\frac{334a^2\sqrt{c-\frac{c}{ax}}}{35x\sqrt{1-ax}\sqrt{1+ax}}-\frac{1336a^2\sqrt{c-\frac{c}{ax}}}{105x\sqrt{1-}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}\sqrt{1+ax}}+\frac{44a\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}\sqrt{1+ax}}+\frac{334a^2\sqrt{c-\frac{c}{ax}}}{35x\sqrt{1-ax}\sqrt{1+ax}}+\frac{2672a^3\sqrt{c-\frac{c}{ax}}}{105\sqrt{1-}}
\end{aligned}$$

Mathematica [A] time = 0.0331991, size = 66, normalized size = 0.31

$$\frac{2\left(1336a^4x^4+668a^3x^3-167a^2x^2+66ax-15\right)\sqrt{c-\frac{c}{ax}}}{105x^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(-15 + 66*a*x - 167*a^2*x^2 + 668*a^3*x^3 + 1336*a^4*x
^4))/(105*x^3*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.082, size = 77, normalized size = 0.4

$$\frac{2672\,x^4a^4+1336\,x^3a^3-334\,a^2x^2+132\,ax-30}{105\,\left(ax+1\right)^2x^3\left(ax-1\right)^2}\sqrt{\frac{c\left(ax-1\right)}{ax}}\left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x)
```

```
[Out] 2/105*(1336*a^4*x^4+668*a^3*x^3-167*a^2*x^2+66*a*x-15)*(c*(a*x-1)/a/x)^(1/2
)*(-a^2*x^2+1)^(3/2)/(a*x+1)^2/x^3/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{\left(ax+1\right)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/((a*x + 1)^3*x^4), x)
```

Fricas [A] time = 2.10802, size = 167, normalized size = 0.78

$$-\frac{2\left(1336\,a^4x^4+668\,a^3x^3-167\,a^2x^2+66\,ax-15\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{105\left(a^2x^5-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="fr
icas")
```

```
[Out] -2/105*(1336*a^4*x^4 + 668*a^3*x^3 - 167*a^2*x^2 + 66*a*x - 15)*sqrt(-a^2*x
^2 + 1)*sqrt((a*c*x - c)/(a*x))/(a^2*x^5 - x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{\left(ax+1\right)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/((a*x + 1)^3*x^4), x)
```

3.616 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=257

$$-\frac{164a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{15x^2\sqrt{1-ax}}+\frac{82a^2\sqrt{c-\frac{c}{ax}}}{9x^2\sqrt{1-ax}\sqrt{ax+1}}-\frac{1312a^4\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{45\sqrt{1-ax}}+\frac{656a^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{45x\sqrt{1-ax}}+\frac{8a\sqrt{c-\frac{c}{ax}}}{9x^3\sqrt{1-ax}\sqrt{ax+}}$$

```
[Out] (-2*Sqrt[c - c/(a*x)])/(9*x^4*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (8*a*Sqrt[c -
c/(a*x)])/(9*x^3*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (82*a^2*Sqrt[c - c/(a*x)])/
(9*x^2*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (1312*a^4*Sqrt[c - c/(a*x)]*Sqrt[1 +
a*x])/(45*Sqrt[1 - a*x]) - (164*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(15*x^
2*Sqrt[1 - a*x]) + (656*a^3*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(45*x*Sqrt[1 -
 a*x])
```

Rubi [A] time = 0.249195, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6134, 6129, 89, 78, 45, 37}

$$-\frac{164a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{15x^2\sqrt{1-ax}}+\frac{82a^2\sqrt{c-\frac{c}{ax}}}{9x^2\sqrt{1-ax}\sqrt{ax+1}}-\frac{1312a^4\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{45\sqrt{1-ax}}+\frac{656a^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{45x\sqrt{1-ax}}+\frac{8a\sqrt{c-\frac{c}{ax}}}{9x^3\sqrt{1-ax}\sqrt{ax+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x^5),x]
```


```
[Out] (-2*Sqrt[c - c/(a*x)])/(9*x^4*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (8*a*Sqrt[c -
c/(a*x)])/(9*x^3*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (82*a^2*Sqrt[c - c/(a*x)])/
(9*x^2*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (1312*a^4*Sqrt[c - c/(a*x)]*Sqrt[1 +
a*x])/(45*Sqrt[1 - a*x]) - (164*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(15*x^
2*Sqrt[1 - a*x]) + (656*a^3*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(45*x*Sqrt[1 -
 a*x])
```


Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
```

```
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{11/2}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^2}{x^{11/2}(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{9x^4\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-14a+\frac{9a^2x}{2}}{x^{9/2}(1+ax)^{3/2}}\,dx}{9\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{9x^4\sqrt{1-ax}\sqrt{1+ax}} + \frac{8a\sqrt{c-\frac{c}{ax}}}{9x^3\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(41a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{x^{7/2}(1+ax)^{3/2}}\,dx}{9\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{9x^4\sqrt{1-ax}\sqrt{1+ax}} + \frac{8a\sqrt{c-\frac{c}{ax}}}{9x^3\sqrt{1-ax}\sqrt{1+ax}} + \frac{82a^2\sqrt{c-\frac{c}{ax}}}{9x^2\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(82a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\cdots\right.}{3\sqrt{1-\cdots}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{9x^4\sqrt{1-ax}\sqrt{1+ax}} + \frac{8a\sqrt{c-\frac{c}{ax}}}{9x^3\sqrt{1-ax}\sqrt{1+ax}} + \frac{82a^2\sqrt{c-\frac{c}{ax}}}{9x^2\sqrt{1-ax}\sqrt{1+ax}} - \frac{164a^2\sqrt{c-\frac{c}{ax}}\sqrt{1\cdots}}{15x^2\sqrt{1-ax\cdots}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{9x^4\sqrt{1-ax}\sqrt{1+ax}} + \frac{8a\sqrt{c-\frac{c}{ax}}}{9x^3\sqrt{1-ax}\sqrt{1+ax}} + \frac{82a^2\sqrt{c-\frac{c}{ax}}}{9x^2\sqrt{1-ax}\sqrt{1+ax}} - \frac{164a^2\sqrt{c-\frac{c}{ax}}\sqrt{1\cdots}}{15x^2\sqrt{1-ax\cdots}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{9x^4\sqrt{1-ax}\sqrt{1+ax}} + \frac{8a\sqrt{c-\frac{c}{ax}}}{9x^3\sqrt{1-ax}\sqrt{1+ax}} + \frac{82a^2\sqrt{c-\frac{c}{ax}}}{9x^2\sqrt{1-ax}\sqrt{1+ax}} - \frac{1312a^4\sqrt{c-\frac{c}{ax}}\sqrt{\cdots}}{45\sqrt{1-ax\cdots}}$$

Mathematica [A] time = 0.0358775, size = 74, normalized size = 0.29

$$-\frac{2\left(656a^5x^5+328a^4x^4-82a^3x^3+41a^2x^2-20ax+5\right)\sqrt{c-\frac{c}{ax}}}{45x^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(5 - 20*a*x + 41*a^2*x^2 - 82*a^3*x^3 + 328*a^4*x^4 +
 656*a^5*x^5))/(45*x^4*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.088, size = 85, normalized size = 0.3

$$-\frac{1312\,x^5a^5+656\,x^4a^4-164\,x^3a^3+82\,a^2x^2-40\,ax+10}{45\,(ax+1)^2\,x^4\,(ax-1)^2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x)
```

```
[Out] -2/45*(656*a^5*x^5+328*a^4*x^4-82*a^3*x^3+41*a^2*x^2-20*a*x+5)*(c*(a*x-1)/a
/x)^(1/2)*(-a^2*x^2+1)^(3/2)/(a*x+1)^2/x^4/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{(ax+1)^3x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/((a*x + 1)^3*x^5), x)
```

Fricas [A] time = 2.14181, size = 178, normalized size = 0.69

$$\frac{2\left(656\,a^5x^5+328\,a^4x^4-82\,a^3x^3+41\,a^2x^2-20\,ax+5\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{45\left(a^2x^6-x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="fr
icas")
```

```
[Out] 2/45*(656*a^5*x^5 + 328*a^4*x^4 - 82*a^3*x^3 + 41*a^2*x^2 - 20*a*x + 5)*sqr
t(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x))/(a^2*x^6 - x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-a^2x^2+1)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{(ax+1)^3x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/((a*x + 1)^3*x^5), x)
```

3.617 $\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=64

$$\frac{x(1-ax)^{-p}\left(c-\frac{c}{ax}\right)^pF_1\left(1-p;\frac{1}{2}(n-2p),-\frac{n}{2};2-p;ax,-ax\right)}{1-p}$$

```
[Out] ((c - c/(a*x))^p*x*AppellF1[1 - p, (n - 2*p)/2, -n/2, 2 - p, a*x, -(a*x)])/
((1 - p)*(1 - a*x)^p)
```

Rubi [A] time = 0.0993379, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6134, 6129, 133}

$$\frac{x(1-ax)^{-p}\left(c-\frac{c}{ax}\right)^pF_1\left(1-p;\frac{1}{2}(n-2p),-\frac{n}{2};2-p;ax,-ax\right)}{1-p}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] ((c - c/(a*x))^p*x*AppellF1[1 - p, (n - 2*p)/2, -n/2, 2 - p, a*x, -(a*x)])/
((1 - p)*(1 - a*x)^p)
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx&=\left(\left(c-\frac{c}{ax}\right)^px^p(1-ax)^{-p}\right)\int e^{n\tanh^{-1}(ax)}x^{-p}(1-ax)^p\,dx\\&=\left(\left(c-\frac{c}{ax}\right)^px^p(1-ax)^{-p}\right)\int x^{-p}(1-ax)^{-\frac{n}{2}+p}(1+ax)^{n/2}\,dx\\&=\frac{\left(c-\frac{c}{ax}\right)^px(1-ax)^{-p}F_1\left(1-p;\frac{1}{2}(n-2p),-\frac{n}{2};2-p;ax,-ax\right)}{1-p}\end{aligned}$$

Mathematica [F] time = 0.618574, size = 0, normalized size = 0.

$$\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a*x))^p, x]
```

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{Artanh}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a/x)^p,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(c-c/a/x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^p,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\left(\frac{acx-c}{ax}\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^p,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)*((a*c*x - c)/(a*x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\right)^p e^{n\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a/x)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p*exp(n*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)^p \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^p,x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.618 $\int e^{-2p\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=54

$$\frac{x(1-ax)^{-p}F_1\left(1-p;-2p,p;2-p;ax,-ax\right)\left(c-\frac{c}{ax}\right)^p}{1-p}$$

```
[Out] ((c - c/(a*x))^p*x*AppellF1[1 - p, -2*p, p, 2 - p, a*x, -(a*x)])/((1 - p)*(
1 - a*x)^p)
```

Rubi [A] time = 0.10718, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6134, 6129, 133}

$$\frac{x(1-ax)^{-p}F_1\left(1-p;-2p,p;2-p;ax,-ax\right)\left(c-\frac{c}{ax}\right)^p}{1-p}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^p/E^(2*p*ArcTanh[a*x]),x]
```

```
[Out] ((c - c/(a*x))^p*x*AppellF1[1 - p, -2*p, p, 2 - p, a*x, -(a*x)])/((1 - p)*(
1 - a*x)^p)
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] && !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2p\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx &= \left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int e^{-2p\tanh^{-1}(ax)}x^{-p}(1-ax)^p\,dx\\
&= \left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int x^{-p}(1-ax)^{2p}(1+ax)^{-p}\,dx\\
&= \frac{\left(c-\frac{c}{ax}\right)^p x(1-ax)^{-p}F_1\left(1-p;-2p,p;2-p;ax,-ax\right)}{1-p}
\end{aligned}$$

Mathematica [F] time = 0.425185, size = 0, normalized size = 0.

$$\int e^{-2p\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c - c/(a*x))^p/E^(2*p*ArcTanh[a*x]),x]
```

```
[Out] Integrate[(c - c/(a*x))^p/E^(2*p*ArcTanh[a*x]), x]
```

Maple [F] time = 0.147, size = 0, normalized size = 0.

$$\int \frac{1}{\mathrm{e}^{2p\mathrm{Artanh}(ax)}}\left(c-\frac{c}{ax}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^p/exp(2*p*arctanh(a*x)),x)
```

```
[Out] int((c-c/a/x)^p/exp(2*p*arctanh(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c-\frac{c}{ax}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/exp(2*p*arctanh(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(\frac{acx-c}{ax}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/exp(2*p*arctanh(a*x)),x, algorithm="fricas")
```

```
[Out] integral(((a*c*x - c)/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**p/exp(2*p*atanh(a*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{ax}\right)^{p}}{\left(\frac{ax+1}{ax-1}\right)^{p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/exp(2*p*arctanh(a*x)),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)
```

3.619 $\int e^{2p\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=50

$$\frac{x(1-ax)^{-p}\left(c-\frac{c}{ax}\right)^p \text{Hypergeometric2F1}(1-p,-p,2-p,-ax)}{1-p}$$

```
[Out] ((c - c/(a*x))^p*x*Hypergeometric2F1[1 - p, -p, 2 - p, -(a*x)])/((1 - p)*(1
 - a*x)^p)
```

Rubi [A] time = 0.0970773, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6134, 6129, 64}

$$\frac{x(1-ax)^{-p}\left(c-\frac{c}{ax}\right)^p {}_2F_1(1-p,-p;2-p;-ax)}{1-p}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*p*ArcTanh[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] ((c - c/(a*x))^p*x*Hypergeometric2F1[1 - p, -p, 2 - p, -(a*x)])/((1 - p)*(1
 - a*x)^p)
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int e^{2p\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx &= \left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int e^{2p\tanh^{-1}(ax)}x^{-p}(1-ax)^p\,dx\\
&= \left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int x^{-p}(1+ax)^p\,dx\\
&= \frac{\left(c-\frac{c}{ax}\right)^p x(1-ax)^{-p}\,{}_2F_1(1-p,-p;2-p;-ax)}{1-p}
\end{aligned}$$

Mathematica [A] time = 0.0198358, size = 50, normalized size = 1.

$$\frac{x(1-ax)^{-p}\left(c-\frac{c}{ax}\right)^p \text{Hypergeometric2F1}(1-p,-p,2-p,-ax)}{1-p}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*p*ArcTanh[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] ((c - c/(a*x))^p*x*Hypergeometric2F1[1 - p, -p, 2 - p, -(a*x)])/((1 - p)*(1
 - a*x)^p)
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int e^{2p\operatorname{Artanh}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arctanh(a*x))*(c-c/a/x)^p,x)
```

```
[Out] int(exp(2*p*arctanh(a*x))*(c-c/a/x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(c-c/a/x)^p,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{ax+1}{ax-1}\right)^p\left(\frac{acx-c}{ax}\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(c-c/a/x)^p,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^p*((a*c*x - c)/(a*x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\right)^p e^{2p\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*atanh(a*x))*(c-c/a/x)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p*exp(2*p*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)^p \left(\frac{ax+1}{ax-1}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(c-c/a/x)^p,x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^p, x)
```

3.620 $\int e^{n \tanh^{-1}(ax)} \left(c - \frac{c}{ax}\right)^2 dx$

Optimal. Leaf size=130

$$\frac{c^2 2^{n/2}(1-ax)^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},2-\frac{n}{2},3-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a(4-n)}+\frac{4c^2(ax+1)^{n/2}(1-ax)^{-n/2}\text{Hypergeometric}}{an}$$

```
[Out] (4*c^2*(1 + a*x)^(n/2)*Hypergeometric2F1[2, n/2, (2 + n)/2, (1 + a*x)/(1 -
a*x)])/(a*n*(1 - a*x)^(n/2)) + (2^(n/2)*c^2*(1 - a*x)^(2 - n/2)*Hypergeomet
ric2F1[1 - n/2, 2 - n/2, 3 - n/2, (1 - a*x)/2])/(a*(4 - n))
```

Rubi [C] time = 0.12373, antiderivative size = 71, normalized size of antiderivative = 0.55, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 136}

$$\frac{c^2 2^{3-\frac{n}{2}}(ax+1)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n-4}{2},2;\frac{n+4}{2};\frac{1}{2}(ax+1),ax+1\right)}{a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] (2^(3 - n/2)*c^2*(1 + a*x)^((2 + n)/2)*AppellF1[(2 + n)/2, (-4 + n)/2, 2, (
4 + n)/2, (1 + a*x)/2, 1 + a*x])/(a*(2 + n))
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2dx = \frac{c^2\int\frac{e^{n\tanh^{-1}(ax)}(1-ax)^2}{x^2}dx}{a^2}$$
$$= \frac{c^2\int\frac{(1-ax)^{2-\frac{n}{2}}(1+ax)^{n/2}}{x^2}dx}{a^2}$$
$$= \frac{2^{3-\frac{n}{2}}c^2(1+ax)^{\frac{2+n}{2}}F_1\left(\frac{2+n}{2};\frac{1}{2}(-4+n),2;\frac{4+n}{2};\frac{1}{2}(1+ax),1+ax\right)}{a(2+n)}$$

Mathematica [B] time = 0.479392, size = 262, normalized size = 2.02

$$-\frac{c^2e^{n\tanh^{-1}(ax)}\left(an^2x\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n}{2}+1,e^{2\tanh^{-1}(ax)}\right)-2anxe^{2\tanh^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n}{2}+1,\ldots\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] -((c^2*E^(n*ArcTanh[a*x])*(2*n + n^2 - 2*a*E^(2*ArcTanh[a*x])*n*x*Hypergeom
etric2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcTanh[a*x])] + a*E^(2*ArcTanh[a*x])*(
-2 + n)*n*x*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcTanh[a*x])] + 4*
a*x*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*ArcTanh[a*x])] + 2*a*n*x*Hyper
geometric2F1[1, n/2, 1 + n/2, -E^(2*ArcTanh[a*x])] - 4*a*x*Hypergeometric2F
1[1, n/2, 1 + n/2, E^(2*ArcTanh[a*x])] + a*n^2*x*Hypergeometric2F1[1, n/2,
1 + n/2, E^(2*ArcTanh[a*x])] - 4*a*E^(2*ArcTanh[a*x])*n*x*Hypergeometric2F1
[2, 1 + n/2, 2 + n/2, -E^(2*ArcTanh[a*x])]))/(a^2*n*(2 + n)*x))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}\left(c-\frac{c}{ax}\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a/x)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(c-c/a/x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{ax}\right)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2c^2x^2 - 2\,ac^2x + c^2\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2
*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2\left(\int a^2e^{n\,\text{atanh}\,(ax)}\,dx + \int \frac{e^{n\,\text{atanh}\,(ax)}}{x^2}\,dx + \int -\frac{2ae^{n\,\text{atanh}\,(ax)}}{x}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a/x)**2,x)
```

```
[Out] c**2*(Integral(a**2*exp(n*atanh(a*x)), x) + Integral(exp(n*atanh(a*x))/x**2
, x) + Integral(-2*a*exp(n*atanh(a*x))/x, x))/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.621 $\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=187

$$-\frac{2c(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-2}{2},\frac{n}{2},\frac{ax+1}{1-ax}\right)}{a(2-n)}+\frac{c2^{n/2}(1-n)(1-ax)^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},2\right.}{a(2-n)(4-n)}$$

```
[Out] (c*(1 - a*x)^(2 - n/2)*(1 + a*x)^((-2 + n)/2))/(a*(2 - n)) - (2*c*(1 - a*x)
^(1 - n/2)*(1 + a*x)^((-2 + n)/2)*Hypergeometric2F1[1, (-2 + n)/2, n/2, (1
+ a*x)/(1 - a*x)])/(a*(2 - n)) + (2^(n/2)*c*(1 - n)*(1 - a*x)^(2 - n/2)*Hyp
ergeometric2F1[(2 - n)/2, 2 - n/2, 3 - n/2, (1 - a*x)/2])/(a*(2 - n)*(4 - n
))
```

Rubi [A] time = 0.132947, antiderivative size = 184, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6131, 6129, 105, 69, 131}

$$-\frac{c2^{\frac{n}{2}+1}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(2-n)}-\frac{2c(ax+1)^{n/2}(1-ax)^{-n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{an}+\frac{c2^{\frac{n}{2}+1}(1-ax)^{-n}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - c/(a*x)),x]
```

```
[Out] (-2*c*(1 + a*x)^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (1 - a*x)/(1 + a*
x)])/(a*n*(1 - a*x)^(n/2)) - (2^(1 + n/2)*c*(1 - a*x)^(1 - n/2)*Hypergeomet
ric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2])/(a*(2 - n)) + (2^(1 + n/2)*c*H
ypergeometric2F1[-n/2, -n/2, 1 - n/2, (1 - a*x)/2])/(a*n*(1 - a*x)^(n/2))
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = -\frac{c\int \frac{e^{n\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}$$
$$= -\frac{c\int \frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{n/2}}{x}\,dx}{a}$$
$$= c\int (1-ax)^{-n/2}(1+ax)^{n/2}\,dx - \frac{c\int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{x}\,dx}{a}$$
$$= -\frac{2^{1+\frac{n}{2}}c(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(2-n)} + c\int (1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}\,dx - \frac{c}{}$$
$$= -\frac{2c(1-ax)^{-n/2}(1+ax)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{1-ax}{1+ax}\right)}{an} - \frac{2^{1+\frac{n}{2}}c(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\right.}{a(2-n)}$$

Mathematica [A] time = 0.2376, size = 180, normalized size = 0.96

$$\frac{ce^{n\tanh^{-1}(ax)}\left(e^{2\tanh^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n}{2}+1,\frac{n}{2}+2,-e^{2\tanh^{-1}(ax)}\right)+e^{2\tanh^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a*x)),x]
```

```
[Out] (c*E^(n*ArcTanh[a*x])*(E^(2*ArcTanh[a*x])*Hypergeometric2F1[1, 1 + n/2, 2 +
 n/2, -E^(2*ArcTanh[a*x])] + E^(2*ArcTanh[a*x])*Hypergeometric2F1[1, 1 + n/
2, 2 + n/2, E^(2*ArcTanh[a*x])] - ((2 + n)*(Hypergeometric2F1[1, n/2, 1 + n
/2, -E^(2*ArcTanh[a*x])] - Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcTanh[
a*x])]))/n + 4*E^(2*ArcTanh[a*x])*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E
^(2*ArcTanh[a*x])]))/(a*(2 + n))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}\left(c-\frac{c}{ax}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a/x),x)
```

[Out] int(exp(n*arctanh(a*x))*(c-c/a/x),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x),x, algorithm="maxima")

[Out] integrate((c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(acx-c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x),x, algorithm="fricas")

[Out] integral((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c\left(\int ae^{n\operatorname{atanh}(ax)}\, dx + \int -\frac{e^{n\operatorname{atanh}(ax)}}{x}\, dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*atanh(a*x))*(c-c/a/x),x)

[Out] c*(Integral(a*exp(n*atanh(a*x)), x) + Integral(-exp(n*atanh(a*x))/x, x))/a

---

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x),x, algorithm="giac")

[Out] integrate((c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)

3.622 $\int \frac{e^{n \tanh^{-1}(ax)}}{c-\frac{c}{ax}} dx$

Optimal. Leaf size=111

$$-\frac{2^{\frac{n}{2}+1}(n+1)(1-ax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{ac(2-n)n}-\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{-n/2}}{acn}$$

```
[Out] -((1 + a*x)^((2 + n)/2)/(a*c*n*(1 - a*x)^(n/2))) - (2^(1 + n/2)*(1 + n)*(1
- a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2])/(a
*c*(2 - n)*n)
```

Rubi [A] time = 0.131442, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6131, 6129, 79, 69}

$$-\frac{2^{\frac{n}{2}+1}(n+1)(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{ac(2-n)n}-\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{-n/2}}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - c/(a*x)),x]
```

```
[Out] -((1 + a*x)^((2 + n)/2)/(a*c*n*(1 - a*x)^(n/2))) - (2^(1 + n/2)*(1 + n)*(1
- a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2])/(a
*c*(2 - n)*n)
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{a\int \frac{e^{n\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}$$
$$= -\frac{a\int x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c}$$
$$= -\frac{(1-ax)^{-n/2}(1+ax)^{\frac{2+n}{2}}}{acn} + \frac{(1+n)\int (1-ax)^{-n/2}(1+ax)^{n/2}\,dx}{cn}$$
$$= -\frac{(1-ax)^{-n/2}(1+ax)^{\frac{2+n}{2}}}{acn} - \frac{2^{1+\frac{n}{2}}(1+n)(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{ac(2-n)n}$$

Mathematica [A] time = 0.0399761, size = 95, normalized size = 0.86

$$\frac{(1-ax)^{-n/2}\left(-2^{\frac{n}{2}+1}(n+1)(ax-1)\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right)-(n-2)(ax+1)^{\frac{n}{2}+1}\right)}{ac(n-2)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - c/(a*x)),x]
```

```
[Out] (-((-2 + n)*(1 + a*x)^(1 + n/2)) - 2^(1 + n/2)*(1 + n)*(-1 + a*x)*Hypergeom
etric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2])/(a*c*(-2 + n)*n*(1 - a*x)^(n
/2))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int e^{n\mathrm{Artanh}(ax)}\left(c-\frac{c}{ax}\right)^{-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a/x),x)
```

```
[Out] int(exp(n*arctanh(a*x))/(c-c/a/x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{c-\frac{c}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ax\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] integral(a*x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\int\frac{xe^{n\,\mathrm{atanh}\,(ax)}}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a/x),x)
```

```
[Out] a*Integral(x*exp(n*atanh(a*x))/(a*x - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{c-\frac{c}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x)), x)
```

3.623 $\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=139

$$-\frac{2^{\frac{n}{2}+1}(n+2)(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{ac^2n}+\frac{(n+3)(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-1}}{ac^2(n+2)}-\frac{x(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-1}}{c^2}$$

```
[Out] ((3 + n)*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^2*(2 + n)) - (x*(
1 - a*x)^(-1 - n/2)*(1 + a*x)^((2 + n)/2))/c^2 - (2^(1 + n/2)*(2 + n)*Hyper
geometric2F1[-n/2, -n/2, 1 - n/2, (1 - a*x)/2])/(a*c^2*n*(1 - a*x)^(n/2))
```

Rubi [A] time = 0.172383, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6131, 6129, 90, 79, 69}

$$-\frac{2^{\frac{n}{2}+1}(n+2)(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{ac^2n}+\frac{(n+3)(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-1}}{ac^2(n+2)}-\frac{x(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-1}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] ((3 + n)*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^2*(2 + n)) - (x*(
1 - a*x)^(-1 - n/2)*(1 + a*x)^((2 + n)/2))/c^2 - (2^(1 + n/2)*(2 + n)*Hyper
geometric2F1[-n/2, -n/2, 1 - n/2, (1 - a*x)/2])/(a*c^2*n*(1 - a*x)^(n/2))
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
```

```
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{e^{n\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}$$

$$= \frac{a^2\int x^2(1-ax)^{-2-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c^2}$$

$$= -\frac{x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{c^2} - \frac{\int (1-ax)^{-2-\frac{n}{2}}(1+ax)^{n/2}(-1-a(2+n)x)\,dx}{c^2}$$

$$= \frac{(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^2(2+n)} - \frac{x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{c^2} - \frac{(2+n)\int (1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c^2}$$

$$= \frac{(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^2(2+n)} - \frac{x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{c^2} - \frac{2^{1+\frac{n}{2}}(2+n)(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2}\right.}{ac^2n}$$

Mathematica [A] time = 0.442551, size = 194, normalized size = 1.4

$$\frac{e^{n\tanh^{-1}(ax)}\left(-2n(ax-1)e^{2\tanh^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n}{2}+1,\frac{n}{2}+2,-e^{2\tanh^{-1}(ax)}\right)-4ne^{2\tanh^{-1}(ax)}\text{Hypergeom}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] (E^(n*ArcTanh[a*x])*(4 + n - 4*a*x - 3*a*n*x - 2*E^(2*ArcTanh[a*x])*n*(-1 +
 a*x)*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcTanh[a*x])] + 2*(2 +
n)*(-1 + a*x)*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*ArcTanh[a*x])] - 4*E
^(2*ArcTanh[a*x])*n*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E^(2*ArcTanh[a*
x])] + 4*a*E^(2*ArcTanh[a*x])*n*x*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E
^(2*ArcTanh[a*x])]))/(a*c^2*n*(2 + n)*(-1 + a*x))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}\left(c-\frac{c}{ax}\right)^{-2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a/x)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))/(c-c/a/x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2c^2x^2-2\,ac^2x+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*x^2 - 2*a*c^2*x + c
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\int\frac{x^2e^{n\,\mathrm{atanh}\,(ax)}}{a^2x^2-2ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2*exp(n*atanh(a*x))/(a**2*x**2 - 2*a*x + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^2, x)
```

3.624 $\int e^{n \tanh^{-1}(ax)} \left(c - \frac{c}{ax}\right)^{3/2} dx$

Optimal. Leaf size=54

$$-\frac{2x\left(c-\frac{c}{ax}\right)^{3/2} F_1\left(-\frac{1}{2};\frac{n-3}{2},-\frac{n}{2};\frac{1}{2};ax,-ax\right)}{(1-ax)^{3/2}}$$

```
[Out] (-2*(c - c/(a*x))^(3/2)*x*AppellF1[-1/2, (-3 + n)/2, -n/2, 1/2, a*x, -(a*x)
])/(1 - a*x)^(3/2)
```

Rubi [A] time = 0.168973, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6134, 6129, 133}

$$-\frac{2x\left(c-\frac{c}{ax}\right)^{3/2} F_1\left(-\frac{1}{2};\frac{n-3}{2},-\frac{n}{2};\frac{1}{2};ax,-ax\right)}{(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (-2*(c - c/(a*x))^(3/2)*x*AppellF1[-1/2, (-3 + n)/2, -n/2, 1/2, a*x, -(a*x)
])/(1 - a*x)^(3/2)
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{e^{n\tanh^{-1}(ax)}(1-ax)^{3/2}}{x^{3/2}}dx}{(1-ax)^{3/2}}$$
$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{(1-ax)^{\frac{3}{2}-\frac{n}{2}}(1+ax)^{n/2}}{x^{3/2}}dx}{(1-ax)^{3/2}}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}xF_1\left(-\frac{1}{2};\frac{1}{2}(-3+n),-\frac{n}{2};\frac{1}{2};ax,-ax\right)}{(1-ax)^{3/2}}$$

Mathematica [F] time = 180.004, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int e^{n\mathrm{Artanh}(ax)}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a/x)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(c-c/a/x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(acx-c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{acx-c}{ax}}}{ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/
(a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)^{\frac{3}{2}} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.625 $\int e^{n \tanh^{-1}(ax)} \sqrt{c - \frac{c}{ax}} \, dx$

Optimal. Leaf size=54

$$\frac{2x\sqrt{c-\frac{c}{ax}}F_1\left(\frac{1}{2};\frac{n-1}{2},-\frac{n}{2};\frac{3}{2};ax,-ax\right)}{\sqrt{1-ax}}$$

```
[Out] (2*Sqrt[c - c/(a*x)]*x*AppellF1[1/2, (-1 + n)/2, -n/2, 3/2, a*x, -(a*x)])/S
qrt[1 - a*x]
```

Rubi [A] time = 0.150397, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6134, 6129, 133}

$$\frac{2x\sqrt{c-\frac{c}{ax}}F_1\left(\frac{1}{2};\frac{n-1}{2},-\frac{n}{2};\frac{3}{2};ax,-ax\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*x*AppellF1[1/2, (-1 + n)/2, -n/2, 3/2, a*x, -(a*x)])/S
qrt[1 - a*x]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{e^{n\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{n/2}}{\sqrt{x}}dx}{\sqrt{1-ax}}$$
$$= \frac{2\sqrt{c-\frac{c}{ax}}xF_1\left(\frac{1}{2};\frac{1}{2}(-1+n),-\frac{n}{2};\frac{3}{2};ax,-ax\right)}{\sqrt{1-ax}}$$

Mathematica [F] time = 180.007, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.122, size = 0, normalized size = 0.

$$\int \mathrm{e}^{n\operatorname{Artanh}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a/x)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(c-c/a/x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{ax}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{acx-c}{ax}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{ax}} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.626 $\int \frac{e^{n \tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}} dx$

Optimal. Leaf size=56

$$\frac{2x\sqrt{1-ax}F_1\left(\frac{3}{2};\frac{n+1}{2},-\frac{n}{2};\frac{5}{2};ax,-ax\right)}{3\sqrt{c-\frac{c}{ax}}}$$

```
[Out] (2*x*Sqrt[1 - a*x]*AppellF1[3/2, (1 + n)/2, -n/2, 5/2, a*x, -(a*x)])/(3*Sqr
t[c - c/(a*x)])
```

Rubi [A] time = 0.154303, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6134, 6129, 133}

$$\frac{2x\sqrt{1-ax}F_1\left(\frac{3}{2};\frac{n+1}{2},-\frac{n}{2};\frac{5}{2};ax,-ax\right)}{3\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (2*x*Sqrt[1 - a*x]*AppellF1[3/2, (1 + n)/2, -n/2, 5/2, a*x, -(a*x)])/(3*Sqr
t[c - c/(a*x)])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-ax}\int\frac{e^{n\tanh^{-1}(ax)}\sqrt{x}}{\sqrt{1-ax}}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$
$$= \frac{\sqrt{1-ax}\int\sqrt{x}(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{n/2}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$
$$= \frac{2x\sqrt{1-ax}F_1\left(\frac{3}{2};\frac{1+n}{2},-\frac{n}{2};\frac{5}{2};ax,-ax\right)}{3\sqrt{c-\frac{c}{ax}}}$$

Mathematica [F] time = 180.006, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int e^{n\operatorname{Artanh}(ax)}\frac{1}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a/x)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/(c-c/a/x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{ax\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{acx-c}{ax}}}{acx-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(a*x*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a*c*x -
 c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{atanh}(ax)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/sqrt(-c*(-1 + 1/(a*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a*x)), x)
```

3.627 $\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=56

$$\frac{2x(1-ax)^{3/2}F_1\left(\frac{5}{2};\frac{n+3}{2},-\frac{n}{2};\frac{7}{2};ax,-ax\right)}{5\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] (2*x*(1 - a*x)^(3/2)*AppellF1[5/2, (3 + n)/2, -n/2, 7/2, a*x, -(a*x)])/(5*(
c - c/(a*x))^(3/2))
```

Rubi [A] time = 0.170338, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6134, 6129, 133}

$$\frac{2x(1-ax)^{3/2}F_1\left(\frac{5}{2};\frac{n+3}{2},-\frac{n}{2};\frac{7}{2};ax,-ax\right)}{5\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] (2*x*(1 - a*x)^(3/2)*AppellF1[5/2, (3 + n)/2, -n/2, 7/2, a*x, -(a*x)])/(5*(
c - c/(a*x))^(3/2))
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{(1-ax)^{3/2}\int \frac{e^{n\tanh^{-1}(ax)}x^{3/2}}{(1-ax)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$
$$= \frac{(1-ax)^{3/2}\int x^{3/2}(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{n/2}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$
$$= \frac{2x(1-ax)^{3/2}F_1\left(\frac{5}{2};\frac{3+n}{2},-\frac{n}{2};\frac{7}{2};ax,-ax\right)}{5\left(c-\frac{c}{ax}\right)^{3/2}}$$

Mathematica [F] time = 180.005, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.122, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{Artanh}(ax)}\left(c-\frac{c}{ax}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a/x)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/(c-c/a/x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^2x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{acx-c}{ax}}}{a^2c^2x^2-2\,ac^2x+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a^2
*c^2*x^2 - 2*a*c^2*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^(3/2), x)
```

3.628 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=169

$$-\frac{c^4(7ax+6)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7}+\frac{c^4(35ax+24)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5}-\frac{c^4(35ax+16)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(16-35ax)\sqrt{1-a^2x^2}}{16a^2x}+\cdots$$

```
[Out] (c^4*(16 - 35*a*x)*Sqrt[1 - a^2*x^2])/(16*a^2*x) - (c^4*(16 + 35*a*x)*(1 - a^2*x^2)^(3/2))/(48*a^4*x^3) + (c^4*(24 + 35*a*x)*(1 - a^2*x^2)^(5/2))/(120*a^6*x^5) - (c^4*(6 + 7*a*x)*(1 - a^2*x^2)^(7/2))/(42*a^8*x^7) + (c^4*ArcSin[a*x])/a + (35*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/(16*a)
```

Rubi [A] time = 0.219308, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6157, 6148, 811, 813, 844, 216, 266, 63, 208}

$$-\frac{c^4(7ax+6)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7}+\frac{c^4(35ax+24)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5}-\frac{c^4(35ax+16)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(16-35ax)\sqrt{1-a^2x^2}}{16a^2x}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (c^4*(16 - 35*a*x)*Sqrt[1 - a^2*x^2])/(16*a^2*x) - (c^4*(16 + 35*a*x)*(1 - a^2*x^2)^(3/2))/(48*a^4*x^3) + (c^4*(24 + 35*a*x)*(1 - a^2*x^2)^(5/2))/(120*a^6*x^5) - (c^4*(6 + 7*a*x)*(1 - a^2*x^2)^(7/2))/(42*a^8*x^7) + (c^4*ArcSin[a*x])/a + (35*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/(16*a)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] && !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4dx=\frac{c^4\int\frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$
$$=\frac{c^4\int\frac{(1+ax)\left(1-a^2x^2\right)^{7/2}}{x^8}dx}{a^8}$$
$$=-\frac{c^4(6+7ax)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7}-\frac{c^4\int\frac{\left(12a^2+14a^3x\right)\left(1-a^2x^2\right)^{5/2}}{x^6}dx}{12a^8}$$
$$=\frac{c^4(24+35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5}-\frac{c^4(6+7ax)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7}+\frac{c^4\int\frac{\left(96a^4+140a^5x\right)\left(1-a^2x^2\right)^{3/2}}{x^4}dx}{96a^8}$$
$$=-\frac{c^4(16+35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(24+35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5}-\frac{c^4(6+7ax)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7}$$
$$=\frac{c^4(16-35ax)\sqrt{1-a^2x^2}}{16a^2x}-\frac{c^4(16+35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(24+35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5}-$$
$$=\frac{c^4(16-35ax)\sqrt{1-a^2x^2}}{16a^2x}-\frac{c^4(16+35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(24+35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5}-$$
$$=\frac{c^4(16-35ax)\sqrt{1-a^2x^2}}{16a^2x}-\frac{c^4(16+35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(24+35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5}-$$
$$=\frac{c^4(16-35ax)\sqrt{1-a^2x^2}}{16a^2x}-\frac{c^4(16+35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(24+35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5}-$$
$$=\frac{c^4(16-35ax)\sqrt{1-a^2x^2}}{16a^2x}-\frac{c^4(16+35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(24+35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5}-$$

Mathematica [C] time = 0.0318889, size = 70, normalized size = 0.41

$$\frac{c^4\left(-7a^7\left(1-a^2x^2\right)^{9/2}\text{Hypergeometric2F1}\left(4,\frac{9}{2},\frac{11}{2},1-a^2x^2\right)-\frac{9\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{7}{2},-\frac{5}{2},a^2x^2\right)}{x^7}\right)}{63a^8}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (c^4*((-9*Hypergeometric2F1[-7/2, -7/2, -5/2, a^2*x^2])/x^7 - 7*a^7*(1 - a^
2*x^2)^(9/2)*Hypergeometric2F1[4, 9/2, 11/2, 1 - a^2*x^2]))/(63*a^8)
```

Maple [A] time = 0.061, size = 233, normalized size = 1.4

$$-\frac{c^4}{a}\sqrt{-a^2x^2+1}+c^4\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{c^4}{6a^7x^6}\sqrt{-a^2x^2+1}+\frac{19c^4}{24a^5x^4}\sqrt{-a^2x^2+1}-\frac{29c^4}{16x^2a^3}\sqrt{-a^2x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^4,x)
```

```
[Out] -c^4*(-a^2*x^2+1)^(1/2)/a+c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)
^(1/2))-1/6*c^4/a^7/x^6*(-a^2*x^2+1)^(1/2)+19/24*c^4/a^5/x^4*(-a^2*x^2+1)^(
```

```
1/2)-29/16*c^4*(-a^2*x^2+1)^(1/2)/x^2/a^3+35/16*c^4/a*arctanh(1/(-a^2*x^2+1
)^(1/2))+176/105*c^4*(-a^2*x^2+1)^(1/2)/a^2/x-1/7*c^4/a^8/x^7*(-a^2*x^2+1)^
(1/2)+22/35*c^4/a^6/x^5*(-a^2*x^2+1)^(1/2)-122/105*c^4*(-a^2*x^2+1)^(1/2)/a
^4/x^3
```

Maxima [B] time = 1.50612, size = 707, normalized size = 4.18

$$\frac{c^4 \arcsin\left(\frac{a^2 x}{\sqrt{a^2}}\right)}{\sqrt{a^2}} + \frac{4\,c^4 \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)}{a} - \frac{\sqrt{-a^2x^2+1}c^4}{a} - \frac{3\left(a^2 \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{\sqrt{-a^2x^2+1}}{x^2}\right)c^4}{a^3} + \frac{4\sqrt{-a^2x^2+1}}{a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] c^4*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + 4*c^4*log(2*sqrt(-a^2*x^2 + 1)/abs(
x) + 2/abs(x))/a - sqrt(-a^2*x^2 + 1)*c^4/a - 3*(a^2*log(2*sqrt(-a^2*x^2 +
1)/abs(x) + 2/abs(x)) + sqrt(-a^2*x^2 + 1)/x^2)*c^4/a^3 + 4*sqrt(-a^2*x^2 +
 1)*c^4/(a^2*x) - 2*(2*sqrt(-a^2*x^2 + 1)*a^2/x + sqrt(-a^2*x^2 + 1)/x^3)*c
^4/a^4 + 1/2*(3*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 3*sqrt(-a
^2*x^2 + 1)*a^2/x^2 + 2*sqrt(-a^2*x^2 + 1)/x^4)*c^4/a^5 + 4/15*(8*sqrt(-a^2
*x^2 + 1)*a^4/x + 4*sqrt(-a^2*x^2 + 1)*a^2/x^3 + 3*sqrt(-a^2*x^2 + 1)/x^5)*
c^4/a^6 - 1/48*(15*a^6*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 15*sqr
t(-a^2*x^2 + 1)*a^4/x^2 + 10*sqrt(-a^2*x^2 + 1)*a^2/x^4 + 8*sqrt(-a^2*x^2 +
 1)/x^6)*c^4/a^7 - 1/35*(16*sqrt(-a^2*x^2 + 1)*a^6/x + 8*sqrt(-a^2*x^2 + 1)
*a^4/x^3 + 6*sqrt(-a^2*x^2 + 1)*a^2/x^5 + 5*sqrt(-a^2*x^2 + 1)/x^7)*c^4/a^8
```

Fricas [A] time = 2.11245, size = 413, normalized size = 2.44

$$-\frac{3360\,a^7c^4x^7 \arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + 3675\,a^7c^4x^7 \log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) + 1680\,a^7c^4x^7 + \left(1680\,a^7c^4x^7 - 2816\,a^6c^4x^6 + 3045\,a^5\right.}{1680\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/1680*(3360*a^7*c^4*x^7*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 3675*a^7
*c^4*x^7*log((sqrt(-a^2*x^2 + 1) - 1)/x) + 1680*a^7*c^4*x^7 + (1680*a^7*c^4
*x^7 - 2816*a^6*c^4*x^6 + 3045*a^5*c^4*x^5 + 1952*a^4*c^4*x^4 - 1330*a^3*c^
4*x^3 - 1056*a^2*c^4*x^2 + 280*a*c^4*x + 240*c^4)*sqrt(-a^2*x^2 + 1))/(a^8*
x^7)
```

Sympy [A] time = 133.666, size = 1119, normalized size = 6.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**4,x)
```

```
[Out] a*c**4*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
 + c**4*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a*
```

```
*2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - 4*c**4*Piecewise((-acosh(1/(a*x)), 1
/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a - 4*c**4*Piecewise((-I*sqr
t(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a
**2 + 6*c**4*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))
/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 -
1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3 + 6*c**
4*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3
*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a*
*2*x**2 + 1)/(3*x**3), True))/a**4 - 4*c**4*Piecewise((-3*a**4*acosh(1/(a*x
))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*
x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*
I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x*
*3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True))/
a**5 - 4*c**4*Piecewise((-8*a**5*sqrt(-1 + 1/(a**2*x**2))/15 - 4*a**3*sqrt(
-1 + 1/(a**2*x**2))/(15*x**2) - a*sqrt(-1 + 1/(a**2*x**2))/(5*x**4), 1/Abs(
a**2*x**2) > 1), (-8*I*a**5*sqrt(1 - 1/(a**2*x**2))/15 - 4*I*a**3*sqrt(1 -
1/(a**2*x**2))/(15*x**2) - I*a*sqrt(1 - 1/(a**2*x**2))/(5*x**4), True))/a**
6 + c**4*Piecewise((-5*a**6*acosh(1/(a*x))/16 + 5*a**5/(16*x*sqrt(-1 + 1/(a
**2*x**2))) - 5*a**3/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) - a/(24*x**5*sqrt(-
1 + 1/(a**2*x**2))) - 1/(6*a*x**7*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**
2) > 1), (5*I*a**6*asin(1/(a*x))/16 - 5*I*a**5/(16*x*sqrt(1 - 1/(a**2*x**2)
)) + 5*I*a**3/(48*x**3*sqrt(1 - 1/(a**2*x**2))) + I*a/(24*x**5*sqrt(1 - 1/(
a**2*x**2))) + I/(6*a*x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**7 + c**4*Pie
cewise((-16*a**7*sqrt(-1 + 1/(a**2*x**2))/35 - 8*a**5*sqrt(-1 + 1/(a**2*x**
2))/(35*x**2) - 6*a**3*sqrt(-1 + 1/(a**2*x**2))/(35*x**4) - a*sqrt(-1 + 1/(
a**2*x**2))/(7*x**6), 1/Abs(a**2*x**2) > 1), (-16*I*a**7*sqrt(1 - 1/(a**2*x
**2))/35 - 8*I*a**5*sqrt(1 - 1/(a**2*x**2))/(35*x**2) - 6*I*a**3*sqrt(1 - 1
/(a**2*x**2))/(35*x**4) - I*a*sqrt(1 - 1/(a**2*x**2))/(7*x**6), True))/a**8
```

Giac [B] time = 1.20922, size = 682, normalized size = 4.04

$$\frac{\left(15\,c^4+\frac{35\left(\sqrt{-a^2x^2+1}|a|+a\right)c^4}{a^2x}-\frac{189\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^4}{a^4x^2}-\frac{525\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c^4}{a^6x^3}+\frac{1295\left(\sqrt{-a^2x^2+1}|a|+a\right)^4c^4}{a^8x^4}+\frac{4935\left(\sqrt{-a^2x^2+1}|a|+a\right)^5c^4}{a^{10}x^5}-\cdots\right.}{13440\left(\sqrt{-a^2x^2+1}|a|+a\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] 1/13440*(15*c^4 + 35*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/(a^2*x) - 189*(sqr
t(-a^2*x^2 + 1)*abs(a) + a)^2*c^4/(a^4*x^2) - 525*(sqrt(-a^2*x^2 + 1)*abs(a
) + a)^3*c^4/(a^6*x^3) + 1295*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^8*x^
4) + 4935*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^10*x^5) - 9765*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)^6*c^4/(a^12*x^6))*a^14*x^7/((sqrt(-a^2*x^2 + 1)*abs(
a) + a)^7*abs(a)) + c^4*arcsin(a*x)*sgn(a)/abs(a) + 35/16*c^4*log(1/2*abs(-
2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1
)*c^4/a + 1/13440*(9765*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^4/x - 4935*(s
qrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^4/x^2 - 1295*(sqrt(-a^2*x^2 + 1)*abs(
a) + a)^3*c^4/x^3 + 525*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^2*x^4) + 1
89*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^4*x^5) - 35*(sqrt(-a^2*x^2 + 1)
*abs(a) + a)^6*c^4/(a^6*x^6) - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7*c^4/(a^
8*x^7))/(a^6*abs(a))
```

3.629 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=136

$$\frac{c^3(5ax+4)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}-\frac{c^3(15ax+8)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3}+\frac{c^3(8-15ax)\sqrt{1-a^2x^2}}{8a^2x}+\frac{15c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{8a}+\frac{c^3\sin^{-1}(ax)}{a}$$

```
[Out] (c^3*(8 - 15*a*x)*Sqrt[1 - a^2*x^2])/(8*a^2*x) - (c^3*(8 + 15*a*x)*(1 - a^2
*x^2)^(3/2))/(24*a^4*x^3) + (c^3*(4 + 5*a*x)*(1 - a^2*x^2)^(5/2))/(20*a^6*x
^5) + (c^3*ArcSin[a*x])/a + (15*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/(8*a)
```

Rubi [A] time = 0.187181, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6157, 6148, 811, 813, 844, 216, 266, 63, 208}

$$\frac{c^3(5ax+4)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}-\frac{c^3(15ax+8)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3}+\frac{c^3(8-15ax)\sqrt{1-a^2x^2}}{8a^2x}+\frac{15c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{8a}+\frac{c^3\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^3,x]
```


```
[Out] (c^3*(8 - 15*a*x)*Sqrt[1 - a^2*x^2])/(8*a^2*x) - (c^3*(8 + 15*a*x)*(1 - a^2
*x^2)^(3/2))/(24*a^4*x^3) + (c^3*(4 + 5*a*x)*(1 - a^2*x^2)^(5/2))/(20*a^6*x
^5) + (c^3*ArcSin[a*x])/a + (15*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/(8*a)
```


Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 && !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
```

```
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{c^3\int\frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^3}{x^6}dx}{a^6}$$

$$= -\frac{c^3\int\frac{(1+ax)\left(1-a^2x^2\right)^{5/2}}{x^6}dx}{a^6}$$

$$= \frac{c^3(4+5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5} + \frac{c^3\int\frac{\left(8a^2+10a^3x\right)\left(1-a^2x^2\right)^{3/2}}{x^4}dx}{8a^6}$$

$$= -\frac{c^3(8+15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4+5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5} - \frac{c^3\int\frac{\left(32a^4+60a^5x\right)\sqrt{1-a^2x^2}}{x^2}dx}{32a^6}$$

$$= \frac{c^3(8-15ax)\sqrt{1-a^2x^2}}{8a^2x} - \frac{c^3(8+15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4+5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5} + \frac{c^3\int\frac{-1\ldots}{x\ldots}}{\ldots}$$

$$= \frac{c^3(8-15ax)\sqrt{1-a^2x^2}}{8a^2x} - \frac{c^3(8+15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4+5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5} + c^3\int\frac{\ldots}{\sqrt{\ldots}}$$

$$= \frac{c^3(8-15ax)\sqrt{1-a^2x^2}}{8a^2x} - \frac{c^3(8+15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4+5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5} + \frac{c^3\sin^{-1}\ldots}{a}$$

$$= \frac{c^3(8-15ax)\sqrt{1-a^2x^2}}{8a^2x} - \frac{c^3(8+15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4+5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5} + \frac{c^3\sin^{-1}\ldots}{a}$$

$$= \frac{c^3(8-15ax)\sqrt{1-a^2x^2}}{8a^2x} - \frac{c^3(8+15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4+5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5} + \frac{c^3\sin^{-1}\ldots}{a}$$

Mathematica [C] time = 0.0263298, size = 70, normalized size = 0.51

$$\frac{c^3\left(5a^5\left(1-a^2x^2\right)^{7/2}\text{Hypergeometric2F1}\left(3,\frac{7}{2},\frac{9}{2},1-a^2x^2\right)+\frac{7\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{2},-\frac{3}{2},a^2x^2\right)}{x^5}\right)}{35a^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (c^3*((7*Hypergeometric2F1[-5/2, -5/2, -3/2, a^2*x^2])/x^5 + 5*a^5*(1 - a^2
*x^2)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, 1 - a^2*x^2]))/(35*a^6)
```

Maple [A] time = 0.053, size = 187, normalized size = 1.4

$$-\frac{c^3}{a}\sqrt{-a^2x^2+1}+c^3\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{c^3}{4a^5x^4}\sqrt{-a^2x^2+1}-\frac{9c^3}{8x^2a^3}\sqrt{-a^2x^2+1}+\frac{15c^3}{8a}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^3,x)
```

```
[Out] -c^3*(-a^2*x^2+1)^(1/2)/a+c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)
^(1/2))+1/4*c^3/a^5/x^4*(-a^2*x^2+1)^(1/2)-9/8*c^3*(-a^2*x^2+1)^(1/2)/x^2/a
^3+15/8*c^3/a*arctanh(1/(-a^2*x^2+1)^(1/2))+23/15*c^3*(-a^2*x^2+1)^(1/2)/a^
```

```
2/x+1/5*c^3/a^6/x^5*(-a^2*x^2+1)^(1/2)-11/15*c^3/a^4/x^3*(-a^2*x^2+1)^(1/2)
```

Maxima [B] time = 1.48176, size = 460, normalized size = 3.38

$$\frac{c^3 \arcsin\left(\frac{a^2 x}{\sqrt{a^2}}\right)}{\sqrt{a^2}} + \frac{3\,c^3 \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)}{a} - \frac{\sqrt{-a^2x^2+1}c^3}{a} - \frac{3\left(a^2 \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{\sqrt{-a^2x^2+1}}{x^2}\right)c^3}{2\,a^3} + \frac{3\sqrt{-a^2x^2}}{a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] c^3*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + 3*c^3*log(2*sqrt(-a^2*x^2 + 1)/abs(
x) + 2/abs(x))/a - sqrt(-a^2*x^2 + 1)*c^3/a - 3/2*(a^2*log(2*sqrt(-a^2*x^2
+ 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*x^2 + 1)/x^2)*c^3/a^3 + 3*sqrt(-a^2*x^2
 + 1)*c^3/(a^2*x) - (2*sqrt(-a^2*x^2 + 1)*a^2/x + sqrt(-a^2*x^2 + 1)/x^3)*c
^3/a^4 + 1/8*(3*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 3*sqrt(-a
^2*x^2 + 1)*a^2/x^2 + 2*sqrt(-a^2*x^2 + 1)/x^4)*c^3/a^5 + 1/15*(8*sqrt(-a^2
*x^2 + 1)*a^4/x + 4*sqrt(-a^2*x^2 + 1)*a^2/x^3 + 3*sqrt(-a^2*x^2 + 1)/x^5)*
c^3/a^6
```

Fricas [A] time = 2.1743, size = 347, normalized size = 2.55

$$-\frac{240\,a^5c^3x^5 \arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + 225\,a^5c^3x^5 \log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) + 120\,a^5c^3x^5 + \left(120\,a^5c^3x^5 - 184\,a^4c^3x^4 + 135\,a^3c^3x^3\right.}{120\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/120*(240*a^5*c^3*x^5*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 225*a^5*c^
3*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + 120*a^5*c^3*x^5 + (120*a^5*c^3*x^5
- 184*a^4*c^3*x^4 + 135*a^3*c^3*x^3 + 88*a^2*c^3*x^2 - 30*a*c^3*x - 24*c^3)
*sqrt(-a^2*x^2 + 1))/(a^6*x^5)
```

Sympy [A] time = 54.0499, size = 687, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**3,x)
```

```
[Out] a*c**3*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
 + c**3*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a*
*2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - 3*c**3*Piecewise((-acosh(1/(a*x)), 1
/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a - 3*c**3*Piecewise((-I*sqr
t(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a
**2 + 3*c**3*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))
/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 -
1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3 + 3*c**
```

```
3*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3
*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a*
*2*x**2 + 1)/(3*x**3), True))/a**4 - c**3*Piecewise((-3*a**4*acosh(1/(a*x))
/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x*
*2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*
a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3
*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True))/a*
*5 - c**3*Piecewise((-8*a**5*sqrt(-1 + 1/(a**2*x**2))/15 - 4*a**3*sqrt(-1 +
 1/(a**2*x**2))/(15*x**2) - a*sqrt(-1 + 1/(a**2*x**2))/(5*x**4), 1/Abs(a**2
*x**2) > 1), (-8*I*a**5*sqrt(1 - 1/(a**2*x**2))/15 - 4*I*a**3*sqrt(1 - 1/(a
**2*x**2))/(15*x**2) - I*a*sqrt(1 - 1/(a**2*x**2))/(5*x**4), True))/a**6
```

Giac [B] time = 1.22412, size = 520, normalized size = 3.82

$$-\frac{\left(6\,c^3+\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)c^3}{a^2x}-\frac{70\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^3}{a^4x^2}-\frac{240\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c^3}{a^6x^3}+\frac{660\left(\sqrt{-a^2x^2+1}|a|+a\right)^4c^3}{a^8x^4}\right)a^{10}x^5}{960\left(\sqrt{-a^2x^2+1}|a|+a\right)^5|a|}+\frac{c^3\arcsin\left(ax\right)\operatorname{sgn}\left(\right.}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -1/960*(6*c^3 + 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3/(a^2*x) - 70*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*c^3/(a^4*x^2) - 240*(sqrt(-a^2*x^2 + 1)*abs(a) +
 a)^3*c^3/(a^6*x^3) + 660*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^3/(a^8*x^4))*
a^10*x^5/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*abs(a)) + c^3*arcsin(a*x)*sgn(a
)/abs(a) + 15/8*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*ab
s(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c^3/a + 1/960*(660*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)*a^2*c^3/x - 240*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3/x^2 - 70*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^3/(a^2*x^3) + 15*(sqrt(-a^2*x^2 + 1)*abs
(a) + a)^4*c^3/(a^4*x^4) + 6*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^3/(a^6*x^5
))/(a^4*abs(a))
```

3.630 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=103

$$-\frac{c^2(3ax+2)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^2(2-3ax)\sqrt{1-a^2x^2}}{2a^2x}+\frac{3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}+\frac{c^2\sin^{-1}(ax)}{a}$$

```
[Out] (c^2*(2 - 3*a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^2*(2 + 3*a*x)*(1 - a^2*x
^2)^(3/2))/(6*a^4*x^3) + (c^2*ArcSin[a*x])/a + (3*c^2*ArcTanh[Sqrt[1 - a^2*
x^2]])/(2*a)
```

Rubi [A] time = 0.158055, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6157, 6148, 811, 813, 844, 216, 266, 63, 208}

$$-\frac{c^2(3ax+2)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^2(2-3ax)\sqrt{1-a^2x^2}}{2a^2x}+\frac{3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}+\frac{c^2\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (c^2*(2 - 3*a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^2*(2 + 3*a*x)*(1 - a^2*x
^2)^(3/2))/(6*a^4*x^3) + (c^2*ArcSin[a*x])/a + (3*c^2*ArcTanh[Sqrt[1 - a^2*
x^2]])/(2*a)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
```

```
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{c^2\int \frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}dx}{a^4}$$
$$= \frac{c^2\int \frac{(1+ax)\left(1-a^2x^2\right)^{3/2}}{x^4}dx}{a^4}$$
$$= -\frac{c^2(2+3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} - \frac{c^2\int \frac{\left(4a^2+6a^3x\right)\sqrt{1-a^2x^2}}{x^2}dx}{4a^4}$$
$$= \frac{c^2(2-3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(2+3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + \frac{c^2\int \frac{-12a^3+8a^4x}{x\sqrt{1-a^2x^2}}dx}{8a^4}$$
$$= \frac{c^2(2-3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(2+3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + c^2\int \frac{1}{\sqrt{1-a^2x^2}}dx - \frac{\left(3c^2\right)\int \frac{1}{x\sqrt{1-}}$$
$$= \frac{c^2(2-3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(2+3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + \frac{c^2\sin^{-1}(ax)}{a} - \frac{\left(3c^2\right)\text{Subst}\left(\int \frac{}{x\sqrt{1}}\right.}{4a}$$
$$= \frac{c^2(2-3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(2+3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + \frac{c^2\sin^{-1}(ax)}{a} + \frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-}\right.}{2}$$
$$= \frac{c^2(2-3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(2+3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + \frac{c^2\sin^{-1}(ax)}{a} + \frac{3c^2\tanh^{-1}\left(\sqrt{1-\right.}{2a}$$

Mathematica [C] time = 0.0321611, size = 70, normalized size = 0.68

$$\frac{c^2\left(-3a^3\left(1-a^2x^2\right)^{5/2}\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},1-a^2x^2\right)-\frac{5\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},a^2x^2\right)}{x^3}\right)}{15a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (c^2*((-5*Hypergeometric2F1[-3/2, -3/2, -1/2, a^2*x^2])/x^3 - 3*a^3*(1 - a^
2*x^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 - a^2*x^2]))/(15*a^4)
```

Maple [A] time = 0.048, size = 141, normalized size = 1.4

$$-\frac{c^2}{a}\sqrt{-a^2x^2+1}+c^2\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{4c^2}{3a^2x}\sqrt{-a^2x^2+1}+\frac{3c^2}{2a}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{c^2}{2x^2a^3}\sqrt{-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^2,x)
```

```
[Out] -c^2*(-a^2*x^2+1)^(1/2)/a+c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)
^(1/2))+4/3*c^2*(-a^2*x^2+1)^(1/2)/a^2/x+3/2*c^2/a*arctanh(1/(-a^2*x^2+1)^(
1/2))-1/2*c^2/a^3/x^2*(-a^2*x^2+1)^(1/2)-1/3*c^2/a^4/x^3*(-a^2*x^2+1)^(1/2)
```

Maxima [B] time = 1.46628, size = 267, normalized size = 2.59

$$\frac{c^2 \arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}} + \frac{2\,c^2 \log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)}{a} - \frac{\sqrt{-a^2x^2+1}c^2}{a} - \frac{\left(a^2 \log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{\sqrt{-a^2x^2+1}}{x^2}\right)c^2}{2\,a^3} + \frac{2\,\sqrt{-a^2x^2+1}c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] c^2*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + 2*c^2*log(2*sqrt(-a^2*x^2 + 1)/abs(
x) + 2/abs(x))/a - sqrt(-a^2*x^2 + 1)*c^2/a - 1/2*(a^2*log(2*sqrt(-a^2*x^2
+ 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*x^2 + 1)/x^2)*c^2/a^3 + 2*sqrt(-a^2*x^2
 + 1)*c^2/(a^2*x) - 1/3*(2*sqrt(-a^2*x^2 + 1)*a^2/x + sqrt(-a^2*x^2 + 1)/x^
3)*c^2/a^4
```

Fricas [A] time = 2.18613, size = 282, normalized size = 2.74

$$-\frac{12\,a^3c^2x^3 \arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + 9\,a^3c^2x^3 \log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) + 6\,a^3c^2x^3 + \left(6\,a^3c^2x^3 - 8\,a^2c^2x^2 + 3\,ac^2x + 2\,c^2\right)\sqrt{-a^2x^2+1}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(12*a^3*c^2*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 9*a^3*c^2*x^3
*log((sqrt(-a^2*x^2 + 1) - 1)/x) + 6*a^3*c^2*x^3 + (6*a^3*c^2*x^3 - 8*a^2*c
^2*x^2 + 3*a*c^2*x + 2*c^2)*sqrt(-a^2*x^2 + 1))/(a^4*x^3)
```

Sympy [A] time = 16.8691, size = 354, normalized size = 3.44

$$ac^2\left(\begin{cases} \frac{x^2}{2} & \text{for}\: a^2 = 0 \\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise} \end{cases}\right) + c^2\left(\begin{cases} \sqrt{\frac{1}{a^2}} \operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for}\: a^2 > 0 \\ \sqrt{-\frac{1}{a^2}} \operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\: a^2 < 0 \end{cases}\right) - \frac{2c^2\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\: \frac{1}{|a^2x^2|} > 1 \\ i \operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise} \end{cases}\right)}{a} - \frac{2c^2 \ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**2,x)
```

```
[Out] a*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
 + c**2*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a*
*2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - 2*c**2*Piecewise((-acosh(1/(a*x)), 1
/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a - 2*c**2*Piecewise((-I*sqr
t(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a
**2 + c**2*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(
2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/
(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3 + c**2*Pi
ecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**
3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x
```

```
**2 + 1)/(3*x**3), True))/a**4
```

Giac [B] time = 1.22426, size = 355, normalized size = 3.45

$$\frac{\left(c^2 + \frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)c^2}{a^2x} - \frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^2}{a^4x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|} + \frac{c^2\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{|a|} + \frac{3\,c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|} - \frac{\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(c^2 + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/(a^2*x) - 15*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^2*c^2/(a^4*x^2))*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*abs(a)) + c^2*arcsin(a*x)*sgn(a)/abs(a) + 3/2*c^2*log(1/2*abs(-2*sqrt(
-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c^2/a
 + 1/24*(15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/x - 3*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)^2*c^2/(a^2*x^2) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^2/(a^4*x^3
))/(a^2*abs(a))
```

3.631 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=58

$$\frac{c\sqrt{1-a^2x^2}(1-ax)}{a^2x}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}+\frac{c\sin^{-1}(ax)}{a}$$

```
[Out] (c*(1 - a*x)*Sqrt[1 - a^2*x^2])/(a^2*x) + (c*ArcSin[a*x])/a + (c*ArcTanh[Sq
rt[1 - a^2*x^2]])/a
```

Rubi [A] time = 0.101278, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6157, 6148, 813, 844, 216, 266, 63, 208}

$$\frac{c\sqrt{1-a^2x^2}(1-ax)}{a^2x}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}+\frac{c\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*(1 - a*x)*Sqrt[1 - a^2*x^2])/(a^2*x) + (c*ArcSin[a*x])/a + (c*ArcTanh[Sq
rt[1 - a^2*x^2]])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx &= -\frac{c\int \frac{e^{\tanh^{-1}(ax)}(1-a^2x^2)}{x^2}\,dx}{a^2}\\
&= -\frac{c\int \frac{(1+ax)\sqrt{1-a^2x^2}}{x^2}\,dx}{a^2}\\
&= \frac{c(1-ax)\sqrt{1-a^2x^2}}{a^2x}+\frac{c\int \frac{-2a+2a^2x}{x\sqrt{1-a^2x^2}}\,dx}{2a^2}\\
&= \frac{c(1-ax)\sqrt{1-a^2x^2}}{a^2x}+c\int \frac{1}{\sqrt{1-a^2x^2}}\,dx-\frac{c\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{a}\\
&= \frac{c(1-ax)\sqrt{1-a^2x^2}}{a^2x}+\frac{c\sin^{-1}(ax)}{a}-\frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2a}\\
&= \frac{c(1-ax)\sqrt{1-a^2x^2}}{a^2x}+\frac{c\sin^{-1}(ax)}{a}+\frac{c\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^3}\\
&= \frac{c(1-ax)\sqrt{1-a^2x^2}}{a^2x}+\frac{c\sin^{-1}(ax)}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0449822, size = 55, normalized size = 0.95

$$\frac{c\left(\sqrt{1-a^2x^2}(1-ax)+ax\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+ax\sin^{-1}(ax)\right)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*((1 - a*x)*Sqrt[1 - a^2*x^2] + a*x*ArcSin[a*x] + a*x*ArcTanh[Sqrt[1 - a^
2*x^2]]))/(a^2*x)
```

Maple [A] time = 0.044, size = 85, normalized size = 1.5

$$-\frac{c}{a}\sqrt{-a^2x^2+1}+c\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{c}{a^2x}\sqrt{-a^2x^2+1}+\frac{c}{a}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2),x)
```

```
[Out] -c*(-a^2*x^2+1)^(1/2)/a+c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/
2))+c*(-a^2*x^2+1)^(1/2)/a^2/x+c/a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.47169, size = 119, normalized size = 2.05

$$\frac{c\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}+\frac{c\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)}{a}-\frac{\sqrt{-a^2x^2+1}c}{a}+\frac{\sqrt{-a^2x^2+1}c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] c*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + c*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2
/abs(x))/a - sqrt(-a^2*x^2 + 1)*c/a + sqrt(-a^2*x^2 + 1)*c/(a^2*x)
```

Fricas [A] time = 2.1198, size = 189, normalized size = 3.26

$$-\frac{2\,acx\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+acx\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+acx+\sqrt{-a^2x^2+1}(acx-c)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] -(2*a*c*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + a*c*x*log((sqrt(-a^2*x^2
 + 1) - 1)/x) + a*c*x + sqrt(-a^2*x^2 + 1)*(a*c*x - c))/(a^2*x)
```

Sympy [A] time = 9.03592, size = 144, normalized size = 2.48

$$ac\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)+c\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)-\frac{c\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}-\frac{c\left(\begin{cases}-\frac{i\sqrt{a}}{} \\ -\frac{\sqrt{-}}{}\end{cases}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2),x)
```

```
[Out] a*c*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) +
c*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*as
inh(x*sqrt(-a**2)), a**2 < 0)) - c*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x
**2) > 1), (I*asin(1/(a*x)), True))/a - c*Piecewise((-I*sqrt(a**2*x**2 - 1)
/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a**2
```

Giac [B] time = 1.19972, size = 173, normalized size = 2.98

$$-\frac{a^2cx}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|}+\frac{c\arcsin(ax)\operatorname{sgn}(a)}{|a|}+\frac{c\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|}-\frac{\sqrt{-a^2x^2+1}c}{a}+\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)c}{2a^2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] -1/2*a^2*c*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) + c*arcsin(a*x)*sgn(a
)/abs(a) + c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/
abs(a) - sqrt(-a^2*x^2 + 1)*c/a + 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c/(a^
2*x*abs(a))
```

3.632 $\int \frac{e^{\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=61

$$-\frac{ax+1}{ac\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac}+\frac{\sin^{-1}(ax)}{ac}$$

```
[Out] -((1 + a*x)/(a*c*Sqrt[1 - a^2*x^2])) - Sqrt[1 - a^2*x^2]/(a*c) + ArcSin[a*x
]/(a*c)
```

Rubi [A] time = 0.128842, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6157, 6148, 797, 641, 216, 637}

$$-\frac{ax+1}{ac\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac}+\frac{\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a^2*x^2)),x]
```

```
[Out] -((1 + a*x)/(a*c*Sqrt[1 - a^2*x^2])) - Sqrt[1 - a^2*x^2]/(a*c) + ArcSin[a*x
]/(a*c)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\frac{a^2\int \frac{e^{\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}\\
&= -\frac{a^2\int \frac{x^2(1+ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c}\\
&= -\frac{\int \frac{1+ax}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c}+\frac{\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{1+ax}{ac\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{1+ax}{ac\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac}+\frac{\sin^{-1}(ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.0280983, size = 53, normalized size = 0.87

$$\frac{a^2x^2+\sqrt{1-a^2x^2}\sin^{-1}(ax)-ax-2}{ac\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a^2*x^2)),x]
```

```
[Out] (-2 - a*x + a^2*x^2 + Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(a*c*Sqrt[1 - a^2*x^2]
)
```

Maple [A] time = 0.039, size = 94, normalized size = 1.5

$$-\frac{1}{ac}\sqrt{-a^2x^2+1}+\frac{1}{c}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{1}{a^2c}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a/c+1/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(
1/2))+1/a^2/c/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))), x)
```

Fricas [A] time = 2.07212, size = 154, normalized size = 2.52

$$-\frac{2\,ax+2\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}(ax-2)-2}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] -(2*a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x - 2) - 2)/(a^2*c*x - a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\int\frac{x^2}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*Integral(x**2/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x)/c
```

Giac [A] time = 1.17708, size = 97, normalized size = 1.59

$$\frac{\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{c|a|}-\frac{\sqrt{-a^2x^2+1}}{ac}-\frac{2}{c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/(c*abs(a)) - sqrt(-a^2*x^2 + 1)/(a*c) - 2/(c*((sqrt(-a^2
*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

3.633 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=96

$$\frac{a^2x^3(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(4ax+3)}{3c^2\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3ac^2}+\frac{\sin^{-1}(ax)}{ac^2}$$

```
[Out] (a^2*x^3*(1 + a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) - (x*(3 + 4*a*x))/(3*c^2*Sq
rt[1 - a^2*x^2]) - (8*Sqrt[1 - a^2*x^2])/(3*a*c^2) + ArcSin[a*x]/(a*c^2)
```

Rubi [A] time = 0.150325, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6157, 6148, 819, 641, 216}

$$\frac{a^2x^3(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(4ax+3)}{3c^2\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3ac^2}+\frac{\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (a^2*x^3*(1 + a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) - (x*(3 + 4*a*x))/(3*c^2*Sq
rt[1 - a^2*x^2]) - (8*Sqrt[1 - a^2*x^2])/(3*a*c^2) + ArcSin[a*x]/(a*c^2)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^2}\,dx}{c^2}$$
$$= \frac{a^4\int \frac{x^4(1+ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}$$
$$= \frac{a^2x^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} - \frac{a^2\int \frac{x^2(3+4ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^2}$$
$$= \frac{a^2x^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x(3+4ax)}{3c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{3+8ax}{\sqrt{1-a^2x^2}}\,dx}{3c^2}$$
$$= \frac{a^2x^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x(3+4ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3ac^2} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^2}$$
$$= \frac{a^2x^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x(3+4ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3ac^2} + \frac{\sin^{-1}(ax)}{ac^2}$$

Mathematica [A] time = 0.0503823, size = 78, normalized size = 0.81

$$\frac{3a^3x^3-7a^2x^2+3(ax-1)\sqrt{1-a^2x^2}\sin^{-1}(ax)-5ax+8}{3ac^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (8 - 5*a*x - 7*a^2*x^2 + 3*a^3*x^3 + 3*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSin[
a*x])/(3*a*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.048, size = 177, normalized size = 1.8

$$-\frac{1}{ac^2}\sqrt{-a^2x^2+1}+\frac{1}{c^2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{1}{6a^3c^2}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-2}+\frac{19}{12a^2c^2}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a/c^2+1/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))+1/6/a^3/c^2/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+19/12/a^
```

```
2/c^2/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/4/a^2/c^2/(x+1/a)*(-a^2*
(x+1/a)^2+2*a*(x+1/a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^2), x)
```

Fricas [A] time = 2.118, size = 297, normalized size = 3.09

$$-\frac{8\,a^3x^3-8\,a^2x^2-8\,ax+6\left(a^3x^3-a^2x^2-ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3\,a^3x^3-7\,a^2x^2-5\,ax+8\right)\sqrt{-a^2x^2+1}+8}{3\left(a^4c^2x^3-a^3c^2x^2-a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(8*a^3*x^3 - 8*a^2*x^2 - 8*a*x + 6*(a^3*x^3 - a^2*x^2 - a*x + 1)*arcta
n((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (3*a^3*x^3 - 7*a^2*x^2 - 5*a*x + 8)*sqr
t(-a^2*x^2 + 1) + 8)/(a^4*c^2*x^3 - a^3*c^2*x^2 - a^2*c^2*x + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^4\int \frac{x^4}{a^3x^3\sqrt{-a^2x^2+1}-a^2x^2\sqrt{-a^2x^2+1}-ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4/(a**3*x**3*sqrt(-a**2*x**2 + 1) - a**2*x**2*sqrt(-a**2*x
**2 + 1) - a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^2), x)
```

3.634 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3} \, dx$

Optimal. Leaf size=129

$$-\frac{a^4x^5(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(6ax+5)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{x(8ax+5)}{5c^3\sqrt{1-a^2x^2}}-\frac{16\sqrt{1-a^2x^2}}{5ac^3}+\frac{\sin^{-1}(ax)}{ac^3}$$

```
[Out] -(a^4*x^5*(1 + a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (a^2*x^3*(5 + 6*a*x))/(1
5*c^3*(1 - a^2*x^2)^(3/2)) - (x*(5 + 8*a*x))/(5*c^3*Sqrt[1 - a^2*x^2]) - (1
6*Sqrt[1 - a^2*x^2])/(5*a*c^3) + ArcSin[a*x]/(a*c^3)
```

Rubi [A] time = 0.196946, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6157, 6148, 819, 641, 216}

$$-\frac{a^4x^5(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(6ax+5)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{x(8ax+5)}{5c^3\sqrt{1-a^2x^2}}-\frac{16\sqrt{1-a^2x^2}}{5ac^3}+\frac{\sin^{-1}(ax)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^3,x]
```

```
[Out] -(a^4*x^5*(1 + a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (a^2*x^3*(5 + 6*a*x))/(1
5*c^3*(1 - a^2*x^2)^(3/2)) - (x*(5 + 8*a*x))/(5*c^3*Sqrt[1 - a^2*x^2]) - (1
6*Sqrt[1 - a^2*x^2])/(5*a*c^3) + ArcSin[a*x]/(a*c^3)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx &= -\frac{a^6\int \frac{e^{\tanh^{-1}(ax)}x^6}{\left(1-a^2x^2\right)^3}\,dx}{c^3}\\
&= -\frac{a^6\int \frac{x^6(1+ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}\\
&= -\frac{a^4x^5(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{a^4\int \frac{x^4(5+6ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^3}\\
&= -\frac{a^4x^5(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(5+6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{a^2\int \frac{x^2(15+24ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^3}\\
&= -\frac{a^4x^5(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(5+6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{x(5+8ax)}{5c^3\sqrt{1-a^2x^2}}+\frac{\int \frac{15+48ax}{\sqrt{1-a^2x^2}}\,dx}{15c^3}\\
&= -\frac{a^4x^5(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(5+6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{x(5+8ax)}{5c^3\sqrt{1-a^2x^2}}-\frac{16\sqrt{1-a^2x^2}}{5ac^3}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^3}\\
&= -\frac{a^4x^5(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(5+6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{x(5+8ax)}{5c^3\sqrt{1-a^2x^2}}-\frac{16\sqrt{1-a^2x^2}}{5ac^3}+\frac{\sin^{-1}(ax)}{ac^3}
\end{aligned}$$

Mathematica [A] time = 0.0761693, size = 108, normalized size = 0.84

$$\frac{15a^5x^5-38a^4x^4-52a^3x^3+87a^2x^2+15(ax-1)^2(ax+1)\sqrt{1-a^2x^2}\sin^{-1}(ax)+33ax-48}{15ac^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^3,x]
```

```
[Out] (-48 + 33*a*x + 87*a^2*x^2 - 52*a^3*x^3 - 38*a^4*x^4 + 15*a^5*x^5 + 15*(-1
+ a*x)^2*(1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(15*a*c^3*(-1 + a*x)^2*(1
 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.053, size = 259, normalized size = 2.

$$-\frac{1}{ac^3}\sqrt{-a^2x^2+1}+\frac{1}{c^3}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{1}{20\,a^4c^3}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}+\frac{23}{60\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^3,x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a/c^3+1/c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))+1/20/a^4/c^3/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+23/60/a
^3/c^3/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+493/240/a^2/c^3/(x-1/a)
*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1/24/a^3/c^3/(x+1/a)^2*(-a^2*(x+1/a)^2+
2*a*(x+1/a))^(1/2)-25/48/a^2/c^3/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^3), x)
```

Fricas [A] time = 2.22016, size = 456, normalized size = 3.53

$$-\frac{48\,a^5x^5-48\,a^4x^4-96\,a^3x^3+96\,a^2x^2+48\,ax+30\left(a^5x^5-a^4x^4-2\,a^3x^3+2\,a^2x^2+ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+(15\ldots}{15\left(a^6c^3x^5-a^5c^3x^4-2\,a^4c^3x^3+2\,a^3c^3x^2+a^2c^3x-ac^3\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(48*a^5*x^5 - 48*a^4*x^4 - 96*a^3*x^3 + 96*a^2*x^2 + 48*a*x + 30*(a^5
*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*arctan((sqrt(-a^2*x^2 + 1
) - 1)/(a*x)) + (15*a^5*x^5 - 38*a^4*x^4 - 52*a^3*x^3 + 87*a^2*x^2 + 33*a*x
 - 48)*sqrt(-a^2*x^2 + 1) - 48)/(a^6*c^3*x^5 - a^5*c^3*x^4 - 2*a^4*c^3*x^3
+ 2*a^3*c^3*x^2 + a^2*c^3*x - a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^6\int \frac{x^6}{a^5x^5\sqrt{-a^2x^2+1}-a^4x^4\sqrt{-a^2x^2+1}-2a^3x^3\sqrt{-a^2x^2+1}+2a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*Integral(x**6/(a**5*x**5*sqrt(-a**2*x**2 + 1) - a**4*x**4*sqrt(-a**2*x
**2 + 1) - 2*a**3*x**3*sqrt(-a**2*x**2 + 1) + 2*a**2*x**2*sqrt(-a**2*x**2 +
 1) + a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^3), x)
```

3.635 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=162

$$\frac{a^6x^7(ax+1)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{a^4x^5(8ax+7)}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(48ax+35)}{105c^4\left(1-a^2x^2\right)^{3/2}}-\frac{x(64ax+35)}{35c^4\sqrt{1-a^2x^2}}-\frac{128\sqrt{1-a^2x^2}}{35ac^4}+\frac{\sin^{-1}(ax)}{ac^4}$$

```
[Out] (a^6*x^7*(1 + a*x))/(7*c^4*(1 - a^2*x^2)^(7/2)) - (a^4*x^5*(7 + 8*a*x))/(35
*c^4*(1 - a^2*x^2)^(5/2)) + (a^2*x^3*(35 + 48*a*x))/(105*c^4*(1 - a^2*x^2)^
(3/2)) - (x*(35 + 64*a*x))/(35*c^4*Sqrt[1 - a^2*x^2]) - (128*Sqrt[1 - a^2*x
^2])/(35*a*c^4) + ArcSin[a*x]/(a*c^4)
```

Rubi [A] time = 0.223133, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6157, 6148, 819, 641, 216}

$$\frac{a^6x^7(ax+1)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{a^4x^5(8ax+7)}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(48ax+35)}{105c^4\left(1-a^2x^2\right)^{3/2}}-\frac{x(64ax+35)}{35c^4\sqrt{1-a^2x^2}}-\frac{128\sqrt{1-a^2x^2}}{35ac^4}+\frac{\sin^{-1}(ax)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^4,x]
```

```
[Out] (a^6*x^7*(1 + a*x))/(7*c^4*(1 - a^2*x^2)^(7/2)) - (a^4*x^5*(7 + 8*a*x))/(35
*c^4*(1 - a^2*x^2)^(5/2)) + (a^2*x^3*(35 + 48*a*x))/(105*c^4*(1 - a^2*x^2)^
(3/2)) - (x*(35 + 64*a*x))/(35*c^4*Sqrt[1 - a^2*x^2]) - (128*Sqrt[1 - a^2*x
^2])/(35*a*c^4) + ArcSin[a*x]/(a*c^4)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{e^{\tanh^{-1}(ax)}x^8}{(1-a^2x^2)^4}\,dx}{c^4}$$

$$= \frac{a^8\int \frac{x^8(1+ax)}{(1-a^2x^2)^{9/2}}\,dx}{c^4}$$

$$= \frac{a^6x^7(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^6\int \frac{x^6(7+8ax)}{(1-a^2x^2)^{7/2}}\,dx}{7c^4}$$

$$= \frac{a^6x^7(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^4x^5(7+8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^4\int \frac{x^4(35+48ax)}{(1-a^2x^2)^{5/2}}\,dx}{35c^4}$$

$$= \frac{a^6x^7(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^4x^5(7+8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(35+48ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} - \frac{a^2\int \frac{x^2(105+192ax)}{(1-a^2x^2)^{3/2}}\,dx}{105c^4}$$

$$= \frac{a^6x^7(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^4x^5(7+8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(35+48ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} - \frac{x(35+64ax)}{35c^4\sqrt{1-a^2x^2}} + \frac{\int \frac{105+384ax}{\sqrt{1-a^2x^2}}\,dx}{105c^4}$$

$$= \frac{a^6x^7(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^4x^5(7+8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(35+48ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} - \frac{x(35+64ax)}{35c^4\sqrt{1-a^2x^2}} - \frac{128\sqrt{1-a^2x^2}}{35ac^4} + \cdots$$

$$= \frac{a^6x^7(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^4x^5(7+8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(35+48ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} - \frac{x(35+64ax)}{35c^4\sqrt{1-a^2x^2}} - \frac{128\sqrt{1-a^2x^2}}{35ac^4} + \cdots$$

Mathematica [A] time = 0.0965116, size = 126, normalized size = 0.78

$$\frac{105a^7x^7-281a^6x^6-559a^5x^5+965a^4x^4+715a^3x^3-1065a^2x^2+105(ax-1)^3(ax+1)^2\sqrt{1-a^2x^2}\sin^{-1}(ax)-279ax \cdots}{105ac^4(ax-1)^3(ax+1)^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^4,x]
```

```
[Out] (384 - 279*a*x - 1065*a^2*x^2 + 715*a^3*x^3 + 965*a^4*x^4 - 559*a^5*x^5 - 2
81*a^6*x^6 + 105*a^7*x^7 + 105*(-1 + a*x)^3*(1 + a*x)^2*Sqrt[1 - a^2*x^2]*A
rcSin[a*x])/(105*a*c^4*(-1 + a*x)^3*(1 + a*x)^2*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.059, size = 341, normalized size = 2.1

$$-\frac{1}{a c^4}\sqrt{-a^2x^2+1}+\frac{1}{c^4}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{17}{112\,a^4c^4}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}+\frac{211}{336\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^4,x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a/c^4+1/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))+17/112/a^4/c^4/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+211/3
36/a^3/c^4/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1657/672/a^2/c^4/(x
-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1/56/a^5/c^4/(x-1/a)^4*(-a^2*(x-1/
a)^2-2*a*(x-1/a))^(1/2)+7/60/a^3/c^4/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))
^(1/2)-379/480/a^2/c^4/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)-1/80/a^4/
c^4/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^4), x)
```

Fricas [A] time = 2.57646, size = 633, normalized size = 3.91

$$-\frac{384\,a^7x^7-384\,a^6x^6-1152\,a^5x^5+1152\,a^4x^4+1152\,a^3x^3-1152\,a^2x^2-384\,ax+210\left(a^7x^7-a^6x^6-3\,a^5x^5+3\,a^4x^4+3\right.}{105\left(a^8c^4x^7-a^7c^4x^6-3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/105*(384*a^7*x^7 - 384*a^6*x^6 - 1152*a^5*x^5 + 1152*a^4*x^4 + 1152*a^3*
x^3 - 1152*a^2*x^2 - 384*a*x + 210*(a^7*x^7 - a^6*x^6 - 3*a^5*x^5 + 3*a^4*x
^4 + 3*a^3*x^3 - 3*a^2*x^2 - a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)
) + (105*a^7*x^7 - 281*a^6*x^6 - 559*a^5*x^5 + 965*a^4*x^4 + 715*a^3*x^3 -
1065*a^2*x^2 - 279*a*x + 384)*sqrt(-a^2*x^2 + 1) + 384)/(a^8*c^4*x^7 - a^7*
c^4*x^6 - 3*a^6*c^4*x^5 + 3*a^5*c^4*x^4 + 3*a^4*c^4*x^3 - 3*a^3*c^4*x^2 - a
^2*c^4*x + a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^8\int\frac{x^8}{a^7x^7\sqrt{-a^2x^2+1}-a^6x^6\sqrt{-a^2x^2+1}-3a^5x^5\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}+3a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}-ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*Integral(x**8/(a**7*x**7*sqrt(-a**2*x**2 + 1) - a**6*x**6*sqrt(-a**2*x
**2 + 1) - 3*a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 +
 1) + 3*a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) -
 a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^4), x)
```

3.636 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$

Optimal. Leaf size=128

$$-\frac{4c^5}{a^3x^2}+\frac{2c^5}{3a^4x^3}+\frac{3c^5}{a^5x^4}+\frac{2c^5}{5a^6x^5}-\frac{4c^5}{3a^7x^6}-\frac{3c^5}{7a^8x^7}+\frac{c^5}{4a^9x^8}+\frac{c^5}{9a^{10}x^9}-\frac{3c^5}{a^2x}-\frac{2c^5\log(x)}{a}+c^5(-x)$$

```
[Out] c^5/(9*a^10*x^9) + c^5/(4*a^9*x^8) - (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) + (2*c^5)/(5*a^6*x^5) + (3*c^5)/(a^5*x^4) + (2*c^5)/(3*a^4*x^3) - (4*c^
5)/(a^3*x^2) - (3*c^5)/(a^2*x) - c^5*x - (2*c^5*Log[x])/a
```

Rubi [A] time = 0.150277, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$-\frac{4c^5}{a^3x^2}+\frac{2c^5}{3a^4x^3}+\frac{3c^5}{a^5x^4}+\frac{2c^5}{5a^6x^5}-\frac{4c^5}{3a^7x^6}-\frac{3c^5}{7a^8x^7}+\frac{c^5}{4a^9x^8}+\frac{c^5}{9a^{10}x^9}-\frac{3c^5}{a^2x}-\frac{2c^5\log(x)}{a}+c^5(-x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] c^5/(9*a^10*x^9) + c^5/(4*a^9*x^8) - (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) + (2*c^5)/(5*a^6*x^5) + (3*c^5)/(a^5*x^4) + (2*c^5)/(3*a^4*x^3) - (4*c^
5)/(a^3*x^2) - (3*c^5)/(a^2*x) - c^5*x - (2*c^5*Log[x])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx = -\frac{c^5\int \frac{e^{2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^5}{x^{10}}dx}{a^{10}}$$
$$= -\frac{c^5\int \frac{(1-ax)^4(1+ax)^6}{x^{10}}dx}{a^{10}}$$
$$= -\frac{c^5\int \left(a^{10}+\frac{1}{x^{10}}+\frac{2a}{x^9}-\frac{3a^2}{x^8}-\frac{8a^3}{x^7}+\frac{2a^4}{x^6}+\frac{12a^5}{x^5}+\frac{2a^6}{x^4}-\frac{8a^7}{x^3}-\frac{3a^8}{x^2}+\frac{2a^9}{x}\right)dx}{a^{10}}$$
$$= \frac{c^5}{9a^{10}x^9}+\frac{c^5}{4a^9x^8}-\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}+\frac{2c^5}{5a^6x^5}+\frac{3c^5}{a^5x^4}+\frac{2c^5}{3a^4x^3}-\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}-c^5x-\frac{2c}{}$$

Mathematica [A] time = 0.0415026, size = 128, normalized size = 1.

$$-\frac{4c^5}{a^3x^2}+\frac{2c^5}{3a^4x^3}+\frac{3c^5}{a^5x^4}+\frac{2c^5}{5a^6x^5}-\frac{4c^5}{3a^7x^6}-\frac{3c^5}{7a^8x^7}+\frac{c^5}{4a^9x^8}+\frac{c^5}{9a^{10}x^9}-\frac{3c^5}{a^2x}-\frac{2c^5\log(x)}{a}+c^5(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] c^5/(9*a^10*x^9) + c^5/(4*a^9*x^8) - (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) + (2*c^5)/(5*a^6*x^5) + (3*c^5)/(a^5*x^4) + (2*c^5)/(3*a^4*x^3) - (4*c^
5)/(a^3*x^2) - (3*c^5)/(a^2*x) - c^5*x - (2*c^5*Log[x])/a
```

Maple [A] time = 0.038, size = 117, normalized size = 0.9

$$\frac{c^5}{9\,a^{10}x^9}+\frac{c^5}{4\,a^9x^8}-\frac{3\,c^5}{7\,a^8x^7}-\frac{4\,c^5}{3\,a^7x^6}+\frac{2\,c^5}{5\,a^6x^5}+3\,\frac{c^5}{a^5x^4}+\frac{2\,c^5}{3\,a^4x^3}-4\,\frac{c^5}{x^2a^3}-3\,\frac{c^5}{a^2x}-c^5x-2\,\frac{c^5\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^5,x)
```

```
[Out] 1/9*c^5/a^10/x^9+1/4*c^5/a^9/x^8-3/7*c^5/a^8/x^7-4/3*c^5/a^7/x^6+2/5*c^5/a^
6/x^5+3*c^5/a^5/x^4+2/3*c^5/a^4/x^3-4*c^5/x^2/a^3-3*c^5/a^2/x-c^5*x-2*c^5*l
n(x)/a
```

Maxima [A] time = 0.964992, size = 155, normalized size = 1.21

$$-c^5x-\frac{2\,c^5\log(x)}{a}-\frac{3780\,a^8c^5x^8+5040\,a^7c^5x^7-840\,a^6c^5x^6-3780\,a^5c^5x^5-504\,a^4c^5x^4+1680\,a^3c^5x^3+540\,a^2c^5x^2}{1260\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^5,x, algorithm="maxima")
```

```
[Out] -c^5*x - 2*c^5*log(x)/a - 1/1260*(3780*a^8*c^5*x^8 + 5040*a^7*c^5*x^7 - 840
*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1680*a^3*c^5*x^3 + 540*
a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

Fricas [A] time = 2.07342, size = 298, normalized size = 2.33

$$-\frac{1260\,a^{10}c^5x^{10}+2520\,a^9c^5x^9\log\left(x\right)+3780\,a^8c^5x^8+5040\,a^7c^5x^7-840\,a^6c^5x^6-3780\,a^5c^5x^5-504\,a^4c^5x^4+1680\,a^3c^5x}{1260\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^5,x, algorithm="fricas")
```

```
[Out] -1/1260*(1260*a^10*c^5*x^10 + 2520*a^9*c^5*x^9*log(x) + 3780*a^8*c^5*x^8 +
5040*a^7*c^5*x^7 - 840*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1
680*a^3*c^5*x^3 + 540*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

Sympy [A] time = 2.0383, size = 126, normalized size = 0.98

$$\frac{-a^{10}c^5x-2a^9c^5\log\left(x\right)-\frac{3780a^8c^5x^8+5040a^7c^5x^7-840a^6c^5x^6-3780a^5c^5x^5-504a^4c^5x^4+1680a^3c^5x^3+540a^2c^5x^2-315ac^5x-140c^5}{1260x^9}}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**5,x)
```

```
[Out] (-a**10*c**5*x - 2*a**9*c**5*log(x) - (3780*a**8*c**5*x**8 + 5040*a**7*c**5
*x**7 - 840*a**6*c**5*x**6 - 3780*a**5*c**5*x**5 - 504*a**4*c**5*x**4 + 168
0*a**3*c**5*x**3 + 540*a**2*c**5*x**2 - 315*a*c**5*x - 140*c**5)/(1260*x**9
))/a**10
```

Giac [A] time = 1.16628, size = 157, normalized size = 1.23

$$-c^5x-\frac{2\,c^5\log\left(|x|\right)}{a}-\frac{3780\,a^8c^5x^8+5040\,a^7c^5x^7-840\,a^6c^5x^6-3780\,a^5c^5x^5-504\,a^4c^5x^4+1680\,a^3c^5x^3+540\,a^2c^5x^2-}{1260\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^5,x, algorithm="giac")
```

```
[Out] -c^5*x - 2*c^5*log(abs(x))/a - 1/1260*(3780*a^8*c^5*x^8 + 5040*a^7*c^5*x^7
- 840*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1680*a^3*c^5*x^3 +
 540*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

3.637 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=91

$$-\frac{3c^4}{a^3x^2}+\frac{3c^4}{2a^5x^4}+\frac{2c^4}{5a^6x^5}-\frac{c^4}{3a^7x^6}-\frac{c^4}{7a^8x^7}-\frac{2c^4}{a^2x}-\frac{2c^4\log(x)}{a}+c^4(-x)$$

```
[Out] -c^4/(7*a^8*x^7) - c^4/(3*a^7*x^6) + (2*c^4)/(5*a^6*x^5) + (3*c^4)/(2*a^5*x
^4) - (3*c^4)/(a^3*x^2) - (2*c^4)/(a^2*x) - c^4*x - (2*c^4*Log[x])/a
```

Rubi [A] time = 0.132113, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$-\frac{3c^4}{a^3x^2}+\frac{3c^4}{2a^5x^4}+\frac{2c^4}{5a^6x^5}-\frac{c^4}{3a^7x^6}-\frac{c^4}{7a^8x^7}-\frac{2c^4}{a^2x}-\frac{2c^4\log(x)}{a}+c^4(-x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] -c^4/(7*a^8*x^7) - c^4/(3*a^7*x^6) + (2*c^4)/(5*a^6*x^5) + (3*c^4)/(2*a^5*x
^4) - (3*c^4)/(a^3*x^2) - (2*c^4)/(a^2*x) - c^4*x - (2*c^4*Log[x])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\int \frac{e^{2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$
$$= \frac{c^4\int \frac{(1-ax)^3(1+ax)^5}{x^8}dx}{a^8}$$
$$= \frac{c^4\int \left(-a^8+\frac{1}{x^8}+\frac{2a}{x^7}-\frac{2a^2}{x^6}-\frac{6a^3}{x^5}+\frac{6a^5}{x^3}+\frac{2a^6}{x^2}-\frac{2a^7}{x}\right)dx}{a^8}$$
$$= -\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}+\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}-\frac{2c^4}{a^2x}-c^4x-\frac{2c^4\log(x)}{a}$$

Mathematica [A] time = 0.0268813, size = 91, normalized size = 1.

$$-\frac{3c^4}{a^3x^2}+\frac{3c^4}{2a^5x^4}+\frac{2c^4}{5a^6x^5}-\frac{c^4}{3a^7x^6}-\frac{c^4}{7a^8x^7}-\frac{2c^4}{a^2x}-\frac{2c^4\log(x)}{a}+c^4(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] -c^4/(7*a^8*x^7) - c^4/(3*a^7*x^6) + (2*c^4)/(5*a^6*x^5) + (3*c^4)/(2*a^5*x
^4) - (3*c^4)/(a^3*x^2) - (2*c^4)/(a^2*x) - c^4*x - (2*c^4*Log[x])/a
```

Maple [A] time = 0.038, size = 84, normalized size = 0.9

$$-\frac{c^4}{7\,a^8x^7}-\frac{c^4}{3\,a^7x^6}+\frac{2\,c^4}{5\,a^6x^5}+\frac{3\,c^4}{2\,a^5x^4}-3\,\frac{c^4}{x^2a^3}-2\,\frac{c^4}{a^2x}-c^4x-2\,\frac{c^4\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^4,x)
```

```
[Out] -1/7*c^4/a^8/x^7-1/3*c^4/a^7/x^6+2/5*c^4/a^6/x^5+3/2*c^4/a^5/x^4-3*c^4/x^2/
a^3-2*c^4/a^2/x-c^4*x-2*c^4*ln(x)/a
```

Maxima [A] time = 0.972166, size = 111, normalized size = 1.22

$$-c^4x-\frac{2\,c^4\log(x)}{a}-\frac{420\,a^6c^4x^6+630\,a^5c^4x^5-315\,a^3c^4x^3-84\,a^2c^4x^2+70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -c^4*x - 2*c^4*log(x)/a - 1/210*(420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 315*a^
3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

Fricas [A] time = 1.98505, size = 208, normalized size = 2.29

$$-\frac{210\,a^8c^4x^8+420\,a^7c^4x^7\log(x)+420\,a^6c^4x^6+630\,a^5c^4x^5-315\,a^3c^4x^3-84\,a^2c^4x^2+70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/210*(210*a^8*c^4*x^8 + 420*a^7*c^4*x^7*log(x) + 420*a^6*c^4*x^6 + 630*a^
5*c^4*x^5 - 315*a^3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^
7)
```

Sympy [A] time = 3.20217, size = 90, normalized size = 0.99

$$\frac{-a^8c^4x - 2a^7c^4\log(x) - \frac{420a^6c^4x^6+630a^5c^4x^5-315a^3c^4x^3-84a^2c^4x^2+70ac^4x+30c^4}{210x^7}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**4,x)
```

```
[Out] (-a**8*c**4*x - 2*a**7*c**4*log(x) - (420*a**6*c**4*x**6 + 630*a**5*c**4*x*
*5 - 315*a**3*c**4*x**3 - 84*a**2*c**4*x**2 + 70*a*c**4*x + 30*c**4)/(210*x
**7))/a**8
```

Giac [A] time = 1.16361, size = 112, normalized size = 1.23

$$-c^4x - \frac{2\,c^4\log(|x|)}{a} - \frac{420\,a^6c^4x^6 + 630\,a^5c^4x^5 - 315\,a^3c^4x^3 - 84\,a^2c^4x^2 + 70\,ac^4x + 30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -c^4*x - 2*c^4*log(abs(x))/a - 1/210*(420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 3
15*a^3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

3.638 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=78

$$-\frac{2c^3}{a^3x^2}-\frac{c^3}{3a^4x^3}+\frac{c^3}{2a^5x^4}+\frac{c^3}{5a^6x^5}-\frac{c^3}{a^2x}-\frac{2c^3\log(x)}{a}+c^3(-x)$$

```
[Out] c^3/(5*a^6*x^5) + c^3/(2*a^5*x^4) - c^3/(3*a^4*x^3) - (2*c^3)/(a^3*x^2) - c
^3/(a^2*x) - c^3*x - (2*c^3*Log[x])/a
```

Rubi [A] time = 0.124213, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$-\frac{2c^3}{a^3x^2}-\frac{c^3}{3a^4x^3}+\frac{c^3}{2a^5x^4}+\frac{c^3}{5a^6x^5}-\frac{c^3}{a^2x}-\frac{2c^3\log(x)}{a}+c^3(-x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] c^3/(5*a^6*x^5) + c^3/(2*a^5*x^4) - c^3/(3*a^4*x^3) - (2*c^3)/(a^3*x^2) - c
^3/(a^2*x) - c^3*x - (2*c^3*Log[x])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3dx = -\frac{c^3\int\frac{e^{2\tanh^{-1}(ax)}(1-a^2x^2)^3}{x^6}dx}{a^6}$$
$$= -\frac{c^3\int\frac{(1-ax)^2(1+ax)^4}{x^6}dx}{a^6}$$
$$= -\frac{c^3\int\left(a^6+\frac{1}{x^6}+\frac{2a}{x^5}-\frac{a^2}{x^4}-\frac{4a^3}{x^3}-\frac{a^4}{x^2}+\frac{2a^5}{x}\right)dx}{a^6}$$
$$= \frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}-\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}-\frac{c^3}{a^2x}-c^3x-\frac{2c^3\log(x)}{a}$$

Mathematica [A] time = 0.0224063, size = 78, normalized size = 1.

$$-\frac{2c^3}{a^3x^2}-\frac{c^3}{3a^4x^3}+\frac{c^3}{2a^5x^4}+\frac{c^3}{5a^6x^5}-\frac{c^3}{a^2x}-\frac{2c^3\log(x)}{a}+c^3(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] c^3/(5*a^6*x^5) + c^3/(2*a^5*x^4) - c^3/(3*a^4*x^3) - (2*c^3)/(a^3*x^2) - c
^3/(a^2*x) - c^3*x - (2*c^3*Log[x])/a
```

Maple [A] time = 0.036, size = 73, normalized size = 0.9

$$\frac{c^3}{5\,a^6x^5}+\frac{c^3}{2\,a^5x^4}-\frac{c^3}{3\,a^4x^3}-2\,\frac{c^3}{x^2a^3}-\frac{c^3}{a^2x}-c^3x-2\,\frac{c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^3,x)
```

```
[Out] 1/5*c^3/a^6/x^5+1/2*c^3/a^5/x^4-1/3*c^3/a^4/x^3-2*c^3/x^2/a^3-c^3/a^2/x-c^3
*x-2*c^3*ln(x)/a
```

Maxima [A] time = 0.959085, size = 96, normalized size = 1.23

$$-c^3x-\frac{2\,c^3\log(x)}{a}-\frac{30\,a^4c^3x^4+60\,a^3c^3x^3+10\,a^2c^3x^2-15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -c^3*x - 2*c^3*log(x)/a - 1/30*(30*a^4*c^3*x^4 + 60*a^3*c^3*x^3 + 10*a^2*c^
3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Fricas [A] time = 1.95947, size = 176, normalized size = 2.26

$$-\frac{30\,a^6c^3x^6+60\,a^5c^3x^5\log(x)+30\,a^4c^3x^4+60\,a^3c^3x^3+10\,a^2c^3x^2-15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/30*(30*a^6*c^3*x^6 + 60*a^5*c^3*x^5*log(x) + 30*a^4*c^3*x^4 + 60*a^3*c^3
*x^3 + 10*a^2*c^3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Sympy [A] time = 2.60921, size = 78, normalized size = 1.

$$\frac{-a^6c^3x - 2a^5c^3\log(x) - \frac{30a^4c^3x^4+60a^3c^3x^3+10a^2c^3x^2-15ac^3x-6c^3}{30x^5}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**3,x)
```

```
[Out] (-a**6*c**3*x - 2*a**5*c**3*log(x) - (30*a**4*c**3*x**4 + 60*a**3*c**3*x**3
 + 10*a**2*c**3*x**2 - 15*a*c**3*x - 6*c**3)/(30*x**5))/a**6
```

Giac [A] time = 1.20588, size = 97, normalized size = 1.24

$$-c^3x - \frac{2\,c^3\log(|x|)}{a} - \frac{30\,a^4c^3x^4 + 60\,a^3c^3x^3 + 10\,a^2c^3x^2 - 15\,ac^3x - 6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -c^3*x - 2*c^3*log(abs(x))/a - 1/30*(30*a^4*c^3*x^4 + 60*a^3*c^3*x^3 + 10*a
^2*c^3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

3.639 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=41

$$-\frac{c^2}{a^3x^2}-\frac{c^2}{3a^4x^3}-\frac{2c^2\log(x)}{a}+c^2(-x)$$

```
[Out] -c^2/(3*a^4*x^3) - c^2/(a^3*x^2) - c^2*x - (2*c^2*Log[x])/a
```

Rubi [A] time = 0.111589, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 75}

$$-\frac{c^2}{a^3x^2}-\frac{c^2}{3a^4x^3}-\frac{2c^2\log(x)}{a}+c^2(-x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -c^2/(3*a^4*x^3) - c^2/(a^3*x^2) - c^2*x - (2*c^2*Log[x])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx &= \frac{c^2\int \frac{e^{2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}\,dx}{a^4}\\
&= \frac{c^2\int \frac{(1-ax)(1+ax)^3}{x^4}\,dx}{a^4}\\
&= \frac{c^2\int \left(-a^4+\frac{1}{x^4}+\frac{2a}{x^3}-\frac{2a^3}{x}\right)dx}{a^4}\\
&= -\frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}-c^2x-\frac{2c^2\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.015539, size = 41, normalized size = 1.

$$-\frac{c^2}{a^3x^2}-\frac{c^2}{3a^4x^3}-\frac{2c^2\log(x)}{a}+c^2(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -c^2/(3*a^4*x^3) - c^2/(a^3*x^2) - c^2*x - (2*c^2*Log[x])/a
```

Maple [A] time = 0.033, size = 40, normalized size = 1.

$$-\frac{c^2}{3\,a^4x^3}-\frac{c^2}{x^2a^3}-xc^2-2\,\frac{c^2\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^2,x)
```

```
[Out] -1/3*c^2/a^4/x^3-c^2/x^2/a^3-x*c^2-2*c^2*ln(x)/a
```

Maxima [A] time = 0.987683, size = 49, normalized size = 1.2

$$-c^2x-\frac{2\,c^2\log(x)}{a}-\frac{3\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -c^2*x - 2*c^2*log(x)/a - 1/3*(3*a*c^2*x + c^2)/(a^4*x^3)
```

Fricas [A] time = 1.96793, size = 99, normalized size = 2.41

$$-\frac{3\,a^4c^2x^4+6\,a^3c^2x^3\log(x)+3\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^4*c^2*x^4 + 6*a^3*c^2*x^3*log(x) + 3*a*c^2*x + c^2)/(a^4*x^3)
```

Sympy [A] time = 0.729716, size = 41, normalized size = 1.

$$\frac{-a^4c^2x-2a^3c^2\log(x)-\frac{3ac^2x+c^2}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**2,x)
```

```
[Out] (-a**4*c**2*x - 2*a**3*c**2*log(x) - (3*a*c**2*x + c**2)/(3*x**3))/a**4
```

Giac [A] time = 1.12534, size = 50, normalized size = 1.22

$$-c^2x - \frac{2\,c^2\log\left(|x|\right)}{a} - \frac{3\,ac^2x + c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] -c^2*x - 2*c^2*log(abs(x))/a - 1/3*(3*a*c^2*x + c^2)/(a^4*x^3)
```

3.640 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$

Optimal. Leaf size=21

$$\frac{c}{a^2x}-\frac{2c\log(x)}{a}+c(-x)$$

```
[Out] c/(a^2*x) - c*x - (2*c*Log[x])/a
```

Rubi [A] time = 0.0679866, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6157, 6150, 43}

$$\frac{c}{a^2x}-\frac{2c\log(x)}{a}+c(-x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] c/(a^2*x) - c*x - (2*c*Log[x])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx &= -\frac{c\int\frac{e^{2\tanh^{-1}(ax)}\left(1-a^2x^2\right)}{x^2}\,dx}{a^2}\\ &= -\frac{c\int\frac{(1+ax)^2}{x^2}\,dx}{a^2}\\ &= -\frac{c\int\left(a^2+\frac{1}{x^2}+\frac{2a}{x}\right)dx}{a^2}\\ &= \frac{c}{a^2x}-cx-\frac{2c\log(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.011725, size = 21, normalized size = 1.

$$\frac{c}{a^2x}-\frac{2c\log(x)}{a}+c(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] c/(a^2*x) - c*x - (2*c*Log[x])/a
```

Maple [A] time = 0.033, size = 22, normalized size = 1.1

$$\frac{c}{a^2x}-cx-2\,\frac{c\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2),x)
```

```
[Out] c/a^2/x-c*x-2*c*ln(x)/a
```

Maxima [A] time = 0.965781, size = 28, normalized size = 1.33

$$-cx-\frac{2\,c\log(x)}{a}+\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -c*x - 2*c*log(x)/a + c/(a^2*x)
```

Fricas [A] time = 1.99785, size = 58, normalized size = 2.76

$$-\frac{a^2cx^2+2\,acx\log(x)-c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] -(a^2*c*x^2 + 2*a*c*x*log(x) - c)/(a^2*x)
```

Sympy [A] time = 0.448533, size = 20, normalized size = 0.95

$$\frac{-a^2cx-2ac\log(x)+\frac{c}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2),x)
```

```
[Out] (-a**2*c*x - 2*a*c*log(x) + c/x)/a**2
```

Giac [A] time = 1.17002, size = 30, normalized size = 1.43

$$-cx - \frac{2\,c\log\left(|x|\right)}{a} + \frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] -c*x - 2*c*log(abs(x))/a + c/(a^2*x)
```

3.641 $\int \frac{e^{2\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{ac(1-ax)}-\frac{2\log(1-ax)}{ac}-\frac{x}{c}$$

```
[Out] -(x/c) - 1/(a*c*(1 - a*x)) - (2*Log[1 - a*x])/(a*c)
```

Rubi [A] time = 0.124496, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 43}

$$-\frac{1}{ac(1-ax)}-\frac{2\log(1-ax)}{ac}-\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] -(x/c) - 1/(a*c*(1 - a*x)) - (2*Log[1 - a*x])/(a*c)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\frac{a^2\int \frac{e^{2\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}\\
&= -\frac{a^2\int \frac{x^2}{(1-ax)^2}\,dx}{c}\\
&= -\frac{a^2\int \left(\frac{1}{a^2}+\frac{1}{a^2(-1+ax)^2}+\frac{2}{a^2(-1+ax)}\right)dx}{c}\\
&= -\frac{x}{c}-\frac{1}{ac(1-ax)}-\frac{2\log(1-ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.0203159, size = 38, normalized size = 1.

$$-\frac{1}{ac(1-ax)}-\frac{2\log(1-ax)}{ac}-\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] -(x/c) - 1/(a*c*(1 - a*x)) - (2*Log[1 - a*x])/(a*c)
```

Maple [A] time = 0.033, size = 36, normalized size = 1.

$$-\frac{x}{c}+\frac{1}{ac\,(ax-1)}-2\,\frac{\ln(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2),x)
```

```
[Out] -x/c+1/a/c/(a*x-1)-2/a/c*ln(a*x-1)
```

Maxima [A] time = 0.962236, size = 46, normalized size = 1.21

$$-\frac{x}{c}+\frac{1}{a^2cx-ac}-\frac{2\,\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -x/c + 1/(a^2*c*x - a*c) - 2*log(a*x - 1)/(a*c)
```

Fricas [A] time = 2.01568, size = 88, normalized size = 2.32

$$-\frac{a^2x^2-ax+2\,(ax-1)\log(ax-1)-1}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] -(a^2*x^2 - a*x + 2*(a*x - 1)*log(a*x - 1) - 1)/(a^2*c*x - a*c)
```

Sympy [A] time = 0.874792, size = 37, normalized size = 0.97

$$-a^2\left(-\frac{1}{a^4cx-a^3c}+\frac{x}{a^2c}+\frac{2\log(ax-1)}{a^3c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2),x)
```

```
[Out] -a**2*(-1/(a**4*c*x - a**3*c) + x/(a**2*c) + 2*log(a*x - 1)/(a**3*c))
```

Giac [A] time = 1.17655, size = 49, normalized size = 1.29

$$-\frac{x}{c} - \frac{2 \log\left(|ax - 1|\right)}{ac} + \frac{1}{(ax - 1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] -x/c - 2*log(abs(a*x - 1))/(a*c) + 1/((a*x - 1)*a*c)
```

3.642 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=76

$$-\frac{7}{4ac^2(1-ax)}+\frac{1}{4ac^2(1-ax)^2}-\frac{17\log(1-ax)}{8ac^2}+\frac{\log(ax+1)}{8ac^2}-\frac{x}{c^2}$$

```
[Out] -(x/c^2) + 1/(4*a*c^2*(1 - a*x)^2) - 7/(4*a*c^2*(1 - a*x)) - (17*Log[1 - a*
x])/(8*a*c^2) + Log[1 + a*x]/(8*a*c^2)
```

Rubi [A] time = 0.145726, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$-\frac{7}{4ac^2(1-ax)}+\frac{1}{4ac^2(1-ax)^2}-\frac{17\log(1-ax)}{8ac^2}+\frac{\log(ax+1)}{8ac^2}-\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] -(x/c^2) + 1/(4*a*c^2*(1 - a*x)^2) - 7/(4*a*c^2*(1 - a*x)) - (17*Log[1 - a*
x])/(8*a*c^2) + Log[1 + a*x]/(8*a*c^2)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{e^{2\tanh^{-1}(ax)}x^4}{(1-a^2x^2)^2}\,dx}{c^2}$$
$$= \frac{a^4\int \frac{x^4}{(1-ax)^3(1+ax)}\,dx}{c^2}$$
$$= \frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{2a^4(-1+ax)^3}-\frac{7}{4a^4(-1+ax)^2}-\frac{17}{8a^4(-1+ax)}+\frac{1}{8a^4(1+ax)}\right)dx}{c^2}$$
$$= -\frac{x}{c^2}+\frac{1}{4ac^2(1-ax)^2}-\frac{7}{4ac^2(1-ax)}-\frac{17\log(1-ax)}{8ac^2}+\frac{\log(1+ax)}{8ac^2}$$

Mathematica [A] time = 0.0405219, size = 69, normalized size = 0.91

$$\frac{-8a^3x^3+16a^2x^2+6ax-17(ax-1)^2\log(1-ax)+(ax-1)^2\log(ax+1)-12}{8ac^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-12 + 6*a*x + 16*a^2*x^2 - 8*a^3*x^3 - 17*(-1 + a*x)^2*Log[1 - a*x] + (-1
+ a*x)^2*Log[1 + a*x])/(8*a*c^2*(-1 + a*x)^2)
```

Maple [A] time = 0.036, size = 66, normalized size = 0.9

$$-\frac{x}{c^2}+\frac{\ln(ax+1)}{8\,ac^2}+\frac{1}{4\,ac^2\,(ax-1)^2}+\frac{7}{4\,ac^2\,(ax-1)}-\frac{17\,\ln(ax-1)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -x/c^2+1/8*ln(a*x+1)/a/c^2+1/4/a/c^2/(a*x-1)^2+7/4/a/c^2/(a*x-1)-17/8/c^2/a
*ln(a*x-1)
```

Maxima [A] time = 0.972944, size = 95, normalized size = 1.25

$$\frac{7\,ax-6}{4\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}-\frac{x}{c^2}+\frac{\log(ax+1)}{8\,ac^2}-\frac{17\,\log(ax-1)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(7*a*x - 6)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) - x/c^2 + 1/8*log(a*x +
 1)/(a*c^2) - 17/8*log(a*x - 1)/(a*c^2)
```

Fricas [A] time = 2.035, size = 212, normalized size = 2.79

$$-\frac{8\,a^3x^3-16\,a^2x^2-6\,ax-\left(a^2x^2-2\,ax+1\right)\log\left(ax+1\right)+17\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+12}{8\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
 17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a
*c^2)
```

Sympy [A] time = 3.91389, size = 75, normalized size = 0.99

$$-a^4\left(-\frac{7ax-6}{4a^7c^2x^2-8a^6c^2x+4a^5c^2}+\frac{x}{a^4c^2}+\frac{\frac{17\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2)**2,x)
```

```
[Out] -a**4*(-(7*a*x - 6)/(4*a**7*c**2*x**2 - 8*a**6*c**2*x + 4*a**5*c**2) + x/(a
**4*c**2) + (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**2))
```

Giac [A] time = 1.18381, size = 78, normalized size = 1.03

$$-\frac{x}{c^2}+\frac{\log\left(|ax+1|\right)}{8\,ac^2}-\frac{17\,\log\left(|ax-1|\right)}{8\,ac^2}+\frac{7\,ax-6}{4\,(ax-1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] -x/c^2 + 1/8*log(abs(a*x + 1))/(a*c^2) - 17/8*log(abs(a*x - 1))/(a*c^2) + 1
/4*(7*a*x - 6)/((a*x - 1)^2*a*c^2)
```

3.643 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=111

$$-\frac{39}{16ac^3(1-ax)}+\frac{1}{16ac^3(ax+1)}+\frac{5}{8ac^3(1-ax)^2}-\frac{1}{12ac^3(1-ax)^3}-\frac{9\log(1-ax)}{4ac^3}+\frac{\log(ax+1)}{4ac^3}-\frac{x}{c^3}$$

```
[Out] -(x/c^3) - 1/(12*a*c^3*(1 - a*x)^3) + 5/(8*a*c^3*(1 - a*x)^2) - 39/(16*a*c^
3*(1 - a*x)) + 1/(16*a*c^3*(1 + a*x)) - (9*Log[1 - a*x])/(4*a*c^3) + Log[1
+ a*x]/(4*a*c^3)
```

Rubi [A] time = 0.168702, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$-\frac{39}{16ac^3(1-ax)}+\frac{1}{16ac^3(ax+1)}+\frac{5}{8ac^3(1-ax)^2}-\frac{1}{12ac^3(1-ax)^3}-\frac{9\log(1-ax)}{4ac^3}+\frac{\log(ax+1)}{4ac^3}-\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] -(x/c^3) - 1/(12*a*c^3*(1 - a*x)^3) + 5/(8*a*c^3*(1 - a*x)^2) - 39/(16*a*c^
3*(1 - a*x)) + 1/(16*a*c^3*(1 + a*x)) - (9*Log[1 - a*x])/(4*a*c^3) + Log[1
+ a*x]/(4*a*c^3)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{a^6\int \frac{e^{2\tanh^{-1}(ax)}x^6}{(1-a^2x^2)^3}\,dx}{c^3}$$
$$= -\frac{a^6\int \frac{x^6}{(1-ax)^4(1+ax)^2}\,dx}{c^3}$$
$$= -\frac{a^6\int \left(\frac{1}{a^6}+\frac{1}{4a^6(-1+ax)^4}+\frac{5}{4a^6(-1+ax)^3}+\frac{39}{16a^6(-1+ax)^2}+\frac{9}{4a^6(-1+ax)}+\frac{1}{16a^6(1+ax)^2}-\frac{1}{4a^6(1+ax)}\right)dx}{c^3}$$
$$= -\frac{x}{c^3}-\frac{1}{12ac^3(1-ax)^3}+\frac{5}{8ac^3(1-ax)^2}-\frac{39}{16ac^3(1-ax)}+\frac{1}{16ac^3(1+ax)}-\frac{9\log(1-ax)}{4ac^3}+\frac{\log(1+ax)}{4ac^3}$$

Mathematica [A] time = 0.0874087, size = 103, normalized size = 0.93

$$\frac{-12a^5x^5+24a^4x^4+30a^3x^3-48a^2x^2-14ax-27(ax-1)^3(ax+1)\log(1-ax)+3(ax-1)^3(ax+1)\log(ax+1)+22}{12ac^3(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] (22 - 14*a*x - 48*a^2*x^2 + 30*a^3*x^3 + 24*a^4*x^4 - 12*a^5*x^5 - 27*(-1 +
 a*x)^3*(1 + a*x)*Log[1 - a*x] + 3*(-1 + a*x)^3*(1 + a*x)*Log[1 + a*x])/(12
*a*c^3*(-1 + a*x)^3*(1 + a*x))
```

Maple [A] time = 0.046, size = 96, normalized size = 0.9

$$-\frac{x}{c^3}+\frac{1}{16\,ac^3\,(ax+1)}+\frac{\ln(ax+1)}{4\,ac^3}+\frac{1}{12\,ac^3\,(ax-1)^3}+\frac{5}{8\,ac^3\,(ax-1)^2}+\frac{39}{16\,ac^3\,(ax-1)}-\frac{9\,\ln(ax-1)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^3,x)
```

```
[Out] -x/c^3+1/16/a/c^3/(a*x+1)+1/4*ln(a*x+1)/a/c^3+1/12/a/c^3/(a*x-1)^3+5/8/a/c^
3/(a*x-1)^2+39/16/a/c^3/(a*x-1)-9/4/a/c^3*ln(a*x-1)
```

Maxima [A] time = 0.973479, size = 132, normalized size = 1.19

$$\frac{15\,a^3x^3-12\,a^2x^2-13\,ax+11}{6\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}-\frac{x}{c^3}+\frac{\log(ax+1)}{4\,ac^3}-\frac{9\,\log(ax-1)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(15*a^3*x^3 - 12*a^2*x^2 - 13*a*x + 11)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 +
2*a^2*c^3*x - a*c^3) - x/c^3 + 1/4*log(a*x + 1)/(a*c^3) - 9/4*log(a*x - 1)/
(a*c^3)
```

Fricas [A] time = 2.00194, size = 308, normalized size = 2.77

$$-\frac{12\,a^{5}x^{5} - 24\,a^{4}x^{4} - 30\,a^{3}x^{3} + 48\,a^{2}x^{2} + 14\,ax - 3\left(a^{4}x^{4} - 2\,a^{3}x^{3} + 2\,ax - 1\right)\log\left(ax+1\right) + 27\left(a^{4}x^{4} - 2\,a^{3}x^{3} + 2\,ax - \right.}{12\left(a^{5}c^{3}x^{4} - 2\,a^{4}c^{3}x^{3} + 2\,a^{2}c^{3}x - ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(12*a^5*x^5 - 24*a^4*x^4 - 30*a^3*x^3 + 48*a^2*x^2 + 14*a*x - 3*(a^4*
x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x + 1) + 27*(a^4*x^4 - 2*a^3*x^3 + 2*a*x
 - 1)*log(a*x - 1) - 22)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3
)
```

Sympy [A] time = 3.64796, size = 104, normalized size = 0.94

$$-a^{6}\left(-\frac{15a^{3}x^{3} - 12a^{2}x^{2} - 13ax + 11}{6a^{11}c^{3}x^{4} - 12a^{10}c^{3}x^{3} + 12a^{8}c^{3}x - 6a^{7}c^{3}} + \frac{x}{a^{6}c^{3}} + \frac{\frac{9\log\left(x-\frac{1}{a}\right)}{4} - \frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^{7}c^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2)**3,x)
```

```
[Out] -a**6*(-(15*a**3*x**3 - 12*a**2*x**2 - 13*a*x + 11)/(6*a**11*c**3*x**4 - 12
*a**10*c**3*x**3 + 12*a**8*c**3*x - 6*a**7*c**3) + x/(a**6*c**3) + (9*log(x
 - 1/a)/4 - log(x + 1/a)/4)/(a**7*c**3))
```

Giac [A] time = 1.15151, size = 109, normalized size = 0.98

$$-\frac{x}{c^{3}} + \frac{\log\left(|ax+1|\right)}{4\,ac^{3}} - \frac{9\,\log\left(|ax-1|\right)}{4\,ac^{3}} + \frac{15\,a^{3}x^{3} - 12\,a^{2}x^{2} - 13\,ax + 11}{6\,(ax+1)(ax-1)^{3}ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -x/c^3 + 1/4*log(abs(a*x + 1))/(a*c^3) - 9/4*log(abs(a*x - 1))/(a*c^3) + 1/
6*(15*a^3*x^3 - 12*a^2*x^2 - 13*a*x + 11)/((a*x + 1)*(a*x - 1)^3*a*c^3)
```

3.644 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=146

$$-\frac{99}{32ac^4(1-ax)}+\frac{11}{64ac^4(ax+1)}+\frac{35}{32ac^4(1-ax)^2}-\frac{1}{64ac^4(ax+1)^2}-\frac{13}{48ac^4(1-ax)^3}+\frac{1}{32ac^4(1-ax)^4}-\frac{303\log(1-ax}{128ac^4}$$

```
[Out] -(x/c^4) + 1/(32*a*c^4*(1 - a*x)^4) - 13/(48*a*c^4*(1 - a*x)^3) + 35/(32*a*
c^4*(1 - a*x)^2) - 99/(32*a*c^4*(1 - a*x)) - 1/(64*a*c^4*(1 + a*x)^2) + 11/
(64*a*c^4*(1 + a*x)) - (303*Log[1 - a*x])/(128*a*c^4) + (47*Log[1 + a*x])/(
128*a*c^4)
```

Rubi [A] time = 0.19897, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$-\frac{99}{32ac^4(1-ax)}+\frac{11}{64ac^4(ax+1)}+\frac{35}{32ac^4(1-ax)^2}-\frac{1}{64ac^4(ax+1)^2}-\frac{13}{48ac^4(1-ax)^3}+\frac{1}{32ac^4(1-ax)^4}-\frac{303\log(1-ax}{128ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^4,x]
```


```
[Out] -(x/c^4) + 1/(32*a*c^4*(1 - a*x)^4) - 13/(48*a*c^4*(1 - a*x)^3) + 35/(32*a*
c^4*(1 - a*x)^2) - 99/(32*a*c^4*(1 - a*x)) - 1/(64*a*c^4*(1 + a*x)^2) + 11/
(64*a*c^4*(1 + a*x)) - (303*Log[1 - a*x])/(128*a*c^4) + (47*Log[1 + a*x])/(
128*a*c^4)
```


Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{e^{2\tanh^{-1}(ax)}x^8}{(1-a^2x^2)^4}\,dx}{c^4}$$

$$= \frac{a^8\int \frac{x^8}{(1-ax)^5(1+ax)^3}\,dx}{c^4}$$

$$= \frac{a^8\int \left(-\frac{1}{a^8}-\frac{1}{8a^8(-1+ax)^5}-\frac{13}{16a^8(-1+ax)^4}-\frac{35}{16a^8(-1+ax)^3}-\frac{99}{32a^8(-1+ax)^2}-\frac{303}{128a^8(-1+ax)}+\frac{1}{32a^8(1+ax)^3}-\frac{}{64a^8(}\right.}{c^4}$$

$$= -\frac{x}{c^4}+\frac{1}{32ac^4(1-ax)^4}-\frac{13}{48ac^4(1-ax)^3}+\frac{35}{32ac^4(1-ax)^2}-\frac{99}{32ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)^2}+\frac{}{64}$$

Mathematica [A] time = 0.0877181, size = 123, normalized size = 0.84

$$\frac{-384a^7x^7+768a^6x^6+1638a^5x^5-2508a^4x^4-1732a^3x^3+2516a^2x^2+550ax-909(ax-1)^4(ax+1)^2\log(1-ax)+1}{384ac^4(ax-1)^4(ax+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] (-800 + 550*a*x + 2516*a^2*x^2 - 1732*a^3*x^3 - 2508*a^4*x^4 + 1638*a^5*x^5
 + 768*a^6*x^6 - 384*a^7*x^7 - 909*(-1 + a*x)^4*(1 + a*x)^2*Log[1 - a*x] +
141*(-1 + a*x)^4*(1 + a*x)^2*Log[1 + a*x])/(384*a*c^4*(-1 + a*x)^4*(1 + a*x
)^2)
```

Maple [A] time = 0.046, size = 126, normalized size = 0.9

$$-\frac{x}{c^4}-\frac{1}{64\,ac^4\,(ax+1)^2}+\frac{11}{64\,ac^4\,(ax+1)}+\frac{47\,\ln(ax+1)}{128\,ac^4}+\frac{1}{32\,ac^4\,(ax-1)^4}+\frac{13}{48\,ac^4\,(ax-1)^3}+\frac{35}{32\,ac^4\,(ax-1)^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^4,x)
```

```
[Out] -x/c^4-1/64/a/c^4/(a*x+1)^2+11/64/a/c^4/(a*x+1)+47/128*ln(a*x+1)/a/c^4+1/32
/a/c^4/(a*x-1)^4+13/48/a/c^4/(a*x-1)^3+35/32/c^4/a/(a*x-1)^2+99/32/c^4/a/(a
*x-1)-303/128/c^4/a*ln(a*x-1)
```

Maxima [A] time = 0.985831, size = 197, normalized size = 1.35

$$\frac{627\,a^5x^5-486\,a^4x^4-1058\,a^3x^3+874\,a^2x^2+467\,ax-400}{192\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4\right)}-\frac{x}{c^4}+\frac{47\,\log(ax+1)}{128\,ac^4}-\frac{303\,\log(ax-1)}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/192*(627*a^5*x^5 - 486*a^4*x^4 - 1058*a^3*x^3 + 874*a^2*x^2 + 467*a*x - 4
00)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^
```

```
2 - 2*a^2*c^4*x + a*c^4) - x/c^4 + 47/128*log(a*x + 1)/(a*c^4) - 303/128*lo
g(a*x - 1)/(a*c^4)
```

Fricas [A] time = 2.07777, size = 510, normalized size = 3.49

$$-\frac{384\,a^7x^7 - 768\,a^6x^6 - 1638\,a^5x^5 + 2508\,a^4x^4 + 1732\,a^3x^3 - 2516\,a^2x^2 - 550\,ax - 141\,\left(a^6x^6 - 2\,a^5x^5 - a^4x^4 + 4\,a^3x^3 - a}{384\,\left(a^7c^4x^6 - 2\,a^6c^4x^5 - a^5c^4x^4 + 4\,a^4c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/384*(384*a^7*x^7 - 768*a^6*x^6 - 1638*a^5*x^5 + 2508*a^4*x^4 + 1732*a^3*
x^3 - 2516*a^2*x^2 - 550*a*x - 141*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x
^3 - a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + 909*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4
 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 800)/(a^7*c^4*x^6 - 2*a^
6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4
)
```

Sympy [A] time = 3.23416, size = 158, normalized size = 1.08

$$-a^8\left(-\frac{627a^5x^5 - 486a^4x^4 - 1058a^3x^3 + 874a^2x^2 + 467ax - 400}{192a^{15}c^4x^6 - 384a^{14}c^4x^5 - 192a^{13}c^4x^4 + 768a^{12}c^4x^3 - 192a^{11}c^4x^2 - 384a^{10}c^4x + 192a^9c^4} + \frac{x}{a^8c^4} + \frac{\frac{303\log\left(x-\frac{1}{a}\right)}{128} - }{a^9c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2)**4,x)
```

```
[Out] -a**8*(-(627*a**5*x**5 - 486*a**4*x**4 - 1058*a**3*x**3 + 874*a**2*x**2 + 4
67*a*x - 400)/(192*a**15*c**4*x**6 - 384*a**14*c**4*x**5 - 192*a**13*c**4*x
**4 + 768*a**12*c**4*x**3 - 192*a**11*c**4*x**2 - 384*a**10*c**4*x + 192*a*
*9*c**4) + x/(a**8*c**4) + (303*log(x - 1/a)/128 - 47*log(x + 1/a)/128)/(a*
*9*c**4))
```

Giac [A] time = 1.23918, size = 131, normalized size = 0.9

$$-\frac{x}{c^4} + \frac{47\,\log\left(|ax+1|\right)}{128\,ac^4} - \frac{303\,\log\left(|ax-1|\right)}{128\,ac^4} + \frac{627\,a^5x^5 - 486\,a^4x^4 - 1058\,a^3x^3 + 874\,a^2x^2 + 467\,ax - 400}{192\,(ax+1)^2(ax-1)^4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -x/c^4 + 47/128*log(abs(a*x + 1))/(a*c^4) - 303/128*log(abs(a*x - 1))/(a*c^
4) + 1/192*(627*a^5*x^5 - 486*a^4*x^4 - 1058*a^3*x^3 + 874*a^2*x^2 + 467*a*
x - 400)/((a*x + 1)^2*(a*x - 1)^4*a*c^4)
```

3.645 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=191

$$-\frac{c^4\left(1-a^2x^2\right)^{7/2}}{2a^7x^6}-\frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7}-\frac{c^4(5ax+24)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5}+\frac{c^4(5ax+16)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3}-\frac{3c^4(16-5ax)\sqrt{1-a^2x^2}}{16a^2x}$$

```
[Out] (-3*c^4*(16 - 5*a*x)*Sqrt[1 - a^2*x^2])/(16*a^2*x) + (c^4*(16 + 5*a*x)*(1 -
 a^2*x^2)^(3/2))/(16*a^4*x^3) - (c^4*(24 + 5*a*x)*(1 - a^2*x^2)^(5/2))/(40*
a^6*x^5) - (c^4*(1 - a^2*x^2)^(7/2))/(7*a^8*x^7) - (c^4*(1 - a^2*x^2)^(7/2)
)/(2*a^7*x^6) - (3*c^4*ArcSin[a*x])/a - (15*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])
/(16*a)
```

Rubi [A] time = 0.366286, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6157, 6148, 1807, 811, 813, 844, 216, 266, 63, 208}

$$-\frac{c^4\left(1-a^2x^2\right)^{7/2}}{2a^7x^6}-\frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7}-\frac{c^4(5ax+24)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5}+\frac{c^4(5ax+16)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3}-\frac{3c^4(16-5ax)\sqrt{1-a^2x^2}}{16a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^4,x]
```


```
[Out] (-3*c^4*(16 - 5*a*x)*Sqrt[1 - a^2*x^2])/(16*a^2*x) + (c^4*(16 + 5*a*x)*(1 -
 a^2*x^2)^(3/2))/(16*a^4*x^3) - (c^4*(24 + 5*a*x)*(1 - a^2*x^2)^(5/2))/(40*
a^6*x^5) - (c^4*(1 - a^2*x^2)^(7/2))/(7*a^8*x^7) - (c^4*(1 - a^2*x^2)^(7/2)
)/(2*a^7*x^6) - (3*c^4*ArcSin[a*x])/a - (15*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])
/(16*a)
```


Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
```

```
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\int \frac{e^{3\tanh^{-1}(ax)}(1-a^2x^2)^4}{x^8}dx}{a^8}$$
$$= \frac{c^4\int \frac{(1+ax)^3(1-a^2x^2)^{5/2}}{x^8}dx}{a^8}$$
$$= -\frac{c^4(1-a^2x^2)^{7/2}}{7a^8x^7} - \frac{c^4\int \frac{(1-a^2x^2)^{5/2}(-21a-21a^2x-7a^3x^2)}{x^7}dx}{7a^8}$$
$$= -\frac{c^4(1-a^2x^2)^{7/2}}{7a^8x^7} - \frac{c^4(1-a^2x^2)^{7/2}}{2a^7x^6} + \frac{c^4\int \frac{(126a^2+21a^3x)(1-a^2x^2)^{5/2}}{x^6}dx}{42a^8}$$
$$= -\frac{c^4(24+5ax)(1-a^2x^2)^{5/2}}{40a^6x^5} - \frac{c^4(1-a^2x^2)^{7/2}}{7a^8x^7} - \frac{c^4(1-a^2x^2)^{7/2}}{2a^7x^6} - \frac{c^4\int \frac{(1008a^4+210a^5x)}{x^4}}{336a^8}$$
$$= \frac{c^4(16+5ax)(1-a^2x^2)^{3/2}}{16a^4x^3} - \frac{c^4(24+5ax)(1-a^2x^2)^{5/2}}{40a^6x^5} - \frac{c^4(1-a^2x^2)^{7/2}}{7a^8x^7} - \frac{c^4(1-a^2}{2a^7x}$$
$$= -\frac{3c^4(16-5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16+5ax)(1-a^2x^2)^{3/2}}{16a^4x^3} - \frac{c^4(24+5ax)(1-a^2x^2)^{5/2}}{40a^6x^5} -$$
$$= -\frac{3c^4(16-5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16+5ax)(1-a^2x^2)^{3/2}}{16a^4x^3} - \frac{c^4(24+5ax)(1-a^2x^2)^{5/2}}{40a^6x^5} -$$
$$= -\frac{3c^4(16-5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16+5ax)(1-a^2x^2)^{3/2}}{16a^4x^3} - \frac{c^4(24+5ax)(1-a^2x^2)^{5/2}}{40a^6x^5} -$$
$$= -\frac{3c^4(16-5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16+5ax)(1-a^2x^2)^{3/2}}{16a^4x^3} - \frac{c^4(24+5ax)(1-a^2x^2)^{5/2}}{40a^6x^5} -$$
$$= -\frac{3c^4(16-5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16+5ax)(1-a^2x^2)^{3/2}}{16a^4x^3} - \frac{c^4(24+5ax)(1-a^2x^2)^{5/2}}{40a^6x^5} -$$

Mathematica [C] time = 0.209166, size = 191, normalized size = 1.

$$\frac{c^4\left(-336a^2x^2\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{2},-\frac{3}{2},a^2x^2\right)-\frac{5\left(16a^7x^7\left(a^2x^2-1\right)^4\text{Hypergeometric2F1}\left(3,\frac{7}{2},\frac{9}{2},1-a^2x^2\right)+16a^8x^8-231a^7x^7-64\right.}{}\right.}{560a^8x^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (c^4*(-336*a^2*x^2*Hypergeometric2F1[-5/2, -5/2, -3/2, a^2*x^2] - (5*(16 +
56*a*x - 64*a^2*x^2 - 238*a^3*x^3 + 96*a^4*x^4 + 413*a^5*x^5 - 64*a^6*x^6 -
 231*a^7*x^7 + 16*a^8*x^8 - 105*a^7*x^7*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 -
a^2*x^2]] + 16*a^7*x^7*(-1 + a^2*x^2)^4*Hypergeometric2F1[3, 7/2, 9/2, 1 -
a^2*x^2]))/Sqrt[1 - a^2*x^2]))/(560*a^8*x^7)
```

Maple [A] time = 0.089, size = 273, normalized size = 1.4

$$-\frac{37c^4}{16x^2a^3}\frac{1}{\sqrt{-a^2x^2+1}} - \frac{218c^4}{35a^2x}\frac{1}{\sqrt{-a^2x^2+1}} - c^4ax^2\frac{1}{\sqrt{-a^2x^2+1}} - 3\frac{c^4}{\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right) - \frac{c^4}{2a^7x^6}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^4,x)
```

```
[Out] -37/16*c^4/a^3/x^2/(-a^2*x^2+1)^(1/2)-218/35*c^4/a^2/x/(-a^2*x^2+1)^(1/2)-c
^4*a*x^2/(-a^2*x^2+1)^(1/2)-3*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^
2+1)^(1/2))-1/2*c^4/a^7/x^6/(-a^2*x^2+1)^(1/2)-1/35*c^4/a^6/x^5/(-a^2*x^2+1
)^(1/2)+15/8*c^4/a^5/x^4/(-a^2*x^2+1)^(1/2)+68/35*c^4/a^4/x^3/(-a^2*x^2+1)^
(1/2)+156/35*c^4*x/(-a^2*x^2+1)^(1/2)-15/16*c^4/a*arctanh(1/(-a^2*x^2+1)^(1
/2))+31/16*c^4/a/(-a^2*x^2+1)^(1/2)-1/7*c^4/a^8/x^7/(-a^2*x^2+1)^(1/2)
```

Maxima [B] time = 1.53111, size = 1022, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="maxima
")
```

```
[Out] -a^3*c^4*(x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4)) + 3*a^
2*c^4*(x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2)
) - 11*c^4*x/sqrt(-a^2*x^2 + 1) - 6*c^4*(1/sqrt(-a^2*x^2 + 1) - log(2*sqrt(
-a^2*x^2 + 1)/abs(x) + 2/abs(x)))/a + 14*(2*a^2*x/sqrt(-a^2*x^2 + 1) - 1/(s
qrt(-a^2*x^2 + 1)*x))*c^4/a^2 - c^4/(sqrt(-a^2*x^2 + 1)*a) - 7*(3*a^2*log(2
*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 3*a^2/sqrt(-a^2*x^2 + 1) + 1/(sqrt
(-a^2*x^2 + 1)*x^2))*c^4/a^3 - 2*(8*a^4*x/sqrt(-a^2*x^2 + 1) - 4*a^2/(sqrt(
-a^2*x^2 + 1)*x) - 1/(sqrt(-a^2*x^2 + 1)*x^3))*c^4/a^4 + 11/8*(15*a^4*log(2
*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 15*a^4/sqrt(-a^2*x^2 + 1) + 5*a^2/
(sqrt(-a^2*x^2 + 1)*x^2) + 2/(sqrt(-a^2*x^2 + 1)*x^4))*c^4/a^5 - 1/5*(16*a^
6*x/sqrt(-a^2*x^2 + 1) - 8*a^4/(sqrt(-a^2*x^2 + 1)*x) - 2*a^2/(sqrt(-a^2*x^
2 + 1)*x^3) - 1/(sqrt(-a^2*x^2 + 1)*x^5))*c^4/a^6 - 1/16*(105*a^6*log(2*sqr
t(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 105*a^6/sqrt(-a^2*x^2 + 1) + 35*a^4/(s
qrt(-a^2*x^2 + 1)*x^2) + 14*a^2/(sqrt(-a^2*x^2 + 1)*x^4) + 8/(sqrt(-a^2*x^2
 + 1)*x^6))*c^4/a^7 + 1/35*(128*a^8*x/sqrt(-a^2*x^2 + 1) - 64*a^6/(sqrt(-a^
2*x^2 + 1)*x) - 16*a^4/(sqrt(-a^2*x^2 + 1)*x^3) - 8*a^2/(sqrt(-a^2*x^2 + 1)
*x^5) - 5/(sqrt(-a^2*x^2 + 1)*x^7))*c^4/a^8
```

Fricas [A] time = 2.22048, size = 398, normalized size = 2.08

$$\frac{3360\,a^7c^4x^7\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+525\,a^7c^4x^7\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+560\,a^7c^4x^7+\left(560\,a^7c^4x^7-2496\,a^6c^4x^6-525\,a^5c^4x^5+\right.}{560\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="fricas
")
```

```
[Out] 1/560*(3360*a^7*c^4*x^7*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 525*a^7*c^
4*x^7*log((sqrt(-a^2*x^2 + 1) - 1)/x) + 560*a^7*c^4*x^7 + (560*a^7*c^4*x^7
- 2496*a^6*c^4*x^6 - 525*a^5*c^4*x^5 + 992*a^4*c^4*x^4 + 770*a^3*c^4*x^3 -
96*a^2*c^4*x^2 - 280*a*c^4*x - 80*c^4)*sqrt(-a^2*x^2 + 1))/(a^8*x^7)
```

Sympy [A] time = 40.3596, size = 935, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**4,x)
```

```
[Out] -a*c**4*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)
) - 3*c**4*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1
/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + 8*c**4*Piecewise((-I*sqrt(a**2*x*
*2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a**2 + 6*c
**4*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1
/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x
**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3 - 6*c**4*Piecewi
se((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**3), A
bs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 +
 1)/(3*x**3), True))/a**4 - 8*c**4*Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*
a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) -
 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*as
in(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1
 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**5 + 3*
c**4*Piecewise((-5*a**6*acosh(1/(a*x))/16 + 5*a**5/(16*x*sqrt(-1 + 1/(a**2*
x**2))) - 5*a**3/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) - a/(24*x**5*sqrt(-1 +
1/(a**2*x**2))) - 1/(6*a*x**7*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) >
 1), (5*I*a**6*asin(1/(a*x))/16 - 5*I*a**5/(16*x*sqrt(1 - 1/(a**2*x**2))) +
 5*I*a**3/(48*x**3*sqrt(1 - 1/(a**2*x**2))) + I*a/(24*x**5*sqrt(1 - 1/(a**2
*x**2))) + I/(6*a*x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**7 + c**4*Piecewi
se((-16*a**7*sqrt(-1 + 1/(a**2*x**2))/35 - 8*a**5*sqrt(-1 + 1/(a**2*x**2))/
(35*x**2) - 6*a**3*sqrt(-1 + 1/(a**2*x**2))/(35*x**4) - a*sqrt(-1 + 1/(a**2
*x**2))/(7*x**6), 1/Abs(a**2*x**2) > 1), (-16*I*a**7*sqrt(1 - 1/(a**2*x**2)
)/35 - 8*I*a**5*sqrt(1 - 1/(a**2*x**2))/(35*x**2) - 6*I*a**3*sqrt(1 - 1/(a*
*2*x**2))/(35*x**4) - I*a*sqrt(1 - 1/(a**2*x**2))/(7*x**6), True))/a**8
```

Giac [B] time = 1.23801, size = 682, normalized size = 3.57

$$\frac{\left(5\,c^4 + \frac{35\left(\sqrt{-a^2x^2+1}|a|+a\right)c^4}{a^2x} + \frac{49\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^4}{a^4x^2} - \frac{245\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c^4}{a^6x^3} - \frac{875\left(\sqrt{-a^2x^2+1}|a|+a\right)^4c^4}{a^8x^4} + \frac{455\left(\sqrt{-a^2x^2+1}|a|+a\right)^5c^4}{a^{10}x^5} + 906\right.}{4480\left(\sqrt{-a^2x^2+1}|a|+a\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] 1/4480*(5*c^4 + 35*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/(a^2*x) + 49*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*c^4/(a^4*x^2) - 245*(sqrt(-a^2*x^2 + 1)*abs(a) +
 a)^3*c^4/(a^6*x^3) - 875*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^8*x^4) +
 455*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^10*x^5) + 9065*(sqrt(-a^2*x^2
 + 1)*abs(a) + a)^6*c^4/(a^12*x^6))*a^14*x^7/((sqrt(-a^2*x^2 + 1)*abs(a) +
a)^7*abs(a)) - 3*c^4*arcsin(a*x)*sgn(a)/abs(a) - 15/16*c^4*log(1/2*abs(-2*s
qrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c
^4/a - 1/4480*(9065*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^4/x + 455*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^4/x^2 - 875*(sqrt(-a^2*x^2 + 1)*abs(a) + a
```

```
)^3*c^4/x^3 - 245*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^2*x^4) + 49*(sqr
t(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^4*x^5) + 35*(sqrt(-a^2*x^2 + 1)*abs(a)
 + a)^6*c^4/(a^6*x^6) + 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7*c^4/(a^8*x^7))/
(a^6*abs(a))
```

3.646 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=157

$$\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{4a^5x^4}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}+\frac{c^3(ax+8)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}-\frac{3c^3(8-ax)\sqrt{1-a^2x^2}}{8a^2x}-\frac{3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{8a}-3c$$

```
[Out] (-3*c^3*(8 - a*x)*Sqrt[1 - a^2*x^2])/(8*a^2*x) + (c^3*(8 + a*x)*(1 - a^2*x^
2)^(3/2))/(8*a^4*x^3) + (c^3*(1 - a^2*x^2)^(5/2))/(5*a^6*x^5) + (3*c^3*(1 -
 a^2*x^2)^(5/2))/(4*a^5*x^4) - (3*c^3*ArcSin[a*x])/a - (3*c^3*ArcTanh[Sqrt[
1 - a^2*x^2]])/(8*a)
```

Rubi [A] time = 0.315477, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6157, 6148, 1807, 811, 813, 844, 216, 266, 63, 208}

$$\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{4a^5x^4}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}+\frac{c^3(ax+8)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}-\frac{3c^3(8-ax)\sqrt{1-a^2x^2}}{8a^2x}-\frac{3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{8a}-3c$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (-3*c^3*(8 - a*x)*Sqrt[1 - a^2*x^2])/(8*a^2*x) + (c^3*(8 + a*x)*(1 - a^2*x^
2)^(3/2))/(8*a^4*x^3) + (c^3*(1 - a^2*x^2)^(5/2))/(5*a^6*x^5) + (3*c^3*(1 -
 a^2*x^2)^(5/2))/(4*a^5*x^4) - (3*c^3*ArcSin[a*x])/a - (3*c^3*ArcTanh[Sqrt[
1 - a^2*x^2]])/(8*a)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
```

```
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3dx=-\frac{c^3\int\frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^3}{x^6}dx}{a^6}$$

$$=-\frac{c^3\int\frac{(1+ax)^3\left(1-a^2x^2\right)^{3/2}}{x^6}dx}{a^6}$$

$$=\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}+\frac{c^3\int\frac{\left(1-a^2x^2\right)^{3/2}\left(-15a-15a^2x-5a^3x^2\right)}{x^5}dx}{5a^6}$$

$$=\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}+\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{4a^5x^4}-\frac{c^3\int\frac{\left(60a^2+5a^3x\right)\left(1-a^2x^2\right)^{3/2}}{x^4}dx}{20a^6}$$

$$=\frac{c^3(8+ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}+\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{4a^5x^4}+\frac{c^3\int\frac{\left(240a^4+30a^5x\right)\sqrt{1-a}}{x^2}}{80a^6}$$

$$=-\frac{3c^3(8-ax)\sqrt{1-a^2x^2}}{8a^2x}+\frac{c^3(8+ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}+\frac{3c^3\left(1-a^2x^2\right)}{4a^5x^4}$$

$$=-\frac{3c^3(8-ax)\sqrt{1-a^2x^2}}{8a^2x}+\frac{c^3(8+ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}+\frac{3c^3\left(1-a^2x^2\right)}{4a^5x^4}$$

$$=-\frac{3c^3(8-ax)\sqrt{1-a^2x^2}}{8a^2x}+\frac{c^3(8+ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}+\frac{3c^3\left(1-a^2x^2\right)}{4a^5x^4}$$

$$=-\frac{3c^3(8-ax)\sqrt{1-a^2x^2}}{8a^2x}+\frac{c^3(8+ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}+\frac{3c^3\left(1-a^2x^2\right)}{4a^5x^4}$$

$$=-\frac{3c^3(8-ax)\sqrt{1-a^2x^2}}{8a^2x}+\frac{c^3(8+ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}+\frac{3c^3\left(1-a^2x^2\right)}{4a^5x^4}$$

Mathematica [C] time = 0.100065, size = 186, normalized size = 1.18

$$\frac{c^3\left(-8a^5x^5\left(a^2x^2-1\right)^3\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},1-a^2x^2\right)+40a^2x^2\sqrt{1-a^2x^2}\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\right.\right.}{40a^6x^5\sqrt{1-}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (c^3*(8 + 30*a*x - 24*a^2*x^2 - 105*a^3*x^3 + 24*a^4*x^4 + 75*a^5*x^5 - 8*a
^6*x^6 + 45*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]] + 40*a^2*x
^2*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[-3/2, -3/2, -1/2, a^2*x^2] - 8*a^5*x
^5*(-1 + a^2*x^2)^3*Hypergeometric2F1[2, 5/2, 7/2, 1 - a^2*x^2]))/(40*a^6*x
^5*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.072, size = 227, normalized size = 1.5

$$-c^3ax^2\frac{1}{\sqrt{-a^2x^2+1}}+\frac{19\,c^3}{8\,a}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{19\,c^3x}{5}\frac{1}{\sqrt{-a^2x^2+1}}-3\,\frac{c^3}{\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+\frac{3\,c^3}{4\,a^5x^4}\frac{1}{\sqrt{-a^2x^2+1}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^3,x)
```

```
[Out] -c^3*a*x^2/(-a^2*x^2+1)^(1/2)+19/8*c^3/a/(-a^2*x^2+1)^(1/2)+19/5*c^3*x/(-a^
2*x^2+1)^(1/2)-3*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+3
/4*c^3/a^5/x^4/(-a^2*x^2+1)^(1/2)-17/8*c^3/a^3/x^2/(-a^2*x^2+1)^(1/2)-3/8*c
^3/a*arctanh(1/(-a^2*x^2+1)^(1/2))-22/5*c^3/a^2/x/(-a^2*x^2+1)^(1/2)+1/5*c^
3/a^6/x^5/(-a^2*x^2+1)^(1/2)+2/5*c^3/a^4/x^3/(-a^2*x^2+1)^(1/2)
```

Maxima [B] time = 1.46734, size = 614, normalized size = 3.91

$$-a^3c^3\left(\frac{x^2}{\sqrt{-a^2x^2+1}a^2}-\frac{2}{\sqrt{-a^2x^2+1}a^4}\right)+3\,a^2c^3\left(\frac{x}{\sqrt{-a^2x^2+1}a^2}-\frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2}\right)-\frac{8\,c^3x}{\sqrt{-a^2x^2+1}}-\frac{6\,c^3\left(\frac{1}{\sqrt{-a^2x^2+1}}-\log\left(2\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="maxima
")
```

```
[Out] -a^3*c^3*(x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4)) + 3*a^
2*c^3*(x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2)
) - 8*c^3*x/sqrt(-a^2*x^2 + 1) - 6*c^3*(1/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-
a^2*x^2 + 1)/abs(x) + 2/abs(x)))/a + 6*(2*a^2*x/sqrt(-a^2*x^2 + 1) - 1/(sqr
t(-a^2*x^2 + 1)*x))*c^3/a^2 - 4*(3*a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/
abs(x)) - 3*a^2/sqrt(-a^2*x^2 + 1) + 1/(sqrt(-a^2*x^2 + 1)*x^2))*c^3/a^3 +
3/8*(15*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 15*a^4/sqrt(-a^2*
x^2 + 1) + 5*a^2/(sqrt(-a^2*x^2 + 1)*x^2) + 2/(sqrt(-a^2*x^2 + 1)*x^4))*c^3
/a^5 - 1/5*(16*a^6*x/sqrt(-a^2*x^2 + 1) - 8*a^4/(sqrt(-a^2*x^2 + 1)*x) - 2*
a^2/(sqrt(-a^2*x^2 + 1)*x^3) - 1/(sqrt(-a^2*x^2 + 1)*x^5))*c^3/a^6
```

Fricas [A] time = 2.27402, size = 338, normalized size = 2.15

$$\frac{240\,a^5c^3x^5\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+15\,a^5c^3x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+40\,a^5c^3x^5+\left(40\,a^5c^3x^5-152\,a^4c^3x^4-55\,a^3c^3x^3+24\,a^2c^3\right.}{40\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="fricas
")
```

```
[Out] 1/40*(240*a^5*c^3*x^5*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 15*a^5*c^3*x
^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + 40*a^5*c^3*x^5 + (40*a^5*c^3*x^5 - 152
*a^4*c^3*x^4 - 55*a^3*c^3*x^3 + 24*a^2*c^3*x^2 + 30*a*c^3*x + 8*c^3)*sqrt(-
a^2*x^2 + 1))/(a^6*x^5)
```

Sympy [A] time = 25.3562, size = 687, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**3,x)
```

```
[Out] -a*c**3*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)
) - 3*c**3*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1
/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - c**3*Piecewise((-acosh(1/(a*x)),
1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a + 5*c**3*Piecewise((-I*sq
rt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/
a**2 + 5*c**3*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2)
)/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 -
 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3 - c**3
*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*
x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**
2*x**2 + 1)/(3*x**3), True))/a**4 - 3*c**3*Piecewise((-3*a**4*acosh(1/(a*x)
)/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x
**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I
*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**
3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True))/a
**5 - c**3*Piecewise((-8*a**5*sqrt(-1 + 1/(a**2*x**2))/15 - 4*a**3*sqrt(-1
+ 1/(a**2*x**2))/(15*x**2) - a*sqrt(-1 + 1/(a**2*x**2))/(5*x**4), 1/Abs(a**
2*x**2) > 1), (-8*I*a**5*sqrt(1 - 1/(a**2*x**2))/15 - 4*I*a**3*sqrt(1 - 1/(
a**2*x**2))/(15*x**2) - I*a*sqrt(1 - 1/(a**2*x**2))/(5*x**4), True))/a**6
```

Giac [B] time = 1.26753, size = 520, normalized size = 3.31

$$-\frac{\left(2\,c^3+\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)c^3}{a^2x}+\frac{30\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^3}{a^4x^2}-\frac{80\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c^3}{a^6x^3}-\frac{580\left(\sqrt{-a^2x^2+1}|a|+a\right)^4c^3}{a^8x^4}\right)a^{10}x^5}{320\left(\sqrt{-a^2x^2+1}|a|+a\right)^5|a|}-\frac{3\,c^3\arcsin\left(ax\right)\mathrm{sg}}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -1/320*(2*c^3 + 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3/(a^2*x) + 30*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*c^3/(a^4*x^2) - 80*(sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*c^3/(a^6*x^3) - 580*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^3/(a^8*x^4))*a
^10*x^5/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*abs(a)) - 3*c^3*arcsin(a*x)*sgn(
a)/abs(a) - 3/8*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*ab
s(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c^3/a - 1/320*(580*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)*a^2*c^3/x + 80*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3/x^2 - 30*(s
qrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^3/(a^2*x^3) - 15*(sqrt(-a^2*x^2 + 1)*abs(
a) + a)^4*c^3/(a^4*x^4) - 2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^3/(a^6*x^5)
)/(a^4*abs(a))
```

3.647 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=124

$$-\frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3}-\frac{c^2(ax+6)\sqrt{1-a^2x^2}}{2a^2x}+\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{3c^2\sin^{-1}(ax)}{a}$$

```
[Out] -(c^2*(6 + a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^2*(1 - a^2*x^2)^(3/2))/(3
*a^4*x^3) - (3*c^2*(1 - a^2*x^2)^(3/2))/(2*a^3*x^2) - (3*c^2*ArcSin[a*x])/a
 + (c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rubi [A] time = 0.270908, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6157, 6148, 1807, 813, 844, 216, 266, 63, 208}

$$-\frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3}-\frac{c^2(ax+6)\sqrt{1-a^2x^2}}{2a^2x}+\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{3c^2\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -(c^2*(6 + a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^2*(1 - a^2*x^2)^(3/2))/(3
*a^4*x^3) - (3*c^2*(1 - a^2*x^2)^(3/2))/(2*a^3*x^2) - (3*c^2*ArcSin[a*x])/a
 + (c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
```

```
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2dx=\frac{c^2\int\frac{e^{3\tanh^{-1}(ax)}(1-a^2x^2)^2}{x^4}dx}{a^4}$$

$$=\frac{c^2\int\frac{(1+ax)^3\sqrt{1-a^2x^2}}{x^4}dx}{a^4}$$

$$=-\frac{c^2(1-a^2x^2)^{3/2}}{3a^4x^3}-\frac{c^2\int\frac{\sqrt{1-a^2x^2}(-9a-9a^2x-3a^3x^2)}{x^3}dx}{3a^4}$$

$$=-\frac{c^2(1-a^2x^2)^{3/2}}{3a^4x^3}-\frac{3c^2(1-a^2x^2)^{3/2}}{2a^3x^2}+\frac{c^2\int\frac{(18a^2-3a^3x)\sqrt{1-a^2x^2}}{x^2}dx}{6a^4}$$

$$=-\frac{c^2(6+ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^2(1-a^2x^2)^{3/2}}{3a^4x^3}-\frac{3c^2(1-a^2x^2)^{3/2}}{2a^3x^2}-\frac{c^2\int\frac{6a^3+36a^4x}{x\sqrt{1-a^2x^2}}dx}{12a^4}$$

$$=-\frac{c^2(6+ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^2(1-a^2x^2)^{3/2}}{3a^4x^3}-\frac{3c^2(1-a^2x^2)^{3/2}}{2a^3x^2}-(3c^2)\int\frac{1}{\sqrt{1-a^2x^2}}dx-\ldots$$

$$=-\frac{c^2(6+ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^2(1-a^2x^2)^{3/2}}{3a^4x^3}-\frac{3c^2(1-a^2x^2)^{3/2}}{2a^3x^2}-\frac{3c^2\sin^{-1}(ax)}{a}-\frac{c^2\,\mathrm{Subst}\ldots}{\ldots}$$

$$=-\frac{c^2(6+ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^2(1-a^2x^2)^{3/2}}{3a^4x^3}-\frac{3c^2(1-a^2x^2)^{3/2}}{2a^3x^2}-\frac{3c^2\sin^{-1}(ax)}{a}+\frac{c^2\,\mathrm{Subst}\ldots}{\ldots}$$

$$=-\frac{c^2(6+ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^2(1-a^2x^2)^{3/2}}{3a^4x^3}-\frac{3c^2(1-a^2x^2)^{3/2}}{2a^3x^2}-\frac{3c^2\sin^{-1}(ax)}{a}+\frac{c^2\tanh^{-1}\ldots}{\ldots}$$

Mathematica [A] time = 0.0651574, size = 128, normalized size = 1.03

$$-\frac{c^2\left(6a^5x^5-16a^4x^4-15a^3x^3+14a^2x^2+18a^3x^3\sqrt{1-a^2x^2}\sin^{-1}(ax)-3a^3x^3\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+9ax+2\right)}{6a^4x^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -(c^2*(2 + 9*a*x + 14*a^2*x^2 - 15*a^3*x^3 - 16*a^4*x^4 + 6*a^5*x^5 + 18*a^
3*x^3*Sqrt[1 - a^2*x^2]*ArcSin[a*x] - 3*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[S
qrt[1 - a^2*x^2]]))/(6*a^4*x^3*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.053, size = 181, normalized size = 1.5

$$-ac^2x^2\frac{1}{\sqrt{-a^2x^2+1}}+\frac{5c^2}{2a}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{8xc^2}{3}\frac{1}{\sqrt{-a^2x^2+1}}-3\frac{c^2}{\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)-\frac{7c^2}{3a^2x}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{c^2}{2a}\mathrm{Art}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^2,x)
```

```
[Out] -c^2*a*x^2/(-a^2*x^2+1)^(1/2)+5/2*c^2/a/(-a^2*x^2+1)^(1/2)+8/3*c^2*x/(-a^2*
x^2+1)^(1/2)-3*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-7/3
*c^2/a^2/x/(-a^2*x^2+1)^(1/2)+1/2*c^2/a*arctanh(1/(-a^2*x^2+1)^(1/2))-3/2*c
```

```
^2/a^3/x^2/(-a^2*x^2+1)^(1/2)-1/3*c^2/a^4/x^3/(-a^2*x^2+1)^(1/2)
```

Maxima [B] time = 1.57591, size = 485, normalized size = 3.91

$$-a^3c^2\left(\frac{x^2}{\sqrt{-a^2x^2+1}a^2}-\frac{2}{\sqrt{-a^2x^2+1}a^4}\right)+3\,a^2c^2\left(\frac{x}{\sqrt{-a^2x^2+1}a^2}-\frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2}\right)-\frac{5\,c^2x}{\sqrt{-a^2x^2+1}}-\frac{5\,c^2\left(\frac{1}{\sqrt{-a^2x^2+1}}-\log\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="maxima
")
```

```
[Out] -a^3*c^2*(x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4)) + 3*a^
2*c^2*(x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2)
) - 5*c^2*x/sqrt(-a^2*x^2 + 1) - 5*c^2*(1/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-
a^2*x^2 + 1)/abs(x) + 2/abs(x)))/a + (2*a^2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(
-a^2*x^2 + 1)*x))*c^2/a^2 + c^2/(sqrt(-a^2*x^2 + 1)*a) - 3/2*(3*a^2*log(2*s
qrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 3*a^2/sqrt(-a^2*x^2 + 1) + 1/(sqrt(-
a^2*x^2 + 1)*x^2))*c^2/a^3 + 1/3*(8*a^4*x/sqrt(-a^2*x^2 + 1) - 4*a^2/(sqrt(
-a^2*x^2 + 1)*x) - 1/(sqrt(-a^2*x^2 + 1)*x^3))*c^2/a^4
```

Fricas [A] time = 2.09998, size = 282, normalized size = 2.27

$$\frac{36\,a^3c^2x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-3\,a^3c^2x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+6\,a^3c^2x^3+\left(6\,a^3c^2x^3-16\,a^2c^2x^2-9\,ac^2x-2\,c^2\right)\sqrt{-a^2x^2}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(36*a^3*c^2*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 3*a^3*c^2*x^3*
log((sqrt(-a^2*x^2 + 1) - 1)/x) + 6*a^3*c^2*x^3 + (6*a^3*c^2*x^3 - 16*a^2*c
^2*x^2 - 9*a*c^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1))/(a^4*x^3)
```

Sympy [A] time = 23.6427, size = 357, normalized size = 2.88

$$-ac^2\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-3c^2\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)-\frac{2c^2\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**2,x)
```

```
[Out] -a*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)
) - 3*c**2*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1
/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - 2*c**2*Piecewise((-acosh(1/(a*x))
, 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a + 2*c**2*Piecewise((-I*
sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True)
)/a**2 + 3*c**2*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**
2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1
 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3 + c*
*2*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(
3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a
**2*x**2 + 1)/(3*x**3), True))/a**4
```

Giac [B] time = 1.18377, size = 354, normalized size = 2.85

$$\frac{\left(c^2+\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)c^2}{a^2x}+\frac{33\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^2}{a^4x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|}-\frac{3\,c^2\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{|a|}+\frac{c^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}+\frac{\sqrt{-a^2x^2+1}c^2}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(c^2 + 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/(a^2*x) + 33*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^2*c^2/(a^4*x^2))*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*abs(a)) - 3*c^2*arcsin(a*x)*sgn(a)/abs(a) + 1/2*c^2*log(1/2*abs(-2*sqr
t(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c^2
/a - 1/24*(33*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/x + 9*(sqrt(-a^2*x^2 + 1)
*abs(a) + a)^2*c^2/(a^2*x^2) + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^2/(a^4*x
^3))/(a^2*abs(a))
```

3.648 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=73

$$\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}+\frac{3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{3c\sin^{-1}(ax)}{a}$$

```
[Out] (c*Sqrt[1 - a^2*x^2])/a + (c*Sqrt[1 - a^2*x^2])/(a^2*x) - (3*c*ArcSin[a*x])
/a + (3*c*ArcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rubi [A] time = 0.209411, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6157, 6148, 1807, 1809, 844, 216, 266, 63, 208}

$$\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}+\frac{3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{3c\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*Sqrt[1 - a^2*x^2])/a + (c*Sqrt[1 - a^2*x^2])/(a^2*x) - (3*c*ArcSin[a*x])
/a + (3*c*ArcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx &= -\frac{c\int \frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)}{x^2}\,dx}{a^2}\\
&= -\frac{c\int \frac{(1+ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= \frac{c\sqrt{1-a^2x^2}}{a^2x}+\frac{c\int \frac{-3a-3a^2x-a^3x^2}{x\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{c\int \frac{3a^3+3a^4x}{x\sqrt{1-a^2x^2}}\,dx}{a^4}\\
&= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-(3c)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx-\frac{(3c)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{a}\\
&= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{3c\sin^{-1}(ax)}{a}-\frac{(3c)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2a}\\
&= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{3c\sin^{-1}(ax)}{a}+\frac{(3c)\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^3}\\
&= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{3c\sin^{-1}(ax)}{a}+\frac{3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0443135, size = 56, normalized size = 0.77

$$\frac{c\left(\sqrt{1-a^2x^2}(ax+1)+3ax\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-3ax\sin^{-1}(ax)\right)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*((1 + a*x)*Sqrt[1 - a^2*x^2] - 3*a*x*ArcSin[a*x] + 3*a*x*ArcTanh[Sqrt[1
- a^2*x^2]]))/(a^2*x)
```

Maple [A] time = 0.043, size = 121, normalized size = 1.7

$$-acx^2\frac{1}{\sqrt{-a^2x^2+1}}+\frac{c}{a}\frac{1}{\sqrt{-a^2x^2+1}}-cx\frac{1}{\sqrt{-a^2x^2+1}}-3\frac{c}{\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+\frac{c}{a^2x}\frac{1}{\sqrt{-a^2x^2+1}}+3\frac{c\mathrm{Artanh}\left(\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2),x)
```

```
[Out] -c*a*x^2/(-a^2*x^2+1)^(1/2)+c/a/(-a^2*x^2+1)^(1/2)-c*x/(-a^2*x^2+1)^(1/2)-3
*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+c/a^2/x/(-a^2*x^2+1
)^(1/2)+3*c/a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [B] time = 1.4669, size = 286, normalized size = 3.92

$$-a^3c\left(\frac{x^2}{\sqrt{-a^2x^2+1}a^2}-\frac{2}{\sqrt{-a^2x^2+1}a^4}\right)+3a^2c\left(\frac{x}{\sqrt{-a^2x^2+1}a^2}-\frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2}\right)-\frac{2cx}{\sqrt{-a^2x^2+1}}-\frac{3c\left(\frac{1}{\sqrt{-a^2x^2+1}}-\log\left(2\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -a^3*c*(x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4)) + 3*a^2*
c*(x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2)) -
2*c*x/sqrt(-a^2*x^2 + 1) - 3*c*(1/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-a^2*x^2
+ 1)/abs(x) + 2/abs(x)))/a - (2*a^2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2
 + 1)*x))*c/a^2 + 2*c/(sqrt(-a^2*x^2 + 1)*a)
```

Fricas [A] time = 2.1226, size = 190, normalized size = 2.6

$$\frac{6\,acx\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-3\,acx\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+acx+\sqrt{-a^2x^2+1}(acx+c)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (6*a*c*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 3*a*c*x*log((sqrt(-a^2*x^
2 + 1) - 1)/x) + a*c*x + sqrt(-a^2*x^2 + 1)*(a*c*x + c))/(a^2*x)
```

Sympy [A] time = 13.1413, size = 150, normalized size = 2.05

$$-ac\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-3c\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\end{cases}\right)-\frac{3c\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for } \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}-\frac{c\left(\begin{cases}\ldots\end{cases}\right)}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2),x)
```

```
[Out] -a*c*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) -
 3*c*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)
*asinh(x*sqrt(-a**2)), a**2 < 0)) - 3*c*Piecewise((-acosh(1/(a*x)), 1/Abs(a
**2*x**2) > 1), (I*asin(1/(a*x)), True))/a - c*Piecewise((-I*sqrt(a**2*x**2
 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a**2
```

Giac [A] time = 1.23643, size = 174, normalized size = 2.38

$$-\frac{a^2cx}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|}-\frac{3\,c\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{|a|}+\frac{3\,c\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\frac{\sqrt{-a^2x^2+1}c}{a}+\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)c}{2\,a^2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] -1/2*a^2*c*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - 3*c*arcsin(a*x)*sgn
(a)/abs(a) + 3*c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x
)))/abs(a) + sqrt(-a^2*x^2 + 1)*c/a + 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c
/(a^2*x*abs(a))
```

3.649 $\int \frac{e^{3\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=95

$$-\frac{(ax+1)^3}{3ac\left(1-a^2x^2\right)^{3/2}}+\frac{2(ax+1)^2}{ac\sqrt{1-a^2x^2}}+\frac{3\sqrt{1-a^2x^2}}{ac}-\frac{3\sin^{-1}(ax)}{ac}$$

```
[Out] -(1 + a*x)^3/(3*a*c*(1 - a^2*x^2)^(3/2)) + (2*(1 + a*x)^2)/(a*c*Sqrt[1 - a^
2*x^2]) + (3*Sqrt[1 - a^2*x^2])/(a*c) - (3*ArcSin[a*x])/(a*c)
```

Rubi [A] time = 0.226167, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6157, 6148, 1635, 21, 669, 641, 216}

$$-\frac{(ax+1)^3}{3ac\left(1-a^2x^2\right)^{3/2}}+\frac{2(ax+1)^2}{ac\sqrt{1-a^2x^2}}+\frac{3\sqrt{1-a^2x^2}}{ac}-\frac{3\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] -(1 + a*x)^3/(3*a*c*(1 - a^2*x^2)^(3/2)) + (2*(1 + a*x)^2)/(a*c*Sqrt[1 - a^
2*x^2]) + (3*Sqrt[1 - a^2*x^2])/(a*c) - (3*ArcSin[a*x])/(a*c)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\frac{a^2\int \frac{e^{3\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}\\
&= -\frac{a^2\int \frac{x^2(1+ax)^3}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c}\\
&= -\frac{(1+ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}}+\frac{a^2\int \frac{\left(\frac{3}{a^2}+\frac{3x}{a}\right)(1+ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c}\\
&= -\frac{(1+ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}}+\frac{\int \frac{(1+ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c}\\
&= -\frac{(1+ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}}+\frac{2(1+ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{3\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{(1+ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}}+\frac{2(1+ax)^2}{ac\sqrt{1-a^2x^2}}+\frac{3\sqrt{1-a^2x^2}}{ac}-\frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{(1+ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}}+\frac{2(1+ax)^2}{ac\sqrt{1-a^2x^2}}+\frac{3\sqrt{1-a^2x^2}}{ac}-\frac{3\sin^{-1}(ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.0756841, size = 78, normalized size = 0.82

$$\frac{-3a^3x^3+16a^2x^2-9(ax-1)\sqrt{1-a^2x^2}\sin^{-1}(ax)+5ax-14}{3ac(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] (-14 + 5*a*x + 16*a^2*x^2 - 3*a^3*x^3 - 9*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcS
in[a*x])/(3*a*c*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.043, size = 168, normalized size = 1.8

$$-\frac{ax^2}{c}\frac{1}{\sqrt{-a^2x^2+1}}+6\frac{1}{ac\sqrt{-a^2x^2+1}}+7\frac{x}{c\sqrt{-a^2x^2+1}}-3\frac{1}{c\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+\frac{4}{3a^2c}\left(x-a^{-1}\right)^{-1}\frac{1}{\sqrt{-a^2\left(x-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2),x)
```

```
[Out] -a/c*x^2/(-a^2*x^2+1)^(1/2)+6/a/c/(-a^2*x^2+1)^(1/2)+7/c*x/(-a^2*x^2+1)^(1/
2)-3/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+4/3/a^2/c/(x-1/
a)/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-8/3/c/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1
/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))), x)
```

Fricas [A] time = 2.12014, size = 236, normalized size = 2.48

$$\frac{14a^2x^2-28ax+18\left(a^2x^2-2ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3a^2x^2-19ax+14\right)\sqrt{-a^2x^2+1}+14}{3\left(a^3cx^2-2a^2cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] 1/3*(14*a^2*x^2 - 28*a*x + 18*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2 +
 1) - 1)/(a*x)) + (3*a^2*x^2 - 19*a*x + 14)*sqrt(-a^2*x^2 + 1) + 14)/(a^3*c
*x^2 - 2*a^2*c*x + a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\left(\int\frac{x^2}{-a^3x^3\sqrt{-a^2x^2+1}+a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int\frac{2ax^3}{-a^3x^3\sqrt{-a^2x^2+1}+a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int\frac{}{-a^3x}\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*(Integral(x**2/(-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2
*x**2 + 1) + a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integra
l(2*a*x**3/(-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1
) + a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(a**2*x*
*4/(-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1) + a*x*
sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

Giac [A] time = 1.23167, size = 170, normalized size = 1.79

$$-\frac{3\arcsin(ax)\,\mathrm{sgn}(a)}{c|a|}+\frac{\sqrt{-a^2x^2+1}}{ac}-\frac{2\left(\frac{24\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}-11\right)}{3c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] -3*arcsin(a*x)*sgn(a)/(c*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c) - 2/3*(24*(sqrt
(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a
^4*x^2) - 11)/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^3*abs(a))
```

3.650 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=125

$$\frac{(ax+1)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}}-\frac{6(ax+1)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}}+\frac{24(ax+1)}{5ac^2\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{3\sin^{-1}(ax)}{ac^2}$$

```
[Out] (1 + a*x)^3/(5*a*c^2*(1 - a^2*x^2)^(5/2)) - (6*(1 + a*x)^2)/(5*a*c^2*(1 - a
^2*x^2)^(3/2)) + (24*(1 + a*x))/(5*a*c^2*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*
x^2]/(a*c^2) - (3*ArcSin[a*x])/(a*c^2)
```

Rubi [A] time = 0.334767, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6157, 6148, 1635, 641, 216}

$$\frac{(ax+1)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}}-\frac{6(ax+1)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}}+\frac{24(ax+1)}{5ac^2\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{3\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (1 + a*x)^3/(5*a*c^2*(1 - a^2*x^2)^(5/2)) - (6*(1 + a*x)^2)/(5*a*c^2*(1 - a
^2*x^2)^(3/2)) + (24*(1 + a*x))/(5*a*c^2*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*
x^2]/(a*c^2) - (3*ArcSin[a*x])/(a*c^2)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{e^{3\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^2}\,dx}{c^2}$$

$$= \frac{a^4\int \frac{x^4(1+ax)^3}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^2}$$

$$= \frac{(1+ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}} - \frac{a^4\int \frac{(1+ax)^2\left(\frac{3}{a^4}+\frac{5x}{a^3}+\frac{5x^2}{a^2}+\frac{5x^3}{a}\right)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^2}$$

$$= \frac{(1+ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}} - \frac{6(1+ax)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^4\int \frac{(1+ax)\left(\frac{27}{a^4}+\frac{30x}{a^3}+\frac{15x^2}{a^2}\right)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^2}$$

$$= \frac{(1+ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}} - \frac{6(1+ax)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}} + \frac{24(1+ax)}{5ac^2\sqrt{1-a^2x^2}} - \frac{a^4\int \frac{\frac{45}{a^4}+\frac{15x}{a^3}}{\sqrt{1-a^2x^2}}\,dx}{15c^2}$$

$$= \frac{(1+ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}} - \frac{6(1+ax)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}} + \frac{24(1+ax)}{5ac^2\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}}{ac^2} - \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^2}$$

$$= \frac{(1+ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}} - \frac{6(1+ax)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}} + \frac{24(1+ax)}{5ac^2\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}}{ac^2} - \frac{3\sin^{-1}(ax)}{ac^2}$$

Mathematica [A] time = 0.0989422, size = 88, normalized size = 0.7

$$\frac{-5a^4x^4+34a^3x^3-18a^2x^2-15(ax-1)^2\sqrt{1-a^2x^2}\sin^{-1}(ax)-33ax+24}{5ac^2(ax-1)^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (24 - 33*a*x - 18*a^2*x^2 + 34*a^3*x^3 - 5*a^4*x^4 - 15*(-1 + a*x)^2*Sqrt[1
 - a^2*x^2]*ArcSin[a*x])/(5*a*c^2*(-1 + a*x)^2*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.05, size = 212, normalized size = 1.7

$$-\frac{ax^2}{c^2}\frac{1}{\sqrt{-a^2x^2+1}}+7\frac{1}{ac^2\sqrt{-a^2x^2+1}}+10\frac{x}{c^2\sqrt{-a^2x^2+1}}-3\frac{1}{c^2\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+\frac{2}{5a^3c^2}\left(x-a^{-1}\right)^{-2}\frac{}{\sqrt{-a^2(x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -a/c^2*x^2/(-a^2*x^2+1)^(1/2)+7/a/c^2/(-a^2*x^2+1)^(1/2)+10*x/c^2/(-a^2*x^2
+1)^(1/2)-3/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+2/5/a^
3/c^2/(x-1/a)^2/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+13/5/a^2/c^2/(x-1/a)/(-a
^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-26/5/c^2/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{(-a^2x^2+1)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^2), x)
```

Fricas [A] time = 2.15279, size = 317, normalized size = 2.54

$$\frac{24\,a^3x^3-72\,a^2x^2+72\,ax+30\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(5\,a^3x^3-39\,a^2x^2+57\,ax-24\right)\sqrt{-a^2}}{5\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas
")
```

```
[Out] 1/5*(24*a^3*x^3 - 72*a^2*x^2 + 72*a*x + 30*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1
)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (5*a^3*x^3 - 39*a^2*x^2 + 57*a*x
 - 24)*sqrt(-a^2*x^2 + 1) - 24)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^4\left(\int \frac{x^4}{-a^4x^4\sqrt{-a^2x^2+1}+2a^3x^3\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^5}{-a^4x^4\sqrt{-a^2x^2+1}+2a^3x^3\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*(Integral(x**4/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a*
*2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Int
egral(a*x**5/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x**2
```

```
+ 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

Giac [A] time = 1.18864, size = 243, normalized size = 1.94

$$-\frac{3\arcsin(ax)\operatorname{sgn}(a)}{c^2|a|}+\frac{\sqrt{-a^2x^2+1}}{ac^2}-\frac{2\left(\frac{80\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{120\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{70\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}-\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4}-\cdots\right)}{5c^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] -3*arcsin(a*x)*sgn(a)/(c^2*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c^2) - 2/5*(80*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 120*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)^2/(a^4*x^2) + 70*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 15*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 19)/(c^2*((sqrt(-a^2*x^2 + 1)*abs(a)
 + a)/(a^2*x) - 1)^5*abs(a))
```

3.651 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=155

$$-\frac{(ax+1)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}}+\frac{38(ax+1)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}}-\frac{137(ax+1)}{35ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{\sqrt{1-a^2x^2}}{ac^3}+\frac{181ax+245}{35ac^3\sqrt{1-a^2x^2}}-\frac{3\sin^{-1}(ax)}{ac^3}$$

```
[Out] -(1 + a*x)^3/(7*a*c^3*(1 - a^2*x^2)^(7/2)) + (38*(1 + a*x)^2)/(35*a*c^3*(1
- a^2*x^2)^(5/2)) - (137*(1 + a*x))/(35*a*c^3*(1 - a^2*x^2)^(3/2)) + (245 +
 181*a*x)/(35*a*c^3*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*x^2]/(a*c^3) - (3*Arc
Sin[a*x])/(a*c^3)
```

Rubi [A] time = 0.439783, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6157, 6148, 1635, 1814, 641, 216}

$$-\frac{(ax+1)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}}+\frac{38(ax+1)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}}-\frac{137(ax+1)}{35ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{\sqrt{1-a^2x^2}}{ac^3}+\frac{181ax+245}{35ac^3\sqrt{1-a^2x^2}}-\frac{3\sin^{-1}(ax)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] -(1 + a*x)^3/(7*a*c^3*(1 - a^2*x^2)^(7/2)) + (38*(1 + a*x)^2)/(35*a*c^3*(1
- a^2*x^2)^(5/2)) - (137*(1 + a*x))/(35*a*c^3*(1 - a^2*x^2)^(3/2)) + (245 +
 181*a*x)/(35*a*c^3*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*x^2]/(a*c^3) - (3*Arc
Sin[a*x])/(a*c^3)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{a^6\int \frac{e^{3\tanh^{-1}(ax)}x^6}{(1-a^2x^2)^3}\,dx}{c^3}$$

$$= -\frac{a^6\int \frac{x^6(1+ax)^3}{(1-a^2x^2)^{9/2}}\,dx}{c^3}$$

$$= -\frac{(1+ax)^3}{7ac^3(1-a^2x^2)^{7/2}} + \frac{a^6\int \frac{(1+ax)^2\left(\frac{3}{a^6}+\frac{7x}{a^5}+\frac{7x^2}{a^4}+\frac{7x^3}{a^3}+\frac{7x^4}{a^2}+\frac{7x^5}{a}\right)}{(1-a^2x^2)^{7/2}}\,dx}{7c^3}$$

$$= -\frac{(1+ax)^3}{7ac^3(1-a^2x^2)^{7/2}} + \frac{38(1+ax)^2}{35ac^3(1-a^2x^2)^{5/2}} - \frac{a^6\int \frac{(1+ax)\left(\frac{61}{a^6}+\frac{140x}{a^5}+\frac{105x^2}{a^4}+\frac{70x^3}{a^3}+\frac{35x^4}{a^2}\right)}{(1-a^2x^2)^{5/2}}\,dx}{35c^3}$$

$$= -\frac{(1+ax)^3}{7ac^3(1-a^2x^2)^{7/2}} + \frac{38(1+ax)^2}{35ac^3(1-a^2x^2)^{5/2}} - \frac{137(1+ax)}{35ac^3(1-a^2x^2)^{3/2}} + \frac{a^6\int \frac{\frac{228}{a^6}+\frac{630x}{a^5}+\frac{315x^2}{a^4}+\frac{105x^3}{a^3}}{(1-a^2x^2)^{3/2}}\,dx}{105c^3}$$

$$= -\frac{(1+ax)^3}{7ac^3(1-a^2x^2)^{7/2}} + \frac{38(1+ax)^2}{35ac^3(1-a^2x^2)^{5/2}} - \frac{137(1+ax)}{35ac^3(1-a^2x^2)^{3/2}} + \frac{245+181ax}{35ac^3\sqrt{1-a^2x^2}} - \frac{a^6\int \frac{\frac{315}{a^6}+\frac{105x}{a^5}}{\sqrt{1-a^2x^2}}\,d}{105c^3}$$

$$= -\frac{(1+ax)^3}{7ac^3(1-a^2x^2)^{7/2}} + \frac{38(1+ax)^2}{35ac^3(1-a^2x^2)^{5/2}} - \frac{137(1+ax)}{35ac^3(1-a^2x^2)^{3/2}} + \frac{245+181ax}{35ac^3\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}}{ac^3} - 3$$

$$= -\frac{(1+ax)^3}{7ac^3(1-a^2x^2)^{7/2}} + \frac{38(1+ax)^2}{35ac^3(1-a^2x^2)^{5/2}} - \frac{137(1+ax)}{35ac^3(1-a^2x^2)^{3/2}} + \frac{245+181ax}{35ac^3\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}}{ac^3} - 3$$

Mathematica [A] time = 0.118628, size = 96, normalized size = 0.62

$$\frac{-35a^5x^5+286a^4x^4-368a^3x^3-125a^2x^2-105(ax-1)^3\sqrt{1-a^2x^2}\sin^{-1}(ax)+423ax-176}{35ac^3(ax-1)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] (-176 + 423*a*x - 125*a^2*x^2 - 368*a^3*x^3 + 286*a^4*x^4 - 35*a^5*x^5 - 10
5*(-1 + a*x)^3*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(35*a*c^3*(-1 + a*x)^3*Sqrt[1
 - a^2*x^2])
```

Maple [A] time = 0.058, size = 256, normalized size = 1.7

$$-\frac{ax^2}{c^3}\frac{1}{\sqrt{-a^2x^2+1}}+8\frac{1}{ac^3\sqrt{-a^2x^2+1}}+13\frac{x}{c^3\sqrt{-a^2x^2+1}}-3\frac{1}{c^3\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+\frac{1}{7a^4c^3}\left(x-a^{-1}\right)^{-3}\frac{}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^3,x)
```

```
[Out] -a/c^3*x^2/(-a^2*x^2+1)^(1/2)+8/a/c^3/(-a^2*x^2+1)^(1/2)+13*x/c^3/(-a^2*x^2
+1)^(1/2)-3/c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/7/a^
4/c^3/(x-1/a)^3/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+38/35/a^3/c^3/(x-1/a)^2/
(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+137/35/a^2/c^3/(x-1/a)/(-a^2*(x-1/a)^2-2
*a*(x-1/a))^(1/2)-274/35/c^3/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^3), x)
```

Fricas [A] time = 2.23451, size = 482, normalized size = 3.11

$$\frac{176\,a^5x^5-528\,a^4x^4+352\,a^3x^3+352\,a^2x^2-528\,ax+210\left(a^5x^5-3\,a^4x^4+2\,a^3x^3+2\,a^2x^2-3\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2}}{ax}\right.}{35\left(a^6c^3x^5-3\,a^5c^3x^4+2\,a^4c^3x^3+2\,a^3c^3x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas
")
```

```
[Out] 1/35*(176*a^5*x^5 - 528*a^4*x^4 + 352*a^3*x^3 + 352*a^2*x^2 - 528*a*x + 210
*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 - 3*a*x + 1)*arctan((sqrt(-a^
2*x^2 + 1) - 1)/(a*x)) + (35*a^5*x^5 - 286*a^4*x^4 + 368*a^3*x^3 + 125*a^2*
x^2 - 423*a*x + 176)*sqrt(-a^2*x^2 + 1) + 176)/(a^6*c^3*x^5 - 3*a^5*c^3*x^4
 + 2*a^4*c^3*x^3 + 2*a^3*c^3*x^2 - 3*a^2*c^3*x + a*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^3), x)
```

3.652 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=185

$$\frac{(ax+1)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}}-\frac{22(ax+1)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}}+\frac{478(ax+1)}{105ac^4\left(1-a^2x^2\right)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{ac^4}+\frac{4(431ax+630)}{315ac^4\sqrt{1-a^2x^2}}-\frac{2(829ax+1155)}{315ac^4\left(1-a^2x^2\right)^3}$$

```
[Out] (1 + a*x)^3/(9*a*c^4*(1 - a^2*x^2)^(9/2)) - (22*(1 + a*x)^2)/(21*a*c^4*(1 -
 a^2*x^2)^(7/2)) + (478*(1 + a*x))/(105*a*c^4*(1 - a^2*x^2)^(5/2)) - (2*(11
55 + 829*a*x))/(315*a*c^4*(1 - a^2*x^2)^(3/2)) + (4*(630 + 431*a*x))/(315*a
*c^4*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*x^2]/(a*c^4) - (3*ArcSin[a*x])/(a*c^
4)
```

Rubi [A] time = 0.564101, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6157, 6148, 1635, 1814, 641, 216}

$$\frac{(ax+1)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}}-\frac{22(ax+1)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}}+\frac{478(ax+1)}{105ac^4\left(1-a^2x^2\right)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{ac^4}+\frac{4(431ax+630)}{315ac^4\sqrt{1-a^2x^2}}-\frac{2(829ax+1155)}{315ac^4\left(1-a^2x^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] (1 + a*x)^3/(9*a*c^4*(1 - a^2*x^2)^(9/2)) - (22*(1 + a*x)^2)/(21*a*c^4*(1 -
 a^2*x^2)^(7/2)) + (478*(1 + a*x))/(105*a*c^4*(1 - a^2*x^2)^(5/2)) - (2*(11
55 + 829*a*x))/(315*a*c^4*(1 - a^2*x^2)^(3/2)) + (4*(630 + 431*a*x))/(315*a
*c^4*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*x^2]/(a*c^4) - (3*ArcSin[a*x])/(a*c^
4)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{e^{3\tanh^{-1}(ax)}x^8}{\left(1-a^2x^2\right)^4}\,dx}{c^4}$$

$$= \frac{a^8\int \frac{x^8(1+ax)^3}{\left(1-a^2x^2\right)^{11/2}}\,dx}{c^4}$$

$$= \frac{(1+ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} - \frac{a^8\int \frac{(1+ax)^2\left(\frac{3}{a^8}+\frac{9x}{a^7}+\frac{9x^2}{a^6}+\frac{9x^3}{a^5}+\frac{9x^4}{a^4}+\frac{9x^5}{a^3}+\frac{9x^6}{a^2}+\frac{9x^7}{a}\right)}{\left(1-a^2x^2\right)^{9/2}}\,dx}{9c^4}$$

$$= \frac{(1+ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} - \frac{22(1+ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{a^8\int \frac{(1+ax)\left(\frac{111}{a^8}+\frac{378x}{a^7}+\frac{315x^2}{a^6}+\frac{252x^3}{a^5}+\frac{189x^4}{a^4}+\frac{126x^5}{a^3}+\frac{63x^6}{a^2}\right)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{63c^4}$$

$$= \frac{(1+ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} - \frac{22(1+ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{478(1+ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{a^8\int \frac{\frac{879}{a^8}+\frac{4725x}{a^7}+\frac{3150x^2}{a^6}+\frac{1890x^3}{a^5}+\frac{945x^4}{a^4}+\frac{315\cdots}{\cdots}}{\left(1-a^2x^2\right)^{5/2}}\cdots}{315c^4}$$

$$= \frac{(1+ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} - \frac{22(1+ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{478(1+ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{2(1155+829ax)}{315ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{a^8\int \frac{\frac{2337}{a^8}+\frac{66\cdots}{\cdots}}{(\cdots}\cdots}{\cdots}$$

$$= \frac{(1+ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} - \frac{22(1+ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{478(1+ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{2(1155+829ax)}{315ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{4(630+43\cdots}{315ac^4\sqrt{1-\cdots}}$$

$$= \frac{(1+ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} - \frac{22(1+ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{478(1+ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{2(1155+829ax)}{315ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{4(630+43\cdots}{315ac^4\sqrt{1-\cdots}}$$

$$= \frac{(1+ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} - \frac{22(1+ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{478(1+ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{2(1155+829ax)}{315ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{4(630+43\cdots}{315ac^4\sqrt{1-\cdots}}$$

Mathematica [A] time = 0.146165, size = 124, normalized size = 0.67

$$\frac{-315a^7x^7+2669a^6x^6-2967a^5x^5-4029a^4x^4+7399a^3x^3-339a^2x^2-945(ax-1)^4(ax+1)\sqrt{1-a^2x^2}\sin^{-1}(ax)-40}{315ac^4(ax-1)^4(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] (1664 - 4047*a*x - 339*a^2*x^2 + 7399*a^3*x^3 - 4029*a^4*x^4 - 2967*a^5*x^5
 + 2669*a^6*x^6 - 315*a^7*x^7 - 945*(-1 + a*x)^4*(1 + a*x)*Sqrt[1 - a^2*x^2
]*ArcSin[a*x])/(315*a*c^4*(-1 + a*x)^4*(1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.061, size = 367, normalized size = 2.

$$-\frac{ax^2}{c^4}\frac{1}{\sqrt{-a^2x^2+1}}+9\frac{1}{ac^4\sqrt{-a^2x^2+1}}+16\frac{x}{c^4\sqrt{-a^2x^2+1}}-3\frac{1}{c^4\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+\frac{125}{252\,a^4c^4}\left(x-a^{-1}\right)^{-3}\frac{}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^4,x)
```

```
[Out] -a/c^4*x^2/(-a^2*x^2+1)^(1/2)+9/a/c^4/(-a^2*x^2+1)^(1/2)+16*x/c^4/(-a^2*x^2
+1)^(1/2)-3/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+125/25
2/a^4/c^4/(x-1/a)^3/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+5111/2520/a^3/c^4/(x
-1/a)^2/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+26633/5040/a^2/c^4/(x-1/a)/(-a^2
*(x-1/a)^2-2*a*(x-1/a))^(1/2)-26633/2520/c^4/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(
1/2)*x+1/18/a^5/c^4/(x-1/a)^4/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/48/a^2/c
^4/(x+1/a)/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1/24/c^4/(-a^2*(x+1/a)^2+2*a*
(x+1/a))^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^4), x)
```

Fricas [A] time = 2.58015, size = 645, normalized size = 3.49

$$\frac{1664\,a^7x^7-4992\,a^6x^6+1664\,a^5x^5+8320\,a^4x^4-8320\,a^3x^3-1664\,a^2x^2+4992\,ax+1890\left(a^7x^7-3\,a^6x^6+a^5x^5+5\,a\right.}{315\left(a^8c^4x^7-3\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas
")
```

```
[Out] 1/315*(1664*a^7*x^7 - 4992*a^6*x^6 + 1664*a^5*x^5 + 8320*a^4*x^4 - 8320*a^3
*x^3 - 1664*a^2*x^2 + 4992*a*x + 1890*(a^7*x^7 - 3*a^6*x^6 + a^5*x^5 + 5*a^
4*x^4 - 5*a^3*x^3 - a^2*x^2 + 3*a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a
*x)) + (315*a^7*x^7 - 2669*a^6*x^6 + 2967*a^5*x^5 + 4029*a^4*x^4 - 7399*a^3
*x^3 + 339*a^2*x^2 + 4047*a*x - 1664)*sqrt(-a^2*x^2 + 1) - 1664)/(a^8*c^4*x
^7 - 3*a^7*c^4*x^6 + a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^4*c^4*x^3 - a^3*c^4*
x^2 + 3*a^2*c^4*x - a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^4), x)
```

3.653 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx$

Optimal. Leaf size=116

$$\frac{4c^5}{a^3x^2}+\frac{14c^5}{3a^4x^3}-\frac{14c^5}{5a^6x^5}-\frac{4c^5}{3a^7x^6}+\frac{3c^5}{7a^8x^7}+\frac{c^5}{2a^9x^8}+\frac{c^5}{9a^{10}x^9}-\frac{3c^5}{a^2x}+\frac{4c^5\log(x)}{a}+c^5x$$

```
[Out] c^5/(9*a^10*x^9) + c^5/(2*a^9*x^8) + (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) - (14*c^5)/(5*a^6*x^5) + (14*c^5)/(3*a^4*x^3) + (4*c^5)/(a^3*x^2) - (3*
c^5)/(a^2*x) + c^5*x + (4*c^5*Log[x])/a
```

Rubi [A] time = 0.14319, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$\frac{4c^5}{a^3x^2}+\frac{14c^5}{3a^4x^3}-\frac{14c^5}{5a^6x^5}-\frac{4c^5}{3a^7x^6}+\frac{3c^5}{7a^8x^7}+\frac{c^5}{2a^9x^8}+\frac{c^5}{9a^{10}x^9}-\frac{3c^5}{a^2x}+\frac{4c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] c^5/(9*a^10*x^9) + c^5/(2*a^9*x^8) + (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) - (14*c^5)/(5*a^6*x^5) + (14*c^5)/(3*a^4*x^3) + (4*c^5)/(a^3*x^2) - (3*
c^5)/(a^2*x) + c^5*x + (4*c^5*Log[x])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=-\frac{c^5\int\frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)^5}{x^{10}}\,dx}{a^{10}}$$
$$=-\frac{c^5\int\frac{(1-ax)^3(1+ax)^7}{x^{10}}\,dx}{a^{10}}$$
$$=-\frac{c^5\int\left(-a^{10}+\frac{1}{x^{10}}+\frac{4a}{x^9}+\frac{3a^2}{x^8}-\frac{8a^3}{x^7}-\frac{14a^4}{x^6}+\frac{14a^6}{x^4}+\frac{8a^7}{x^3}-\frac{3a^8}{x^2}-\frac{4a^9}{x}\right)dx}{a^{10}}$$
$$=\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+c^5x+\frac{4c^5\log(x)}{a}$$

Mathematica [A] time = 0.0355639, size = 116, normalized size = 1.

$$\frac{4c^5}{a^3x^2}+\frac{14c^5}{3a^4x^3}-\frac{14c^5}{5a^6x^5}-\frac{4c^5}{3a^7x^6}+\frac{3c^5}{7a^8x^7}+\frac{c^5}{2a^9x^8}+\frac{c^5}{9a^{10}x^9}-\frac{3c^5}{a^2x}+\frac{4c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] c^5/(9*a^10*x^9) + c^5/(2*a^9*x^8) + (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) - (14*c^5)/(5*a^6*x^5) + (14*c^5)/(3*a^4*x^3) + (4*c^5)/(a^3*x^2) - (3*
c^5)/(a^2*x) + c^5*x + (4*c^5*Log[x])/a
```

Maple [A] time = 0.036, size = 105, normalized size = 0.9

$$\frac{c^5}{9\,a^{10}x^9}+\frac{c^5}{2\,a^9x^8}+\frac{3\,c^5}{7\,a^8x^7}-\frac{4\,c^5}{3\,a^7x^6}-\frac{14\,c^5}{5\,a^6x^5}+\frac{14\,c^5}{3\,a^4x^3}+4\,\frac{c^5}{x^2a^3}-3\,\frac{c^5}{a^2x}+c^5x+4\,\frac{c^5\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^5,x)
```

```
[Out] 1/9*c^5/a^10/x^9+1/2*c^5/a^9/x^8+3/7*c^5/a^8/x^7-4/3*c^5/a^7/x^6-14/5*c^5/a
^6/x^5+14/3*c^5/a^4/x^3+4*c^5/x^2/a^3-3*c^5/a^2/x+c^5*x+4*c^5*ln(x)/a
```

Maxima [A] time = 0.981, size = 139, normalized size = 1.2

$$c^5x+\frac{4\,c^5\log(x)}{a}-\frac{1890\,a^8c^5x^8-2520\,a^7c^5x^7-2940\,a^6c^5x^6+1764\,a^4c^5x^4+840\,a^3c^5x^3-270\,a^2c^5x^2-315\,ac^5x-70\,c^5}{630\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^5,x, algorithm="maxima")
```

```
[Out] c^5*x + 4*c^5*log(x)/a - 1/630*(1890*a^8*c^5*x^8 - 2520*a^7*c^5*x^7 - 2940*
a^6*c^5*x^6 + 1764*a^4*c^5*x^4 + 840*a^3*c^5*x^3 - 270*a^2*c^5*x^2 - 315*a*
c^5*x - 70*c^5)/(a^10*x^9)
```

Fricas [A] time = 2.08493, size = 269, normalized size = 2.32

$$\frac{630\,a^{10}c^5x^{10} + 2520\,a^9c^5x^9\log\left(x\right) - 1890\,a^8c^5x^8 + 2520\,a^7c^5x^7 + 2940\,a^6c^5x^6 - 1764\,a^4c^5x^4 - 840\,a^3c^5x^3 + 270\,a^2c^5x}{630\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/630*(630*a^10*c^5*x^10 + 2520*a^9*c^5*x^9*log(x) - 1890*a^8*c^5*x^8 + 252
0*a^7*c^5*x^7 + 2940*a^6*c^5*x^6 - 1764*a^4*c^5*x^4 - 840*a^3*c^5*x^3 + 270
*a^2*c^5*x^2 + 315*a*c^5*x + 70*c^5)/(a^10*x^9)
```

Sympy [A] time = 3.89744, size = 112, normalized size = 0.97

$$\frac{a^{10}c^5x + 4a^9c^5\log\left(x\right) - \frac{1890a^8c^5x^8 - 2520a^7c^5x^7 - 2940a^6c^5x^6 + 1764a^4c^5x^4 + 840a^3c^5x^3 - 270a^2c^5x^2 - 315ac^5x - 70c^5}{630x^9}}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a**2/x**2)**5,x)
```

```
[Out] (a**10*c**5*x + 4*a**9*c**5*log(x) - (1890*a**8*c**5*x**8 - 2520*a**7*c**5*
x**7 - 2940*a**6*c**5*x**6 + 1764*a**4*c**5*x**4 + 840*a**3*c**5*x**3 - 270
*a**2*c**5*x**2 - 315*a*c**5*x - 70*c**5)/(630*x**9))/a**10
```

Giac [A] time = 1.14537, size = 140, normalized size = 1.21

$$c^5x + \frac{4\,c^5\log\left(|x|\right)}{a} - \frac{1890\,a^8c^5x^8 - 2520\,a^7c^5x^7 - 2940\,a^6c^5x^6 + 1764\,a^4c^5x^4 + 840\,a^3c^5x^3 - 270\,a^2c^5x^2 - 315\,ac^5x - 7}{630\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^5,x, algorithm="giac")
```

```
[Out] c^5*x + 4*c^5*log(abs(x))/a - 1/630*(1890*a^8*c^5*x^8 - 2520*a^7*c^5*x^7 -
2940*a^6*c^5*x^6 + 1764*a^4*c^5*x^4 + 840*a^3*c^5*x^3 - 270*a^2*c^5*x^2 - 3
15*a*c^5*x - 70*c^5)/(a^10*x^9)
```

3.654 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=100

$$\frac{2c^4}{a^3x^2}+\frac{10c^4}{3a^4x^3}+\frac{c^4}{a^5x^4}-\frac{4c^4}{5a^6x^5}-\frac{2c^4}{3a^7x^6}-\frac{c^4}{7a^8x^7}-\frac{4c^4}{a^2x}+\frac{4c^4\log(x)}{a}+c^4x$$

```
[Out] -c^4/(7*a^8*x^7) - (2*c^4)/(3*a^7*x^6) - (4*c^4)/(5*a^6*x^5) + c^4/(a^5*x^4
) + (10*c^4)/(3*a^4*x^3) + (2*c^4)/(a^3*x^2) - (4*c^4)/(a^2*x) + c^4*x + (4
*c^4*Log[x])/a
```

Rubi [A] time = 0.136699, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$\frac{2c^4}{a^3x^2}+\frac{10c^4}{3a^4x^3}+\frac{c^4}{a^5x^4}-\frac{4c^4}{5a^6x^5}-\frac{2c^4}{3a^7x^6}-\frac{c^4}{7a^8x^7}-\frac{4c^4}{a^2x}+\frac{4c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] -c^4/(7*a^8*x^7) - (2*c^4)/(3*a^7*x^6) - (4*c^4)/(5*a^6*x^5) + c^4/(a^5*x^4
) + (10*c^4)/(3*a^4*x^3) + (2*c^4)/(a^3*x^2) - (4*c^4)/(a^2*x) + c^4*x + (4
*c^4*Log[x])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\int \frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$
$$= \frac{c^4\int \frac{(1-ax)^2(1+ax)^6}{x^8}dx}{a^8}$$
$$= \frac{c^4\int \left(a^8+\frac{1}{x^8}+\frac{4a}{x^7}+\frac{4a^2}{x^6}-\frac{4a^3}{x^5}-\frac{10a^4}{x^4}-\frac{4a^5}{x^3}+\frac{4a^6}{x^2}+\frac{4a^7}{x}\right)dx}{a^8}$$
$$= -\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+c^4x+\frac{4c^4\log(x)}{a}$$

Mathematica [A] time = 0.0278238, size = 100, normalized size = 1.

$$\frac{2c^4}{a^3x^2}+\frac{10c^4}{3a^4x^3}+\frac{c^4}{a^5x^4}-\frac{4c^4}{5a^6x^5}-\frac{2c^4}{3a^7x^6}-\frac{c^4}{7a^8x^7}-\frac{4c^4}{a^2x}+\frac{4c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] -c^4/(7*a^8*x^7) - (2*c^4)/(3*a^7*x^6) - (4*c^4)/(5*a^6*x^5) + c^4/(a^5*x^4
) + (10*c^4)/(3*a^4*x^3) + (2*c^4)/(a^3*x^2) - (4*c^4)/(a^2*x) + c^4*x + (4
*c^4*Log[x])/a
```

Maple [A] time = 0.037, size = 93, normalized size = 0.9

$$-\frac{c^4}{7\,a^8x^7}-\frac{2\,c^4}{3\,a^7x^6}-\frac{4\,c^4}{5\,a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10\,c^4}{3\,a^4x^3}+2\,\frac{c^4}{x^2a^3}-4\,\frac{c^4}{a^2x}+c^4x+4\,\frac{c^4\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^4,x)
```

```
[Out] -1/7*c^4/a^8/x^7-2/3*c^4/a^7/x^6-4/5*c^4/a^6/x^5+c^4/a^5/x^4+10/3*c^4/a^4/x
^3+2*c^4/x^2/a^3-4*c^4/a^2/x+c^4*x+4*c^4*ln(x)/a
```

Maxima [A] time = 1.00682, size = 124, normalized size = 1.24

$$c^4x+\frac{4\,c^4\log(x)}{a}-\frac{420\,a^6c^4x^6-210\,a^5c^4x^5-350\,a^4c^4x^4-105\,a^3c^4x^3+84\,a^2c^4x^2+70\,ac^4x+15\,c^4}{105\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] c^4*x + 4*c^4*log(x)/a - 1/105*(420*a^6*c^4*x^6 - 210*a^5*c^4*x^5 - 350*a^4
*c^4*x^4 - 105*a^3*c^4*x^3 + 84*a^2*c^4*x^2 + 70*a*c^4*x + 15*c^4)/(a^8*x^7
)
```

Fricas [A] time = 2.27309, size = 231, normalized size = 2.31

$$\frac{105\,a^8c^4x^8 + 420\,a^7c^4x^7\log(x) - 420\,a^6c^4x^6 + 210\,a^5c^4x^5 + 350\,a^4c^4x^4 + 105\,a^3c^4x^3 - 84\,a^2c^4x^2 - 70\,ac^4x - 15\,c^4}{105\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^8*c^4*x^8 + 420*a^7*c^4*x^7*log(x) - 420*a^6*c^4*x^6 + 210*a^5
*c^4*x^5 + 350*a^4*c^4*x^4 + 105*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x
- 15*c^4)/(a^8*x^7)
```

Sympy [A] time = 3.26825, size = 100, normalized size = 1.

$$\frac{a^8c^4x + 4a^7c^4\log(x) - \frac{420a^6c^4x^6 - 210a^5c^4x^5 - 350a^4c^4x^4 - 105a^3c^4x^3 + 84a^2c^4x^2 + 70ac^4x + 15c^4}{105x^7}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a**2/x**2)**4,x)
```

```
[Out] (a**8*c**4*x + 4*a**7*c**4*log(x) - (420*a**6*c**4*x**6 - 210*a**5*c**4*x**
5 - 350*a**4*c**4*x**4 - 105*a**3*c**4*x**3 + 84*a**2*c**4*x**2 + 70*a*c**4
*x + 15*c**4)/(105*x**7))/a**8
```

Giac [A] time = 1.21732, size = 126, normalized size = 1.26

$$c^4x + \frac{4\,c^4\log(|x|)}{a} - \frac{420\,a^6c^4x^6 - 210\,a^5c^4x^5 - 350\,a^4c^4x^4 - 105\,a^3c^4x^3 + 84\,a^2c^4x^2 + 70\,ac^4x + 15\,c^4}{105\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] c^4*x + 4*c^4*log(abs(x))/a - 1/105*(420*a^6*c^4*x^6 - 210*a^5*c^4*x^5 - 35
0*a^4*c^4*x^4 - 105*a^3*c^4*x^3 + 84*a^2*c^4*x^2 + 70*a*c^4*x + 15*c^4)/(a^
8*x^7)
```

3.655 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=63

$$\frac{5c^3}{3a^4x^3}+\frac{c^3}{a^5x^4}+\frac{c^3}{5a^6x^5}-\frac{5c^3}{a^2x}+\frac{4c^3\log(x)}{a}+c^3x$$

```
[Out] c^3/(5*a^6*x^5) + c^3/(a^5*x^4) + (5*c^3)/(3*a^4*x^3) - (5*c^3)/(a^2*x) + c
^3*x + (4*c^3*Log[x])/a
```

Rubi [A] time = 0.123084, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 75}

$$\frac{5c^3}{3a^4x^3}+\frac{c^3}{a^5x^4}+\frac{c^3}{5a^6x^5}-\frac{5c^3}{a^2x}+\frac{4c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] c^3/(5*a^6*x^5) + c^3/(a^5*x^4) + (5*c^3)/(3*a^4*x^3) - (5*c^3)/(a^2*x) + c
^3*x + (4*c^3*Log[x])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{c^3\int \frac{e^{4\tanh^{-1}(ax)}(1-a^2x^2)^3}{x^6}dx}{a^6}$$
$$= -\frac{c^3\int \frac{(1-ax)(1+ax)^5}{x^6}dx}{a^6}$$
$$= -\frac{c^3\int \left(-a^6+\frac{1}{x^6}+\frac{4a}{x^5}+\frac{5a^2}{x^4}-\frac{5a^4}{x^2}-\frac{4a^5}{x}\right)dx}{a^6}$$
$$= \frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+c^3x+\frac{4c^3\log(x)}{a}$$

Mathematica [A] time = 0.0206176, size = 63, normalized size = 1.

$$\frac{5c^3}{3a^4x^3}+\frac{c^3}{a^5x^4}+\frac{c^3}{5a^6x^5}-\frac{5c^3}{a^2x}+\frac{4c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] c^3/(5*a^6*x^5) + c^3/(a^5*x^4) + (5*c^3)/(3*a^4*x^3) - (5*c^3)/(a^2*x) + c
^3*x + (4*c^3*Log[x])/a
```

Maple [A] time = 0.036, size = 60, normalized size = 1.

$$\frac{c^3}{5\,a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5\,c^3}{3\,a^4x^3}-5\,\frac{c^3}{a^2x}+c^3x+4\,\frac{c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^3,x)
```

```
[Out] 1/5*c^3/a^6/x^5+c^3/a^5/x^4+5/3*c^3/a^4/x^3-5*c^3/a^2/x+c^3*x+4*c^3*ln(x)/a
```

Maxima [A] time = 0.977965, size = 80, normalized size = 1.27

$$c^3x+\frac{4\,c^3\log(x)}{a}-\frac{75\,a^4c^3x^4-25\,a^2c^3x^2-15\,ac^3x-3\,c^3}{15\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] c^3*x + 4*c^3*log(x)/a - 1/15*(75*a^4*c^3*x^4 - 25*a^2*c^3*x^2 - 15*a*c^3*x
 - 3*c^3)/(a^6*x^5)
```

Fricas [A] time = 2.30919, size = 151, normalized size = 2.4

$$\frac{15\,a^6c^3x^6+60\,a^5c^3x^5\log(x)-75\,a^4c^3x^4+25\,a^2c^3x^2+15\,ac^3x+3\,c^3}{15\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^6*c^3*x^6 + 60*a^5*c^3*x^5*log(x) - 75*a^4*c^3*x^4 + 25*a^2*c^3*
x^2 + 15*a*c^3*x + 3*c^3)/(a^6*x^5)
```

Sympy [A] time = 0.741336, size = 65, normalized size = 1.03

$$\frac{a^6c^3x + 4a^5c^3\log(x) - \frac{75a^4c^3x^4 - 25a^2c^3x^2 - 15ac^3x - 3c^3}{15x^5}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a**2/x**2)**3,x)
```

```
[Out] (a**6*c**3*x + 4*a**5*c**3*log(x) - (75*a**4*c**3*x**4 - 25*a**2*c**3*x**2
- 15*a*c**3*x - 3*c**3)/(15*x**5))/a**6
```

Giac [A] time = 1.15911, size = 81, normalized size = 1.29

$$c^3x + \frac{4\,c^3\log(|x|)}{a} - \frac{75\,a^4c^3x^4 - 25\,a^2c^3x^2 - 15\,ac^3x - 3\,c^3}{15\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] c^3*x + 4*c^3*log(abs(x))/a - 1/15*(75*a^4*c^3*x^4 - 25*a^2*c^3*x^2 - 15*a*
c^3*x - 3*c^3)/(a^6*x^5)
```

3.656 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=51

$$-\frac{2c^2}{a^3x^2}-\frac{c^2}{3a^4x^3}-\frac{6c^2}{a^2x}+\frac{4c^2\log(x)}{a}+c^2x$$

```
[Out] -c^2/(3*a^4*x^3) - (2*c^2)/(a^3*x^2) - (6*c^2)/(a^2*x) + c^2*x + (4*c^2*Log
[x])/a
```

Rubi [A] time = 0.11878, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 43}

$$-\frac{2c^2}{a^3x^2}-\frac{c^2}{3a^4x^3}-\frac{6c^2}{a^2x}+\frac{4c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -c^2/(3*a^4*x^3) - (2*c^2)/(a^3*x^2) - (6*c^2)/(a^2*x) + c^2*x + (4*c^2*Log
[x])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{c^2\int \frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}dx}{a^4}$$
$$= \frac{c^2\int \frac{(1+ax)^4}{x^4}dx}{a^4}$$
$$= \frac{c^2\int \left(a^4+\frac{1}{x^4}+\frac{4a}{x^3}+\frac{6a^2}{x^2}+\frac{4a^3}{x}\right)dx}{a^4}$$
$$= -\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}$$

Mathematica [A] time = 0.0169839, size = 51, normalized size = 1.

$$-\frac{2c^2}{a^3x^2}-\frac{c^2}{3a^4x^3}-\frac{6c^2}{a^2x}+\frac{4c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -c^2/(3*a^4*x^3) - (2*c^2)/(a^3*x^2) - (6*c^2)/(a^2*x) + c^2*x + (4*c^2*Log
[x])/a
```

Maple [A] time = 0.036, size = 50, normalized size = 1.

$$-\frac{c^2}{3\,a^4x^3}-2\,\frac{c^2}{x^2a^3}-6\,\frac{c^2}{a^2x}+xc^2+4\,\frac{c^2\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^2,x)
```

```
[Out] -1/3*c^2/a^4/x^3-2*c^2/x^2/a^3-6*c^2/a^2/x+x*c^2+4*c^2*ln(x)/a
```

Maxima [A] time = 0.984707, size = 62, normalized size = 1.22

$$c^2x+\frac{4\,c^2\log(x)}{a}-\frac{18\,a^2c^2x^2+6\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] c^2*x + 4*c^2*log(x)/a - 1/3*(18*a^2*c^2*x^2 + 6*a*c^2*x + c^2)/(a^4*x^3)
```

Fricas [A] time = 2.24394, size = 122, normalized size = 2.39

$$\frac{3\,a^4c^2x^4+12\,a^3c^2x^3\log(x)-18\,a^2c^2x^2-6\,ac^2x-c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^2*x^4 + 12*a^3*c^2*x^3*log(x) - 18*a^2*c^2*x^2 - 6*a*c^2*x - c
^2)/(a^4*x^3)
```

Sympy [A] time = 0.622003, size = 51, normalized size = 1.

$$\frac{a^4c^2x + 4a^3c^2\log(x) - \frac{18a^2c^2x^2+6ac^2x+c^2}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a**2/x**2)**2,x)
```

```
[Out] (a**4*c**2*x + 4*a**3*c**2*log(x) - (18*a**2*c**2*x**2 + 6*a*c**2*x + c**2)
/(3*x**3))/a**4
```

Giac [A] time = 1.21416, size = 63, normalized size = 1.24

$$c^2x + \frac{4\,c^2\log(|x|)}{a} - \frac{18\,a^2c^2x^2 + 6\,ac^2x + c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] c^2*x + 4*c^2*log(abs(x))/a - 1/3*(18*a^2*c^2*x^2 + 6*a*c^2*x + c^2)/(a^4*x
^3)
```

3.657 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=33

$$\frac{c}{a^2x}-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}+cx$$

```
[Out] c/(a^2*x) + c*x - (4*c*Log[x])/a + (8*c*Log[1 - a*x])/a
```

Rubi [A] time = 0.0768802, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6157, 6150, 88}

$$\frac{c}{a^2x}-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] c/(a^2*x) + c*x - (4*c*Log[x])/a + (8*c*Log[1 - a*x])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx &= -\frac{c\int\frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)}{x^2}\,dx}{a^2}\\ &= -\frac{c\int\frac{(1+ax)^3}{x^2(1-ax)}\,dx}{a^2}\\ &= -\frac{c\int\left(-a^2+\frac{1}{x^2}+\frac{4a}{x}-\frac{8a^2}{-1+ax}\right)\,dx}{a^2}\\ &= \frac{c}{a^2x}+cx-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.018009, size = 33, normalized size = 1.

$$\frac{c}{a^2x}-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] c/(a^2*x) + c*x - (4*c*Log[x])/a + (8*c*Log[1 - a*x])/a
```

Maple [A] time = 0.043, size = 33, normalized size = 1.

$$cx+\frac{c}{a^2x}-4\,\frac{c\ln(x)}{a}+8\,\frac{c\ln(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2),x)
```

```
[Out] c*x+c/a^2/x-4*c*ln(x)/a+8*c/a*ln(a*x-1)
```

Maxima [A] time = 0.965423, size = 43, normalized size = 1.3

$$cx+\frac{8\,c\log(ax-1)}{a}-\frac{4\,c\log(x)}{a}+\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] c*x + 8*c*log(a*x - 1)/a - 4*c*log(x)/a + c/(a^2*x)
```

Fricas [A] time = 1.94795, size = 88, normalized size = 2.67

$$\frac{a^2cx^2+8\,acx\log(ax-1)-4\,acx\log(x)+c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (a^2*c*x^2 + 8*a*c*x*log(a*x - 1) - 4*a*c*x*log(x) + c)/(a^2*x)
```

Sympy [A] time = 0.828424, size = 26, normalized size = 0.79

$$cx+\frac{4c\left(-\log(x)+2\log\left(x-\frac{1}{a}\right)\right)}{a}+\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a**2/x**2),x)
```

```
[Out] c*x + 4*c*(-log(x) + 2*log(x - 1/a))/a + c/(a**2*x)
```

Giac [A] time = 1.15249, size = 46, normalized size = 1.39

$$cx + \frac{8\,c\log\left(|ax-1|\right)}{a} - \frac{4\,c\log\left(|x|\right)}{a} + \frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] c*x + 8*c*log(abs(a*x - 1))/a - 4*c*log(abs(x))/a + c/(a^2*x)
```

3.658 $\int \frac{e^{4\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{5}{ac(1-ax)}-\frac{1}{ac(1-ax)^2}+\frac{4\log(1-ax)}{ac}+\frac{x}{c}$$

```
[Out] x/c - 1/(a*c*(1 - a*x)^2) + 5/(a*c*(1 - a*x)) + (4*Log[1 - a*x])/(a*c)
```

Rubi [A] time = 0.133577, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 77}

$$\frac{5}{ac(1-ax)}-\frac{1}{ac(1-ax)^2}+\frac{4\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] x/c - 1/(a*c*(1 - a*x)^2) + 5/(a*c*(1 - a*x)) + (4*Log[1 - a*x])/(a*c)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{a^2\int \frac{e^{4\tanh^{-1}(ax)}x^2}{1-a^2x^2}dx}{c}$$
$$= -\frac{a^2\int \frac{x^2(1+ax)}{(1-ax)^3}dx}{c}$$
$$= -\frac{a^2\int \left(-\frac{1}{a^2}-\frac{2}{a^2(-1+ax)^3}-\frac{5}{a^2(-1+ax)^2}-\frac{4}{a^2(-1+ax)}\right)dx}{c}$$
$$= \frac{x}{c}-\frac{1}{ac(1-ax)^2}+\frac{5}{ac(1-ax)}+\frac{4\log(1-ax)}{ac}$$

Mathematica [A] time = 0.0303221, size = 53, normalized size = 1.

$$\frac{5}{ac(1-ax)}-\frac{1}{ac(1-ax)^2}+\frac{4\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] x/c - 1/(a*c*(1 - a*x)^2) + 5/(a*c*(1 - a*x)) + (4*Log[1 - a*x])/(a*c)
```

Maple [A] time = 0.039, size = 51, normalized size = 1.

$$\frac{x}{c}-\frac{1}{ac\,(ax-1)^2}-5\,\frac{1}{ac\,(ax-1)}+4\,\frac{\ln(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2),x)
```

```
[Out] x/c-1/a/c/(a*x-1)^2-5/a/c/(a*x-1)+4/a/c*ln(a*x-1)
```

Maxima [A] time = 0.958677, size = 66, normalized size = 1.25

$$-\frac{5\,ax-4}{a^3cx^2-2\,a^2cx+ac}+\frac{x}{c}+\frac{4\,\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -(5*a*x - 4)/(a^3*c*x^2 - 2*a^2*c*x + a*c) + x/c + 4*log(a*x - 1)/(a*c)
```

Fricas [A] time = 1.79228, size = 140, normalized size = 2.64

$$\frac{a^3x^3-2\,a^2x^2-4\,ax+4\left(a^2x^2-2\,ax+1\right)\log(ax-1)+4}{a^3cx^2-2\,a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - 2*a^2*x^2 - 4*a*x + 4*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 4)/(a
^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [A] time = 1.22986, size = 41, normalized size = 0.77

$$-\frac{5ax-4}{a^3cx^2-2a^2cx+ac}+\frac{x}{c}+\frac{4\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(c-c/a**2/x**2),x)
```

```
[Out] -(5*a*x - 4)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + x/c + 4*log(a*x - 1)/(a*c)
```

Giac [A] time = 1.18507, size = 57, normalized size = 1.08

$$\frac{x}{c}+\frac{4\log(|ax-1|)}{ac}-\frac{5ax-4}{(ax-1)^2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] x/c + 4*log(abs(a*x - 1))/(a*c) - (5*a*x - 4)/((a*x - 1)^2*a*c)
```

3.659 $\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} dx$

Optimal. Leaf size=71

$$\frac{6}{ac^2(1-ax)}-\frac{2}{ac^2(1-ax)^2}+\frac{1}{3ac^2(1-ax)^3}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

```
[Out] x/c^2 + 1/(3*a*c^2*(1 - a*x)^3) - 2/(a*c^2*(1 - a*x)^2) + 6/(a*c^2*(1 - a*x
)) + (4*Log[1 - a*x])/(a*c^2)
```

Rubi [A] time = 0.136199, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 43}

$$\frac{6}{ac^2(1-ax)}-\frac{2}{ac^2(1-ax)^2}+\frac{1}{3ac^2(1-ax)^3}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] x/c^2 + 1/(3*a*c^2*(1 - a*x)^3) - 2/(a*c^2*(1 - a*x)^2) + 6/(a*c^2*(1 - a*x
)) + (4*Log[1 - a*x])/(a*c^2)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{e^{4\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^2}\,dx}{c^2}$$
$$= \frac{a^4\int \frac{x^4}{(1-ax)^4}\,dx}{c^2}$$
$$= \frac{a^4\int \left(\frac{1}{a^4}+\frac{1}{a^4(-1+ax)^4}+\frac{4}{a^4(-1+ax)^3}+\frac{6}{a^4(-1+ax)^2}+\frac{4}{a^4(-1+ax)}\right)dx}{c^2}$$
$$= \frac{x}{c^2}+\frac{1}{3ac^2(1-ax)^3}-\frac{2}{ac^2(1-ax)^2}+\frac{6}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}$$

Mathematica [A] time = 0.0345264, size = 63, normalized size = 0.89

$$\frac{3a^4x^4-9a^3x^3-9a^2x^2+27ax+12(ax-1)^3\log(1-ax)-13}{3ac^2(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-13 + 27*a*x - 9*a^2*x^2 - 9*a^3*x^3 + 3*a^4*x^4 + 12*(-1 + a*x)^3*Log[1 -
 a*x])/(3*a*c^2*(-1 + a*x)^3)
```

Maple [A] time = 0.034, size = 66, normalized size = 0.9

$$\frac{x}{c^2}-2\frac{1}{ac^2(ax-1)^2}-6\frac{1}{ac^2(ax-1)}-\frac{1}{3ac^2(ax-1)^3}+4\frac{\ln(ax-1)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^2,x)
```

```
[Out] x/c^2-2/a/c^2/(a*x-1)^2-6/a/c^2/(a*x-1)-1/3/a/c^2/(a*x-1)^3+4/c^2/a*ln(a*x-
1)
```

Maxima [A] time = 0.98477, size = 101, normalized size = 1.42

$$-\frac{18a^2x^2-30ax+13}{3\left(a^4c^2x^3-3a^3c^2x^2+3a^2c^2x-ac^2\right)}+\frac{x}{c^2}+\frac{4\log(ax-1)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(18*a^2*x^2 - 30*a*x + 13)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2) + x/c^2 + 4*log(a*x - 1)/(a*c^2)
```

Fricas [A] time = 1.87894, size = 215, normalized size = 3.03

$$\frac{3\,a^4x^4 - 9\,a^3x^3 - 9\,a^2x^2 + 27\,ax + 12\,\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\log\left(ax - 1\right) - 13}{3\,\left(a^4c^2x^3 - 3\,a^3c^2x^2 + 3\,a^2c^2x - ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*x^4 - 9*a^3*x^3 - 9*a^2*x^2 + 27*a*x + 12*(a^3*x^3 - 3*a^2*x^2 +
 3*a*x - 1)*log(a*x - 1) - 13)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x -
 a*c^2)
```

Sympy [A] time = 2.37611, size = 83, normalized size = 1.17

$$a^4\left(-\frac{18a^2x^2 - 30ax + 13}{3a^8c^2x^3 - 9a^7c^2x^2 + 9a^6c^2x - 3a^5c^2} + \frac{x}{a^4c^2} + \frac{4\log\left(ax - 1\right)}{a^5c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*(-(18*a**2*x**2 - 30*a*x + 13)/(3*a**8*c**2*x**3 - 9*a**7*c**2*x**2 +
9*a**6*c**2*x - 3*a**5*c**2) + x/(a**4*c**2) + 4*log(a*x - 1)/(a**5*c**2))
```

Giac [A] time = 1.17194, size = 68, normalized size = 0.96

$$\frac{x}{c^2} + \frac{4\,\log\left(|ax - 1|\right)}{ac^2} - \frac{18\,a^2x^2 - 30\,ax + 13}{3\,(ax - 1)^3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] x/c^2 + 4*log(abs(a*x - 1))/(a*c^2) - 1/3*(18*a^2*x^2 - 30*a*x + 13)/((a*x
- 1)^3*a*c^2)
```

3.660 $\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=111

$$\frac{111}{16ac^3(1-ax)}-\frac{49}{16ac^3(1-ax)^2}+\frac{11}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(ax+1)}{32ac^3}+\frac{x}{c^3}$$

```
[Out] x/c^3 - 1/(8*a*c^3*(1 - a*x)^4) + 11/(12*a*c^3*(1 - a*x)^3) - 49/(16*a*c^3*
(1 - a*x)^2) + 111/(16*a*c^3*(1 - a*x)) + (129*Log[1 - a*x])/(32*a*c^3) - L
og[1 + a*x]/(32*a*c^3)
```

Rubi [A] time = 0.166164, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$\frac{111}{16ac^3(1-ax)}-\frac{49}{16ac^3(1-ax)^2}+\frac{11}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(ax+1)}{32ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] x/c^3 - 1/(8*a*c^3*(1 - a*x)^4) + 11/(12*a*c^3*(1 - a*x)^3) - 49/(16*a*c^3*
(1 - a*x)^2) + 111/(16*a*c^3*(1 - a*x)) + (129*Log[1 - a*x])/(32*a*c^3) - L
og[1 + a*x]/(32*a*c^3)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{a^6\int\frac{e^{4\tanh^{-1}(ax)}x^6}{(1-a^2x^2)^3}\,dx}{c^3}$$
$$= -\frac{a^6\int\frac{x^6}{(1-ax)^5(1+ax)}\,dx}{c^3}$$
$$= -\frac{a^6\int\left(-\frac{1}{a^6}-\frac{1}{2a^6(-1+ax)^5}-\frac{11}{4a^6(-1+ax)^4}-\frac{49}{8a^6(-1+ax)^3}-\frac{111}{16a^6(-1+ax)^2}-\frac{129}{32a^6(-1+ax)}+\frac{1}{32a^6(1+ax)}\right)dx}{c^3}$$
$$= \frac{x}{c^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{11}{12ac^3(1-ax)^3}-\frac{49}{16ac^3(1-ax)^2}+\frac{111}{16ac^3(1-ax)}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(1}{32}$$

Mathematica [A] time = 0.0647984, size = 89, normalized size = 0.8

$$\frac{2\left(48a^5x^5-192a^4x^4-45a^3x^3+660a^2x^2-701ax+224\right)+387(ax-1)^4\log(1-ax)-3(ax-1)^4\log(ax+1)}{96ac^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] (2*(224 - 701*a*x + 660*a^2*x^2 - 45*a^3*x^3 - 192*a^4*x^4 + 48*a^5*x^5) +
387*(-1 + a*x)^4*Log[1 - a*x] - 3*(-1 + a*x)^4*Log[1 + a*x])/(96*a*c^3*(-1
+ a*x)^4)
```

Maple [A] time = 0.039, size = 95, normalized size = 0.9

$$\frac{x}{c^3}-\frac{\ln(ax+1)}{32\,ac^3}-\frac{1}{8\,ac^3\,(ax-1)^4}-\frac{11}{12\,ac^3\,(ax-1)^3}-\frac{49}{16\,ac^3\,(ax-1)^2}-\frac{111}{16\,ac^3\,(ax-1)}+\frac{129\,\ln(ax-1)}{32\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^3,x)
```

```
[Out] x/c^3-1/32*ln(a*x+1)/a/c^3-1/8/a/c^3/(a*x-1)^4-11/12/a/c^3/(a*x-1)^3-49/16/
a/c^3/(a*x-1)^2-111/16/a/c^3/(a*x-1)+129/32/a/c^3*ln(a*x-1)
```

Maxima [A] time = 1.01198, size = 144, normalized size = 1.3

$$-\frac{333\,a^3x^3-852\,a^2x^2+749\,ax-224}{48\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}+\frac{x}{c^3}-\frac{\log(ax+1)}{32\,ac^3}+\frac{129\,\log(ax-1)}{32\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(333*a^3*x^3 - 852*a^2*x^2 + 749*a*x - 224)/(a^5*c^3*x^4 - 4*a^4*c^3*
x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + x/c^3 - 1/32*log(a*x + 1)/(a*c
^3) + 129/32*log(a*x - 1)/(a*c^3)
```

Fricas [A] time = 1.86458, size = 370, normalized size = 3.33

$$\frac{96\,a^5x^5-384\,a^4x^4-90\,a^3x^3+1320\,a^2x^2-1402\,ax-3\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(ax+1\right)+387\left(a^4x^4-4\,a^3 \cdots\right.}{96\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(96*a^5*x^5 - 384*a^4*x^4 - 90*a^3*x^3 + 1320*a^2*x^2 - 1402*a*x - 3*(
a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x + 1) + 387*(a^4*x^4 -
4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 448)/(a^5*c^3*x^4 - 4*a^4
*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [A] time = 2.86011, size = 114, normalized size = 1.03

$$a^6\left(-\frac{333a^3x^3-852a^2x^2+749ax-224}{48a^{11}c^3x^4-192a^{10}c^3x^3+288a^9c^3x^2-192a^8c^3x+48a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{129\log\left(x-\frac{1}{a}\right)}{32}-\frac{\log\left(x+\frac{1}{a}\right)}{32}}{a^7c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*(-(333*a**3*x**3 - 852*a**2*x**2 + 749*a*x - 224)/(48*a**11*c**3*x**4
- 192*a**10*c**3*x**3 + 288*a**9*c**3*x**2 - 192*a**8*c**3*x + 48*a**7*c**3
) + x/(a**6*c**3) + (129*log(x - 1/a)/32 - log(x + 1/a)/32)/(a**7*c**3))
```

Giac [A] time = 1.17464, size = 99, normalized size = 0.89

$$\frac{x}{c^3}-\frac{\log\left(|ax+1|\right)}{32\,ac^3}+\frac{129\,\log\left(|ax-1|\right)}{32\,ac^3}-\frac{333\,a^3x^3-852\,a^2x^2+749\,ax-224}{48\,(ax-1)^4ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] x/c^3 - 1/32*log(abs(a*x + 1))/(a*c^3) + 129/32*log(abs(a*x - 1))/(a*c^3) -
 1/48*(333*a^3*x^3 - 852*a^2*x^2 + 749*a*x - 224)/((a*x - 1)^4*a*c^3)
```

3.661 $\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=146

$$\frac{501}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}-\frac{67}{16ac^4(1-ax)^2}+\frac{83}{48ac^4(1-ax)^3}-\frac{7}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{261\log(1-a}{64ac^4}$$

```
[Out] x/c^4 + 1/(20*a*c^4*(1 - a*x)^5) - 7/(16*a*c^4*(1 - a*x)^4) + 83/(48*a*c^4*
(1 - a*x)^3) - 67/(16*a*c^4*(1 - a*x)^2) + 501/(64*a*c^4*(1 - a*x)) - 1/(64
*a*c^4*(1 + a*x)) + (261*Log[1 - a*x])/(64*a*c^4) - (5*Log[1 + a*x])/(64*a*
c^4)
```

Rubi [A] time = 0.192352, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$\frac{501}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}-\frac{67}{16ac^4(1-ax)^2}+\frac{83}{48ac^4(1-ax)^3}-\frac{7}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{261\log(1-a}{64ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] x/c^4 + 1/(20*a*c^4*(1 - a*x)^5) - 7/(16*a*c^4*(1 - a*x)^4) + 83/(48*a*c^4*
(1 - a*x)^3) - 67/(16*a*c^4*(1 - a*x)^2) + 501/(64*a*c^4*(1 - a*x)) - 1/(64
*a*c^4*(1 + a*x)) + (261*Log[1 - a*x])/(64*a*c^4) - (5*Log[1 + a*x])/(64*a*
c^4)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{e^{4\tanh^{-1}(ax)}x^8}{(1-a^2x^2)^4}\,dx}{c^4}$$

$$= \frac{a^8\int \frac{x^8}{(1-ax)^6(1+ax)^2}\,dx}{c^4}$$

$$= \frac{a^8\int \left(\frac{1}{a^8}+\frac{1}{4a^8(-1+ax)^6}+\frac{7}{4a^8(-1+ax)^5}+\frac{83}{16a^8(-1+ax)^4}+\frac{67}{8a^8(-1+ax)^3}+\frac{501}{64a^8(-1+ax)^2}+\frac{261}{64a^8(-1+ax)}+\frac{1}{64a^8(1+ax)^2}\right.}{c^4}$$

$$= \frac{x}{c^4}+\frac{1}{20ac^4(1-ax)^5}-\frac{7}{16ac^4(1-ax)^4}+\frac{83}{48ac^4(1-ax)^3}-\frac{67}{16ac^4(1-ax)^2}+\frac{501}{64ac^4(1-ax)}-\frac{1}{64ac^4(1}$$

Mathematica [A] time = 0.0969648, size = 98, normalized size = 0.67

$$\frac{\frac{2(480a^7x^7-1920a^6x^6-1365a^5x^5+9300a^4x^4-6800a^3x^3-4900a^2x^2+7541ax-2384)}{(ax-1)^5(ax+1)}+3915\log(1-ax)-75\log(ax+1)}{960ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] ((2*(-2384 + 7541*a*x - 4900*a^2*x^2 - 6800*a^3*x^3 + 9300*a^4*x^4 - 1365*a
^5*x^5 - 1920*a^6*x^6 + 480*a^7*x^7))/((-1 + a*x)^5*(1 + a*x)) + 3915*Log[1
 - a*x] - 75*Log[1 + a*x])/(960*a*c^4)
```

Maple [A] time = 0.043, size = 125, normalized size = 0.9

$$\frac{x}{c^4}-\frac{1}{64\,ac^4\,(ax+1)}-\frac{5\,\ln(ax+1)}{64\,ac^4}-\frac{1}{20\,ac^4\,(ax-1)^5}-\frac{7}{16\,ac^4\,(ax-1)^4}-\frac{83}{48\,ac^4\,(ax-1)^3}-\frac{67}{16\,ac^4\,(ax-1)^2}-\frac{50}{64\,ac^4\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^4,x)
```

```
[Out] x/c^4-1/64/a/c^4/(a*x+1)-5/64*ln(a*x+1)/a/c^4-1/20/a/c^4/(a*x-1)^5-7/16/a/c
^4/(a*x-1)^4-83/48/a/c^4/(a*x-1)^3-67/16/c^4/a/(a*x-1)^2-501/64/c^4/a/(a*x-
1)+261/64/c^4/a*ln(a*x-1)
```

Maxima [A] time = 1.00635, size = 182, normalized size = 1.25

$$-\frac{3765\,a^5x^5-9300\,a^4x^4+4400\,a^3x^3+6820\,a^2x^2-8021\,ax+2384}{480\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4\right)}+\frac{x}{c^4}-\frac{5\,\log(ax+1)}{64\,ac^4}+\frac{261\,\log(ax-1)}{64\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/480*(3765*a^5*x^5 - 9300*a^4*x^4 + 4400*a^3*x^3 + 6820*a^2*x^2 - 8021*a*
x + 2384)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*
a^2*c^4*x - a*c^4) + x/c^4 - 5/64*log(a*x + 1)/(a*c^4) + 261/64*log(a*x - 1
```

```
)/(a*c^4)
```

Fricas [A] time = 1.92101, size = 479, normalized size = 3.28

$$\frac{960\,a^7x^7-3840\,a^6x^6-2730\,a^5x^5+18600\,a^4x^4-13600\,a^3x^3-9800\,a^2x^2+15082\,ax-75\left(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\right.}{960\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/960*(960*a^7*x^7 - 3840*a^6*x^6 - 2730*a^5*x^5 + 18600*a^4*x^4 - 13600*a^
3*x^3 - 9800*a^2*x^2 + 15082*a*x - 75*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*
a^2*x^2 + 4*a*x - 1)*log(a*x + 1) + 3915*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 -
 5*a^2*x^2 + 4*a*x - 1)*log(a*x - 1) - 4768)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 +
 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

Sympy [A] time = 2.24824, size = 144, normalized size = 0.99

$$a^8\left(-\frac{3765a^5x^5-9300a^4x^4+4400a^3x^3+6820a^2x^2-8021ax+2384}{480a^{15}c^4x^6-1920a^{14}c^4x^5+2400a^{13}c^4x^4-2400a^{11}c^4x^2+1920a^{10}c^4x-480a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{261\log\left(x-\frac{1}{a}\right)}{64}-\frac{5\log\left(x+\right.}{64}}{a^9c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*(-(3765*a**5*x**5 - 9300*a**4*x**4 + 4400*a**3*x**3 + 6820*a**2*x**2 -
 8021*a*x + 2384)/(480*a**15*c**4*x**6 - 1920*a**14*c**4*x**5 + 2400*a**13*
c**4*x**4 - 2400*a**11*c**4*x**2 + 1920*a**10*c**4*x - 480*a**9*c**4) + x/(
a**8*c**4) + (261*log(x - 1/a)/64 - 5*log(x + 1/a)/64)/(a**9*c**4))
```

Giac [A] time = 1.31479, size = 130, normalized size = 0.89

$$\frac{x}{c^4}-\frac{5\log\left(|ax+1|\right)}{64\,ac^4}+\frac{261\log\left(|ax-1|\right)}{64\,ac^4}-\frac{3765\,a^5x^5-9300\,a^4x^4+4400\,a^3x^3+6820\,a^2x^2-8021\,ax+2384}{480\,(ax+1)(ax-1)^5ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] x/c^4 - 5/64*log(abs(a*x + 1))/(a*c^4) + 261/64*log(abs(a*x - 1))/(a*c^4) -
 1/480*(3765*a^5*x^5 - 9300*a^4*x^4 + 4400*a^3*x^3 + 6820*a^2*x^2 - 8021*a*
x + 2384)/((a*x + 1)*(a*x - 1)^5*a*c^4)
```

3.662 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$

Optimal. Leaf size=169

$$-\frac{c^4(6-7ax)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7}+\frac{c^4(24-35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5}-\frac{c^4(16-35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(35ax+16)\sqrt{1-a^2x^2}}{16a^2x}-\frac{35c^4}{}$$

```
[Out] (c^4*(16 + 35*a*x)*Sqrt[1 - a^2*x^2])/(16*a^2*x) - (c^4*(16 - 35*a*x)*(1 -
a^2*x^2)^(3/2))/(48*a^4*x^3) + (c^4*(24 - 35*a*x)*(1 - a^2*x^2)^(5/2))/(120
*a^6*x^5) - (c^4*(6 - 7*a*x)*(1 - a^2*x^2)^(7/2))/(42*a^8*x^7) + (c^4*ArcSi
n[a*x])/a - (35*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/(16*a)
```

Rubi [A] time = 0.232407, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6157, 6149, 811, 813, 844, 216, 266, 63, 208}

$$-\frac{c^4(6-7ax)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7}+\frac{c^4(24-35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5}-\frac{c^4(16-35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(35ax+16)\sqrt{1-a^2x^2}}{16a^2x}-\frac{35c^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^4/E^ArcTanh[a*x],x]
```


```
[Out] (c^4*(16 + 35*a*x)*Sqrt[1 - a^2*x^2])/(16*a^2*x) - (c^4*(16 - 35*a*x)*(1 -
a^2*x^2)^(3/2))/(48*a^4*x^3) + (c^4*(24 - 35*a*x)*(1 - a^2*x^2)^(5/2))/(120
*a^6*x^5) - (c^4*(6 - 7*a*x)*(1 - a^2*x^2)^(7/2))/(42*a^8*x^7) + (c^4*ArcSi
n[a*x])/a - (35*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/(16*a)
```


Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\int \frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$
$$= \frac{c^4\int \frac{(1-ax)\left(1-a^2x^2\right)^{7/2}}{x^8}dx}{a^8}$$
$$= -\frac{c^4(6-7ax)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7} - \frac{c^4\int \frac{\left(12a^2-14a^3x\right)\left(1-a^2x^2\right)^{5/2}}{x^6}dx}{12a^8}$$
$$= \frac{c^4(24-35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5} - \frac{c^4(6-7ax)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7} + \frac{c^4\int \frac{\left(96a^4-140a^5x\right)\left(1-a^2x^2\right)^{3/2}}{x^4}dx}{96a^8}$$
$$= -\frac{c^4(16-35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3} + \frac{c^4(24-35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5} - \frac{c^4(6-7ax)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7} -$$
$$= \frac{c^4(16+35ax)\sqrt{1-a^2x^2}}{16a^2x} - \frac{c^4(16-35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3} + \frac{c^4(24-35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5} - \frac{c}{}$$
$$= \frac{c^4(16+35ax)\sqrt{1-a^2x^2}}{16a^2x} - \frac{c^4(16-35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3} + \frac{c^4(24-35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5} - \frac{c}{}$$
$$= \frac{c^4(16+35ax)\sqrt{1-a^2x^2}}{16a^2x} - \frac{c^4(16-35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3} + \frac{c^4(24-35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5} - \frac{c}{}$$
$$= \frac{c^4(16+35ax)\sqrt{1-a^2x^2}}{16a^2x} - \frac{c^4(16-35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3} + \frac{c^4(24-35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5} - \frac{c}{}$$
$$= \frac{c^4(16+35ax)\sqrt{1-a^2x^2}}{16a^2x} - \frac{c^4(16-35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3} + \frac{c^4(24-35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5} - \frac{c}{}$$

Mathematica [C] time = 0.0276227, size = 70, normalized size = 0.41

$$\frac{c^4\left(7a^7\left(1-a^2x^2\right)^{9/2}\text{Hypergeometric2F1}\left(4,\frac{9}{2},\frac{11}{2},1-a^2x^2\right)-\frac{9\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{7}{2},-\frac{5}{2},a^2x^2\right)}{x^7}\right)}{63a^8}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^ArcTanh[a*x],x]
```

```
[Out] (c^4*((-9*Hypergeometric2F1[-7/2, -7/2, -5/2, a^2*x^2])/x^7 + 7*a^7*(1 - a^
2*x^2)^(9/2)*Hypergeometric2F1[4, 9/2, 11/2, 1 - a^2*x^2]))/(63*a^8)
```

Maple [A] time = 0.085, size = 249, normalized size = 1.5

$$-\frac{5c^4}{8a^5x^4}\left(-a^2x^2+1\right)^{\frac{3}{2}}+\frac{19c^4}{16x^2a^3}\left(-a^2x^2+1\right)^{\frac{3}{2}}+\frac{35c^4}{16a}\sqrt{-a^2x^2+1}-\frac{35c^4}{16a}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{c^4}{a^2x}\left(-a^2x^2+1\right)^{\frac{3}{2}}+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -5/8*c^4/a^5/x^4*(-a^2*x^2+1)^(3/2)+19/16*c^4*(-a^2*x^2+1)^(3/2)/x^2/a^3+35
/16*c^4*(-a^2*x^2+1)^(1/2)/a-35/16*c^4/a*arctanh(1/(-a^2*x^2+1)^(1/2))+c^4/
```

```
a^2/x*(-a^2*x^2+1)^(3/2)+c^4*x*(-a^2*x^2+1)^(1/2)+c^4/(a^2)^(1/2)*arctan((a
^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-1/7*c^4/a^8/x^7*(-a^2*x^2+1)^(3/2)+17/35*c^
4/a^6/x^5*(-a^2*x^2+1)^(3/2)-71/105*c^4*(-a^2*x^2+1)^(3/2)/a^4/x^3+1/6*c^4/
a^7/x^6*(-a^2*x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$c^4\left(\frac{\arcsin(ax)}{a} + \frac{\sqrt{-a^2x^2+1}}{a}\right) - \int \frac{\left(4\,a^6c^4x^6 - 6\,a^4c^4x^4 + 4\,a^2c^4x^2 - c^4\right)\sqrt{ax+1}\sqrt{-ax+1}}{a^9x^9 + a^8x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] c^4*(arcsin(a*x)/a + sqrt(-a^2*x^2 + 1)/a) - integrate((4*a^6*c^4*x^6 - 6*a
^4*c^4*x^4 + 4*a^2*c^4*x^2 - c^4)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^9*x^9 + a
^8*x^8), x)
```

Fricas [A] time = 1.98607, size = 413, normalized size = 2.44

$$-\frac{3360\,a^7c^4x^7\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - 3675\,a^7c^4x^7\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - 1680\,a^7c^4x^7 - \left(1680\,a^7c^4x^7 + 2816\,a^6c^4x^6 + 3045\right.}{1680\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/1680*(3360*a^7*c^4*x^7*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 3675*a^7
*c^4*x^7*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 1680*a^7*c^4*x^7 - (1680*a^7*c^4
*x^7 + 2816*a^6*c^4*x^6 + 3045*a^5*c^4*x^5 - 1952*a^4*c^4*x^4 - 1330*a^3*c^
4*x^3 + 1056*a^2*c^4*x^2 + 280*a*c^4*x - 240*c^4)*sqrt(-a^2*x^2 + 1))/(a^8*
x^7)
```

Sympy [C] time = 25.6365, size = 1110, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**4/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c**4*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asi
n(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 -
 log(sqrt(-a**2*x**2 + 1) + 1), True))/a - c**4*Piecewise((-I*a**2*x/sqrt(a
**2*x**2 - 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2)
> 1), (a**2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2 + 1
)), True))/a**2 - 3*c**4*Piecewise((a**2*acosh(1/(a*x))/2 + a/(2*x*sqrt(-1
+ 1/(a**2*x**2))) - 1/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2)
 > 1), (-I*a**2*asin(1/(a*x))/2 - I*a*sqrt(1 - 1/(a**2*x**2))/(2*x), True))
/a**3 + 3*c**4*Piecewise((a**3*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(-1 + 1/(
a**2*x**2))/(3*x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(1 - 1/(a**2*x**2)
```

```
)/3 - I*a*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True))/a**4 + 3*c**4*Piecewise(
(a**4*acosh(1/(a*x))/8 - a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*a/(8*x**3*
sqrt(-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a*
*2*x**2) > 1), (-I*a**4*asin(1/(a*x))/8 + I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2
))) - 3*I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2
*x**2))), True))/a**5 - 3*c**4*Piecewise((2*I*a**4*sqrt(a**2*x**2 - 1)/(15*
x) + I*a**2*sqrt(a**2*x**2 - 1)/(15*x**3) - I*sqrt(a**2*x**2 - 1)/(5*x**5),
 Abs(a**2*x**2) > 1), (2*a**4*sqrt(-a**2*x**2 + 1)/(15*x) + a**2*sqrt(-a**2
*x**2 + 1)/(15*x**3) - sqrt(-a**2*x**2 + 1)/(5*x**5), True))/a**6 - c**4*Pi
ecewise((a**6*acosh(1/(a*x))/16 - a**5/(16*x*sqrt(-1 + 1/(a**2*x**2))) + a*
*3/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) + 5*a/(24*x**5*sqrt(-1 + 1/(a**2*x**2
))) - 1/(6*a*x**7*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*a**
6*asin(1/(a*x))/16 + I*a**5/(16*x*sqrt(1 - 1/(a**2*x**2))) - I*a**3/(48*x**
3*sqrt(1 - 1/(a**2*x**2))) - 5*I*a/(24*x**5*sqrt(1 - 1/(a**2*x**2))) + I/(6
*a*x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**7 + c**4*Piecewise((8*a**7*sqrt
(-1 + 1/(a**2*x**2))/105 + 4*a**5*sqrt(-1 + 1/(a**2*x**2))/(105*x**2) + a**
3*sqrt(-1 + 1/(a**2*x**2))/(35*x**4) - a*sqrt(-1 + 1/(a**2*x**2))/(7*x**6),
 1/Abs(a**2*x**2) > 1), (8*I*a**7*sqrt(1 - 1/(a**2*x**2))/105 + 4*I*a**5*sq
rt(1 - 1/(a**2*x**2))/(105*x**2) + I*a**3*sqrt(1 - 1/(a**2*x**2))/(35*x**4)
 - I*a*sqrt(1 - 1/(a**2*x**2))/(7*x**6), True))/a**8
```

Giac [B] time = 1.28195, size = 680, normalized size = 4.02

$$\frac{\left(15\,c^4 - \frac{35\left(\sqrt{-a^2x^2+1}|a|+a\right)c^4}{a^2x} - \frac{189\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^4}{a^4x^2} + \frac{525\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c^4}{a^6x^3} + \frac{1295\left(\sqrt{-a^2x^2+1}|a|+a\right)^4c^4}{a^8x^4} - \frac{4935\left(\sqrt{-a^2x^2+1}|a|+a\right)^5c^4}{a^{10}x^5} - \cdots\right.}{13440\left(\sqrt{-a^2x^2+1}|a|+a\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/13440*(15*c^4 - 35*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/(a^2*x) - 189*(sqr
t(-a^2*x^2 + 1)*abs(a) + a)^2*c^4/(a^4*x^2) + 525*(sqrt(-a^2*x^2 + 1)*abs(a
) + a)^3*c^4/(a^6*x^3) + 1295*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^8*x^
4) - 4935*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^10*x^5) - 9765*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)^6*c^4/(a^12*x^6))*a^14*x^7/((sqrt(-a^2*x^2 + 1)*abs(
a) + a)^7*abs(a)) + c^4*arcsin(a*x)*sgn(a)/abs(a) - 35/16*c^4*log(1/2*abs(-
2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1
)*c^4/a + 1/13440*(9765*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^4/x + 4935*(s
qrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^4/x^2 - 1295*(sqrt(-a^2*x^2 + 1)*abs(
a) + a)^3*c^4/x^3 - 525*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^2*x^4) + 1
89*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^4*x^5) + 35*(sqrt(-a^2*x^2 + 1)
*abs(a) + a)^6*c^4/(a^6*x^6) - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7*c^4/(a^
8*x^7))/(a^6*abs(a))
```

3.663 $\int e^{-\tanh^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right)^3 dx$

Optimal. Leaf size=136

$$\frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}-\frac{c^3(8-15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3}+\frac{c^3(15ax+8)\sqrt{1-a^2x^2}}{8a^2x}-\frac{15c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{8a}+\frac{c^3\sin^{-1}}{a}$$

```
[Out] (c^3*(8 + 15*a*x)*Sqrt[1 - a^2*x^2])/(8*a^2*x) - (c^3*(8 - 15*a*x)*(1 - a^2
*x^2)^(3/2))/(24*a^4*x^3) + (c^3*(4 - 5*a*x)*(1 - a^2*x^2)^(5/2))/(20*a^6*x
^5) + (c^3*ArcSin[a*x])/a - (15*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/(8*a)
```

Rubi [A] time = 0.223287, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6157, 6149, 811, 813, 844, 216, 266, 63, 208}

$$\frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}-\frac{c^3(8-15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3}+\frac{c^3(15ax+8)\sqrt{1-a^2x^2}}{8a^2x}-\frac{15c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{8a}+\frac{c^3\sin^{-1}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^3/E^ArcTanh[a*x],x]
```

```
[Out] (c^3*(8 + 15*a*x)*Sqrt[1 - a^2*x^2])/(8*a^2*x) - (c^3*(8 - 15*a*x)*(1 - a^2
*x^2)^(3/2))/(24*a^4*x^3) + (c^3*(4 - 5*a*x)*(1 - a^2*x^2)^(5/2))/(20*a^6*x
^5) + (c^3*ArcSin[a*x])/a - (15*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/(8*a)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
```

```
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{c^3\int \frac{e^{-\tanh^{-1}(ax)}(1-a^2x^2)^3}{x^6}dx}{a^6}$$

$$= -\frac{c^3\int \frac{(1-ax)(1-a^2x^2)^{5/2}}{x^6}dx}{a^6}$$

$$= \frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5} + \frac{c^3\int \frac{(8a^2-10a^3x)(1-a^2x^2)^{3/2}}{x^4}dx}{8a^6}$$

$$= -\frac{c^3(8-15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5} - \frac{c^3\int \frac{(32a^4-60a^5x)\sqrt{1-a^2x^2}}{x^2}dx}{32a^6}$$

$$= \frac{c^3(8+15ax)\sqrt{1-a^2x^2}}{8a^2x} - \frac{c^3(8-15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5} + \frac{c^3}{}$$

$$= \frac{c^3(8+15ax)\sqrt{1-a^2x^2}}{8a^2x} - \frac{c^3(8-15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5} + c^3$$

$$= \frac{c^3(8+15ax)\sqrt{1-a^2x^2}}{8a^2x} - \frac{c^3(8-15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5} + \frac{c^3}{}$$

$$= \frac{c^3(8+15ax)\sqrt{1-a^2x^2}}{8a^2x} - \frac{c^3(8-15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5} + \frac{c^3}{}$$

$$= \frac{c^3(8+15ax)\sqrt{1-a^2x^2}}{8a^2x} - \frac{c^3(8-15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5} + \frac{c^3}{}$$

Mathematica [C] time = 0.0252719, size = 70, normalized size = 0.51

$$\frac{c^3\left(\frac{7\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{2},-\frac{3}{2},a^2x^2\right)}{x^5} - 5a^5\left(1-a^2x^2\right)^{7/2}\text{Hypergeometric2F1}\left(3,\frac{7}{2},\frac{9}{2},1-a^2x^2\right)\right)}{35a^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^ArcTanh[a*x],x]
```

```
[Out] (c^3*((7*Hypergeometric2F1[-5/2, -5/2, -3/2, a^2*x^2])/x^5 - 5*a^5*(1 - a^2
*x^2)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, 1 - a^2*x^2]))/(35*a^6)
```

Maple [A] time = 0.056, size = 203, normalized size = 1.5

$$-\frac{c^3}{4a^5x^4}\left(-a^2x^2+1\right)^{\frac{3}{2}} + \frac{7c^3}{8x^2a^3}\left(-a^2x^2+1\right)^{\frac{3}{2}} - \frac{15c^3}{8a}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) + \frac{15c^3}{8a}\sqrt{-a^2x^2+1} + \frac{c^3}{a^2x}\left(-a^2x^2+1\right)^{\frac{3}{2}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/4*c^3/a^5/x^4*(-a^2*x^2+1)^(3/2)+7/8*c^3*(-a^2*x^2+1)^(3/2)/x^2/a^3-15/8
*c^3/a*arctanh(1/(-a^2*x^2+1)^(1/2))+15/8*c^3*(-a^2*x^2+1)^(1/2)/a+c^3/a^2/
x*(-a^2*x^2+1)^(3/2)+c^3*x*(-a^2*x^2+1)^(1/2)+c^3/(a^2)^(1/2)*arctan((a^2)^
(1/2)*x/(-a^2*x^2+1)^(1/2))+1/5*c^3/a^6/x^5*(-a^2*x^2+1)^(3/2)-8/15*c^3/a^4
```

```
/x^3*(-a^2*x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$c^3\left(\frac{\arcsin(ax)}{a}+\frac{\sqrt{-a^2x^2+1}}{a}\right)-\int\frac{(3a^4c^3x^4-3a^2c^3x^2+c^3)\sqrt{ax+1}\sqrt{-ax+1}}{a^7x^7+a^6x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] c^3*(arcsin(a*x)/a + sqrt(-a^2*x^2 + 1)/a) - integrate((3*a^4*c^3*x^4 - 3*a
^2*c^3*x^2 + c^3)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^7*x^7 + a^6*x^6), x)
```

Fricas [A] time = 2.05432, size = 347, normalized size = 2.55

$$-\frac{240a^5c^3x^5\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-225a^5c^3x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-120a^5c^3x^5-(120a^5c^3x^5+184a^4c^3x^4+135a^3c^3x^3-8}{120a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(240*a^5*c^3*x^5*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 225*a^5*c^
3*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 120*a^5*c^3*x^5 - (120*a^5*c^3*x^5
+ 184*a^4*c^3*x^4 + 135*a^3*c^3*x^3 - 88*a^2*c^3*x^2 - 30*a*c^3*x + 24*c^3)
*sqrt(-a^2*x^2 + 1))/(a^6*x^5)
```

Sympy [C] time = 36.5803, size = 692, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**3/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c**3*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asi
n(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 -
 log(sqrt(-a**2*x**2 + 1) + 1), True))/a - c**3*Piecewise((-I*a**2*x/sqrt(a
**2*x**2 - 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2)
> 1), (a**2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2 + 1
)), True))/a**2 - 2*c**3*Piecewise((a**2*acosh(1/(a*x))/2 + a/(2*x*sqrt(-1
+ 1/(a**2*x**2))) - 1/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2)
 > 1), (-I*a**2*asin(1/(a*x))/2 - I*a*sqrt(1 - 1/(a**2*x**2))/(2*x), True))
/a**3 + 2*c**3*Piecewise((a**3*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(-1 + 1/(
a**2*x**2))/(3*x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(1 - 1/(a**2*x**2)
)/3 - I*a*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True))/a**4 + c**3*Piecewise((a
**4*acosh(1/(a*x))/8 - a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*a/(8*x**3*sq
rt(-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2
*x**2) > 1), (-I*a**4*asin(1/(a*x))/8 + I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))
) - 3*I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x
```

```
**2))), True))/a**5 - c**3*Piecewise((2*I*a**4*sqrt(a**2*x**2 - 1)/(15*x) +
 I*a**2*sqrt(a**2*x**2 - 1)/(15*x**3) - I*sqrt(a**2*x**2 - 1)/(5*x**5), Abs
(a**2*x**2) > 1), (2*a**4*sqrt(-a**2*x**2 + 1)/(15*x) + a**2*sqrt(-a**2*x**
2 + 1)/(15*x**3) - sqrt(-a**2*x**2 + 1)/(5*x**5), True))/a**6
```

Giac [B] time = 1.21877, size = 518, normalized size = 3.81

$$-\frac{\left(6\,c^3-\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)c^3}{a^2x}-\frac{70\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^3}{a^4x^2}+\frac{240\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c^3}{a^6x^3}+\frac{660\left(\sqrt{-a^2x^2+1}|a|+a\right)^4c^3}{a^8x^4}\right)a^{10}x^5}{960\left(\sqrt{-a^2x^2+1}|a|+a\right)^5|a|}+\frac{c^3\arcsin\left(ax\right)\mathrm{sg}}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/960*(6*c^3 - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3/(a^2*x) - 70*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*c^3/(a^4*x^2) + 240*(sqrt(-a^2*x^2 + 1)*abs(a) +
 a)^3*c^3/(a^6*x^3) + 660*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^3/(a^8*x^4))*
a^10*x^5/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*abs(a)) + c^3*arcsin(a*x)*sgn(a
)/abs(a) - 15/8*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*ab
s(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c^3/a + 1/960*(660*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)*a^2*c^3/x + 240*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3/x^2 - 70*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^3/(a^2*x^3) - 15*(sqrt(-a^2*x^2 + 1)*abs
(a) + a)^4*c^3/(a^4*x^4) + 6*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^3/(a^6*x^5
))/(a^4*abs(a))
```

3.664 $\int e^{-\tanh^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^2 dx$

Optimal. Leaf size=103

$$-\frac{c^2(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^2(3ax+2)\sqrt{1-a^2x^2}}{2a^2x}-\frac{3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}+\frac{c^2\sin^{-1}(ax)}{a}$$

```
[Out] (c^2*(2 + 3*a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^2*(2 - 3*a*x)*(1 - a^2*x
^2)^(3/2))/(6*a^4*x^3) + (c^2*ArcSin[a*x])/a - (3*c^2*ArcTanh[Sqrt[1 - a^2*
x^2]])/(2*a)
```

Rubi [A] time = 0.169505, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6157, 6149, 811, 813, 844, 216, 266, 63, 208}

$$-\frac{c^2(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^2(3ax+2)\sqrt{1-a^2x^2}}{2a^2x}-\frac{3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}+\frac{c^2\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^2/E^ArcTanh[a*x],x]
```

```
[Out] (c^2*(2 + 3*a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^2*(2 - 3*a*x)*(1 - a^2*x
^2)^(3/2))/(6*a^4*x^3) + (c^2*ArcSin[a*x])/a - (3*c^2*ArcTanh[Sqrt[1 - a^2*
x^2]])/(2*a)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
```

```
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{c^2\int \frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}dx}{a^4}$$
$$= \frac{c^2\int \frac{(1-ax)\left(1-a^2x^2\right)^{3/2}}{x^4}dx}{a^4}$$
$$= -\frac{c^2(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} - \frac{c^2\int \frac{\left(4a^2-6a^3x\right)\sqrt{1-a^2x^2}}{x^2}dx}{4a^4}$$
$$= \frac{c^2(2+3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + \frac{c^2\int \frac{12a^3+8a^4x}{x\sqrt{1-a^2x^2}}dx}{8a^4}$$
$$= \frac{c^2(2+3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + c^2\int \frac{1}{\sqrt{1-a^2x^2}}dx + \frac{\left(3c^2\right)\int \frac{1}{x\sqrt{1-a^2}}}{2a}$$
$$= \frac{c^2(2+3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + \frac{c^2\sin^{-1}(ax)}{a} + \frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-}}\right.}{4a}$$
$$= \frac{c^2(2+3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + \frac{c^2\sin^{-1}(ax)}{a} - \frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\right.}{2a^3}$$
$$= \frac{c^2(2+3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + \frac{c^2\sin^{-1}(ax)}{a} - \frac{3c^2\tanh^{-1}\left(\sqrt{1-a^2}\right.}{2a}$$

Mathematica [C] time = 0.0245761, size = 70, normalized size = 0.68

$$\frac{c^2\left(3a^3\left(1-a^2x^2\right)^{5/2}\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},1-a^2x^2\right)-\frac{5\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},a^2x^2\right)}{x^3}\right)}{15a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^ArcTanh[a*x],x]
```

```
[Out] (c^2*((-5*Hypergeometric2F1[-3/2, -3/2, -1/2, a^2*x^2])/x^3 + 3*a^3*(1 - a^
2*x^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 - a^2*x^2]))/(15*a^4)
```

Maple [A] time = 0.046, size = 157, normalized size = 1.5

$$\frac{c^2}{a^2x}\left(-a^2x^2+1\right)^{\frac{3}{2}}+c^2x\sqrt{-a^2x^2+1}+c^2\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{3c^2}{2a}\sqrt{-a^2x^2+1}-\frac{3c^2}{2a}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] c^2/a^2/x*(-a^2*x^2+1)^(3/2)+c^2*x*(-a^2*x^2+1)^(1/2)+c^2/(a^2)^(1/2)*arcta
n((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+3/2*c^2*(-a^2*x^2+1)^(1/2)/a-3/2*c^2/a*
arctanh(1/(-a^2*x^2+1)^(1/2))+1/2*c^2*(-a^2*x^2+1)^(3/2)/x^2/a^3-1/3*c^2*(-
a^2*x^2+1)^(3/2)/a^4/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\frac{\arcsin(ax)}{a} + \frac{\sqrt{-a^2x^2+1}}{a}\right) - \int \frac{(2a^2c^2x^2 - c^2)\sqrt{ax+1}\sqrt{-ax+1}}{a^5x^5 + a^4x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] c^2*(arcsin(a*x)/a + sqrt(-a^2*x^2 + 1)/a) - integrate((2*a^2*c^2*x^2 - c^2
)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^5*x^5 + a^4*x^4), x)
```

Fricas [A] time = 1.82589, size = 282, normalized size = 2.74

$$-\frac{12a^3c^2x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - 9a^3c^2x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - 6a^3c^2x^3 - (6a^3c^2x^3 + 8a^2c^2x^2 + 3ac^2x - 2c^2)\sqrt{-a^2x^2}}{6a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(12*a^3*c^2*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 9*a^3*c^2*x^3
*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 6*a^3*c^2*x^3 - (6*a^3*c^2*x^3 + 8*a^2*c
^2*x^2 + 3*a*c^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1))/(a^4*x^3)
```

Sympy [C] time = 15.0891, size = 381, normalized size = 3.7

$$\frac{c^2\left(\begin{cases} i\sqrt{a^2x^2-1} - \log(ax) + \frac{\log(a^2x^2)}{2} + i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{for } |a^2x^2| > 1 \\ \sqrt{-a^2x^2+1} + \frac{\log(a^2x^2)}{2} - \log\left(\sqrt{-a^2x^2+1}+1\right) & \text{otherwise} \end{cases}\right)}{a} - \frac{c^2\left(\begin{cases} -\frac{ia^2x}{\sqrt{a^2x^2-1}} + ia\operatorname{acosh}(ax) + \frac{i}{x\sqrt{a^2x^2-1}} & \text{f} \\ \frac{a^2x}{\sqrt{-a^2x^2+1}} - a\operatorname{asin}(ax) - \frac{1}{x\sqrt{-a^2x^2+1}} & \text{c} \end{cases}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**2/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c**2*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asi
n(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 -
 log(sqrt(-a**2*x**2 + 1) + 1), True))/a - c**2*Piecewise((-I*a**2*x/sqrt(a
**2*x**2 - 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2)
> 1), (a**2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2 + 1
)), True))/a**2 - c**2*Piecewise((a**2*acosh(1/(a*x))/2 + a/(2*x*sqrt(-1 +
1/(a**2*x**2))) - 1/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) >
 1), (-I*a**2*asin(1/(a*x))/2 - I*a*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a
**3 + c**2*Piecewise((a**3*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(-1 + 1/(a**2
*x**2))/(3*x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(1 - 1/(a**2*x**2))/3
- I*a*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True))/a**4
```

Giac [B] time = 1.2255, size = 354, normalized size = 3.44

$$\frac{\left(c^2-\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)c^2}{a^2x}-\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^2}{a^4x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|}+\frac{c^2\arcsin(ax)\operatorname{sgn}(a)}{|a|}-\frac{3c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{2|a|}+\frac{\sqrt{-a^2x^2+1}c^2}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(c^2 - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/(a^2*x) - 15*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^2*c^2/(a^4*x^2))*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*abs(a)) + c^2*arcsin(a*x)*sgn(a)/abs(a) - 3/2*c^2*log(1/2*abs(-2*sqrt(
-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c^2/a
 + 1/24*(15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/x + 3*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)^2*c^2/(a^2*x^2) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^2/(a^4*x^3
))/(a^2*abs(a))
```

3.665 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=58

$$\frac{c\sqrt{1-a^2x^2}(ax+1)}{a^2x}-\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}+\frac{c\sin^{-1}(ax)}{a}$$

```
[Out] (c*(1 + a*x)*Sqrt[1 - a^2*x^2])/(a^2*x) + (c*ArcSin[a*x])/a - (c*ArcTanh[Sq
rt[1 - a^2*x^2]])/a
```

Rubi [A] time = 0.112489, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6157, 6149, 813, 844, 216, 266, 63, 208}

$$\frac{c\sqrt{1-a^2x^2}(ax+1)}{a^2x}-\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}+\frac{c\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))/E^ArcTanh[a*x],x]
```


```
[Out] (c*(1 + a*x)*Sqrt[1 - a^2*x^2])/(a^2*x) + (c*ArcSin[a*x])/a - (c*ArcTanh[Sq
rt[1 - a^2*x^2]])/a
```


Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx &= -\frac{c\int \frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)}{x^2}\,dx}{a^2}\\
&= -\frac{c\int \frac{(1-ax)\sqrt{1-a^2x^2}}{x^2}\,dx}{a^2}\\
&= \frac{c(1+ax)\sqrt{1-a^2x^2}}{a^2x}+\frac{c\int \frac{2a+2a^2x}{x\sqrt{1-a^2x^2}}\,dx}{2a^2}\\
&= \frac{c(1+ax)\sqrt{1-a^2x^2}}{a^2x}+c\int \frac{1}{\sqrt{1-a^2x^2}}\,dx+\frac{c\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{a}\\
&= \frac{c(1+ax)\sqrt{1-a^2x^2}}{a^2x}+\frac{c\sin^{-1}(ax)}{a}+\frac{c\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2a}\\
&= \frac{c(1+ax)\sqrt{1-a^2x^2}}{a^2x}+\frac{c\sin^{-1}(ax)}{a}-\frac{c\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^3}\\
&= \frac{c(1+ax)\sqrt{1-a^2x^2}}{a^2x}+\frac{c\sin^{-1}(ax)}{a}-\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0375332, size = 55, normalized size = 0.95

$$
\frac{c\left(\sqrt{1-a^2x^2}(ax+1)-ax\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+ax\sin^{-1}(ax)\right)}{a^2x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))/E^ArcTanh[a*x],x]
```

```
[Out] (c*((1 + a*x)*Sqrt[1 - a^2*x^2] + a*x*ArcSin[a*x] - a*x*ArcTanh[Sqrt[1 - a^
2*x^2]]))/(a^2*x)
```

Maple [A] time = 0.039, size = 100, normalized size = 1.7

$$\frac{c}{a^2x}\left(-a^2x^2+1\right)^{\frac{3}{2}}+cx\sqrt{-a^2x^2+1}+c\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{c}{a}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{c}{a}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] c/a^2/x*(-a^2*x^2+1)^(3/2)+c*x*(-a^2*x^2+1)^(1/2)+c/(a^2)^(1/2)*arctan((a^2
)^(1/2)*x/(-a^2*x^2+1)^(1/2))-c/a*arctanh(1/(-a^2*x^2+1)^(1/2))+c*(-a^2*x^2
+1)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$c\left(\frac{\arcsin\left(ax\right)}{a}+\frac{\sqrt{-a^2x^2+1}}{a}\right)-c\int\frac{\sqrt{ax+1}\sqrt{-ax+1}}{a^3x^3+a^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] c*(arcsin(a*x)/a + sqrt(-a^2*x^2 + 1)/a) - c*integrate(sqrt(a*x + 1)*sqrt(-
a*x + 1)/(a^3*x^3 + a^2*x^2), x)
```

Fricas [A] time = 2.24217, size = 189, normalized size = 3.26

$$-\frac{2\,acx\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-acx\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-acx-\sqrt{-a^2x^2+1}(acx+c)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a*c*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - a*c*x*log((sqrt(-a^2*x^2
 + 1) - 1)/x) - a*c*x - sqrt(-a^2*x^2 + 1)*(a*c*x + c))/(a^2*x)
```

Sympy [C] time = 7.11426, size = 177, normalized size = 3.05

$$\frac{c\left(\begin{cases}i\sqrt{a^2x^2-1}-\log\left(ax\right)+\frac{\log\left(a^2x^2\right)}{2}+i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{for}\ \left|a^2x^2\right|>1\\ \sqrt{-a^2x^2+1}+\frac{\log\left(a^2x^2\right)}{2}-\log\left(\sqrt{-a^2x^2+1}+1\right) & \text{otherwise}\end{cases}\right)}{a}-\frac{c\left(\begin{cases}-\frac{ia^2x}{\sqrt{a^2x^2-1}}+ia\,\mathrm{acosh}\left(ax\right)+\frac{i}{x\sqrt{a^2x^2-1}} & \text{for}\\ \frac{a^2x}{\sqrt{-a^2x^2+1}}-a\,\mathrm{asin}\left(ax\right)-\frac{1}{x\sqrt{-a^2x^2+1}} & \text{oth}\end{cases}\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asin(1
/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 - lo
g(sqrt(-a**2*x**2 + 1) + 1), True))/a - c*Piecewise((-I*a**2*x/sqrt(a**2*x*
*2 - 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2) > 1),
(a**2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2 + 1)), Tr
ue))/a**2
```

Giac [B] time = 1.19562, size = 173, normalized size = 2.98

$$-\frac{a^2cx}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|}+\frac{c\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{|a|}-\frac{c\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\frac{\sqrt{-a^2x^2+1}c}{a}+\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)c}{2\,a^2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a^2*c*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) + c*arcsin(a*x)*sgn(a
)/abs(a) - c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/
abs(a) + sqrt(-a^2*x^2 + 1)*c/a + 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c/(a^
2*x*abs(a))
```

3.666 $\int \frac{e^{-\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=60

$$\frac{1-ax}{ac\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac}+\frac{\sin^{-1}(ax)}{ac}$$

```
[Out] (1 - a*x)/(a*c*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*x^2]/(a*c) + ArcSin[a*x]/(
a*c)
```

Rubi [A] time = 0.140548, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6157, 6149, 797, 641, 216, 637}

$$\frac{1-ax}{ac\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac}+\frac{\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))),x]
```


```
[Out] (1 - a*x)/(a*c*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*x^2]/(a*c) + ArcSin[a*x]/(
a*c)
```


Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\frac{a^2\int \frac{e^{-\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}\\ &= -\frac{a^2\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx}{c}\\ &= -\frac{\int \frac{1-ax}{(1-a^2x^2)^{3/2}}\,dx}{c}+\frac{\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx}{c}\\ &= \frac{1-ax}{ac\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\ &= \frac{1-ax}{ac\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac}+\frac{\sin^{-1}(ax)}{ac}\end{aligned}$$

Mathematica [A] time = 0.0288781, size = 54, normalized size = 0.9

$$\frac{-a^2x^2+\sqrt{1-a^2x^2}\sin^{-1}(ax)-ax+2}{ac\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))),x]
```

```
[Out] (2 - a*x - a^2*x^2 + Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(a*c*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.047, size = 192, normalized size = 3.2

$$\frac{1}{2a^3c\left(x+a^{-1}\right)^2}\left(-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)\right)^{\frac{3}{2}}+\frac{5}{4ac}\sqrt{-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)}+\frac{5}{4c}\arctan\left(x\sqrt{a^2}\frac{}{\sqrt{-a^2\left(x+\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2),x)
```

```
[Out] 1/2/a^3/c/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+5/4/a/c*(-a^2*(x+1/a
)^2+2*a*(x+1/a))^(1/2)+5/4/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)
^2+2*a*(x+1/a))^(1/2))+1/4/a/c*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/4/c/(a^
2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))), x)
```

Fricas [A] time = 2.00184, size = 153, normalized size = 2.55

$$\frac{2\,ax - 2\,(ax + 1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2 + 1}(ax + 2) + 2}{a^2cx + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (2*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2
 + 1)*(a*x + 2) + 2)/(a^2*c*x + a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \int \frac{x^2\sqrt{-a^2x^2+1}}{a^3x^3+a^2x^2-ax-1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*Integral(x**2*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + a**2*x**2 - a*x - 1),
x)/c
```

Giac [A] time = 1.26101, size = 96, normalized size = 1.6

$$\frac{\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{c|a|} + \frac{\sqrt{-a^2x^2 + 1}}{ac} - \frac{2}{c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} + 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/(c*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c) - 2/(c*((sqrt(-a^2
*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

3.667 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=97

$$\frac{a^2x^3(1-ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(3-4ax)}{3c^2\sqrt{1-a^2x^2}}+\frac{8\sqrt{1-a^2x^2}}{3ac^2}+\frac{\sin^{-1}(ax)}{ac^2}$$

```
[Out] (a^2*x^3*(1 - a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) - (x*(3 - 4*a*x))/(3*c^2*Sq
rt[1 - a^2*x^2]) + (8*Sqrt[1 - a^2*x^2])/(3*a*c^2) + ArcSin[a*x]/(a*c^2)
```

Rubi [A] time = 0.163424, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6157, 6149, 819, 641, 216}

$$\frac{a^2x^3(1-ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(3-4ax)}{3c^2\sqrt{1-a^2x^2}}+\frac{8\sqrt{1-a^2x^2}}{3ac^2}+\frac{\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^2),x]
```

```
[Out] (a^2*x^3*(1 - a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) - (x*(3 - 4*a*x))/(3*c^2*Sq
rt[1 - a^2*x^2]) + (8*Sqrt[1 - a^2*x^2])/(3*a*c^2) + ArcSin[a*x]/(a*c^2)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx &= \frac{a^4\int \frac{e^{-\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^2}\,dx}{c^2}\\
&= \frac{a^4\int \frac{x^4(1-ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}\\
&= \frac{a^2x^3(1-ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{a^2\int \frac{x^2(3-4ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^2}\\
&= \frac{a^2x^3(1-ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(3-4ax)}{3c^2\sqrt{1-a^2x^2}}+\frac{\int \frac{3-8ax}{\sqrt{1-a^2x^2}}\,dx}{3c^2}\\
&= \frac{a^2x^3(1-ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(3-4ax)}{3c^2\sqrt{1-a^2x^2}}+\frac{8\sqrt{1-a^2x^2}}{3ac^2}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^2}\\
&= \frac{a^2x^3(1-ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(3-4ax)}{3c^2\sqrt{1-a^2x^2}}+\frac{8\sqrt{1-a^2x^2}}{3ac^2}+\frac{\sin^{-1}(ax)}{ac^2}
\end{aligned}$$

Mathematica [A] time = 0.0535168, size = 78, normalized size = 0.8

$$\frac{-3a^3x^3-7a^2x^2+3(ax+1)\sqrt{1-a^2x^2}\sin^{-1}(ax)+5ax+8}{3ac^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^2),x]
```

```
[Out] (8 + 5*a*x - 7*a^2*x^2 - 3*a^3*x^3 + 3*(1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSin[a
*x])/(3*a*c^2*(1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.053, size = 274, normalized size = 2.8

$$\frac{1}{8\,a^3c^2}\left(-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)\right)^{\frac{3}{2}}\left(x-a^{-1}\right)^{-2}+\frac{7}{16\,ac^2}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}-\frac{7}{16\,c^2}\arctan\left(x\sqrt{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^2,x)
```

```
[Out] 1/8/a^3/c^2/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+7/16/a/c^2*(-a^2*(
x-1/a)^2-2*a*(x-1/a))^(1/2)-7/16/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2
*(x-1/a)^2-2*a*(x-1/a))^(1/2))+3/4/a^3/c^2/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x
+1/a))^(3/2)+23/16/a/c^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+23/16/c^2/(a^2)
^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-1/12/a^4/c^
2/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^2), x)
```

Fricas [A] time = 1.83865, size = 296, normalized size = 3.05

$$\frac{8\,a^3x^3+8\,a^2x^2-8\,ax-6\left(a^3x^3+a^2x^2-ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3\,a^3x^3+7\,a^2x^2-5\,ax-8\right)\sqrt{-a^2x^2+1}-8}{3\left(a^4c^2x^3+a^3c^2x^2-a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas
")
```

```
[Out] 1/3*(8*a^3*x^3 + 8*a^2*x^2 - 8*a*x - 6*(a^3*x^3 + a^2*x^2 - a*x - 1)*arctan
((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (3*a^3*x^3 + 7*a^2*x^2 - 5*a*x - 8)*sqrt
(-a^2*x^2 + 1) - 8)/(a^4*c^2*x^3 + a^3*c^2*x^2 - a^2*c^2*x - a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^4\int\frac{x^4\sqrt{-a^2x^2+1}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4*sqrt(-a**2*x**2 + 1)/(a**5*x**5 + a**4*x**4 - 2*a**3*x**
3 - 2*a**2*x**2 + a*x + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^2), x)
```

3.668 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=130

$$-\frac{a^4x^5(1-ax)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(5-6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{x(5-8ax)}{5c^3\sqrt{1-a^2x^2}}+\frac{16\sqrt{1-a^2x^2}}{5ac^3}+\frac{\sin^{-1}(ax)}{ac^3}$$

```
[Out] -(a^4*x^5*(1 - a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (a^2*x^3*(5 - 6*a*x))/(1
5*c^3*(1 - a^2*x^2)^(3/2)) - (x*(5 - 8*a*x))/(5*c^3*Sqrt[1 - a^2*x^2]) + (1
6*Sqrt[1 - a^2*x^2])/(5*a*c^3) + ArcSin[a*x]/(a*c^3)
```

Rubi [A] time = 0.19856, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6157, 6149, 819, 641, 216}

$$-\frac{a^4x^5(1-ax)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(5-6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{x(5-8ax)}{5c^3\sqrt{1-a^2x^2}}+\frac{16\sqrt{1-a^2x^2}}{5ac^3}+\frac{\sin^{-1}(ax)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^3),x]
```

```
[Out] -(a^4*x^5*(1 - a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (a^2*x^3*(5 - 6*a*x))/(1
5*c^3*(1 - a^2*x^2)^(3/2)) - (x*(5 - 8*a*x))/(5*c^3*Sqrt[1 - a^2*x^2]) + (1
6*Sqrt[1 - a^2*x^2])/(5*a*c^3) + ArcSin[a*x]/(a*c^3)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx &= -\frac{a^6\int \frac{e^{-\tanh^{-1}(ax)}x^6}{\left(1-a^2x^2\right)^3}\,dx}{c^3}\\
&= -\frac{a^6\int \frac{x^6(1-ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}\\
&= -\frac{a^4x^5(1-ax)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{a^4\int \frac{x^4(5-6ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^3}\\
&= -\frac{a^4x^5(1-ax)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(5-6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{a^2\int \frac{x^2(15-24ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^3}\\
&= -\frac{a^4x^5(1-ax)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(5-6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{x(5-8ax)}{5c^3\sqrt{1-a^2x^2}}+\frac{\int \frac{15-48ax}{\sqrt{1-a^2x^2}}\,dx}{15c^3}\\
&= -\frac{a^4x^5(1-ax)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(5-6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{x(5-8ax)}{5c^3\sqrt{1-a^2x^2}}+\frac{16\sqrt{1-a^2x^2}}{5ac^3}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^3}\\
&= -\frac{a^4x^5(1-ax)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(5-6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{x(5-8ax)}{5c^3\sqrt{1-a^2x^2}}+\frac{16\sqrt{1-a^2x^2}}{5ac^3}+\frac{\sin^{-1}(ax)}{ac^3}
\end{aligned}$$

Mathematica [A] time = 0.0763804, size = 108, normalized size = 0.83

$$\frac{-15a^5x^5-38a^4x^4+52a^3x^3+87a^2x^2+15(ax-1)(ax+1)^2\sqrt{1-a^2x^2}\sin^{-1}(ax)-33ax-48}{15ac^3(ax-1)(ax+1)^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^3),x]
```

```
[Out] (-48 - 33*a*x + 87*a^2*x^2 + 52*a^3*x^3 - 38*a^4*x^4 - 15*a^5*x^5 + 15*(-1
+ a*x)*(1 + a*x)^2*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(15*a*c^3*(-1 + a*x)*(1 +
 a*x)^2*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.062, size = 356, normalized size = 2.7

$$\frac{1}{40\,a^5c^3\left(x+a^{-1}\right)^4}\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right)^{\frac{3}{2}}-\frac{43}{240\,a^4c^3\left(x+a^{-1}\right)^3}\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right)^{\frac{3}{2}}+\frac{1}{48\,a^4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^3,x)
```

```
[Out] 1/40/a^5/c^3/(x+1/a)^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)-43/240/a^4/c^3/(x
+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+1/48/a^4/c^3/(x-1/a)^3*(-a^2*(x-
1/a)^2-2*a*(x-1/a))^(3/2)+1/4/a^3/c^3/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a)
)^(3/2)+19/32/a/c^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-19/32/c^3/(a^2)^(1/2
)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))+15/16/a^3/c^3/(x
+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+51/32/a/c^3*(-a^2*(x+1/a)^2+2*a*
(x+1/a))^(1/2)+51/32/c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2
*a*(x+1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^3), x)
```

Fricas [A] time = 1.90871, size = 455, normalized size = 3.5

$$\frac{48\,a^5x^5+48\,a^4x^4-96\,a^3x^3-96\,a^2x^2+48\,ax-30\left(a^5x^5+a^4x^4-2\,a^3x^3-2\,a^2x^2+ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(15\,a^5\right.}{15\left(a^6c^3x^5+a^5c^3x^4-2\,a^4c^3x^3-2\,a^3c^3x^2+a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas
")
```

```
[Out] 1/15*(48*a^5*x^5 + 48*a^4*x^4 - 96*a^3*x^3 - 96*a^2*x^2 + 48*a*x - 30*(a^5*
x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*arctan((sqrt(-a^2*x^2 + 1)
 - 1)/(a*x)) + (15*a^5*x^5 + 38*a^4*x^4 - 52*a^3*x^3 - 87*a^2*x^2 + 33*a*x
+ 48)*sqrt(-a^2*x^2 + 1) + 48)/(a^6*c^3*x^5 + a^5*c^3*x^4 - 2*a^4*c^3*x^3 -
 2*a^3*c^3*x^2 + a^2*c^3*x + a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^6\int \frac{x^6\sqrt{-a^2x^2+1}}{a^7x^7+a^6x^6-3a^5x^5-3a^4x^4+3a^3x^3+3a^2x^2-ax-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*Integral(x**6*sqrt(-a**2*x**2 + 1)/(a**7*x**7 + a**6*x**6 - 3*a**5*x**
5 - 3*a**4*x**4 + 3*a**3*x**3 + 3*a**2*x**2 - a*x - 1), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^3), x)
```

3.669 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=163

$$\frac{a^6x^7(1-ax)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{a^4x^5(7-8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(35-48ax)}{105c^4\left(1-a^2x^2\right)^{3/2}}-\frac{x(35-64ax)}{35c^4\sqrt{1-a^2x^2}}+\frac{128\sqrt{1-a^2x^2}}{35ac^4}+\frac{\sin^{-1}(ax)}{ac^4}$$

```
[Out] (a^6*x^7*(1 - a*x))/(7*c^4*(1 - a^2*x^2)^(7/2)) - (a^4*x^5*(7 - 8*a*x))/(35
*c^4*(1 - a^2*x^2)^(5/2)) + (a^2*x^3*(35 - 48*a*x))/(105*c^4*(1 - a^2*x^2)^
(3/2)) - (x*(35 - 64*a*x))/(35*c^4*Sqrt[1 - a^2*x^2]) + (128*Sqrt[1 - a^2*x
^2])/(35*a*c^4) + ArcSin[a*x]/(a*c^4)
```

Rubi [A] time = 0.240775, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6157, 6149, 819, 641, 216}

$$\frac{a^6x^7(1-ax)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{a^4x^5(7-8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(35-48ax)}{105c^4\left(1-a^2x^2\right)^{3/2}}-\frac{x(35-64ax)}{35c^4\sqrt{1-a^2x^2}}+\frac{128\sqrt{1-a^2x^2}}{35ac^4}+\frac{\sin^{-1}(ax)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^4),x]
```


```
[Out] (a^6*x^7*(1 - a*x))/(7*c^4*(1 - a^2*x^2)^(7/2)) - (a^4*x^5*(7 - 8*a*x))/(35
*c^4*(1 - a^2*x^2)^(5/2)) + (a^2*x^3*(35 - 48*a*x))/(105*c^4*(1 - a^2*x^2)^
(3/2)) - (x*(35 - 64*a*x))/(35*c^4*Sqrt[1 - a^2*x^2]) + (128*Sqrt[1 - a^2*x
^2])/(35*a*c^4) + ArcSin[a*x]/(a*c^4)
```


Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{e^{-\tanh^{-1}(ax)}x^8}{(1-a^2x^2)^4}\,dx}{c^4}$$

$$= \frac{a^8\int \frac{x^8(1-ax)}{(1-a^2x^2)^{9/2}}\,dx}{c^4}$$

$$= \frac{a^6x^7(1-ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^6\int \frac{x^6(7-8ax)}{(1-a^2x^2)^{7/2}}\,dx}{7c^4}$$

$$= \frac{a^6x^7(1-ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^4x^5(7-8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^4\int \frac{x^4(35-48ax)}{(1-a^2x^2)^{5/2}}\,dx}{35c^4}$$

$$= \frac{a^6x^7(1-ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^4x^5(7-8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(35-48ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} - \frac{a^2\int \frac{x^2(105-192ax)}{(1-a^2x^2)^{3/2}}\,dx}{105c^4}$$

$$= \frac{a^6x^7(1-ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^4x^5(7-8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(35-48ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} - \frac{x(35-64ax)}{35c^4\sqrt{1-a^2x^2}} + \frac{\int \frac{105-384ax}{\sqrt{1-a^2x^2}}\,dx}{105c^4}$$

$$= \frac{a^6x^7(1-ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^4x^5(7-8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(35-48ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} - \frac{x(35-64ax)}{35c^4\sqrt{1-a^2x^2}} + \frac{128\sqrt{1-a^2x^2}}{35ac^4} +$$

$$= \frac{a^6x^7(1-ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^4x^5(7-8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(35-48ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} - \frac{x(35-64ax)}{35c^4\sqrt{1-a^2x^2}} + \frac{128\sqrt{1-a^2x^2}}{35ac^4} +$$

Mathematica [A] time = 0.0961641, size = 126, normalized size = 0.77

$$\frac{-105a^7x^7-281a^6x^6+559a^5x^5+965a^4x^4-715a^3x^3-1065a^2x^2+105(ax-1)^2(ax+1)^3\sqrt{1-a^2x^2}\sin^{-1}(ax)+279ax}{105ac^4(ax-1)^2(ax+1)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^4),x]
```

```
[Out] (384 + 279*a*x - 1065*a^2*x^2 - 715*a^3*x^3 + 965*a^4*x^4 + 559*a^5*x^5 - 2
81*a^6*x^6 - 105*a^7*x^7 + 105*(-1 + a*x)^2*(1 + a*x)^3*Sqrt[1 - a^2*x^2]*A
rcSin[a*x])/(105*a*c^4*(-1 + a*x)^2*(1 + a*x)^3*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.069, size = 438, normalized size = 2.7

$$\frac{1}{14\,a^5c^4\left(x+a^{-1}\right)^4}\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right)^{\frac{3}{2}}-\frac{187}{672\,a^4c^4\left(x+a^{-1}\right)^3}\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right)^{\frac{3}{2}}+\frac{53}{960\,a^4c^4}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^4,x)
```

```
[Out] 1/14/a^5/c^4/(x+1/a)^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)-187/672/a^4/c^4/(
x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+53/960/a^4/c^4/(x-1/a)^3*(-a^2*
(x-1/a)^2-2*a*(x-1/a))^(3/2)+47/128/a^3/c^4/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(
x-1/a))^(3/2)+187/256/a/c^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-187/256/c^4/
(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))+1/160/
a^5/c^4/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+35/32/a^3/c^4/(x+1/a)^
2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+443/256/a/c^4*(-a^2*(x+1/a)^2+2*a*(x+1
/a))^(1/2)+443/256/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a
*(x+1/a))^(1/2))-1/112/a^6/c^4/(x+1/a)^5*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^4), x)
```

Fricas [A] time = 2.38672, size = 632, normalized size = 3.88

$$\frac{384\,a^7x^7+384\,a^6x^6-1152\,a^5x^5-1152\,a^4x^4+1152\,a^3x^3+1152\,a^2x^2-384\,ax-210\left(a^7x^7+a^6x^6-3\,a^5x^5-3\,a^4x^4+3\,a\right.}{105\left(a^8c^4x^7+a^7c^4x^6-3\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas
")
```

```
[Out] 1/105*(384*a^7*x^7 + 384*a^6*x^6 - 1152*a^5*x^5 - 1152*a^4*x^4 + 1152*a^3*x
^3 + 1152*a^2*x^2 - 384*a*x - 210*(a^7*x^7 + a^6*x^6 - 3*a^5*x^5 - 3*a^4*x^
4 + 3*a^3*x^3 + 3*a^2*x^2 - a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x))
 + (105*a^7*x^7 + 281*a^6*x^6 - 559*a^5*x^5 - 965*a^4*x^4 + 715*a^3*x^3 + 1
065*a^2*x^2 - 279*a*x - 384)*sqrt(-a^2*x^2 + 1) - 384)/(a^8*c^4*x^7 + a^7*c
^4*x^6 - 3*a^6*c^4*x^5 - 3*a^5*c^4*x^4 + 3*a^4*c^4*x^3 + 3*a^3*c^4*x^2 - a^
2*c^4*x - a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^8 \int \frac{x^8\sqrt{-a^2x^2+1}}{a^9x^9+a^8x^8-4a^7x^7-4a^6x^6+6a^5x^5+6a^4x^4-4a^3x^3-4a^2x^2+ax+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*Integral(x**8*sqrt(-a**2*x**2 + 1)/(a**9*x**9 + a**8*x**8 - 4*a**7*x**
7 - 4*a**6*x**6 + 6*a**5*x**5 + 6*a**4*x**4 - 4*a**3*x**3 - 4*a**2*x**2 + a
*x + 1), x)/c**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^4), x)
```

3.670 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=91

$$\frac{3c^4}{a^3x^2}-\frac{3c^4}{2a^5x^4}+\frac{2c^4}{5a^6x^5}+\frac{c^4}{3a^7x^6}-\frac{c^4}{7a^8x^7}-\frac{2c^4}{a^2x}+\frac{2c^4\log(x)}{a}+c^4(-x)$$

```
[Out] -c^4/(7*a^8*x^7) + c^4/(3*a^7*x^6) + (2*c^4)/(5*a^6*x^5) - (3*c^4)/(2*a^5*x
^4) + (3*c^4)/(a^3*x^2) - (2*c^4)/(a^2*x) - c^4*x + (2*c^4*Log[x])/a
```

Rubi [A] time = 0.139025, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$\frac{3c^4}{a^3x^2}-\frac{3c^4}{2a^5x^4}+\frac{2c^4}{5a^6x^5}+\frac{c^4}{3a^7x^6}-\frac{c^4}{7a^8x^7}-\frac{2c^4}{a^2x}+\frac{2c^4\log(x)}{a}+c^4(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^4/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -c^4/(7*a^8*x^7) + c^4/(3*a^7*x^6) + (2*c^4)/(5*a^6*x^5) - (3*c^4)/(2*a^5*x
^4) + (3*c^4)/(a^3*x^2) - (2*c^4)/(a^2*x) - c^4*x + (2*c^4*Log[x])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx &= \frac{c^4\int\frac{e^{-2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}\,dx}{a^8}\\
&= \frac{c^4\int\frac{(1-ax)^5(1+ax)^3}{x^8}\,dx}{a^8}\\
&= \frac{c^4\int\left(-a^8+\frac{1}{x^8}-\frac{2a}{x^7}-\frac{2a^2}{x^6}+\frac{6a^3}{x^5}-\frac{6a^5}{x^3}+\frac{2a^6}{x^2}+\frac{2a^7}{x}\right)dx}{a^8}\\
&= -\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}+\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}-\frac{2c^4}{a^2x}-c^4x+\frac{2c^4\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0264886, size = 91, normalized size = 1.

$$\frac{3c^4}{a^3x^2}-\frac{3c^4}{2a^5x^4}+\frac{2c^4}{5a^6x^5}+\frac{c^4}{3a^7x^6}-\frac{c^4}{7a^8x^7}-\frac{2c^4}{a^2x}+\frac{2c^4\log(x)}{a}+c^4(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -c^4/(7*a^8*x^7) + c^4/(3*a^7*x^6) + (2*c^4)/(5*a^6*x^5) - (3*c^4)/(2*a^5*x
^4) + (3*c^4)/(a^3*x^2) - (2*c^4)/(a^2*x) - c^4*x + (2*c^4*Log[x])/a
```

Maple [A] time = 0.042, size = 84, normalized size = 0.9

$$-\frac{c^4}{7\,a^8x^7}+\frac{c^4}{3\,a^7x^6}+\frac{2\,c^4}{5\,a^6x^5}-\frac{3\,c^4}{2\,a^5x^4}+3\,\frac{c^4}{x^2a^3}-2\,\frac{c^4}{a^2x}-c^4x+2\,\frac{c^4\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^4/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/7*c^4/a^8/x^7+1/3*c^4/a^7/x^6+2/5*c^4/a^6/x^5-3/2*c^4/a^5/x^4+3*c^4/x^2/
a^3-2*c^4/a^2/x-c^4*x+2*c^4*ln(x)/a
```

Maxima [A] time = 0.961714, size = 111, normalized size = 1.22

$$-c^4x+\frac{2\,c^4\log(x)}{a}-\frac{420\,a^6c^4x^6-630\,a^5c^4x^5+315\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -c^4*x + 2*c^4*log(x)/a - 1/210*(420*a^6*c^4*x^6 - 630*a^5*c^4*x^5 + 315*a^
3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

Fricas [A] time = 1.92597, size = 208, normalized size = 2.29

$$-\frac{210\,a^8c^4x^8-420\,a^7c^4x^7\log(x)+420\,a^6c^4x^6-630\,a^5c^4x^5+315\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/210*(210*a^8*c^4*x^8 - 420*a^7*c^4*x^7*log(x) + 420*a^6*c^4*x^6 - 630*a^
5*c^4*x^5 + 315*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^
7)
```

Sympy [A] time = 3.56531, size = 88, normalized size = 0.97

$$\frac{-a^8c^4x+2a^7c^4\log(x)-\frac{420a^6c^4x^6-630a^5c^4x^5+315a^3c^4x^3-84a^2c^4x^2-70ac^4x+30c^4}{210x^7}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**4/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] (-a**8*c**4*x + 2*a**7*c**4*log(x) - (420*a**6*c**4*x**6 - 630*a**5*c**4*x*
*5 + 315*a**3*c**4*x**3 - 84*a**2*c**4*x**2 - 70*a*c**4*x + 30*c**4)/(210*x
**7))/a**8
```

Giac [A] time = 1.17059, size = 216, normalized size = 2.37

$$-\frac{2\,c^4\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}+\frac{2\,c^4\log\left(\left|-\frac{1}{ax+1}+1\right|\right)}{a}+\frac{\left(210\,c^4-\frac{719\,c^4}{ax+1}-\frac{427\,c^4}{(ax+1)^2}+\frac{5271\,c^4}{(ax+1)^3}-\frac{9485\,c^4}{(ax+1)^4}+\frac{7490\,c^4}{(ax+1)^5}-\frac{2730\,c^4}{(ax+1)^6}+\frac{420\,c^4}{(ax+1)^7}\right)(a\cdots}{210\,a\left(\frac{1}{ax+1}-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -2*c^4*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a + 2*c^4*log(abs(-1/(a*x + 1
) + 1))/a + 1/210*(210*c^4 - 719*c^4/(a*x + 1) - 427*c^4/(a*x + 1)^2 + 5271
*c^4/(a*x + 1)^3 - 9485*c^4/(a*x + 1)^4 + 7490*c^4/(a*x + 1)^5 - 2730*c^4/(
a*x + 1)^6 + 420*c^4/(a*x + 1)^7)*(a*x + 1)/(a*(1/(a*x + 1) - 1)^7)
```

3.671 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$

Optimal. Leaf size=78

$$\frac{2c^3}{a^3x^2}-\frac{c^3}{3a^4x^3}-\frac{c^3}{2a^5x^4}+\frac{c^3}{5a^6x^5}-\frac{c^3}{a^2x}+\frac{2c^3\log(x)}{a}+c^3(-x)$$

```
[Out] c^3/(5*a^6*x^5) - c^3/(2*a^5*x^4) - c^3/(3*a^4*x^3) + (2*c^3)/(a^3*x^2) - c
^3/(a^2*x) - c^3*x + (2*c^3*Log[x])/a
```

Rubi [A] time = 0.128178, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$\frac{2c^3}{a^3x^2}-\frac{c^3}{3a^4x^3}-\frac{c^3}{2a^5x^4}+\frac{c^3}{5a^6x^5}-\frac{c^3}{a^2x}+\frac{2c^3\log(x)}{a}+c^3(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] c^3/(5*a^6*x^5) - c^3/(2*a^5*x^4) - c^3/(3*a^4*x^3) + (2*c^3)/(a^3*x^2) - c
^3/(a^2*x) - c^3*x + (2*c^3*Log[x])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{c^3\int \frac{e^{-2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^3}{x^6}dx}{a^6}$$
$$= -\frac{c^3\int \frac{(1-ax)^4(1+ax)^2}{x^6}dx}{a^6}$$
$$= -\frac{c^3\int \left(a^6+\frac{1}{x^6}-\frac{2a}{x^5}-\frac{a^2}{x^4}+\frac{4a^3}{x^3}-\frac{a^4}{x^2}-\frac{2a^5}{x}\right)dx}{a^6}$$
$$= \frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}-\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}-\frac{c^3}{a^2x}-c^3x+\frac{2c^3\log(x)}{a}$$

Mathematica [A] time = 0.0217449, size = 78, normalized size = 1.

$$\frac{2c^3}{a^3x^2}-\frac{c^3}{3a^4x^3}-\frac{c^3}{2a^5x^4}+\frac{c^3}{5a^6x^5}-\frac{c^3}{a^2x}+\frac{2c^3\log(x)}{a}+c^3(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] c^3/(5*a^6*x^5) - c^3/(2*a^5*x^4) - c^3/(3*a^4*x^3) + (2*c^3)/(a^3*x^2) - c
^3/(a^2*x) - c^3*x + (2*c^3*Log[x])/a
```

Maple [A] time = 0.037, size = 73, normalized size = 0.9

$$\frac{c^3}{5\,a^6x^5}-\frac{c^3}{2\,a^5x^4}-\frac{c^3}{3\,a^4x^3}+2\,\frac{c^3}{x^2a^3}-\frac{c^3}{a^2x}-c^3x+2\,\frac{c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^3/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/5*c^3/a^6/x^5-1/2*c^3/a^5/x^4-1/3*c^3/a^4/x^3+2*c^3/x^2/a^3-c^3/a^2/x-c^3
*x+2*c^3*ln(x)/a
```

Maxima [A] time = 0.967265, size = 96, normalized size = 1.23

$$-c^3x+\frac{2\,c^3\log(x)}{a}-\frac{30\,a^4c^3x^4-60\,a^3c^3x^3+10\,a^2c^3x^2+15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -c^3*x + 2*c^3*log(x)/a - 1/30*(30*a^4*c^3*x^4 - 60*a^3*c^3*x^3 + 10*a^2*c^
3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Fricas [A] time = 1.85926, size = 176, normalized size = 2.26

$$-\frac{30\,a^6c^3x^6-60\,a^5c^3x^5\log(x)+30\,a^4c^3x^4-60\,a^3c^3x^3+10\,a^2c^3x^2+15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/30*(30*a^6*c^3*x^6 - 60*a^5*c^3*x^5*log(x) + 30*a^4*c^3*x^4 - 60*a^3*c^3
*x^3 + 10*a^2*c^3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Sympy [A] time = 2.21957, size = 76, normalized size = 0.97

$$\frac{-a^6c^3x + 2a^5c^3\log(x) - \frac{30a^4c^3x^4 - 60a^3c^3x^3 + 10a^2c^3x^2 + 15ac^3x - 6c^3}{30x^5}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**3/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] (-a**6*c**3*x + 2*a**5*c**3*log(x) - (30*a**4*c**3*x**4 - 60*a**3*c**3*x**3
 + 10*a**2*c**3*x**2 + 15*a*c**3*x - 6*c**3)/(30*x**5))/a**6
```

Giac [A] time = 1.23353, size = 184, normalized size = 2.36

$$-\frac{2c^3\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a} + \frac{2c^3\log\left(\left|-\frac{1}{ax+1}+1\right|\right)}{a} + \frac{\left(30c^3 - \frac{71c^3}{ax+1} - \frac{65c^3}{(ax+1)^2} + \frac{310c^3}{(ax+1)^3} - \frac{270c^3}{(ax+1)^4} + \frac{60c^3}{(ax+1)^5}\right)(ax+1)}{30a\left(\frac{1}{ax+1}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -2*c^3*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a + 2*c^3*log(abs(-1/(a*x + 1
) + 1))/a + 1/30*(30*c^3 - 71*c^3/(a*x + 1) - 65*c^3/(a*x + 1)^2 + 310*c^3/
(a*x + 1)^3 - 270*c^3/(a*x + 1)^4 + 60*c^3/(a*x + 1)^5)*(a*x + 1)/(a*(1/(a*
x + 1) - 1)^5)
```

3.672 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=40

$$\frac{c^2}{a^3x^2}-\frac{c^2}{3a^4x^3}+\frac{2c^2\log(x)}{a}+c^2(-x)$$

```
[Out] -c^2/(3*a^4*x^3) + c^2/(a^3*x^2) - c^2*x + (2*c^2*Log[x])/a
```

Rubi [A] time = 0.116405, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 75}

$$\frac{c^2}{a^3x^2}-\frac{c^2}{3a^4x^3}+\frac{2c^2\log(x)}{a}+c^2(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^2/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -c^2/(3*a^4*x^3) + c^2/(a^3*x^2) - c^2*x + (2*c^2*Log[x])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx &= \frac{c^2\int\frac{e^{-2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}\,dx}{a^4}\\
&= \frac{c^2\int\frac{(1-ax)^3(1+ax)}{x^4}\,dx}{a^4}\\
&= \frac{c^2\int\left(-a^4+\frac{1}{x^4}-\frac{2a}{x^3}+\frac{2a^3}{x}\right)dx}{a^4}\\
&= -\frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}-c^2x+\frac{2c^2\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.015661, size = 40, normalized size = 1.

$$\frac{c^2}{a^3x^2}-\frac{c^2}{3a^4x^3}+\frac{2c^2\log(x)}{a}+c^2(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -c^2/(3*a^4*x^3) + c^2/(a^3*x^2) - c^2*x + (2*c^2*Log[x])/a
```

Maple [A] time = 0.035, size = 39, normalized size = 1.

$$-\frac{c^2}{3\,a^4x^3}+\frac{c^2}{x^2a^3}-xc^2+2\,\frac{c^2\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^2/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/3*c^2/a^4/x^3+c^2/x^2/a^3-x*c^2+2*c^2*ln(x)/a
```

Maxima [A] time = 0.959533, size = 51, normalized size = 1.27

$$-c^2x+\frac{2\,c^2\log(x)}{a}+\frac{3\,ac^2x-c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -c^2*x + 2*c^2*log(x)/a + 1/3*(3*a*c^2*x - c^2)/(a^4*x^3)
```

Fricas [A] time = 1.73587, size = 99, normalized size = 2.48

$$-\frac{3\,a^4c^2x^4-6\,a^3c^2x^3\log(x)-3\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^4*c^2*x^4 - 6*a^3*c^2*x^3*log(x) - 3*a*c^2*x + c^2)/(a^4*x^3)
```

Sympy [A] time = 1.93487, size = 39, normalized size = 0.98

$$\frac{-a^4c^2x+2a^3c^2\log(x)+\frac{3ac^2x-c^2}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**2/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] (-a**4*c**2*x + 2*a**3*c**2*log(x) + (3*a*c**2*x - c**2)/(3*x**3))/a**4
```

Giac [B] time = 1.34723, size = 151, normalized size = 3.78

$$-\frac{2\,c^2\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}+\frac{2\,c^2\log\left(\left|-\frac{1}{ax+1}+1\right|\right)}{a}+\frac{\left(3\,c^2-\frac{5\,c^2}{ax+1}-\frac{3\,c^2}{(ax+1)^2}+\frac{6\,c^2}{(ax+1)^3}\right)(ax+1)}{3\,a\left(\frac{1}{ax+1}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -2*c^2*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a + 2*c^2*log(abs(-1/(a*x + 1
) + 1))/a + 1/3*(3*c^2 - 5*c^2/(a*x + 1) - 3*c^2/(a*x + 1)^2 + 6*c^2/(a*x +
 1)^3)*(a*x + 1)/(a*(1/(a*x + 1) - 1)^3)
```

3.673 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=21

$$\frac{c}{a^2x}+\frac{2c\log(x)}{a}+c(-x)$$

```
[Out] c/(a^2*x) - c*x + (2*c*Log[x])/a
```

Rubi [A] time = 0.0703377, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6157, 6150, 43}

$$\frac{c}{a^2x}+\frac{2c\log(x)}{a}+c(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))/E^(2*ArcTanh[a*x]),x]
```

```
[Out] c/(a^2*x) - c*x + (2*c*Log[x])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx &= -\frac{c\int \frac{e^{-2\tanh^{-1}(ax)}\left(1-a^2x^2\right)}{x^2}\,dx}{a^2}\\
&= -\frac{c\int \frac{(1-ax)^2}{x^2}\,dx}{a^2}\\
&= -\frac{c\int \left(a^2+\frac{1}{x^2}-\frac{2a}{x}\right)\,dx}{a^2}\\
&= \frac{c}{a^2x}-cx+\frac{2c\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0118523, size = 21, normalized size = 1.

$$\frac{c}{a^2x}+\frac{2c\log(x)}{a}+c(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))/E^(2*ArcTanh[a*x]),x]
```

```
[Out] c/(a^2*x) - c*x + (2*c*Log[x])/a
```

Maple [A] time = 0.034, size = 22, normalized size = 1.1

$$\frac{c}{a^2x}-cx+2\frac{c\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] c/a^2/x-c*x+2*c*ln(x)/a
```

Maxima [A] time = 0.967262, size = 28, normalized size = 1.33

$$-cx+\frac{2c\log(x)}{a}+\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -c*x + 2*c*log(x)/a + c/(a^2*x)
```

Fricas [A] time = 1.9188, size = 58, normalized size = 2.76

$$-\frac{a^2cx^2-2acx\log(x)-c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -(a^2*c*x^2 - 2*a*c*x*log(x) - c)/(a^2*x)
```

Sympy [A] time = 0.565182, size = 20, normalized size = 0.95

$$\frac{-a^2cx+2ac\log(x)+\frac{c}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] (-a**2*c*x + 2*a*c*log(x) + c/x)/a**2
```

Giac [B] time = 1.2327, size = 119, normalized size = 5.67

$$-\frac{2\,c\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}+\frac{2\,c\log\left(\left|-\frac{1}{ax+1}+1\right|\right)}{a}-\frac{c-\frac{2\,c}{ax+1}}{a^2\left(\frac{1}{(ax+1)a}-\frac{1}{(ax+1)^2a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -2*c*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a + 2*c*log(abs(-1/(a*x + 1) +
1))/a - (c - 2*c/(a*x + 1))/(a^2*(1/((a*x + 1)*a) - 1/((a*x + 1)^2*a)))
```

3.674 $\int \frac{e^{-2\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{1}{ac(ax+1)}+\frac{2\log(ax+1)}{ac}-\frac{x}{c}$$

[Out] -(x/c) + 1/(a*c*(1 + a*x)) + (2*Log[1 + a*x])/(a*c)

Rubi [A] time = 0.132358, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 43}

$$\frac{1}{ac(ax+1)}+\frac{2\log(ax+1)}{ac}-\frac{x}{c}$$

Antiderivative was successfully verified.

[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))),x]

[Out] -(x/c) + 1/(a*c*(1 + a*x)) + (2*Log[1 + a*x])/(a*c)

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\frac{a^2\int \frac{e^{-2\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}\\
&= -\frac{a^2\int \frac{x^2}{(1+ax)^2}\,dx}{c}\\
&= -\frac{a^2\int \left(\frac{1}{a^2}+\frac{1}{a^2(1+ax)^2}-\frac{2}{a^2(1+ax)}\right)dx}{c}\\
&= -\frac{x}{c}+\frac{1}{ac(1+ax)}+\frac{2\log(1+ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.0268276, size = 28, normalized size = 0.8

$$\frac{\frac{1}{a^2x+a}+\frac{2\log(ax+1)}{a}-x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] (-x + (a + a^2*x)^(-1) + (2*Log[1 + a*x])/a)/c
```

Maple [A] time = 0.035, size = 36, normalized size = 1.

$$-\frac{x}{c}+\frac{1}{ac\,(ax+1)}+2\,\frac{\ln(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2),x)
```

```
[Out] -x/c+1/a/c/(a*x+1)+2*ln(a*x+1)/a/c
```

Maxima [A] time = 0.963086, size = 45, normalized size = 1.29

$$-\frac{x}{c}+\frac{1}{a^2cx+ac}+\frac{2\,\log(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -x/c + 1/(a^2*c*x + a*c) + 2*log(a*x + 1)/(a*c)
```

Fricas [A] time = 1.79469, size = 88, normalized size = 2.51

$$-\frac{a^2x^2+ax-2\,(ax+1)\log(ax+1)-1}{a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] -(a^2*x^2 + a*x - 2*(a*x + 1)*log(a*x + 1) - 1)/(a^2*c*x + a*c)
```

Sympy [A] time = 0.733326, size = 37, normalized size = 1.06

$$-a^2\left(-\frac{1}{a^4cx+a^3c}+\frac{x}{a^2c}-\frac{2\log(ax+1)}{a^3c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a**2/x**2),x)
```

```
[Out] -a**2*(-1/(a**4*c*x + a**3*c) + x/(a**2*c) - 2*log(a*x + 1)/(a**3*c))
```

Giac [A] time = 1.31822, size = 74, normalized size = 2.11

$$-\frac{ax+1}{ac}-\frac{2\,\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{ac}+\frac{1}{(ax+1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] -(a*x + 1)/(a*c) - 2*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/(a*c) + 1/((a*x
 + 1)*a*c)
```

3.675 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=74

$$\frac{7}{4ac^2(ax+1)}-\frac{1}{4ac^2(ax+1)^2}-\frac{\log(1-ax)}{8ac^2}+\frac{17\log(ax+1)}{8ac^2}-\frac{x}{c^2}$$

```
[Out] -(x/c^2) - 1/(4*a*c^2*(1 + a*x)^2) + 7/(4*a*c^2*(1 + a*x)) - Log[1 - a*x]/(
8*a*c^2) + (17*Log[1 + a*x])/(8*a*c^2)
```

Rubi [A] time = 0.144847, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$\frac{7}{4ac^2(ax+1)}-\frac{1}{4ac^2(ax+1)^2}-\frac{\log(1-ax)}{8ac^2}+\frac{17\log(ax+1)}{8ac^2}-\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] -(x/c^2) - 1/(4*a*c^2*(1 + a*x)^2) + 7/(4*a*c^2*(1 + a*x)) - Log[1 - a*x]/(
8*a*c^2) + (17*Log[1 + a*x])/(8*a*c^2)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{a^4\int \frac{e^{-2\tanh^{-1}(ax)}x^4}{(1-a^2x^2)^2}\,dx}{c^2}$$
$$=\frac{a^4\int \frac{x^4}{(1-ax)(1+ax)^3}\,dx}{c^2}$$
$$=\frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{8a^4(-1+ax)}+\frac{1}{2a^4(1+ax)^3}-\frac{7}{4a^4(1+ax)^2}+\frac{17}{8a^4(1+ax)}\right)dx}{c^2}$$
$$=-\frac{x}{c^2}-\frac{1}{4ac^2(1+ax)^2}+\frac{7}{4ac^2(1+ax)}-\frac{\log(1-ax)}{8ac^2}+\frac{17\log(1+ax)}{8ac^2}$$

Mathematica [A] time = 0.0437949, size = 68, normalized size = 0.92

$$\frac{-8a^3x^3-16a^2x^2+6ax-(ax+1)^2\log(1-ax)+17(ax+1)^2\log(ax+1)+12}{8a(acx+c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] (12 + 6*a*x - 16*a^2*x^2 - 8*a^3*x^3 - (1 + a*x)^2*Log[1 - a*x] + 17*(1 + a
*x)^2*Log[1 + a*x])/(8*a*(c + a*c*x)^2)
```

Maple [A] time = 0.039, size = 66, normalized size = 0.9

$$-\frac{x}{c^2}-\frac{1}{4\,ac^2\,(ax+1)^2}+\frac{7}{4\,ac^2\,(ax+1)}+\frac{17\,\ln(ax+1)}{8\,ac^2}-\frac{\ln(ax-1)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -x/c^2-1/4/a/c^2/(a*x+1)^2+7/4/a/c^2/(a*x+1)+17/8*ln(a*x+1)/a/c^2-1/8/c^2/a
*ln(a*x-1)
```

Maxima [A] time = 0.970696, size = 95, normalized size = 1.28

$$\frac{7\,ax+6}{4\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}-\frac{x}{c^2}+\frac{17\,\log(ax+1)}{8\,ac^2}-\frac{\log(ax-1)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(7*a*x + 6)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2) - x/c^2 + 17/8*log(a*x
+ 1)/(a*c^2) - 1/8*log(a*x - 1)/(a*c^2)
```

Fricas [A] time = 1.87532, size = 212, normalized size = 2.86

$$-\frac{8\,a^3x^3+16\,a^2x^2-6\,ax-17\left(a^2x^2+2\,ax+1\right)\log\left(ax+1\right)+\left(a^2x^2+2\,ax+1\right)\log\left(ax-1\right)-12}{8\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a^3*x^3 + 16*a^2*x^2 - 6*a*x - 17*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1
) + (a^2*x^2 + 2*a*x + 1)*log(a*x - 1) - 12)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a
*c^2)
```

Sympy [A] time = 2.00849, size = 75, normalized size = 1.01

$$-a^4\left(-\frac{7ax+6}{4a^7c^2x^2+8a^6c^2x+4a^5c^2}+\frac{x}{a^4c^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{17\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a**2/x**2)**2,x)
```

```
[Out] -a**4*(-(7*a*x + 6)/(4*a**7*c**2*x**2 + 8*a**6*c**2*x + 4*a**5*c**2) + x/(a
**4*c**2) + (log(x - 1/a)/8 - 17*log(x + 1/a)/8)/(a**5*c**2))
```

Giac [A] time = 1.2179, size = 136, normalized size = 1.84

$$-\frac{ax+1}{ac^2}-\frac{2\,\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{ac^2}-\frac{\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{8\,ac^2}+\frac{\frac{7\,a^5c^2}{ax+1}-\frac{a^5c^2}{(ax+1)^2}}{4\,a^6c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] -(a*x + 1)/(a*c^2) - 2*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/(a*c^2) - 1/8
*log(abs(-2/(a*x + 1) + 1))/(a*c^2) + 1/4*(7*a^5*c^2/(a*x + 1) - a^5*c^2/(a
*x + 1)^2)/(a^6*c^4)
```

3.676 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=109

$$-\frac{1}{16ac^3(1-ax)}+\frac{39}{16ac^3(ax+1)}-\frac{5}{8ac^3(ax+1)^2}+\frac{1}{12ac^3(ax+1)^3}-\frac{\log(1-ax)}{4ac^3}+\frac{9\log(ax+1)}{4ac^3}-\frac{x}{c^3}$$

```
[Out] -(x/c^3) - 1/(16*a*c^3*(1 - a*x)) + 1/(12*a*c^3*(1 + a*x)^3) - 5/(8*a*c^3*(
1 + a*x)^2) + 39/(16*a*c^3*(1 + a*x)) - Log[1 - a*x]/(4*a*c^3) + (9*Log[1 +
 a*x])/(4*a*c^3)
```

Rubi [A] time = 0.16565, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$-\frac{1}{16ac^3(1-ax)}+\frac{39}{16ac^3(ax+1)}-\frac{5}{8ac^3(ax+1)^2}+\frac{1}{12ac^3(ax+1)^3}-\frac{\log(1-ax)}{4ac^3}+\frac{9\log(ax+1)}{4ac^3}-\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] -(x/c^3) - 1/(16*a*c^3*(1 - a*x)) + 1/(12*a*c^3*(1 + a*x)^3) - 5/(8*a*c^3*(
1 + a*x)^2) + 39/(16*a*c^3*(1 + a*x)) - Log[1 - a*x]/(4*a*c^3) + (9*Log[1 +
 a*x])/(4*a*c^3)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{a^6\int \frac{e^{-2\tanh^{-1}(ax)}x^6}{(1-a^2x^2)^3}\,dx}{c^3}$$
$$= -\frac{a^6\int \frac{x^6}{(1-ax)^2(1+ax)^4}\,dx}{c^3}$$
$$= -\frac{a^6\int \left(\frac{1}{a^6}+\frac{1}{16a^6(-1+ax)^2}+\frac{1}{4a^6(-1+ax)}+\frac{1}{4a^6(1+ax)^4}-\frac{5}{4a^6(1+ax)^3}+\frac{39}{16a^6(1+ax)^2}-\frac{9}{4a^6(1+ax)}\right)dx}{c^3}$$
$$= -\frac{x}{c^3}-\frac{1}{16ac^3(1-ax)}+\frac{1}{12ac^3(1+ax)^3}-\frac{5}{8ac^3(1+ax)^2}+\frac{39}{16ac^3(1+ax)}-\frac{\log(1-ax)}{4ac^3}+\frac{9\log(1}{4a}$$

Mathematica [A] time = 0.0623581, size = 104, normalized size = 0.95

$$\frac{-2\left(6a^5x^5+12a^4x^4-15a^3x^3-24a^2x^2+7ax+11\right)-3(ax-1)(ax+1)^3\log(1-ax)+27(ax-1)(ax+1)^3\log(ax+1)}{12a(ax-1)(acx+c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] (-2*(11 + 7*a*x - 24*a^2*x^2 - 15*a^3*x^3 + 12*a^4*x^4 + 6*a^5*x^5) - 3*(-1
 + a*x)*(1 + a*x)^3*Log[1 - a*x] + 27*(-1 + a*x)*(1 + a*x)^3*Log[1 + a*x])/
(12*a*(-1 + a*x)*(c + a*c*x)^3)
```

Maple [A] time = 0.041, size = 96, normalized size = 0.9

$$-\frac{x}{c^3}+\frac{1}{12\,ac^3\,(ax+1)^3}-\frac{5}{8\,ac^3\,(ax+1)^2}+\frac{39}{16\,ac^3\,(ax+1)}+\frac{9\,\ln(ax+1)}{4\,ac^3}+\frac{1}{16\,ac^3\,(ax-1)}-\frac{\ln(ax-1)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^3,x)
```

```
[Out] -x/c^3+1/12/a/c^3/(a*x+1)^3-5/8/a/c^3/(a*x+1)^2+39/16/a/c^3/(a*x+1)+9/4*ln(
a*x+1)/a/c^3+1/16/a/c^3/(a*x-1)-1/4/a/c^3*ln(a*x-1)
```

Maxima [A] time = 0.972072, size = 132, normalized size = 1.21

$$\frac{15\,a^3x^3+12\,a^2x^2-13\,ax-11}{6\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}-\frac{x}{c^3}+\frac{9\,\log(ax+1)}{4\,ac^3}-\frac{\log(ax-1)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(15*a^3*x^3 + 12*a^2*x^2 - 13*a*x - 11)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 -
2*a^2*c^3*x - a*c^3) - x/c^3 + 9/4*log(a*x + 1)/(a*c^3) - 1/4*log(a*x - 1)/
(a*c^3)
```

Fricas [A] time = 1.86269, size = 308, normalized size = 2.83

$$-\frac{12\,a^5x^5+24\,a^4x^4-30\,a^3x^3-48\,a^2x^2+14\,ax-27\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log\left(ax+1\right)+3\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)}{12\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(12*a^5*x^5 + 24*a^4*x^4 - 30*a^3*x^3 - 48*a^2*x^2 + 14*a*x - 27*(a^4
*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x + 1) + 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x
 - 1)*log(a*x - 1) + 22)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3
)
```

Sympy [A] time = 3.70591, size = 104, normalized size = 0.95

$$-a^6\left(-\frac{15a^3x^3+12a^2x^2-13ax-11}{6a^{11}c^3x^4+12a^{10}c^3x^3-12a^8c^3x-6a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{4}-\frac{9\log\left(x+\frac{1}{a}\right)}{4}}{a^7c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a**2/x**2)**3,x)
```

```
[Out] -a**6*(-(15*a**3*x**3 + 12*a**2*x**2 - 13*a*x - 11)/(6*a**11*c**3*x**4 + 12
*a**10*c**3*x**3 - 12*a**8*c**3*x - 6*a**7*c**3) + x/(a**6*c**3) + (log(x -
 1/a)/4 - 9*log(x + 1/a)/4)/(a**7*c**3))
```

Giac [A] time = 1.19834, size = 189, normalized size = 1.73

$$-\frac{(ax+1)\left(\frac{65}{ax+1}-32\right)}{32\,ac^3\left(\frac{2}{ax+1}-1\right)}-\frac{2\,\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{ac^3}-\frac{\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{4\,ac^3}+\frac{\frac{117\,a^{11}c^6}{ax+1}-\frac{30\,a^{11}c^6}{(ax+1)^2}+\frac{4\,a^{11}c^6}{(ax+1)^3}}{48\,a^{12}c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -1/32*(a*x + 1)*(65/(a*x + 1) - 32)/(a*c^3*(2/(a*x + 1) - 1)) - 2*log(abs(a
*x + 1)/((a*x + 1)^2*abs(a)))/(a*c^3) - 1/4*log(abs(-2/(a*x + 1) + 1))/(a*c
^3) + 1/48*(117*a^11*c^6/(a*x + 1) - 30*a^11*c^6/(a*x + 1)^2 + 4*a^11*c^6/(
a*x + 1)^3)/(a^12*c^9)
```

3.677 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=144

$$-\frac{11}{64ac^4(1-ax)}+\frac{99}{32ac^4(ax+1)}+\frac{1}{64ac^4(1-ax)^2}-\frac{35}{32ac^4(ax+1)^2}+\frac{13}{48ac^4(ax+1)^3}-\frac{1}{32ac^4(ax+1)^4}-\frac{47\log(1-}{128ac^4}$$

```
[Out] -(x/c^4) + 1/(64*a*c^4*(1 - a*x)^2) - 11/(64*a*c^4*(1 - a*x)) - 1/(32*a*c^4
*(1 + a*x)^4) + 13/(48*a*c^4*(1 + a*x)^3) - 35/(32*a*c^4*(1 + a*x)^2) + 99/
(32*a*c^4*(1 + a*x)) - (47*Log[1 - a*x])/(128*a*c^4) + (303*Log[1 + a*x])/(
128*a*c^4)
```

Rubi [A] time = 0.200986, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$-\frac{11}{64ac^4(1-ax)}+\frac{99}{32ac^4(ax+1)}+\frac{1}{64ac^4(1-ax)^2}-\frac{35}{32ac^4(ax+1)^2}+\frac{13}{48ac^4(ax+1)^3}-\frac{1}{32ac^4(ax+1)^4}-\frac{47\log(1-}{128ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^4),x]
```


```
[Out] -(x/c^4) + 1/(64*a*c^4*(1 - a*x)^2) - 11/(64*a*c^4*(1 - a*x)) - 1/(32*a*c^4
*(1 + a*x)^4) + 13/(48*a*c^4*(1 + a*x)^3) - 35/(32*a*c^4*(1 + a*x)^2) + 99/
(32*a*c^4*(1 + a*x)) - (47*Log[1 - a*x])/(128*a*c^4) + (303*Log[1 + a*x])/(
128*a*c^4)
```


Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{e^{-2\tanh^{-1}(ax)}x^8}{\left(1-a^2x^2\right)^4}\,dx}{c^4}$$

$$= \frac{a^8\int \frac{x^8}{(1-ax)^3(1+ax)^5}\,dx}{c^4}$$

$$= \frac{a^8\int \left(-\frac{1}{a^8}-\frac{1}{32a^8(-1+ax)^3}-\frac{11}{64a^8(-1+ax)^2}-\frac{47}{128a^8(-1+ax)}+\frac{1}{8a^8(1+ax)^5}-\frac{13}{16a^8(1+ax)^4}+\frac{35}{16a^8(1+ax)^3}-\frac{99}{32a^8(1+ax)}\right.}{c^4}$$

$$= -\frac{x}{c^4}+\frac{1}{64ac^4(1-ax)^2}-\frac{11}{64ac^4(1-ax)}-\frac{1}{32ac^4(1+ax)^4}+\frac{13}{48ac^4(1+ax)^3}-\frac{35}{32ac^4(1+ax)^2}+\frac{}{32a}$$

Mathematica [A] time = 0.0899573, size = 121, normalized size = 0.84

$$\frac{-384a^7x^7-768a^6x^6+1638a^5x^5+2508a^4x^4-1732a^3x^3-2516a^2x^2+550ax-141(ax-1)^2(ax+1)^4\log(1-ax)+909(}{384a(ax-1)^2(acx+c)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] (800 + 550*a*x - 2516*a^2*x^2 - 1732*a^3*x^3 + 2508*a^4*x^4 + 1638*a^5*x^5
- 768*a^6*x^6 - 384*a^7*x^7 - 141*(-1 + a*x)^2*(1 + a*x)^4*Log[1 - a*x] + 9
09*(-1 + a*x)^2*(1 + a*x)^4*Log[1 + a*x])/(384*a*(-1 + a*x)^2*(c + a*c*x)^4
)
```

Maple [A] time = 0.047, size = 126, normalized size = 0.9

$$-\frac{x}{c^4}-\frac{1}{32\,ac^4\,(ax+1)^4}+\frac{13}{48\,ac^4\,(ax+1)^3}-\frac{35}{32\,ac^4\,(ax+1)^2}+\frac{99}{32\,ac^4\,(ax+1)}+\frac{303\,\ln(ax+1)}{128\,ac^4}+\frac{1}{64\,ac^4\,(ax-1)^2}+\frac{}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^4,x)
```

```
[Out] -x/c^4-1/32/a/c^4/(a*x+1)^4+13/48/a/c^4/(a*x+1)^3-35/32/a/c^4/(a*x+1)^2+99/
32/a/c^4/(a*x+1)+303/128*ln(a*x+1)/a/c^4+1/64/c^4/a/(a*x-1)^2+11/64/c^4/a/(
a*x-1)-47/128/c^4/a*ln(a*x-1)
```

Maxima [A] time = 0.991712, size = 197, normalized size = 1.37

$$\frac{627\,a^5x^5+486\,a^4x^4-1058\,a^3x^3-874\,a^2x^2+467\,ax+400}{192\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4\right)}-\frac{x}{c^4}+\frac{303\,\log(ax+1)}{128\,ac^4}-\frac{47\,\log(ax-1)}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/192*(627*a^5*x^5 + 486*a^4*x^4 - 1058*a^3*x^3 - 874*a^2*x^2 + 467*a*x + 4
00)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^
```

```
2 + 2*a^2*c^4*x + a*c^4) - x/c^4 + 303/128*log(a*x + 1)/(a*c^4) - 47/128*lo
g(a*x - 1)/(a*c^4)
```

Fricas [A] time = 1.77335, size = 510, normalized size = 3.54

$$-\frac{384\,a^7x^7+768\,a^6x^6-1638\,a^5x^5-2508\,a^4x^4+1732\,a^3x^3+2516\,a^2x^2-550\,ax-909\left(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3\cdots\right.}{384\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/384*(384*a^7*x^7 + 768*a^6*x^6 - 1638*a^5*x^5 - 2508*a^4*x^4 + 1732*a^3*
x^3 + 2516*a^2*x^2 - 550*a*x - 909*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x
^3 - a^2*x^2 + 2*a*x + 1)*log(a*x + 1) + 141*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4
 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x - 1) - 800)/(a^7*c^4*x^6 + 2*a^
6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4
)
```

Sympy [A] time = 5.42726, size = 158, normalized size = 1.1

$$-a^8\left(-\frac{627a^5x^5+486a^4x^4-1058a^3x^3-874a^2x^2+467ax+400}{192a^{15}c^4x^6+384a^{14}c^4x^5-192a^{13}c^4x^4-768a^{12}c^4x^3-192a^{11}c^4x^2+384a^{10}c^4x+192a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{47\log\left(x-\frac{1}{a}\right)}{128}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a**2/x**2)**4,x)
```

```
[Out] -a**8*(-(627*a**5*x**5 + 486*a**4*x**4 - 1058*a**3*x**3 - 874*a**2*x**2 + 4
67*a*x + 400)/(192*a**15*c**4*x**6 + 384*a**14*c**4*x**5 - 192*a**13*c**4*x
**4 - 768*a**12*c**4*x**3 - 192*a**11*c**4*x**2 + 384*a**10*c**4*x + 192*a*
*9*c**4) + x/(a**8*c**4) + (47*log(x - 1/a)/128 - 303*log(x + 1/a)/128)/(a*
*9*c**4))
```

Giac [A] time = 1.21461, size = 221, normalized size = 1.53

$$-\frac{2\,\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{ac^4}-\frac{47\,\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{128\,ac^4}+\frac{(ax+1)\left(\frac{1045}{ax+1}-\frac{1064}{(ax+1)^2}-256\right)}{256\,ac^4\left(\frac{2}{ax+1}-1\right)^2}+\frac{\frac{297\,a^{19}c^{12}}{ax+1}-\frac{105\,a^{19}c^{12}}{(ax+1)^2}+\frac{26\,a^{19}c^{12}}{(ax+1)^3}-\frac{3\,a^{19}c^{12}}{(ax+1)^4}}{96\,a^{20}c^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -2*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/(a*c^4) - 47/128*log(abs(-2/(a*x
+ 1) + 1))/(a*c^4) + 1/256*(a*x + 1)*(1045/(a*x + 1) - 1064/(a*x + 1)^2 - 2
56)/(a*c^4*(2/(a*x + 1) - 1)^2) + 1/96*(297*a^19*c^12/(a*x + 1) - 105*a^19*
c^12/(a*x + 1)^2 + 26*a^19*c^12/(a*x + 1)^3 - 3*a^19*c^12/(a*x + 1)^4)/(a^2
0*c^16)
```

3.678 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=191

$$\frac{c^4\left(1-a^2x^2\right)^{7/2}}{2a^7x^6}-\frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7}-\frac{c^4(24-5ax)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5}+\frac{c^4(16-5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3}-\frac{3c^4(5ax+16)\sqrt{1-a^2x^2}}{16a^2x}+$$

```
[Out] (-3*c^4*(16 + 5*a*x)*Sqrt[1 - a^2*x^2])/(16*a^2*x) + (c^4*(16 - 5*a*x)*(1 -
 a^2*x^2)^(3/2))/(16*a^4*x^3) - (c^4*(24 - 5*a*x)*(1 - a^2*x^2)^(5/2))/(40*
a^6*x^5) - (c^4*(1 - a^2*x^2)^(7/2))/(7*a^8*x^7) + (c^4*(1 - a^2*x^2)^(7/2)
)/(2*a^7*x^6) - (3*c^4*ArcSin[a*x])/a + (15*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])
/(16*a)
```

Rubi [A] time = 0.361692, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6157, 6149, 1807, 811, 813, 844, 216, 266, 63, 208}

$$\frac{c^4\left(1-a^2x^2\right)^{7/2}}{2a^7x^6}-\frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7}-\frac{c^4(24-5ax)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5}+\frac{c^4(16-5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3}-\frac{3c^4(5ax+16)\sqrt{1-a^2x^2}}{16a^2x}+$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^4/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-3*c^4*(16 + 5*a*x)*Sqrt[1 - a^2*x^2])/(16*a^2*x) + (c^4*(16 - 5*a*x)*(1 -
 a^2*x^2)^(3/2))/(16*a^4*x^3) - (c^4*(24 - 5*a*x)*(1 - a^2*x^2)^(5/2))/(40*
a^6*x^5) - (c^4*(1 - a^2*x^2)^(7/2))/(7*a^8*x^7) + (c^4*(1 - a^2*x^2)^(7/2)
)/(2*a^7*x^6) - (3*c^4*ArcSin[a*x])/a + (15*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])
/(16*a)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
```

```
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\int\frac{e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$

$$= \frac{c^4\int\frac{(1-ax)^3\left(1-a^2x^2\right)^{5/2}}{x^8}dx}{a^8}$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7} - \frac{c^4\int\frac{\left(1-a^2x^2\right)^{5/2}\left(21a-21a^2x+7a^3x^2\right)}{x^7}dx}{7a^8}$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7} + \frac{c^4\left(1-a^2x^2\right)^{7/2}}{2a^7x^6} + \frac{c^4\int\frac{\left(126a^2-21a^3x\right)\left(1-a^2x^2\right)^{5/2}}{x^6}dx}{42a^8}$$

$$= -\frac{c^4(24-5ax)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5} - \frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7} + \frac{c^4\left(1-a^2x^2\right)^{7/2}}{2a^7x^6} - \frac{c^4\int\frac{\left(1008a^4-210a^5x\right)(1-}{x^4}}{336a^8}$$

$$= \frac{c^4(16-5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24-5ax)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5} - \frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7} + \frac{c^4\left(1-a^2x^2\right.}{2a^7x^6}$$

$$= -\frac{3c^4(16+5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16-5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24-5ax)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5} - \frac{c}{}$$

$$= -\frac{3c^4(16+5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16-5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24-5ax)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5} - \frac{c}{}$$

$$= -\frac{3c^4(16+5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16-5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24-5ax)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5} - \frac{c}{}$$

$$= -\frac{3c^4(16+5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16-5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24-5ax)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5} - \frac{c}{}$$

$$= -\frac{3c^4(16+5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16-5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24-5ax)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5} - \frac{c}{}$$

Mathematica [C] time = 0.18322, size = 191, normalized size = 1.

$$\frac{c^4\left(\frac{5\left(16a^7x^7\left(a^2x^2-1\right)^4\text{Hypergeometric2F1}\left(3,\frac{7}{2},\frac{9}{2},1-a^2x^2\right)-16a^8x^8-231a^7x^7+64a^6x^6+413a^5x^5-96a^4x^4-238a^3x^3+64a^2x^2-105a^7x^7\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x\right.}{\sqrt{1-a^2x^2}}\right.}{560a^8x^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^4*(-336*a^2*x^2*Hypergeometric2F1[-5/2, -5/2, -3/2, a^2*x^2] + (5*(-16 +
 56*a*x + 64*a^2*x^2 - 238*a^3*x^3 - 96*a^4*x^4 + 413*a^5*x^5 + 64*a^6*x^6
- 231*a^7*x^7 - 16*a^8*x^8 - 105*a^7*x^7*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 -
 a^2*x^2]] + 16*a^7*x^7*(-1 + a^2*x^2)^4*Hypergeometric2F1[3, 7/2, 9/2, 1 -
 a^2*x^2]))/Sqrt[1 - a^2*x^2]))/(560*a^8*x^7)
```

Maple [A] time = 0.104, size = 289, normalized size = 1.5

$$-\frac{3c^4}{8a^5x^4}\left(-a^2x^2+1\right)^{\frac{5}{2}} - \frac{5c^4}{16x^2a^3}\left(-a^2x^2+1\right)^{\frac{5}{2}} - \frac{5c^4}{16a}\left(-a^2x^2+1\right)^{\frac{3}{2}} - \frac{15c^4}{16a}\sqrt{-a^2x^2+1} + \frac{15c^4}{16a}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^4/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -3/8*c^4/a^5/x^4*(-a^2*x^2+1)^(5/2)-5/16*c^4/a^3/x^2*(-a^2*x^2+1)^(5/2)-5/1
6*c^4*(-a^2*x^2+1)^(3/2)/a-15/16*c^4*(-a^2*x^2+1)^(1/2)/a+15/16*c^4/a*arcta
nh(1/(-a^2*x^2+1)^(1/2))-1/7*c^4/a^8/x^7*(-a^2*x^2+1)^(5/2)-16/35*c^4/a^6/x
^5*(-a^2*x^2+1)^(5/2)+1/2*c^4/a^7/x^6*(-a^2*x^2+1)^(5/2)+c^4/a^4/x^3*(-a^2*
x^2+1)^(5/2)-2*c^4/a^2/x*(-a^2*x^2+1)^(5/2)-2*c^4*x*(-a^2*x^2+1)^(3/2)-3*c^
4*x*(-a^2*x^2+1)^(1/2)-3*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^4}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^4/(a*x + 1)^3, x)
```

Fricas [A] time = 2.21505, size = 398, normalized size = 2.08

$$\frac{3360\,a^7c^4x^7\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-525\,a^7c^4x^7\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-560\,a^7c^4x^7-\left(560\,a^7c^4x^7+2496\,a^6c^4x^6-525\,a^5c^4x}{560\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/560*(3360*a^7*c^4*x^7*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 525*a^7*c^
4*x^7*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 560*a^7*c^4*x^7 - (560*a^7*c^4*x^7
+ 2496*a^6*c^4*x^6 - 525*a^5*c^4*x^5 - 992*a^4*c^4*x^4 + 770*a^3*c^4*x^3 +
96*a^2*c^4*x^2 - 280*a*c^4*x + 80*c^4)*sqrt(-a^2*x^2 + 1))/(a^8*x^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**4/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25757, size = 683, normalized size = 3.58

$$\frac{\left(5\,c^4-\frac{35\left(\sqrt{-a^2x^2+1}|a|+a\right)c^4}{a^2x}+\frac{49\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^4}{a^4x^2}+\frac{245\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c^4}{a^6x^3}-\frac{875\left(\sqrt{-a^2x^2+1}|a|+a\right)^4c^4}{a^8x^4}-\frac{455\left(\sqrt{-a^2x^2+1}|a|+a\right)^5c^4}{a^{10}x^5}+\frac{9065\left(\sqrt{\cdots}\right.}{\cdots}\right.}{4480\left(\sqrt{-a^2x^2+1}|a|+a\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4480*(5*c^4 - 35*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/(a^2*x) + 49*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*c^4/(a^4*x^2) + 245*(sqrt(-a^2*x^2 + 1)*abs(a) +
 a)^3*c^4/(a^6*x^3) - 875*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^8*x^4) -
 455*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^10*x^5) + 9065*(sqrt(-a^2*x^2
 + 1)*abs(a) + a)^6*c^4/(a^12*x^6))*a^14*x^7/((sqrt(-a^2*x^2 + 1)*abs(a) +
a)^7*abs(a)) - 3*c^4*arcsin(a*x)*sgn(a)/abs(a) + 15/16*c^4*log(1/2*abs(-2*s
qrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c
^4/a - 1/4480*(9065*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^4/x - 455*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^4/x^2 - 875*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)^3*c^4/x^3 + 245*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^2*x^4) + 49*(sqr
t(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^4*x^5) - 35*(sqrt(-a^2*x^2 + 1)*abs(a)
 + a)^6*c^4/(a^6*x^6) + 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7*c^4/(a^8*x^7))/
(a^6*abs(a))
```

3.679 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=157

$$-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{4a^5x^4}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}+\frac{c^3(8-ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}-\frac{3c^3(ax+8)\sqrt{1-a^2x^2}}{8a^2x}+\frac{3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{8a}-$$

```
[Out] (-3*c^3*(8 + a*x)*Sqrt[1 - a^2*x^2])/(8*a^2*x) + (c^3*(8 - a*x)*(1 - a^2*x^
2)^(3/2))/(8*a^4*x^3) + (c^3*(1 - a^2*x^2)^(5/2))/(5*a^6*x^5) - (3*c^3*(1 -
 a^2*x^2)^(5/2))/(4*a^5*x^4) - (3*c^3*ArcSin[a*x])/a + (3*c^3*ArcTanh[Sqrt[
1 - a^2*x^2]])/(8*a)
```

Rubi [A] time = 0.321341, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6157, 6149, 1807, 811, 813, 844, 216, 266, 63, 208}

$$-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{4a^5x^4}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}+\frac{c^3(8-ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}-\frac{3c^3(ax+8)\sqrt{1-a^2x^2}}{8a^2x}+\frac{3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{8a}-$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-3*c^3*(8 + a*x)*Sqrt[1 - a^2*x^2])/(8*a^2*x) + (c^3*(8 - a*x)*(1 - a^2*x^
2)^(3/2))/(8*a^4*x^3) + (c^3*(1 - a^2*x^2)^(5/2))/(5*a^6*x^5) - (3*c^3*(1 -
 a^2*x^2)^(5/2))/(4*a^5*x^4) - (3*c^3*ArcSin[a*x])/a + (3*c^3*ArcTanh[Sqrt[
1 - a^2*x^2]])/(8*a)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
```

```
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3dx=-\frac{c^3\int\frac{e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^3}{x^6}dx}{a^6}$$
$$=-\frac{c^3\int\frac{(1-ax)^3\left(1-a^2x^2\right)^{3/2}}{x^6}dx}{a^6}$$
$$=\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}+\frac{c^3\int\frac{\left(1-a^2x^2\right)^{3/2}\left(15a-15a^2x+5a^3x^2\right)}{x^5}dx}{5a^6}$$
$$=\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{4a^5x^4}-\frac{c^3\int\frac{\left(60a^2-5a^3x\right)\left(1-a^2x^2\right)^{3/2}}{x^4}dx}{20a^6}$$
$$=\frac{c^3(8-ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{4a^5x^4}+\frac{c^3\int\frac{\left(240a^4-30a^5x\right)\sqrt{1-}}{x^2}}{80a^6}$$
$$=-\frac{3c^3(8+ax)\sqrt{1-a^2x^2}}{8a^2x}+\frac{c^3(8-ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}-\frac{3c^3\left(1-a^2x^2\right.}{4a^5x^4}$$
$$=-\frac{3c^3(8+ax)\sqrt{1-a^2x^2}}{8a^2x}+\frac{c^3(8-ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}-\frac{3c^3\left(1-a^2x^2\right.}{4a^5x^4}$$
$$=-\frac{3c^3(8+ax)\sqrt{1-a^2x^2}}{8a^2x}+\frac{c^3(8-ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}-\frac{3c^3\left(1-a^2x^2\right.}{4a^5x^4}$$
$$=-\frac{3c^3(8+ax)\sqrt{1-a^2x^2}}{8a^2x}+\frac{c^3(8-ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}-\frac{3c^3\left(1-a^2x^2\right.}{4a^5x^4}$$
$$=-\frac{3c^3(8+ax)\sqrt{1-a^2x^2}}{8a^2x}+\frac{c^3(8-ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}-\frac{3c^3\left(1-a^2x^2\right.}{4a^5x^4}$$

Mathematica [C] time = 0.0988295, size = 186, normalized size = 1.18

$$\frac{c^3\left(8a^5x^5\left(a^2x^2-1\right)^3\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},1-a^2x^2\right)+40a^2x^2\sqrt{1-a^2x^2}\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\right.\right.}{40a^6x^5\sqrt{1-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^3*(8 - 30*a*x - 24*a^2*x^2 + 105*a^3*x^3 + 24*a^4*x^4 - 75*a^5*x^5 - 8*a
^6*x^6 - 45*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]] + 40*a^2*x
^2*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[-3/2, -3/2, -1/2, a^2*x^2] + 8*a^5*x
^5*(-1 + a^2*x^2)^3*Hypergeometric2F1[2, 5/2, 7/2, 1 - a^2*x^2]))/(40*a^6*x
^5*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.066, size = 243, normalized size = 1.6

$$-\frac{3c^3}{4a^5x^4}\left(-a^2x^2+1\right)^{\frac{5}{2}}-\frac{c^3}{8x^2a^3}\left(-a^2x^2+1\right)^{\frac{5}{2}}-\frac{c^3}{8a}\left(-a^2x^2+1\right)^{\frac{3}{2}}-\frac{3c^3}{8a}\sqrt{-a^2x^2+1}+\frac{3c^3}{8a}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -3/4*c^3*(-a^2*x^2+1)^(5/2)/a^5/x^4-1/8*c^3/a^3/x^2*(-a^2*x^2+1)^(5/2)-1/8*
c^3*(-a^2*x^2+1)^(3/2)/a-3/8*c^3*(-a^2*x^2+1)^(1/2)/a+3/8*c^3/a*arctanh(1/(
-a^2*x^2+1)^(1/2))+1/5*c^3*(-a^2*x^2+1)^(5/2)/a^6/x^5+c^3/a^4/x^3*(-a^2*x^2
+1)^(5/2)-2*c^3/a^2/x*(-a^2*x^2+1)^(5/2)-2*c^3*x*(-a^2*x^2+1)^(3/2)-3*c^3*x
*(-a^2*x^2+1)^(1/2)-3*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/
2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^3}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^3/(a*x + 1)^3, x)
```

Fricas [A] time = 2.1836, size = 338, normalized size = 2.15

$$\frac{240\,a^5c^3x^5\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-15\,a^5c^3x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-40\,a^5c^3x^5-\left(40\,a^5c^3x^5+152\,a^4c^3x^4-55\,a^3c^3x^3-24\,a^2c^3}{40\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/40*(240*a^5*c^3*x^5*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 15*a^5*c^3*x
^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 40*a^5*c^3*x^5 - (40*a^5*c^3*x^5 + 152
*a^4*c^3*x^4 - 55*a^3*c^3*x^3 - 24*a^2*c^3*x^2 + 30*a*c^3*x - 8*c^3)*sqrt(-
a^2*x^2 + 1))/(a^6*x^5)
```

Sympy [C] time = 45.8976, size = 695, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**3/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -c**3*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*as
in(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2
- log(sqrt(-a**2*x**2 + 1) + 1), True))/a + 3*c**3*Piecewise((-I*a**2*x/sqr
t(a**2*x**2 - 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**
2) > 1), (a**2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2
+ 1)), True))/a**2 - 2*c**3*Piecewise((a**2*acosh(1/(a*x))/2 + a/(2*x*sqrt(
-1 + 1/(a**2*x**2))) - 1/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x*
*2) > 1), (-I*a**2*asin(1/(a*x))/2 - I*a*sqrt(1 - 1/(a**2*x**2))/(2*x), Tru
```

```
e))/a**3 - 2*c**3*Piecewise((a**3*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(-1 +
1/(a**2*x**2))/(3*x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(1 - 1/(a**2*x*
*2))/3 - I*a*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True))/a**4 + 3*c**3*Piecewi
se((a**4*acosh(1/(a*x))/8 - a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*a/(8*x*
*3*sqrt(-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs
(a**2*x**2) > 1), (-I*a**4*asin(1/(a*x))/8 + I*a**3/(8*x*sqrt(1 - 1/(a**2*x
**2))) - 3*I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a
**2*x**2))), True))/a**5 - c**3*Piecewise((2*I*a**4*sqrt(a**2*x**2 - 1)/(15
*x) + I*a**2*sqrt(a**2*x**2 - 1)/(15*x**3) - I*sqrt(a**2*x**2 - 1)/(5*x**5)
, Abs(a**2*x**2) > 1), (2*a**4*sqrt(-a**2*x**2 + 1)/(15*x) + a**2*sqrt(-a**
2*x**2 + 1)/(15*x**3) - sqrt(-a**2*x**2 + 1)/(5*x**5), True))/a**6
```

Giac [B] time = 1.27207, size = 521, normalized size = 3.32

$$-\frac{\left(2\,c^3-\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)c^3}{a^2x}+\frac{30\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^3}{a^4x^2}+\frac{80\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c^3}{a^6x^3}-\frac{580\left(\sqrt{-a^2x^2+1}|a|+a\right)^4c^3}{a^8x^4}\right)a^{10}x^5}{320\left(\sqrt{-a^2x^2+1}|a|+a\right)^5|a|}-\frac{3\,c^3\arcsin\left(ax\right)\mathrm{sg}}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/320*(2*c^3 - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3/(a^2*x) + 30*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*c^3/(a^4*x^2) + 80*(sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*c^3/(a^6*x^3) - 580*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^3/(a^8*x^4))*a
^10*x^5/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*abs(a)) - 3*c^3*arcsin(a*x)*sgn(
a)/abs(a) + 3/8*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*ab
s(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c^3/a - 1/320*(580*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)*a^2*c^3/x - 80*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3/x^2 - 30*(s
qrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^3/(a^2*x^3) + 15*(sqrt(-a^2*x^2 + 1)*abs(
a) + a)^4*c^3/(a^4*x^4) - 2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^3/(a^6*x^5)
)/(a^4*abs(a))
```

3.680 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=125

$$\frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3}-\frac{c^2(6-ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{3c^2\sin^{-1}(ax)}{a}$$

```
[Out] -(c^2*(6 - a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^2*(1 - a^2*x^2)^(3/2))/(3
*a^4*x^3) + (3*c^2*(1 - a^2*x^2)^(3/2))/(2*a^3*x^2) - (3*c^2*ArcSin[a*x])/a
 - (c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rubi [A] time = 0.275947, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6157, 6149, 1807, 813, 844, 216, 266, 63, 208}

$$\frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3}-\frac{c^2(6-ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{3c^2\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(c^2*(6 - a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^2*(1 - a^2*x^2)^(3/2))/(3
*a^4*x^3) + (3*c^2*(1 - a^2*x^2)^(3/2))/(2*a^3*x^2) - (3*c^2*ArcSin[a*x])/a
 - (c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
```

```
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{c^2\int \frac{e^{-3\tanh^{-1}(ax)}(1-a^2x^2)^2}{x^4}dx}{a^4}$$
$$= \frac{c^2\int \frac{(1-ax)^3\sqrt{1-a^2x^2}}{x^4}dx}{a^4}$$
$$= -\frac{c^2(1-a^2x^2)^{3/2}}{3a^4x^3} - \frac{c^2\int \frac{\sqrt{1-a^2x^2}(9a-9a^2x+3a^3x^2)}{x^3}dx}{3a^4}$$
$$= -\frac{c^2(1-a^2x^2)^{3/2}}{3a^4x^3} + \frac{3c^2(1-a^2x^2)^{3/2}}{2a^3x^2} + \frac{c^2\int \frac{(18a^2+3a^3x)\sqrt{1-a^2x^2}}{x^2}dx}{6a^4}$$
$$= -\frac{c^2(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(1-a^2x^2)^{3/2}}{3a^4x^3} + \frac{3c^2(1-a^2x^2)^{3/2}}{2a^3x^2} - \frac{c^2\int \frac{-6a^3+36a^4x}{x\sqrt{1-a^2x^2}}dx}{12a^4}$$
$$= -\frac{c^2(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(1-a^2x^2)^{3/2}}{3a^4x^3} + \frac{3c^2(1-a^2x^2)^{3/2}}{2a^3x^2} - (3c^2)\int \frac{1}{\sqrt{1-a^2x^2}}dx +$$
$$= -\frac{c^2(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(1-a^2x^2)^{3/2}}{3a^4x^3} + \frac{3c^2(1-a^2x^2)^{3/2}}{2a^3x^2} - \frac{3c^2\sin^{-1}(ax)}{a} + \frac{c^2\,\text{Subs}}{}$$
$$= -\frac{c^2(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(1-a^2x^2)^{3/2}}{3a^4x^3} + \frac{3c^2(1-a^2x^2)^{3/2}}{2a^3x^2} - \frac{3c^2\sin^{-1}(ax)}{a} - \frac{c^2\,\text{Subst}}{}$$
$$= -\frac{c^2(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(1-a^2x^2)^{3/2}}{3a^4x^3} + \frac{3c^2(1-a^2x^2)^{3/2}}{2a^3x^2} - \frac{3c^2\sin^{-1}(ax)}{a} - \frac{c^2\tanh}{}$$

Mathematica [A] time = 0.0807866, size = 128, normalized size = 1.02

$$-\frac{c^2\left(-6a^5x^5-16a^4x^4+15a^3x^3+14a^2x^2+18a^3x^3\sqrt{1-a^2x^2}\sin^{-1}(ax)+3a^3x^3\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-9ax+2\right)}{6a^4x^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(c^2*(2 - 9*a*x + 14*a^2*x^2 + 15*a^3*x^3 - 16*a^4*x^4 - 6*a^5*x^5 + 18*a^
3*x^3*Sqrt[1 - a^2*x^2]*ArcSin[a*x] + 3*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[S
qrt[1 - a^2*x^2]]))/(6*a^4*x^3*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.055, size = 299, normalized size = 2.4

$$-\frac{10c^2}{3a^2x}\left(-a^2x^2+1\right)^{\frac{5}{2}}-\frac{10xc^2}{3}\left(-a^2x^2+1\right)^{\frac{3}{2}}-5c^2x\sqrt{-a^2x^2+1}-5\frac{c^2}{\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)+\frac{c^2}{6a}\left(-a^2x^2+1\right)^{\frac{3}{2}}+\frac{c^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -10/3*c^2/a^2/x*(-a^2*x^2+1)^(5/2)-10/3*c^2*x*(-a^2*x^2+1)^(3/2)-5*c^2*x*(-
a^2*x^2+1)^(1/2)-5*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
+1/6*c^2*(-a^2*x^2+1)^(3/2)/a+1/2*c^2*(-a^2*x^2+1)^(1/2)/a-1/2*c^2/a*arctan
```

```
h(1/(-a^2*x^2+1)^(1/2))+4/3*c^2/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+2*c^2*
(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x+2*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x
/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))+3/2*c^2/a^3/x^2*(-a^2*x^2+1)^(5/2)-1/3
*c^2/a^4/x^3*(-a^2*x^2+1)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^2}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^2/(a*x + 1)^3, x)
```

Fricas [A] time = 2.29584, size = 282, normalized size = 2.26

$$\frac{36\,a^3c^2x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+3\,a^3c^2x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-6\,a^3c^2x^3-\left(6\,a^3c^2x^3+16\,a^2c^2x^2-9\,ac^2x+2\,c^2\right)\sqrt{-a^2x^2}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/6*(36*a^3*c^2*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 3*a^3*c^2*x^3*
log((sqrt(-a^2*x^2 + 1) - 1)/x) - 6*a^3*c^2*x^3 - (6*a^3*c^2*x^3 + 16*a^2*c
^2*x^2 - 9*a*c^2*x + 2*c^2)*sqrt(-a^2*x^2 + 1))/(a^4*x^3)
```

Sympy [C] time = 33.2566, size = 384, normalized size = 3.07

$$-\frac{c^2\left(\begin{cases} i\sqrt{a^2x^2-1}-\log(ax)+\frac{\log(a^2x^2)}{2}+i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{for } \left|a^2x^2\right|>1 \\ \sqrt{-a^2x^2+1}+\frac{\log(a^2x^2)}{2}-\log\left(\sqrt{-a^2x^2+1}+1\right) & \text{otherwise} \end{cases}\right)}{a}+\frac{3c^2\left(\begin{cases} -\frac{ia^2x}{\sqrt{a^2x^2-1}}+ia\,\mathrm{acosh}(ax)+\frac{i}{x\sqrt{a^2x^2-1}} \\ \frac{a^2x}{\sqrt{-a^2x^2+1}}-a\,\mathrm{asin}(ax)-\frac{1}{x\sqrt{-a^2x^2+1}} \end{cases}\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**2/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -c**2*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*as
in(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2
- log(sqrt(-a**2*x**2 + 1) + 1), True))/a + 3*c**2*Piecewise((-I*a**2*x/sqr
t(a**2*x**2 - 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**
2) > 1), (a**2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2
+ 1)), True))/a**2 - 3*c**2*Piecewise((a**2*acosh(1/(a*x))/2 + a/(2*x*sqrt(
```

```
-1 + 1/(a**2*x**2))) - 1/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x*
*2) > 1), (-I*a**2*asin(1/(a*x))/2 - I*a*sqrt(1 - 1/(a**2*x**2))/(2*x), Tru
e))/a**3 + c**2*Piecewise((a**3*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(-1 + 1/
(a**2*x**2))/(3*x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(1 - 1/(a**2*x**2
))/3 - I*a*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True))/a**4
```

Giac [B] time = 1.22321, size = 355, normalized size = 2.84

$$\frac{\left(c^2 - \frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)c^2}{a^2x} + \frac{33\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c^2}{a^4x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|} - \frac{3\,c^2\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{|a|} - \frac{c^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|} - \frac{\sqrt{-a^2x^2+1}c^2}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*(c^2 - 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/(a^2*x) + 33*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^2*c^2/(a^4*x^2))*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*abs(a)) - 3*c^2*arcsin(a*x)*sgn(a)/abs(a) - 1/2*c^2*log(1/2*abs(-2*sqr
t(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c^2
/a - 1/24*(33*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/x - 9*(sqrt(-a^2*x^2 + 1)
*abs(a) + a)^2*c^2/(a^2*x^2) + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^2/(a^4*x
^3))/(a^2*abs(a))
```

3.681 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$

Optimal. Leaf size=74

$$-\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{3c\sin^{-1}(ax)}{a}$$

```
[Out] -((c*Sqrt[1 - a^2*x^2])/a) + (c*Sqrt[1 - a^2*x^2])/(a^2*x) - (3*c*ArcSin[a*
x])/a - (3*c*ArcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rubi [A] time = 0.207967, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6157, 6149, 1807, 1809, 844, 216, 266, 63, 208}

$$-\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{3c\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((c*Sqrt[1 - a^2*x^2])/a) + (c*Sqrt[1 - a^2*x^2])/(a^2*x) - (3*c*ArcSin[a*
x])/a - (3*c*ArcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx &= -\frac{c\int \frac{e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)}{x^2}\,dx}{a^2}\\
&= -\frac{c\int \frac{(1-ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= \frac{c\sqrt{1-a^2x^2}}{a^2x}+\frac{c\int \frac{3a-3a^2x+a^3x^2}{x\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= -\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{c\int \frac{-3a^3+3a^4x}{x\sqrt{1-a^2x^2}}\,dx}{a^4}\\
&= -\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-(3c)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx+\frac{(3c)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{a}\\
&= -\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{3c\sin^{-1}(ax)}{a}+\frac{(3c)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2a}\\
&= -\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{3c\sin^{-1}(ax)}{a}-\frac{(3c)\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^3}\\
&= -\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{3c\sin^{-1}(ax)}{a}-\frac{3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0628591, size = 57, normalized size = 0.77

$$-\frac{c\left(\sqrt{1-a^2x^2}(ax-1)+3ax\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+3ax\sin^{-1}(ax)\right)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((c*((-1 + a*x)*Sqrt[1 - a^2*x^2] + 3*a*x*ArcSin[a*x] + 3*a*x*ArcTanh[Sqrt
[1 - a^2*x^2]]))/(a^2*x))
```

Maple [B] time = 0.057, size = 266, normalized size = 3.6

$$\frac{c}{a^2x}\left(-a^2x^2+1\right)^{\frac{5}{2}}+cx\left(-a^2x^2+1\right)^{\frac{3}{2}}+\frac{3cx}{2}\sqrt{-a^2x^2+1}+\frac{3c}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-2\frac{c\left(-a^2\left(x+a^{-1}\right)^2+\right.}{a^3\left(x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] c/a^2/x*(-a^2*x^2+1)^(5/2)+c*x*(-a^2*x^2+1)^(3/2)+3/2*c*x*(-a^2*x^2+1)^(1/2
)+3/2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-2*c/a^3/(x+1/a
)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-3*c/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(
3/2)-9/2*c*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-9/2*c/(a^2)^(1/2)*arctan((a
^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))+c*(-a^2*x^2+1)^(3/2)/a-3*c/
a*arctanh(1/(-a^2*x^2+1)^(1/2))+3*c*(-a^2*x^2+1)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))/(a*x + 1)^3, x)
```

Fricas [A] time = 2.1854, size = 190, normalized size = 2.57

$$\frac{6\,acx\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+3\,acx\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-acx-\sqrt{-a^2x^2+1}(acx-c)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (6*a*c*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 3*a*c*x*log((sqrt(-a^2*x^
2 + 1) - 1)/x) - a*c*x - sqrt(-a^2*x^2 + 1)*(a*c*x - c))/(a^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c\left(\int -\frac{\sqrt{-a^2x^2+1}}{a^2x^4+2ax^3+x^2}\,dx + \int \frac{ax\sqrt{-a^2x^2+1}}{a^2x^4+2ax^3+x^2}\,dx + \int \frac{a^2x^2\sqrt{-a^2x^2+1}}{a^2x^4+2ax^3+x^2}\,dx + \int -\frac{a^3x^3\sqrt{-a^2x^2+1}}{a^2x^4+2ax^3+x^2}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] c*(Integral(-sqrt(-a**2*x**2 + 1)/(a**2*x**4 + 2*a*x**3 + x**2), x) + Integ
ral(a*x*sqrt(-a**2*x**2 + 1)/(a**2*x**4 + 2*a*x**3 + x**2), x) + Integral(a
**2*x**2*sqrt(-a**2*x**2 + 1)/(a**2*x**4 + 2*a*x**3 + x**2), x) + Integral(
-a**3*x**3*sqrt(-a**2*x**2 + 1)/(a**2*x**4 + 2*a*x**3 + x**2), x))/a**2
```

Giac [A] time = 1.25703, size = 176, normalized size = 2.38

$$-\frac{a^2cx}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|} - \frac{3\,c\arcsin(ax)\operatorname{sgn}(a)}{|a|} - \frac{3\,c\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|} - \frac{\sqrt{-a^2x^2+1}c}{a} + \frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)c}{2\,a^2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*a^2*c*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - 3*c*arcsin(a*x)*sgn
(a)/abs(a) - 3*c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x
)))/abs(a) - sqrt(-a^2*x^2 + 1)*c/a + 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c
/(a^2*x*abs(a))
```

3.682 $\int \frac{e^{-3\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=97

$$\frac{(1-ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}}-\frac{2(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{ac}-\frac{3\sin^{-1}(ax)}{ac}$$

```
[Out] (1 - a*x)^3/(3*a*c*(1 - a^2*x^2)^(3/2)) - (2*(1 - a*x)^2)/(a*c*Sqrt[1 - a^2
*x^2]) - (3*Sqrt[1 - a^2*x^2])/(a*c) - (3*ArcSin[a*x])/(a*c)
```

Rubi [A] time = 0.233686, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6157, 6149, 1635, 21, 669, 641, 216}

$$\frac{(1-ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}}-\frac{2(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{ac}-\frac{3\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] (1 - a*x)^3/(3*a*c*(1 - a^2*x^2)^(3/2)) - (2*(1 - a*x)^2)/(a*c*Sqrt[1 - a^2
*x^2]) - (3*Sqrt[1 - a^2*x^2])/(a*c) - (3*ArcSin[a*x])/(a*c)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\frac{a^2\int \frac{e^{-3\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}\\
&= -\frac{a^2\int \frac{x^2(1-ax)^3}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c}\\
&= \frac{(1-ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}}+\frac{a^2\int \frac{\left(\frac{3}{a^2}-\frac{3x}{a}\right)(1-ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c}\\
&= \frac{(1-ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}}+\frac{\int \frac{(1-ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c}\\
&= \frac{(1-ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}}-\frac{2(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{3\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= \frac{(1-ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}}-\frac{2(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{ac}-\frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= \frac{(1-ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}}-\frac{2(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{ac}-\frac{3\sin^{-1}(ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.0794545, size = 78, normalized size = 0.8

$$\frac{3a^3x^3+16a^2x^2-9(ax+1)\sqrt{1-a^2x^2}\sin^{-1}(ax)-5ax-14}{3ac(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] (-14 - 5*a*x + 16*a^2*x^2 + 3*a^3*x^3 - 9*(1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSi
n[a*x])/(3*a*c*(1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.062, size = 330, normalized size = 3.4

$$\frac{1}{6\,a^{5}c\left(x+a^{-1}\right)^{4}}\left(-a^{2}\left(x+a^{-1}\right)^{2}+2\,a\left(x+a^{-1}\right)\right)^{\frac{5}{2}}-\frac{11}{12\,a^{4}c\left(x+a^{-1}\right)^{3}}\left(-a^{2}\left(x+a^{-1}\right)^{2}+2\,a\left(x+a^{-1}\right)\right)^{\frac{5}{2}}-\frac{47}{24\,a^{3}c\left(x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2),x)
```

```
[Out] 1/6/a^5/c/(x+1/a)^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-11/12/a^4/c/(x+1/a)^
3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-47/24/a^3/c/(x+1/a)^2*(-a^2*(x+1/a)^2+
2*a*(x+1/a))^(5/2)-95/48/c/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)-95/32/c*(-a
^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-95/32/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/
(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))+1/48/a/c*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(
3/2)-1/32/c*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x-1/32/c/(a^2)^(1/2)*arctan(
(a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{(ax+1)^{3}\left(c-\frac{c}{a^{2}x^{2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))), x)
```

Fricas [A] time = 2.15619, size = 238, normalized size = 2.45

$$-\frac{14\,a^{2}x^{2}+28\,ax-18\left(a^{2}x^{2}+2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)+\left(3\,a^{2}x^{2}+19\,ax+14\right)\sqrt{-a^{2}x^{2}+1}+14}{3\left(a^{3}cx^{2}+2\,a^{2}cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2),x, algorithm="fricas
")
```

```
[Out] -1/3*(14*a^2*x^2 + 28*a*x - 18*(a^2*x^2 + 2*a*x + 1)*arctan((sqrt(-a^2*x^2
+ 1) - 1)/(a*x)) + (3*a^2*x^2 + 19*a*x + 14)*sqrt(-a^2*x^2 + 1) + 14)/(a^3*
c*x^2 + 2*a^2*c*x + a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^{2}\left(\int \frac{x^{2}\sqrt{-a^{2}x^{2}+1}}{a^{5}x^{5}+3a^{4}x^{4}+2a^{3}x^{3}-2a^{2}x^{2}-3ax-1}\,dx+\int -\frac{a^{2}x^{4}\sqrt{-a^{2}x^{2}+1}}{a^{5}x^{5}+3a^{4}x^{4}+2a^{3}x^{3}-2a^{2}x^{2}-3ax-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*(Integral(x**2*sqrt(-a**2*x**2 + 1)/(a**5*x**5 + 3*a**4*x**4 + 2*a**3*
x**3 - 2*a**2*x**2 - 3*a*x - 1), x) + Integral(-a**2*x**4*sqrt(-a**2*x**2 +
 1)/(a**5*x**5 + 3*a**4*x**4 + 2*a**3*x**3 - 2*a**2*x**2 - 3*a*x - 1), x))/
c
```

Giac [A] time = 1.24862, size = 171, normalized size = 1.76

$$-\frac{3\arcsin(ax)\operatorname{sgn}(a)}{c|a|}-\frac{\sqrt{-a^2x^2+1}}{ac}+\frac{2\left(\frac{24\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}+\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+11\right)}{3c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] -3*arcsin(a*x)*sgn(a)/(c*abs(a)) - sqrt(-a^2*x^2 + 1)/(a*c) + 2/3*(24*(sqrt
(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a
^4*x^2) + 11)/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)^3*abs(a))
```

3.683 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=129

$$-\frac{(1-ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}}+\frac{6(1-ax)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}}-\frac{24(1-ax)}{5ac^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{3\sin^{-1}(ax)}{ac^2}$$

```
[Out] -(1 - a*x)^3/(5*a*c^2*(1 - a^2*x^2)^(5/2)) + (6*(1 - a*x)^2)/(5*a*c^2*(1 -
a^2*x^2)^(3/2)) - (24*(1 - a*x))/(5*a*c^2*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2
*x^2]/(a*c^2) - (3*ArcSin[a*x])/(a*c^2)
```

Rubi [A] time = 0.363405, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6157, 6149, 1635, 641, 216}

$$-\frac{(1-ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}}+\frac{6(1-ax)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}}-\frac{24(1-ax)}{5ac^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{3\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] -(1 - a*x)^3/(5*a*c^2*(1 - a^2*x^2)^(5/2)) + (6*(1 - a*x)^2)/(5*a*c^2*(1 -
a^2*x^2)^(3/2)) - (24*(1 - a*x))/(5*a*c^2*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2
*x^2]/(a*c^2) - (3*ArcSin[a*x])/(a*c^2)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx &= \frac{a^4\int \frac{e^{-3\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^2}\,dx}{c^2}\\
&= \frac{a^4\int \frac{x^4(1-ax)^3}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^2}\\
&= -\frac{(1-ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}}-\frac{a^4\int \frac{(1-ax)^2\left(\frac{3}{a^4}-\frac{5x}{a^3}+\frac{5x^2}{a^2}-\frac{5x^3}{a}\right)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^2}\\
&= -\frac{(1-ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}}+\frac{6(1-ax)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}}+\frac{a^4\int \frac{(1-ax)\left(\frac{27}{a^4}-\frac{30x}{a^3}+\frac{15x^2}{a^2}\right)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^2}\\
&= -\frac{(1-ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}}+\frac{6(1-ax)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}}-\frac{24(1-ax)}{5ac^2\sqrt{1-a^2x^2}}-\frac{a^4\int \frac{\frac{45}{a^4}-\frac{15x}{a^3}}{\sqrt{1-a^2x^2}}\,dx}{15c^2}\\
&= -\frac{(1-ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}}+\frac{6(1-ax)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}}-\frac{24(1-ax)}{5ac^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^2}\\
&= -\frac{(1-ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}}+\frac{6(1-ax)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}}-\frac{24(1-ax)}{5ac^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{3\sin^{-1}(ax)}{ac^2}
\end{aligned}$$

Mathematica [A] time = 0.101475, size = 86, normalized size = 0.67

$$\frac{5a^4x^4+34a^3x^3+18a^2x^2-15(ax+1)^2\sqrt{1-a^2x^2}\sin^{-1}(ax)-33ax-24}{5a\sqrt{1-a^2x^2}(acx+c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] (-24 - 33*a*x + 18*a^2*x^2 + 34*a^3*x^3 + 5*a^4*x^4 - 15*(1 + a*x)^2*Sqrt[1
 - a^2*x^2]*ArcSin[a*x])/(5*a*(c + a*c*x)^2*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.066, size = 412, normalized size = 3.2

$$-\frac{1}{32a^3c^2}\left(-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)\right)^{\frac{5}{2}}\left(x-a^{-1}\right)^{-2}+\frac{3x}{128c^2}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}+\frac{3}{128c^2}\arctan\left(x\sqrt{a^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -1/32/a^3/c^2/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)+3/128/c^2*(-a^2*
(x-1/a)^2-2*a*(x-1/a))^(1/2)*x+3/128/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(
-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))+1/4/a^5/c^2/(x+1/a)^4*(-a^2*(x+1/a)^2+2*
a*(x+1/a))^(5/2)-15/16/a^4/c^2/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)
-2/a^3/c^2/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-387/128/c^2*(-a^2*(
x+1/a)^2+2*a*(x+1/a))^(1/2)*x-387/128/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/
(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-1/20/a^6/c^2/(x+1/a)^5*(-a^2*(x+1/a)^2+
2*a*(x+1/a))^(5/2)-1/64/a/c^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)-129/64/a/c
^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^2), x)
```

Fricas [A] time = 2.36615, size = 319, normalized size = 2.47

$$-\frac{24\,a^3x^3+72\,a^2x^2+72\,ax-30\left(a^3x^3+3\,a^2x^2+3\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(5\,a^3x^3+39\,a^2x^2+57\,ax+24\right)\sqrt{-}}{5\left(a^4c^2x^3+3\,a^3c^2x^2+3\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="fric
as")
```

```
[Out] -1/5*(24*a^3*x^3 + 72*a^2*x^2 + 72*a*x - 30*(a^3*x^3 + 3*a^2*x^2 + 3*a*x +
1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (5*a^3*x^3 + 39*a^2*x^2 + 57*a*
x + 24)*sqrt(-a^2*x^2 + 1) + 24)/(a^4*c^2*x^3 + 3*a^3*c^2*x^2 + 3*a^2*c^2*x
 + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^4\left(\int \frac{x^4\sqrt{-a^2x^2+1}}{a^7x^7+3a^6x^6+a^5x^5-5a^4x^4-5a^3x^3+a^2x^2+3ax+1}\,dx+\int -\frac{a^2x^6\sqrt{-a^2x^2+1}}{a^7x^7+3a^6x^6+a^5x^5-5a^4x^4-5a^3x^3+a^2x^2+3ax+1}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*(Integral(x**4*sqrt(-a**2*x**2 + 1)/(a**7*x**7 + 3*a**6*x**6 + a**5*x*
*5 - 5*a**4*x**4 - 5*a**3*x**3 + a**2*x**2 + 3*a*x + 1), x) + Integral(-a**
2*x**6*sqrt(-a**2*x**2 + 1)/(a**7*x**7 + 3*a**6*x**6 + a**5*x**5 - 5*a**4*x
**4 - 5*a**3*x**3 + a**2*x**2 + 3*a*x + 1), x))/c**2
```

Giac [A] time = 1.21655, size = 244, normalized size = 1.89

$$-\frac{3\arcsin(ax)\,\mathrm{sgn}(a)}{c^2|a|}-\frac{\sqrt{-a^2x^2+1}}{ac^2}+\frac{2\left(\frac{80\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}+\frac{120\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{70\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}+\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4}+\right.}{5c^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="giac
")
```

```
[Out] -3*arcsin(a*x)*sgn(a)/(c^2*abs(a)) - sqrt(-a^2*x^2 + 1)/(a*c^2) + 2/5*(80*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 120*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)^2/(a^4*x^2) + 70*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) + 15*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 19)/(c^2*((sqrt(-a^2*x^2 + 1)*abs(a)
 + a)/(a^2*x) + 1)^5*abs(a))
```

3.684 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=159

$$\frac{(1-ax)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}}-\frac{38(1-ax)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}}+\frac{137(1-ax)}{35ac^3\left(1-a^2x^2\right)^{3/2}}-\frac{\sqrt{1-a^2x^2}}{ac^3}-\frac{245-181ax}{35ac^3\sqrt{1-a^2x^2}}-\frac{3\sin^{-1}(ax)}{ac^3}$$

```
[Out] (1 - a*x)^3/(7*a*c^3*(1 - a^2*x^2)^(7/2)) - (38*(1 - a*x)^2)/(35*a*c^3*(1 -
 a^2*x^2)^(5/2)) + (137*(1 - a*x))/(35*a*c^3*(1 - a^2*x^2)^(3/2)) - (245 -
181*a*x)/(35*a*c^3*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(a*c^3) - (3*ArcS
in[a*x])/(a*c^3)
```

Rubi [A] time = 0.450863, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6157, 6149, 1635, 1814, 641, 216}

$$\frac{(1-ax)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}}-\frac{38(1-ax)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}}+\frac{137(1-ax)}{35ac^3\left(1-a^2x^2\right)^{3/2}}-\frac{\sqrt{1-a^2x^2}}{ac^3}-\frac{245-181ax}{35ac^3\sqrt{1-a^2x^2}}-\frac{3\sin^{-1}(ax)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] (1 - a*x)^3/(7*a*c^3*(1 - a^2*x^2)^(7/2)) - (38*(1 - a*x)^2)/(35*a*c^3*(1 -
 a^2*x^2)^(5/2)) + (137*(1 - a*x))/(35*a*c^3*(1 - a^2*x^2)^(3/2)) - (245 -
181*a*x)/(35*a*c^3*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(a*c^3) - (3*ArcS
in[a*x])/(a*c^3)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{a^6\int \frac{e^{-3\tanh^{-1}(ax)}x^6}{\left(1-a^2x^2\right)^3}\,dx}{c^3}$$

$$= -\frac{a^6\int \frac{x^6(1-ax)^3}{\left(1-a^2x^2\right)^{9/2}}\,dx}{c^3}$$

$$= \frac{(1-ax)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}} + \frac{a^6\int \frac{(1-ax)^2\left(\frac{3}{a^6}-\frac{7x}{a^5}+\frac{7x^2}{a^4}-\frac{7x^3}{a^3}+\frac{7x^4}{a^2}-\frac{7x^5}{a}\right)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{7c^3}$$

$$= \frac{(1-ax)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}} - \frac{38(1-ax)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{a^6\int \frac{(1-ax)\left(\frac{61}{a^6}-\frac{140x}{a^5}+\frac{105x^2}{a^4}-\frac{70x^3}{a^3}+\frac{35x^4}{a^2}\right)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{35c^3}$$

$$= \frac{(1-ax)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}} - \frac{38(1-ax)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}} + \frac{137(1-ax)}{35ac^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^6\int \frac{\frac{228}{a^6}-\frac{630x}{a^5}+\frac{315x^2}{a^4}-\frac{105x^3}{a^3}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{105c^3}$$

$$= \frac{(1-ax)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}} - \frac{38(1-ax)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}} + \frac{137(1-ax)}{35ac^3\left(1-a^2x^2\right)^{3/2}} - \frac{245-181ax}{35ac^3\sqrt{1-a^2x^2}} - \frac{a^6\int \frac{\frac{315}{a^6}-\frac{105x}{a^5}}{\sqrt{1-a^2x^2}}\,dx}{105c^3}$$

$$= \frac{(1-ax)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}} - \frac{38(1-ax)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}} + \frac{137(1-ax)}{35ac^3\left(1-a^2x^2\right)^{3/2}} - \frac{245-181ax}{35ac^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{ac^3} - \frac{3}{}$$

$$= \frac{(1-ax)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}} - \frac{38(1-ax)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}} + \frac{137(1-ax)}{35ac^3\left(1-a^2x^2\right)^{3/2}} - \frac{245-181ax}{35ac^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{ac^3} - \frac{3\,\mathrm{s}}{}$$

Mathematica [A] time = 0.126437, size = 94, normalized size = 0.59

$$\frac{35a^5x^5+286a^4x^4+368a^3x^3-125a^2x^2-105(ax+1)^3\sqrt{1-a^2x^2}\sin^{-1}(ax)-423ax-176}{35a\sqrt{1-a^2x^2}(acx+c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] (-176 - 423*a*x - 125*a^2*x^2 + 368*a^3*x^3 + 286*a^4*x^4 + 35*a^5*x^5 - 10
5*(1 + a*x)^3*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(35*a*(c + a*c*x)^3*Sqrt[1 - a
^2*x^2])
```

Maple [B] time = 0.076, size = 494, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^3,x)
```

```
[Out] 1/64/a^4/c^3/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)-5/64/a^3/c^3/(x-1
/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)+51/512/c^3*(-a^2*(x-1/a)^2-2*a*(x-
1/a))^(1/2)*x+51/512/c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2
*a*(x-1/a))^(1/2))-17/256/a/c^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+1/56/a^7
/c^3/(x+1/a)^6*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-61/560/a^6/c^3/(x+1/a)^5*
(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)+5/16/a^5/c^3/(x+1/a)^4*(-a^2*(x+1/a)^2+2
*a*(x+1/a))^(5/2)-31/32/a^4/c^3/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2
)-263/128/a^3/c^3/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-1587/512/c^3
*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-1587/512/c^3/(a^2)^(1/2)*arctan((a^2)
^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-529/256/a/c^3*(-a^2*(x+1/a)^2+
2*a*(x+1/a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^3), x)
```

Fricas [A] time = 2.63076, size = 483, normalized size = 3.04

$$-\frac{176\,a^5x^5+528\,a^4x^4+352\,a^3x^3-352\,a^2x^2-528\,ax-210\left(a^5x^5+3\,a^4x^4+2\,a^3x^3-2\,a^2x^2-3\,ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2 \cdots}}{a \cdots}\right)\cdots}{35\left(a^6c^3x^5+3\,a^5c^3x^4+2\,a^4c^3x^3-2\,a^3c^3x^2-\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="fric
as")
```

```
[Out] -1/35*(176*a^5*x^5 + 528*a^4*x^4 + 352*a^3*x^3 - 352*a^2*x^2 - 528*a*x - 21
0*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 - 3*a*x - 1)*arctan((sqrt(-a
^2*x^2 + 1) - 1)/(a*x)) + (35*a^5*x^5 + 286*a^4*x^4 + 368*a^3*x^3 - 125*a^2
*x^2 - 423*a*x - 176)*sqrt(-a^2*x^2 + 1) - 176)/(a^6*c^3*x^5 + 3*a^5*c^3*x^
4 + 2*a^4*c^3*x^3 - 2*a^3*c^3*x^2 - 3*a^2*c^3*x - a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^6\left(\int \frac{x^6\sqrt{-a^2x^2+1}}{a^9x^9+3a^8x^8-8a^6x^6-6a^5x^5+6a^4x^4+8a^3x^3-3ax-1}\,dx+\int -\frac{a^2x^8\sqrt{-a^2x^2+1}}{a^9x^9+3a^8x^8-8a^6x^6-6a^5x^5+6a^4x^4+8a^3x^3-3ax-1}\,dx\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*(Integral(x**6*sqrt(-a**2*x**2 + 1)/(a**9*x**9 + 3*a**8*x**8 - 8*a**6*
x**6 - 6*a**5*x**5 + 6*a**4*x**4 + 8*a**3*x**3 - 3*a*x - 1), x) + Integral(
-a**2*x**8*sqrt(-a**2*x**2 + 1)/(a**9*x**9 + 3*a**8*x**8 - 8*a**6*x**6 - 6*
a**5*x**5 + 6*a**4*x**4 + 8*a**3*x**3 - 3*a*x - 1), x))/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^3), x)
```

3.685 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=189

$$-\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}}+\frac{22(1-ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}}-\frac{478(1-ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}}-\frac{\sqrt{1-a^2x^2}}{ac^4}-\frac{4(630-431ax)}{315ac^4\sqrt{1-a^2x^2}}+\frac{2(1155-829ax)}{315ac^4\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] -(1 - a*x)^3/(9*a*c^4*(1 - a^2*x^2)^(9/2)) + (22*(1 - a*x)^2)/(21*a*c^4*(1
- a^2*x^2)^(7/2)) - (478*(1 - a*x))/(105*a*c^4*(1 - a^2*x^2)^(5/2)) + (2*(1
155 - 829*a*x))/(315*a*c^4*(1 - a^2*x^2)^(3/2)) - (4*(630 - 431*a*x))/(315*
a*c^4*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(a*c^4) - (3*ArcSin[a*x])/(a*c
^4)
```

Rubi [A] time = 0.636255, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6157, 6149, 1635, 1814, 641, 216}

$$-\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}}+\frac{22(1-ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}}-\frac{478(1-ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}}-\frac{\sqrt{1-a^2x^2}}{ac^4}-\frac{4(630-431ax)}{315ac^4\sqrt{1-a^2x^2}}+\frac{2(1155-829ax)}{315ac^4\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] -(1 - a*x)^3/(9*a*c^4*(1 - a^2*x^2)^(9/2)) + (22*(1 - a*x)^2)/(21*a*c^4*(1
- a^2*x^2)^(7/2)) - (478*(1 - a*x))/(105*a*c^4*(1 - a^2*x^2)^(5/2)) + (2*(1
155 - 829*a*x))/(315*a*c^4*(1 - a^2*x^2)^(3/2)) - (4*(630 - 431*a*x))/(315*
a*c^4*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(a*c^4) - (3*ArcSin[a*x])/(a*c
^4)
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{e^{-3\tanh^{-1}(ax)}x^8}{\left(1-a^2x^2\right)^4}\,dx}{c^4}$$

$$= \frac{a^8\int \frac{x^8(1-ax)^3}{\left(1-a^2x^2\right)^{11/2}}\,dx}{c^4}$$

$$= -\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} - \frac{a^8\int \frac{(1-ax)^2\left(\frac{3}{a^8}-\frac{9x}{a^7}+\frac{9x^2}{a^6}-\frac{9x^3}{a^5}+\frac{9x^4}{a^4}-\frac{9x^5}{a^3}+\frac{9x^6}{a^2}-\frac{9x^7}{a}\right)}{\left(1-a^2x^2\right)^{9/2}}\,dx}{9c^4}$$

$$= -\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} + \frac{22(1-ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{a^8\int \frac{(1-ax)\left(\frac{111}{a^8}-\frac{378x}{a^7}+\frac{315x^2}{a^6}-\frac{252x^3}{a^5}+\frac{189x^4}{a^4}-\frac{126x^5}{a^3}+\frac{63x^6}{a^2}\right)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{63c^4}$$

$$= -\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} + \frac{22(1-ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} - \frac{478(1-ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{a^8\int \frac{\frac{879}{a^8}-\frac{4725x}{a^7}+\frac{3150x^2}{a^6}-\frac{1890x^3}{a^5}+\frac{945x^4}{a^4}-\cdots}{\left(1-a^2x^2\right)^{5/2}}\,dx}{315c^4}$$

$$= -\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} + \frac{22(1-ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} - \frac{478(1-ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}} + \frac{2(1155-829ax)}{315ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{a^8\int \frac{\frac{2337}{a^8}\cdots}{\cdots}}{\cdots}$$

$$= -\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} + \frac{22(1-ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} - \frac{478(1-ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}} + \frac{2(1155-829ax)}{315ac^4\left(1-a^2x^2\right)^{3/2}} - \frac{4(630-\cdots}{315ac^4\sqrt{\cdots}}$$

$$= -\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} + \frac{22(1-ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} - \frac{478(1-ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}} + \frac{2(1155-829ax)}{315ac^4\left(1-a^2x^2\right)^{3/2}} - \frac{4(630-\cdots}{315ac^4\sqrt{\cdots}}$$

$$= -\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} + \frac{22(1-ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} - \frac{478(1-ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}} + \frac{2(1155-829ax)}{315ac^4\left(1-a^2x^2\right)^{3/2}} - \frac{4(630-\cdots}{315ac^4\sqrt{\cdots}}$$

Mathematica [A] time = 0.148053, size = 122, normalized size = 0.65

$$\frac{315a^7x^7 + 2669a^6x^6 + 2967a^5x^5 - 4029a^4x^4 - 7399a^3x^3 - 339a^2x^2 - 945(ax-1)(ax+1)^4\sqrt{1-a^2x^2}\sin^{-1}(ax) + 404}{315a(ax-1)\sqrt{1-a^2x^2}(acx+c)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] (1664 + 4047*a*x - 339*a^2*x^2 - 7399*a^3*x^3 - 4029*a^4*x^4 + 2967*a^5*x^5
 + 2669*a^6*x^6 + 315*a^7*x^7 - 945*(-1 + a*x)*(1 + a*x)^4*Sqrt[1 - a^2*x^2
]*ArcSin[a*x])/(315*a*(-1 + a*x)*(c + a*c*x)^4*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.095, size = 576, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^4,x)
```

```
[Out] -1723/10080/a^6/c^4/(x+1/a)^5*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-31/256/a/c
^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+1/384/a^5/c^4/(x-1/a)^4*(-a^2*(x-1/a)
^2-2*a*(x-1/a))^(5/2)-1/144/a^8/c^4/(x+1/a)^7*(-a^2*(x+1/a)^2+2*a*(x+1/a))^
(5/2)+13/252/a^7/c^4/(x+1/a)^6*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)+35/96/a^5
/c^4/(x+1/a)^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-769/768/a^4/c^4/(x+1/a)^3
*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-811/384/a^3/c^4/(x+1/a)^2*(-a^2*(x+1/a)
^2+2*a*(x+1/a))^(5/2)-1629/512/c^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-162
9/512/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/
2))+29/768/a^4/c^4/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)-25/192/a^3/
c^4/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)+93/512/c^4*(-a^2*(x-1/a)^2
-2*a*(x-1/a))^(1/2)*x+93/512/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-
1/a)^2-2*a*(x-1/a))^(1/2))-543/256/a/c^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^4), x)
```

Fricas [A] time = 2.63303, size = 647, normalized size = 3.42

$$-\frac{1664\,a^7x^7 + 4992\,a^6x^6 + 1664\,a^5x^5 - 8320\,a^4x^4 - 8320\,a^3x^3 + 1664\,a^2x^2 + 4992\,ax - 1890\left(a^7x^7 + 3\,a^6x^6 + a^5x^5 - 5\right.}{315\left(a^8c^4x^7 + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="fric
as")
```

```
[Out] -1/315*(1664*a^7*x^7 + 4992*a^6*x^6 + 1664*a^5*x^5 - 8320*a^4*x^4 - 8320*a^
3*x^3 + 1664*a^2*x^2 + 4992*a*x - 1890*(a^7*x^7 + 3*a^6*x^6 + a^5*x^5 - 5*a
^4*x^4 - 5*a^3*x^3 + a^2*x^2 + 3*a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(
a*x)) + (315*a^7*x^7 + 2669*a^6*x^6 + 2967*a^5*x^5 - 4029*a^4*x^4 - 7399*a^
3*x^3 - 339*a^2*x^2 + 4047*a*x + 1664)*sqrt(-a^2*x^2 + 1) + 1664)/(a^8*c^4*
x^7 + 3*a^7*c^4*x^6 + a^6*c^4*x^5 - 5*a^5*c^4*x^4 - 5*a^4*c^4*x^3 + a^3*c^4
*x^2 + 3*a^2*c^4*x + a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^4), x)
```

3.686 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal. Leaf size=374

$$\frac{a^9x^{10}\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{4a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{2a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{2a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{2\left(1-a^2x^2\right)^{9/2}}+\frac{4a^3x^4\left(c-\right.}{5\left(1-a^2\right.}$$

```
[Out] -((c - c/(a^2*x^2))^(9/2)*x)/(8*(1 - a^2*x^2)^(9/2)) - (a*(c - c/(a^2*x^2))
^(9/2)*x^2)/(7*(1 - a^2*x^2)^(9/2)) + (2*a^2*(c - c/(a^2*x^2))^(9/2)*x^3)/(
3*(1 - a^2*x^2)^(9/2)) + (4*a^3*(c - c/(a^2*x^2))^(9/2)*x^4)/(5*(1 - a^2*x^
2)^(9/2)) - (3*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(2*(1 - a^2*x^2)^(9/2)) - (
2*a^5*(c - c/(a^2*x^2))^(9/2)*x^6)/(1 - a^2*x^2)^(9/2) + (2*a^6*(c - c/(a^2
*x^2))^(9/2)*x^7)/(1 - a^2*x^2)^(9/2) + (4*a^7*(c - c/(a^2*x^2))^(9/2)*x^8)
/(1 - a^2*x^2)^(9/2) + (a^9*(c - c/(a^2*x^2))^(9/2)*x^10)/(1 - a^2*x^2)^(9/
2) + (a^8*(c - c/(a^2*x^2))^(9/2)*x^9*Log[x])/(1 - a^2*x^2)^(9/2)
```

Rubi [A] time = 0.204493, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 88}

$$\frac{a^9x^{10}\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{4a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{2a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{2a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{2\left(1-a^2x^2\right)^{9/2}}+\frac{4a^3x^4\left(c-\right.}{5\left(1-a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] -((c - c/(a^2*x^2))^(9/2)*x)/(8*(1 - a^2*x^2)^(9/2)) - (a*(c - c/(a^2*x^2))
^(9/2)*x^2)/(7*(1 - a^2*x^2)^(9/2)) + (2*a^2*(c - c/(a^2*x^2))^(9/2)*x^3)/(
3*(1 - a^2*x^2)^(9/2)) + (4*a^3*(c - c/(a^2*x^2))^(9/2)*x^4)/(5*(1 - a^2*x^
2)^(9/2)) - (3*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(2*(1 - a^2*x^2)^(9/2)) - (
2*a^5*(c - c/(a^2*x^2))^(9/2)*x^6)/(1 - a^2*x^2)^(9/2) + (2*a^6*(c - c/(a^2
*x^2))^(9/2)*x^7)/(1 - a^2*x^2)^(9/2) + (4*a^7*(c - c/(a^2*x^2))^(9/2)*x^8)
/(1 - a^2*x^2)^(9/2) + (a^9*(c - c/(a^2*x^2))^(9/2)*x^10)/(1 - a^2*x^2)^(9/
2) + (a^8*(c - c/(a^2*x^2))^(9/2)*x^9*Log[x])/(1 - a^2*x^2)^(9/2)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{9/2}}{x^9}dx}{\left(1-a^2x^2\right)^{9/2}}$$

$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^4(1+ax)^5}{x^9}dx}{\left(1-a^2x^2\right)^{9/2}}$$

$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\left(a^9+\frac{1}{x^9}+\frac{a}{x^8}-\frac{4a^2}{x^7}-\frac{4a^3}{x^6}+\frac{6a^4}{x^5}+\frac{6a^5}{x^4}-\frac{4a^6}{x^3}-\frac{4a^7}{x^2}+\frac{a^8}{x}\right)dx}{\left(1-a^2x^2\right)^{9/2}}$$

$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x}{8\left(1-a^2x^2\right)^{9/2}}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{7\left(1-a^2x^2\right)^{9/2}}+\frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3}{3\left(1-a^2x^2\right)^{9/2}}+\frac{4a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4}{5\left(1-a^2x^2\right)^{9/2}}-\frac{3a^4\left(c\right.}{2\left(1\right.}$$

Mathematica [A] time = 0.101706, size = 114, normalized size = 0.3

$$\frac{x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(\frac{2a^6}{x^2}-\frac{2a^5}{x^3}-\frac{3a^4}{2x^4}+\frac{4a^3}{5x^5}+\frac{2a^2}{3x^6}+a^9x+\frac{4a^7}{x}+a^8\log(x)-\frac{a}{7x^7}-\frac{1}{8x^8}\right)}{\left(1-a^2x^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(9/2)*x^9*(-1/(8*x^8) - a/(7*x^7) + (2*a^2)/(3*x^6) + (4
*a^3)/(5*x^5) - (3*a^4)/(2*x^4) - (2*a^5)/x^3 + (2*a^6)/x^2 + (4*a^7)/x + a
^9*x + a^8*Log[x]))/(1 - a^2*x^2)^(9/2)
```

Maple [A] time = 0.213, size = 118, normalized size = 0.3

$$-\frac{x\left(840\,a^9x^9+840\,a^8\ln(x)\,x^8+3360\,a^7x^7+1680\,x^6a^6-1680\,x^5a^5-1260\,x^4a^4+672\,x^3a^3+560\,a^2x^2-120\,ax-105\right)}{840\,\left(a^2x^2-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(9/2),x)
```

```
[Out] -1/840*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/(a^2*x^2-1)^5*(-a^2*x^2+1)^(1/2)*(84
0*a^9*x^9+840*a^8*ln(x)*x^8+3360*a^7*x^7+1680*x^6*a^6-1680*x^5*a^5-1260*x^4
*a^4+672*x^3*a^3+560*a^2*x^2-120*a*x-105)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(9/2)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [A] time = 2.59853, size = 1355, normalized size = 3.62

$$\left[\frac{420\left(a^{9}c^{4}x^{9}-a^{7}c^{4}x^{7}\right)\sqrt{-c}\log\left(\frac{a^{2}cx^{6}+a^{2}cx^{2}-cx^{4}+\left(ax^{5}-ax\right)\sqrt{-a^{2}x^{2}+1}\sqrt{-c}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c}{a^{2}x^{4}-x^{2}}\right)-\left(840\,a^{9}c^{4}x^{9}+3360\,a^{7}c^{4}x^{7}+1680\,a^{6}c^{4}\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="fric
as")
```

```
[Out] [1/840*(420*(a^9*c^4*x^9 - a^7*c^4*x^7)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2
 - c*x^4 + (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)) - c)/(a^2*x^4 - x^2)) - (840*a^9*c^4*x^9 + 3360*a^7*c^4*x^7 + 168
0*a^6*c^4*x^6 - 1680*a^5*c^4*x^5 - (840*a^9 + 3360*a^7 + 1680*a^6 - 1680*a^
5 - 1260*a^4 + 672*a^3 + 560*a^2 - 120*a - 105)*c^4*x^8 - 1260*a^4*c^4*x^4
+ 672*a^3*c^4*x^3 + 560*a^2*c^4*x^2 - 120*a*c^4*x - 105*c^4)*sqrt(-a^2*x^2
+ 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^10*x^9 - a^8*x^7), -1/840*(840*(a^
9*c^4*x^9 - a^7*c^4*x^7)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sq
rt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) +
(840*a^9*c^4*x^9 + 3360*a^7*c^4*x^7 + 1680*a^6*c^4*x^6 - 1680*a^5*c^4*x^5 -
 (840*a^9 + 3360*a^7 + 1680*a^6 - 1680*a^5 - 1260*a^4 + 672*a^3 + 560*a^2 -
 120*a - 105)*c^4*x^8 - 1260*a^4*c^4*x^4 + 672*a^3*c^4*x^3 + 560*a^2*c^4*x^
2 - 120*a*c^4*x - 105*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)))/(a^10*x^9 - a^8*x^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{9}{2}}}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(9/2)/sqrt(-a^2*x^2 + 1), x)
```

3.687 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=299

$$-\frac{a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}-\frac{3a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}-\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{2\left(1-a^2x^2\right)^{7/2}}+\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{3a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{4\left(1-a^2x^2\right)^{7/2}}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\cdots\right.}{5\left(1-a^2x^2\cdots\right.}$$

```
[Out] -((c - c/(a^2*x^2))^(7/2)*x)/(6*(1 - a^2*x^2)^(7/2)) - (a*(c - c/(a^2*x^2))
^(7/2)*x^2)/(5*(1 - a^2*x^2)^(7/2)) + (3*a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(
4*(1 - a^2*x^2)^(7/2)) + (a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(1 - a^2*x^2)^(7
/2) - (3*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(2*(1 - a^2*x^2)^(7/2)) - (3*a^5*
(c - c/(a^2*x^2))^(7/2)*x^6)/(1 - a^2*x^2)^(7/2) - (a^7*(c - c/(a^2*x^2))^(
7/2)*x^8)/(1 - a^2*x^2)^(7/2) - (a^6*(c - c/(a^2*x^2))^(7/2)*x^7*Log[x])/(1
 - a^2*x^2)^(7/2)
```

Rubi [A] time = 0.190384, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 88}

$$-\frac{a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}-\frac{3a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}-\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{2\left(1-a^2x^2\right)^{7/2}}+\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{3a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{4\left(1-a^2x^2\right)^{7/2}}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\cdots\right.}{5\left(1-a^2x^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] -((c - c/(a^2*x^2))^(7/2)*x)/(6*(1 - a^2*x^2)^(7/2)) - (a*(c - c/(a^2*x^2))
^(7/2)*x^2)/(5*(1 - a^2*x^2)^(7/2)) + (3*a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(
4*(1 - a^2*x^2)^(7/2)) + (a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(1 - a^2*x^2)^(7
/2) - (3*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(2*(1 - a^2*x^2)^(7/2)) - (3*a^5*
(c - c/(a^2*x^2))^(7/2)*x^6)/(1 - a^2*x^2)^(7/2) - (a^7*(c - c/(a^2*x^2))^(
7/2)*x^8)/(1 - a^2*x^2)^(7/2) - (a^6*(c - c/(a^2*x^2))^(7/2)*x^7*Log[x])/(1
 - a^2*x^2)^(7/2)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{e^{\tanh^{-1}(ax)}(1-a^2x^2)^{7/2}}{x^7}dx}{(1-a^2x^2)^{7/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^3(1+ax)^4}{x^7}dx}{(1-a^2x^2)^{7/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\left(-a^7+\frac{1}{x^7}+\frac{a}{x^6}-\frac{3a^2}{x^5}-\frac{3a^3}{x^4}+\frac{3a^4}{x^3}+\frac{3a^5}{x^2}-\frac{a^6}{x}\right)dx}{(1-a^2x^2)^{7/2}}$$
$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}{6(1-a^2x^2)^{7/2}}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{5(1-a^2x^2)^{7/2}}+\frac{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{4(1-a^2x^2)^{7/2}}+\frac{a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{(1-a^2x^2)^{7/2}}-\frac{3a^4\left(c-\right.}{2(1-}$$

Mathematica [A] time = 0.0586676, size = 98, normalized size = 0.33

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(60a^7x^7+180a^5x^5+90a^4x^4-60a^3x^3-45a^2x^2+60a^6x^6\log(x)+12ax+10\right)}{60a^6x^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)]*(10 + 12*a*x - 45*a^2*x^2 - 60*a^3*x^3 + 90*a^4*
x^4 + 180*a^5*x^5 + 60*a^7*x^7 + 60*a^6*x^6*Log[x]))/(60*a^6*x^5*Sqrt[1 - a
^2*x^2])
```

Maple [A] time = 0.164, size = 102, normalized size = 0.3

$$-\frac{x\left(60\,a^7x^7+60\,a^6\ln(x)\,x^6+180\,x^5a^5+90\,x^4a^4-60\,x^3a^3-45\,a^2x^2+12\,ax+10\right)}{60\,(a^2x^2-1)^4}\left(\frac{c\,(a^2x^2-1)}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] -1/60*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a^2*x^2-1)^4*(-a^2*x^2+1)^(1/2)*(60*
a^7*x^7+60*a^6*ln(x)*x^6+180*x^5*a^5+90*x^4*a^4-60*x^3*a^3-45*a^2*x^2+12*a*
x+10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(7/2)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [A] time = 2.56219, size = 1142, normalized size = 3.82

$$\left[\frac{30\left(a^{7}c^{3}x^{7}-a^{5}c^{3}x^{5}\right)\sqrt{-c}\log\left(\frac{a^{2}cx^{6}+a^{2}cx^{2}-cx^{4}+\left(ax^{5}-ax\right)\sqrt{-a^{2}x^{2}+1}\sqrt{-c}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c}{a^{2}x^{4}-x^{2}}\right)-\left(60\,a^{7}c^{3}x^{7}+180\,a^{5}c^{3}x^{5}+90\,a^{4}c^{3}x^{4}-(6}{60\left(a^{8}x^{7}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="fric
as")
```

```
[Out] [1/60*(30*(a^7*c^3*x^7 - a^5*c^3*x^5)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 -
 c*x^4 + (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^
2*x^2)) - c)/(a^2*x^4 - x^2)) - (60*a^7*c^3*x^7 + 180*a^5*c^3*x^5 + 90*a^4*
c^3*x^4 - (60*a^7 + 180*a^5 + 90*a^4 - 60*a^3 - 45*a^2 + 12*a + 10)*c^3*x^6
 - 60*a^3*c^3*x^3 - 45*a^2*c^3*x^2 + 12*a*c^3*x + 10*c^3)*sqrt(-a^2*x^2 + 1
)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^8*x^7 - a^6*x^5), -1/60*(60*(a^7*c^3*
x^7 - a^5*c^3*x^5)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqrt(c)*
sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) + (60*a^
7*c^3*x^7 + 180*a^5*c^3*x^5 + 90*a^4*c^3*x^4 - (60*a^7 + 180*a^5 + 90*a^4 -
 60*a^3 - 45*a^2 + 12*a + 10)*c^3*x^6 - 60*a^3*c^3*x^3 - 45*a^2*c^3*x^2 + 1
2*a*c^3*x + 10*c^3)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^
8*x^7 - a^6*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{7}{2}}}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(7/2)/sqrt(-a^2*x^2 + 1), x)
```

3.688 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=219

$$\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{3\left(1-a^2x^2\right)^{5/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4\left(1-a^2x^2\right)^{5/2}}+\frac{a^4x^5\log(x)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] -((c - c/(a^2*x^2))^(5/2)*x)/(4*(1 - a^2*x^2)^(5/2)) - (a*(c - c/(a^2*x^2))
^(5/2)*x^2)/(3*(1 - a^2*x^2)^(5/2)) + (a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(1
- a^2*x^2)^(5/2) + (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)/(1 - a^2*x^2)^(5/2)
+ (a^5*(c - c/(a^2*x^2))^(5/2)*x^6)/(1 - a^2*x^2)^(5/2) + (a^4*(c - c/(a^2*
x^2))^(5/2)*x^5*Log[x])/(1 - a^2*x^2)^(5/2)
```

Rubi [A] time = 0.178567, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 88}

$$\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{3\left(1-a^2x^2\right)^{5/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4\left(1-a^2x^2\right)^{5/2}}+\frac{a^4x^5\log(x)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] -((c - c/(a^2*x^2))^(5/2)*x)/(4*(1 - a^2*x^2)^(5/2)) - (a*(c - c/(a^2*x^2))
^(5/2)*x^2)/(3*(1 - a^2*x^2)^(5/2)) + (a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(1
- a^2*x^2)^(5/2) + (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)/(1 - a^2*x^2)^(5/2)
+ (a^5*(c - c/(a^2*x^2))^(5/2)*x^6)/(1 - a^2*x^2)^(5/2) + (a^4*(c - c/(a^2*
x^2))^(5/2)*x^5*Log[x])/(1 - a^2*x^2)^(5/2)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{\tanh^{-1}(ax)}(1-a^2x^2)^{5/2}}{x^5}dx}{(1-a^2x^2)^{5/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^2(1+ax)^3}{x^5}dx}{(1-a^2x^2)^{5/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\left(a^5+\frac{1}{x^5}+\frac{a}{x^4}-\frac{2a^2}{x^3}-\frac{2a^3}{x^2}+\frac{a^4}{x}\right)dx}{(1-a^2x^2)^{5/2}}$$
$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}{4(1-a^2x^2)^{5/2}}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{3(1-a^2x^2)^{5/2}}+\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{(1-a^2x^2)^{5/2}}+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-a^2x^2)^{5/2}}+\frac{a^5\left(c-\right.}{(1-}$$

Mathematica [A] time = 0.0462039, size = 82, normalized size = 0.37

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(12a^5x^5+24a^3x^3+12a^2x^2+12a^4x^4\log(x)-4ax-3\right)}{12a^4x^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)]*(-3 - 4*a*x + 12*a^2*x^2 + 24*a^3*x^3 + 12*a^5*x
^5 + 12*a^4*x^4*Log[x]))/(12*a^4*x^3*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.153, size = 86, normalized size = 0.4

$$-\frac{x\left(12x^5a^5+12a^4\ln(x)x^4+24x^3a^3+12a^2x^2-4ax-3\right)}{12\left(a^2x^2-1\right)^3}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] -1/12*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/(a^2*x^2-1)^3*(-a^2*x^2+1)^(1/2)*(12*
x^5*a^5+12*a^4*ln(x)*x^4+24*x^3*a^3+12*a^2*x^2-4*a*x-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\sqrt{-a^2x^2+1}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(5/2)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [A] time = 2.63352, size = 984, normalized size = 4.49

$$\left[\frac{6\left(a^5c^2x^5 - a^3c^2x^3\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right) - \left(12\,a^5c^2x^5 + 24\,a^3c^2x^3 - \left(12\,a^5 + 24\,a^3 + \right.\right.}{12\left(a^6x^5 - a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="fric
as")
```

```
[Out] [1/12*(6*(a^5*c^2*x^5 - a^3*c^2*x^3)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 -
c*x^4 + (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2
*x^2)) - c)/(a^2*x^4 - x^2)) - (12*a^5*c^2*x^5 + 24*a^3*c^2*x^3 - (12*a^5 +
 24*a^3 + 12*a^2 - 4*a - 3)*c^2*x^4 + 12*a^2*c^2*x^2 - 4*a*c^2*x - 3*c^2)*s
qrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5 - a^4*x^3), -1/
12*(12*(a^5*c^2*x^5 - a^3*c^2*x^3)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3
 + a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^
2 + c)) + (12*a^5*c^2*x^5 + 24*a^3*c^2*x^3 - (12*a^5 + 24*a^3 + 12*a^2 - 4*
a - 3)*c^2*x^4 + 12*a^2*c^2*x^2 - 4*a*c^2*x - 3*c^2)*sqrt(-a^2*x^2 + 1)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5 - a^4*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c - \frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(5/2)/sqrt(-a^2*x^2 + 1), x)
```

3.689 $\int e^{\tanh^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^{3/2} \, dx$

Optimal. Leaf size=146

$$-\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2\left(1-a^2x^2\right)^{3/2}}-\frac{a^2x^3\log(x)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] -((c - c/(a^2*x^2))^(3/2)*x)/(2*(1 - a^2*x^2)^(3/2)) - (a*(c - c/(a^2*x^2))
^(3/2)*x^2)/(1 - a^2*x^2)^(3/2) - (a^3*(c - c/(a^2*x^2))^(3/2)*x^4)/(1 - a^
2*x^2)^(3/2) - (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*Log[x])/(1 - a^2*x^2)^(3/2)
```

Rubi [A] time = 0.167457, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 75}

$$-\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2\left(1-a^2x^2\right)^{3/2}}-\frac{a^2x^3\log(x)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -((c - c/(a^2*x^2))^(3/2)*x)/(2*(1 - a^2*x^2)^(3/2)) - (a*(c - c/(a^2*x^2))
^(3/2)*x^2)/(1 - a^2*x^2)^(3/2) - (a^3*(c - c/(a^2*x^2))^(3/2)*x^4)/(1 - a^
2*x^2)^(3/2) - (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*Log[x])/(1 - a^2*x^2)^(3/2)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{(1-ax)(1+ax)^2}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \left(-a^3+\frac{1}{x^3}+\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}{2\left(1-a^2x^2\right)^{3/2}} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{\left(1-a^2x^2\right)^{3/2}} - \frac{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^4}{\left(1-a^2x^2\right)^{3/2}} - \frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(1-a^2x^2\right)^{3/2}}$$

Mathematica [A] time = 0.034949, size = 72, normalized size = 0.49

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(2a^3x^3+3a^2x^2+2a^2x^2\log(x)+2ax+1\right)}{2a^2x\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a^2*x^2)]*(1 + 2*a*x + 3*a^2*x^2 + 2*a^3*x^3 + 2*a^2*x^2*Log
[x]))/(2*a^2*x*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.157, size = 70, normalized size = 0.5

$$-\frac{x\left(2x^3a^3+2a^2\ln(x)x^2+2ax+1\right)}{2\left(a^2x^2-1\right)^2}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a^2*x^2-1)^2*(-a^2*x^2+1)^(1/2)*(2*x^
3*a^3+2*a^2*ln(x)*x^2+2*a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\sqrt{-a^2x^2+1}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(3/2)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [A] time = 2.55127, size = 778, normalized size = 5.33

$$\left[\frac{\left(a^3cx^3 - acx\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right) - \left(2\,a^3cx^3 - \left(2\,a^3 + 2\,a + 1\right)cx^2 + 2\,acx + c\right)\sqrt{-a^2x^2}}{2\left(a^4x^3 - a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="fric
as")
```

```
[Out] [1/2*((a^3*c*x^3 - a*c*x)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 + (a*
x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c)
/(a^2*x^4 - x^2)) - (2*a^3*c*x^3 - (2*a^3 + 2*a + 1)*c*x^2 + 2*a*c*x + c)*s
qrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3 - a^2*x), -1/2*
(2*(a^3*c*x^3 - a*c*x)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqrt
(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) + (2
*a^3*c*x^3 - (2*a^3 + 2*a + 1)*c*x^2 + 2*a*c*x + c)*sqrt(-a^2*x^2 + 1)*sqrt
((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3 - a^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-1 + \frac{1}{ax}\right)\left(1 + \frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2)*(a*x + 1)/sqrt(-(a*x - 1)
*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c - \frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(3/2)/sqrt(-a^2*x^2 + 1), x)
```

3.690 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=68

$$\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

```
[Out] (a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.143545, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 43}

$$\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1+ax}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(a+\frac{1}{x}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{a\sqrt{c-\frac{c}{a^2x^2}}x^2}{\sqrt{1-a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0206847, size = 37, normalized size = 0.54

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}(ax+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(a*x + Log[x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.15, size = 52, normalized size = 0.8

$$-\frac{x(ax+\ln(x))}{a^2x^2-1}\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(a*x+ln(x))*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)
```

Maxima [C] time = 1.18066, size = 23, normalized size = 0.34

$$-i\sqrt{c}x - \frac{i\sqrt{c}\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -I*sqrt(c)*x - I*sqrt(c)*log(x)/a
```

Fricas [B] time = 2.42968, size = 648, normalized size = 9.53

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)-2\left(a^2x^2-a^2x\right)\sqrt{-a^2x^2+1}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^3x^2-a\right)},-\frac{\left(a^2x^2-1\right)\sqrt{c}\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 + (a*x^5 -
a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c)/(a^2*
x^4 - x^2)) - 2*(a^2*x^2 - a^2*x)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^3*x^2 - a), -((a^2*x^2 - 1)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)
*(a*x^3 + a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 +
1)*c*x^2 + c)) + (a^2*x^2 - a^2*x)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)))/(a^3*x^2 - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x - 1)*(a
*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/sqrt(-a^2*x^2 + 1), x)
```

3.691 $\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] -(Sqrt[1 - a^2*x^2]/(a*Sqrt[c - c/(a^2*x^2)])) - (Sqrt[1 - a^2*x^2]*Log[1 -
 a*x])/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Rubi [A] time = 0.122805, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 43}

$$-\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/(a*Sqrt[c - c/(a^2*x^2)])) - (Sqrt[1 - a^2*x^2]*Log[1 -
 a*x])/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}x}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x}{1-ax}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}x}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{a}-\frac{1}{a(-1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}x}$$
$$= -\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}$$

Mathematica [A] time = 0.0296023, size = 48, normalized size = 0.61

$$-\frac{\sqrt{1-a^2x^2}(ax+\log(1-ax))}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(a*x + Log[1 - a*x]))/(a^2*Sqrt[c - c/(a^2*x^2)]*x))
```

Maple [A] time = 0.151, size = 50, normalized size = 0.6

$$-\frac{ax+\ln(ax-1)}{a^2x}\sqrt{-a^2x^2+1}\frac{1}{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)*(a*x+ln(a*x-1))/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a^2
```

Maxima [C] time = 1.34733, size = 28, normalized size = 0.35

$$-\frac{ix}{\sqrt{c}} - \frac{i\log(ax-1)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -I*x/sqrt(c) - I*log(a*x - 1)/(a*sqrt(c))
```

Fricas [B] time = 2.5501, size = 765, normalized size = 9.68

$$\left[-\frac{2\sqrt{-a^2x^2+1}a^2x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+(a^2x^2-1)\sqrt{-c}\log\left(\frac{a^6cx^6-4a^5cx^5+5a^4cx^4-4a^2cx^2+4acx-(a^5x^5-4a^4x^4+6a^3x^3-4a^2x^2)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2}{a^2x}}}{a^4x^4-2a^3x^3+2ax-1}\right.}{2(a^3cx^2-ac)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

```
[Out] [-1/2*(2*sqrt(-a^2*x^2 + 1)*a^2*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + (a^2*
x^2 - 1)*sqrt(-c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2
+ 4*a*c*x - (a^5*x^5 - 4*a^4*x^4 + 6*a^3*x^3 - 4*a^2*x^2)*sqrt(-a^2*x^2 + 1
)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*
a*x - 1)))/(a^3*c*x^2 - a*c), -(sqrt(-a^2*x^2 + 1)*a^2*x^2*sqrt((a^2*c*x^2
- c)/(a^2*x^2)) + (a^2*x^2 - 1)*sqrt(c)*arctan((a^2*x^2 - 2*a*x + 2)*sqrt(-
a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 - 2*a^2*c*x
^2 - a*c*x + 2*c)))/(a^3*c*x^2 - a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-(ax-1)(ax+1)}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 +
1/(a*x)))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))), x)
```

3.692 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=175

$$\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4x^3(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}}{a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{5\left(1-a^2x^2\right)^{3/2}\log(1-ax)}{4a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{\left(1-a^2x^2\right)^{3/2}\log(ax+1)}{4a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[Out] (1 - a^2*x^2)^(3/2)/(a^3*(c - c/(a^2*x^2))^(3/2)*x^2) + (1 - a^2*x^2)^(3/2)
/(2*a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 - a*x)) + (5*(1 - a^2*x^2)^(3/2)*Log
[1 - a*x])/(4*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) - ((1 - a^2*x^2)^(3/2)*Log[1
 + a*x])/(4*a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rubi [A] time = 0.184058, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 88}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4x^3(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}}{a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{5\left(1-a^2x^2\right)^{3/2}\log(1-ax)}{4a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{\left(1-a^2x^2\right)^{3/2}\log(ax+1)}{4a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(a^3*(c - c/(a^2*x^2))^(3/2)*x^2) + (1 - a^2*x^2)^(3/2)
/(2*a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 - a*x)) + (5*(1 - a^2*x^2)^(3/2)*Log
[1 - a*x])/(4*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) - ((1 - a^2*x^2)^(3/2)*Log[1
 + a*x])/(4*a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{x^3}{(1-ax)^2(1+ax)}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3(1-ax)}+\frac{5\left(1-a^2x^2\right)^{3/2}\log(1-ax)}{4a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}-\frac{\left(1-a^2x^2\right)^{3/2}\log(1+}{4a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}$$

Mathematica [A] time = 0.0516773, size = 91, normalized size = 0.52

$$-\frac{\sqrt{1-a^2x^2}\left(a^2x^2-1\right)\left(\frac{x}{a^3}+\frac{1}{2a^4(1-ax)}+\frac{5\log(1-ax)}{4a^4}-\frac{\log(ax+1)}{4a^4}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(-1 + a^2*x^2)*(x/a^3 + 1/(2*a^4*(1 - a*x)) + (5*Log[1
 - a*x])/(4*a^4) - Log[1 + a*x]/(4*a^4)))/((c - c/(a^2*x^2))^(3/2)*x^3))
```

Maple [A] time = 0.174, size = 94, normalized size = 0.5

$$\frac{\left(-4\,a^2x^2+ax\ln(ax+1)-5\,\ln(ax-1)\,xa+4\,ax-\ln(ax+1)+5\,\ln(ax-1)+2\right)(ax+1)}{4\,a^4x^3}\sqrt{-a^2x^2+1}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/4*(-4*a^2*x^2+a*x*ln(a*x+1)-5*ln(a*x-1)*x*a+4*a*x-ln(a*x+1)+5*ln(a*x-1)+2
)*(a*x+1)*(-a^2*x^2+1)^(1/2)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}a^4x^4\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^5c^2x^5-a^4c^2x^4-2\,a^3c^2x^3+2\,a^2c^2x^2+ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*a^4*x^4*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^5*c
^2*x^5 - a^4*c^2*x^4 - 2*a^3*c^2*x^3 + 2*a^2*c^2*x^2 + a*c^2*x - c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax+1}{\sqrt{-(ax-1)(ax+1)}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(-1 + 1/(a*x))*(1 + 1/(a
*x)))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

3.693 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=267

$$-\frac{\left(1-a^2x^2\right)^{5/2}}{a^6x^5(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{23\left(1-a^2x^2\right.}{16a^6x^5}$$

```
[Out] -((1 - a^2*x^2)^(5/2)/(a^5*(c - c/(a^2*x^2))^(5/2)*x^4)) + (1 - a^2*x^2)^(5
/2)/(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)^2) - (1 - a^2*x^2)^(5/2)/(
a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)) + (1 - a^2*x^2)^(5/2)/(8*a^6*(c
- c/(a^2*x^2))^(5/2)*x^5*(1 + a*x)) - (23*(1 - a^2*x^2)^(5/2)*Log[1 - a*x])
/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) + (7*(1 - a^2*x^2)^(5/2)*Log[1 + a*x]
)/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rubi [A] time = 0.204935, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 88}

$$-\frac{\left(1-a^2x^2\right)^{5/2}}{a^6x^5(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{23\left(1-a^2x^2\right.}{16a^6x^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] -((1 - a^2*x^2)^(5/2)/(a^5*(c - c/(a^2*x^2))^(5/2)*x^4)) + (1 - a^2*x^2)^(5
/2)/(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)^2) - (1 - a^2*x^2)^(5/2)/(
a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)) + (1 - a^2*x^2)^(5/2)/(8*a^6*(c
- c/(a^2*x^2))^(5/2)*x^5*(1 + a*x)) - (23*(1 - a^2*x^2)^(5/2)*Log[1 - a*x])
/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) + (7*(1 - a^2*x^2)^(5/2)*Log[1 + a*x]
)/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{e^{\tanh^{-1}(ax)}x^5}{\left(1-a^2x^2\right)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{x^5}{(1-ax)^3(1+ax)^2}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}\int \left(-\frac{1}{a^5}-\frac{1}{4a^5(-1+ax)^3}-\frac{1}{a^5(-1+ax)^2}-\frac{23}{16a^5(-1+ax)}-\frac{1}{8a^5(1+ax)^2}+\frac{7}{16a^5(1+ax)}\right)dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$
$$= -\frac{\left(1-a^2x^2\right)^{5/2}}{a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5(1-ax)^2}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5(1-ax)}+\frac{\left(1-a^2x\right.}{8a^6\left(c-\frac{c}{a^2x^2}\right)^{5/}}$$

Mathematica [A] time = 0.132773, size = 87, normalized size = 0.33

$$\frac{\left(1-a^2x^2\right)^{5/2}\left(2\left(-8ax+\frac{8}{ax-1}+\frac{1}{ax+1}+\frac{1}{(ax-1)^2}\right)-23\log(1-ax)+7\log(ax+1)\right)}{16a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((1 - a^2*x^2)^(5/2)*(2*(-8*a*x + (-1 + a*x)^(-2) + 8/(-1 + a*x) + (1 + a*x
)^(-1)) - 23*Log[1 - a*x] + 7*Log[1 + a*x]))/(16*a^6*(c - c/(a^2*x^2))^(5/2
)*x^5)
```

Maple [A] time = 0.178, size = 167, normalized size = 0.6

$$\frac{(ax+1)\left(-16\,x^4a^4+7\,a^3x^3\ln(ax+1)-23\,\ln(ax-1)\,x^3a^3+16\,x^3a^3-7\,\ln(ax+1)\,a^2x^2+23\,\ln(ax-1)\,a^2x^2+34\,a\right.}{16\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(a*x+1)*(-16*x^4*a^4+7*a^3*x^3*ln(a*x+1)-23*ln(a*x-
1)*x^3*a^3+16*x^3*a^3-7*ln(a*x+1)*a^2*x^2+23*ln(a*x-1)*a^2*x^2+34*a^2*x^2-7
*a*x*ln(a*x+1)+23*ln(a*x-1)*x*a-18*a*x+7*ln(a*x+1)-23*ln(a*x-1)-12)/a^6/x^5
/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}a^6x^6\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^7c^3x^7-a^6c^3x^6-3\,a^5c^3x^5+3\,a^4c^3x^4+3\,a^3c^3x^3-3\,a^2c^3x^2-ac^3x+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*a^6*x^6*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^7*c
^3*x^7 - a^6*c^3*x^6 - 3*a^5*c^3*x^5 + 3*a^4*c^3*x^4 + 3*a^3*c^3*x^3 - 3*a^
2*c^3*x^2 - a*c^3*x + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

3.694 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=361

$$\frac{3\left(1-a^2x^2\right)^{7/2}}{2a^8x^7(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{5\left(1-a^2x^2\right)^{7/2}}{16a^8x^7(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{11\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

```
[Out] (1 - a^2*x^2)^(7/2)/(a^7*(c - c/(a^2*x^2))^(7/2)*x^6) + (1 - a^2*x^2)^(7/2)
/(24*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^3) - (11*(1 - a^2*x^2)^(7/2)
)/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^2) + (3*(1 - a^2*x^2)^(7/2)
)/(2*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)) + (1 - a^2*x^2)^(7/2)/(32*a
^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^2) - (5*(1 - a^2*x^2)^(7/2))/(16*a
^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)) + (51*(1 - a^2*x^2)^(7/2)*Log[1 -
 a*x])/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) - (19*(1 - a^2*x^2)^(7/2)*Log[1
 + a*x])/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```

Rubi [A] time = 0.297853, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 88}

$$\frac{3\left(1-a^2x^2\right)^{7/2}}{2a^8x^7(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{5\left(1-a^2x^2\right)^{7/2}}{16a^8x^7(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{11\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^(7/2),x]
```



```
[Out] (1 - a^2*x^2)^(7/2)/(a^7*(c - c/(a^2*x^2))^(7/2)*x^6) + (1 - a^2*x^2)^(7/2)
/(24*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^3) - (11*(1 - a^2*x^2)^(7/2)
)/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^2) + (3*(1 - a^2*x^2)^(7/2)
)/(2*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)) + (1 - a^2*x^2)^(7/2)/(32*a
^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^2) - (5*(1 - a^2*x^2)^(7/2))/(16*a
^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)) + (51*(1 - a^2*x^2)^(7/2)*Log[1 -
 a*x])/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) - (19*(1 - a^2*x^2)^(7/2)*Log[1
 + a*x])/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left(1-a^2x^2\right)^{7/2}\int \frac{e^{\tanh^{-1}(ax)}x^7}{\left(1-a^2x^2\right)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left(1-a^2x^2\right)^{7/2}\int \frac{x^7}{(1-ax)^4(1+ax)^3}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left(1-a^2x^2\right)^{7/2}\int \left(\frac{1}{a^7}+\frac{1}{8a^7(-1+ax)^4}+\frac{11}{16a^7(-1+ax)^3}+\frac{3}{2a^7(-1+ax)^2}+\frac{51}{32a^7(-1+ax)}-\frac{1}{16a^7(1+ax)^3}+\frac{5}{16a^7(1+ax)^2}-\right.}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left(1-a^2x^2\right)^{7/2}}{a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}+\frac{\left(1-a^2x^2\right)^{7/2}}{24a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7(1-ax)^3}-\frac{11\left(1-a^2x^2\right)^{7/2}}{32a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7(1-ax)^2}+\frac{3\left(1-a^2\right.}{2a^8\left(c-\frac{c}{a^2x^2}\right)^{7/}}$$

Mathematica [A] time = 0.117451, size = 147, normalized size = 0.41

$$\frac{\sqrt{1-a^2x^2}\left(-96a^6x^6+96a^5x^5+366a^4x^4-222a^3x^3-338a^2x^2+122ax-153(ax-1)^3(ax+1)^2\log(1-ax)+57(ax-1)^3\right.}{96a^2c^3x(ax-1)^3(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(88 + 122*a*x - 338*a^2*x^2 - 222*a^3*x^3 + 366*a^4*x^4
+ 96*a^5*x^5 - 96*a^6*x^6 - 153*(-1 + a*x)^3*(1 + a*x)^2*Log[1 - a*x] + 57*
(-1 + a*x)^3*(1 + a*x)^2*Log[1 + a*x]))/(96*a^2*c^3*Sqrt[c - c/(a^2*x^2)]*x
*(-1 + a*x)^3*(1 + a*x)^2)
```

Maple [A] time = 0.174, size = 239, normalized size = 0.7

$$\frac{(ax+1)\left(-96x^6a^6+57\ln(ax+1)x^5a^5-153\ln(ax-1)x^5a^5+96x^5a^5-57\ln(ax+1)a^4x^4+153\ln(ax-1)a^4x^4+36\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/96*(-a^2*x^2+1)^(1/2)*(a*x+1)*(-96*x^6*a^6+57*ln(a*x+1)*x^5*a^5-153*ln(a*
x-1)*x^5*a^5+96*x^5*a^5-57*ln(a*x+1)*a^4*x^4+153*ln(a*x-1)*a^4*x^4+366*x^4*
a^4-114*a^3*x^3*ln(a*x+1)+306*ln(a*x-1)*x^3*a^3-222*x^3*a^3+114*ln(a*x+1)*a
^2*x^2-306*ln(a*x-1)*a^2*x^2-338*a^2*x^2+57*a*x*ln(a*x+1)-153*ln(a*x-1)*x*a
+122*a*x-57*ln(a*x+1)+153*ln(a*x-1)+88)/a^8/x^7/(c*(a^2*x^2-1)/a^2/x^2)^(7/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}a^8x^8\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^9c^4x^9-a^8c^4x^8-4\,a^7c^4x^7+4\,a^6c^4x^6+6\,a^5c^4x^5-6\,a^4c^4x^4-4\,a^3c^4x^3+4\,a^2c^4x^2+ac^4x-c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*a^8*x^8*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^9*c
^4*x^9 - a^8*c^4*x^8 - 4*a^7*c^4*x^7 + 4*a^6*c^4*x^6 + 6*a^5*c^4*x^5 - 6*a^
4*c^4*x^4 - 4*a^3*c^4*x^3 + 4*a^2*c^4*x^2 + a*c^4*x - c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

3.695 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx$

Optimal. Leaf size=450

$$-\frac{501a^8x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{128(1-ax)^4(ax+1)^4}+\frac{373a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{192(1-ax)^4(ax+1)^3}+\frac{501a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{640(1-ax)^4(ax+1)^2}+\frac{661a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{1680(1-ax)^4(ax+1)}+\frac{295a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{1344(1-ax)^4}$$

```
[Out] (295*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(1344*(1 - a*x)^4) - (501*a^8*(c - c/
(a^2*x^2))^(9/2)*x^9)/(128*(1 - a*x)^4*(1 + a*x)^4) + (373*a^7*(c - c/(a^2*
x^2))^(9/2)*x^8)/(192*(1 - a*x)^4*(1 + a*x)^3) + (501*a^6*(c - c/(a^2*x^2))
^(9/2)*x^7)/(640*(1 - a*x)^4*(1 + a*x)^2) + (661*a^5*(c - c/(a^2*x^2))^(9/2
)*x^6)/(1680*(1 - a*x)^4*(1 + a*x)) - (127*a^3*(c - c/(a^2*x^2))^(9/2)*x^4*
(1 + a*x))/(420*(1 - a*x)^4) + (71*a^2*(c - c/(a^2*x^2))^(9/2)*x^3*(1 + a*x
))/(336*(1 - a*x)^3) - (a*(c - c/(a^2*x^2))^(9/2)*x^2*(1 + a*x))/(28*(1 - a
*x)^2) - ((c - c/(a^2*x^2))^(9/2)*x*(1 + a*x))/(8*(1 - a*x)) + (2*a^8*(c -
c/(a^2*x^2))^(9/2)*x^9*ArcSin[a*x])/((1 - a*x)^(9/2)*(1 + a*x)^(9/2)) + (24
5*a^8*(c - c/(a^2*x^2))^(9/2)*x^9*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(12
8*(1 - a*x)^(9/2)*(1 + a*x)^(9/2))
```

Rubi [A] time = 0.55677, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$-\frac{501a^8x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{128(1-ax)^4(ax+1)^4}+\frac{373a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{192(1-ax)^4(ax+1)^3}+\frac{501a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{640(1-ax)^4(ax+1)^2}+\frac{661a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{1680(1-ax)^4(ax+1)}+\frac{295a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{1344(1-ax)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```


```
[Out] (295*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(1344*(1 - a*x)^4) - (501*a^8*(c - c/
(a^2*x^2))^(9/2)*x^9)/(128*(1 - a*x)^4*(1 + a*x)^4) + (373*a^7*(c - c/(a^2*
x^2))^(9/2)*x^8)/(192*(1 - a*x)^4*(1 + a*x)^3) + (501*a^6*(c - c/(a^2*x^2))
^(9/2)*x^7)/(640*(1 - a*x)^4*(1 + a*x)^2) + (661*a^5*(c - c/(a^2*x^2))^(9/2
)*x^6)/(1680*(1 - a*x)^4*(1 + a*x)) - (127*a^3*(c - c/(a^2*x^2))^(9/2)*x^4*
(1 + a*x))/(420*(1 - a*x)^4) + (71*a^2*(c - c/(a^2*x^2))^(9/2)*x^3*(1 + a*x
))/(336*(1 - a*x)^3) - (a*(c - c/(a^2*x^2))^(9/2)*x^2*(1 + a*x))/(28*(1 - a
*x)^2) - ((c - c/(a^2*x^2))^(9/2)*x*(1 + a*x))/(8*(1 - a*x)) + (2*a^8*(c -
c/(a^2*x^2))^(9/2)*x^9*ArcSin[a*x])/((1 - a*x)^(9/2)*(1 + a*x)^(9/2)) + (24
5*a^8*(c - c/(a^2*x^2))^(9/2)*x^9*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(12
8*(1 - a*x)^(9/2)*(1 + a*x)^(9/2))
```


Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^{9/2}(1+ax)^{9/2}}{x^9}dx}{(1-ax)^{9/2}(1+ax)^{9/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^{7/2}(1+ax)^{11/2}}{x^9}dx}{(1-ax)^{9/2}(1+ax)^{9/2}}$$

$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x(1+ax)}{8(1-ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^{5/2}(1+ax)^{9/2}\left(2a-9a^2x\right)}{x^8}dx}{8(1-ax)^{9/2}(1+ax)^{9/2}}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2(1+ax)}{28(1-ax)^2}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x(1+ax)}{8(1-ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^{3/2}(1+ax)^{9/}}{x^{}}}{56(1-ax)^{9/2}(1+ax}$$

$$=\frac{71a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3(1+ax)}{336(1-ax)^3}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2(1+ax)}{28(1-ax)^2}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x(1+ax)}{8(1-ax)}+\frac{((c}{}$$

$$=-\frac{127a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4(1+ax)}{420(1-ax)^4}+\frac{71a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3(1+ax)}{336(1-ax)^3}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2(1+a}{28(1-ax)^2}$$

$$=\frac{295a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{1344(1-ax)^4}-\frac{127a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4(1+ax)}{420(1-ax)^4}+\frac{71a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3(1+ax)}{336(1-ax)^3}-$$

$$=\frac{295a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{1344(1-ax)^4}+\frac{661a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^6}{1680(1-ax)^4(1+ax)}-\frac{127a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4(1+ax)}{420(1-ax)^4}+\frac{71a^2(}{}$$

$$=\frac{295a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{1344(1-ax)^4}+\frac{501a^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^7}{640(1-ax)^4(1+ax)^2}+\frac{661a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^6}{1680(1-ax)^4(1+ax)}-\frac{127a^3\left(c-\frac{c}{a^2}\right.}{420(}$$

$$=\frac{295a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{1344(1-ax)^4}+\frac{373a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{192(1-ax)^4(1+ax)^3}+\frac{501a^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^7}{640(1-ax)^4(1+ax)^2}+\frac{661a^5\left(c-\frac{}{a^2}\right.}{1680(1-ax)}$$

$$=\frac{295a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{1344(1-ax)^4}-\frac{501a^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}{128(1-ax)^4(1+ax)^4}+\frac{373a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{192(1-ax)^4(1+ax)^3}+\frac{501a^6\left(c-\frac{}{a^2}\right.}{640(1-ax)^4(}$$

$$=\frac{295a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{1344(1-ax)^4}-\frac{501a^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}{128(1-ax)^4(1+ax)^4}+\frac{373a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{192(1-ax)^4(1+ax)^3}+\frac{501a^6\left(c-\frac{}{a^2}\right.}{640(1-ax)^4(}$$

$$=\frac{295a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{1344(1-ax)^4}-\frac{501a^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}{128(1-ax)^4(1+ax)^4}+\frac{373a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{192(1-ax)^4(1+ax)^3}+\frac{501a^6\left(c-\frac{}{a^2}\right.}{640(1-ax)^4(}$$

$$=\frac{295a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{1344(1-ax)^4}-\frac{501a^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}{128(1-ax)^4(1+ax)^4}+\frac{373a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{192(1-ax)^4(1+ax)^3}+\frac{501a^6\left(c-\frac{}{a^2}\right.}{640(1-ax)^4(}$$

Mathematica [A] time = 0.185555, size = 166, normalized size = 0.37

$$-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(13440a^8x^8-45056a^7x^7+14595a^6x^6+31232a^5x^5+770a^4x^4-16896a^3x^3-4760a^2x^2+3840\right.\right.}{13440a^8x^7\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] -(c^4*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1680 + 3840*a*x - 4760*a^2
*x^2 - 16896*a^3*x^3 + 770*a^4*x^4 + 31232*a^5*x^5 + 14595*a^6*x^6 - 45056*
a^7*x^7 + 13440*a^8*x^8) + 25725*a^8*x^8*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 268
80*a^8*x^8*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(13440*a^8*x^7*Sqrt[-1 + a^2*x^2
])
```

Maple [B] time = 0.274, size = 965, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(9/2),x)
```

```
[Out] -1/40320*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/a^2*(58590*(-c/a^2)^(1/2)*ln(x*c^(
1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*c^(11/2)*x^8*a+22050*(-c/a^2)^(1/2)*ln((((a
*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*c^(11/2)*x^8*a-58590*(c*(a
^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*x^9*a^3*c^5+77175*(c*(a^2*x^2-1)/a^2)^(
1/2)*(-c/a^2)^(1/2)*x^8*a^2*c^5+23808*(c*(a^2*x^2-1)/a^2)^(11/2)*(-c/a^2)^(
1/2)*x^7*a^11-17535*(c*(a^2*x^2-1)/a^2)^(11/2)*(-c/a^2)^(1/2)*x^6*a^10-1305
6*(c*(a^2*x^2-1)/a^2)^(11/2)*(-c/a^2)^(1/2)*x^5*a^9-6510*(c*(a^2*x^2-1)/a^2
)^(11/2)*(-c/a^2)^(1/2)*x^4*a^8-6912*(c*(a^2*x^2-1)/a^2)^(11/2)*(-c/a^2)^(1
/2)*x^3*a^7-10920*(c*(a^2*x^2-1)/a^2)^(11/2)*(-c/a^2)^(1/2)*x^2*a^6-11520*(
c*(a^2*x^2-1)/a^2)^(11/2)*(-c/a^2)^(1/2)*x*a^5-5040*a^4*(c*(a^2*x^2-1)/a^2)
^(11/2)*(-c/a^2)^(1/2)+77175*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)
*a^2-c)/x/a^2)*x^8*c^6-23808*(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*x^9*a
^11*c+8960*((a*x-1)*(a*x+1)*c/a^2)^(9/2)*(-c/a^2)^(1/2)*x^8*a^10*c+8575*(c*
(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*x^8*a^10*c+10080*((a*x-1)*(a*x+1)*c/a
^2)^(7/2)*(-c/a^2)^(1/2)*x^9*a^9*c^2+26784*(c*(a^2*x^2-1)/a^2)^(7/2)*(-c/a^
2)^(1/2)*x^9*a^9*c^2-11025*(c*(a^2*x^2-1)/a^2)^(7/2)*(-c/a^2)^(1/2)*x^8*a^8
*c^2-11760*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*(-c/a^2)^(1/2)*x^9*a^7*c^3-31248*(
c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)*x^9*a^7*c^3+15435*(c*(a^2*x^2-1)/a^
2)^(5/2)*(-c/a^2)^(1/2)*x^8*a^6*c^3+14700*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*(-c
/a^2)^(1/2)*x^9*a^5*c^4+39060*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x^9*
a^5*c^4-25725*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x^8*a^4*c^4-22050*((
a*x-1)*(a*x+1)*c/a^2)^(1/2)*(-c/a^2)^(1/2)*x^9*a^3*c^5)/(c*(a^2*x^2-1)/a^2)
^(9/2)/(-c/a^2)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(9/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(c - c/(a^2*x^2))^(9/2)/(a^2*x^2 - 1), x)
```

Fricas [A] time = 2.43752, size = 1129, normalized size = 2.51

$$\left[\frac{53760\,a^7\sqrt{-c}c^4x^7\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+25725\,a^7\sqrt{-c}c^4x^7\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(13440\,a^8c^4x^8-4505\cdots\right.}{26880\,a\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/26880*(53760*a^7*sqrt(-c)*c^4*x^7*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^
2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 25725*a^7*sqrt(-c)*c^4*x^7*log(-(a^2*c
*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(1344
0*a^8*c^4*x^8 - 45056*a^7*c^4*x^7 + 14595*a^6*c^4*x^6 + 31232*a^5*c^4*x^5 +
 770*a^4*c^4*x^4 - 16896*a^3*c^4*x^3 - 4760*a^2*c^4*x^2 + 3840*a*c^4*x + 16
80*c^4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^8*x^7), -1/13440*(25725*a^7*c^(
9/2)*x^7*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)
) - 13440*a^7*c^(9/2)*x^7*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x
^2 - c)/(a^2*x^2)) - c) + (13440*a^8*c^4*x^8 - 45056*a^7*c^4*x^7 + 14595*a^
6*c^4*x^6 + 31232*a^5*c^4*x^5 + 770*a^4*c^4*x^4 - 16896*a^3*c^4*x^3 - 4760*
a^2*c^4*x^2 + 3840*a*c^4*x + 1680*c^4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^
8*x^7)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(9/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 57.1708, size = 954, normalized size = 2.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(9/2),x, algorithm="giac")
```

```
[Out] 1/6720*(25725*c^(9/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x)/a^2 + 13440*c^(9/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))
*sgn(x)/(a*abs(a)) - 6720*sqrt(a^2*c*x^2 - c)*c^4*sgn(x)/a^2 + (14595*(sqrt
(a^2*c)*x - sqrt(a^2*c*x^2 - c))^15*c^5*abs(a)*sgn(x) + 107520*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 - c))^14*a*c^(11/2)*sgn(x) + 76055*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^13*c^6*abs(a)*sgn(x) + 430080*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))^12*a*c^(13/2)*sgn(x) + 64435*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))^11*c^7*abs(a)*sgn(x) + 1111040*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^10*
a*c^(15/2)*sgn(x) + 110495*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*c^8*abs(
a)*sgn(x) + 1576960*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^8*a*c^(17/2)*sgn(
```

```
x) - 110495*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^9*abs(a)*sgn(x) + 141
2096*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(19/2)*sgn(x) - 64435*(sqr
t(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^10*abs(a)*sgn(x) + 831488*(sqrt(a^2*c
)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(21/2)*sgn(x) - 76055*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*c^11*abs(a)*sgn(x) + 252928*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))^2*a*c^(23/2)*sgn(x) - 14595*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c)
)*c^12*abs(a)*sgn(x) + 45056*a*c^(25/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^2 + c)^8*a^2*abs(a)))*abs(a)
```

3.696 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx$

Optimal. Leaf size=372

$$\frac{57a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}-\frac{41a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{24(1-ax)^3(ax+1)^2}-\frac{57a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{80(1-ax)^3(ax+1)}-\frac{11a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{30(1-ax)^3}+\frac{13a^2x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{40(1-ax)^3}-$$

```
[Out] (-11*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(30*(1 - a*x)^3) + (57*a^6*(c - c/(a^
2*x^2))^(7/2)*x^7)/(16*(1 - a*x)^3*(1 + a*x)^3) - (41*a^5*(c - c/(a^2*x^2))
^(7/2)*x^6)/(24*(1 - a*x)^3*(1 + a*x)^2) - (57*a^4*(c - c/(a^2*x^2))^(7/2)*
x^5)/(80*(1 - a*x)^3*(1 + a*x)) + (13*a^2*(c - c/(a^2*x^2))^(7/2)*x^3*(1 +
a*x))/(40*(1 - a*x)^3) - (a*(c - c/(a^2*x^2))^(7/2)*x^2*(1 + a*x))/(15*(1 -
 a*x)^2) - ((c - c/(a^2*x^2))^(7/2)*x*(1 + a*x))/(6*(1 - a*x)) - (2*a^6*(c
- c/(a^2*x^2))^(7/2)*x^7*ArcSin[a*x])/((1 - a*x)^(7/2)*(1 + a*x)^(7/2)) - (
25*a^6*(c - c/(a^2*x^2))^(7/2)*x^7*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(1
6*(1 - a*x)^(7/2)*(1 + a*x)^(7/2))
```

Rubi [A] time = 0.496865, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$\frac{57a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}-\frac{41a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{24(1-ax)^3(ax+1)^2}-\frac{57a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{80(1-ax)^3(ax+1)}-\frac{11a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{30(1-ax)^3}+\frac{13a^2x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{40(1-ax)^3}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```


```
[Out] (-11*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(30*(1 - a*x)^3) + (57*a^6*(c - c/(a^
2*x^2))^(7/2)*x^7)/(16*(1 - a*x)^3*(1 + a*x)^3) - (41*a^5*(c - c/(a^2*x^2))
^(7/2)*x^6)/(24*(1 - a*x)^3*(1 + a*x)^2) - (57*a^4*(c - c/(a^2*x^2))^(7/2)*
x^5)/(80*(1 - a*x)^3*(1 + a*x)) + (13*a^2*(c - c/(a^2*x^2))^(7/2)*x^3*(1 +
a*x))/(40*(1 - a*x)^3) - (a*(c - c/(a^2*x^2))^(7/2)*x^2*(1 + a*x))/(15*(1 -
 a*x)^2) - ((c - c/(a^2*x^2))^(7/2)*x*(1 + a*x))/(6*(1 - a*x)) - (2*a^6*(c
- c/(a^2*x^2))^(7/2)*x^7*ArcSin[a*x])/((1 - a*x)^(7/2)*(1 + a*x)^(7/2)) - (
25*a^6*(c - c/(a^2*x^2))^(7/2)*x^7*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(1
6*(1 - a*x)^(7/2)*(1 + a*x)^(7/2))
```


Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^{7/2}(1+ax)^{7/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{5/2}(1+ax)^{9/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{3/2}(1+ax)^{7/2}(2a-7a^2x)}{x^6}dx}{6(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{\sqrt{1-ax}(1+ax)^{7/2}(\cdots}{x^5}\cdots}{30(1-ax)^{7/2}(1+ax)\cdots}$$

$$= \frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)} + \frac{((c\cdots}{\cdots}$$

$$= -\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3} + \frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2} - \frac{(c-\cdots}{\cdots}$$

$$= -\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3} - \frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)} + \frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3} - \frac{a\left(c-\cdots\right.}{15\cdots}$$

$$= -\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3} - \frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2} - \frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)} + \frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/\cdots}}{40(1-\cdots}$$

$$= -\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3} + \frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} - \frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2} - \frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7\cdots}}{80(1-ax)^3(1+\cdots}$$

$$= -\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3} + \frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} - \frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2} - \frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7\cdots}}{80(1-ax)^3(1+\cdots}$$

$$= -\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3} + \frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} - \frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2} - \frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7\cdots}}{80(1-ax)^3(1+\cdots}$$

$$= -\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3} + \frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} - \frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2} - \frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7\cdots}}{80(1-ax)^3(1+\cdots}$$

Mathematica [A] time = 0.152684, size = 150, normalized size = 0.4

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(240a^6x^6-736a^5x^5+105a^4x^4+352a^3x^3+70a^2x^2-96ax-40\right)+480a^6x^6\log\left(\sqrt{a^2x^2-1}+a\cdots\right.\right.}{240a^6x^5\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] -(c^3*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-40 - 96*a*x + 70*a^2*x^2
+ 352*a^3*x^3 + 105*a^4*x^4 - 736*a^5*x^5 + 240*a^6*x^6) + 375*a^6*x^6*ArcT
an[1/Sqrt[-1 + a^2*x^2]] + 480*a^6*x^6*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(240
*a^6*x^5*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.163, size = 795, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] -1/1680*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/a^2*(2016*(-c/a^2)^(1/2)*(c*(a^2*x^
2-1)/a^2)^(7/2)*x^7*a^9*c-2016*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x^5
*a^9+480*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^6*a^8*c-375*(-c/a^2
)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^6*a^8*c+560*(-c/a^2)^(1/2)*((a*x-1)*(a*
x+1)*c/a^2)^(5/2)*x^7*a^7*c^2-105*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*
x^4*a^8-2352*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^7*a^7*c^2-224*(-c/a
^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x^3*a^7+525*(-c/a^2)^(1/2)*(c*(a^2*x^2-
1)/a^2)^(5/2)*x^6*a^6*c^2-700*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*
x^7*a^5*c^3+2940*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^7*a^5*c^3-630*(
-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x^2*a^6-875*(-c/a^2)^(1/2)*(c*(a^2*
x^2-1)/a^2)^(3/2)*x^6*a^4*c^3+1050*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(
1/2)*x^7*a^3*c^4-672*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x*a^5-4410*(-
c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^7*a^3*c^4+4410*(-c/a^2)^(1/2)*c^(9
/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^6*a-1050*(-c/a^2)^(1/2)*c^(9/
2)*ln((((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*x^6*a-280*a^4*(c
*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)+2625*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a
^2)^(1/2)*x^6*a^2*c^4+2625*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a
^2-c)/x/a^2)*x^6*c^5)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(7/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(c - c/(a^2*x^2))^(7/2)/(a^2*x^2 - 1), x)
```

Fricas [A] time = 2.1754, size = 972, normalized size = 2.61

$$\left[\frac{960\,a^5\sqrt{-c}c^3x^5\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+375\,a^5\sqrt{-c}c^3x^5\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(240\,a^6c^3x^6-736\,a^5c^3x\right.}{480\,a^6x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/480*(960*a^5*sqrt(-c)*c^3*x^5*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 375*a^5*sqrt(-c)*c^3*x^5*log(-(a^2*c*x^2 +
 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(240*a^6*c^
3*x^6 - 736*a^5*c^3*x^5 + 105*a^4*c^3*x^4 + 352*a^3*c^3*x^3 + 70*a^2*c^3*x^
2 - 96*a*c^3*x - 40*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5), -1/240
*(375*a^5*c^(7/2)*x^5*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a
```

```
^2*c*x^2 - c)) - 240*a^5*c^(7/2)*x^5*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (240*a^6*c^3*x^6 - 736*a^5*c^3*x^5 + 1
05*a^4*c^3*x^4 + 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 - 96*a*c^3*x - 40*c^3)*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 84.5367, size = 757, normalized size = 2.03

$$\frac{1}{120}\left(\frac{375\,c^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}+\frac{240\,c^{\frac{7}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{120\,\sqrt{a^2cx^2-c}c^3\operatorname{sgn}(x)}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/120*(375*c^(7/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*s
gn(x)/a^2 + 240*c^(7/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(
x)/(a*abs(a)) - 120*sqrt(a^2*c*x^2 - c)*c^3*sgn(x)/a^2 + (105*(sqrt(a^2*c)*
x - sqrt(a^2*c*x^2 - c))^11*c^4*abs(a)*sgn(x) + 1440*(sqrt(a^2*c)*x - sqrt(
a^2*c*x^2 - c))^10*a*c^(9/2)*sgn(x) + 595*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 c))^9*c^5*abs(a)*sgn(x) + 4320*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^8*a*c
^(11/2)*sgn(x) - 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^6*abs(a)*sgn
(x) + 7360*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(13/2)*sgn(x) + 150*
(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^7*abs(a)*sgn(x) + 6720*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(15/2)*sgn(x) - 595*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^3*c^8*abs(a)*sgn(x) + 2976*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - c))^2*a*c^(17/2)*sgn(x) - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^9*
abs(a)*sgn(x) + 736*a*c^(19/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
c))^2 + c)^6*a^2*abs(a)))*abs(a)
```

3.697 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=294

$$-\frac{25a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}+\frac{17a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{12(1-ax)^2(ax+1)}+\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2}-\frac{ax^2(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(1-ax)^2}-\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(1-ax)}$$

```
[Out] (5*a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(8*(1 - a*x)^2) - (25*a^4*(c - c/(a^2*x
^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(1 + a*x)^2) + (17*a^3*(c - c/(a^2*x^2))^(5/
2)*x^4)/(12*(1 - a*x)^2*(1 + a*x)) - (a*(c - c/(a^2*x^2))^(5/2)*x^2*(1 + a*
x))/(6*(1 - a*x)^2) - ((c - c/(a^2*x^2))^(5/2)*x*(1 + a*x))/(4*(1 - a*x)) +
 (2*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcSin[a*x])/((1 - a*x)^(5/2)*(1 + a*x)
^(5/2)) + (9*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 +
 a*x]])/(8*(1 - a*x)^(5/2)*(1 + a*x)^(5/2))
```

Rubi [A] time = 0.454069, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$-\frac{25a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}+\frac{17a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{12(1-ax)^2(ax+1)}+\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2}-\frac{ax^2(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(1-ax)^2}-\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(1-ax)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```


```
[Out] (5*a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(8*(1 - a*x)^2) - (25*a^4*(c - c/(a^2*x
^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(1 + a*x)^2) + (17*a^3*(c - c/(a^2*x^2))^(5/
2)*x^4)/(12*(1 - a*x)^2*(1 + a*x)) - (a*(c - c/(a^2*x^2))^(5/2)*x^2*(1 + a*
x))/(6*(1 - a*x)^2) - ((c - c/(a^2*x^2))^(5/2)*x*(1 + a*x))/(4*(1 - a*x)) +
 (2*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcSin[a*x])/((1 - a*x)^(5/2)*(1 + a*x)
^(5/2)) + (9*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 +
 a*x]])/(8*(1 - a*x)^(5/2)*(1 + a*x)^(5/2))
```


Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^{5/2}(1+ax)^{5/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{3/2}(1+ax)^{7/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$
$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{\sqrt{1-ax}(1+ax)^{5/2}(2a-5a^2x)}{x^4}dx}{4(1-ax)^{5/2}(1+ax)^{5/2}}$$
$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1+ax)^{5/2}(-1}{x^3\sqrt{1}}}{12(1-ax)^{5/2}(1+ax)}$$
$$= \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right.\right.}{}$$
$$= \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} + \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/}}{4(1-}$$
$$= \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} - \frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} + \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1-ax)}$$
$$= \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} - \frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} + \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1-ax)}$$
$$= \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} - \frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} + \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1-ax)}$$
$$= \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} - \frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} + \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1-ax)}$$

Mathematica [A] time = 0.125283, size = 134, normalized size = 0.46

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(24a^4x^4-64a^3x^3-3a^2x^2+16ax+6\right)+48a^4x^4\log\left(\sqrt{a^2x^2-1}+ax\right)+27a^4x^4\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2}}\right.\right.}{24a^4x^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] -(c^2*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 + 16*a*x - 3*a^2*x^2 - 6
4*a^3*x^3 + 24*a^4*x^4) + 27*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 48*a^4*
x^4*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^4*x^3*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.15, size = 625, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] -1/120*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/a^2*(-80*(-c/a^2)^(1/2)*(c*(a^2*x^2-
1)/a^2)^(5/2)*x^5*a^7*c+80*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^3*a^7
```

```
+48*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^4*a^6*c+27*(-c/a^2)^(1/2
)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^4*a^6*c+60*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/
a^2)^(3/2)*x^5*a^5*c^2-75*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^2*a^6+
100*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^5*a^5*c^2-80*(-c/a^2)^(1/2)*
(c*(a^2*x^2-1)/a^2)^(7/2)*x*a^5-45*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)
*x^4*a^4*c^2-90*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^5*a^3*c^3-15
0*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^5*a^3*c^3-30*a^4*(c*(a^2*x^2-1
)/a^2)^(7/2)*(-c/a^2)^(1/2)+150*(-c/a^2)^(1/2)*c^(7/2)*ln(x*c^(1/2)+(c*(a^2
*x^2-1)/a^2)^(1/2))*x^4*a+90*(-c/a^2)^(1/2)*c^(7/2)*ln((((a*x-1)*(a*x+1)*c/
a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*x^4*a+135*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^
2)^(1/2)*x^4*a^2*c^3+135*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2
-c)/x/a^2)*x^4*c^4)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(5/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(c - c/(a^2*x^2))^(5/2)/(a^2*x^2 - 1), x)
```

Fricas [A] time = 2.16081, size = 856, normalized size = 2.91

$$\left[\frac{96\,a^3\sqrt{-c}c^2x^3\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+27\,a^3\sqrt{-c}c^2x^3\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(24\,a^4c^2x^4-64\,a^3c^2x^3-3\,a^2c\right.}{48\,a^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(96*a^3*sqrt(-c)*c^2*x^3*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)
/(a^2*x^2))/(a^2*c*x^2 - c)) + 27*a^3*sqrt(-c)*c^2*x^3*log(-(a^2*c*x^2 + 2*
a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(24*a^4*c^2*x^
4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)))/(a^4*x^3), -1/24*(27*a^3*c^(5/2)*x^3*arctan(a*sqrt(c)*x*sqrt
((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 24*a^3*c^(5/2)*x^3*log(2*a^2
*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (24*a^4*c
^2*x^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x
^2 - c)/(a^2*x^2)))/(a^4*x^3)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 3.58452, size = 562, normalized size = 1.91

$$\frac{1}{12}\left(\frac{27\,c^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}+\frac{24\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{12\,\sqrt{a^2cx^2-c}c^2\operatorname{sgn}(x)}{a^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*(27*c^(5/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn
(x)/a^2 + 24*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/
(a*abs(a)) - 12*sqrt(a^2*c*x^2 - c)*c^2*sgn(x)/a^2 - (3*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^7*c^3*abs(a)*sgn(x) - 96*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - c))^6*a*c^(7/2)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^4*
abs(a)*sgn(x) - 192*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(9/2)*sgn(x
) + 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*c^5*abs(a)*sgn(x) - 160*(sqr
t(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(11/2)*sgn(x) - 3*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))*c^6*abs(a)*sgn(x) - 64*a*c^(13/2)*sgn(x))/(((sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^4*a^2*abs(a)))*abs(a)
```

3.698 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=214

$$\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{1-ax}-\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)}-\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\sin^{-1}(ax)}{(1-ax)^{3/2}(ax+1)^{3/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\tanh^{-1}}{2(1-ax)^{3/2}(a}$$

```
[Out] -((a*(c - c/(a^2*x^2))^(3/2)*x^2)/(1 - a*x)) + (5*a^2*(c - c/(a^2*x^2))^(3/
2)*x^3)/(2*(1 - a*x)*(1 + a*x)) - ((c - c/(a^2*x^2))^(3/2)*x*(1 + a*x))/(2*
(1 - a*x)) - (2*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcSin[a*x])/((1 - a*x)^(3/
2)*(1 + a*x)^(3/2)) - (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcTanh[Sqrt[1 - a*x
]*Sqrt[1 + a*x]])/(2*(1 - a*x)^(3/2)*(1 + a*x)^(3/2))
```

Rubi [A] time = 0.394797, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{1-ax}-\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)}-\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\sin^{-1}(ax)}{(1-ax)^{3/2}(ax+1)^{3/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\tanh^{-1}}{2(1-ax)^{3/2}(a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```


```
[Out] -((a*(c - c/(a^2*x^2))^(3/2)*x^2)/(1 - a*x)) + (5*a^2*(c - c/(a^2*x^2))^(3/
2)*x^3)/(2*(1 - a*x)*(1 + a*x)) - ((c - c/(a^2*x^2))^(3/2)*x*(1 + a*x))/(2*
(1 - a*x)) - (2*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcSin[a*x])/((1 - a*x)^(3/
2)*(1 + a*x)^(3/2)) - (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcTanh[Sqrt[1 - a*x
]*Sqrt[1 + a*x]])/(2*(1 - a*x)^(3/2)*(1 + a*x)^(3/2))
```


Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)^{3/2}(1+ax)^{3/2}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{\sqrt{1-ax}(1+ax)^{5/2}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$
$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{(1+ax)^{3/2}(2a-3a^2x)}{x^2\sqrt{1-ax}}dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$
$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{\sqrt{1+ax}(a^2-5a^3x)}{x\sqrt{1-ax}}dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$
$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} + \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int}{2a(1-ax)^{3/2}(}$$
$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} + \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} + \frac{\left(a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right.}{2(1-ax)^{3/2}}$$
$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} + \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} - \frac{\left(a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)}{2}$$
$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} + \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} - \frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{(1-ax)^{3/2}(1+}$$

Mathematica [A] time = 0.107572, size = 115, normalized size = 0.54

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(2a^2x^2-4ax-1\right)+4a^2x^2\log\left(\sqrt{a^2x^2-1}+ax\right)+a^2x^2\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{2a^2x\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -(c*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-1 - 4*a*x + 2*a^2*x^2) + a^
2*x^2*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 4*a^2*x^2*Log[a*x + Sqrt[-1 + a^2*x^2]
]))/(2*a^2*x*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.142, size = 455, normalized size = 2.1

$$-\frac{x}{6a^2c}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}\left(12\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}x^3a^5c-12\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{5/2}xa^5+4\sqrt{-\frac{c}{a^2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] -1/6*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/a^2*(12*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/
a^2)^(3/2)*x^3*a^5*c-12*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x*a^5+4*(-
c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^2*a^4*c-(-c/a^2)^(1/2)*(c*(a^2
*x^2-1)/a^2)^(3/2)*x^2*a^4*c+6*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)
*x^3*a^3*c^2-3*a^4*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)-18*(-c/a^2)^(1/
```

```
2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^3*a^3*c^2+18*c^(5/2)*ln(x*c^(1/2)+(c*(a^2*x^
2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*x^2*a-6*c^(5/2)*(-c/a^2)^(1/2)*ln((((a*x-1)
*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*x^2*a+3*(-c/a^2)^(1/2)*(c*(a^2*
x^2-1)/a^2)^(1/2)*x^2*a^2*c^2+3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1
/2)*a^2-c)/x/a^2)*x^2*c^3)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(3/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(c - c/(a^2*x^2))^(3/2)/(a^2*x^2 - 1), x)
```

Fricas [A] time = 1.96883, size = 694, normalized size = 3.24

$$\left[\frac{8\,a\sqrt{-c}cx\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+a\sqrt{-c}cx\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(2\,a^2cx^2-4\,acx-c\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,a^2x},-\frac{ac^{\frac{3}{2}}x\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(8*a*sqrt(-c)*c*x*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^2*c*x^2 - c)) + a*sqrt(-c)*c*x*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt
((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(2*a^2*c*x^2 - 4*a*c*x - c)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x), -1/2*(a*c^(3/2)*x*arctan(a*sqrt(c)*x
*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 2*a*c^(3/2)*x*log(2*a^2
*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (2*a^2*c*
x^2 - 4*a*c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.60852, size = 358, normalized size = 1.67

$$\left(\frac{c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}+\frac{2\,c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{\sqrt{a^2cx^2-c}c\operatorname{sgn}(x)}{a^2}-\frac{\left(\sqrt{a^2c}x-\sqrt{a^2c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] (c^(3/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^2
+ 2*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(a)
) - sqrt(a^2*c*x^2 - c)*c*sgn(x)/a^2 - ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))^3*c^2*abs(a)*sgn(x) - 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(5/2
)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^3*abs(a)*sgn(x) - 4*a*c^
(7/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^2*a^2*abs(a)))
*abs(a)
```

3.699 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=118

$$-x\sqrt{c-\frac{c}{a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x) + (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[
1 - a*x]*Sqrt[1 + a*x]) - (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.305355, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6159, 6129, 102, 157, 41, 216, 92, 208}

$$-x\sqrt{c-\frac{c}{a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x) + (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[
1 - a*x]*Sqrt[1 + a*x]) - (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
```

```
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{(1+ax)^{3/2}}{x\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}x-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{-a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}x+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}x-\frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int\frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\cdots}{\sqrt{1-ax}\sqrt{\cdots}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}x+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.0785072, size = 81, normalized size = 0.69

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}+2\log\left(\sqrt{a^2x^2-1}+ax\right)-\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

[Out] -((Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2
]] + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2])

Maple [A] time = 0.144, size = 198, normalized size = 1.7

$$-\frac{x}{a^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(2\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}a^2\sqrt{-\frac{c}{a^2}}+2\sqrt{c}\ln\left(\frac{1}{\sqrt{c}}\left(\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\sqrt{c}+cx\right)\right)a\sqrt{-\frac{c}{a^2}}-\sqrt{-\frac{c}{a^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2),x)

[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a^2*(-c/a
^2)^(1/2)+2*c^(1/2)*ln((((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))
a(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/
a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2))/(c*(a^2*x^2-1)/a^2)^(1/
2)/a^2/(-c/a^2)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")

[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a^2*x^2))/(a^2*x^2 - 1), x)

Fricas [A] time = 1.95154, size = 579, normalized size = 4.91

$$\left[-\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-4\,\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\sqrt{-c}\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},-\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\sqrt{c}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")

[Out] [-1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 4*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - sqrt(-c)*log(-(a^
2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, -(
a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*
c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sq
rt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx-\int \frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] -Integral(sqrt(c - c/(a**2*x**2))/(a*x - 1), x) - Integral(a*x*sqrt(c - c/(
a**2*x**2))/(a*x - 1), x)
```

Giac [A] time = 1.24046, size = 208, normalized size = 1.76

$$-\left(\frac{2\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{2\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}+\frac{\sqrt{a^2cx^2-c}\operatorname{sgn}(x)}{a^2}-\frac{\left(2\sqrt{c}|a|\operatorname{arc}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
^2 - 2*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs
(a)) + sqrt(a^2*c*x^2 - c)*sgn(x)/a^2 - (2*sqrt(c)*abs(a)*arctan(sqrt(-c)/s
qrt(c)) - a*sqrt(c)*log(abs(c)) + sqrt(-c)*abs(a))*sgn(x)/(a^2*abs(a)))*abs
(a)
```

3.700 $\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=110

$$\frac{(ax+1)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2(1-ax)(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{1-ax}\sqrt{ax+1}\sin^{-1}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (2*(1 - a*x)*(1 + a*x))/(a^2*Sqrt[c - c/(a^2*x^2)]*x) + (1 + a*x)^2/(a^2*Sq
rt[c - c/(a^2*x^2)]*x) - (2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcSin[a*x])/(a^2*S
qrt[c - c/(a^2*x^2)]*x)
```

Rubi [A] time = 0.244246, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6159, 6129, 78, 50, 41, 216}

$$\frac{(ax+1)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2(1-ax)(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{1-ax}\sqrt{ax+1}\sin^{-1}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (2*(1 - a*x)*(1 + a*x))/(a^2*Sqrt[c - c/(a^2*x^2)]*x) + (1 + a*x)^2/(a^2*Sq
rt[c - c/(a^2*x^2)]*x) - (2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcSin[a*x])/(a^2*S
qrt[c - c/(a^2*x^2)]*x)
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx &= \frac{\left(\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{e^{2\tanh^{-1}(ax)}x}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}x}\\
&= \frac{\left(\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{x\sqrt{1+ax}}{(1-ax)^{3/2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}x}\\
&= \frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}-\frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}x}\\
&= \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}+\frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}-\frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}x}\\
&= \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}+\frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}-\frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}x}\\
&= \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}+\frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}-\frac{2\sqrt{1-ax}\sqrt{1+ax}\sin^{-1}(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}
\end{aligned}$$

Mathematica [A] time = 0.068285, size = 69, normalized size = 0.63

$$\frac{-a^2x^2-2\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)+2ax+3}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (3 + 2*a*x - a^2*x^2 - 2*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/
(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Maple [A] time = 0.145, size = 178, normalized size = 1.6

$$-\frac{1}{a(ax-1)x}\sqrt{\frac{c(a^2x^2-1)}{a^2}}\left(\sqrt{c}\sqrt{\frac{c(a^2x^2-1)}{a^2}}xa^2+2\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)xac-2a\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\sqrt{c}-\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2)^(1/2)*(c^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a^2+2*ln(x*
c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x*a*c-2*a*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*
c^(1/2)-(c*(a^2*x^2-1)/a^2)^(1/2)*a*c^(1/2)-2*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a
^2)^(1/2))*c)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/c^(3/2)/a/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(ax+1)^2}{(a^2x^2-1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))), x)
```

Fricas [A] time = 2.0197, size = 447, normalized size = 4.06

$$\left[\frac{(ax-1)\sqrt{c}\log\left(2a^2cx^2-2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-(a^2x^2-3ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx-ac}, \frac{2(ax-1)\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(\ldots}{a^2cx-ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [((a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) - c) - (a^2*x^2 - 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2
*c*x - a*c), (2*(a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2
- c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (a^2*x^2 - 3*a*x)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^2*c*x - a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{ax}{ax\sqrt{c-\frac{c}{a^2x^2}}-\sqrt{c-\frac{c}{a^2x^2}}}\,dx-\int\frac{1}{ax\sqrt{c-\frac{c}{a^2x^2}}-\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x*sqrt(c - c/(a**2*x**2)) - sqrt(c - c/(a**2*x**2))), x) -
 Integral(1/(a*x*sqrt(c - c/(a**2*x**2)) - sqrt(c - c/(a**2*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)^2}{(a^2x^2-1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2/((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))), x)
```

3.701 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{2(5-2ax)(1-ax)(ax+1)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{(ax+1)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{2(1-ax)^{3/2}(ax+1)^{3/2}\sin^{-1}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[Out] (1 + a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(3/2)*x) - (2*(5 - 2*a*x)*(1 - a*x)*(1
 + a*x)^2)/(3*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) + (2*(1 - a*x)^(3/2)*(1 + a*
x)^(3/2)*ArcSin[a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rubi [A] time = 0.373212, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6159, 6129, 98, 143, 41, 216}

$$-\frac{2(5-2ax)(1-ax)(ax+1)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{(ax+1)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{2(1-ax)^{3/2}(ax+1)^{3/2}\sin^{-1}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (1 + a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(3/2)*x) - (2*(5 - 2*a*x)*(1 - a*x)*(1
 + a*x)^2)/(3*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) + (2*(1 - a*x)^(3/2)*(1 + a*
x)^(3/2)*ArcSin[a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx &= \frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(1-ax)^{3/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= \frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x^3}{(1-ax)^{5/2}\sqrt{1+ax}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= \frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}-\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x(2+4ax)}{(1-ax)^{3/2}\sqrt{1+ax}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= \frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}-\frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}+\frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= \frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}-\frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}+\frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= \frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}-\frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}+\frac{2(1-ax)^{3/2}(1+ax)^{3/2}\sin^{-1}(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}
\end{aligned}
$$

Mathematica [A] time = 0.0833405, size = 95, normalized size = 0.77

$$\frac{-3a^3x^3+11a^2x^2-6(ax-1)\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)+4ax-10}{3a^2cx(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (-10 + 4*a*x + 11*a^2*x^2 - 3*a^3*x^3 - 6*(-1 + a*x)*Sqrt[-1 + a^2*x^2]*Log
[a*x + Sqrt[-1 + a^2*x^2]])/(3*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x))
```

Maple [B] time = 0.134, size = 326, normalized size = 2.7

$$-\frac{ax+1}{3x^3a^4}\left(3c^{3/2}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}x^3a^3-15x^2a^2c^{3/2}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+4c^{3/2}\sqrt{\frac{c(a^2x^2-1)}{a^2}}x^2a^2+6\ln\left(x\sqrt{c}+\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] -1/3*(3*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^3*a^3-15*x^2*a^2*c^(3/2)*((
a*x-1)*(a*x+1)*c/a^2)^(1/2)+4*c^(3/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^2*a^2+6*l
n(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*(c*(a^
2*x^2-1)/a^2)^(1/2)*x*a^2*c-4*c^(3/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a-6*ln(x*
c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*(c*(a^2*x^
2-1)/a^2)^(1/2)*a*c+12*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)-2*(c*(a^2*x^2-
1)/a^2)^(1/2)*c^(3/2))*(a*x+1)/((a*x-1)*(a*x+1)*c/a^2)^(1/2)/x^3/(c*(a^2*x^
2-1)/a^2/x^2)^(3/2)/a^4/c^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(ax+1)^2}{(a^2x^2-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

Fricas [A] time = 1.94541, size = 590, normalized size = 4.8

$$\left[\frac{3\left(a^2x^2-2ax+1\right)\sqrt{c}\log\left(2a^2cx^2-2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-\left(3a^3x^3-14a^2x^2+10ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\left(a^3c^2x^2-2a^2c^2x+ac^2\right)},\frac{6\left(a^2x^2-2ax+1\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) - (3*a^3*x^3 - 14*a^2*x^2 + 10*a*x)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), 1/3*(6*(
a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/
(a^2*x^2))/(a^2*c*x^2 - c)) - (3*a^3*x^3 - 14*a^2*x^2 + 10*a*x)*sqrt((a^2*c
*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{acx\sqrt{c-\frac{c}{a^2x^2}}-c\sqrt{c-\frac{c}{a^2x^2}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{ax}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2}}\,dx-\int \frac{1}{acx\sqrt{c-\frac{c}{a^2x^2}}-c\sqrt{c-\frac{c}{a^2x^2}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{ax}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] -Integral(a*x/(a*c*x*sqrt(c - c/(a**2*x**2)) - c*sqrt(c - c/(a**2*x**2)) -
c*sqrt(c - c/(a**2*x**2))/(a*x) + c*sqrt(c - c/(a**2*x**2))/(a**2*x**2)), x
) - Integral(1/(a*c*x*sqrt(c - c/(a**2*x**2)) - c*sqrt(c - c/(a**2*x**2)) -
 c*sqrt(c - c/(a**2*x**2))/(a*x) + c*sqrt(c - c/(a**2*x**2))/(a**2*x**2)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)^2}{(a^2x^2-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

3.702 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=203

$$\frac{2(ax+1)^2(43ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{58(ax+1)^2(1-ax)^2}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^2(1-ax)}{3a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{(ax+1)^2}{5a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^{5/2}(1-a}{a^6x^5\left(c-\right.}$$

```
[Out] (1 + a*x)^2/(5*a^2*(c - c/(a^2*x^2))^(5/2)*x) - (2*(1 - a*x)*(1 + a*x)^2)/(
3*a^3*(c - c/(a^2*x^2))^(5/2)*x^2) + (58*(1 - a*x)^2*(1 + a*x)^2)/(15*a^4*(
c - c/(a^2*x^2))^(5/2)*x^3) + (2*(1 - a*x)^3*(1 + a*x)^2*(28 + 43*a*x))/(15
*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) - (2*(1 - a*x)^(5/2)*(1 + a*x)^(5/2)*ArcS
in[a*x])/(a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rubi [A] time = 0.405363, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6159, 6129, 98, 150, 143, 41, 216}

$$\frac{2(ax+1)^2(43ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{58(ax+1)^2(1-ax)^2}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^2(1-ax)}{3a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{(ax+1)^2}{5a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^{5/2}(1-a}{a^6x^5\left(c-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (1 + a*x)^2/(5*a^2*(c - c/(a^2*x^2))^(5/2)*x) - (2*(1 - a*x)*(1 + a*x)^2)/(
3*a^3*(c - c/(a^2*x^2))^(5/2)*x^2) + (58*(1 - a*x)^2*(1 + a*x)^2)/(15*a^4*(
c - c/(a^2*x^2))^(5/2)*x^3) + (2*(1 - a*x)^3*(1 + a*x)^2*(28 + 43*a*x))/(15
*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) - (2*(1 - a*x)^(5/2)*(1 + a*x)^(5/2)*ArcS
in[a*x])/(a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx = \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^5}{(1-ax)^{5/2}(1+ax)^{5/2}}dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^5}{(1-ax)^{7/2}(1+ax)^{3/2}}dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^3(4+6ax)}{(1-ax)^{5/2}(1+ax)^{3/2}}dx}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^2(-30a-28a^2x)}{(1-ax)^{3/2}(1+ax)^{3/2}}dx}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x(116}{\sqrt{1-a}}}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{2(1-ax)^3(1+ax)^2(28+43ax}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{2(1-ax)^3(1+ax)^2(28+43ax}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{2(1-ax)^3(1+ax)^2(28+43ax}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

Mathematica [A] time = 0.0978351, size = 105, normalized size = 0.52

$$\frac{-15a^4x^4+76a^3x^3-32a^2x^2-30(ax-1)^2\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)-82ax+56}{15a^2c^2x(ax-1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (56 - 82*a*x - 32*a^2*x^2 + 76*a^3*x^3 - 15*a^4*x^4 - 30*(-1 + a*x)^2*Sqrt[
-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(15*a^2*c^2*Sqrt[c - c/(a^2*x^
2)]*x*(-1 + a*x)^2)
```

Maple [B] time = 0.138, size = 462, normalized size = 2.3

$$-\frac{ax+1}{15x^5a^6}\left(15c^{5/2}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{3/2}x^5a^5-45x^4c^{5/2}a^4\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{3/2}-16c^{5/2}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}x^4a^4-60\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] -1/15*(15*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^5*a^5-45*x^4*c^(5/2)*a^4*
((a*x-1)*(a*x+1)*c/a^2)^(3/2)-16*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^4*a^4-
```

```
60*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^3*a^3+16*c^(5/2)*(c*(a^2*x^2-1)/
a^2)^(3/2)*x^3*a^3+30*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x
+1)*c/a^2)^(3/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a^4*c+90*c^(5/2)*((a*x-1)*(a*x
+1)*c/a^2)^(3/2)*x^2*a^2+24*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2*a^2-30*ln
(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*(c*(a^2
*x^2-1)/a^2)^(3/2)*a^3*c+50*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x*a-24*c^
(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a-50*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2
)-6*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2))*(a*x+1)/((a*x-1)*(a*x+1)*c/a^2)^(3/2
)/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)/a^6/c^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2}{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

Fricas [A] time = 2.23527, size = 744, normalized size = 3.67

$$\left[\frac{15\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-\left(15\,a^5x^5-76\,a^4x^4+32\,a^3x^3+82\,a^2x^2-56\,ax\right)}{15\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2
*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) - (15*a^5*x^5 - 76*a^4*x^
4 + 32*a^3*x^3 + 82*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5
*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3), 1/15*(30*(a^4*x^4 - 2*a^3*
x^3 + 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2
*x^2))/(a^2*c*x^2 - c)) - (15*a^5*x^5 - 76*a^4*x^4 + 32*a^3*x^3 + 82*a^2*x^
2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 +
 2*a^2*c^3*x - a*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{ac^2x\sqrt{c-\frac{c}{a^2x^2}}-c^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{ax}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^3}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^4}}\,dx-\int \frac{}{ac^2x\sqrt{c-\frac{c}{a^2x^2}}-c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] -Integral(a*x/(a*c**2*x*sqrt(c - c/(a**2*x**2)) - c**2*sqrt(c - c/(a**2*x**
2)) - 2*c**2*sqrt(c - c/(a**2*x**2))/(a*x) + 2*c**2*sqrt(c - c/(a**2*x**2))
/(a**2*x**2) + c**2*sqrt(c - c/(a**2*x**2))/(a**3*x**3) - c**2*sqrt(c - c/(
a**2*x**2))/(a**4*x**4)), x) - Integral(1/(a*c**2*x*sqrt(c - c/(a**2*x**2))
 - c**2*sqrt(c - c/(a**2*x**2)) - 2*c**2*sqrt(c - c/(a**2*x**2))/(a*x) + 2*
c**2*sqrt(c - c/(a**2*x**2))/(a**2*x**2) + c**2*sqrt(c - c/(a**2*x**2))/(a*
*3*x**3) - c**2*sqrt(c - c/(a**2*x**2))/(a**4*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)^2}{(a^2x^2-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

3.703 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=283

$$-\frac{142(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{2(ax+1)^3(107ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{782(ax+1)^2(1-ax)^3}{105a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{124(ax+1)^2(1-ax)^2}{105a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{2(ax+1)^2}{5a^3x^2\left(c-\right.}$$

```
[Out] (1 + a*x)^2/(7*a^2*(c - c/(a^2*x^2))^(7/2)*x) - (2*(1 - a*x)*(1 + a*x)^2)/(
5*a^3*(c - c/(a^2*x^2))^(7/2)*x^2) + (124*(1 - a*x)^2*(1 + a*x)^2)/(105*a^4
*(c - c/(a^2*x^2))^(7/2)*x^3) - (782*(1 - a*x)^3*(1 + a*x)^2)/(105*a^5*(c -
 c/(a^2*x^2))^(7/2)*x^4) - (142*(1 - a*x)^4*(1 + a*x)^2)/(35*a^6*(c - c/(a^
2*x^2))^(7/2)*x^5) - (2*(1 - a*x)^4*(1 + a*x)^3*(72 + 107*a*x))/(35*a^8*(c
- c/(a^2*x^2))^(7/2)*x^7) + (2*(1 - a*x)^(7/2)*(1 + a*x)^(7/2)*ArcSin[a*x])
/(a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```

Rubi [A] time = 0.427405, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6159, 6129, 98, 150, 143, 41, 216}

$$-\frac{142(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{2(ax+1)^3(107ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{782(ax+1)^2(1-ax)^3}{105a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{124(ax+1)^2(1-ax)^2}{105a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{2(ax+1)^2}{5a^3x^2\left(c-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```


```
[Out] (1 + a*x)^2/(7*a^2*(c - c/(a^2*x^2))^(7/2)*x) - (2*(1 - a*x)*(1 + a*x)^2)/(
5*a^3*(c - c/(a^2*x^2))^(7/2)*x^2) + (124*(1 - a*x)^2*(1 + a*x)^2)/(105*a^4
*(c - c/(a^2*x^2))^(7/2)*x^3) - (782*(1 - a*x)^3*(1 + a*x)^2)/(105*a^5*(c -
 c/(a^2*x^2))^(7/2)*x^4) - (142*(1 - a*x)^4*(1 + a*x)^2)/(35*a^6*(c - c/(a^
2*x^2))^(7/2)*x^5) - (2*(1 - a*x)^4*(1 + a*x)^3*(72 + 107*a*x))/(35*a^8*(c
- c/(a^2*x^2))^(7/2)*x^7) + (2*(1 - a*x)^(7/2)*(1 + a*x)^(7/2)*ArcSin[a*x])
/(a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```


Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^7}{(1-ax)^{7/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^7}{(1-ax)^{9/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^5(6+8ax)}{(1-ax)^{7/2}(1+ax)^{5/2}}\,dx}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^4(-70a-54a^2x)}{(1-ax)^{5/2}(1+ax)^{5/2}}\,dx}{35a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^3(496a^2}{(1-ax)^{3/2}}}{105a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} - \frac{((1-ax}{}$$

$$= \frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} - \frac{142(1-}{35a^6(}$$

$$= \frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} - \frac{142(1-}{35a^6(}$$

$$= \frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} - \frac{142(1-}{35a^6(}$$

$$= \frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} - \frac{142(1-}{35a^6(}$$

Mathematica [A] time = 0.115456, size = 133, normalized size = 0.47

$$\frac{-105a^6x^6+562a^5x^5-74a^4x^4-1226a^3x^3+636a^2x^2-210(ax-1)^3(ax+1)\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)+654ax-4}{105a^2c^3x(ax-1)^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (-432 + 654*a*x + 636*a^2*x^2 - 1226*a^3*x^3 - 74*a^4*x^4 + 562*a^5*x^5 - 1
05*a^6*x^6 - 210*(-1 + a*x)^3*(1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-
1 + a^2*x^2]])/(105*a^2*c^3*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)^3*(1 + a*x))
```

Maple [B] time = 0.151, size = 572, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] -1/105*(105*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^7*a^7-553*x^6*c^(7/2)*a
^6*((a*x-1)*(a*x+1)*c/a^2)^(5/2)+96*c^(7/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^6*a
^6-392*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^5*a^5-96*c^(7/2)*(c*(a^2*x^2
-1)/a^2)^(5/2)*x^5*a^5+1540*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^4*a^4-2
40*c^(7/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^4*a^4+210*ln(x*c^(1/2)+(c*(a^2*x^2-1
)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x*a^6
*c+350*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^3*a^3+240*c^(7/2)*(c*(a^2*x^
2-1)/a^2)^(5/2)*x^3*a^3-210*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1
)*(a*x+1)*c/a^2)^(5/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*a^5*c-1470*c^(7/2)*((a*x-1
)*(a*x+1)*c/a^2)^(5/2)*x^2*a^2+180*c^(7/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^2*a^
2-42*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x*a-180*c^(7/2)*(c*(a^2*x^2-1)/a
^2)^(5/2)*x*a+462*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)-30*c^(7/2)*(c*(a^2*
x^2-1)/a^2)^(5/2))*(a*x+1)/((a*x-1)*(a*x+1)*c/a^2)^(5/2)/x^7/(c*(a^2*x^2-1)
/a^2/x^2)^(7/2)/a^8/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2}{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

Fricas [A] time = 3.0785, size = 1044, normalized size = 3.69

$$\left[\frac{105\left(a^6x^6-2a^5x^5-a^4x^4+4a^3x^3-a^2x^2-2ax+1\right)\sqrt{c}\log\left(2a^2cx^2-2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-\left(105a^7x^7-562a^6x^6\right.}{105\left(a^7c^4x^6-2a^6c^4x^5-a^5c^4x^4+4a^4c^4x^3-a^3c^4x^2-2a^2c^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x +
1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)) - c) - (105*a^7*x^7 - 562*a^6*x^6 + 74*a^5*x^5 + 1226*a^4*x^4 - 636*a^3
*x^3 - 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^7*c^4*x^6
 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x
+ a*c^4), 1/105*(210*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 -
 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
)/(a^2*c*x^2 - c)) - (105*a^7*x^7 - 562*a^6*x^6 + 74*a^5*x^5 + 1226*a^4*x^4
 - 636*a^3*x^3 - 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a
^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*
a^2*c^4*x + a*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{ac^3x\sqrt{c-\frac{c}{a^2x^2}} - c^3\sqrt{c-\frac{c}{a^2x^2}} - \frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{ax} + \frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2} + \frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^3} - \frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^4} - \frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^5x^5} + \frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^6x^6}}\,dx - \int \frac{}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] -Integral(a*x/(a*c**3*x*sqrt(c - c/(a**2*x**2)) - c**3*sqrt(c - c/(a**2*x**
2)) - 3*c**3*sqrt(c - c/(a**2*x**2))/(a*x) + 3*c**3*sqrt(c - c/(a**2*x**2))
/(a**2*x**2) + 3*c**3*sqrt(c - c/(a**2*x**2))/(a**3*x**3) - 3*c**3*sqrt(c -
 c/(a**2*x**2))/(a**4*x**4) - c**3*sqrt(c - c/(a**2*x**2))/(a**5*x**5) + c*
*3*sqrt(c - c/(a**2*x**2))/(a**6*x**6)), x) - Integral(1/(a*c**3*x*sqrt(c -
 c/(a**2*x**2)) - c**3*sqrt(c - c/(a**2*x**2)) - 3*c**3*sqrt(c - c/(a**2*x*
*2))/(a*x) + 3*c**3*sqrt(c - c/(a**2*x**2))/(a**2*x**2) + 3*c**3*sqrt(c - c
/(a**2*x**2))/(a**3*x**3) - 3*c**3*sqrt(c - c/(a**2*x**2))/(a**4*x**4) - c*
*3*sqrt(c - c/(a**2*x**2))/(a**5*x**5) + c**3*sqrt(c - c/(a**2*x**2))/(a**6
*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)^2}{(a^2x^2-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

3.704 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{9/2}}\,dx$

Optimal. Leaf size=363

$$\frac{1334(ax+1)^3(1-ax)^5}{315a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}+\frac{2458(ax+1)^2(1-ax)^5}{315a^7x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}+\frac{2(ax+1)^4(1019ax+704)(1-ax)^5}{315a^{10}x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}}+\frac{302(ax+1)^2(1-ax)^4}{21a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}-\frac{64\cdots}{31\cdots}$$

```
[Out] (1 + a*x)^2/(9*a^2*(c - c/(a^2*x^2))^(9/2)*x) - (2*(1 - a*x)*(1 + a*x)^2)/(
7*a^3*(c - c/(a^2*x^2))^(9/2)*x^2) + (214*(1 - a*x)^2*(1 + a*x)^2)/(315*a^4
*(c - c/(a^2*x^2))^(9/2)*x^3) - (646*(1 - a*x)^3*(1 + a*x)^2)/(315*a^5*(c -
 c/(a^2*x^2))^(9/2)*x^4) + (302*(1 - a*x)^4*(1 + a*x)^2)/(21*a^6*(c - c/(a^
2*x^2))^(9/2)*x^5) + (2458*(1 - a*x)^5*(1 + a*x)^2)/(315*a^7*(c - c/(a^2*x^
2))^(9/2)*x^6) + (1334*(1 - a*x)^5*(1 + a*x)^3)/(315*a^8*(c - c/(a^2*x^2))^
(9/2)*x^7) + (2*(1 - a*x)^5*(1 + a*x)^4*(704 + 1019*a*x))/(315*a^10*(c - c/
(a^2*x^2))^(9/2)*x^9) - (2*(1 - a*x)^(9/2)*(1 + a*x)^(9/2)*ArcSin[a*x])/(a^
10*(c - c/(a^2*x^2))^(9/2)*x^9)
```

Rubi [A] time = 0.475402, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6159, 6129, 98, 150, 143, 41, 216}

$$\frac{1334(ax+1)^3(1-ax)^5}{315a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}+\frac{2458(ax+1)^2(1-ax)^5}{315a^7x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}+\frac{2(ax+1)^4(1019ax+704)(1-ax)^5}{315a^{10}x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}}+\frac{302(ax+1)^2(1-ax)^4}{21a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}-\frac{64\cdots}{31\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^(9/2),x]
```


```
[Out] (1 + a*x)^2/(9*a^2*(c - c/(a^2*x^2))^(9/2)*x) - (2*(1 - a*x)*(1 + a*x)^2)/(
7*a^3*(c - c/(a^2*x^2))^(9/2)*x^2) + (214*(1 - a*x)^2*(1 + a*x)^2)/(315*a^4
*(c - c/(a^2*x^2))^(9/2)*x^3) - (646*(1 - a*x)^3*(1 + a*x)^2)/(315*a^5*(c -
 c/(a^2*x^2))^(9/2)*x^4) + (302*(1 - a*x)^4*(1 + a*x)^2)/(21*a^6*(c - c/(a^
2*x^2))^(9/2)*x^5) + (2458*(1 - a*x)^5*(1 + a*x)^2)/(315*a^7*(c - c/(a^2*x^
2))^(9/2)*x^6) + (1334*(1 - a*x)^5*(1 + a*x)^3)/(315*a^8*(c - c/(a^2*x^2))^
(9/2)*x^7) + (2*(1 - a*x)^5*(1 + a*x)^4*(704 + 1019*a*x))/(315*a^10*(c - c/
(a^2*x^2))^(9/2)*x^9) - (2*(1 - a*x)^(9/2)*(1 + a*x)^(9/2)*ArcSin[a*x])/(a^
10*(c - c/(a^2*x^2))^(9/2)*x^9)
```


Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{9/2}}\,dx = \frac{\left((1-ax)^{9/2}(1+ax)^{9/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^9}{(1-ax)^{9/2}(1+ax)^{9/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}$$

$$= \frac{\left((1-ax)^{9/2}(1+ax)^{9/2}\right)\int \frac{x^9}{(1-ax)^{11/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{\left((1-ax)^{9/2}(1+ax)^{9/2}\right)\int \frac{x^7(8+10ax)}{(1-ax)^{9/2}(1+ax)^{7/2}}\,dx}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} - \frac{\left((1-ax)^{9/2}(1+ax)^{9/2}\right)\int \frac{x^6(-126a-88a^2x)}{(1-ax)^{7/2}(1+ax)^{7/2}}\,dx}{63a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{\left((1-ax)^{9/2}(1+ax)^{9/2}\right)\int \frac{x^5(1\cdots}{(1-\cdots}}{315a^6\left(c-\frac{c}{a^2x^2}\right)^{9/\cdots}}$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{646(1-ax)^3(1+ax)^2}{315a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4} - \frac{((1 \cdots}{\cdots}$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{646(1-ax)^3(1+ax)^2}{315a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4} + \frac{302\cdots}{21a\cdots}$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{646(1-ax)^3(1+ax)^2}{315a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4} + \frac{302\cdots}{21a\cdots}$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{646(1-ax)^3(1+ax)^2}{315a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4} + \frac{302\cdots}{21a\cdots}$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{646(1-ax)^3(1+ax)^2}{315a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4} + \frac{302\cdots}{21a\cdots}$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{646(1-ax)^3(1+ax)^2}{315a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4} + \frac{302\cdots}{21a\cdots}$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{646(1-ax)^3(1+ax)^2}{315a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4} + \frac{302\cdots}{21a\cdots}$$

Mathematica [A] time = 0.157839, size = 151, normalized size = 0.42

$$\frac{-315a^8x^8+1756a^7x^7+268a^6x^6-5784a^5x^5+2060a^4x^4+6200a^3x^3-3372a^2x^2-630(ax-1)^4(ax+1)^2\sqrt{a^2x^2-1}\,\mathrm{lo}\cdots}{315a^2c^4x(ax-1)^4(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] (1408 - 2186*a*x - 3372*a^2*x^2 + 6200*a^3*x^3 + 2060*a^4*x^4 - 5784*a^5*x^
5 + 268*a^6*x^6 + 1756*a^7*x^7 - 315*a^8*x^8 - 630*(-1 + a*x)^4*(1 + a*x)^2
*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(315*a^2*c^4*Sqrt[c - c/
```

```
(a^2*x^2)]*x*(-1 + a*x)^4*(1 + a*x)^2)
```

Maple [B] time = 0.195, size = 682, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(9/2),x)
```

```
[Out] -1/315*(315*c^(9/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^9*a^9-1185*x^8*c^(9/2)*
a^8*((a*x-1)*(a*x+1)*c/a^2)^(7/2)-256*c^(9/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^8
*a^8-2280*c^(9/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^7*a^7+256*c^(9/2)*(c*(a^2
*x^2-1)/a^2)^(7/2)*x^7*a^7+4620*c^(9/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^6*a
^6+896*c^(9/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^6*a^6+4620*c^(9/2)*((a*x-1)*(a*x
+1)*c/a^2)^(7/2)*x^5*a^5-896*c^(9/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^5*a^5+630*
ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*(c*(a
^2*x^2-1)/a^2)^(7/2)*x*a^8*c-7140*c^(9/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^4
*a^4-1120*c^(9/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^4*a^4-630*ln(x*c^(1/2)+(c*(a^
2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*(c*(a^2*x^2-1)/a^2)^(7/2
)*a^7*c-3948*c^(9/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^3*a^3+1120*c^(9/2)*(c*
(a^2*x^2-1)/a^2)^(7/2)*x^3*a^3+4998*c^(9/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x
^2*a^2+560*c^(9/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^2*a^2+1338*c^(9/2)*((a*x-1)*
(a*x+1)*c/a^2)^(7/2)*x*a-560*c^(9/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x*a-1338*c^(
9/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)-70*c^(9/2)*(c*(a^2*x^2-1)/a^2)^(7/2))*(a
*x+1)/((a*x-1)*(a*x+1)*c/a^2)^(7/2)/x^9/(c*(a^2*x^2-1)/a^2/x^2)^(9/2)/a^10/
c^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2}{(a^2x^2-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(9/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(9/2)), x)
```

Fricas [A] time = 4.27769, size = 1235, normalized size = 3.4

$$\left[\frac{315\left(a^8x^8-2a^7x^7-2a^6x^6+6a^5x^5-6a^3x^3+2a^2x^2+2ax-1\right)\sqrt{c}\log\left(2a^2cx^2-2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-\left(315a^9x^9-\right.}{315\left(a^9c^5x^8-2a^8c^5x^7-2a^7c^5x^6+6a^6c^5x^5-6\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/315*(315*(a^8*x^8 - 2*a^7*x^7 - 2*a^6*x^6 + 6*a^5*x^5 - 6*a^3*x^3 + 2*a^
2*x^2 + 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*
x^2 - c)/(a^2*x^2)) - c) - (315*a^9*x^9 - 1756*a^8*x^8 - 268*a^7*x^7 + 5784
*a^6*x^6 - 2060*a^5*x^5 - 6200*a^4*x^4 + 3372*a^3*x^3 + 2186*a^2*x^2 - 1408
*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^9*c^5*x^8 - 2*a^8*c^5*x^7 - 2*a^7
*c^5*x^6 + 6*a^6*c^5*x^5 - 6*a^4*c^5*x^3 + 2*a^3*c^5*x^2 + 2*a^2*c^5*x - a*
c^5), 1/315*(630*(a^8*x^8 - 2*a^7*x^7 - 2*a^6*x^6 + 6*a^5*x^5 - 6*a^3*x^3 +
 2*a^2*x^2 + 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (315*a^9*x^9 - 1756*a^8*x^8 - 268*a^7*x^7
+ 5784*a^6*x^6 - 2060*a^5*x^5 - 6200*a^4*x^4 + 3372*a^3*x^3 + 2186*a^2*x^2
- 1408*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^9*c^5*x^8 - 2*a^8*c^5*x^7 -
 2*a^7*c^5*x^6 + 6*a^6*c^5*x^5 - 6*a^4*c^5*x^3 + 2*a^3*c^5*x^2 + 2*a^2*c^5*
x - a*c^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{ac^4x\sqrt{c-\frac{c}{a^2x^2}} - c^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{ax} + \frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2} + \frac{6c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^3} - \frac{6c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^4} - \frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^5x^5} + \frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^6x^6} + \frac{c^4\sqrt{c-}}{a^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2)**(9/2),x)
```

```
[Out] -Integral(a*x/(a*c**4*x*sqrt(c - c/(a**2*x**2)) - c**4*sqrt(c - c/(a**2*x**
2)) - 4*c**4*sqrt(c - c/(a**2*x**2))/(a*x) + 4*c**4*sqrt(c - c/(a**2*x**2))
/(a**2*x**2) + 6*c**4*sqrt(c - c/(a**2*x**2))/(a**3*x**3) - 6*c**4*sqrt(c -
 c/(a**2*x**2))/(a**4*x**4) - 4*c**4*sqrt(c - c/(a**2*x**2))/(a**5*x**5) +
4*c**4*sqrt(c - c/(a**2*x**2))/(a**6*x**6) + c**4*sqrt(c - c/(a**2*x**2))/(
a**7*x**7) - c**4*sqrt(c - c/(a**2*x**2))/(a**8*x**8)), x) - Integral(1/(a*
c**4*x*sqrt(c - c/(a**2*x**2)) - c**4*sqrt(c - c/(a**2*x**2)) - 4*c**4*sqrt
(c - c/(a**2*x**2))/(a*x) + 4*c**4*sqrt(c - c/(a**2*x**2))/(a**2*x**2) + 6*
c**4*sqrt(c - c/(a**2*x**2))/(a**3*x**3) - 6*c**4*sqrt(c - c/(a**2*x**2))/(
a**4*x**4) - 4*c**4*sqrt(c - c/(a**2*x**2))/(a**5*x**5) + 4*c**4*sqrt(c - c
/(a**2*x**2))/(a**6*x**6) + c**4*sqrt(c - c/(a**2*x**2))/(a**7*x**7) - c**4
*sqrt(c - c/(a**2*x**2))/(a**8*x**8)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)^2}{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(9/2)), x)
```

3.705 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx$

Optimal. Leaf size=300

$$-\frac{a^9x^{10}\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{4a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{2a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{2\left(1-a^2x^2\right)^{9/2}}+\frac{8a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{5\left(1-a^2x^2\right)^{9/2}}-\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{7\left(1-a^2x^2\right)^{9/2}}$$

```
[Out] -((c - c/(a^2*x^2))^(9/2)*x)/(8*(1 - a^2*x^2)^(9/2)) - (3*a*(c - c/(a^2*x^2
))^(9/2)*x^2)/(7*(1 - a^2*x^2)^(9/2)) + (8*a^3*(c - c/(a^2*x^2))^(9/2)*x^4)
/(5*(1 - a^2*x^2)^(9/2)) + (3*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(2*(1 - a^2*
x^2)^(9/2)) - (2*a^5*(c - c/(a^2*x^2))^(9/2)*x^6)/(1 - a^2*x^2)^(9/2) - (4*
a^6*(c - c/(a^2*x^2))^(9/2)*x^7)/(1 - a^2*x^2)^(9/2) - (a^9*(c - c/(a^2*x^2
))^(9/2)*x^10)/(1 - a^2*x^2)^(9/2) - (3*a^8*(c - c/(a^2*x^2))^(9/2)*x^9*Log
[x])/(1 - a^2*x^2)^(9/2)
```

Rubi [A] time = 0.194142, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$-\frac{a^9x^{10}\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{4a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{2a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{2\left(1-a^2x^2\right)^{9/2}}+\frac{8a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{5\left(1-a^2x^2\right)^{9/2}}-\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{7\left(1-a^2x^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```


```
[Out] -((c - c/(a^2*x^2))^(9/2)*x)/(8*(1 - a^2*x^2)^(9/2)) - (3*a*(c - c/(a^2*x^2
))^(9/2)*x^2)/(7*(1 - a^2*x^2)^(9/2)) + (8*a^3*(c - c/(a^2*x^2))^(9/2)*x^4)
/(5*(1 - a^2*x^2)^(9/2)) + (3*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(2*(1 - a^2*
x^2)^(9/2)) - (2*a^5*(c - c/(a^2*x^2))^(9/2)*x^6)/(1 - a^2*x^2)^(9/2) - (4*
a^6*(c - c/(a^2*x^2))^(9/2)*x^7)/(1 - a^2*x^2)^(9/2) - (a^9*(c - c/(a^2*x^2
))^(9/2)*x^10)/(1 - a^2*x^2)^(9/2) - (3*a^8*(c - c/(a^2*x^2))^(9/2)*x^9*Log
[x])/(1 - a^2*x^2)^(9/2)
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{9/2}}{x^9}dx}{\left(1-a^2x^2\right)^{9/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^3(1+ax)^6}{x^9}dx}{\left(1-a^2x^2\right)^{9/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\left(-a^9+\frac{1}{x^9}+\frac{3a}{x^8}-\frac{8a^3}{x^6}-\frac{6a^4}{x^5}+\frac{6a^5}{x^4}+\frac{8a^6}{x^3}-\frac{3a^8}{x}\right)dx}{\left(1-a^2x^2\right)^{9/2}}$$

$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x}{8\left(1-a^2x^2\right)^{9/2}}-\frac{3a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{7\left(1-a^2x^2\right)^{9/2}}+\frac{8a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4}{5\left(1-a^2x^2\right)^{9/2}}+\frac{3a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{2\left(1-a^2x^2\right)^{9/2}}-$$

Mathematica [A] time = 0.0653958, size = 98, normalized size = 0.33

$$-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\left(280a^9x^9+1120a^6x^6+560a^5x^5-420a^4x^4-448a^3x^3+840a^8x^8\log(x)+120ax+35\right)}{280a^8x^7\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] -(c^4*Sqrt[c - c/(a^2*x^2)]*(35 + 120*a*x - 448*a^3*x^3 - 420*a^4*x^4 + 560
*a^5*x^5 + 1120*a^6*x^6 + 280*a^9*x^9 + 840*a^8*x^8*Log[x]))/(280*a^8*x^7*S
qrt[1 - a^2*x^2])
```

Maple [A] time = 0.16, size = 102, normalized size = 0.3

$$\frac{x\left(280\,a^9x^9+840\,a^8\ln(x)\,x^8+1120\,x^6a^6+560\,x^5a^5-420\,x^4a^4-448\,x^3a^3+120\,ax+35\right)}{280\,\left(a^2x^2-1\right)^5}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{9}{2}}\sqrt{-a^2x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(9/2),x)
```

```
[Out] 1/280*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/(a^2*x^2-1)^5*(-a^2*x^2+1)^(1/2)*(280
*a^9*x^9+840*a^8*ln(x)*x^8+1120*x^6*a^6+560*x^5*a^5-420*x^4*a^4-448*x^3*a^3
+120*a*x+35)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^(9/2)/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 2.3707, size = 1181, normalized size = 3.94

$$\left[\frac{420\left(a^9c^4x^9 - a^7c^4x^7\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right) + \left(280\,a^9c^4x^9 + 1120\,a^6c^4x^6 + 560\,a^5c^4x^5 - \right.}{280}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="fr
icas")
```

```
[Out] [1/280*(420*(a^9*c^4*x^9 - a^7*c^4*x^7)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2
 - c*x^4 - (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)) - c)/(a^2*x^4 - x^2)) + (280*a^9*c^4*x^9 + 1120*a^6*c^4*x^6 + 560
*a^5*c^4*x^5 - (280*a^9 + 1120*a^6 + 560*a^5 - 420*a^4 - 448*a^3 + 120*a +
35)*c^4*x^8 - 420*a^4*c^4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x + 35*c^4)*sqr
t(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^10*x^9 - a^8*x^7), 1/28
0*(840*(a^9*c^4*x^9 - a^7*c^4*x^7)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3
 + a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^
2 + c)) + (280*a^9*c^4*x^9 + 1120*a^6*c^4*x^6 + 560*a^5*c^4*x^5 - (280*a^9
+ 1120*a^6 + 560*a^5 - 420*a^4 - 448*a^3 + 120*a + 35)*c^4*x^8 - 420*a^4*c^
4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x + 35*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a^
2*c*x^2 - c)/(a^2*x^2)))/(a^10*x^9 - a^8*x^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(c - \frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^(9/2)/(-a^2*x^2 + 1)^(3/2), x)
```

3.706 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx$

Optimal. Leaf size=301

$$\frac{a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}-\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{5a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{2\left(1-a^2x^2\right)^{7/2}}+\frac{5a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{3\left(1-a^2x^2\right)^{7/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{4\left(1-a^2x^2\right)^{7/2}}-\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{5\left(1-a^2x^2\right)^{7/2}}$$

```
[Out] -((c - c/(a^2*x^2))^(7/2)*x)/(6*(1 - a^2*x^2)^(7/2)) - (3*a*(c - c/(a^2*x^2
))^(7/2)*x^2)/(5*(1 - a^2*x^2)^(7/2)) - (a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(
4*(1 - a^2*x^2)^(7/2)) + (5*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(3*(1 - a^2*x^
2)^(7/2)) + (5*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(2*(1 - a^2*x^2)^(7/2)) - (
a^5*(c - c/(a^2*x^2))^(7/2)*x^6)/(1 - a^2*x^2)^(7/2) + (a^7*(c - c/(a^2*x^2
))^(7/2)*x^8)/(1 - a^2*x^2)^(7/2) + (3*a^6*(c - c/(a^2*x^2))^(7/2)*x^7*Log[
x])/(1 - a^2*x^2)^(7/2)
```

Rubi [A] time = 0.196597, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$\frac{a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}-\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{5a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{2\left(1-a^2x^2\right)^{7/2}}+\frac{5a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{3\left(1-a^2x^2\right)^{7/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{4\left(1-a^2x^2\right)^{7/2}}-\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{5\left(1-a^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```


```
[Out] -((c - c/(a^2*x^2))^(7/2)*x)/(6*(1 - a^2*x^2)^(7/2)) - (3*a*(c - c/(a^2*x^2
))^(7/2)*x^2)/(5*(1 - a^2*x^2)^(7/2)) - (a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(
4*(1 - a^2*x^2)^(7/2)) + (5*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(3*(1 - a^2*x^
2)^(7/2)) + (5*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(2*(1 - a^2*x^2)^(7/2)) - (
a^5*(c - c/(a^2*x^2))^(7/2)*x^6)/(1 - a^2*x^2)^(7/2) + (a^7*(c - c/(a^2*x^2
))^(7/2)*x^8)/(1 - a^2*x^2)^(7/2) + (3*a^6*(c - c/(a^2*x^2))^(7/2)*x^7*Log[
x])/(1 - a^2*x^2)^(7/2)
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{7/2}}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{(1-ax)^2(1+ax)^5}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int \left(a^7+\frac{1}{x^7}+\frac{3a}{x^6}+\frac{a^2}{x^5}-\frac{5a^3}{x^4}-\frac{5a^4}{x^3}+\frac{a^5}{x^2}+\frac{3a^6}{x}\right)dx}{\left(1-a^2x^2\right)^{7/2}}$$
$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}{6\left(1-a^2x^2\right)^{7/2}}-\frac{3a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{5\left(1-a^2x^2\right)^{7/2}}-\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{4\left(1-a^2x^2\right)^{7/2}}+\frac{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3\left(1-a^2x^2\right)^{7/2}}+\frac{5}{-}$$

Mathematica [A] time = 0.0625543, size = 98, normalized size = 0.33

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(60a^7x^7-60a^5x^5+150a^4x^4+100a^3x^3-15a^2x^2+180a^6x^6\log(x)-36ax-10\right)}{60a^6x^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] -(c^3*Sqrt[c - c/(a^2*x^2)]*(-10 - 36*a*x - 15*a^2*x^2 + 100*a^3*x^3 + 150*
a^4*x^4 - 60*a^5*x^5 + 60*a^7*x^7 + 180*a^6*x^6*Log[x]))/(60*a^6*x^5*Sqrt[1
 - a^2*x^2])
```

Maple [A] time = 0.16, size = 102, normalized size = 0.3

$$\frac{x\left(60\,a^7x^7+180\,a^6\ln(x)\,x^6-60\,x^5a^5+150\,x^4a^4+100\,x^3a^3-15\,a^2x^2-36\,ax-10\right)}{60\,\left(a^2x^2-1\right)^4}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/60*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a^2*x^2-1)^4*(-a^2*x^2+1)^(1/2)*(60*a
^7*x^7+180*a^6*ln(x)*x^6-60*x^5*a^5+150*x^4*a^4+100*x^3*a^3-15*a^2*x^2-36*a
*x-10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^(7/2)/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 2.31201, size = 1148, normalized size = 3.81

$$\left[\frac{90\left(a^7c^3x^7 - a^5c^3x^5\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right) + \left(60\,a^7c^3x^7 - 60\,a^5c^3x^5 + 150\,a^4c^3x^4 - \left(60\,a\right.\right.}{60\left(a^8x^7 - \right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="fr
icas")
```

```
[Out] [1/60*(90*(a^7*c^3*x^7 - a^5*c^3*x^5)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 -
 c*x^4 - (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^
2*x^2)) - c)/(a^2*x^4 - x^2)) + (60*a^7*c^3*x^7 - 60*a^5*c^3*x^5 + 150*a^4*
c^3*x^4 - (60*a^7 - 60*a^5 + 150*a^4 + 100*a^3 - 15*a^2 - 36*a - 10)*c^3*x^
6 + 100*a^3*c^3*x^3 - 15*a^2*c^3*x^2 - 36*a*c^3*x - 10*c^3)*sqrt(-a^2*x^2 +
 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^8*x^7 - a^6*x^5), 1/60*(180*(a^7*c^
3*x^7 - a^5*c^3*x^5)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqrt(c
)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) + (60*
a^7*c^3*x^7 - 60*a^5*c^3*x^5 + 150*a^4*c^3*x^4 - (60*a^7 - 60*a^5 + 150*a^4
 + 100*a^3 - 15*a^2 - 36*a - 10)*c^3*x^6 + 100*a^3*c^3*x^3 - 15*a^2*c^3*x^2
 - 36*a*c^3*x - 10*c^3)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))
/(a^8*x^7 - a^6*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^(7/2)/(-a^2*x^2 + 1)^(3/2), x)
```

3.707 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=220

$$-\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4\left(1-a^2x^2\right)^{5/2}}-\frac{3a^4x^5\log(x)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] -((c - c/(a^2*x^2))^(5/2)*x)/(4*(1 - a^2*x^2)^(5/2)) - (a*(c - c/(a^2*x^2))
^(5/2)*x^2)/(1 - a^2*x^2)^(5/2) - (a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(1 - a^
2*x^2)^(5/2) + (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)/(1 - a^2*x^2)^(5/2) - (a
^5*(c - c/(a^2*x^2))^(5/2)*x^6)/(1 - a^2*x^2)^(5/2) - (3*a^4*(c - c/(a^2*x^
2))^(5/2)*x^5*Log[x])/(1 - a^2*x^2)^(5/2)
```

Rubi [A] time = 0.18485, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 75}

$$-\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4\left(1-a^2x^2\right)^{5/2}}-\frac{3a^4x^5\log(x)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] -((c - c/(a^2*x^2))^(5/2)*x)/(4*(1 - a^2*x^2)^(5/2)) - (a*(c - c/(a^2*x^2))
^(5/2)*x^2)/(1 - a^2*x^2)^(5/2) - (a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(1 - a^
2*x^2)^(5/2) + (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)/(1 - a^2*x^2)^(5/2) - (a
^5*(c - c/(a^2*x^2))^(5/2)*x^6)/(1 - a^2*x^2)^(5/2) - (3*a^4*(c - c/(a^2*x^
2))^(5/2)*x^5*Log[x])/(1 - a^2*x^2)^(5/2)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{3\tanh^{-1}(ax)}(1-a^2x^2)^{5/2}}{x^5}dx}{(1-a^2x^2)^{5/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{(1-ax)(1+ax)^4}{x^5}dx}{(1-a^2x^2)^{5/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \left(-a^5+\frac{1}{x^5}+\frac{3a}{x^4}+\frac{2a^2}{x^3}-\frac{2a^3}{x^2}-\frac{3a^4}{x}\right)dx}{(1-a^2x^2)^{5/2}}$$
$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}{4(1-a^2x^2)^{5/2}} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{(1-a^2x^2)^{5/2}} - \frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{(1-a^2x^2)^{5/2}} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-a^2x^2)^{5/2}} - \frac{a^5}{(}$$

Mathematica [A] time = 0.0457743, size = 90, normalized size = 0.41

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(4a^5x^5+5a^4x^4-8a^3x^3+4a^2x^2+12a^4x^4\log(x)+4ax+1\right)}{4a^4x^3\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] -(c^2*Sqrt[c - c/(a^2*x^2)]*(1 + 4*a*x + 4*a^2*x^2 - 8*a^3*x^3 + 5*a^4*x^4
+ 4*a^5*x^5 + 12*a^4*x^4*Log[x]))/(4*a^4*x^3*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.16, size = 86, normalized size = 0.4

$$\frac{x\left(4x^5a^5+12a^4\ln(x)x^4-8x^3a^3+4a^2x^2+4ax+1\right)}{4\left(a^2x^2-1\right)^3}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/4*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/(a^2*x^2-1)^3*(-a^2*x^2+1)^(1/2)*(4*x^5
*a^5+12*a^4*ln(x)*x^4-8*x^3*a^3+4*a^2*x^2+4*a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^(5/2)/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 2.34976, size = 959, normalized size = 4.36

$$\left[\frac{6\left(a^5c^2x^5-a^3c^2x^3\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)+\left(4\,a^5c^2x^5-8\,a^3c^2x^3-\left(4\,a^5-8\,a^3+4\,a^2+4\right.\right.}{4\left(a^6x^5-a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="fr
icas")
```

```
[Out] [1/4*(6*(a^5*c^2*x^5 - a^3*c^2*x^3)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c
*x^4 - (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)) - c)/(a^2*x^4 - x^2)) + (4*a^5*c^2*x^5 - 8*a^3*c^2*x^3 - (4*a^5 - 8*a
^3 + 4*a^2 + 4*a + 1)*c^2*x^4 + 4*a^2*c^2*x^2 + 4*a*c^2*x + c^2)*sqrt(-a^2*
x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5 - a^4*x^3), 1/4*(12*(a^5
*c^2*x^5 - a^3*c^2*x^3)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqr
t(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) + (
4*a^5*c^2*x^5 - 8*a^3*c^2*x^3 - (4*a^5 - 8*a^3 + 4*a^2 + 4*a + 1)*c^2*x^4 +
 4*a^2*c^2*x^2 + 4*a*c^2*x + c^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^6*x^5 - a^4*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^(5/2)/(-a^2*x^2 + 1)^(3/2), x)
```

3.708 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=145

$$\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2\left(1-a^2x^2\right)^{3/2}}+\frac{3a^2x^3\log(x)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] -((c - c/(a^2*x^2))^(3/2)*x)/(2*(1 - a^2*x^2)^(3/2)) - (3*a*(c - c/(a^2*x^2
))^(3/2)*x^2)/(1 - a^2*x^2)^(3/2) + (a^3*(c - c/(a^2*x^2))^(3/2)*x^4)/(1 -
a^2*x^2)^(3/2) + (3*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*Log[x])/(1 - a^2*x^2)^(
3/2)
```

Rubi [A] time = 0.170008, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 43}

$$\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2\left(1-a^2x^2\right)^{3/2}}+\frac{3a^2x^3\log(x)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -((c - c/(a^2*x^2))^(3/2)*x)/(2*(1 - a^2*x^2)^(3/2)) - (3*a*(c - c/(a^2*x^2
))^(3/2)*x^2)/(1 - a^2*x^2)^(3/2) + (a^3*(c - c/(a^2*x^2))^(3/2)*x^4)/(1 -
a^2*x^2)^(3/2) + (3*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*Log[x])/(1 - a^2*x^2)^(
3/2)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{e^{3\tanh^{-1}(ax)}(1-a^2x^2)^{3/2}}{x^3}dx}{(1-a^2x^2)^{3/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1+ax)^3}{x^3}dx}{(1-a^2x^2)^{3/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\left(a^3+\frac{1}{x^3}+\frac{3a}{x^2}+\frac{3a^2}{x}\right)dx}{(1-a^2x^2)^{3/2}}$$
$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}{2(1-a^2x^2)^{3/2}} - \frac{3a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{(1-a^2x^2)^{3/2}} + \frac{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^4}{(1-a^2x^2)^{3/2}} + \frac{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\log(x)}{(1-a^2x^2)^{3/2}}$$

Mathematica [A] time = 0.038023, size = 64, normalized size = 0.44

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(2a^3x^3+6a^2x^2\log(x)-6ax-1\right)}{2a^2x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -(c*Sqrt[c - c/(a^2*x^2)]*(-1 - 6*a*x + 2*a^3*x^3 + 6*a^2*x^2*Log[x]))/(2*a
^2*x*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.156, size = 70, normalized size = 0.5

$$\frac{x\left(2x^3a^3+6a^2\ln(x)x^2-6ax-1\right)}{2\left(a^2x^2-1\right)^2}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/2*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a^2*x^2-1)^2*(-a^2*x^2+1)^(1/2)*(2*x^3
*a^3+6*a^2*ln(x)*x^2-6*a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^(3/2)/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 2.23374, size = 779, normalized size = 5.37

$$\left[\frac{3\left(a^{3}cx^{3}-acx\right)\sqrt{-c}\log\left(\frac{a^{2}cx^{6}+a^{2}cx^{2}-cx^{4}-\left(ax^{5}-ax\right)\sqrt{-a^{2}x^{2}+1}\sqrt{-c}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c}{a^{2}x^{4}-x^{2}}\right)+\left(2\,a^{3}cx^{3}-\left(2\,a^{3}-6\,a-1\right)cx^{2}-6\,acx-c\right)\sqrt{-c}}{2\left(a^{4}x^{3}-a^{2}x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="fr
icas")
```

```
[Out] [1/2*(3*(a^3*c*x^3 - a*c*x)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 - (
a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) -
c)/(a^2*x^4 - x^2)) + (2*a^3*c*x^3 - (2*a^3 - 6*a - 1)*c*x^2 - 6*a*c*x - c)
*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3 - a^2*x), 1/2
*(6*(a^3*c*x^3 - a*c*x)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqr
t(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) + (
2*a^3*c*x^3 - (2*a^3 - 6*a - 1)*c*x^2 - 6*a*c*x - c)*sqrt(-a^2*x^2 + 1)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3 - a^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)^{3}}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2)*(a*x + 1)**3/(-(a*x - 1)*
(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^{3}\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{3}{2}}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^(3/2)/(-a^2*x^2 + 1)^(3/2), x)
```

3.709 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=108

$$-\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -((a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2]) + (Sqrt[c - c/(a^2*x^2)]
*x*Log[x])/Sqrt[1 - a^2*x^2] - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sqr
t[1 - a^2*x^2]
```

Rubi [A] time = 0.151778, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 72}

$$-\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -((a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2]) + (Sqrt[c - c/(a^2*x^2)]
*x*Log[x])/Sqrt[1 - a^2*x^2] - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sqr
t[1 - a^2*x^2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^2}{x(1-ax)}dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(-a+\frac{1}{x}-\frac{4a}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\sqrt{1-a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}} - \frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0304213, size = 47, normalized size = 0.44

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}(-ax-4\log(1-ax)+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-(a*x) + Log[x] - 4*Log[1 - a*x]))/Sqrt[1 - a^2*x
^2]
```

Maple [A] time = 0.158, size = 61, normalized size = 0.6

$$-\frac{x(-ax+\ln(x)-4\ln(ax-1))}{a^2x^2-1}\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a*x+ln(x)-4*ln(a*x-1))*(-a^2*x^2+1)^(1/2
)/(a^2*x^2-1)
```

Maxima [C] time = 1.3531, size = 200, normalized size = 1.85

$$-\frac{1}{2}a^3\left(-\frac{2i\sqrt{c}x}{a^3}+\frac{i\sqrt{c}\log(ax+1)}{a^4}-\frac{i\sqrt{c}\log(ax-1)}{a^4}\right)-\frac{3}{2}a^2\left(-\frac{i\sqrt{c}\log(ax+1)}{a^3}-\frac{i\sqrt{c}\log(ax-1)}{a^3}\right)-\frac{3}{2}a\left(\frac{i\sqrt{c}\log}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/2*a^3*(-2*I*sqrt(c)*x/a^3 + I*sqrt(c)*log(a*x + 1)/a^4 - I*sqrt(c)*log(a
*x - 1)/a^4) - 3/2*a^2*(-I*sqrt(c)*log(a*x + 1)/a^3 - I*sqrt(c)*log(a*x - 1
)/a^3) - 3/2*a*(I*sqrt(c)*log(a*x + 1)/a^2 - I*sqrt(c)*log(a*x - 1)/a^2) +
```

```
1/2*I*sqrt(c)*log(a*x + 1)/a + 1/2*I*sqrt(c)*log(a*x - 1)/a - I*sqrt(c)*log
(x)/a
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}(ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*
x^2 - 2*a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1)*(a*
x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))/(-a^2*x^2 + 1)^(3/2), x)
```

3.710 $\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=123

$$\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-a^2x^2}}{a^2x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-a^2x^2}\log(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(a*Sqrt[c - c/(a^2*x^2)]) + (2*Sqrt[1 - a^2*x^2])/(a^2*Sq
rt[c - c/(a^2*x^2)]*x*(1 - a*x)) + (3*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(a^2*
Sqrt[c - c/(a^2*x^2)]*x)
```

Rubi [A] time = 0.139521, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 77}

$$\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-a^2x^2}}{a^2x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-a^2x^2}\log(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a*Sqrt[c - c/(a^2*x^2)]) + (2*Sqrt[1 - a^2*x^2])/(a^2*Sq
rt[c - c/(a^2*x^2)]*x*(1 - a*x)) + (3*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(a^2*
Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{3\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x(1+ax)}{(1-ax)^2}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{a}+\frac{2}{a(-1+ax)^2}+\frac{3}{a(-1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-a^2x^2}}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}+\frac{3\sqrt{1-a^2x^2}\log(1-ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.0445779, size = 59, normalized size = 0.48

$$\frac{\sqrt{1-a^2x^2}\left(ax+\frac{2}{1-ax}+3\log(1-ax)\right)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(a*x + 2/(1 - a*x) + 3*Log[1 - a*x]))/(a^2*Sqrt[c - c/(a
^2*x^2)]*x)
```

Maple [A] time = 0.151, size = 77, normalized size = 0.6

$$\frac{a^2x^2+3\ln(ax-1)xa-ax-3\ln(ax-1)-2}{a^2x(ax-1)}\sqrt{-a^2x^2+1}\frac{1}{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-a^2*x^2+1)^(1/2)/a^2*(a^2*x^2+3*ln(a*x-
1)*x*a-a*x-3*ln(a*x-1)-2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{i}{2(ax+1)(ax-1)a\sqrt{c}}-\int \frac{a^4x^4+3a^3x^3+3a^2x^2}{\left(ia^2\sqrt{c}x^2-i\sqrt{c}\right)(ax+1)(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/2*I/((a*x + 1)*(a*x - 1)*a*sqrt(c)) - integrate((a^4*x^4 + 3*a^3*x^3 + 3
*a^2*x^2)/((I*a^2*sqrt(c)*x^2 - I*sqrt(c))*(a*x + 1)*(a*x - 1)), x)
```

Fricas [A] time = 2.05115, size = 910, normalized size = 7.4

$$\left[-\frac{3\left(a^3x^3 - a^2x^2 - ax + 1\right)\sqrt{-c}\log\left(\frac{a^6cx^6 - 4a^5cx^5 + 5a^4cx^4 - 4a^2cx^2 + 4acx + \left(a^5x^5 - 4a^4x^4 + 6a^3x^3 - 4a^2x^2\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}} - 2c}{a^4x^4 - 2a^3x^3 + 2ax - 1}\right) - 2\left(\right.}{2\left(a^4cx^3 - a^3cx^2 - a^2cx + ac\right)} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fr
icas")
```

```
[Out] [-1/2*(3*(a^3*x^3 - a^2*x^2 - a*x + 1)*sqrt(-c)*log((a^6*c*x^6 - 4*a^5*c*x^
5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 + 4*a*c*x + (a^5*x^5 - 4*a^4*x^4 + 6*a^3*x^3
- 4*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) -
2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)) - 2*(a^3*x^3 - 3*a^2*x^2)*sqrt(-a^2
*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*c*x^3 - a^3*c*x^2 - a^2*c*x
 + a*c), (3*(a^3*x^3 - a^2*x^2 - a*x + 1)*sqrt(c)*arctan((a^2*x^2 - 2*a*x +
 2)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 -
 2*a^2*c*x^2 - a*c*x + 2*c)) + (a^3*x^3 - 3*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*c*x^3 - a^3*c*x^2 - a^2*c*x + a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(-1 + 1/(a*x))
*(1 + 1/(a*x)))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))), x)
```

3.711 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{3\left(1-a^2x^2\right)^{3/2}}{a^4x^3(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4x^3(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{3\left(1-a^2x^2\right)^{3/2}\log(1-ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[Out] -((1 - a^2*x^2)^(3/2)/(a^3*(c - c/(a^2*x^2))^(3/2)*x^2)) + (1 - a^2*x^2)^(3
/2)/(2*a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 - a*x)^2) - (3*(1 - a^2*x^2)^(3/2
))/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 - a*x)) - (3*(1 - a^2*x^2)^(3/2)*Log
[1 - a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rubi [A] time = 0.191601, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 43}

$$-\frac{3\left(1-a^2x^2\right)^{3/2}}{a^4x^3(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4x^3(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{3\left(1-a^2x^2\right)^{3/2}\log(1-ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```


```
[Out] -((1 - a^2*x^2)^(3/2)/(a^3*(c - c/(a^2*x^2))^(3/2)*x^2)) + (1 - a^2*x^2)^(3
/2)/(2*a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 - a*x)^2) - (3*(1 - a^2*x^2)^(3/2
))/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 - a*x)) - (3*(1 - a^2*x^2)^(3/2)*Log
[1 - a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{e^{3\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{x^3}{(1-ax)^3}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}\int \left(-\frac{1}{a^3}-\frac{1}{a^3(-1+ax)^3}-\frac{3}{a^3(-1+ax)^2}-\frac{3}{a^3(-1+ax)}\right)dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$
$$= -\frac{\left(1-a^2x^2\right)^{3/2}}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2} + \frac{\left(1-a^2x^2\right)^{3/2}}{2a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3(1-ax)^2} - \frac{3\left(1-a^2x^2\right)^{3/2}}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3(1-ax)} - \frac{3\left(1-a^2x^2\right)^{3/2}}{a^4\left(c-\frac{c}{a^2x^2}\right.}$$

Mathematica [A] time = 0.0556207, size = 87, normalized size = 0.5

$$\frac{\sqrt{1-a^2x^2}\left(2a^3x^3-4a^2x^2-4ax+6(ax-1)^2\log(1-ax)+5\right)}{2a^2cx(ax-1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(5 - 4*a*x - 4*a^2*x^2 + 2*a^3*x^3 + 6*(-1 + a*x)^2*Log[
1 - a*x]))/(2*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)^2)
```

Maple [A] time = 0.182, size = 106, normalized size = 0.6

$$\frac{\left(2x^3a^3+6\ln(ax-1)a^2x^2-4a^2x^2-12\ln(ax-1)xa-4ax+6\ln(ax-1)+5\right)(ax+1)}{(2ax-2)a^4x^3}\sqrt{-a^2x^2+1}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/2*(2*x^3*a^3+6*ln(a*x-1)*a^2*x^2-4*a^2*x^2-12*ln(a*x-1)*x*a-4*a*x+6*ln(a*
x-1)+5)*(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^
(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(3/2)), x)
```

Fricas [A] time = 2.18, size = 994, normalized size = 5.71

$$\left[-\frac{3\left(a^4x^4-2a^3x^3+2ax-1\right)\sqrt{-c}\log\left(\frac{a^6cx^6-4a^5cx^5+5a^4cx^4-4a^2cx^2+4acx+\left(a^5x^5-4a^4x^4+6a^3x^3-4a^2x^2\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{a^4x^4-2a^3x^3+2ax-1}\right)-(2\ldots}{2\left(a^5c^2x^4-2a^4c^2x^3+2a^2c^2x-ac^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fr
icas")
```

```
[Out] [-1/2*(3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*sqrt(-c)*log((a^6*c*x^6 - 4*a^5*
c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 + 4*a*c*x + (a^5*x^5 - 4*a^4*x^4 + 6*a^3*
x^3 - 4*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)) - (2*a^4*x^4 - 9*a^3*x^3 + 6*a^
2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^2*x^4 - 2
*a^4*c^2*x^3 + 2*a^2*c^2*x - a*c^2), 1/2*(6*(a^4*x^4 - 2*a^3*x^3 + 2*a*x -
1)*sqrt(c)*arctan((a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^
2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 - 2*a^2*c*x^2 - a*c*x + 2*c)) + (2*a^4*x
^4 - 9*a^3*x^3 + 6*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)))/(a^5*c^2*x^4 - 2*a^4*c^2*x^3 + 2*a^2*c^2*x - a*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(-1 + 1/(a*x))*(1
+ 1/(a*x)))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(3/2)), x)
```

3.712 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=269

$$\frac{31\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{9\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{6a^6x^5(1-ax)^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{49\left(1-\right.}{16a}$$

```
[Out] (1 - a^2*x^2)^(5/2)/(a^5*(c - c/(a^2*x^2))^(5/2)*x^4) + (1 - a^2*x^2)^(5/2)
/(6*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)^3) - (9*(1 - a^2*x^2)^(5/2))/
(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)^2) + (31*(1 - a^2*x^2)^(5/2))/
(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)) + (49*(1 - a^2*x^2)^(5/2)*Log
[1 - a*x])/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) - ((1 - a^2*x^2)^(5/2)*Log[
1 + a*x])/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rubi [A] time = 0.218608, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$\frac{31\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{9\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{6a^6x^5(1-ax)^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{49\left(1-\right.}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```


```
[Out] (1 - a^2*x^2)^(5/2)/(a^5*(c - c/(a^2*x^2))^(5/2)*x^4) + (1 - a^2*x^2)^(5/2)
/(6*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)^3) - (9*(1 - a^2*x^2)^(5/2))/
(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)^2) + (31*(1 - a^2*x^2)^(5/2))/
(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)) + (49*(1 - a^2*x^2)^(5/2)*Log
[1 - a*x])/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) - ((1 - a^2*x^2)^(5/2)*Log[
1 + a*x])/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{e^{3\tanh^{-1}(ax)}x^5}{\left(1-a^2x^2\right)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{x^5}{(1-ax)^4(1+ax)}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}\int \left(\frac{1}{a^5}+\frac{1}{2a^5(-1+ax)^4}+\frac{9}{4a^5(-1+ax)^3}+\frac{31}{8a^5(-1+ax)^2}+\frac{49}{16a^5(-1+ax)}-\frac{1}{16a^5(1+ax)}\right)dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}}{a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}+\frac{\left(1-a^2x^2\right)^{5/2}}{6a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5(1-ax)^3}-\frac{9\left(1-a^2x^2\right)^{5/2}}{8a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5(1-ax)^2}+\frac{31\left(1-a^2x^2\right.}{8a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}$$

Mathematica [A] time = 0.0817772, size = 113, normalized size = 0.42

$$\frac{\sqrt{1-a^2x^2}\left(2\left(24a^4x^4-72a^3x^3-21a^2x^2+135ax-70\right)+147(ax-1)^3\log(1-ax)-3(ax-1)^3\log(ax+1)\right)}{48a^2c^2x(ax-1)^3\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*(-70 + 135*a*x - 21*a^2*x^2 - 72*a^3*x^3 + 24*a^4*x^4
) + 147*(-1 + a*x)^3*Log[1 - a*x] - 3*(-1 + a*x)^3*Log[1 + a*x]))/(48*a^2*c
^2*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)^3)
```

Maple [A] time = 0.161, size = 176, normalized size = 0.7

$$-\frac{\left(-48x^4a^4+3a^3x^3\ln(ax+1)-147\ln(ax-1)x^3a^3+144x^3a^3-9\ln(ax+1)a^2x^2+441\ln(ax-1)a^2x^2+42a^2x^2+\right.}{(48ax-48)a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] -1/48*(-48*x^4*a^4+3*a^3*x^3*ln(a*x+1)-147*ln(a*x-1)*x^3*a^3+144*x^3*a^3-9*
ln(a*x+1)*a^2*x^2+441*ln(a*x-1)*a^2*x^2+42*a^2*x^2+9*a*x*ln(a*x+1)-441*ln(a
*x-1)*x*a-270*a*x-3*ln(a*x+1)+147*ln(a*x-1)+140)*(a*x+1)^2*(-a^2*x^2+1)^(1/
2)/(a*x-1)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}a^6x^6\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^7c^3x^7-3\,a^6c^3x^6+a^5c^3x^5+5\,a^4c^3x^4-5\,a^3c^3x^3-a^2c^3x^2+3\,ac^3x-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*a^6*x^6*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^7*c^
3*x^7 - 3*a^6*c^3*x^6 + a^5*c^3*x^5 + 5*a^4*c^3*x^4 - 5*a^3*c^3*x^3 - a^2*c
^3*x^2 + 3*a*c^3*x - c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(5/2)), x)
```

3.713 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=363

$$-\frac{75\left(1-a^2x^2\right)^{7/2}}{16a^8x^7(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{59\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{\left(1-a^2x^2\right)^{7/2}}{2a^8x^7(1-ax)^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

```
[Out] -((1 - a^2*x^2)^(7/2)/(a^7*(c - c/(a^2*x^2))^(7/2)*x^6)) + (1 - a^2*x^2)^(7
/2)/(16*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^4) - (1 - a^2*x^2)^(7/2)/
(2*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^3) + (59*(1 - a^2*x^2)^(7/2))/
(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^2) - (75*(1 - a^2*x^2)^(7/2))
/(16*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)) + (1 - a^2*x^2)^(7/2)/(32*a
^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)) - (201*(1 - a^2*x^2)^(7/2)*Log[1
- a*x])/(64*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) + (9*(1 - a^2*x^2)^(7/2)*Log[1
 + a*x])/(64*a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```

Rubi [A] time = 0.2468, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$-\frac{75\left(1-a^2x^2\right)^{7/2}}{16a^8x^7(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{59\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{\left(1-a^2x^2\right)^{7/2}}{2a^8x^7(1-ax)^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```


```
[Out] -((1 - a^2*x^2)^(7/2)/(a^7*(c - c/(a^2*x^2))^(7/2)*x^6)) + (1 - a^2*x^2)^(7
/2)/(16*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^4) - (1 - a^2*x^2)^(7/2)/
(2*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^3) + (59*(1 - a^2*x^2)^(7/2))/
(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^2) - (75*(1 - a^2*x^2)^(7/2))
/(16*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)) + (1 - a^2*x^2)^(7/2)/(32*a
^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)) - (201*(1 - a^2*x^2)^(7/2)*Log[1
- a*x])/(64*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) + (9*(1 - a^2*x^2)^(7/2)*Log[1
 + a*x])/(64*a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left(1-a^2x^2\right)^{7/2}\int \frac{e^{3\tanh^{-1}(ax)}x^7}{\left(1-a^2x^2\right)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$
$$= \frac{\left(1-a^2x^2\right)^{7/2}\int \frac{x^7}{(1-ax)^5(1+ax)^2}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$
$$= \frac{\left(1-a^2x^2\right)^{7/2}\int \left(-\frac{1}{a^7}-\frac{1}{4a^7(-1+ax)^5}-\frac{3}{2a^7(-1+ax)^4}-\frac{59}{16a^7(-1+ax)^3}-\frac{75}{16a^7(-1+ax)^2}-\frac{201}{64a^7(-1+ax)}-\frac{1}{32a^7(1+a\ldots}\right.}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$
$$= -\frac{\left(1-a^2x^2\right)^{7/2}}{a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}+\frac{\left(1-a^2x^2\right)^{7/2}}{16a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7(1-ax)^4}-\frac{\left(1-a^2x^2\right)^{7/2}}{2a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7(1-ax)^3}+\frac{59\,(1\ldots}{32a^8\left(c-\ldots\right.}$$

Mathematica [A] time = 0.115289, size = 146, normalized size = 0.4

$$\frac{\sqrt{1-a^2x^2}\left(2\left(32a^6x^6-96a^5x^5-87a^4x^4+309a^3x^3-59a^2x^2-207ax+104\right)+201(ax+1)(ax-1)^4\log(1-ax)-9(a\ldots\right.}{64a^2c^3x(ax-1)^4(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*(104 - 207*a*x - 59*a^2*x^2 + 309*a^3*x^3 - 87*a^4*x^
4 - 96*a^5*x^5 + 32*a^6*x^6) + 201*(-1 + a*x)^4*(1 + a*x)*Log[1 - a*x] - 9*
(-1 + a*x)^4*(1 + a*x)*Log[1 + a*x]))/(64*a^2*c^3*Sqrt[c - c/(a^2*x^2)]*x*(
-1 + a*x)^4*(1 + a*x))
```

Maple [A] time = 0.161, size = 248, normalized size = 0.7

$$-\frac{(ax+1)^2\left(-64x^6a^6+9\ln(ax+1)x^5a^5-201\ln(ax-1)x^5a^5+192x^5a^5-27\ln(ax+1)a^4x^4+603\ln(ax-1)a^4x\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] -1/64*(-a^2*x^2+1)^(1/2)*(a*x+1)^2*(-64*x^6*a^6+9*ln(a*x+1)*x^5*a^5-201*ln(
a*x-1)*x^5*a^5+192*x^5*a^5-27*ln(a*x+1)*a^4*x^4+603*ln(a*x-1)*a^4*x^4+174*x
^4*a^4+18*a^3*x^3*ln(a*x+1)-402*ln(a*x-1)*x^3*a^3-618*x^3*a^3+18*ln(a*x+1)*
a^2*x^2-402*ln(a*x-1)*a^2*x^2+118*a^2*x^2-27*a*x*ln(a*x+1)+603*ln(a*x-1)*x*
a+414*a*x+9*ln(a*x+1)-201*ln(a*x-1)-208)/(a*x-1)/a^8/x^7/(c*(a^2*x^2-1)/a^2
/x^2)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}a^8x^8\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^9c^4x^9-3\,a^8c^4x^8+8\,a^6c^4x^6-6\,a^5c^4x^5-6\,a^4c^4x^4+8\,a^3c^4x^3-3\,ac^4x+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*a^8*x^8*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^9*c^
4*x^9 - 3*a^8*c^4*x^8 + 8*a^6*c^4*x^6 - 6*a^5*c^4*x^5 - 6*a^4*c^4*x^4 + 8*a
^3*c^4*x^3 - 3*a*c^4*x + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(7/2)), x)
```

3.714 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal. Leaf size=375

$$-\frac{a^9x^{10}\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{4a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{2a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{2a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{2\left(1-a^2x^2\right)^{9/2}}-\frac{4a^3x^4\left(c-\right.}{5\left(1-a\right.}$$

```
[Out] -((c - c/(a^2*x^2))^(9/2)*x)/(8*(1 - a^2*x^2)^(9/2)) + (a*(c - c/(a^2*x^2))
^(9/2)*x^2)/(7*(1 - a^2*x^2)^(9/2)) + (2*a^2*(c - c/(a^2*x^2))^(9/2)*x^3)/(
3*(1 - a^2*x^2)^(9/2)) - (4*a^3*(c - c/(a^2*x^2))^(9/2)*x^4)/(5*(1 - a^2*x^
2)^(9/2)) - (3*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(2*(1 - a^2*x^2)^(9/2)) + (
2*a^5*(c - c/(a^2*x^2))^(9/2)*x^6)/(1 - a^2*x^2)^(9/2) + (2*a^6*(c - c/(a^2
*x^2))^(9/2)*x^7)/(1 - a^2*x^2)^(9/2) - (4*a^7*(c - c/(a^2*x^2))^(9/2)*x^8)
/(1 - a^2*x^2)^(9/2) - (a^9*(c - c/(a^2*x^2))^(9/2)*x^10)/(1 - a^2*x^2)^(9/
2) + (a^8*(c - c/(a^2*x^2))^(9/2)*x^9*Log[x])/(1 - a^2*x^2)^(9/2)
```

Rubi [A] time = 0.207394, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$-\frac{a^9x^{10}\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{4a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{2a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{2a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{2\left(1-a^2x^2\right)^{9/2}}-\frac{4a^3x^4\left(c-\right.}{5\left(1-a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(9/2)/E^ArcTanh[a*x],x]
```


```
[Out] -((c - c/(a^2*x^2))^(9/2)*x)/(8*(1 - a^2*x^2)^(9/2)) + (a*(c - c/(a^2*x^2))
^(9/2)*x^2)/(7*(1 - a^2*x^2)^(9/2)) + (2*a^2*(c - c/(a^2*x^2))^(9/2)*x^3)/(
3*(1 - a^2*x^2)^(9/2)) - (4*a^3*(c - c/(a^2*x^2))^(9/2)*x^4)/(5*(1 - a^2*x^
2)^(9/2)) - (3*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(2*(1 - a^2*x^2)^(9/2)) + (
2*a^5*(c - c/(a^2*x^2))^(9/2)*x^6)/(1 - a^2*x^2)^(9/2) + (2*a^6*(c - c/(a^2
*x^2))^(9/2)*x^7)/(1 - a^2*x^2)^(9/2) - (4*a^7*(c - c/(a^2*x^2))^(9/2)*x^8)
/(1 - a^2*x^2)^(9/2) - (a^9*(c - c/(a^2*x^2))^(9/2)*x^10)/(1 - a^2*x^2)^(9/
2) + (a^8*(c - c/(a^2*x^2))^(9/2)*x^9*Log[x])/(1 - a^2*x^2)^(9/2)
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{9/2}}{x^9}dx}{\left(1-a^2x^2\right)^{9/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^5(1+ax)^4}{x^9}dx}{\left(1-a^2x^2\right)^{9/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\left(-a^9+\frac{1}{x^9}-\frac{a}{x^8}-\frac{4a^2}{x^7}+\frac{4a^3}{x^6}+\frac{6a^4}{x^5}-\frac{6a^5}{x^4}-\frac{4a^6}{x^3}+\frac{4a^7}{x^2}+\frac{a^8}{x}\right)dx}{\left(1-a^2x^2\right)^{9/2}}$$
$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x}{8\left(1-a^2x^2\right)^{9/2}}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{7\left(1-a^2x^2\right)^{9/2}}+\frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3}{3\left(1-a^2x^2\right)^{9/2}}-\frac{4a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4}{5\left(1-a^2x^2\right)^{9/2}}-\frac{3a^4(}{2(}$$

Mathematica [A] time = 0.105585, size = 115, normalized size = 0.31

$$\frac{x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(\frac{2a^6}{x^2}+\frac{2a^5}{x^3}-\frac{3a^4}{2x^4}-\frac{4a^3}{5x^5}+\frac{2a^2}{3x^6}+a^9(-x)-\frac{4a^7}{x}+a^8\log(x)+\frac{a}{7x^7}-\frac{1}{8x^8}\right)}{\left(1-a^2x^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(9/2)/E^ArcTanh[a*x],x]
```

```
[Out] ((c - c/(a^2*x^2))^(9/2)*x^9*(-1/(8*x^8) + a/(7*x^7) + (2*a^2)/(3*x^6) - (4
*a^3)/(5*x^5) - (3*a^4)/(2*x^4) + (2*a^5)/x^3 + (2*a^6)/x^2 - (4*a^7)/x - a
^9*x + a^8*Log[x]))/(1 - a^2*x^2)^(9/2)
```

Maple [A] time = 0.161, size = 118, normalized size = 0.3

$$-\frac{x\left(-840\,a^9x^9+840\,a^8\ln(x)\,x^8-3360\,a^7x^7+1680\,x^6a^6+1680\,x^5a^5-1260\,x^4a^4-672\,x^3a^3+560\,a^2x^2+120\,ax-105\right)}{840\,\left(a^2x^2-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/840*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/(a^2*x^2-1)^5*(-a^2*x^2+1)^(1/2)*(-8
40*a^9*x^9+840*a^8*ln(x)*x^8-3360*a^7*x^7+1680*x^6*a^6+1680*x^5*a^5-1260*x^
4*a^4-672*x^3*a^3+560*a^2*x^2+120*a*x-105)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(9/2)/(a*x + 1), x)
```

Fricas [A] time = 2.26478, size = 1355, normalized size = 3.61

$$\left[\frac{420\left(a^9c^4x^9-a^7c^4x^7\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)+\left(840\,a^9c^4x^9+3360\,a^7c^4x^7-1680\,a^6c^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/840*(420*(a^9*c^4*x^9 - a^7*c^4*x^7)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2
 - c*x^4 + (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)) - c)/(a^2*x^4 - x^2)) + (840*a^9*c^4*x^9 + 3360*a^7*c^4*x^7 - 168
0*a^6*c^4*x^6 - 1680*a^5*c^4*x^5 - (840*a^9 + 3360*a^7 - 1680*a^6 - 1680*a^
5 + 1260*a^4 + 672*a^3 - 560*a^2 - 120*a + 105)*c^4*x^8 + 1260*a^4*c^4*x^4
+ 672*a^3*c^4*x^3 - 560*a^2*c^4*x^2 - 120*a*c^4*x + 105*c^4)*sqrt(-a^2*x^2
+ 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^10*x^9 - a^8*x^7), -1/840*(840*(a^
9*c^4*x^9 - a^7*c^4*x^7)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sq
rt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) -
(840*a^9*c^4*x^9 + 3360*a^7*c^4*x^7 - 1680*a^6*c^4*x^6 - 1680*a^5*c^4*x^5 -
 (840*a^9 + 3360*a^7 - 1680*a^6 - 1680*a^5 + 1260*a^4 + 672*a^3 - 560*a^2 -
 120*a + 105)*c^4*x^8 + 1260*a^4*c^4*x^4 + 672*a^3*c^4*x^3 - 560*a^2*c^4*x^
2 - 120*a*c^4*x + 105*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)))/(a^10*x^9 - a^8*x^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(9/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(9/2)/(a*x + 1), x)
```

3.715 $\int e^{-\tanh^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^{7/2} dx$

Optimal. Leaf size=299

$$\frac{a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{3a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}-\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{2\left(1-a^2x^2\right)^{7/2}}-\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{3a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{4\left(1-a^2x^2\right)^{7/2}}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{5\left(1-a^2x^2\right)^{7/2}}$$

```
[Out] -((c - c/(a^2*x^2))^(7/2)*x)/(6*(1 - a^2*x^2)^(7/2)) + (a*(c - c/(a^2*x^2))
^(7/2)*x^2)/(5*(1 - a^2*x^2)^(7/2)) + (3*a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(
4*(1 - a^2*x^2)^(7/2)) - (a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(1 - a^2*x^2)^(7
/2) - (3*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(2*(1 - a^2*x^2)^(7/2)) + (3*a^5*
(c - c/(a^2*x^2))^(7/2)*x^6)/(1 - a^2*x^2)^(7/2) + (a^7*(c - c/(a^2*x^2))^(
7/2)*x^8)/(1 - a^2*x^2)^(7/2) - (a^6*(c - c/(a^2*x^2))^(7/2)*x^7*Log[x])/(1
 - a^2*x^2)^(7/2)
```

Rubi [A] time = 0.192087, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$\frac{a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{3a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}-\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{2\left(1-a^2x^2\right)^{7/2}}-\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{3a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{4\left(1-a^2x^2\right)^{7/2}}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{5\left(1-a^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^ArcTanh[a*x],x]
```


```
[Out] -((c - c/(a^2*x^2))^(7/2)*x)/(6*(1 - a^2*x^2)^(7/2)) + (a*(c - c/(a^2*x^2))
^(7/2)*x^2)/(5*(1 - a^2*x^2)^(7/2)) + (3*a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(
4*(1 - a^2*x^2)^(7/2)) - (a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(1 - a^2*x^2)^(7
/2) - (3*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(2*(1 - a^2*x^2)^(7/2)) + (3*a^5*
(c - c/(a^2*x^2))^(7/2)*x^6)/(1 - a^2*x^2)^(7/2) + (a^7*(c - c/(a^2*x^2))^(
7/2)*x^8)/(1 - a^2*x^2)^(7/2) - (a^6*(c - c/(a^2*x^2))^(7/2)*x^7*Log[x])/(1
 - a^2*x^2)^(7/2)
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{7/2}}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^4(1+ax)^3}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\left(a^7+\frac{1}{x^7}-\frac{a}{x^6}-\frac{3a^2}{x^5}+\frac{3a^3}{x^4}+\frac{3a^4}{x^3}-\frac{3a^5}{x^2}-\frac{a^6}{x}\right)dx}{\left(1-a^2x^2\right)^{7/2}}$$
$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}{6\left(1-a^2x^2\right)^{7/2}}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{5\left(1-a^2x^2\right)^{7/2}}+\frac{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{4\left(1-a^2x^2\right)^{7/2}}-\frac{a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{\left(1-a^2x^2\right)^{7/2}}-\frac{3a^4\left(c\right.}{2\left(1\right.}$$

Mathematica [A] time = 0.0565689, size = 98, normalized size = 0.33

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(60a^7x^7+180a^5x^5-90a^4x^4-60a^3x^3+45a^2x^2-60a^6x^6\log(x)+12ax-10\right)}{60a^6x^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^ArcTanh[a*x],x]
```

```
[Out] -(c^3*Sqrt[c - c/(a^2*x^2)]*(-10 + 12*a*x + 45*a^2*x^2 - 60*a^3*x^3 - 90*a^
4*x^4 + 180*a^5*x^5 + 60*a^7*x^7 - 60*a^6*x^6*Log[x]))/(60*a^6*x^5*Sqrt[1 -
 a^2*x^2])
```

Maple [A] time = 0.155, size = 102, normalized size = 0.3

$$-\frac{x\left(-60\,a^7x^7+60\,a^6\ln(x)\,x^6-180\,x^5a^5+90\,x^4a^4+60\,x^3a^3-45\,a^2x^2-12\,ax+10\right)}{60\,\left(a^2x^2-1\right)^4}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/60*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a^2*x^2-1)^4*(-a^2*x^2+1)^(1/2)*(-60
*a^7*x^7+60*a^6*ln(x)*x^6-180*x^5*a^5+90*x^4*a^4+60*x^3*a^3-45*a^2*x^2-12*a
*x+10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(7/2)/(a*x + 1), x)
```

Fricas [A] time = 2.27702, size = 1142, normalized size = 3.82

$$\left[\frac{30\left(a^7c^3x^7 - a^5c^3x^5\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right) + \left(60\,a^7c^3x^7 + 180\,a^5c^3x^5 - 90\,a^4c^3x^4 - (6\right.}{60\left(a^8x^7\right.} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/60*(30*(a^7*c^3*x^7 - a^5*c^3*x^5)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 -
 c*x^4 + (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^
2*x^2)) - c)/(a^2*x^4 - x^2)) + (60*a^7*c^3*x^7 + 180*a^5*c^3*x^5 - 90*a^4*
c^3*x^4 - (60*a^7 + 180*a^5 - 90*a^4 - 60*a^3 + 45*a^2 + 12*a - 10)*c^3*x^6
 - 60*a^3*c^3*x^3 + 45*a^2*c^3*x^2 + 12*a*c^3*x - 10*c^3)*sqrt(-a^2*x^2 + 1
)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^8*x^7 - a^6*x^5), -1/60*(60*(a^7*c^3*
x^7 - a^5*c^3*x^5)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqrt(c)*
sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) - (60*a^
7*c^3*x^7 + 180*a^5*c^3*x^5 - 90*a^4*c^3*x^4 - (60*a^7 + 180*a^5 - 90*a^4 -
 60*a^3 + 45*a^2 + 12*a - 10)*c^3*x^6 - 60*a^3*c^3*x^3 + 45*a^2*c^3*x^2 + 1
2*a*c^3*x - 10*c^3)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^
8*x^7 - a^6*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(7/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(c - \frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(7/2)/(a*x + 1), x)
```

3.716 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx$

Optimal. Leaf size=220

$$-\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{3\left(1-a^2x^2\right)^{5/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4\left(1-a^2x^2\right)^{5/2}}+\frac{a^4x^5\log(x)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] -((c - c/(a^2*x^2))^(5/2)*x)/(4*(1 - a^2*x^2)^(5/2)) + (a*(c - c/(a^2*x^2))
^(5/2)*x^2)/(3*(1 - a^2*x^2)^(5/2)) + (a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(1
- a^2*x^2)^(5/2) - (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)/(1 - a^2*x^2)^(5/2)
- (a^5*(c - c/(a^2*x^2))^(5/2)*x^6)/(1 - a^2*x^2)^(5/2) + (a^4*(c - c/(a^2*
x^2))^(5/2)*x^5*Log[x])/(1 - a^2*x^2)^(5/2)
```

Rubi [A] time = 0.178944, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$-\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{3\left(1-a^2x^2\right)^{5/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4\left(1-a^2x^2\right)^{5/2}}+\frac{a^4x^5\log(x)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^ArcTanh[a*x],x]
```


```
[Out] -((c - c/(a^2*x^2))^(5/2)*x)/(4*(1 - a^2*x^2)^(5/2)) + (a*(c - c/(a^2*x^2))
^(5/2)*x^2)/(3*(1 - a^2*x^2)^(5/2)) + (a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(1
- a^2*x^2)^(5/2) - (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)/(1 - a^2*x^2)^(5/2)
- (a^5*(c - c/(a^2*x^2))^(5/2)*x^6)/(1 - a^2*x^2)^(5/2) + (a^4*(c - c/(a^2*
x^2))^(5/2)*x^5*Log[x])/(1 - a^2*x^2)^(5/2)
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{-\tanh^{-1}(ax)}(1-a^2x^2)^{5/2}}{x^5}dx}{(1-a^2x^2)^{5/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^3(1+ax)^2}{x^5}dx}{(1-a^2x^2)^{5/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\left(-a^5+\frac{1}{x^5}-\frac{a}{x^4}-\frac{2a^2}{x^3}+\frac{2a^3}{x^2}+\frac{a^4}{x}\right)dx}{(1-a^2x^2)^{5/2}}$$
$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}{4(1-a^2x^2)^{5/2}}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{3(1-a^2x^2)^{5/2}}+\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{(1-a^2x^2)^{5/2}}-\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-a^2x^2)^{5/2}}-\frac{a^5\left(c\right.}{(1\,}$$

Mathematica [A] time = 0.0483715, size = 82, normalized size = 0.37

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(12a^5x^5+24a^3x^3-12a^2x^2-12a^4x^4\log(x)-4ax+3\right)}{12a^4x^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^ArcTanh[a*x],x]
```

```
[Out] -(c^2*Sqrt[c - c/(a^2*x^2)]*(3 - 4*a*x - 12*a^2*x^2 + 24*a^3*x^3 + 12*a^5*x
^5 - 12*a^4*x^4*Log[x]))/(12*a^4*x^3*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.158, size = 86, normalized size = 0.4

$$-\frac{x\left(-12x^5a^5+12a^4\ln(x)x^4-24x^3a^3+12a^2x^2+4ax-3\right)}{12(a^2x^2-1)^3}\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/12*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/(a^2*x^2-1)^3*(-a^2*x^2+1)^(1/2)*(-12
*x^5*a^5+12*a^4*ln(x)*x^4-24*x^3*a^3+12*a^2*x^2+4*a*x-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{ax+1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(5/2)/(a*x + 1), x)
```

Fricas [A] time = 2.12639, size = 984, normalized size = 4.47

$$\left[\frac{6\left(a^5c^2x^5-a^3c^2x^3\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)+\left(12\,a^5c^2x^5+24\,a^3c^2x^3-\left(12\,a^5+24\,a^3-\right.\right.}{12\left(a^6x^5-a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/12*(6*(a^5*c^2*x^5 - a^3*c^2*x^3)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 -
c*x^4 + (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2
*x^2)) - c)/(a^2*x^4 - x^2)) + (12*a^5*c^2*x^5 + 24*a^3*c^2*x^3 - (12*a^5 +
 24*a^3 - 12*a^2 - 4*a + 3)*c^2*x^4 - 12*a^2*c^2*x^2 - 4*a*c^2*x + 3*c^2)*s
qrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5 - a^4*x^3), -1/
12*(12*(a^5*c^2*x^5 - a^3*c^2*x^3)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3
 + a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^
2 + c)) - (12*a^5*c^2*x^5 + 24*a^3*c^2*x^3 - (12*a^5 + 24*a^3 - 12*a^2 - 4*
a + 3)*c^2*x^4 - 12*a^2*c^2*x^2 - 4*a*c^2*x + 3*c^2)*sqrt(-a^2*x^2 + 1)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5 - a^4*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(5/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(5/2)/(a*x + 1), x)
```

3.717 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=144

$$\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2\left(1-a^2x^2\right)^{3/2}}-\frac{a^2x^3\log(x)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] -((c - c/(a^2*x^2))^(3/2)*x)/(2*(1 - a^2*x^2)^(3/2)) + (a*(c - c/(a^2*x^2))
^(3/2)*x^2)/(1 - a^2*x^2)^(3/2) + (a^3*(c - c/(a^2*x^2))^(3/2)*x^4)/(1 - a^
2*x^2)^(3/2) - (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*Log[x])/(1 - a^2*x^2)^(3/2)
```

Rubi [A] time = 0.16735, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 75}

$$\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2\left(1-a^2x^2\right)^{3/2}}-\frac{a^2x^3\log(x)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^ArcTanh[a*x],x]
```

```
[Out] -((c - c/(a^2*x^2))^(3/2)*x)/(2*(1 - a^2*x^2)^(3/2)) + (a*(c - c/(a^2*x^2))
^(3/2)*x^2)/(1 - a^2*x^2)^(3/2) + (a^3*(c - c/(a^2*x^2))^(3/2)*x^4)/(1 - a^
2*x^2)^(3/2) - (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*Log[x])/(1 - a^2*x^2)^(3/2)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$

$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{(1-ax)^2(1+ax)}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$

$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \left(a^3+\frac{1}{x^3}-\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{\left(1-a^2x^2\right)^{3/2}}$$

$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}{2\left(1-a^2x^2\right)^{3/2}}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{\left(1-a^2x^2\right)^{3/2}}+\frac{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^4}{\left(1-a^2x^2\right)^{3/2}}-\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(1-a^2x^2\right)^{3/2}}$$

Mathematica [A] time = 0.0346892, size = 72, normalized size = 0.5

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(2a^3x^3-3a^2x^2-2a^2x^2\log(x)+2ax-1\right)}{2a^2x\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^ArcTanh[a*x],x]
```

```
[Out] -(c*Sqrt[c - c/(a^2*x^2)]*(-1 + 2*a*x - 3*a^2*x^2 + 2*a^3*x^3 - 2*a^2*x^2*L
og[x]))/(2*a^2*x*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.161, size = 70, normalized size = 0.5

$$-\frac{x\left(-2x^3a^3+2a^2\ln(x)x^2-2ax+1\right)}{2\left(a^2x^2-1\right)^2}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a^2*x^2-1)^2*(-a^2*x^2+1)^(1/2)*(-2*x
^3*a^3+2*a^2*ln(x)*x^2-2*a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(3/2)/(a*x + 1), x)
```

Fricas [A] time = 2.19078, size = 778, normalized size = 5.4

$$\left[\frac{\left(a^3cx^3 - acx\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right) + \left(2\,a^3cx^3 - \left(2\,a^3 + 2\,a - 1\right)cx^2 + 2\,acx - c\right)\sqrt{-a^2x^2}}{2\left(a^4x^3 - a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/2*((a^3*c*x^3 - a*c*x)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 + (a*
x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c)
/(a^2*x^4 - x^2)) + (2*a^3*c*x^3 - (2*a^3 + 2*a - 1)*c*x^2 + 2*a*c*x - c)*s
qrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3 - a^2*x), -1/2*
(2*(a^3*c*x^3 - a*c*x)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqrt
(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) - (2
*a^3*c*x^3 - (2*a^3 + 2*a - 1)*c*x^2 + 2*a*c*x - c)*sqrt(-a^2*x^2 + 1)*sqrt
((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3 - a^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(3/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2
)/(a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(3/2)/(a*x + 1), x)
```

3.718 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=69

$$\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

```
[Out] -((a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2]) + (Sqrt[c - c/(a^2*x^2)]
*x*Log[x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.143345, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 43}

$$\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^ArcTanh[a*x],x]
```

```
[Out] -((a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2]) + (Sqrt[c - c/(a^2*x^2)]
*x*Log[x])/Sqrt[1 - a^2*x^2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{e^{-\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1-ax}{x}dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\left(-a+\frac{1}{x}\right)dx}{\sqrt{1-a^2x^2}}$$
$$=-\frac{a\sqrt{c-\frac{c}{a^2x^2}}x^2}{\sqrt{1-a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0210885, size = 38, normalized size = 0.55

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}(\log(x)-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-(a*x) + Log[x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.143, size = 53, normalized size = 0.8

$$-\frac{x(-ax+\ln(x))}{a^2x^2-1}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a*x+ln(x))*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

Fricas [B] time = 2.17055, size = 648, normalized size = 9.39

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)+2\left(a^2x^2-a^2x\right)\sqrt{-a^2x^2+1}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^3x^2-a\right)},-\frac{\left(a^2x^2-1\right)\sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 + (a*x^5 -
a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c)/(a^2*
x^4 - x^2)) + 2*(a^2*x^2 - a^2*x)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^3*x^2 - a), -((a^2*x^2 - 1)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)
*(a*x^3 + a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 +
1)*c*x^2 + c)) - (a^2*x^2 - a^2*x)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)))/(a^3*x^2 - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/(
a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

3.719 $\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-a^2x^2}\log(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(a*Sqrt[c - c/(a^2*x^2)]) - (Sqrt[1 - a^2*x^2]*Log[1 + a*
x])/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Rubi [A] time = 0.127999, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 43}

$$\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-a^2x^2}\log(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a*Sqrt[c - c/(a^2*x^2)]) - (Sqrt[1 - a^2*x^2]*Log[1 + a*
x])/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x}{1+ax}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{a}-\frac{1}{a(1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-a^2x^2}\log(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.029466, size = 50, normalized size = 0.65

$$\frac{\sqrt{1-a^2x^2}\left(\frac{x}{a}-\frac{\log(ax+1)}{a^2}\right)}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(x/a - Log[1 + a*x]/a^2))/(Sqrt[c - c/(a^2*x^2)]*x)
```

Maple [A] time = 0.148, size = 51, normalized size = 0.7

$$-\frac{-ax+\ln(ax+1)}{a^2x}\sqrt{-a^2x^2+1}\frac{1}{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)*(-a*x+ln(a*x+1))/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a^2
```

Maxima [C] time = 1.11338, size = 28, normalized size = 0.36

$$\frac{i\,x}{\sqrt{c}} - \frac{i\log(ax+1)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="ma
xima")
```

```
[Out] I*x/sqrt(c) - I*log(a*x + 1)/(a*sqrt(c))
```

Fricas [B] time = 2.24806, size = 763, normalized size = 9.91

$$\left[\frac{2\sqrt{-a^2x^2+1}a^2x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\left(a^2x^2-1\right)\sqrt{-c}\log\left(\frac{a^6cx^6+4a^5cx^5+5a^4cx^4-4a^2cx^2-4acx-\left(a^5x^5+4a^4x^4+6a^3x^3+4a^2x^2\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^4x^4+2a^3x^3-2ax-1}\right.}{2\left(a^3cx^2-ac\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fr
icas")
```

```
[Out] [1/2*(2*sqrt(-a^2*x^2 + 1)*a^2*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - (a^2*x
^2 - 1)*sqrt(-c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 -
 4*a*c*x - (a^5*x^5 + 4*a^4*x^4 + 6*a^3*x^3 + 4*a^2*x^2)*sqrt(-a^2*x^2 + 1)
*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a
*x - 1)))/(a^3*c*x^2 - a*c), (sqrt(-a^2*x^2 + 1)*a^2*x^2*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)) - (a^2*x^2 - 1)*sqrt(c)*arctan((a^2*x^2 + 2*a*x + 2)*sqrt(-a^
2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 + 2*a^2*c*x^2
 - a*c*x - 2*c)))/(a^3*c*x^2 - a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*
(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}}{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*sqrt(c - c/(a^2*x^2))), x)
```

3.720 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=175

$$\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{\left(1-a^2x^2\right)^{3/2}\log(1-ax)}{4a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{5\left(1-a^2x^2\right)^{3/2}\log(ax+1)}{4a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[Out] -((1 - a^2*x^2)^(3/2)/(a^3*(c - c/(a^2*x^2))^(3/2)*x^2)) + (1 - a^2*x^2)^(3
/2)/(2*a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 + a*x)) - ((1 - a^2*x^2)^(3/2)*Lo
g[1 - a*x])/(4*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) + (5*(1 - a^2*x^2)^(3/2)*Lo
g[1 + a*x])/(4*a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rubi [A] time = 0.185951, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{\left(1-a^2x^2\right)^{3/2}\log(1-ax)}{4a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{5\left(1-a^2x^2\right)^{3/2}\log(ax+1)}{4a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
```


```
[Out] -((1 - a^2*x^2)^(3/2)/(a^3*(c - c/(a^2*x^2))^(3/2)*x^2)) + (1 - a^2*x^2)^(3
/2)/(2*a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 + a*x)) - ((1 - a^2*x^2)^(3/2)*Lo
g[1 - a*x])/(4*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) + (5*(1 - a^2*x^2)^(3/2)*Lo
g[1 + a*x])/(4*a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{e^{-\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{x^3}{(1-ax)(1+ax)^2}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}\int \left(-\frac{1}{a^3}-\frac{1}{4a^3(-1+ax)}-\frac{1}{2a^3(1+ax)^2}+\frac{5}{4a^3(1+ax)}\right)dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$
$$= -\frac{\left(1-a^2x^2\right)^{3/2}}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3(1+ax)}-\frac{\left(1-a^2x^2\right)^{3/2}\log(1-ax)}{4a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}+\frac{5\left(1-a^2x^2\right)^{3/2}\log(1}{4a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Mathematica [A] time = 0.0534217, size = 91, normalized size = 0.52

$$-\frac{\sqrt{1-a^2x^2}\left(a^2x^2-1\right)\left(-\frac{x}{a^3}+\frac{1}{2a^4(ax+1)}-\frac{\log(1-ax)}{4a^4}+\frac{5\log(ax+1)}{4a^4}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(-1 + a^2*x^2)*(-(x/a^3) + 1/(2*a^4*(1 + a*x)) - Log[1
 - a*x]/(4*a^4) + (5*Log[1 + a*x])/(4*a^4)))/((c - c/(a^2*x^2))^(3/2)*x^3))
```

Maple [A] time = 0.168, size = 95, normalized size = 0.5

$$-\frac{\left(-4\,a^2x^2+5\,ax\ln(ax+1)-\ln(ax-1)\,xa-4\,ax+5\,\ln(ax+1)-\ln(ax-1)+2\right)(ax-1)}{4\,a^4x^3}\sqrt{-a^2x^2+1}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] -1/4*(-4*a^2*x^2+5*a*x*ln(a*x+1)-ln(a*x-1)*x*a-4*a*x+5*ln(a*x+1)-ln(a*x-1)+
2)*(a*x-1)*(-a^2*x^2+1)^(1/2)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}a^4x^4\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^5c^2x^5+a^4c^2x^4-2\,a^3c^2x^3-2\,a^2c^2x^2+ac^2x+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*a^4*x^4*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^5*c^
2*x^5 + a^4*c^2*x^4 - 2*a^3*c^2*x^3 - 2*a^2*c^2*x^2 + a*c^2*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/
2)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

3.721 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}} \, dx$

Optimal. Leaf size=265

$$\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^6x^5(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{7\left(1-a^2x^2\right)^{5}}{16a^6x^5\left(c\right.}$$

```
[Out] (1 - a^2*x^2)^(5/2)/(a^5*(c - c/(a^2*x^2))^(5/2)*x^4) + (1 - a^2*x^2)^(5/2)
/(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)) + (1 - a^2*x^2)^(5/2)/(8*a^6
*(c - c/(a^2*x^2))^(5/2)*x^5*(1 + a*x)^2) - (1 - a^2*x^2)^(5/2)/(a^6*(c - c
/(a^2*x^2))^(5/2)*x^5*(1 + a*x)) + (7*(1 - a^2*x^2)^(5/2)*Log[1 - a*x])/(16
*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) - (23*(1 - a^2*x^2)^(5/2)*Log[1 + a*x])/(
16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rubi [A] time = 0.211101, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^6x^5(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{7\left(1-a^2x^2\right)^{5}}{16a^6x^5\left(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(5/2)),x]
```


```
[Out] (1 - a^2*x^2)^(5/2)/(a^5*(c - c/(a^2*x^2))^(5/2)*x^4) + (1 - a^2*x^2)^(5/2)
/(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)) + (1 - a^2*x^2)^(5/2)/(8*a^6
*(c - c/(a^2*x^2))^(5/2)*x^5*(1 + a*x)^2) - (1 - a^2*x^2)^(5/2)/(a^6*(c - c
/(a^2*x^2))^(5/2)*x^5*(1 + a*x)) + (7*(1 - a^2*x^2)^(5/2)*Log[1 - a*x])/(16
*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) - (23*(1 - a^2*x^2)^(5/2)*Log[1 + a*x])/(
16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{e^{-\tanh^{-1}(ax)}x^5}{\left(1-a^2x^2\right)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{x^5}{(1-ax)^2(1+ax)^3}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}\int \left(\frac{1}{a^5}+\frac{1}{8a^5(-1+ax)^2}+\frac{7}{16a^5(-1+ax)}-\frac{1}{4a^5(1+ax)^3}+\frac{1}{a^5(1+ax)^2}-\frac{23}{16a^5(1+ax)}\right)dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}}{a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5(1-ax)}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5(1+ax)^2}-\frac{\left(1-a^2x^2\right.}{a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Mathematica [A] time = 0.124133, size = 88, normalized size = 0.33

$$\frac{\left(1-a^2x^2\right)^{5/2}\left(2\left(8ax+\frac{1}{1-ax}-\frac{8}{ax+1}+\frac{1}{(ax+1)^2}\right)+7\log(1-ax)-23\log(ax+1)\right)}{16a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] ((1 - a^2*x^2)^(5/2)*(2*(8*a*x + (1 - a*x)^(-1) + (1 + a*x)^(-2) - 8/(1 + a
*x)) + 7*Log[1 - a*x] - 23*Log[1 + a*x]))/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x
^5)
```

Maple [A] time = 0.161, size = 167, normalized size = 0.6

$$-\frac{(ax-1)\left(-16x^4a^4+23a^3x^3\ln(ax+1)-7\ln(ax-1)x^3a^3-16x^3a^3+23\ln(ax+1)a^2x^2-7\ln(ax-1)a^2x^2+34\right.}{16a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] -1/16*(-a^2*x^2+1)^(1/2)*(a*x-1)*(-16*x^4*a^4+23*a^3*x^3*ln(a*x+1)-7*ln(a*x
-1)*x^3*a^3-16*x^3*a^3+23*ln(a*x+1)*a^2*x^2-7*ln(a*x-1)*a^2*x^2+34*a^2*x^2-
23*a*x*ln(a*x+1)+7*ln(a*x-1)*x*a+18*a*x-23*ln(a*x+1)+7*ln(a*x-1)-12)/a^6/x^
5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}a^6x^6\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^7c^3x^7+a^6c^3x^6-3\,a^5c^3x^5-3\,a^4c^3x^4+3\,a^3c^3x^3+3\,a^2c^3x^2-ac^3x-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*a^6*x^6*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^7*c^
3*x^7 + a^6*c^3*x^6 - 3*a^5*c^3*x^5 - 3*a^4*c^3*x^4 + 3*a^3*c^3*x^3 + 3*a^2
*c^3*x^2 - a*c^3*x - c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

3.722 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=361

$$-\frac{5\left(1-a^2x^2\right)^{7/2}}{16a^8x^7(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{3\left(1-a^2x^2\right)^{7/2}}{2a^8x^7(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{11\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(ax+1)^2\left(c-\frac{c}{a^2x^2}\right.}$$

```
[Out] -((1 - a^2*x^2)^(7/2)/(a^7*(c - c/(a^2*x^2))^(7/2)*x^6)) + (1 - a^2*x^2)^(7
/2)/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^2) - (5*(1 - a^2*x^2)^(7/
2))/(16*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)) + (1 - a^2*x^2)^(7/2)/(2
4*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^3) - (11*(1 - a^2*x^2)^(7/2))/(
32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^2) + (3*(1 - a^2*x^2)^(7/2))/(
2*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)) - (19*(1 - a^2*x^2)^(7/2)*Log[
1 - a*x])/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) + (51*(1 - a^2*x^2)^(7/2)*Lo
g[1 + a*x])/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```

Rubi [A] time = 0.240329, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$-\frac{5\left(1-a^2x^2\right)^{7/2}}{16a^8x^7(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{3\left(1-a^2x^2\right)^{7/2}}{2a^8x^7(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{11\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(ax+1)^2\left(c-\frac{c}{a^2x^2}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] -((1 - a^2*x^2)^(7/2)/(a^7*(c - c/(a^2*x^2))^(7/2)*x^6)) + (1 - a^2*x^2)^(7
/2)/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^2) - (5*(1 - a^2*x^2)^(7/
2))/(16*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)) + (1 - a^2*x^2)^(7/2)/(2
4*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^3) - (11*(1 - a^2*x^2)^(7/2))/(
32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^2) + (3*(1 - a^2*x^2)^(7/2))/(
2*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)) - (19*(1 - a^2*x^2)^(7/2)*Log[
1 - a*x])/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) + (51*(1 - a^2*x^2)^(7/2)*Lo
g[1 + a*x])/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left(1-a^2x^2\right)^{7/2}\int \frac{e^{-\tanh^{-1}(ax)}x^7}{\left(1-a^2x^2\right)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$
$$= \frac{\left(1-a^2x^2\right)^{7/2}\int \frac{x^7}{(1-ax)^3(1+ax)^4}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$
$$= \frac{\left(1-a^2x^2\right)^{7/2}\int \left(-\frac{1}{a^7}-\frac{1}{16a^7(-1+ax)^3}-\frac{5}{16a^7(-1+ax)^2}-\frac{19}{32a^7(-1+ax)}-\frac{1}{8a^7(1+ax)^4}+\frac{11}{16a^7(1+ax)^3}-\frac{3}{2a^7(1+ax)^2}+\right.}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$
$$= -\frac{\left(1-a^2x^2\right)^{7/2}}{a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}+\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7(1-ax)^2}-\frac{5\left(1-a^2x^2\right)^{7/2}}{16a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7(1-ax)}+\frac{\left(1-a^{2}\right.}{24a^8\left(c-\frac{c}{a^2x^2}\right)}$$

Mathematica [A] time = 0.117548, size = 145, normalized size = 0.4

$$\frac{\sqrt{1-a^2x^2}\left(96a^6x^6+96a^5x^5-366a^4x^4-222a^3x^3+338a^2x^2+122ax+57(ax-1)^2(ax+1)^3\log(1-ax)-153(ax-1)^2(\right.}{96a^2x(ax-1)^2\sqrt{c-\frac{c}{a^2x^2}}(acx+c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-88 + 122*a*x + 338*a^2*x^2 - 222*a^3*x^3 - 366*a^4*x^4
 + 96*a^5*x^5 + 96*a^6*x^6 + 57*(-1 + a*x)^2*(1 + a*x)^3*Log[1 - a*x] - 153
*(-1 + a*x)^2*(1 + a*x)^3*Log[1 + a*x]))/(96*a^2*Sqrt[c - c/(a^2*x^2)]*x*(-
1 + a*x)^2*(c + a*c*x)^3)
```

Maple [A] time = 0.168, size = 239, normalized size = 0.7

$$-\frac{(ax-1)\left(-96x^6a^6+153\ln(ax+1)x^5a^5-57\ln(ax-1)x^5a^5-96x^5a^5+153\ln(ax+1)a^4x^4-57\ln(ax-1)a^4x^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] -1/96*(-a^2*x^2+1)^(1/2)*(a*x-1)*(-96*x^6*a^6+153*ln(a*x+1)*x^5*a^5-57*ln(a
*x-1)*x^5*a^5-96*x^5*a^5+153*ln(a*x+1)*a^4*x^4-57*ln(a*x-1)*a^4*x^4+366*x^4
*a^4-306*a^3*x^3*ln(a*x+1)+114*ln(a*x-1)*x^3*a^3+222*x^3*a^3-306*ln(a*x+1)*
a^2*x^2+114*ln(a*x-1)*a^2*x^2-338*a^2*x^2+153*a*x*ln(a*x+1)-57*ln(a*x-1)*x*
a-122*a*x+153*ln(a*x+1)-57*ln(a*x-1)+88)/a^8/x^7/(c*(a^2*x^2-1)/a^2/x^2)^(7
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}a^8x^8\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^9c^4x^9+a^8c^4x^8-4\,a^7c^4x^7-4\,a^6c^4x^6+6\,a^5c^4x^5+6\,a^4c^4x^4-4\,a^3c^4x^3-4\,a^2c^4x^2+ac^4x+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*a^8*x^8*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^9*c^
4*x^9 + a^8*c^4*x^8 - 4*a^7*c^4*x^7 - 4*a^6*c^4*x^6 + 6*a^5*c^4*x^5 + 6*a^4
*c^4*x^4 - 4*a^3*c^4*x^3 - 4*a^2*c^4*x^2 + a*c^4*x + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

3.723 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx$

Optimal. Leaf size=455

$$\frac{11a^8x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{128(1-ax)^4(ax+1)^4}+\frac{39a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{64(1-ax)^4(ax+1)^3}-\frac{11a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{640(1-ax)^4(ax+1)^2}-\frac{103a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{160(1-ax)^4(ax+1)}+\frac{629a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{960(1-ax)^3(ax+1)}-$$

```
[Out] (11*a^8*(c - c/(a^2*x^2))^(9/2)*x^9)/(128*(1 - a*x)^4*(1 + a*x)^4) + (39*a^
7*(c - c/(a^2*x^2))^(9/2)*x^8)/(64*(1 - a*x)^4*(1 + a*x)^3) - (11*a^6*(c -
c/(a^2*x^2))^(9/2)*x^7)/(640*(1 - a*x)^4*(1 + a*x)^2) + (a*(c - c/(a^2*x^2)
)^(9/2)*x^2)/(28*(1 + a*x)) - (103*a^5*(c - c/(a^2*x^2))^(9/2)*x^6)/(160*(1
 - a*x)^4*(1 + a*x)) + (629*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(960*(1 - a*x)
^3*(1 + a*x)) - (2*a^3*(c - c/(a^2*x^2))^(9/2)*x^4)/(5*(1 - a*x)^2*(1 + a*x
)) + (47*a^2*(c - c/(a^2*x^2))^(9/2)*x^3)/(336*(1 - a*x)*(1 + a*x)) - ((c -
 c/(a^2*x^2))^(9/2)*x*(1 - a*x))/(8*(1 + a*x)) - (2*a^8*(c - c/(a^2*x^2))^(
9/2)*x^9*ArcSin[a*x])/((1 - a*x)^(9/2)*(1 + a*x)^(9/2)) + (245*a^8*(c - c/(
a^2*x^2))^(9/2)*x^9*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(128*(1 - a*x)^(9
/2)*(1 + a*x)^(9/2))
```

Rubi [A] time = 0.537151, antiderivative size = 455, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$\frac{11a^8x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{128(1-ax)^4(ax+1)^4}+\frac{39a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{64(1-ax)^4(ax+1)^3}-\frac{11a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{640(1-ax)^4(ax+1)^2}-\frac{103a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{160(1-ax)^4(ax+1)}+\frac{629a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{960(1-ax)^3(ax+1)}-$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(9/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (11*a^8*(c - c/(a^2*x^2))^(9/2)*x^9)/(128*(1 - a*x)^4*(1 + a*x)^4) + (39*a^
7*(c - c/(a^2*x^2))^(9/2)*x^8)/(64*(1 - a*x)^4*(1 + a*x)^3) - (11*a^6*(c -
c/(a^2*x^2))^(9/2)*x^7)/(640*(1 - a*x)^4*(1 + a*x)^2) + (a*(c - c/(a^2*x^2)
)^(9/2)*x^2)/(28*(1 + a*x)) - (103*a^5*(c - c/(a^2*x^2))^(9/2)*x^6)/(160*(1
 - a*x)^4*(1 + a*x)) + (629*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(960*(1 - a*x)
^3*(1 + a*x)) - (2*a^3*(c - c/(a^2*x^2))^(9/2)*x^4)/(5*(1 - a*x)^2*(1 + a*x
)) + (47*a^2*(c - c/(a^2*x^2))^(9/2)*x^3)/(336*(1 - a*x)*(1 + a*x)) - ((c -
 c/(a^2*x^2))^(9/2)*x*(1 - a*x))/(8*(1 + a*x)) - (2*a^8*(c - c/(a^2*x^2))^(
9/2)*x^9*ArcSin[a*x])/((1 - a*x)^(9/2)*(1 + a*x)^(9/2)) + (245*a^8*(c - c/(
a^2*x^2))^(9/2)*x^9*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(128*(1 - a*x)^(9
/2)*(1 + a*x)^(9/2))
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{9/2}(1+ax)^{9/2}}{x^9}dx}{(1-ax)^{9/2}(1+ax)^{9/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^{11/2}(1+ax)^{7/2}}{x^9}dx}{(1-ax)^{9/2}(1+ax)^{9/2}}$$
$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x(1-ax)}{8(1+ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^{9/2}(1+ax)^{5/2}(-2a-9a^2x)}{x^8}dx}{8(1-ax)^{9/2}(1+ax)^{9/2}}$$
$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{28(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x(1-ax)}{8(1+ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^{7/2}(1+ax)^{5/2}(-47a^2+6}{x^7}}{56(1-ax)^{9/2}(1+ax)^{9/2}}$$
$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{28(1+ax)}+\frac{47a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3}{336(1-ax)(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x(1-ax)}{8(1+ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int}{336(1}$$
$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{28(1+ax)}-\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4}{5(1-ax)^2(1+ax)}+\frac{47a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3}{336(1-ax)(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x(1-ax)}{8(1+ax)}$$
$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{28(1+ax)}+\frac{629a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{960(1-ax)^3(1+ax)}-\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4}{5(1-ax)^2(1+ax)}+\frac{47a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3}{336(1-ax)(1+ax)}$$
$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{28(1+ax)}-\frac{103a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^6}{160(1-ax)^4(1+ax)}+\frac{629a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{960(1-ax)^3(1+ax)}-\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x}{5(1-ax)^2(1+ax}$$
$$=-\frac{11a^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^7}{640(1-ax)^4(1+ax)^2}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{28(1+ax)}-\frac{103a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^6}{160(1-ax)^4(1+ax)}+\frac{629a^4\left(c-\frac{c}{a^2x^2}\right.}{960(1-ax)^3(1}$$
$$=\frac{39a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{64(1-ax)^4(1+ax)^3}-\frac{11a^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^7}{640(1-ax)^4(1+ax)^2}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{28(1+ax)}-\frac{103a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{160(1-ax)^4(1+}$$
$$=\frac{11a^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}{128(1-ax)^4(1+ax)^4}+\frac{39a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{64(1-ax)^4(1+ax)^3}-\frac{11a^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^7}{640(1-ax)^4(1+ax)^2}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/}}{28(1+ax}$$
$$=\frac{11a^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}{128(1-ax)^4(1+ax)^4}+\frac{39a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{64(1-ax)^4(1+ax)^3}-\frac{11a^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^7}{640(1-ax)^4(1+ax)^2}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/}}{28(1+ax}$$
$$=\frac{11a^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}{128(1-ax)^4(1+ax)^4}+\frac{39a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{64(1-ax)^4(1+ax)^3}-\frac{11a^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^7}{640(1-ax)^4(1+ax)^2}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/}}{28(1+ax}$$
$$=\frac{11a^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}{128(1-ax)^4(1+ax)^4}+\frac{39a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{64(1-ax)^4(1+ax)^3}-\frac{11a^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^7}{640(1-ax)^4(1+ax)^2}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/}}{28(1+ax}$$

Mathematica [A] time = 0.199254, size = 166, normalized size = 0.36

$$-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(13440a^8x^8+45056a^7x^7+14595a^6x^6-31232a^5x^5+770a^4x^4+16896a^3x^3-4760a^2x^2-3840\right.\right.}{13440a^8x^7\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(9/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c^4*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1680 - 3840*a*x - 4760*a^2
*x^2 + 16896*a^3*x^3 + 770*a^4*x^4 - 31232*a^5*x^5 + 14595*a^6*x^6 + 45056*
a^7*x^7 + 13440*a^8*x^8) + 25725*a^8*x^8*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 268
80*a^8*x^8*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(13440*a^8*x^7*Sqrt[-1 + a^2*x^2
])
```

Maple [B] time = 0.26, size = 965, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(9/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/40320*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/a^2*(58590*(-c/a^2)^(1/2)*ln(x*c^(1
/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*c^(11/2)*x^8*a+22050*(-c/a^2)^(1/2)*ln((((a*
x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*c^(11/2)*x^8*a-58590*(c*(a^
2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*x^9*a^3*c^5-77175*(c*(a^2*x^2-1)/a^2)^(1
/2)*(-c/a^2)^(1/2)*x^8*a^2*c^5+23808*(c*(a^2*x^2-1)/a^2)^(11/2)*(-c/a^2)^(1
/2)*x^7*a^11+17535*(c*(a^2*x^2-1)/a^2)^(11/2)*(-c/a^2)^(1/2)*x^6*a^10-13056
*(c*(a^2*x^2-1)/a^2)^(11/2)*(-c/a^2)^(1/2)*x^5*a^9+6510*(c*(a^2*x^2-1)/a^2)
^(11/2)*(-c/a^2)^(1/2)*x^4*a^8-6912*(c*(a^2*x^2-1)/a^2)^(11/2)*(-c/a^2)^(1/
2)*x^3*a^7+10920*(c*(a^2*x^2-1)/a^2)^(11/2)*(-c/a^2)^(1/2)*x^2*a^6-11520*(c
*(a^2*x^2-1)/a^2)^(11/2)*(-c/a^2)^(1/2)*x*a^5+5040*a^4*(c*(a^2*x^2-1)/a^2)^
(11/2)*(-c/a^2)^(1/2)-77175*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*
a^2-c)/x/a^2)*x^8*c^6-23808*(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*x^9*a^
11*c-8960*((a*x-1)*(a*x+1)*c/a^2)^(9/2)*(-c/a^2)^(1/2)*x^8*a^10*c-8575*(c*(
a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*x^8*a^10*c+10080*((a*x-1)*(a*x+1)*c/a^
2)^(7/2)*(-c/a^2)^(1/2)*x^9*a^9*c^2+26784*(c*(a^2*x^2-1)/a^2)^(7/2)*(-c/a^2
)^(1/2)*x^9*a^9*c^2+11025*(c*(a^2*x^2-1)/a^2)^(7/2)*(-c/a^2)^(1/2)*x^8*a^8*
c^2-11760*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*(-c/a^2)^(1/2)*x^9*a^7*c^3-31248*(c
*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)*x^9*a^7*c^3-15435*(c*(a^2*x^2-1)/a^2
)^(5/2)*(-c/a^2)^(1/2)*x^8*a^6*c^3+14700*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*(-c/
a^2)^(1/2)*x^9*a^5*c^4+39060*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x^9*a
^5*c^4+25725*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x^8*a^4*c^4-22050*((a
*x-1)*(a*x+1)*c/a^2)^(1/2)*(-c/a^2)^(1/2)*x^9*a^3*c^5)/(-c/a^2)^(1/2)/(c*(a
^2*x^2-1)/a^2)^(9/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(9/2)/(a*x + 1)^2, x)
```

Fricas [A] time = 2.48006, size = 1130, normalized size = 2.48

$$\left[-\frac{53760\,a^7\sqrt{-c}c^4x^7\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-25725\,a^7\sqrt{-c}c^4x^7\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(13440\,a^8c^4x^8+450\right.}{26880}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/26880*(53760*a^7*sqrt(-c)*c^4*x^7*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x
^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 25725*a^7*sqrt(-c)*c^4*x^7*log(-(a^2*
c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(134
40*a^8*c^4*x^8 + 45056*a^7*c^4*x^7 + 14595*a^6*c^4*x^6 - 31232*a^5*c^4*x^5
+ 770*a^4*c^4*x^4 + 16896*a^3*c^4*x^3 - 4760*a^2*c^4*x^2 - 3840*a*c^4*x + 1
680*c^4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^8*x^7), -1/13440*(25725*a^7*c^
(9/2)*x^7*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c
)) - 13440*a^7*c^(9/2)*x^7*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*
x^2 - c)/(a^2*x^2)) - c) + (13440*a^8*c^4*x^8 + 45056*a^7*c^4*x^7 + 14595*a
^6*c^4*x^6 - 31232*a^5*c^4*x^5 + 770*a^4*c^4*x^4 + 16896*a^3*c^4*x^3 - 4760
*a^2*c^4*x^2 - 3840*a*c^4*x + 1680*c^4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a
^8*x^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(9/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [A] time = 57.7623, size = 954, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6720*(25725*c^(9/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x)/a^2 - 13440*c^(9/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))
*sgn(x)/(a*abs(a)) - 6720*sqrt(a^2*c*x^2 - c)*c^4*sgn(x)/a^2 + (14595*(sqrt
(a^2*c)*x - sqrt(a^2*c*x^2 - c))^15*c^5*abs(a)*sgn(x) - 107520*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 - c))^14*a*c^(11/2)*sgn(x) + 76055*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^13*c^6*abs(a)*sgn(x) - 430080*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))^12*a*c^(13/2)*sgn(x) + 64435*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))^11*c^7*abs(a)*sgn(x) - 1111040*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^10*
a*c^(15/2)*sgn(x) + 110495*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*c^8*abs(
a)*sgn(x) - 1576960*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^8*a*c^(17/2)*sgn(
```

```
x) - 110495*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^9*abs(a)*sgn(x) - 141
2096*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(19/2)*sgn(x) - 64435*(sqr
t(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^10*abs(a)*sgn(x) - 831488*(sqrt(a^2*c
)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(21/2)*sgn(x) - 76055*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*c^11*abs(a)*sgn(x) - 252928*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))^2*a*c^(23/2)*sgn(x) - 14595*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c)
)*c^12*abs(a)*sgn(x) - 45056*a*c^(25/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^2 + c)^8*a^2*abs(a)))*abs(a)
```

3.724 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx$

Optimal. Leaf size=375

$$-\frac{7a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}-\frac{3a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{8(1-ax)^3(ax+1)^2}+\frac{19a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)}-\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{3(1-ax)^2(ax+1)}+\frac{23a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{120(1-ax)(ax+1)}+\frac{ax^2\left(c-\right.}{15(a}$$

```
[Out] (-7*a^6*(c - c/(a^2*x^2))^(7/2)*x^7)/(16*(1 - a*x)^3*(1 + a*x)^3) - (3*a^5*
(c - c/(a^2*x^2))^(7/2)*x^6)/(8*(1 - a*x)^3*(1 + a*x)^2) + (a*(c - c/(a^2*x
^2))^(7/2)*x^2)/(15*(1 + a*x)) + (19*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(16*(
1 - a*x)^3*(1 + a*x)) - (2*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(3*(1 - a*x)^2*
(1 + a*x)) + (23*a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(120*(1 - a*x)*(1 + a*x))
 - ((c - c/(a^2*x^2))^(7/2)*x*(1 - a*x))/(6*(1 + a*x)) + (2*a^6*(c - c/(a^2
*x^2))^(7/2)*x^7*ArcSin[a*x])/((1 - a*x)^(7/2)*(1 + a*x)^(7/2)) - (25*a^6*(
c - c/(a^2*x^2))^(7/2)*x^7*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(16*(1 - a
*x)^(7/2)*(1 + a*x)^(7/2))
```

Rubi [A] time = 0.467053, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$-\frac{7a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}-\frac{3a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{8(1-ax)^3(ax+1)^2}+\frac{19a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)}-\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{3(1-ax)^2(ax+1)}+\frac{23a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{120(1-ax)(ax+1)}+\frac{ax^2\left(c-\right.}{15(a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^(2*ArcTanh[a*x]),x]
```



```
[Out] (-7*a^6*(c - c/(a^2*x^2))^(7/2)*x^7)/(16*(1 - a*x)^3*(1 + a*x)^3) - (3*a^5*
(c - c/(a^2*x^2))^(7/2)*x^6)/(8*(1 - a*x)^3*(1 + a*x)^2) + (a*(c - c/(a^2*x
^2))^(7/2)*x^2)/(15*(1 + a*x)) + (19*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(16*(
1 - a*x)^3*(1 + a*x)) - (2*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(3*(1 - a*x)^2*
(1 + a*x)) + (23*a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(120*(1 - a*x)*(1 + a*x))
 - ((c - c/(a^2*x^2))^(7/2)*x*(1 - a*x))/(6*(1 + a*x)) + (2*a^6*(c - c/(a^2
*x^2))^(7/2)*x^7*ArcSin[a*x])/((1 - a*x)^(7/2)*(1 + a*x)^(7/2)) - (25*a^6*(
c - c/(a^2*x^2))^(7/2)*x^7*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(16*(1 - a
*x)^(7/2)*(1 + a*x)^(7/2))
```


Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{7/2}(1+ax)^{7/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{(1-ax)^{9/2}(1+ax)^{5/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{(1-ax)^{7/2}(1+ax)^{3/2}(-2a-7a^2x)}{x^6}dx}{6(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{(1-ax)^{5/2}(1+ax)^{3/2}(-23a^2+3}{x^5}}{30(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} + \frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int}{120(1}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)} + \frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} + \frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)} - \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)} + \frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)} -$$

$$= -\frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} + \frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)} - \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)}$$

$$= -\frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} - \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} + \frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}{16(1-ax)^3(1+ax}$$

$$= -\frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} - \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} + \frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}{16(1-ax)^3(1+ax}$$

$$= -\frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} - \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} + \frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}{16(1-ax)^3(1+ax}$$

$$= -\frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} - \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} + \frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}{16(1-ax)^3(1+ax}$$

Mathematica [A] time = 0.158363, size = 150, normalized size = 0.4

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(240a^6x^6+736a^5x^5+105a^4x^4-352a^3x^3+70a^2x^2+96ax-40\right)-480a^6x^6\log\left(\sqrt{a^2x^2-1}+a\right.\right.}{240a^6x^5\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c^3*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-40 + 96*a*x + 70*a^2*x^2
- 352*a^3*x^3 + 105*a^4*x^4 + 736*a^5*x^5 + 240*a^6*x^6) + 375*a^6*x^6*ArcT
an[1/Sqrt[-1 + a^2*x^2]] - 480*a^6*x^6*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(240
*a^6*x^5*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.161, size = 795, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/1680*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/a^2*(2016*(-c/a^2)^(1/2)*(c*(a^2*x^2
-1)/a^2)^(7/2)*x^7*a^9*c-2016*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x^5*
a^9-480*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^6*a^8*c+375*(-c/a^2)
^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^6*a^8*c+560*(-c/a^2)^(1/2)*((a*x-1)*(a*x
+1)*c/a^2)^(5/2)*x^7*a^7*c^2+105*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x
^4*a^8-2352*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^7*a^7*c^2-224*(-c/a^
2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x^3*a^7-525*(-c/a^2)^(1/2)*(c*(a^2*x^2-1
)/a^2)^(5/2)*x^6*a^6*c^2-700*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x
^7*a^5*c^3+2940*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^7*a^5*c^3+630*(-
c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x^2*a^6+875*(-c/a^2)^(1/2)*(c*(a^2*x
^2-1)/a^2)^(3/2)*x^6*a^4*c^3+1050*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1
/2)*x^7*a^3*c^4-672*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x*a^5-4410*(-c
/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^7*a^3*c^4+4410*(-c/a^2)^(1/2)*c^(9/
2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^6*a-1050*(-c/a^2)^(1/2)*c^(9/2
)*ln((((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*x^6*a+280*a^4*(c*
(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)-2625*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^
2)^(1/2)*x^6*a^2*c^4-2625*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^
2-c)/x/a^2)*x^6*c^5)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(7/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(7/2)/(a*x + 1)^2, x)
```

Fricas [A] time = 2.17425, size = 973, normalized size = 2.59

$$\left[-\frac{960\,a^5\sqrt{-c}c^3x^5\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-375\,a^5\sqrt{-c}c^3x^5\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(240\,a^6c^3x^6+736\,a^5c^3\right.}{480\,a^6x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/480*(960*a^5*sqrt(-c)*c^3*x^5*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 375*a^5*sqrt(-c)*c^3*x^5*log(-(a^2*c*x^2
+ 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(240*a^6*c
^3*x^6 + 736*a^5*c^3*x^5 + 105*a^4*c^3*x^4 - 352*a^3*c^3*x^3 + 70*a^2*c^3*x
^2 + 96*a*c^3*x - 40*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5), -1/24
0*(375*a^5*c^(7/2)*x^5*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(
```

```
a^2*c*x^2 - c)) - 240*a^5*c^(7/2)*x^5*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (240*a^6*c^3*x^6 + 736*a^5*c^3*x^5 +
105*a^4*c^3*x^4 - 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 + 96*a*c^3*x - 40*c^3)*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5)]
```

Sympy [C] time = 39.1809, size = 1059, normalized size = 2.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(7/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -c**3*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*s
qrt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
 (I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqr
t(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**3*Piecewise((-a*sqrt(c)
*x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
 1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*
asin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + c**3*Piecewise
((I*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) -
 I*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (
-a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/
a**2 - 4*c**3*Piecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*
c), True))/a**3 + c**3*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*
sqrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a
**2*x**2))) - I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*
x**2) > 1), (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(
a**2*x**2))) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2
*x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**4 + 2*c**3*Piecewise((2*a**3*sqrt
(c)*sqrt(a**2*x**2 - 1)/(15*x) + a*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x**3) -
sqrt(c)*sqrt(a**2*x**2 - 1)/(5*a*x**5), Abs(a**2*x**2) > 1), (2*I*a**3*sqrt
(c)*sqrt(-a**2*x**2 + 1)/(15*x) + I*a*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3
) - I*sqrt(c)*sqrt(-a**2*x**2 + 1)/(5*a*x**5), True))/a**5 - c**3*Piecewise
((I*a**5*sqrt(c)*acosh(1/(a*x))/16 - I*a**4*sqrt(c)/(16*x*sqrt(-1 + 1/(a**2
*x**2))) + I*a**2*sqrt(c)/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) + 5*I*sqrt(c)/
(24*x**5*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)/(6*a**2*x**7*sqrt(-1 + 1/(a*
*2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**5*sqrt(c)*asin(1/(a*x))/16 + a**4*s
qrt(c)/(16*x*sqrt(1 - 1/(a**2*x**2))) - a**2*sqrt(c)/(48*x**3*sqrt(1 - 1/(a
**2*x**2))) - 5*sqrt(c)/(24*x**5*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(6*a**2
*x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**6
```

Giac [A] time = 86.447, size = 757, normalized size = 2.02

$$\frac{1}{120}\left(\frac{375\,c^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{240\,c^{\frac{7}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{120\,\sqrt{a^2cx^2-c}c^3\operatorname{sgn}(x)}{a^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/120*(375*c^(7/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*s
gn(x)/a^2 - 240*c^(7/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(
```

```
x)/(a*abs(a)) - 120*sqrt(a^2*c*x^2 - c)*c^3*sgn(x)/a^2 + (105*(sqrt(a^2*c)*
x - sqrt(a^2*c*x^2 - c))^11*c^4*abs(a)*sgn(x) - 1440*(sqrt(a^2*c)*x - sqrt(
a^2*c*x^2 - c))^10*a*c^(9/2)*sgn(x) + 595*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 c))^9*c^5*abs(a)*sgn(x) - 4320*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^8*a*c
^(11/2)*sgn(x) - 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^6*abs(a)*sgn
(x) - 7360*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(13/2)*sgn(x) + 150*
(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^7*abs(a)*sgn(x) - 6720*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(15/2)*sgn(x) - 595*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^3*c^8*abs(a)*sgn(x) - 2976*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - c))^2*a*c^(17/2)*sgn(x) - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^9*
abs(a)*sgn(x) - 736*a*c^(19/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
c))^2 + c)^6*a^2*abs(a)))*abs(a)
```

3.725 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=293

$$\frac{7a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}-\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{(1-ax)^2(ax+1)}+\frac{7a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{24(1-ax)(ax+1)}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(ax+1)}-\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(ax+1)}-\frac{2a^4x^5\left(c-\right.}{(1-ax)}$$

```
[Out] (7*a^4*(c - c/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(1 + a*x)^2) + (a*(c - c
/(a^2*x^2))^(5/2)*x^2)/(6*(1 + a*x)) - (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)/
((1 - a*x)^2*(1 + a*x)) + (7*a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(24*(1 - a*x)
*(1 + a*x)) - ((c - c/(a^2*x^2))^(5/2)*x*(1 - a*x))/(4*(1 + a*x)) - (2*a^4*
(c - c/(a^2*x^2))^(5/2)*x^5*ArcSin[a*x])/((1 - a*x)^(5/2)*(1 + a*x)^(5/2))
+ (9*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/
(8*(1 - a*x)^(5/2)*(1 + a*x)^(5/2))
```

Rubi [A] time = 0.426757, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$\frac{7a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}-\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{(1-ax)^2(ax+1)}+\frac{7a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{24(1-ax)(ax+1)}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(ax+1)}-\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(ax+1)}-\frac{2a^4x^5\left(c-\right.}{(1-ax)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (7*a^4*(c - c/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(1 + a*x)^2) + (a*(c - c
/(a^2*x^2))^(5/2)*x^2)/(6*(1 + a*x)) - (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)/
((1 - a*x)^2*(1 + a*x)) + (7*a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(24*(1 - a*x)
*(1 + a*x)) - ((c - c/(a^2*x^2))^(5/2)*x*(1 - a*x))/(4*(1 + a*x)) - (2*a^4*
(c - c/(a^2*x^2))^(5/2)*x^5*ArcSin[a*x])/((1 - a*x)^(5/2)*(1 + a*x)^(5/2))
+ (9*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/
(8*(1 - a*x)^(5/2)*(1 + a*x)^(5/2))
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{5/2}(1+ax)^{5/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{(1-ax)^{7/2}(1+ax)^{3/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{(1-ax)^{5/2}\sqrt{1+ax}\left(-2a-5a^2x\right)}{x^4}dx}{4(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{(1-ax)^{3/2}\sqrt{1+ax}\left(-7a^2+17a^3\ldots\right.}{x^3}}{12(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \ldots}{24(1-ax\ldots}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} - \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} + \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} \ldots$$

$$= \frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} - \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} + \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} - \ldots$$

$$= \frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} - \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} + \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} - \ldots$$

$$= \frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} - \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} + \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} - \ldots$$

$$= \frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} - \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} + \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} - \ldots$$

Mathematica [A] time = 0.13476, size = 134, normalized size = 0.46

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(24a^4x^4+64a^3x^3-3a^2x^2-16ax+6\right)-48a^4x^4\log\left(\sqrt{a^2x^2-1}+ax\right)+27a^4x^4\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\ldots}{24a^4x^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c^2*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 - 16*a*x - 3*a^2*x^2 + 6
4*a^3*x^3 + 24*a^4*x^4) + 27*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 48*a^4*
x^4*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^4*x^3*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.147, size = 625, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/120*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/a^2*(-80*(-c/a^2)^(1/2)*(c*(a^2*x^2-1
)/a^2)^(5/2)*x^5*a^7*c+80*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^3*a^7-
```

```
48*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^4*a^6*c-27*(-c/a^2)^(1/2)
*(c*(a^2*x^2-1)/a^2)^(5/2)*x^4*a^6*c+60*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a
^2)^(3/2)*x^5*a^5*c^2+75*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^2*a^6+1
00*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^5*a^5*c^2-80*(-c/a^2)^(1/2)*(
c*(a^2*x^2-1)/a^2)^(7/2)*x*a^5+45*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*
x^4*a^4*c^2-90*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^5*a^3*c^3-150
*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^5*a^3*c^3+30*a^4*(c*(a^2*x^2-1)
/a^2)^(7/2)*(-c/a^2)^(1/2)+150*(-c/a^2)^(1/2)*c^(7/2)*ln(x*c^(1/2)+(c*(a^2*
x^2-1)/a^2)^(1/2))*x^4*a+90*(-c/a^2)^(1/2)*c^(7/2)*ln((((a*x-1)*(a*x+1)*c/a
^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*x^4*a-135*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2
)^(1/2)*x^4*a^2*c^3-135*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-
c)/x/a^2)*x^4*c^4)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(5/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(5/2)/(a*x + 1)^2, x)
```

Fricas [A] time = 2.08861, size = 857, normalized size = 2.92

$$\left[-\frac{96\,a^3\sqrt{-c}c^2x^3\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-27\,a^3\sqrt{-c}c^2x^3\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(24\,a^4c^2x^4+64\,a^3c^2x^3-\right.}{48\,a^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/48*(96*a^3*sqrt(-c)*c^2*x^3*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2))/(a^2*c*x^2 - c)) - 27*a^3*sqrt(-c)*c^2*x^3*log(-(a^2*c*x^2 + 2
*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*c^2*x
^4 + 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)))/(a^4*x^3), -1/24*(27*a^3*c^(5/2)*x^3*arctan(a*sqrt(c)*x*sqr
t((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 24*a^3*c^(5/2)*x^3*log(2*a^
2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (24*a^4*
c^2*x^4 + 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*
x^2 - c)/(a^2*x^2)))/(a^4*x^3)]
```

Sympy [C] time = 25.5076, size = 500, normalized size = 1.71

$$-c^2\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}+\frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } |a^2x^2|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right)+\frac{2c^2\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}(ax)+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}}\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}(ax)-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}}\end{cases}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(5/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -c**2*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*s
qrt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
 (I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqr
t(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**2*Piecewise((-a*sqrt(c)
*x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
 1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*
asin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - 2*c**2*Piecewi
se((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c), True))/a**3 + c**
2*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*sqrt(c)/(8*x*sqrt(-1
+ 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(
c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**3*sq
rt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2))) - 3*sqrt
(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*x**5*sqrt(1 - 1/(a**
2*x**2))), True))/a**4
```

Giac [A] time = 3.39111, size = 562, normalized size = 1.92

$$\frac{1}{12}\left(\frac{27\,c^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{24\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{12\,\sqrt{a^2cx^2-c}c^2\operatorname{sgn}(x)}{a^2}-\frac{3(\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*(27*c^(5/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn
(x)/a^2 - 24*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/
(a*abs(a)) - 12*sqrt(a^2*c*x^2 - c)*c^2*sgn(x)/a^2 - (3*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^7*c^3*abs(a)*sgn(x) + 96*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - c))^6*a*c^(7/2)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^4*
abs(a)*sgn(x) + 192*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(9/2)*sgn(x
) + 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*c^5*abs(a)*sgn(x) + 160*(sqr
t(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(11/2)*sgn(x) - 3*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))*c^6*abs(a)*sgn(x) + 64*a*c^(13/2)*sgn(x))/(((sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^4*a^2*abs(a)))*abs(a)
```

3.726 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=212

$$\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{ax+1}-\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(ax+1)}+\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\sin^{-1}(ax)}{(1-ax)^{3/2}(ax+1)^{3/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\tan}{2(1-ax)^{3/}}$$

```
[Out] (a*(c - c/(a^2*x^2))^(3/2)*x^2)/(1 + a*x) + (5*a^2*(c - c/(a^2*x^2))^(3/2)*
x^3)/(2*(1 - a*x)*(1 + a*x)) - ((c - c/(a^2*x^2))^(3/2)*x*(1 - a*x))/(2*(1
+ a*x)) + (2*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcSin[a*x])/((1 - a*x)^(3/2)*
(1 + a*x)^(3/2)) - (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcTanh[Sqrt[1 - a*x]*S
qrt[1 + a*x]])/(2*(1 - a*x)^(3/2)*(1 + a*x)^(3/2))
```

Rubi [A] time = 0.383582, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{ax+1}-\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(ax+1)}+\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\sin^{-1}(ax)}{(1-ax)^{3/2}(ax+1)^{3/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\tan}{2(1-ax)^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^(2*ArcTanh[a*x]),x]
```


```
[Out] (a*(c - c/(a^2*x^2))^(3/2)*x^2)/(1 + a*x) + (5*a^2*(c - c/(a^2*x^2))^(3/2)*
x^3)/(2*(1 - a*x)*(1 + a*x)) - ((c - c/(a^2*x^2))^(3/2)*x*(1 - a*x))/(2*(1
+ a*x)) + (2*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcSin[a*x])/((1 - a*x)^(3/2)*
(1 + a*x)^(3/2)) - (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcTanh[Sqrt[1 - a*x]*S
qrt[1 + a*x]])/(2*(1 - a*x)^(3/2)*(1 + a*x)^(3/2))
```


Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{3/2}(1+ax)^{3/2}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1-ax)^{5/2}\sqrt{1+ax}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$
$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1-ax)^{3/2}(-2a-3a^2x)}{x^2\sqrt{1+ax}}dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$
$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{\sqrt{1-ax}(a^2+5a^3x)}{x\sqrt{1+ax}}dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$
$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax} + \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right.}{2a(1-ax)^{3}}$$
$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax} + \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} + \frac{\left(a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}\right.}{2(1-ax)}$$
$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax} + \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} - \frac{\left(a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\right.}{}$$
$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax} + \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} + \frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{(1-ax)^{3/2}(1}$$

Mathematica [A] time = 0.125709, size = 115, normalized size = 0.54

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(2a^2x^2+4ax-1\right)-4a^2x^2\log\left(\sqrt{a^2x^2-1}+ax\right)+a^2x^2\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{2a^2x\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-1 + 4*a*x + 2*a^2*x^2) + a^
2*x^2*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 4*a^2*x^2*Log[a*x + Sqrt[-1 + a^2*x^2]
]))/(2*a^2*x*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.135, size = 454, normalized size = 2.1

$$\frac{x}{6a^2c}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}\left(12\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}x^3a^5c-12\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{5/2}xa^5-4\sqrt{-\frac{c}{a^2}}\left(\frac{(ax-1)(ax+1)}{a^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/6*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/a^2*(12*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a
^2)^(3/2)*x^3*a^5*c-12*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x*a^5-4*(-c
/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^2*a^4*c+(-c/a^2)^(1/2)*(c*(a^2*
x^2-1)/a^2)^(3/2)*x^2*a^4*c+6*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*
x^3*a^3*c^2+3*a^4*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)-18*(-c/a^2)^(1/2
```

```
)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^3*a^3*c^2+18*c^(5/2)*ln(x*c^(1/2)+(c*(a^2*x^2
-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*x^2*a-6*c^(5/2)*(-c/a^2)^(1/2)*ln((((a*x-1)*
(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*x^2*a-3*(-c/a^2)^(1/2)*(c*(a^2*x
^2-1)/a^2)^(1/2)*x^2*a^2*c^2-3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/
2)*a^2-c)/x/a^2)*x^2*c^3)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(3/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(a^2x^2-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(3/2)/(a*x + 1)^2, x)
```

Fricas [A] time = 1.93294, size = 695, normalized size = 3.28

$$\left[-\frac{8\,a\sqrt{-c}cx\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-a\sqrt{-c}cx\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(2\,a^2cx^2+4\,acx-c\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,a^2x},\ -\frac{ac^{\frac{3}{2}}x\,\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/4*(8*a*sqrt(-c)*c*x*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x
^2))/(a^2*c*x^2 - c)) - a*sqrt(-c)*c*x*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(2*a^2*c*x^2 + 4*a*c*x - c)*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x), -1/2*(a*c^(3/2)*x*arctan(a*sqrt(c)*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 2*a*c^(3/2)*x*log(2*a^
2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (2*a^2*c
*x^2 + 4*a*c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x)]
```

Sympy [C] time = 14.8001, size = 376, normalized size = 1.77

$$-c\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a}+\frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } \left|a^2x^2\right|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right)+\frac{2c\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}(ax)+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \ldots\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}(ax)-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \ldots\end{cases}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(3/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -c*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sqrt
(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1), (I
*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt(c
)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c*Piecewise((-a*sqrt(c)*x/sqr
t(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 - 1)),
Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*asin(a
*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - c*Piecewise((I*a*sqr
t(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c
)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a*sqrt(c
)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a**2
```

Giac [A] time = 1.684, size = 358, normalized size = 1.69

$$\left(\frac{c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{2c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{\sqrt{a^2cx^2-c}c\operatorname{sgn}(x)}{a^2}-\frac{\left(\sqrt{a^2c}x-\sqrt{\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] (c^(3/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^2
- 2*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(a)
) - sqrt(a^2*c*x^2 - c)*c*sgn(x)/a^2 - ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))^3*c^2*abs(a)*sgn(x) + 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(5/2
)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^3*abs(a)*sgn(x) + 4*a*c^
(7/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^2*a^2*abs(a)))
*abs(a)
```

3.727 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=118

$$-x\sqrt{c-\frac{c}{a^2x^2}}-\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x) - (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[
1 - a*x]*Sqrt[1 + a*x]) - (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.295744, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6159, 6129, 102, 157, 41, 216, 92, 208}

$$-x\sqrt{c-\frac{c}{a^2x^2}}-\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x) - (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[
1 - a*x]*Sqrt[1 + a*x]) - (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
```

```
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{(1-ax)^{3/2}}{x\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}x+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}x+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}x-\frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int\frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x}}\right.}{\sqrt{1-a}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}x-\frac{2\sqrt{c-\frac{c}{a^2x^2}}x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.0887921, size = 80, normalized size = 0.68

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\sqrt{a^2x^2-1}+2\log\left(\sqrt{a^2x^2-1}+ax\right)+\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-Sqrt[-1 + a^2*x^2] + ArcTan[1/Sqrt[-1 + a^2*x^2]
] + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [A] time = 0.138, size = 198, normalized size = 1.7

$$-\frac{x}{a^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(2\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}a^2\sqrt{-\frac{c}{a^2}}-2\sqrt{c}\ln\left(\frac{1}{\sqrt{c}}\left(\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\sqrt{c}+cx\right)\right)a\sqrt{-\frac{c}{a^2}}-\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c(}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a^2*(-c/a
^2)^(1/2)-2*c^(1/2)*ln((((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))
*a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/
a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2))/(c*(a^2*x^2-1)/a^2)^(1/
2)/a^2/(-c/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1)^2, x)
```

Fricas [A] time = 2.02227, size = 579, normalized size = 4.91

$$\left[-\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+4\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\sqrt{-c}\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},-\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\sqrt{c}\arctan\left(\frac{a\sqrt{c}x\cdots}{a^2c\cdots}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 4*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - sqrt(-c)*log(-(a^
2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, -(
a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*
c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sq
rt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx-\int \frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-sqrt(c - c/(a**2*x**2))/(a*x + 1), x) - Integral(a*x*sqrt(c - c/
(a**2*x**2))/(a*x + 1), x)
```

Giac [A] time = 1.27075, size = 207, normalized size = 1.75

$$-\left(\frac{2\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}+\frac{2\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}+\frac{\sqrt{a^2cx^2-c}\operatorname{sgn}(x)}{a^2}-\frac{\left(2\sqrt{c}|a|\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -(2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
^2 + 2*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs
(a)) + sqrt(a^2*c*x^2 - c)*sgn(x)/a^2 - (2*sqrt(c)*abs(a)*arctan(sqrt(-c)/s
qrt(c)) + a*sqrt(c)*log(abs(c)) + sqrt(-c)*abs(a))*sgn(x)/(a^2*abs(a)))*abs
(a)
```

3.728 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=111

$$\frac{(1-ax)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2(ax+1)(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{ax+1}\sqrt{1-ax}\sin^{-1}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (1 - a*x)^2/(a^2*Sqrt[c - c/(a^2*x^2)]*x) + (2*(1 - a*x)*(1 + a*x))/(a^2*Sq
rt[c - c/(a^2*x^2)]*x) + (2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcSin[a*x])/(a^2*S
qrt[c - c/(a^2*x^2)]*x)
```

Rubi [A] time = 0.24115, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6159, 6129, 78, 50, 41, 216}

$$\frac{(1-ax)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2(ax+1)(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{ax+1}\sqrt{1-ax}\sin^{-1}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (1 - a*x)^2/(a^2*Sqrt[c - c/(a^2*x^2)]*x) + (2*(1 - a*x)*(1 + a*x))/(a^2*Sq
rt[c - c/(a^2*x^2)]*x) + (2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcSin[a*x])/(a^2*S
qrt[c - c/(a^2*x^2)]*x)
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx &= \frac{\left(\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= \frac{\left(\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{x\sqrt{1-ax}}{(1+ax)^{3/2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= \frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}+\frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= \frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}+\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}+\frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= \frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}+\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}+\frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= \frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}+\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}+\frac{2\sqrt{1-ax}\sqrt{1+ax}\sin^{-1}(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}
\end{aligned}
$$

Mathematica [A] time = 0.0692871, size = 69, normalized size = 0.62

$$
\frac{-a^2x^2+2\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)-2ax+3}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (3 - 2*a*x - a^2*x^2 + 2*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/
(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Maple [A] time = 0.145, size = 177, normalized size = 1.6

$$-\frac{1}{(ax+1)\,ax}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\left(\sqrt{c}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}xa^{2}-2\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\right)xac+\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}a\sqrt{c}+2\,a\sqrt{\frac{(ax-1}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2)^(1/2)*(c^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a^2-2*ln(x*
c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x*a*c+(c*(a^2*x^2-1)/a^2)^(1/2)*a*c^(1/2
)+2*a*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)-2*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a
^2)^(1/2))*c)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/c^(3/2)/a/(a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{a^{2}x^{2}-1}{(ax+1)^{2}\sqrt{c-\frac{c}{a^{2}x^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*sqrt(c - c/(a^2*x^2))), x)
```

Fricas [A] time = 1.97899, size = 448, normalized size = 4.04

$$\left[\frac{(ax+1)\sqrt{c}\log\left(2\,a^{2}cx^{2}+2\,a^{2}\sqrt{c}x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c\right)-\left(a^{2}x^{2}+3\,ax\right)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx+ac},-\frac{2\,(ax+1)\sqrt{-c}\arctan\left(\frac{a^{2}\sqrt{-c}x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)+(}{a^{2}cx+ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] [((a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) - c) - (a^2*x^2 + 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2
*c*x + a*c), -(2*(a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2
 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (a^2*x^2 + 3*a*x)*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)))/(a^2*c*x + a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{ax}{ax\sqrt{c-\frac{c}{a^{2}x^{2}}}+\sqrt{c-\frac{c}{a^{2}x^{2}}}}\,dx-\int-\frac{1}{ax\sqrt{c-\frac{c}{a^{2}x^{2}}}+\sqrt{c-\frac{c}{a^{2}x^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x*sqrt(c - c/(a**2*x**2)) + sqrt(c - c/(a**2*x**2))), x) -
 Integral(-1/(a*x*sqrt(c - c/(a**2*x**2)) + sqrt(c - c/(a**2*x**2))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.729 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{2(ax+1)(2ax+5)(1-ax)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{(1-ax)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{2(ax+1)^{3/2}(1-ax)^{3/2}\sin^{-1}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[Out] (1 - a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(3/2)*x) - (2*(1 - a*x)^2*(1 + a*x)*(5
 + 2*a*x))/(3*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) - (2*(1 - a*x)^(3/2)*(1 + a*
x)^(3/2)*ArcSin[a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rubi [A] time = 0.366143, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6159, 6129, 98, 143, 41, 216}

$$-\frac{2(ax+1)(2ax+5)(1-ax)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{(1-ax)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{2(ax+1)^{3/2}(1-ax)^{3/2}\sin^{-1}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```


```
[Out] (1 - a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(3/2)*x) - (2*(1 - a*x)^2*(1 + a*x)*(5
 + 2*a*x))/(3*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) - (2*(1 - a*x)^(3/2)*(1 + a*
x)^(3/2)*ArcSin[a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```


Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx &= \frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x^3}{(1-ax)^{3/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= \frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x^3}{\sqrt{1-ax}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} - \frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x(2-4ax)}{\sqrt{1-ax}(1+ax)^{3/2}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} - \frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} - \frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} - \frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} - \frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} - \frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} - \frac{2(1-ax)^{3/2}(1+ax)^{3/2}\sin^{-1}(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}
\end{aligned}
$$

Mathematica [A] time = 0.0862678, size = 95, normalized size = 0.77

$$
\frac{-3a^3x^3-11a^2x^2+6(ax+1)\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)+4ax+10}{3a^2cx(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] (10 + 4*a*x - 11*a^2*x^2 - 3*a^3*x^3 + 6*(1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[a
*x + Sqrt[-1 + a^2*x^2]])/(3*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))
```

Maple [B] time = 0.134, size = 326, normalized size = 2.6

$$-\frac{ax-1}{3\,x^{3}a^{4}}\left(3\,c^{3/2}\sqrt{\frac{(ax-1)\,(ax+1)\,c}{a^{2}}}x^{3}a^{3}+15\,x^{2}a^{2}c^{3/2}\sqrt{\frac{(ax-1)\,(ax+1)\,c}{a^{2}}}-4\,c^{3/2}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}x^{2}a^{2}-6\ln\left(x\sqrt{c}+\sqrt{\frac{c(}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] -1/3*(3*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^3*a^3+15*x^2*a^2*c^(3/2)*((
a*x-1)*(a*x+1)*c/a^2)^(1/2)-4*c^(3/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^2*a^2-6*l
n(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*(c*(a^
2*x^2-1)/a^2)^(1/2)*x*a^2*c-4*c^(3/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a-6*ln(x*
c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*(c*(a^2*x^
2-1)/a^2)^(1/2)*a*c-12*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+2*(c*(a^2*x^2-
1)/a^2)^(1/2)*c^(3/2))*(a*x-1)/((a*x-1)*(a*x+1)*c/a^2)^(1/2)/x^3/(c*(a^2*x^
2-1)/a^2/x^2)^(3/2)/a^4/c^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{a^{2}x^{2}-1}{(ax+1)^{2}\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima
")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a^2*x^2))^(3/2)), x)
```

Fricas [A] time = 2.12467, size = 591, normalized size = 4.77

$$\left[\frac{3\left(a^{2}x^{2}+2\,ax+1\right)\sqrt{c}\log\left(2\,a^{2}cx^{2}+2\,a^{2}\sqrt{c}x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c\right)-\left(3\,a^{3}x^{3}+14\,a^{2}x^{2}+10\,ax\right)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{3\left(a^{3}c^{2}x^{2}+2\,a^{2}c^{2}x+ac^{2}\right)},-\frac{6\left(a^{2}x^{2}+2\,ax+1\right)\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/3*(3*(a^2*x^2 + 2*a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) - (3*a^3*x^3 + 14*a^2*x^2 + 10*a*x)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2), -1/3*(6*
(a^2*x^2 + 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)
/(a^2*x^2))/(a^2*c*x^2 - c)) + (3*a^3*x^3 + 14*a^2*x^2 + 10*a*x)*sqrt((a^2*
c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{acx\sqrt{c-\frac{c}{a^2x^2}}+c\sqrt{c-\frac{c}{a^2x^2}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{ax}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2}}\,dx-\int -\frac{1}{acx\sqrt{c-\frac{c}{a^2x^2}}+c\sqrt{c-\frac{c}{a^2x^2}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{ax}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] -Integral(a*x/(a*c*x*sqrt(c - c/(a**2*x**2)) + c*sqrt(c - c/(a**2*x**2)) -
c*sqrt(c - c/(a**2*x**2))/(a*x) - c*sqrt(c - c/(a**2*x**2))/(a**2*x**2)), x
) - Integral(-1/(a*c*x*sqrt(c - c/(a**2*x**2)) + c*sqrt(c - c/(a**2*x**2))
- c*sqrt(c - c/(a**2*x**2))/(a*x) - c*sqrt(c - c/(a**2*x**2))/(a**2*x**2)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a^2x^2-1}{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a^2*x^2))^(3/2)), x)
```

3.730 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=194

$$-\frac{2(ax+1)(1-ax)^3}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^2(13ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(1-ax)^3}{5a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{(1-ax)^2}{a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^{5/2}(1-ax)^{5/2}}{a^6x^5\left(c-\frac{c}{a^2x^2}\right)}$$

```
[Out] (1 - a*x)^2/(a^2*(c - c/(a^2*x^2))^(5/2)*x) + (2*(1 - a*x)^3)/(5*a^3*(c - c
/(a^2*x^2))^(5/2)*x^2) - (2*(1 - a*x)^3*(1 + a*x))/(15*a^4*(c - c/(a^2*x^2)
)^(5/2)*x^3) + (2*(1 - a*x)^3*(1 + a*x)^2*(28 + 13*a*x))/(15*a^6*(c - c/(a^
2*x^2))^(5/2)*x^5) + (2*(1 - a*x)^(5/2)*(1 + a*x)^(5/2)*ArcSin[a*x])/(a^6*(
c - c/(a^2*x^2))^(5/2)*x^5)
```

Rubi [A] time = 0.398065, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6159, 6129, 98, 150, 143, 41, 216}

$$-\frac{2(ax+1)(1-ax)^3}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^2(13ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(1-ax)^3}{5a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{(1-ax)^2}{a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^{5/2}(1-ax)^{5/2}}{a^6x^5\left(c-\frac{c}{a^2x^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (1 - a*x)^2/(a^2*(c - c/(a^2*x^2))^(5/2)*x) + (2*(1 - a*x)^3)/(5*a^3*(c - c
/(a^2*x^2))^(5/2)*x^2) - (2*(1 - a*x)^3*(1 + a*x))/(15*a^4*(c - c/(a^2*x^2)
)^(5/2)*x^3) + (2*(1 - a*x)^3*(1 + a*x)^2*(28 + 13*a*x))/(15*a^6*(c - c/(a^
2*x^2))^(5/2)*x^5) + (2*(1 - a*x)^(5/2)*(1 + a*x)^(5/2)*ArcSin[a*x])/(a^6*(
c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x^5}{(1-ax)^{5/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^5}{(1-ax)^{3/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^3(4+2ax)}{\sqrt{1-ax}(1+ax)^{7/2}}\,dx}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^2(6a+8a^2x)}{\sqrt{1-ax}(1+ax)^{5/2}}\,dx}{5a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x\left(-4a^2+26a\right.}{\sqrt{1-ax}(1+ax}}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{2(1-ax)^3(1+ax)^2(28+13ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} + \cdots$$

$$= \frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{2(1-ax)^3(1+ax)^2(28+13ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} + \cdots$$

$$= \frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{2(1-ax)^3(1+ax)^2(28+13ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} + \cdots$$

Mathematica [A] time = 0.101071, size = 105, normalized size = 0.54

$$\frac{-15a^4x^4-76a^3x^3-32a^2x^2+30(ax+1)^2\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)+82ax+56}{15a^2c^2x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (56 + 82*a*x - 32*a^2*x^2 - 76*a^3*x^3 - 15*a^4*x^4 + 30*(1 + a*x)^2*Sqrt[-
1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(15*a^2*c^2*Sqrt[c - c/(a^2*x^2
)]*x*(1 + a*x)^2)
```

Maple [B] time = 0.144, size = 462, normalized size = 2.4

$$-\frac{ax-1}{15\,x^5a^6}\left(15\,c^{5/2}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{3/2}x^5a^5+45\,x^4c^{5/2}a^4\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{3/2}+16\,c^{5/2}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}x^4a^4-60\,c^{5/}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] -1/15*(15*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^5*a^5+45*x^4*c^(5/2)*a^4*
((a*x-1)*(a*x+1)*c/a^2)^(3/2)+16*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^4*a^4-
60*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^3*a^3+16*c^(5/2)*(c*(a^2*x^2-1)/
a^2)^(3/2)*x^3*a^3-30*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x
+1)*c/a^2)^(3/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a^4*c-90*c^(5/2)*((a*x-1)*(a*x
+1)*c/a^2)^(3/2)*x^2*a^2-24*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2*a^2-30*ln
(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*(c*(a^2
*x^2-1)/a^2)^(3/2)*a^3*c+50*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x*a-24*c^
(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a+50*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2
)+6*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2))*(a*x-1)/((a*x-1)*(a*x+1)*c/a^2)^(3/2
)/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)/a^6/c^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^2x^2-1}{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima
")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a^2*x^2))^(5/2)), x)
```

Fricas [A] time = 2.28098, size = 745, normalized size = 3.84

$$\left[\frac{15\left(a^4x^4+2a^3x^3-2ax-1\right)\sqrt{c}\log\left(2a^2cx^2+2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-\left(15a^5x^5+76a^4x^4+32a^3x^3-82a^2x^2-56a\right.}{15\left(a^5c^3x^4+2a^4c^3x^3-2a^2c^3x-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas
")
```

```
[Out] [1/15*(15*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2
*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) - (15*a^5*x^5 + 76*a^4*x^
4 + 32*a^3*x^3 - 82*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5
*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3), -1/15*(30*(a^4*x^4 + 2*a^3
*x^3 - 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^
2*x^2))/(a^2*c*x^2 - c)) + (15*a^5*x^5 + 76*a^4*x^4 + 32*a^3*x^3 - 82*a^2*x
^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3
- 2*a^2*c^3*x - a*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{ac^2x\sqrt{c-\frac{c}{a^2x^2}}+c^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{ax}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^4}}\,dx-\int -\frac{}{ac^2x\sqrt{c-\frac{c}{a^2x^2}}+c^2\sqrt{c-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] -Integral(a*x/(a*c**2*x*sqrt(c - c/(a**2*x**2)) + c**2*sqrt(c - c/(a**2*x**
2)) - 2*c**2*sqrt(c - c/(a**2*x**2))/(a*x) - 2*c**2*sqrt(c - c/(a**2*x**2))
/(a**2*x**2) + c**2*sqrt(c - c/(a**2*x**2))/(a**3*x**3) + c**2*sqrt(c - c/(
a**2*x**2))/(a**4*x**4)), x) - Integral(-1/(a*c**2*x*sqrt(c - c/(a**2*x**2)
) + c**2*sqrt(c - c/(a**2*x**2)) - 2*c**2*sqrt(c - c/(a**2*x**2))/(a*x) - 2
*c**2*sqrt(c - c/(a**2*x**2))/(a**2*x**2) + c**2*sqrt(c - c/(a**2*x**2))/(a
**3*x**3) + c**2*sqrt(c - c/(a**2*x**2))/(a**4*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a^2x^2-1}{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a^2*x^2))^(5/2)), x)
```

3.731 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{2(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{82(ax+1)(1-ax)^4}{105a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{2(ax+1)^3(37ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{12(1-ax)^4}{7a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{10(1-ax)^3}{3a^3x^2\left(c-\frac{c}{a^2x^2}\right.}$$

```
[Out] (1 - a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(7/2)*x) - (10*(1 - a*x)^3)/(3*a^3*(c
- c/(a^2*x^2))^(7/2)*x^2) - (12*(1 - a*x)^4)/(7*a^4*(c - c/(a^2*x^2))^(7/2)
*x^3) - (82*(1 - a*x)^4*(1 + a*x))/(105*a^5*(c - c/(a^2*x^2))^(7/2)*x^4) -
(2*(1 - a*x)^4*(1 + a*x)^2)/(35*a^6*(c - c/(a^2*x^2))^(7/2)*x^5) - (2*(1 -
a*x)^4*(1 + a*x)^3*(72 + 37*a*x))/(35*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) - (2
*(1 - a*x)^(7/2)*(1 + a*x)^(7/2)*ArcSin[a*x])/(a^8*(c - c/(a^2*x^2))^(7/2)*
x^7)
```

Rubi [A] time = 0.42703, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6159, 6129, 98, 150, 143, 41, 216}

$$-\frac{2(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{82(ax+1)(1-ax)^4}{105a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{2(ax+1)^3(37ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{12(1-ax)^4}{7a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{10(1-ax)^3}{3a^3x^2\left(c-\frac{c}{a^2x^2}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```


```
[Out] (1 - a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(7/2)*x) - (10*(1 - a*x)^3)/(3*a^3*(c
- c/(a^2*x^2))^(7/2)*x^2) - (12*(1 - a*x)^4)/(7*a^4*(c - c/(a^2*x^2))^(7/2)
*x^3) - (82*(1 - a*x)^4*(1 + a*x))/(105*a^5*(c - c/(a^2*x^2))^(7/2)*x^4) -
(2*(1 - a*x)^4*(1 + a*x)^2)/(35*a^6*(c - c/(a^2*x^2))^(7/2)*x^5) - (2*(1 -
a*x)^4*(1 + a*x)^3*(72 + 37*a*x))/(35*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) - (2
*(1 - a*x)^(7/2)*(1 + a*x)^(7/2)*ArcSin[a*x])/(a^8*(c - c/(a^2*x^2))^(7/2)*
x^7)
```


Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x^7}{(1-ax)^{7/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^7}{(1-ax)^{5/2}(1+ax)^{9/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^5(6+4ax)}{(1-ax)^{3/2}(1+ax)^{9/2}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^4(-50a-14a^2x)}{\sqrt{1-ax}(1+ax)^{9/2}}\,dx}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^3(-144}{\sqrt{1-ax}}}{21a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} - \frac{((1-ax}{}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} - \frac{2(1-a}{35a^6(c}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} - \frac{2(1-a}{35a^6(c}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} - \frac{2(1-a}{35a^6(c}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} - \frac{2(1-a}{35a^6(c}$$

Mathematica [A] time = 0.120931, size = 131, normalized size = 0.49

$$\frac{-105a^6x^6-562a^5x^5-74a^4x^4+1226a^3x^3+636a^2x^2+210(ax-1)(ax+1)^3\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)-654ax}{105a^2x(ax-1)\sqrt{c-\frac{c}{a^2x^2}}(acx+c)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (-432 - 654*a*x + 636*a^2*x^2 + 1226*a^3*x^3 - 74*a^4*x^4 - 562*a^5*x^5 - 1
05*a^6*x^6 + 210*(-1 + a*x)*(1 + a*x)^3*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-
1 + a^2*x^2]])/(105*a^2*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)*(c + a*c*x)^3)
```

Maple [B] time = 0.15, size = 572, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] -1/105*(105*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^7*a^7+553*x^6*c^(7/2)*a
^6*((a*x-1)*(a*x+1)*c/a^2)^(5/2)-96*c^(7/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^6*a
^6-392*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^5*a^5-96*c^(7/2)*(c*(a^2*x^2
-1)/a^2)^(5/2)*x^5*a^5-1540*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^4*a^4+2
40*c^(7/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^4*a^4-210*ln(x*c^(1/2)+(c*(a^2*x^2-1
)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x*a^6
*c+350*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^3*a^3+240*c^(7/2)*(c*(a^2*x^
2-1)/a^2)^(5/2)*x^3*a^3-210*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1
)*(a*x+1)*c/a^2)^(5/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*a^5*c+1470*c^(7/2)*((a*x-1
)*(a*x+1)*c/a^2)^(5/2)*x^2*a^2-180*c^(7/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^2*a^
2-42*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x*a-180*c^(7/2)*(c*(a^2*x^2-1)/a
^2)^(5/2)*x*a-462*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)+30*c^(7/2)*(c*(a^2*
x^2-1)/a^2)^(5/2))*(a*x-1)/((a*x-1)*(a*x+1)*c/a^2)^(5/2)/x^7/(c*(a^2*x^2-1)
/a^2/x^2)^(7/2)/a^8/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^2x^2-1}{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima
")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a^2*x^2))^(7/2)), x)
```

Fricas [A] time = 2.97124, size = 1045, normalized size = 3.87

$$\left[\frac{105\left(a^6x^6+2a^5x^5-a^4x^4-4a^3x^3-a^2x^2+2ax+1\right)\sqrt{c}\log\left(2a^2cx^2+2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-\left(105a^7x^7+562a^6x^6+\right.}{105\left(a^7c^4x^6+2a^6c^4x^5-a^5c^4x^4-4a^4c^4x^3-a^3c^4x^2+2a^2c^4x+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas
")
```

```
[Out] [1/105*(105*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x +
1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)) - c) - (105*a^7*x^7 + 562*a^6*x^6 + 74*a^5*x^5 - 1226*a^4*x^4 - 636*a^3
*x^3 + 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^7*c^4*x^6
 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x
+ a*c^4), -1/105*(210*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2
+ 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2
))/(a^2*c*x^2 - c)) + (105*a^7*x^7 + 562*a^6*x^6 + 74*a^5*x^5 - 1226*a^4*x^
4 - 636*a^3*x^3 + 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(
a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2
```

```
*a^2*c^4*x + a*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{ac^3x\sqrt{c-\frac{c}{a^2x^2}}+c^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{ax}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^3}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^5x^5}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^6x^6}}\,dx-\int -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] -Integral(a*x/(a*c**3*x*sqrt(c - c/(a**2*x**2)) + c**3*sqrt(c - c/(a**2*x**
2)) - 3*c**3*sqrt(c - c/(a**2*x**2))/(a*x) - 3*c**3*sqrt(c - c/(a**2*x**2))
/(a**2*x**2) + 3*c**3*sqrt(c - c/(a**2*x**2))/(a**3*x**3) + 3*c**3*sqrt(c -
 c/(a**2*x**2))/(a**4*x**4) - c**3*sqrt(c - c/(a**2*x**2))/(a**5*x**5) - c*
*3*sqrt(c - c/(a**2*x**2))/(a**6*x**6)), x) - Integral(-1/(a*c**3*x*sqrt(c
- c/(a**2*x**2)) + c**3*sqrt(c - c/(a**2*x**2)) - 3*c**3*sqrt(c - c/(a**2*x
**2))/(a*x) - 3*c**3*sqrt(c - c/(a**2*x**2))/(a**2*x**2) + 3*c**3*sqrt(c -
c/(a**2*x**2))/(a**3*x**3) + 3*c**3*sqrt(c - c/(a**2*x**2))/(a**4*x**4) - c
**3*sqrt(c - c/(a**2*x**2))/(a**5*x**5) - c**3*sqrt(c - c/(a**2*x**2))/(a**
6*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a^2x^2-1}{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a^2*x^2))^(7/2)), x)
```

3.732 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx$

Optimal. Leaf size=299

$$\frac{a^9x^{10}\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{4a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{2a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{2\left(1-a^2x^2\right)^{9/2}}-\frac{8a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{5\left(1-a^2x^2\right)^{9/2}}+\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right.}{7\left(1-a^2x^2\right)}$$

```
[Out] -((c - c/(a^2*x^2))^(9/2)*x)/(8*(1 - a^2*x^2)^(9/2)) + (3*a*(c - c/(a^2*x^2
))^(9/2)*x^2)/(7*(1 - a^2*x^2)^(9/2)) - (8*a^3*(c - c/(a^2*x^2))^(9/2)*x^4)
/(5*(1 - a^2*x^2)^(9/2)) + (3*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(2*(1 - a^2*
x^2)^(9/2)) + (2*a^5*(c - c/(a^2*x^2))^(9/2)*x^6)/(1 - a^2*x^2)^(9/2) - (4*
a^6*(c - c/(a^2*x^2))^(9/2)*x^7)/(1 - a^2*x^2)^(9/2) + (a^9*(c - c/(a^2*x^2
))^(9/2)*x^10)/(1 - a^2*x^2)^(9/2) - (3*a^8*(c - c/(a^2*x^2))^(9/2)*x^9*Log
[x])/(1 - a^2*x^2)^(9/2)
```

Rubi [A] time = 0.201024, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$\frac{a^9x^{10}\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{4a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{2a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{2\left(1-a^2x^2\right)^{9/2}}-\frac{8a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{5\left(1-a^2x^2\right)^{9/2}}+\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right.}{7\left(1-a^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(9/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((c - c/(a^2*x^2))^(9/2)*x)/(8*(1 - a^2*x^2)^(9/2)) + (3*a*(c - c/(a^2*x^2
))^(9/2)*x^2)/(7*(1 - a^2*x^2)^(9/2)) - (8*a^3*(c - c/(a^2*x^2))^(9/2)*x^4)
/(5*(1 - a^2*x^2)^(9/2)) + (3*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(2*(1 - a^2*
x^2)^(9/2)) + (2*a^5*(c - c/(a^2*x^2))^(9/2)*x^6)/(1 - a^2*x^2)^(9/2) - (4*
a^6*(c - c/(a^2*x^2))^(9/2)*x^7)/(1 - a^2*x^2)^(9/2) + (a^9*(c - c/(a^2*x^2
))^(9/2)*x^10)/(1 - a^2*x^2)^(9/2) - (3*a^8*(c - c/(a^2*x^2))^(9/2)*x^9*Log
[x])/(1 - a^2*x^2)^(9/2)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{9/2}}{x^9}dx}{\left(1-a^2x^2\right)^{9/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^6(1+ax)^3}{x^9}dx}{\left(1-a^2x^2\right)^{9/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\left(a^9+\frac{1}{x^9}-\frac{3a}{x^8}+\frac{8a^3}{x^6}-\frac{6a^4}{x^5}-\frac{6a^5}{x^4}+\frac{8a^6}{x^3}-\frac{3a^8}{x}\right)dx}{\left(1-a^2x^2\right)^{9/2}}$$
$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x}{8\left(1-a^2x^2\right)^{9/2}}+\frac{3a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{7\left(1-a^2x^2\right)^{9/2}}-\frac{8a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4}{5\left(1-a^2x^2\right)^{9/2}}+\frac{3a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{2\left(1-a^2x^2\right)^{9/2}}+$$

Mathematica [A] time = 0.0930783, size = 97, normalized size = 0.32

$$\frac{x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(-\frac{4a^6}{x^2}+\frac{2a^5}{x^3}+\frac{3a^4}{2x^4}-\frac{8a^3}{5x^5}+a^9x-3a^8\log(x)+\frac{3a}{7x^7}-\frac{1}{8x^8}\right)}{\left(1-a^2x^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(9/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^(9/2)*x^9*(-1/(8*x^8) + (3*a)/(7*x^7) - (8*a^3)/(5*x^5)
+ (3*a^4)/(2*x^4) + (2*a^5)/x^3 - (4*a^6)/x^2 + a^9*x - 3*a^8*Log[x]))/(1 -
 a^2*x^2)^(9/2)
```

Maple [A] time = 0.16, size = 102, normalized size = 0.3

$$\frac{x\left(-280\,a^9x^9+840\,a^8\ln(x)\,x^8+1120\,x^6a^6-560\,x^5a^5-420\,x^4a^4+448\,x^3a^3-120\,ax+35\right)}{280\left(a^2x^2-1\right)^5}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{9}{2}}\sqrt{-a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/280*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/(a^2*x^2-1)^5*(-a^2*x^2+1)^(1/2)*(-28
0*a^9*x^9+840*a^8*ln(x)*x^8+1120*x^6*a^6-560*x^5*a^5-420*x^4*a^4+448*x^3*a^
3-120*a*x+35)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{(ax+1)^3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(9/2)/(a*x + 1)^3, x)
```

Fricas [A] time = 2.1566, size = 1181, normalized size = 3.95

$$\left[\frac{420\left(a^{9}c^{4}x^{9}-a^{7}c^{4}x^{7}\right)\sqrt{-c}\log\left(\frac{a^{2}cx^{6}+a^{2}cx^{2}-cx^{4}-\left(ax^{5}-ax\right)\sqrt{-a^{2}x^{2}+1}\sqrt{-c}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c}{a^{2}x^{4}-x^{2}}\right)-\left(280\,a^{9}c^{4}x^{9}-1120\,a^{6}c^{4}x^{6}+560\,a^{5}c^{4}x^{5}-\cdots\right.}{280\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fr
icas")
```

```
[Out] [1/280*(420*(a^9*c^4*x^9 - a^7*c^4*x^7)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2
 - c*x^4 - (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)) - c)/(a^2*x^4 - x^2)) - (280*a^9*c^4*x^9 - 1120*a^6*c^4*x^6 + 560
*a^5*c^4*x^5 - (280*a^9 - 1120*a^6 + 560*a^5 + 420*a^4 - 448*a^3 + 120*a -
35)*c^4*x^8 + 420*a^4*c^4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x - 35*c^4)*sqr
t(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^10*x^9 - a^8*x^7), 1/28
0*(840*(a^9*c^4*x^9 - a^7*c^4*x^7)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3
 + a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^
2 + c)) - (280*a^9*c^4*x^9 - 1120*a^6*c^4*x^6 + 560*a^5*c^4*x^5 - (280*a^9
- 1120*a^6 + 560*a^5 + 420*a^4 - 448*a^3 + 120*a - 35)*c^4*x^8 + 420*a^4*c^
4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x - 35*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a^
2*c*x^2 - c)/(a^2*x^2)))/(a^10*x^9 - a^8*x^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(9/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{9}{2}}}{(ax+1)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(9/2)/(a*x + 1)^3, x)
```

3.733 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=301

$$-\frac{a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{5a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{2\left(1-a^2x^2\right)^{7/2}}-\frac{5a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{3\left(1-a^2x^2\right)^{7/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{4\left(1-a^2x^2\right)^{7/2}}+\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{5\left(1-a^2x^2\right)^{7/2}}$$

```
[Out] -((c - c/(a^2*x^2))^(7/2)*x)/(6*(1 - a^2*x^2)^(7/2)) + (3*a*(c - c/(a^2*x^2
))^(7/2)*x^2)/(5*(1 - a^2*x^2)^(7/2)) - (a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(
4*(1 - a^2*x^2)^(7/2)) - (5*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(3*(1 - a^2*x^
2)^(7/2)) + (5*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(2*(1 - a^2*x^2)^(7/2)) + (
a^5*(c - c/(a^2*x^2))^(7/2)*x^6)/(1 - a^2*x^2)^(7/2) - (a^7*(c - c/(a^2*x^2
))^(7/2)*x^8)/(1 - a^2*x^2)^(7/2) + (3*a^6*(c - c/(a^2*x^2))^(7/2)*x^7*Log[
x])/(1 - a^2*x^2)^(7/2)
```

Rubi [A] time = 0.189591, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$-\frac{a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{5a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{2\left(1-a^2x^2\right)^{7/2}}-\frac{5a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{3\left(1-a^2x^2\right)^{7/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{4\left(1-a^2x^2\right)^{7/2}}+\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{5\left(1-a^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((c - c/(a^2*x^2))^(7/2)*x)/(6*(1 - a^2*x^2)^(7/2)) + (3*a*(c - c/(a^2*x^2
))^(7/2)*x^2)/(5*(1 - a^2*x^2)^(7/2)) - (a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(
4*(1 - a^2*x^2)^(7/2)) - (5*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(3*(1 - a^2*x^
2)^(7/2)) + (5*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(2*(1 - a^2*x^2)^(7/2)) + (
a^5*(c - c/(a^2*x^2))^(7/2)*x^6)/(1 - a^2*x^2)^(7/2) - (a^7*(c - c/(a^2*x^2
))^(7/2)*x^8)/(1 - a^2*x^2)^(7/2) + (3*a^6*(c - c/(a^2*x^2))^(7/2)*x^7*Log[
x])/(1 - a^2*x^2)^(7/2)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{7/2}}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^5(1+ax)^2}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\left(-a^7+\frac{1}{x^7}-\frac{3a}{x^6}+\frac{a^2}{x^5}+\frac{5a^3}{x^4}-\frac{5a^4}{x^3}-\frac{a^5}{x^2}+\frac{3a^6}{x}\right)dx}{\left(1-a^2x^2\right)^{7/2}}$$
$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}{6\left(1-a^2x^2\right)^{7/2}}+\frac{3a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{5\left(1-a^2x^2\right)^{7/2}}-\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{4\left(1-a^2x^2\right)^{7/2}}-\frac{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3\left(1-a^2x^2\right)^{7/2}}+$$

Mathematica [A] time = 0.0599441, size = 98, normalized size = 0.33

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(60a^7x^7-60a^5x^5-150a^4x^4+100a^3x^3+15a^2x^2-180a^6x^6\log(x)-36ax+10\right)}{60a^6x^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)]*(10 - 36*a*x + 15*a^2*x^2 + 100*a^3*x^3 - 150*a^
4*x^4 - 60*a^5*x^5 + 60*a^7*x^7 - 180*a^6*x^6*Log[x]))/(60*a^6*x^5*Sqrt[1 -
 a^2*x^2])
```

Maple [A] time = 0.166, size = 102, normalized size = 0.3

$$\frac{x\left(-60\,a^7x^7+180\,a^6\ln(x)\,x^6+60\,x^5a^5+150\,x^4a^4-100\,x^3a^3-15\,a^2x^2+36\,ax-10\right)}{60\,\left(a^2x^2-1\right)^4}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/60*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a^2*x^2-1)^4*(-a^2*x^2+1)^(1/2)*(-60*
a^7*x^7+180*a^6*ln(x)*x^6+60*x^5*a^5+150*x^4*a^4-100*x^3*a^3-15*a^2*x^2+36*
a*x-10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(7/2)/(a*x + 1)^3, x)
```

Fricas [A] time = 2.30348, size = 1148, normalized size = 3.81

$$\left[\frac{90\left(a^7c^3x^7 - a^5c^3x^5\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right) - \left(60\,a^7c^3x^7 - 60\,a^5c^3x^5 - 150\,a^4c^3x^4 - \left(60\,a^7\right.\right.}{60\left(a^8x^7 - \right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fr
icas")
```

```
[Out] [1/60*(90*(a^7*c^3*x^7 - a^5*c^3*x^5)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 -
 c*x^4 - (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^
2*x^2)) - c)/(a^2*x^4 - x^2)) - (60*a^7*c^3*x^7 - 60*a^5*c^3*x^5 - 150*a^4*
c^3*x^4 - (60*a^7 - 60*a^5 - 150*a^4 + 100*a^3 + 15*a^2 - 36*a + 10)*c^3*x^
6 + 100*a^3*c^3*x^3 + 15*a^2*c^3*x^2 - 36*a*c^3*x + 10*c^3)*sqrt(-a^2*x^2 +
 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^8*x^7 - a^6*x^5), 1/60*(180*(a^7*c^
3*x^7 - a^5*c^3*x^5)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqrt(c
)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) - (60*
a^7*c^3*x^7 - 60*a^5*c^3*x^5 - 150*a^4*c^3*x^4 - (60*a^7 - 60*a^5 - 150*a^4
 + 100*a^3 + 15*a^2 - 36*a + 10)*c^3*x^6 + 100*a^3*c^3*x^3 + 15*a^2*c^3*x^2
 - 36*a*c^3*x + 10*c^3)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))
/(a^8*x^7 - a^6*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(7/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(7/2)/(a*x + 1)^3, x)
```

3.734 $\int e^{-3\tanh^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^{5/2} dx$

Optimal. Leaf size=218

$$\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4\left(1-a^2x^2\right)^{5/2}}-\frac{3a^4x^5\log(x)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] -((c - c/(a^2*x^2))^(5/2)*x)/(4*(1 - a^2*x^2)^(5/2)) + (a*(c - c/(a^2*x^2))
^(5/2)*x^2)/(1 - a^2*x^2)^(5/2) - (a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(1 - a^
2*x^2)^(5/2) - (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)/(1 - a^2*x^2)^(5/2) + (a
^5*(c - c/(a^2*x^2))^(5/2)*x^6)/(1 - a^2*x^2)^(5/2) - (3*a^4*(c - c/(a^2*x^
2))^(5/2)*x^5*Log[x])/(1 - a^2*x^2)^(5/2)
```

Rubi [A] time = 0.184716, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 75}

$$\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4\left(1-a^2x^2\right)^{5/2}}-\frac{3a^4x^5\log(x)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^(3*ArcTanh[a*x]),x]
```


```
[Out] -((c - c/(a^2*x^2))^(5/2)*x)/(4*(1 - a^2*x^2)^(5/2)) + (a*(c - c/(a^2*x^2))
^(5/2)*x^2)/(1 - a^2*x^2)^(5/2) - (a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(1 - a^
2*x^2)^(5/2) - (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)/(1 - a^2*x^2)^(5/2) + (a
^5*(c - c/(a^2*x^2))^(5/2)*x^6)/(1 - a^2*x^2)^(5/2) - (3*a^4*(c - c/(a^2*x^
2))^(5/2)*x^5*Log[x])/(1 - a^2*x^2)^(5/2)
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}}{x^5}dx}{\left(1-a^2x^2\right)^{5/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{(1-ax)^4(1+ax)}{x^5}dx}{\left(1-a^2x^2\right)^{5/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \left(a^5+\frac{1}{x^5}-\frac{3a}{x^4}+\frac{2a^2}{x^3}+\frac{2a^3}{x^2}-\frac{3a^4}{x}\right)dx}{\left(1-a^2x^2\right)^{5/2}}$$
$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}{4\left(1-a^2x^2\right)^{5/2}}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{\left(1-a^2x^2\right)^{5/2}}-\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{\left(1-a^2x^2\right)^{5/2}}-\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{\left(1-a^2x^2\right)^{5/2}}+\frac{a}{}$$

Mathematica [A] time = 0.053199, size = 90, normalized size = 0.41

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(4a^5x^5-5a^4x^4-8a^3x^3-4a^2x^2-12a^4x^4\log(x)+4ax-1\right)}{4a^4x^3\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)]*(-1 + 4*a*x - 4*a^2*x^2 - 8*a^3*x^3 - 5*a^4*x^4
+ 4*a^5*x^5 - 12*a^4*x^4*Log[x]))/(4*a^4*x^3*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.152, size = 86, normalized size = 0.4

$$\frac{x\left(-4x^5a^5+12a^4\ln(x)x^4+8x^3a^3+4a^2x^2-4ax+1\right)}{4\left(a^2x^2-1\right)^3}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/4*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/(a^2*x^2-1)^3*(-a^2*x^2+1)^(1/2)*(-4*x^
5*a^5+12*a^4*ln(x)*x^4+8*x^3*a^3+4*a^2*x^2-4*a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(5/2)/(a*x + 1)^3, x)
```

Fricas [A] time = 2.09093, size = 959, normalized size = 4.4

$$\left[\frac{6\left(a^5c^2x^5 - a^3c^2x^3\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right) - \left(4\,a^5c^2x^5 - 8\,a^3c^2x^3 - \left(4\,a^5 - 8\,a^3 - 4\,a^2 + 4\right.\right.}{4\left(a^6x^5 - a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fr
icas")
```

```
[Out] [1/4*(6*(a^5*c^2*x^5 - a^3*c^2*x^3)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c
*x^4 - (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)) - c)/(a^2*x^4 - x^2)) - (4*a^5*c^2*x^5 - 8*a^3*c^2*x^3 - (4*a^5 - 8*a
^3 - 4*a^2 + 4*a - 1)*c^2*x^4 - 4*a^2*c^2*x^2 + 4*a*c^2*x - c^2)*sqrt(-a^2*
x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5 - a^4*x^3), 1/4*(12*(a^5
*c^2*x^5 - a^3*c^2*x^3)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqr
t(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) - (
4*a^5*c^2*x^5 - 8*a^3*c^2*x^3 - (4*a^5 - 8*a^3 - 4*a^2 + 4*a - 1)*c^2*x^4 -
 4*a^2*c^2*x^2 + 4*a*c^2*x - c^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^6*x^5 - a^4*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(5/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(5/2)/(a*x + 1)^3, x)
```

3.735 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=146

$$-\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}+\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2\left(1-a^2x^2\right)^{3/2}}+\frac{3a^2x^3\log(x)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] -((c - c/(a^2*x^2))^(3/2)*x)/(2*(1 - a^2*x^2)^(3/2)) + (3*a*(c - c/(a^2*x^2
))^(3/2)*x^2)/(1 - a^2*x^2)^(3/2) - (a^3*(c - c/(a^2*x^2))^(3/2)*x^4)/(1 -
a^2*x^2)^(3/2) + (3*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*Log[x])/(1 - a^2*x^2)^(
3/2)
```

Rubi [A] time = 0.167182, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 43}

$$-\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}+\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2\left(1-a^2x^2\right)^{3/2}}+\frac{3a^2x^3\log(x)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((c - c/(a^2*x^2))^(3/2)*x)/(2*(1 - a^2*x^2)^(3/2)) + (3*a*(c - c/(a^2*x^2
))^(3/2)*x^2)/(1 - a^2*x^2)^(3/2) - (a^3*(c - c/(a^2*x^2))^(3/2)*x^4)/(1 -
a^2*x^2)^(3/2) + (3*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*Log[x])/(1 - a^2*x^2)^(
3/2)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{(1-ax)^3}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \left(-a^3+\frac{1}{x^3}-\frac{3a}{x^2}+\frac{3a^2}{x}\right)dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}{2\left(1-a^2x^2\right)^{3/2}}+\frac{3a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{\left(1-a^2x^2\right)^{3/2}}-\frac{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^4}{\left(1-a^2x^2\right)^{3/2}}+\frac{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(1-a^2x^2\right)^{3/2}}$$

Mathematica [A] time = 0.0369828, size = 64, normalized size = 0.44

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(2a^3x^3-6a^2x^2\log(x)-6ax+1\right)}{2a^2x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a^2*x^2)]*(1 - 6*a*x + 2*a^3*x^3 - 6*a^2*x^2*Log[x]))/(2*a^2
*x*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.162, size = 70, normalized size = 0.5

$$\frac{x\left(-2\,x^3a^3+6\,a^2\ln(x)\,x^2+6\,ax-1\right)}{2\,\left(a^2x^2-1\right)^2}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/2*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a^2*x^2-1)^2*(-a^2*x^2+1)^(1/2)*(-2*x^
3*a^3+6*a^2*ln(x)*x^2+6*a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{(ax+1)^3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(3/2)/(a*x + 1)^3, x)
```

Fricas [A] time = 2.08172, size = 779, normalized size = 5.34

$$\left[\frac{3\left(a^3cx^3-acx\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)-\left(2a^3cx^3-\left(2a^3-6a+1\right)cx^2-6acx+c\right)\sqrt{-\ldots}}{2\left(a^4x^3-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fr
icas")
```

```
[Out] [1/2*(3*(a^3*c*x^3 - a*c*x)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 - (
a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) -
c)/(a^2*x^4 - x^2)) - (2*a^3*c*x^3 - (2*a^3 - 6*a + 1)*c*x^2 - 6*a*c*x + c)
*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3 - a^2*x), 1/2
*(6*(a^3*c*x^3 - a*c*x)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqr
t(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) - (
2*a^3*c*x^3 - (2*a^3 - 6*a + 1)*c*x^2 - 6*a*c*x + c)*sqrt(-a^2*x^2 + 1)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3 - a^2*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(3/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(3/2)/(a*x + 1)^3, x)
```

3.736 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=106

$$\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] (a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2] - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[1
 - a^2*x^2]
```

Rubi [A] time = 0.147256, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 72}

$$\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2] - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[1
 - a^2*x^2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^2}{x(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{a\sqrt{c-\frac{c}{a^2x^2}}x^2}{\sqrt{1-a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x\log(x)}{\sqrt{1-a^2x^2}} - \frac{4\sqrt{c-\frac{c}{a^2x^2}}x\log(1+ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0260811, size = 45, normalized size = 0.42

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}(ax-4\log(ax+1)+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(a*x + Log[x] - 4*Log[1 + a*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.154, size = 60, normalized size = 0.6

$$-\frac{x\left(ax+\ln(x)-4\ln(ax+1)\right)}{a^2x^2-1}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(a*x+ln(x)-4*ln(a*x+1))*(-a^2*x^2+1)^(1/2)
/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/(a*x + 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}(ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2x^2+2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2
*x^2 + 2*a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))
)/(a*x + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/(a*x + 1)^3, x)
```

3.737 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-a^2x^2}}{a^2x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-a^2x^2}\log(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] -(Sqrt[1 - a^2*x^2]/(a*Sqrt[c - c/(a^2*x^2)])) + (2*Sqrt[1 - a^2*x^2])/(a^2
*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)) + (3*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(a
^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Rubi [A] time = 0.136777, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 77}

$$-\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-a^2x^2}}{a^2x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-a^2x^2}\log(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```


```
[Out] -(Sqrt[1 - a^2*x^2]/(a*Sqrt[c - c/(a^2*x^2)])) + (2*Sqrt[1 - a^2*x^2])/(a^2
*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)) + (3*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(a
^2*Sqrt[c - c/(a^2*x^2)]*x)
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-3\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x(1-ax)}{(1+ax)^2}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{a}-\frac{2}{a(1+ax)^2}+\frac{3}{a(1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= -\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-a^2x^2}}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}+\frac{3\sqrt{1-a^2x^2}\log(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.0475552, size = 58, normalized size = 0.48

$$\frac{\sqrt{1-a^2x^2}\left(-ax+\frac{2}{ax+1}+3\log(ax+1)\right)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-(a*x) + 2/(1 + a*x) + 3*Log[1 + a*x]))/(a^2*Sqrt[c - c
/(a^2*x^2)]*x)
```

Maple [A] time = 0.158, size = 78, normalized size = 0.6

$$\frac{-a^2x^2+3\,ax\ln(ax+1)-ax+3\,\ln(ax+1)+2}{a^2x\,(ax+1)}\sqrt{-a^2x^2+1}\frac{1}{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-a^2*x^2+1)^(1/2)/a^2*(-a^2*x^2+3*a*x*ln
(a*x+1)-a*x+3*ln(a*x+1)+2)/(a*x+1)
```

Maxima [C] time = 1.05672, size = 72, normalized size = 0.59

$$-\frac{i\,a^2\sqrt{c}x^2+i\,a\sqrt{c}x-2i\,\sqrt{c}}{a^2cx+ac}+\frac{3i\,\log(ax+1)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="
maxima")
```

```
[Out] -(I*a^2*sqrt(c)*x^2 + I*a*sqrt(c)*x - 2*I*sqrt(c))/(a^2*c*x + a*c) + 3*I*lo
g(a*x + 1)/(a*sqrt(c))
```

Fricas [A] time = 2.20916, size = 910, normalized size = 7.46

$$\left[-\frac{3\left(a^3x^3 + a^2x^2 - ax - 1\right)\sqrt{-c}\log\left(\frac{a^6cx^6+4a^5cx^5+5a^4cx^4-4a^2cx^2-4acx+\left(a^5x^5+4a^4x^4+6a^3x^3+4a^2x^2\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{a^4x^4+2a^3x^3-2ax-1}\right) + 2\left(\right.}{2\left(a^4cx^3 + a^3cx^2 - a^2cx - ac\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="
fricas")
```

```
[Out] [-1/2*(3*(a^3*x^3 + a^2*x^2 - a*x - 1)*sqrt(-c)*log((a^6*c*x^6 + 4*a^5*c*x^
5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 - 4*a*c*x + (a^5*x^5 + 4*a^4*x^4 + 6*a^3*x^3
+ 4*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) -
2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)) + 2*(a^3*x^3 + 3*a^2*x^2)*sqrt(-a^2
*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*c*x^3 + a^3*c*x^2 - a^2*c*x
 - a*c), (3*(a^3*x^3 + a^2*x^2 - a*x - 1)*sqrt(c)*arctan((a^2*x^2 + 2*a*x +
 2)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 +
 2*a^2*c*x^2 - a*c*x - 2*c)) - (a^3*x^3 + 3*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*c*x^3 + a^3*c*x^2 - a^2*c*x - a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(ax - 1\right)\left(ax + 1\right)\right)^{\frac{3}{2}}}{\sqrt{-c\left(-1 + \frac{1}{ax}\right)\left(1 + \frac{1}{ax}\right)}\left(ax + 1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)
))*(a*x + 1)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2 + 1\right)^{\frac{3}{2}}}{\left(ax + 1\right)^3\sqrt{c - \frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*sqrt(c - c/(a^2*x^2))), x)
```

3.738 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{3\left(1-a^2x^2\right)^{3/2}}{a^4x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}}{a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4x^3(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{3\left(1-a^2x^2\right)^{3/2}\log(ax+1)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[Out] (1 - a^2*x^2)^(3/2)/(a^3*(c - c/(a^2*x^2))^(3/2)*x^2) + (1 - a^2*x^2)^(3/2)
/(2*a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 + a*x)^2) - (3*(1 - a^2*x^2)^(3/2))/
(a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 + a*x)) - (3*(1 - a^2*x^2)^(3/2)*Log[1
+ a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rubi [A] time = 0.197348, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 43}

$$-\frac{3\left(1-a^2x^2\right)^{3/2}}{a^4x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}}{a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4x^3(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{3\left(1-a^2x^2\right)^{3/2}\log(ax+1)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(a^3*(c - c/(a^2*x^2))^(3/2)*x^2) + (1 - a^2*x^2)^(3/2)
/(2*a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 + a*x)^2) - (3*(1 - a^2*x^2)^(3/2))/
(a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 + a*x)) - (3*(1 - a^2*x^2)^(3/2)*Log[1
+ a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{e^{-3\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{x^3}{(1+ax)^3}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}\int \left(\frac{1}{a^3}-\frac{1}{a^3(1+ax)^3}+\frac{3}{a^3(1+ax)^2}-\frac{3}{a^3(1+ax)}\right)dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$
$$= \frac{\left(1-a^2x^2\right)^{3/2}}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3(1+ax)^2}-\frac{3\left(1-a^2x^2\right)^{3/2}}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3(1+ax)}-\frac{3\left(1-a^2x^2\right)^{3/2}\log}{a^4\left(c-\frac{c}{a^2x^2}\right.}$$

Mathematica [A] time = 0.0557708, size = 86, normalized size = 0.51

$$-\frac{\sqrt{1-a^2x^2}\left(a^2x^2-1\right)\left(\frac{x}{a^3}-\frac{3}{a^4(ax+1)}+\frac{1}{2a^4(ax+1)^2}-\frac{3\log(ax+1)}{a^4}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(-1 + a^2*x^2)*(x/a^3 + 1/(2*a^4*(1 + a*x)^2) - 3/(a^4
*(1 + a*x)) - (3*Log[1 + a*x])/a^4))/((c - c/(a^2*x^2))^(3/2)*x^3))
```

Maple [A] time = 0.177, size = 106, normalized size = 0.6

$$\frac{\left(-2x^3a^3+6\ln(ax+1)a^2x^2-4a^2x^2+12ax\ln(ax+1)+4ax+6\ln(ax+1)+5\right)(ax-1)}{(2ax+2)a^4x^3}\sqrt{-a^2x^2+1}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/2*(-2*x^3*a^3+6*ln(a*x+1)*a^2*x^2-4*a^2*x^2+12*a*x*ln(a*x+1)+4*a*x+6*ln(a
*x+1)+5)*(a*x-1)*(-a^2*x^2+1)^(1/2)/(a*x+1)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)
^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^(3/2)), x)
```

Fricas [A] time = 2.1977, size = 994, normalized size = 5.85

$$\left[-\frac{3\left(a^4x^4+2a^3x^3-2ax-1\right)\sqrt{-c}\log\left(\frac{a^6cx^6+4a^5cx^5+5a^4cx^4-4a^2cx^2-4acx+\left(a^5x^5+4a^4x^4+6a^3x^3+4a^2x^2\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{a^4x^4+2a^3x^3-2ax-1}\right)+(2}{2\left(a^5c^2x^4+2a^4c^2x^3-2a^2c^2x-ac^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="
fricas")
```

```
[Out] [-1/2*(3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*sqrt(-c)*log((a^6*c*x^6 + 4*a^5*
c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 - 4*a*c*x + (a^5*x^5 + 4*a^4*x^4 + 6*a^3*
x^3 + 4*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)) + (2*a^4*x^4 + 9*a^3*x^3 + 6*a^
2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^2*x^4 + 2
*a^4*c^2*x^3 - 2*a^2*c^2*x - a*c^2), 1/2*(6*(a^4*x^4 + 2*a^3*x^3 - 2*a*x -
1)*sqrt(c)*arctan((a^2*x^2 + 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^
2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 + 2*a^2*c*x^2 - a*c*x - 2*c)) - (2*a^4*x
^4 + 9*a^3*x^3 + 6*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)))/(a^5*c^2*x^4 + 2*a^4*c^2*x^3 - 2*a^2*c^2*x - a*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^(3/2)), x)
```

3.739 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=267

$$\frac{31\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{9\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{6a^6x^5(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{\left(1-a^2\right.}{16a^6}$$

```
[Out] -((1 - a^2*x^2)^(5/2)/(a^5*(c - c/(a^2*x^2))^(5/2)*x^4)) + (1 - a^2*x^2)^(5
/2)/(6*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 + a*x)^3) - (9*(1 - a^2*x^2)^(5/2
))/(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 + a*x)^2) + (31*(1 - a^2*x^2)^(5/2
))/(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 + a*x)) - ((1 - a^2*x^2)^(5/2)*Log
[1 - a*x])/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) + (49*(1 - a^2*x^2)^(5/2)*L
og[1 + a*x])/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rubi [A] time = 0.220454, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$\frac{31\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{9\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{6a^6x^5(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{\left(1-a^2\right.}{16a^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] -((1 - a^2*x^2)^(5/2)/(a^5*(c - c/(a^2*x^2))^(5/2)*x^4)) + (1 - a^2*x^2)^(5
/2)/(6*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 + a*x)^3) - (9*(1 - a^2*x^2)^(5/2
))/(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 + a*x)^2) + (31*(1 - a^2*x^2)^(5/2
))/(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 + a*x)) - ((1 - a^2*x^2)^(5/2)*Log
[1 - a*x])/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) + (49*(1 - a^2*x^2)^(5/2)*L
og[1 + a*x])/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{e^{-3\tanh^{-1}(ax)}x^5}{\left(1-a^2x^2\right)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{x^5}{(1-ax)(1+ax)^4}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}\int \left(-\frac{1}{a^5}-\frac{1}{16a^5(-1+ax)}-\frac{1}{2a^5(1+ax)^4}+\frac{9}{4a^5(1+ax)^3}-\frac{31}{8a^5(1+ax)^2}+\frac{49}{16a^5(1+ax)}\right)dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$
$$= -\frac{\left(1-a^2x^2\right)^{5/2}}{a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}+\frac{\left(1-a^2x^2\right)^{5/2}}{6a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5(1+ax)^3}-\frac{9\left(1-a^2x^2\right)^{5/2}}{8a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5(1+ax)^2}+\frac{31\left(1-a^2\right.}{8a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Mathematica [A] time = 0.0906552, size = 110, normalized size = 0.41

$$\frac{\sqrt{1-a^2x^2}\left(-48a^4x^4-144a^3x^3+42a^2x^2+270ax-3(ax+1)^3\log(1-ax)+147(ax+1)^3\log(ax+1)+140\right)}{48a^2c^2x(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(140 + 270*a*x + 42*a^2*x^2 - 144*a^3*x^3 - 48*a^4*x^4 -
 3*(1 + a*x)^3*Log[1 - a*x] + 147*(1 + a*x)^3*Log[1 + a*x]))/(48*a^2*c^2*Sq
rt[c - c/(a^2*x^2)]*x*(1 + a*x)^3)
```

Maple [A] time = 0.187, size = 176, normalized size = 0.7

$$\frac{\left(-48x^4a^4+147a^3x^3\ln(ax+1)-3\ln(ax-1)x^3a^3-144x^3a^3+441\ln(ax+1)a^2x^2-9\ln(ax-1)a^2x^2+42a^2x^2+44\right.}{(48ax+48)a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/48*(-48*x^4*a^4+147*a^3*x^3*ln(a*x+1)-3*ln(a*x-1)*x^3*a^3-144*x^3*a^3+441
*ln(a*x+1)*a^2*x^2-9*ln(a*x-1)*a^2*x^2+42*a^2*x^2+441*a*x*ln(a*x+1)-9*ln(a*
x-1)*x*a+270*a*x+147*ln(a*x+1)-3*ln(a*x-1)+140)*(a*x-1)^2*(-a^2*x^2+1)^(1/2
)/(a*x+1)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}a^6x^6\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^7c^3x^7+3\,a^6c^3x^6+a^5c^3x^5-5\,a^4c^3x^4-5\,a^3c^3x^3+a^2c^3x^2+3\,ac^3x+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="
fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*a^6*x^6*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^7*c
^3*x^7 + 3*a^6*c^3*x^6 + a^5*c^3*x^5 - 5*a^4*c^3*x^4 - 5*a^3*c^3*x^3 + a^2*
c^3*x^2 + 3*a*c^3*x + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^(5/2)), x)
```

3.740 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=359

$$\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{75\left(1-a^2x^2\right)^{7/2}}{16a^8x^7(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{59\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{\left(1-a^2x^2\right)^{7/2}}{2a^8x^7(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

```
[Out] (1 - a^2*x^2)^(7/2)/(a^7*(c - c/(a^2*x^2))^(7/2)*x^6) + (1 - a^2*x^2)^(7/2)
/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)) + (1 - a^2*x^2)^(7/2)/(16*a
^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^4) - (1 - a^2*x^2)^(7/2)/(2*a^8*(c
 - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^3) + (59*(1 - a^2*x^2)^(7/2))/(32*a^8*(
c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^2) - (75*(1 - a^2*x^2)^(7/2))/(16*a^8*
(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)) + (9*(1 - a^2*x^2)^(7/2)*Log[1 - a*x
])/(64*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) - (201*(1 - a^2*x^2)^(7/2)*Log[1 +
a*x])/(64*a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```

Rubi [A] time = 0.251201, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{75\left(1-a^2x^2\right)^{7/2}}{16a^8x^7(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{59\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{\left(1-a^2x^2\right)^{7/2}}{2a^8x^7(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```


```
[Out] (1 - a^2*x^2)^(7/2)/(a^7*(c - c/(a^2*x^2))^(7/2)*x^6) + (1 - a^2*x^2)^(7/2)
/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)) + (1 - a^2*x^2)^(7/2)/(16*a
^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^4) - (1 - a^2*x^2)^(7/2)/(2*a^8*(c
 - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^3) + (59*(1 - a^2*x^2)^(7/2))/(32*a^8*(
c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^2) - (75*(1 - a^2*x^2)^(7/2))/(16*a^8*
(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)) + (9*(1 - a^2*x^2)^(7/2)*Log[1 - a*x
])/(64*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) - (201*(1 - a^2*x^2)^(7/2)*Log[1 +
a*x])/(64*a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left(1-a^2x^2\right)^{7/2}\int \frac{e^{-3\tanh^{-1}(ax)}x^7}{\left(1-a^2x^2\right)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left(1-a^2x^2\right)^{7/2}\int \frac{x^7}{(1-ax)^2(1+ax)^5}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left(1-a^2x^2\right)^{7/2}\int \left(\frac{1}{a^7}+\frac{1}{32a^7(-1+ax)^2}+\frac{9}{64a^7(-1+ax)}-\frac{1}{4a^7(1+ax)^5}+\frac{3}{2a^7(1+ax)^4}-\frac{59}{16a^7(1+ax)^3}+\frac{75}{16a^7(1+ax)^2}-\right.}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left(1-a^2x^2\right)^{7/2}}{a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}+\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7(1-ax)}+\frac{\left(1-a^2x^2\right)^{7/2}}{16a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7(1+ax)^4}-\frac{\left(1-\right.}{2a^8\left(c-\frac{c}{a^2x^2}\right.}$$

Mathematica [A] time = 0.121583, size = 146, normalized size = 0.41

$$\frac{\sqrt{1-a^2x^2}\left(-2\left(32a^6x^6+96a^5x^5-87a^4x^4-309a^3x^3-59a^2x^2+207ax+104\right)-9(ax-1)(ax+1)^4\log(1-ax)+201\right.}{64a^2c^3x(ax-1)(ax+1)^4\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2*(104 + 207*a*x - 59*a^2*x^2 - 309*a^3*x^3 - 87*a^4*x
^4 + 96*a^5*x^5 + 32*a^6*x^6) - 9*(-1 + a*x)*(1 + a*x)^4*Log[1 - a*x] + 201
*(-1 + a*x)*(1 + a*x)^4*Log[1 + a*x]))/(64*a^2*c^3*Sqrt[c - c/(a^2*x^2)]*x*
(-1 + a*x)*(1 + a*x)^4)
```

Maple [A] time = 0.173, size = 248, normalized size = 0.7

$$\frac{(ax-1)^2\left(-64x^6a^6+201\ln(ax+1)x^5a^5-9\ln(ax-1)x^5a^5-192x^5a^5+603\ln(ax+1)a^4x^4-27\ln(ax-1)a^4x^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/64*(-a^2*x^2+1)^(1/2)*(a*x-1)^2*(-64*x^6*a^6+201*ln(a*x+1)*x^5*a^5-9*ln(a
*x-1)*x^5*a^5-192*x^5*a^5+603*ln(a*x+1)*a^4*x^4-27*ln(a*x-1)*a^4*x^4+174*x^
4*a^4+402*a^3*x^3*ln(a*x+1)-18*ln(a*x-1)*x^3*a^3+618*x^3*a^3-402*ln(a*x+1)*
a^2*x^2+18*ln(a*x-1)*a^2*x^2+118*a^2*x^2-603*a*x*ln(a*x+1)+27*ln(a*x-1)*x*a
-414*a*x-201*ln(a*x+1)+9*ln(a*x-1)-208)/(a*x+1)/a^8/x^7/(c*(a^2*x^2-1)/a^2/
x^2)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}a^8x^8\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^9c^4x^9+3\,a^8c^4x^8-8\,a^6c^4x^6-6\,a^5c^4x^5+6\,a^4c^4x^4+8\,a^3c^4x^3-3\,ac^4x-c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="
fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*a^8*x^8*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^9*c
^4*x^9 + 3*a^8*c^4*x^8 - 8*a^6*c^4*x^6 - 6*a^5*c^4*x^5 + 6*a^4*c^4*x^4 + 8*
a^3*c^4*x^3 - 3*a*c^4*x - c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^(7/2)), x)
```

3.741 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx$

Optimal. Leaf size=80

$$\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{m\sqrt{1-a^2x^2}}+\frac{ax^{m+2}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-a^2x^2}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^(1 + m))/(m*Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - c/(a^
2*x^2)]*x^(2 + m))/((1 + m)*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.220095, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6160, 6150, 43}

$$\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{m\sqrt{1-a^2x^2}}+\frac{ax^{m+2}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)]*x^m,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^(1 + m))/(m*Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - c/(a^
2*x^2)]*x^(2 + m))/((1 + m)*Sqrt[1 - a^2*x^2])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{\tanh^{-1}(ax)}x^{-1+m}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int x^{-1+m}(1+ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(x^{-1+m}+ax^m\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x^{1+m}}{m\sqrt{1-a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}x^{2+m}}{(1+m)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0311057, size = 51, normalized size = 0.64

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax^{m+1}}{m+1}+\frac{x^m}{m}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)]*x^m,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(x^m/m + (a*x^(1 + m))/(1 + m)))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.087, size = 53, normalized size = 0.7

$$\frac{x^{1+m}(axm+m+1)}{(1+m)\,m}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] x^(1+m)*(a*m*x+m+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(1+m)/m/(-a^2*x^2+1)^(1/2
)
```

Maxima [C] time = 1.16039, size = 41, normalized size = 0.51

$$\frac{\sqrt{c}xx^m}{i\,m+i}-\frac{i\,\sqrt{c}x^m}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="
maxima")
```

```
[Out] sqrt(c)*x*x^m/(I*m + I) - I*sqrt(c)*x^m/(a*m)
```

Fricas [A] time = 1.87706, size = 153, normalized size = 1.91

$$-\frac{\sqrt{-a^2x^2+1}\left(amx^2+(m+1)x\right)x^m\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\left(a^2m^2+a^2m\right)x^2-m^2-m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="
fricas")
```

```
[Out] -sqrt(-a^2*x^2 + 1)*(a*m*x^2 + (m + 1)*x)*x^m*sqrt((a^2*c*x^2 - c)/(a^2*x^2
))/((a^2*m^2 + a^2*m)*x^2 - m^2 - m)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x -
1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}\,x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^m/sqrt(-a^2*x^2 + 1), x)
```

3.742 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal. Leaf size=74

$$\frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-a^2x^2}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^3)/(2*Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - c/(a^2*x^2)
]*x^4)/(3*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.212168, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6160, 6150, 43}

$$\frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-a^2x^2}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^3)/(2*Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - c/(a^2*x^2)
]*x^4)/(3*Sqrt[1 - a^2*x^2])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int x(1+ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(x+ax^2\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{2\sqrt{1-a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}x^4}{3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0195994, size = 42, normalized size = 0.57

$$\frac{x^3(2ax+3)\sqrt{c-\frac{c}{a^2x^2}}}{6\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^3*(3 + 2*a*x))/(6*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.105, size = 43, normalized size = 0.6

$$\frac{x^3\,(2\,ax+3)}{6}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/6*x^3*(2*a*x+3)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Maxima [C] time = 1.14674, size = 27, normalized size = 0.36

$$-\frac{1}{3}i\,\sqrt{c}x^3 - \frac{i\,\sqrt{c}x^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="
maxima")
```

```
[Out] -1/3*I*sqrt(c)*x^3 - 1/2*I*sqrt(c)*x^2/a
```

Fricas [A] time = 1.7899, size = 120, normalized size = 1.62

$$-\frac{\left(2\,ax^{4}+3\,x^{3}\right)\sqrt{-a^{2}x^{2}+1}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{6\left(a^{2}x^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="
fricas")
```

```
[Out] -1/6*(2*a*x^4 + 3*x^3)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(
a^2*x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{2}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x -
1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\sqrt{c-\frac{c}{a^{2}x^{2}}}\,x^{2}}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^2/sqrt(-a^2*x^2 + 1), x)
```

3.743 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal. Leaf size=71

$$\frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}+\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (a*Sqrt[c - c/(a^2*x^2)]*x^
3)/(2*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.127907, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6160, 6140}

$$\frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}+\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (a*Sqrt[c - c/(a^2*x^2)]*x^
3)/(2*Sqrt[1 - a^2*x^2])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx&=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}\\&=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int(1+ax)\,dx}{\sqrt{1-a^2x^2}}\\&=\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}x^3}{2\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0198959, size = 41, normalized size = 0.58

$$\frac{x\left(\frac{ax^2}{2}+x\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(x + (a*x^2)/2))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.092, size = 42, normalized size = 0.6

$$\frac{x^{2}\left(ax+2\right)}{2}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\frac{1}{\sqrt{-a^{2}x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/2*x^2*(a*x+2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Maxima [C] time = 1.13777, size = 24, normalized size = 0.34

$$-\frac{1}{2}i\sqrt{c}x^{2}-\frac{i\sqrt{c}x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/2*I*sqrt(c)*x^2 - I*sqrt(c)*x/a
```

Fricas [A] time = 1.87118, size = 117, normalized size = 1.65

$$-\frac{\sqrt{-a^{2}x^{2}+1}\left(ax^{3}+2\,x^{2}\right)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{2\left(a^{2}x^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(a*x^3 + 2*x^2)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^
2*x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x - 1)*
(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x/sqrt(-a^2*x^2 + 1), x)
```

3.744 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=68

$$\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

```
[Out] (a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.129959, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 43}

$$\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1+ax}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(a+\frac{1}{x}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{a\sqrt{c-\frac{c}{a^2x^2}}x^2}{\sqrt{1-a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0179108, size = 37, normalized size = 0.54

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}(ax+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(a*x + Log[x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.166, size = 52, normalized size = 0.8

$$-\frac{x(ax+\ln(x))}{a^2x^2-1}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(a*x+ln(x))*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)
```

Maxima [C] time = 1.15479, size = 23, normalized size = 0.34

$$-i\sqrt{c}x - \frac{i\sqrt{c}\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -I*sqrt(c)*x - I*sqrt(c)*log(x)/a
```

Fricas [B] time = 2.0638, size = 648, normalized size = 9.53

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)-2\left(a^2x^2-a^2x\right)\sqrt{-a^2x^2+1}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^3x^2-a\right)},-\frac{\left(a^2x^2-1\right)\sqrt{c}\,\mathrm{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 + (a*x^5 -
a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c)/(a^2*
x^4 - x^2)) - 2*(a^2*x^2 - a^2*x)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^3*x^2 - a), -((a^2*x^2 - 1)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)
*(a*x^3 + a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 +
1)*c*x^2 + c)) + (a^2*x^2 - a^2*x)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)))/(a^3*x^2 - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x - 1)*(a
*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/sqrt(-a^2*x^2 + 1), x)
```

3.745 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=66

$$\frac{ax\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - c/(a^2*x^2)]*x*Log
[x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.222997, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6160, 6150, 43}

$$\frac{ax\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - c/(a^2*x^2)]*x*Log
[x])/Sqrt[1 - a^2*x^2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1+ax}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(\frac{1}{x^2}+\frac{a}{x}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}x\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0210531, size = 37, normalized size = 0.56

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax\log(x)-1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1 + a*x*Log[x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.182, size = 52, normalized size = 0.8

$$-\frac{a\ln(x)x-1}{a^2x^2-1}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*ln(x)*x-1)*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)
```

Maxima [C] time = 1.14641, size = 26, normalized size = 0.39

$$-i\sqrt{c}\log(x) + \frac{i\sqrt{c}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="ma
xima")
```

```
[Out] -I*sqrt(c)*log(x) + I*sqrt(c)/(a*x)
```

Fricas [B] time = 2.03989, size = 621, normalized size = 9.41

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)-2\sqrt{-a^2x^2+1}(x-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^2x^2-1\right)},-\frac{\left(a^2x^2-1\right)\sqrt{c}\operatorname{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fr
icas")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 + (a*x^5 -
a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c)/(a^2*
x^4 - x^2)) - 2*sqrt(-a^2*x^2 + 1)*(x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))
/(a^2*x^2 - 1), -((a^2*x^2 - 1)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 +
a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 +
 c)) + sqrt(-a^2*x^2 + 1)*(x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^2
 - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x*sqrt(-(a*x - 1)
*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/(sqrt(-a^2*x^2 + 1)*x), x)
```

3.746 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=43

$$-\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}$$

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(2*x*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.21235, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6160, 6150, 37}

$$-\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(2*x*Sqrt[1 - a^2*x^2])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^3}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1+ax}{x^3}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{2x\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0185749, size = 42, normalized size = 0.98

$$-\frac{(2ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*(1 + 2*a*x))/(2*x*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.091, size = 43, normalized size = 1.

$$-\frac{2\,ax+1}{2\,x}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x)
```

```
[Out] -1/2*(2*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/(-a^2*x^2+1)^(1/2)
```

Maxima [C] time = 1.15945, size = 27, normalized size = 0.63

$$\frac{i\,\sqrt{c}}{x}+\frac{i\,\sqrt{c}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
maxima")
```

```
[Out] I*sqrt(c)/x + 1/2*I*sqrt(c)/(a*x^2)
```

Fricas [A] time = 1.8587, size = 134, normalized size = 3.12

$$-\frac{\sqrt{-a^2x^2+1}\left((2\,a+1)x^2-2\,ax-1\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^2x^3-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
fricas")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*((2*a + 1)*x^2 - 2*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a
^2*x^2))/(a^2*x^3 - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**2*sqrt(-(a*x -
 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
giac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

3.747 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

Optimal. Leaf size=160

$$-\frac{x^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}-\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}-\frac{7x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{8a^3\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (-7*Sqrt[c - c/(a^2*x^2)]*x)/(8*a^3) - (7*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)
)/(24*a^3) - (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)^2)/(6*a^3) - (Sqrt[c - c/(a
^2*x^2)]*x^2*(1 + a*x)^2)/(4*a^2) + (7*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])
/(8*a^3*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.408155, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6159, 6129, 90, 80, 50, 41, 216}

$$-\frac{x^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}-\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}-\frac{7x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{8a^3\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```


```
[Out] (-7*Sqrt[c - c/(a^2*x^2)]*x)/(8*a^3) - (7*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)
)/(24*a^3) - (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)^2)/(6*a^3) - (Sqrt[c - c/(a
^2*x^2)]*x^2*(1 + a*x)^2)/(4*a^2) + (7*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])
/(8*a^3*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```


Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{2\tanh^{-1}(ax)}x^2\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{x^2(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(-1-2ax)(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{4a^2\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{6a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2} + \frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{12a^2\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{6a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2} + \frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}x\right)}{8a^2\sqrt{1-ax}} \\
&= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{6a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2} + \\
&= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{6a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2} + \\
&= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{6a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2} +
\end{aligned}$$

Mathematica [A] time = 0.0897146, size = 93, normalized size = 0.58

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(6a^3x^3+16a^2x^2+21ax+32\right)+21\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{24a^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(32 + 21*a*x + 16*a^2*x^2 + 6
*a^3*x^3) + 21*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^3*Sqrt[-1 + a^2*x^2])
```

Maple [A] time = 0.148, size = 196, normalized size = 1.2

$$\frac{x}{24\,ca^4}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-6\,x\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}a^4-16\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}a^3-27\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}xa^2c+27\,c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-6*x*(c*(a^2*x^2-1)/a^2)^(3/2)*a^4-16
*(c*(a^2*x^2-1)/a^2)^(3/2)*a^3-27*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a^2*c+27*c^(3
/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))-48*c^(3/2)*ln((((a*x-1)*(a*x+1)
*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))-48*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a*c)/(
c*(a^2*x^2-1)/a^2)^(1/2)/c/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}x^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a^2*x^2))*x^3/(a^2*x^2 - 1), x)
```

Fricas [A] time = 2.02267, size = 490, normalized size = 3.06

$$\left[-\frac{2\left(6\,a^4x^4+16\,a^3x^3+21\,a^2x^2+32\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-21\,\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{48\,a^4},-\frac{\left(6\,a^4x^4+16\,a^3x^3+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

```
[Out] [-1/48*(2*(6*a^4*x^4 + 16*a^3*x^3 + 21*a^2*x^2 + 32*a*x)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)) - 21*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*
```

```
x^2 - c)/(a^2*x^2)) - c))/a^4, -1/24*((6*a^4*x^4 + 16*a^3*x^3 + 21*a^2*x^2
+ 32*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 21*sqrt(-c)*arctan(a^2*sqrt(-c)
*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx - \int \frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] -Integral(x**3*sqrt(c - c/(a**2*x**2))/(a*x - 1), x) - Integral(a*x**4*sqrt
(c - c/(a**2*x**2))/(a*x - 1), x)
```

Giac [A] time = 1.22639, size = 173, normalized size = 1.08

$$-\frac{1}{48}\left(2\sqrt{a^2cx^2-c}\left(\left(2x\left(\frac{3x\operatorname{sgn}(x)}{a^2}+\frac{8\operatorname{sgn}(x)}{a^3}\right)+\frac{21\operatorname{sgn}(x)}{a^4}\right)x+\frac{32\operatorname{sgn}(x)}{a^5}\right)-\frac{42\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}}{a^4|a|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/48*(2*sqrt(a^2*c*x^2 - c)*((2*x*(3*x*sgn(x)/a^2 + 8*sgn(x)/a^3) + 21*sgn
(x)/a^4)*x + 32*sgn(x)/a^5) - 42*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*
c*x^2 - c)))*sgn(x)/(a^4*abs(a)) + (21*a*sqrt(c)*log(abs(c)) - 64*sqrt(-c)*
abs(a))*sgn(x)/(a^5*abs(a)))*abs(a)
```

3.748 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal. Leaf size=123

$$-\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}-\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{a^2\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x)/a^2) - (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))/(3*a
^2) - (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)^2)/(3*a^2) + (Sqrt[c - c/(a^2*x^2)
]*x*ArcSin[a*x])/(a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.330686, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6159, 6129, 80, 50, 41, 216}

$$-\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}-\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{a^2\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x)/a^2) - (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))/(3*a
^2) - (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)^2)/(3*a^2) + (Sqrt[c - c/(a^2*x^2)
]*x*ArcSin[a*x])/(a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{2\tanh^{-1}(ax)}x\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{x(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2}+\frac{\left(2\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{3a\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{3a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{3a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}}{a\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{3a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{3a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\sin^{-1}(ax)}{a^2\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.0626633, size = 84, normalized size = 0.68

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(a^2x^2+3ax+5\right)+3\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{3a^2\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(5 + 3*a*x + a^2*x^2) + 3*Log
[a*x + Sqrt[-1 + a^2*x^2]]))/(3*a^2*Sqrt[-1 + a^2*x^2])
```

Maple [A] time = 0.124, size = 174, normalized size = 1.4

$$\frac{x}{3a^3c}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^3-3\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}xa^2c+3c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)-6c^{3/2}\ln\left(\frac{1}{\sqrt{c}}\left(\sqrt{(a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-(c*(a^2*x^2-1)/a^2)^(3/2)*a^3-3*(c*(a
^2*x^2-1)/a^2)^(1/2)*x*a^2*c+3*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/
2))-6*c^(3/2)*ln((((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))-6*((a
*x-1)*(a*x+1)*c/a^2)^(1/2)*a*c)/(c*(a^2*x^2-1)/a^2)^(1/2)/a^3/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}x^2}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a^2*x^2))*x^2/(a^2*x^2 - 1), x)
```

Fricas [A] time = 1.905, size = 439, normalized size = 3.57

$$\left[-\frac{2\left(a^3x^3+3a^2x^2+5ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\sqrt{c}\log\left(2a^2cx^2-2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{6a^3}, -\frac{\left(a^3x^3+3a^2x^2+5ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3}{3a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

```
[Out] [-1/6*(2*(a^3*x^3 + 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*
sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
 - c))/a^3, -1/3*((a^3*x^3 + 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x
^2)) - 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(
a^2*c*x^2 - c)))/a^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx-\int \frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] -Integral(x**2*sqrt(c - c/(a**2*x**2))/(a*x - 1), x) - Integral(a*x**3*sqrt
(c - c/(a**2*x**2))/(a*x - 1), x)
```

Giac [A] time = 1.26156, size = 157, normalized size = 1.28

$$-\frac{1}{6}\left(2\sqrt{a^2cx^2-c}\left(x\left(\frac{x\operatorname{sgn}(x)}{a^2}+\frac{3\operatorname{sgn}(x)}{a^3}\right)+\frac{5\operatorname{sgn}(x)}{a^4}\right)-\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a^3|a|}+\frac{\left(3a\sqrt{c}\log(|c|)-\right.}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/6*(2*sqrt(a^2*c*x^2 - c)*(x*(x*sgn(x)/a^2 + 3*sgn(x)/a^3) + 5*sgn(x)/a^4
) - 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^3*ab
s(a)) + (3*a*sqrt(c)*log(abs(c)) - 10*sqrt(-c)*abs(a))*sgn(x)/(a^4*abs(a)))
*abs(a)
```

3.749 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal. Leaf size=98

$$-\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{2a\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x)/(2*a) - (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))/(2
*a) + (3*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(2*a*Sqrt[1 - a*x]*Sqrt[1 + a
*x])
```

Rubi [A] time = 0.210241, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6159, 6129, 50, 41, 216}

$$-\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{2a\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x)/(2*a) - (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))/(2
*a) + (3*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(2*a*Sqrt[1 - a*x]*Sqrt[1 + a
*x])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
```

```
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{2a}+\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{2a}+\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{2a}+\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{2a}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}x\sin^{-1}(ax)}{2a\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] time = 0.0621817, size = 77, normalized size = 0.79

$$
-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{1-a^2x^2}(ax+4)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 -
a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.112, size = 147, normalized size = 1.5

$$
-\frac{x}{2\,a^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}a^2-\sqrt{c}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)+4\sqrt{c}\ln\left(\frac{1}{\sqrt{c}}\left(\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\sqrt{c}+cx\right)\right)+
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(x*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c^(1/
2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))+4*c^(1/2)*ln((((a*x-1)*(a*x+1)*c
```

```
/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))+4*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a)/(c*(a^
2*x^2-1)/a^2)^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a^2*x^2))*x/(a^2*x^2 - 1), x)
```

Fricas [A] time = 1.94417, size = 406, normalized size = 4.14

$$\left[-\frac{2\left(a^2x^2+4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{4\,a^2},-\frac{\left(a^2x^2+4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{-c}\arctan\left(\frac{a^2\sqrt{\cdots}}{\cdots}\right)}{2\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] [-1/4*(2*(a^2*x^2 + 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(c)*log(
2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^2,
-1/2*((a^2*x^2 + 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(-c)*arctan
(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx-\int \frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] -Integral(x*sqrt(c - c/(a**2*x**2))/(a*x - 1), x) - Integral(a*x**2*sqrt(c
- c/(a**2*x**2))/(a*x - 1), x)
```

Giac [A] time = 1.21421, size = 143, normalized size = 1.46

$$-\frac{1}{4}\left(2\,\sqrt{a^2cx^2-c}\left(\frac{x\mathrm{sgn}\,(x)}{a^2}+\frac{4\,\mathrm{sgn}\,(x)}{a^3}\right)-\frac{6\,\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}\,(x)}{a^2|a|}+\frac{\left(3\,a\sqrt{c}\log\left(|c|\right)-8\,\sqrt{-c}|a|\right)\mathrm{sg}\cdots}{a^3|a|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(a^2*c*x^2 - c)*(x*sgn(x)/a^2 + 4*sgn(x)/a^3) - 6*sqrt(c)*log(a
bs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^2*abs(a)) + (3*a*sqrt(c
)*log(abs(c)) - 8*sqrt(-c)*abs(a))*sgn(x)/(a^3*abs(a)))*abs(a)
```

3.750 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=118

$$-x\sqrt{c-\frac{c}{a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x) + (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[
1 - a*x]*Sqrt[1 + a*x]) - (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.287734, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6159, 6129, 102, 157, 41, 216, 92, 208}

$$-x\sqrt{c-\frac{c}{a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```


```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x) + (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[
1 - a*x]*Sqrt[1 + a*x]) - (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```


Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
```

```
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}\,x-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}\,x+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}\,x-\frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\ldots}{\sqrt{1-ax}\sqrt{\ldots}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}\,x+\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] time = 0.084807, size = 81, normalized size = 0.69

$$
-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}+2\log\left(\sqrt{a^2x^2-1}+ax\right)-\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2
]] + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2])
```

Maple [A] time = 0.118, size = 198, normalized size = 1.7

$$-\frac{x}{a^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(2\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}a^2\sqrt{-\frac{c}{a^2}}+2\sqrt{c}\ln\left(\frac{1}{\sqrt{c}}\left(\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\sqrt{c}+cx\right)\right)a\sqrt{-\frac{c}{a^2}}-\sqrt{-\frac{c}{a^2}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a^2*(-c/a
^2)^(1/2)+2*c^(1/2)*ln((((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))
*a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/
a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2))/(c*(a^2*x^2-1)/a^2)^(1/
2)/a^2/(-c/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a^2*x^2))/(a^2*x^2 - 1), x)
```

Fricas [A] time = 1.97262, size = 579, normalized size = 4.91

$$\left[-\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-4\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\sqrt{-c}\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},-\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\sqrt{c}\arctan\left(\frac{a\sqrt{}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 4*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - sqrt(-c)*log(-(a^
2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, -(
a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*
c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sq
rt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx-\int \frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] -Integral(sqrt(c - c/(a**2*x**2))/(a*x - 1), x) - Integral(a*x*sqrt(c - c/(
a**2*x**2))/(a*x - 1), x)
```

Giac [A] time = 1.21925, size = 208, normalized size = 1.76

$$-\left(\frac{2\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{2\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}+\frac{\sqrt{a^2cx^2-c}\operatorname{sgn}(x)}{a^2}-\frac{\left(2\sqrt{c}|a|\operatorname{arc}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
^2 - 2*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs
(a)) + sqrt(a^2*c*x^2 - c)*sgn(x)/a^2 - (2*sqrt(c)*abs(a)*arctan(sqrt(-c)/s
qrt(c)) - a*sqrt(c)*log(abs(c)) + sqrt(-c)*abs(a))*sgn(x)/(a^2*abs(a)))*abs
(a)
```

3.751 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=118

$$-\sqrt{c-\frac{c}{a^2x^2}}+\frac{ax\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -Sqrt[c - c/(a^2*x^2)] + (a*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
a*x]*Sqrt[1 + a*x]) - (2*a*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.383307, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6159, 6129, 98, 157, 41, 216, 92, 208}

$$-\sqrt{c-\frac{c}{a^2x^2}}+\frac{ax\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] -Sqrt[c - c/(a^2*x^2)] + (a*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
a*x]*Sqrt[1 + a*x]) - (2*a*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^2}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^2\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-2a-a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}+\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}+\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-\right.}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] time = 0.0697894, size = 85, normalized size = 0.72

$$
\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\sqrt{a^2x^2-1}-ax\log\left(\sqrt{a^2x^2-1}+ax\right)+2ax\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-Sqrt[-1 + a^2*x^2] + 2*a*x*ArcTan[1/Sqrt[-1 + a^2*
x^2]] - a*x*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [B] time = 0.122, size = 305, normalized size = 2.6

$$\frac{1}{ac}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}x^2a^3c+a^3\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}+c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)\sqrt{-\frac{c}{a^2}}xa -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x,x)
```

```
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)/a*(-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*
x^2*a^3*c+a^3*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)+c^(3/2)*ln(x*c^(1/2)
+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*x*a-2*c^(3/2)*(-c/a^2)^(1/2)*ln(
(((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*x*a-2*(-c/a^2)^(1/2)*(
(a*x-1)*(a*x+1)*c/a^2)^(1/2)*x*a^2*c+2*(c*(a^2*x^2-1)/a^2)^(1/2)*c*x*a^2*(-
c/a^2)^(1/2)+2*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2)
*x*c^2)/(c*(a^2*x^2-1)/a^2)^(1/2)/c/(-c/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{ax+1}\sqrt{ax-1}\sqrt{c}}{ax}-\int\frac{\left(a\sqrt{c}x+2\sqrt{c}\right)\sqrt{ax+1}\sqrt{ax-1}}{a^2x^3-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="maxima
")
```

```
[Out] -sqrt(a*x + 1)*sqrt(a*x - 1)*sqrt(c)/(a*x) - integrate((a*sqrt(c)*x + 2*sqr
t(c))*sqrt(a*x + 1)*sqrt(a*x - 1)/(a^2*x^3 - x), x)
```

Fricas [A] time = 1.86809, size = 552, normalized size = 4.68

$$\left[\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2-2a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{x^2}\right)-\sqrt{\frac{a^2cx^2-c}{a^2x^2}},2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x\sqrt{\cdots}}{a^2cx^2\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fricas
")
```

```
[Out] [sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^
2 - c)) + sqrt(-c)*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a
^2*x^2)) - 2*c)/x^2) - sqrt((a^2*c*x^2 - c)/(a^2*x^2)), 2*sqrt(c)*arctan(a*
```

```
sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 1/2*sqrt(c)*lo
g(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) - sq
rt((a^2*c*x^2 - c)/(a^2*x^2))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax^2-x}\,dx-\int \frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax^2-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(1/2)/x,x)
```

```
[Out] -Integral(sqrt(c - c/(a**2*x**2))/(a*x**2 - x), x) - Integral(a*x*sqrt(c -
c/(a**2*x**2))/(a*x**2 - x), x)
```

Giac [A] time = 1.41099, size = 173, normalized size = 1.47

$$-\left(\frac{4\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a}-\frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{|a|}+\frac{2c^{\frac{3}{2}}\operatorname{sgn}(x)}{\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^2+c\right)|a|}\right)|a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -(4*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
 - sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/abs(a) + 2
*c^(3/2)*sgn(x)/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)*abs(a)))*abs
(a)
```

3.752 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=111

$$-\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}-\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2x}-\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{2\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (-3*a*Sqrt[c - c/(a^2*x^2)])/2 - (Sqrt[c - c/(a^2*x^2)]*(1 + a*x))/(2*x) -
(3*a^2*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(2*Sqr
t[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.381735, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6159, 6129, 94, 92, 208}

$$-\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}-\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2x}-\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{2\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (-3*a*Sqrt[c - c/(a^2*x^2)])/2 - (Sqrt[c - c/(a^2*x^2)]*(1 + a*x))/(2*x) -
(3*a^2*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(2*Sqr
t[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^3}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^3\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{2x} + \frac{\left(3a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1+ax}}{x^2\sqrt{1-ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{2x} + \frac{\left(3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{2x} - \frac{\left(3a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+\right.}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{2x} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{2\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.0682801, size = 79, normalized size = 0.71

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(3a^2x^2\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-(4ax+1)\sqrt{a^2x^2-1}\right)}{2x\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-((1 + 4*a*x)*Sqrt[-1 + a^2*x^2]) + 3*a^2*x^2*ArcTa
n[1/Sqrt[-1 + a^2*x^2]]))/(2*x*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.119, size = 347, normalized size = 3.1

$$\frac{1}{2\,cx}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-4\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}x^3a^3c+4\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}xa^3+4\,c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^2,x)
```

```
[Out] 1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^
(1/2)*x^3*a^3*c+4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a^3+4*c^(3/2)*
ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*x^2*a-4*c^(3/2)*(-c/
a^2)^(1/2)*ln((((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*x^2*a-4*
(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^2*a^2*c+3*(-c/a^2)^(1/2)*(c*
(a^2*x^2-1)/a^2)^(1/2)*x^2*a^2*c+a^2*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/
2)+3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2)*x^2*c^2)/
(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{(a^2x^2-1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="maxi
ma")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a^2*x^2))/((a^2*x^2 - 1)*x^2), x)
```

Fricas [A] time = 1.96044, size = 392, normalized size = 3.53

$$\left[\frac{3\,a\sqrt{-c}x\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\,(4\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,x},\ \frac{3\,a\sqrt{c}x\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(4\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="fric
as")
```

```
[Out] [1/4*(3*a*sqrt(-c)*x*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)) - 2*c)/x^2) - 2*(4*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x,
1/2*(3*a*sqrt(c)*x*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*
c*x^2 - c)) - (4*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax^3-x^2}\,dx-\int \frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax^3-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

```
[Out] -Integral(sqrt(c - c/(a**2*x**2))/(a*x**3 - x**2), x) - Integral(a*x*sqrt(c
 - c/(a**2*x**2))/(a*x**3 - x**2), x)
```

Giac [B] time = 1.72293, size = 263, normalized size = 2.37

$$-\left(3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)-\frac{\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^3ac\operatorname{sgn}(x)-4\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^2c^{\frac{3}{2}}|a|\operatorname{sgn}(x}{\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="giac
")
```

```
[Out] -(3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) -
 ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*a*c*sgn(x) - 4*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^2*c^(3/2)*abs(a)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x
^2 - c))*a*c^2*sgn(x) - 4*c^(5/2)*abs(a)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^2 + c)^2*a))*abs(a)
```

3.753 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=139

$$-a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3x}-\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -(a^2*Sqrt[c - c/(a^2*x^2)]) - (a*Sqrt[c - c/(a^2*x^2)]*(1 + a*x))/(3*x) -
(Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(3*x^2) - (a^3*Sqrt[c - c/(a^2*x^2)]*x*
ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.388239, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6159, 6129, 96, 94, 92, 208}

$$-a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3x}-\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] -(a^2*Sqrt[c - c/(a^2*x^2)]) - (a*Sqrt[c - c/(a^2*x^2)]*(1 + a*x))/(3*x) -
(Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(3*x^2) - (a^3*Sqrt[c - c/(a^2*x^2)]*x*
ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^4}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{x^4\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2}+\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{x^3\sqrt{1-ax}}\,dx}{3\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{3x}-\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2}+\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1+ax}}{x^2\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{3x}-\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2}+\frac{\left(a^3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+a}}}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{3x}-\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2}-\frac{\left(a^4\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{a-a}\right.}{\sqrt{1-ax}\sqrt{}}\\
&= -a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{3x}-\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2}-\frac{a^3\sqrt{c-\frac{c}{a^2x^2}}x\tanh^{-1}\left(\sqrt{1-a}\right.}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.0825959, size = 71, normalized size = 0.51

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-5a^2x^2+\frac{3a^3x^3\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{\sqrt{a^2x^2-1}}-3ax-1\right)}{3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1 - 3*a*x - 5*a^2*x^2 + (3*a^3*x^3*ArcTan[1/Sqrt[-
1 + a^2*x^2]])/Sqrt[-1 + a^2*x^2]))/(3*x^2)
```

Maple [B] time = 0.125, size = 378, normalized size = 2.7

$$\frac{a}{3\,c x^{2}}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\left(-6\sqrt{-\frac{c}{a^{2}}}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}x^{4}a^{3}c+6\sqrt{-\frac{c}{a^{2}}}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}\right)^{3/2}x^{2}a^{3}+6\,c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^3,x)
```

```
[Out] 1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^2*a*(-6*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a
^2)^(1/2)*x^4*a^3*c+6*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2*a^3+6*c^
(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*x^3*a-6*c^(3/2
)*(-c/a^2)^(1/2)*ln((((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*x^
3*a-6*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^3*a^2*c+3*(-c/a^2)^(1/
2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^3*a^2*c+3*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)
^(3/2)*x*a^2+3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2)
*x^3*c^2+a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(-c/a^2)^(1/2)/(c*(a^2
*x^2-1)/a^2)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(2\,a^{2}\sqrt{c}x^{2}+\sqrt{c}\right)\sqrt{ax+1}\sqrt{ax-1}}{3\,ax^{3}}-\int\frac{\left(a\sqrt{c}x+2\,\sqrt{c}\right)\sqrt{ax+1}\sqrt{ax-1}}{a^{2}x^{5}-x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="maxi
ma")
```

```
[Out] -1/3*(2*a^2*sqrt(c)*x^2 + sqrt(c))*sqrt(a*x + 1)*sqrt(a*x - 1)/(a*x^3) - in
tegrate((a*sqrt(c)*x + 2*sqrt(c))*sqrt(a*x + 1)*sqrt(a*x - 1)/(a^2*x^5 - x^
3), x)
```

Fricas [A] time = 1.946, size = 440, normalized size = 3.17

$$\left[\frac{3\,a^{2}\sqrt{-c}x^{2}\log\left(-\frac{a^{2}cx^{2}-2\,a\sqrt{-c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{x^{2}}\right)-2\left(5\,a^{2}x^{2}+3\,ax+1\right)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{6\,x^{2}},\ \frac{3\,a^{2}\sqrt{c}x^{2}\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)-\left(5\,a^{2}x^{2}\right.}{3\,x^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="fric
as")
```

```
[Out] [1/6*(3*a^2*sqrt(-c)*x^2*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)) - 2*c)/x^2) - 2*(5*a^2*x^2 + 3*a*x + 1)*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)))/x^2, 1/3*(3*a^2*sqrt(c)*x^2*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (5*a^2*x^2 + 3*a*x + 1)*sqrt((a^2*c*x^2
```

```
- c)/(a^2*x^2)))/x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax^4-x^3}\,dx-\int \frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax^4-x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(1/2)/x**3,x)
```

```
[Out] -Integral(sqrt(c - c/(a**2*x**2))/(a*x**4 - x**3), x) - Integral(a*x*sqrt(c
 - c/(a**2*x**2))/(a*x**4 - x**3), x)
```

Giac [A] time = 2.17066, size = 312, normalized size = 2.24

$$-\frac{2}{3}\left(3\,a\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)-\frac{3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^5 ac\operatorname{sgn}(x)-3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^4 c^{\frac{3}{2}}|a|\mathrm{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="giac
")
```

```
[Out] -2/3*(3*a*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sg
n(x) - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a*c*sgn(x) - 3*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - c))^4*c^(3/2)*abs(a)*sgn(x) - 12*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^2*c^(5/2)*abs(a)*sgn(x) - 3*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))*a*c^3*sgn(x) - 5*c^(7/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - c))^2 + c)^3)*abs(a)
```

3.754 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=156

$$-\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (-4*a^3*Sqrt[c - c/(a^2*x^2)])/3 - Sqrt[c - c/(a^2*x^2)]/(4*x^3) - (2*a*Sqr
t[c - c/(a^2*x^2)])/(3*x^2) - (7*a^2*Sqrt[c - c/(a^2*x^2)])/(8*x) - (7*a^4*
Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(8*Sqrt[1 - a
*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.410949, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6159, 6129, 98, 151, 12, 92, 208}

$$-\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (-4*a^3*Sqrt[c - c/(a^2*x^2)])/3 - Sqrt[c - c/(a^2*x^2)]/(4*x^3) - (2*a*Sqr
t[c - c/(a^2*x^2)])/(3*x^2) - (7*a^2*Sqrt[c - c/(a^2*x^2)])/(8*x) - (7*a^4*
Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(8*Sqrt[1 - a
*x]*Sqrt[1 + a*x])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^5}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^5\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-8a-7a^2x}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{21a^2+16a^3x}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx}{12\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-32a^3-21a^4x}{x^2\sqrt{1-ax}\sqrt{1+ax}}\,dx}{24\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{21a^4}{x\sqrt{1-ax}\sqrt{1+a}}}{24\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{\left(7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}}}{8\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{\left(7a^5\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\mathrm{Subst}\left(\int\right.}{8\sqrt{1-}}\\
&= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1}\right.}{8\sqrt{1-ax}\sqrt{1+}}
\end{aligned}
$$

Mathematica [A] time = 0.0862713, size = 95, normalized size = 0.61

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(21a^4x^4\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-\sqrt{a^2x^2-1}\left(32a^3x^3+21a^2x^2+16ax+6\right)\right)}{24x^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-(Sqrt[-1 + a^2*x^2]*(6 + 16*a*x + 21*a^2*x^2 + 32*
a^3*x^3)) + 21*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(24*x^3*Sqrt[-1 + a^2
*x^2])
```

Maple [B] time = 0.13, size = 410, normalized size = 2.6

$$\frac{a^2}{24\,cx^3}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-48\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}x^5a^3c+48\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}x^3a^3+48\sqrt{-\frac{c}{a^2}}c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^4,x)
```

```
[Out] 1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^3*a^2*(-48*(-c/a^2)^(1/2)*(c*(a^2*x^2-
1)/a^2)^(1/2)*x^5*a^3*c+48*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^3*a^3
+48*(-c/a^2)^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^4*a-48
*(-c/a^2)^(1/2)*c^(3/2)*ln((((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1
/2))*x^4*a-48*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^4*a^2*c+21*(-c
/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^4*a^2*c+27*(-c/a^2)^(1/2)*(c*(a^2*x
^2-1)/a^2)^(3/2)*x^2*a^2+21*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*
a^2-c)/x/a^2)*x^4*c^2+16*a*(c*(a^2*x^2-1)/a^2)^(3/2)*x*(-c/a^2)^(1/2)+6*(c*
(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(
1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{\left(a^2x^2-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="maxi
ma")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a^2*x^2))/((a^2*x^2 - 1)*x^4), x)
```

Fricas [A] time = 2.28952, size = 486, normalized size = 3.12

$$\left[\frac{21\,a^{3}\sqrt{-c}x^{3}\log\left(-\frac{a^{2}cx^{2}-2\,a\sqrt{-c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{x^{2}}\right)-2\left(32\,a^{3}x^{3}+21\,a^{2}x^{2}+16\,ax+6\right)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{48\,x^{3}},\ \frac{21\,a^{3}\sqrt{c}x^{3}\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="fric
as")
```

```
[Out] [1/48*(21*a^3*sqrt(-c)*x^3*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/x^3, 1/24*(21*a^3*sqrt(c)*x^3*arctan(a*sqrt(c
)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (32*a^3*x^3 + 21*a^2
*x^2 + 16*a*x + 6)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}}{ax^{5}-x^{4}}\,dx-\int \frac{ax\sqrt{c-\frac{c}{a^{2}x^{2}}}}{ax^{5}-x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(1/2)/x**4,x)
```

```
[Out] -Integral(sqrt(c - c/(a**2*x**2))/(a*x**5 - x**4), x) - Integral(a*x*sqrt(c
 - c/(a**2*x**2))/(a*x**5 - x**4), x)
```

Giac [B] time = 3.74437, size = 427, normalized size = 2.74

$$-\frac{1}{12}\left(21\,a^{2}\sqrt{c}\arctan\left(-\frac{\sqrt{a^{2}c}x-\sqrt{a^{2}cx^{2}-c}}{\sqrt{c}}\right)\mathrm{sgn}\left(x\right)-\frac{21\left(\sqrt{a^{2}c}x-\sqrt{a^{2}cx^{2}-c}\right)^{7}a^{2}c\,\mathrm{sgn}\left(x\right)+45\left(\sqrt{a^{2}c}x-\sqrt{a^{2}cx^{2}-c}\right)^{5}\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="giac
")
```

```
[Out] -1/12*(21*a^2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x) - (21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^2*c*sgn(x) + 45*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a^2*c^2*sgn(x) - 96*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^4*a*c^(5/2)*abs(a)*sgn(x) - 45*(sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - c))^3*a^2*c^3*sgn(x) - 128*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))
^2*a*c^(7/2)*abs(a)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^2*c
^4*sgn(x) - 32*a*c^(9/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
c))^2 + c)^4)*abs(a)
```

3.755 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=181

$$-\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}-\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}-\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{4\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (-6*a^4*Sqrt[c - c/(a^2*x^2)])/5 - Sqrt[c - c/(a^2*x^2)]/(5*x^4) - (a*Sqrt[
c - c/(a^2*x^2)])/(2*x^3) - (3*a^2*Sqrt[c - c/(a^2*x^2)])/(5*x^2) - (3*a^3*
Sqrt[c - c/(a^2*x^2)])/(4*x) - (3*a^5*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[
1 - a*x]*Sqrt[1 + a*x]])/(4*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.430333, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6159, 6129, 98, 151, 12, 92, 208}

$$-\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}-\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}-\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{4\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (-6*a^4*Sqrt[c - c/(a^2*x^2)])/5 - Sqrt[c - c/(a^2*x^2)]/(5*x^4) - (a*Sqrt[
c - c/(a^2*x^2)])/(2*x^3) - (3*a^2*Sqrt[c - c/(a^2*x^2)])/(5*x^2) - (3*a^3*
Sqrt[c - c/(a^2*x^2)])/(4*x) - (3*a^5*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[
1 - a*x]*Sqrt[1 + a*x]])/(4*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^6}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{x^6\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{-10a-9a^2x}{x^5\sqrt{1-ax}\sqrt{1+ax}}\,dx}{5\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{36a^2+30a^3x}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{20\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{-90a^3-72a^4x}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx}{60\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{144a^4+90a^5}{x^2\sqrt{1-ax}\sqrt{1+}}}{120\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} - \frac{\left(\sqrt{c-\right.}}{12}$$

$$= -\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} + \frac{\left(3a^5\sqrt{c\right.}}{}$$

$$= -\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} - \frac{\left(3a^6\sqrt{c\right.}}{}$$

$$= -\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} - \frac{3a^5\sqrt{c-}}{}$$

Mathematica [A] time = 0.0871152, size = 103, normalized size = 0.57

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(15a^5x^5\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-\sqrt{a^2x^2-1}\left(24a^4x^4+15a^3x^3+12a^2x^2+10ax+4\right)\right)}{20x^4\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-(Sqrt[-1 + a^2*x^2]*(4 + 10*a*x + 12*a^2*x^2 + 15*
a^3*x^3 + 24*a^4*x^4)) + 15*a^5*x^5*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(20*x^4*
Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.13, size = 447, normalized size = 2.5

$$\frac{a^2}{20\,cx^4}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-40\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}x^6a^4c+40\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}x^4a^4+40\sqrt{-\frac{c}{a^2}}c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^5,x)
```

```
[Out] 1/20*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^4*a^2*(-40*(-c/a^2)^(1/2)*(c*(a^2*x^2-
1)/a^2)^(1/2)*x^6*a^4*c+40*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^4*a^4
+40*(-c/a^2)^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^5*a^2-
40*(-c/a^2)^(1/2)*c^(3/2)*ln((((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^
(1/2))*x^5*a^2-40*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^5*a^3*c+15
*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^5*a^3*c+25*(-c/a^2)^(1/2)*(c*(a
^2*x^2-1)/a^2)^(3/2)*x^3*a^3+15*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1
/2)*a^2-c)/x/a^2)*x^5*a*c^2+16*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2
*a^2+10*a*(c*(a^2*x^2-1)/a^2)^(3/2)*x*(-c/a^2)^(1/2)+4*(c*(a^2*x^2-1)/a^2)^
(3/2)*(-c/a^2)^(1/2))/(c*(a^2*x^2-1)/a^2)^(1/2)/c/(-c/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(8\,a^{4}\sqrt{c}x^{4}+4\,a^{2}\sqrt{c}x^{2}+3\,\sqrt{c}\right)\sqrt{ax+1}\sqrt{ax-1}}{15\,ax^{5}}-\int\frac{\left(a\sqrt{c}x+2\,\sqrt{c}\right)\sqrt{ax+1}\sqrt{ax-1}}{a^{2}x^{7}-x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="maxi
ma")
```

```
[Out] -1/15*(8*a^4*sqrt(c)*x^4 + 4*a^2*sqrt(c)*x^2 + 3*sqrt(c))*sqrt(a*x + 1)*sqr
t(a*x - 1)/(a*x^5) - integrate((a*sqrt(c)*x + 2*sqrt(c))*sqrt(a*x + 1)*sqrt
(a*x - 1)/(a^2*x^7 - x^5), x)
```

Fricas [A] time = 2.60579, size = 521, normalized size = 2.88

$$\left[\frac{15\,a^{4}\sqrt{-c}x^{4}\log\left(-\frac{a^{2}cx^{2}-2\,a\sqrt{-c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{x^{2}}\right)-2\left(24\,a^{4}x^{4}+15\,a^{3}x^{3}+12\,a^{2}x^{2}+10\,ax+4\right)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{40\,x^{4}},\frac{15\,a^{4}\sqrt{c}x^{4}\arctan\left(\frac{a\sqrt{}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="fric
as")
```

```
[Out] [1/40*(15*a^4*sqrt(-c)*x^4*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(24*a^4*x^4 + 15*a^3*x^3 + 12*a^2*x^2 + 10
*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4, 1/20*(15*a^4*sqrt(c)*x^4*ar
ctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (24*a^4
*x^4 + 15*a^3*x^3 + 12*a^2*x^2 + 10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
))/x^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}}{ax^{6}-x^{5}}\,dx-\int\frac{ax\sqrt{c-\frac{c}{a^{2}x^{2}}}}{ax^{6}-x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(1/2)/x**5,x)
```

```
[Out] -Integral(sqrt(c - c/(a**2*x**2))/(a*x**6 - x**5), x) - Integral(a*x*sqrt(c
 - c/(a**2*x**2))/(a*x**6 - x**5), x)
```

Giac [B] time = 3.73588, size = 489, normalized size = 2.7

$$-\frac{1}{10}\left(15\,a^3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}\left(x\right)-\frac{15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^9a^3c\mathrm{sgn}\left(x\right)+70\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="giac
")
```

```
[Out] -1/10*(15*a^3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x) - (15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*a^3*c*sgn(x) + 70*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^3*c^2*sgn(x) - 40*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^6*a^2*c^(5/2)*abs(a)*sgn(x) - 200*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^4*a^2*c^(7/2)*abs(a)*sgn(x) - 70*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^3*a^3*c^4*sgn(x) - 120*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
2*a^2*c^(9/2)*abs(a)*sgn(x) - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^3*
c^5*sgn(x) - 24*a^2*c^(11/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^2 + c)^5)*abs(a)
```

3.756 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

Optimal. Leaf size=188

$$-\frac{ax^5\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-a^2x^2}}-\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{2x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-a^2x^2}}-\frac{4x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^3\sqrt{1-a^2x^2}}$$

```
[Out] (-4*Sqrt[c - c/(a^2*x^2)]*x^2)/(a^2*Sqrt[1 - a^2*x^2]) - (2*Sqrt[c - c/(a^2
*x^2)]*x^3)/(a*Sqrt[1 - a^2*x^2]) - (Sqrt[c - c/(a^2*x^2)]*x^4)/Sqrt[1 - a^
2*x^2] - (a*Sqrt[c - c/(a^2*x^2)]*x^5)/(4*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c -
c/(a^2*x^2)]*x*Log[1 - a*x])/(a^3*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.264748, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{ax^5\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-a^2x^2}}-\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{2x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-a^2x^2}}-\frac{4x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (-4*Sqrt[c - c/(a^2*x^2)]*x^2)/(a^2*Sqrt[1 - a^2*x^2]) - (2*Sqrt[c - c/(a^2
*x^2)]*x^3)/(a*Sqrt[1 - a^2*x^2]) - (Sqrt[c - c/(a^2*x^2)]*x^4)/Sqrt[1 - a^
2*x^2] - (a*Sqrt[c - c/(a^2*x^2)]*x^5)/(4*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c -
c/(a^2*x^2)]*x*Log[1 - a*x])/(a^3*Sqrt[1 - a^2*x^2])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{3\tanh^{-1}(ax)}x^2\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{x^2(1+ax)^2}{1-ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(-\frac{4}{a^2}-\frac{4x}{a}-3x^2-ax^3-\frac{4}{a^2(-1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4\sqrt{c-\frac{c}{a^2x^2}}x^2}{a^2\sqrt{1-a^2x^2}} - \frac{2\sqrt{c-\frac{c}{a^2x^2}}x^3}{a\sqrt{1-a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x^4}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}x^5}{4\sqrt{1-a^2x^2}} - \frac{4\sqrt{c-\frac{c}{a^2x^2}}x\log(1}{a^3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0477428, size = 71, normalized size = 0.38

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4x}{a^2}-\frac{4\log(1-ax)}{a^3}-\frac{ax^4}{4}-\frac{2x^2}{a}-x^3\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*((-4*x)/a^2 - (2*x^2)/a - x^3 - (a*x^4)/4 - (4*Log
[1 - a*x])/a^3))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.144, size = 85, normalized size = 0.5

$$\frac{x\left(x^4a^4+4x^3a^3+8a^2x^2+16ax+16\ln(ax-1)\right)}{\left(4a^2x^2-4\right)a^3}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/4*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(x^4*a^4+4*x^3*a^3+8
*a^2*x^2+16*a*x+16*ln(a*x-1))/(a^2*x^2-1)/a^3
```

Maxima [C] time = 1.32435, size = 265, normalized size = 1.41

$$\frac{1}{4}a^3\left(\frac{ia^2\sqrt{c}x^4+2i\sqrt{c}x^2}{a^5}+\frac{2i\sqrt{c}\log(ax+1)}{a^7}+\frac{2i\sqrt{c}\log(ax-1)}{a^7}\right)+\frac{1}{2}a^2\left(\frac{2\left(ia^2\sqrt{c}x^3+3i\sqrt{c}x\right)}{a^5}-\frac{3i\sqrt{c}\log(ax+1)}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm
="maxima")
```

```
[Out] 1/4*a^3*((I*a^2*sqrt(c)*x^4 + 2*I*sqrt(c)*x^2)/a^5 + 2*I*sqrt(c)*log(a*x +
1)/a^7 + 2*I*sqrt(c)*log(a*x - 1)/a^7) + 1/2*a^2*(2*(I*a^2*sqrt(c)*x^3 + 3*
```

```
I*sqrt(c)*x)/a^5 - 3*I*sqrt(c)*log(a*x + 1)/a^6 + 3*I*sqrt(c)*log(a*x - 1)/
a^6) - 3/2*a*(-I*sqrt(c)*x^2/a^3 - I*sqrt(c)*log(a*x + 1)/a^5 - I*sqrt(c)*l
og(a*x - 1)/a^5) + I*sqrt(c)*x/a^3 - 1/2*I*sqrt(c)*log(a*x + 1)/a^4 + 1/2*I
*sqrt(c)*log(a*x - 1)/a^4
```

Fricas [A] time = 2.63082, size = 872, normalized size = 4.64

$$\left[\frac{8\left(a^2x^2-1\right)\sqrt{-c}\log\left(\frac{a^6cx^6-4a^5cx^5+5a^4cx^4-4a^2cx^2+4acx-\left(a^5x^5-4a^4x^4+6a^3x^3-4a^2x^2\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{a^4x^4-2a^3x^3+2ax-1}\right)+\left(a^5x^5+4a^4x^4+8a\right.}{4\left(a^6x^2-a^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/4*(8*(a^2*x^2 - 1)*sqrt(-c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 -
 4*a^2*c*x^2 + 4*a*c*x - (a^5*x^5 - 4*a^4*x^4 + 6*a^3*x^3 - 4*a^2*x^2)*sqrt
(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/(a^4*x^4 - 2
*a^3*x^3 + 2*a*x - 1)) + (a^5*x^5 + 4*a^4*x^4 + 8*a^3*x^3 + 16*a^2*x^2)*sqr
t(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^2 - a^4), 1/4*(16*(
a^2*x^2 - 1)*sqrt(c)*arctan((a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(c
)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 - 2*a^2*c*x^2 - a*c*x + 2*c))
+ (a^5*x^5 + 4*a^4*x^4 + 8*a^3*x^3 + 16*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a
^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^2 - a^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**3*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1
)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))*x^3/(-a^2*x^2 + 1)^(3/2), x)
```

3.757 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal. Leaf size=153

$$-\frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-a^2x^2}}-\frac{3x^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}-\frac{4x^2\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^2\sqrt{1-a^2x^2}}$$

```
[Out] (-4*Sqrt[c - c/(a^2*x^2)]*x^2)/(a*Sqrt[1 - a^2*x^2]) - (3*Sqrt[c - c/(a^2*x
^2)]*x^3)/(2*Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - c/(a^2*x^2)]*x^4)/(3*Sqrt[1 -
 a^2*x^2]) - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/(a^2*Sqrt[1 - a^2*x^2
])
```

Rubi [A] time = 0.2323, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 77}

$$-\frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-a^2x^2}}-\frac{3x^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}-\frac{4x^2\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (-4*Sqrt[c - c/(a^2*x^2)]*x^2)/(a*Sqrt[1 - a^2*x^2]) - (3*Sqrt[c - c/(a^2*x
^2)]*x^3)/(2*Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - c/(a^2*x^2)]*x^4)/(3*Sqrt[1 -
 a^2*x^2]) - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/(a^2*Sqrt[1 - a^2*x^2
])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{3\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{x(1+ax)^2}{1-ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(-\frac{4}{a}-3x-ax^2-\frac{4}{a(-1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4\sqrt{c-\frac{c}{a^2x^2}}x^2}{a\sqrt{1-a^2x^2}} - \frac{3\sqrt{c-\frac{c}{a^2x^2}}x^3}{2\sqrt{1-a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}x^4}{3\sqrt{1-a^2x^2}} - \frac{4\sqrt{c-\frac{c}{a^2x^2}}x\log(1-ax)}{a^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0424301, size = 65, normalized size = 0.42

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4\log(1-ax)}{a^2}-\frac{ax^3}{3}-\frac{4x}{a}-\frac{3x^2}{2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*((-4*x)/a - (3*x^2)/2 - (a*x^3)/3 - (4*Log[1 - a*x
])/a^2))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.141, size = 78, normalized size = 0.5

$$\frac{x\left(2\,x^3a^3+9\,a^2x^2+24\,ax+24\,\ln(ax-1)\right)}{\left(6\,a^2x^2-6\right)a^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3+9*a^2*x^2
+24*a*x+24*ln(a*x-1))/(a^2*x^2-1)/a^2
```

Maxima [C] time = 1.32941, size = 232, normalized size = 1.52

$$\frac{1}{6}a^3\left(\frac{2\left(i\,a^2\sqrt{c}x^3+3i\,\sqrt{c}x\right)}{a^5}-\frac{3i\,\sqrt{c}\log(ax+1)}{a^6}+\frac{3i\,\sqrt{c}\log(ax-1)}{a^6}\right)-\frac{3}{2}a^2\left(-\frac{i\,\sqrt{c}x^2}{a^3}-\frac{i\,\sqrt{c}\log(ax+1)}{a^5}-\frac{i\,\sqrt{c}\log(\ldots}{a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm
="maxima")
```

```
[Out] 1/6*a^3*(2*(I*a^2*sqrt(c)*x^3 + 3*I*sqrt(c)*x)/a^5 - 3*I*sqrt(c)*log(a*x +
1)/a^6 + 3*I*sqrt(c)*log(a*x - 1)/a^6) - 3/2*a^2*(-I*sqrt(c)*x^2/a^3 - I*sq
```

```
rt(c)*log(a*x + 1)/a^5 - I*sqrt(c)*log(a*x - 1)/a^5) - 3/2*a*(-2*I*sqrt(c)*
x/a^3 + I*sqrt(c)*log(a*x + 1)/a^4 - I*sqrt(c)*log(a*x - 1)/a^4) + 1/2*I*sq
rt(c)*log(a*x + 1)/a^3 + 1/2*I*sqrt(c)*log(a*x - 1)/a^3
```

Fricas [A] time = 2.52007, size = 846, normalized size = 5.53

$$\left[\frac{12\left(a^2x^2-1\right)\sqrt{-c}\log\left(\frac{a^6cx^6-4a^5cx^5+5a^4cx^4-4a^2cx^2+4acx-\left(a^5x^5-4a^4x^4+6a^3x^3-4a^2x^2\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{a^4x^4-2a^3x^3+2ax-1}\right)+\left(2a^4x^4+9a^3x^3+\right.}{6\left(a^5x^2-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/6*(12*(a^2*x^2 - 1)*sqrt(-c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4
- 4*a^2*c*x^2 + 4*a*c*x - (a^5*x^5 - 4*a^4*x^4 + 6*a^3*x^3 - 4*a^2*x^2)*sqr
t(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/(a^4*x^4 -
2*a^3*x^3 + 2*a*x - 1)) + (2*a^4*x^4 + 9*a^3*x^3 + 24*a^2*x^2)*sqrt(-a^2*x^
2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*x^2 - a^3), 1/6*(24*(a^2*x^2 -
 1)*sqrt(c)*arctan((a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a
^2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 - 2*a^2*c*x^2 - a*c*x + 2*c)) + (2*a^4*
x^4 + 9*a^3*x^3 + 24*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)))/(a^5*x^2 - a^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1
)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))*x^2/(-a^2*x^2 + 1)^(3/2), x)
```

3.758 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal. Leaf size=114

$$-\frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-a^2x^2}}$$

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] - (a*Sqrt[c - c/(a^2*x^2)]
*x^3)/(2*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/(a*S
qrt[1 - a^2*x^2])
```

Rubi [A] time = 0.147705, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6160, 6140, 43}

$$-\frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] - (a*Sqrt[c - c/(a^2*x^2)]
*x^3)/(2*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/(a*S
qrt[1 - a^2*x^2])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^2}{1-ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(-3-ax+\frac{4}{1-ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^2}{\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}x^3}{2\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}x\log(1-ax)}{a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0261204, size = 55, normalized size = 0.48

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}-3x\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a))/Sqrt[1 -
a^2*x^2]
```

Maple [A] time = 0.143, size = 69, normalized size = 0.6

$$\frac{x\left(a^2x^2+6\,ax+8\,\ln(ax-1)\right)}{\left(2\,a^2x^2-2\right)a}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(a^2*x^2+6*a*x+8*ln(
a*x-1))/(a^2*x^2-1)/a
```

Maxima [C] time = 1.30492, size = 201, normalized size = 1.76

$$-\frac{1}{2}a^3\left(-\frac{i\sqrt{c}x^2}{a^3}-\frac{i\sqrt{c}\log(ax+1)}{a^5}-\frac{i\sqrt{c}\log(ax-1)}{a^5}\right)-\frac{3}{2}a^2\left(-\frac{2i\sqrt{c}x}{a^3}+\frac{i\sqrt{c}\log(ax+1)}{a^4}-\frac{i\sqrt{c}\log(ax-1)}{a^4}\right)-\frac{3}{2}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="
maxima")
```

```
[Out] -1/2*a^3*(-I*sqrt(c)*x^2/a^3 - I*sqrt(c)*log(a*x + 1)/a^5 - I*sqrt(c)*log(a
*x - 1)/a^5) - 3/2*a^2*(-2*I*sqrt(c)*x/a^3 + I*sqrt(c)*log(a*x + 1)/a^4 - I
```

```
*sqrt(c)*log(a*x - 1)/a^4) - 3/2*a*(-I*sqrt(c)*log(a*x + 1)/a^3 - I*sqrt(c)
*log(a*x - 1)/a^3) - 1/2*I*sqrt(c)*log(a*x + 1)/a^2 + 1/2*I*sqrt(c)*log(a*x
 - 1)/a^2
```

Fricas [A] time = 2.57683, size = 803, normalized size = 7.04

$$\left[\frac{4\left(a^{2}x^{2}-1\right)\sqrt{-c}\log\left(\frac{a^{6}cx^{6}-4\,a^{5}cx^{5}+5\,a^{4}cx^{4}-4\,a^{2}cx^{2}+4\,acx-\left(a^{5}x^{5}-4\,a^{4}x^{4}+6\,a^{3}x^{3}-4\,a^{2}x^{2}\right)\sqrt{-a^{2}x^{2}+1}\sqrt{-c}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{a^{4}x^{4}-2\,a^{3}x^{3}+2\,ax-1}\right)+\left(a^{3}x^{3}+6\,a^{2}x^{2}\right)\sqrt{-a}}{2\left(a^{4}x^{2}-a^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/2*(4*(a^2*x^2 - 1)*sqrt(-c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 -
 4*a^2*c*x^2 + 4*a*c*x - (a^5*x^5 - 4*a^4*x^4 + 6*a^3*x^3 - 4*a^2*x^2)*sqrt
(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/(a^4*x^4 - 2
*a^3*x^3 + 2*a*x - 1)) + (a^3*x^3 + 6*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2
*c*x^2 - c)/(a^2*x^2)))/(a^4*x^2 - a^2), 1/2*(8*(a^2*x^2 - 1)*sqrt(c)*arcta
n((a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^
2*x^2))/(a^3*c*x^3 - 2*a^2*c*x^2 - a*c*x + 2*c)) + (a^3*x^3 + 6*a^2*x^2)*sq
rt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^2 - a^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)^{3}}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1)*(
a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^{3}\sqrt{c-\frac{c}{a^{2}x^{2}}}\,x}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))*x/(-a^2*x^2 + 1)^(3/2), x)
```

3.759 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=108

$$-\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -((a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2]) + (Sqrt[c - c/(a^2*x^2)]
*x*Log[x])/Sqrt[1 - a^2*x^2] - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sqr
t[1 - a^2*x^2]
```

Rubi [A] time = 0.147306, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 72}

$$-\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -((a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2]) + (Sqrt[c - c/(a^2*x^2)]
*x*Log[x])/Sqrt[1 - a^2*x^2] - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sqr
t[1 - a^2*x^2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^2}{x(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(-a+\frac{1}{x}-\frac{4a}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{a\sqrt{c-\frac{c}{a^2x^2}}x^2}{\sqrt{1-a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\log(x)}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}x\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0289368, size = 47, normalized size = 0.44

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}(-ax-4\log(1-ax)+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-(a*x) + Log[x] - 4*Log[1 - a*x]))/Sqrt[1 - a^2*x
^2]
```

Maple [A] time = 0.142, size = 61, normalized size = 0.6

$$-\frac{x\left(-ax+\ln(x)-4\ln(ax-1)\right)}{a^2x^2-1}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a*x+ln(x)-4*ln(a*x-1))*(-a^2*x^2+1)^(1/2
)/(a^2*x^2-1)
```

Maxima [C] time = 1.31057, size = 200, normalized size = 1.85

$$-\frac{1}{2}a^3\left(-\frac{2i\sqrt{c}x}{a^3}+\frac{i\sqrt{c}\log(ax+1)}{a^4}-\frac{i\sqrt{c}\log(ax-1)}{a^4}\right)-\frac{3}{2}a^2\left(-\frac{i\sqrt{c}\log(ax+1)}{a^3}-\frac{i\sqrt{c}\log(ax-1)}{a^3}\right)-\frac{3}{2}a\left(\frac{i\sqrt{c}\log(ax}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/2*a^3*(-2*I*sqrt(c)*x/a^3 + I*sqrt(c)*log(a*x + 1)/a^4 - I*sqrt(c)*log(a
*x - 1)/a^4) - 3/2*a^2*(-I*sqrt(c)*log(a*x + 1)/a^3 - I*sqrt(c)*log(a*x - 1
)/a^3) - 3/2*a*(I*sqrt(c)*log(a*x + 1)/a^2 - I*sqrt(c)*log(a*x - 1)/a^2) +
```

```
1/2*I*sqrt(c)*log(a*x + 1)/a + 1/2*I*sqrt(c)*log(a*x - 1)/a - I*sqrt(c)*log
(x)/a
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}(ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*
x^2 - 2*a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)^3}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1)*(a*
x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))/(-a^2*x^2 + 1)^(3/2), x)
```

3.760 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{3ax\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4ax\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - c/(a^2*x^2)]*x*L
og[x])/Sqrt[1 - a^2*x^2] - (4*a*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[
1 - a^2*x^2]
```

Rubi [A] time = 0.244818, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{3ax\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4ax\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - c/(a^2*x^2)]*x*L
og[x])/Sqrt[1 - a^2*x^2] - (4*a*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[
1 - a^2*x^2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^2}{x^2(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(\frac{1}{x^2}+\frac{3a}{x}-\frac{4a^2}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-\frac{c}{a^2x^2}}x\log(x)}{\sqrt{1-a^2x^2}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}x\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0328704, size = 49, normalized size = 0.46

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(3ax\log(x)-4ax\log(1-ax)-1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1 + 3*a*x*Log[x] - 4*a*x*Log[1 - a*x]))/Sqrt[1 - a
^2*x^2]
```

Maple [A] time = 0.148, size = 63, normalized size = 0.6

$$-\frac{3\,a\ln(x)\,x-4\,\ln(ax-1)\,xa-1}{a^2x^2-1}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(3*a*ln(x)*x-4*ln(a*x-1)*x*a-1)*(-a^2*x^2+1)
^(1/2)/(a^2*x^2-1)
```

Maxima [C] time = 1.32433, size = 194, normalized size = 1.81

$$-\frac{1}{2}a^3\left(-\frac{i\sqrt{c}\log(ax+1)}{a^3}-\frac{i\sqrt{c}\log(ax-1)}{a^3}\right)-\frac{3}{2}a^2\left(\frac{i\sqrt{c}\log(ax+1)}{a^2}-\frac{i\sqrt{c}\log(ax-1)}{a^2}\right)-\frac{3}{2}a\left(-\frac{i\sqrt{c}\log(ax+1)}{a}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="
maxima")
```

```
[Out] -1/2*a^3*(-I*sqrt(c)*log(a*x + 1)/a^3 - I*sqrt(c)*log(a*x - 1)/a^3) - 3/2*a
^2*(I*sqrt(c)*log(a*x + 1)/a^2 - I*sqrt(c)*log(a*x - 1)/a^2) - 3/2*a*(-I*sq
rt(c)*log(a*x + 1)/a - I*sqrt(c)*log(a*x - 1)/a + 2*I*sqrt(c)*log(x)/a) - 1
```

```
/2*I*sqrt(c)*log(a*x + 1) + 1/2*I*sqrt(c)*log(a*x - 1) + I*sqrt(c)/(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}(ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2x^3-2\,ax^2+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="
fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*
x^3 - 2*a*x^2 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)^3}{x\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(x*(-(a*x - 1)*
(a*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="
giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))/((-a^2*x^2 + 1)^(3/2)*x), x)
```

3.761 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=148

$$-\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}+\frac{4a^2x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^2x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] (-3*a*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(2*x
*Sqrt[1 - a^2*x^2]) + (4*a^2*Sqrt[c - c/(a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x
^2] - (4*a^2*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.24951, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}+\frac{4a^2x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^2x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (-3*a*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(2*x
*Sqrt[1 - a^2*x^2]) + (4*a^2*Sqrt[c - c/(a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x
^2] - (4*a^2*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[1 - a^2*x^2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^3}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^2}{x^3(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{1}{x^3}+\frac{3a}{x^2}+\frac{4a^2}{x}-\frac{4a^3}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}} + \frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}} - \frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0446526, size = 64, normalized size = 0.43

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(4a^2\log(x)-4a^2\log(1-ax)-\frac{3a}{x}-\frac{1}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-1/(2*x^2) - (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1
 - a*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.148, size = 78, normalized size = 0.5

$$-\frac{8\,a^2\ln(x)\,x^2-8\,\ln(ax-1)\,a^2x^2-6\,ax-1}{2\,x\left(a^2x^2-1\right)}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-a^2*x^2+1)^(1/2)*(8*a^2*ln(x)*x^2-8*
ln(a*x-1)*a^2*x^2-6*a*x-1)/(a^2*x^2-1)
```

Maxima [C] time = 1.32495, size = 215, normalized size = 1.45

$$-\frac{1}{2}\,a^3\left(\frac{i\,\sqrt{c}\log(ax+1)}{a^2}-\frac{i\,\sqrt{c}\log(ax-1)}{a^2}\right)-\frac{3}{2}\,a^2\left(-\frac{i\,\sqrt{c}\log(ax+1)}{a}-\frac{i\,\sqrt{c}\log(ax-1)}{a}+\frac{2i\,\sqrt{c}\log(x)}{a}\right)+\frac{1}{2}i\,a\sqrt{c}\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm
="maxima")
```

```
[Out] -1/2*a^3*(I*sqrt(c)*log(a*x + 1)/a^2 - I*sqrt(c)*log(a*x - 1)/a^2) - 3/2*a^
2*(-I*sqrt(c)*log(a*x + 1)/a - I*sqrt(c)*log(a*x - 1)/a + 2*I*sqrt(c)*log(x
```

```
)/a) + 1/2*I*a*sqrt(c)*log(a*x + 1) + 1/2*I*a*sqrt(c)*log(a*x - 1) - I*a*sq
rt(c)*log(x) - 3/2*(I*sqrt(c)*log(a*x + 1) - I*sqrt(c)*log(a*x - 1) - 2*I*s
qrt(c)/(a*x))*a + 1/2*I*sqrt(c)/(a*x^2)
```

Fricas [A] time = 2.67676, size = 1023, normalized size = 6.91

$$\left[\frac{4\left(a^{3}x^{3}-ax\right)\sqrt{-c}\log\left(-\frac{4\,a^{5}cx^{5}-\left(2\,a^{6}-4\,a^{5}+6\,a^{4}-4\,a^{3}+a^{2}\right)cx^{6}-\left(4\,a^{4}+4\,a^{3}-6\,a^{2}+4\,a-1\right)cx^{4}+5\,a^{2}cx^{2}-4\,acx-\left(4\,a^{4}x^{4}-6\,a^{3}x^{3}-\left(4\,a^{4}-6\,a^{3}+4\,a^{2}-a\right)x^{5}+\cdots}{a^{4}x^{6}-2\,a^{3}x^{5}+2\,ax^{3}-x^{2}}\right.}{2\left(a^{2}x^{3}-x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm
="fricas")
```

```
[Out] [1/2*(4*(a^3*x^3 - a*x)*sqrt(-c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6*a^4
 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2*c*x
^2 - 4*a*c*x - (4*a^4*x^4 - 6*a^3*x^3 - (4*a^4 - 6*a^3 + 4*a^2 - a)*x^5 + 4
*a^2*x^2 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
 + c)/(a^4*x^6 - 2*a^3*x^5 + 2*a*x^3 - x^2)) - sqrt(-a^2*x^2 + 1)*((6*a + 1
)*x^2 - 6*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^3 - x), 1/2*(8*(
a^3*x^3 - a*x)*sqrt(c)*arctan((2*a^2*x^2 - (2*a^3 - 2*a^2 + a)*x^3 - a*x)*s
qrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(2*a^3*c*x^3 - (2
*a^3 - a^2)*c*x^4 - (a^2 - 2*a + 1)*c*x^2 - 2*a*c*x + c)) - sqrt(-a^2*x^2 +
 1)*((6*a + 1)*x^2 - 6*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^3 -
 x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)^{3}}{x^{2}\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(x**2*(-(a*x -
1)*(a*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^{3}\sqrt{c-\frac{c}{a^{2}x^{2}}}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm
="giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

3.762 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=187

$$-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}+\frac{4a^3x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^3x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] (-4*a^2*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(3
*x^2*Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[c - c/(a^2*x^2)])/(2*x*Sqrt[1 - a^2*x^2
]) + (4*a^3*Sqrt[c - c/(a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^3*Sqrt
[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.257147, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}+\frac{4a^3x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^3x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] (-4*a^2*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(3
*x^2*Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[c - c/(a^2*x^2)])/(2*x*Sqrt[1 - a^2*x^2
]) + (4*a^3*Sqrt[c - c/(a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^3*Sqrt
[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[1 - a^2*x^2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^4}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^2}{x^4(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{1}{x^4}+\frac{3a}{x^3}+\frac{4a^2}{x^2}+\frac{4a^3}{x}-\frac{4a^4}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}} - \frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}} + \frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}} - \frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\,\mathrm{l}}{\sqrt{1-a^2}}$$

Mathematica [A] time = 0.0497132, size = 74, normalized size = 0.4

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4a^2}{x}+4a^3\log(x)-4a^3\log(1-ax)-\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-1/(3*x^3) - (3*a)/(2*x^2) - (4*a^2)/x + 4*a^3*Lo
g[x] - 4*a^3*Log[1 - a*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.146, size = 86, normalized size = 0.5

$$\frac{24\ln(ax-1)x^3a^3-24a^3\ln(x)x^3+24a^2x^2+9ax+2}{6x^2\left(a^2x^2-1\right)}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x)
```

```
[Out] 1/6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^2*(-a^2*x^2+1)^(1/2)*(24*ln(a*x-1)*x^3*
a^3-24*a^3*ln(x)*x^3+24*a^2*x^2+9*a*x+2)/(a^2*x^2-1)
```

Maxima [C] time = 1.32784, size = 248, normalized size = 1.33

$$-\frac{1}{2}a^3\left(-\frac{i\sqrt{c}\log(ax+1)}{a}-\frac{i\sqrt{c}\log(ax-1)}{a}+\frac{2i\sqrt{c}\log(x)}{a}\right)-\frac{1}{2}ia^2\sqrt{c}\log(ax+1)+\frac{1}{2}ia^2\sqrt{c}\log(ax-1)-\frac{3}{2}\left(i\sqrt{c}\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm
="maxima")
```

```
[Out] -1/2*a^3*(-I*sqrt(c)*log(a*x + 1)/a - I*sqrt(c)*log(a*x - 1)/a + 2*I*sqrt(c
)*log(x)/a) - 1/2*I*a^2*sqrt(c)*log(a*x + 1) + 1/2*I*a^2*sqrt(c)*log(a*x -
```

```
1) - 3/2*(I*sqrt(c)*log(a*x + 1) - I*sqrt(c)*log(a*x - 1) - 2*I*sqrt(c)/(a*
x))*a^2 - 3/2*(-I*a*sqrt(c)*log(a*x + 1) - I*a*sqrt(c)*log(a*x - 1) + 2*I*a
*sqrt(c)*log(x) - I*sqrt(c)/(a*x^2))*a + 1/3*(3*I*a^2*sqrt(c)*x^2 + I*sqrt(
c))/(a*x^3)
```

Fricas [A] time = 2.66592, size = 1102, normalized size = 5.89

$$\left[\frac{12\left(a^4x^4-a^2x^2\right)\sqrt{-c}\log\left(-\frac{4a^5cx^5-\left(2a^6-4a^5+6a^4-4a^3+a^2\right)cx^6-\left(4a^4+4a^3-6a^2+4a-1\right)cx^4+5a^2cx^2-4acx-\left(4a^4x^4-6a^3x^3-\left(4a^4-6a^3+4a^2-a\right)\right.}{a^4x^6-2a^3x^5+2ax^3-x^2}\right.}{6\left(a^2x^4-x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm
="fricas")
```

```
[Out] [1/6*(12*(a^4*x^4 - a^2*x^2)*sqrt(-c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 +
6*a^4 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^
2*c*x^2 - 4*a*c*x - (4*a^4*x^4 - 6*a^3*x^3 - (4*a^4 - 6*a^3 + 4*a^2 - a)*x^
5 + 4*a^2*x^2 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)) + c)/(a^4*x^6 - 2*a^3*x^5 + 2*a*x^3 - x^2)) + (24*a^2*x^2 - (24*a^2 +
 9*a + 2)*x^3 + 9*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)))/(a^2*x^4 - x^2), 1/6*(24*(a^4*x^4 - a^2*x^2)*sqrt(c)*arctan((2*a^2*x^2
- (2*a^3 - 2*a^2 + a)*x^3 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2
 - c)/(a^2*x^2))/(2*a^3*c*x^3 - (2*a^3 - a^2)*c*x^4 - (a^2 - 2*a + 1)*c*x^2
 - 2*a*c*x + c)) + (24*a^2*x^2 - (24*a^2 + 9*a + 2)*x^3 + 9*a*x + 2)*sqrt(-
a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^4 - x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)^3}{x^3\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(x**3*(-(a*x -
1)*(a*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm
="giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

3.763 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=222

$$-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}+\frac{4a^4x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] (-4*a^3*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(4
*x^3*Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - c/(a^2*x^2)])/(x^2*Sqrt[1 - a^2*x^2])
 - (2*a^2*Sqrt[c - c/(a^2*x^2)])/(x*Sqrt[1 - a^2*x^2]) + (4*a^4*Sqrt[c - c/
(a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^4*Sqrt[c - c/(a^2*x^2)]*x*Log
[1 - a*x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.267187, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}+\frac{4a^4x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```


```
[Out] (-4*a^3*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(4
*x^3*Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - c/(a^2*x^2)])/(x^2*Sqrt[1 - a^2*x^2])
 - (2*a^2*Sqrt[c - c/(a^2*x^2)])/(x*Sqrt[1 - a^2*x^2]) + (4*a^4*Sqrt[c - c/
(a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^4*Sqrt[c - c/(a^2*x^2)]*x*Log
[1 - a*x])/Sqrt[1 - a^2*x^2]
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^5}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^2}{x^5(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(\frac{1}{x^5}+\frac{3a}{x^4}+\frac{4a^2}{x^3}+\frac{4a^3}{x^2}+\frac{4a^4}{x}-\frac{4a^5}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-a^2x^2}} - \frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}} + \frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}x\log(x)}{\sqrt{1-a^2x^2}} -$$

Mathematica [A] time = 0.0548248, size = 80, normalized size = 0.36

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{2a^2}{x^2}-\frac{4a^3}{x}+4a^4\log(x)-4a^4\log(1-ax)-\frac{a}{x^3}-\frac{1}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-1/(4*x^4) - a/x^3 - (2*a^2)/x^2 - (4*a^3)/x + 4*
a^4*Log[x] - 4*a^4*Log[1 - a*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.154, size = 94, normalized size = 0.4

$$-\frac{16\,a^4\ln(x)\,x^4-16\,\ln(ax-1)\,a^4x^4-16\,x^3a^3-8\,a^2x^2-4\,ax-1}{4\,x^3\left(a^2x^2-1\right)}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x)
```

```
[Out] -1/4*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^3*(-a^2*x^2+1)^(1/2)*(16*a^4*ln(x)*x^4
-16*ln(a*x-1)*a^4*x^4-16*x^3*a^3-8*a^2*x^2-4*a*x-1)/(a^2*x^2-1)
```

Maxima [C] time = 1.36656, size = 282, normalized size = 1.27

$$\frac{1}{2}i\,a^3\sqrt{c}\log(ax+1)+\frac{1}{2}i\,a^3\sqrt{c}\log(ax-1)-i\,a^3\sqrt{c}\log(x)-\frac{1}{2}\left(i\sqrt{c}\log(ax+1)-i\sqrt{c}\log(ax-1)-\frac{2i\sqrt{c}}{ax}\right)a^3-\frac{3}{2}\left(-i\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm
="maxima")
```

```
[Out] 1/2*I*a^3*sqrt(c)*log(a*x + 1) + 1/2*I*a^3*sqrt(c)*log(a*x - 1) - I*a^3*sqr
t(c)*log(x) - 1/2*(I*sqrt(c)*log(a*x + 1) - I*sqrt(c)*log(a*x - 1) - 2*I*sq
```

```
rt(c)/(a*x))*a^3 - 3/2*(-I*a*sqrt(c)*log(a*x + 1) - I*a*sqrt(c)*log(a*x - 1
) + 2*I*a*sqrt(c)*log(x) - I*sqrt(c)/(a*x^2))*a^2 - 1/2*(3*I*a^2*sqrt(c)*lo
g(a*x + 1) - 3*I*a^2*sqrt(c)*log(a*x - 1) - 2*(3*I*a^2*sqrt(c)*x^2 + I*sqrt
(c))/(a*x^3))*a + 1/4*(2*I*a^2*sqrt(c)*x^2 + I*sqrt(c))/(a*x^4)
```

Fricas [A] time = 2.68817, size = 1154, normalized size = 5.2

$$\left[\frac{8\left(a^5x^5 - a^3x^3\right)\sqrt{-c}\log\left(-\frac{4\,a^5cx^5-\left(2\,a^6-4\,a^5+6\,a^4-4\,a^3+a^2\right)cx^6-\left(4\,a^4+4\,a^3-6\,a^2+4\,a-1\right)cx^4+5\,a^2cx^2-4\,acx-\left(4\,a^4x^4-6\,a^3x^3-\left(4\,a^4-6\,a^3+4\,a^2-a\right)x\right.}{a^4x^6-2\,a^3x^5+2\,ax^3-x^2}\right.}{4\left(a^2x^5 -\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm
="fricas")
```

```
[Out] [1/4*(8*(a^5*x^5 - a^3*x^3)*sqrt(-c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6
*a^4 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2
*c*x^2 - 4*a*c*x - (4*a^4*x^4 - 6*a^3*x^3 - (4*a^4 - 6*a^3 + 4*a^2 - a)*x^5
 + 4*a^2*x^2 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x
^2)) + c)/(a^4*x^6 - 2*a^3*x^5 + 2*a*x^3 - x^2)) + (16*a^3*x^3 - (16*a^3 +
8*a^2 + 4*a + 1)*x^4 + 8*a^2*x^2 + 4*a*x + 1)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*
c*x^2 - c)/(a^2*x^2)))/(a^2*x^5 - x^3), 1/4*(16*(a^5*x^5 - a^3*x^3)*sqrt(c)
*arctan((2*a^2*x^2 - (2*a^3 - 2*a^2 + a)*x^3 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt
(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(2*a^3*c*x^3 - (2*a^3 - a^2)*c*x^4 - (a
^2 - 2*a + 1)*c*x^2 - 2*a*c*x + c)) + (16*a^3*x^3 - (16*a^3 + 8*a^2 + 4*a +
 1)*x^4 + 8*a^2*x^2 + 4*a*x + 1)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a
^2*x^2)))/(a^2*x^5 - x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)^3}{x^4\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(x**4*(-(a*x -
1)*(a*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm
="giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))/((-a^2*x^2 + 1)^(3/2)*x^4), x)
```

3.764 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=263

$$-\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{2a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}-\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4\sqrt{1-a^2x^2}}+\frac{4a^5x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^5x\sqrt{c}}{}$$

```
[Out] (-4*a^4*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(5
*x^4*Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[c - c/(a^2*x^2)])/(4*x^3*Sqrt[1 - a^2*x
^2]) - (4*a^2*Sqrt[c - c/(a^2*x^2)])/(3*x^2*Sqrt[1 - a^2*x^2]) - (2*a^3*Sqr
t[c - c/(a^2*x^2)])/(x*Sqrt[1 - a^2*x^2]) + (4*a^5*Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2] - (4*a^5*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sq
rt[1 - a^2*x^2]
```

Rubi [A] time = 0.273588, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{2a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}-\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4\sqrt{1-a^2x^2}}+\frac{4a^5x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^5x\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```


```
[Out] (-4*a^4*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(5
*x^4*Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[c - c/(a^2*x^2)])/(4*x^3*Sqrt[1 - a^2*x
^2]) - (4*a^2*Sqrt[c - c/(a^2*x^2)])/(3*x^2*Sqrt[1 - a^2*x^2]) - (2*a^3*Sqr
t[c - c/(a^2*x^2)])/(x*Sqrt[1 - a^2*x^2]) + (4*a^5*Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2] - (4*a^5*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sq
rt[1 - a^2*x^2]
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^6}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^2}{x^6(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(\frac{1}{x^6}+\frac{3a}{x^5}+\frac{4a^2}{x^4}+\frac{4a^3}{x^3}+\frac{4a^4}{x^2}+\frac{4a^5}{x}-\frac{4a^6}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4\sqrt{1-a^2x^2}} - \frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}} - \frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}} - \frac{2a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}} + \frac{4a^5\sqrt{\cdots}}{\cdots}$$

Mathematica [A] time = 0.0669908, size = 92, normalized size = 0.35

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{2a^3}{x^2}-\frac{4a^2}{3x^3}-\frac{4a^4}{x}+4a^5\log(x)-4a^5\log(1-ax)-\frac{3a}{4x^4}-\frac{1}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-1/(5*x^5) - (3*a)/(4*x^4) - (4*a^2)/(3*x^3) - (2
*a^3)/x^2 - (4*a^4)/x + 4*a^5*Log[x] - 4*a^5*Log[1 - a*x]))/Sqrt[1 - a^2*x^
2]
```

Maple [A] time = 0.151, size = 102, normalized size = 0.4

$$-\frac{240\,a^5\ln(x)\,x^5-240\,\ln(ax-1)\,x^5a^5-240\,x^4a^4-120\,x^3a^3-80\,a^2x^2-45\,ax-12}{60\,x^4\left(a^2x^2-1\right)}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x)
```

```
[Out] -1/60*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^4*(-a^2*x^2+1)^(1/2)*(240*a^5*ln(x)*x
^5-240*ln(a*x-1)*x^5*a^5-240*x^4*a^4-120*x^3*a^3-80*a^2*x^2-45*a*x-12)/(a^2
*x^2-1)
```

Maxima [C] time = 1.33493, size = 324, normalized size = 1.23

$$-\frac{1}{2}i\,a^4\sqrt{c}\log(ax+1)+\frac{1}{2}i\,a^4\sqrt{c}\log(ax-1)-\frac{1}{2}\left(-i\,a\sqrt{c}\log(ax+1)-i\,a\sqrt{c}\log(ax-1)+2i\,a\sqrt{c}\log(x)-\frac{i\sqrt{c}}{ax^2}\right)a^3-\frac{1}{2}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm
="maxima")
```

```
[Out] -1/2*I*a^4*sqrt(c)*log(a*x + 1) + 1/2*I*a^4*sqrt(c)*log(a*x - 1) - 1/2*(-I*
a*sqrt(c)*log(a*x + 1) - I*a*sqrt(c)*log(a*x - 1) + 2*I*a*sqrt(c)*log(x) -
I*sqrt(c)/(a*x^2))*a^3 - 1/2*(3*I*a^2*sqrt(c)*log(a*x + 1) - 3*I*a^2*sqrt(c
)*log(a*x - 1) - 2*(3*I*a^2*sqrt(c)*x^2 + I*sqrt(c))/(a*x^3))*a^2 - 3/4*(-2
*I*a^3*sqrt(c)*log(a*x + 1) - 2*I*a^3*sqrt(c)*log(a*x - 1) + 4*I*a^3*sqrt(c
)*log(x) - (2*I*a^2*sqrt(c)*x^2 + I*sqrt(c))/(a*x^4))*a + 1/15*I*(15*a^4*sq
rt(c)*x^4 + 5*a^2*sqrt(c)*x^2 + 3*sqrt(c))/(a*x^5)
```

Fricas [A] time = 2.72963, size = 1247, normalized size = 4.74

$$\left[\frac{120\left(a^6x^6-a^4x^4\right)\sqrt{-c}\log\left(-\frac{4a^5cx^5-\left(2a^6-4a^5+6a^4-4a^3+a^2\right)cx^6-\left(4a^4+4a^3-6a^2+4a-1\right)cx^4+5a^2cx^2-4acx-\left(4a^4x^4-6a^3x^3-\left(4a^4-6a^3+4a^2-\right.\right.}{a^4x^6-2a^3x^5+2ax^3-x^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm
="fricas")
```

```
[Out] [1/60*(120*(a^6*x^6 - a^4*x^4)*sqrt(-c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5
+ 6*a^4 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*
a^2*c*x^2 - 4*a*c*x - (4*a^4*x^4 - 6*a^3*x^3 - (4*a^4 - 6*a^3 + 4*a^2 - a)*
x^5 + 4*a^2*x^2 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^
2*x^2)) + c)/(a^4*x^6 - 2*a^3*x^5 + 2*a*x^3 - x^2)) + (240*a^4*x^4 + 120*a^
3*x^3 - (240*a^4 + 120*a^3 + 80*a^2 + 45*a + 12)*x^5 + 80*a^2*x^2 + 45*a*x
+ 12)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^6 - x^4),
1/60*(240*(a^6*x^6 - a^4*x^4)*sqrt(c)*arctan((2*a^2*x^2 - (2*a^3 - 2*a^2 +
a)*x^3 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(2
*a^3*c*x^3 - (2*a^3 - a^2)*c*x^4 - (a^2 - 2*a + 1)*c*x^2 - 2*a*c*x + c)) +
(240*a^4*x^4 + 120*a^3*x^3 - (240*a^4 + 120*a^3 + 80*a^2 + 45*a + 12)*x^5 +
 80*a^2*x^2 + 45*a*x + 12)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)))/(a^2*x^6 - x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)^3}{x^5\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(x**5*(-(a*x -
1)*(a*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm
="giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))/((-a^2*x^2 + 1)^(3/2)*x^5), x)
```

3.765 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx$

Optimal. Leaf size=81

$$\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{m\sqrt{1-a^2x^2}}-\frac{ax^{m+2}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-a^2x^2}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^(1 + m))/(m*Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - c/(a^
2*x^2)]*x^(2 + m))/((1 + m)*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.234089, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 43}

$$\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{m\sqrt{1-a^2x^2}}-\frac{ax^{m+2}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^m)/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^(1 + m))/(m*Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - c/(a^
2*x^2)]*x^(2 + m))/((1 + m)*Sqrt[1 - a^2*x^2])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{-\tanh^{-1}(ax)}x^{-1+m}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int x^{-1+m}(1-ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\left(x^{-1+m}-ax^m\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x^{1+m}}{m\sqrt{1-a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}x^{2+m}}{(1+m)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0387454, size = 52, normalized size = 0.64

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{x^m}{m}-\frac{ax^{m+1}}{m+1}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^m)/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(x^m/m - (a*x^(1 + m))/(1 + m)))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.08, size = 69, normalized size = 0.9

$$\frac{x^{1+m}(axm-m-1)}{(1+m)\,m\,(ax-1)\,(ax+1)}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] x^(1+m)*(a*m*x-m-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(-a^2*x^2+1)^(1/2)/(1+m)/
m/(a*x-1)/(a*x+1)
```

Maxima [C] time = 1.14975, size = 74, normalized size = 0.91

$$\frac{\left(i\,a\sqrt{c}mx+\sqrt{c}(-i\,m-i)\right)(ax+1)(ax-1)x^m}{\left(m^2+m\right)a^3x^2-\left(m^2+m\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] (I*a*sqrt(c)*m*x + sqrt(c)*(-I*m - I))*(a*x + 1)*(a*x - 1)*x^m/((m^2 + m)*a
^3*x^2 - (m^2 + m)*a)
```

Fricas [A] time = 2.20564, size = 151, normalized size = 1.86

$$\frac{\sqrt{-a^2x^2+1}\left(amx^2-(m+1)x\right)x^m\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\left(a^2m^2+a^2m\right)x^2-m^2-m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] sqrt(-a^2*x^2 + 1)*(a*m*x^2 - (m + 1)*x)*x^m*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
)/((a^2*m^2 + a^2*m)*x^2 - m^2 - m)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^m\sqrt{-(ax-1)(ax+1)}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c-c/a**2/x**2)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x
)))/(a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{a^2x^2}}x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))*x^m/(a*x + 1), x)
```

3.766 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=74

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}-\frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-a^2x^2}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^3)/(2*Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - c/(a^2*x^2)
]*x^4)/(3*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.20854, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 43}

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}-\frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^3)/(2*Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - c/(a^2*x^2)
]*x^4)/(3*Sqrt[1 - a^2*x^2])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{-\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int x(1-ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(x-ax^2\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{2\sqrt{1-a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}x^4}{3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0221022, size = 42, normalized size = 0.57

$$-\frac{x^3(2ax-3)\sqrt{c-\frac{c}{a^2x^2}}}{6\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^ArcTanh[a*x],x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x^3*(-3 + 2*a*x))/(6*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.079, size = 57, normalized size = 0.8

$$\frac{(2\,ax-3)\,x^3}{(6\,ax-6)\,(ax+1)}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/6*x^3*(2*a*x-3)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/
(a*x+1)
```

Maxima [C] time = 1.1374, size = 58, normalized size = 0.78

$$\frac{\left(2i\,a\sqrt{c}x^3-3i\,\sqrt{c}x^2\right)(ax+1)(ax-1)}{6\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/6*(2*I*a*sqrt(c)*x^3 - 3*I*sqrt(c)*x^2)*(a*x + 1)*(a*x - 1)/(a^3*x^2 - a)
```

Fricas [A] time = 2.15906, size = 119, normalized size = 1.61

$$\frac{\left(2\,ax^4-3\,x^3\right)\sqrt{-a^2x^2+1}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{6\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/6*(2*a*x^4 - 3*x^3)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a
^2*x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\sqrt{-(ax-1)(ax+1)}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x
)))/(a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{a^2x^2}}x^2}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))*x^2/(a*x + 1), x)
```

3.767 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal. Leaf size=71

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] - (a*Sqrt[c - c/(a^2*x^2)]*x^
3)/(2*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.131313, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6160, 6140}

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] - (a*Sqrt[c - c/(a^2*x^2)]*x^
3)/(2*Sqrt[1 - a^2*x^2])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{-\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int(1-ax)\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}x^3}{2\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0191384, size = 41, normalized size = 0.58

$$\frac{x\left(x-\frac{ax^2}{2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(x - (a*x^2)/2))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.082, size = 56, normalized size = 0.8

$$\frac{(ax-2)\,x^2}{(2\,ax-2)\,(ax+1)}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*x^2*(a*x-2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(a
*x+1)
```

Maxima [C] time = 1.12499, size = 69, normalized size = 0.97

$$\frac{\left(i\,a^2\sqrt{c}x^2-2i\,a\sqrt{c}x+2i\,\sqrt{c}\right)(ax+1)(ax-1)}{2\left(a^4x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 1/2*(I*a^2*sqrt(c)*x^2 - 2*I*a*sqrt(c)*x + 2*I*sqrt(c))*(a*x + 1)*(a*x - 1)
/(a^4*x^2 - a^2)
```

Fricas [A] time = 2.09807, size = 116, normalized size = 1.63

$$\frac{\sqrt{-a^2x^2+1}\left(ax^3-2\,x^2\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*(a*x^3 - 2*x^2)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2
*x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\sqrt{-(ax-1)\,(ax+1)}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))
/(a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{a^2x^2}}\,x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))*x/(a*x + 1), x)
```

3.768 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=69

$$\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

```
[Out] -((a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2]) + (Sqrt[c - c/(a^2*x^2)]
*x*Log[x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.136392, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 43}

$$\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^ArcTanh[a*x],x]
```

```
[Out] -((a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2]) + (Sqrt[c - c/(a^2*x^2)]
*x*Log[x])/Sqrt[1 - a^2*x^2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1-ax}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(-a+\frac{1}{x}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{a\sqrt{c-\frac{c}{a^2x^2}}x^2}{\sqrt{1-a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0198208, size = 38, normalized size = 0.55

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}(\log(x)-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-(a*x) + Log[x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.135, size = 53, normalized size = 0.8

$$-\frac{x\left(-ax+\ln(x)\right)}{a^2x^2-1}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a*x+ln(x))*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

Fricas [B] time = 2.44497, size = 648, normalized size = 9.39

$$\left[\frac{\left(a^{2}x^{2}-1\right)\sqrt{-c}\log\left(\frac{a^{2}cx^{6}+a^{2}cx^{2}-cx^{4}+\left(ax^{5}-ax\right)\sqrt{-a^{2}x^{2}+1}\sqrt{-c}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c}{a^{2}x^{4}-x^{2}}\right)+2\left(a^{2}x^{2}-a^{2}x\right)\sqrt{-a^{2}x^{2}+1}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{2\left(a^{3}x^{2}-a\right)},-\frac{\left(a^{2}x^{2}-1\right)\sqrt{c}\,\mathrm{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 + (a*x^5 -
a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c)/(a^2*
x^4 - x^2)) + 2*(a^2*x^2 - a^2*x)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^3*x^2 - a), -((a^2*x^2 - 1)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)
*(a*x^3 + a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 +
1)*c*x^2 + c)) - (a^2*x^2 - a^2*x)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)))/(a^3*x^2 - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/(
a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^{2}x^{2}+1}\sqrt{c-\frac{c}{a^{2}x^{2}}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

3.769 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{ax\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - c/(a^2*x^2)]*x*Log
[x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.230661, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 43}

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{ax\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^ArcTanh[a*x]*x),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - c/(a^2*x^2)]*x*Log
[x])/Sqrt[1 - a^2*x^2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1-ax}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(\frac{1}{x^2}-\frac{a}{x}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}x\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0214352, size = 38, normalized size = 0.57

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax\log(x)+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^ArcTanh[a*x]*x),x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*(1 + a*x*Log[x]))/Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.141, size = 51, normalized size = 0.8

$$\frac{a\ln(x)x+1}{a^2x^2-1}\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(-a^2*x^2+1)^(1/2)*(a*ln(x)*x+1)/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x), x)
```

Fricas [B] time = 2.37658, size = 620, normalized size = 9.25

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)-2\sqrt{-a^2x^2+1}(x-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^2x^2-1\right)}, \frac{\left(a^2x^2-1\right)\sqrt{c}\arctan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="fr
icas")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 - (a*x^5 -
a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c)/(a^2*
x^4 - x^2)) - 2*sqrt(-a^2*x^2 + 1)*(x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))
/(a^2*x^2 - 1), ((a^2*x^2 - 1)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a
*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 +
c)) - sqrt(-a^2*x^2 + 1)*(x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^2
- 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/(
x*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x), x)
```

3.770 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=44

$$-\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}$$

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(2*x*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.221966, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 37}

$$-\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(2*x*Sqrt[1 - a^2*x^2])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^3}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1-ax}{x^3}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{2x\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0202038, size = 44, normalized size = 1.

$$\frac{x\left(\frac{a}{x}-\frac{1}{2x^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/(2*x^2) + a/x)*x)/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.078, size = 57, normalized size = 1.3

$$-\frac{2\,ax-1}{(2\,ax+2)\,(ax-1)\,x}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] -1/2*(2*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)/(a*
x-1)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^2), x)
```

Fricas [A] time = 2.11657, size = 132, normalized size = 3.

$$\frac{\sqrt{-a^2x^2+1}\left((2\,a-1)x^2-2\,ax+1\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^2x^3-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="
fricas")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*((2*a - 1)*x^2 - 2*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^
2*x^2))/(a^2*x^3 - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x^2(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/(
x**2*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^2), x)
```

3.771 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

Optimal. Leaf size=163

$$-\frac{x^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}+\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}+\frac{7x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{8a^3\sqrt{ax+1}\sqrt{1-ax}}$$

```
[Out] (7*Sqrt[c - c/(a^2*x^2)]*x)/(8*a^3) + (7*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))
/(24*a^3) + (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^2)/(6*a^3) - (Sqrt[c - c/(a^
2*x^2)]*x^2*(1 - a*x)^2)/(4*a^2) + (7*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/
(8*a^3*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.403114, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6159, 6129, 90, 80, 50, 41, 216}

$$-\frac{x^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}+\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}+\frac{7x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{8a^3\sqrt{ax+1}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(2*ArcTanh[a*x]),x]
```


```
[Out] (7*Sqrt[c - c/(a^2*x^2)]*x)/(8*a^3) + (7*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))
/(24*a^3) + (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^2)/(6*a^3) - (Sqrt[c - c/(a^
2*x^2)]*x^2*(1 - a*x)^2)/(4*a^2) + (7*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/
(8*a^3*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```


Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{x^2(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1-ax)^2}{4a^2}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}(-1+2ax)}{\sqrt{1+ax}}\,dx}{4a^2\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{6a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1-ax)^2}{4a^2}+\frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{12a^2\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{24a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{6a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1-ax)^2}{4a^2}+\frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}x\right)\int}{8a^2\sqrt{1-ax}}\\
&= \frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3}+\frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{24a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{6a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1-ax)^2}{4a^2}+\\
&= \frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3}+\frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{24a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{6a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1-ax)^2}{4a^2}+\\
&= \frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3}+\frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{24a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{6a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1-ax)^2}{4a^2}+
\end{aligned}
$$

Mathematica [A] time = 0.0815096, size = 93, normalized size = 0.57

$$
-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(6a^3x^3-16a^2x^2+21ax-32\right)+21\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{24a^3\sqrt{a^2x^2-1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(-32 + 21*a*x - 16*a^2*x^2 +
6*a^3*x^3) + 21*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^3*Sqrt[-1 + a^2*x^2])
```

Maple [A] time = 0.141, size = 196, normalized size = 1.2

$$\frac{x}{24\,ca^4}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-6\,x\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}a^4+16\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}a^3-27\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}xa^2c+27\,c^{3/2}\ln\left(x\sqrt{c}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-6*x*(c*(a^2*x^2-1)/a^2)^(3/2)*a^4+16
*(c*(a^2*x^2-1)/a^2)^(3/2)*a^3-27*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a^2*c+27*c^(3
/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))-48*c^(3/2)*ln((((a*x-1)*(a*x+1)
*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))+48*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a*c)/(
c*(a^2*x^2-1)/a^2)^(1/2)/c/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}x^3}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxi
ma")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))*x^3/(a*x + 1)^2, x)
```

Fricas [A] time = 2.23512, size = 490, normalized size = 3.01

$$\left[-\frac{2\left(6\,a^4x^4-16\,a^3x^3+21\,a^2x^2-32\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-21\,\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{48\,a^4},-\frac{\left(6\,a^4x^4-16\,a^3x^3+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fric
as")
```

```
[Out] [-1/48*(2*(6*a^4*x^4 - 16*a^3*x^3 + 21*a^2*x^2 - 32*a*x)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)) - 21*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*
x^2 - c)/(a^2*x^2)) - c))/a^4, -1/24*((6*a^4*x^4 - 16*a^3*x^3 + 21*a^2*x^2
- 32*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 21*sqrt(-c)*arctan(a^2*sqrt(-c)
*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx-\int \frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x**3*sqrt(c - c/(a**2*x**2))/(a*x + 1), x) - Integral(a*x**4*sqr
t(c - c/(a**2*x**2))/(a*x + 1), x)
```

Giac [A] time = 1.25207, size = 173, normalized size = 1.06

$$-\frac{1}{48}\left(2\sqrt{a^2cx^2-c}\left(\left(2x\left(\frac{3x\,\mathrm{sgn}(x)}{a^2}-\frac{8\,\mathrm{sgn}(x)}{a^3}\right)+\frac{21\,\mathrm{sgn}(x)}{a^4}\right)x-\frac{32\,\mathrm{sgn}(x)}{a^5}\right)-\frac{42\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}}{a^4|a|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac
")
```

```
[Out] -1/48*(2*sqrt(a^2*c*x^2 - c)*((2*x*(3*x*sgn(x)/a^2 - 8*sgn(x)/a^3) + 21*sgn
(x)/a^4)*x - 32*sgn(x)/a^5) - 42*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*
c*x^2 - c)))*sgn(x)/(a^4*abs(a)) + (21*a*sqrt(c)*log(abs(c)) + 64*sqrt(-c)*
abs(a))*sgn(x)/(a^5*abs(a)))*abs(a)
```

3.772 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal. Leaf size=126

$$-\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}-\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{a^2\sqrt{ax+1}\sqrt{1-ax}}$$

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x)/a^2) - (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))/(3*a
^2) - (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^2)/(3*a^2) - (Sqrt[c - c/(a^2*x^2)
]*x*ArcSin[a*x])/(a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.326863, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6159, 6129, 80, 50, 41, 216}

$$-\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}-\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{a^2\sqrt{ax+1}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x)/a^2) - (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))/(3*a
^2) - (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^2)/(3*a^2) - (Sqrt[c - c/(a^2*x^2)
]*x*ArcSin[a*x])/(a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{-2\tanh^{-1}(ax)}x\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{x(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2}-\frac{\left(2\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{3a\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{3a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{3a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{3a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{3a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x\sin^{-1}(ax)}{a^2\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.0666597, size = 84, normalized size = 0.67

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(a^2x^2-3ax+5\right)-3\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{3a^2\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(5 - 3*a*x + a^2*x^2) - 3*Log
[a*x + Sqrt[-1 + a^2*x^2]]))/(3*a^2*Sqrt[-1 + a^2*x^2])
```

Maple [A] time = 0.112, size = 173, normalized size = 1.4

$$-\frac{x}{3a^3c}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^3-3\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}xa^2c+3c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)-6c^{3/2}\ln\left(\frac{1}{\sqrt{c}}\left(\sqrt{(a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*((c*(a^2*x^2-1)/a^2)^(3/2)*a^3-3*(c*(a
^2*x^2-1)/a^2)^(1/2)*x*a^2*c+3*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/
2))-6*c^(3/2)*ln((((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))+6*((a
*x-1)*(a*x+1)*c/a^2)^(1/2)*a*c)/(c*(a^2*x^2-1)/a^2)^(1/2)/a^3/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}x^2}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxi
ma")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))*x^2/(a*x + 1)^2, x)
```

Fricas [A] time = 2.19807, size = 439, normalized size = 3.48

$$\left[-\frac{2\left(a^3x^3-3\,a^2x^2+5\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{6\,a^3},\ -\frac{\left(a^3x^3-3\,a^2x^2+5\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3}{3\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fric
as")
```

```
[Out] [-1/6*(2*(a^3*x^3 - 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*
sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
 - c))/a^3, -1/3*((a^3*x^3 - 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x
^2)) + 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(
a^2*c*x^2 - c)))/a^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx-\int \frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x**2*sqrt(c - c/(a**2*x**2))/(a*x + 1), x) - Integral(a*x**3*sqr
t(c - c/(a**2*x**2))/(a*x + 1), x)
```

Giac [A] time = 1.33203, size = 158, normalized size = 1.25

$$-\frac{1}{6}\left(2\sqrt{a^2cx^2-c}\left(x\left(\frac{x\operatorname{sgn}(x)}{a^2}-\frac{3\operatorname{sgn}(x)}{a^3}\right)+\frac{5\operatorname{sgn}(x)}{a^4}\right)+\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a^3|a|}-\frac{\left(3a\sqrt{c}\log(|c|)+\right.}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac
")
```

```
[Out] -1/6*(2*sqrt(a^2*c*x^2 - c)*(x*(x*sgn(x)/a^2 - 3*sgn(x)/a^3) + 5*sgn(x)/a^4
) + 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^3*ab
s(a)) - (3*a*sqrt(c)*log(abs(c)) + 10*sqrt(-c)*abs(a))*sgn(x)/(a^4*abs(a)))
*abs(a)
```

3.773 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal. Leaf size=99

$$\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{2a\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (3*Sqrt[c - c/(a^2*x^2)]*x)/(2*a) + (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))/(2*
a) + (3*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(2*a*Sqrt[1 - a*x]*Sqrt[1 + a*
x])
```

Rubi [A] time = 0.213416, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6159, 6129, 50, 41, 216}

$$\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{2a\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (3*Sqrt[c - c/(a^2*x^2)]*x)/(2*a) + (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))/(2*
a) + (3*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(2*a*Sqrt[1 - a*x]*Sqrt[1 + a*
x])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
```

```
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{2a}+\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{2a}+\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{2a}+\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{2a}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}x\sin^{-1}(ax)}{2a\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] time = 0.0564665, size = 100, normalized size = 1.01

$$
\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{ax+1}\left(a^2x^2-5ax+4\right)-6\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2) - 6*Sqrt[1 -
a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.109, size = 147, normalized size = 1.5

$$
-\frac{x}{2\,a^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}a^2-\sqrt{c}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)+4\sqrt{c}\ln\left(\frac{1}{\sqrt{c}}\left(\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\sqrt{c}+cx\right)\right)-
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(x*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c^(1/
2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))+4*c^(1/2)*ln((((a*x-1)*(a*x+1)*c
```

```
/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))-4*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a)/(c*(a^
2*x^2-1)/a^2)^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}\,x}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima
")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))*x/(a*x + 1)^2, x)
```

Fricas [A] time = 2.28018, size = 406, normalized size = 4.1

$$\left[-\frac{2\left(a^2x^2-4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{4\,a^2},-\frac{\left(a^2x^2-4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-}}{}\right.}{2\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas
")
```

```
[Out] [-1/4*(2*(a^2*x^2 - 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(c)*log(
2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^2,
-1/2*((a^2*x^2 - 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(-c)*arctan
(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx-\int \frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x*sqrt(c - c/(a**2*x**2))/(a*x + 1), x) - Integral(a*x**2*sqrt(c
 - c/(a**2*x**2))/(a*x + 1), x)
```

Giac [A] time = 1.22271, size = 143, normalized size = 1.44

$$-\frac{1}{4}\left(2\,\sqrt{a^2cx^2-c}\left(\frac{x\mathrm{sgn}\left(x\right)}{a^2}-\frac{4\,\mathrm{sgn}\left(x\right)}{a^3}\right)-\frac{6\,\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}\left(x\right)}{a^2|a|}+\frac{\left(3\,a\sqrt{c}\log\left(|c|\right)+8\,\sqrt{-c}|a|\right)\mathrm{sg}}{a^3|a|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(a^2*c*x^2 - c)*(x*sgn(x)/a^2 - 4*sgn(x)/a^3) - 6*sqrt(c)*log(a
bs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^2*abs(a)) + (3*a*sqrt(c
)*log(abs(c)) + 8*sqrt(-c)*abs(a))*sgn(x)/(a^3*abs(a)))*abs(a)
```

3.774 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=118

$$-x\sqrt{c-\frac{c}{a^2x^2}}-\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x) - (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[
1 - a*x]*Sqrt[1 + a*x]) - (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.290068, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6159, 6129, 102, 157, 41, 216, 92, 208}

$$-x\sqrt{c-\frac{c}{a^2x^2}}-\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x) - (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[
1 - a*x]*Sqrt[1 + a*x]) - (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
```

```
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}\,x+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}\,x+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}\,x-\frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)}{\sqrt{1-ax}\ldots}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}\,x-\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.0798359, size = 80, normalized size = 0.68

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\sqrt{a^2x^2-1}+2\log\left(\sqrt{a^2x^2-1}+ax\right)+\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-Sqrt[-1 + a^2*x^2] + ArcTan[1/Sqrt[-1 + a^2*x^2]
] + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [A] time = 0.116, size = 198, normalized size = 1.7

$$-\frac{x}{a^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(2\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}a^2\sqrt{-\frac{c}{a^2}}-2\sqrt{c}\ln\left(\frac{1}{\sqrt{c}}\left(\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\sqrt{c}+cx\right)\right)a\sqrt{-\frac{c}{a^2}}-\sqrt{-\frac{c}{a^2}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a^2*(-c/a
^2)^(1/2)-2*c^(1/2)*ln((((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))
*a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/
a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2))/(c*(a^2*x^2-1)/a^2)^(1/
2)/a^2/(-c/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1)^2, x)
```

Fricas [A] time = 2.29886, size = 579, normalized size = 4.91

$$\left[-\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+4\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\sqrt{-c}\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},-\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\sqrt{c}\arctan\left(\frac{a\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 4*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - sqrt(-c)*log(-(a^
2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, -(
a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*
c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sq
rt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx-\int \frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-sqrt(c - c/(a**2*x**2))/(a*x + 1), x) - Integral(a*x*sqrt(c - c/
(a**2*x**2))/(a*x + 1), x)
```

Giac [A] time = 1.24405, size = 207, normalized size = 1.75

$$-\left(\frac{2\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}+\frac{2\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}+\frac{\sqrt{a^2cx^2-c}\operatorname{sgn}(x)}{a^2}-\frac{\left(2\sqrt{c}|a|\,\mathrm{arc}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -(2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
^2 + 2*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs
(a)) + sqrt(a^2*c*x^2 - c)*sgn(x)/a^2 - (2*sqrt(c)*abs(a)*arctan(sqrt(-c)/s
qrt(c)) + a*sqrt(c)*log(abs(c)) + sqrt(-c)*abs(a))*sgn(x)/(a^2*abs(a)))*abs
(a)
```

3.775 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=118

$$-\sqrt{c-\frac{c}{a^2x^2}}+\frac{ax\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -Sqrt[c - c/(a^2*x^2)] + (a*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
a*x]*Sqrt[1 + a*x]) + (2*a*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.417812, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6159, 6129, 98, 157, 41, 216, 92, 208}

$$-\sqrt{c-\frac{c}{a^2x^2}}+\frac{ax\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] -Sqrt[c - c/(a^2*x^2)] + (a*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
a*x]*Sqrt[1 + a*x]) + (2*a*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^2}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^2\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{2a-a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}+\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(2a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\operatorname{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-\right.}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\sqrt{c-\frac{c}{a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.0663151, size = 83, normalized size = 0.7

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}+ax\log\left(\sqrt{a^2x^2-1}+ax\right)+2ax\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2] + 2*a*x*ArcTan[1/Sqrt[-1 + a^2
*x^2]] + a*x*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.115, size = 307, normalized size = 2.6

$$-\frac{1}{ac}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}x^2a^3c-a^3\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}-2\sqrt{-\frac{c}{a^2}}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}xa^2c+2c^{3/}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/a*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*
x^2*a^3*c-a^3*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)-2*(-c/a^2)^(1/2)*((a
*x-1)*(a*x+1)*c/a^2)^(1/2)*x*a^2*c+2*c^(3/2)*(-c/a^2)^(1/2)*ln((((a*x-1)*(a
*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*x*a+2*(c*(a^2*x^2-1)/a^2)^(1/2)*c*
x*a^2*(-c/a^2)^(1/2)-c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^
2)^(1/2)*x*a+2*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2)
*x*c^2)/(c*(a^2*x^2-1)/a^2)^(1/2)/c/(-c/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="maxima
")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^2*x), x)
```

Fricas [A] time = 2.24231, size = 554, normalized size = 4.69

$$\left[\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{x^2}\right)-\sqrt{\frac{a^2cx^2-c}{a^2x^2}},-2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x\sqrt{}}{a^2cx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="fricas
")
```

```
[Out] [sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^
2 - c)) + sqrt(-c)*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a
^2*x^2)) - 2*c)/x^2) - sqrt((a^2*c*x^2 - c)/(a^2*x^2)), -2*sqrt(c)*arctan(a
*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 1/2*sqrt(c)*l
```

```
og(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) - s
qrt((a^2*c*x^2 - c)/(a^2*x^2))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax^2+x}\,dx-\int \frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax^2+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x,x)
```

```
[Out] -Integral(-sqrt(c - c/(a**2*x**2))/(a*x**2 + x), x) - Integral(a*x*sqrt(c -
 c/(a**2*x**2))/(a*x**2 + x), x)
```

Giac [A] time = 1.41336, size = 170, normalized size = 1.44

$$\left(\frac{4\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a}+\frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{|a|}-\frac{2\,c^{\frac{3}{2}}\operatorname{sgn}(x)}{\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^2+c\right)|a|}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="giac")
```

```
[Out] (4*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
+ sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/abs(a) - 2*
c^(3/2)*sgn(x)/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)*abs(a)))*abs(
a)
```

3.776 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=112

$$\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}-\frac{(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2x}-\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{2\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (3*a*Sqrt[c - c/(a^2*x^2)])/2 - (Sqrt[c - c/(a^2*x^2)]*(1 - a*x))/(2*x) - (
3*a^2*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(2*Sqrt
[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.390893, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6159, 6129, 94, 92, 208}

$$\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}-\frac{(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2x}-\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{2\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] (3*a*Sqrt[c - c/(a^2*x^2)])/2 - (Sqrt[c - c/(a^2*x^2)]*(1 - a*x))/(2*x) - (
3*a^2*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(2*Sqrt
[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^3}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^3\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{2x} - \frac{\left(3a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1-ax}}{x^2\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{2x} + \frac{\left(3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{2x} - \frac{\left(3a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ \right.}{2\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{2x} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{2\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.0652253, size = 78, normalized size = 0.7

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left((4ax-1)\sqrt{a^2x^2-1}+3a^2x^2\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{2x\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*((-1 + 4*a*x)*Sqrt[-1 + a^2*x^2] + 3*a^2*x^2*ArcTan[
1/Sqrt[-1 + a^2*x^2]]))/(2*x*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.124, size = 348, normalized size = 3.1

$$-\frac{1}{2cx}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-4\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}x^3a^3c+4\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}xa^3+4c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)
^(1/2)*x^3*a^3*c+4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a^3+4*c^(3/2)
*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*x^2*a-4*c^(3/2)*(-c
/a^2)^(1/2)*ln((((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*x^2*a+4
*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^2*a^2*c-3*(-c/a^2)^(1/2)*(c
*(a^2*x^2-1)/a^2)^(1/2)*x^2*a^2*c-a^2*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1
/2)-3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2)*x^2*c^2)
/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="maxi
ma")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^2*x^2), x)
```

Fricas [A] time = 2.22041, size = 392, normalized size = 3.5

$$\left[\frac{3\,a\sqrt{-c}x\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(4\,ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,x},\ \frac{3\,a\sqrt{c}x\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+(4\,ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="fric
as")
```

```
[Out] [1/4*(3*a*sqrt(-c)*x*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)) - 2*c)/x^2) + 2*(4*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x,
1/2*(3*a*sqrt(c)*x*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*
c*x^2 - c)) + (4*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax^3+x^2}\,dx-\int \frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax^3+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**2,x)
```

```
[Out] -Integral(-sqrt(c - c/(a**2*x**2))/(a*x**3 + x**2), x) - Integral(a*x*sqrt(
c - c/(a**2*x**2))/(a*x**3 + x**2), x)
```

Giac [B] time = 1.60445, size = 263, normalized size = 2.35

$$-\left(3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)-\frac{\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^3ac\operatorname{sgn}(x)+4\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^2c^{\frac{3}{2}}|a|\operatorname{sgn}(x}{\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="giac
")
```

```
[Out] -(3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) -
 ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*a*c*sgn(x) + 4*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^2*c^(3/2)*abs(a)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x
^2 - c))*a*c^2*sgn(x) + 4*c^(5/2)*abs(a)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^2 + c)^2*a))*abs(a)
```

3.777 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=140

$$-a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3x}-\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -(a^2*Sqrt[c - c/(a^2*x^2)]) + (a*Sqrt[c - c/(a^2*x^2)]*(1 - a*x))/(3*x) -
(Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(3*x^2) + (a^3*Sqrt[c - c/(a^2*x^2)]*x*
ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.393428, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6159, 6129, 96, 94, 92, 208}

$$-a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3x}-\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] -(a^2*Sqrt[c - c/(a^2*x^2)]) + (a*Sqrt[c - c/(a^2*x^2)]*(1 - a*x))/(3*x) -
(Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(3*x^2) + (a^3*Sqrt[c - c/(a^2*x^2)]*x*
ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^4}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^4\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^3\sqrt{1+ax}}\,dx}{3\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{3x}-\frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2}+\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1-ax}}{x^2\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{3x}-\frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2}-\frac{\left(a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+}}}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{3x}-\frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2}+\frac{\left(a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\operatorname{Subst}\left(\int \frac{}{a-}\right.}{\sqrt{1-ax}}\\
&= -a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{3x}-\frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2}+\frac{a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-}\right.}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] time = 0.0730487, size = 86, normalized size = 0.61

$$
-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(5a^2x^2-3ax+1\right)+3a^3x^3\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{3x^2\sqrt{a^2x^2-1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1 - 3*a*x + 5*a^2*x^2) + 3*a^3
*x^3*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(3*x^2*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.129, size = 378, normalized size = 2.7

$$\frac{a}{3\,c x^{2}}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\left(-6\sqrt{-\frac{c}{a^{2}}}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}x^{4}a^{3}c+6\sqrt{-\frac{c}{a^{2}}}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}\right)^{3/2}x^{2}a^{3}+6\,c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x)
```

```
[Out] 1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^2*a*(-6*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a
^2)^(1/2)*x^4*a^3*c+6*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2*a^3+6*c^
(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*x^3*a-6*c^(3/2
)*(-c/a^2)^(1/2)*ln((((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*x^
3*a+6*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^3*a^2*c-3*(-c/a^2)^(1/
2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^3*a^2*c-3*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)
^(3/2)*x*a^2-3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2)
*x^3*c^2+a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(-c/a^2)^(1/2)/(c*(a^2
*x^2-1)/a^2)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^{2}x^{2}-1\right)\sqrt{c-\frac{c}{a^{2}x^{2}}}}{\left(ax+1\right)^{2}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="maxi
ma")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^2*x^3), x)
```

Fricas [A] time = 2.35167, size = 441, normalized size = 3.15

$$\left[\frac{3\,a^{2}\sqrt{-c}x^{2}\log\left(-\frac{a^{2}cx^{2}+2\,a\sqrt{-c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{x^{2}}\right)-2\left(5\,a^{2}x^{2}-3\,ax+1\right)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{6\,x^{2}},-\frac{3\,a^{2}\sqrt{c}x^{2}\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)+\left(5\,a^{2}x^{2}\right.}{3\,x^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="fric
as")
```

```
[Out] [1/6*(3*a^2*sqrt(-c)*x^2*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)) - 2*c)/x^2) - 2*(5*a^2*x^2 - 3*a*x + 1)*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)))/x^2, -1/3*(3*a^2*sqrt(c)*x^2*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^
2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (5*a^2*x^2 - 3*a*x + 1)*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)))/x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax^4+x^3}\,dx-\int \frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax^4+x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**3,x)
```

```
[Out] -Integral(-sqrt(c - c/(a**2*x**2))/(a*x**4 + x**3), x) - Integral(a*x*sqrt(
c - c/(a**2*x**2))/(a*x**4 + x**3), x)
```

Giac [A] time = 1.98771, size = 312, normalized size = 2.23

$$\frac{2}{3}\left(3\,a\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)-\frac{3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^5 ac\operatorname{sgn}(x)+3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^4 c^{\frac{3}{2}}|a|\text{sgn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="giac
")
```

```
[Out] 2/3*(3*a*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn
(x) - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a*c*sgn(x) + 3*(sqrt(a^2*c
)*x - sqrt(a^2*c*x^2 - c))^4*c^(3/2)*abs(a)*sgn(x) + 12*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*c^(5/2)*abs(a)*sgn(x) - 3*(sqrt(a^2*c)*x - sqrt(a^2*c*
x^2 - c))*a*c^3*sgn(x) + 5*c^(7/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^2 + c)^3)*abs(a)
```

3.778 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=156

$$\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (4*a^3*Sqrt[c - c/(a^2*x^2)])/3 - Sqrt[c - c/(a^2*x^2)]/(4*x^3) + (2*a*Sqrt
[c - c/(a^2*x^2)])/(3*x^2) - (7*a^2*Sqrt[c - c/(a^2*x^2)])/(8*x) - (7*a^4*S
qrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(8*Sqrt[1 - a*
x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.415311, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6159, 6129, 98, 151, 12, 92, 208}

$$\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x^4),x]
```


```
[Out] (4*a^3*Sqrt[c - c/(a^2*x^2)])/3 - Sqrt[c - c/(a^2*x^2)]/(4*x^3) + (2*a*Sqrt
[c - c/(a^2*x^2)])/(3*x^2) - (7*a^2*Sqrt[c - c/(a^2*x^2)])/(8*x) - (7*a^4*S
qrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(8*Sqrt[1 - a*
x]*Sqrt[1 + a*x])
```


Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^5}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^5\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{8a-7a^2x}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{21a^2-16a^3x}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx}{12\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{32a^3-21a^4x}{x^2\sqrt{1-ax}\sqrt{1+ax}}\,dx}{24\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{21a^4}{x\sqrt{1-ax}\sqrt{1+a}}}{24\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{\left(7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{}}}{8\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{\left(7a^5\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int\right.}{8\sqrt{1-a}}\\
&= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1}\right.}{8\sqrt{1-ax}\sqrt{1+}}
\end{aligned}
$$

Mathematica [A] time = 0.0788166, size = 94, normalized size = 0.6

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(32a^3x^3-21a^2x^2+16ax-6\right)+21a^4x^4\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{24x^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-6 + 16*a*x - 21*a^2*x^2 + 32*a
^3*x^3) + 21*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(24*x^3*Sqrt[-1 + a^2*x
^2])
```

Maple [B] time = 0.126, size = 410, normalized size = 2.6

$$-\frac{a^2}{24\,cx^3}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-48\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}x^5a^3c+48\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}x^3a^3+48\sqrt{-\frac{c}{a^2}}c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x)
```

```
[Out] -1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^3*a^2*(-48*(-c/a^2)^(1/2)*(c*(a^2*x^2
-1)/a^2)^(1/2)*x^5*a^3*c+48*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^3*a^
3+48*(-c/a^2)^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^4*a-4
8*(-c/a^2)^(1/2)*c^(3/2)*ln((((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^(
1/2))*x^4*a+48*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^4*a^2*c-21*(-
c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^4*a^2*c-27*(-c/a^2)^(1/2)*(c*(a^2*
x^2-1)/a^2)^(3/2)*x^2*a^2-21*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)
*a^2-c)/x/a^2)*x^4*c^2+16*a*(c*(a^2*x^2-1)/a^2)^(3/2)*x*(-c/a^2)^(1/2)-6*(c
*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^
(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="maxi
ma")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^2*x^4), x)
```

Fricas [A] time = 2.31462, size = 486, normalized size = 3.12

$$\left[\frac{21\,a^{3}\sqrt{-c}x^{3}\log\left(-\frac{a^{2}cx^{2}-2\,a\sqrt{-c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{x^{2}}\right)+2\left(32\,a^{3}x^{3}-21\,a^{2}x^{2}+16\,ax-6\right)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{48\,x^{3}},\ \frac{21\,a^{3}\sqrt{c}x^{3}\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="fric
as")
```

```
[Out] [1/48*(21*a^3*sqrt(-c)*x^3*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/x^3, 1/24*(21*a^3*sqrt(c)*x^3*arctan(a*sqrt(c
)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (32*a^3*x^3 - 21*a^2
*x^2 + 16*a*x - 6)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}}{ax^{5}+x^{4}}\,dx-\int \frac{ax\sqrt{c-\frac{c}{a^{2}x^{2}}}}{ax^{5}+x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**4,x)
```

```
[Out] -Integral(-sqrt(c - c/(a**2*x**2))/(a*x**5 + x**4), x) - Integral(a*x*sqrt(
c - c/(a**2*x**2))/(a*x**5 + x**4), x)
```

Giac [B] time = 3.49046, size = 427, normalized size = 2.74

$$-\frac{1}{12}\left(21\,a^{2}\sqrt{c}\arctan\left(-\frac{\sqrt{a^{2}c}x-\sqrt{a^{2}cx^{2}-c}}{\sqrt{c}}\right)\mathrm{sgn}\left(x\right)-\frac{21\left(\sqrt{a^{2}c}x-\sqrt{a^{2}cx^{2}-c}\right)^{7}a^{2}c\,\mathrm{sgn}\left(x\right)+45\left(\sqrt{a^{2}c}x-\sqrt{a^{2}cx^{2}-c}\right)^{5}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="giac
")
```

```
[Out] -1/12*(21*a^2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x) - (21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^2*c*sgn(x) + 45*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a^2*c^2*sgn(x) + 96*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^4*a*c^(5/2)*abs(a)*sgn(x) - 45*(sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - c))^3*a^2*c^3*sgn(x) + 128*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))
^2*a*c^(7/2)*abs(a)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^2*c
^4*sgn(x) + 32*a*c^(9/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
c))^2 + c)^4)*abs(a)
```

3.779 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=181

$$-\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}-\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+}\right.}{4\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (-6*a^4*Sqrt[c - c/(a^2*x^2)])/5 - Sqrt[c - c/(a^2*x^2)]/(5*x^4) + (a*Sqrt[
c - c/(a^2*x^2)])/(2*x^3) - (3*a^2*Sqrt[c - c/(a^2*x^2)])/(5*x^2) + (3*a^3*
Sqrt[c - c/(a^2*x^2)])/(4*x) + (3*a^5*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[
1 - a*x]*Sqrt[1 + a*x]])/(4*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.440128, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6159, 6129, 98, 151, 12, 92, 208}

$$-\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}-\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+}\right.}{4\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x^5),x]
```

```
[Out] (-6*a^4*Sqrt[c - c/(a^2*x^2)])/5 - Sqrt[c - c/(a^2*x^2)]/(5*x^4) + (a*Sqrt[
c - c/(a^2*x^2)])/(2*x^3) - (3*a^2*Sqrt[c - c/(a^2*x^2)])/(5*x^2) + (3*a^3*
Sqrt[c - c/(a^2*x^2)])/(4*x) + (3*a^5*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[
1 - a*x]*Sqrt[1 + a*x]])/(4*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^6}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{x^6\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{10a-9a^2x}{x^5\sqrt{1-ax}\sqrt{1+ax}}\,dx}{5\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{36a^2-30a^3x}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{20\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{90a^3-72a^4x}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx}{60\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{144a^4-90}{x^2\sqrt{1-ax}\sqrt{}}}{120\sqrt{1-ax}\sqrt{1+a}}$$

$$= -\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} - \frac{\left(\sqrt{c-\right.}{12}$$

$$= -\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} - \left(3a^5\sqrt{\right.$$

$$= -\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} + \left(3a^6\sqrt{\right.$$

$$= -\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} + 3a^5\sqrt{c$$

Mathematica [A] time = 0.0838015, size = 102, normalized size = 0.56

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(24a^4x^4-15a^3x^3+12a^2x^2-10ax+4\right)+15a^5x^5\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{20x^4\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x^5),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(4 - 10*a*x + 12*a^2*x^2 - 15*a
^3*x^3 + 24*a^4*x^4) + 15*a^5*x^5*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(20*x^4*Sq
rt[-1 + a^2*x^2])
```

Maple [B] time = 0.126, size = 447, normalized size = 2.5

$$\frac{a^2}{20\,cx^4}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-40\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}x^6a^4c+40\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}x^4a^4+40\sqrt{-\frac{c}{a^2}}c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x)
```

```
[Out] 1/20*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^4*a^2*(-40*(-c/a^2)^(1/2)*(c*(a^2*x^2-
1)/a^2)^(1/2)*x^6*a^4*c+40*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^4*a^4
+40*(-c/a^2)^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^5*a^2-
40*(-c/a^2)^(1/2)*c^(3/2)*ln((((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)+c*x)/c^
(1/2))*x^5*a^2+40*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^5*a^3*c-15
*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^5*a^3*c-25*(-c/a^2)^(1/2)*(c*(a
^2*x^2-1)/a^2)^(3/2)*x^3*a^3-15*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1
/2)*a^2-c)/x/a^2)*x^5*a*c^2+16*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2
*a^2-10*a*(c*(a^2*x^2-1)/a^2)^(3/2)*x*(-c/a^2)^(1/2)+4*(c*(a^2*x^2-1)/a^2)^
(3/2)*(-c/a^2)^(1/2))/(c*(a^2*x^2-1)/a^2)^(1/2)/c/(-c/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}}{\left(ax+1\right)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="maxi
ma")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^2*x^5), x)
```

Fricas [A] time = 2.29492, size = 522, normalized size = 2.88

$$\left[\frac{15\,a^4\sqrt{-c}x^4\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(24\,a^4x^4-15\,a^3x^3+12\,a^2x^2-10\,ax+4\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{40\,x^4},-\frac{15\,a^4\sqrt{c}x^4\arctan\left(\cdots\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="fric
as")
```

```
[Out] [1/40*(15*a^4*sqrt(-c)*x^4*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(24*a^4*x^4 - 15*a^3*x^3 + 12*a^2*x^2 - 10
*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4, -1/20*(15*a^4*sqrt(c)*x^4*a
rctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (24*a^
4*x^4 - 15*a^3*x^3 + 12*a^2*x^2 - 10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)))/x^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax^6+x^5}\,dx-\int \frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax^6+x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**5,x)
```

```
[Out] -Integral(-sqrt(c - c/(a**2*x**2))/(a*x**6 + x**5), x) - Integral(a*x*sqrt(
c - c/(a**2*x**2))/(a*x**6 + x**5), x)
```

Giac [B] time = 3.69881, size = 489, normalized size = 2.7

$$\frac{1}{10}\left(15\,a^3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}\left(x\right)-\frac{15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^9a^3c\mathrm{sgn}\left(x\right)+70\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="giac
")
```

```
[Out] 1/10*(15*a^3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x) - (15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*a^3*c*sgn(x) + 70*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^3*c^2*sgn(x) + 40*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^6*a^2*c^(5/2)*abs(a)*sgn(x) + 200*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^4*a^2*c^(7/2)*abs(a)*sgn(x) - 70*(sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^3*a^3*c^4*sgn(x) + 120*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2
*a^2*c^(9/2)*abs(a)*sgn(x) - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^3*c
^5*sgn(x) + 24*a^2*c^(11/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - c))^2 + c)^5)*abs(a)
```

3.780 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

Optimal. Leaf size=187

$$\frac{ax^5\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-a^2x^2}}-\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{2x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-a^2x^2}}-\frac{4x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-a^2x^2}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^3\sqrt{1-a^2x^2}}$$

```
[Out] (-4*Sqrt[c - c/(a^2*x^2)]*x^2)/(a^2*Sqrt[1 - a^2*x^2]) + (2*Sqrt[c - c/(a^2
*x^2)]*x^3)/(a*Sqrt[1 - a^2*x^2]) - (Sqrt[c - c/(a^2*x^2)]*x^4)/Sqrt[1 - a^
2*x^2] + (a*Sqrt[c - c/(a^2*x^2)]*x^5)/(4*Sqrt[1 - a^2*x^2]) + (4*Sqrt[c -
c/(a^2*x^2)]*x*Log[1 + a*x])/(a^3*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.265933, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$\frac{ax^5\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-a^2x^2}}-\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{2x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-a^2x^2}}-\frac{4x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-a^2x^2}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-4*Sqrt[c - c/(a^2*x^2)]*x^2)/(a^2*Sqrt[1 - a^2*x^2]) + (2*Sqrt[c - c/(a^2
*x^2)]*x^3)/(a*Sqrt[1 - a^2*x^2]) - (Sqrt[c - c/(a^2*x^2)]*x^4)/Sqrt[1 - a^
2*x^2] + (a*Sqrt[c - c/(a^2*x^2)]*x^5)/(4*Sqrt[1 - a^2*x^2]) + (4*Sqrt[c -
c/(a^2*x^2)]*x*Log[1 + a*x])/(a^3*Sqrt[1 - a^2*x^2])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{-3\tanh^{-1}(ax)}x^2\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{x^2(1-ax)^2}{1+ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(-\frac{4}{a^2}+\frac{4x}{a}-3x^2+ax^3+\frac{4}{a^2(1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4\sqrt{c-\frac{c}{a^2x^2}}x^2}{a^2\sqrt{1-a^2x^2}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x^3}{a\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^4}{\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}x^5}{4\sqrt{1-a^2x^2}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}x\log(}{a^3\sqrt{1-a^2x}}$$

Mathematica [A] time = 0.0494406, size = 70, normalized size = 0.37

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4x}{a^2}+\frac{4\log(ax+1)}{a^3}+\frac{ax^4}{4}+\frac{2x^2}{a}-x^3\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*((-4*x)/a^2 + (2*x^2)/a - x^3 + (a*x^4)/4 + (4*Log
[1 + a*x])/a^3))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.138, size = 85, normalized size = 0.5

$$-\frac{x\left(x^4a^4-4x^3a^3+8a^2x^2-16ax+16\ln(ax+1)\right)}{\left(4a^2x^2-4\right)a^3}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/4*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(x^4*a^4-4*x^3*a^3+
8*a^2*x^2-16*a*x+16*ln(a*x+1))/(a^2*x^2-1)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}x^3}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))*x^3/(a*x + 1)^3, x)
```

Fricas [A] time = 2.51729, size = 873, normalized size = 4.67

$$\left[\frac{8\left(a^2x^2-1\right)\sqrt{-c}\log\left(\frac{a^6cx^6+4a^5cx^5+5a^4cx^4-4a^2cx^2-4acx+\left(a^5x^5+4a^4x^4+6a^3x^3+4a^2x^2\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{a^4x^4+2a^3x^3-2ax-1}\right)-\left(a^5x^5-4a^4x^4+8a\right.}{4\left(a^6x^2-a^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/4*(8*(a^2*x^2 - 1)*sqrt(-c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 -
 4*a^2*c*x^2 - 4*a*c*x + (a^5*x^5 + 4*a^4*x^4 + 6*a^3*x^3 + 4*a^2*x^2)*sqrt
(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/(a^4*x^4 + 2
*a^3*x^3 - 2*a*x - 1)) - (a^5*x^5 - 4*a^4*x^4 + 8*a^3*x^3 - 16*a^2*x^2)*sqr
t(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^2 - a^4), -1/4*(16*
(a^2*x^2 - 1)*sqrt(c)*arctan((a^2*x^2 + 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(
c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 + 2*a^2*c*x^2 - a*c*x - 2*c))
 + (a^5*x^5 - 4*a^4*x^4 + 8*a^3*x^3 - 16*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((
a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^2 - a^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}x^3}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))*x^3/(a*x + 1)^3, x)
```

3.781 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal. Leaf size=152

$$\frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-a^2x^2}}-\frac{3x^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}+\frac{4x^2\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^2\sqrt{1-a^2x^2}}$$

```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x^2)/(a*Sqrt[1 - a^2*x^2]) - (3*Sqrt[c - c/(a^2*x^
2)]*x^3)/(2*Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - c/(a^2*x^2)]*x^4)/(3*Sqrt[1 -
a^2*x^2]) - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/(a^2*Sqrt[1 - a^2*x^2]
)
```

Rubi [A] time = 0.240186, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 77}

$$\frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-a^2x^2}}-\frac{3x^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}+\frac{4x^2\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x^2)/(a*Sqrt[1 - a^2*x^2]) - (3*Sqrt[c - c/(a^2*x^
2)]*x^3)/(2*Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - c/(a^2*x^2)]*x^4)/(3*Sqrt[1 -
a^2*x^2]) - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/(a^2*Sqrt[1 - a^2*x^2]
)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{-3\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{x(1-ax)^2}{1+ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(\frac{4}{a}-3x+ax^2-\frac{4}{a(1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{4\sqrt{c-\frac{c}{a^2x^2}}x^2}{a\sqrt{1-a^2x^2}}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^3}{2\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}x^4}{3\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}x\log(1+ax)}{a^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0462516, size = 64, normalized size = 0.42

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4\log(ax+1)}{a^2}+\frac{ax^3}{3}+\frac{4x}{a}-\frac{3x^2}{2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*((4*x)/a - (3*x^2)/2 + (a*x^3)/3 - (4*Log[1 + a*x]
)/a^2))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.139, size = 78, normalized size = 0.5

$$\frac{x\left(-2x^3a^3+9a^2x^2-24ax+24\ln(ax+1)\right)}{\left(6a^2x^2-6\right)a^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(-2*x^3*a^3+9*a^2*x^
2-24*a*x+24*ln(a*x+1))/(a^2*x^2-1)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}x^2}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))*x^2/(a*x + 1)^3, x)
```

Fricas [A] time = 2.45616, size = 846, normalized size = 5.57

$$\left[\frac{12\left(a^2x^2-1\right)\sqrt{-c}\log\left(\frac{a^6cx^6+4a^5cx^5+5a^4cx^4-4a^2cx^2-4acx-\left(a^5x^5+4a^4x^4+6a^3x^3+4a^2x^2\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{a^4x^4+2a^3x^3-2ax-1}\right)-\left(2a^4x^4-9a^3x^3\right.}{6\left(a^5x^2-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/6*(12*(a^2*x^2 - 1)*sqrt(-c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4
- 4*a^2*c*x^2 - 4*a*c*x - (a^5*x^5 + 4*a^4*x^4 + 6*a^3*x^3 + 4*a^2*x^2)*sqr
t(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/(a^4*x^4 +
2*a^3*x^3 - 2*a*x - 1)) - (2*a^4*x^4 - 9*a^3*x^3 + 24*a^2*x^2)*sqrt(-a^2*x^
2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*x^2 - a^3), 1/6*(24*(a^2*x^2 -
 1)*sqrt(c)*arctan((a^2*x^2 + 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a
^2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 + 2*a^2*c*x^2 - a*c*x - 2*c)) - (2*a^4*
x^4 - 9*a^3*x^3 + 24*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)))/(a^5*x^2 - a^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))*x^2/(a*x + 1)^3, x)
```

3.782 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal. Leaf size=113

$$\frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a\sqrt{1-a^2x^2}}$$

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (a*Sqrt[c - c/(a^2*x^2)]
*x^3)/(2*Sqrt[1 - a^2*x^2]) + (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/(a*S
qrt[1 - a^2*x^2])
```

Rubi [A] time = 0.175669, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6160, 6140, 43}

$$\frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (a*Sqrt[c - c/(a^2*x^2)]
*x^3)/(2*Sqrt[1 - a^2*x^2]) + (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/(a*S
qrt[1 - a^2*x^2])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^2}{1+ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(-3+ax+\frac{4}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^2}{\sqrt{1-a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}x^3}{2\sqrt{1-a^2x^2}} + \frac{4\sqrt{c-\frac{c}{a^2x^2}}x\log(1+ax)}{a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0328475, size = 54, normalized size = 0.48

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax^2}{2}+\frac{4\log(ax+1)}{a}-3x\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-3*x + (a*x^2)/2 + (4*Log[1 + a*x])/a))/Sqrt[1 -
a^2*x^2]
```

Maple [A] time = 0.148, size = 69, normalized size = 0.6

$$-\frac{x\left(a^2x^2-6\,ax+8\,\ln(ax+1)\right)}{\left(2\,a^2x^2-2\right)a}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(a^2*x^2-6*a*x+8*ln
(a*x+1))/(a^2*x^2-1)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}x}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))*x/(a*x + 1)^3, x)
```

Fricas [A] time = 2.57151, size = 805, normalized size = 7.12

$$\left[\frac{4\left(a^2x^2-1\right)\sqrt{-c}\log\left(\frac{a^6cx^6+4a^5cx^5+5a^4cx^4-4a^2cx^2-4acx+\left(a^5x^5+4a^4x^4+6a^3x^3+4a^2x^2\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{a^4x^4+2a^3x^3-2ax-1}\right)-\left(a^3x^3-6a^2x^2\right)\sqrt{-a}}{2\left(a^4x^2-a^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/2*(4*(a^2*x^2 - 1)*sqrt(-c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 -
 4*a^2*c*x^2 - 4*a*c*x + (a^5*x^5 + 4*a^4*x^4 + 6*a^3*x^3 + 4*a^2*x^2)*sqrt
(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/(a^4*x^4 + 2
*a^3*x^3 - 2*a*x - 1)) - (a^3*x^3 - 6*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2
*c*x^2 - c)/(a^2*x^2)))/(a^4*x^2 - a^2), -1/2*(8*(a^2*x^2 - 1)*sqrt(c)*arct
an((a^2*x^2 + 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a
^2*x^2))/(a^3*c*x^3 + 2*a^2*c*x^2 - a*c*x - 2*c)) + (a^3*x^3 - 6*a^2*x^2)*s
qrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^2 - a^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*(-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x
)))/(a*x + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}\,x}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))*x/(a*x + 1)^3, x)
```

3.783 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=106

$$\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] (a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2] - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[1
 - a^2*x^2]
```

Rubi [A] time = 0.169183, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 72}

$$\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2] - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[1
 - a^2*x^2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^2}{x(1+ax)}dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{a\sqrt{c-\frac{c}{a^2x^2}}x^2}{\sqrt{1-a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x\log(x)}{\sqrt{1-a^2x^2}} - \frac{4\sqrt{c-\frac{c}{a^2x^2}}x\log(1+ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0250886, size = 45, normalized size = 0.42

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}(ax-4\log(ax+1)+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(a*x + Log[x] - 4*Log[1 + a*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.141, size = 60, normalized size = 0.6

$$-\frac{x(ax+\ln(x)-4\ln(ax+1))}{a^2x^2-1}\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(a*x+ln(x)-4*ln(a*x+1))*(-a^2*x^2+1)^(1/2)
/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-a^2x^2+1)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/(a*x + 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}(ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2x^2+2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2
*x^2 + 2*a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))
)/(a*x + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/(a*x + 1)^3, x)
```

3.784 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=106

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{3ax\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{4ax\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[c - c/(a^2*x^2)]*x*L
og[x])/Sqrt[1 - a^2*x^2] + (4*a*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[
1 - a^2*x^2]
```

Rubi [A] time = 0.26123, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{3ax\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{4ax\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[c - c/(a^2*x^2)]*x*L
og[x])/Sqrt[1 - a^2*x^2] + (4*a*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[
1 - a^2*x^2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^2}{x^2(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}} + \frac{4a\sqrt{c-\frac{c}{a^2x^2}}\,x\log(1+ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0326525, size = 48, normalized size = 0.45

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(-3ax\log(x)+4ax\log(ax+1)-1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1 - 3*a*x*Log[x] + 4*a*x*Log[1 + a*x]))/Sqrt[1 - a
^2*x^2]
```

Maple [A] time = 0.145, size = 62, normalized size = 0.6

$$\frac{3\,a\ln(x)\,x-4\,ax\ln(ax+1)+1}{a^2x^2-1}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x)
```

```
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(-a^2*x^2+1)^(1/2)*(3*a*ln(x)*x-4*a*x*ln(a*x+
1)+1)/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="
maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}(ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2x^3+2\,ax^2+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="
fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2
*x^3 + 2*a*x^2 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))
)/(x*(a*x + 1)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="
giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x), x)
```

3.785 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=147

$$\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}+\frac{4a^2x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^2x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] (3*a*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(2*x*
Sqrt[1 - a^2*x^2]) + (4*a^2*Sqrt[c - c/(a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x^
2] - (4*a^2*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.268698, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}+\frac{4a^2x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^2x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x^2),x]
```


```
[Out] (3*a*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(2*x*
Sqrt[1 - a^2*x^2]) + (4*a^2*Sqrt[c - c/(a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x^
2] - (4*a^2*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[1 - a^2*x^2]
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^3}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^2}{x^3(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{1}{x^3}-\frac{3a}{x^2}+\frac{4a^2}{x}-\frac{4a^3}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}} + \frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}} - \frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\log(1+ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.040549, size = 63, normalized size = 0.43

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(4a^2\log(x)-4a^2\log(ax+1)+\frac{3a}{x}-\frac{1}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-1/(2*x^2) + (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1
 + a*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.151, size = 78, normalized size = 0.5

$$-\frac{8\,a^2\ln(x)\,x^2-8\,\ln(ax+1)\,a^2x^2+6\,ax-1}{2\,x\left(a^2x^2-1\right)}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-a^2*x^2+1)^(1/2)*(8*a^2*ln(x)*x^2-8*
ln(a*x+1)*a^2*x^2+6*a*x-1)/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm
="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x^2), x)
```

Fricas [A] time = 2.70146, size = 1025, normalized size = 6.97

$$\left[\frac{4\left(a^3x^3-ax\right)\sqrt{-c}\log\left(\frac{4a^5cx^5+\left(2a^6+4a^5+6a^4+4a^3+a^2\right)cx^6+\left(4a^4-4a^3-6a^2-4a-1\right)cx^4-5a^2cx^2-4acx-\left(4a^4x^4+6a^3x^3-\left(4a^4+6a^3+4a^2+a\right)x^5+4\right.}{a^4x^6+2a^3x^5-2ax^3-x^2}\right.}{2\left(a^2x^3-x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm
="fricas")
```

```
[Out] [1/2*(4*(a^3*x^3 - a*x)*sqrt(-c)*log((4*a^5*c*x^5 + (2*a^6 + 4*a^5 + 6*a^4
+ 4*a^3 + a^2)*c*x^6 + (4*a^4 - 4*a^3 - 6*a^2 - 4*a - 1)*c*x^4 - 5*a^2*c*x^
2 - 4*a*c*x - (4*a^4*x^4 + 6*a^3*x^3 - (4*a^4 + 6*a^3 + 4*a^2 + a)*x^5 + 4*
a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
- c)/(a^4*x^6 + 2*a^3*x^5 - 2*a*x^3 - x^2)) + sqrt(-a^2*x^2 + 1)*((6*a - 1)
*x^2 - 6*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^3 - x), -1/2*(8*(
a^3*x^3 - a*x)*sqrt(c)*arctan(-(2*a^2*x^2 + (2*a^3 + 2*a^2 + a)*x^3 + a*x)*
sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(2*a^3*c*x^3 - (
2*a^3 + a^2)*c*x^4 + (a^2 + 2*a + 1)*c*x^2 - 2*a*c*x - c)) - sqrt(-a^2*x^2
+ 1)*((6*a - 1)*x^2 - 6*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^3
- x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm
="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x^2), x)
```

3.786 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=186

$$-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}-\frac{4a^3x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{4a^3x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] (-4*a^2*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(3
*x^2*Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - c/(a^2*x^2)])/(2*x*Sqrt[1 - a^2*x^2
]) - (4*a^3*Sqrt[c - c/(a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x^2] + (4*a^3*Sqrt
[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.281378, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}-\frac{4a^3x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{4a^3x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] (-4*a^2*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(3
*x^2*Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - c/(a^2*x^2)])/(2*x*Sqrt[1 - a^2*x^2
]) - (4*a^3*Sqrt[c - c/(a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x^2] + (4*a^3*Sqrt
[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[1 - a^2*x^2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^4}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^2}{x^4(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(\frac{1}{x^4}-\frac{3a}{x^3}+\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}} + \frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}} - \frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}x\log(x)}{\sqrt{1-a^2x^2}} + \frac{4a^3\sqrt{c-\frac{c}{a^2}}}{\sqrt{1}}$$

Mathematica [A] time = 0.0444212, size = 73, normalized size = 0.39

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4a^2}{x}-4a^3\log(x)+4a^3\log(ax+1)+\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-1/(3*x^3) + (3*a)/(2*x^2) - (4*a^2)/x - 4*a^3*Lo
g[x] + 4*a^3*Log[1 + a*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.15, size = 86, normalized size = 0.5

$$\frac{24\,a^3\ln(x)\,x^3-24\,a^3x^3\ln(ax+1)+24\,a^2x^2-9\,ax+2}{6\,x^2\left(a^2x^2-1\right)}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x)
```

```
[Out] 1/6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^2*(-a^2*x^2+1)^(1/2)*(24*a^3*ln(x)*x^3-
24*a^3*x^3*ln(a*x+1)+24*a^2*x^2-9*a*x+2)/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm
="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x^3), x)
```

Fricas [A] time = 2.68808, size = 1102, normalized size = 5.92

$$\left[\frac{12\left(a^4x^4-a^2x^2\right)\sqrt{-c}\log\left(\frac{4\,a^5cx^5+\left(2\,a^6+4\,a^5+6\,a^4+4\,a^3+a^2\right)cx^6+\left(4\,a^4-4\,a^3-6\,a^2-4\,a-1\right)cx^4-5\,a^2cx^2-4\,acx+\left(4\,a^4x^4+6\,a^3x^3-\left(4\,a^4+6\,a^3+4\,a^2+a\right)x^5+4}{a^4x^6+2\,a^3x^5-2\,ax^3-x^2}\right.}{6\left(a^2x^4-x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm
="fricas")
```

```
[Out] [1/6*(12*(a^4*x^4 - a^2*x^2)*sqrt(-c)*log((4*a^5*c*x^5 + (2*a^6 + 4*a^5 + 6
*a^4 + 4*a^3 + a^2)*c*x^6 + (4*a^4 - 4*a^3 - 6*a^2 - 4*a - 1)*c*x^4 - 5*a^2
*c*x^2 - 4*a*c*x + (4*a^4*x^4 + 6*a^3*x^3 - (4*a^4 + 6*a^3 + 4*a^2 + a)*x^5
 + 4*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x
^2)) - c)/(a^4*x^6 + 2*a^3*x^5 - 2*a*x^3 - x^2)) + (24*a^2*x^2 - (24*a^2 -
9*a + 2)*x^3 - 9*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
))/(a^2*x^4 - x^2), 1/6*(24*(a^4*x^4 - a^2*x^2)*sqrt(c)*arctan(-(2*a^2*x^2
+ (2*a^3 + 2*a^2 + a)*x^3 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2
 - c)/(a^2*x^2))/(2*a^3*c*x^3 - (2*a^3 + a^2)*c*x^4 + (a^2 + 2*a + 1)*c*x^2
 - 2*a*c*x - c)) + (24*a^2*x^2 - (24*a^2 - 9*a + 2)*x^3 - 9*a*x + 2)*sqrt(-
a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^4 - x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{\left(ax+1\right)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm
="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x^3), x)
```

3.787 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=220

$$\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}+\frac{4a^4x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^4x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] (4*a^3*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(4*
x^3*Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - c/(a^2*x^2)])/(x^2*Sqrt[1 - a^2*x^2])
- (2*a^2*Sqrt[c - c/(a^2*x^2)])/(x*Sqrt[1 - a^2*x^2]) + (4*a^4*Sqrt[c - c/(
a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^4*Sqrt[c - c/(a^2*x^2)]*x*Log[
1 + a*x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.258756, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}+\frac{4a^4x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^4x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] (4*a^3*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(4*
x^3*Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - c/(a^2*x^2)])/(x^2*Sqrt[1 - a^2*x^2])
- (2*a^2*Sqrt[c - c/(a^2*x^2)])/(x*Sqrt[1 - a^2*x^2]) + (4*a^4*Sqrt[c - c/(
a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^4*Sqrt[c - c/(a^2*x^2)]*x*Log[
1 + a*x])/Sqrt[1 - a^2*x^2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^5}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^2}{x^5(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(\frac{1}{x^5}-\frac{3a}{x^4}+\frac{4a^2}{x^3}-\frac{4a^3}{x^2}+\frac{4a^4}{x}-\frac{4a^5}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-a^2x^2}} - \frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}} + \frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}x\log(x)}{\sqrt{1-a^2x^2}} -$$

Mathematica [A] time = 0.0547849, size = 78, normalized size = 0.35

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{2a^2}{x^2}+\frac{4a^3}{x}+4a^4\log(x)-4a^4\log(ax+1)+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-1/(4*x^4) + a/x^3 - (2*a^2)/x^2 + (4*a^3)/x + 4*
a^4*Log[x] - 4*a^4*Log[1 + a*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.146, size = 94, normalized size = 0.4

$$-\frac{16\,a^4\ln(x)\,x^4-16\,\ln(ax+1)\,a^4x^4+16\,x^3a^3-8\,a^2x^2+4\,ax-1}{4\,x^3\left(a^2x^2-1\right)}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x)
```

```
[Out] -1/4*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^3*(-a^2*x^2+1)^(1/2)*(16*a^4*ln(x)*x^4
-16*ln(a*x+1)*a^4*x^4+16*x^3*a^3-8*a^2*x^2+4*a*x-1)/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm
="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x^4), x)
```

Fricas [A] time = 2.71627, size = 1156, normalized size = 5.25

$$\left[\frac{8\left(a^5x^5-a^3x^3\right)\sqrt{-c}\log\left(\frac{4a^5cx^5+\left(2a^6+4a^5+6a^4+4a^3+a^2\right)cx^6+\left(4a^4-4a^3-6a^2-4a-1\right)cx^4-5a^2cx^2-4acx-\left(4a^4x^4+6a^3x^3-\left(4a^4+6a^3+4a^2+a\right)x^5\right.}{a^4x^6+2a^3x^5-2ax^3-x^2}\right.}{4\left(a^2x^5-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm
="fricas")
```

```
[Out] [1/4*(8*(a^5*x^5 - a^3*x^3)*sqrt(-c)*log((4*a^5*c*x^5 + (2*a^6 + 4*a^5 + 6*
a^4 + 4*a^3 + a^2)*c*x^6 + (4*a^4 - 4*a^3 - 6*a^2 - 4*a - 1)*c*x^4 - 5*a^2*
c*x^2 - 4*a*c*x - (4*a^4*x^4 + 6*a^3*x^3 - (4*a^4 + 6*a^3 + 4*a^2 + a)*x^5
+ 4*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)) - c)/(a^4*x^6 + 2*a^3*x^5 - 2*a*x^3 - x^2)) - (16*a^3*x^3 - (16*a^3 - 8
*a^2 + 4*a - 1)*x^4 - 8*a^2*x^2 + 4*a*x - 1)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c
*x^2 - c)/(a^2*x^2)))/(a^2*x^5 - x^3), -1/4*(16*(a^5*x^5 - a^3*x^3)*sqrt(c)
*arctan(-(2*a^2*x^2 + (2*a^3 + 2*a^2 + a)*x^3 + a*x)*sqrt(-a^2*x^2 + 1)*sqr
t(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(2*a^3*c*x^3 - (2*a^3 + a^2)*c*x^4 + (
a^2 + 2*a + 1)*c*x^2 - 2*a*c*x - c)) + (16*a^3*x^3 - (16*a^3 - 8*a^2 + 4*a
- 1)*x^4 - 8*a^2*x^2 + 4*a*x - 1)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^2*x^5 - x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{\left(ax+1\right)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm
="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x^4), x)
```

3.788 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=262

$$-\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{2a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4\sqrt{1-a^2x^2}}-\frac{4a^5x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{4a^5x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

```
[Out] (-4*a^4*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(5
*x^4*Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - c/(a^2*x^2)])/(4*x^3*Sqrt[1 - a^2*x
^2]) - (4*a^2*Sqrt[c - c/(a^2*x^2)])/(3*x^2*Sqrt[1 - a^2*x^2]) + (2*a^3*Sqr
t[c - c/(a^2*x^2)])/(x*Sqrt[1 - a^2*x^2]) - (4*a^5*Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2] + (4*a^5*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sq
rt[1 - a^2*x^2]
```

Rubi [A] time = 0.262267, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{2a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4\sqrt{1-a^2x^2}}-\frac{4a^5x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{4a^5x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x^5),x]
```


```
[Out] (-4*a^4*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(5
*x^4*Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - c/(a^2*x^2)])/(4*x^3*Sqrt[1 - a^2*x
^2]) - (4*a^2*Sqrt[c - c/(a^2*x^2)])/(3*x^2*Sqrt[1 - a^2*x^2]) + (2*a^3*Sqr
t[c - c/(a^2*x^2)])/(x*Sqrt[1 - a^2*x^2]) - (4*a^5*Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2] + (4*a^5*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sq
rt[1 - a^2*x^2]
```


Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^6}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^2}{x^6(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \left(\frac{1}{x^6}-\frac{3a}{x^5}+\frac{4a^2}{x^4}-\frac{4a^3}{x^3}+\frac{4a^4}{x^2}-\frac{4a^5}{x}+\frac{4a^6}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}+\frac{2a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}-\frac{4}{}$$

Mathematica [A] time = 0.0641368, size = 87, normalized size = 0.33

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{240a^4x^4-120a^3x^3+80a^2x^2-45ax+12}{60x^5}-4a^5\log(x)+4a^5\log(ax+1)\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-(12 - 45*a*x + 80*a^2*x^2 - 120*a^3*x^3 + 240*a^
4*x^4)/(60*x^5) - 4*a^5*Log[x] + 4*a^5*Log[1 + a*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.148, size = 102, normalized size = 0.4

$$\frac{240\,a^5\ln(x)\,x^5-240\,\ln(ax+1)\,x^5a^5+240\,x^4a^4-120\,x^3a^3+80\,a^2x^2-45\,ax+12}{60\,x^4\left(a^2x^2-1\right)}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x)
```

```
[Out] 1/60*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^4*(-a^2*x^2+1)^(1/2)*(240*a^5*ln(x)*x^
5-240*ln(a*x+1)*x^5*a^5+240*x^4*a^4-120*x^3*a^3+80*a^2*x^2-45*a*x+12)/(a^2*
x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^3x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm
="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x^5), x)
```

Fricas [A] time = 2.70083, size = 1247, normalized size = 4.76

$$\left[\frac{120\left(a^{6}x^{6}-a^{4}x^{4}\right)\sqrt{-c}\log\left(\frac{4a^{5}cx^{5}+\left(2a^{6}+4a^{5}+6a^{4}+4a^{3}+a^{2}\right)cx^{6}+\left(4a^{4}-4a^{3}-6a^{2}-4a-1\right)cx^{4}-5a^{2}cx^{2}-4acx+\left(4a^{4}x^{4}+6a^{3}x^{3}-\left(4a^{4}+6a^{3}+4a^{2}+a\right)x^{5}+\right.}{a^{4}x^{6}+2a^{3}x^{5}-2ax^{3}-x^{2}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm
="fricas")
```

```
[Out] [1/60*(120*(a^6*x^6 - a^4*x^4)*sqrt(-c)*log((4*a^5*c*x^5 + (2*a^6 + 4*a^5 +
 6*a^4 + 4*a^3 + a^2)*c*x^6 + (4*a^4 - 4*a^3 - 6*a^2 - 4*a - 1)*c*x^4 - 5*a
^2*c*x^2 - 4*a*c*x + (4*a^4*x^4 + 6*a^3*x^3 - (4*a^4 + 6*a^3 + 4*a^2 + a)*x
^5 + 4*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2
*x^2)) - c)/(a^4*x^6 + 2*a^3*x^5 - 2*a*x^3 - x^2)) + (240*a^4*x^4 - 120*a^3
*x^3 - (240*a^4 - 120*a^3 + 80*a^2 - 45*a + 12)*x^5 + 80*a^2*x^2 - 45*a*x +
 12)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^6 - x^4), 1
/60*(240*(a^6*x^6 - a^4*x^4)*sqrt(c)*arctan(-(2*a^2*x^2 + (2*a^3 + 2*a^2 +
a)*x^3 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(2
*a^3*c*x^3 - (2*a^3 + a^2)*c*x^4 + (a^2 + 2*a + 1)*c*x^2 - 2*a*c*x - c)) +
(240*a^4*x^4 - 120*a^3*x^3 - (240*a^4 - 120*a^3 + 80*a^2 - 45*a + 12)*x^5 +
 80*a^2*x^2 - 45*a*x + 12)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)))/(a^2*x^6 - x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^{2}x^{2}}}}{(ax+1)^{3}x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm
="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x^5), x)
```

3.789 $\int e^{-2p\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$

Optimal. Leaf size=53

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p \text{Hypergeometric2F1}(1-2p,-2p,2-2p,ax)}{1-2p}$$

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[1 - 2*p, -2*p, 2 - 2*p, a*x])/((1
- 2*p)*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.115895, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6160, 6150, 64}

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1(1-2p,-2p;2-2p;ax)}{1-2p}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^p/E^(2*p*ArcTanh[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[1 - 2*p, -2*p, 2 - 2*p, a*x])/((1
- 2*p)*(1 - a^2*x^2)^p)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int e^{-2p\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx &= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int e^{-2p\tanh^{-1}(ax)}x^{-2p}\left(1-a^2x^2\right)^p dx\\
&= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}(1-ax)^{2p} dx\\
&= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x\left(1-a^2x^2\right)^{-p} {}_2F_1(1-2p,-2p;2-2p;ax)}{1-2p}
\end{aligned}$$

Mathematica [A] time = 0.0193244, size = 53, normalized size = 1.

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p \text{Hypergeometric2F1}(1-2p,-2p,2-2p,ax)}{1-2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^p/E^(2*p*ArcTanh[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[1 - 2*p, -2*p, 2 - 2*p, a*x])/((1
- 2*p)*(1 - a^2*x^2)^p)
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int \frac{1}{\mathrm{e}^{2p\operatorname{Artanh}(ax)}}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^p/exp(2*p*arctanh(a*x)),x)
```

```
[Out] int((c-c/a^2/x^2)^p/exp(2*p*arctanh(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c-\frac{c}{a^2x^2}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arctanh(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arctanh(a*x)),x, algorithm="fricas")
```

```
[Out] integral(((a^2*c*x^2 - c)/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**p/exp(2*p*atanh(a*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{a^2x^2}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arctanh(a*x)),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

3.790 $\int e^{2p\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=54

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\text{Hypergeometric2F1}(1-2p,-2p,2-2p,-ax)}{1-2p}$$

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[1 - 2*p, -2*p, 2 - 2*p, -(a*x)])/(
(1 - 2*p)*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.116221, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6160, 6150, 64}

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1(1-2p,-2p;2-2p;-ax)}{1-2p}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*p*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[1 - 2*p, -2*p, 2 - 2*p, -(a*x)])/(
(1 - 2*p)*(1 - a^2*x^2)^p)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int e^{2p\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx&=\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int e^{2p\tanh^{-1}(ax)}x^{-2p}\left(1-a^2x^2\right)^p\,dx\\&=\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}(1+ax)^{2p}\,dx\\&=\frac{\left(c-\frac{c}{a^2x^2}\right)^p x\left(1-a^2x^2\right)^{-p}\,{}_2F_1(1-2p,-2p;2-2p;-ax)}{1-2p}\end{aligned}$$

Mathematica [A] time = 0.0141907, size = 54, normalized size = 1.

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p \text{Hypergeometric2F1}(1-2p,-2p,2-2p,-ax)}{1-2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*p*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[1 - 2*p, -2*p, 2 - 2*p, -(a*x)])/(
(1 - 2*p)*(1 - a^2*x^2)^p)
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \mathrm{e}^{2p\,\mathrm{Artanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arctanh(a*x))*(c-c/a^2/x^2)^p,x)
```

```
[Out] int(exp(2*p*arctanh(a*x))*(c-c/a^2/x^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(c-c/a^2/x^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{ax+1}{ax-1}\right)^p\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(c-c/a^2/x^2)^p,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^p*((a^2*c*x^2 - c)/(a^2*x^2))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p e^{2p\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*atanh(a*x))*(c-c/a**2/x**2)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(2*p*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c-\frac{c}{a^2x^2}\right)^p \left(\frac{ax+1}{ax-1}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(c-c/a^2/x^2)^p,x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^p, x)
```

3.791 $\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=331

$$-\frac{c^2n\left(10-n^2\right)(ax+1)^{\frac{n-4}{2}}(1-ax)^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-4}{2},\frac{n-2}{2},\frac{ax+1}{1-ax}\right)}{3a(4-n)}+\frac{c^22^{\frac{n}{2}-1}n(1-ax)^{3-\frac{n}{2}}\text{Hypergeometr}}{a\left(n^2-1\right.}$$

```
[Out] (-4*c^2*(1 - a*x)^(3 - n/2)*(1 + a*x)^((-4 + n)/2))/(a*(4 - n)) - (c^2*(1 -
 a*x)^(3 - n/2)*(1 + a*x)^((-4 + n)/2))/(3*a^4*x^3) - (c^2*(10 + n)*(1 - a*
x)^(3 - n/2)*(1 + a*x)^((-4 + n)/2))/(6*a^3*x^2) - (c^2*(14 + 5*n + n^2)*(1
 - a*x)^(3 - n/2)*(1 + a*x)^((-4 + n)/2))/(6*a^2*x) - (c^2*n*(10 - n^2)*(1
- a*x)^(2 - n/2)*(1 + a*x)^((-4 + n)/2)*Hypergeometric2F1[1, (-4 + n)/2, (-
2 + n)/2, (1 + a*x)/(1 - a*x)])/(3*a*(4 - n)) + (2^(-1 + n/2)*c^2*n*(1 - a*
x)^(3 - n/2)*Hypergeometric2F1[(4 - n)/2, 3 - n/2, 4 - n/2, (1 - a*x)/2])/(
a*(24 - 10*n + n^2))
```

Rubi [C] time = 0.133327, antiderivative size = 71, normalized size of antiderivative = 0.21, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 136}

$$\frac{c^22^{3-\frac{n}{2}}(ax+1)^{\frac{n+6}{2}}F_1\left(\frac{n+6}{2};\frac{n-4}{2},4;\frac{n+8}{2};\frac{1}{2}(ax+1),ax+1\right)}{a(n+6)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (2^(3 - n/2)*c^2*(1 + a*x)^((6 + n)/2)*AppellF1[(6 + n)/2, (-4 + n)/2, 4, (
8 + n)/2, (1 + a*x)/2, 1 + a*x])/(a*(6 + n))
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2dx = \frac{c^2\int\frac{e^{n\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}dx}{a^4}$$
$$= \frac{c^2\int\frac{(1-ax)^{2-\frac{n}{2}}(1+ax)^{2+\frac{n}{2}}}{x^4}dx}{a^4}$$
$$= \frac{2^{3-\frac{n}{2}}c^2(1+ax)^{\frac{6+n}{2}}F_1\left(\frac{6+n}{2};\frac{1}{2}(-4+n),4;\frac{8+n}{2};\frac{1}{2}(1+ax),1+ax\right)}{a(6+n)}$$

Mathematica [A] time = 0.800958, size = 229, normalized size = 0.69

$$-\frac{c^2e^{n\tanh^{-1}(ax)}\left(a^3\left(n^2-10\right)nx^3e^{2\tanh^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n}{2}+1,\frac{n}{2}+2,e^{2\tanh^{-1}(ax)}\right)+a^3\left(n^3+2n^2-10n-20\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -(c^2*E^(n*ArcTanh[a*x])*(4 + 2*n + 2*a*n*x + a*n^2*x - 24*a^2*x^2 - 12*a^2
*n*x^2 + 2*a^2*n^2*x^2 + a^2*n^3*x^2 - 2*a^3*n*x^3 - a^3*n^2*x^3 + a^3*E^(2
*ArcTanh[a*x])*n*(-10 + n^2)*x^3*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(
2*ArcTanh[a*x])] + a^3*(-20 - 10*n + 2*n^2 + n^3)*x^3*Hypergeometric2F1[1,
n/2, 1 + n/2, E^(2*ArcTanh[a*x])] - 24*a^3*E^(2*ArcTanh[a*x])*x^3*Hypergeom
etric2F1[2, 1 + n/2, 2 + n/2, -E^(2*ArcTanh[a*x])]))/(6*a^4*(2 + n)*x^3)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{a^2x^2}\right)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a^2*x^2))^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4c^2x^4 - 2\,a^2c^2x^2 + c^2\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*((a*x + 1)/(a*x - 1))^(1/2*n)/
(a^4*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2\left(\int a^4e^{n\operatorname{atanh}(ax)}\,dx + \int\frac{e^{n\operatorname{atanh}(ax)}}{x^4}\,dx + \int -\frac{2a^2e^{n\operatorname{atanh}(ax)}}{x^2}\,dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a**2/x**2)**2,x)
```

```
[Out] c**2*(Integral(a**4*exp(n*atanh(a*x)), x) + Integral(exp(n*atanh(a*x))/x**4
, x) + Integral(-2*a**2*exp(n*atanh(a*x))/x**2, x))/a**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c - \frac{c}{a^2x^2}\right)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.792 $\int e^{n \tanh^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right) dx$

Optimal. Leaf size=137

$$\frac{4c(1-ax)^{1-\frac{n}{2}}(ax+1)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-ax}{ax+1}\right)}{a(2-n)} - \frac{c2^{\frac{n}{2}+1}(1-ax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\right.}{a(2-n)}$$

```
[Out] (4*c*(1 - a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/
2, 2 - n/2, (1 - a*x)/(1 + a*x)])/(a*(2 - n)) - (2^(1 + n/2)*c*(1 - a*x)^(1
 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2])/(a*(2 - n))
```

Rubi [C] time = 0.0968612, antiderivative size = 70, normalized size of antiderivative = 0.51, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6157, 6150, 136}

$$-\frac{c2^{2-\frac{n}{2}}(ax+1)^{\frac{n+4}{2}}F_1\left(\frac{n+4}{2};\frac{n-2}{2},2;\frac{n+6}{2};\frac{1}{2}(ax+1),ax+1\right)}{a(n+4)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] -((2^(2 - n/2)*c*(1 + a*x)^((4 + n)/2)*AppellF1[(4 + n)/2, (-2 + n)/2, 2, (
6 + n)/2, (1 + a*x)/2, 1 + a*x])/(a*(4 + n)))
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx=-\frac{c\int\frac{e^{n\tanh^{-1}(ax)}(1-a^2x^2)}{x^2}dx}{a^2}$$
$$=-\frac{c\int\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{1+\frac{n}{2}}}{x^2}dx}{a^2}$$
$$=-\frac{2^{2-\frac{n}{2}}c(1+ax)^{\frac{4+n}{2}}F_1\left(\frac{4+n}{2};\frac{1}{2}(-2+n),2;\frac{6+n}{2};\frac{1}{2}(1+ax),1+ax\right)}{a(4+n)}$$

Mathematica [A] time = 0.229702, size = 126, normalized size = 0.92

$$\frac{ce^{n\tanh^{-1}(ax)}\left(anxe^{2\tanh^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n}{2}+1,\frac{n}{2}+2,e^{2\tanh^{-1}(ax)}\right)+a(n+2)x\text{Hypergeometric2F1}\left(1\right.\right.}{a^2(n+2)x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*E^(n*ArcTanh[a*x])*(2 + n + a*E^(2*ArcTanh[a*x])*n*x*Hypergeometric2F1[1
, 1 + n/2, 2 + n/2, E^(2*ArcTanh[a*x])] + a*(2 + n)*x*Hypergeometric2F1[1,
n/2, 1 + n/2, E^(2*ArcTanh[a*x])] + 4*a*E^(2*ArcTanh[a*x])*x*Hypergeometric
2F1[2, 1 + n/2, 2 + n/2, -E^(2*ArcTanh[a*x])]))/(a^2*(2 + n)*x)
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{a^2x^2}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{2}cx^{2}-c\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^{2}x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c\left(\int a^{2}e^{n\,\text{atanh}\,(ax)}\,dx+\int -\frac{e^{n\,\text{atanh}\,(ax)}}{x^{2}}\,dx\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a**2/x**2),x)
```

```
[Out] c*(Integral(a**2*exp(n*atanh(a*x)), x) + Integral(-exp(n*atanh(a*x))/x**2,
x))/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{a^{2}x^{2}}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.793 $\int \frac{e^{n\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=125

$$-\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{ac}+\frac{x(ax+1)^{n/2}(1-ax)^{-n/2}}{c}-\frac{(1-n)(ax+1)^{n/2}(1-}{acn}$$

```
[Out] -(((1 - n)*(1 + a*x)^(n/2))/(a*c*n*(1 - a*x)^(n/2))) + (x*(1 + a*x)^(n/2))/
(c*(1 - a*x)^(n/2)) - (2^(1 + n/2)*Hypergeometric2F1[-n/2, -n/2, 1 - n/2, (
1 - a*x)/2])/(a*c*(1 - a*x)^(n/2))
```

Rubi [A] time = 0.171389, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6157, 6150, 90, 79, 69}

$$-\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{ac}+\frac{x(ax+1)^{n/2}(1-ax)^{-n/2}}{c}-\frac{(1-n)(ax+1)^{n/2}(1-ax)^{-n/2}}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] -(((1 - n)*(1 + a*x)^(n/2))/(a*c*n*(1 - a*x)^(n/2))) + (x*(1 + a*x)^(n/2))/
(c*(1 - a*x)^(n/2)) - (2^(1 + n/2)*Hypergeometric2F1[-n/2, -n/2, 1 - n/2, (
1 - a*x)/2])/(a*c*(1 - a*x)^(n/2))
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
```

```
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\frac{a^2\int \frac{e^{n\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}\\
&= -\frac{a^2\int x^2(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}\,dx}{c}\\
&= \frac{x(1-ax)^{-n/2}(1+ax)^{n/2}}{c}+\frac{\int (1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}(-1-anx)\,dx}{c}\\
&= -\frac{(1-n)(1-ax)^{-n/2}(1+ax)^{n/2}}{acn}+\frac{x(1-ax)^{-n/2}(1+ax)^{n/2}}{c}-\frac{n\int (1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c}\\
&= -\frac{(1-n)(1-ax)^{-n/2}(1+ax)^{n/2}}{acn}+\frac{x(1-ax)^{-n/2}(1+ax)^{n/2}}{c}-\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1\right.}{ac}
\end{aligned}$$

Mathematica [A] time = 0.104928, size = 82, normalized size = 0.66

$$\frac{(1-ax)^{-n/2}\left((ax+1)^{n/2}(anx+n-1)-2^{\frac{n}{2}+1}n\,\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}(1-ax)\right)\right)}{acn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] ((1 + a*x)^(n/2)*(-1 + n + a*n*x) - 2^(1 + n/2)*n*Hypergeometric2F1[-n/2, -
n/2, 1 - n/2, (1 - a*x)/2])/(a*c*n*(1 - a*x)^(n/2))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{Artanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{c-\frac{c}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\int \frac{x^2e^{n\,\text{atanh}\,(ax)}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a**2/x**2),x)
```

```
[Out] a**2*Integral(x**2*exp(n*atanh(a*x))/(a**2*x**2 - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{c-\frac{c}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2)), x)
```

3.794 $\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=373

$$-\frac{2^{n/2}n(1-ax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{ac^2(2-n)}-\frac{a^2x^3(ax+1)^{\frac{n-2}{2}}(1-ax)^{-\frac{n}{2}-1}}{c^2}-\frac{(n+3)\left(2-n^2\right)(}{ac^2}$$

```
[Out] ((1 - n)*(3 + n)*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a*c^2*(2 - n
)) + ((3 + n)*x*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/c^2 - (a^2*x^3
*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/c^2 + ((1 - a*x)^(1 - n/2)*(1
 + a*x)^((-2 + n)/2))/(a*c^2*(2 - n)) - (1 + a*x)^((-2 + n)/2)/(a*c^2*(1 -
a*x)^(n/2)) - ((3 + n)*(2 - n^2)*(1 - a*x)^(-1 - n/2)*(1 + a*x)^(n/2))/(a*c
^2*(4 - n^2)) - ((3 + n)*(2 - n^2)*(1 + a*x)^(n/2))/(a*c^2*n*(4 - n^2)*(1 -
 a*x)^(n/2)) - (2^(n/2)*n*(1 - a*x)^(1 - n/2)*Hypergeometric2F1[(2 - n)/2,
1 - n/2, 2 - n/2, (1 - a*x)/2])/(a*c^2*(2 - n))
```

Rubi [A] time = 0.423005, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6157, 6150, 100, 159, 89, 79, 69, 90, 45, 37}

$$-\frac{a^2x^3(ax+1)^{\frac{n-2}{2}}(1-ax)^{-\frac{n}{2}-1}}{c^2}-\frac{2^{n/2}n(1-ax)^{1-\frac{n}{2}}{}_2F_1\left(\frac{2-n}{2},1-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{ac^2(2-n)}-\frac{(n+3)\left(2-n^2\right)(ax+1)^{n/2}(1-ax)^{-}}{ac^2\left(4-n^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] ((1 - n)*(3 + n)*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a*c^2*(2 - n
)) + ((3 + n)*x*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/c^2 - (a^2*x^3
*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/c^2 + ((1 - a*x)^(1 - n/2)*(1
 + a*x)^((-2 + n)/2))/(a*c^2*(2 - n)) - (1 + a*x)^((-2 + n)/2)/(a*c^2*(1 -
a*x)^(n/2)) - ((3 + n)*(2 - n^2)*(1 - a*x)^(-1 - n/2)*(1 + a*x)^(n/2))/(a*c
^2*(4 - n^2)) - ((3 + n)*(2 - n^2)*(1 + a*x)^(n/2))/(a*c^2*n*(4 - n^2)*(1 -
 a*x)^(n/2)) - (2^(n/2)*n*(1 - a*x)^(1 - n/2)*Hypergeometric2F1[(2 - n)/2,
1 - n/2, 2 - n/2, (1 - a*x)/2])/(a*c^2*(2 - n))
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{e^{n\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^2}\,dx}{c^2}$$

$$= \frac{a^4\int x^4(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\,dx}{c^2}$$

$$= -\frac{a^2x^3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2} - \frac{a^2\int x^2(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}(-3-anx)\,dx}{c^2}$$

$$= -\frac{a^2x^3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2} - \frac{\left(a^2n\right)\int x^2(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\,dx}{c^2} + \frac{\left(a^2(3+n)\right)\int x^2(1-a}{c}$$

$$= \frac{(3+n)x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2} - \frac{a^2x^3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2} - \frac{(1-ax)^{-n/2}(1+ax)^{\frac{1}{2}(-2+n)}}{ac^2}$$

$$= \frac{(1-n)(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)} + \frac{(3+n)x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2} - \frac{a^2x^3(1-ax)^{-1-\frac{n}{2}}}{c^2}$$

$$= \frac{(1-n)(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)} + \frac{(3+n)x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2} - \frac{a^2x^3(1-ax)^{-1-\frac{n}{2}}}{c^2}$$

$$= \frac{(1-n)(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)} + \frac{(3+n)x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2} - \frac{a^2x^3(1-ax)^{-1-\frac{n}{2}}}{c^2}$$

Mathematica [A] time = 0.156713, size = 178, normalized size = 0.48

$$-\frac{(1-ax)^{-\frac{n}{2}-1}\left((ax+1)^{n/2}\left(n^2\left(1-2a^2x^2\right)+n\left(-4a^3x^3+4a^2x^2+6ax-4\right)+6a^2x^2+n^3(ax-1)^2(ax+1)-6\right)-2^{n/2}n^2(n}{ac^2(n-2)n(n+2)(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] -(((1 - a*x)^(-1 - n/2)*((1 + a*x)^(n/2)*(-6 + 6*a^2*x^2 + n^3*(-1 + a*x)^2
*(1 + a*x) + n^2*(1 - 2*a^2*x^2) + n*(-4 + 6*a*x + 4*a^2*x^2 - 4*a^3*x^3))
- 2^(n/2)*n^2*(2 + n)*(-1 + a*x)^2*(1 + a*x)*Hypergeometric2F1[1 - n/2, 1 -
 n/2, 2 - n/2, (1 - a*x)/2]))/(a*c^2*(-2 + n)*n*(2 + n)*(1 + a*x)))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int e^{n\operatorname{Artanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{-2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4x^4\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^4-2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] integral(a^4*x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^4 - 2*a^2*c^2*x^2
 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^4\int\frac{x^4e^{n\,\text{atanh}\,(ax)}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4*exp(n*atanh(a*x))/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^2, x)
```

3.795 $\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=430

$$-\frac{a^2\left(3-n^2\right)x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{3-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-3}{2},\frac{n-1}{2},\frac{ax+1}{1-ax}\right)}{(3-n)\left(1-a^2x^2\right)^{3/2}}+\frac{a^2 2^{\frac{n-1}{2}}nx^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{}$$

```
[Out] -((c - c/(a^2*x^2))^(3/2)*x*(1 - a*x)^((5 - n)/2)*(1 + a*x)^((-3 + n)/2))/(
2*(1 - a^2*x^2)^(3/2)) - (a*(4 + n)*(c - c/(a^2*x^2))^(3/2)*x^2*(1 - a*x)^(
(5 - n)/2)*(1 + a*x)^((-3 + n)/2))/(2*(1 - a^2*x^2)^(3/2)) - (3*a^2*(c - c/
(a^2*x^2))^(3/2)*x^3*(1 - a*x)^((5 - n)/2)*(1 + a*x)^((-3 + n)/2))/((3 - n)
*(1 - a^2*x^2)^(3/2)) - (a^2*(3 - n^2)*(c - c/(a^2*x^2))^(3/2)*x^3*(1 - a*x
)^((3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Hypergeometric2F1[1, (-3 + n)/2, (-1 +
 n)/2, (1 + a*x)/(1 - a*x)])/((3 - n)*(1 - a^2*x^2)^(3/2)) + (2^((-1 + n)/2
)*a^2*n*(c - c/(a^2*x^2))^(3/2)*x^3*(1 - a*x)^((5 - n)/2)*Hypergeometric2F1
[(3 - n)/2, (5 - n)/2, (7 - n)/2, (1 - a*x)/2])/((3 - n)*(5 - n)*(1 - a^2*x
^2)^(3/2))
```

Rubi [C] time = 0.206055, antiderivative size = 103, normalized size of antiderivative = 0.24, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 136}

$$-\frac{a^2 2^{\frac{5}{2}-\frac{n}{2}}x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ax+1)^{\frac{n+5}{2}}F_1\left(\frac{n+5}{2};\frac{n-3}{2},3;\frac{n+7}{2};\frac{1}{2}(ax+1),ax+1\right)}{(n+5)\left(1-a^2x^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -((2^(5/2 - n/2)*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*(1 + a*x)^((5 + n)/2)*Appe
llF1[(5 + n)/2, (-3 + n)/2, 3, (7 + n)/2, (1 + a*x)/2, 1 + a*x])/((5 + n)*(
1 - a^2*x^2)^(3/2)))
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
```

```
0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{e^{n\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1-ax)^{\frac{3}{2}-\frac{n}{2}}(1+ax)^{\frac{3}{2}+\frac{n}{2}}}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$=-\frac{2^{\frac{5}{2}-\frac{n}{2}}a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3(1+ax)^{\frac{5+n}{2}}F_1\left(\frac{5+n}{2};\frac{1}{2}(-3+n),3;\frac{7+n}{2};\frac{1}{2}(1+ax),1+ax\right)}{(5+n)\left(1-a^2x^2\right)^{3/2}}$$

Mathematica [A] time = 1.68723, size = 190, normalized size = 0.44

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}e^{n\tanh^{-1}(ax)}\text{csch}\left(\frac{1}{2}\tanh^{-1}(ax)\right)\text{sech}\left(\frac{1}{2}\tanh^{-1}(ax)\right)\left(-4a\left(n^2-3\right)xe^{\tanh^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (c*E^(n*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x*Csch[ArcTanh[a*x]/2]*Sech[Arc
Tanh[a*x]/2]*(8*a*E^ArcTanh[a*x]*n*x*Hypergeometric2F1[1, (1 + n)/2, (3 + n
)/2, -E^(2*ArcTanh[a*x])] - 4*a*E^ArcTanh[a*x]*(-3 + n^2)*x*Hypergeometric2
F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcTanh[a*x])] - (1 + n)*(a*x*(n + a*x) +
(1 - a^2*x^2)*Cosh[2*ArcTanh[a*x]])*Csch[ArcTanh[a*x]/2]*Sech[ArcTanh[a*x]/
2]))/(8*(1 + n)*(-1 + a^2*x^2))
```

Maple [F] time = 0.133, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2-c\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)
/(a^2*x^2))/(a^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.796 $\int e^{n \tanh^{-1}(ax)} \sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=272

$$\frac{2^{\frac{n+1}{2}} n x \sqrt{c-\frac{c}{a^2x^2}}(1-ax)^{\frac{3-n}{2}} \text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-ax)\right)}{\left(n^2-4n+3\right)\sqrt{1-a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\text{Hyper}}{(1-n)\sqrt{1-}}$$

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((-1 + n)/2))/((
1 - n)*Sqrt[1 - a^2*x^2])) + (2*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^((1 - n)/
2)*(1 + a*x)^((-1 + n)/2)*Hypergeometric2F1[1, (-1 + n)/2, (1 + n)/2, (1 +
a*x)/(1 - a*x)])/((1 - n)*Sqrt[1 - a^2*x^2]) + (2^((1 + n)/2)*n*Sqrt[c - c/
(a^2*x^2)]*x*(1 - a*x)^((3 - n)/2)*Hypergeometric2F1[(1 - n)/2, (3 - n)/2,
(5 - n)/2, (1 - a*x)/2])/((3 - 4*n + n^2)*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.259879, antiderivative size = 302, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6160, 6150, 105, 69, 131}

$$\frac{2x\sqrt{c-\frac{c}{a^2x^2}}(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1}{2}(-n-1)}\,{}_2F_1\left(1,\frac{1}{2}(-n-1);\frac{1-n}{2};\frac{1-ax}{ax+1}\right)}{(n+1)\sqrt{1-a^2x^2}}-\frac{2^{\frac{n+3}{2}}x\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^{\frac{1}{2}(-n-1)}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{1}{2}(-n\right.}{(n+1)\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (2*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((1 + n)/2)*Hyp
ergeometric2F1[1, (-1 - n)/2, (1 - n)/2, (1 - a*x)/(1 + a*x)])/((1 + n)*Sqr
t[1 - a^2*x^2]) - (2^((3 + n)/2)*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^((-1 - n
)/2)*Hypergeometric2F1[(-1 - n)/2, (-1 - n)/2, (1 - n)/2, (1 - a*x)/2])/((1
 + n)*Sqrt[1 - a^2*x^2]) + (2^((3 + n)/2)*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)
^((1 - n)/2)*Hypergeometric2F1[(-1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - a*x)/
2])/((1 - n)*Sqrt[1 - a^2*x^2])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
```

```
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{e^{n\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}}{x}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}}{x}\,dx}{\sqrt{1-a^2x^2}} - \frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{2^{\frac{3+n}{2}}\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-1-n),\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{(1-n)\sqrt{1-a^2x^2}} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{(1-ax)}{}}{\sqrt{1-a^2}}$$

$$= \frac{2\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1+n}{2}}\,{}_2F_1\left(1,\frac{1}{2}(-1-n);\frac{1-n}{2};\frac{1-ax}{1+ax}\right)}{(1+n)\sqrt{1-a^2x^2}} - \frac{2^{\frac{3+n}{2}}\sqrt{c-\frac{c}{a^2x^2}}x(}{}$$

Mathematica [A] time = 0.172003, size = 208, normalized size = 0.76

$$\frac{2x\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^{\frac{1}{2}(-n-1)}\left((n-1)(ax+1)^{\frac{n+1}{2}}\text{Hypergeometric2F1}\left(1,-\frac{n}{2}-\frac{1}{2},\frac{1}{2}-\frac{n}{2},\frac{1-ax}{ax+1}\right)+2^{\frac{n+1}{2}}\left((n+1)(ax-1)\text{H}\right.\right.}{\left(n^2-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (2*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^((-1 - n)/2)*((-1 + n)*(1 + a*x)^((1 +
 n)/2)*Hypergeometric2F1[1, -1/2 - n/2, 1/2 - n/2, (1 - a*x)/(1 + a*x)] + 2
^((1 + n)/2)*(-((-1 + n)*Hypergeometric2F1[-1/2 - n/2, -1/2 - n/2, 1/2 - n/
2, 1/2 - (a*x)/2]) + (1 + n)*(-1 + a*x)*Hypergeometric2F1[-1/2 - n/2, 1/2 -
 n/2, 3/2 - n/2, 1/2 - (a*x)/2])))/((-1 + n^2)*Sqrt[1 - a^2*x^2])
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int e^{n\operatorname{Artanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{a^2cx^2-c}{a^2x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.797 $\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=182

$$-\frac{2^{\frac{n+3}{2}}n\sqrt{1-a^2x^2}(1-ax)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-1),\frac{1-n}{2},\frac{3-n}{2},\frac{1}{2}(1-ax)\right)}{a^2\left(1-n^2\right)x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-a^2x^2}(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{a^2(n+1)x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] -(((1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(a^2*(1 +
 n)*Sqrt[c - c/(a^2*x^2)]*x)) - (2^((3 + n)/2)*n*(1 - a*x)^((1 - n)/2)*Sqrt
[1 - a^2*x^2]*Hypergeometric2F1[(-1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - a*x)
/2])/(a^2*(1 - n^2)*Sqrt[c - c/(a^2*x^2)]*x)
```

Rubi [A] time = 0.189559, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6160, 6150, 79, 69}

$$-\frac{2^{\frac{n+3}{2}}n\sqrt{1-a^2x^2}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a^2\left(1-n^2\right)x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-a^2x^2}(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{a^2(n+1)x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -(((1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(a^2*(1 +
 n)*Sqrt[c - c/(a^2*x^2)]*x)) - (2^((3 + n)/2)*n*(1 - a*x)^((1 - n)/2)*Sqrt
[1 - a^2*x^2]*Hypergeometric2F1[(-1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - a*x)
/2])/(a^2*(1 - n^2)*Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}x}$$
$$= \frac{\sqrt{1-a^2x^2}\int x(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}x}$$
$$= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{a^2(1+n)\sqrt{c-\frac{c}{a^2x^2}}x} + \frac{\left(n\sqrt{1-a^2x^2}\right)\int (1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1+n}{2}}\,dx}{a(1+n)\sqrt{c-\frac{c}{a^2x^2}}x}$$
$$= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{a^2(1+n)\sqrt{c-\frac{c}{a^2x^2}}x} - \frac{2^{\frac{3+n}{2}}n(1-ax)^{\frac{1-n}{2}}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1}{2}(-1-n),\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax\right)}{a^2\left(1-n^2\right)\sqrt{c-\frac{c}{a^2x^2}}x}$$

Mathematica [A] time = 0.0981287, size = 130, normalized size = 0.71

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}-\frac{n}{2}}\left(2^{\frac{n+3}{2}}n\text{Hypergeometric2F1}\left(-\frac{n}{2}-\frac{1}{2},\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},\frac{1}{2}-\frac{ax}{2}\right)-(n-1)(ax+1)^{\frac{n+1}{2}}\right)}{a^2(n-1)(n+1)x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] ((1 - a*x)^(1/2 - n/2)*Sqrt[1 - a^2*x^2]*(-((-1 + n)*(1 + a*x)^((1 + n)/2))
 + 2^((3 + n)/2)*n*Hypergeometric2F1[-1/2 - n/2, 1/2 - n/2, 3/2 - n/2, 1/2
- (a*x)/2]))/(a^2*(-1 + n)*(1 + n)*Sqrt[c - c/(a^2*x^2)]*x)
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}\frac{1}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a^2*x^2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^2x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^2*c*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n\,\mathrm{atanh}\,(ax)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a^2*x^2)), x)
```

3.798 $\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=267

$$-\frac{2^{\frac{n-1}{2}}n\left(1-a^2x^2\right)^{3/2}(1-ax)^{\frac{3-n}{2}}\text{Hypergeometric2F1}\left(\frac{3-n}{2},\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-ax)\right)}{a^4(3-n)x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}(ax+1)^{\frac{n-1}{2}}\left(-a(2n+\right.}{a^4\left(1-n^2\right)x}$$

```
[Out] -(((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*(1 - a^2*x^2)^(3/2))/(a^2*
(c - c/(a^2*x^2))^(3/2)*x)) + ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2
)*(2 + 2*n + n^2 - a*n*(3 + 2*n)*x)*(1 - a^2*x^2)^(3/2))/(a^4*(1 - n^2)*(c
- c/(a^2*x^2))^(3/2)*x^3) - (2^((-1 + n)/2)*n*(1 - a*x)^((3 - n)/2)*(1 - a^
2*x^2)^(3/2)*Hypergeometric2F1[(3 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2
])/(a^4*(3 - n)*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rubi [A] time = 0.291179, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6160, 6150, 100, 145, 69}

$$-\frac{2^{\frac{n-1}{2}}n\left(1-a^2x^2\right)^{3/2}(1-ax)^{\frac{3-n}{2}}{}_2F_1\left(\frac{3-n}{2},\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{a^4(3-n)x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}(ax+1)^{\frac{n-1}{2}}\left(-a(2n+3)nx+n^2+2n+\right.}{a^4\left(1-n^2\right)x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -(((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*(1 - a^2*x^2)^(3/2))/(a^2*
(c - c/(a^2*x^2))^(3/2)*x)) + ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2
)*(2 + 2*n + n^2 - a*n*(3 + 2*n)*x)*(1 - a^2*x^2)^(3/2))/(a^4*(1 - n^2)*(c
- c/(a^2*x^2))^(3/2)*x^3) - (2^((-1 + n)/2)*n*(1 - a*x)^((3 - n)/2)*(1 - a^
2*x^2)^(3/2)*Hypergeometric2F1[(3 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2
])/(a^4*(3 - n)*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
```

```
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx &= \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}\int x^3(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\left(1-a^2x^2\right)^{3/2}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} - \frac{\left(1-a^2x^2\right)^{3/2}\int x(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}(-2-anx}{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\left(1-a^2x^2\right)^{3/2}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\left(2+2n+n^2-an(3+\right.}{a^4\left(1-n^2\right)\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\left(1-a^2x^2\right)^{3/2}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\left(2+2n+n^2-an(3+\right.}{a^4\left(1-n^2\right)\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}
\end{aligned}
$$

Mathematica [A] time = 0.244913, size = 186, normalized size = 0.7

$$
\frac{\left(1-a^2x^2\right)^{3/2}(1-ax)^{\frac{1}{2}(-n-1)}\left(\frac{a^2 2^{\frac{n+3}{2}}n(ax-1)^2\text{Hypergeometric2F1}\left(\frac{3}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},\frac{5}{2}-\frac{n}{2},\frac{1}{2}-\frac{ax}{2}\right)}{n-3}+\frac{4a^2\left(n^2(2ax-1)+n(3ax-2)-2\right)(ax+1)^{\frac{n-1}{2}}}{n^2-1}-4a^4x^2(a\right.}{4a^6x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] ((1 - a*x)^((-1 - n)/2)*(1 - a^2*x^2)^(3/2)*(-4*a^4*x^2*(1 + a*x)^((-1 + n)
/2) + (4*a^2*(1 + a*x)^((-1 + n)/2)*(-2 + n^2*(-1 + 2*a*x) + n*(-2 + 3*a*x)
))/(-1 + n^2) + (2^((3 + n)/2)*a^2*n*(-1 + a*x)^2*Hypergeometric2F1[3/2 - n
/2, 3/2 - n/2, 5/2 - n/2, 1/2 - (a*x)/2])/(-3 + n)))/(4*a^6*(c - c/(a^2*x^2
))^(3/2)*x^3)
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int e^{n\operatorname{Artanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{a^4x^4\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^4c^2x^4-2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(a^4*x^4*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^(3/2), x)
```

3.799 $\int \frac{e^{n \tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=1039

result too large to display

```
[Out] ((4 + n)*(1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*(1 - a^2*x^2)^(5/2))
/(a^3*(3 + n)*(c - c/(a^2*x^2))^(5/2)*x^2) - ((1 - a*x)^((-3 - n)/2)*(1 + a
*x)^((-3 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^2*(c - c/(a^2*x^2))^(5/2)*x) + (n*
(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(1
+ n)*(c - c/(a^2*x^2))^(5/2)*x^5) - (3*(2 - n)*(4 + n)*(1 - a*x)^((-1 - n)/
2)*(1 + a*x)^((-3 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(9 - n^2)*(c - c/(a^2*x
^2))^(5/2)*x^5) - (3*(4 + n)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*
(1 - a^2*x^2)^(5/2))/(a^5*(3 + n)*(c - c/(a^2*x^2))^(5/2)*x^4) - (2*n*(1 -
a*x)^((1 - n)/2)*(1 + a*x)^((-3 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(1 - n^2)
*(c - c/(a^2*x^2))^(5/2)*x^5) + (2*n*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((-3 +
 n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(1 + n)*(3 - 4*n + n^2)*(c - c/(a^2*x^2))^
(5/2)*x^5) - (3*n*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*(1 - a^2*x^
2)^(5/2))/(a^6*(1 + n)*(c - c/(a^2*x^2))^(5/2)*x^5) + (3*(4 + n)*(1 + 2*n -
 n^2)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*(1 - a^2*x^2)^(5/2))/(a
^6*(3 - n)*(1 + n)*(3 + n)*(c - c/(a^2*x^2))^(5/2)*x^5) + (3*n*(1 - a*x)^((
1 - n)/2)*(1 + a*x)^((-1 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(1 - n^2)*(c - c
/(a^2*x^2))^(5/2)*x^5) - (3*(4 + n)*(1 + 2*n - n^2)*(1 - a*x)^((1 - n)/2)*(
1 + a*x)^((-1 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(9 - 10*n^2 + n^4)*(c - c/(
a^2*x^2))^(5/2)*x^5) + (3*n*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((1 + n)/2)*(1
 - a^2*x^2)^(5/2))/(a^6*(1 + n)*(c - c/(a^2*x^2))^(5/2)*x^5) - (2^((3 + n)/
2)*n*(1 - a*x)^((-1 - n)/2)*(1 - a^2*x^2)^(5/2)*Hypergeometric2F1[(-1 - n)/
2, (-1 - n)/2, (1 - n)/2, (1 - a*x)/2])/(a^6*(1 + n)*(c - c/(a^2*x^2))^(5/2
)*x^5)
```

Rubi [A] time = 0.770349, antiderivative size = 1039, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6160, 6150, 100, 159, 128, 45, 37, 69, 94, 90, 79}

$$-\frac{(ax+1)^{\frac{n-3}{2}}\left(1-a^2x^2\right)^{5/2}(1-ax)^{\frac{1}{2}(-n-3)}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}+\frac{(n+4)(ax+1)^{\frac{n-3}{2}}\left(1-a^2x^2\right)^{5/2}(1-ax)^{\frac{1}{2}(-n-3)}}{a^3(n+3)\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}-\frac{3(n+4)(ax+1)^{\frac{n-3}{2}}\left(1-\cdots\right.}{a^5(n+3)\left(c\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```


```
[Out] ((4 + n)*(1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*(1 - a^2*x^2)^(5/2))
/(a^3*(3 + n)*(c - c/(a^2*x^2))^(5/2)*x^2) - ((1 - a*x)^((-3 - n)/2)*(1 + a
*x)^((-3 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^2*(c - c/(a^2*x^2))^(5/2)*x) + (n*
(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(1
+ n)*(c - c/(a^2*x^2))^(5/2)*x^5) - (3*(2 - n)*(4 + n)*(1 - a*x)^((-1 - n)/
2)*(1 + a*x)^((-3 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(9 - n^2)*(c - c/(a^2*x
^2))^(5/2)*x^5) - (3*(4 + n)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*
(1 - a^2*x^2)^(5/2))/(a^5*(3 + n)*(c - c/(a^2*x^2))^(5/2)*x^4) - (2*n*(1 -
a*x)^((1 - n)/2)*(1 + a*x)^((-3 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(1 - n^2)
*(c - c/(a^2*x^2))^(5/2)*x^5) + (2*n*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((-3 +
 n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(1 + n)*(3 - 4*n + n^2)*(c - c/(a^2*x^2))^
(5/2)*x^5) - (3*n*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*(1 - a^2*x^
```

```
2)^(5/2))/(a^6*(1 + n)*(c - c/(a^2*x^2))^(5/2)*x^5) + (3*(4 + n)*(1 + 2*n -
 n^2)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*(1 - a^2*x^2)^(5/2))/(a
^6*(3 - n)*(1 + n)*(3 + n)*(c - c/(a^2*x^2))^(5/2)*x^5) + (3*n*(1 - a*x)^((
1 - n)/2)*(1 + a*x)^((-1 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(1 - n^2)*(c - c
/(a^2*x^2))^(5/2)*x^5) - (3*(4 + n)*(1 + 2*n - n^2)*(1 - a*x)^((1 - n)/2)*(
1 + a*x)^((-1 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(9 - 10*n^2 + n^4)*(c - c/(
a^2*x^2))^(5/2)*x^5) + (3*n*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((1 + n)/2)*(1
 - a^2*x^2)^(5/2))/(a^6*(1 + n)*(c - c/(a^2*x^2))^(5/2)*x^5) - (2^((3 + n)/
2)*n*(1 - a*x)^((-1 - n)/2)*(1 - a^2*x^2)^(5/2)*Hypergeometric2F1[(-1 - n)/
2, (-1 - n)/2, (1 - n)/2, (1 - a*x)/2])/(a^6*(1 + n)*(c - c/(a^2*x^2))^(5/2
)*x^5)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 128

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (IGtQ[m, 0] || (ILtQ[
m, 0] && ILtQ[n, 0]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{e^{n\tanh^{-1}(ax)}x^5}{\left(1-a^2x^2\right)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{\left(1-a^2x^2\right)^{5/2}\int x^5(1-ax)^{-\frac{5}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{\left(1-a^2x^2\right)^{5/2}\int x^3(1-ax)^{-\frac{5}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}(-4-an}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{\left(n\left(1-a^2x^2\right)^{5/2}\right)\int x^3(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}\,dx}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} +$$

$$= \frac{(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^3(3+n)\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{(}{}$$

$$= \frac{(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^3(3+n)\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{3}{}$$

$$= \frac{(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^3(3+n)\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{n}{}$$

$$= \frac{(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^3(3+n)\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{n}{}$$

$$= \frac{(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^3(3+n)\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{n}{}$$

Mathematica [A] time = 6.31678, size = 227, normalized size = 0.22

$$\frac{\left(a^2x^2-1\right)^2\left(-\frac{4\left(a^2x^2-1\right)\left(\frac{2ne^{(n+1)\tanh^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-e^{2\tanh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}-(n+1)e^{n\tanh^{-1}(ax)}\right)}{n+1} - \frac{e^{n\tanh^{-1}(ax)}\left(3\left(n^2-1\right)\sqrt{1-a^2x^2}\cosh(3\tanh\right.}{}\right.}{4a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((-1 + a^2*x^2)^2*((-8*E^(n*ArcTanh[a*x])*(-1 + a*n*x))/(-1 + n^2) - (E^(n*
ArcTanh[a*x])*(-9 + n^2 + 10*a*n*x - 2*a*n^3*x - 2*a*n*(-1 + n^2)*x*Cosh[2*
ArcTanh[a*x]] + 3*(-1 + n^2)*Sqrt[1 - a^2*x^2]*Cosh[3*ArcTanh[a*x]]))/(9 -
10*n^2 + n^4) - (4*(-1 + a^2*x^2)*(-(E^(n*ArcTanh[a*x])*(1 + n)) + (2*E^((1
 + n)*ArcTanh[a*x])*n*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -E^(2*ArcT
anh[a*x])])/Sqrt[1 - a^2*x^2]))/(1 + n)))/(4*a^6*(c - c/(a^2*x^2))^(5/2)*x^
5)
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{Artanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^6x^6\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(a^6*x^6*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^(5/2), x)
```

3.800 $\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=72

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p F_1\left(1-2p;\frac{1}{2}(n-2p),-\frac{n}{2}-p;2-2p;ax,-ax\right)}{1-2p}$$

```
[Out] ((c - c/(a^2*x^2))^p*x*AppellF1[1 - 2*p, (n - 2*p)/2, -n/2 - p, 2 - 2*p, a*
x, -(a*x)])/((1 - 2*p)*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.128311, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 133}

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p F_1\left(1-2p;\frac{1}{2}(n-2p),-\frac{n}{2}-p;2-2p;ax,-ax\right)}{1-2p}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*AppellF1[1 - 2*p, (n - 2*p)/2, -n/2 - p, 2 - 2*p, a*
x, -(a*x)])/((1 - 2*p)*(1 - a^2*x^2)^p)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx &= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int e^{n\tanh^{-1}(ax)}x^{-2p}\left(1-a^2x^2\right)^p\,dx\\
&= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}(1-ax)^{-\frac{n}{2}+p}(1+ax)^{\frac{n}{2}+p}\,dx\\
&= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x\left(1-a^2x^2\right)^{-p}F_1\left(1-2p;\frac{1}{2}(n-2p),-\frac{n}{2}-p;2-2p;ax,-ax\right)}{1-2p}
\end{aligned}$$

Mathematica [F] time = 0.35542, size = 0, normalized size = 0.

$$\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2))^p, x]
```

Maple [F] time = 0.163, size = 0, normalized size = 0.

$$\int e^{n\operatorname{Artanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^p,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^p,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)*((a^2*c*x^2 - c)/(a^2*x^2))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p e^{n\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a**2/x**2)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(n*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c-\frac{c}{a^2x^2}\right)^p \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^p,x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.801 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=339

$$\frac{x(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2}(1-2p),2-p,\frac{1}{2}(3-2p),a^2x^2\right)}{1-2p}+\frac{6a^2x^3(1-ax)^{-p}(ax+1)^{-p}\left(c\right.}{}$$

```
[Out] (2*a*(c - c/(a^2*x^2))^p*x^2)/((1 - p)*(1 - a*x)*(1 + a*x)) + ((c - c/(a^2*
x^2))^p*x*Hypergeometric2F1[(1 - 2*p)/2, 2 - p, (3 - 2*p)/2, a^2*x^2])/((1
- 2*p)*(1 - a*x)^p*(1 + a*x)^p) + (6*a^2*(c - c/(a^2*x^2))^p*x^3*Hypergeome
tric2F1[(3 - 2*p)/2, 2 - p, (5 - 2*p)/2, a^2*x^2])/((3 - 2*p)*(1 - a*x)^p*(
1 + a*x)^p) + (a^4*(c - c/(a^2*x^2))^p*x^5*Hypergeometric2F1[(5 - 2*p)/2, 2
 - p, (7 - 2*p)/2, a^2*x^2])/((5 - 2*p)*(1 - a*x)^p*(1 + a*x)^p) + (2*a^3*(
c - c/(a^2*x^2))^p*x^4*Hypergeometric2F1[2 - p, 2 - p, 3 - p, a^2*x^2])/((2
 - p)*(1 - a*x)^p*(1 + a*x)^p)
```

Rubi [A] time = 0.342747, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6159, 6129, 127, 95, 125, 364}

$$\frac{x(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1\left(\frac{1}{2}(1-2p),2-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}+\frac{6a^2x^3(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1\left(\frac{1}{2}(3\right.}{3-2p}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] (2*a*(c - c/(a^2*x^2))^p*x^2)/((1 - p)*(1 - a*x)*(1 + a*x)) + ((c - c/(a^2*
x^2))^p*x*Hypergeometric2F1[(1 - 2*p)/2, 2 - p, (3 - 2*p)/2, a^2*x^2])/((1
- 2*p)*(1 - a*x)^p*(1 + a*x)^p) + (6*a^2*(c - c/(a^2*x^2))^p*x^3*Hypergeome
tric2F1[(3 - 2*p)/2, 2 - p, (5 - 2*p)/2, a^2*x^2])/((3 - 2*p)*(1 - a*x)^p*(
1 + a*x)^p) + (a^4*(c - c/(a^2*x^2))^p*x^5*Hypergeometric2F1[(5 - 2*p)/2, 2
 - p, (7 - 2*p)/2, a^2*x^2])/((5 - 2*p)*(1 - a*x)^p*(1 + a*x)^p) + (2*a^3*(
c - c/(a^2*x^2))^p*x^4*Hypergeometric2F1[2 - p, 2 - p, 3 - p, a^2*x^2])/((2
 - p)*(1 - a*x)^p*(1 + a*x)^p)
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^n*(c + d*x)^n*(f*x)^p, (a + b
```

```
*x)^(m - n), x], x] /; FreeQ[{a, b, c, d, f, m, n, p}, x] && EqQ[b*c + a*d,
 0] && IGtQ[m - n, 0] && NeQ[m + n + p + 2, 0]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 125

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0] && GtQ[a, 0] && GtQ[c, 0
]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx &= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int e^{4\tanh^{-1}(ax)}x^{-2p}(1-ax)^p(1+ax)^p\,dx\\
&= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}(1-ax)^{-2+p}(1+ax)^{2+p}\,dx\\
&= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int \left(4ax^{1-2p}(1-ax)^{-2+p}(1+ax)^{-2+p}+6a^2x^{2-2p}\right.\\
&= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}(1-ax)^{-2+p}(1+ax)^{-2+p}\,dx+\left(4a\left(c-\frac{}{a}\right.\right.\\
&= \frac{2a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{(1-p)(1-ax)(1+ax)}+\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}\left(1-a^2x^2\right)^{-2+p}\\
&= \frac{2a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{(1-p)(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^p x(1-ax)^{-p}(1+ax)^{-p}\,{}_2F_1\left(\frac{1}{2}(1-2p),2-p;\frac{1}{2}(3-\right.}{1-2p}\end{aligned}$$

Mathematica [C] time = 0.199298, size = 217, normalized size = 0.64

$$-\frac{x(1-ax)^{-p}\left(-\left(a^2x^2-1\right)^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(4(ax-1)^p(ax+1)^{2p}\left(1-a^2x^2\right)^p\text{Hypergeometric2F1}\left(1-2p,2-p,2-2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] -(((c - c/(a^2*x^2))^p*x*(-4*(-1 + a*x)^p*(1 + a*x)*(1 - a^2*x^2)^p*AppellF
1[1 - 2*p, 1 - p, -p, 2 - 2*p, a*x, -(a*x)] + 4*(-1 + a*x)^p*(1 + a*x)^(2*p
)*(1 - a^2*x^2)^p*Hypergeometric2F1[1 - 2*p, 2 - p, 2 - 2*p, (2*a*x)/(1 + a
*x)] + (1 - a*x)^p*(1 + a*x)*(-1 + a^2*x^2)^p*Hypergeometric2F1[1/2 - p, -p
```

```
, 3/2 - p, a^2*x^2]))/((-1 + 2*p)*(1 - a*x)^p*(1 + a*x)*(-(-1 + a^2*x^2)^2)
^p))
```

Maple [F] time = 0.332, size = 0, normalized size = 0.

$$\int \frac{(ax+1)^4}{(-a^2x^2+1)^2}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^p,x)
```

```
[Out] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^4\left(c-\frac{c}{a^2x^2}\right)^p}{(a^2x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^4*(c - c/(a^2*x^2))^p/(a^2*x^2 - 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(a^2x^2+2\,ax+1)\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{a^2x^2-2\,ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*((a^2*c*x^2 - c)/(a^2*x^2))^p/(a^2*x^2 - 2*a
*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p (ax+1)^2}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a**2/x**2)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*(a*x + 1)**2/(a*x - 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^4\left(c-\frac{c}{a^2x^2}\right)^p}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^4*(c - c/(a^2*x^2))^p/(a^2*x^2 - 1)^2, x)
```

3.802 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=217

$$\frac{3a^2x^3\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2}(3-2p),\frac{3}{2}-p,\frac{1}{2}(5-2p),a^2x^2\right)}{3-2p}+\frac{a(5-2p)x^2\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right.}{}$$

```
[Out] ((c - c/(a^2*x^2))^p*x)/((1 - 2*p)*Sqrt[1 - a^2*x^2]) - (a*(c - c/(a^2*x^2)
)^p*x^2)/Sqrt[1 - a^2*x^2] + (3*a^2*(c - c/(a^2*x^2))^p*x^3*Hypergeometric2
F1[(3 - 2*p)/2, 3/2 - p, (5 - 2*p)/2, a^2*x^2])/((3 - 2*p)*(1 - a^2*x^2)^p)
 + (a*(5 - 2*p)*(c - c/(a^2*x^2))^p*x^2*Hypergeometric2F1[1 - p, 3/2 - p, 2
 - p, a^2*x^2])/(2*(1 - p)*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.330087, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6160, 6148, 1809, 1808, 364, 807}

$$\frac{3a^2x^3\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1\left(\frac{1}{2}(3-2p),\frac{3}{2}-p;\frac{1}{2}(5-2p);a^2x^2\right)}{3-2p}+\frac{a(5-2p)x^2\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1\left(1-p,\frac{3}{2}\right.}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x)/((1 - 2*p)*Sqrt[1 - a^2*x^2]) - (a*(c - c/(a^2*x^2)
)^p*x^2)/Sqrt[1 - a^2*x^2] + (3*a^2*(c - c/(a^2*x^2))^p*x^3*Hypergeometric2
F1[(3 - 2*p)/2, 3/2 - p, (5 - 2*p)/2, a^2*x^2])/((3 - 2*p)*(1 - a^2*x^2)^p)
 + (a*(5 - 2*p)*(c - c/(a^2*x^2))^p*x^2*Hypergeometric2F1[1 - p, 3/2 - p, 2
 - p, a^2*x^2])/(2*(1 - p)*(1 - a^2*x^2)^p)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 1808

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> With
[{q = Expon[Pq, x]}, Dist[Coeff[Pq, x, q]/c^q, Int[(c*x)^(m + q)*(a + b*x^2
)^p, x], x] + Dist[1/c^q, Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[c^q*Pq - Co
eff[Pq, x, q]*(c*x)^q, x], x], x] /; EqQ[q, 1] || EqQ[m + q + 2*p + 1, 0]]
/; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] &&  !(IGtQ[m, 0] && ILtQ[p + 1
/2, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx &= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int e^{3\tanh^{-1}(ax)}x^{-2p}\left(1-a^2x^2\right)^p dx\\
&= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p} dx\\
&= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{\sqrt{1-a^2x^2}}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\left(-a^2-a^3(5-\right.}{a^2}\\
&= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{\sqrt{1-a^2x^2}}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}\left(-a^2-a^3(5-2p)x\right)\left(1-a^2x^2\right.}{a^2}\\
&= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x}{(1-2p)\sqrt{1-a^2x^2}}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{\sqrt{1-a^2x^2}}+\frac{3a^2\left(c-\frac{c}{a^2x^2}\right)^p x^3\left(1-a^2x^2\right)^{-p}\ {}_2F_1\left(\frac{1}{2}(3-2p),\right.}{3-2p}\\
&= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x}{(1-2p)\sqrt{1-a^2x^2}}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{\sqrt{1-a^2x^2}}+\frac{3a^2\left(c-\frac{c}{a^2x^2}\right)^p x^3\left(1-a^2x^2\right)^{-p}\ {}_2F_1\left(\frac{1}{2}(3-2p),\right.}{3-2p}
\end{aligned}
$$

Mathematica [A] time = 0.15768, size = 175, normalized size = 0.81

$$
\frac{1}{2}x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(-\frac{3ax\text{Hypergeometric2F1}\left(1-p,\frac{3}{2}-p,2-p,a^2x^2\right)}{p-1}+\frac{6a^2x^2\text{Hypergeometric2F1}\left(\frac{3}{2}\right.}{3-2p}\right.
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*((2*(1 - a^2*x^2)^(-1/2 + p))/(1 - 2*p) - (3*a*x*Hyp
ergeometric2F1[1 - p, 3/2 - p, 2 - p, a^2*x^2])/(-1 + p) + (6*a^2*x^2*Hyper
```

```
geometric2F1[3/2 - p, 3/2 - p, 5/2 - p, a^2*x^2])/(3 - 2*p) + (a^3*x^3*Hype
rgeometric2F1[3/2 - p, 2 - p, 3 - p, a^2*x^2])/(2 - p)))/(2*(1 - a^2*x^2)^p
)
```

Maple [F] time = 0.298, size = 0, normalized size = 0.

$$\int (ax+1)^3 \left(c-\frac{c}{a^2x^2}\right)^p \left(-a^2x^2+1\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^p,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3 \left(c-\frac{c}{a^2x^2}\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^p,x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^p/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}(ax+1)\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{a^2x^2-2\,ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^p,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*((a^2*c*x^2 - c)/(a^2*x^2))^p/(a^2*x^
2 - 2*a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p (ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*(a*x + 1)**3/(-(a*x - 1)*(a*x
 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^p/(-a^2*x^2 + 1)^(3/2), x)
```

3.803 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=217

$$\frac{x(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2}(1-2p),1-p,\frac{1}{2}(3-2p),a^2x^2\right)}{1-2p}+\frac{a^2x^3(1-ax)^{-p}(ax+1)^{-p}\left(c-\right.}{}$$

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[(1 - 2*p)/2, 1 - p, (3 - 2*p)/2, a
^2*x^2])/((1 - 2*p)*(1 - a*x)^p*(1 + a*x)^p) + (a^2*(c - c/(a^2*x^2))^p*x^3
*Hypergeometric2F1[(3 - 2*p)/2, 1 - p, (5 - 2*p)/2, a^2*x^2])/((3 - 2*p)*(1
 - a*x)^p*(1 + a*x)^p) + (a*(c - c/(a^2*x^2))^p*x^2*Hypergeometric2F1[1 - p
, 1 - p, 2 - p, a^2*x^2])/((1 - p)*(1 - a*x)^p*(1 + a*x)^p)
```

Rubi [A] time = 0.260579, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6159, 6129, 127, 125, 364}

$$\frac{x(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\, {}_2F_1\left(\frac{1}{2}(1-2p),1-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}+\frac{a^2x^3(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\, {}_2F_1\left(\frac{1}{2}(3-\right.}{3-2p}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[(1 - 2*p)/2, 1 - p, (3 - 2*p)/2, a
^2*x^2])/((1 - 2*p)*(1 - a*x)^p*(1 + a*x)^p) + (a^2*(c - c/(a^2*x^2))^p*x^3
*Hypergeometric2F1[(3 - 2*p)/2, 1 - p, (5 - 2*p)/2, a^2*x^2])/((3 - 2*p)*(1
 - a*x)^p*(1 + a*x)^p) + (a*(c - c/(a^2*x^2))^p*x^2*Hypergeometric2F1[1 - p
, 1 - p, 2 - p, a^2*x^2])/((1 - p)*(1 - a*x)^p*(1 + a*x)^p)
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^n*(c + d*x)^n*(f*x)^p, (a + b
*x)^(m - n), x], x] /; FreeQ[{a, b, c, d, f, m, n, p}, x] && EqQ[b*c + a*d,
 0] && IGtQ[m - n, 0] && NeQ[m + n + p + 2, 0]
```

Rule 125

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
```

```
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0] && GtQ[a, 0] && GtQ[c, 0
]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx=\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int e^{2\tanh^{-1}(ax)}x^{-2p}(1-ax)^p(1+ax)^p\,dx$$
$$=\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}(1-ax)^{-1+p}(1+ax)^{1+p}\,dx$$
$$=\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int\left(2ax^{1-2p}(1-ax)^{-1+p}(1+ax)^{-1+p}+a^2x^{2-2p}(\right.$$
$$=\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}(1-ax)^{-1+p}(1+ax)^{-1+p}\,dx+\left(2a\left(c-\frac{}{a}\right.\right.$$
$$=\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}\left(1-a^2x^2\right)^{-1+p}dx+\left(2a\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\right.$$
$$=\frac{\left(c-\frac{c}{a^2x^2}\right)^p x(1-ax)^{-p}(1+ax)^{-p}\,{}_2F_1\left(\frac{1}{2}(1-2p),1-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}+\frac{a^2\left(c-\frac{c}{a^2x^2}\right.}{}$$

Mathematica [C] time = 0.0983864, size = 142, normalized size = 0.65

$$\frac{x(1-ax)^{-p}\left(-\left(a^2x^2-1\right)^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left((1-ax)^p\left(a^2x^2-1\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2}-p,-p,\frac{3}{2}-p,a^2x^2\right)-2(ax-\right.}{2p-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*(-2*(-1 + a*x)^p*(1 - a^2*x^2)^p*AppellF1[1 - 2*p, 1
 - p, -p, 2 - 2*p, a*x, -(a*x)] + (1 - a*x)^p*(-1 + a^2*x^2)^p*Hypergeometr
ic2F1[1/2 - p, -p, 3/2 - p, a^2*x^2]))/((-1 + 2*p)*(1 - a*x)^p*(-(-1 + a^2*
x^2)^2)^p)
```

Maple [F] time = 0.317, size = 0, normalized size = 0.

$$\int\frac{(ax+1)^2}{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^p,x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^p}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^p,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(c - c/(a^2*x^2))^p/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ax+1)\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^p,x, algorithm="fricas")
```

```
[Out] integral(-(a*x + 1)*((a^2*c*x^2 - c)/(a^2*x^2))^p/(a*x - 1), x)
```

Sympy [C] time = 11.4331, size = 697, normalized size = 3.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**p,x)
```

```
[Out] -a*Piecewise((0**p*x/a - 0**p*log(1/(a**2*x**2))/(2*a**2) + 0**p*log(-1 + 1
/(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 + a*a**(-2*p)*c**p*p*x**3
*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(p - 3/2)*hyper((1 - p, 3/2 - p), (5/2
 - p,), a**2*x**2)/(2*gamma(p - 1/2)*gamma(p + 1)) - a**(-2*p)*c**p*p*x**2*
x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,),
 a**2*x**2)/(2*gamma(2 - p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a
 - 0**p*log(1/(a**2*x**2))/(2*a**2) + 0**p*log(1 - 1/(a**2*x**2))/(2*a**2)
- 0**p*atanh(1/(a*x))/a**2 + a*a**(-2*p)*c**p*p*x**3*x**(-2*p)*exp(I*pi*p)*
gamma(p)*gamma(p - 3/2)*hyper((1 - p, 3/2 - p), (5/2 - p,), a**2*x**2)/(2*g
amma(p - 1/2)*gamma(p + 1)) - a**(-2*p)*c**p*p*x**2*x**(-2*p)*exp(I*pi*p)*g
amma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a**2*x**2)/(2*gamma(2
- p)*gamma(p + 1)), True)) - Piecewise((0**p*log(a**2*x**2 - 1)/(2*a) - 0**
p*acoth(a*x)/a - a*a**(-2*p)*c**p*p*x**2*x**(-2*p)*exp(I*pi*p)*gamma(p)*gam
ma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a**2*x**2)/(2*gamma(2 - p)*gamma(
p + 1)) + a**(-2*p)*c**p*p*x*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(p - 1/2)*
hyper((1 - p, 1/2 - p), (3/2 - p,), a**2*x**2)/(2*gamma(p + 1/2)*gamma(p +
1)), Abs(a**2*x**2) > 1), (0**p*log(-a**2*x**2 + 1)/(2*a) - 0**p*atanh(a*x)
/a - a*a**(-2*p)*c**p*p*x**2*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hy
per((1 - p, 1 - p), (2 - p,), a**2*x**2)/(2*gamma(2 - p)*gamma(p + 1)) + a*
*(-2*p)*c**p*p*x*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(p - 1/2)*hyper((1 - p
```

```
, 1/2 - p), (3/2 - p,), a**2*x**2)/(2*gamma(p + 1/2)*gamma(p + 1)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^p}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^p,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2*(c - c/(a^2*x^2))^p/(a^2*x^2 - 1), x)
```

3.804 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=137

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2}(1-2p),\frac{1}{2}-p,\frac{1}{2}(3-2p),a^2x^2\right)}{1-2p}+\frac{ax^2\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\text{Hyper}}{2}$$

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[(1 - 2*p)/2, 1/2 - p, (3 - 2*p)/2,
 a^2*x^2])/((1 - 2*p)*(1 - a^2*x^2)^p) + (a*(c - c/(a^2*x^2))^p*x^2*Hyperge
ometric2F1[1/2 - p, 1 - p, 2 - p, a^2*x^2])/(2*(1 - p)*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.132928, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6160, 6148, 808, 364}

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1\left(\frac{1}{2}(1-2p),\frac{1}{2}-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}+\frac{ax^2\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1\left(\frac{1}{2}-p,1-p;2-p;a^2\right.}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^p,x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[(1 - 2*p)/2, 1/2 - p, (3 - 2*p)/2,
 a^2*x^2])/((1 - 2*p)*(1 - a^2*x^2)^p) + (a*(c - c/(a^2*x^2))^p*x^2*Hyperge
ometric2F1[1/2 - p, 1 - p, 2 - p, a^2*x^2])/(2*(1 - p)*(1 - a^2*x^2)^p)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int e^{\tanh^{-1}(ax)}x^{-2p}\left(1-a^2x^2\right)^p dx$$
$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p} dx$$
$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}\left(1-a^2x^2\right)^{-\frac{1}{2}+p} dx + \left(a\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right.\right.$$
$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x\left(1-a^2x^2\right)^{-p}\ {}_2F_1\left(\frac{1}{2}(1-2p),\frac{1}{2}-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2\left(1-\right.}{}$$

Mathematica [A] time = 0.0418854, size = 112, normalized size = 0.82

$$-\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(2(p-1)\text{Hypergeometric2F1}\left(\frac{1}{2}-p,\frac{1}{2}-p,\frac{3}{2}-p,a^2x^2\right)+a(2p-1)x\text{Hypergeometric2}\right.}{2(p-1)(2p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^p,x]
```

```
[Out] -((c - c/(a^2*x^2))^p*x*(2*(-1 + p)*Hypergeometric2F1[1/2 - p, 1/2 - p, 3/2
 - p, a^2*x^2] + a*(-1 + 2*p)*x*Hypergeometric2F1[1/2 - p, 1 - p, 2 - p, a^
2*x^2]))/(2*(-1 + p)*(-1 + 2*p)*(1 - a^2*x^2)^p)
```

Maple [F] time = 0.211, size = 0, normalized size = 0.

$$\int (ax+1)\left(c-\frac{c}{a^2x^2}\right)^p \frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^p,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^p/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^p,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*((a^2*c*x^2 - c)/(a^2*x^2))^p/(a*x - 1), x)
```

Sympy [C] time = 16.9293, size = 178, normalized size = 1.3

$$\frac{ac^px^2\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,1\\2,p+1\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\Gamma\left(p+1\right)}+\frac{c^px\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}-\frac{1}{2},1,-p\\\frac{1}{2},\frac{1}{2}\end{matrix}\middle|\frac{e^{2i\pi}}{a^2x^2}\right)}{\sqrt{\pi}\Gamma\left(p+1\right)}+\frac{c^px\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},\frac{1}{2},1\\\frac{3}{2},p+1\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{\sqrt{\pi}\Gamma\left(p+1\right)}-\frac{c^pG_{3,}^{2,}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**p,x)
```

```
[Out] a*c**p*x**2*gamma(p + 1/2)*hyper((1/2, 1, 1), (2, p + 1), a**2*x**2*exp_pol
ar(2*I*pi))/(2*sqrt(pi)*gamma(p + 1)) + c**p*x*gamma(p + 1/2)*hyper((-1/2,
1, -p), (1/2, 1/2), exp_polar(2*I*pi)/(a**2*x**2))/(sqrt(pi)*gamma(p + 1))
+ c**p*x*gamma(p + 1/2)*hyper((1/2, 1/2, 1), (3/2, p + 1), a**2*x**2*exp_po
lar(2*I*pi))/(sqrt(pi)*gamma(p + 1)) - c**p*meijerg(((-1, p), (1,)), ((-1,
0), (-1/2,)), exp_polar(I*pi)/(a**2*x**2))*gamma(p + 1/2)/(2*a*gamma(-p)*ga
mma(p + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^p/sqrt(-a^2*x^2 + 1), x)
```

3.805 $\int e^{-\tanh^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right)^p dx$

Optimal. Leaf size=137

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2}(1-2p),\frac{1}{2}-p,\frac{1}{2}(3-2p),a^2x^2\right)}{1-2p}-\frac{ax^2\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p \text{Hyp}}{}$$

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[(1 - 2*p)/2, 1/2 - p, (3 - 2*p)/2,
 a^2*x^2])/((1 - 2*p)*(1 - a^2*x^2)^p) - (a*(c - c/(a^2*x^2))^p*x^2*Hyperge
ometric2F1[1/2 - p, 1 - p, 2 - p, a^2*x^2])/(2*(1 - p)*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.13672, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6160, 6149, 808, 364}

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1\left(\frac{1}{2}(1-2p),\frac{1}{2}-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}-\frac{ax^2\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1\left(\frac{1}{2}-p,1-p;2-p;\right.}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^p/E^ArcTanh[a*x],x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[(1 - 2*p)/2, 1/2 - p, (3 - 2*p)/2,
 a^2*x^2])/((1 - 2*p)*(1 - a^2*x^2)^p) - (a*(c - c/(a^2*x^2))^p*x^2*Hyperge
ometric2F1[1/2 - p, 1 - p, 2 - p, a^2*x^2])/(2*(1 - p)*(1 - a^2*x^2)^p)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int e^{-\tanh^{-1}(ax)}x^{-2p}\left(1-a^2x^2\right)^p dx$$
$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p} dx$$
$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}\left(1-a^2x^2\right)^{-\frac{1}{2}+p} dx - \left(a\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-}\right.$$
$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x\left(1-a^2x^2\right)^{-p} {}_2F_1\left(\frac{1}{2}(1-2p),\frac{1}{2}-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2\left(1-a\right.}{}$$

Mathematica [A] time = 0.0291181, size = 112, normalized size = 0.82

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(a(2p-1)x\text{Hypergeometric2F1}\left(\frac{1}{2}-p,1-p,2-p,a^2x^2\right)-2(p-1)\text{Hypergeometric2F1}\left(\frac{1}{2}\right.\right.}{2(p-1)(2p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^p/E^ArcTanh[a*x],x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*(-2*(-1 + p)*Hypergeometric2F1[1/2 - p, 1/2 - p, 3/2
 - p, a^2*x^2] + a*(-1 + 2*p)*x*Hypergeometric2F1[1/2 - p, 1 - p, 2 - p, a^
2*x^2]))/(2*(-1 + p)*(-1 + 2*p)*(1 - a^2*x^2)^p)
```

Maple [F] time = 0.324, size = 0, normalized size = 0.

$$\int \frac{1}{ax+1}\left(c-\frac{c}{a^2x^2}\right)^p\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int((c-c/a^2/x^2)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^p/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*((a^2*c*x^2 - c)/(a^2*x^2))^p/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p/(a
*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^p/(a*x + 1), x)
```

3.806 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=218

$$\frac{x(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2}(1-2p),1-p,\frac{1}{2}(3-2p),a^2x^2\right)}{1-2p}+\frac{a^2x^3(1-ax)^{-p}(ax+1)^{-p}\left(c-\right.}{}$$

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[(1 - 2*p)/2, 1 - p, (3 - 2*p)/2, a
^2*x^2])/((1 - 2*p)*(1 - a*x)^p*(1 + a*x)^p) + (a^2*(c - c/(a^2*x^2))^p*x^3
*Hypergeometric2F1[(3 - 2*p)/2, 1 - p, (5 - 2*p)/2, a^2*x^2])/((3 - 2*p)*(1
 - a*x)^p*(1 + a*x)^p) - (a*(c - c/(a^2*x^2))^p*x^2*Hypergeometric2F1[1 - p
, 1 - p, 2 - p, a^2*x^2])/((1 - p)*(1 - a*x)^p*(1 + a*x)^p)
```

Rubi [A] time = 0.263655, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6159, 6129, 127, 125, 364}

$$\frac{x(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1\left(\frac{1}{2}(1-2p),1-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}+\frac{a^2x^3(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1\left(\frac{1}{2}(3-\right.}{3-2p}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^p/E^(2*ArcTanh[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[(1 - 2*p)/2, 1 - p, (3 - 2*p)/2, a
^2*x^2])/((1 - 2*p)*(1 - a*x)^p*(1 + a*x)^p) + (a^2*(c - c/(a^2*x^2))^p*x^3
*Hypergeometric2F1[(3 - 2*p)/2, 1 - p, (5 - 2*p)/2, a^2*x^2])/((3 - 2*p)*(1
 - a*x)^p*(1 + a*x)^p) - (a*(c - c/(a^2*x^2))^p*x^2*Hypergeometric2F1[1 - p
, 1 - p, 2 - p, a^2*x^2])/((1 - p)*(1 - a*x)^p*(1 + a*x)^p)
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^n*(c + d*x)^n*(f*x)^p, (a + b
*x)^(m - n), x], x] /; FreeQ[{a, b, c, d, f, m, n, p}, x] && EqQ[b*c + a*d,
 0] && IGtQ[m - n, 0] && NeQ[m + n + p + 2, 0]
```

Rule 125

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
```

```
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0] && GtQ[a, 0] && GtQ[c, 0
]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int e^{-2\tanh^{-1}(ax)}x^{-2p}(1-ax)^p(1+ax)^p\,dx$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}(1-ax)^{1+p}(1+ax)^{-1+p}\,dx$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int \left(-2ax^{1-2p}(1-ax)^{-1+p}(1+ax)^{-1+p}+a^2x^{2-2}\right.$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}(1-ax)^{-1+p}(1+ax)^{-1+p}\,dx-\left(2a\left(c-\right.\right.$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}\left(1-a^2x^2\right)^{-1+p}dx-\left(2a\left(c-\frac{c}{a^2x^2}\right)^p x^2\right.$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x(1-ax)^{-p}(1+ax)^{-p}\,{}_2F_1\left(\frac{1}{2}(1-2p),1-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}+\frac{a^2\left(c-\frac{c}{a^2x^2}\right.}{}$$

Mathematica [C] time = 0.101709, size = 142, normalized size = 0.65

$$\frac{x(1-ax)^{-p}\left(-\left(a^2x^2-1\right)^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left((1-ax)^p\left(a^2x^2-1\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2}-p,-p,\frac{3}{2}-p,a^2x^2\right)-2(ax-\right.}{2p-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^p/E^(2*ArcTanh[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*(-2*(-1 + a*x)^p*(1 - a^2*x^2)^p*AppellF1[1 - 2*p, -
p, 1 - p, 2 - 2*p, a*x, -(a*x)] + (1 - a*x)^p*(-1 + a^2*x^2)^p*Hypergeometr
ic2F1[1/2 - p, -p, 3/2 - p, a^2*x^2]))/((-1 + 2*p)*(1 - a*x)^p*(-(-1 + a^2*
x^2)^2)^p)
```

Maple [F] time = 0.219, size = 0, normalized size = 0.

$$\int \frac{-a^2x^2+1}{(ax+1)^2}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^p/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] int((c-c/a^2/x^2)^p/(a*x+1)^2*(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^p}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a^2*x^2))^p/(a*x + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(ax-1)\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-(a*x - 1)*((a^2*c*x^2 - c)/(a^2*x^2))^p/(a*x + 1), x)
```

Sympy [C] time = 12.5377, size = 695, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**p/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -a*Piecewise((0**p*x/a + 0**p*log(1/(a**2*x**2))/(2*a**2) - 0**p*log(-1 + 1
/(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 + a*a**(-2*p)*c**p*p*x**3
*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(p - 3/2)*hyper((1 - p, 3/2 - p), (5/2
 - p,), a**2*x**2)/(2*gamma(p - 1/2)*gamma(p + 1)) + a**(-2*p)*c**p*p*x**2*
x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,),
 a**2*x**2)/(2*gamma(2 - p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a
 + 0**p*log(1/(a**2*x**2))/(2*a**2) - 0**p*log(1 - 1/(a**2*x**2))/(2*a**2)
- 0**p*atanh(1/(a*x))/a**2 + a*a**(-2*p)*c**p*p*x**3*x**(-2*p)*exp(I*pi*p)*
gamma(p)*gamma(p - 3/2)*hyper((1 - p, 3/2 - p), (5/2 - p,), a**2*x**2)/(2*g
amma(p - 1/2)*gamma(p + 1)) + a**(-2*p)*c**p*p*x**2*x**(-2*p)*exp(I*pi*p)*g
amma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a**2*x**2)/(2*gamma(2
- p)*gamma(p + 1)), True)) + Piecewise((0**p*log(a**2*x**2 - 1)/(2*a) + 0**
p*acoth(a*x)/a - a*a**(-2*p)*c**p*p*x**2*x**(-2*p)*exp(I*pi*p)*gamma(p)*gam
ma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a**2*x**2)/(2*gamma(2 - p)*gamma(
p + 1)) - a**(-2*p)*c**p*p*x*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(p - 1/2)*
hyper((1 - p, 1/2 - p), (3/2 - p,), a**2*x**2)/(2*gamma(p + 1/2)*gamma(p +
1)), Abs(a**2*x**2) > 1), (0**p*log(-a**2*x**2 + 1)/(2*a) + 0**p*atanh(a*x)
/a - a*a**(-2*p)*c**p*p*x**2*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hy
per((1 - p, 1 - p), (2 - p,), a**2*x**2)/(2*gamma(2 - p)*gamma(p + 1)) - a*
*(-2*p)*c**p*p*x*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(p - 1/2)*hyper((1 - p
```

```
, 1/2 - p), (3/2 - p,), a**2*x**2)/(2*gamma(p + 1/2)*gamma(p + 1)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^p}{\left(ax+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*(c - c/(a^2*x^2))^p/(a*x + 1)^2, x)
```

3.807 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=216

$$\frac{3a^2x^3\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2}(3-2p),\frac{3}{2}-p,\frac{1}{2}(5-2p),a^2x^2\right)}{3-2p}-\frac{a(5-2p)x^2\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2\right.}}{}$$

```
[Out] ((c - c/(a^2*x^2))^p*x)/((1 - 2*p)*Sqrt[1 - a^2*x^2]) + (a*(c - c/(a^2*x^2)
)^p*x^2)/Sqrt[1 - a^2*x^2] + (3*a^2*(c - c/(a^2*x^2))^p*x^3*Hypergeometric2
F1[(3 - 2*p)/2, 3/2 - p, (5 - 2*p)/2, a^2*x^2])/((3 - 2*p)*(1 - a^2*x^2)^p)
 - (a*(5 - 2*p)*(c - c/(a^2*x^2))^p*x^2*Hypergeometric2F1[1 - p, 3/2 - p, 2
 - p, a^2*x^2])/(2*(1 - p)*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.321789, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6160, 6149, 1809, 1808, 364, 807}

$$\frac{3a^2x^3\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p{}_2F_1\left(\frac{1}{2}(3-2p),\frac{3}{2}-p;\frac{1}{2}(5-2p);a^2x^2\right)}{3-2p}-\frac{a(5-2p)x^2\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p{}_2F_1\left(1-p,\frac{3}{2}\right.}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^p/E^(3*ArcTanh[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x)/((1 - 2*p)*Sqrt[1 - a^2*x^2]) + (a*(c - c/(a^2*x^2)
)^p*x^2)/Sqrt[1 - a^2*x^2] + (3*a^2*(c - c/(a^2*x^2))^p*x^3*Hypergeometric2
F1[(3 - 2*p)/2, 3/2 - p, (5 - 2*p)/2, a^2*x^2])/((3 - 2*p)*(1 - a^2*x^2)^p)
 - (a*(5 - 2*p)*(c - c/(a^2*x^2))^p*x^2*Hypergeometric2F1[1 - p, 3/2 - p, 2
 - p, a^2*x^2])/(2*(1 - p)*(1 - a^2*x^2)^p)
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 1808

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> With
[{q = Expon[Pq, x]}, Dist[Coeff[Pq, x, q]/c^q, Int[(c*x)^(m + q)*(a + b*x^2
)^p, x], x] + Dist[1/c^q, Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[c^q*Pq - Co
eff[Pq, x, q]*(c*x)^q, x], x], x] /; EqQ[q, 1] || EqQ[m + q + 2*p + 1, 0]]
/; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] &&  !(IGtQ[m, 0] && ILtQ[p + 1
/2, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rubi steps

$$\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx &= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int e^{-3\tanh^{-1}(ax)}x^{-2p}\left(1-a^2x^2\right)^p dx\\
&= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}(1-ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p} dx\\
&= \frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{\sqrt{1-a^2x^2}}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\left(-a^2+a^3(5-\right.}{a^2}\\
&= \frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{\sqrt{1-a^2x^2}}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}\left(-a^2+a^3(5-2p)x\right)\left(1-a^2x^2\right.}{a^2}\\
&= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x}{(1-2p)\sqrt{1-a^2x^2}}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{\sqrt{1-a^2x^2}}+\frac{3a^2\left(c-\frac{c}{a^2x^2}\right)^p x^3\left(1-a^2x^2\right)^{-p}\ {}_2F_1\left(\frac{1}{2}(3-2p\right.}{3-2p}\\
&= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x}{(1-2p)\sqrt{1-a^2x^2}}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{\sqrt{1-a^2x^2}}+\frac{3a^2\left(c-\frac{c}{a^2x^2}\right)^p x^3\left(1-a^2x^2\right)^{-p}\ {}_2F_1\left(\frac{1}{2}(3-2p\right.}{3-2p}
\end{aligned}$$

Mathematica [A] time = 0.138352, size = 173, normalized size = 0.8

$$\frac{1}{2}x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{3ax\text{Hypergeometric2F1}\left(1-p,\frac{3}{2}-p,2-p,a^2x^2\right)}{p-1}+\frac{6a^2x^2\text{Hypergeometric2F1}\left(\frac{3}{2}-\right.}{3-2p}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^p/E^(3*ArcTanh[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*((2*(1 - a^2*x^2)^(-1/2 + p))/(1 - 2*p) + (3*a*x*Hyp
ergeometric2F1[1 - p, 3/2 - p, 2 - p, a^2*x^2])/(-1 + p) + (6*a^2*x^2*Hyper
```

```
geometric2F1[3/2 - p, 3/2 - p, 5/2 - p, a^2*x^2])/(3 - 2*p) + (a^3*x^3*Hype
rgeometric2F1[3/2 - p, 2 - p, 3 - p, a^2*x^2])/(-2 + p)))/(2*(1 - a^2*x^2)^
p)
```

Maple [F] time = 0.336, size = 0, normalized size = 0.

$$\int \frac{1}{(ax+1)^3}\left(c-\frac{c}{a^2x^2}\right)^p\left(-a^2x^2+1\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int((c-c/a^2/x^2)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^p}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^p/(a*x + 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}(ax-1)\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{a^2x^2+2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(a*x - 1)*((a^2*c*x^2 - c)/(a^2*x^2))^p/(a^2*x
^2 + 2*a*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**p/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^p}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^p/(a*x + 1)^3, x)
```

3.808 $\int e^{\tanh^{-1}(x)}x\sqrt{1+x}\sin(x)\,dx$

Optimal. Leaf size=240

$$-2\sqrt{2\pi}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\frac{3}{2}\sqrt{\frac{\pi}{2}}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-3\sqrt{\frac{\pi}{2}}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-3$$

```
[Out] 3*Sqrt[1 - x]*Cos[x] - (1 - x)^(3/2)*Cos[x] - 3*Sqrt[Pi/2]*Cos[1]*FresnelC[
Sqrt[2/Pi]*Sqrt[1 - x]] - (3*Sqrt[Pi/2]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 -
 x]])/2 + 2*Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]] + (3*Sqrt[Pi
/2]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]]*Sin[1])/2 - 2*Sqrt[2*Pi]*FresnelC[Sqrt
[2/Pi]*Sqrt[1 - x]]*Sin[1] - 3*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]]*
Sin[1] - (3*Sqrt[1 - x]*Sin[x])/2
```

Rubi [A] time = 0.396753, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6129, 6742, 3353, 3352, 3351, 3385, 3354, 3386}

$$-2\sqrt{2\pi}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\frac{3}{2}\sqrt{\frac{\pi}{2}}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-3\sqrt{\frac{\pi}{2}}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-3$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*x*Sqrt[1 + x]*Sin[x],x]
```

```
[Out] 3*Sqrt[1 - x]*Cos[x] - (1 - x)^(3/2)*Cos[x] - 3*Sqrt[Pi/2]*Cos[1]*FresnelC[
Sqrt[2/Pi]*Sqrt[1 - x]] - (3*Sqrt[Pi/2]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 -
 x]])/2 + 2*Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]] + (3*Sqrt[Pi
/2]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]]*Sin[1])/2 - 2*Sqrt[2*Pi]*FresnelC[Sqrt
[2/Pi]*Sqrt[1 - x]]*Sin[1] - 3*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]]*
Sin[1] - (3*Sqrt[1 - x]*Sin[x])/2
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(x)} x\sqrt{1+x}\sin(x)\,dx &= \int \frac{x(1+x)\sin(x)}{\sqrt{1-x}}\,dx\\
&= -\left(2\,\text{Subst}\left(\int \left(-2+x^2\right)\left(-1+x^2\right)\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)\right)\\
&= -\left(2\,\text{Subst}\left(\int \left(2\sin\left(1-x^2\right)-3x^2\sin\left(1-x^2\right)+x^4\sin\left(1-x^2\right)\right)\,dx, x, \sqrt{1-x}\right)\right)\\
&= -\left(2\,\text{Subst}\left(\int x^4\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)\right)-4\,\text{Subst}\left(\int \sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right.\\
&= 3\sqrt{1-x}\cos(x)-(1-x)^{3/2}\cos(x)-3\,\text{Subst}\left(\int \cos\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)+3\,\text{Subs}\\
&= 3\sqrt{1-x}\cos(x)-(1-x)^{3/2}\cos(x)+2\sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-2\sqrt{2\pi}C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-}\right.\\
&= 3\sqrt{1-x}\cos(x)-(1-x)^{3/2}\cos(x)-3\sqrt{\frac{\pi}{2}}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+2\sqrt{2\pi}\cos(1)S\left(\sqrt{\right.\\
&= 3\sqrt{1-x}\cos(x)-(1-x)^{3/2}\cos(x)-3\sqrt{\frac{\pi}{2}}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\frac{3}{2}\sqrt{\frac{\pi}{2}}\cos(1)S\left(\sqrt{\right.
\end{aligned}$$

Mathematica [C] time = 8.95077, size = 184, normalized size = 0.77

$$-\frac{\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{x+1}\left((\cos(x+1)-i\sin(x+1))\left((2+2i)\left(2x^2+(2-3i)x-(4-3i)\right)(\cos(1)+i\sin(1))-(6+5i)\sqrt{2\pi}\sqrt{.}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*x*Sqrt[1 + x]*Sin[x],x]
```

```
[Out] ((-1/16 + I/16)*Sqrt[1 + x]*((-6 + 5*I)*Sqrt[2*Pi]*Sqrt[-1 + x]*Erfi[((1 +
I)*Sqrt[-1 + x])/Sqrt[2]]*(Cos[1] + I*Sin[1]) + (2 + 2*I)*((-4 - 3*I) + (2
```

```
+ 3*I)*x + 2*x^2)*(Cos[x] + I*Sin[x]) + ((2 + 2*I)*((-4 + 3*I) + (2 - 3*I)*
x + 2*x^2)*(Cos[1] + I*Sin[1]) - (6 + 5*I)*Sqrt[2*Pi]*Sqrt[-1 + x]*Erf[((1
+ I)*Sqrt[-1 + x])/Sqrt[2]]*(Cos[x] + I*Sin[x]))*(Cos[1 + x] - I*Sin[1 + x]
)))/Sqrt[1 - x^2]
```

Maple [F] time = 0.276, size = 0, normalized size = 0.

$$\int x \sin(x) (1+x)^{\frac{3}{2}} \frac{1}{\sqrt{-x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(-x^2+1)^(1/2)*x*sin(x),x)

[Out] int((1+x)^(3/2)/(-x^2+1)^(1/2)*x*sin(x),x)
```

Maxima [C] time = 1.31122, size = 863, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="maxima")

[Out] -1/2*((((2*I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) - 2*I*sqrt(pi)*(erf(sqrt(-I*
x + I)) - 1))*cos(1) + 2*(sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + sqrt(pi)*(erf
(sqrt(-I*x + I)) - 1))*sin(1))*cos(1/2*arctan2(x - 1, 0)) + (2*(sqrt(pi)*(e
rf(sqrt(I*x - I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*cos(1) + (-2*I
*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*x + I)) - 1)
)*sin(1))*sin(1/2*arctan2(x - 1, 0)))*(x - 1)^2 + (((-I*cos(1) - sin(1))*ga
mma(5/2, I*x - I) + (I*cos(1) - sin(1))*gamma(5/2, -I*x + I))*cos(5/2*arcta
n2(x - 1, 0)) - ((cos(1) - I*sin(1))*gamma(5/2, I*x - I) + (cos(1) + I*sin(
1))*gamma(5/2, -I*x + I))*sin(5/2*arctan2(x - 1, 0)))*x^2 - ((3*((I*cos(1)
+ sin(1))*gamma(3/2, I*x - I) + (-I*cos(1) + sin(1))*gamma(3/2, -I*x + I))*
cos(3/2*arctan2(x - 1, 0)) + ((3*cos(1) - 3*I*sin(1))*gamma(3/2, I*x - I) +
 (3*cos(1) + 3*I*sin(1))*gamma(3/2, -I*x + I))*sin(3/2*arctan2(x - 1, 0)))*
abs(x - 1) + 2*((-I*cos(1) - sin(1))*gamma(5/2, I*x - I) + (I*cos(1) - sin(
1))*gamma(5/2, -I*x + I))*cos(5/2*arctan2(x - 1, 0)) - ((2*cos(1) - 2*I*sin
(1))*gamma(5/2, I*x - I) + (2*cos(1) + 2*I*sin(1))*gamma(5/2, -I*x + I))*si
n(5/2*arctan2(x - 1, 0)))*x - (3*((-I*cos(1) - sin(1))*gamma(3/2, I*x - I)
+ (I*cos(1) - sin(1))*gamma(3/2, -I*x + I))*cos(3/2*arctan2(x - 1, 0)) - ((
3*cos(1) - 3*I*sin(1))*gamma(3/2, I*x - I) + (3*cos(1) + 3*I*sin(1))*gamma(
3/2, -I*x + I))*sin(3/2*arctan2(x - 1, 0)))*abs(x - 1) + ((-I*cos(1) - sin(
1))*gamma(5/2, I*x - I) + (I*cos(1) - sin(1))*gamma(5/2, -I*x + I))*cos(5/2
*arctan2(x - 1, 0)) - ((cos(1) - I*sin(1))*gamma(5/2, I*x - I) + (cos(1) +
I*sin(1))*gamma(5/2, -I*x + I))*sin(5/2*arctan2(x - 1, 0)))*sqrt(-x + 1)/((
x - 1)^2*sqrt(abs(x - 1)))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^2+1}\sqrt{x+1}x\sin(x)}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*sqrt(x + 1)*x*sin(x)/(x - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(-x**2+1)**(1/2)*x*sin(x),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.23661, size = 167, normalized size = 0.7

$$-\left(\frac{11}{16}i-\frac{1}{16}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{i}+\left(\frac{11}{16}i+\frac{1}{16}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{(-i)}-\frac{1}{4}i\left(-2i(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="giac")
```

```
[Out] -(11/16*I - 1/16)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(-x + 1))
*e^I + (11/16*I + 1/16)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(-x
+ 1))*e^(-I) - 1/4*I*(-2*I*(-x + 1)^(3/2) + (4*I - 3)*sqrt(-x + 1))*e^(I*x)
 - 1/4*I*(-2*I*(-x + 1)^(3/2) + (4*I + 3)*sqrt(-x + 1))*e^(-I*x) + 1/2*sqrt
(-x + 1)*e^(I*x) + 1/2*sqrt(-x + 1)*e^(-I*x) + 1.79526793396
```

3.809 $\int e^{\tanh^{-1}(x)}\sqrt{1+x}\sin(x)\,dx$

Optimal. Leaf size=141

$$-2\sqrt{2\pi}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{\frac{\pi}{2}}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+2\sqrt{2\pi}\cos(1)S$$

```
[Out] Sqrt[1 - x]*Cos[x] - Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]] + 2
*Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]] - 2*Sqrt[2*Pi]*FresnelC
[Sqrt[2/Pi]*Sqrt[1 - x]]*Sin[1] - Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x
]]*Sin[1]
```

Rubi [A] time = 0.170036, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6129, 6742, 3353, 3352, 3351, 3385, 3354}

$$-2\sqrt{2\pi}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{\frac{\pi}{2}}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+2\sqrt{2\pi}\cos(1)S$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*Sqrt[1 + x]*Sin[x],x]
```

```
[Out] Sqrt[1 - x]*Cos[x] - Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]] + 2
*Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]] - 2*Sqrt[2*Pi]*FresnelC
[Sqrt[2/Pi]*Sqrt[1 - x]]*Sin[1] - Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x
]]*Sin[1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(x)}\sqrt{1+x}\sin(x)\,dx &= \int \frac{(1+x)\sin(x)}{\sqrt{1-x}}\,dx \\
&= 2\,\text{Subst}\left(\int \left(-2+x^2\right)\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right) \\
&= 2\,\text{Subst}\left(\int \left(-2\sin\left(1-x^2\right)+x^2\sin\left(1-x^2\right)\right)\,dx, x, \sqrt{1-x}\right) \\
&= 2\,\text{Subst}\left(\int x^2\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)-4\,\text{Subst}\left(\int \sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right) \\
&= \sqrt{1-x}\cos(x)+(4\cos(1))\,\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{1-x}\right)-(4\sin(1))\,\text{Subst}\left(\int \cos
\right. \\
&= \sqrt{1-x}\cos(x)+2\sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-2\sqrt{2\pi}C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)\sin(1)-\cos(1)\,\text{S} \\
&= \sqrt{1-x}\cos(x)-\sqrt{\frac{\pi}{2}}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+2\sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-2\sqrt{2\pi}C\left(\right.
\end{aligned}$$

Mathematica [C] time = 7.98664, size = 134, normalized size = 0.95

$$\left(\frac{1}{8}+\frac{i}{8}\right)\left(\frac{e^{-ix}\sqrt{1-x^2}\left((4+i)\sqrt{2\pi}e^{i(x+1)}\text{Erfi}\left(\frac{(1+i)\sqrt{x-1}}{\sqrt{2}}\right)+(2-2i)\sqrt{x-1}\left(1+e^{2ix}\right)\right)}{\sqrt{x-1}\sqrt{x+1}}-(4-i)e^{-i}\sqrt{2\pi}\text{Erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{2}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*Sqrt[1 + x]*Sin[x],x]
```

```
[Out] (1/8 + I/8)*(((-4 + I)*Sqrt[2*Pi]*Erfi[(1/2 + I/2)*Sqrt[2 - 2*x]])/E^I + (S
qrt[1 - x^2]*((2 - 2*I)*(1 + E^((2*I)*x))*Sqrt[-1 + x] + (4 + I)*E^(I*(1 +
x))*Sqrt[2*Pi]*Erfi[((1 + I)*Sqrt[-1 + x])/Sqrt[2]]))/(E^(I*x)*Sqrt[-1 + x]
*Sqrt[1 + x]))
```

Maple [F] time = 0.277, size = 0, normalized size = 0.

$$\int \sin(x)\,(1+x)^{\frac{3}{2}}\frac{1}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(-x^2+1)^(1/2)*sin(x),x)
```

```
[Out] int((1+x)^(3/2)/(-x^2+1)^(1/2)*sin(x),x)
```

Maxima [C] time = 1.21529, size = 471, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*((((-I*cos(1) - sin(1))*gamma(3/2, I*x - I) + (I*cos(1) - sin(1))*gamm
a(3/2, -I*x + I))*cos(3/2*arctan2(x - 1, 0)) - ((cos(1) - I*sin(1))*gamma(3
/2, I*x - I) + (cos(1) + I*sin(1))*gamma(3/2, -I*x + I))*sin(3/2*arctan2(x
- 1, 0)))*x + (((2*I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) - 2*I*sqrt(pi)*(erf(
sqrt(-I*x + I)) - 1))*cos(1) + 2*(sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + sqrt(
pi)*(erf(sqrt(-I*x + I)) - 1))*sin(1))*cos(1/2*arctan2(x - 1, 0)) + (2*(sqr
t(pi)*(erf(sqrt(I*x - I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*cos(1)
 + (-2*I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*x +
I)) - 1))*sin(1))*sin(1/2*arctan2(x - 1, 0)))*abs(x - 1) + ((I*cos(1) + sin
(1))*gamma(3/2, I*x - I) + (-I*cos(1) + sin(1))*gamma(3/2, -I*x + I))*cos(3
/2*arctan2(x - 1, 0)) + ((cos(1) - I*sin(1))*gamma(3/2, I*x - I) + (cos(1)
+ I*sin(1))*gamma(3/2, -I*x + I))*sin(3/2*arctan2(x - 1, 0)))*sqrt(-x + 1)*
sqrt(abs(x - 1))/(x - 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-x^2+1}\sqrt{x+1}\sin(x)}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*sqrt(x + 1)*sin(x)/(x - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(-x**2+1)**(1/2)*sin(x),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.224, size = 100, normalized size = 0.71

$$-\left(\frac{5}{8}i+\frac{3}{8}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{i}+\left(\frac{5}{8}i-\frac{3}{8}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{(-i)}+\frac{1}{2}\sqrt{-x+1}e^{(ix)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="giac")
```

```
[Out] -(5/8*I + 3/8)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(-x + 1))*e^
I + (5/8*I - 3/8)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(-x + 1))*
e^(-I) + 1/2*sqrt(-x + 1)*e^(I*x) + 1/2*sqrt(-x + 1)*e^(-I*x) + 1.992845037
43
```

3.810 $\int e^{\tanh^{-1}(x)}\sqrt{1-xx}\sin(x)\,dx$

Optimal. Leaf size=163

$$\frac{3}{2}\sqrt{\frac{\pi}{2}}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\sqrt{\frac{\pi}{2}}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\frac{3}{2}\sqrt{\frac{\pi}{2}}\cos(1)S$$

```
[Out] Sqrt[1 + x]*Cos[x] - (1 + x)^(3/2)*Cos[x] - Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt
[2/Pi]*Sqrt[1 + x]] - (3*Sqrt[Pi/2]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]]
)/2 + (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[1 + x]]*Sin[1])/2 - Sqrt[Pi/2]
*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]]*Sin[1] + (3*Sqrt[1 + x]*Sin[x])/2
```

Rubi [A] time = 0.297927, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {6129, 6742, 3385, 3354, 3352, 3351, 3386, 3353}

$$\frac{3}{2}\sqrt{\frac{\pi}{2}}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\sqrt{\frac{\pi}{2}}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\frac{3}{2}\sqrt{\frac{\pi}{2}}\cos(1)S$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*Sqrt[1 - x]*x*Sin[x],x]
```


```
[Out] Sqrt[1 + x]*Cos[x] - (1 + x)^(3/2)*Cos[x] - Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt
[2/Pi]*Sqrt[1 + x]] - (3*Sqrt[Pi/2]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]]
)/2 + (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[1 + x]]*Sin[1])/2 - Sqrt[Pi/2]
*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]]*Sin[1] + (3*Sqrt[1 + x]*Sin[x])/2
```


Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\int e^{\tanh^{-1}(x)}\sqrt{1-x}x\sin(x)\,dx = \int x\sqrt{1+x}\sin(x)\,dx$$
$$= -\left(2\,\text{Subst}\left(\int x^2\left(-1+x^2\right)\sin\left(1-x^2\right)\,dx, x, \sqrt{1+x}\right)\right)$$
$$= -\left(2\,\text{Subst}\left(\int\left(-x^2\sin\left(1-x^2\right)+x^4\sin\left(1-x^2\right)\right)\,dx, x, \sqrt{1+x}\right)\right)$$
$$= 2\,\text{Subst}\left(\int x^2\sin\left(1-x^2\right)\,dx, x, \sqrt{1+x}\right) - 2\,\text{Subst}\left(\int x^4\sin\left(1-x^2\right)\,dx, x, \sqrt{1+x}\right.$$
$$= \sqrt{1+x}\cos(x) - (1+x)^{3/2}\cos(x) + 3\,\text{Subst}\left(\int x^2\cos\left(1-x^2\right)\,dx, x, \sqrt{1+x}\right) - \text{Subs}$$
$$= \sqrt{1+x}\cos(x) - (1+x)^{3/2}\cos(x) + \frac{3}{2}\sqrt{1+x}\sin(x) + \frac{3}{2}\,\text{Subst}\left(\int\sin\left(1-x^2\right)\,dx, x,\right.$$
$$= \sqrt{1+x}\cos(x) - (1+x)^{3/2}\cos(x) - \sqrt{\frac{\pi}{2}}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right) - \sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right)$$
$$= \sqrt{1+x}\cos(x) - (1+x)^{3/2}\cos(x) - \sqrt{\frac{\pi}{2}}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right) - \frac{3}{2}\sqrt{\frac{\pi}{2}}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\right.$$

Mathematica [C] time = 8.52325, size = 168, normalized size = 1.03

$$\frac{\left(\frac{1}{16}+\frac{i}{16}\right)e^{-i(x+1)}\sqrt{1-x}\left(e^i\left((3-2i)\sqrt{2\pi}e^{i(x+1)}\sqrt{-x-1}\text{Erfi}\left(\frac{(1+i)\sqrt{-x-1}}{\sqrt{2}}\right)+(2+2i)\left(e^{2ix}(-3+2ix)+2ix+3\right)(x+1)\right)-}{\sqrt{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*Sqrt[1 - x]*x*Sin[x],x]
```

```
[Out] ((1/16 + I/16)*Sqrt[1 - x]*((-3 - 2*I)*E^(I*x)*Sqrt[2*Pi]*Sqrt[-1 - x]*Erf[
((1 + I)*Sqrt[-1 - x])/Sqrt[2]] + E^I*((2 + 2*I)*(3 + E^((2*I)*x)*(-3 + (2*
I)*x) + (2*I)*x)*(1 + x) + (3 - 2*I)*E^(I*(1 + x))*Sqrt[2*Pi]*Sqrt[-1 - x]*
Erfi[((1 + I)*Sqrt[-1 - x])/Sqrt[2]])))/(E^(I*(1 + x))*Sqrt[1 - x^2])
```

Maple [F] time = 0.305, size = 0, normalized size = 0.

$$\int x(1+x)\sin(x)\sqrt{1-x}\frac{1}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x*sin(x),x)
```

```
[Out] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x*sin(x),x)
```

Maxima [C] time = 1.46475, size = 1227, normalized size = 7.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(((I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*x - I
)) - 1))*cos(1) - (sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(-
I*x - I)) - 1))*sin(1))*abs(x + 1)*cos(1/2*arctan2(x + 1, 0)) + ((sqrt(pi)*
(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1) + (I*
sqrt(pi)*(erf(sqrt(I*x + I)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*s
in(1))*abs(x + 1)*sin(1/2*arctan2(x + 1, 0)) + (((I*cos(1) - sin(1))*gamma(
3/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(3/2, -I*x - I))*x + (I*cos(1) -
sin(1))*gamma(3/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(3/2, -I*x - I))*co
s(3/2*arctan2(x + 1, 0)) + (((cos(1) + I*sin(1))*gamma(3/2, I*x + I) + (cos
(1) - I*sin(1))*gamma(3/2, -I*x - I))*x + (cos(1) + I*sin(1))*gamma(3/2, I*
x + I) + (cos(1) - I*sin(1))*gamma(3/2, -I*x - I))*sin(3/2*arctan2(x + 1, 0
)))*sqrt(abs(x + 1))/(x + 1)^(3/2) - 1/2*(((-I*sqrt(pi)*(erf(sqrt(I*x + I))
 - 1) + I*sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1) + (sqrt(pi)*(erf(sqrt(
I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin(1))*(x + 1)^2*cos(
1/2*arctan2(x + 1, 0)) - ((sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(er
f(sqrt(-I*x - I)) - 1))*cos(1) - (-I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + I*
sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin(1))*(x + 1)^2*sin(1/2*arctan2(x + 1
, 0)) - 2*(((I*cos(1) - sin(1))*gamma(3/2, I*x + I) + (-I*cos(1) - sin(1))*
gamma(3/2, -I*x - I))*x + (I*cos(1) - sin(1))*gamma(3/2, I*x + I) + (-I*cos
(1) - sin(1))*gamma(3/2, -I*x - I))*abs(x + 1)*cos(3/2*arctan2(x + 1, 0)) -
 (((2*cos(1) + 2*I*sin(1))*gamma(3/2, I*x + I) + (2*cos(1) - 2*I*sin(1))*ga
mma(3/2, -I*x - I))*x + (2*cos(1) + 2*I*sin(1))*gamma(3/2, I*x + I) + (2*co
s(1) - 2*I*sin(1))*gamma(3/2, -I*x - I))*abs(x + 1)*sin(3/2*arctan2(x + 1,
0)) + (((I*cos(1) - sin(1))*gamma(5/2, I*x + I) + (-I*cos(1) - sin(1))*gamm
a(5/2, -I*x - I))*x^2 - 2*((-I*cos(1) + sin(1))*gamma(5/2, I*x + I) + (I*co
s(1) + sin(1))*gamma(5/2, -I*x - I))*x + (I*cos(1) - sin(1))*gamma(5/2, I*x
 + I) + (-I*cos(1) - sin(1))*gamma(5/2, -I*x - I))*cos(5/2*arctan2(x + 1, 0
)) + (((cos(1) + I*sin(1))*gamma(5/2, I*x + I) + (cos(1) - I*sin(1))*gamma(
5/2, -I*x - I))*x^2 + ((2*cos(1) + 2*I*sin(1))*gamma(5/2, I*x + I) + (2*cos
(1) - 2*I*sin(1))*gamma(5/2, -I*x - I))*x + (cos(1) + I*sin(1))*gamma(5/2,
I*x + I) + (cos(1) - I*sin(1))*gamma(5/2, -I*x - I))*sin(5/2*arctan2(x + 1,
 0)))/((x + 1)^(3/2)*sqrt(abs(x + 1)))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^2+1}x\sqrt{-x+1}\sin(x)}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x*sin(x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*x*sqrt(-x + 1)*sin(x)/(x - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*(1-x)**(1/2)*x*sin(x),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.2461, size = 146, normalized size = 0.9

$$\left(\frac{1}{16}i+\frac{5}{16}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{i}-\left(\frac{1}{16}i-\frac{5}{16}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{(-i)}+\frac{1}{4}i\left(2i\,(x+1)^{\frac{3}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x*sin(x),x, algorithm="giac")
```

```
[Out] (1/16*I + 5/16)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x + 1))*e^
I - (1/16*I - 5/16)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x + 1))
*e^(-I) + 1/4*I*(2*I*(x + 1)^(3/2) - (4*I + 3)*sqrt(x + 1))*e^(I*x) + 1/4*I
*(2*I*(x + 1)^(3/2) - (4*I - 3)*sqrt(x + 1))*e^(-I*x) - 1/2*sqrt(x + 1)*e^(
I*x) - 1/2*sqrt(x + 1)*e^(-I*x) - 0.537182832596
```

3.811 $\int e^{\tanh^{-1}(x)}\sqrt{1-x}\sin(x)\,dx$

Optimal. Leaf size=72

$$\sqrt{\frac{\pi}{2}}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\sqrt{x+1}\cos(x)$$

```
[Out] -(Sqrt[1 + x]*Cos[x]) + Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 + x]]
+ Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]]*Sin[1]
```

Rubi [A] time = 0.122283, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6129, 3296, 3306, 3305, 3351, 3304, 3352}

$$\sqrt{\frac{\pi}{2}}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\sqrt{x+1}\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*Sqrt[1 - x]*Sin[x],x]
```

```
[Out] -(Sqrt[1 + x]*Cos[x]) + Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 + x]]
+ Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]]*Sin[1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(x)}\sqrt{1-x}\sin(x)\,dx &= \int \sqrt{1+x}\sin(x)\,dx \\
&= -\sqrt{1+x}\cos(x)+\frac{1}{2}\int \frac{\cos(x)}{\sqrt{1+x}}\,dx \\
&= -\sqrt{1+x}\cos(x)+\frac{1}{2}\cos(1)\int \frac{\cos(1+x)}{\sqrt{1+x}}\,dx+\frac{1}{2}\sin(1)\int \frac{\sin(1+x)}{\sqrt{1+x}}\,dx \\
&= -\sqrt{1+x}\cos(x)+\cos(1)\,\text{Subst}\left(\int \cos\left(x^2\right)\,dx,x,\sqrt{1+x}\right)+\sin(1)\,\text{Subst}\left(\int \sin\left(x^2\right)\right. \\
&= -\sqrt{1+x}\cos(x)+\sqrt{\frac{\pi}{2}}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right)+\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right)\sin(1)
\end{aligned}$$

Mathematica [C] time = 0.0235258, size = 77, normalized size = 1.07

$$-\frac{e^{-i}\sqrt{x+1}\text{Gamma}\left(\frac{3}{2},-i(x+1)\right)}{2\sqrt{-i(x+1)}}-\frac{e^{i}\sqrt{x+1}\text{Gamma}\left(\frac{3}{2},i(x+1)\right)}{2\sqrt{i(x+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*Sqrt[1 - x]*Sin[x],x]

[Out] -(Sqrt[1 + x]*Gamma[3/2, (-I)*(1 + x)])/(2*E^I*Sqrt[(-I)*(1 + x)]) - (E^I*S
qrt[1 + x]*Gamma[3/2, I*(1 + x)])/(2*Sqrt[I*(1 + x)])
```

Maple [F] time = 0.302, size = 0, normalized size = 0.

$$\int (1+x)\sin(x)\sqrt{1-x}\frac{1}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*sin(x),x)

[Out] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*sin(x),x)
```

Maxima [C] time = 1.29654, size = 672, normalized size = 9.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(((-I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + I*sqrt(pi)*(erf(sqrt(-I*x -
I)) - 1))*cos(1) + (sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(
-I*x - I)) - 1))*sin(1))*cos(1/2*arctan2(x + 1, 0)) - ((sqrt(pi)*(erf(sqrt(
I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1) - (-I*sqrt(pi)*
(erf(sqrt(I*x + I)) - 1) + I*sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin(1))*si
n(1/2*arctan2(x + 1, 0)))*sqrt(x + 1)/sqrt(abs(x + 1)) - 1/2*(((I*sqrt(pi)*
(erf(sqrt(I*x + I)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1) - (
sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin
(1))*abs(x + 1)*cos(1/2*arctan2(x + 1, 0)) + ((sqrt(pi)*(erf(sqrt(I*x + I))
 - 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1) + (I*sqrt(pi)*(erf(sqrt(
I*x + I)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin(1))*abs(x + 1)*s
in(1/2*arctan2(x + 1, 0)) + (((I*cos(1) - sin(1))*gamma(3/2, I*x + I) + (-I
*cos(1) - sin(1))*gamma(3/2, -I*x - I))*x + (I*cos(1) - sin(1))*gamma(3/2,
I*x + I) + (-I*cos(1) - sin(1))*gamma(3/2, -I*x - I))*cos(3/2*arctan2(x + 1
, 0)) + (((cos(1) + I*sin(1))*gamma(3/2, I*x + I) + (cos(1) - I*sin(1))*gam
ma(3/2, -I*x - I))*x + (cos(1) + I*sin(1))*gamma(3/2, I*x + I) + (cos(1) -
I*sin(1))*gamma(3/2, -I*x - I))*sin(3/2*arctan2(x + 1, 0)))*sqrt(abs(x + 1)
)/(x + 1)^(3/2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^2+1}\sqrt{-x+1}\sin(x)}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*sin(x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*sqrt(-x + 1)*sin(x)/(x - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-x}(x+1)\sin(x)}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*(1-x)**(1/2)*sin(x),x)
```

```
[Out] Integral(sqrt(1 - x)*(x + 1)*sin(x)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [C] time = 1.1806, size = 89, normalized size = 1.24

$$\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{i}-\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{(-i)}-\frac{1}{2}\sqrt{x+1}e^{(ix)}-\frac{1}{2}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*sin(x),x, algorithm="giac")
```

```
[Out] (1/8*I - 1/8)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x + 1))*e^I
- (1/8*I + 1/8)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x + 1))*e^(
-I) - 1/2*sqrt(x + 1)*e^(I*x) - 1/2*sqrt(x + 1)*e^(-I*x) - 0.339605729125
```

3.812 $\int e^{\tanh^{-1}(x)}x(1+x)^{3/2}\sin(x)\,dx$

Optimal. Leaf size=335

$$-4\sqrt{2\pi}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\frac{15}{2}\sqrt{\frac{\pi}{2}}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-4\sqrt{2\pi}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+$$

```
[Out] (17*Sqrt[1 - x]*Cos[x])/4 - 5*(1 - x)^(3/2)*Cos[x] + (1 - x)^(5/2)*Cos[x] +
 (15*Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]])/4 - 4*Sqrt[2*Pi]*C
os[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]] - (15*Sqrt[Pi/2]*Cos[1]*FresnelS[Sqr
t[2/Pi]*Sqrt[1 - x]])/2 + 4*Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 -
x]] + (15*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]]*Sin[1])/2 - 4*Sqrt[2*
Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]]*Sin[1] + (15*Sqrt[Pi/2]*FresnelS[Sqrt[
2/Pi]*Sqrt[1 - x]]*Sin[1])/4 - 4*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]
]*Sin[1] - (15*Sqrt[1 - x]*Sin[x])/2 + (5*(1 - x)^(3/2)*Sin[x])/2
```

Rubi [A] time = 0.475661, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6129, 6742, 3353, 3352, 3351, 3385, 3354, 3386}

$$-4\sqrt{2\pi}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\frac{15}{2}\sqrt{\frac{\pi}{2}}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-4\sqrt{2\pi}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*x*(1 + x)^(3/2)*Sin[x],x]
```


```
[Out] (17*Sqrt[1 - x]*Cos[x])/4 - 5*(1 - x)^(3/2)*Cos[x] + (1 - x)^(5/2)*Cos[x] +
 (15*Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]])/4 - 4*Sqrt[2*Pi]*C
os[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]] - (15*Sqrt[Pi/2]*Cos[1]*FresnelS[Sqr
t[2/Pi]*Sqrt[1 - x]])/2 + 4*Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 -
x]] + (15*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]]*Sin[1])/2 - 4*Sqrt[2*
Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]]*Sin[1] + (15*Sqrt[Pi/2]*FresnelS[Sqrt[
2/Pi]*Sqrt[1 - x]]*Sin[1])/4 - 4*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]
]*Sin[1] - (15*Sqrt[1 - x]*Sin[x])/2 + (5*(1 - x)^(3/2)*Sin[x])/2
```


Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(x)}x(1+x)^{3/2}\sin(x)\,dx &= \int \frac{x(1+x)^2\sin(x)}{\sqrt{1-x}}\,dx \\
&= 2\,\text{Subst}\left(\int \left(-2+x^2\right)^2\left(-1+x^2\right)\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right) \\
&= 2\,\text{Subst}\left(\int \left(-4\sin\left(1-x^2\right)+8x^2\sin\left(1-x^2\right)-5x^4\sin\left(1-x^2\right)+x^6\sin\left(1-x^2\right)\right) \\
&= 2\,\text{Subst}\left(\int x^6\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)-8\,\text{Subst}\left(\int \sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right) \\
&= 8\sqrt{1-x}\cos(x)-5(1-x)^{3/2}\cos(x)+(1-x)^{5/2}\cos(x)-5\,\text{Subst}\left(\int x^4\cos\left(1-x^2\right) \\
&= 8\sqrt{1-x}\cos(x)-5(1-x)^{3/2}\cos(x)+(1-x)^{5/2}\cos(x)+4\sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x} \\
&= \frac{17}{4}\sqrt{1-x}\cos(x)-5(1-x)^{3/2}\cos(x)+(1-x)^{5/2}\cos(x)-4\sqrt{2\pi}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-} \\
&= \frac{17}{4}\sqrt{1-x}\cos(x)-5(1-x)^{3/2}\cos(x)+(1-x)^{5/2}\cos(x)-4\sqrt{2\pi}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-} \\
&= \frac{17}{4}\sqrt{1-x}\cos(x)-5(1-x)^{3/2}\cos(x)+(1-x)^{5/2}\cos(x)+\frac{15}{4}\sqrt{\frac{\pi}{2}}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1}
\end{aligned}$$

Mathematica [C] time = 9.47678, size = 200, normalized size = 0.6

$$\left(\frac{1}{32}+\frac{i}{32}\right)\sqrt{x+1}\left((\cos(x+1)-i\sin(x+1))\left((17+2i)\sqrt{2\pi}\sqrt{x-1}\text{Erf}\left(\frac{(1+i)\sqrt{x-1}}{\sqrt{2}}\right)(\sin(x)-i\cos(x))+(2+2i)\left(4ix^3+\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*x*(1 + x)^(3/2)*Sin[x],x]
```

```
[Out] ((1/32 + I/32)*Sqrt[1 + x]*((-2 - 17*I)*Sqrt[2*Pi]*Sqrt[-1 + x]*Erfi[((1 +
I)*Sqrt[-1 + x])/Sqrt[2]]*(Cos[1] + I*Sin[1]) - (2 - 2*I)*((-1 - 20*I) - (1
1 - 10*I)*x + (8 + 10*I)*x^2 + 4*x^3)*(Cos[x] + I*Sin[x]) + ((2 + 2*I)*((-2
0 - I) + (10 - 11*I)*x + (10 + 8*I)*x^2 + (4*I)*x^3)*(Cos[1] + I*Sin[1]) +
(17 + 2*I)*Sqrt[2*Pi]*Sqrt[-1 + x]*Erf[((1 + I)*Sqrt[-1 + x])/Sqrt[2]]*((-I
)*Cos[x] + Sin[x]))*(Cos[1 + x] - I*Sin[1 + x])))/Sqrt[1 - x^2]
```

Maple [F] time = 0.28, size = 0, normalized size = 0.

$$\int x \sin(x) (1 + x)^{\frac{5}{2}} \frac{1}{\sqrt{-x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(-x^2+1)^(1/2)*x*sin(x),x)
```

```
[Out] int((1+x)^(5/2)/(-x^2+1)^(1/2)*x*sin(x),x)
```

Maxima [C] time = 1.40317, size = 1364, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*((((-I*cos(1) - sin(1))*gamma(7/2, I*x - I) + (I*cos(1) - sin(1))*gamm
a(7/2, -I*x + I))*cos(7/2*arctan2(x - 1, 0)) - ((cos(1) - I*sin(1))*gamma(7
/2, I*x - I) + (cos(1) + I*sin(1))*gamma(7/2, -I*x + I))*sin(7/2*arctan2(x
- 1, 0)))*x^3 + (((4*I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) - 4*I*sqrt(pi)*(er
f(sqrt(-I*x + I)) - 1))*cos(1) + 4*(sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + sqr
t(pi)*(erf(sqrt(-I*x + I)) - 1))*sin(1))*cos(1/2*arctan2(x - 1, 0)) + (4*(s
qrt(pi)*(erf(sqrt(I*x - I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*cos(
1) + (-4*I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + 4*I*sqrt(pi)*(erf(sqrt(-I*x
+ I)) - 1))*sin(1))*sin(1/2*arctan2(x - 1, 0)))*(x - 1)^2*abs(x - 1) - (8*(
(-I*cos(1) - sin(1))*gamma(3/2, I*x - I) + (I*cos(1) - sin(1))*gamma(3/2, -
I*x + I))*cos(3/2*arctan2(x - 1, 0)) - ((8*cos(1) - 8*I*sin(1))*gamma(3/2,
I*x - I) + (8*cos(1) + 8*I*sin(1))*gamma(3/2, -I*x + I))*sin(3/2*arctan2(x
- 1, 0)))*(x - 1)^2 - ((5*((I*cos(1) + sin(1))*gamma(5/2, I*x - I) + (-I*co
s(1) + sin(1))*gamma(5/2, -I*x + I))*cos(5/2*arctan2(x - 1, 0)) + ((5*cos(1
) - 5*I*sin(1))*gamma(5/2, I*x - I) + (5*cos(1) + 5*I*sin(1))*gamma(5/2, -I
*x + I))*sin(5/2*arctan2(x - 1, 0)))*abs(x - 1) + 3*((-I*cos(1) - sin(1))*g
amma(7/2, I*x - I) + (I*cos(1) - sin(1))*gamma(7/2, -I*x + I))*cos(7/2*arct
an2(x - 1, 0)) - ((3*cos(1) - 3*I*sin(1))*gamma(7/2, I*x - I) + (3*cos(1) +
 3*I*sin(1))*gamma(7/2, -I*x + I))*sin(7/2*arctan2(x - 1, 0)))*x^2 - ((8*((
I*cos(1) + sin(1))*gamma(3/2, I*x - I) + (-I*cos(1) + sin(1))*gamma(3/2, -I
*x + I))*cos(3/2*arctan2(x - 1, 0)) + ((8*cos(1) - 8*I*sin(1))*gamma(3/2, I
*x - I) + (8*cos(1) + 8*I*sin(1))*gamma(3/2, -I*x + I))*sin(3/2*arctan2(x -
 1, 0)))*(x - 1)^2 + (10*((-I*cos(1) - sin(1))*gamma(5/2, I*x - I) + (I*cos
(1) - sin(1))*gamma(5/2, -I*x + I))*cos(5/2*arctan2(x - 1, 0)) - ((10*cos(1
) - 10*I*sin(1))*gamma(5/2, I*x - I) + (10*cos(1) + 10*I*sin(1))*gamma(5/2,
 -I*x + I))*sin(5/2*arctan2(x - 1, 0)))*abs(x - 1) + 3*((I*cos(1) + sin(1))
```

```
*gamma(7/2, I*x - I) + (-I*cos(1) + sin(1))*gamma(7/2, -I*x + I))*cos(7/2*a
rctan2(x - 1, 0)) + ((3*cos(1) - 3*I*sin(1))*gamma(7/2, I*x - I) + (3*cos(1
) + 3*I*sin(1))*gamma(7/2, -I*x + I))*sin(7/2*arctan2(x - 1, 0)))*x - (5*((
I*cos(1) + sin(1))*gamma(5/2, I*x - I) + (-I*cos(1) + sin(1))*gamma(5/2, -I
*x + I))*cos(5/2*arctan2(x - 1, 0)) + ((5*cos(1) - 5*I*sin(1))*gamma(5/2, I
*x - I) + (5*cos(1) + 5*I*sin(1))*gamma(5/2, -I*x + I))*sin(5/2*arctan2(x -
 1, 0)))*abs(x - 1) + ((I*cos(1) + sin(1))*gamma(7/2, I*x - I) + (-I*cos(1)
 + sin(1))*gamma(7/2, -I*x + I))*cos(7/2*arctan2(x - 1, 0)) + ((cos(1) - I*
sin(1))*gamma(7/2, I*x - I) + (cos(1) + I*sin(1))*gamma(7/2, -I*x + I))*sin
(7/2*arctan2(x - 1, 0)))*sqrt(-x + 1)*sqrt(abs(x - 1))/(x - 1)^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(x^2+x\right)\sqrt{-x^2+1}\sqrt{x+1}\sin\left(x\right)}{x-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="fricas")
```

```
[Out] integral(-(x^2 + x)*sqrt(-x^2 + 1)*sqrt(x + 1)*sin(x)/(x - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(-x**2+1)**(1/2)*x*sin(x),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.28932, size = 273, normalized size = 0.81

$$-\left(\frac{19}{32}i-\frac{15}{32}\right)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{i}+\left(\frac{19}{32}i+\frac{15}{32}\right)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{(-i)}-\frac{1}{8}i\left(4i(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="giac")
```

```
[Out] -(19/32*I - 15/32)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(-x + 1)
)*e^I + (19/32*I + 15/32)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(-
x + 1))*e^(-I) - 1/8*I*(4*I*(x - 1)^2*sqrt(-x + 1) - (12*I - 10)*(-x + 1)^(
3/2) - (3*I + 18)*sqrt(-x + 1))*e^(I*x) - 1/2*I*(-2*I*(-x + 1)^(3/2) + (4*I
 - 3)*sqrt(-x + 1))*e^(I*x) - 1/8*I*(4*I*(x - 1)^2*sqrt(-x + 1) - (12*I + 1
0)*(-x + 1)^(3/2) - (3*I - 18)*sqrt(-x + 1))*e^(-I*x) - 1/2*I*(-2*I*(-x + 1
)^(3/2) + (4*I + 3)*sqrt(-x + 1))*e^(-I*x) + 1/2*sqrt(-x + 1)*e^(I*x) + 1/2
*sqrt(-x + 1)*e^(-I*x) + 3.25954715712
```

3.813 $\int e^{\tanh^{-1}(x)}(1+x)^{3/2}\sin(x)\,dx$

Optimal. Leaf size=236

$$-4\sqrt{2\pi}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\frac{3}{2}\sqrt{\frac{\pi}{2}}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-2\sqrt{2\pi}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-2$$

```
[Out] 4*Sqrt[1 - x]*Cos[x] - (1 - x)^(3/2)*Cos[x] - 2*Sqrt[2*Pi]*Cos[1]*FresnelC[
Sqrt[2/Pi]*Sqrt[1 - x]] - (3*Sqrt[Pi/2]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 -
 x]])/2 + 4*Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]] + (3*Sqrt[Pi
/2]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]]*Sin[1])/2 - 4*Sqrt[2*Pi]*FresnelC[Sqrt
[2/Pi]*Sqrt[1 - x]]*Sin[1] - 2*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]]*
Sin[1] - (3*Sqrt[1 - x]*Sin[x])/2
```

Rubi [A] time = 0.262805, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6129, 6742, 3353, 3352, 3351, 3385, 3354, 3386}

$$-4\sqrt{2\pi}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\frac{3}{2}\sqrt{\frac{\pi}{2}}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-2\sqrt{2\pi}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-2$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*(1 + x)^(3/2)*Sin[x],x]
```

```
[Out] 4*Sqrt[1 - x]*Cos[x] - (1 - x)^(3/2)*Cos[x] - 2*Sqrt[2*Pi]*Cos[1]*FresnelC[
Sqrt[2/Pi]*Sqrt[1 - x]] - (3*Sqrt[Pi/2]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 -
 x]])/2 + 4*Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]] + (3*Sqrt[Pi
/2]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]]*Sin[1])/2 - 4*Sqrt[2*Pi]*FresnelC[Sqrt
[2/Pi]*Sqrt[1 - x]]*Sin[1] - 2*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]]*
Sin[1] - (3*Sqrt[1 - x]*Sin[x])/2
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(x)}(1+x)^{3/2}\sin(x)\,dx &= \int \frac{(1+x)^2\sin(x)}{\sqrt{1-x}}\,dx\\
&= -\left(2\,\text{Subst}\left(\int \left(-2+x^2\right)^2\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)\right)\\
&= -\left(2\,\text{Subst}\left(\int \left(4\sin\left(1-x^2\right)-4x^2\sin\left(1-x^2\right)+x^4\sin\left(1-x^2\right)\right)\,dx, x, \sqrt{1-x}\right)\right)\\
&= -\left(2\,\text{Subst}\left(\int x^4\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)\right)-8\,\text{Subst}\left(\int \sin\left(1-x^2\right)\,dx, x, \sqrt{1-\right.\\
&= 4\sqrt{1-x}\cos(x)-(1-x)^{3/2}\cos(x)+3\,\text{Subst}\left(\int x^2\cos\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)-4\,\text{S}\\
&= 4\sqrt{1-x}\cos(x)-(1-x)^{3/2}\cos(x)+4\sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-4\sqrt{2\pi}C\left(\sqrt{\frac{2}{\pi}}\sqrt{1}\right.\\
&= 4\sqrt{1-x}\cos(x)-(1-x)^{3/2}\cos(x)-2\sqrt{2\pi}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+4\sqrt{2\pi}\cos(1)S\left(\right.\\
&= 4\sqrt{1-x}\cos(x)-(1-x)^{3/2}\cos(x)-2\sqrt{2\pi}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\frac{3}{2}\sqrt{\frac{\pi}{2}}\cos(1)S\left(\right.
\end{aligned}$$

Mathematica [C] time = 8.35558, size = 178, normalized size = 0.75

$$\frac{i\sqrt{1-x^2}\left(-(\cos(x+1)-i\sin(x+1))\left((21+5i)\sqrt{\frac{\pi}{2}}\text{Erf}\left(\frac{(1+i)\sqrt{x-1}}{\sqrt{2}}\right)(\sin(x)-i\cos(x))+2\sqrt{x-1}(2ix+(3+6i))(\cos(1)\right.\right.}{8\sqrt{x-1}\sqrt{x+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*(1 + x)^(3/2)*Sin[x],x]
```

```
[Out] ((I/8)*Sqrt[1 - x^2]*((5 + 21*I)*Sqrt[Pi/2]*Erfi[((1 + I)*Sqrt[-1 + x])/Sqr
t[2]]*((-I)*Cos[1] + Sin[1]) + 2*Sqrt[-1 + x]*((6 + 3*I) + 2*x)*((-I)*Cos[x
```

```
] + Sin[x]) - (2*((3 + 6*I) + (2*I)*x)*Sqrt[-1 + x]*(Cos[1] + I*Sin[1]) + (
21 + 5*I)*Sqrt[Pi/2]*Erf[((1 + I)*Sqrt[-1 + x])/Sqrt[2]]*((-I)*Cos[x] + Sin
[x]))*(Cos[1 + x] - I*Sin[1 + x])))/(Sqrt[-1 + x]*Sqrt[1 + x])
```

Maple [F] time = 0.27, size = 0, normalized size = 0.

$$\int \sin(x)(1+x)^{\frac{5}{2}}\frac{1}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(-x^2+1)^(1/2)*sin(x),x)
```

```
[Out] int((1+x)^(5/2)/(-x^2+1)^(1/2)*sin(x),x)
```

Maxima [C] time = 1.3232, size = 863, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*((((4*I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) - 4*I*sqrt(pi)*(erf(sqrt(-I*
x + I)) - 1))*cos(1) + 4*(sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + sqrt(pi)*(erf
(sqrt(-I*x + I)) - 1))*sin(1))*cos(1/2*arctan2(x - 1, 0)) + (4*(sqrt(pi)*(e
rf(sqrt(I*x - I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*cos(1) + (-4*I
*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + 4*I*sqrt(pi)*(erf(sqrt(-I*x + I)) - 1)
)*sin(1))*sin(1/2*arctan2(x - 1, 0)))*(x - 1)^2 + (((-I*cos(1) - sin(1))*ga
mma(5/2, I*x - I) + (I*cos(1) - sin(1))*gamma(5/2, -I*x + I))*cos(5/2*arcta
n2(x - 1, 0)) - ((cos(1) - I*sin(1))*gamma(5/2, I*x - I) + (cos(1) + I*sin(
1))*gamma(5/2, -I*x + I))*sin(5/2*arctan2(x - 1, 0)))*x^2 - ((4*((I*cos(1)
+ sin(1))*gamma(3/2, I*x - I) + (-I*cos(1) + sin(1))*gamma(3/2, -I*x + I))*
cos(3/2*arctan2(x - 1, 0)) + ((4*cos(1) - 4*I*sin(1))*gamma(3/2, I*x - I) +
 (4*cos(1) + 4*I*sin(1))*gamma(3/2, -I*x + I))*sin(3/2*arctan2(x - 1, 0)))*
abs(x - 1) + 2*((-I*cos(1) - sin(1))*gamma(5/2, I*x - I) + (I*cos(1) - sin(
1))*gamma(5/2, -I*x + I))*cos(5/2*arctan2(x - 1, 0)) - ((2*cos(1) - 2*I*sin
(1))*gamma(5/2, I*x - I) + (2*cos(1) + 2*I*sin(1))*gamma(5/2, -I*x + I))*si
n(5/2*arctan2(x - 1, 0)))*x - (4*((-I*cos(1) - sin(1))*gamma(3/2, I*x - I)
+ (I*cos(1) - sin(1))*gamma(3/2, -I*x + I))*cos(3/2*arctan2(x - 1, 0)) - ((
4*cos(1) - 4*I*sin(1))*gamma(3/2, I*x - I) + (4*cos(1) + 4*I*sin(1))*gamma(
3/2, -I*x + I))*sin(3/2*arctan2(x - 1, 0)))*abs(x - 1) + ((-I*cos(1) - sin(
1))*gamma(5/2, I*x - I) + (I*cos(1) - sin(1))*gamma(5/2, -I*x + I))*cos(5/2
*arctan2(x - 1, 0)) - ((cos(1) - I*sin(1))*gamma(5/2, I*x - I) + (cos(1) +
I*sin(1))*gamma(5/2, -I*x + I))*sin(5/2*arctan2(x - 1, 0)))*sqrt(-x + 1)/((
x - 1)^2*sqrt(abs(x - 1)))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^2+1}(x+1)^{\frac{3}{2}}\sin(x)}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*(x + 1)^(3/2)*sin(x)/(x - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(-x**2+1)**(1/2)*sin(x),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.26874, size = 165, normalized size = 0.7

$$-\left(\frac{21}{16}i+\frac{5}{16}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{i}+\left(\frac{21}{16}i-\frac{5}{16}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{(-i)}-\frac{1}{4}i\left(-2i(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="giac")
```

```
[Out] -(21/16*I + 5/16)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(-x + 1))
*e^I + (21/16*I - 5/16)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(-x
+ 1))*e^(-I) - 1/4*I*(-2*I*(-x + 1)^(3/2) + (4*I - 3)*sqrt(-x + 1))*e^(I*x)
 - 1/4*I*(-2*I*(-x + 1)^(3/2) + (4*I + 3)*sqrt(-x + 1))*e^(-I*x) + sqrt(-x
+ 1)*e^(I*x) + sqrt(-x + 1)*e^(-I*x) + 3.78811297138
```

3.814 $\int e^{\tanh^{-1}(x)}(1-x)^{3/2}x\sin(x)\,dx$

Optimal. Leaf size=193

$$\frac{9}{2}\sqrt{\frac{\pi}{2}}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\frac{7}{4}\sqrt{\frac{\pi}{2}}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\frac{7}{4}\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\frac{9}{2}\sqrt{\frac{\pi}{2}}\cos$$

```
[Out] (-7*Sqrt[1 + x]*Cos[x])/4 - 3*(1 + x)^(3/2)*Cos[x] + (1 + x)^(5/2)*Cos[x] +
 (7*Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 + x]])/4 - (9*Sqrt[Pi/2]*C
os[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]])/2 + (9*Sqrt[Pi/2]*FresnelC[Sqrt[2/P
i]*Sqrt[1 + x]]*Sin[1])/2 + (7*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]]*
Sin[1])/4 + (9*Sqrt[1 + x]*Sin[x])/2 - (5*(1 + x)^(3/2)*Sin[x])/2
```

Rubi [A] time = 0.40128, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {6129, 6742, 3385, 3354, 3352, 3351, 3386, 3353}

$$\frac{9}{2}\sqrt{\frac{\pi}{2}}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\frac{7}{4}\sqrt{\frac{\pi}{2}}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\frac{7}{4}\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\frac{9}{2}\sqrt{\frac{\pi}{2}}\cos$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*(1 - x)^(3/2)*x*Sin[x],x]
```



```
[Out] (-7*Sqrt[1 + x]*Cos[x])/4 - 3*(1 + x)^(3/2)*Cos[x] + (1 + x)^(5/2)*Cos[x] +
 (7*Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 + x]])/4 - (9*Sqrt[Pi/2]*C
os[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]])/2 + (9*Sqrt[Pi/2]*FresnelC[Sqrt[2/P
i]*Sqrt[1 + x]]*Sin[1])/2 + (7*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]]*
Sin[1])/4 + (9*Sqrt[1 + x]*Sin[x])/2 - (5*(1 + x)^(3/2)*Sin[x])/2
```


Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\int e^{\tanh^{-1}(x)}(1-x)^{3/2}x\sin(x)\,dx = \int(1-x)x\sqrt{1+x}\sin(x)\,dx$$
$$= 2\,\text{Subst}\left(\int x^2\left(-2+x^2\right)\left(-1+x^2\right)\sin\left(1-x^2\right)dx, x, \sqrt{1+x}\right)$$
$$= 2\,\text{Subst}\left(\int\left(2x^2\sin\left(1-x^2\right)-3x^4\sin\left(1-x^2\right)+x^6\sin\left(1-x^2\right)\right)dx, x, \sqrt{1+x}\right)$$
$$= 2\,\text{Subst}\left(\int x^6\sin\left(1-x^2\right)dx, x, \sqrt{1+x}\right)+4\,\text{Subst}\left(\int x^2\sin\left(1-x^2\right)dx, x, \sqrt{1+\right.$$
$$= 2\sqrt{1+x}\cos(x)-3(1+x)^{3/2}\cos(x)+(1+x)^{5/2}\cos(x)-2\,\text{Subst}\left(\int\cos\left(1-x^2\right)d\right.$$
$$= 2\sqrt{1+x}\cos(x)-3(1+x)^{3/2}\cos(x)+(1+x)^{5/2}\cos(x)+\frac{9}{2}\sqrt{1+x}\sin(x)-\frac{5}{2}(1+x)$$
$$= -\frac{7}{4}\sqrt{1+x}\cos(x)-3(1+x)^{3/2}\cos(x)+(1+x)^{5/2}\cos(x)-\sqrt{2\pi}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1+\right.$$
$$= -\frac{7}{4}\sqrt{1+x}\cos(x)-3(1+x)^{3/2}\cos(x)+(1+x)^{5/2}\cos(x)-\sqrt{2\pi}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1+\right.$$
$$= -\frac{7}{4}\sqrt{1+x}\cos(x)-3(1+x)^{3/2}\cos(x)+(1+x)^{5/2}\cos(x)+\frac{15}{4}\sqrt{\frac{\pi}{2}}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{\right.$$

Mathematica [C] time = 8.27329, size = 193, normalized size = 1.

$$\left(\frac{1}{32}+\frac{i}{32}\right)\sqrt{1-x}\left((\cos(1)-i\sin(1))\left((2+2i)\left(-4ix^3+10x^2+(2+19i)x-(8-15i)\right)(\cos(x+1)+i\sin(x+1))-(18\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*(1 - x)^(3/2)*x*Sin[x],x]
```

```
[Out] ((1/32 + I/32)*Sqrt[1 - x]*(((2 + 2*I)*((8 + 15*I) - (2 - 19*I)*x - 10*x^2
- (4*I)*x^3))/E^(I*x) + (18 + 7*I)*E^I*Sqrt[2*Pi]*Sqrt[-1 - x]*Erfi[((1 + I
```

```
)*Sqrt[-1 - x])/Sqrt[2]] + (Cos[1] - I*Sin[1])*((-18 + 7*I)*Sqrt[2*Pi]*Sqrt
[-1 - x]*Erf[((1 + I)*Sqrt[-1 - x])/Sqrt[2]] + (2 + 2*I)*((-8 + 15*I) + (2
+ 19*I)*x + 10*x^2 - (4*I)*x^3)*(Cos[1 + x] + I*Sin[1 + x]))))/Sqrt[1 - x^2
]
```

Maple [F] time = 0.295, size = 0, normalized size = 0.

$$\int (1+x)\, x \sin(x)\, (1-x)^{\frac{3}{2}} \frac{1}{\sqrt{-x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x*sin(x),x)

[Out] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x*sin(x),x)
```

Maxima [C] time = 1.69048, size = 2009, normalized size = 10.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x*sin(x),x, algorithm="maxima")

[Out] 1/2*(((-2*I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*x
 - I)) - 1))*cos(1) + 2*(sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(
sqrt(-I*x - I)) - 1))*sin(1))*(x + 1)^2*cos(1/2*arctan2(x + 1, 0)) - (2*(sq
rt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1
) - (-2*I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*x -
 I)) - 1))*sin(1))*(x + 1)^2*sin(1/2*arctan2(x + 1, 0)) - 3*(((I*cos(1) - s
in(1))*gamma(3/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(3/2, -I*x - I))*x +
 (I*cos(1) - sin(1))*gamma(3/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(3/2,
-I*x - I))*abs(x + 1)*cos(3/2*arctan2(x + 1, 0)) - (((3*cos(1) + 3*I*sin(1)
)*gamma(3/2, I*x + I) + (3*cos(1) - 3*I*sin(1))*gamma(3/2, -I*x - I))*x + (
3*cos(1) + 3*I*sin(1))*gamma(3/2, I*x + I) + (3*cos(1) - 3*I*sin(1))*gamma(
3/2, -I*x - I))*abs(x + 1)*sin(3/2*arctan2(x + 1, 0)) + (((I*cos(1) - sin(1
))*gamma(5/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(5/2, -I*x - I))*x^2 - 2
*((-I*cos(1) + sin(1))*gamma(5/2, I*x + I) + (I*cos(1) + sin(1))*gamma(5/2,
 -I*x - I))*x + (I*cos(1) - sin(1))*gamma(5/2, I*x + I) + (-I*cos(1) - sin(
1))*gamma(5/2, -I*x - I))*cos(5/2*arctan2(x + 1, 0)) + (((cos(1) + I*sin(1)
)*gamma(5/2, I*x + I) + (cos(1) - I*sin(1))*gamma(5/2, -I*x - I))*x^2 + ((2
*cos(1) + 2*I*sin(1))*gamma(5/2, I*x + I) + (2*cos(1) - 2*I*sin(1))*gamma(5
/2, -I*x - I))*x + (cos(1) + I*sin(1))*gamma(5/2, I*x + I) + (cos(1) - I*si
n(1))*gamma(5/2, -I*x - I))*sin(5/2*arctan2(x + 1, 0)))/((x + 1)^(3/2)*sqrt
(abs(x + 1))) + 1/2*(((2*I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) - 2*I*sqrt(pi)
*(erf(sqrt(-I*x - I)) - 1))*cos(1) - 2*(sqrt(pi)*(erf(sqrt(I*x + I)) - 1) +
 sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin(1))*(x + 1)^2*abs(x + 1)*cos(1/2*a
rctan2(x + 1, 0)) + (2*(sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(s
qrt(-I*x - I)) - 1))*cos(1) + (2*I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) - 2*I*
sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin(1))*(x + 1)^2*abs(x + 1)*sin(1/2*ar
ctan2(x + 1, 0)) - 5*(((-I*cos(1) + sin(1))*gamma(3/2, I*x + I) + (I*cos(1)
 + sin(1))*gamma(3/2, -I*x - I))*x + (-I*cos(1) + sin(1))*gamma(3/2, I*x +
I) + (I*cos(1) + sin(1))*gamma(3/2, -I*x - I))*(x + 1)^2*cos(3/2*arctan2(x
+ 1, 0)) + (((5*cos(1) + 5*I*sin(1))*gamma(3/2, I*x + I) + (5*cos(1) - 5*I*
sin(1))*gamma(3/2, -I*x - I))*x + (5*cos(1) + 5*I*sin(1))*gamma(3/2, I*x +
```

```
I) + (5*cos(1) - 5*I*sin(1))*gamma(3/2, -I*x - I))*(x + 1)^2*sin(3/2*arctan
2(x + 1, 0)) - 4*(((I*cos(1) - sin(1))*gamma(5/2, I*x + I) + (-I*cos(1) - s
in(1))*gamma(5/2, -I*x - I))*x^2 + 2*((I*cos(1) - sin(1))*gamma(5/2, I*x +
I) + (-I*cos(1) - sin(1))*gamma(5/2, -I*x - I))*x + (I*cos(1) - sin(1))*gam
ma(5/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(5/2, -I*x - I))*abs(x + 1)*co
s(5/2*arctan2(x + 1, 0)) - (((4*cos(1) + 4*I*sin(1))*gamma(5/2, I*x + I) +
(4*cos(1) - 4*I*sin(1))*gamma(5/2, -I*x - I))*x^2 + ((8*cos(1) + 8*I*sin(1)
)*gamma(5/2, I*x + I) + (8*cos(1) - 8*I*sin(1))*gamma(5/2, -I*x - I))*x + (
4*cos(1) + 4*I*sin(1))*gamma(5/2, I*x + I) + (4*cos(1) - 4*I*sin(1))*gamma(
5/2, -I*x - I))*abs(x + 1)*sin(5/2*arctan2(x + 1, 0)) + (((I*cos(1) - sin(1
))*gamma(7/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(7/2, -I*x - I))*x^3 - 3
*((-I*cos(1) + sin(1))*gamma(7/2, I*x + I) + (I*cos(1) + sin(1))*gamma(7/2,
 -I*x - I))*x^2 - 3*((-I*cos(1) + sin(1))*gamma(7/2, I*x + I) + (I*cos(1) +
 sin(1))*gamma(7/2, -I*x - I))*x + (I*cos(1) - sin(1))*gamma(7/2, I*x + I)
+ (-I*cos(1) - sin(1))*gamma(7/2, -I*x - I))*cos(7/2*arctan2(x + 1, 0)) + (
((cos(1) + I*sin(1))*gamma(7/2, I*x + I) + (cos(1) - I*sin(1))*gamma(7/2, -
I*x - I))*x^3 + ((3*cos(1) + 3*I*sin(1))*gamma(7/2, I*x + I) + (3*cos(1) -
3*I*sin(1))*gamma(7/2, -I*x - I))*x^2 + ((3*cos(1) + 3*I*sin(1))*gamma(7/2,
 I*x + I) + (3*cos(1) - 3*I*sin(1))*gamma(7/2, -I*x - I))*x + (cos(1) + I*s
in(1))*gamma(7/2, I*x + I) + (cos(1) - I*sin(1))*gamma(7/2, -I*x - I))*sin(
7/2*arctan2(x + 1, 0)))*sqrt(abs(x + 1))/(x + 1)^(7/2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^2+1}x\sqrt{-x+1}\sin(x),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x*sin(x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)*x*sqrt(-x + 1)*sin(x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*(1-x)**(3/2)*x*sin(x),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.27134, size = 165, normalized size = 0.85

$$\left(\frac{25}{32}i+\frac{11}{32}\right)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{i}-\left(\frac{25}{32}i-\frac{11}{32}\right)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{(-i)}-\frac{1}{8}i\left(4i\,(x+1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x*sin(x),x, algorithm="giac")
```

```
[Out] (25/32*I + 11/32)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x + 1))*
e^I - (25/32*I - 11/32)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x +
 1))*e^(-I) - 1/8*I*(4*I*(x + 1)^(5/2) - (12*I + 10)*(x + 1)^(3/2) - (3*I -
 18)*sqrt(x + 1))*e^(I*x) - 1/8*I*(4*I*(x + 1)^(5/2) - (12*I - 10)*(x + 1)^
(3/2) - (3*I + 18)*sqrt(x + 1))*e^(-I*x) - 1/2*sqrt(x + 1)*e^(I*x) - 1/2*sq
rt(x + 1)*e^(-I*x) - 0.330988710799
```

3.815 $\int e^{\tanh^{-1}(x)}(1-x)^{3/2}\sin(x)\,dx$

Optimal. Leaf size=157

$$-\frac{3}{2}\sqrt{\frac{\pi}{2}}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\sqrt{2\pi}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\sqrt{2\pi}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{co}$$

```
[Out] -2*Sqrt[1 + x]*Cos[x] + (1 + x)^(3/2)*Cos[x] + Sqrt[2*Pi]*Cos[1]*FresnelC[S
qrt[2/Pi]*Sqrt[1 + x]] + (3*Sqrt[Pi/2]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 +
x]])/2 - (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[1 + x]]*Sin[1])/2 + Sqrt[2*
Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]]*Sin[1] - (3*Sqrt[1 + x]*Sin[x])/2
```

Rubi [A] time = 0.232529, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6129, 6742, 3385, 3354, 3352, 3351, 3386, 3353}

$$-\frac{3}{2}\sqrt{\frac{\pi}{2}}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\sqrt{2\pi}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\sqrt{2\pi}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{co}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*(1 - x)^(3/2)*Sin[x],x]
```

```
[Out] -2*Sqrt[1 + x]*Cos[x] + (1 + x)^(3/2)*Cos[x] + Sqrt[2*Pi]*Cos[1]*FresnelC[S
qrt[2/Pi]*Sqrt[1 + x]] + (3*Sqrt[Pi/2]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 +
x]])/2 - (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[1 + x]]*Sin[1])/2 + Sqrt[2*
Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]]*Sin[1] - (3*Sqrt[1 + x]*Sin[x])/2
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(x)}(1-x)^{3/2}\sin(x)\,dx &= \int (1-x)\sqrt{1+x}\sin(x)\,dx\\
&= 2\,\text{Subst}\left(\int x^2\left(-2+x^2\right)\sin\left(1-x^2\right)\,dx, x, \sqrt{1+x}\right)\\
&= 2\,\text{Subst}\left(\int \left(-2x^2\sin\left(1-x^2\right)+x^4\sin\left(1-x^2\right)\right)\,dx, x, \sqrt{1+x}\right)\\
&= 2\,\text{Subst}\left(\int x^4\sin\left(1-x^2\right)\,dx, x, \sqrt{1+x}\right) - 4\,\text{Subst}\left(\int x^2\sin\left(1-x^2\right)\,dx, x, \sqrt{1+x}\right)\\
&= -2\sqrt{1+x}\cos(x)+(1+x)^{3/2}\cos(x)+2\,\text{Subst}\left(\int \cos\left(1-x^2\right)\,dx, x, \sqrt{1+x}\right) - 3\,\text{Subst}\\
&= -2\sqrt{1+x}\cos(x)+(1+x)^{3/2}\cos(x)-\frac{3}{2}\sqrt{1+x}\sin(x)-\frac{3}{2}\,\text{Subst}\left(\int \sin\left(1-x^2\right)\,dx, x,\right.\\
&= -2\sqrt{1+x}\cos(x)+(1+x)^{3/2}\cos(x)+\sqrt{2\pi}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right)+\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right.\\
&= -2\sqrt{1+x}\cos(x)+(1+x)^{3/2}\cos(x)+\sqrt{2\pi}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right)+\frac{3}{2}\sqrt{\frac{\pi}{2}}\cos(1)S\left(\sqrt{\right.
\end{aligned}$$

Mathematica [C] time = 7.01448, size = 176, normalized size = 1.12

$$\frac{\left(\frac{1}{16}+\frac{i}{16}\right)e^{-ix}\sqrt{1-x^2}\left(-(3+4i)\sqrt{2\pi}e^{ix}\text{Erf}\left(\frac{(1+i)\sqrt{-x-1}}{\sqrt{2}}\right)(\cos(1)-i\sin(1))+(4+3i)\sqrt{2\pi}e^{ix}\text{Erfi}\left(\frac{(1+i)\sqrt{-x-1}}{\sqrt{2}}\right)(\sin(1)-i\cos\right.}{\sqrt{-x-1}\sqrt{1-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*(1 - x)^(3/2)*Sin[x],x]
```

```
[Out] ((1/16 + I/16)*Sqrt[1 - x^2]*((2 + 2*I)*Sqrt[-1 - x]*((-3 + 2*I) + E^((2*I)
*x)*((3 + 2*I) - (2*I)*x) - (2*I)*x) - (3 + 4*I)*E^(I*x)*Sqrt[2*Pi]*Erf[((1
 + I)*Sqrt[-1 - x])/Sqrt[2]]*(Cos[1] - I*Sin[1]) + (4 + 3*I)*E^(I*x)*Sqrt[2
*Pi]*Erfi[((1 + I)*Sqrt[-1 - x])/Sqrt[2]]*((-I)*Cos[1] + Sin[1])))/(E^(I*x)
*Sqrt[-1 - x]*Sqrt[1 - x])
```

Maple [F] time = 0.284, size = 0, normalized size = 0.

$$\int (1+x)\sin(x)(1-x)^{\frac{3}{2}} \frac{1}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*sin(x),x)
```

```
[Out] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*sin(x),x)
```

Maxima [C] time = 1.48217, size = 1231, normalized size = 7.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*sin(x),x, algorithm="maxima")
```

```
[Out] 1/2*(((2*I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) - 2*I*sqrt(pi)*(erf(sqrt(-I*x
- I)) - 1))*cos(1) - 2*(sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(s
qrt(-I*x - I)) - 1))*sin(1))*abs(x + 1)*cos(1/2*arctan2(x + 1, 0)) + (2*(sq
rt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1
) + (2*I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) - 2*I*sqrt(pi)*(erf(sqrt(-I*x -
I)) - 1))*sin(1))*abs(x + 1)*sin(1/2*arctan2(x + 1, 0)) + (((I*cos(1) - sin
(1))*gamma(3/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(3/2, -I*x - I))*x + (
I*cos(1) - sin(1))*gamma(3/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(3/2, -I
*x - I))*cos(3/2*arctan2(x + 1, 0)) + (((cos(1) + I*sin(1))*gamma(3/2, I*x
+ I) + (cos(1) - I*sin(1))*gamma(3/2, -I*x - I))*x + (cos(1) + I*sin(1))*ga
mma(3/2, I*x + I) + (cos(1) - I*sin(1))*gamma(3/2, -I*x - I))*sin(3/2*arcta
n2(x + 1, 0)))*sqrt(abs(x + 1))/(x + 1)^(3/2) + 1/2*(((-2*I*sqrt(pi)*(erf(s
qrt(I*x + I)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1) + 2*(sq
rt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin(1
))*(x + 1)^2*cos(1/2*arctan2(x + 1, 0)) - (2*(sqrt(pi)*(erf(sqrt(I*x + I))
- 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1) - (-2*I*sqrt(pi)*(erf(sqr
t(I*x + I)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin(1))*(x + 1)^
2*sin(1/2*arctan2(x + 1, 0)) - 3*(((I*cos(1) - sin(1))*gamma(3/2, I*x + I)
+ (-I*cos(1) - sin(1))*gamma(3/2, -I*x - I))*x + (I*cos(1) - sin(1))*gamma(
3/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(3/2, -I*x - I))*abs(x + 1)*cos(3
/2*arctan2(x + 1, 0)) - (((3*cos(1) + 3*I*sin(1))*gamma(3/2, I*x + I) + (3*
cos(1) - 3*I*sin(1))*gamma(3/2, -I*x - I))*x + (3*cos(1) + 3*I*sin(1))*gamm
a(3/2, I*x + I) + (3*cos(1) - 3*I*sin(1))*gamma(3/2, -I*x - I))*abs(x + 1)*
sin(3/2*arctan2(x + 1, 0)) + (((I*cos(1) - sin(1))*gamma(5/2, I*x + I) + (-
I*cos(1) - sin(1))*gamma(5/2, -I*x - I))*x^2 - 2*((-I*cos(1) + sin(1))*gamm
a(5/2, I*x + I) + (I*cos(1) + sin(1))*gamma(5/2, -I*x - I))*x + (I*cos(1) -
 sin(1))*gamma(5/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(5/2, -I*x - I))*c
os(5/2*arctan2(x + 1, 0)) + (((cos(1) + I*sin(1))*gamma(5/2, I*x + I) + (co
s(1) - I*sin(1))*gamma(5/2, -I*x - I))*x^2 + ((2*cos(1) + 2*I*sin(1))*gamma
(5/2, I*x + I) + (2*cos(1) - 2*I*sin(1))*gamma(5/2, -I*x - I))*x + (cos(1)
+ I*sin(1))*gamma(5/2, I*x + I) + (cos(1) - I*sin(1))*gamma(5/2, -I*x - I))
*sin(5/2*arctan2(x + 1, 0)))/((x + 1)^(3/2)*sqrt(abs(x + 1)))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^2+1}\sqrt{-x+1}\sin\left(x\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*sin(x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)*sqrt(-x + 1)*sin(x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*(1-x)**(3/2)*sin(x),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.20836, size = 116, normalized size = 0.74

$$\left(\frac{1}{16}i-\frac{7}{16}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{i}-\left(\frac{1}{16}i+\frac{7}{16}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{(-i)}-\frac{1}{4}i\left(2i\left(x+1\right)^{\frac{3}{2}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*sin(x),x, algorithm="giac")
```

```
[Out] (1/16*I - 7/16)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x + 1))*e^
I - (1/16*I + 7/16)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x + 1))
*e^(-I) - 1/4*I*(2*I*(x + 1)^(3/2) - (4*I + 3)*sqrt(x + 1))*e^(I*x) - 1/4*I
*(2*I*(x + 1)^(3/2) - (4*I - 3)*sqrt(x + 1))*e^(-I*x) + 0.19757710347
```

3.816 $\int \frac{e^{\tanh^{-1}(x)} x \sin(x)}{\sqrt{1+x}} \, dx$

Optimal. Leaf size=140

$$-\sqrt{2\pi}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{\frac{\pi}{2}}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\sqrt{2\pi}\cos(1)$$

```
[Out] Sqrt[1 - x]*Cos[x] - Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]] + S
qrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]] - Sqrt[2*Pi]*FresnelC[Sqr
t[2/Pi]*Sqrt[1 - x]]*Sin[1] - Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]]*S
in[1]
```

Rubi [A] time = 0.187591, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6129, 6742, 3353, 3352, 3351, 3385, 3354}

$$-\sqrt{2\pi}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{\frac{\pi}{2}}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\sqrt{2\pi}\cos(1)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[x]*x*Sin[x])/Sqrt[1 + x],x]
```


```
[Out] Sqrt[1 - x]*Cos[x] - Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]] + S
qrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]] - Sqrt[2*Pi]*FresnelC[Sqr
t[2/Pi]*Sqrt[1 - x]]*Sin[1] - Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]]*S
in[1]
```


Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(x)}x\sin(x)}{\sqrt{1+x}}\,dx &= \int \frac{x\sin(x)}{\sqrt{1-x}}\,dx\\
&= 2\,\text{Subst}\left(\int \left(-1+x^2\right)\sin\left(1-x^2\right)\,dx,x,\sqrt{1-x}\right)\\
&= 2\,\text{Subst}\left(\int \left(-\sin\left(1-x^2\right)+x^2\sin\left(1-x^2\right)\right)\,dx,x,\sqrt{1-x}\right)\\
&= -\left(2\,\text{Subst}\left(\int \sin\left(1-x^2\right)\,dx,x,\sqrt{1-x}\right)\right)+2\,\text{Subst}\left(\int x^2\sin\left(1-x^2\right)\,dx,x,\sqrt{1-x}\right)\\
&= \sqrt{1-x}\cos(x)+(2\cos(1))\,\text{Subst}\left(\int \sin\left(x^2\right)\,dx,x,\sqrt{1-x}\right)-(2\sin(1))\,\text{Subst}\left(\int \cos\left(x^2\right)\,dx,\right.\\
&= \sqrt{1-x}\cos(x)+\sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{2\pi}C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)\sin(1)-\cos(1)\,\text{Subst}\left(\int \text{co}\right.\\
&= \sqrt{1-x}\cos(x)-\sqrt{\frac{\pi}{2}}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{2\pi}C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right.
\end{aligned}$$

Mathematica [C] time = 7.88062, size = 162, normalized size = 1.16

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\sqrt{x+1}\left((\cos(x+1)-i\sin(x+1))\left((1+2i)\sqrt{2\pi}\sqrt{x-1}\text{Erf}\left(\frac{(1+i)\sqrt{x-1}}{\sqrt{2}}\right)(\sin(x)-i\cos(x))-(2-2i)(x-1)(\cos(1)\right.\right.}{\sqrt{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[x]*x*Sin[x])/Sqrt[1 + x],x]
```

```
[Out] ((1/8 + I/8)*Sqrt[1 + x]*((-2 - I)*Sqrt[2*Pi]*Sqrt[-1 + x]*Erfi[((1 + I)*Sq
rt[-1 + x])/Sqrt[2]]*(Cos[1] + I*Sin[1]) - (2 - 2*I)*(-1 + x)*(Cos[x] + I*S
in[x]) + ((-2 + 2*I)*(-1 + x)*(Cos[1] + I*Sin[1]) + (1 + 2*I)*Sqrt[2*Pi]*Sq
rt[-1 + x]*Erf[((1 + I)*Sqrt[-1 + x])/Sqrt[2]]*((-I)*Cos[x] + Sin[x]))*(Cos
[1 + x] - I*Sin[1 + x])))/Sqrt[1 - x^2]
```

Maple [F] time = 0.27, size = 0, normalized size = 0.

$$\int x\sin(x)\sqrt{1+x}\frac{1}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(-x^2+1)^(1/2)*x*sin(x),x)
```

```
[Out] int((1+x)^(1/2)/(-x^2+1)^(1/2)*x*sin(x),x)
```

Maxima [C] time = 1.21693, size = 468, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*((((-I*cos(1) - sin(1))*gamma(3/2, I*x - I) + (I*cos(1) - sin(1))*gamm
a(3/2, -I*x + I))*cos(3/2*arctan2(x - 1, 0)) - ((cos(1) - I*sin(1))*gamma(3
/2, I*x - I) + (cos(1) + I*sin(1))*gamma(3/2, -I*x + I))*sin(3/2*arctan2(x
- 1, 0)))*x + (((I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) - I*sqrt(pi)*(erf(sqrt
(-I*x + I)) - 1))*cos(1) + (sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + sqrt(pi)*(e
rf(sqrt(-I*x + I)) - 1))*sin(1))*cos(1/2*arctan2(x - 1, 0)) + ((sqrt(pi)*(e
rf(sqrt(I*x - I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*cos(1) + (-I*s
qrt(pi)*(erf(sqrt(I*x - I)) - 1) + I*sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*si
n(1))*sin(1/2*arctan2(x - 1, 0)))*abs(x - 1) + ((I*cos(1) + sin(1))*gamma(3
/2, I*x - I) + (-I*cos(1) + sin(1))*gamma(3/2, -I*x + I))*cos(3/2*arctan2(x
 - 1, 0)) + ((cos(1) - I*sin(1))*gamma(3/2, I*x - I) + (cos(1) + I*sin(1))*
gamma(3/2, -I*x + I))*sin(3/2*arctan2(x - 1, 0)))*sqrt(-x + 1)*sqrt(abs(x -
 1))/(x - 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-x^2+1}\sqrt{x+1}x\sin(x)}{x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*sqrt(x + 1)*x*sin(x)/(x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{x+1}\sin(x)}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(-x**2+1)**(1/2)*x*sin(x),x)
```

```
[Out] Integral(x*sqrt(x + 1)*sin(x)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [C] time = 1.21159, size = 100, normalized size = 0.71

$$-\left(\frac{3}{8}i+\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{i}+\left(\frac{3}{8}i-\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{(-i)}+\frac{1}{2}\sqrt{-x+1}e^{(i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="giac")
```

```
[Out] -(3/8*I + 1/8)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(-x + 1))*e^
I + (3/8*I - 1/8)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(-x + 1))*
e^(-I) + 1/2*sqrt(-x + 1)*e^(I*x) + 1/2*sqrt(-x + 1)*e^(-I*x) + 1.166225383
276
```

3.817 $\int \frac{e^{\tanh^{-1}(x)}\sin(x)}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=62

$$\sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{2\pi}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)$$

```
[Out] Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]] - Sqrt[2*Pi]*FresnelC[Sq
rt[2/Pi]*Sqrt[1 - x]]*Sin[1]
```

Rubi [A] time = 0.110165, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6129, 3306, 3305, 3351, 3304, 3352}

$$\sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{2\pi}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[x]*Sin[x])/Sqrt[1 + x],x]
```


```
[Out] Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]] - Sqrt[2*Pi]*FresnelC[Sq
rt[2/Pi]*Sqrt[1 - x]]*Sin[1]
```


Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(x)}\sin(x)}{\sqrt{1+x}}\,dx &= \int \frac{\sin(x)}{\sqrt{1-x}}\,dx \\
&= -\left(\cos(1)\int \frac{\sin(1-x)}{\sqrt{1-x}}\,dx\right)+\sin(1)\int \frac{\cos(1-x)}{\sqrt{1-x}}\,dx \\
&= (2\cos(1))\,\text{Subst}\left(\int \sin\left(x^2\right)dx,x,\sqrt{1-x}\right)-(2\sin(1))\,\text{Subst}\left(\int \cos\left(x^2\right)dx,x,\sqrt{1-x}\right) \\
&= \sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{2\pi}C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)\sin(1)
\end{aligned}
$$

Mathematica [C] time = 0.0274356, size = 70, normalized size = 1.13

$$
-\frac{e^{-i}\left(e^{2i}\sqrt{-i(x-1)}\,\text{Gamma}\left(\frac{1}{2},-i(x-1)\right)+\sqrt{i(x-1)}\,\text{Gamma}\left(\frac{1}{2},i(x-1)\right)\right)}{2\sqrt{1-x}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[x]*Sin[x])/Sqrt[1 + x],x]
```

```
[Out] -(E^(2*I)*Sqrt[(-I)*(-1 + x)]*Gamma[1/2, (-I)*(-1 + x)] + Sqrt[I*(-1 + x)]*
Gamma[1/2, I*(-1 + x)])/(2*E^I*Sqrt[1 - x])
```

Maple [F] time = 0.281, size = 0, normalized size = 0.

$$
\int \sin(x)\sqrt{1+x}\frac{1}{\sqrt{-x^2+1}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(-x^2+1)^(1/2)*sin(x),x)
```

```
[Out] int((1+x)^(1/2)/(-x^2+1)^(1/2)*sin(x),x)
```

Maxima [C] time = 1.1369, size = 227, normalized size = 3.66

$$
-\frac{\left(\left(\left(i\sqrt{\pi}\left(\text{erf}\left(\sqrt{ix-i}\right)-1\right)-i\sqrt{\pi}\left(\text{erf}\left(\sqrt{-ix+i}\right)-1\right)\right)\cos(1)+\left(\sqrt{\pi}\left(\text{erf}\left(\sqrt{ix-i}\right)-1\right)+\sqrt{\pi}\left(\text{erf}\left(\sqrt{-ix+i}\right)-1\right)\right)\sin\right.\right.}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(((I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*x + I
)) - 1))*cos(1) + (sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + sqrt(pi)*(erf(sqrt(-
```

```
I*x + I)) - 1))*sin(1))*cos(1/2*arctan2(x - 1, 0)) + ((sqrt(pi)*(erf(sqrt(I
*x - I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*cos(1) + (-I*sqrt(pi)*(
erf(sqrt(I*x - I)) - 1) + I*sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*sin(1))*sin
(1/2*arctan2(x - 1, 0)))*sqrt(-x + 1)/sqrt(abs(x - 1))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^2+1}\sqrt{x+1}\sin(x)}{x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*sqrt(x + 1)*sin(x)/(x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+1}\sin(x)}{\sqrt{-(x-1)(x+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(-x**2+1)**(1/2)*sin(x),x)
```

```
[Out] Integral(sqrt(x + 1)*sin(x)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [C] time = 1.20691, size = 65, normalized size = 1.05

$$-\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{i}+\left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{(-i)}+0.82661965\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="giac")
```

```
[Out] -(1/4*I + 1/4)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(-x + 1))*e^
I + (1/4*I - 1/4)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(-x + 1))*
e^(-I) + 0.82661965415
```

3.818 $\int e^{\tanh^{-1}(a+bx)} x^3 \, dx$

Optimal. Leaf size=156

$$-\frac{\sqrt{-a-bx+1}(a+bx+1)^{3/2}\left(18a^2+2(1-6a)bx-10a+7\right)}{24b^4}-\frac{\left(-8a^3+12a^2-12a+3\right)\sqrt{-a-bx+1}\sqrt{a+bx+1}}{8b^4}+\frac{(-8}{}$$

```
[Out] -((3 - 12*a + 12*a^2 - 8*a^3)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(8*b^4)
- (x^2*Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2))/(4*b^2) - (Sqrt[1 - a - b*x]*
(1 + a + b*x)^(3/2)*(7 - 10*a + 18*a^2 + 2*(1 - 6*a)*b*x))/(24*b^4) + ((3 -
 12*a + 12*a^2 - 8*a^3)*ArcSin[a + b*x])/(8*b^4)
```

Rubi [A] time = 0.159706, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6163, 100, 147, 50, 53, 619, 216}

$$-\frac{\sqrt{-a-bx+1}(a+bx+1)^{3/2}\left(18a^2+2(1-6a)bx-10a+7\right)}{24b^4}-\frac{\left(-8a^3+12a^2-12a+3\right)\sqrt{-a-bx+1}\sqrt{a+bx+1}}{8b^4}+\frac{(-8}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]*x^3,x]
```


```
[Out] -((3 - 12*a + 12*a^2 - 8*a^3)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(8*b^4)
- (x^2*Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2))/(4*b^2) - (Sqrt[1 - a - b*x]*
(1 + a + b*x)^(3/2)*(7 - 10*a + 18*a^2 + 2*(1 - 6*a)*b*x))/(24*b^4) + ((3 -
 12*a + 12*a^2 - 8*a^3)*ArcSin[a + b*x])/(8*b^4)
```


Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(a+bx)}x^3\,dx = \int \frac{x^3\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\,dx$$

$$= -\frac{x^2\sqrt{1-a-bx}(1+a+bx)^{3/2}}{4b^2} - \frac{\int \frac{x\sqrt{1+a+bx}\left(-2\left(1-a^2\right)-(1-6a)bx\right)}{\sqrt{1-a-bx}}\,dx}{4b^2}$$

$$= -\frac{x^2\sqrt{1-a-bx}(1+a+bx)^{3/2}}{4b^2} - \frac{\sqrt{1-a-bx}(1+a+bx)^{3/2}\left(7-10a+18a^2+2(1-6a)bx\right)}{24b^4} + \cdots$$

$$= -\frac{\left(3-12a+12a^2-8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} - \frac{x^2\sqrt{1-a-bx}(1+a+bx)^{3/2}}{4b^2} - \frac{\sqrt{1-a-b\cdots}}{\cdots}$$

$$= -\frac{\left(3-12a+12a^2-8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} - \frac{x^2\sqrt{1-a-bx}(1+a+bx)^{3/2}}{4b^2} - \frac{\sqrt{1-a-b\cdots}}{\cdots}$$

$$= -\frac{\left(3-12a+12a^2-8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} - \frac{x^2\sqrt{1-a-bx}(1+a+bx)^{3/2}}{4b^2} - \frac{\sqrt{1-a-b\cdots}}{\cdots}$$

$$= -\frac{\left(3-12a+12a^2-8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} - \frac{x^2\sqrt{1-a-bx}(1+a+bx)^{3/2}}{4b^2} - \frac{\sqrt{1-a-b\cdots}}{\cdots}$$

Mathematica [A] time = 0.392637, size = 149, normalized size = 0.96

$$-\frac{\sqrt{-a^2-2abx-b^2x^2+1}\left(a^2(6bx+44)-6a^3-a\left(6b^2x^2+20bx+39\right)+6b^3x^3+8b^2x^2+9bx+16\right)}{24b^4} - \frac{\left(8a^3-12a^2+\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]*x^3,x]
```

```
[Out] -(Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*(16 - 6*a^3 + 9*b*x + 8*b^2*x^2 + 6*b^3
*x^3 + a^2*(44 + 6*b*x) - a*(39 + 20*b*x + 6*b^2*x^2)))/(24*b^4) - ((-3 + 1
2*a - 12*a^2 + 8*a^3)*Sqrt[-b]*ArcSinh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2
]*Sqrt[b])])/(4*b^(9/2))
```

Maple [B] time = 0.067, size = 487, normalized size = 3.1

$$-\frac{x^3}{4b}\sqrt{-b^2x^2-2xab-a^2+1}+\frac{ax^2}{4b^2}\sqrt{-b^2x^2-2xab-a^2+1}-\frac{a^2x}{4b^3}\sqrt{-b^2x^2-2xab-a^2+1}+\frac{a^3}{4b^4}\sqrt{-b^2x^2-2xab-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^3,x)
```

```
[Out] -1/4*x^3/b*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/4*a/b^2*x^2*(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2)-1/4*a^2/b^3*x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/4*a^3/b^4*(-b^2*
x^2-2*a*b*x-a^2+1)^(1/2)+3/2*a^2/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)
/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+13/8*a/b^4*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-
3/8/b^3*x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/8/b^3/(b^2)^(1/2)*arctan((b^2)^(
1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/3*x^2/b^2*(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2)+5/6*a/b^3*x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-11/6*a^2/b^4*(-b^2*
x^2-2*a*b*x-a^2+1)^(1/2)-a^3/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b
^2*x^2-2*a*b*x-a^2+1)^(1/2))-3/2*a/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/
b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-2/3/b^4*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.14665, size = 338, normalized size = 2.17

$$\frac{3\left(8a^3-12a^2+12a-3\right)\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}(bx+a)}{b^2x^2+2abx+a^2-1}\right)-\left(6b^3x^3-2(3a-4)b^2x^2-6a^3+\left(6a^2-20a+9\right)bx+44a^2\right.}{24b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*a^3 - 12*a^2 + 12*a - 3)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (6*b^3*x^3 - 2*(3*a - 4)*b^2*
x^2 - 6*a^3 + (6*a^2 - 20*a + 9)*b*x + 44*a^2 - 39*a + 16)*sqrt(-b^2*x^2 -
2*a*b*x - a^2 + 1))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\,(a+bx+1)}{\sqrt{-\,(a+bx-1)\,(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)*x**3,x)
```

```
[Out] Integral(x**3*(a + b*x + 1)/sqrt(-(a + b*x - 1)*(a + b*x + 1)), x)
```

Giac [A] time = 1.21645, size = 188, normalized size = 1.21

$$-\frac{1}{24}\sqrt{-(bx+a)^2+1}\left(\left(2x\left(\frac{3x}{b}-\frac{3ab^5-4b^5}{b^7}\right)+\frac{6a^2b^4-20ab^4+9b^4}{b^7}\right)x-\frac{6a^3b^3-44a^2b^3+39ab^3-16b^3}{b^7}\right)+\frac{(8a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^3,x, algorithm="giac")
```

```
[Out] -1/24*sqrt(-(b*x + a)^2 + 1)*((2*x*(3*x/b - (3*a*b^5 - 4*b^5)/b^7) + (6*a^2
*b^4 - 20*a*b^4 + 9*b^4)/b^7)*x - (6*a^3*b^3 - 44*a^2*b^3 + 39*a*b^3 - 16*b
^3)/b^7) + 1/8*(8*a^3 - 12*a^2 + 12*a - 3)*arcsin(-b*x - a)*sgn(b)/(b^3*abs
(b))
```

3.819 $\int e^{\tanh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=130

$$-\frac{(2a^2-2a+1)\sqrt{-a-bx+1}\sqrt{a+bx+1}}{2b^3}+\frac{(2a^2-2a+1)\sin^{-1}(a+bx)}{2b^3}-\frac{x\sqrt{-a-bx+1}(a+bx+1)^{3/2}}{3b^2}-\frac{(1-4a)\sqrt{-a}}{}$$

```
[Out] -((1 - 2*a + 2*a^2)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^3) - ((1 - 4*
a)*Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2))/(6*b^3) - (x*Sqrt[1 - a - b*x]*(1
 + a + b*x)^(3/2))/(3*b^2) + ((1 - 2*a + 2*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rubi [A] time = 0.166757, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6163, 90, 80, 50, 53, 619, 216}

$$-\frac{(2a^2-2a+1)\sqrt{-a-bx+1}\sqrt{a+bx+1}}{2b^3}+\frac{(2a^2-2a+1)\sin^{-1}(a+bx)}{2b^3}-\frac{x\sqrt{-a-bx+1}(a+bx+1)^{3/2}}{3b^2}-\frac{(1-4a)\sqrt{-a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]*x^2,x]
```

```
[Out] -((1 - 2*a + 2*a^2)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^3) - ((1 - 4*
a)*Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2))/(6*b^3) - (x*Sqrt[1 - a - b*x]*(1
 + a + b*x)^(3/2))/(3*b^2) + ((1 - 2*a + 2*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(a+bx)}x^2\,dx &= \int \frac{x^2\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\,dx\\
&= -\frac{x\sqrt{1-a-bx}(1+a+bx)^{3/2}}{3b^2}-\frac{\int \frac{\sqrt{1+a+bx}\left(-1+a^2-(1-4a)bx\right)}{\sqrt{1-a-bx}}\,dx}{3b^2}\\
&= -\frac{(1-4a)\sqrt{1-a-bx}(1+a+bx)^{3/2}}{6b^3}-\frac{x\sqrt{1-a-bx}(1+a+bx)^{3/2}}{3b^2}+\frac{\left(1-2a+2a^2\right)\int \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}}{2b^2}\\
&= -\frac{\left(1-2a+2a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3}-\frac{(1-4a)\sqrt{1-a-bx}(1+a+bx)^{3/2}}{6b^3}-\frac{x\sqrt{1-a-bx}(}{3b}\\
&= -\frac{\left(1-2a+2a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3}-\frac{(1-4a)\sqrt{1-a-bx}(1+a+bx)^{3/2}}{6b^3}-\frac{x\sqrt{1-a-bx}(}{3b}\\
&= -\frac{\left(1-2a+2a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3}-\frac{(1-4a)\sqrt{1-a-bx}(1+a+bx)^{3/2}}{6b^3}-\frac{x\sqrt{1-a-bx}(}{3b}\\
&= -\frac{\left(1-2a+2a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3}-\frac{(1-4a)\sqrt{1-a-bx}(1+a+bx)^{3/2}}{6b^3}-\frac{x\sqrt{1-a-bx}(}{3b}
\end{aligned}
$$

Mathematica [A] time = 0.174695, size = 159, normalized size = 1.22

$$
\frac{-\sqrt{b}\sqrt{-a^2-2abx-b^2x^2+1}\left(2a^2-a(2bx+9)+2b^2x^2+3bx+4\right)+6\left(2a^2+1\right)\sqrt{-b}\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{b}}\right)+12a\sqrt{-b}}{6b^{7/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]*x^2,x]
```

```
[Out] (-(Sqrt[b]*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*(4 + 2*a^2 + 3*b*x + 2*b^2*x^2
 - a*(9 + 2*b*x))) + 6*(1 + 2*a^2)*Sqrt[-b]*ArcSinh[(Sqrt[-b]*Sqrt[1 - a -
b*x])/(Sqrt[2]*Sqrt[b])] + 12*a*Sqrt[-b]*ArcSinh[(Sqrt[b]*Sqrt[1 - a - b*x]
)/(Sqrt[2]*Sqrt[-b])])/(6*b^(7/2))
```

Maple [B] time = 0.062, size = 315, normalized size = 2.4

$$-\frac{x^2}{3b}\sqrt{-b^2x^2-2xab-a^2+1}+\frac{ax}{3b^2}\sqrt{-b^2x^2-2xab-a^2+1}-\frac{a^2}{3b^3}\sqrt{-b^2x^2-2xab-a^2+1}-\frac{a}{b^2}\arctan\left(\sqrt{b^2}\left(x+\frac{a}{b}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^2,x)
```

```
[Out] -1/3*x^2/b*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/3*a/b^2*x*(-b^2*x^2-2*a*b*x-a^2
+1)^(1/2)-1/3*a^2/b^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a/b^2/(b^2)^(1/2)*arct
an((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-2/3/b^3*(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2)-1/2*x/b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/2*a/b^3*(-b^2
*x^2-2*a*b*x-a^2+1)^(1/2)+a^2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-
b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b
)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.16545, size = 267, normalized size = 2.05

$$-\frac{3\left(2a^2-2a+1\right)\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}(bx+a)}{b^2x^2+2abx+a^2-1}\right)+\left(2b^2x^2-(2a-3)bx+2a^2-9a+4\right)\sqrt{-b^2x^2-2abx-a^2+1}}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*(2*a^2 - 2*a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x +
a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (2*b^2*x^2 - (2*a - 3)*b*x + 2*a^2 - 9*
a + 4)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2(a+bx+1)}{\sqrt{-(a+bx-1)(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)*x**2,x)
```

```
[Out] Integral(x**2*(a + b*x + 1)/sqrt(-(a + b*x - 1)*(a + b*x + 1)), x)
```

Giac [A] time = 1.17233, size = 131, normalized size = 1.01

$$-\frac{1}{6}\sqrt{-(bx+a)^2+1}\left(x\left(\frac{2x}{b}-\frac{2ab^3-3b^3}{b^5}\right)+\frac{2a^2b^2-9ab^2+4b^2}{b^5}\right)-\frac{\left(2a^2-2a+1\right)\arcsin\left(-bx-a\right)\operatorname{sgn}\left(b\right)}{2b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^2,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-(b*x + a)^2 + 1)*(x*(2*x/b - (2*a*b^3 - 3*b^3)/b^5) + (2*a^2*b^2
 - 9*a*b^2 + 4*b^2)/b^5) - 1/2*(2*a^2 - 2*a + 1)*arcsin(-b*x - a)*sgn(b)/(b
^2*abs(b))
```

3.820 $\int e^{\tanh^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{-a-bx+1}(a+bx+1)^{3/2}}{2b^2}-\frac{(1-2a)\sqrt{-a-bx+1}\sqrt{a+bx+1}}{2b^2}+\frac{(1-2a)\sin^{-1}(a+bx)}{2b^2}$$

```
[Out] -((1 - 2*a)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^2) - (Sqrt[1 - a - b*
x]*(1 + a + b*x)^(3/2))/(2*b^2) + ((1 - 2*a)*ArcSin[a + b*x])/(2*b^2)
```

Rubi [A] time = 0.0639097, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6163, 80, 50, 53, 619, 216}

$$-\frac{\sqrt{-a-bx+1}(a+bx+1)^{3/2}}{2b^2}-\frac{(1-2a)\sqrt{-a-bx+1}\sqrt{a+bx+1}}{2b^2}+\frac{(1-2a)\sin^{-1}(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]*x,x]
```

```
[Out] -((1 - 2*a)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^2) - (Sqrt[1 - a - b*
x]*(1 + a + b*x)^(3/2))/(2*b^2) + ((1 - 2*a)*ArcSin[a + b*x])/(2*b^2)
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(a+bx)}x\,dx = \int \frac{x\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\,dx$$
$$= -\frac{\sqrt{1-a-bx}(1+a+bx)^{3/2}}{2b^2} + \frac{(1-2a)\int \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\,dx}{2b}$$
$$= -\frac{(1-2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2} - \frac{\sqrt{1-a-bx}(1+a+bx)^{3/2}}{2b^2} + \frac{(1-2a)\int \frac{1}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{2b}$$
$$= -\frac{(1-2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2} - \frac{\sqrt{1-a-bx}(1+a+bx)^{3/2}}{2b^2} + \frac{(1-2a)\int \frac{1}{\sqrt{(1-a)(1+a)-2abx-b^2x^2}}$$
$$= -\frac{(1-2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2} - \frac{\sqrt{1-a-bx}(1+a+bx)^{3/2}}{2b^2} - \frac{(1-2a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx,\right.}{4b^3}$$
$$= -\frac{(1-2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2} - \frac{\sqrt{1-a-bx}(1+a+bx)^{3/2}}{2b^2} + \frac{(1-2a)\sin^{-1}(a+bx)}{2b^2}$$

Mathematica [A] time = 0.0909827, size = 130, normalized size = 1.55

$$\frac{\sqrt{b}\sqrt{-a^2-2abx-b^2x^2+1}(a-bx-2)+2\sqrt{-b}\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{b}}\right)+4a\sqrt{-b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{-b}}\right)}{2b^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]*x,x]
```

```
[Out] (Sqrt[b]*(-2 + a - b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] + 2*Sqrt[-b]*ArcS
inh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[b])] + 4*a*Sqrt[-b]*ArcSinh[
(Sqrt[b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[-b])])/(2*b^(5/2))
```

Maple [B] time = 0.058, size = 178, normalized size = 2.1

$$-\frac{x}{2b}\sqrt{-b^2x^2-2xab-a^2+1}+\frac{a}{2b^2}\sqrt{-b^2x^2-2xab-a^2+1}+\frac{1}{2b}\arctan\left(\sqrt{b^2}\left(x+\frac{a}{b}\right)\frac{1}{\sqrt{-b^2x^2-2xab-a^2+1}}\right)\frac{1}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x,x)
```

```
[Out] -1/2*x/b*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/2*a/b^2*(-b^2*x^2-2*a*b*x-a^2+1)^
(1/2)+1/2/b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)
^(1/2))-1/b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a/b/(b^2)^(1/2)*arctan((b^2)^(
1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.15044, size = 209, normalized size = 2.49

$$\frac{(2\,a-1)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)-\sqrt{-b^2x^2-2\,abx-a^2+1}(bx-a+2)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*((2*a - 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2
 + 2*a*b*x + a^2 - 1)) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x - a + 2))/
b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,(a+bx+1)}{\sqrt{-\,(a+bx-1)\,(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)*x,x)
```

```
[Out] Integral(x*(a + b*x + 1)/sqrt(-(a + b*x - 1)*(a + b*x + 1)), x)
```

Giac [A] time = 1.20743, size = 80, normalized size = 0.95

$$-\frac{1}{2}\sqrt{-(bx+a)^2+1}\left(\frac{x}{b}-\frac{ab-2\,b}{b^3}\right)+\frac{(2\,a-1)\arcsin\left(-bx-a\right)\operatorname{sgn}\left(b\right)}{2\,b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-(b*x + a)^2 + 1)*(x/b - (a*b - 2*b)/b^3) + 1/2*(2*a - 1)*arcsin(
-b*x - a)*sgn(b)/(b*abs(b))
```

3.821 $\int e^{\tanh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=39

$$\frac{\sin^{-1}(a+bx)}{b}-\frac{\sqrt{-a-bx+1}\sqrt{a+bx+1}}{b}$$

```
[Out] -((Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/b) + ArcSin[a + b*x]/b
```

Rubi [A] time = 0.0264667, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6161, 50, 53, 619, 216}

$$\frac{\sin^{-1}(a+bx)}{b}-\frac{\sqrt{-a-bx+1}\sqrt{a+bx+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x],x]
```

```
[Out] -((Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/b) + ArcSin[a + b*x]/b
```

Rule 6161

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 + a*c
 + b*c*x)^(n/2)/(1 - a*c - b*c*x)^(n/2), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(a+bx)}\,dx &= \int \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\,dx\\
&= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{b} + \int \frac{1}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx\\
&= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{b} + \int \frac{1}{\sqrt{(1-a)(1+a)-2abx-b^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{b} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2ab-2b^2x\right)}{2b^2}\\
&= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{b} + \frac{\sin^{-1}(a+bx)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0161056, size = 28, normalized size = 0.72

$$\frac{\sin^{-1}(a+bx)-\sqrt{1-(a+bx)^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a + b*x],x]
```

```
[Out] (-Sqrt[1 - (a + b*x)^2] + ArcSin[a + b*x])/b
```

Maple [A] time = 0.059, size = 71, normalized size = 1.8

$$-\frac{1}{b}\sqrt{-b^2x^2-2\,xab-a^2+1}+\arctan\left(\sqrt{b^2}\left(x+\frac{a}{b}\right)\frac{1}{\sqrt{-b^2x^2-2\,xab-a^2+1}}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2),x)
```

```
[Out] -1/b*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b
)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.26897, size = 170, normalized size = 4.36

$$-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}+\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1) + arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2
 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+bx+1}{\sqrt{-(a+bx-1)(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral((a + b*x + 1)/sqrt(-(a + b*x - 1)*(a + b*x + 1)), x)
```

Giac [A] time = 1.20283, size = 49, normalized size = 1.26

$$-\frac{\arcsin\left(-bx-a\right)\operatorname{sgn}\left(b\right)}{|b|}-\frac{\sqrt{-(bx+a)^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(-b*x - a)*sgn(b)/abs(b) - sqrt(-(b*x + a)^2 + 1)/b
```

3.822 $\int \frac{e^{\tanh^{-1}(a+bx)}}{x} dx$

Optimal. Leaf size=64

$$\sin^{-1}(a+bx) - \frac{2(a+1)\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{\sqrt{1-a^2}}$$

```
[Out] ArcSin[a + b*x] - (2*(1 + a)*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[
1 + a]*Sqrt[1 - a - b*x])])/Sqrt[1 - a^2]
```

Rubi [A] time = 0.0754015, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6163, 105, 53, 619, 216, 93, 208}

$$\sin^{-1}(a+bx) - \frac{2(a+1)\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]/x,x]
```

```
[Out] ArcSin[a + b*x] - (2*(1 + a)*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[
1 + a]*Sqrt[1 - a - b*x])])/Sqrt[1 - a^2]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(a+bx)}}{x}\,dx &= \int \frac{\sqrt{1+a+bx}}{x\sqrt{1-a-bx}}\,dx\\ &= -\left((-1-a)\int \frac{1}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx\right)+b\int \frac{1}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx\\ &= (2(1+a))\,\text{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx,x,\frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\right)+b\int \frac{1}{\sqrt{(1-a)(1+a)-2abx-b^2x^2}}\,dx\\ &= -\frac{2(1+a)\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{\sqrt{1-a^2}}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx,x,-2ab-2b^2x\right)}{2b}\\ &= \sin^{-1}(a+bx)-\frac{2(1+a)\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{\sqrt{1-a^2}}\end{aligned}$$

Mathematica [A] time = 0.10338, size = 100, normalized size = 1.56

$$\frac{2\tan^{-1}\left(\frac{\sqrt{-a-bx+1}}{\sqrt{\frac{a-1}{a+1}}\sqrt{a+bx+1}}\right)}{\sqrt{\frac{a-1}{a+1}}}+\frac{2\sqrt{-b}\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]/x,x]
```

```
[Out] (2*Sqrt[-b]*ArcSinh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[b])])/Sqrt[b
] + (2*ArcTan[Sqrt[1 - a - b*x]/(Sqrt[(-1 + a)/(1 + a)]*Sqrt[1 + a + b*x])]
)/Sqrt[(-1 + a)/(1 + a)]
```

Maple [B] time = 0.057, size = 168, normalized size = 2.6

$$b\arctan\left(\sqrt{b^2}\left(x+\frac{a}{b}\right)\frac{1}{\sqrt{-b^2x^2-2xab-a^2+1}}\right)\frac{1}{\sqrt{b^2}}-\ln\left(\frac{1}{x}\left(-2a^2+2-2xab+2\sqrt{-a^2+1}\sqrt{-b^2x^2-2xab-a^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x,x)
```

```
[Out] b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/
(-a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+
1)^(1/2))/x)-1/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x
^2-2*a*b*x-a^2+1)^(1/2))/x)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.63945, size = 717, normalized size = 11.2

$$\left[\frac{1}{2}\sqrt{-\frac{a+1}{a-1}}\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx-4a^2-2\sqrt{-b^2x^2-2abx-a^2+1}\left(a^3+\left(a^2-a\right)bx-a^2-a+1\right)\sqrt{}}{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-(a + 1)/(a - 1))*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*
b*x - 4*a^2 - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^3 + (a^2 - a)*b*x - a
^2 - a + 1)*sqrt(-(a + 1)/(a - 1)) + 2)/x^2) - arctan(sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)), sqrt((a + 1)/(a - 1
))*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt((a + 1)
/(a - 1))/((a + 1)*b^2*x^2 + a^3 + 2*(a^2 + a)*b*x + a^2 - a - 1)) - arctan
(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx+1}{x\sqrt{-(a+bx-1)(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)/x,x)
```

```
[Out] Integral((a + b*x + 1)/(x*sqrt(-(a + b*x - 1)*(a + b*x + 1))), x)
```

Giac [A] time = 1.20983, size = 107, normalized size = 1.67

$$-\frac{b\arcsin\left(-bx-a\right)\operatorname{sgn}\left(b\right)}{|b|}+\frac{2\left(ab+b\right)\arctan\left(\frac{\frac{\left(\sqrt{-(bx+a)^2+1}|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -b*arcsin(-b*x - a)*sgn(b)/abs(b) + 2*(a*b + b)*arctan(((sqrt(-(b*x + a)^2
+ 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/(sqrt(a^2 - 1)*abs(b))
```

3.823 $\int \frac{e^{\tanh^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=98

$$-\frac{2b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)\sqrt{1-a^2}}-\frac{\sqrt{-a-bx+1}\sqrt{a+bx+1}}{(1-a)x}$$

```
[Out] -((Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/((1 - a)*x)) - (2*b*ArcTanh[(Sqrt[1
 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 - a)*Sqrt[1
- a^2])
```

Rubi [A] time = 0.0559057, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6163, 94, 93, 208}

$$-\frac{2b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)\sqrt{1-a^2}}-\frac{\sqrt{-a-bx+1}\sqrt{a+bx+1}}{(1-a)x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]/x^2,x]
```

```
[Out] -((Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/((1 - a)*x)) - (2*b*ArcTanh[(Sqrt[1
 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 - a)*Sqrt[1
- a^2])
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{1+a+bx}}{x^2\sqrt{1-a-bx}}\,dx$$
$$= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{(1-a)x} + \frac{b\int \frac{1}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{1-a}$$
$$= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{(1-a)x} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx, x, \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\right)}{1-a}$$
$$= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{(1-a)x} - \frac{2b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{(1-a)\sqrt{1-a^2}}$$

Mathematica [A] time = 0.0499374, size = 83, normalized size = 0.85

$$\frac{\sqrt{-(a+bx-1)(a+bx+1)}}{(a-1)x} - \frac{2b\tan^{-1}\left(\frac{\sqrt{-a-bx+1}}{\sqrt{\frac{a-1}{a+1}}\sqrt{a+bx+1}}\right)}{(a-1)^{3/2}\sqrt{a+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]/x^2,x]
```

```
[Out] Sqrt[-((-1 + a + b*x)*(1 + a + b*x))]/((-1 + a)*x) - (2*b*ArcTan[Sqrt[1 - a
 - b*x]/(Sqrt[(-1 + a)/(1 + a)]*Sqrt[1 + a + b*x])])/((-1 + a)^(3/2)*Sqrt[1
 + a])
```

Maple [B] time = 0.063, size = 265, normalized size = 2.7

$$-\frac{1}{\left(-a^2+1\right)x}\sqrt{-b^2x^2-2\,xab-a^2+1} - ab\ln\left(\frac{1}{x}\left(-2\,a^2+2-2\,xab+2\,\sqrt{-a^2+1}\sqrt{-b^2x^2-2\,xab-a^2+1}\right)\right)\left(-a^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^2,x)
```

```
[Out] -1/(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a*b/(-a^2+1)^(3/2)*ln((-2*a^2+
2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-a/(-a^2+1)/x*
(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a^2*b/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*
(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-b/(-a^2+1)^(1/2)*ln((-2*a
^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 2.66297, size = 633, normalized size = 6.46

$$\left[-\frac{\sqrt{-a^2+1}bx\log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx-2\sqrt{-b^2x^2-2abx-a^2+1}(abx+a^2-1)\sqrt{-a^2+1}-4a^2+2}{x^2}\right)-2\sqrt{-b^2x^2-2abx-a^2+1}(a^2-1)}{2(a^3-a^2-a+1)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + 1)*b*x*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*
x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1) -
 4*a^2 + 2)/x^2) - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1))/((a^3 -
a^2 - a + 1)*x), -(sqrt(a^2 - 1)*b*x*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*
b*x - 2*a^2 + 1)) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1))/((a^3 - a
^2 - a + 1)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+bx+1}{x^2\sqrt{-(a+bx-1)(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)/x**2,x)
```

```
[Out] Integral((a + b*x + 1)/(x**2*sqrt(-(a + b*x - 1)*(a + b*x + 1))), x)
```

Giac [B] time = 1.27024, size = 257, normalized size = 2.62

$$-\frac{2b^2\arctan\left(\frac{\frac{\left(\sqrt{-(bx+a)^2+1}|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}(a|b|-|b|)}+\frac{2\left(ab^2-\frac{\left(\sqrt{-(bx+a)^2+1}|b|+b\right)b^2}{b^2x+ab}\right)}{\left(a^2|b|-a|b|\right)\left(\frac{\left(\sqrt{-(bx+a)^2+1}|b|+b\right)^2a}{(b^2x+ab)^2}+a-\frac{2\left(\sqrt{-(bx+a)^2+1}|b|+b\right)}{b^2x+ab}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -2*b^2*arctan(((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)/sqr
t(a^2 - 1))/(sqrt(a^2 - 1)*(a*abs(b) - abs(b))) + 2*(a*b^2 - (sqrt(-(b*x +
a)^2 + 1)*abs(b) + b)*b^2/(b^2*x + a*b))/((a^2*abs(b) - a*abs(b))*((sqrt(-(
b*x + a)^2 + 1)*abs(b) + b)^2*a/(b^2*x + a*b)^2 + a - 2*(sqrt(-(b*x + a)^2
+ 1)*abs(b) + b)/(b^2*x + a*b)))
```

3.824 $\int \frac{e^{\tanh^{-1}(a+bx)}}{x^3} dx$

Optimal. Leaf size=162

$$-\frac{(2a+1)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)^2(a+1)\sqrt{1-a^2}}-\frac{\sqrt{-a-bx+1}(a+bx+1)^{3/2}}{2\left(1-a^2\right)x^2}-\frac{(2a+1)b\sqrt{-a-bx+1}\sqrt{a+bx+1}}{2(1-a)^2(a+1)x}$$

```
[Out] -((1 + 2*a)*b*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*(1 - a)^2*(1 + a)*x)
- (Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2))/(2*(1 - a^2)*x^2) - ((1 + 2*a)*b^
2*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])
/((1 - a)^2*(1 + a)*Sqrt[1 - a^2])
```

Rubi [A] time = 0.108395, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6163, 96, 94, 93, 208}

$$-\frac{(2a+1)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)^2(a+1)\sqrt{1-a^2}}-\frac{\sqrt{-a-bx+1}(a+bx+1)^{3/2}}{2\left(1-a^2\right)x^2}-\frac{(2a+1)b\sqrt{-a-bx+1}\sqrt{a+bx+1}}{2(1-a)^2(a+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]/x^3,x]
```

```
[Out] -((1 + 2*a)*b*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*(1 - a)^2*(1 + a)*x)
- (Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2))/(2*(1 - a^2)*x^2) - ((1 + 2*a)*b^
2*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])
/((1 - a)^2*(1 + a)*Sqrt[1 - a^2])
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(a+bx)}}{x^3}\,dx = \int \frac{\sqrt{1+a+bx}}{x^3\sqrt{1-a-bx}}\,dx$$

$$= -\frac{\sqrt{1-a-bx}(1+a+bx)^{3/2}}{2\left(1-a^2\right)x^2} + \frac{((1+2a)b)\int \frac{\sqrt{1+a+bx}}{x^2\sqrt{1-a-bx}}\,dx}{2\left(1-a^2\right)}$$

$$= -\frac{(1+2a)b\sqrt{1-a-bx}\sqrt{1+a+bx}}{2(1-a)^2(1+a)x} - \frac{\sqrt{1-a-bx}(1+a+bx)^{3/2}}{2\left(1-a^2\right)x^2} + \frac{\left((1+2a)b^2\right)\int \frac{1}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,d}{2(1-a)^2(1+a)}$$

$$= -\frac{(1+2a)b\sqrt{1-a-bx}\sqrt{1+a+bx}}{2(1-a)^2(1+a)x} - \frac{\sqrt{1-a-bx}(1+a+bx)^{3/2}}{2\left(1-a^2\right)x^2} + \frac{\left((1+2a)b^2\right)\text{Subst}\left(\int \frac{1}{-1-a-(-1+}\right.}{(1-a)^2(1+a)}$$

$$= -\frac{(1+2a)b\sqrt{1-a-bx}\sqrt{1+a+bx}}{2(1-a)^2(1+a)x} - \frac{\sqrt{1-a-bx}(1+a+bx)^{3/2}}{2\left(1-a^2\right)x^2} - \frac{(1+2a)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{(1-a)^2(1+a)\sqrt{1-a^2}}$$

Mathematica [A] time = 0.112476, size = 117, normalized size = 0.72

$$\frac{\left(a^2-abx-2bx-1\right)\sqrt{-a^2-2abx-b^2x^2+1}}{2(a-1)^2(a+1)x^2} + \frac{(2a+1)b^2\tan^{-1}\left(\frac{\sqrt{-a-bx+1}}{\sqrt{\frac{a-1}{a+1}}\sqrt{a+bx+1}}\right)}{(a-1)^{5/2}(a+1)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]/x^3,x]
```

```
[Out] ((-1 + a^2 - 2*b*x - a*b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/(2*(-1 + a)^
2*(1 + a)*x^2) + ((1 + 2*a)*b^2*ArcTan[Sqrt[1 - a - b*x]/(Sqrt[(-1 + a)/(1
+ a)]*Sqrt[1 + a + b*x])])/((-1 + a)^(5/2)*(1 + a)^(3/2))
```

Maple [B] time = 0.072, size = 453, normalized size = 2.8

$$-\frac{b}{\left(-a^2+1\right)x}\sqrt{-b^2x^2-2\,xab-a^2+1} - \frac{3\,ab^2}{2}\ln\left(\frac{1}{x}\left(-2\,a^2+2-2\,xab+2\,\sqrt{-a^2+1}\sqrt{-b^2x^2-2\,xab-a^2+1}\right)\right)\left(-a^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^3,x)
```

```
[Out] -b/(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/2*a*b^2/(-a^2+1)^(3/2)*ln((-
2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/2/(-a
^2+1)/x^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/2*a*b/(-a^2+1)^2/x*(-b^2*x^2-2*a
*b*x-a^2+1)^(1/2)-3/2*a^2*b^2/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1
)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/2*b^2/(-a^2+1)^(3/2)*ln((-2*a^
2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/2*a/(-a^2
+1)/x^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/2*a^2*b/(-a^2+1)^2/x*(-b^2*x^2-2*a
*b*x-a^2+1)^(1/2)-3/2*a^3*b^2/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1
)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.63096, size = 821, normalized size = 5.07

$$\left[-\frac{\sqrt{-a^2+1}(2a+1)b^2x^2\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx+2\sqrt{-b^2x^2-2abx-a^2+1}\left(abx+a^2-1\right)\sqrt{-a^2+1}-4a^2+2}{x^2}\right)-2\left(a^4-\left(a^3+2a^2-\right.\right.}{4\left(a^5-a^4-2a^3+2a^2+a-1\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a^2 + 1)*(2*a + 1)*b^2*x^2*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 +
4*(a^3 - a)*b*x + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sq
rt(-a^2 + 1) - 4*a^2 + 2)/x^2) - 2*(a^4 - (a^3 + 2*a^2 - a - 2)*b*x - 2*a^2
 + 1)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^5 - a^4 - 2*a^3 + 2*a^2 + a -
 1)*x^2), 1/2*(sqrt(a^2 - 1)*(2*a + 1)*b^2*x^2*arctan(sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*
(a^3 - a)*b*x - 2*a^2 + 1)) + (a^4 - (a^3 + 2*a^2 - a - 2)*b*x - 2*a^2 + 1)
*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^5 - a^4 - 2*a^3 + 2*a^2 + a - 1)*x
^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx+1}{x^3\sqrt{-(a+bx-1)(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)/x**3,x)
```

```
[Out] Integral((a + b*x + 1)/(x**3*sqrt(-(a + b*x - 1)*(a + b*x + 1))), x)
```

Giac [B] time = 1.30846, size = 840, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (2*a*b^3 + b^3)*arctan(((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b)
 - 1)/sqrt(a^2 - 1))/((a^3*abs(b) - a^2*abs(b) - a*abs(b) + abs(b))*sqrt(a^
2 - 1)) - (2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^4*b^3/(b^2*x + a*b)^2
+ 2*a^4*b^3 - 5*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a^3*b^3/(b^2*x + a*b) +
 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^3*b^3/(b^2*x + a*b)^2 - 3*(sqrt(
-(b*x + a)^2 + 1)*abs(b) + b)^3*a^3*b^3/(b^2*x + a*b)^3 + 2*a^3*b^3 - 6*(sq
rt(-(b*x + a)^2 + 1)*abs(b) + b)*a^2*b^3/(b^2*x + a*b) + 3*(sqrt(-(b*x + a)
^2 + 1)*abs(b) + b)^2*a^2*b^3/(b^2*x + a*b)^2 - 2*(sqrt(-(b*x + a)^2 + 1)*a
bs(b) + b)^3*a^2*b^3/(b^2*x + a*b)^3 - a^2*b^3 + 2*(sqrt(-(b*x + a)^2 + 1)*
abs(b) + b)*a*b^3/(b^2*x + a*b) + 4*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a
*b^3/(b^2*x + a*b)^2 + 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a*b^3/(b^2*x
 + a*b)^3 - 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*b^3/(b^2*x + a*b)^2)/((
a^5*abs(b) - a^4*abs(b) - a^3*abs(b) + a^2*abs(b))*((sqrt(-(b*x + a)^2 + 1)
*abs(b) + b)^2*a/(b^2*x + a*b)^2 + a - 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b
)/(b^2*x + a*b))^2)
```

3.825 $\int \frac{e^{\tanh^{-1}(a+bx)}}{x^4} dx$

Optimal. Leaf size=213

$$-\frac{(2a^2+2a+1)\,b^3\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)\left(1-a^2\right)^{5/2}}-\frac{(a+4)(2a+1)b^2\sqrt{-a-bx+1}\sqrt{a+bx+1}}{6(1-a)^3(a+1)^2x}-\frac{(2a+3)b\sqrt{-a-bx+1}\sqrt{a}}{6(1-a)^2(a+1)x^2}$$

```
[Out] -(Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(3*(1 - a)*x^3) - ((3 + 2*a)*b*Sqrt[
1 - a - b*x]*Sqrt[1 + a + b*x])/(6*(1 - a)^2*(1 + a)*x^2) - ((4 + a)*(1 + 2
*a)*b^2*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(6*(1 - a)^3*(1 + a)^2*x) - ((
1 + 2*a + 2*a^2)*b^3*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*S
qrt[1 - a - b*x])])/((1 - a)*(1 - a^2)^(5/2))
```

Rubi [A] time = 0.182627, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6163, 99, 151, 12, 93, 208}

$$-\frac{(2a^2+2a+1)\,b^3\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)\left(1-a^2\right)^{5/2}}-\frac{(a+4)(2a+1)b^2\sqrt{-a-bx+1}\sqrt{a+bx+1}}{6(1-a)^3(a+1)^2x}-\frac{(2a+3)b\sqrt{-a-bx+1}\sqrt{a}}{6(1-a)^2(a+1)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]/x^4,x]
```

```
[Out] -(Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(3*(1 - a)*x^3) - ((3 + 2*a)*b*Sqrt[
1 - a - b*x]*Sqrt[1 + a + b*x])/(6*(1 - a)^2*(1 + a)*x^2) - ((4 + a)*(1 + 2
*a)*b^2*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(6*(1 - a)^3*(1 + a)^2*x) - ((
1 + 2*a + 2*a^2)*b^3*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*S
qrt[1 - a - b*x])])/((1 - a)*(1 - a^2)^(5/2))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(a+bx)}}{x^4} dx = \int \frac{\sqrt{1+a+bx}}{x^4\sqrt{1-a-bx}} dx$$

$$= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{3(1-a)x^3} + \frac{\int \frac{(3+2a)b+2b^2x}{x^3\sqrt{1-a-bx}\sqrt{1+a+bx}} dx}{3(1-a)}$$

$$= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{3(1-a)x^3} - \frac{(3+2a)b\sqrt{1-a-bx}\sqrt{1+a+bx}}{6(1-a)^2(1+a)x^2} - \frac{\int \frac{-(4+a)(1+2a)b^2-(3+2a)b^3x}{x^2\sqrt{1-a-bx}\sqrt{1+a+bx}} dx}{6(1-a)^2(1+a)}$$

$$= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{3(1-a)x^3} - \frac{(3+2a)b\sqrt{1-a-bx}\sqrt{1+a+bx}}{6(1-a)^2(1+a)x^2} - \frac{(4+a)(1+2a)b^2\sqrt{1-a-bx}\sqrt{1+}}{6(1-a)^3(1+a)^2x}$$

$$= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{3(1-a)x^3} - \frac{(3+2a)b\sqrt{1-a-bx}\sqrt{1+a+bx}}{6(1-a)^2(1+a)x^2} - \frac{(4+a)(1+2a)b^2\sqrt{1-a-bx}\sqrt{1+}}{6(1-a)^3(1+a)^2x}$$

$$= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{3(1-a)x^3} - \frac{(3+2a)b\sqrt{1-a-bx}\sqrt{1+a+bx}}{6(1-a)^2(1+a)x^2} - \frac{(4+a)(1+2a)b^2\sqrt{1-a-bx}\sqrt{1+}}{6(1-a)^3(1+a)^2x}$$

$$= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{3(1-a)x^3} - \frac{(3+2a)b\sqrt{1-a-bx}\sqrt{1+a+bx}}{6(1-a)^2(1+a)x^2} - \frac{(4+a)(1+2a)b^2\sqrt{1-a-bx}\sqrt{1+}}{6(1-a)^3(1+a)^2x}$$

Mathematica [A] time = 0.271107, size = 186, normalized size = 0.87

$$\frac{\frac{3(2a^2+2a+1)b^2x^2\left(\sqrt{a-1}\sqrt{a+1}\sqrt{-(a+bx-1)(a+bx+1)}-2bx\tan^{-1}\left(\frac{\sqrt{-a-bx+1}}{\sqrt{\frac{a-1}{a+1}}\sqrt{a+bx+1}}\right)\right)}{(a-1)^{3/2}\sqrt{a+1}} - (4a+1)bx\sqrt{-a-bx+1}(a+bx+1)^{3/2} + 2(a-1)(a+1)}{6\left(a^2-1\right)^2 x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]/x^4,x]
```

```
[Out] (2*(-1 + a)*(1 + a)*Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2) - (1 + 4*a)*b*x*S
qrt[1 - a - b*x]*(1 + a + b*x)^(3/2) + (3*(1 + 2*a + 2*a^2)*b^2*x^2*(Sqrt[-
1 + a]*Sqrt[1 + a]*Sqrt[-((-1 + a + b*x)*(1 + a + b*x))] - 2*b*x*ArcTan[Sqr
t[1 - a - b*x]/(Sqrt[(-1 + a)/(1 + a)]*Sqrt[1 + a + b*x])]))/((-1 + a)^(3/2
)*Sqrt[1 + a]))/(6*(-1 + a^2)^2*x^3)
```

Maple [B] time = 0.067, size = 683, normalized size = 3.2

$$-\frac{b}{\left(-2\,a^2+2\right)x^2}\sqrt{-b^2x^2-2\,xab-a^2+1}-\frac{13\,ab^2}{6\left(-a^2+1\right)^2x}\sqrt{-b^2x^2-2\,xab-a^2+1}-3\,\frac{a^2b^3}{\left(-a^2+1\right)^{5/2}}\ln\left(\frac{-2\,a^2+2-2\,\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^4,x)
```

```
[Out] -1/2*b/(-a^2+1)/x^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-13/6*a*b^2/(-a^2+1)^2/x*
(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3*a^2*b^3/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*x*a*
b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/2*b^3/(-a^2+1)^(3/2
)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-
1/3/(-a^2+1)/x^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-5/6*a*b/(-a^2+1)^2/x^2*(-b^
2*x^2-2*a*b*x-a^2+1)^(1/2)-5/2*a^2*b^2/(-a^2+1)^3/x*(-b^2*x^2-2*a*b*x-a^2+1
)^(1/2)-5/2*a^3*b^3/(-a^2+1)^(7/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-
b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-3/2*a*b^3/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*x*
a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-2/3*b^2/(-a^2+1)^2/
x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/3*a/(-a^2+1)/x^3*(-b^2*x^2-2*a*b*x-a^2+1
)^(1/2)-5/6*a^2*b/(-a^2+1)^2/x^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-5/2*a^3*b^2
/(-a^2+1)^3/x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-5/2*a^4*b^3/(-a^2+1)^(7/2)*ln(
(-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.28506, size = 1094, normalized size = 5.14

$$\left[-\frac{3\left(2\,a^2+2\,a+1\right)\sqrt{-a^2+1}\,b^3x^3\log\left(\frac{\left(2\,a^2-1\right)b^2x^2+2\,a^4+4\left(a^3-a\right)bx-2\,\sqrt{-b^2x^2-2\,abx-a^2+1}\left(abx+a^2-1\right)\sqrt{-a^2+1}-4\,a^2+2}{x^2}\right)-2\left(2\,a^6+\left(2\cdots\right.\right.}{12\left(a^7-a^6-3\,a^5+3\,a^4\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*a^2 + 2*a + 1)*sqrt(-a^2 + 1)*b^3*x^3*log(((2*a^2 - 1)*b^2*x^2
 + 2*a^4 + 4*(a^3 - a)*b*x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x +
```

```
a^2 - 1)*sqrt(-a^2 + 1) - 4*a^2 + 2)/x^2) - 2*(2*a^6 + (2*a^4 + 9*a^3 + 2*a
^2 - 9*a - 4)*b^2*x^2 - 6*a^4 - (2*a^5 + 3*a^4 - 4*a^3 - 6*a^2 + 2*a + 3)*b
*x + 6*a^2 - 2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^7 - a^6 - 3*a^5 + 3
*a^4 + 3*a^3 - 3*a^2 - a + 1)*x^3), -1/6*(3*(2*a^2 + 2*a + 1)*sqrt(a^2 - 1)
*b^3*x^3*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a
^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) - (2*a^6 +
 (2*a^4 + 9*a^3 + 2*a^2 - 9*a - 4)*b^2*x^2 - 6*a^4 - (2*a^5 + 3*a^4 - 4*a^3
 - 6*a^2 + 2*a + 3)*b*x + 6*a^2 - 2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((
a^7 - a^6 - 3*a^5 + 3*a^4 + 3*a^3 - 3*a^2 - a + 1)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx+1}{x^4\sqrt{-(a+bx-1)(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)/x**4,x)
```

```
[Out] Integral((a + b*x + 1)/(x**4*sqrt(-(a + b*x - 1)*(a + b*x + 1))), x)
```

Giac [B] time = 1.33715, size = 1858, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -(2*a^2*b^4 + 2*a*b^4 + b^4)*arctan(((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a/
(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/((a^5*abs(b) - a^4*abs(b) - 2*a^3*abs(b)
+ 2*a^2*abs(b) + a*abs(b) - abs(b))*sqrt(a^2 - 1)) + 1/3*(12*(sqrt(-(b*x +
a)^2 + 1)*abs(b) + b)^2*a^7*b^4/(b^2*x + a*b)^2 + 6*(sqrt(-(b*x + a)^2 + 1)
*abs(b) + b)^4*a^7*b^4/(b^2*x + a*b)^4 + 6*a^7*b^4 - 24*(sqrt(-(b*x + a)^2
+ 1)*abs(b) + b)*a^6*b^4/(b^2*x + a*b) + 24*(sqrt(-(b*x + a)^2 + 1)*abs(b)
+ b)^2*a^6*b^4/(b^2*x + a*b)^2 - 36*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a
^6*b^4/(b^2*x + a*b)^3 + 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^4*a^6*b^4/(
b^2*x + a*b)^4 - 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^5*a^6*b^4/(b^2*x +
a*b)^5 + 12*a^6*b^4 - 57*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a^5*b^4/(b^2*x
 + a*b) + 36*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^5*b^4/(b^2*x + a*b)^2
- 72*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a^5*b^4/(b^2*x + a*b)^3 + 30*(sq
rt(-(b*x + a)^2 + 1)*abs(b) + b)^4*a^5*b^4/(b^2*x + a*b)^4 - 15*(sqrt(-(b*x
 + a)^2 + 1)*abs(b) + b)^5*a^5*b^4/(b^2*x + a*b)^5 - 2*a^5*b^4 + 84*(sqrt(-
(b*x + a)^2 + 1)*abs(b) + b)^2*a^4*b^4/(b^2*x + a*b)^2 - 12*(sqrt(-(b*x + a
)^2 + 1)*abs(b) + b)^3*a^4*b^4/(b^2*x + a*b)^3 + 51*(sqrt(-(b*x + a)^2 + 1)
*abs(b) + b)^4*a^4*b^4/(b^2*x + a*b)^4 + 12*(sqrt(-(b*x + a)^2 + 1)*abs(b)
+ b)^5*a^4*b^4/(b^2*x + a*b)^5 - 3*a^4*b^4 + 12*(sqrt(-(b*x + a)^2 + 1)*abs
(b) + b)*a^3*b^4/(b^2*x + a*b) - 30*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a
^3*b^4/(b^2*x + a*b)^3 - 18*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^4*a^3*b^4/(
b^2*x + a*b)^4 + 6*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^5*a^3*b^4/(b^2*x + a
*b)^5 + 2*a^3*b^4 - 6*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a^2*b^4/(b^2*x +
a*b) - 18*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^2*b^4/(b^2*x + a*b)^2 - 4
*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a^2*b^4/(b^2*x + a*b)^3 - 18*(sqrt(-
(b*x + a)^2 + 1)*abs(b) + b)^4*a^2*b^4/(b^2*x + a*b)^4 - 6*(sqrt(-(b*x + a)
^2 + 1)*abs(b) + b)^5*a^2*b^4/(b^2*x + a*b)^5 + 12*(sqrt(-(b*x + a)^2 + 1)*
```

```
abs(b) + b)^2*a*b^4/(b^2*x + a*b)^2 + 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b
)^3*a*b^4/(b^2*x + a*b)^3 + 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^4*a*b^4/
(b^2*x + a*b)^4 - 8*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*b^4/(b^2*x + a*b)
^3)/((a^8*abs(b) - a^7*abs(b) - 2*a^6*abs(b) + 2*a^5*abs(b) + a^4*abs(b) -
a^3*abs(b))*((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a/(b^2*x + a*b)^2 + a -
2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b))^3)
```

3.826 $\int e^{2\tanh^{-1}(a+bx)}x^4\,dx$

Optimal. Leaf size=83

$$-\frac{2(1-a)x^3}{3b^2}-\frac{(1-a)^2x^2}{b^3}-\frac{2(1-a)^3x}{b^4}-\frac{2(1-a)^4\log(-a-bx+1)}{b^5}-\frac{x^4}{2b}-\frac{x^5}{5}$$

```
[Out] (-2*(1 - a)^3*x)/b^4 - ((1 - a)^2*x^2)/b^3 - (2*(1 - a)*x^3)/(3*b^2) - x^4/
(2*b) - x^5/5 - (2*(1 - a)^4*Log[1 - a - b*x])/b^5
```

Rubi [A] time = 0.0834926, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{2(1-a)x^3}{3b^2}-\frac{(1-a)^2x^2}{b^3}-\frac{2(1-a)^3x}{b^4}-\frac{2(1-a)^4\log(-a-bx+1)}{b^5}-\frac{x^4}{2b}-\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x])*x^4,x]
```

```
[Out] (-2*(1 - a)^3*x)/b^4 - ((1 - a)^2*x^2)/b^3 - (2*(1 - a)*x^3)/(3*b^2) - x^4/
(2*b) - x^5/5 - (2*(1 - a)^4*Log[1 - a - b*x])/b^5
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(a+bx)}x^4\,dx &= \int \frac{x^4(1+a+bx)}{1-a-bx}\,dx\\ &= \int \left(\frac{2(-1+a)^3}{b^4}-\frac{2(-1+a)^2x}{b^3}+\frac{2(-1+a)x^2}{b^2}-\frac{2x^3}{b}-x^4-\frac{2(-1+a)^4}{b^4(-1+a+bx)}\right)dx\\ &= -\frac{2(1-a)^3x}{b^4}-\frac{(1-a)^2x^2}{b^3}-\frac{2(1-a)x^3}{3b^2}-\frac{x^4}{2b}-\frac{x^5}{5}-\frac{2(1-a)^4\log(1-a-bx)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.0636001, size = 75, normalized size = 0.9

$$\frac{2(a-1)x^3}{3b^2}-\frac{(a-1)^2x^2}{b^3}+\frac{2(a-1)^3x}{b^4}-\frac{2(a-1)^4\log(-a-bx+1)}{b^5}-\frac{x^4}{2b}-\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x])*x^4,x]
```

```
[Out] (2*(-1 + a)^3*x)/b^4 - ((-1 + a)^2*x^2)/b^3 + (2*(-1 + a)*x^3)/(3*b^2) - x^
4/(2*b) - x^5/5 - (2*(-1 + a)^4*Log[1 - a - b*x])/b^5
```

Maple [B] time = 0.03, size = 161, normalized size = 1.9

$$-\frac{x^5}{5}-\frac{x^4}{2\,b}+\frac{2\,x^3a}{3\,b^2}-\frac{2\,x^3}{3\,b^2}-\frac{a^2x^2}{b^3}+2\,\frac{ax^2}{b^3}+2\,\frac{xa^3}{b^4}-\frac{x^2}{b^3}-6\,\frac{a^2x}{b^4}+6\,\frac{ax}{b^4}-2\,\frac{x}{b^4}-2\,\frac{\ln\left(bx+a-1\right)a^4}{b^5}+8\,\frac{\ln\left(bx+a-1\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2)*x^4,x)
```

```
[Out] -1/5*x^5-1/2*x^4/b+2/3/b^2*x^3*a-2/3/b^2*x^3-1/b^3*x^2*a^2+2/b^3*x^2*a+2/b^
4*x*a^3-1/b^3*x^2-6/b^4*x*a^2+6/b^4*a*x-2/b^4*x-2/b^5*ln(b*x+a-1)*a^4+8/b^5
*ln(b*x+a-1)*a^3-12/b^5*ln(b*x+a-1)*a^2+8/b^5*ln(b*x+a-1)*a-2/b^5*ln(b*x+a-
1)
```

Maxima [A] time = 0.961459, size = 127, normalized size = 1.53

$$-\frac{6\,b^4x^5+15\,b^3x^4-20\,(a-1)b^2x^3+30\left(a^2-2\,a+1\right)bx^2-60\left(a^3-3\,a^2+3\,a-1\right)x}{30\,b^4}-\frac{2\left(a^4-4\,a^3+6\,a^2-4\,a+1\right)\log}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^4,x, algorithm="maxima")
```

```
[Out] -1/30*(6*b^4*x^5 + 15*b^3*x^4 - 20*(a - 1)*b^2*x^3 + 30*(a^2 - 2*a + 1)*b*x
^2 - 60*(a^3 - 3*a^2 + 3*a - 1)*x)/b^4 - 2*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*
log(b*x + a - 1)/b^5
```

Fricas [A] time = 1.88907, size = 234, normalized size = 2.82

$$-\frac{6\,b^5x^5+15\,b^4x^4-20\,(a-1)b^3x^3+30\left(a^2-2\,a+1\right)b^2x^2-60\left(a^3-3\,a^2+3\,a-1\right)bx+60\left(a^4-4\,a^3+6\,a^2-4\,a+1\right)}{30\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^4,x, algorithm="fricas")
```

```
[Out] -1/30*(6*b^5*x^5 + 15*b^4*x^4 - 20*(a - 1)*b^3*x^3 + 30*(a^2 - 2*a + 1)*b^2
*x^2 - 60*(a^3 - 3*a^2 + 3*a - 1)*b*x + 60*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*
log(b*x + a - 1))/b^5
```

Sympy [A] time = 0.444106, size = 78, normalized size = 0.94

$$-\frac{x^5}{5}-\frac{x^4}{2b}+\frac{x^3\,(2a-2)}{3b^2}-\frac{x^2\left(a^2-2a+1\right)}{b^3}+\frac{x\left(2a^3-6a^2+6a-2\right)}{b^4}-\frac{2\,(a-1)^4\log\left(a+bx-1\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2)*x**4,x)
```

```
[Out] -x**5/5 - x**4/(2*b) + x**3*(2*a - 2)/(3*b**2) - x**2*(a**2 - 2*a + 1)/b**3
 + x*(2*a**3 - 6*a**2 + 6*a - 2)/b**4 - 2*(a - 1)**4*log(a + b*x - 1)/b**5
```

Giac [A] time = 1.15296, size = 165, normalized size = 1.99

$$-\frac{2\left(a^4-4a^3+6a^2-4a+1\right)\log\left(|bx+a-1|\right)}{b^5}-\frac{6b^5x^5+15b^4x^4-20ab^3x^3+30a^2b^2x^2+20b^3x^3-60a^3bx-60ab^2x^2}{30b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^4,x, algorithm="giac")
```

```
[Out] -2*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log(abs(b*x + a - 1))/b^5 - 1/30*(6*b^5*
x^5 + 15*b^4*x^4 - 20*a*b^3*x^3 + 30*a^2*b^2*x^2 + 20*b^3*x^3 - 60*a^3*b*x
- 60*a*b^2*x^2 + 180*a^2*b*x + 30*b^2*x^2 - 180*a*b*x + 60*b*x)/b^5
```

3.827 $\int e^{2\tanh^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=66

$$-\frac{(1-a)x^2}{b^2}-\frac{2(1-a)^2x}{b^3}-\frac{2(1-a)^3\log(-a-bx+1)}{b^4}-\frac{2x^3}{3b}-\frac{x^4}{4}$$

```
[Out] (-2*(1 - a)^2*x)/b^3 - ((1 - a)*x^2)/b^2 - (2*x^3)/(3*b) - x^4/4 - (2*(1 -
a)^3*Log[1 - a - b*x])/b^4
```

Rubi [A] time = 0.0600754, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{(1-a)x^2}{b^2}-\frac{2(1-a)^2x}{b^3}-\frac{2(1-a)^3\log(-a-bx+1)}{b^4}-\frac{2x^3}{3b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x])*x^3,x]
```

```
[Out] (-2*(1 - a)^2*x)/b^3 - ((1 - a)*x^2)/b^2 - (2*x^3)/(3*b) - x^4/4 - (2*(1 -
a)^3*Log[1 - a - b*x])/b^4
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(a+bx)}x^3\,dx&=\int\frac{x^3(1+a+bx)}{1-a-bx}\,dx\\&=\int\left(-\frac{2(-1+a)^2}{b^3}+\frac{2(-1+a)x}{b^2}-\frac{2x^2}{b}-x^3+\frac{2(-1+a)^3}{b^3(-1+a+bx)}\right)dx\\&=-\frac{2(1-a)^2x}{b^3}-\frac{(1-a)x^2}{b^2}-\frac{2x^3}{3b}-\frac{x^4}{4}-\frac{2(1-a)^3\log(1-a-bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.0426213, size = 66, normalized size = 1.

$$-\frac{(1-a)x^2}{b^2}-\frac{2(1-a)^2x}{b^3}-\frac{2(1-a)^3\log(-a-bx+1)}{b^4}-\frac{2x^3}{3b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x])*x^3,x]
```

```
[Out] (-2*(1 - a)^2*x)/b^3 - ((1 - a)*x^2)/b^2 - (2*x^3)/(3*b) - x^4/4 - (2*(1 -
a)^3*Log[1 - a - b*x])/b^4
```

Maple [A] time = 0.027, size = 108, normalized size = 1.6

$$-\frac{x^4}{4} - \frac{2\,x^3}{3\,b} + \frac{a x^2}{b^2} - \frac{x^2}{b^2} - 2\,\frac{a^2 x}{b^3} + 4\,\frac{a x}{b^3} - 2\,\frac{x}{b^3} + 2\,\frac{\ln\left(bx + a - 1\right)a^3}{b^4} - 6\,\frac{\ln\left(bx + a - 1\right)a^2}{b^4} + 6\,\frac{\ln\left(bx + a - 1\right)a}{b^4} - 2\,\frac{\ln\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2)*x^3,x)
```

```
[Out] -1/4*x^4-2/3*x^3/b+1/b^2*x^2*a-1/b^2*x^2-2/b^3*x*a^2+4/b^3*a*x-2/b^3*x+2/b^
4*ln(b*x+a-1)*a^3-6/b^4*ln(b*x+a-1)*a^2+6/b^4*ln(b*x+a-1)*a-2/b^4*ln(b*x+a-
1)
```

Maxima [A] time = 0.966067, size = 92, normalized size = 1.39

$$-\frac{3\,b^3 x^4 + 8\,b^2 x^3 - 12\,(a-1) b x^2 + 24\,\left(a^2 - 2\,a + 1\right)x}{12\,b^3} + \frac{2\,\left(a^3 - 3\,a^2 + 3\,a - 1\right)\log\left(bx + a - 1\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^3*x^4 + 8*b^2*x^3 - 12*(a - 1)*b*x^2 + 24*(a^2 - 2*a + 1)*x)/b^3
 + 2*(a^3 - 3*a^2 + 3*a - 1)*log(b*x + a - 1)/b^4
```

Fricas [A] time = 1.83136, size = 171, normalized size = 2.59

$$-\frac{3\,b^4 x^4 + 8\,b^3 x^3 - 12\,(a-1) b^2 x^2 + 24\,\left(a^2 - 2\,a + 1\right)bx - 24\,\left(a^3 - 3\,a^2 + 3\,a - 1\right)\log\left(bx + a - 1\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^4*x^4 + 8*b^3*x^3 - 12*(a - 1)*b^2*x^2 + 24*(a^2 - 2*a + 1)*b*x
- 24*(a^3 - 3*a^2 + 3*a - 1)*log(b*x + a - 1))/b^4
```

Sympy [A] time = 0.40406, size = 56, normalized size = 0.85

$$-\frac{x^4}{4} - \frac{2x^3}{3b} + \frac{x^2\,(a-1)}{b^2} - \frac{x\left(2a^2 - 4a + 2\right)}{b^3} + \frac{2\,(a-1)^3\log\left(a + bx - 1\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2)*x**3,x)
```

```
[Out] -x**4/4 - 2*x**3/(3*b) + x**2*(a - 1)/b**2 - x*(2*a**2 - 4*a + 2)/b**3 + 2*
(a - 1)**3*log(a + b*x - 1)/b**4
```

Giac [A] time = 1.21377, size = 111, normalized size = 1.68

$$\frac{2\left(a^3-3\,a^2+3\,a-1\right)\log\left(|bx+a-1|\right)}{b^4}-\frac{3\,b^4x^4+8\,b^3x^3-12\,ab^2x^2+24\,a^2bx+12\,b^2x^2-48\,abx+24\,bx}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^3,x, algorithm="giac")
```

```
[Out] 2*(a^3 - 3*a^2 + 3*a - 1)*log(abs(b*x + a - 1))/b^4 - 1/12*(3*b^4*x^4 + 8*b
^3*x^3 - 12*a*b^2*x^2 + 24*a^2*b*x + 12*b^2*x^2 - 48*a*b*x + 24*b*x)/b^4
```

3.828 $\int e^{2\tanh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=49

$$-\frac{2(1-a)x}{b^2}-\frac{2(1-a)^2\log(-a-bx+1)}{b^3}-\frac{x^2}{b}-\frac{x^3}{3}$$

```
[Out] (-2*(1 - a)*x)/b^2 - x^2/b - x^3/3 - (2*(1 - a)^2*Log[1 - a - b*x])/b^3
```

Rubi [A] time = 0.0506131, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{2(1-a)x}{b^2}-\frac{2(1-a)^2\log(-a-bx+1)}{b^3}-\frac{x^2}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x])*x^2,x]
```

```
[Out] (-2*(1 - a)*x)/b^2 - x^2/b - x^3/3 - (2*(1 - a)^2*Log[1 - a - b*x])/b^3
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(a+bx)}x^2\,dx &= \int \frac{x^2(1+a+bx)}{1-a-bx}\,dx\\ &= \int \left(\frac{2(-1+a)}{b^2}-\frac{2x}{b}-x^2-\frac{2(-1+a)^2}{b^2(-1+a+bx)}\right)dx\\ &= -\frac{2(1-a)x}{b^2}-\frac{x^2}{b}-\frac{x^3}{3}-\frac{2(1-a)^2\log(1-a-bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0310277, size = 44, normalized size = 0.9

$$-\frac{bx\left(-6a+b^2x^2+3bx+6\right)+6(a-1)^2\log(-a-bx+1)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x])*x^2,x]
```

```
[Out] -(b*x*(6 - 6*a + 3*b*x + b^2*x^2) + 6*(-1 + a)^2*Log[1 - a - b*x])/(3*b^3)
```

Maple [A] time = 0.028, size = 68, normalized size = 1.4

$$-\frac{x^3}{3}-\frac{x^2}{b}+2\,\frac{ax}{b^2}-2\,\frac{x}{b^2}-2\,\frac{\ln(bx+a-1)\,a^2}{b^3}+4\,\frac{\ln(bx+a-1)\,a}{b^3}-2\,\frac{\ln(bx+a-1)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2)*x^2,x)
```

```
[Out] -1/3*x^3-x^2/b+2/b^2*a*x-2/b^2*x-2/b^3*ln(b*x+a-1)*a^2+4/b^3*ln(b*x+a-1)*a-
2/b^3*ln(b*x+a-1)
```

Maxima [A] time = 0.940435, size = 62, normalized size = 1.27

$$-\frac{b^2x^3+3\,bx^2-6\,(a-1)x}{3\,b^2}-\frac{2\left(a^2-2\,a+1\right)\log(bx+a-1)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b^2*x^3 + 3*b*x^2 - 6*(a - 1)*x)/b^2 - 2*(a^2 - 2*a + 1)*log(b*x + a
- 1)/b^3
```

Fricas [A] time = 1.73783, size = 115, normalized size = 2.35

$$-\frac{b^3x^3+3\,b^2x^2-6\,(a-1)bx+6\left(a^2-2\,a+1\right)\log(bx+a-1)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 + 3*b^2*x^2 - 6*(a - 1)*b*x + 6*(a^2 - 2*a + 1)*log(b*x + a -
 1))/b^3
```

Sympy [A] time = 0.361109, size = 37, normalized size = 0.76

$$-\frac{x^3}{3}-\frac{x^2}{b}+\frac{x\,(2a-2)}{b^2}-\frac{2\,(a-1)^2\log(a+bx-1)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2)*x**2,x)
```

```
[Out] -x**3/3 - x**2/b + x*(2*a - 2)/b**2 - 2*(a - 1)**2*log(a + b*x - 1)/b**3
```

Giac [A] time = 1.15478, size = 70, normalized size = 1.43

$$-\frac{2\left(a^{2}-2\,a+1\right)\log\left(|bx+a-1|\right)}{b^{3}}-\frac{b^{3}x^{3}+3\,b^{2}x^{2}-6\,abx+6\,bx}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^2,x, algorithm="giac")
```

```
[Out] -2*(a^2 - 2*a + 1)*log(abs(b*x + a - 1))/b^3 - 1/3*(b^3*x^3 + 3*b^2*x^2 - 6
*a*b*x + 6*b*x)/b^3
```

3.829 $\int e^{2\tanh^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=34

$$-\frac{2(1-a)\log(-a-bx+1)}{b^2}-\frac{2x}{b}-\frac{x^2}{2}$$

```
[Out] (-2*x)/b - x^2/2 - (2*(1 - a)*Log[1 - a - b*x])/b^2
```

Rubi [A] time = 0.0306465, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6163, 77}

$$-\frac{2(1-a)\log(-a-bx+1)}{b^2}-\frac{2x}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x])*x,x]

[Out] (-2*x)/b - x^2/2 - (2*(1 - a)*Log[1 - a - b*x])/b^2
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(a+bx)}x\,dx&=\int\frac{x(1+a+bx)}{1-a-bx}\,dx\\&=\int\left(-\frac{2}{b}-x+\frac{2(-1+a)}{b(-1+a+bx)}\right)dx\\&=-\frac{2x}{b}-\frac{x^2}{2}-\frac{2(1-a)\log(1-a-bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0171643, size = 34, normalized size = 1.

$$-\frac{2(1-a)\log(-a-bx+1)}{b^2}-\frac{2x}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x])*x,x]
```

```
[Out] (-2*x)/b - x^2/2 - (2*(1 - a)*Log[1 - a - b*x])/b^2
```

Maple [A] time = 0.03, size = 38, normalized size = 1.1

$$-\frac{x^2}{2} - 2\frac{x}{b} + 2\frac{\ln(bx + a - 1)a}{b^2} - 2\frac{\ln(bx + a - 1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2)*x,x)
```

```
[Out] -1/2*x^2-2*x/b+2/b^2*ln(b*x+a-1)*a-2/b^2*ln(b*x+a-1)
```

Maxima [A] time = 0.950921, size = 41, normalized size = 1.21

$$-\frac{bx^2 + 4x}{2b} + \frac{2(a - 1)\log(bx + a - 1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + 4*x)/b + 2*(a - 1)*log(b*x + a - 1)/b^2
```

Fricas [A] time = 1.71797, size = 77, normalized size = 2.26

$$-\frac{b^2x^2 + 4bx - 4(a - 1)\log(bx + a - 1)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 + 4*b*x - 4*(a - 1)*log(b*x + a - 1))/b^2
```

Sympy [A] time = 0.319083, size = 26, normalized size = 0.76

$$-\frac{x^2}{2} - \frac{2x}{b} + \frac{2(a - 1)\log(a + bx - 1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2)*x,x)
```

```
[Out] -x**2/2 - 2*x/b + 2*(a - 1)*log(a + b*x - 1)/b**2
```

Giac [A] time = 1.18514, size = 46, normalized size = 1.35

$$\frac{2(a - 1)\log(|bx + a - 1|)}{b^2} - \frac{b^2x^2 + 4bx}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x,x, algorithm="giac")
```

```
[Out] 2*(a - 1)*log(abs(b*x + a - 1))/b^2 - 1/2*(b^2*x^2 + 4*b*x)/b^2
```

3.830 $\int e^{2\tanh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=19

$$-\frac{2\log(-a-bx+1)}{b}-x$$

```
[Out] -x - (2*Log[1 - a - b*x])/b
```

Rubi [A] time = 0.0124833, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6161, 43}

$$-\frac{2\log(-a-bx+1)}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x]),x]

[Out] -x - (2*Log[1 - a - b*x])/b
```

Rule 6161

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 + a*c
 + b*c*x)^(n/2)/(1 - a*c - b*c*x)^(n/2), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(a+bx)}\,dx&=\int\frac{1+a+bx}{1-a-bx}\,dx\\&=\int\left(-1-\frac{2}{-1+a+bx}\right)dx\\&=-x-\frac{2\log(1-a-bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0099302, size = 19, normalized size = 1.

$$-\frac{2\log(-a-bx+1)}{b}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x]),x]

[Out] -x - (2*Log[1 - a - b*x])/b
```

Maple [A] time = 0.03, size = 17, normalized size = 0.9

$$-x-2\,\frac{\ln\left(bx+a-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2),x)
```

```
[Out] -x-2/b*ln(b*x+a-1)
```

Maxima [A] time = 0.952147, size = 22, normalized size = 1.16

$$-x-\frac{2\,\log\left(bx+a-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -x - 2*log(b*x + a - 1)/b
```

Fricas [A] time = 1.87019, size = 42, normalized size = 2.21

$$-\frac{bx+2\,\log\left(bx+a-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -(b*x + 2*log(b*x + a - 1))/b
```

Sympy [A] time = 0.122734, size = 14, normalized size = 0.74

$$-x-\frac{2\log\left(a+bx-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2),x)
```

```
[Out] -x - 2*log(a + b*x - 1)/b
```

Giac [A] time = 1.16458, size = 23, normalized size = 1.21

$$-x-\frac{2\,\log\left(|bx+a-1|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -x - 2*log(abs(b*x + a - 1))/b
```

3.831 $\int \frac{e^{2\tanh^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=33

$$\frac{(a+1)\log(x)}{1-a}-\frac{2\log(-a-bx+1)}{1-a}$$

```
[Out] ((1 + a)*Log[x])/(1 - a) - (2*Log[1 - a - b*x])/(1 - a)
```

Rubi [A] time = 0.0377747, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 72}

$$\frac{(a+1)\log(x)}{1-a}-\frac{2\log(-a-bx+1)}{1-a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x])/x,x]
```

```
[Out] ((1 + a)*Log[x])/(1 - a) - (2*Log[1 - a - b*x])/(1 - a)
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(a+bx)}}{x}\,dx &= \int \frac{1+a+bx}{x(1-a-bx)}\,dx\\ &= \int \left(\frac{-1-a}{(-1+a)x}+\frac{2b}{(-1+a)(-1+a+bx)}\right)dx\\ &= \frac{(1+a)\log(x)}{1-a}-\frac{2\log(1-a-bx)}{1-a}\end{aligned}$$

Mathematica [A] time = 0.0173799, size = 26, normalized size = 0.79

$$\frac{2\log(-a-bx+1)-(a+1)\log(x)}{a-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x])/x,x]
```

```
[Out] (-((1 + a)*Log[x]) + 2*Log[1 - a - b*x])/(-1 + a)
```

Maple [A] time = 0.033, size = 35, normalized size = 1.1

$$-\frac{\ln(x)}{a-1}-\frac{a\ln(x)}{a-1}+2\,\frac{\ln(bx+a-1)}{a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2)/x,x)
```

```
[Out] -1/(a-1)*ln(x)-1/(a-1)*ln(x)*a+2/(a-1)*ln(b*x+a-1)
```

Maxima [A] time = 0.953696, size = 36, normalized size = 1.09

$$-\frac{(a+1)\log(x)}{a-1}+\frac{2\,\log(bx+a-1)}{a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] -(a + 1)*log(x)/(a - 1) + 2*log(b*x + a - 1)/(a - 1)
```

Fricas [A] time = 1.88123, size = 65, normalized size = 1.97

$$-\frac{(a+1)\log(x)-2\,\log(bx+a-1)}{a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x,x, algorithm="fricas")
```

```
[Out] -((a + 1)*log(x) - 2*log(b*x + a - 1))/(a - 1)
```

Sympy [B] time = 0.532849, size = 88, normalized size = 2.67

$$-\frac{(a+1)\log\left(x+\frac{a^2-\frac{a^2(a+1)}{a-1}+\frac{2a(a+1)}{a-1}-1-\frac{a+1}{a-1}}{ab+3b}\right)}{a-1}+\frac{2\log\left(x+\frac{a^2+\frac{2a^2}{a-1}-\frac{4a}{a-1}-1+\frac{2}{a-1}}{ab+3b}\right)}{a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2)/x,x)
```

```
[Out] -(a + 1)*log(x + (a**2 - a**2*(a + 1)/(a - 1) + 2*a*(a + 1)/(a - 1) - 1 - (
a + 1)/(a - 1))/(a*b + 3*b))/(a - 1) + 2*log(x + (a**2 + 2*a**2/(a - 1) - 4
*a/(a - 1) - 1 + 2/(a - 1))/(a*b + 3*b))/(a - 1)
```

Giac [A] time = 1.20083, size = 46, normalized size = 1.39

$$\frac{2 b \log \left(|bx + a - 1|\right)}{ab - b} - \frac{(a + 1) \log \left(|x|\right)}{a - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x,x, algorithm="giac")
```

```
[Out] 2*b*log(abs(b*x + a - 1))/(a*b - b) - (a + 1)*log(abs(x))/(a - 1)
```

3.832 $\int \frac{e^{2\tanh^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=48

$$\frac{2b\log(x)}{(1-a)^2}-\frac{2b\log(-a-bx+1)}{(1-a)^2}-\frac{a+1}{(1-a)x}$$

```
[Out] -((1 + a)/((1 - a)*x)) + (2*b*Log[x])/(1 - a)^2 - (2*b*Log[1 - a - b*x])/(1
 - a)^2
```

Rubi [A] time = 0.0408594, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$\frac{2b\log(x)}{(1-a)^2}-\frac{2b\log(-a-bx+1)}{(1-a)^2}-\frac{a+1}{(1-a)x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x])/x^2,x]
```

```
[Out] -((1 + a)/((1 - a)*x)) + (2*b*Log[x])/(1 - a)^2 - (2*b*Log[1 - a - b*x])/(1
 - a)^2
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{1+a+bx}{x^2(1-a-bx)}\,dx\\ &= \int \left(\frac{-1-a}{(-1+a)x^2}+\frac{2b}{(-1+a)^2x}-\frac{2b^2}{(-1+a)^2(-1+a+bx)}\right)dx\\ &= -\frac{1+a}{(1-a)x}+\frac{2b\log(x)}{(1-a)^2}-\frac{2b\log(1-a-bx)}{(1-a)^2}\end{aligned}$$

Mathematica [A] time = 0.0221775, size = 34, normalized size = 0.71

$$\frac{a^2-2bx\log(-a-bx+1)+2bx\log(x)-1}{(a-1)^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x])/x^2,x]
```

```
[Out] (-1 + a^2 + 2*b*x*Log[x] - 2*b*x*Log[1 - a - b*x])/((-1 + a)^2*x)
```

Maple [A] time = 0.05, size = 46, normalized size = 1.

$$\frac{1}{(a-1)x}+\frac{a}{(a-1)x}+2\frac{b\ln(x)}{(a-1)^2}-2\frac{b\ln(bx+a-1)}{(a-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2)/x^2,x)
```

```
[Out] 1/(a-1)/x+1/(a-1)/x*a+2*b/(a-1)^2*ln(x)-2*b/(a-1)^2*ln(b*x+a-1)
```

Maxima [A] time = 0.961006, size = 65, normalized size = 1.35

$$-\frac{2\,b\log(bx+a-1)}{a^2-2\,a+1}+\frac{2\,b\log(x)}{a^2-2\,a+1}+\frac{a+1}{(a-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] -2*b*log(b*x + a - 1)/(a^2 - 2*a + 1) + 2*b*log(x)/(a^2 - 2*a + 1) + (a + 1
)/((a - 1)*x)
```

Fricas [A] time = 1.83429, size = 97, normalized size = 2.02

$$-\frac{2\,bx\log(bx+a-1)-2\,bx\log(x)-a^2+1}{\left(a^2-2\,a+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*b*x*log(b*x + a - 1) - 2*b*x*log(x) - a^2 + 1)/((a^2 - 2*a + 1)*x)
```

Sympy [B] time = 0.538894, size = 143, normalized size = 2.98

$$\frac{2b\log\left(x+\frac{-\frac{2a^3b}{(a-1)^2}+\frac{6a^2b}{(a-1)^2}+2ab-\frac{6ab}{(a-1)^2}-2b+\frac{2b}{(a-1)^2}}{4b^2}\right)}{(a-1)^2}-\frac{2b\log\left(x+\frac{\frac{2a^3b}{(a-1)^2}-\frac{6a^2b}{(a-1)^2}+2ab+\frac{6ab}{(a-1)^2}-2b-\frac{2b}{(a-1)^2}}{4b^2}\right)}{(a-1)^2}+\frac{a+1}{x(a-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2)/x**2,x)
```

```
[Out] 2*b*log(x + (-2*a**3*b/(a - 1)**2 + 6*a**2*b/(a - 1)**2 + 2*a*b - 6*a*b/(a
- 1)**2 - 2*b + 2*b/(a - 1)**2)/(4*b**2))/(a - 1)**2 - 2*b*log(x + (2*a**3*
b/(a - 1)**2 - 6*a**2*b/(a - 1)**2 + 2*a*b + 6*a*b/(a - 1)**2 - 2*b - 2*b/(
a - 1)**2)/(4*b**2))/(a - 1)**2 + (a + 1)/(x*(a - 1))
```

Giac [A] time = 1.16605, size = 77, normalized size = 1.6

$$-\frac{2\,b^2\log\left(|bx+a-1|\right)}{a^2b-2\,ab+b}+\frac{2\,b\log\left(|x|\right)}{a^2-2\,a+1}+\frac{a^2-1}{(a-1)^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^2,x, algorithm="giac")
```

```
[Out] -2*b^2*log(abs(b*x + a - 1))/(a^2*b - 2*a*b + b) + 2*b*log(abs(x))/(a^2 - 2
*a + 1) + (a^2 - 1)/((a - 1)^2*x)
```

3.833 $\int \frac{e^{2\tanh^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=67

$$\frac{2b^2\log(x)}{(1-a)^3}-\frac{2b^2\log(-a-bx+1)}{(1-a)^3}-\frac{2b}{(1-a)^2x}-\frac{a+1}{2(1-a)x^2}$$

```
[Out] -(1 + a)/(2*(1 - a)*x^2) - (2*b)/((1 - a)^2*x) + (2*b^2*Log[x])/(1 - a)^3 -
 (2*b^2*Log[1 - a - b*x])/(1 - a)^3
```

Rubi [A] time = 0.050526, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$\frac{2b^2\log(x)}{(1-a)^3}-\frac{2b^2\log(-a-bx+1)}{(1-a)^3}-\frac{2b}{(1-a)^2x}-\frac{a+1}{2(1-a)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x])/x^3,x]
```

```
[Out] -(1 + a)/(2*(1 - a)*x^2) - (2*b)/((1 - a)^2*x) + (2*b^2*Log[x])/(1 - a)^3 -
 (2*b^2*Log[1 - a - b*x])/(1 - a)^3
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(a+bx)}}{x^3}\,dx &= \int \frac{1+a+bx}{x^3(1-a-bx)}\,dx\\ &= \int \left(\frac{-1-a}{(-1+a)x^3}+\frac{2b}{(-1+a)^2x^2}-\frac{2b^2}{(-1+a)^3x}+\frac{2b^3}{(-1+a)^3(-1+a+bx)}\right)dx\\ &= -\frac{1+a}{2(1-a)x^2}-\frac{2b}{(1-a)^2x}+\frac{2b^2\log(x)}{(1-a)^3}-\frac{2b^2\log(1-a-bx)}{(1-a)^3}\end{aligned}$$

Mathematica [A] time = 0.0333783, size = 54, normalized size = 0.81

$$\frac{(a-1)\left(a^2-4bx-1\right)+4b^2x^2\log(-a-bx+1)-4b^2x^2\log(x)}{2(a-1)^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x])/x^3,x]
```

```
[Out] ((-1 + a)*(-1 + a^2 - 4*b*x) - 4*b^2*x^2*Log[x] + 4*b^2*x^2*Log[1 - a - b*x
])/(2*(-1 + a)^3*x^2)
```

Maple [A] time = 0.037, size = 63, normalized size = 0.9

$$\frac{1}{(2a-2)x^2}+\frac{a}{(2a-2)x^2}-2\frac{b}{(a-1)^2x}-2\frac{b^2\ln(x)}{(a-1)^3}+2\frac{b^2\ln(bx+a-1)}{(a-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2)/x^3,x)
```

```
[Out] 1/2/(a-1)/x^2+1/2/(a-1)/x^2*a-2*b/(a-1)^2/x-2/(a-1)^3*b^2*ln(x)+2/(a-1)^3*b
^2*ln(b*x+a-1)
```

Maxima [A] time = 0.958169, size = 100, normalized size = 1.49

$$\frac{2b^2\log(bx+a-1)}{a^3-3a^2+3a-1}-\frac{2b^2\log(x)}{a^3-3a^2+3a-1}+\frac{a^2-4bx-1}{2\left(a^2-2a+1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^3,x, algorithm="maxima")
```

```
[Out] 2*b^2*log(b*x + a - 1)/(a^3 - 3*a^2 + 3*a - 1) - 2*b^2*log(x)/(a^3 - 3*a^2
+ 3*a - 1) + 1/2*(a^2 - 4*b*x - 1)/((a^2 - 2*a + 1)*x^2)
```

Fricas [A] time = 1.88922, size = 161, normalized size = 2.4

$$\frac{4b^2x^2\log(bx+a-1)-4b^2x^2\log(x)+a^3-4(a-1)bx-a^2-a+1}{2\left(a^3-3a^2+3a-1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b^2*x^2*log(b*x + a - 1) - 4*b^2*x^2*log(x) + a^3 - 4*(a - 1)*b*x -
a^2 - a + 1)/((a^3 - 3*a^2 + 3*a - 1)*x^2)
```

Sympy [B] time = 0.669218, size = 209, normalized size = 3.12

$$-\frac{2b^2\log\left(x+\frac{-\frac{2a^4b^2}{(a-1)^3}+\frac{8a^3b^2}{(a-1)^3}-\frac{12a^2b^2}{(a-1)^3}+2ab^2+\frac{8ab^2}{(a-1)^3}-2b^2-\frac{2b^2}{(a-1)^3}}{4b^3}\right)}{(a-1)^3}+\frac{2b^2\log\left(x+\frac{\frac{2a^4b^2}{(a-1)^3}-\frac{8a^3b^2}{(a-1)^3}+\frac{12a^2b^2}{(a-1)^3}+2ab^2-\frac{8ab^2}{(a-1)^3}-2b^2+\frac{2b^2}{(a-1)^3}}{4b^3}\right)}{(a-1)^3}-\frac{-a^2+4}{x^2\left(2a^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2)/x**3,x)
```

```
[Out] -2*b**2*log(x + (-2*a**4*b**2/(a - 1)**3 + 8*a**3*b**2/(a - 1)**3 - 12*a**2
*b**2/(a - 1)**3 + 2*a*b**2 + 8*a*b**2/(a - 1)**3 - 2*b**2 - 2*b**2/(a - 1)
**3)/(4*b**3))/(a - 1)**3 + 2*b**2*log(x + (2*a**4*b**2/(a - 1)**3 - 8*a**3
*b**2/(a - 1)**3 + 12*a**2*b**2/(a - 1)**3 + 2*a*b**2 - 8*a*b**2/(a - 1)**3
 - 2*b**2 + 2*b**2/(a - 1)**3)/(4*b**3))/(a - 1)**3 - (-a**2 + 4*b*x + 1)/(
x**2*(2*a**2 - 4*a + 2))
```

Giac [A] time = 1.1441, size = 123, normalized size = 1.84

$$\frac{2\,b^3\log\left(|bx+a-1|\right)}{a^3b-3\,a^2b+3\,ab-b}-\frac{2\,b^2\log\left(|x|\right)}{a^3-3\,a^2+3\,a-1}+\frac{a^3-a^2-4\,(ab-b)x-a+1}{2\,(a-1)^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^3,x, algorithm="giac")
```

```
[Out] 2*b^3*log(abs(b*x + a - 1))/(a^3*b - 3*a^2*b + 3*a*b - b) - 2*b^2*log(abs(x
))/(a^3 - 3*a^2 + 3*a - 1) + 1/2*(a^3 - a^2 - 4*(a*b - b)*x - a + 1)/((a -
1)^3*x^2)
```

3.834 $\int \frac{e^{2\tanh^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=82

$$-\frac{2b^2}{(1-a)^3x}+\frac{2b^3\log(x)}{(1-a)^4}-\frac{2b^3\log(-a-bx+1)}{(1-a)^4}-\frac{b}{(1-a)^2x^2}-\frac{a+1}{3(1-a)x^3}$$

```
[Out] -(1 + a)/(3*(1 - a)*x^3) - b/((1 - a)^2*x^2) - (2*b^2)/((1 - a)^3*x) + (2*b
^3*Log[x])/(1 - a)^4 - (2*b^3*Log[1 - a - b*x])/(1 - a)^4
```

Rubi [A] time = 0.0616321, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{2b^2}{(1-a)^3x}+\frac{2b^3\log(x)}{(1-a)^4}-\frac{2b^3\log(-a-bx+1)}{(1-a)^4}-\frac{b}{(1-a)^2x^2}-\frac{a+1}{3(1-a)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x])/x^4,x]
```

```
[Out] -(1 + a)/(3*(1 - a)*x^3) - b/((1 - a)^2*x^2) - (2*b^2)/((1 - a)^3*x) + (2*b
^3*Log[x])/(1 - a)^4 - (2*b^3*Log[1 - a - b*x])/(1 - a)^4
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(a+bx)}}{x^4}\,dx &= \int \frac{1+a+bx}{x^4(1-a-bx)}\,dx\\
&= \int \left(\frac{-1-a}{(-1+a)x^4}+\frac{2b}{(-1+a)^2x^3}-\frac{2b^2}{(-1+a)^3x^2}+\frac{2b^3}{(-1+a)^4x}-\frac{2b^4}{(-1+a)^4(-1+a+bx)}\right)dx\\
&= -\frac{1+a}{3(1-a)x^3}-\frac{b}{(1-a)^2x^2}-\frac{2b^2}{(1-a)^3x}+\frac{2b^3\log(x)}{(1-a)^4}-\frac{2b^3\log(1-a-bx)}{(1-a)^4}
\end{aligned}$$

Mathematica [A] time = 0.0469672, size = 75, normalized size = 0.91

$$\frac{(a-1)\left(a^3-a^2-3abx-a+6b^2x^2+3bx+1\right)-6b^3x^3\log(-a-bx+1)+6b^3x^3\log(x)}{3(a-1)^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x])/x^4,x]
```

```
[Out] ((-1 + a)*(1 - a - a^2 + a^3 + 3*b*x - 3*a*b*x + 6*b^2*x^2) + 6*b^3*x^3*Log
[x] - 6*b^3*x^3*Log[1 - a - b*x])/(3*(-1 + a)^4*x^3)
```

Maple [A] time = 0.038, size = 76, normalized size = 0.9

$$\frac{1}{(3a-3)x^3}+\frac{a}{(3a-3)x^3}-\frac{b}{(a-1)^2x^2}+2\frac{b^3\ln(x)}{(a-1)^4}+2\frac{b^2}{(a-1)^3x}-2\frac{b^3\ln(bx+a-1)}{(a-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2)/x^4,x)
```

```
[Out] 1/3/(a-1)/x^3+1/3/(a-1)/x^3*a-b/(a-1)^2/x^2+2/(a-1)^4*b^3*ln(x)+2/(a-1)^3*b
^2/x-2/(a-1)^4*b^3*ln(b*x+a-1)
```

Maxima [A] time = 0.956396, size = 146, normalized size = 1.78

$$-\frac{2b^3\log(bx+a-1)}{a^4-4a^3+6a^2-4a+1}+\frac{2b^3\log(x)}{a^4-4a^3+6a^2-4a+1}+\frac{6b^2x^2+a^3-3(a-1)bx-a^2-a+1}{3\left(a^3-3a^2+3a-1\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^4,x, algorithm="maxima")
```

```
[Out] -2*b^3*log(b*x + a - 1)/(a^4 - 4*a^3 + 6*a^2 - 4*a + 1) + 2*b^3*log(x)/(a^4
 - 4*a^3 + 6*a^2 - 4*a + 1) + 1/3*(6*b^2*x^2 + a^3 - 3*(a - 1)*b*x - a^2 -
a + 1)/((a^3 - 3*a^2 + 3*a - 1)*x^3)
```

Fricas [A] time = 1.87461, size = 216, normalized size = 2.63

$$-\frac{6b^3x^3\log(bx+a-1)-6b^3x^3\log(x)-6(a-1)b^2x^2-a^4+2a^3+3\left(a^2-2a+1\right)bx-2a+1}{3\left(a^4-4a^3+6a^2-4a+1\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*b^3*x^3*log(b*x + a - 1) - 6*b^3*x^3*log(x) - 6*(a - 1)*b^2*x^2 - a
^4 + 2*a^3 + 3*(a^2 - 2*a + 1)*b*x - 2*a + 1)/((a^4 - 4*a^3 + 6*a^2 - 4*a +
 1)*x^3)
```

Sympy [B] time = 0.79484, size = 260, normalized size = 3.17

$$\frac{2b^3\log\left(x+\frac{-\frac{2a^5b^3}{(a-1)^4}+\frac{10a^4b^3}{(a-1)^4}-\frac{20a^3b^3}{(a-1)^4}+\frac{20a^2b^3}{(a-1)^4}+2ab^3-\frac{10ab^3}{(a-1)^4}-2b^3+\frac{2b^3}{(a-1)^4}}{4b^4}\right)}{(a-1)^4}-\frac{2b^3\log\left(x+\frac{\frac{2a^5b^3}{(a-1)^4}-\frac{10a^4b^3}{(a-1)^4}+\frac{20a^3b^3}{(a-1)^4}-\frac{20a^2b^3}{(a-1)^4}+2ab^3+\frac{10ab^3}{(a-1)^4}-2b^3-\frac{2b^3}{(a-1)^{\cdot}}\right.}{(a-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2)/x**4,x)
```

```
[Out] 2*b**3*log(x + (-2*a**5*b**3/(a - 1)**4 + 10*a**4*b**3/(a - 1)**4 - 20*a**3
*b**3/(a - 1)**4 + 20*a**2*b**3/(a - 1)**4 + 2*a*b**3 - 10*a*b**3/(a - 1)**
4 - 2*b**3 + 2*b**3/(a - 1)**4)/(4*b**4))/(a - 1)**4 - 2*b**3*log(x + (2*a*
*5*b**3/(a - 1)**4 - 10*a**4*b**3/(a - 1)**4 + 20*a**3*b**3/(a - 1)**4 - 20
*a**2*b**3/(a - 1)**4 + 2*a*b**3 + 10*a*b**3/(a - 1)**4 - 2*b**3 - 2*b**3/(
a - 1)**4)/(4*b**4))/(a - 1)**4 + (a**3 - a**2 - a + 6*b**2*x**2 + x*(-3*a*
b + 3*b) + 1)/(x**3*(3*a**3 - 9*a**2 + 9*a - 3))
```

Giac [A] time = 1.18719, size = 162, normalized size = 1.98

$$-\frac{2\,b^4\log\left(|bx+a-1|\right)}{a^4b-4\,a^3b+6\,a^2b-4\,ab+b}+\frac{2\,b^3\log\left(|x|\right)}{a^4-4\,a^3+6\,a^2-4\,a+1}+\frac{a^4-2\,a^3+6\left(ab^2-b^2\right)x^2-3\left(a^2b-2\,ab+b\right)x+2\,a-1}{3\,(a-1)^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^4,x, algorithm="giac")
```

```
[Out] -2*b^4*log(abs(b*x + a - 1))/(a^4*b - 4*a^3*b + 6*a^2*b - 4*a*b + b) + 2*b^
3*log(abs(x))/(a^4 - 4*a^3 + 6*a^2 - 4*a + 1) + 1/3*(a^4 - 2*a^3 + 6*(a*b^2
 - b^2)*x^2 - 3*(a^2*b - 2*a*b + b)*x + 2*a - 1)/((a - 1)^4*x^3)
```

3.835 $\int e^{3\tanh^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=187

$$\frac{\sqrt{-a-bx+1}(a+bx+1)^{3/2}\left(22a^2+2(11-10a)bx-54a+29\right)}{8b^4}+\frac{3\left(-8a^3+36a^2-44a+17\right)\sqrt{-a-bx+1}\sqrt{a+bx+}}{8b^4}$$

```
[Out] (3*(17 - 44*a + 36*a^2 - 8*a^3)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(8*b^4
) + (2*x^3*(1 + a + b*x)^(3/2))/(b*Sqrt[1 - a - b*x]) + (9*x^2*Sqrt[1 - a -
 b*x]*(1 + a + b*x)^(3/2))/(4*b^2) + (Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2)
*(29 - 54*a + 22*a^2 + 2*(11 - 10*a)*b*x))/(8*b^4) - (3*(17 - 44*a + 36*a^2
 - 8*a^3)*ArcSin[a + b*x])/(8*b^4)
```

Rubi [A] time = 0.182205, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6163, 97, 153, 147, 50, 53, 619, 216}

$$\frac{\sqrt{-a-bx+1}(a+bx+1)^{3/2}\left(22a^2+2(11-10a)bx-54a+29\right)}{8b^4}+\frac{3\left(-8a^3+36a^2-44a+17\right)\sqrt{-a-bx+1}\sqrt{a+bx+}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a + b*x])*x^3,x]
```

```
[Out] (3*(17 - 44*a + 36*a^2 - 8*a^3)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(8*b^4
) + (2*x^3*(1 + a + b*x)^(3/2))/(b*Sqrt[1 - a - b*x]) + (9*x^2*Sqrt[1 - a -
 b*x]*(1 + a + b*x)^(3/2))/(4*b^2) + (Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2)
*(29 - 54*a + 22*a^2 + 2*(11 - 10*a)*b*x))/(8*b^4) - (3*(17 - 44*a + 36*a^2
 - 8*a^3)*ArcSin[a + b*x])/(8*b^4)
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(a+bx)}x^3\,dx = \int \frac{x^3(1+a+bx)^{3/2}}{(1-a-bx)^{3/2}}\,dx$$
$$= \frac{2x^3(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} - \frac{2\int \frac{x^2\sqrt{1+a+bx}\left(3(1+a)+\frac{9bx}{2}\right)}{\sqrt{1-a-bx}}\,dx}{b}$$
$$= \frac{2x^3(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} + \frac{9x^2\sqrt{1-a-bx}(1+a+bx)^{3/2}}{4b^2} + \frac{\int \frac{x\sqrt{1+a+bx}\left(-9(1-a)(1+a)b-\frac{3}{2}(11-10a)b^2x\right)}{\sqrt{1-a-bx}}\,dx}{2b^3}$$
$$= \frac{2x^3(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} + \frac{9x^2\sqrt{1-a-bx}(1+a+bx)^{3/2}}{4b^2} + \frac{\sqrt{1-a-bx}(1+a+bx)^{3/2}\left(29-54a+\right.}{8b^4}$$
$$= \frac{3\left(17-44a+36a^2-8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} + \frac{2x^3(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} + \frac{9x^2\sqrt{1-a-bx}(1-}{4b^2}$$
$$= \frac{3\left(17-44a+36a^2-8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} + \frac{2x^3(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} + \frac{9x^2\sqrt{1-a-bx}(1-}{4b^2}$$

$$= \frac{3\left(17-44a+36a^2-8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} + \frac{2x^3(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} + \frac{9x^2\sqrt{1-a-bx}(1-}{4b^2}$$
$$= \frac{3\left(17-44a+36a^2-8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} + \frac{2x^3(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} + \frac{9x^2\sqrt{1-a-bx}(1-}{4b^2}$$

Mathematica [A] time = 0.284744, size = 203, normalized size = 1.09

$$\frac{-\frac{\sqrt{b}\sqrt{a+bx+1}\left(a^2(22bx-233)-2a^4+78a^3+a\left(-10b^2x^2-54bx+237\right)+2b^4x^4+6b^3x^3+11b^2x^2+29bx-80\right)}{\sqrt{-a-bx+1}} + 24a\left(2a^2+11\right)\sqrt{-b}\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-a-bx+}}{\sqrt{2}\sqrt{b}}\right.}{8b^{9/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a + b*x])*x^3,x]
```

```
[Out] (-((Sqrt[b]*Sqrt[1 + a + b*x]*(-80 + 78*a^3 - 2*a^4 + 29*b*x + 11*b^2*x^2 +
 6*b^3*x^3 + 2*b^4*x^4 + a^2*(-233 + 22*b*x) + a*(237 - 54*b*x - 10*b^2*x^2
)))/Sqrt[1 - a - b*x]) + 24*a*(11 + 2*a^2)*Sqrt[-b]*ArcSinh[(Sqrt[-b]*Sqrt[
1 - a - b*x])/(Sqrt[2]*Sqrt[b])] + 6*(17 + 36*a^2)*Sqrt[-b]*ArcSinh[(Sqrt[b
]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[-b])])/(8*b^(9/2))
```

Maple [B] time = 0.088, size = 756, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x^3,x)
```

```
[Out] -157/8/b^4*a/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/4*b*x^5/(-b^2*x^2-2*a*b*x-a^2
+1)^(1/2)-1/4*a*x^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-17/8/b*x^3/(-b^2*x^2-2*a
*b*x-a^2+1)^(1/2)+51/8/b^3*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/4/b^4*a^5/(-b
^2*x^2-2*a*b*x-a^2+1)^(1/2)+155/8/b^4*a^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3*
```

```
a^3/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/
2))-1/2*a^2/b^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-19/2*a^4/b^4/(-b^2*x^2-2*a*b
*x-a^2+1)^(1/2)+33/2*a/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2
-2*a*b*x-a^2+1)^(1/2))+1/2/b*a*x^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/2*x^2/b
^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^2-x^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-53
/2*a/b^3*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-25/2*a^3/b^3/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2)*x-27/2*a^2/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2))-51/8/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2
*x^2-2*a*b*x-a^2+1)^(1/2))-5*x^2/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+265/8/b
^3*a^2*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+53/8/b^2*a*x^2/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2)+1/4/b^3*a^4*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+10/b^4/(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.59418, size = 456, normalized size = 2.44

$$-\frac{3\left(8a^4-44a^3+\left(8a^3-36a^2+44a-17\right)bx+80a^2-61a+17\right)\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}(bx+a)}{b^2x^2+2abx+a^2-1}\right)-\left(2b^4x^4+6b^3x^3-(\right.}{8\left(b^5x+(a-1)b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*(8*a^4 - 44*a^3 + (8*a^3 - 36*a^2 + 44*a - 17)*b*x + 80*a^2 - 61*a
+ 17)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*
x + a^2 - 1)) - (2*b^4*x^4 + 6*b^3*x^3 - (10*a - 11)*b^2*x^2 - 2*a^4 + 78*a
^3 + (22*a^2 - 54*a + 29)*b*x - 233*a^2 + 237*a - 80)*sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1))/(b^5*x + (a - 1)*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\,(a+bx+1)^3}{\left(-\left(a+bx-1\right)\left(a+bx+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**3/(1-(b*x+a)**2)**(3/2)*x**3,x)
```

```
[Out] Integral(x**3*(a + b*x + 1)**3/(-(a + b*x - 1)*(a + b*x + 1))**(3/2), x)
```

Giac [A] time = 1.26128, size = 261, normalized size = 1.4

$$\frac{1}{8}\sqrt{-(bx+a)^2+1}\left(\left(2x\left(\frac{x}{b}-\frac{ab^{11}-4b^{11}}{b^{13}}\right)+\frac{2a^2b^{10}-20ab^{10}+19b^{10}}{b^{13}}\right)x-\frac{2a^3b^9-44a^2b^9+93ab^9-48b^9}{b^{13}}\right)-\frac{3(8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(-(b*x + a)^2 + 1)*((2*x*(x/b - (a*b^11 - 4*b^11)/b^13) + (2*a^2*b^
10 - 20*a*b^10 + 19*b^10)/b^13)*x - (2*a^3*b^9 - 44*a^2*b^9 + 93*a*b^9 - 48
*b^9)/b^13) - 3/8*(8*a^3 - 36*a^2 + 44*a - 17)*arcsin(-b*x - a)*sgn(b)/(b^3
*abs(b)) - 8*(a^3 - 3*a^2 + 3*a - 1)/(b^3*((sqrt(-(b*x + a)^2 + 1)*abs(b) +
 b)/(b^2*x + a*b) - 1)*abs(b))
```

3.836 $\int e^{3\tanh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=168

$$\frac{\left(6a^2-18a+11\right)\sqrt{-a-bx+1}(a+bx+1)^{3/2}}{6b^3}+\frac{\left(6a^2-18a+11\right)\sqrt{-a-bx+1}\sqrt{a+bx+1}}{2b^3}-\frac{\left(6a^2-18a+11\right)\sin^{-1}(a+$$

```
[Out] ((11 - 18*a + 6*a^2)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^3) + ((11 -
18*a + 6*a^2)*Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2))/(6*b^3) + ((1 - a)^2*(
1 + a + b*x)^(5/2))/(b^3*Sqrt[1 - a - b*x]) + (Sqrt[1 - a - b*x]*(1 + a + b
*x)^(5/2))/(3*b^3) - ((11 - 18*a + 6*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rubi [A] time = 0.191308, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6163, 89, 80, 50, 53, 619, 216}

$$\frac{\left(6a^2-18a+11\right)\sqrt{-a-bx+1}(a+bx+1)^{3/2}}{6b^3}+\frac{\left(6a^2-18a+11\right)\sqrt{-a-bx+1}\sqrt{a+bx+1}}{2b^3}-\frac{\left(6a^2-18a+11\right)\sin^{-1}(a+$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a + b*x])*x^2,x]
```


```
[Out] ((11 - 18*a + 6*a^2)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^3) + ((11 -
18*a + 6*a^2)*Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2))/(6*b^3) + ((1 - a)^2*(
1 + a + b*x)^(5/2))/(b^3*Sqrt[1 - a - b*x]) + (Sqrt[1 - a - b*x]*(1 + a + b
*x)^(5/2))/(3*b^3) - ((11 - 18*a + 6*a^2)*ArcSin[a + b*x])/(2*b^3)
```


Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(a+bx)}x^2\,dx = \int \frac{x^2(1+a+bx)^{3/2}}{(1-a-bx)^{3/2}}\,dx$$

$$= \frac{(1-a)^2(1+a+bx)^{5/2}}{b^3\sqrt{1-a-bx}} - \frac{\int \frac{(1+a+bx)^{3/2}\left((3-2a)(1-a)b+b^2x\right)}{\sqrt{1-a-bx}}\,dx}{b^3}$$

$$= \frac{(1-a)^2(1+a+bx)^{5/2}}{b^3\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}(1+a+bx)^{5/2}}{3b^3} - \frac{\left(11-18a+6a^2\right)\int \frac{(1+a+bx)^{3/2}}{\sqrt{1-a-bx}}\,dx}{3b^2}$$

$$= \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}(1+a+bx)^{3/2}}{6b^3} + \frac{(1-a)^2(1+a+bx)^{5/2}}{b^3\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}(1+a+b}{3b^3}$$

$$= \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3} + \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}(1+a+bx)^{3/2}}{6b^3} + \frac{(1}{}$$

$$= \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3} + \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}(1+a+bx)^{3/2}}{6b^3} + \frac{(1}{}$$

$$= \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3} + \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}(1+a+bx)^{3/2}}{6b^3} + \frac{(1}{}$$

$$= \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3} + \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}(1+a+bx)^{3/2}}{6b^3} + \frac{(1}{}$$

Mathematica [A] time = 0.217007, size = 170, normalized size = 1.01

$$\frac{-\frac{\sqrt{b}\sqrt{a+bx+1}\left(2a^3-53a^2+a(103-16bx)+2b^3x^3+7b^2x^2+19bx-52\right)}{\sqrt{-a-bx+1}} + 6\left(6a^2+11\right)\sqrt{-b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{-b}}\right) + 108a\sqrt{-b}\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-}}{\sqrt{2}}\right.}{6b^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a + b*x])*x^2,x]
```

```
[Out] (-((Sqrt[b]*Sqrt[1 + a + b*x]*(-52 - 53*a^2 + 2*a^3 + 19*b*x + 7*b^2*x^2 +
2*b^3*x^3 + a*(103 - 16*b*x)))/Sqrt[1 - a - b*x]) + 108*a*Sqrt[-b]*ArcSinh[
(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[b])] + 6*(11 + 6*a^2)*Sqrt[-b]*A
rcSinh[(Sqrt[b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[-b])])/(6*b^(7/2))
```

Maple [B] time = 0.051, size = 552, normalized size = 3.3

$$\frac{23\,a^2x}{2\,b^2}\frac{1}{\sqrt{-b^2x^2-2\,xab-a^2+1}}-\frac{3\,x^3}{2}\frac{1}{\sqrt{-b^2x^2-2\,xab-a^2+1}}-\frac{bx^4}{3}\frac{1}{\sqrt{-b^2x^2-2\,xab-a^2+1}}-\frac{x^3a}{3}\frac{1}{\sqrt{-b^2x^2-2\,xab-a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x^2,x)
```

```
[Out] 23/2*a^2/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-3/2*x^3/(-b^2*x^2-2*a*b*x-a^2
+1)^(1/2)-1/3*b*x^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/3*a*x^3/(-b^2*x^2-2*a*
b*x-a^2+1)^(1/2)-1/3/b^3*a^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-25/3/b^3*a^2/(-
b^2*x^2-2*a*b*x-a^2+1)^(1/2)-13/3/b*x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+9*a/
b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-
11/2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1
/2))-3*a^2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2
+1)^(1/2))-17/2*a/b^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+17/2*a^3/b^3/(-b^2*x^2
-2*a*b*x-a^2+1)^(1/2)-1/3/b^2*a^3*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-53/3/b^2
*a*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+26/3/b^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
+11/2*x/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/2/b*a*x^2/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.28028, size = 375, normalized size = 2.23

$$\frac{3\left(6\,a^3+\left(6\,a^2-18\,a+11\right)bx-24\,a^2+29\,a-11\right)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\left(2\,b^3x^3+7\,b^2x^2+2\,a^3-(16\,a-19)b\right.}{6\left(b^4x+(a-1)b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(6*a^3 + (6*a^2 - 18*a + 11)*b*x - 24*a^2 + 29*a - 11)*arctan(sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (2*
b^3*x^3 + 7*b^2*x^2 + 2*a^3 - (16*a - 19)*b*x - 53*a^2 + 103*a - 52)*sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1))/(b^4*x + (a - 1)*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (a+bx+1)^3}{(-(a+bx-1)(a+bx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**3/(1-(b*x+a)**2)**(3/2)*x**2,x)
```

```
[Out] Integral(x**2*(a + b*x + 1)**3/(-(a + b*x - 1)*(a + b*x + 1))**(3/2), x)
```

Giac [A] time = 1.18672, size = 200, normalized size = 1.19

$$\frac{1}{6}\sqrt{-(bx+a)^2+1}\left(x\left(\frac{2x}{b}-\frac{2ab^6-9b^6}{b^8}\right)+\frac{2a^2b^5-27ab^5+28b^5}{b^8}\right)+\frac{\left(6a^2-18a+11\right)\arcsin\left(-bx-a\right)\mathrm{sgn}\left(b\right)}{2b^2|b|}+ -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(-(b*x + a)^2 + 1)*(x*(2*x/b - (2*a*b^6 - 9*b^6)/b^8) + (2*a^2*b^5
- 27*a*b^5 + 28*b^5)/b^8) + 1/2*(6*a^2 - 18*a + 11)*arcsin(-b*x - a)*sgn(b)
/(b^2*abs(b)) + 8*(a^2 - 2*a + 1)/(b^2*((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)
/(b^2*x + a*b) - 1)*abs(b))
```

3.837 $\int e^{3\tanh^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=121

$$\frac{(1-a)(a+bx+1)^{5/2}}{b^2\sqrt{-a-bx+1}}+\frac{(3-2a)\sqrt{-a-bx+1}(a+bx+1)^{3/2}}{2b^2}+\frac{3(3-2a)\sqrt{-a-bx+1}\sqrt{a+bx+1}}{2b^2}-\frac{3(3-2a)\sin^{-1}(a+}{2b^2}$$

```
[Out] (3*(3 - 2*a)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^2) + ((3 - 2*a)*Sqrt
[1 - a - b*x]*(1 + a + b*x)^(3/2))/(2*b^2) + ((1 - a)*(1 + a + b*x)^(5/2))/
(b^2*Sqrt[1 - a - b*x]) - (3*(3 - 2*a)*ArcSin[a + b*x])/(2*b^2)
```

Rubi [A] time = 0.0955477, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6163, 78, 50, 53, 619, 216}

$$\frac{(1-a)(a+bx+1)^{5/2}}{b^2\sqrt{-a-bx+1}}+\frac{(3-2a)\sqrt{-a-bx+1}(a+bx+1)^{3/2}}{2b^2}+\frac{3(3-2a)\sqrt{-a-bx+1}\sqrt{a+bx+1}}{2b^2}-\frac{3(3-2a)\sin^{-1}(a+}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a + b*x])*x,x]
```

```
[Out] (3*(3 - 2*a)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^2) + ((3 - 2*a)*Sqrt
[1 - a - b*x]*(1 + a + b*x)^(3/2))/(2*b^2) + ((1 - a)*(1 + a + b*x)^(5/2))/
(b^2*Sqrt[1 - a - b*x]) - (3*(3 - 2*a)*ArcSin[a + b*x])/(2*b^2)
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(a+bx)}x\,dx = \int \frac{x(1+a+bx)^{3/2}}{(1-a-bx)^{3/2}}\,dx$$

$$= \frac{(1-a)(1+a+bx)^{5/2}}{b^2\sqrt{1-a-bx}} - \frac{(3-2a)\int \frac{(1+a+bx)^{3/2}}{\sqrt{1-a-bx}}\,dx}{b}$$

$$= \frac{(3-2a)\sqrt{1-a-bx}(1+a+bx)^{3/2}}{2b^2} + \frac{(1-a)(1+a+bx)^{5/2}}{b^2\sqrt{1-a-bx}} - \frac{(3(3-2a))\int \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\,dx}{2b}$$

$$= \frac{3(3-2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2} + \frac{(3-2a)\sqrt{1-a-bx}(1+a+bx)^{3/2}}{2b^2} + \frac{(1-a)(1+a+bx)^{5/2}}{b^2\sqrt{1-a-bx}}$$

$$= \frac{3(3-2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2} + \frac{(3-2a)\sqrt{1-a-bx}(1+a+bx)^{3/2}}{2b^2} + \frac{(1-a)(1+a+bx)^{5/2}}{b^2\sqrt{1-a-bx}}$$

$$= \frac{3(3-2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2} + \frac{(3-2a)\sqrt{1-a-bx}(1+a+bx)^{3/2}}{2b^2} + \frac{(1-a)(1+a+bx)^{5/2}}{b^2\sqrt{1-a-bx}}$$

$$= \frac{3(3-2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2} + \frac{(3-2a)\sqrt{1-a-bx}(1+a+bx)^{3/2}}{2b^2} + \frac{(1-a)(1+a+bx)^{5/2}}{b^2\sqrt{1-a-bx}}$$

Mathematica [A] time = 0.15619, size = 142, normalized size = 1.17

$$\frac{\frac{\sqrt{b}\sqrt{a+bx+1}\left(a^2-15a-b^2x^2-5bx+14\right)}{\sqrt{-a-bx+1}} + 12a\sqrt{-b}\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{b}}\right) + 18\sqrt{-b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{-b}}\right)}{2b^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a + b*x])*x,x]
```

```
[Out] ((Sqrt[b]*Sqrt[1 + a + b*x]*(14 - 15*a + a^2 - 5*b*x - b^2*x^2))/Sqrt[1 - a
 - b*x] + 12*a*Sqrt[-b]*ArcSinh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[
b])] + 18*Sqrt[-b]*ArcSinh[(Sqrt[b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[-b])])
/(2*b^(5/2))
```

Maple [B] time = 0.042, size = 381, normalized size = 3.2

$$-\frac{bx^3}{2}\frac{1}{\sqrt{-b^2x^2-2xab-a^2+1}} + \frac{a^2x}{2b}\frac{1}{\sqrt{-b^2x^2-2xab-a^2+1}} - \frac{9}{2b}\arctan\left(\sqrt{b^2}\left(x+\frac{a}{b}\right)\frac{1}{\sqrt{-b^2x^2-2xab-a^2+1}}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x,x)
```

```
[Out] -1/2*b*x^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/2/b*a^2*x/(-b^2*x^2-2*a*b*x-a^2
+1)^(1/2)-9/2/b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2))+7/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-10*a/b/(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2)*x+3*a/b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2))-7*a^2/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+9/2/b*x/(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2)-1/2*a*x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/2/b^2*a^3/(
-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/2/b^2*a/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3*x^
2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.18579, size = 300, normalized size = 2.48

$$-\frac{3\left((2a-3)bx+2a^2-5a+3\right)\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}(bx+a)}{b^2x^2+2abx+a^2-1}\right)-\left(b^2x^2-a^2+5bx+15a-14\right)\sqrt{-b^2x^2-2abx-a^2+1}}{2\left(b^3x+(a-1)b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] -1/2*(3*((2*a - 3)*b*x + 2*a^2 - 5*a + 3)*arctan(sqrt(-b^2*x^2 - 2*a*b*x -
a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (b^2*x^2 - a^2 + 5*b*x
+ 15*a - 14)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/(b^3*x + (a - 1)*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x(a+bx+1)^3}{(-(a+bx-1)(a+bx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**3/(1-(b*x+a)**2)**(3/2)*x,x)
```

```
[Out] Integral(x*(a + b*x + 1)**3/(-(a + b*x - 1)*(a + b*x + 1))**(3/2), x)
```

Giac [A] time = 1.2323, size = 147, normalized size = 1.21

$$\frac{1}{2}\sqrt{-(bx+a)^2+1}\left(\frac{x}{b}-\frac{ab^2-6\,b^2}{b^4}\right)-\frac{3\,(2\,a-3)\arcsin\left(-bx-a\right)\operatorname{sgn}\left(b\right)}{2\,b|b|}-\frac{8\,(a-1)}{b\left(\frac{\sqrt{-(bx+a)^2+1}|b|+b}{b^2x+ab}-1\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-(b*x + a)^2 + 1)*(x/b - (a*b^2 - 6*b^2)/b^4) - 3/2*(2*a - 3)*arcs
in(-b*x - a)*sgn(b)/(b*abs(b)) - 8*(a - 1)/(b*((sqrt(-(b*x + a)^2 + 1)*abs(
b) + b)/(b^2*x + a*b) - 1)*abs(b))
```

3.838 $\int e^{3\tanh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=68

$$\frac{2(a+bx+1)^{3/2}}{b\sqrt{-a-bx+1}}+\frac{3\sqrt{-a-bx+1}\sqrt{a+bx+1}}{b}-\frac{3\sin^{-1}(a+bx)}{b}$$

```
[Out] (3*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/b + (2*(1 + a + b*x)^(3/2))/(b*Sqrt
[1 - a - b*x]) - (3*ArcSin[a + b*x])/b
```

Rubi [A] time = 0.0343045, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6161, 47, 50, 53, 619, 216}

$$\frac{2(a+bx+1)^{3/2}}{b\sqrt{-a-bx+1}}+\frac{3\sqrt{-a-bx+1}\sqrt{a+bx+1}}{b}-\frac{3\sin^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a + b*x]),x]
```

```
[Out] (3*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/b + (2*(1 + a + b*x)^(3/2))/(b*Sqrt
[1 - a - b*x]) - (3*ArcSin[a + b*x])/b
```

Rule 6161

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 + a*c
 + b*c*x)^(n/2)/(1 - a*c - b*c*x)^(n/2), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{3\tanh^{-1}(a+bx)}\,dx &= \int \frac{(1+a+bx)^{3/2}}{(1-a-bx)^{3/2}}\,dx\\
&= \frac{2(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} - 3\int \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\,dx\\
&= \frac{3\sqrt{1-a-bx}\sqrt{1+a+bx}}{b} + \frac{2(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} - 3\int \frac{1}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx\\
&= \frac{3\sqrt{1-a-bx}\sqrt{1+a+bx}}{b} + \frac{2(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} - 3\int \frac{1}{\sqrt{(1-a)(1+a)-2abx-b^2x^2}}\,dx\\
&= \frac{3\sqrt{1-a-bx}\sqrt{1+a+bx}}{b} + \frac{2(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2ab-2b^2x\right)}{2b^2}\\
&= \frac{3\sqrt{1-a-bx}\sqrt{1+a+bx}}{b} + \frac{2(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} - \frac{3\sin^{-1}(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0385456, size = 43, normalized size = 0.63

$$\frac{\left(1-\frac{4}{a+bx-1}\right)\sqrt{1-(a+bx)^2}}{b} - \frac{3\sin^{-1}(a+bx)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a + b*x]),x]
```

```
[Out] ((1 - 4/(-1 + a + b*x))*Sqrt[1 - (a + b*x)^2])/b - (3*ArcSin[a + b*x])/b
```

Maple [B] time = 0.037, size = 388, normalized size = 5.7

$$3\frac{x}{\sqrt{-b^2x^2-2xab-a^2+1}} + 3\frac{a}{b\sqrt{-b^2x^2-2xab-a^2+1}} - 3\frac{a^2x}{\sqrt{-b^2x^2-2xab-a^2+1}} - 3\frac{a^3}{b\sqrt{-b^2x^2-2xab-a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2),x)
```

```
[Out] 3*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/b*a/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3*a
^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-3/b*a^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+
2*(1+a)^3*(-2*b^2*x-2*a*b)/(-4*b^2*(-a^2+1)-4*a^2*b^2)/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2)-1/b*a^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-5*a*x/(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2)-4/b*a^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a^3/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2)*x-b*x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/(b^2)^(1/2)*arctan((b^2
```

```
)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+5/b/(-b^2*x^2-2*a*b*x-a^2+1
)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99537, size = 234, normalized size = 3.44

$$\frac{3\,(bx+a-1)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\sqrt{-b^2x^2-2\,abx-a^2+1}(bx+a-5)}{b^2x+(a-1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (3*(b*x + a - 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x
^2 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a - 5)
)/(b^2*x + (a - 1)*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx+1)^3}{(-(a+bx-1)(a+bx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**3/(1-(b*x+a)**2)**(3/2),x)
```

```
[Out] Integral((a + b*x + 1)**3/(-(a + b*x - 1)*(a + b*x + 1))**(3/2), x)
```

Giac [A] time = 1.24711, size = 101, normalized size = 1.49

$$\frac{3\,\arcsin(-bx-a)\,\mathrm{sgn}(b)}{|b|}+\frac{\sqrt{-(bx+a)^2+1}}{b}+\frac{8}{\left(\frac{\sqrt{-(bx+a)^2+1}|b|+b}{b^2x+ab}-1\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3*arcsin(-b*x - a)*sgn(b)/abs(b) + sqrt(-(b*x + a)^2 + 1)/b + 8/(((sqrt(-(b
*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 1)*abs(b))
```

3.839 $\int \frac{e^{3\tanh^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=107

$$-\frac{2(a+1)^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)\sqrt{1-a^2}}+\frac{4\sqrt{a+bx+1}}{(1-a)\sqrt{-a-bx+1}}-\sin^{-1}(a+bx)$$

```
[Out] (4*Sqrt[1 + a + b*x])/((1 - a)*Sqrt[1 - a - b*x]) - ArcSin[a + b*x] - (2*(1
 + a)^2*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b
*x])])/((1 - a)*Sqrt[1 - a^2])
```

Rubi [A] time = 0.0908619, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6163, 98, 157, 53, 619, 216, 93, 208}

$$-\frac{2(a+1)^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)\sqrt{1-a^2}}+\frac{4\sqrt{a+bx+1}}{(1-a)\sqrt{-a-bx+1}}-\sin^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a + b*x])/x,x]
```


```
[Out] (4*Sqrt[1 + a + b*x])/((1 - a)*Sqrt[1 - a - b*x]) - ArcSin[a + b*x] - (2*(1
 + a)^2*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b
*x])])/((1 - a)*Sqrt[1 - a^2])
```


Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
```

```
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(a+bx)}}{x}\,dx &= \int \frac{(1+a+bx)^{3/2}}{x(1-a-bx)^{3/2}}\,dx\\
&= \frac{4\sqrt{1+a+bx}}{(1-a)\sqrt{1-a-bx}} - \frac{2\int \frac{-\frac{1}{2}(1+a)^2b+\frac{1}{2}(1-a)b^2x}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{(1-a)b}\\
&= \frac{4\sqrt{1+a+bx}}{(1-a)\sqrt{1-a-bx}} + \frac{(1+a)^2\int \frac{1}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{1-a} - b\int \frac{1}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx\\
&= \frac{4\sqrt{1+a+bx}}{(1-a)\sqrt{1-a-bx}} + \frac{\left(2(1+a)^2\right)\text{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx, x, \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\right)}{1-a} - b\int \frac{1}{\sqrt{(1-a)(1+a)-2a}}
\\
&= \frac{4\sqrt{1+a+bx}}{(1-a)\sqrt{1-a-bx}} - \frac{2(1+a)^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{(1-a)\sqrt{1-a^2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2ab-2b^2x\right)}{2b}\\
&= \frac{4\sqrt{1+a+bx}}{(1-a)\sqrt{1-a-bx}} - \sin^{-1}(a+bx) - \frac{2(1+a)^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{(1-a)\sqrt{1-a^2}}
\end{aligned}
$$

Mathematica [A] time = 0.486515, size = 160, normalized size = 1.5

$$
\frac{2\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{-b}} - \frac{2\left(2\sqrt{a-1}\sqrt{a+1}\sqrt{a+bx+1}+(a+1)^2\sqrt{-a-bx+1}\tan^{-1}\left(\frac{\sqrt{-a-bx+1}}{\sqrt{\frac{a-1}{a+1}}\sqrt{a+bx+1}}\right)\right)}{(a-1)^{3/2}\sqrt{a+1}\sqrt{-a-bx+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a + b*x])/x,x]
```

```
[Out] (2*Sqrt[b]*ArcSinh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[b])])/Sqrt[-b
] - (2*(2*Sqrt[-1 + a]*Sqrt[1 + a]*Sqrt[1 + a + b*x] + (1 + a)^2*Sqrt[1 - a
 - b*x]*ArcTan[Sqrt[1 - a - b*x]/(Sqrt[(-1 + a)/(1 + a)]*Sqrt[1 + a + b*x])
]))/((-1 + a)^(3/2)*Sqrt[1 + a]*Sqrt[1 - a - b*x])
```

Maple [B] time = 0.042, size = 1019, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x,x)
```

```
[Out] 3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+6*a^2*b*(-2*b^2*x-2*a*b)/(-4*b^2*(-a^2+1)-
4*a^2*b^2)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3*a^2*b/(-a^2+1)/(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2)*x+a*b/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+6*b*(-2*b^2*
x-2*a*b)/(-4*b^2*(-a^2+1)-4*a^2*b^2)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-2*b*a^2
/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+12*a*b*(-2*b^2*x-2*a*b)/(-4*b^2*(-a^2+1)-
4*a^2*b^2)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+a^4*b/(-a^2+1)/(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2)*x+3*a^3*b/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-3*b*a/(-b^
2*x^2-2*a*b*x-a^2+1)^(1/2)*x+3/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a+a^
5/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3*a^4/(-a^2+1)/(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2)+4/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^3-3/(-a^2+1)^(3/2)
*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)*a
^2-3/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x
-a^2+1)^(1/2))/x)*a+4*a^2/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+b*x/(-b^2
*x^2-2*a*b*x-a^2+1)^(1/2)-b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^
2-2*a*b*x-a^2+1)^(1/2))-1/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1
/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)*a^3+1/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2
+1)^(1/2)-1/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2))/x)-3*a^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+2*a/(-b^2*x^2
-2*a*b*x-a^2+1)^(1/2)-2*a^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14404, size = 1058, normalized size = 9.89

$$\left[\frac{\left((a+1)bx+a^2-1\right)\sqrt{-\frac{a+1}{a-1}}\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx-4a^2+2\sqrt{-b^2x^2-2abx-a^2+1}\left(a^3+\left(a^2-a\right)bx-a^2-a+1\right)\sqrt{-\frac{a+1}{a-1}}+2}{x^2}\right)+2\left((a-1\right.}{2\left((a-1)bx+a^2-2a+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(((a + 1)*b*x + a^2 - 1)*sqrt(-(a + 1)/(a - 1))*log(((2*a^2 - 1)*b^2*x
^2 + 2*a^4 + 4*(a^3 - a)*b*x - 4*a^2 + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
*(a^3 + (a^2 - a)*b*x - a^2 - a + 1)*sqrt(-(a + 1)/(a - 1)) + 2)/x^2) + 2*(
(a - 1)*b*x + a^2 - 2*a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x
 + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + 8*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
)/((a - 1)*b*x + a^2 - 2*a + 1), -(((a + 1)*b*x + a^2 - 1)*sqrt((a + 1)/(a
- 1))*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt((a +
 1)/(a - 1))/((a + 1)*b^2*x^2 + a^3 + 2*(a^2 + a)*b*x + a^2 - a - 1)) - ((a
 - 1)*b*x + a^2 - 2*a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x +
 a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 4*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/
((a - 1)*b*x + a^2 - 2*a + 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx + 1)^3}{x\left(-(a + bx - 1)(a + bx + 1)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**3/(1-(b*x+a)**2)**(3/2)/x,x)
```

```
[Out] Integral((a + b*x + 1)**3/(x*(-(a + b*x - 1)*(a + b*x + 1))**(3/2)), x)
```

Giac [A] time = 1.25198, size = 188, normalized size = 1.76

$$\frac{b \arcsin\left(-bx - a\right) \operatorname{sgn}\left(b\right)}{|b|} - \frac{2\left(a^2 b + 2\,ab + b\right) \arctan\left(\frac{\frac{\left(\sqrt{-(bx+a)^2+1}|b|+b\right)a}{b^2x+ab} - 1}{\sqrt{a^2-1}}\right)}{\sqrt{a^2 - 1}(a|b| - |b|)} - \frac{8\,b}{(a|b| - |b|)\left(\frac{\sqrt{-(bx+a)^2+1}|b|+b}{b^2x+ab} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] b*arcsin(-b*x - a)*sgn(b)/abs(b) - 2*(a^2*b + 2*a*b + b)*arctan(((sqrt(-(b*
x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/(sqrt(a^2 - 1
)*(a*abs(b) - abs(b))) - 8*b/((a*abs(b) - abs(b))*((sqrt(-(b*x + a)^2 + 1)*
abs(b) + b)/(b^2*x + a*b) - 1))
```

3.840 $\int \frac{e^{3\tanh^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=134

$$-\frac{6(a+1)b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)^2\sqrt{1-a^2}}-\frac{(a+bx+1)^{3/2}}{(1-a)x\sqrt{-a-bx+1}}+\frac{6b\sqrt{a+bx+1}}{(1-a)^2\sqrt{-a-bx+1}}$$

```
[Out] (6*b*Sqrt[1 + a + b*x])/((1 - a)^2*Sqrt[1 - a - b*x]) - (1 + a + b*x)^(3/2)
/((1 - a)*x*Sqrt[1 - a - b*x]) - (6*(1 + a)*b*ArcTanh[(Sqrt[1 - a]*Sqrt[1 +
 a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 - a)^2*Sqrt[1 - a^2])
```

Rubi [A] time = 0.0753951, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6163, 94, 93, 208}

$$-\frac{6(a+1)b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)^2\sqrt{1-a^2}}-\frac{(a+bx+1)^{3/2}}{(1-a)x\sqrt{-a-bx+1}}+\frac{6b\sqrt{a+bx+1}}{(1-a)^2\sqrt{-a-bx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a + b*x])/x^2,x]
```

```
[Out] (6*b*Sqrt[1 + a + b*x])/((1 - a)^2*Sqrt[1 - a - b*x]) - (1 + a + b*x)^(3/2)
/((1 - a)*x*Sqrt[1 - a - b*x]) - (6*(1 + a)*b*ArcTanh[(Sqrt[1 - a]*Sqrt[1 +
 a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 - a)^2*Sqrt[1 - a^2])
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(a+bx)}}{x^2}\,dx = \int \frac{(1+a+bx)^{3/2}}{x^2(1-a-bx)^{3/2}}\,dx$$
$$= -\frac{(1+a+bx)^{3/2}}{(1-a)x\sqrt{1-a-bx}} + \frac{(3b)\int \frac{\sqrt{1+a+bx}}{x(1-a-bx)^{3/2}}\,dx}{1-a}$$
$$= \frac{6b\sqrt{1+a+bx}}{(1-a)^2\sqrt{1-a-bx}} - \frac{(1+a+bx)^{3/2}}{(1-a)x\sqrt{1-a-bx}} + \frac{(3(1+a)b)\int \frac{1}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{(1-a)^2}$$
$$= \frac{6b\sqrt{1+a+bx}}{(1-a)^2\sqrt{1-a-bx}} - \frac{(1+a+bx)^{3/2}}{(1-a)x\sqrt{1-a-bx}} + \frac{(6(1+a)b)\,\text{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx, x, \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\right)}{(1-a)^2}$$
$$= \frac{6b\sqrt{1+a+bx}}{(1-a)^2\sqrt{1-a-bx}} - \frac{(1+a+bx)^{3/2}}{(1-a)x\sqrt{1-a-bx}} - \frac{6(1+a)b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{(1-a)^2\sqrt{1-a^2}}$$

Mathematica [A] time = 0.146294, size = 101, normalized size = 0.75

$$\frac{\sqrt{a+bx+1}\left(a^2+abx+5bx-1\right)}{(a-1)^2x\sqrt{-a-bx+1}} + \frac{6\sqrt{a+1}b\tan^{-1}\left(\frac{\sqrt{-a-bx+1}}{\sqrt{\frac{a-1}{a+1}}\sqrt{a+bx+1}}\right)}{(a-1)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a + b*x])/x^2,x]
```

```
[Out] (Sqrt[1 + a + b*x]*(-1 + a^2 + 5*b*x + a*b*x))/((-1 + a)^2*x*Sqrt[1 - a - b
*x]) + (6*Sqrt[1 + a]*b*ArcTan[Sqrt[1 - a - b*x]/(Sqrt[(-1 + a)/(1 + a)]*Sq
rt[1 + a + b*x])])/(-1 + a)^(5/2)
```

Maple [B] time = 0.042, size = 1520, normalized size = 11.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^2,x)
```

```
[Out] 6*a*b^2*(-2*b^2*x-2*a*b)/(-4*b^2*(-a^2+1)-4*a^2*b^2)/(-b^2*x^2-2*a*b*x-a^2+
1)^(1/2)+3*a^2*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-9*a^2*b/(-a^
2+1)^(5/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(
1/2))/x)+12*a^4*b/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3*a*b/(-a^2+1)^
(5/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))
/x)+9*a^5*b/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3*a^4*b/(-a^2+1)^(5/2
)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+
12/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^2*b-b*a^2/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2)-1/(-a^2+1)/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^3-3/(-a^2+1)/x/(-b
^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^2-3/(-a^2+1)/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
*a+5/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^4*b+12/(-a^2+1)/(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2)*a^3*b+9*a^2*b/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+
3*a^6*b/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/(-a^2+1)/x/(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2)+3*b/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3*b/(-a^2+1)
```

```
^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
)/x)-3*a^2*b/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2
-2*a*b*x-a^2+1)^(1/2))/x)-6*a*b/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2
+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-9*a^3*b/(-a^2+1)^(5/2)*ln((-2*
a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+2/(-a^2+1
)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x*b^2+8/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^
(1/2)*a*b+6*b^2*(-2*b^2*x-2*a*b)/(-4*b^2*(-a^2+1)-4*a^2*b^2)/(-b^2*x^2-2*a*
b*x-a^2+1)^(1/2)+3*a*b/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+12*a^3*b/(
-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-b^2*a/(-b^2*x^2-2*a*b*x-a^2+1)^(1/
2)*x+9*a^3*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+5/(-a^2+1)/(-b^2
*x^2-2*a*b*x-a^2+1)^(1/2)*x*b^2*a^3+3*a^5*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2)*x+9*a^4*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+12/(-a
^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x*b^2*a^2+9/(-a^2+1)/(-b^2*x^2-2*a*b*x
-a^2+1)^(1/2)*x*b^2*a+b/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 2.11742, size = 871, normalized size = 6.5

$$\left[\frac{3\left(b^2x^2+(a-1)bx\right)\sqrt{-\frac{a+1}{a-1}}\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx-4a^2-2\sqrt{-b^2x^2-2abx-a^2+1}\left(a^3+\left(a^2-a\right)bx-a^2-a+1\right)\sqrt{-\frac{a+1}{a-1}}+2}{x^2}\right)-2\sqrt{-b^2x^2 \ldots}}{2\left(\left(a^2-2a+1\right)bx^2+\left(a^3-3a^2+3a-1\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^2,x, algorithm="fricas")

[Out] [1/2*(3*(b^2*x^2 + (a - 1)*b*x)*sqrt(-(a + 1)/(a - 1))*log(((2*a^2 - 1)*b^2
*x^2 + 2*a^4 + 4*(a^3 - a)*b*x - 4*a^2 - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
1)*(a^3 + (a^2 - a)*b*x - a^2 - a + 1)*sqrt(-(a + 1)/(a - 1)) + 2)/x^2) - 2
*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*((a + 5)*b*x + a^2 - 1))/((a^2 - 2*a +
1)*b*x^2 + (a^3 - 3*a^2 + 3*a - 1)*x), (3*(b^2*x^2 + (a - 1)*b*x)*sqrt((a +
 1)/(a - 1))*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sq
rt((a + 1)/(a - 1))/((a + 1)*b^2*x^2 + a^3 + 2*(a^2 + a)*b*x + a^2 - a - 1)
) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*((a + 5)*b*x + a^2 - 1))/((a^2 - 2*a
 + 1)*b*x^2 + (a^3 - 3*a^2 + 3*a - 1)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx+1)^3}{x^2\left(-(a+bx-1)(a+bx+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**3/(1-(b*x+a)**2)**(3/2)/x**2,x)
```

```
[Out] Integral((a + b*x + 1)**3/(x**2*(-(a + b*x - 1)*(a + b*x + 1))**(3/2)), x)
```

Giac [B] time = 1.35117, size = 672, normalized size = 5.01

$$\frac{6\left(ab^2+b^2\right)\arctan\left(\frac{\frac{\left(\sqrt{-(bx+a)^2+1}|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\left(a^2|b|-2\,a|b|+|b|\right)\sqrt{a^2-1}}-\frac{2\left(\frac{\left(\sqrt{-(bx+a)^2+1}|b|+b\right)a^2b^2}{b^2x+ab}-\frac{4\left(\sqrt{-(bx+a)^2+1}|b|+b\right)^2a^2b^2}{\left(b^2x+ab\right)^2}-5\,a^2b^2+\frac{10\left(\sqrt{-(bx+a)^2+1}|b|+b\right)ab}{b^2x+ab}\right.}{\left(a^3|b|-2\,a^2|b|+a|b|\right)\left(\frac{\left(\sqrt{-(bx+a)^2+1}|b|+b\right)a}{b^2x+ab}-\frac{\left(\sqrt{-(bx+a)^2+1}|b|+b\right)^2a}{\left(b^2x+ab\right)^2}+\frac{\left(\sqrt{-\right.}{}}{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 6*(a*b^2 + b^2)*arctan(((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b)
 - 1)/sqrt(a^2 - 1))/((a^2*abs(b) - 2*a*abs(b) + abs(b))*sqrt(a^2 - 1)) - 2
*((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a^2*b^2/(b^2*x + a*b) - 4*(sqrt(-(b*x
 + a)^2 + 1)*abs(b) + b)^2*a^2*b^2/(b^2*x + a*b)^2 - 5*a^2*b^2 + 10*(sqrt(-
(b*x + a)^2 + 1)*abs(b) + b)*a*b^2/(b^2*x + a*b) - (sqrt(-(b*x + a)^2 + 1)*
abs(b) + b)^2*a*b^2/(b^2*x + a*b)^2 - a*b^2 + (sqrt(-(b*x + a)^2 + 1)*abs(b
) + b)*b^2/(b^2*x + a*b) - (sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*b^2/(b^2*x
 + a*b)^2)/((a^3*abs(b) - 2*a^2*abs(b) + a*abs(b))*((sqrt(-(b*x + a)^2 + 1)
*abs(b) + b)*a/(b^2*x + a*b) - (sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a/(b^2
*x + a*b)^2 + (sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a/(b^2*x + a*b)^3 - a +
 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 2*(sqrt(-(b*x + a)^2
 + 1)*abs(b) + b)^2/(b^2*x + a*b)^2))
```

3.841 $\int \frac{e^{3\tanh^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=202

$$-\frac{3(2a+3)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)^3\sqrt{1-a^2}}-\frac{(a+bx+1)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{-a-bx+1}}+\frac{3(2a+3)b^2\sqrt{a+bx+1}}{(1-a)^3(a+1)\sqrt{-a-bx+1}}-\frac{(2a+3)b(a+}{2(1-a)^2(a+1)x}$$

```
[Out] (3*(3 + 2*a)*b^2*Sqrt[1 + a + b*x])/((1 - a)^3*(1 + a)*Sqrt[1 - a - b*x]) -
 ((3 + 2*a)*b*(1 + a + b*x)^(3/2))/(2*(1 - a)^2*(1 + a)*x*Sqrt[1 - a - b*x]
) - (1 + a + b*x)^(5/2)/(2*(1 - a^2)*x^2*Sqrt[1 - a - b*x]) - (3*(3 + 2*a)*
b^2*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])
])/((1 - a)^3*Sqrt[1 - a^2])
```

Rubi [A] time = 0.12217, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6163, 96, 94, 93, 208}

$$-\frac{3(2a+3)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)^3\sqrt{1-a^2}}-\frac{(a+bx+1)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{-a-bx+1}}+\frac{3(2a+3)b^2\sqrt{a+bx+1}}{(1-a)^3(a+1)\sqrt{-a-bx+1}}-\frac{(2a+3)b(a+}{2(1-a)^2(a+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a + b*x])/x^3,x]
```

```
[Out] (3*(3 + 2*a)*b^2*Sqrt[1 + a + b*x])/((1 - a)^3*(1 + a)*Sqrt[1 - a - b*x]) -
 ((3 + 2*a)*b*(1 + a + b*x)^(3/2))/(2*(1 - a)^2*(1 + a)*x*Sqrt[1 - a - b*x]
) - (1 + a + b*x)^(5/2)/(2*(1 - a^2)*x^2*Sqrt[1 - a - b*x]) - (3*(3 + 2*a)*
b^2*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])
])/((1 - a)^3*Sqrt[1 - a^2])
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(a+bx)}}{x^3}\,dx = \int \frac{(1+a+bx)^{3/2}}{x^3(1-a-bx)^{3/2}}\,dx$$

$$= -\frac{(1+a+bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1-a-bx}} + \frac{((3+2a)b)\int \frac{(1+a+bx)^{3/2}}{x^2(1-a-bx)^{3/2}}\,dx}{2\left(1-a^2\right)}$$

$$= -\frac{(3+2a)b(1+a+bx)^{3/2}}{2(1-a)^2(1+a)x\sqrt{1-a-bx}} - \frac{(1+a+bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1-a-bx}} + \frac{\left(3(3+2a)b^2\right)\int \frac{\sqrt{1+a+bx}}{x(1-a-bx)^{3/2}}\,dx}{2(1-a)^2(1+a)}$$

$$= \frac{3(3+2a)b^2\sqrt{1+a+bx}}{(1-a)^3(1+a)\sqrt{1-a-bx}} - \frac{(3+2a)b(1+a+bx)^{3/2}}{2(1-a)^2(1+a)x\sqrt{1-a-bx}} - \frac{(1+a+bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1-a-bx}} + \frac{\left(3(3+2a\right.}{}$$

$$= \frac{3(3+2a)b^2\sqrt{1+a+bx}}{(1-a)^3(1+a)\sqrt{1-a-bx}} - \frac{(3+2a)b(1+a+bx)^{3/2}}{2(1-a)^2(1+a)x\sqrt{1-a-bx}} - \frac{(1+a+bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1-a-bx}} + \frac{\left(3(3+2a\right.}{}$$

$$= \frac{3(3+2a)b^2\sqrt{1+a+bx}}{(1-a)^3(1+a)\sqrt{1-a-bx}} - \frac{(3+2a)b(1+a+bx)^{3/2}}{2(1-a)^2(1+a)x\sqrt{1-a-bx}} - \frac{(1+a+bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1-a-bx}} - \frac{3(3+2a)b}{(}$$

Mathematica [A] time = 0.134306, size = 136, normalized size = 0.67

$$\frac{\sqrt{a+bx+1}\left(a^3-a^2-a\left(b^2x^2+5bx+1\right)-14b^2x^2+5bx+1\right)}{2(a-1)^3x^2\sqrt{-a-bx+1}} - \frac{3(2a+3)b^2\tan^{-1}\left(\frac{\sqrt{-a-bx+1}}{\sqrt{\frac{a-1}{a+1}}\sqrt{a+bx+1}}\right)}{(a-1)^{7/2}\sqrt{a+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a + b*x])/x^3,x]
```

```
[Out] (Sqrt[1 + a + b*x]*(1 - a^2 + a^3 + 5*b*x - 14*b^2*x^2 - a*(1 + 5*b*x + b^2
*x^2)))/(2*(-1 + a)^3*x^2*Sqrt[1 - a - b*x]) - (3*(3 + 2*a)*b^2*ArcTan[Sqrt
[1 - a - b*x]/(Sqrt[(-1 + a)/(1 + a)]*Sqrt[1 + a + b*x])])/((-1 + a)^(7/2)*
Sqrt[1 + a])
```

Maple [B] time = 0.045, size = 2194, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^3,x)
```

```
[Out] -21/2*a^3*b^2/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^
2-2*a*b*x-a^2+1)^(1/2))/x)-3*a*b^2/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-
a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-45/2*a^2*b^2/(-a^2+1)^(5/2)
*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-3
*b^2/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x
-a^2+1)^(1/2))/x)-3/2/(-a^2+1)/x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a+30*a^5*
b^2/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+45/2*a^3*b^2/(-a^2+1)^3/(-b^2
*x^2-2*a*b*x-a^2+1)^(1/2)-45/2*a^4*b^2/(-a^2+1)^(7/2)*ln((-2*a^2+2-2*x*a*b+
2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-45/2*a^3*b^2/(-a^2+1)^(
7/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/
x)+15/2*a^7*b^2/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+45/2*a^6*b^2/(-a^
2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-15/2*a^2*b^2/(-a^2+1)^(7/2)*ln((-2*a^
2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-3/2*b^2/(-a
^2+1)^(5/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^
(1/2))/x)+3/2*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3*b^2/(-a^2+1)/
(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/2/(-a^2+1)/x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1
/2)-3*b/(-a^2+1)/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+39*a^3*b^2/(-a^2+1)^2/(-b
^2*x^2-2*a*b*x-a^2+1)^(1/2)+29*a^2*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^
(1/2)+27/2*a*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+31/2*a^5*b^2/(-a
^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+75/2*a^4*b^2/(-a^2+1)^2/(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2)+15/2*a^2*b^2/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3
0*a^4*b^2/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+2*b^3*(-2*b^2*x-2*a*b)/
(-4*b^2*(-a^2+1)-4*a^2*b^2)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-27/2*a*b^2/(-a^2
+1)^(5/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1
/2))/x)+6/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x*b^3+9/(-a^2+1)/(-b^2*x^
2-2*a*b*x-a^2+1)^(1/2)*a^3*b^2+15/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a
^2*b^2+9/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a*b^2-15/2*a^5*b^2/(-a^2+1
)^(7/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2
))/x)-1/2/(-a^2+1)/x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^3-3/2/(-a^2+1)/x^2/
(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^2+31/2*a^4*b^3/(-a^2+1)^2/(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2)*x+75/2*a^3*b^3/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+5
7/2*a^2*b^3/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+9/(-a^2+1)/(-b^2*x^
2-2*a*b*x-a^2+1)^(1/2)*x*b^3*a^2+15/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
*x*b^3*a-5/2*a*b/(-a^2+1)^2/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+15/2*a^3*b^3/(
-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+13/2*a*b^3/(-a^2+1)^2/(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2)*x-5/2*a^4*b/(-a^2+1)^2/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2
)-15/2*a^3*b/(-a^2+1)^2/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-15/2*a^2*b/(-a^2+1
)^2/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+15/2*a^6*b^3/(-a^2+1)^3/(-b^2*x^2-2*a*
b*x-a^2+1)^(1/2)*x+45/2*a^5*b^3/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x
+45/2*a^4*b^3/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-3*b/(-a^2+1)/x/(-
b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^2-6*b/(-a^2+1)/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1
/2)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.515, size = 1184, normalized size = 5.86

$$\left[-\frac{3\left((2a+3)b^3x^3+\left(2a^2+a-3\right)b^2x^2\right)\sqrt{-a^2+1}\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx-2\sqrt{-b^2x^2-2abx-a^2+1}\left(abx+a^2-1\right)\sqrt{-a^2+1}-4a^2+2}{x^2}\right)+}{4\left(\left(a^5-3a^4+2a^3+2a^2-3a+1\right)bx^3+\left(a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(3*((2*a + 3)*b^3*x^3 + (2*a^2 + a - 3)*b^2*x^2)*sqrt(-a^2 + 1)*log((
(2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*x - 2*sqrt(-b^2*x^2 - 2*a*b*x -
 a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1) - 4*a^2 + 2)/x^2) + 2*(a^5 - (a^
3 + 14*a^2 - a - 14)*b^2*x^2 - a^4 - 2*a^3 - 5*(a^3 - a^2 - a + 1)*b*x + 2*
a^2 + a - 1)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^5 - 3*a^4 + 2*a^3 + 2*
a^2 - 3*a + 1)*b*x^3 + (a^6 - 4*a^5 + 5*a^4 - 5*a^2 + 4*a - 1)*x^2), -1/2*(
3*((2*a + 3)*b^3*x^3 + (2*a^2 + a - 3)*b^2*x^2)*sqrt(a^2 - 1)*arctan(sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2
*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + (a^5 - (a^3 + 14*a^2 - a - 14)
*b^2*x^2 - a^4 - 2*a^3 - 5*(a^3 - a^2 - a + 1)*b*x + 2*a^2 + a - 1)*sqrt(-b
^2*x^2 - 2*a*b*x - a^2 + 1))/((a^5 - 3*a^4 + 2*a^3 + 2*a^2 - 3*a + 1)*b*x^3
 + (a^6 - 4*a^5 + 5*a^4 - 5*a^2 + 4*a - 1)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx+1)^3}{x^3\left(-(a+bx-1)(a+bx+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**3/(1-(b*x+a)**2)**(3/2)/x**3,x)
```

```
[Out] Integral((a + b*x + 1)**3/(x**3*(-(a + b*x - 1)*(a + b*x + 1))**(3/2)), x)
```

Giac [B] time = 1.28377, size = 933, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -8*b^3/((a^3*abs(b) - 3*a^2*abs(b) + 3*a*abs(b) - abs(b))*((sqrt(-(b*x + a)
^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 1)) - 3*(2*a*b^3 + 3*b^3)*arctan(((sqrt
(-(b*x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/((a^3*ab
s(b) - 3*a^2*abs(b) + 3*a*abs(b) - abs(b))*sqrt(a^2 - 1)) + (2*(sqrt(-(b*x
+ a)^2 + 1)*abs(b) + b)^2*a^4*b^3/(b^2*x + a*b)^2 + 2*a^4*b^3 - 5*(sqrt(-(b
*x + a)^2 + 1)*abs(b) + b)*a^3*b^3/(b^2*x + a*b) + 6*(sqrt(-(b*x + a)^2 + 1
)*abs(b) + b)^2*a^3*b^3/(b^2*x + a*b)^2 - 3*(sqrt(-(b*x + a)^2 + 1)*abs(b)
+ b)^3*a^3*b^3/(b^2*x + a*b)^3 + 6*a^3*b^3 - 18*(sqrt(-(b*x + a)^2 + 1)*abs
(b) + b)*a^2*b^3/(b^2*x + a*b) + 3*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^
```

```
2*b^3/(b^2*x + a*b)^2 - 6*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a^2*b^3/(b^
2*x + a*b)^3 - a^2*b^3 + 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a*b^3/(b^2*x
 + a*b) + 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a*b^3/(b^2*x + a*b)^2 +
2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a*b^3/(b^2*x + a*b)^3 - 2*(sqrt(-(b
*x + a)^2 + 1)*abs(b) + b)^2*b^3/(b^2*x + a*b)^2)/((a^5*abs(b) - 3*a^4*abs(
b) + 3*a^3*abs(b) - a^2*abs(b))*((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a/(b
^2*x + a*b)^2 + a - 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b))^2)
```

3.842 $\int \frac{e^{3\tanh^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=260

$$\frac{(2a^2+51a+52)\,b^3\sqrt{a+bx+1}}{6(1-a)^4(a+1)\sqrt{-a-bx+1}}-\frac{(6a^2+18a+11)\,b^3\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)^4(a+1)\sqrt{1-a^2}}-\frac{(16a+19)b^2\sqrt{a+bx+1}}{6(1-a)^3(a+1)x\sqrt{-a-bx+1}}-\frac{7}{6(1-}$$

```
[Out] ((52 + 51*a + 2*a^2)*b^3*Sqrt[1 + a + b*x])/(6*(1 - a)^4*(1 + a)*Sqrt[1 - a
 - b*x]) - ((1 + a)*Sqrt[1 + a + b*x])/(3*(1 - a)*x^3*Sqrt[1 - a - b*x]) -
(7*b*Sqrt[1 + a + b*x])/(6*(1 - a)^2*x^2*Sqrt[1 - a - b*x]) - ((19 + 16*a)*
b^2*Sqrt[1 + a + b*x])/(6*(1 - a)^3*(1 + a)*x*Sqrt[1 - a - b*x]) - ((11 + 1
8*a + 6*a^2)*b^3*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[
1 - a - b*x])])/((1 - a)^4*(1 + a)*Sqrt[1 - a^2])
```

Rubi [A] time = 0.209521, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6163, 98, 151, 152, 12, 93, 208}

$$\frac{(2a^2+51a+52)\,b^3\sqrt{a+bx+1}}{6(1-a)^4(a+1)\sqrt{-a-bx+1}}-\frac{(6a^2+18a+11)\,b^3\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)^4(a+1)\sqrt{1-a^2}}-\frac{(16a+19)b^2\sqrt{a+bx+1}}{6(1-a)^3(a+1)x\sqrt{-a-bx+1}}-\frac{7}{6(1-}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a + b*x])/x^4,x]
```


```
[Out] ((52 + 51*a + 2*a^2)*b^3*Sqrt[1 + a + b*x])/(6*(1 - a)^4*(1 + a)*Sqrt[1 - a
 - b*x]) - ((1 + a)*Sqrt[1 + a + b*x])/(3*(1 - a)*x^3*Sqrt[1 - a - b*x]) -
(7*b*Sqrt[1 + a + b*x])/(6*(1 - a)^2*x^2*Sqrt[1 - a - b*x]) - ((19 + 16*a)*
b^2*Sqrt[1 + a + b*x])/(6*(1 - a)^3*(1 + a)*x*Sqrt[1 - a - b*x]) - ((11 + 1
8*a + 6*a^2)*b^3*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[
1 - a - b*x])])/((1 - a)^4*(1 + a)*Sqrt[1 - a^2])
```


Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(a+bx)}}{x^4}dx = \int \frac{(1+a+bx)^{3/2}}{x^4(1-a-bx)^{3/2}}dx$$

$$= -\frac{(1+a)\sqrt{1+a+bx}}{3(1-a)x^3\sqrt{1-a-bx}} - \frac{\int \frac{-7(1+a)b-6b^2x}{x^3(1-a-bx)^{3/2}\sqrt{1+a+bx}}dx}{3(1-a)}$$

$$= -\frac{(1+a)\sqrt{1+a+bx}}{3(1-a)x^3\sqrt{1-a-bx}} - \frac{7b\sqrt{1+a+bx}}{6(1-a)^2x^2\sqrt{1-a-bx}} + \frac{\int \frac{(1+a)(19+16a)b^2+14(1+a)b^3x}{x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}dx}{6(1-a)^2(1+a)}$$

$$= -\frac{(1+a)\sqrt{1+a+bx}}{3(1-a)x^3\sqrt{1-a-bx}} - \frac{7b\sqrt{1+a+bx}}{6(1-a)^2x^2\sqrt{1-a-bx}} - \frac{(19+16a)b^2\sqrt{1+a+bx}}{6(1-a)^3(1+a)x\sqrt{1-a-bx}} - \frac{\int \frac{-3(1+a)(11+18}{x(1-}}{6(}$$

$$= \frac{(52+51a+2a^2)b^3\sqrt{1+a+bx}}{6(1-a)^4(1+a)\sqrt{1-a-bx}} - \frac{(1+a)\sqrt{1+a+bx}}{3(1-a)x^3\sqrt{1-a-bx}} - \frac{7b\sqrt{1+a+bx}}{6(1-a)^2x^2\sqrt{1-a-bx}} - \frac{(19+16a)}{6(1-a)^3(1-}$$

$$= \frac{(52+51a+2a^2)b^3\sqrt{1+a+bx}}{6(1-a)^4(1+a)\sqrt{1-a-bx}} - \frac{(1+a)\sqrt{1+a+bx}}{3(1-a)x^3\sqrt{1-a-bx}} - \frac{7b\sqrt{1+a+bx}}{6(1-a)^2x^2\sqrt{1-a-bx}} - \frac{(19+16a)}{6(1-a)^3(1-}$$

$$= \frac{(52+51a+2a^2)b^3\sqrt{1+a+bx}}{6(1-a)^4(1+a)\sqrt{1-a-bx}} - \frac{(1+a)\sqrt{1+a+bx}}{3(1-a)x^3\sqrt{1-a-bx}} - \frac{7b\sqrt{1+a+bx}}{6(1-a)^2x^2\sqrt{1-a-bx}} - \frac{(19+16a)}{6(1-a)^3(1-}$$

$$= \frac{(52+51a+2a^2)b^3\sqrt{1+a+bx}}{6(1-a)^4(1+a)\sqrt{1-a-bx}} - \frac{(1+a)\sqrt{1+a+bx}}{3(1-a)x^3\sqrt{1-a-bx}} - \frac{7b\sqrt{1+a+bx}}{6(1-a)^2x^2\sqrt{1-a-bx}} - \frac{(19+16a)}{6(1-a)^3(1-}$$

Mathematica [A] time = 0.337401, size = 194, normalized size = 0.75

$$-\frac{-(6a^2+18a+11)b^2x^2\left(\sqrt{a-1}\sqrt{a+bx+1}(a^2+abx+5bx-1)+6\sqrt{a+1}bx\sqrt{-a-bx+1}\tan^{-1}\left(\frac{\sqrt{-a-bx+1}}{\sqrt{\frac{a-1}{a+1}}\sqrt{a+bx+1}}\right)\right)-2(}{6(a-1)^{5/2}(a^2-1)^2x^3\sqrt{-a-bx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a + b*x])/x^4,x]
```

```
[Out] -(-2*(-1 + a)^(7/2)*(1 + a)*(1 + a + b*x)^(5/2) + (-1 + a)^(5/2)*(3 + 4*a)*
b*x*(1 + a + b*x)^(5/2) - (11 + 18*a + 6*a^2)*b^2*x^2*(Sqrt[-1 + a]*Sqrt[1
+ a + b*x]*(-1 + a^2 + 5*b*x + a*b*x) + 6*Sqrt[1 + a]*b*x*Sqrt[1 - a - b*x]
*ArcTan[Sqrt[1 - a - b*x]/(Sqrt[(-1 + a)/(1 + a)]*Sqrt[1 + a + b*x])]))/(6*
(-1 + a)^(5/2)*(-1 + a^2)^2*x^3*Sqrt[1 - a - b*x])
```

Maple [B] time = 0.047, size = 2947, normalized size = 11.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^4,x)
```

```
[Out] -b^3/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x
-a^2+1)^(1/2))/x)-35/2*a^6*b^3/(-a^2+1)^(9/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+
1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-105/2*a^5*b^3/(-a^2+1)^(9/2)*ln
```

```
((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-105/
2*a^4*b^3/(-a^2+1)^(9/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2))/x)-1/3/(-a^2+1)/x^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^3-
1/(-a^2+1)/x^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^2-1/(-a^2+1)/x^3/(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2)*a+70*a^6*b^3/(-a^2+1)^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
+105/2*a^4*b^3/(-a^2+1)^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+35/2*a^8*b^3/(-a^2
+1)^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-4/3*b^2/(-a^2+1)^2/x/(-b^2*x^2-2*a*b*x
-a^2+1)^(1/2)+8/3*b^4/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+35/2*a^3*
b^3/(-a^2+1)^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+70*a^5*b^3/(-a^2+1)^4/(-b^2*x
^2-2*a*b*x-a^2+1)^(1/2)+15/2*a*b^3/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2
)+45*a^2*b^3/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+110*a^4*b^3/(-a^2+1)
^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/2*b/(-a^2+1)/x^2/(-b^2*x^2-2*a*b*x-a^2+
1)^(1/2)+260/3*a^3*b^3/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+125/3*a^6*
b^3/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+205/2*a^5*b^3/(-a^2+1)^3/(-b^
2*x^2-2*a*b*x-a^2+1)^(1/2)-45*a^2*b^3/(-a^2+1)^(7/2)*ln((-2*a^2+2-2*x*a*b+2
*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-9/2*b^3/(-a^2+1)^(5/2)*l
n((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+9/2
*b^3/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+b^3/(-a^2+1)/(-b^2*x^2-2*a*b
*x-a^2+1)^(1/2)-1/3/(-a^2+1)/x^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+41*b^3*a^2/
(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+187/6*b^3*a^4/(-a^2+1)^2/(-b^2*x^
2-2*a*b*x-a^2+1)^(1/2)+7*b^3/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^2-3*
b^2/(-a^2+1)/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+62/3*b^3*a/(-a^2+1)^2/(-b^2*x
^2-2*a*b*x-a^2+1)^(1/2)+56*b^3*a^3/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2
)+6*b^4/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+6*b^3/(-a^2+1)/(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2)*a-35/2*a^3*b^3/(-a^2+1)^(9/2)*ln((-2*a^2+2-2*x*a*b+2*(
-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-15/2*a*b^3/(-a^2+1)^(7/2)*
ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+10
5/2*a^7*b^3/(-a^2+1)^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-23/2*a*b^2/(-a^2+1)^2
/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/2*b/(-a^2+1)/x^2/(-b^2*x^2-2*a*b*x-a^2+
1)^(1/2)*a^2-30*a^4*b^3/(-a^2+1)^(7/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2
)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-27/2*a^2*b^3/(-a^2+1)^(5/2)*ln((-2*a^2
+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-135/2*a^3*b^
3/(-a^2+1)^(7/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2))/x)-18*a*b^3/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2
)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-3*b/(-a^2+1)/x^2/(-b^2*x^2-2*a*b*x-a^2
+1)^(1/2)*a-53/6*a^3*b^2/(-a^2+1)^2/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-19*a^2
*b^2/(-a^2+1)^2/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+125/3*a^5*b^4/(-a^2+1)^3/(
-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+205/2*a^4*b^4/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2)*x+55/2*a*b^4/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+80*a^
3*b^4/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+35/2*a^7*b^4/(-a^2+1)^4/(
-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+105/2*a^6*b^4/(-a^2+1)^4/(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2)*x-7/2*a^3*b/(-a^2+1)^2/x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-7/2*
a^2*b/(-a^2+1)^2/x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-35/6*a^5*b^2/(-a^2+1)^3
/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-35/2*a^4*b^2/(-a^2+1)^3/x/(-b^2*x^2-2*a*b
*x-a^2+1)^(1/2)-35/2*a^3*b^2/(-a^2+1)^3/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+11
5/6*a^2*b^4/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-7/6*a*b/(-a^2+1)^2/
x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-35/6*a^2*b^2/(-a^2+1)^3/x/(-b^2*x^2-2*a*
b*x-a^2+1)^(1/2)+35/2*a^4*b^4/(-a^2+1)^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+7
*b^4*a/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+56*b^4*a^2/(-a^2+1)^2/(-b^
2*x^2-2*a*b*x-a^2+1)^(1/2)*x-3*b^2*a/(-a^2+1)/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1
/2)+187/6*b^4*a^3/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+105/2*a^5*b^4
/(-a^2+1)^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-7/6*a^4*b/(-a^2+1)^2/x^2/(-b^2
*x^2-2*a*b*x-a^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.76915, size = 1611, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((6*a^2 + 18*a + 11)*b^4*x^4 + (6*a^3 + 12*a^2 - 7*a - 11)*b^3*x^
3)*sqrt(-a^2 + 1)*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*x + 2*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1) - 4*a^2 +
 2)/x^2) + 2*(2*a^7 + (2*a^4 + 51*a^3 + 50*a^2 - 51*a - 52)*b^3*x^3 - 2*a^6
 - 6*a^5 + (16*a^4 + 3*a^3 - 35*a^2 - 3*a + 19)*b^2*x^2 + 6*a^4 + 6*a^3 - 7
*(a^5 - a^4 - 2*a^3 + 2*a^2 + a - 1)*b*x - 6*a^2 - 2*a + 2)*sqrt(-b^2*x^2 -
 2*a*b*x - a^2 + 1))/((a^7 - 3*a^6 + a^5 + 5*a^4 - 5*a^3 - a^2 + 3*a - 1)*b
*x^4 + (a^8 - 4*a^7 + 4*a^6 + 4*a^5 - 10*a^4 + 4*a^3 + 4*a^2 - 4*a + 1)*x^3
), 1/6*(3*((6*a^2 + 18*a + 11)*b^4*x^4 + (6*a^3 + 12*a^2 - 7*a - 11)*b^3*x^
3)*sqrt(a^2 - 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1
)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) -
(2*a^7 + (2*a^4 + 51*a^3 + 50*a^2 - 51*a - 52)*b^3*x^3 - 2*a^6 - 6*a^5 + (1
6*a^4 + 3*a^3 - 35*a^2 - 3*a + 19)*b^2*x^2 + 6*a^4 + 6*a^3 - 7*(a^5 - a^4 -
 2*a^3 + 2*a^2 + a - 1)*b*x - 6*a^2 - 2*a + 2)*sqrt(-b^2*x^2 - 2*a*b*x - a^
2 + 1))/((a^7 - 3*a^6 + a^5 + 5*a^4 - 5*a^3 - a^2 + 3*a - 1)*b*x^4 + (a^8 -
 4*a^7 + 4*a^6 + 4*a^5 - 10*a^4 + 4*a^3 + 4*a^2 - 4*a + 1)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx+1)^3}{x^4\left(-(a+bx-1)(a+bx+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**3/(1-(b*x+a)**2)**(3/2)/x**4,x)
```

```
[Out] Integral((a + b*x + 1)**3/(x**4*(-(a + b*x - 1)*(a + b*x + 1))**(3/2)), x)
```

Giac [B] time = 1.44147, size = 2053, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 8*b^4/((a^4*abs(b) - 4*a^3*abs(b) + 6*a^2*abs(b) - 4*a*abs(b) + abs(b))*((s
qrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 1)) + (6*a^2*b^4 + 18*a*b
```

```
^4 + 11*b^4)*arctan(((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) -
1)/sqrt(a^2 - 1))/((a^5*abs(b) - 3*a^4*abs(b) + 2*a^3*abs(b) + 2*a^2*abs(b)
 - 3*a*abs(b) + abs(b))*sqrt(a^2 - 1)) - 1/3*(12*(sqrt(-(b*x + a)^2 + 1)*ab
s(b) + b)^2*a^7*b^4/(b^2*x + a*b)^2 + 6*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)
^4*a^7*b^4/(b^2*x + a*b)^4 + 6*a^7*b^4 - 24*(sqrt(-(b*x + a)^2 + 1)*abs(b)
+ b)*a^6*b^4/(b^2*x + a*b) + 72*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^6*b
^4/(b^2*x + a*b)^2 - 36*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a^6*b^4/(b^2*
x + a*b)^3 + 36*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^4*a^6*b^4/(b^2*x + a*b)
^4 - 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^5*a^6*b^4/(b^2*x + a*b)^5 + 36*
a^6*b^4 - 171*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a^5*b^4/(b^2*x + a*b) + 8
4*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^5*b^4/(b^2*x + a*b)^2 - 216*(sqrt
(-(b*x + a)^2 + 1)*abs(b) + b)^3*a^5*b^4/(b^2*x + a*b)^3 + 54*(sqrt(-(b*x +
 a)^2 + 1)*abs(b) + b)^4*a^5*b^4/(b^2*x + a*b)^4 - 45*(sqrt(-(b*x + a)^2 +
1)*abs(b) + b)^5*a^5*b^4/(b^2*x + a*b)^5 + 22*a^5*b^4 - 120*(sqrt(-(b*x + a
)^2 + 1)*abs(b) + b)*a^4*b^4/(b^2*x + a*b) + 252*(sqrt(-(b*x + a)^2 + 1)*ab
s(b) + b)^2*a^4*b^4/(b^2*x + a*b)^2 - 156*(sqrt(-(b*x + a)^2 + 1)*abs(b) +
b)^3*a^4*b^4/(b^2*x + a*b)^3 + 153*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^4*a^
4*b^4/(b^2*x + a*b)^4 - 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^5*a^4*b^4/(b
^2*x + a*b)^5 - 9*a^4*b^4 + 36*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a^3*b^4/
(b^2*x + a*b) + 192*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^3*b^4/(b^2*x +
a*b)^2 - 90*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a^3*b^4/(b^2*x + a*b)^3 +
 78*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^4*a^3*b^4/(b^2*x + a*b)^4 + 18*(sqr
t(-(b*x + a)^2 + 1)*abs(b) + b)^5*a^3*b^4/(b^2*x + a*b)^5 + 2*a^3*b^4 - 6*(
sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a^2*b^4/(b^2*x + a*b) - 54*(sqrt(-(b*x +
 a)^2 + 1)*abs(b) + b)^2*a^2*b^4/(b^2*x + a*b)^2 - 100*(sqrt(-(b*x + a)^2 +
 1)*abs(b) + b)^3*a^2*b^4/(b^2*x + a*b)^3 - 54*(sqrt(-(b*x + a)^2 + 1)*abs(
b) + b)^4*a^2*b^4/(b^2*x + a*b)^4 - 6*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^5
*a^2*b^4/(b^2*x + a*b)^5 + 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a*b^4/(
b^2*x + a*b)^2 + 36*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a*b^4/(b^2*x + a*
b)^3 + 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^4*a*b^4/(b^2*x + a*b)^4 - 8*(
sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*b^4/(b^2*x + a*b)^3)/((a^8*abs(b) - 3*
a^7*abs(b) + 2*a^6*abs(b) + 2*a^5*abs(b) - 3*a^4*abs(b) + a^3*abs(b))*((sqr
t(-(b*x + a)^2 + 1)*abs(b) + b)^2*a/(b^2*x + a*b)^2 + a - 2*(sqrt(-(b*x + a
)^2 + 1)*abs(b) + b)/(b^2*x + a*b))^3)
```

3.843 $\int e^{-\tanh^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=156

$$-\frac{(-a-bx+1)^{3/2}\sqrt{a+bx+1}\left(18a^2-2(6a+1)bx+10a+7\right)}{24b^4}-\frac{\left(8a^3+12a^2+12a+3\right)\sqrt{-a-bx+1}\sqrt{a+bx+1}}{8b^4}-\frac{(8a^3}{}$$

```
[Out] -((3 + 12*a + 12*a^2 + 8*a^3)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(8*b^4)
- (x^2*(1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x])/(4*b^2) - ((1 - a - b*x)^(3/2
)*Sqrt[1 + a + b*x]*(7 + 10*a + 18*a^2 - 2*(1 + 6*a)*b*x))/(24*b^4) - ((3 +
 12*a + 12*a^2 + 8*a^3)*ArcSin[a + b*x])/(8*b^4)
```

Rubi [A] time = 0.166045, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6163, 100, 147, 50, 53, 619, 216}

$$-\frac{(-a-bx+1)^{3/2}\sqrt{a+bx+1}\left(18a^2-2(6a+1)bx+10a+7\right)}{24b^4}-\frac{\left(8a^3+12a^2+12a+3\right)\sqrt{-a-bx+1}\sqrt{a+bx+1}}{8b^4}-\frac{(8a^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^ArcTanh[a + b*x],x]
```

```
[Out] -((3 + 12*a + 12*a^2 + 8*a^3)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(8*b^4)
- (x^2*(1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x])/(4*b^2) - ((1 - a - b*x)^(3/2
)*Sqrt[1 + a + b*x]*(7 + 10*a + 18*a^2 - 2*(1 + 6*a)*b*x))/(24*b^4) - ((3 +
 12*a + 12*a^2 + 8*a^3)*ArcSin[a + b*x])/(8*b^4)
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(a+bx)}x^3\,dx = \int \frac{x^3\sqrt{1-a-bx}}{\sqrt{1+a+bx}}\,dx$$

$$= -\frac{x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}{4b^2} - \frac{\int \frac{x\sqrt{1-a-bx}\left(-2\left(1-a^2\right)+(1+6a)bx\right)}{\sqrt{1+a+bx}}\,dx}{4b^2}$$

$$= -\frac{x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}{4b^2} - \frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}\left(7+10a+18a^2-2(1+6a)bx\right)}{24b^4} -$$

$$= -\frac{\left(3+12a+12a^2+8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} - \frac{x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}{4b^2} - \frac{(1-a-b}{}$$

$$= -\frac{\left(3+12a+12a^2+8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} - \frac{x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}{4b^2} - \frac{(1-a-b}{}$$

$$= -\frac{\left(3+12a+12a^2+8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} - \frac{x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}{4b^2} - \frac{(1-a-b}{}$$

$$= -\frac{\left(3+12a+12a^2+8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} - \frac{x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}{4b^2} - \frac{(1-a-b}{}$$

Mathematica [A] time = 0.438158, size = 160, normalized size = 1.03

$$\frac{\frac{\sqrt{a+bx+1}\left(5a^2(6bx-1)+6a^4+38a^3+a\left(-18b^2x^2+50bx-23\right)-6b^4x^4+14b^3x^3-17b^2x^2+25bx-16\right)}{\sqrt{-a-bx+1}}+\frac{6\left(8a^3+12a^2+12a+3\right)\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{-b}}}{24b^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^ArcTanh[a + b*x],x]
```

```
[Out] ((Sqrt[1 + a + b*x]*(-16 + 38*a^3 + 6*a^4 + 25*b*x - 17*b^2*x^2 + 14*b^3*x^
3 - 6*b^4*x^4 + 5*a^2*(-1 + 6*b*x) + a*(-23 + 50*b*x - 18*b^2*x^2)))/Sqrt[1
 - a - b*x] + (6*(3 + 12*a + 12*a^2 + 8*a^3)*Sqrt[b]*ArcSinh[(Sqrt[-b]*Sqrt
[1 - a - b*x])/(Sqrt[2]*Sqrt[b])])/Sqrt[-b])/(24*b^4)
```

Maple [B] time = 0.043, size = 809, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x)
```

```
[Out] 3/2*a/b^3*x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/2*a/b^3/(b^2)^(1/2)*arctan((b^
2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/b^4*(-(x+(1+a)/b)^2*b^2+
2*b*(x+(1+a)/b))^(1/2)*a^3-3/b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)
*a^2-3/b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a-1/b^3/(b^2)^(1/2)*a
rctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2
))+1/3/b^4*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-1/4/b^3*x*(-b^2*x^2-2*a*b*x-a^2+1
)^(3/2)+3/4/b^4*a*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-1/b^3/(b^2)^(1/2)*arctan((
b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))*a^3-
3/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*
b*(x+(1+a)/b))^(1/2))*a^2-3/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1
/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))*a+3/2*a^2/b^3*x*(-b^2*x^2-2
*a*b*x-a^2+1)^(1/2)+3/2*a^2/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^
2*x^2-2*a*b*x-a^2+1)^(1/2))+3/2*a^2/b^4*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/2*
a^3/b^4*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+5/8*a/b^4*(-b^2*x^2-2*a*b*x-a^2+1)^(
1/2)+5/8/b^3*x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+5/8/b^3/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/b^4*(-(x+(1+a)/b)^2*b^2
+2*b*(x+(1+a)/b))^(1/2)
```

Maxima [B] time = 1.47801, size = 456, normalized size = 2.92

$$\frac{3\sqrt{-b^2x^2-2\,abx-a^2+1}a^2x}{2\,b^3}-\frac{a^3\arcsin\left(bx+a\right)}{b^4}+\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}a^3}{2\,b^4}-\frac{\left(-b^2x^2-2\,abx-a^2+1\right)^{\frac{3}{2}}x}{4\,b^3}+\frac{3\sqrt{-b^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^2*x/b^3 - a^3*arcsin(b*x + a)/b^4
+ 1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^3/b^4 - 1/4*(-b^2*x^2 - 2*a*b*x
- a^2 + 1)^(3/2)*x/b^3 + 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a*x/b^3 - 3
/2*a^2*arcsin(b*x + a)/b^4 + 3/4*(-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)*a/b^4
 - 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^2/b^4 + 5/8*sqrt(-b^2*x^2 - 2*a
*b*x - a^2 + 1)*x/b^3 - 3/2*a*arcsin(b*x + a)/b^4 + 1/3*(-b^2*x^2 - 2*a*b*x
 - a^2 + 1)^(3/2)/b^4 - 19/8*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/b^4 - 3/8
*arcsin(b*x + a)/b^4 - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/b^4
```

Fricas [A] time = 2.03707, size = 338, normalized size = 2.17

$$\frac{3\left(8\,a^{3}+12\,a^{2}+12\,a+3\right)\arctan\left(\frac{\sqrt{-b^{2}x^{2}-2\,abx-a^{2}+1}(bx+a)}{b^{2}x^{2}+2\,abx+a^{2}-1}\right)+\left(6\,b^{3}x^{3}-2\,(3\,a+4)b^{2}x^{2}-6\,a^{3}+\left(6\,a^{2}+20\,a+9\right)bx-4}{24\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*a^3 + 12*a^2 + 12*a + 3)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (6*b^3*x^3 - 2*(3*a + 4)*b^2*
x^2 - 6*a^3 + (6*a^2 + 20*a + 9)*b*x - 44*a^2 - 39*a - 16)*sqrt(-b^2*x^2 -
2*a*b*x - a^2 + 1))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}\sqrt{-(a+bx-1)(a+bx+1)}}{a+bx+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a+1)*(1-(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-(a + b*x - 1)*(a + b*x + 1))/(a + b*x + 1), x)
```

Giac [A] time = 1.22074, size = 200, normalized size = 1.28

$$\frac{1}{24}\sqrt{-b^{2}x^{2}-2\,abx-a^{2}+1}\left(\left(2\,x\left(\frac{3\,x}{b}-\frac{3\,ab^{5}+4\,b^{5}}{b^{7}}\right)+\frac{6\,a^{2}b^{4}+20\,ab^{4}+9\,b^{4}}{b^{7}}\right)x-\frac{6\,a^{3}b^{3}+44\,a^{2}b^{3}+39\,ab^{3}+16\,b^{3}}{b^{7}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*((2*x*(3*x/b - (3*a*b^5 + 4*b^5)/b^
7) + (6*a^2*b^4 + 20*a*b^4 + 9*b^4)/b^7)*x - (6*a^3*b^3 + 44*a^2*b^3 + 39*a
*b^3 + 16*b^3)/b^7) + 1/8*(8*a^3 + 12*a^2 + 12*a + 3)*arcsin(-b*x - a)*sgn(
b)/(b^3*abs(b))
```

3.844 $\int e^{-\tanh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=130

$$\frac{(2a^2+2a+1)\sqrt{a+bx+1}\sqrt{-a-bx+1}}{2b^3}+\frac{(2a^2+2a+1)\sin^{-1}(a+bx)}{2b^3}-\frac{x\sqrt{a+bx+1}(-a-bx+1)^{3/2}}{3b^2}+\frac{(4a+1)\sqrt{a+}}{}$$

```
[Out] ((1 + 2*a + 2*a^2)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^3) + ((1 + 4*a
)*(1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x])/(6*b^3) - (x*(1 - a - b*x)^(3/2)*S
qrt[1 + a + b*x])/(3*b^2) + ((1 + 2*a + 2*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rubi [A] time = 0.145756, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6163, 90, 80, 50, 53, 619, 216}

$$\frac{(2a^2+2a+1)\sqrt{a+bx+1}\sqrt{-a-bx+1}}{2b^3}+\frac{(2a^2+2a+1)\sin^{-1}(a+bx)}{2b^3}-\frac{x\sqrt{a+bx+1}(-a-bx+1)^{3/2}}{3b^2}+\frac{(4a+1)\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^ArcTanh[a + b*x],x]
```



```
[Out] ((1 + 2*a + 2*a^2)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^3) + ((1 + 4*a
)*(1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x])/(6*b^3) - (x*(1 - a - b*x)^(3/2)*S
qrt[1 + a + b*x])/(3*b^2) + ((1 + 2*a + 2*a^2)*ArcSin[a + b*x])/(2*b^3)
```


Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(a+bx)}x^2\,dx = \int \frac{x^2\sqrt{1-a-bx}}{\sqrt{1+a+bx}}\,dx$$

$$= -\frac{x(1-a-bx)^{3/2}\sqrt{1+a+bx}}{3b^2} - \frac{\int \frac{\sqrt{1-a-bx}\left(-1+a^2+(1+4a)bx\right)}{\sqrt{1+a+bx}}\,dx}{3b^2}$$

$$= \frac{(1+4a)(1-a-bx)^{3/2}\sqrt{1+a+bx}}{6b^3} - \frac{x(1-a-bx)^{3/2}\sqrt{1+a+bx}}{3b^2} + \frac{\left(1+2a+2a^2\right)\int \frac{\sqrt{1-a-bx}}{\sqrt{1+a+bx}}$$

$$= \frac{\left(1+2a+2a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3} + \frac{(1+4a)(1-a-bx)^{3/2}\sqrt{1+a+bx}}{6b^3} - \frac{x(1-a-bx)^{3}}{3}$$

$$= \frac{\left(1+2a+2a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3} + \frac{(1+4a)(1-a-bx)^{3/2}\sqrt{1+a+bx}}{6b^3} - \frac{x(1-a-bx)^{3}}{3}$$

$$= \frac{\left(1+2a+2a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3} + \frac{(1+4a)(1-a-bx)^{3/2}\sqrt{1+a+bx}}{6b^3} - \frac{x(1-a-bx)^{3}}{3}$$

$$= \frac{\left(1+2a+2a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3} + \frac{(1+4a)(1-a-bx)^{3/2}\sqrt{1+a+bx}}{6b^3} - \frac{x(1-a-bx)^{3}}{3}$$

Mathematica [A] time = 0.156885, size = 126, normalized size = 0.97

$$\frac{\left(2a^2+2a+1\right)\sqrt{-b}\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{b}}\right)}{b^{7/2}} - \frac{\sqrt{a+bx+1}\left(2a^3+7a^2+a(8bx-5)+2b^3x^3-5b^2x^2+7bx-4\right)}{6b^3\sqrt{-a-bx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^ArcTanh[a + b*x],x]
```

```
[Out] -(Sqrt[1 + a + b*x]*(-4 + 7*a^2 + 2*a^3 + 7*b*x - 5*b^2*x^2 + 2*b^3*x^3 + a
*(-5 + 8*b*x)))/(6*b^3*Sqrt[1 - a - b*x]) + ((1 + 2*a + 2*a^2)*Sqrt[-b]*Arc
Sinh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[b])])/b^(7/2)
```

Maple [B] time = 0.04, size = 535, normalized size = 4.1

$$-\frac{1}{3b^3}\left(-b^2x^2-2xab-a^2+1\right)^{\frac{3}{2}}-\frac{ax}{b^2}\sqrt{-b^2x^2-2xab-a^2+1}-\frac{a^2}{b^3}\sqrt{-b^2x^2-2xab-a^2+1}-\frac{a}{b^2}\arctan\left(\sqrt{b^2}\left(x+\frac{a}{b}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x)
```

```
[Out] -1/3/b^3*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-a/b^2*x*(-b^2*x^2-2*a*b*x-a^2+1)^(1
/2)-a^2/b^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a/b^2/(b^2)^(1/2)*arctan((b^2)^(
1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/2*x/b^2*(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2)-1/2*a/b^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/2/b^2/(b^2)^(1/2)*arc
tan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/b^3*(-(x+(1+a)/b)
^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a^2+2/b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b)
)^(1/2)*a+1/b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)+1/b^2/(b^2)^(1/2
)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(
1/2))*a^2+2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b
)^2*b^2+2*b*(x+(1+a)/b))^(1/2))*a+1/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(
1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))
```

Maxima [A] time = 1.46141, size = 235, normalized size = 1.81

$$-\frac{\sqrt{-b^2x^2-2abx-a^2+1}ax}{b^2}+\frac{a^2\arcsin(bx+a)}{b^3}-\frac{\sqrt{-b^2x^2-2abx-a^2+1}x}{2b^2}+\frac{a\arcsin(bx+a)}{b^3}-\frac{\left(-b^2x^2-2abx-a^2\right.}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a*x/b^2 + a^2*arcsin(b*x + a)/b^3 - 1/2
*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x/b^2 + a*arcsin(b*x + a)/b^3 - 1/3*(-b
^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)/b^3 + 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
 1)*a/b^3 + 1/2*arcsin(b*x + a)/b^3 + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/b^
3
```

Fricas [A] time = 1.89475, size = 267, normalized size = 2.05

$$-\frac{3\left(2a^2+2a+1\right)\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}(bx+a)}{b^2x^2+2abx+a^2-1}\right)-\left(2b^2x^2-(2a+3)bx+2a^2+9a+4\right)\sqrt{-b^2x^2-2abx-a^2+1}}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(2*a^2 + 2*a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x +
a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (2*b^2*x^2 - (2*a + 3)*b*x + 2*a^2 + 9*
a + 4)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-\left(a+bx-1\right)\left(a+bx+1\right)}}{a+bx+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a+1)*(1-(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a + b*x - 1)*(a + b*x + 1))/(a + b*x + 1), x)
```

Giac [A] time = 1.19987, size = 143, normalized size = 1.1

$$\frac{1}{6}\sqrt{-b^2x^2-2\,abx-a^2+1}\left(x\left(\frac{2\,x}{b}-\frac{2\,ab^3+3\,b^3}{b^5}\right)+\frac{2\,a^2b^2+9\,ab^2+4\,b^2}{b^5}\right)-\frac{\left(2\,a^2+2\,a+1\right)\arcsin\left(-bx-a\right)\operatorname{sgn}\left(b\right)}{2\,b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(x*(2*x/b - (2*a*b^3 + 3*b^3)/b^5) +
 (2*a^2*b^2 + 9*a*b^2 + 4*b^2)/b^5) - 1/2*(2*a^2 + 2*a + 1)*arcsin(-b*x - a
)*sgn(b)/(b^2*abs(b))
```

3.845 $\int e^{-\tanh^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{a+bx+1}(-a-bx+1)^{3/2}}{2b^2}-\frac{(2a+1)\sqrt{a+bx+1}\sqrt{-a-bx+1}}{2b^2}-\frac{(2a+1)\sin^{-1}(a+bx)}{2b^2}$$

```
[Out] -((1 + 2*a)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^2) - ((1 - a - b*x)^(
3/2)*Sqrt[1 + a + b*x])/(2*b^2) - ((1 + 2*a)*ArcSin[a + b*x])/(2*b^2)
```

Rubi [A] time = 0.0631693, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6163, 80, 50, 53, 619, 216}

$$-\frac{\sqrt{a+bx+1}(-a-bx+1)^{3/2}}{2b^2}-\frac{(2a+1)\sqrt{a+bx+1}\sqrt{-a-bx+1}}{2b^2}-\frac{(2a+1)\sin^{-1}(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^ArcTanh[a + b*x],x]
```

```
[Out] -((1 + 2*a)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^2) - ((1 - a - b*x)^(
3/2)*Sqrt[1 + a + b*x])/(2*b^2) - ((1 + 2*a)*ArcSin[a + b*x])/(2*b^2)
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(a+bx)}x\,dx &= \int \frac{x\sqrt{1-a-bx}}{\sqrt{1+a+bx}}\,dx\\
&= -\frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2b^2}-\frac{(1+2a)\int \frac{\sqrt{1-a-bx}}{\sqrt{1+a+bx}}\,dx}{2b}\\
&= -\frac{(1+2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2}-\frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2b^2}-\frac{(1+2a)\int \frac{1}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{2b}\\
&= -\frac{(1+2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2}-\frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2b^2}-\frac{(1+2a)\int \frac{1}{\sqrt{(1-a)(1+a)-2abx-b^2x^2}}\,dx}{2b}\\
&= -\frac{(1+2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2}-\frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2b^2}+\frac{(1+2a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx\right.}{4b^3}\\
&= -\frac{(1+2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2}-\frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2b^2}-\frac{(1+2a)\sin^{-1}(a+bx)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.108946, size = 99, normalized size = 1.18

$$\frac{\sqrt{a+bx+1}\left(a^2+a-b^2x^2+3bx-2\right)}{2b^2\sqrt{-a-bx+1}}+\frac{(2a+1)\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{b}}\right)}{(-b)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^ArcTanh[a + b*x],x]
```

```
[Out] (Sqrt[1 + a + b*x]*(-2 + a + a^2 + 3*b*x - b^2*x^2))/(2*b^2*Sqrt[1 - a - b*
x]) + ((1 + 2*a)*Sqrt[b]*ArcSinh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt
[b])])/(-b)^(5/2)
```

Maple [B] time = 0.036, size = 302, normalized size = 3.6

$$\frac{x}{2\,b}\sqrt{-b^2x^2-2\,xab-a^2+1}+\frac{a}{2\,b^2}\sqrt{-b^2x^2-2\,xab-a^2+1}+\frac{1}{2\,b}\arctan\left(\sqrt{b^2}\left(x+\frac{a}{b}\right)\frac{1}{\sqrt{-b^2x^2-2\,xab-a^2+1}}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x)
```

```
[Out] 1/2*x/b*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/2*a/b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(
1/2)+1/2/b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^
```

```
(1/2))-1/b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)-1/b/(b^2)^(1/2)*arc
tan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))
-1/b^2*a*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)-1/b*a/(b^2)^(1/2)*arcta
n((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))
```

Maxima [A] time = 1.44329, size = 144, normalized size = 1.71

$$\frac{\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}x}{2\,b} - \frac{a\arcsin\left(bx + a\right)}{b^2} - \frac{\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}a}{2\,b^2} - \frac{\arcsin\left(bx + a\right)}{2\,b^2} - \frac{\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x/b - a*arcsin(b*x + a)/b^2 - 1/2*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/b^2 - 1/2*arcsin(b*x + a)/b^2 - sqrt(-b^
2*x^2 - 2*a*b*x - a^2 + 1)/b^2
```

Fricas [A] time = 1.89695, size = 209, normalized size = 2.49

$$\frac{\left(2\,a + 1\right)\arctan\left(\frac{\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}(bx + a)}{b^2x^2 + 2\,abx + a^2 - 1}\right) + \sqrt{-b^2x^2 - 2\,abx - a^2 + 1}(bx - a - 2)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((2*a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2
 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x - a - 2))/
b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-\left(a + bx - 1\right)\left(a + bx + 1\right)}}{a + bx + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)*(1-(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a + b*x - 1)*(a + b*x + 1))/(a + b*x + 1), x)
```

Giac [A] time = 1.19213, size = 92, normalized size = 1.1

$$\frac{1}{2}\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}\left(\frac{x}{b} - \frac{ab + 2\,b}{b^3}\right) + \frac{\left(2\,a + 1\right)\arcsin\left(-bx - a\right)\mathrm{sgn}\left(b\right)}{2\,b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(x/b - (a*b + 2*b)/b^3) + 1/2*(2*a +
 1)*arcsin(-b*x - a)*sgn(b)/(b*abs(b))
```

3.846 $\int e^{-\tanh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=38

$$\frac{\sqrt{-a-bx+1}\sqrt{a+bx+1}}{b}+\frac{\sin^{-1}(a+bx)}{b}$$

```
[Out] (Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/b + ArcSin[a + b*x]/b
```

Rubi [A] time = 0.0281589, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6161, 50, 53, 619, 216}

$$\frac{\sqrt{-a-bx+1}\sqrt{a+bx+1}}{b}+\frac{\sin^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-ArcTanh[a + b*x]),x]
```

```
[Out] (Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/b + ArcSin[a + b*x]/b
```

Rule 6161

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 + a*c
 + b*c*x)^(n/2)/(1 - a*c - b*c*x)^(n/2), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(a+bx)}\,dx &= \int \frac{\sqrt{1-a-bx}}{\sqrt{1+a+bx}}\,dx \\
&= \frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{b}+\int \frac{1}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx \\
&= \frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{b}+\int \frac{1}{\sqrt{(1-a)(1+a)-2abx-b^2x^2}}\,dx \\
&= \frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{b}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx,x,-2ab-2b^2x\right)}{2b^2} \\
&= \frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{b}+\frac{\sin^{-1}(a+bx)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0180475, size = 26, normalized size = 0.68

$$\frac{\sqrt{1-(a+bx)^2}+\sin^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-ArcTanh[a + b*x]),x]
```

```
[Out] (Sqrt[1 - (a + b*x)^2] + ArcSin[a + b*x])/b
```

Maple [B] time = 0.03, size = 95, normalized size = 2.5

$$\frac{1}{b}\sqrt{-\left(x+\frac{1+a}{b}\right)^2 b^2+2\,b\left(x+\frac{1+a}{b}\right)}+\arctan\left(\sqrt{b^2}\left(x+\frac{1+a}{b}-b^{-1}\right)\frac{1}{\sqrt{-\left(x+\frac{1+a}{b}\right)^2 b^2+2\,b\left(x+\frac{1+a}{b}\right)}}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x)
```

```
[Out] 1/b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)+1/(b^2)^(1/2)*arctan((b^2)^(
1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))
```

Maxima [A] time = 1.43452, size = 50, normalized size = 1.32

$$\frac{\arcsin(bx+a)}{b}+\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(b*x + a)/b + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/b
```

Fricas [B] time = 1.84697, size = 169, normalized size = 4.45

$$\frac{\sqrt{-b^2x^2 - 2\,abx - a^2 + 1} - \arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1) - arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2
+ 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(a + bx - 1)(a + bx + 1)}}{a + bx + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a + b*x - 1)*(a + b*x + 1))/(a + b*x + 1), x)
```

Giac [A] time = 1.29321, size = 59, normalized size = 1.55

$$-\frac{\arcsin(-bx - a)\,\mathrm{sgn}(b)}{|b|} + \frac{\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(-b*x - a)*sgn(b)/abs(b) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/b
```

3.847 $\int \frac{e^{-\tanh^{-1}(a+bx)}}{x} dx$

Optimal. Leaf size=68

$$-\frac{2(1-a)\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{\sqrt{1-a^2}}-\sin^{-1}(a+bx)$$

```
[Out] -ArcSin[a + b*x] - (2*(1 - a)*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt
[1 + a]*Sqrt[1 - a - b*x])])/Sqrt[1 - a^2]
```

Rubi [A] time = 0.0601839, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6163, 105, 53, 619, 216, 93, 208}

$$-\frac{2(1-a)\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{\sqrt{1-a^2}}-\sin^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a + b*x]*x),x]
```


```
[Out] -ArcSin[a + b*x] - (2*(1 - a)*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt
[1 + a]*Sqrt[1 - a - b*x])])/Sqrt[1 - a^2]
```


Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\tanh^{-1}(a+bx)}}{x}\,dx &= \int \frac{\sqrt{1-a-bx}}{x\sqrt{1+a+bx}}\,dx \\
&= -\left((-1+a)\int \frac{1}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx\right) - b\int \frac{1}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx \\
&= (2(1-a))\,\text{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx, x, \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\right) - b\int \frac{1}{\sqrt{(1-a)(1+a)-2abx-b^2x^2}} \\
&= -\frac{2(1-a)\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{\sqrt{1-a^2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2ab-2b^2x\right)}{2b} \\
&= -\sin^{-1}(a+bx) - \frac{2(1-a)\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{\sqrt{1-a^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0668553, size = 100, normalized size = 1.47

$$
\frac{2\sqrt{-b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{-b}}\right)}{\sqrt{b}} - 2\sqrt{\frac{a-1}{a+1}}\tan^{-1}\left(\frac{\sqrt{-a-bx+1}}{\sqrt{\frac{a-1}{a+1}}\sqrt{a+bx+1}}\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a + b*x]*x),x]
```

```
[Out] (2*Sqrt[-b]*ArcSinh[(Sqrt[b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[-b])])/Sqrt[b
] - 2*Sqrt[(-1 + a)/(1 + a)]*ArcTan[Sqrt[1 - a - b*x]/(Sqrt[(-1 + a)/(1 + a
)]*Sqrt[1 + a + b*x])]
```

Maple [B] time = 0.042, size = 249, normalized size = 3.7

$$
-\frac{1}{1+a}\sqrt{-\left(x+\frac{1+a}{b}\right)^2 b^2 + 2b\left(x+\frac{1+a}{b}\right)} - \frac{b}{1+a}\arctan\left(\sqrt{b^2}\left(x+\frac{1+a}{b}-b^{-1}\right)\frac{1}{\sqrt{-\left(x+\frac{1+a}{b}\right)^2 b^2 + 2b\left(x+\frac{1+a}{b}\right)}}\right) -
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x,x)
```

```
[Out] -1/(1+a)*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)-1/(1+a)*b/(b^2)^(1/2)*a
rctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2
))+1/(1+a)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/(1+a)*a*b/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/(1+a)*(-a^2+1)^(1/2)*ln
((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.91138, size = 718, normalized size = 10.56

$$\left[\frac{1}{2}\sqrt{-\frac{a-1}{a+1}}\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx-4a^2+2\sqrt{-b^2x^2-2abx-a^2+1}\left(a^3+\left(a^2+a\right)bx+a^2-a-1\right.}{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-(a - 1)/(a + 1))*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*
b*x - 4*a^2 + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^3 + (a^2 + a)*b*x + a
^2 - a - 1)*sqrt(-(a - 1)/(a + 1)) + 2)/x^2) + arctan(sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)), -sqrt((a - 1)/(a +
1))*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt((a - 1
)/(a + 1))/((a - 1)*b^2*x^2 + a^3 + 2*(a^2 - a)*b*x - a^2 - a + 1)) + arcta
n(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1
))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(a+bx-1)(a+bx+1)}}{x(a+bx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-(a + b*x - 1)*(a + b*x + 1))/(x*(a + b*x + 1)), x)
```

Giac [A] time = 1.30562, size = 120, normalized size = 1.76

$$\frac{b\arcsin\left(-bx-a\right)\operatorname{sgn}\left(b\right)}{|b|}-\frac{2\left(ab-b\right)\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] b*arcsin(-b*x - a)*sgn(b)/abs(b) - 2*(a*b - b)*arctan(((sqrt(-b^2*x^2 - 2*a
*b*x - a^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/(sqrt(a^2 -
 1)*abs(b))
```

3.848 $\int \frac{e^{-\tanh^{-1}(a+bx)}}{x^2} \, dx$

Optimal. Leaf size=94

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(a+1)\sqrt{1-a^2}}-\frac{\sqrt{-a-bx+1}\sqrt{a+bx+1}}{(a+1)x}$$

```
[Out] -((Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/((1 + a)*x)) + (2*b*ArcTanh[(Sqrt[1
 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 + a)*Sqrt[1
- a^2])
```

Rubi [A] time = 0.0567481, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6163, 94, 93, 208}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(a+1)\sqrt{1-a^2}}-\frac{\sqrt{-a-bx+1}\sqrt{a+bx+1}}{(a+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a + b*x]*x^2),x]
```

```
[Out] -((Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/((1 + a)*x)) + (2*b*ArcTanh[(Sqrt[1
 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 + a)*Sqrt[1
- a^2])
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tanh^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{\sqrt{1-a-bx}}{x^2\sqrt{1+a+bx}}\,dx\\ &= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{(1+a)x}-\frac{b\int \frac{1}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{1+a}\\ &= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{(1+a)x}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx,x,\frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\right)}{1+a}\\ &= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{(1+a)x}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{(1+a)\sqrt{1-a^2}}\end{aligned}$$

Mathematica [A] time = 0.0619592, size = 89, normalized size = 0.95

$$-\frac{\sqrt{-a^2-2abx-b^2x^2+1}}{ax+x}-\frac{2b\tan^{-1}\left(\frac{\sqrt{-a-bx+1}}{\sqrt{\frac{a-1}{a+1}}\sqrt{a+bx+1}}\right)}{\sqrt{a-1}(a+1)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a + b*x]*x^2),x]
```

```
[Out] -(Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]/(x + a*x)) - (2*b*ArcTan[Sqrt[1 - a - b
*x]/(Sqrt[(-1 + a)/(1 + a)]*Sqrt[1 + a + b*x])])/(Sqrt[-1 + a]*(1 + a)^(3/2
))
```

Maple [B] time = 0.083, size = 565, normalized size = 6.

$$-\frac{1}{(1+a)\left(-a^2+1\right)x}\left(-b^2x^2-2\,xab-a^2+1\right)^{\frac{3}{2}}-2\,\frac{ab\sqrt{-b^2x^2-2\,xab-a^2+1}}{(1+a)\left(-a^2+1\right)}+\frac{a^2b^2}{(1+a)\left(-a^2+1\right)}\arctan\left(\sqrt{b^2}\left(x+\frac{a}{b}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^2,x)
```

```
[Out] -1/(1+a)/(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-2/(1+a)*a*b/(-a^2+1)*(-b
^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/(1+a)*a^2*b^2/(-a^2+1)/(b^2)^(1/2)*arctan((b^
2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/(1+a)*a*b/(-a^2+1)^(1/2)
*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1
/(1+a)*b^2/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-1/(1+a)*b^2/(-a^2+1)/(
b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/(1+
a)^2*b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)+1/(1+a)^2*b^2/(b^2)^(1/2)
*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1
/2))-1/(1+a)^2*b*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/(1+a)^2*b^2*a/(b^2)^(1/2)
*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/(1+a)^2*b*(-a
^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^
(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(bx+a)^2+1}}{(bx+a+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(b*x + a)^2 + 1)/((b*x + a + 1)*x^2), x)
```

Fricas [A] time = 1.70272, size = 633, normalized size = 6.73

$$\left[-\frac{\sqrt{-a^2+1}bx\log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx-2\sqrt{-b^2x^2-2abx-a^2+1}(abx+a^2-1)\sqrt{-a^2+1}-4a^2+2}{x^2}\right)+2\sqrt{-b^2x^2-2abx-a^2+1}(a^2-}{2(a^3+a^2-a-1)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + 1)*b*x*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*
x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1) -
 4*a^2 + 2)/x^2) + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1))/((a^3 +
a^2 - a - 1)*x), -(sqrt(a^2 - 1)*b*x*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*
b*x - 2*a^2 + 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1))/((a^3 + a
^2 - a - 1)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(a+bx-1)(a+bx+1)}}{x^2(a+bx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(a + b*x - 1)*(a + b*x + 1))/(x**2*(a + b*x + 1)), x)
```

Giac [B] time = 1.27779, size = 301, normalized size = 3.2

$$-\frac{2b^2\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2abx-a^2+1}|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}(a|b|+|b|)}-\frac{2\left(ab^2-\frac{\left(\sqrt{-b^2x^2-2abx-a^2+1}|b|+b\right)b^2}{b^2x+ab}\right)}{\left(a^2|b|+a|b|\right)\left(\frac{\left(\sqrt{-b^2x^2-2abx-a^2+1}|b|+b\right)^2a}{\left(b^2x+ab\right)^2}+a-\frac{2\left(\sqrt{-b^2x^2-2abx-a^2+1}|b|+b\right)}{b^2x+ab}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -2*b^2*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(b^2*x + a
*b) - 1)/sqrt(a^2 - 1))/(sqrt(a^2 - 1)*(a*abs(b) + abs(b))) - 2*(a*b^2 - (s
qrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*b^2/(b^2*x + a*b))/((a^2*abs(
b) + a*abs(b))*((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a/(b^2*x
+ a*b)^2 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a
*b)))
```

3.849 $\int \frac{e^{-\tanh^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=162

$$-\frac{(1-2a)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)(a+1)^2\sqrt{1-a^2}}-\frac{(-a-bx+1)^{3/2}\sqrt{a+bx+1}}{2\left(1-a^2\right)x^2}+\frac{(1-2a)b\sqrt{-a-bx+1}\sqrt{a+bx+1}}{2(1-a)(a+1)^2x}$$

```
[Out] ((1 - 2*a)*b*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*(1 - a)*(1 + a)^2*x) -
 ((1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x])/(2*(1 - a^2)*x^2) - ((1 - 2*a)*b^2
*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/
((1 - a)*(1 + a)^2*Sqrt[1 - a^2])
```

Rubi [A] time = 0.0979128, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6163, 96, 94, 93, 208}

$$-\frac{(1-2a)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)(a+1)^2\sqrt{1-a^2}}-\frac{(-a-bx+1)^{3/2}\sqrt{a+bx+1}}{2\left(1-a^2\right)x^2}+\frac{(1-2a)b\sqrt{-a-bx+1}\sqrt{a+bx+1}}{2(1-a)(a+1)^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a + b*x]*x^3),x]
```

```
[Out] ((1 - 2*a)*b*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*(1 - a)*(1 + a)^2*x) -
 ((1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x])/(2*(1 - a^2)*x^2) - ((1 - 2*a)*b^2
*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/
((1 - a)*(1 + a)^2*Sqrt[1 - a^2])
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(a+bx)}}{x^3}\,dx = \int \frac{\sqrt{1-a-bx}}{x^3\sqrt{1+a+bx}}\,dx$$

$$= -\frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2\left(1-a^2\right)x^2} - \frac{((1-2a)b)\int \frac{\sqrt{1-a-bx}}{x^2\sqrt{1+a+bx}}\,dx}{2\left(1-a^2\right)}$$

$$= \frac{(1-2a)b\sqrt{1-a-bx}\sqrt{1+a+bx}}{2(1-a)(1+a)^2x} - \frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2\left(1-a^2\right)x^2} + \frac{\left((1-2a)b^2\right)\int \frac{1}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,d}{2(1-a)(1+a)^2}$$

$$= \frac{(1-2a)b\sqrt{1-a-bx}\sqrt{1+a+bx}}{2(1-a)(1+a)^2x} - \frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2\left(1-a^2\right)x^2} + \frac{\left((1-2a)b^2\right)\text{Subst}\left(\int \frac{1}{-1-a-(-1+a}\right.}{(1-a)(1+a)^2}$$

$$= \frac{(1-2a)b\sqrt{1-a-bx}\sqrt{1+a+bx}}{2(1-a)(1+a)^2x} - \frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2\left(1-a^2\right)x^2} - \frac{(1-2a)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{(1-a)(1+a)^2\sqrt{1-a^2}}$$

Mathematica [A] time = 0.124291, size = 117, normalized size = 0.72

$$\frac{(2a-1)b^2\tan^{-1}\left(\frac{\sqrt{-a-bx+1}}{\sqrt{\frac{a-1}{a+1}}\sqrt{a+bx+1}}\right)}{(a-1)^{3/2}(a+1)^{5/2}} - \frac{\left(a^2-abx+2bx-1\right)\sqrt{-a^2-2abx-b^2x^2+1}}{2(a-1)(a+1)^2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a + b*x]*x^3),x]
```

```
[Out] -((-1 + a^2 + 2*b*x - a*b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/(2*(-1 + a)
*(1 + a)^2*x^2) + ((-1 + 2*a)*b^2*ArcTan[Sqrt[1 - a - b*x]/(Sqrt[(-1 + a)/(
1 + a)]*Sqrt[1 + a + b*x])])/((-1 + a)^(3/2)*(1 + a)^(5/2))
```

Maple [B] time = 0.049, size = 1116, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^3,x)
```

```
[Out] 1/(1+a)^2*b/(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+2/(1+a)^2*b^2*a/(-a^2
+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/(1+a)^2*b^3*a^2/(-a^2+1)/(b^2)^(1/2)*a
rctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/(1+a)^2*b^2*a/(
-a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1
)^(1/2))/x)+1/(1+a)^2*b^3/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+1/(1+a)
^2*b^3/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2))-1/(1+a)^3*b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)-1/(1+a
)^3*b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+
2*b*(x+(1+a)/b))^(1/2))+1/(1+a)^3*b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/(1+a
)^3*b^3*a/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(
1/2))-1/(1+a)^3*b^2*(-a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-
b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/2/(1+a)/(-a^2+1)/x^2*(-b^2*x^2-2*a*b*x-a
^2+1)^(3/2)-1/2/(1+a)*a*b/(-a^2+1)^2/x*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-1/(1+
a)*a^2*b^2/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/2/(1+a)*a^3*b^3/(-a^
2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2
))+1/2/(1+a)*a^2*b^2/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(
-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/2/(1+a)*a*b^3/(-a^2+1)^2*(-b^2*x^2-2*a*
b*x-a^2+1)^(1/2)*x-1/2/(1+a)*a*b^3/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2
)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/2/(1+a)*b^2/(-a^2+1)*(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2)+1/2/(1+a)*b^3/(-a^2+1)*a/(b^2)^(1/2)*arctan((b^2)^(1/2
)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/(1+a)*b^2/(-a^2+1)^(1/2)*ln((
-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(bx+a)^2+1}}{(bx+a+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(b*x + a)^2 + 1)/((b*x + a + 1)*x^3), x)
```

Fricas [A] time = 1.7765, size = 821, normalized size = 5.07

$$\left[-\frac{\sqrt{-a^2+1}(2a-1)b^2x^2\log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx+2\sqrt{-b^2x^2-2abx-a^2+1}(abx+a^2-1)\sqrt{-a^2+1}-4a^2+2}{x^2}\right)+2\left(a^4-\left(a^3-2a^2-\right.\right.}{4\left(a^5+a^4-2a^3-2a^2+a+1\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a^2 + 1)*(2*a - 1)*b^2*x^2*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 +
4*(a^3 - a)*b*x + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sq
rt(-a^2 + 1) - 4*a^2 + 2)/x^2) + 2*(a^4 - (a^3 - 2*a^2 - a + 2)*b*x - 2*a^2
 + 1)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^5 + a^4 - 2*a^3 - 2*a^2 + a +
 1)*x^2), 1/2*(sqrt(a^2 - 1)*(2*a - 1)*b^2*x^2*arctan(sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*
(a^3 - a)*b*x - 2*a^2 + 1)) - (a^4 - (a^3 - 2*a^2 - a + 2)*b*x - 2*a^2 + 1)
*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^5 + a^4 - 2*a^3 - 2*a^2 + a + 1)*x
^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(a+bx-1)(a+bx+1)}}{x^3(a+bx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-(a + b*x - 1)*(a + b*x + 1))/(x**3*(a + b*x + 1)), x)
```

Giac [B] time = 1.27263, size = 1013, normalized size = 6.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (2*a*b^3 - b^3)*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(
b^2*x + a*b) - 1)/sqrt(a^2 - 1))/((a^3*abs(b) + a^2*abs(b) - a*abs(b) - abs
(b))*sqrt(a^2 - 1)) + (2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*
a^4*b^3/(b^2*x + a*b)^2 + 2*a^4*b^3 - 5*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
*abs(b) + b)*a^3*b^3/(b^2*x + a*b) - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*
abs(b) + b)^2*a^3*b^3/(b^2*x + a*b)^2 - 3*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
1)*abs(b) + b)^3*a^3*b^3/(b^2*x + a*b)^3 - 2*a^3*b^3 + 6*(sqrt(-b^2*x^2 - 2
*a*b*x - a^2 + 1)*abs(b) + b)*a^2*b^3/(b^2*x + a*b) + 3*(sqrt(-b^2*x^2 - 2*
a*b*x - a^2 + 1)*abs(b) + b)^2*a^2*b^3/(b^2*x + a*b)^2 + 2*(sqrt(-b^2*x^2 -
 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a^2*b^3/(b^2*x + a*b)^3 - a^2*b^3 + 2*(sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a*b^3/(b^2*x + a*b) - 4*(sqrt(
-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a*b^3/(b^2*x + a*b)^2 + 2*(sqrt
(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a*b^3/(b^2*x + a*b)^3 - 2*(sqr
t(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*b^3/(b^2*x + a*b)^2)/((a^5*ab
s(b) + a^4*abs(b) - a^3*abs(b) - a^2*abs(b))*((sqrt(-b^2*x^2 - 2*a*b*x - a^
2 + 1)*abs(b) + b)^2*a/(b^2*x + a*b)^2 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1)*abs(b) + b)/(b^2*x + a*b))^2)
```

3.850 $\int \frac{e^{-\tanh^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=210

$$\frac{(2a^2-2a+1)\,b^3\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(a+1)\left(1-a^2\right)^{5/2}}-\frac{(1-2a)(4-a)b^2\sqrt{-a-bx+1}\sqrt{a+bx+1}}{6(1-a)^2(a+1)^3x}+\frac{(3-2a)b\sqrt{-a-bx+1}\sqrt{a+}}{6(1-a)(a+1)^2x^2}$$

```
[Out] -(Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(3*(1 + a)*x^3) + ((3 - 2*a)*b*Sqrt[
1 - a - b*x]*Sqrt[1 + a + b*x])/(6*(1 - a)*(1 + a)^2*x^2) - ((1 - 2*a)*(4 -
 a)*b^2*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(6*(1 - a)^2*(1 + a)^3*x) + ((
1 - 2*a + 2*a^2)*b^3*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*S
qrt[1 - a - b*x])])/((1 + a)*(1 - a^2)^(5/2))
```

Rubi [A] time = 0.167932, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6163, 99, 151, 12, 93, 208}

$$\frac{(2a^2-2a+1)\,b^3\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(a+1)\left(1-a^2\right)^{5/2}}-\frac{(1-2a)(4-a)b^2\sqrt{-a-bx+1}\sqrt{a+bx+1}}{6(1-a)^2(a+1)^3x}+\frac{(3-2a)b\sqrt{-a-bx+1}\sqrt{a+}}{6(1-a)(a+1)^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a + b*x]*x^4),x]
```

```
[Out] -(Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(3*(1 + a)*x^3) + ((3 - 2*a)*b*Sqrt[
1 - a - b*x]*Sqrt[1 + a + b*x])/(6*(1 - a)*(1 + a)^2*x^2) - ((1 - 2*a)*(4 -
 a)*b^2*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(6*(1 - a)^2*(1 + a)^3*x) + ((
1 - 2*a + 2*a^2)*b^3*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*S
qrt[1 - a - b*x])])/((1 + a)*(1 - a^2)^(5/2))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(a+bx)}}{x^4}\,dx = \int \frac{\sqrt{1-a-bx}}{x^4\sqrt{1+a+bx}}\,dx$$

$$= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{3(1+a)x^3} + \frac{\int \frac{-(3-2a)b+2b^2x}{x^3\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{3(1+a)}$$

$$= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{3(1+a)x^3} + \frac{(3-2a)b\sqrt{1-a-bx}\sqrt{1+a+bx}}{6(1-a)(1+a)^2x^2} - \frac{\int \frac{-(1-2a)(4-a)b^2+(3-2a)b^3x}{x^2\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{6(1-a)(1+a)^2}$$

$$= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{3(1+a)x^3} + \frac{(3-2a)b\sqrt{1-a-bx}\sqrt{1+a+bx}}{6(1-a)(1+a)^2x^2} - \frac{(1-2a)(4-a)b^2\sqrt{1-a-bx}\sqrt{1+}}{6(1-a)^2(1+a)^3x}$$

$$= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{3(1+a)x^3} + \frac{(3-2a)b\sqrt{1-a-bx}\sqrt{1+a+bx}}{6(1-a)(1+a)^2x^2} - \frac{(1-2a)(4-a)b^2\sqrt{1-a-bx}\sqrt{1+}}{6(1-a)^2(1+a)^3x}$$

$$= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{3(1+a)x^3} + \frac{(3-2a)b\sqrt{1-a-bx}\sqrt{1+a+bx}}{6(1-a)(1+a)^2x^2} - \frac{(1-2a)(4-a)b^2\sqrt{1-a-bx}\sqrt{1+}}{6(1-a)^2(1+a)^3x}$$

$$= -\frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{3(1+a)x^3} + \frac{(3-2a)b\sqrt{1-a-bx}\sqrt{1+a+bx}}{6(1-a)(1+a)^2x^2} - \frac{(1-2a)(4-a)b^2\sqrt{1-a-bx}\sqrt{1+}}{6(1-a)^2(1+a)^3x}$$

Mathematica [A] time = 0.273431, size = 187, normalized size = 0.89

$$\frac{-\frac{3(2a^2-2a+1)b^2x^2\left(\sqrt{a-1}\sqrt{a+1}\sqrt{-(a+bx-1)(a+bx+1)}+2bx\tan^{-1}\left(\frac{\sqrt{-a-bx+1}}{\sqrt{\frac{a-1}{a+1}}\sqrt{a+bx+1}}\right)\right)}{\sqrt{a-1}(a+1)^{3/2}} + (1-4a)bx(-a-bx+1)^{3/2}\sqrt{a+bx+1} - 2(1-a)(a+1}{6\left(a^2-1\right)^2x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a + b*x]*x^4),x]
```

```
[Out] (-2*(1 - a)*(1 + a)*(1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x] + (1 - 4*a)*b*x*(
1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x] - (3*(1 - 2*a + 2*a^2)*b^2*x^2*(Sqrt[-
1 + a]*Sqrt[1 + a]*Sqrt[-((-1 + a + b*x)*(1 + a + b*x))] + 2*b*x*ArcTan[Sqr
t[1 - a - b*x]/(Sqrt[(-1 + a)/(1 + a)]*Sqrt[1 + a + b*x])]))/(Sqrt[-1 + a]*
(1 + a)^(3/2)))/(6*(-1 + a^2)^2*x^3)
```

Maple [B] time = 0.05, size = 1711, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^4,x)
```

```
[Out] -1/(1+a)^4*b^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/(1+a)^4*b^3*(-(x+(1+a)/b)^2
*b^2+2*b*(x+(1+a)/b))^(1/2)-1/2/(1+a)*a*b/(-a^2+1)^2/x^2*(-b^2*x^2-2*a*b*x-
a^2+1)^(3/2)-1/2/(1+a)*a^2*b^2/(-a^2+1)^3/x*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+
1/2/(1+a)*a^4*b^4/(-a^2+1)^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x
^2-2*a*b*x-a^2+1)^(1/2))-1/2/(1+a)*a^2*b^4/(-a^2+1)^3*(-b^2*x^2-2*a*b*x-a^2
+1)^(1/2)*x-1/2/(1+a)*a^2*b^4/(-a^2+1)^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+
a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/(1+a)*a^2*b^4/(-a^2+1)^2/(b^2)^(1/
2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/(1+a)^2*b
^4*a/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2))-1/2/(1+a)^2*b^4/(-a^2+1)*a/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/
b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/(1+a)^2*b^3/(-a^2+1)*(-b^2*x^2-2*a*b
*x-a^2+1)^(1/2)-1/2/(1+a)^2*b^3/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2
+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/3/(1+a)/(-a^2+1)/x^3*(-b^2*x
^2-2*a*b*x-a^2+1)^(3/2)+1/(1+a)^4*b^3*(-a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2
*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+1/(1+a)^4*b^4/(b^2)^(1/2
)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(
1/2))+1/(1+a)^3*b^4*a^2/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b
^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/(1+a)^2*b^2*a/(-a^2+1)^2/x*(-b^2*x^2-2*a*b
*x-a^2+1)^(3/2)-1/2/(1+a)^2*b^4*a^3/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/
2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/(1+a)^2*b^4*a/(-a^2+1)^2*(-b
^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-1/(1+a)^3*b^4/(-a^2+1)/(b^2)^(1/2)*arctan((b^
2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/(1+a)^4*b^4*a/(b^2)^(1/2
)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/(1+a)^3*b^2/
(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-1/(1+a)*a^3*b^3/(-a^2+1)^3*(-b^2*
x^2-2*a*b*x-a^2+1)^(1/2)+1/2/(1+a)*a^3*b^3/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*x*
a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/2/(1+a)*a*b^3/(-a
^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/2/(1+a)*a*b^3/(-a^2+1)^(3/2)*ln((-
2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/2/(1+
a)^2*b^3*a^2/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2
-2*a*b*x-a^2+1)^(1/2))/x)+1/2/(1+a)^2*b/(-a^2+1)/x^2*(-b^2*x^2-2*a*b*x-a^2+
1)^(3/2)+1/(1+a)^2*b^3*a^2/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-2/(1+a
)^3*b^3*a/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/(1+a)^3*b^3*a/(-a^2+1)^
(1/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))
/x)-1/(1+a)^3*b^4/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(bx+a)^2+1}}{(bx+a+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(b*x + a)^2 + 1)/((b*x + a + 1)*x^4), x)
```

Fricas [A] time = 1.86134, size = 1094, normalized size = 5.21

$$\left[-\frac{3\left(2a^2-2a+1\right)\sqrt{-a^2+1}b^3x^3\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx-2\sqrt{-b^2x^2-2abx-a^2+1}\left(abx+a^2-1\right)\sqrt{-a^2+1}-4a^2+2}{x^2}\right)+2\left(2a^6+\left(2a^4\right.\right.}{12\left(a^7+a^6-3a^5-3a^4+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*a^2 - 2*a + 1)*sqrt(-a^2 + 1)*b^3*x^3*log(((2*a^2 - 1)*b^2*x^2
 + 2*a^4 + 4*(a^3 - a)*b*x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x +
a^2 - 1)*sqrt(-a^2 + 1) - 4*a^2 + 2)/x^2) + 2*(2*a^6 + (2*a^4 - 9*a^3 + 2*a
^2 + 9*a - 4)*b^2*x^2 - 6*a^4 - (2*a^5 - 3*a^4 - 4*a^3 + 6*a^2 + 2*a - 3)*b
*x + 6*a^2 - 2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^7 + a^6 - 3*a^5 - 3
*a^4 + 3*a^3 + 3*a^2 - a - 1)*x^3), -1/6*(3*(2*a^2 - 2*a + 1)*sqrt(a^2 - 1)
*b^3*x^3*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a
^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + (2*a^6 +
 (2*a^4 - 9*a^3 + 2*a^2 + 9*a - 4)*b^2*x^2 - 6*a^4 - (2*a^5 - 3*a^4 - 4*a^3
 + 6*a^2 + 2*a - 3)*b*x + 6*a^2 - 2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((
a^7 + a^6 - 3*a^5 - 3*a^4 + 3*a^3 + 3*a^2 - a - 1)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(a+bx-1)(a+bx+1)}}{x^4(a+bx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-(a + b*x - 1)*(a + b*x + 1))/(x**4*(a + b*x + 1)), x)
```

Giac [B] time = 1.36367, size = 2240, normalized size = 10.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -(2*a^2*b^4 - 2*a*b^4 + b^4)*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*ab
s(b) + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/((a^5*abs(b) + a^4*abs(b) - 2
*a^3*abs(b) - 2*a^2*abs(b) + a*abs(b) + abs(b))*sqrt(a^2 - 1)) - 1/3*(12*(s
qrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^7*b^4/(b^2*x + a*b)^2 + 6
*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a^7*b^4/(b^2*x + a*b)^4
```

+ 6*a^7*b^4 - 24*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a^6*b^4/(b^2*x + a*b) - 24*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^6*b^4/(b^2*x + a*b)^2 - 36*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a^6*b^4/(b^2*x + a*b)^3 - 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a^6*b^4/(b^2*x + a*b)^4 - 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^5*a^6*b^4/(b^2*x + a*b)^5 - 12*a^6*b^4 + 57*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a^5*b^4/(b^2*x + a*b) + 36*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^5*b^4/(b^2*x + a*b)^2 + 72*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a^5*b^4/(b^2*x + a*b)^3 + 30*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a^5*b^4/(b^2*x + a*b)^4 + 15*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^5*a^5*b^4/(b^2*x + a*b)^5 - 2*a^5*b^4 - 84*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^4*b^4/(b^2*x + a*b)^2 - 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a^4*b^4/(b^2*x + a*b)^3 - 51*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a^4*b^4/(b^2*x + a*b)^4 + 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^5*a^4*b^4/(b^2*x + a*b)^5 + 3*a^4*b^4 - 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a^3*b^4/(b^2*x + a*b) + 30*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a^3*b^4/(b^2*x + a*b)^3 - 18*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a^3*b^4/(b^2*x + a*b)^4 - 6*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^5*a^3*b^4/(b^2*x + a*b)^5 + 2*a^3*b^4 - 6*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a^2*b^4/(b^2*x + a*b) + 18*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^2*b^4/(b^2*x + a*b)^2 - 4*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a^2*b^4/(b^2*x + a*b)^3 + 18*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a^2*b^4/(b^2*x + a*b)^4 - 6*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^5*a^2*b^4/(b^2*x + a*b)^5 + 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a*b^4/(b^2*x + a*b)^2 - 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a*b^4/(b^2*x + a*b)^3 + 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a*b^4/(b^2*x + a*b)^4 - 8*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*b^4/(b^2*x + a*b)^3)/((a^8*abs(b) + a^7*abs(b) - 2*a^6*abs(b) - 2*a^5*abs(b) + a^4*abs(b) + a^3*abs(b))*((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a/(b^2*x + a*b)^2 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a*b))^3)

3.851 $\int e^{-2\tanh^{-1}(a+bx)}x^4\,dx$

Optimal. Leaf size=71

$$-\frac{2(a+1)x^3}{3b^2}+\frac{(a+1)^2x^2}{b^3}-\frac{2(a+1)^3x}{b^4}+\frac{2(a+1)^4\log(a+bx+1)}{b^5}+\frac{x^4}{2b}-\frac{x^5}{5}$$

```
[Out] (-2*(1 + a)^3*x)/b^4 + ((1 + a)^2*x^2)/b^3 - (2*(1 + a)*x^3)/(3*b^2) + x^4/
(2*b) - x^5/5 + (2*(1 + a)^4*Log[1 + a + b*x])/b^5
```

Rubi [A] time = 0.0785129, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{2(a+1)x^3}{3b^2}+\frac{(a+1)^2x^2}{b^3}-\frac{2(a+1)^3x}{b^4}+\frac{2(a+1)^4\log(a+bx+1)}{b^5}+\frac{x^4}{2b}-\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^(2*ArcTanh[a + b*x]),x]
```

```
[Out] (-2*(1 + a)^3*x)/b^4 + ((1 + a)^2*x^2)/b^3 - (2*(1 + a)*x^3)/(3*b^2) + x^4/
(2*b) - x^5/5 + (2*(1 + a)^4*Log[1 + a + b*x])/b^5
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(a+bx)}x^4\,dx &= \int \frac{x^4(1-a-bx)}{1+a+bx}\,dx\\
&= \int \left(-\frac{2(1+a)^3}{b^4}+\frac{2(1+a)^2x}{b^3}-\frac{2(1+a)x^2}{b^2}+\frac{2x^3}{b}-x^4+\frac{2(1+a)^4}{b^4(1+a+bx)}\right)dx\\
&= -\frac{2(1+a)^3x}{b^4}+\frac{(1+a)^2x^2}{b^3}-\frac{2(1+a)x^3}{3b^2}+\frac{x^4}{2b}-\frac{x^5}{5}+\frac{2(1+a)^4\log(1+a+bx)}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.0570517, size = 71, normalized size = 1.

$$-\frac{2(a+1)x^3}{3b^2}+\frac{(a+1)^2x^2}{b^3}-\frac{2(a+1)^3x}{b^4}+\frac{2(a+1)^4\log(a+bx+1)}{b^5}+\frac{x^4}{2b}-\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/E^(2*ArcTanh[a + b*x]),x]
```

```
[Out] (-2*(1 + a)^3*x)/b^4 + ((1 + a)^2*x^2)/b^3 - (2*(1 + a)*x^3)/(3*b^2) + x^4/
(2*b) - x^5/5 + (2*(1 + a)^4*Log[1 + a + b*x])/b^5
```

Maple [B] time = 0.029, size = 159, normalized size = 2.2

$$-\frac{x^5}{5}+\frac{x^4}{2b}-\frac{2x^3a}{3b^2}-\frac{2x^3}{3b^2}+\frac{a^2x^2}{b^3}+2\frac{ax^2}{b^3}-2\frac{xa^3}{b^4}+\frac{x^2}{b^3}-6\frac{a^2x}{b^4}-6\frac{ax}{b^4}-2\frac{x}{b^4}+2\frac{\ln(bx+a+1)a^4}{b^5}+8\frac{\ln(bx+a+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a+1)^2*(1-(b*x+a)^2),x)
```

```
[Out] -1/5*x^5+1/2*x^4/b-2/3/b^2*x^3*a-2/3/b^2*x^3+1/b^3*x^2*a^2+2/b^3*x^2*a-2/b^
4*x*a^3+1/b^3*x^2-6/b^4*x*a^2-6/b^4*a*x-2/b^4*x+2/b^5*ln(b*x+a+1)*a^4+8/b^5
*ln(b*x+a+1)*a^3+12/b^5*ln(b*x+a+1)*a^2+8/b^5*ln(b*x+a+1)*a+2/b^5*ln(b*x+a+
1)
```

Maxima [A] time = 0.955928, size = 127, normalized size = 1.79

$$-\frac{6b^4x^5-15b^3x^4+20(a+1)b^2x^3-30\left(a^2+2a+1\right)bx^2+60\left(a^3+3a^2+3a+1\right)x}{30b^4}+\frac{2\left(a^4+4a^3+6a^2+4a+1\right)\mathrm{l}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/30*(6*b^4*x^5 - 15*b^3*x^4 + 20*(a + 1)*b^2*x^3 - 30*(a^2 + 2*a + 1)*b*x
^2 + 60*(a^3 + 3*a^2 + 3*a + 1)*x)/b^4 + 2*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*
log(b*x + a + 1)/b^5
```

Fricas [A] time = 1.41104, size = 234, normalized size = 3.3

$$-\frac{6b^5x^5-15b^4x^4+20(a+1)b^3x^3-30\left(a^2+2a+1\right)b^2x^2+60\left(a^3+3a^2+3a+1\right)bx-60\left(a^4+4a^3+6a^2+4a+1\right.}{30b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/30*(6*b^5*x^5 - 15*b^4*x^4 + 20*(a + 1)*b^3*x^3 - 30*(a^2 + 2*a + 1)*b^2
*x^2 + 60*(a^3 + 3*a^2 + 3*a + 1)*b*x - 60*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*
log(b*x + a + 1))/b^5
```

Sympy [A] time = 0.447723, size = 78, normalized size = 1.1

$$-\frac{x^5}{5}+\frac{x^4}{2b}-\frac{x^3(2a+2)}{3b^2}+\frac{x^2\left(a^2+2a+1\right)}{b^3}-\frac{x\left(2a^3+6a^2+6a+2\right)}{b^4}+\frac{2(a+1)^4\log(a+bx+1)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a+1)**2*(1-(b*x+a)**2),x)
```

```
[Out] -x**5/5 + x**4/(2*b) - x**3*(2*a + 2)/(3*b**2) + x**2*(a**2 + 2*a + 1)/b**3
 - x*(2*a**3 + 6*a**2 + 6*a + 2)/b**4 + 2*(a + 1)**4*log(a + b*x + 1)/b**5
```

<u>Giac</u> [B] time = 1.20884, size = 273, normalized size = 3.85

$$\frac{(bx+a+1)^5\left(\frac{15\,(2\,ab+3\,b)}{(bx+a+1)b} - \frac{20\left(3\,a^2b^2+10\,ab^2+7\,b^2\right)}{(bx+a+1)^2b^2} + \frac{60\left(a^3b^3+6\,a^2b^3+9\,ab^3+4\,b^3\right)}{(bx+a+1)^3b^3} - \frac{30\left(a^4b^4+12\,a^3b^4+30\,a^2b^4+28\,ab^4+9\,b^4\right)}{(bx+a+1)^4b^4} - 6\right)}{30\,b^5} - \frac{2\left(a^4+4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="giac")
```

```
[Out] 1/30*(b*x + a + 1)^5*(15*(2*a*b + 3*b)/((b*x + a + 1)*b) - 20*(3*a^2*b^2 +
10*a*b^2 + 7*b^2)/((b*x + a + 1)^2*b^2) + 60*(a^3*b^3 + 6*a^2*b^3 + 9*a*b^3
 + 4*b^3)/((b*x + a + 1)^3*b^3) - 30*(a^4*b^4 + 12*a^3*b^4 + 30*a^2*b^4 + 2
8*a*b^4 + 9*b^4)/((b*x + a + 1)^4*b^4) - 6)/b^5 - 2*(a^4 + 4*a^3 + 6*a^2 +
4*a + 1)*log(abs(b*x + a + 1)/((b*x + a + 1)^2*abs(b)))/b^5
```

3.852 $\int e^{-2\tanh^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=57

$$-\frac{(a+1)x^2}{b^2}+\frac{2(a+1)^2x}{b^3}-\frac{2(a+1)^3\log(a+bx+1)}{b^4}+\frac{2x^3}{3b}-\frac{x^4}{4}$$

```
[Out] (2*(1 + a)^2*x)/b^3 - ((1 + a)*x^2)/b^2 + (2*x^3)/(3*b) - x^4/4 - (2*(1 + a
)^3*Log[1 + a + b*x])/b^4
```

Rubi [A] time = 0.0559445, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{(a+1)x^2}{b^2}+\frac{2(a+1)^2x}{b^3}-\frac{2(a+1)^3\log(a+bx+1)}{b^4}+\frac{2x^3}{3b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(2*ArcTanh[a + b*x]),x]
```

```
[Out] (2*(1 + a)^2*x)/b^3 - ((1 + a)*x^2)/b^2 + (2*x^3)/(3*b) - x^4/4 - (2*(1 + a
)^3*Log[1 + a + b*x])/b^4
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(a+bx)}x^3\,dx &= \int \frac{x^3(1-a-bx)}{1+a+bx}\,dx\\ &= \int \left(\frac{2(1+a)^2}{b^3}-\frac{2(1+a)x}{b^2}+\frac{2x^2}{b}-x^3-\frac{2(1+a)^3}{b^3(1+a+bx)}\right)dx\\ &= \frac{2(1+a)^2x}{b^3}-\frac{(1+a)x^2}{b^2}+\frac{2x^3}{3b}-\frac{x^4}{4}-\frac{2(1+a)^3\log(1+a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.0411919, size = 57, normalized size = 1.

$$-\frac{(a+1)x^2}{b^2}+\frac{2(a+1)^2x}{b^3}-\frac{2(a+1)^3\log(a+bx+1)}{b^4}+\frac{2x^3}{3b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^(2*ArcTanh[a + b*x]),x]
```

```
[Out] (2*(1 + a)^2*x)/b^3 - ((1 + a)*x^2)/b^2 + (2*x^3)/(3*b) - x^4/4 - (2*(1 + a
)^3*Log[1 + a + b*x])/b^4
```

Maple [B] time = 0.026, size = 109, normalized size = 1.9

$$-\frac{x^4}{4}+\frac{2x^3}{3b}-\frac{ax^2}{b^2}-\frac{x^2}{b^2}+2\frac{a^2x}{b^3}+4\frac{ax}{b^3}+2\frac{x}{b^3}-2\frac{\ln(bx+a+1)a^3}{b^4}-6\frac{\ln(bx+a+1)a^2}{b^4}-6\frac{\ln(bx+a+1)a}{b^4}-2\frac{\ln(b\text{:}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a+1)^2*(1-(b*x+a)^2),x)
```

```
[Out] -1/4*x^4+2/3*x^3/b-1/b^2*x^2*a-1/b^2*x^2+2/b^3*x*a^2+4/b^3*a*x+2/b^3*x-2/b^
4*ln(b*x+a+1)*a^3-6/b^4*ln(b*x+a+1)*a^2-6/b^4*ln(b*x+a+1)*a-2/b^4*ln(b*x+a+
1)
```

Maxima [A] time = 0.967719, size = 92, normalized size = 1.61

$$-\frac{3b^3x^4-8b^2x^3+12(a+1)bx^2-24(a^2+2a+1)x}{12b^3}-\frac{2(a^3+3a^2+3a+1)\log(bx+a+1)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^3*x^4 - 8*b^2*x^3 + 12*(a + 1)*b*x^2 - 24*(a^2 + 2*a + 1)*x)/b^3
 - 2*(a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1)/b^4
```

Fricas [A] time = 1.41636, size = 171, normalized size = 3.

$$-\frac{3b^4x^4-8b^3x^3+12(a+1)b^2x^2-24(a^2+2a+1)bx+24(a^3+3a^2+3a+1)\log(bx+a+1)}{12b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^4*x^4 - 8*b^3*x^3 + 12*(a + 1)*b^2*x^2 - 24*(a^2 + 2*a + 1)*b*x
+ 24*(a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1))/b^4
```

Sympy [A] time = 0.383628, size = 56, normalized size = 0.98

$$-\frac{x^4}{4}+\frac{2x^3}{3b}-\frac{x^2(a+1)}{b^2}+\frac{x(2a^2+4a+2)}{b^3}-\frac{2(a+1)^3\log(a+bx+1)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a+1)**2*(1-(b*x+a)**2),x)
```

```
[Out] -x**4/4 + 2*x**3/(3*b) - x**2*(a + 1)/b**2 + x*(2*a**2 + 4*a + 2)/b**3 - 2*
(a + 1)**3*log(a + b*x + 1)/b**4
```

Giac [B] time = 1.13757, size = 200, normalized size = 3.51

$$\frac{(bx+a+1)^4\left(\frac{4\,(3\,ab+5\,b)}{(bx+a+1)b}-\frac{18\,\left(a^2b^2+4\,ab^2+3\,b^2\right)}{(bx+a+1)^2b^2}+\frac{12\,\left(a^3b^3+9\,a^2b^3+15\,ab^3+7\,b^3\right)}{(bx+a+1)^3b^3}-3\right)}{12\,b^4}+\frac{2\left(a^3+3\,a^2+3\,a+1\right)\log\left(\frac{|bx+a+1|}{(bx+a+1)^2|b|}\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="giac")
```

```
[Out] 1/12*(b*x + a + 1)^4*(4*(3*a*b + 5*b)/((b*x + a + 1)*b) - 18*(a^2*b^2 + 4*a
*b^2 + 3*b^2)/((b*x + a + 1)^2*b^2) + 12*(a^3*b^3 + 9*a^2*b^3 + 15*a*b^3 +
7*b^3)/((b*x + a + 1)^3*b^3) - 3)/b^4 + 2*(a^3 + 3*a^2 + 3*a + 1)*log(abs(b
*x + a + 1)/((b*x + a + 1)^2*abs(b)))/b^4
```

3.853 $\int e^{-2\tanh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=41

$$-\frac{2(a+1)x}{b^2}+\frac{2(a+1)^2\log(a+bx+1)}{b^3}+\frac{x^2}{b}-\frac{x^3}{3}$$

```
[Out] (-2*(1 + a)*x)/b^2 + x^2/b - x^3/3 + (2*(1 + a)^2*Log[1 + a + b*x])/b^3
```

Rubi [A] time = 0.0456562, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{2(a+1)x}{b^2}+\frac{2(a+1)^2\log(a+bx+1)}{b^3}+\frac{x^2}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(2*ArcTanh[a + b*x]),x]
```

```
[Out] (-2*(1 + a)*x)/b^2 + x^2/b - x^3/3 + (2*(1 + a)^2*Log[1 + a + b*x])/b^3
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(a+bx)}x^2\,dx&=\int \frac{x^2(1-a-bx)}{1+a+bx}\,dx\\&=\int \left(-\frac{2(1+a)}{b^2}+\frac{2x}{b}-x^2+\frac{2(1+a)^2}{b^2(1+a+bx)}\right)dx\\&=-\frac{2(1+a)x}{b^2}+\frac{x^2}{b}-\frac{x^3}{3}+\frac{2(1+a)^2\log(1+a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0296369, size = 41, normalized size = 1.

$$-\frac{2(a+1)x}{b^2}+\frac{2(a+1)^2\log(a+bx+1)}{b^3}+\frac{x^2}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^(2*ArcTanh[a + b*x]),x]
```

```
[Out] (-2*(1 + a)*x)/b^2 + x^2/b - x^3/3 + (2*(1 + a)^2*Log[1 + a + b*x])/b^3
```

Maple [A] time = 0.028, size = 67, normalized size = 1.6

$$-\frac{x^3}{3}+\frac{x^2}{b}-2\,\frac{ax}{b^2}-2\,\frac{x}{b^2}+2\,\frac{\ln\left(bx+a+1\right)a^2}{b^3}+4\,\frac{\ln\left(bx+a+1\right)a}{b^3}+2\,\frac{\ln\left(bx+a+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a+1)^2*(1-(b*x+a)^2),x)
```

```
[Out] -1/3*x^3+x^2/b-2/b^2*a*x-2/b^2*x+2/b^3*ln(b*x+a+1)*a^2+4/b^3*ln(b*x+a+1)*a+
2/b^3*ln(b*x+a+1)
```

Maxima [A] time = 0.966852, size = 62, normalized size = 1.51

$$-\frac{b^2x^3-3\,bx^2+6\,(a+1)x}{3\,b^2}+\frac{2\left(a^2+2\,a+1\right)\log\left(bx+a+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(b^2*x^3 - 3*b*x^2 + 6*(a + 1)*x)/b^2 + 2*(a^2 + 2*a + 1)*log(b*x + a
+ 1)/b^3
```

Fricas [A] time = 1.44515, size = 115, normalized size = 2.8

$$-\frac{b^3x^3-3\,b^2x^2+6\,(a+1)bx-6\left(a^2+2\,a+1\right)\log\left(bx+a+1\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 - 3*b^2*x^2 + 6*(a + 1)*b*x - 6*(a^2 + 2*a + 1)*log(b*x + a +
 1))/b^3
```

Sympy [A] time = 0.353981, size = 37, normalized size = 0.9

$$-\frac{x^3}{3}+\frac{x^2}{b}-\frac{x\,(2a+2)}{b^2}+\frac{2\,(a+1)^2\log\left(a+bx+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a+1)**2*(1-(b*x+a)**2),x)
```

```
[Out] -x**3/3 + x**2/b - x*(2*a + 2)/b**2 + 2*(a + 1)**2*log(a + b*x + 1)/b**3
```

Giac [B] time = 1.19284, size = 138, normalized size = 3.37

$$\frac{(bx+a+1)^3\left(\frac{3\,(ab+2\,b)}{(bx+a+1)b}-\frac{3\,(a^2b^2+6\,ab^2+5\,b^2)}{(bx+a+1)^2b^2}-1\right)}{3\,b^3}-\frac{2\,(a^2+2\,a+1)\log\left(\frac{|bx+a+1|}{(bx+a+1)^2|b|}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="giac")
```

```
[Out] 1/3*(b*x + a + 1)^3*(3*(a*b + 2*b)/((b*x + a + 1)*b) - 3*(a^2*b^2 + 6*a*b^2
 + 5*b^2)/((b*x + a + 1)^2*b^2) - 1)/b^3 - 2*(a^2 + 2*a + 1)*log(abs(b*x +
a + 1)/((b*x + a + 1)^2*abs(b)))/b^3
```

3.854 $\int e^{-2\tanh^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=29

$$-\frac{2(a+1)\log(a+bx+1)}{b^2}+\frac{2x}{b}-\frac{x^2}{2}$$

```
[Out] (2*x)/b - x^2/2 - (2*(1 + a)*Log[1 + a + b*x])/b^2
```

Rubi [A] time = 0.0317985, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6163, 77}

$$-\frac{2(a+1)\log(a+bx+1)}{b^2}+\frac{2x}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(2*ArcTanh[a + b*x]),x]
```

```
[Out] (2*x)/b - x^2/2 - (2*(1 + a)*Log[1 + a + b*x])/b^2
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(a+bx)}x\,dx &= \int \frac{x(1-a-bx)}{1+a+bx}\,dx\\
&= \int \left(\frac{2}{b}-x-\frac{2(1+a)}{b(1+a+bx)}\right)dx\\
&= \frac{2x}{b}-\frac{x^2}{2}-\frac{2(1+a)\log(1+a+bx)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.0172854, size = 29, normalized size = 1.

$$-\frac{2(a+1)\log(a+bx+1)}{b^2}+\frac{2x}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^(2*ArcTanh[a + b*x]),x]
```

```
[Out] (2*x)/b - x^2/2 - (2*(1 + a)*Log[1 + a + b*x])/b^2
```

Maple [A] time = 0.028, size = 38, normalized size = 1.3

$$-\frac{x^2}{2}+2\,\frac{x}{b}-2\,\frac{\ln\left(bx+a+1\right)a}{b^2}-2\,\frac{\ln\left(bx+a+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a+1)^2*(1-(b*x+a)^2),x)
```

```
[Out] -1/2*x^2+2*x/b-2/b^2*ln(b*x+a+1)*a-2/b^2*ln(b*x+a+1)
```

Maxima [A] time = 0.958947, size = 41, normalized size = 1.41

$$-\frac{bx^2-4\,x}{2\,b}-\frac{2\,(a+1)\log\left(bx+a+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 - 4*x)/b - 2*(a + 1)*log(b*x + a + 1)/b^2
```

Fricas [A] time = 1.48488, size = 77, normalized size = 2.66

$$-\frac{b^2x^2-4\,bx+4\,(a+1)\log\left(bx+a+1\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - 4*b*x + 4*(a + 1)*log(b*x + a + 1))/b^2
```

Sympy [A] time = 0.315509, size = 26, normalized size = 0.9

$$-\frac{x^2}{2}+\frac{2x}{b}-\frac{2\,(a+1)\log\left(a+bx+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)**2*(1-(b*x+a)**2),x)
```

```
[Out] -x**2/2 + 2*x/b - 2*(a + 1)*log(a + b*x + 1)/b**2
```

Giac [B] time = 1.13369, size = 93, normalized size = 3.21

$$\frac{\frac{(bx+a+1)^2\left(\frac{2\,(ab+3\,b)}{(bx+a+1)b}-1\right)}{b}+\frac{4\,(a+1)\log\left(\frac{|bx+a+1|}{(bx+a+1)^2|b|}\right)}{b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="giac")
```

```
[Out] 1/2*((b*x + a + 1)^2*(2*(a*b + 3*b)/((b*x + a + 1)*b) - 1)/b + 4*(a + 1)*lo
g(abs(b*x + a + 1)/((b*x + a + 1)^2*abs(b)))/b)/b
```

3.855 $\int e^{-2\tanh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=16

$$\frac{2\log(a+bx+1)}{b}-x$$

```
[Out] -x + (2*Log[1 + a + b*x])/b
```

Rubi [A] time = 0.0121516, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6161, 43}

$$\frac{2\log(a+bx+1)}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[E^(-2*ArcTanh[a + b*x]),x]
```

```
[Out] -x + (2*Log[1 + a + b*x])/b
```

Rule 6161

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 + a*c
 + b*c*x)^(n/2)/(1 - a*c - b*c*x)^(n/2), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(a+bx)}\,dx &= \int \frac{1-a-bx}{1+a+bx}\,dx\\ &= \int \left(-1+\frac{2}{1+a+bx}\right)dx\\ &= -x+\frac{2\log(1+a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0111211, size = 16, normalized size = 1.

$$\frac{2\log(a+bx+1)}{b}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-2*ArcTanh[a + b*x]),x]
```

```
[Out] -x + (2*Log[1 + a + b*x])/b
```

Maple [A] time = 0.029, size = 17, normalized size = 1.1

$$-x+2\,\frac{\ln\left(bx+a+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^2*(1-(b*x+a)^2),x)
```

```
[Out] -x+2*ln(b*x+a+1)/b
```

Maxima [A] time = 0.959308, size = 22, normalized size = 1.38

$$-x+\frac{2\,\log\left(bx+a+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -x + 2*log(b*x + a + 1)/b
```

Fricas [A] time = 1.44847, size = 42, normalized size = 2.62

$$-\frac{bx-2\,\log\left(bx+a+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -(b*x - 2*log(b*x + a + 1))/b
```

Sympy [A] time = 0.123091, size = 12, normalized size = 0.75

$$-x+\frac{2\log\left(a+bx+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**2*(1-(b*x+a)**2),x)
```

```
[Out] -x + 2*log(a + b*x + 1)/b
```

Giac [B] time = 1.17688, size = 51, normalized size = 3.19

$$-\frac{bx+a+1}{b}-\frac{2\,\log\left(\frac{|bx+a+1|}{(bx+a+1)^2|b|}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -(b*x + a + 1)/b - 2*log(abs(b*x + a + 1)/((b*x + a + 1)^2*abs(b)))/b
```

3.856 $\int \frac{e^{-2\tanh^{-1}(a+bx)}}{x} dx$

Optimal. Leaf size=28

$$\frac{(1-a)\log(x)}{a+1} - \frac{2\log(a+bx+1)}{a+1}$$

```
[Out] ((1 - a)*Log[x])/(1 + a) - (2*Log[1 + a + b*x])/(1 + a)
```

Rubi [A] time = 0.0346215, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 72}

$$\frac{(1-a)\log(x)}{a+1} - \frac{2\log(a+bx+1)}{a+1}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a + b*x])*x),x]
```

```
[Out] ((1 - a)*Log[x])/(1 + a) - (2*Log[1 + a + b*x])/(1 + a)
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(a+bx)}}{x} dx &= \int \frac{1-a-bx}{x(1+a+bx)} dx \\
&= \int \left(\frac{1-a}{(1+a)x} - \frac{2b}{(1+a)(1+a+bx)}\right) dx \\
&= \frac{(1-a)\log(x)}{1+a} - \frac{2\log(1+a+bx)}{1+a}
\end{aligned}$$

Mathematica [A] time = 0.0160087, size = 23, normalized size = 0.82

$$\frac{-2\log(a+bx+1) - a\log(x) + \log(x)}{a+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a + b*x])*x),x]
```

```
[Out] (Log[x] - a*Log[x] - 2*Log[1 + a + b*x])/(1 + a)
```

Maple [A] time = 0.033, size = 34, normalized size = 1.2

$$-2\,\frac{\ln(bx+a+1)}{1+a}+\frac{\ln(x)}{1+a}-\frac{a\ln(x)}{1+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x,x)
```

```
[Out] -2*ln(b*x+a+1)/(1+a)+1/(1+a)*ln(x)-1/(1+a)*ln(x)*a
```

Maxima [A] time = 0.953797, size = 36, normalized size = 1.29

$$-\frac{(a-1)\log(x)}{a+1}-\frac{2\,\log(bx+a+1)}{a+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] -(a - 1)*log(x)/(a + 1) - 2*log(b*x + a + 1)/(a + 1)
```

Fricas [A] time = 1.52606, size = 65, normalized size = 2.32

$$-\frac{(a-1)\log(x)+2\,\log(bx+a+1)}{a+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x,x, algorithm="fricas")
```

```
[Out] -((a - 1)*log(x) + 2*log(b*x + a + 1))/(a + 1)
```

Sympy [B] time = 0.523708, size = 90, normalized size = 3.21

$$-\frac{(a-1)\log\left(x+\frac{-\frac{a^2(a-1)}{a+1}+a^2-\frac{2a(a-1)}{a+1}-\frac{a-1}{a+1}-1}{ab-3b}\right)}{a+1}-\frac{2\log\left(x+\frac{a^2-\frac{2a^2}{a+1}-\frac{4a}{a+1}-1-\frac{2}{a+1}}{ab-3b}\right)}{a+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**2*(1-(b*x+a)**2)/x,x)
```

```
[Out] -(a - 1)*log(x + (-a**2*(a - 1)/(a + 1) + a**2 - 2*a*(a - 1)/(a + 1) - (a -
 1)/(a + 1) - 1)/(a*b - 3*b))/(a + 1) - 2*log(x + (a**2 - 2*a**2/(a + 1) -
4*a/(a + 1) - 1 - 2/(a + 1))/(a*b - 3*b))/(a + 1)
```

Giac [B] time = 1.1946, size = 89, normalized size = 3.18

$$-b\left(\frac{(a-1)\log\left(\left|-\frac{a}{bx+a+1}-\frac{1}{bx+a+1}+1\right|\right)}{ab+b}-\frac{\log\left(\frac{|bx+a+1|}{(bx+a+1)^2|b|}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x,x, algorithm="giac")
```

```
[Out] -b*((a - 1)*log(abs(-a/(b*x + a + 1) - 1/(b*x + a + 1) + 1))/(a*b + b) - lo
g(abs(b*x + a + 1)/((b*x + a + 1)^2*abs(b)))/b)
```

3.857 $\int \frac{e^{-2\tanh^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=41

$$-\frac{2b\log(x)}{(a+1)^2}+\frac{2b\log(a+bx+1)}{(a+1)^2}-\frac{1-a}{(a+1)x}$$

```
[Out] -((1 - a)/((1 + a)*x)) - (2*b*Log[x])/(1 + a)^2 + (2*b*Log[1 + a + b*x])/(1
 + a)^2
```

Rubi [A] time = 0.043266, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{2b\log(x)}{(a+1)^2}+\frac{2b\log(a+bx+1)}{(a+1)^2}-\frac{1-a}{(a+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a + b*x])*x^2),x]
```

```
[Out] -((1 - a)/((1 + a)*x)) - (2*b*Log[x])/(1 + a)^2 + (2*b*Log[1 + a + b*x])/(1
 + a)^2
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{1-a-bx}{x^2(1+a+bx)}\,dx\\ &= \int \left(\frac{1-a}{(1+a)x^2}-\frac{2b}{(1+a)^2x}+\frac{2b^2}{(1+a)^2(1+a+bx)}\right)dx\\ &= -\frac{1-a}{(1+a)x}-\frac{2b\log(x)}{(1+a)^2}+\frac{2b\log(1+a+bx)}{(1+a)^2}\end{aligned}$$

Mathematica [A] time = 0.0206532, size = 31, normalized size = 0.76

$$\frac{a^2+2bx\log(a+bx+1)-2bx\log(x)-1}{(a+1)^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a + b*x])*x^2),x]
```

```
[Out] (-1 + a^2 - 2*b*x*Log[x] + 2*b*x*Log[1 + a + b*x])/((1 + a)^2*x)
```

Maple [A] time = 0.035, size = 47, normalized size = 1.2

$$2\,{\frac {b\ln \left(bx+a+1 \right) }{ \left(1+a \right) ^{2}}}-{\frac {1}{ \left(1+a \right) x}}+{\frac {a}{ \left(1+a \right) x}}-2\,{\frac {b\ln \left(x \right) }{ \left(1+a \right) ^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^2,x)
```

```
[Out] 2*b*ln(b*x+a+1)/(1+a)^2-1/(1+a)/x+1/(1+a)/x*a-2*b*ln(x)/(1+a)^2
```

Maxima [A] time = 0.954293, size = 65, normalized size = 1.59

$$\frac {2 \, b \log \left (b x + a + 1\right)}{a^{2} + 2 \, a + 1} - \frac {2 \, b \log \left (x\right)}{a^{2} + 2 \, a + 1} + \frac {a - 1}{{\left (a + 1\right)} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] 2*b*log(b*x + a + 1)/(a^2 + 2*a + 1) - 2*b*log(x)/(a^2 + 2*a + 1) + (a - 1)
/((a + 1)*x)
```

Fricas [A] time = 1.55313, size = 96, normalized size = 2.34

$$\frac {2 \, b x \log \left (b x + a + 1\right) - 2 \, b x \log \left (x\right) + a^{2} - 1}{{\left (a^{2} + 2 \, a + 1\right)} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*b*x*log(b*x + a + 1) - 2*b*x*log(x) + a^2 - 1)/((a^2 + 2*a + 1)*x)
```

Sympy [B] time = 0.51789, size = 143, normalized size = 3.49

$$- \frac {2 b \log {\left (x + \frac {- \frac {2 a^{3} b}{\left (a + 1\right)^{2}} - \frac {6 a^{2} b}{\left (a + 1\right)^{2}} + 2 a b - \frac {6 a b}{\left (a + 1\right)^{2}} + 2 b - \frac {2 b}{\left (a + 1\right)^{2}}}{4 b^{2}} \right)}}{\left (a + 1\right)^{2}} + \frac {2 b \log {\left (x + \frac {\frac {2 a^{3} b}{\left (a + 1\right)^{2}} + \frac {6 a^{2} b}{\left (a + 1\right)^{2}} + 2 a b + \frac {6 a b}{\left (a + 1\right)^{2}} + 2 b + \frac {2 b}{\left (a + 1\right)^{2}}}{4 b^{2}} \right)}}{\left (a + 1\right)^{2}} + \frac {a - 1}{x \left (a + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**2*(1-(b*x+a)**2)/x**2,x)
```

```
[Out] -2*b*log(x + (-2*a**3*b/(a + 1)**2 - 6*a**2*b/(a + 1)**2 + 2*a*b - 6*a*b/(a
 + 1)**2 + 2*b - 2*b/(a + 1)**2)/(4*b**2))/(a + 1)**2 + 2*b*log(x + (2*a**3
*b/(a + 1)**2 + 6*a**2*b/(a + 1)**2 + 2*a*b + 6*a*b/(a + 1)**2 + 2*b + 2*b/
(a + 1)**2)/(4*b**2))/(a + 1)**2 + (a - 1)/(x*(a + 1))
```

Giac [B] time = 1.16359, size = 108, normalized size = 2.63

$$-\frac{2\,b^2\log\left(\left|-\frac{a}{bx+a+1}-\frac{1}{bx+a+1}+1\right|\right)}{a^2b+2\,ab+b}-\frac{ab-b}{(a+1)^2\left(\frac{a}{bx+a+1}+\frac{1}{bx+a+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^2,x, algorithm="giac")
```

```
[Out] -2*b^2*log(abs(-a/(b*x + a + 1) - 1/(b*x + a + 1) + 1))/(a^2*b + 2*a*b + b)
 - (a*b - b)/((a + 1)^2*(a/(b*x + a + 1) + 1/(b*x + a + 1) - 1))
```

3.858 $\int \frac{e^{-2\tanh^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=58

$$\frac{2b^2\log(x)}{(a+1)^3}-\frac{2b^2\log(a+bx+1)}{(a+1)^3}+\frac{2b}{(a+1)^2x}-\frac{1-a}{2(a+1)x^2}$$

```
[Out] -(1 - a)/(2*(1 + a)*x^2) + (2*b)/((1 + a)^2*x) + (2*b^2*Log[x])/(1 + a)^3 -
 (2*b^2*Log[1 + a + b*x])/(1 + a)^3
```

Rubi [A] time = 0.0498517, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$\frac{2b^2\log(x)}{(a+1)^3}-\frac{2b^2\log(a+bx+1)}{(a+1)^3}+\frac{2b}{(a+1)^2x}-\frac{1-a}{2(a+1)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a + b*x])*x^3),x]
```

```
[Out] -(1 - a)/(2*(1 + a)*x^2) + (2*b)/((1 + a)^2*x) + (2*b^2*Log[x])/(1 + a)^3 -
 (2*b^2*Log[1 + a + b*x])/(1 + a)^3
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(a+bx)}}{x^3}\,dx &= \int \frac{1-a-bx}{x^3(1+a+bx)}\,dx\\ &= \int \left(\frac{1-a}{(1+a)x^3}-\frac{2b}{(1+a)^2x^2}+\frac{2b^2}{(1+a)^3x}-\frac{2b^3}{(1+a)^3(1+a+bx)}\right)dx\\ &= -\frac{1-a}{2(1+a)x^2}+\frac{2b}{(1+a)^2x}+\frac{2b^2\log(x)}{(1+a)^3}-\frac{2b^2\log(1+a+bx)}{(1+a)^3}\end{aligned}$$

Mathematica [A] time = 0.0330436, size = 51, normalized size = 0.88

$$\frac{(a+1)\left(a^2+4bx-1\right)-4b^2x^2\log(a+bx+1)+4b^2x^2\log(x)}{2(a+1)^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a + b*x])*x^3),x]
```

```
[Out] ((1 + a)*(-1 + a^2 + 4*b*x) + 4*b^2*x^2*Log[x] - 4*b^2*x^2*Log[1 + a + b*x]
)/(2*(1 + a)^3*x^2)
```

Maple [A] time = 0.034, size = 63, normalized size = 1.1

$$-2\,{\frac{{b}^{2}\ln \left(bx+a+1 \right) }{ \left(1+a \right) ^{3}}}-{\frac{1}{ \left(2+2\,a \right) {x}^{2}}}+{\frac{a}{ \left(2+2\,a \right) {x}^{2}}}+2\,{\frac{b}{ \left(1+a \right) ^{2}x}}+2\,{\frac{{b}^{2}\ln \left(x \right) }{ \left(1+a \right) ^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^3,x)
```

```
[Out] -2*b^2*ln(b*x+a+1)/(1+a)^3-1/2/(1+a)/x^2+1/2/(1+a)/x^2*a+2*b/(1+a)^2/x+2*b^
2*ln(x)/(1+a)^3
```

Maxima [A] time = 0.95116, size = 100, normalized size = 1.72

$$-\frac{2\,b^{2}\log\left(bx+a+1\right)}{a^{3}+3\,a^{2}+3\,a+1}+\frac{2\,b^{2}\log\left(x\right)}{a^{3}+3\,a^{2}+3\,a+1}+\frac{a^{2}+4\,bx-1}{2\,{\left(a^{2}+2\,a+1\right)}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^3,x, algorithm="maxima")
```

```
[Out] -2*b^2*log(b*x + a + 1)/(a^3 + 3*a^2 + 3*a + 1) + 2*b^2*log(x)/(a^3 + 3*a^2
 + 3*a + 1) + 1/2*(a^2 + 4*b*x - 1)/((a^2 + 2*a + 1)*x^2)
```

Fricas [A] time = 1.5681, size = 162, normalized size = 2.79

$$-\frac{4\,b^{2}x^{2}\log\left(bx+a+1\right)-4\,b^{2}x^{2}\log\left(x\right)-a^{3}-4\,{\left(a+1\right)}bx-a^{2}+a+1}{2\,{\left(a^{3}+3\,a^{2}+3\,a+1\right)}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b^2*x^2*log(b*x + a + 1) - 4*b^2*x^2*log(x) - a^3 - 4*(a + 1)*b*x -
 a^2 + a + 1)/((a^3 + 3*a^2 + 3*a + 1)*x^2)
```

Sympy [B] time = 0.651894, size = 209, normalized size = 3.6

$$\frac{2b^{2}\log{\left(x + \frac{- \frac{2a^{4}b^{2}}{(a+1)^{3}} - \frac{8a^{3}b^{2}}{(a+1)^{3}} - \frac{12a^{2}b^{2}}{(a+1)^{3}} + 2ab^{2} - \frac{8ab^{2}}{(a+1)^{3}} + 2b^{2} - \frac{2b^{2}}{(a+1)^{3}}}{4b^{3}} \right)}}{\left(a + 1\right)^{3}} - \frac{2b^{2}\log{\left(x + \frac{\frac{2a^{4}b^{2}}{(a+1)^{3}} + \frac{8a^{3}b^{2}}{(a+1)^{3}} + \frac{12a^{2}b^{2}}{(a+1)^{3}} + 2ab^{2} + \frac{8ab^{2}}{(a+1)^{3}} + 2b^{2} + \frac{2b^{2}}{(a+1)^{3}}}{4b^{3}} \right)}}{\left(a + 1\right)^{3}} + \frac{a^{2} + 4b \cdots}{x^{2}\left(2a^{2} + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**2*(1-(b*x+a)**2)/x**3,x)
```

```
[Out] 2*b**2*log(x + (-2*a**4*b**2/(a + 1)**3 - 8*a**3*b**2/(a + 1)**3 - 12*a**2*
b**2/(a + 1)**3 + 2*a*b**2 - 8*a*b**2/(a + 1)**3 + 2*b**2 - 2*b**2/(a + 1)*
*3)/(4*b**3))/(a + 1)**3 - 2*b**2*log(x + (2*a**4*b**2/(a + 1)**3 + 8*a**3*
b**2/(a + 1)**3 + 12*a**2*b**2/(a + 1)**3 + 2*a*b**2 + 8*a*b**2/(a + 1)**3
+ 2*b**2 + 2*b**2/(a + 1)**3)/(4*b**3))/(a + 1)**3 + (a**2 + 4*b*x - 1)/(x*
*2*(2*a**2 + 4*a + 2))
```

Giac [B] time = 1.19617, size = 163, normalized size = 2.81

$$\frac{2\,b^3\log\left(\left|-\frac{a}{bx+a+1}-\frac{1}{bx+a+1}+1\right|\right)}{a^3b+3\,a^2b+3\,ab+b}-\frac{\frac{ab^2-5\,b^2}{a+1}-\frac{2\left(ab^3-3\,b^3\right)}{(bx+a+1)b}}{2\,(a+1)^2\left(\frac{a}{bx+a+1}+\frac{1}{bx+a+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^3,x, algorithm="giac")
```

```
[Out] 2*b^3*log(abs(-a/(b*x + a + 1) - 1/(b*x + a + 1) + 1))/(a^3*b + 3*a^2*b + 3
*a*b + b) - 1/2*((a*b^2 - 5*b^2)/(a + 1) - 2*(a*b^3 - 3*b^3)/((b*x + a + 1)
*b))/((a + 1)^2*(a/(b*x + a + 1) + 1/(b*x + a + 1) - 1)^2)
```

3.859 $\int \frac{e^{-2\tanh^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=70

$$-\frac{2b^2}{(a+1)^3x}-\frac{2b^3\log(x)}{(a+1)^4}+\frac{2b^3\log(a+bx+1)}{(a+1)^4}+\frac{b}{(a+1)^2x^2}-\frac{1-a}{3(a+1)x^3}$$

```
[Out] -(1 - a)/(3*(1 + a)*x^3) + b/((1 + a)^2*x^2) - (2*b^2)/((1 + a)^3*x) - (2*b
^3*Log[x])/(1 + a)^4 + (2*b^3*Log[1 + a + b*x])/(1 + a)^4
```

Rubi [A] time = 0.0582411, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{2b^2}{(a+1)^3x}-\frac{2b^3\log(x)}{(a+1)^4}+\frac{2b^3\log(a+bx+1)}{(a+1)^4}+\frac{b}{(a+1)^2x^2}-\frac{1-a}{3(a+1)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a + b*x])*x^4),x]
```

```
[Out] -(1 - a)/(3*(1 + a)*x^3) + b/((1 + a)^2*x^2) - (2*b^2)/((1 + a)^3*x) - (2*b
^3*Log[x])/(1 + a)^4 + (2*b^3*Log[1 + a + b*x])/(1 + a)^4
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(a+bx)}}{x^4}\,dx &= \int \frac{1-a-bx}{x^4(1+a+bx)}\,dx\\ &= \int \left(\frac{1-a}{(1+a)x^4}-\frac{2b}{(1+a)^2x^3}+\frac{2b^2}{(1+a)^3x^2}-\frac{2b^3}{(1+a)^4x}+\frac{2b^4}{(1+a)^4(1+a+bx)}\right)dx\\ &= -\frac{1-a}{3(1+a)x^3}+\frac{b}{(1+a)^2x^2}-\frac{2b^2}{(1+a)^3x}-\frac{2b^3\log(x)}{(1+a)^4}+\frac{2b^3\log(1+a+bx)}{(1+a)^4}\end{aligned}$$

Mathematica [A] time = 0.0464527, size = 70, normalized size = 1.

$$-\frac{2b^2}{(a+1)^3x}-\frac{2b^3\log(x)}{(a+1)^4}+\frac{2b^3\log(a+bx+1)}{(a+1)^4}+\frac{b}{(a+1)^2x^2}-\frac{1-a}{3(a+1)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a + b*x])*x^4),x]
```

```
[Out] -(1 - a)/(3*(1 + a)*x^3) + b/((1 + a)^2*x^2) - (2*b^2)/((1 + a)^3*x) - (2*b
^3*Log[x])/(1 + a)^4 + (2*b^3*Log[1 + a + b*x])/(1 + a)^4
```

Maple [A] time = 0.036, size = 75, normalized size = 1.1

$$2\,{\frac{{b}^{3}\ln \left(bx+a+1 \right) }{ \left(1+a \right) ^{4}}}-{\frac{1}{ \left(3+3\,a \right) {x}^{3}}}+{\frac{a}{ \left(3+3\,a \right) {x}^{3}}}+{\frac{b}{ \left(1+a \right) ^{2}{x}^{2}}}-2\,{\frac{{b}^{3}\ln \left(x \right) }{ \left(1+a \right) ^{4}}}-2\,{\frac{{b}^{2}}{ \left(1+a \right) ^{3}x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^4,x)
```

```
[Out] 2*b^3*ln(b*x+a+1)/(1+a)^4-1/3/(1+a)/x^3+1/3/(1+a)/x^3*a+b/(1+a)^2/x^2-2*b^3
*ln(x)/(1+a)^4-2*b^2/(1+a)^3/x
```

Maxima [A] time = 0.95605, size = 146, normalized size = 2.09

$$\frac{2\,b^{3}\log\left(bx+a+1\right)}{a^{4}+4\,a^{3}+6\,a^{2}+4\,a+1}-\frac{2\,b^{3}\log\left(x\right)}{a^{4}+4\,a^{3}+6\,a^{2}+4\,a+1}-\frac{6\,b^{2}x^{2}-a^{3}-3\,(a+1)bx-a^{2}+a+1}{3\,\left(a^{3}+3\,a^{2}+3\,a+1\right)x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^4,x, algorithm="maxima")
```

```
[Out] 2*b^3*log(b*x + a + 1)/(a^4 + 4*a^3 + 6*a^2 + 4*a + 1) - 2*b^3*log(x)/(a^4
+ 4*a^3 + 6*a^2 + 4*a + 1) - 1/3*(6*b^2*x^2 - a^3 - 3*(a + 1)*b*x - a^2 + a
 + 1)/((a^3 + 3*a^2 + 3*a + 1)*x^3)
```

Fricas [A] time = 1.54742, size = 215, normalized size = 3.07

$$\frac{6\,b^{3}x^{3}\log\left(bx+a+1\right)-6\,b^{3}x^{3}\log\left(x\right)-6\,(a+1)b^{2}x^{2}+a^{4}+2\,a^{3}+3\,\left(a^{2}+2\,a+1\right)bx-2\,a-1}{3\,\left(a^{4}+4\,a^{3}+6\,a^{2}+4\,a+1\right)x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(6*b^3*x^3*log(b*x + a + 1) - 6*b^3*x^3*log(x) - 6*(a + 1)*b^2*x^2 + a^
4 + 2*a^3 + 3*(a^2 + 2*a + 1)*b*x - 2*a - 1)/((a^4 + 4*a^3 + 6*a^2 + 4*a +
1)*x^3)
```

Sympy [B] time = 0.76786, size = 262, normalized size = 3.74

$$-\frac{2b^{3}\log\left(x+\frac{-\frac{2a^{5}b^{3}}{(a+1)^{4}}-\frac{10a^{4}b^{3}}{(a+1)^{4}}-\frac{20a^{3}b^{3}}{(a+1)^{4}}-\frac{20a^{2}b^{3}}{(a+1)^{4}}+2ab^{3}-\frac{10ab^{3}}{(a+1)^{4}}+2b^{3}-\frac{2b^{3}}{(a+1)^{4}}}{4b^{4}}\right)}{(a+1)^{4}}+\frac{2b^{3}\log\left(x+\frac{\frac{2a^{5}b^{3}}{(a+1)^{4}}+\frac{10a^{4}b^{3}}{(a+1)^{4}}+\frac{20a^{3}b^{3}}{(a+1)^{4}}+\frac{20a^{2}b^{3}}{(a+1)^{4}}+2ab^{3}+\frac{10ab^{3}}{(a+1)^{4}}+2b^{3}+\cdots}{4b^{4}}\right.}{(a+1)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**2*(1-(b*x+a)**2)/x**4,x)
```

```
[Out] -2*b**3*log(x + (-2*a**5*b**3/(a + 1)**4 - 10*a**4*b**3/(a + 1)**4 - 20*a**
3*b**3/(a + 1)**4 - 20*a**2*b**3/(a + 1)**4 + 2*a*b**3 - 10*a*b**3/(a + 1)*
*4 + 2*b**3 - 2*b**3/(a + 1)**4)/(4*b**4))/(a + 1)**4 + 2*b**3*log(x + (2*a
**5*b**3/(a + 1)**4 + 10*a**4*b**3/(a + 1)**4 + 20*a**3*b**3/(a + 1)**4 + 2
0*a**2*b**3/(a + 1)**4 + 2*a*b**3 + 10*a*b**3/(a + 1)**4 + 2*b**3 + 2*b**3/
(a + 1)**4)/(4*b**4))/(a + 1)**4 - (-a**3 - a**2 + a + 6*b**2*x**2 + x*(-3*
a*b - 3*b) + 1)/(x**3*(3*a**3 + 9*a**2 + 9*a + 3))
```

Giac [B] time = 1.17191, size = 215, normalized size = 3.07

$$-\frac{2\,b^4\log\left(\left|-\frac{a}{bx+a+1}-\frac{1}{bx+a+1}+1\right|\right)}{a^4b+4\,a^3b+6\,a^2b+4\,ab+b}-\frac{\frac{ab^3-10\,b^3}{a+1}-\frac{3\left(ab^4-8\,b^4\right)}{(bx+a+1)b}+\frac{3\left(a^2b^5-4\,ab^5-5\,b^5\right)}{(bx+a+1)^2b^2}}{3\,(a+1)^3\left(\frac{a}{bx+a+1}+\frac{1}{bx+a+1}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^4,x, algorithm="giac")
```

```
[Out] -2*b^4*log(abs(-a/(b*x + a + 1) - 1/(b*x + a + 1) + 1))/(a^4*b + 4*a^3*b +
6*a^2*b + 4*a*b + b) - 1/3*((a*b^3 - 10*b^3)/(a + 1) - 3*(a*b^4 - 8*b^4)/((
b*x + a + 1)*b) + 3*(a^2*b^5 - 4*a*b^5 - 5*b^5)/((b*x + a + 1)^2*b^2))/((a
+ 1)^3*(a/(b*x + a + 1) + 1/(b*x + a + 1) - 1)^3)
```

3.860 $\int e^{-3\tanh^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=187

$$\frac{(-a-bx+1)^{3/2}\sqrt{a+bx+1}\left(22a^2-2(10a+11)bx+54a+29\right)}{8b^4}+\frac{3\left(8a^3+36a^2+44a+17\right)\sqrt{-a-bx+1}\sqrt{a+bx+}}{8b^4}$$

```
[Out] (-2*x^3*(1 - a - b*x)^(3/2))/(b*Sqrt[1 + a + b*x]) + (3*(17 + 44*a + 36*a^2
 + 8*a^3)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(8*b^4) + (9*x^2*(1 - a - b*
x)^(3/2)*Sqrt[1 + a + b*x])/(4*b^2) + ((1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x
]*(29 + 54*a + 22*a^2 - 2*(11 + 10*a)*b*x))/(8*b^4) + (3*(17 + 44*a + 36*a^
2 + 8*a^3)*ArcSin[a + b*x])/(8*b^4)
```

Rubi [A] time = 0.187765, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6163, 97, 153, 147, 50, 53, 619, 216}

$$\frac{(-a-bx+1)^{3/2}\sqrt{a+bx+1}\left(22a^2-2(10a+11)bx+54a+29\right)}{8b^4}+\frac{3\left(8a^3+36a^2+44a+17\right)\sqrt{-a-bx+1}\sqrt{a+bx+}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(3*ArcTanh[a + b*x]),x]
```

```
[Out] (-2*x^3*(1 - a - b*x)^(3/2))/(b*Sqrt[1 + a + b*x]) + (3*(17 + 44*a + 36*a^2
 + 8*a^3)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(8*b^4) + (9*x^2*(1 - a - b*
x)^(3/2)*Sqrt[1 + a + b*x])/(4*b^2) + ((1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x
]*(29 + 54*a + 22*a^2 - 2*(11 + 10*a)*b*x))/(8*b^4) + (3*(17 + 44*a + 36*a^
2 + 8*a^3)*ArcSin[a + b*x])/(8*b^4)
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(a+bx)}x^3\,dx = \int \frac{x^3(1-a-bx)^{3/2}}{(1+a+bx)^{3/2}}\,dx$$

$$= -\frac{2x^3(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} + \frac{2\int \frac{x^2\left(3(1-a)-\frac{9bx}{2}\right)\sqrt{1-a-bx}}{\sqrt{1+a+bx}}\,dx}{b}$$

$$= -\frac{2x^3(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} + \frac{9x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}{4b^2} - \frac{\int \frac{x\sqrt{1-a-bx}\left(9(1-a)(1+a)b-\frac{3}{2}(11+10a)b^2x\right)}{\sqrt{1+a+bx}}\,dx}{2b^3}$$

$$= -\frac{2x^3(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} + \frac{9x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}{4b^2} + \frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}\,(29+54a}{8b^4}$$

$$= -\frac{2x^3(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} + \frac{3\left(17+44a+36a^2+8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} + \frac{9x^2(1-a-bx)^3}{4b^2}$$

$$= -\frac{2x^3(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} + \frac{3\left(17+44a+36a^2+8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} + \frac{9x^2(1-a-bx)^3}{4b^2}$$

$$= -\frac{2x^3(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} + \frac{3\left(17+44a+36a^2+8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} + \frac{9x^2(1-a-bx)^3}{4b^2}$$

$$= -\frac{2x^3(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} + \frac{3\left(17+44a+36a^2+8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} + \frac{9x^2(1-a-bx)^3}{4b^2}$$

Mathematica [A] time = 0.206157, size = 231, normalized size = 1.24

$$\frac{\sqrt{-b}\left(-a^2\left(12b^2x^2+265bx+4\right)-2a^4(bx+38)-5a^3(20bx+31)-2a^5+a\left(2b^4x^4+4b^3x^3-53b^2x^2-212bx+157\right)+\right.}{8(-b)^{9/2}\sqrt{-(a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^(3*ArcTanh[a + b*x]),x]
```

```
[Out] (Sqrt[-b]*(80 - 2*a^5 - 51*b*x - 40*b^2*x^2 + 17*b^3*x^3 - 8*b^4*x^4 + 2*b^
5*x^5 - 2*a^4*(38 + b*x) - 5*a^3*(31 + 20*b*x) - a^2*(4 + 265*b*x + 12*b^2*
x^2) + a*(157 - 212*b*x - 53*b^2*x^2 + 4*b^3*x^3 + 2*b^4*x^4)) + 6*(17 + 44
*a + 36*a^2 + 8*a^3)*Sqrt[b]*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*ArcSinh[(Sqr
t[b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[-b])])/(8*(-b)^(9/2)*Sqrt[-((-1 + a +
 b*x)*(1 + a + b*x))])
```

Maple [B] time = 0.051, size = 1271, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x)
```

```
[Out] 3/b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a^4+1/b^7/(x+1/b+a/b)^3*(-
(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+2/b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(
1+a)/b))^(3/2)*a^3+5/b^6/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))
```

```
^(5/2)+9/b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)*a^2+11/b^4*(-(x+(1+
a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)*a+6/b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)
/b))^(1/2)*x+27/2/b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a^3+33/2/b
^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a^2+6/b^4*(-(x+(1+a)/b)^2*b^2
+2*b*(x+(1+a)/b))^(1/2)*a+6/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1
/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))+1/4/b^3*x*(-b^2*x^2-2*a*b*x
-a^2+1)^(3/2)+1/4/b^4*a*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+33/2/b^3/(b^2)^(1/2)
*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1
/2))*a+3/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2
*b^2+2*b*(x+(1+a)/b))^(1/2))*a^3+27/2/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x
+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))*a^2+3/8/b^3*x*(-b
^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/8/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/
(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+3/8*a/b^4*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+27
/2/b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x*a^2+33/2/b^3*(-(x+(1+a)
/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x*a+1/b^7/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^
2+2*b*(x+(1+a)/b))^(5/2)*a^3+3/b^7/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x
+(1+a)/b))^(5/2)*a^2+3/b^7/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b
))^(5/2)*a+2/b^6/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)*a
^3+3/b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x*a^3+9/b^6/(x+1/b+a/b)
^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)*a^2+12/b^6/(x+1/b+a/b)^2*(-(x
+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)*a+4/b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(
1+a)/b))^(3/2)
```

Maxima [C] time = 1.51733, size = 1330, normalized size = 7.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)*a^3/(b^6*x^2 + 2*a*b^5*x + a^2*b^4 +
2*b^5*x + 2*a*b^4 + b^4) - 3*(-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)*a^2/(b^6*
x^2 + 2*a*b^5*x + a^2*b^4 + 2*b^5*x + 2*a*b^4 + b^4) + 3/2*(-b^2*x^2 - 2*a*
b*x - a^2 + 1)^(3/2)*a^2/(b^5*x + a*b^4 + b^4) + 6*sqrt(-b^2*x^2 - 2*a*b*x
- a^2 + 1)*a^3/(b^5*x + a*b^4 + b^4) - 3*(-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/
2)*a/(b^6*x^2 + 2*a*b^5*x + a^2*b^4 + 2*b^5*x + 2*a*b^4 + b^4) + 3*(-b^2*x^
2 - 2*a*b*x - a^2 + 1)^(3/2)*a/(b^5*x + a*b^4 + b^4) + 18*sqrt(-b^2*x^2 - 2
*a*b*x - a^2 + 1)*a^2/(b^5*x + a*b^4 + b^4) - (-b^2*x^2 - 2*a*b*x - a^2 + 1
)^(3/2)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4 + 2*b^5*x + 2*a*b^4 + b^4) + 3/2*(-b
^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)/(b^5*x + a*b^4 + b^4) + 18*sqrt(-b^2*x^2
- 2*a*b*x - a^2 + 1)*a/(b^5*x + a*b^4 + b^4) + 3*a^3*arcsin(b*x + a)/b^4 +
6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/(b^5*x + a*b^4 + b^4) + 1/4*(-b^2*x^2
- 2*a*b*x - a^2 + 1)^(3/2)*x/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 4*b*x
 + 4*a + 3)*a*x/b^3 + 27/2*a^2*arcsin(b*x + a)/b^4 - 3/4*(-b^2*x^2 - 2*a*b*
x - a^2 + 1)^(3/2)*a/b^4 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 4*b*x + 4*a +
 3)*a^2/b^4 + 9/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^2/b^4 - 3/2*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 4*b*x + 4*a + 3)*x/b^3 + 3/8*sqrt(-b^2*x^2 - 2*a*b*x
 - a^2 + 1)*x/b^3 + 3/2*I*a*arcsin(b*x + a + 2)/b^4 + 18*a*arcsin(b*x + a)/
b^4 - (-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)/b^4 - 9/2*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 4*b*x + 4*a + 3)*a/b^4 + 75/8*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a
/b^4 + 3/2*I*arcsin(b*x + a + 2)/b^4 + 63/8*arcsin(b*x + a)/b^4 - 3*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 4*b*x + 4*a + 3)/b^4 + 9/2*sqrt(-b^2*x^2 - 2*a*b*x
- a^2 + 1)/b^4
```

Fricas [A] time = 2.09733, size = 456, normalized size = 2.44

$$-\frac{3\left(8a^4+44a^3+\left(8a^3+36a^2+44a+17\right)bx+80a^2+61a+17\right)\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}(bx+a)}{b^2x^2+2abx+a^2-1}\right)+\left(2b^4x^4-6b^3x^3\right.}{8\left(b^5x+(a+1)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(3*(8*a^4 + 44*a^3 + (8*a^3 + 36*a^2 + 44*a + 17)*b*x + 80*a^2 + 61*a
+ 17)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*
x + a^2 - 1)) + (2*b^4*x^4 - 6*b^3*x^3 + (10*a + 11)*b^2*x^2 - 2*a^4 - 78*a
^3 - (22*a^2 + 54*a + 29)*b*x - 233*a^2 - 237*a - 80)*sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1))/(b^5*x + (a + 1)*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a+1)**3*(1-(b*x+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2352, size = 285, normalized size = 1.52

$$-\frac{1}{8}\sqrt{-b^2x^2-2abx-a^2+1}\left(\left(2x\left(\frac{x}{b}-\frac{ab^{11}+4b^{11}}{b^{13}}\right)+\frac{2a^2b^{10}+20ab^{10}+19b^{10}}{b^{13}}\right)x-\frac{2a^3b^9+44a^2b^9+93ab^9+48b}{b^{13}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*((2*x*(x/b - (a*b^11 + 4*b^11)/b^13
) + (2*a^2*b^10 + 20*a*b^10 + 19*b^10)/b^13)*x - (2*a^3*b^9 + 44*a^2*b^9 +
93*a*b^9 + 48*b^9)/b^13) - 3/8*(8*a^3 + 36*a^2 + 44*a + 17)*arcsin(-b*x - a
)*sgn(b)/(b^3*abs(b)) - 8*(a^3 + 3*a^2 + 3*a + 1)/(b^3*((sqrt(-b^2*x^2 - 2*
a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a*b) + 1)*abs(b))
```

3.861 $\int e^{-3\tanh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=167

$$-\frac{\left(6a^2+18a+11\right)\sqrt{a+bx+1}(-a-bx+1)^{3/2}}{6b^3}-\frac{\left(6a^2+18a+11\right)\sqrt{a+bx+1}\sqrt{-a-bx+1}}{2b^3}-\frac{\left(6a^2+18a+11\right)\sin^{-1}(a}{2b^3}$$

```
[Out] -(((1 + a)^2*(1 - a - b*x)^(5/2))/(b^3*Sqrt[1 + a + b*x])) - ((11 + 18*a +
6*a^2)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^3) - ((11 + 18*a + 6*a^2)*
(1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x])/(6*b^3) - ((1 - a - b*x)^(5/2)*Sqrt[
1 + a + b*x])/(3*b^3) - ((11 + 18*a + 6*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rubi [A] time = 0.200624, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6163, 89, 80, 50, 53, 619, 216}

$$-\frac{\left(6a^2+18a+11\right)\sqrt{a+bx+1}(-a-bx+1)^{3/2}}{6b^3}-\frac{\left(6a^2+18a+11\right)\sqrt{a+bx+1}\sqrt{-a-bx+1}}{2b^3}-\frac{\left(6a^2+18a+11\right)\sin^{-1}(a}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(3*ArcTanh[a + b*x]),x]
```

```
[Out] -(((1 + a)^2*(1 - a - b*x)^(5/2))/(b^3*Sqrt[1 + a + b*x])) - ((11 + 18*a +
6*a^2)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^3) - ((11 + 18*a + 6*a^2)*
(1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x])/(6*b^3) - ((1 - a - b*x)^(5/2)*Sqrt[
1 + a + b*x])/(3*b^3) - ((11 + 18*a + 6*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(a+bx)}x^2\,dx = \int \frac{x^2(1-a-bx)^{3/2}}{(1+a+bx)^{3/2}}\,dx$$

$$= -\frac{(1+a)^2(1-a-bx)^{5/2}}{b^3\sqrt{1+a+bx}} + \frac{\int \frac{(1-a-bx)^{3/2}\left(-(1+a)(3+2a)b+b^2x\right)}{\sqrt{1+a+bx}}\,dx}{b^3}$$

$$= -\frac{(1+a)^2(1-a-bx)^{5/2}}{b^3\sqrt{1+a+bx}} - \frac{(1-a-bx)^{5/2}\sqrt{1+a+bx}}{3b^3} - \frac{\left(11+18a+6a^2\right)\int \frac{(1-a-bx)^{3/2}}{\sqrt{1+a+bx}}\,dx}{3b^2}$$

$$= -\frac{(1+a)^2(1-a-bx)^{5/2}}{b^3\sqrt{1+a+bx}} - \frac{\left(11+18a+6a^2\right)(1-a-bx)^{3/2}\sqrt{1+a+bx}}{6b^3} - \frac{(1-a-bx)^{5/2}\sqrt{1+}}{3b^3}$$

$$= -\frac{(1+a)^2(1-a-bx)^{5/2}}{b^3\sqrt{1+a+bx}} - \frac{\left(11+18a+6a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3} - \frac{\left(11+18a+6a^2\right)(1-}{6}$$

$$= -\frac{(1+a)^2(1-a-bx)^{5/2}}{b^3\sqrt{1+a+bx}} - \frac{\left(11+18a+6a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3} - \frac{\left(11+18a+6a^2\right)(1-}{6}$$

$$= -\frac{(1+a)^2(1-a-bx)^{5/2}}{b^3\sqrt{1+a+bx}} - \frac{\left(11+18a+6a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3} - \frac{\left(11+18a+6a^2\right)(1-}{6}$$

$$= -\frac{(1+a)^2(1-a-bx)^{5/2}}{b^3\sqrt{1+a+bx}} - \frac{\left(11+18a+6a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3} - \frac{\left(11+18a+6a^2\right)(1-}{6}$$

Mathematica [A] time = 0.169403, size = 190, normalized size = 1.14

$$-\frac{\sqrt{-b}\left(a^3(2bx+51)+a^2(69bx+50)+2a^4+a\left(2b^3x^3+9b^2x^2+106bx-51\right)+2b^4x^4-9b^3x^3+26b^2x^2+33bx-52\right)}{6(-b)^{7/2}\sqrt{-(a+bx-1)(a+bx+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(3*ArcTanh[a + b*x]),x]
```

```
[Out] -(Sqrt[-b]*(-52 + 2*a^4 + 33*b*x + 26*b^2*x^2 - 9*b^3*x^3 + 2*b^4*x^4 + a^3
*(51 + 2*b*x) + a^2*(50 + 69*b*x) + a*(-51 + 106*b*x + 9*b^2*x^2 + 2*b^3*x^
3)) + 6*(11 + 18*a + 6*a^2)*Sqrt[b]*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*ArcSi
nh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[b])])/(6*(-b)^(7/2)*Sqrt[-((-
1 + a + b*x)*(1 + a + b*x))])
```

Maple [B] time = 0.048, size = 830, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x)
```

```
[Out] -3/b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a^3-1/b^6/(x+1/b+a/b)^3*(
-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)-2/b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+
(1+a)/b))^(3/2)*a^2-6/b^3*a*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)-4/b^
5/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)-11/2/b^2*(-(x+(1
+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x-9/b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a
)/b))^(1/2)*a^2-11/2/b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a-11/2/
b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*
(x+(1+a)/b))^(1/2))-6/b^5*a/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/
b))^(5/2)-9/b^2*a*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x-9/b^2/(b^2)^
(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b
))^(1/2))*a-3/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)
/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))*a^2-1/b^6/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b
^2+2*b*(x+(1+a)/b))^(5/2)*a^2-2/b^6/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(
x+(1+a)/b))^(5/2)*a-2/b^5/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b)
)^(5/2)*a^2-3/b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x*a^2-11/3/b^3
*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)
```

Maxima [C] time = 1.49084, size = 840, normalized size = 5.03

$$\frac{\left(-b^2x^2-2\,abx-a^2+1\right)^{\frac{3}{2}}a^2}{b^5x^2+2\,ab^4x+a^2b^3+2\,b^4x+2\,ab^3+b^3}+\frac{2\left(-b^2x^2-2\,abx-a^2+1\right)^{\frac{3}{2}}a}{b^5x^2+2\,ab^4x+a^2b^3+2\,b^4x+2\,ab^3+b^3}-\frac{\left(-b^2x^2-2\,abx-a^2+1\right)^{\frac{3}{2}}a}{b^4x+ab^3+b^3}-\frac{6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)*a^2/(b^5*x^2 + 2*a*b^4*x + a^2*b^3 + 2
*b^4*x + 2*a*b^3 + b^3) + 2*(-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)*a/(b^5*x^2
 + 2*a*b^4*x + a^2*b^3 + 2*b^4*x + 2*a*b^3 + b^3) - (-b^2*x^2 - 2*a*b*x - a
^2 + 1)^(3/2)*a/(b^4*x + a*b^3 + b^3) - 6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1
)*a^2/(b^4*x + a*b^3 + b^3) + (-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)/(b^5*x^2
 + 2*a*b^4*x + a^2*b^3 + 2*b^4*x + 2*a*b^3 + b^3) - (-b^2*x^2 - 2*a*b*x - a
^2 + 1)^(3/2)/(b^4*x + a*b^3 + b^3) - 12*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
*a/(b^4*x + a*b^3 + b^3) - 3*a^2*arcsin(b*x + a)/b^3 - 6*sqrt(-b^2*x^2 - 2*
a*b*x - a^2 + 1)/(b^4*x + a*b^3 + b^3) + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
```

```
 4*b*x + 4*a + 3)*x/b^2 - 9*a*arcsin(b*x + a)/b^3 + 1/3*(-b^2*x^2 - 2*a*b*x
 - a^2 + 1)^(3/2)/b^3 + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 4*b*x + 4*a + 3)
*a/b^3 - 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/b^3 - 1/2*I*arcsin(b*x + a
+ 2)/b^3 - 6*arcsin(b*x + a)/b^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 4*b*x + 4
*a + 3)/b^3 - 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/b^3
```

Fricas [A] time = 2.01453, size = 375, normalized size = 2.25

$$\frac{3\left(6a^3+\left(6a^2+18a+11\right)bx+24a^2+29a+11\right)\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}(bx+a)}{b^2x^2+2abx+a^2-1}\right)-\left(2b^3x^3-7b^2x^2+2a^3+(16a+1\right.}{6\left(b^4x+(a+1)b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(6*a^3 + (6*a^2 + 18*a + 11)*b*x + 24*a^2 + 29*a + 11)*arctan(sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (2*
b^3*x^3 - 7*b^2*x^2 + 2*a^3 + (16*a + 19)*b*x + 53*a^2 + 103*a + 52)*sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1))/(b^4*x + (a + 1)*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a+1)**3*(1-(b*x+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19749, size = 224, normalized size = 1.34

$$-\frac{1}{6}\sqrt{-b^2x^2-2abx-a^2+1}\left(x\left(\frac{2x}{b}-\frac{2ab^6+9b^6}{b^8}\right)+\frac{2a^2b^5+27ab^5+28b^5}{b^8}\right)+\frac{\left(6a^2+18a+11\right)\arcsin(-bx-a)\,\mathrm{sg}}{2b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(x*(2*x/b - (2*a*b^6 + 9*b^6)/b^8)
+ (2*a^2*b^5 + 27*a*b^5 + 28*b^5)/b^8) + 1/2*(6*a^2 + 18*a + 11)*arcsin(-b*
x - a)*sgn(b)/(b^2*abs(b)) + 8*(a^2 + 2*a + 1)/(b^2*((sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1)*abs(b) + b)/(b^2*x + a*b) + 1)*abs(b))
```

3.862 $\int e^{-3\tanh^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=119

$$\frac{(a+1)(-a-bx+1)^{5/2}}{b^2\sqrt{a+bx+1}}+\frac{(2a+3)\sqrt{a+bx+1}(-a-bx+1)^{3/2}}{2b^2}+\frac{3(2a+3)\sqrt{a+bx+1}\sqrt{-a-bx+1}}{2b^2}+\frac{3(2a+3)\sin^{-1}(a}{2b^2}$$

```
[Out] ((1 + a)*(1 - a - b*x)^(5/2))/(b^2*Sqrt[1 + a + b*x]) + (3*(3 + 2*a)*Sqrt[1
 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^2) + ((3 + 2*a)*(1 - a - b*x)^(3/2)*Sqr
t[1 + a + b*x])/(2*b^2) + (3*(3 + 2*a)*ArcSin[a + b*x])/(2*b^2)
```

Rubi [A] time = 0.0921294, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6163, 78, 50, 53, 619, 216}

$$\frac{(a+1)(-a-bx+1)^{5/2}}{b^2\sqrt{a+bx+1}}+\frac{(2a+3)\sqrt{a+bx+1}(-a-bx+1)^{3/2}}{2b^2}+\frac{3(2a+3)\sqrt{a+bx+1}\sqrt{-a-bx+1}}{2b^2}+\frac{3(2a+3)\sin^{-1}(a}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(3*ArcTanh[a + b*x]),x]
```

```
[Out] ((1 + a)*(1 - a - b*x)^(5/2))/(b^2*Sqrt[1 + a + b*x]) + (3*(3 + 2*a)*Sqrt[1
 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^2) + ((3 + 2*a)*(1 - a - b*x)^(3/2)*Sqr
t[1 + a + b*x])/(2*b^2) + (3*(3 + 2*a)*ArcSin[a + b*x])/(2*b^2)
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(a+bx)}x\,dx = \int \frac{x(1-a-bx)^{3/2}}{(1+a+bx)^{3/2}}\,dx$$

$$= \frac{(1+a)(1-a-bx)^{5/2}}{b^2\sqrt{1+a+bx}} + \frac{(3+2a)\int \frac{(1-a-bx)^{3/2}}{\sqrt{1+a+bx}}\,dx}{b}$$

$$= \frac{(1+a)(1-a-bx)^{5/2}}{b^2\sqrt{1+a+bx}} + \frac{(3+2a)(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2b^2} + \frac{(3(3+2a))\int \frac{\sqrt{1-a-bx}}{\sqrt{1+a+bx}}\,dx}{2b}$$

$$= \frac{(1+a)(1-a-bx)^{5/2}}{b^2\sqrt{1+a+bx}} + \frac{3(3+2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2} + \frac{(3+2a)(1-a-bx)^{3/2}\sqrt{1+a+b}}{2b^2}$$

$$= \frac{(1+a)(1-a-bx)^{5/2}}{b^2\sqrt{1+a+bx}} + \frac{3(3+2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2} + \frac{(3+2a)(1-a-bx)^{3/2}\sqrt{1+a+b}}{2b^2}$$

$$= \frac{(1+a)(1-a-bx)^{5/2}}{b^2\sqrt{1+a+bx}} + \frac{3(3+2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2} + \frac{(3+2a)(1-a-bx)^{3/2}\sqrt{1+a+b}}{2b^2}$$

$$= \frac{(1+a)(1-a-bx)^{5/2}}{b^2\sqrt{1+a+bx}} + \frac{3(3+2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2} + \frac{(3+2a)(1-a-bx)^{3/2}\sqrt{1+a+b}}{2b^2}$$

Mathematica [A] time = 0.135023, size = 157, normalized size = 1.32

$$\frac{\sqrt{-b}\left(-a^2(bx+14)-a^3+a\left(b^2x^2-20bx+1\right)+b^3x^3-6b^2x^2-9bx+14\right)+6(2a+3)\sqrt{b}\sqrt{-a^2-2abx-b^2x^2+1}\,\text{sinh}}{2(-b)^{5/2}\sqrt{-(a+bx-1)(a+bx+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(3*ArcTanh[a + b*x]),x]
```

```
[Out] (Sqrt[-b]*(14 - a^3 - 9*b*x - 6*b^2*x^2 + b^3*x^3 - a^2*(14 + b*x) + a*(1 -
 20*b*x + b^2*x^2)) + 6*(3 + 2*a)*Sqrt[b]*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]
*ArcSinh[(Sqrt[b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[-b])])/(2*(-b)^(5/2)*Sqr
t[-((-1 + a + b*x)*(1 + a + b*x))])
```

Maple [B] time = 0.043, size = 543, normalized size = 4.6

$$3\,\frac{1}{b^4}\left(-\left(x+\frac{1+a}{b}\right)^2 b^2+2\,b\left(x+\frac{1+a}{b}\right)\right)^{5/2}\left(x+b^{-1}+\frac{a}{b}\right)^{-2}+3\,\frac{1}{b^2}\left(-\left(x+\frac{1+a}{b}\right)^2 b^2+2\,b\left(x+\frac{1+a}{b}\right)\right)^{3/2}+\frac{9\,x}{2\,b}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x)
```

```
[Out] 3/b^4/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+3/b^2*(-(x+(
1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)+9/2/b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)
/b))^(1/2)*x+9/2/b^2*a*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)+9/2/b/(b^
2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a
)/b))^(1/2))+1/b^5/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)
+1/b^5*a/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+2/b^4*a/(
x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+2/b^2*a*(-(x+(1+a)/
b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)+3/b*a*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^
(1/2)*x+3/b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a^2+3/b*a/(b^2)^(1
/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))
^(1/2))
```

Maxima [B] time = 1.45439, size = 404, normalized size = 3.39

$$-\frac{\left(-b^2x^2-2\,abx-a^2+1\right)^{\frac{3}{2}}a}{b^4x^2+2\,ab^3x+a^2b^2+2\,b^3x+2\,ab^2+b^2}-\frac{\left(-b^2x^2-2\,abx-a^2+1\right)^{\frac{3}{2}}}{b^4x^2+2\,ab^3x+a^2b^2+2\,b^3x+2\,ab^2+b^2}+\frac{\left(-b^2x^2-2\,abx-a^2+1\right)^{\frac{3}{2}}}{2\left(b^3x+ab^2+b^2\right)}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)*a/(b^4*x^2 + 2*a*b^3*x + a^2*b^2 + 2*
b^3*x + 2*a*b^2 + b^2) - (-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)/(b^4*x^2 + 2*
a*b^3*x + a^2*b^2 + 2*b^3*x + 2*a*b^2 + b^2) + 1/2*(-b^2*x^2 - 2*a*b*x - a^
2 + 1)^(3/2)/(b^3*x + a*b^2 + b^2) + 6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a
/(b^3*x + a*b^2 + b^2) + 3*a*arcsin(b*x + a)/b^2 + 6*sqrt(-b^2*x^2 - 2*a*b*
x - a^2 + 1)/(b^3*x + a*b^2 + b^2) + 9/2*arcsin(b*x + a)/b^2 + 3/2*sqrt(-b^
2*x^2 - 2*a*b*x - a^2 + 1)/b^2
```

Fricas [A] time = 1.8327, size = 300, normalized size = 2.52

$$-\frac{3\left((2\,a+3)bx+2\,a^2+5\,a+3\right)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\left(b^2x^2-a^2-5\,bx-15\,a-14\right)\sqrt{-b^2x^2-2\,abx-a^2+1}}{2\left(b^3x+(a+1)b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(3*((2*a + 3)*b*x + 2*a^2 + 5*a + 3)*arctan(sqrt(-b^2*x^2 - 2*a*b*x -
a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (b^2*x^2 - a^2 - 5*b*x
- 15*a - 14)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/(b^3*x + (a + 1)*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)**3*(1-(b*x+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22523, size = 171, normalized size = 1.44

$$-\frac{1}{2}\sqrt{-b^2x^2-2\,abx-a^2+1}\left(\frac{x}{b}-\frac{ab^2+6\,b^2}{b^4}\right)-\frac{3\,(2\,a+3)\arcsin\left(-bx-a\right)\operatorname{sgn}\left(b\right)}{2\,b|b|}-\frac{8\,(a+1)}{b\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b}{b^2x+ab}+1\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(x/b - (a*b^2 + 6*b^2)/b^4) - 3/2*(
2*a + 3)*arcsin(-b*x - a)*sgn(b)/(b*abs(b)) - 8*(a + 1)/(b*((sqrt(-b^2*x^2
- 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a*b) + 1)*abs(b))
```

3.863 $\int e^{-3\tanh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=68

$$-\frac{2(-a-bx+1)^{3/2}}{b\sqrt{a+bx+1}}-\frac{3\sqrt{a+bx+1}\sqrt{-a-bx+1}}{b}-\frac{3\sin^{-1}(a+bx)}{b}$$

```
[Out] (-2*(1 - a - b*x)^(3/2))/(b*Sqrt[1 + a + b*x]) - (3*Sqrt[1 - a - b*x]*Sqrt[
1 + a + b*x])/b - (3*ArcSin[a + b*x])/b
```

Rubi [A] time = 0.0337114, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6161, 47, 50, 53, 619, 216}

$$-\frac{2(-a-bx+1)^{3/2}}{b\sqrt{a+bx+1}}-\frac{3\sqrt{a+bx+1}\sqrt{-a-bx+1}}{b}-\frac{3\sin^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-3*ArcTanh[a + b*x]),x]
```

```
[Out] (-2*(1 - a - b*x)^(3/2))/(b*Sqrt[1 + a + b*x]) - (3*Sqrt[1 - a - b*x]*Sqrt[
1 + a + b*x])/b - (3*ArcSin[a + b*x])/b
```

Rule 6161

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 + a*c
 + b*c*x)^(n/2)/(1 - a*c - b*c*x)^(n/2), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{-3\tanh^{-1}(a+bx)}\,dx &= \int \frac{(1-a-bx)^{3/2}}{(1+a+bx)^{3/2}}\,dx\\
&= -\frac{2(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} - 3\int \frac{\sqrt{1-a-bx}}{\sqrt{1+a+bx}}\,dx\\
&= -\frac{2(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} - \frac{3\sqrt{1-a-bx}\sqrt{1+a+bx}}{b} - 3\int \frac{1}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx\\
&= -\frac{2(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} - \frac{3\sqrt{1-a-bx}\sqrt{1+a+bx}}{b} - 3\int \frac{1}{\sqrt{(1-a)(1+a)-2abx-b^2x^2}}\,dx\\
&= -\frac{2(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} - \frac{3\sqrt{1-a-bx}\sqrt{1+a+bx}}{b} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2ab-2b^2x\right)}{2b^2}\\
&= -\frac{2(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} - \frac{3\sqrt{1-a-bx}\sqrt{1+a+bx}}{b} - \frac{3\sin^{-1}(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0379708, size = 43, normalized size = 0.63

$$\frac{\sqrt{1-(a+bx)^2}\left(-\frac{4}{a+bx+1}-1\right)}{b} - \frac{3\sin^{-1}(a+bx)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-3*ArcTanh[a + b*x]),x]
```

```
[Out] (Sqrt[1 - (a + b*x)^2]*(-1 - 4/(1 + a + b*x)))/b - (3*ArcSin[a + b*x])/b
```

Maple [B] time = 0.035, size = 264, normalized size = 3.9

$$-\frac{1}{b^4}\left(-\left(x+\frac{1+a}{b}\right)^2 b^2 + 2b\left(x+\frac{1+a}{b}\right)\right)^{\frac{5}{2}}\left(x+b^{-1}+\frac{a}{b}\right)^{-3} - 2\frac{1}{b^3}\left(-\left(x+\frac{1+a}{b}\right)^2 b^2 + 2b\left(x+\frac{1+a}{b}\right)\right)^{5/2}\left(x+b^{-1}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x)
```

```
[Out] -1/b^4/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)-2/b^3/(x+1/
b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)-2/b*(-(x+(1+a)/b)^2*b^2
+2*b*(x+(1+a)/b))^(3/2)-3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x-3/b*
(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a-3/(b^2)^(1/2)*arctan((b^2)^(1/
```

```
2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))
```

Maxima [A] time = 1.45365, size = 140, normalized size = 2.06

$$\frac{\left(-b^2x^2 - 2\,abx - a^2 + 1\right)^{\frac{3}{2}}}{b^3x^2 + 2\,ab^2x + a^2b + 2\,b^2x + 2\,ab + b} - \frac{3\,\arcsin\left(bx + a\right)}{b} - \frac{6\,\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}}{b^2x + ab + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)/(b^3*x^2 + 2*a*b^2*x + a^2*b + 2*b^2*x
 + 2*a*b + b) - 3*arcsin(b*x + a)/b - 6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/
(b^2*x + a*b + b)
```

Fricas [A] time = 1.63828, size = 234, normalized size = 3.44

$$\frac{3\left(bx + a + 1\right)\arctan\left(\frac{\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}(bx + a)}{b^2x^2 + 2\,abx + a^2 - 1}\right) - \sqrt{-b^2x^2 - 2\,abx - a^2 + 1}(bx + a + 5)}{b^2x + (a + 1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (3*(b*x + a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x
^2 + 2*a*b*x + a^2 - 1)) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a + 5)
)/(b^2*x + (a + 1)*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(a + bx - 1\right)\left(a + bx + 1\right)\right)^{\frac{3}{2}}}{\left(a + bx + 1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**3*(1-(b*x+a)**2)**(3/2),x)
```

```
[Out] Integral((-(a + b*x - 1)*(a + b*x + 1))**(3/2)/(a + b*x + 1)**3, x)
```

Giac [A] time = 1.1759, size = 127, normalized size = 1.87

$$\frac{3\,\arcsin\left(-bx - a\right)\mathrm{sgn}\left(b\right)}{|b|} - \frac{\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}}{b} + \frac{8}{\left(\frac{\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}|b| + b}{b^2x + ab} + 1\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3*arcsin(-b*x - a)*sgn(b)/abs(b) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/b + 8
/(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a*b) + 1)*abs(b
))
```

3.864 $\int \frac{e^{-3\tanh^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=103

$$-\frac{2(1-a)^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(a+1)\sqrt{1-a^2}}+\frac{4\sqrt{-a-bx+1}}{(a+1)\sqrt{a+bx+1}}+\sin^{-1}(a+bx)$$

```
[Out] (4*Sqrt[1 - a - b*x])/((1 + a)*Sqrt[1 + a + b*x]) + ArcSin[a + b*x] - (2*(1
 - a)^2*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b
*x])])/((1 + a)*Sqrt[1 - a^2])
```

Rubi [A] time = 0.0853212, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6163, 98, 157, 53, 619, 216, 93, 208}

$$-\frac{2(1-a)^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(a+1)\sqrt{1-a^2}}+\frac{4\sqrt{-a-bx+1}}{(a+1)\sqrt{a+bx+1}}+\sin^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a + b*x])*x),x]
```

```
[Out] (4*Sqrt[1 - a - b*x])/((1 + a)*Sqrt[1 + a + b*x]) + ArcSin[a + b*x] - (2*(1
 - a)^2*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b
*x])])/((1 + a)*Sqrt[1 - a^2])
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
```

```
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(a+bx)}}{x}\,dx &= \int \frac{(1-a-bx)^{3/2}}{x(1+a+bx)^{3/2}}\,dx\\
&= \frac{4\sqrt{1-a-bx}}{(1+a)\sqrt{1+a+bx}} + \frac{2\int \frac{\frac{1}{2}(1-a)^2b+\frac{1}{2}(1+a)b^2x}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{(1+a)b}\\
&= \frac{4\sqrt{1-a-bx}}{(1+a)\sqrt{1+a+bx}} + \frac{(1-a)^2\int \frac{1}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{1+a} + b\int \frac{1}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx\\
&= \frac{4\sqrt{1-a-bx}}{(1+a)\sqrt{1+a+bx}} + \frac{\left(2(1-a)^2\right)\text{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx, x, \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\right)}{1+a} + b\int \frac{1}{\sqrt{(1-a)(1+a)}}\\
&= \frac{4\sqrt{1-a-bx}}{(1+a)\sqrt{1+a+bx}} - \frac{2(1-a)^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{(1+a)\sqrt{1-a^2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2ab-2b^2x\right)}{2b}\\
&= \frac{4\sqrt{1-a-bx}}{(1+a)\sqrt{1+a+bx}} + \sin^{-1}(a+bx) - \frac{2(1-a)^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{(1+a)\sqrt{1-a^2}}
\end{aligned}$$

Mathematica [A] time = 0.442148, size = 131, normalized size = 1.27

$$-\frac{4(a+bx-1)}{(a+1)\sqrt{-(a+bx-1)(a+bx+1)}} + 2\left(\frac{a-1}{a+1}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{-a-bx+1}}{\sqrt{\frac{a-1}{a+1}}\sqrt{a+bx+1}}\right) + \frac{2\sqrt{-b}\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a + b*x])*x),x]
```

```
[Out] (-4*(-1 + a + b*x))/((1 + a)*Sqrt[-((-1 + a + b*x)*(1 + a + b*x))]) + (2*Sq
rt[-b]*ArcSinh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[b])])/Sqrt[b] + 2
*((-1 + a)/(1 + a))^(3/2)*ArcTan[Sqrt[1 - a - b*x]/(Sqrt[(-1 + a)/(1 + a)]*
Sqrt[1 + a + b*x])]
```

Maple [B] time = 0.066, size = 1062, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x,x)
```

```
[Out] -1/(1+a)^2/b^2/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)-1/(
1+a)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)-3/2/(1+a)^2*b*(-(x+(1+a)/
b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x-3/2/(1+a)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1
+a)/b))^(1/2)*a-3/2/(1+a)^2*b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b
)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))-1/3/(1+a)^3*(-(x+(1+a)/b)^2*b
^2+2*b*(x+(1+a)/b))^(3/2)-1/2/(1+a)^3*b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b)
)^(1/2)*x-1/2/(1+a)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a-1/2/(1+a
)^3*b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*
b*(x+(1+a)/b))^(1/2))+1/(1+a)/b^3/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+
(1+a)/b))^(5/2)+2/(1+a)/b^2/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/
b))^(5/2)+2/(1+a)*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)+3/(1+a)*b*(-(x
+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x+3/(1+a)*(-(x+(1+a)/b)^2*b^2+2*b*(x
+(1+a)/b))^(1/2)*a+3/(1+a)*b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)
/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))+1/3/(1+a)^3*(-b^2*x^2-2*a*b*x-
a^2+1)^(3/2)-1/2/(1+a)^3*a*b*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-3/2/(1+a)^3*a
^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/2/(1+a)^3*a*b/(b^2)^(1/2)*arctan((b^2)^
(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/(1+a)^3*(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2)+1/(1+a)^3*a^3*b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^
2-2*a*b*x-a^2+1)^(1/2))+1/(1+a)^3*(-a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2*(-a
^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)*a^2-1/(1+a)^3*(-a^2+1)^(1/2)
*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-(bx+a)^2+1\right)^{\frac{3}{2}}}{(bx+a+1)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((-(b*x + a)^2 + 1)^(3/2)/((b*x + a + 1)^3*x), x)
```

Fricas [B] time = 1.83458, size = 1057, normalized size = 10.26

$$\left[\frac{\left((a-1)bx+a^2-1\right)\sqrt{-\frac{a-1}{a+1}}\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx-4a^2-2\sqrt{-b^2x^2-2abx-a^2+1}\left(a^3+\left(a^2+a\right)bx+a^2-a-1\right)\sqrt{-\frac{a-1}{a+1}}+2}{x^2}\right)-2\left((a+1\right.}{2\left((a+1)bx+a^2+2a+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(((a - 1)*b*x + a^2 - 1)*sqrt(-(a - 1)/(a + 1))*log(((2*a^2 - 1)*b^2*x
^2 + 2*a^4 + 4*(a^3 - a)*b*x - 4*a^2 - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
*(a^3 + (a^2 + a)*b*x + a^2 - a - 1)*sqrt(-(a - 1)/(a + 1)) + 2)/x^2) - 2*(
(a + 1)*b*x + a^2 + 2*a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x
 + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + 8*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
)/((a + 1)*b*x + a^2 + 2*a + 1), (((a - 1)*b*x + a^2 - 1)*sqrt((a - 1)/(a +
 1))*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt((a -
1)/(a + 1))/((a - 1)*b^2*x^2 + a^3 + 2*(a^2 - a)*b*x - a^2 - a + 1)) - ((a
+ 1)*b*x + a^2 + 2*a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x +
a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + 4*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/(
(a + 1)*b*x + a^2 + 2*a + 1)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**3*(1-(b*x+a)**2)**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21332, size = 208, normalized size = 2.02

$$-\frac{b\arcsin\left(-bx-a\right)\mathrm{sgn}\left(b\right)}{|b|}+\frac{2\left(a^2b-2ab+b\right)\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2abx-a^2+1}|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}(a|b|+|b|)}-\frac{8b}{(a|b|+|b|)\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}|b|+b}{b^2x+ab}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -b*arcsin(-b*x - a)*sgn(b)/abs(b) + 2*(a^2*b - 2*a*b + b)*arctan(((sqrt(-b^
2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/
(sqrt(a^2 - 1)*(a*abs(b) + abs(b))) - 8*b/((a*abs(b) + abs(b))*((sqrt(-b^2*
x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a*b) + 1))
```

3.865 $\int \frac{e^{-3\tanh^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=130

$$\frac{6(1-a)b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(a+1)^2\sqrt{1-a^2}}-\frac{(-a-bx+1)^{3/2}}{(a+1)x\sqrt{a+bx+1}}-\frac{6b\sqrt{-a-bx+1}}{(a+1)^2\sqrt{a+bx+1}}$$

```
[Out] (-6*b*Sqrt[1 - a - b*x])/((1 + a)^2*Sqrt[1 + a + b*x]) - (1 - a - b*x)^(3/2
)/((1 + a)*x*Sqrt[1 + a + b*x]) + (6*(1 - a)*b*ArcTanh[(Sqrt[1 - a]*Sqrt[1
+ a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 + a)^2*Sqrt[1 - a^2])
```

Rubi [A] time = 0.0768065, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6163, 94, 93, 208}

$$\frac{6(1-a)b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(a+1)^2\sqrt{1-a^2}}-\frac{(-a-bx+1)^{3/2}}{(a+1)x\sqrt{a+bx+1}}-\frac{6b\sqrt{-a-bx+1}}{(a+1)^2\sqrt{a+bx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a + b*x])*x^2),x]
```


```
[Out] (-6*b*Sqrt[1 - a - b*x])/((1 + a)^2*Sqrt[1 + a + b*x]) - (1 - a - b*x)^(3/2
)/((1 + a)*x*Sqrt[1 + a + b*x]) + (6*(1 - a)*b*ArcTanh[(Sqrt[1 - a]*Sqrt[1
+ a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 + a)^2*Sqrt[1 - a^2])
```


Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(a+bx)}}{x^2}\,dx = \int \frac{(1-a-bx)^{3/2}}{x^2(1+a+bx)^{3/2}}\,dx$$

$$= -\frac{(1-a-bx)^{3/2}}{(1+a)x\sqrt{1+a+bx}} - \frac{(3b)\int \frac{\sqrt{1-a-bx}}{x(1+a+bx)^{3/2}}\,dx}{1+a}$$

$$= -\frac{6b\sqrt{1-a-bx}}{(1+a)^2\sqrt{1+a+bx}} - \frac{(1-a-bx)^{3/2}}{(1+a)x\sqrt{1+a+bx}} - \frac{(3(1-a)b)\int \frac{1}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{(1+a)^2}$$

$$= -\frac{6b\sqrt{1-a-bx}}{(1+a)^2\sqrt{1+a+bx}} - \frac{(1-a-bx)^{3/2}}{(1+a)x\sqrt{1+a+bx}} - \frac{(6(1-a)b)\operatorname{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx, x, \frac{\sqrt{1+a+}}{\sqrt{1-a-}}\right.}{(1+a)^2}$$

$$= -\frac{6b\sqrt{1-a-bx}}{(1+a)^2\sqrt{1+a+bx}} - \frac{(1-a-bx)^{3/2}}{(1+a)x\sqrt{1+a+bx}} + \frac{6(1-a)b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{(1+a)^2\sqrt{1-a^2}}$$

Mathematica [A] time = 0.0984897, size = 101, normalized size = 0.78

$$\frac{\sqrt{-a-bx+1}\left(a^2+abx-5bx-1\right)}{(a+1)^2x\sqrt{a+bx+1}} + \frac{6\sqrt{a-1}b\tan^{-1}\left(\frac{\sqrt{-a-bx+1}}{\sqrt{\frac{a-1}{a+1}}\sqrt{a+bx+1}}\right)}{(a+1)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a + b*x])*x^2),x]
```

```
[Out] (Sqrt[1 - a - b*x]*(-1 + a^2 - 5*b*x + a*b*x))/((1 + a)^2*x*Sqrt[1 + a + b*
x]) + (6*Sqrt[-1 + a]*b*ArcTan[Sqrt[1 - a - b*x]/(Sqrt[(-1 + a)/(1 + a)]*Sq
rt[1 + a + b*x])])/(1 + a)^(5/2)
```

Maple [B] time = 0.08, size = 1710, normalized size = 13.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^2,x)
```

```
[Out] -3/(1+a)^2*b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)
^2*b^2+2*b*(x+(1+a)/b))^(1/2))-2/(1+a)^2*b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)
/b))^(3/2)+2/(1+a)^3*b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)+1/(1+a)^4
*b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)-1/(1+a)^4*b*(-b^2*x^2-2*a*b*x
-a^2+1)^(3/2)-3/(1+a)^4*b*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+9/2/(1+a)^3*a^2*b^
2/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^
(1/2))-3/(1+a)^3*a^4*b^2/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-
b^2*x^2-2*a*b*x-a^2+1)^(1/2))+3/2/(1+a)^3*a^2*b^2/(-a^2+1)*(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2)*x+3/(1+a)^3*b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x
+3/(1+a)^3*b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a+3/(1+a)^3*b^2/(b^
2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a
)/b))^(1/2))+3/2/(1+a)^4*b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x+3
/2/(1+a)^4*b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a+3/2/(1+a)^4*b^2/(
b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1
```

```
+a)/b))^(1/2))-1/(1+a)^3/(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(5/2)+2/(1+a)^
3/b/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+9/2/(1+a)^4*b*
a^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/(1+a)^4*b*(-a^2+1)^(1/2)*ln((-2*a^2+2-
2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/(1+a)^2/b^2/(
x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)-2/(1+a)^2/b/(x+1/b+
a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)-3/(1+a)^2*b^2*(-(x+(1+a)/
b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x-3/(1+a)^2*b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1
+a)/b))^(1/2)*a+3/2/(1+a)^4*b^2*a*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+9/2/(1+a
)^4*b^2*a/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(
1/2))-3/(1+a)^4*b^2*a^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2))-3/(1+a)^4*b*(-a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2
+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)*a^2-9/2/(1+a)^3*a*b/(-a^2+1)*(
-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/(1+a)^3*a^3*b/(-a^2+1)^(1/2)*ln((-2*a^2+2-2
*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+3/(1+a)^3*a*b/(-
a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)
^(1/2))/x)-1/(1+a)^3*b^2/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)*x-3/2/(1+a
)^3*b^2/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-3/2/(1+a)^3*b^2/(-a^2+1)/
(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-2/(1
+a)^3*a*b/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+9/2/(1+a)^3*a^3*b/(-a^2+1
)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-(bx+a)^2+1\right)^{\frac{3}{2}}}{(bx+a+1)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((-(b*x + a)^2 + 1)^(3/2)/((b*x + a + 1)^3*x^2), x)
```

Fricas [A] time = 1.82437, size = 871, normalized size = 6.7

$$\left[\frac{3\left(b^2x^2+(a+1)bx\right)\sqrt{-\frac{a-1}{a+1}}\log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx-4a^2-2\sqrt{-b^2x^2-2abx-a^2+1}\left(a^3+(a^2+a)bx+a^2-a-1\right)\sqrt{-\frac{a-1}{a+1}}+2}{x^2}\right)+2\sqrt{-b^2x^2-\cdots}}{2\left(\left(a^2+2a+1\right)bx^2+\left(a^3+3a^2+3a+1\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^2 + (a + 1)*b*x)*sqrt(-(a - 1)/(a + 1))*log(((2*a^2 - 1)*b^2
*x^2 + 2*a^4 + 4*(a^3 - a)*b*x - 4*a^2 - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
1)*(a^3 + (a^2 + a)*b*x + a^2 - a - 1)*sqrt(-(a - 1)/(a + 1)) + 2)/x^2) + 2
*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*((a - 5)*b*x + a^2 - 1))/((a^2 + 2*a +
1)*b*x^2 + (a^3 + 3*a^2 + 3*a + 1)*x), (3*(b^2*x^2 + (a + 1)*b*x)*sqrt((a -
 1)/(a + 1))*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sq
rt((a - 1)/(a + 1))/((a - 1)*b^2*x^2 + a^3 + 2*(a^2 - a)*b*x - a^2 - a + 1)
) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*((a - 5)*b*x + a^2 - 1))/((a^2 + 2*a
 + 1)*b*x^2 + (a^3 + 3*a^2 + 3*a + 1)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**3*(1-(b*x+a)**2)**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22405, size = 815, normalized size = 6.27

$$\frac{6\left(ab^2-b^2\right)\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\left(a^2|b|+2\,a|b|+|b|\right)\sqrt{a^2-1}}+\frac{2\left(\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)a^2b^2}{b^2x+ab}+\frac{4\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)^2a^2b^2}{\left(b^2x+ab\right)^2}+5\,a^2b^2-\frac{10}{}\right.}{\left(a^3|b|+2\,a^2|b|+a|b|\right)\left(\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)a}{b^2x+ab}+\frac{\left(\sqrt{-b^2x^2-2}\right.}{(}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 6*(a*b^2 - b^2)*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(
b^2*x + a*b) - 1)/sqrt(a^2 - 1))/((a^2*abs(b) + 2*a*abs(b) + abs(b))*sqrt(a
^2 - 1)) + 2*((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a^2*b^2/(b^2*
x + a*b) + 4*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^2*b^2/(b^2
*x + a*b)^2 + 5*a^2*b^2 - 10*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b
)*a*b^2/(b^2*x + a*b) - (sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a
*b^2/(b^2*x + a*b)^2 - a*b^2 + (sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) +
 b)*b^2/(b^2*x + a*b) + (sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*b
^2/(b^2*x + a*b)^2)/((a^3*abs(b) + 2*a^2*abs(b) + a*abs(b))*((sqrt(-b^2*x^2
 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) + (sqrt(-b^2*x^2 - 2*a*b*
x - a^2 + 1)*abs(b) + b)^2*a/(b^2*x + a*b)^2 + (sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1)*abs(b) + b)^3*a/(b^2*x + a*b)^3 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x -
a^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*
abs(b) + b)^2/(b^2*x + a*b)^2))
```

3.866 $\int \frac{e^{-3\tanh^{-1}(a+bx)}}{x^3} dx$

Optimal. Leaf size=200

$$-\frac{3(3-2a)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(a+1)^3\sqrt{1-a^2}}-\frac{(-a-bx+1)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{a+bx+1}}+\frac{3(3-2a)b^2\sqrt{-a-bx+1}}{(1-a)(a+1)^3\sqrt{a+bx+1}}+\frac{(3-2a)b(-a-bx+1}{2(1-a)(a+1)^2x\sqrt{a+b}}$$

```
[Out] (3*(3 - 2*a)*b^2*Sqrt[1 - a - b*x])/((1 - a)*(1 + a)^3*Sqrt[1 + a + b*x]) +
 ((3 - 2*a)*b*(1 - a - b*x)^(3/2))/(2*(1 - a)*(1 + a)^2*x*Sqrt[1 + a + b*x]
) - (1 - a - b*x)^(5/2)/(2*(1 - a^2)*x^2*Sqrt[1 + a + b*x]) - (3*(3 - 2*a)*
b^2*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])
])/((1 + a)^3*Sqrt[1 - a^2])
```

Rubi [A] time = 0.129499, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6163, 96, 94, 93, 208}

$$-\frac{3(3-2a)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(a+1)^3\sqrt{1-a^2}}-\frac{(-a-bx+1)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{a+bx+1}}+\frac{3(3-2a)b^2\sqrt{-a-bx+1}}{(1-a)(a+1)^3\sqrt{a+bx+1}}+\frac{(3-2a)b(-a-bx+1}{2(1-a)(a+1)^2x\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a + b*x])*x^3),x]
```


```
[Out] (3*(3 - 2*a)*b^2*Sqrt[1 - a - b*x])/((1 - a)*(1 + a)^3*Sqrt[1 + a + b*x]) +
 ((3 - 2*a)*b*(1 - a - b*x)^(3/2))/(2*(1 - a)*(1 + a)^2*x*Sqrt[1 + a + b*x]
) - (1 - a - b*x)^(5/2)/(2*(1 - a^2)*x^2*Sqrt[1 + a + b*x]) - (3*(3 - 2*a)*
b^2*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])
])/((1 + a)^3*Sqrt[1 - a^2])
```


Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(a+bx)}}{x^3}\,dx = \int \frac{(1-a-bx)^{3/2}}{x^3(1+a+bx)^{3/2}}\,dx$$
$$= -\frac{(1-a-bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1+a+bx}} - \frac{((3-2a)b)\int \frac{(1-a-bx)^{3/2}}{x^2(1+a+bx)^{3/2}}\,dx}{2\left(1-a^2\right)}$$
$$= \frac{(3-2a)b(1-a-bx)^{3/2}}{2(1-a)(1+a)^2x\sqrt{1+a+bx}} - \frac{(1-a-bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1+a+bx}} + \frac{\left(3(3-2a)b^2\right)\int \frac{\sqrt{1-a-bx}}{x(1+a+bx)^{3/2}}\,dx}{2(1-a)(1+a)^2}$$
$$= \frac{3(3-2a)b^2\sqrt{1-a-bx}}{(1-a)(1+a)^3\sqrt{1+a+bx}} + \frac{(3-2a)b(1-a-bx)^{3/2}}{2(1-a)(1+a)^2x\sqrt{1+a+bx}} - \frac{(1-a-bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1+a+bx}} + \frac{(3(3}{}$$
$$= \frac{3(3-2a)b^2\sqrt{1-a-bx}}{(1-a)(1+a)^3\sqrt{1+a+bx}} + \frac{(3-2a)b(1-a-bx)^{3/2}}{2(1-a)(1+a)^2x\sqrt{1+a+bx}} - \frac{(1-a-bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1+a+bx}} + \frac{(3(3}{}$$
$$= \frac{3(3-2a)b^2\sqrt{1-a-bx}}{(1-a)(1+a)^3\sqrt{1+a+bx}} + \frac{(3-2a)b(1-a-bx)^{3/2}}{2(1-a)(1+a)^2x\sqrt{1+a+bx}} - \frac{(1-a-bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1+a+bx}} - \frac{3(3}{}$$

Mathematica [A] time = 0.153857, size = 134, normalized size = 0.67

$$\frac{\sqrt{-a-bx+1}\left(a^3+a^2-a\left(b^2x^2-5bx+1\right)+14b^2x^2+5bx-1\right)}{2(a+1)^3x^2\sqrt{a+bx+1}} - \frac{3(2a-3)b^2\tan^{-1}\left(\frac{\sqrt{-a-bx+1}}{\sqrt{\frac{a-1}{a+1}}\sqrt{a+bx+1}}\right)}{\sqrt{a-1}(a+1)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a + b*x])*x^3),x]
```

```
[Out] (Sqrt[1 - a - b*x]*(-1 + a^2 + a^3 + 5*b*x + 14*b^2*x^2 - a*(1 - 5*b*x + b^
2*x^2)))/(2*(1 + a)^3*x^2*Sqrt[1 + a + b*x]) - (3*(-3 + 2*a)*b^2*ArcTan[Sqr
t[1 - a - b*x]/(Sqrt[(-1 + a)/(1 + a)]*Sqrt[1 + a + b*x])])/(Sqrt[-1 + a]*(
1 + a)^(7/2))
```

Maple [B] time = 0.057, size = 2848, normalized size = 14.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^3,x)
```

```
[Out] 2/(1+a)^3/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+2/(1+a)^
3*b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)-3/(1+a)^4/(x+1/b+a/b)^2*(-
(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)-3/(1+a)^4*b^2*(-(x+(1+a)/b)^2*b^2+
2*b*(x+(1+a)/b))^(3/2)-2/(1+a)^5*b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(
3/2)+2/(1+a)^5*b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+6/(1+a)^5*b^2*(-b^2*x^2-2
*a*b*x-a^2+1)^(1/2)-1/2/(1+a)^3/(-a^2+1)/x^2*(-b^2*x^2-2*a*b*x-a^2+1)^(5/2)
-1/2/(1+a)^3*b^2/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-3/2/(1+a)^3*b^2/(-
a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/2/(1+a)^3*b^2/(-a^2+1)^(1/2)*ln((-2
*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-9/(1+a)^
5*b^2*a^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-6/(1+a)^5*b^2*(-a^2+1)^(1/2)*ln((-
2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-9/2/(1+
a)^4*b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x-9/2/(1+a)^4*b^2*(-(x+
(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a-9/2/(1+a)^4*b^3/(b^2)^(1/2)*arctan(
(b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))+3/(
1+a)^3*b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a-3/(1+a)^5*b^3*(-(x+
(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x-3/(1+a)^5*b^2*(-(x+(1+a)/b)^2*b^2+2
*b*(x+(1+a)/b))^(1/2)*a-3/(1+a)^5*b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+
a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))+1/(1+a)^3/b/(x+1/b+a/
b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+3/(1+a)^3*b^3*(-(x+(1+a)/b)
^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x+1/2/(1+a)^3*a*b/(-a^2+1)^2/x*(-b^2*x^2-2*a*
b*x-a^2+1)^(5/2)-3/4/(1+a)^3*a^3*b^3/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1
/2)*x-9/4/(1+a)^3*a^3*b^3/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)
/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+3/2/(1+a)^3*a^5*b^3/(-a^2+1)^2/(b^2)^(1/2)
*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-9/2/(1+a)^4*b^3
*a^2/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-27/2/(1+a)^4*b^3*a^2/(-a^2+1
)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+9/
(1+a)^4*b^3*a^4/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2
*a*b*x-a^2+1)^(1/2))+1/2/(1+a)^3*a*b^3/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^
(3/2)*x+3/4/(1+a)^3*a*b^3/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+3/4/(
1+a)^3*a*b^3/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2))+3/4/(1+a)^3*b^3/(-a^2+1)*a*(-b^2*x^2-2*a*b*x-a^2+1)^(1/
2)*x+9/4/(1+a)^3*b^3/(-a^2+1)*a/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^
2*x^2-2*a*b*x-a^2+1)^(1/2))-3/2/(1+a)^3*b^3/(-a^2+1)*a^3/(b^2)^(1/2)*arctan
((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+3/(1+a)^3*b^3/(b^2)^(1
/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))
^(1/2))+6/(1+a)^5*b^3*a^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2))+6/(1+a)^5*b^2*(-a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2*(
-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)*a^2-3/(1+a)^5*b^3*a*(-b^2*
x^2-2*a*b*x-a^2+1)^(1/2)*x-9/(1+a)^5*b^3*a/(b^2)^(1/2)*arctan((b^2)^(1/2)*(
x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+3/(1+a)^4*b/(-a^2+1)/x*(-b^2*x^2-2*a
*b*x-a^2+1)^(5/2)+6/(1+a)^4*b^2*a/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-2
7/2/(1+a)^4*b^2*a^3/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+27/2/(1+a)^4*b^
2*a/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+9/(1+a)^4*b^2*a^3/(-a^2+1)^(1/2
)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-
9/(1+a)^4*b^2*a/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*
x^2-2*a*b*x-a^2+1)^(1/2))/x)+3/(1+a)^4*b^3/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1
)^(3/2)*x+9/2/(1+a)^4*b^3/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+9/2/(1+
a)^4*b^3/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2))+1/(1+a)^3*a^2*b^2/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-9
/4/(1+a)^3*a^4*b^2/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+9/4/(1+a)^3*a^
2*b^2/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/2/(1+a)^3*a^4*b^2/(-a^2+1
)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2
))/x)-3/2/(1+a)^3*a^2*b^2/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1
/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+9/4/(1+a)^3*b^2/(-a^2+1)*a^2*(-b^2*x
^2-2*a*b*x-a^2+1)^(1/2)-3/2/(1+a)^3*b^2/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b
```

```
+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)*a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-(bx+a)^2+1\right)^{\frac{3}{2}}}{(bx+a+1)^3 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((-(b*x + a)^2 + 1)^(3/2)/((b*x + a + 1)^3*x^3), x)
```

Fricas [A] time = 2.07525, size = 1184, normalized size = 5.92

$$\left[-\frac{3\left((2a-3)b^3x^3+\left(2a^2-a-3\right)b^2x^2\right)\sqrt{-a^2+1}\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx-2\sqrt{-b^2x^2-2abx-a^2+1}\left(abx+a^2-1\right)\sqrt{-a^2+1}-4a^2+2}{x^2}\right.}{4\left(\left(a^5+3a^4+2a^3-2a^2-3a-1\right)bx^3+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(3*((2*a - 3)*b^3*x^3 + (2*a^2 - a - 3)*b^2*x^2)*sqrt(-a^2 + 1)*log((
(2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*x - 2*sqrt(-b^2*x^2 - 2*a*b*x -
 a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1) - 4*a^2 + 2)/x^2) - 2*(a^5 - (a^
3 - 14*a^2 - a + 14)*b^2*x^2 + a^4 - 2*a^3 + 5*(a^3 + a^2 - a - 1)*b*x - 2*
a^2 + a + 1)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^5 + 3*a^4 + 2*a^3 - 2*
a^2 - 3*a - 1)*b*x^3 + (a^6 + 4*a^5 + 5*a^4 - 5*a^2 - 4*a - 1)*x^2), -1/2*(
3*((2*a - 3)*b^3*x^3 + (2*a^2 - a - 3)*b^2*x^2)*sqrt(a^2 - 1)*arctan(sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2
*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) - (a^5 - (a^3 - 14*a^2 - a + 14)
*b^2*x^2 + a^4 - 2*a^3 + 5*(a^3 + a^2 - a - 1)*b*x - 2*a^2 + a + 1)*sqrt(-b
^2*x^2 - 2*a*b*x - a^2 + 1))/((a^5 + 3*a^4 + 2*a^3 - 2*a^2 - 3*a - 1)*b*x^3
 + (a^6 + 4*a^5 + 5*a^4 - 5*a^2 - 4*a - 1)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**3*(1-(b*x+a)**2)**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21796, size = 1110, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -8*b^3/((a^3*abs(b) + 3*a^2*abs(b) + 3*a*abs(b) + abs(b))*((sqrt(-b^2*x^2 -
 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a*b) + 1)) - 3*(2*a*b^3 - 3*b^3)*a
rctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)
/sqrt(a^2 - 1))/((a^3*abs(b) + 3*a^2*abs(b) + 3*a*abs(b) + abs(b))*sqrt(a^2
 - 1)) - (2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^4*b^3/(b^2*
x + a*b)^2 + 2*a^4*b^3 - 5*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*
a^3*b^3/(b^2*x + a*b) - 6*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2
*a^3*b^3/(b^2*x + a*b)^2 - 3*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b
)^3*a^3*b^3/(b^2*x + a*b)^3 - 6*a^3*b^3 + 18*(sqrt(-b^2*x^2 - 2*a*b*x - a^2
 + 1)*abs(b) + b)*a^2*b^3/(b^2*x + a*b) + 3*(sqrt(-b^2*x^2 - 2*a*b*x - a^2
+ 1)*abs(b) + b)^2*a^2*b^3/(b^2*x + a*b)^2 + 6*(sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1)*abs(b) + b)^3*a^2*b^3/(b^2*x + a*b)^3 - a^2*b^3 + 2*(sqrt(-b^2*x^2
- 2*a*b*x - a^2 + 1)*abs(b) + b)*a*b^3/(b^2*x + a*b) - 12*(sqrt(-b^2*x^2 -
2*a*b*x - a^2 + 1)*abs(b) + b)^2*a*b^3/(b^2*x + a*b)^2 + 2*(sqrt(-b^2*x^2 -
 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a*b^3/(b^2*x + a*b)^3 - 2*(sqrt(-b^2*x^2
- 2*a*b*x - a^2 + 1)*abs(b) + b)^2*b^3/(b^2*x + a*b)^2)/((a^5*abs(b) + 3*a^
4*abs(b) + 3*a^3*abs(b) + a^2*abs(b))*((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*
abs(b) + b)^2*a/(b^2*x + a*b)^2 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
*abs(b) + b)/(b^2*x + a*b))^2)
```

3.867 $\int \frac{e^{-3\tanh^{-1}(a+bx)}}{x^4} dx$

Optimal. Leaf size=257

$$-\frac{\left(2a^2-51a+52\right)b^3\sqrt{-a-bx+1}}{6(1-a)(a+1)^4\sqrt{a+bx+1}}+\frac{\left(6a^2-18a+11\right)b^3\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)(a+1)^4\sqrt{1-a^2}}-\frac{(19-16a)b^2\sqrt{-a-bx+1}}{6(1-a)(a+1)^3x\sqrt{a+bx+1}}+-\frac{}{6}$$

```
[Out] -((52 - 51*a + 2*a^2)*b^3*Sqrt[1 - a - b*x])/(6*(1 - a)*(1 + a)^4*Sqrt[1 +
a + b*x]) - ((1 - a)*Sqrt[1 - a - b*x])/(3*(1 + a)*x^3*Sqrt[1 + a + b*x]) +
 (7*b*Sqrt[1 - a - b*x])/(6*(1 + a)^2*x^2*Sqrt[1 + a + b*x]) - ((19 - 16*a)
*b^2*Sqrt[1 - a - b*x])/(6*(1 - a)*(1 + a)^3*x*Sqrt[1 + a + b*x]) + ((11 -
18*a + 6*a^2)*b^3*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt
[1 - a - b*x])])/((1 - a)*(1 + a)^4*Sqrt[1 - a^2])
```

Rubi [A] time = 0.216336, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6163, 98, 151, 152, 12, 93, 208}

$$-\frac{\left(2a^2-51a+52\right)b^3\sqrt{-a-bx+1}}{6(1-a)(a+1)^4\sqrt{a+bx+1}}+\frac{\left(6a^2-18a+11\right)b^3\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)(a+1)^4\sqrt{1-a^2}}-\frac{(19-16a)b^2\sqrt{-a-bx+1}}{6(1-a)(a+1)^3x\sqrt{a+bx+1}}+-\frac{}{6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a + b*x])*x^4),x]
```

```
[Out] -((52 - 51*a + 2*a^2)*b^3*Sqrt[1 - a - b*x])/(6*(1 - a)*(1 + a)^4*Sqrt[1 +
a + b*x]) - ((1 - a)*Sqrt[1 - a - b*x])/(3*(1 + a)*x^3*Sqrt[1 + a + b*x]) +
 (7*b*Sqrt[1 - a - b*x])/(6*(1 + a)^2*x^2*Sqrt[1 + a + b*x]) - ((19 - 16*a)
*b^2*Sqrt[1 - a - b*x])/(6*(1 - a)*(1 + a)^3*x*Sqrt[1 + a + b*x]) + ((11 -
18*a + 6*a^2)*b^3*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt
[1 - a - b*x])])/((1 - a)*(1 + a)^4*Sqrt[1 - a^2])
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(a+bx)}}{x^4}\,dx = \int \frac{(1-a-bx)^{3/2}}{x^4(1+a+bx)^{3/2}}\,dx$$

$$= -\frac{(1-a)\sqrt{1-a-bx}}{3(1+a)x^3\sqrt{1+a+bx}} - \frac{\int \frac{7(1-a)b-6b^2x}{x^3\sqrt{1-a-bx}(1+a+bx)^{3/2}}\,dx}{3(1+a)}$$

$$= -\frac{(1-a)\sqrt{1-a-bx}}{3(1+a)x^3\sqrt{1+a+bx}} + \frac{7b\sqrt{1-a-bx}}{6(1+a)^2x^2\sqrt{1+a+bx}} + \frac{\int \frac{(19-16a)(1-a)b^2-14(1-a)b^3x}{x^2\sqrt{1-a-bx}(1+a+bx)^{3/2}}\,dx}{6(1-a)(1+a)^2}$$

$$= -\frac{(1-a)\sqrt{1-a-bx}}{3(1+a)x^3\sqrt{1+a+bx}} + \frac{7b\sqrt{1-a-bx}}{6(1+a)^2x^2\sqrt{1+a+bx}} - \frac{(19-16a)b^2\sqrt{1-a-bx}}{6(1-a)(1+a)^3x\sqrt{1+a+bx}} - \frac{\int \frac{3(1-a)(}{}}{}$$

$$= -\frac{\left(52-51a+2a^2\right)b^3\sqrt{1-a-bx}}{6(1-a)(1+a)^4\sqrt{1+a+bx}} - \frac{(1-a)\sqrt{1-a-bx}}{3(1+a)x^3\sqrt{1+a+bx}} + \frac{7b\sqrt{1-a-bx}}{6(1+a)^2x^2\sqrt{1+a+bx}} - \frac{(19}{6(1-}$$

$$= -\frac{\left(52-51a+2a^2\right)b^3\sqrt{1-a-bx}}{6(1-a)(1+a)^4\sqrt{1+a+bx}} - \frac{(1-a)\sqrt{1-a-bx}}{3(1+a)x^3\sqrt{1+a+bx}} + \frac{7b\sqrt{1-a-bx}}{6(1+a)^2x^2\sqrt{1+a+bx}} - \frac{(19}{6(1-}$$

$$= -\frac{\left(52-51a+2a^2\right)b^3\sqrt{1-a-bx}}{6(1-a)(1+a)^4\sqrt{1+a+bx}} - \frac{(1-a)\sqrt{1-a-bx}}{3(1+a)x^3\sqrt{1+a+bx}} + \frac{7b\sqrt{1-a-bx}}{6(1+a)^2x^2\sqrt{1+a+bx}} - \frac{(19}{6(1-}$$

$$= -\frac{\left(52-51a+2a^2\right)b^3\sqrt{1-a-bx}}{6(1-a)(1+a)^4\sqrt{1+a+bx}} - \frac{(1-a)\sqrt{1-a-bx}}{3(1+a)x^3\sqrt{1+a+bx}} + \frac{7b\sqrt{1-a-bx}}{6(1+a)^2x^2\sqrt{1+a+bx}} - \frac{(19}{6(1-}$$

Mathematica [A] time = 0.3427, size = 200, normalized size = 0.78

$$\frac{-\left(6a^2-18a+11\right)b^2x^2\left(-\sqrt{a+1}\sqrt{-a-bx+1}\left(a^2+abx-5bx-1\right)-6\sqrt{a-1}bx\sqrt{a+bx+1}\tan^{-1}\left(\frac{\sqrt{-a-bx+1}}{\sqrt{\frac{a-1}{a+1}}\sqrt{a+bx+1}}\right)\right)-}{6(a+1)^{5/2}\left(a^2-1\right)^2x^3\sqrt{a+bx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a + b*x])*x^4),x]
```

```
[Out] (-2*(1 - a)*(1 + a)^(7/2)*(1 - a - b*x)^(5/2) + (3 - 4*a)*(1 + a)^(5/2)*b*x
*(1 - a - b*x)^(5/2) - (11 - 18*a + 6*a^2)*b^2*x^2*(-(Sqrt[1 + a]*Sqrt[1 -
a - b*x]*(-1 + a^2 - 5*b*x + a*b*x)) - 6*Sqrt[-1 + a]*b*x*Sqrt[1 + a + b*x]
*ArcTan[Sqrt[1 - a - b*x]/(Sqrt[(-1 + a)/(1 + a)]*Sqrt[1 + a + b*x])]))/(6*
(1 + a)^(5/2)*(-1 + a^2)^2*x^3*Sqrt[1 + a + b*x])
```

Maple [B] time = 0.067, size = 4212, normalized size = 16.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^4,x)
```

```
[Out] -10/3/(1+a)^6*b^3*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-10/(1+a)^6*b^3*(-b^2*x^2-2
*a*b*x-a^2+1)^(1/2)+10/3/(1+a)^6*b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(
3/2)-1/(1+a)^4/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)-2/(
```

1+a)^4*b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)+4/(1+a)^5*b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)+5/(1+a)^6*b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x+5/(1+a)^6*b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a+4/(1+a)^5*b/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+6/(1+a)^5*b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x+6/(1+a)^5*b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a+6/(1+a)^5*b^4/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))+15/(1+a)^6*b^3*a^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+10/(1+a)^6*b^3*(-a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/3/(1+a)^3/(-a^2+1)/x^3*(-b^2*x^2-2*a*b*x-a^2+1)^(5/2)+5/(1+a)^6*b^4/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))-2/(1+a)^4*b/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+3/2/(1+a)^4*b^3/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+9/2/(1+a)^4*b^3/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-9/2/(1+a)^4*b^3/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+7/6/(1+a)^3*a*b^3/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+1/2/(1+a)^3*a^5*b^3/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+2/3/(1+a)^3*b^2/(-a^2+1)^2/x*(-b^2*x^2-2*a*b*x-a^2+1)^(5/2)+15/(1+a)^6*b^4*a/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-10/(1+a)^6*b^4*a^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-10/(1+a)^6*b^3*(-a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)*a^2-3/(1+a)^4*b^4/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))-3/4/(1+a)^3*a^4*b^4/(-a^2+1)^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/(1+a)^3*a^6*b^4/(-a^2+1)^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/6/(1+a)^3*a^2*b^4/(-a^2+1)^3*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)*x-3/2/(1+a)^4*b^2*a/(-a^2+1)^2/x*(-b^2*x^2-2*a*b*x-a^2+1)^(5/2)+9/4/(1+a)^4*b^4*a^3/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+27/4/(1+a)^4*b^4*a^3/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-9/2/(1+a)^4*b^4*a^5/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-3/2/(1+a)^4*b^4*a/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)*x-9/4/(1+a)^4*b^4*a/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-9/4/(1+a)^4*b^4*a/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-9/4/(1+a)^4*b^4/(-a^2+1)*a*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-27/4/(1+a)^4*b^4/(-a^2+1)*a/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+9/2/(1+a)^4*b^4/(-a^2+1)*a^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+27/(1+a)^5*b^4*a^2/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-18/(1+a)^5*b^4*a^4/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+9/(1+a)^5*b^4*a^2/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+1/4/(1+a)^3*a^2*b^4/(-a^2+1)^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-1/6/(1+a)^3*a*b/(-a^2+1)^2/x^2*(-b^2*x^2-2*a*b*x-a^2+1)^(5/2)+1/4/(1+a)^3*a^2*b^4/(-a^2+1)^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-3/4/(1+a)^3*a^2*b^4/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-9/4/(1+a)^3*a^2*b^4/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+3/2/(1+a)^3*a^4*b^4/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/4/(1+a)^3*a^4*b^4/(-a^2+1)^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-3/(1+a)^4*b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x-3/(1+a)^4*b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a+3/2/(1+a)^3*a^3*b^3/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-3/2/(1+a)^3*a*b^3/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-9/4/(1+a)^3*a^3*b^3/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+5/2/(1+a)^3*a*b^3/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/3/(1+a)^3*a^3*b^3/(-a^2+1)^3*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-3/4/(1+a)^3*a^5*b^3/(-a^2+1)^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/4/(1+a)^3*a^3*b^3/(-a^2+1)^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-18/(1+a)^5*b^3*a^3/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+3/2/(1+a)^4*b/(-a^2+1)/x^2*(-b^2*x^2-2*a

```
*b*x-a^2+1)^(5/2)-3/(1+a)^4*b^3*a^2/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(3/
2)+27/4/(1+a)^4*b^3*a^4/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-27/4/(1+a
)^4*b^3*a^2/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-9/2/(1+a)^4*b^3*a^4/(
-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1
)^(1/2))/x)+9/2/(1+a)^4*b^3*a^2/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2
+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-27/4/(1+a)^4*b^3/(-a^2+1)*a^2*
(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+9/2/(1+a)^4*b^3/(-a^2+1)^(1/2)*ln((-2*a^2+2-
2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)*a^2-6/(1+a)^5*b
^4/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)*x-9/(1+a)^5*b^4/(-a^2+1)*(-b^2*x
^2-2*a*b*x-a^2+1)^(1/2)*x-9/(1+a)^5*b^4/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(
1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+2/3/(1+a)^3*b^4/(-a^2+1)^2*(-b
^2*x^2-2*a*b*x-a^2+1)^(3/2)*x+1/(1+a)^3*b^4/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2)*x+1/(1+a)^3*b^4/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b
)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+18/(1+a)^5*b^3*a/(-a^2+1)^(1/2)*ln((-2*a^
2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-6/(1+a)^5*b
^2/(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(5/2)-12/(1+a)^5*b^3*a/(-a^2+1)*(-b^
2*x^2-2*a*b*x-a^2+1)^(3/2)+27/(1+a)^5*b^3*a^3/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2)-27/(1+a)^5*b^3*a/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/2/(1+
a)^3*a^3*b^3/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2
-2*a*b*x-a^2+1)^(1/2))/x)+5/(1+a)^6*b^4*a*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+
1/6/(1+a)^3*a^2*b^2/(-a^2+1)^3/x*(-b^2*x^2-2*a*b*x-a^2+1)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-(bx+a)^2+1\right)^{\frac{3}{2}}}{(bx+a+1)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((-(b*x + a)^2 + 1)^(3/2)/((b*x + a + 1)^3*x^4), x)
```

Fricas [A] time = 2.38553, size = 1611, normalized size = 6.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((6*a^2 - 18*a + 11)*b^4*x^4 + (6*a^3 - 12*a^2 - 7*a + 11)*b^3*x^
3)*sqrt(-a^2 + 1)*log((((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*x + 2*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1) - 4*a^2 +
 2)/x^2) - 2*(2*a^7 + (2*a^4 - 51*a^3 + 50*a^2 + 51*a - 52)*b^3*x^3 + 2*a^6
 - 6*a^5 - (16*a^4 - 3*a^3 - 35*a^2 + 3*a + 19)*b^2*x^2 - 6*a^4 + 6*a^3 + 7
*(a^5 + a^4 - 2*a^3 - 2*a^2 + a + 1)*b*x + 6*a^2 - 2*a - 2)*sqrt(-b^2*x^2 -
 2*a*b*x - a^2 + 1))/((a^7 + 3*a^6 + a^5 - 5*a^4 - 5*a^3 + a^2 + 3*a + 1)*b
*x^4 + (a^8 + 4*a^7 + 4*a^6 - 4*a^5 - 10*a^4 - 4*a^3 + 4*a^2 + 4*a + 1)*x^3
), 1/6*(3*((6*a^2 - 18*a + 11)*b^4*x^4 + (6*a^3 - 12*a^2 - 7*a + 11)*b^3*x^
3)*sqrt(a^2 - 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1
)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) +
(2*a^7 + (2*a^4 - 51*a^3 + 50*a^2 + 51*a - 52)*b^3*x^3 + 2*a^6 - 6*a^5 - (1
6*a^4 - 3*a^3 - 35*a^2 + 3*a + 19)*b^2*x^2 - 6*a^4 + 6*a^3 + 7*(a^5 + a^4 -
```

```
2*a^3 - 2*a^2 + a + 1)*b*x + 6*a^2 - 2*a - 2)*sqrt(-b^2*x^2 - 2*a*b*x - a^
2 + 1))/((a^7 + 3*a^6 + a^5 - 5*a^4 - 5*a^3 + a^2 + 3*a + 1)*b*x^4 + (a^8 +
 4*a^7 + 4*a^6 - 4*a^5 - 10*a^4 - 4*a^3 + 4*a^2 + 4*a + 1)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**3*(1-(b*x+a)**2)**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32481, size = 2483, normalized size = 9.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 8*b^4/((a^4*abs(b) + 4*a^3*abs(b) + 6*a^2*abs(b) + 4*a*abs(b) + abs(b))*((s
qrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a*b) + 1)) + (6*a^2*
b^4 - 18*a*b^4 + 11*b^4)*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b)
 + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/((a^5*abs(b) + 3*a^4*abs(b) + 2*a
^3*abs(b) - 2*a^2*abs(b) - 3*a*abs(b) - abs(b))*sqrt(a^2 - 1)) + 1/3*(12*(s
qrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^7*b^4/(b^2*x + a*b)^2 + 6
*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a^7*b^4/(b^2*x + a*b)^4
+ 6*a^7*b^4 - 24*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a^6*b^4/(b
^2*x + a*b) - 72*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^6*b^4/
(b^2*x + a*b)^2 - 36*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a^6*
b^4/(b^2*x + a*b)^3 - 36*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*
a^6*b^4/(b^2*x + a*b)^4 - 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b
)^5*a^6*b^4/(b^2*x + a*b)^5 - 36*a^6*b^4 + 171*(sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1)*abs(b) + b)*a^5*b^4/(b^2*x + a*b) + 84*(sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1)*abs(b) + b)^2*a^5*b^4/(b^2*x + a*b)^2 + 216*(sqrt(-b^2*x^2 - 2*a*b*
x - a^2 + 1)*abs(b) + b)^3*a^5*b^4/(b^2*x + a*b)^3 + 54*(sqrt(-b^2*x^2 - 2*
a*b*x - a^2 + 1)*abs(b) + b)^4*a^5*b^4/(b^2*x + a*b)^4 + 45*(sqrt(-b^2*x^2
- 2*a*b*x - a^2 + 1)*abs(b) + b)^5*a^5*b^4/(b^2*x + a*b)^5 + 22*a^5*b^4 - 1
20*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a^4*b^4/(b^2*x + a*b) -
252*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^4*b^4/(b^2*x + a*b)
^2 - 156*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a^4*b^4/(b^2*x +
 a*b)^3 - 153*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a^4*b^4/(b^
2*x + a*b)^4 - 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^5*a^4*b^4
/(b^2*x + a*b)^5 + 9*a^4*b^4 - 36*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b
) + b)*a^3*b^4/(b^2*x + a*b) + 192*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(
b) + b)^2*a^3*b^4/(b^2*x + a*b)^2 + 90*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*
abs(b) + b)^3*a^3*b^4/(b^2*x + a*b)^3 + 78*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
 1)*abs(b) + b)^4*a^3*b^4/(b^2*x + a*b)^4 - 18*(sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1)*abs(b) + b)^5*a^3*b^4/(b^2*x + a*b)^5 + 2*a^3*b^4 - 6*(sqrt(-b^2*x^
2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a^2*b^4/(b^2*x + a*b) + 54*(sqrt(-b^2*x^
2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^2*b^4/(b^2*x + a*b)^2 - 100*(sqrt(-b
^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a^2*b^4/(b^2*x + a*b)^3 + 54*(sqr
```

```
t(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a^2*b^4/(b^2*x + a*b)^4 - 6*(
sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^5*a^2*b^4/(b^2*x + a*b)^5 +
12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a*b^4/(b^2*x + a*b)^2
- 36*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a*b^4/(b^2*x + a*b)^
3 + 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a*b^4/(b^2*x + a*b
)^4 - 8*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*b^4/(b^2*x + a*b)
^3)/((a^8*abs(b) + 3*a^7*abs(b) + 2*a^6*abs(b) - 2*a^5*abs(b) - 3*a^4*abs(b
) - a^3*abs(b))*((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a/(b^2*x
 + a*b)^2 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x +
a*b))^3)
```

3.868 $\int \frac{e^{\tanh^{-1}(1+bx)}}{2+bx}\,dx$

Optimal. Leaf size=10

$$\frac{\sin^{-1}(bx+1)}{b}$$

```
[Out] ArcSin[1 + b*x]/b
```

Rubi [A] time = 0.0353483, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6163, 53, 619, 216}

$$\frac{\sin^{-1}(bx+1)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[1 + b*x]/(2 + b*x),x]
```

```
[Out] ArcSin[1 + b*x]/b
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(1+bx)}}{2+bx}\,dx = \int \frac{1}{\sqrt{-bx}\sqrt{2+bx}}\,dx$$
$$= \int \frac{1}{\sqrt{-2bx-b^2x^2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2b-2b^2x\right)}{2b^2}$$
$$= \frac{\sin^{-1}(1+bx)}{b}$$

Mathematica [B] time = 0.0120201, size = 37, normalized size = 3.7

$$-\frac{2\sqrt{-bx}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}\sqrt{x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[1 + b*x]/(2 + b*x),x]
```

```
[Out] (-2*Sqrt[-(b*x)]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(b^(3/2)*Sqrt[x])
```

Maple [B] time = 0.032, size = 34, normalized size = 3.4

$$\arctan\left((x+b^{-1})\sqrt{b^2}\frac{1}{\sqrt{-b^2x^2-2\,bx}}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(b*x+1)^2+1)^(1/2),x)
```

```
[Out] 1/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+1/b)/(-b^2*x^2-2*b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(b*x+1)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.4805, size = 58, normalized size = 5.8

$$-\frac{2\arctan\left(\frac{\sqrt{-b^2x^2-2\,bx}}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(b*x+1)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-b^2*x^2 - 2*b*x)/(b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{1-(bx+1)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(b*x+1)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - (b*x + 1)**2), x)
```

Giac [A] time = 1.20334, size = 20, normalized size = 2.

$$-\frac{\arcsin\left(-bx-1\right)\operatorname{sgn}\left(b\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(b*x+1)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(-b*x - 1)*sgn(b)/abs(b)
```

3.869 $\int \frac{e^{\tanh^{-1}(a+bx)}x^3}{1-a^2-2abx-b^2x^2}\,dx$

Optimal. Leaf size=109

$$-\frac{3\left(2a^2-2a+1\right)\sin^{-1}(a+bx)}{2b^4}+\frac{(1-a)x^2\sqrt{a+bx+1}}{b^2\sqrt{-a-bx+1}}+\frac{\sqrt{-a-bx+1}\sqrt{a+bx+1}((3-2a)bx+(1-2a)(4-a))}{2b^4}$$

```
[Out] ((1 - a)*x^2*Sqrt[1 + a + b*x])/(b^2*Sqrt[1 - a - b*x]) + (Sqrt[1 - a - b*x
]*Sqrt[1 + a + b*x]*((1 - 2*a)*(4 - a) + (3 - 2*a)*b*x))/(2*b^4) - (3*(1 -
2*a + 2*a^2)*ArcSin[a + b*x])/(2*b^4)
```

Rubi [A] time = 0.168588, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6164, 98, 147, 53, 619, 216}

$$-\frac{3\left(2a^2-2a+1\right)\sin^{-1}(a+bx)}{2b^4}+\frac{(1-a)x^2\sqrt{a+bx+1}}{b^2\sqrt{-a-bx+1}}+\frac{\sqrt{-a-bx+1}\sqrt{a+bx+1}((3-2a)bx+(1-2a)(4-a))}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a + b*x]*x^3)/(1 - a^2 - 2*a*b*x - b^2*x^2),x]
```

```
[Out] ((1 - a)*x^2*Sqrt[1 + a + b*x])/(b^2*Sqrt[1 - a - b*x]) + (Sqrt[1 - a - b*x
]*Sqrt[1 + a + b*x]*((1 - 2*a)*(4 - a) + (3 - 2*a)*b*x))/(2*b^4) - (3*(1 -
2*a + 2*a^2)*ArcSin[a + b*x])/(2*b^4)
```

Rule 6164

```
Int[E^(ArcTanh[(a_) + (b_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> Dist[(c/(1 - a^2))^p, Int[u*(1 - a - b*x)^(p - n
/2)*(1 + a + b*x)^(p + n/2), x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && E
qQ[b*d - 2*a*e, 0] && EqQ[b^2*c + e*(1 - a^2), 0] && (IntegerQ[p] || GtQ[c/
(1 - a^2), 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
```

```
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(a+bx)}x^3}{1-a^2-2abx-b^2x^2}\,dx = \int \frac{x^3}{(1-a-bx)^{3/2}\sqrt{1+a+bx}}\,dx$$
$$= \frac{(1-a)x^2\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} - \frac{\int \frac{x\left(2\left(1-a^2\right)+(3-2a)bx\right)}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{b^2}$$
$$= \frac{(1-a)x^2\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\sqrt{1+a+bx}((1-2a)(4-a)+(3-2a)bx)}{2b^4} - \frac{\left(3\left(1-2a\right.\right.}{}$$
$$= \frac{(1-a)x^2\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\sqrt{1+a+bx}((1-2a)(4-a)+(3-2a)bx)}{2b^4} - \frac{\left(3\left(1-2a\right.\right.}{}$$
$$= \frac{(1-a)x^2\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\sqrt{1+a+bx}((1-2a)(4-a)+(3-2a)bx)}{2b^4} + \frac{\left(3\left(1-2a\right.\right.}{}$$
$$= \frac{(1-a)x^2\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\sqrt{1+a+bx}((1-2a)(4-a)+(3-2a)bx)}{2b^4} - \frac{3\left(1-2a\right.}{}$$

Mathematica [A] time = 0.338711, size = 90, normalized size = 0.83

$$-\frac{3\left(2a^2-2a+1\right)\sin^{-1}(a+bx)-\frac{\sqrt{-a^2-2abx-b^2x^2+1}\left(2a^3-11a^2+a(13-4bx)+b^2x^2+bx-4\right)}{a+bx-1}}{2b^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a + b*x]*x^3)/(1 - a^2 - 2*a*b*x - b^2*x^2),x]
```

```
[Out] -(-((Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*(-4 - 11*a^2 + 2*a^3 + b*x + b^2*x^2
 + a*(13 - 4*b*x)))/(-1 + a + b*x)) + 3*(1 - 2*a + 2*a^2)*ArcSin[a + b*x])/
(2*b^4)
```

Maple [B] time = 0.043, size = 499, normalized size = 4.6

$$-\frac{x^{3}}{2b}\frac{1}{\sqrt{-b^{2}x^{2}-2xab-a^{2}+1}}+\frac{3ax^{2}}{2b^{2}}\frac{1}{\sqrt{-b^{2}x^{2}-2xab-a^{2}+1}}+\frac{15a^{2}x}{2b^{3}}\frac{1}{\sqrt{-b^{2}x^{2}-2xab-a^{2}+1}}+\frac{9a^{3}}{2b^{4}}\frac{1}{\sqrt{-b^{2}x^{2}-2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^3/(-b^2*x^2-2*a*b*x-a^2+1),x)
```

```
[Out] -1/2/b*x^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/2/b^2*a*x^2/(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2)+15/2/b^3*a^2*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+9/2/b^4*a^3/(-b^2
*x^2-2*a*b*x-a^2+1)^(1/2)-3*a^2/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/
(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-9/2/b^4*a/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/
2/b^3*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/2/b^3/(b^2)^(1/2)*arctan((b^2)^(1/
2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-x^2/b^2/(-b^2*x^2-2*a*b*x-a^2+1)
^(1/2)-5*a/b^3*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a^2/b^4/(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2)-a^3/b^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-a^4/b^4/(-b^2*x^2-2*a*
b*x-a^2+1)^(1/2)+3*a/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2
*a*b*x-a^2+1)^(1/2))+2/b^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
```

Maxima [B] time = 2.1036, size = 1358, normalized size = 12.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^3/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="maxima")
```

```
[Out] -1/2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^3/(b^6*x + a*b^5 + sqrt(b^2)*b^4
) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^3/(b^6*x + a*b^5 - sqrt(b^2)*b^4)
- sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^3/(sqrt(b^2)*b^5*x + a*sqrt(b^2)*b^4
 + b^5) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^3/(sqrt(b^2)*b^5*x + a*sqrt(
b^2)*b^4 - b^5) + 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^2*sqrt(b^2)/(b^7*x
 + a*b^6 + sqrt(b^2)*b^5) + 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^2*sqrt(b
^2)/(b^7*x + a*b^6 - sqrt(b^2)*b^5) - 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*
a^2/(b^6*x + a*b^5 + sqrt(b^2)*b^4) + 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*
a^2/(b^6*x + a*b^5 - sqrt(b^2)*b^4) + 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*
a/(b^6*x + a*b^5 + sqrt(b^2)*b^4) - 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/
(b^6*x + a*b^5 - sqrt(b^2)*b^4) - 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/(s
qrt(b^2)*b^5*x + a*sqrt(b^2)*b^4 + b^5) - 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
 1)*a/(sqrt(b^2)*b^5*x + a*sqrt(b^2)*b^4 - b^5) + sqrt(-b^2*x^2 - 2*a*b*x -
 a^2 + 1)*sqrt(b^2)/(b^7*x + a*b^6 + sqrt(b^2)*b^5) + sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1)*sqrt(b^2)/(b^7*x + a*b^6 - sqrt(b^2)*b^5) - sqrt(-b^2*x^2 - 2
*a*b*x - a^2 + 1)/(b^6*x + a*b^5 + sqrt(b^2)*b^4) + sqrt(-b^2*x^2 - 2*a*b*x
 - a^2 + 1)/(b^6*x + a*b^5 - sqrt(b^2)*b^4) + 6*a^2*arcsin(sqrt(b^2)*x + a*
sqrt(b^2)/b)/b^5 - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*sqrt(b^2)*x/b^5 - 6*a
*arcsin(sqrt(b^2)*x + a*sqrt(b^2)/b)/b^5 + 5*sqrt(-b^2*x^2 - 2*a*b*x - a^2
+ 1)*a*sqrt(b^2)/b^6 + 3*arcsin(sqrt(b^2)*x + a*sqrt(b^2)/b)/b^5 - 2*sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1)*sqrt(b^2)/b^6)*b^2/sqrt(a^2*b^2 - (a^2 - 1)*b^
2)
```

Fricas [A] time = 1.84489, size = 344, normalized size = 3.16

$$\frac{3\left(2a^3+\left(2a^2-2a+1\right)bx-4a^2+3a-1\right)\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}(bx+a)}{b^2x^2+2abx+a^2-1}\right)+\left(b^2x^2+2a^3-(4a-1)bx-11a^2+13a-\right.}{2\left(b^5x+(a-1)b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^3/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="fricas")
```

```
[Out] 1/2*(3*(2*a^3 + (2*a^2 - 2*a + 1)*b*x - 4*a^2 + 3*a - 1)*arctan(sqrt(-b^2*x
^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (b^2*x^2
 + 2*a^3 - (4*a - 1)*b*x - 11*a^2 + 13*a - 4)*sqrt(-b^2*x^2 - 2*a*b*x - a^2
 + 1))/(b^5*x + (a - 1)*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3}{a\sqrt{-a^2-2abx-b^2x^2+1}+bx\sqrt{-a^2-2abx-b^2x^2+1}-\sqrt{-a^2-2abx-b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)*x**3/(-b**2*x**2-2*a*b*x-a**2+1),
x)
```

```
[Out] -Integral(x**3/(a*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1) + b*x*sqrt(-a**2 -
2*a*b*x - b**2*x**2 + 1) - sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)), x)
```

Giac [A] time = 1.26801, size = 169, normalized size = 1.55

$$\frac{1}{2}\sqrt{-(bx+a)^2+1}\left(\frac{x}{b^3}-\frac{5ab^6-2b^6}{b^{10}}\right)+\frac{3\left(2a^2-2a+1\right)\arcsin\left(-bx-a\right)\operatorname{sgn}\left(b\right)}{2b^3|b|}-\frac{2\left(a^3-3a^2+3a-1\right)}{b^3\left(\frac{\sqrt{-(bx+a)^2+1}|b|+b}{b^2x+ab}-1\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^3/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="giac")
```

```
[Out] 1/2*sqrt(-(b*x + a)^2 + 1)*(x/b^3 - (5*a*b^6 - 2*b^6)/b^10) + 3/2*(2*a^2 -
2*a + 1)*arcsin(-b*x - a)*sgn(b)/(b^3*abs(b)) - 2*(a^3 - 3*a^2 + 3*a - 1)/(
b^3*((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 1)*abs(b))
```

3.870 $\int \frac{e^{\tanh^{-1}(a+bx)}x^2}{1-a^2-2abx-b^2x^2}\,dx$

Optimal. Leaf size=78

$$\frac{(1-a)^2\sqrt{a+bx+1}}{b^3\sqrt{-a-bx+1}}+\frac{\sqrt{-a-bx+1}\sqrt{a+bx+1}}{b^3}-\frac{(1-2a)\sin^{-1}(a+bx)}{b^3}$$

```
[Out] ((1 - a)^2*Sqrt[1 + a + b*x])/(b^3*Sqrt[1 - a - b*x]) + (Sqrt[1 - a - b*x]*
Sqrt[1 + a + b*x])/b^3 - ((1 - 2*a)*ArcSin[a + b*x])/b^3
```

Rubi [A] time = 0.137006, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6164, 89, 80, 53, 619, 216}

$$\frac{(1-a)^2\sqrt{a+bx+1}}{b^3\sqrt{-a-bx+1}}+\frac{\sqrt{-a-bx+1}\sqrt{a+bx+1}}{b^3}-\frac{(1-2a)\sin^{-1}(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a + b*x]*x^2)/(1 - a^2 - 2*a*b*x - b^2*x^2),x]
```

```
[Out] ((1 - a)^2*Sqrt[1 + a + b*x])/(b^3*Sqrt[1 - a - b*x]) + (Sqrt[1 - a - b*x]*
Sqrt[1 + a + b*x])/b^3 - ((1 - 2*a)*ArcSin[a + b*x])/b^3
```

Rule 6164

```
Int[E^(ArcTanh[(a_) + (b_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> Dist[(c/(1 - a^2))^p, Int[u*(1 - a - b*x)^(p - n
/2)*(1 + a + b*x)^(p + n/2), x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && E
qQ[b*d - 2*a*e, 0] && EqQ[b^2*c + e*(1 - a^2), 0] && (IntegerQ[p] || GtQ[c/
(1 - a^2), 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
```

```
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(a+bx)}x^2}{1-a^2-2abx-b^2x^2}\,dx &= \int \frac{x^2}{(1-a-bx)^{3/2}\sqrt{1+a+bx}}\,dx \\
&= \frac{(1-a)^2\sqrt{1+a+bx}}{b^3\sqrt{1-a-bx}} - \frac{\int \frac{(1-a)b+b^2x}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{b^3} \\
&= \frac{(1-a)^2\sqrt{1+a+bx}}{b^3\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{b^3} - \frac{(1-2a)\int \frac{1}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{b^2} \\
&= \frac{(1-a)^2\sqrt{1+a+bx}}{b^3\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{b^3} - \frac{(1-2a)\int \frac{1}{\sqrt{(1-a)(1+a)-2abx-b^2x^2}}\,dx}{b^2} \\
&= \frac{(1-a)^2\sqrt{1+a+bx}}{b^3\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{b^3} + \frac{(1-2a)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2ab-2\right.}{2b^4} \\
&= \frac{(1-a)^2\sqrt{1+a+bx}}{b^3\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{b^3} - \frac{(1-2a)\sin^{-1}(a+bx)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.215419, size = 64, normalized size = 0.82

$$-\frac{\frac{\left(a^2-3a-bx+2\right)\sqrt{-a^2-2abx-b^2x^2+1}}{a+bx-1}-(2a-1)\sin^{-1}(a+bx)}{b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a + b*x]*x^2)/(1 - a^2 - 2*a*b*x - b^2*x^2),x]
```

```
[Out] -((((2 - 3*a + a^2 - b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/(-1 + a + b*x)
 - (-1 + 2*a)*ArcSin[a + b*x])/b^3)
```

Maple [B] time = 0.04, size = 325, normalized size = 4.2

$$-\frac{x^2}{b}\frac{1}{\sqrt{-b^2x^2-2\,xab-a^2+1}}-4\,\frac{ax}{b^2\sqrt{-b^2x^2-2\,xab-a^2+1}}-2\,\frac{a^2}{b^3\sqrt{-b^2x^2-2\,xab-a^2+1}}+2\,\frac{a}{b^2\sqrt{b^2}}\arctan\left(\frac{}{\sqrt{-b^2x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^2/(-b^2*x^2-2*a*b*x-a^2+1),x)
```

```
[Out] -1/b*x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-4/b^2*a*x/(-b^2*x^2-2*a*b*x-a^2+1)^
(1/2)-2/b^3*a^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+2*a/b^2/(b^2)^(1/2)*arctan((
b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+2/b^3/(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2)+x/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a/b^3/(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2)+a^2/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+a^3/b^3/(-b^2*x^2-2*a*
b*x-a^2+1)^(1/2)-1/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a
*b*x-a^2+1)^(1/2))
```

Maxima [B] time = 1.81177, size = 954, normalized size = 12.23

$$\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}a^2}{b^5x+ab^4+\sqrt{b^2}b^3}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}a^2}{b^5x+ab^4-\sqrt{b^2}b^3}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}a^2}{\sqrt{b^2}b^4x+a\sqrt{b^2}b^3+b^4}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}a^2}{\sqrt{b^2}b^4x+a\sqrt{b^2}b^3-b^4}+\frac{2\,\sqrt{-b^2x^2-2\,abx-a^2+1}a\sqrt{b^2}}{b^6x+ab^5+\sqrt{b^2}b^4}+\frac{2\,\sqrt{-b^2x^2-2 \cdots}}{b^6x+a \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^2/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^2/(b^5*x + a*b^4 + sqrt(b^2)*b^3)
 - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^2/(b^5*x + a*b^4 - sqrt(b^2)*b^3) -
 sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^2/(sqrt(b^2)*b^4*x + a*sqrt(b^2)*b^3
+ b^4) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^2/(sqrt(b^2)*b^4*x + a*sqrt(b
^2)*b^3 - b^4) + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a*sqrt(b^2)/(b^6*x +
a*b^5 + sqrt(b^2)*b^4) + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a*sqrt(b^2)/(
b^6*x + a*b^5 - sqrt(b^2)*b^4) - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/(b^
5*x + a*b^4 + sqrt(b^2)*b^3) + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/(b^5*
x + a*b^4 - sqrt(b^2)*b^3) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/(b^5*x + a*
b^4 + sqrt(b^2)*b^3) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/(b^5*x + a*b^4 -
sqrt(b^2)*b^3) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/(sqrt(b^2)*b^4*x + a*sq
rt(b^2)*b^3 + b^4) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/(sqrt(b^2)*b^4*x +
a*sqrt(b^2)*b^3 - b^4) + 4*a*arcsin(sqrt(b^2)*x + a*sqrt(b^2)/b)/b^4 - 2*ar
csin(sqrt(b^2)*x + a*sqrt(b^2)/b)/b^4 + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1
)*sqrt(b^2)/b^5)*b^2/sqrt(a^2*b^2 - (a^2 - 1)*b^2)
```

Fricas [A] time = 1.72606, size = 273, normalized size = 3.5

$$-\frac{\left((2\,a-1)bx+2\,a^2-3\,a+1\right)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\sqrt{-b^2x^2-2\,abx-a^2+1}\left(a^2-bx-3\,a+2\right)}{b^4x+(a-1)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^2/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="fricas")
```

```
[Out] -(((2*a - 1)*b*x + 2*a^2 - 3*a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2
 + 1)*(a^2 - b*x - 3*a + 2))/(b^4*x + (a - 1)*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{a\sqrt{-a^2-2abx-b^2x^2+1}+bx\sqrt{-a^2-2abx-b^2x^2+1}-\sqrt{-a^2-2abx-b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)*x**2/(-b**2*x**2-2*a*b*x-a**2+1),
x)
```

```
[Out] -Integral(x**2/(a*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1) + b*x*sqrt(-a**2 -
2*a*b*x - b**2*x**2 + 1) - sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)), x)
```

Giac [A] time = 1.22262, size = 127, normalized size = 1.63

$$-\frac{(2a-1)\arcsin(-bx-a)\operatorname{sgn}(b)}{b^2|b|}+\frac{\sqrt{-(bx+a)^2+1}}{b^3}+\frac{2\left(a^2-2a+1\right)}{b^2\left(\frac{\sqrt{-(bx+a)^2+1}|b|+b}{b^2x+ab}-1\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^2/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="giac")
```

```
[Out] -(2*a - 1)*arcsin(-b*x - a)*sgn(b)/(b^2*abs(b)) + sqrt(-(b*x + a)^2 + 1)/b^
3 + 2*(a^2 - 2*a + 1)/(b^2*((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*
b) - 1)*abs(b))
```

3.871 $\int \frac{e^{\tanh^{-1}(a+bx)}x}{1-a^2-2abx-b^2x^2}\,dx$

Optimal. Leaf size=44

$$\frac{(1-a)\sqrt{a+bx+1}}{b^2\sqrt{-a-bx+1}}-\frac{\sin^{-1}(a+bx)}{b^2}$$

```
[Out] ((1 - a)*Sqrt[1 + a + b*x])/(b^2*Sqrt[1 - a - b*x]) - ArcSin[a + b*x]/b^2
```

Rubi [A] time = 0.0822346, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {6164, 78, 53, 619, 216}

$$\frac{(1-a)\sqrt{a+bx+1}}{b^2\sqrt{-a-bx+1}}-\frac{\sin^{-1}(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a + b*x]*x)/(1 - a^2 - 2*a*b*x - b^2*x^2),x]
```

```
[Out] ((1 - a)*Sqrt[1 + a + b*x])/(b^2*Sqrt[1 - a - b*x]) - ArcSin[a + b*x]/b^2
```

Rule 6164

```
Int[E^(ArcTanh[(a_) + (b_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> Dist[(c/(1 - a^2))^p, Int[u*(1 - a - b*x)^(p - n
/2)*(1 + a + b*x)^(p + n/2), x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && E
qQ[b*d - 2*a*e, 0] && EqQ[b^2*c + e*(1 - a^2), 0] && (IntegerQ[p] || GtQ[c/
(1 - a^2), 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(a+bx)}x}{1-a^2-2abx-b^2x^2}\,dx &= \int \frac{x}{(1-a-bx)^{3/2}\sqrt{1+a+bx}}\,dx\\
&= \frac{(1-a)\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} - \frac{\int \frac{1}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{b}\\
&= \frac{(1-a)\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} - \frac{\int \frac{1}{\sqrt{(1-a)(1+a)-2abx-b^2x^2}}\,dx}{b}\\
&= \frac{(1-a)\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2ab-2b^2x\right)}{2b^3}\\
&= \frac{(1-a)\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} - \frac{\sin^{-1}(a+bx)}{b^2}
\end{aligned}
$$

Mathematica [A] time = 0.145452, size = 49, normalized size = 1.11

$$
-\frac{\sin^{-1}(a+bx)-\frac{(a-1)\sqrt{-a^2-2abx-b^2x^2+1}}{a+bx-1}}{b^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a + b*x]*x)/(1 - a^2 - 2*a*b*x - b^2*x^2),x]
```

```
[Out] -((-(((-1 + a)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/(-1 + a + b*x)) + ArcSin[
a + b*x])/b^2)
```

Maple [B] time = 0.037, size = 160, normalized size = 3.6

$$
\frac{x}{b}\frac{1}{\sqrt{-b^2x^2-2\,xab-a^2+1}}-\frac{1}{b}\arctan\left(\sqrt{b^2}\left(x+\frac{a}{b}\right)\frac{1}{\sqrt{-b^2x^2-2\,xab-a^2+1}}\right)\frac{1}{\sqrt{b^2}}+\frac{1}{b^2}\frac{1}{\sqrt{-b^2x^2-2\,xab-a^2+1}}-\frac{ax}{b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x/(-b^2*x^2-2*a*b*x-a^2+1),x)
```

```
[Out] x/b/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/
b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a/b
/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-a^2/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
```

Maxima [B] time = 1.6145, size = 594, normalized size = 13.5

$$
-\frac{b^2\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}a}{b^4x+ab^3+\sqrt{b^2}b^2}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}a}{b^4x+ab^3-\sqrt{b^2}b^2}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}a}{\sqrt{b^2}b^3x+a\sqrt{b^2}b^2+b^3}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}a}{\sqrt{b^2}b^3x+a\sqrt{b^2}b^2-b^3}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}}{b^4x+ab^3+\sqrt{b^2}b^2}+\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}}{b^4x+ab^3-\sqrt{b^2}b^2}+\right.}{2\sqrt{a^2b^2-\left(a^2-1\right)b^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x/(-b^2*x^2-2*a*b*x-a^2+1),x, algor
ithm="maxima")
```

```
[Out] -1/2*b^2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/(b^4*x + a*b^3 + sqrt(b^2)*b
^2) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/(b^4*x + a*b^3 - sqrt(b^2)*b^2)
- sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/(sqrt(b^2)*b^3*x + a*sqrt(b^2)*b^2 +
 b^3) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/(sqrt(b^2)*b^3*x + a*sqrt(b^2)
*b^2 - b^3) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/(b^4*x + a*b^3 + sqrt(b^2)
*b^2) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/(b^4*x + a*b^3 - sqrt(b^2)*b^2)
+ sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/(sqrt(b^2)*b^3*x + a*sqrt(b^2)*b^2 + b
^3) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/(sqrt(b^2)*b^3*x + a*sqrt(b^2)*b^2
 - b^3) + 2*arcsin(sqrt(b^2)*x + a*sqrt(b^2)/b)/b^3)/sqrt(a^2*b^2 - (a^2 -
1)*b^2)
```

Fricas [B] time = 1.63749, size = 225, normalized size = 5.11

$$\frac{(bx+a-1)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\sqrt{-b^2x^2-2\,abx-a^2+1}(a-1)}{b^3x+(a-1)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x/(-b^2*x^2-2*a*b*x-a^2+1),x, algor
ithm="fricas")
```

```
[Out] ((b*x + a - 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2
 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a - 1))/(b^3*x
 + (a - 1)*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{a\sqrt{-a^2-2abx-b^2x^2+1}+bx\sqrt{-a^2-2abx-b^2x^2+1}-\sqrt{-a^2-2abx-b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)*x/(-b**2*x**2-2*a*b*x-a**2+1),x)
```

```
[Out] -Integral(x/(a*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1) + b*x*sqrt(-a**2 - 2*a
*b*x - b**2*x**2 + 1) - sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)), x)
```

Giac [A] time = 1.19682, size = 89, normalized size = 2.02

$$\frac{\arcsin\left(-bx-a\right)\mathrm{sgn}\left(b\right)}{b|b|}-\frac{2\,(a-1)}{b\left(\frac{\sqrt{-(bx+a)^2+1}|b|+b}{b^2x+ab}-1\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x/(-b^2*x^2-2*a*b*x-a^2+1),x, algor
ithm="giac")
```

```
[Out] arcsin(-b*x - a)*sgn(b)/(b*abs(b)) - 2*(a - 1)/(b*((sqrt(-(b*x + a)^2 + 1)*
abs(b) + b)/(b^2*x + a*b) - 1)*abs(b))
```

3.872 $\int \frac{e^{\tanh^{-1}(a+bx)}}{1-a^2-2abx-b^2x^2}\,dx$

Optimal. Leaf size=27

$$\frac{\sqrt{a+bx+1}}{b\sqrt{-a-bx+1}}$$

```
[Out] Sqrt[1 + a + b*x]/(b*Sqrt[1 - a - b*x])
```

Rubi [A] time = 0.0371005, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {6164, 37}

$$\frac{\sqrt{a+bx+1}}{b\sqrt{-a-bx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]/(1 - a^2 - 2*a*b*x - b^2*x^2),x]
```

```
[Out] Sqrt[1 + a + b*x]/(b*Sqrt[1 - a - b*x])
```

Rule 6164

```
Int[E^(ArcTanh[(a_) + (b_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> Dist[(c/(1 - a^2))^p, Int[u*(1 - a - b*x)^(p - n
/2)*(1 + a + b*x)^(p + n/2), x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && E
qQ[b*d - 2*a*e, 0] && EqQ[b^2*c + e*(1 - a^2), 0] && (IntegerQ[p] || GtQ[c/
(1 - a^2), 0])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(a+bx)}}{1-a^2-2abx-b^2x^2}\,dx &= \int \frac{1}{(1-a-bx)^{3/2}\sqrt{1+a+bx}}\,dx\\ &= \frac{\sqrt{1+a+bx}}{b\sqrt{1-a-bx}}\end{aligned}$$

Mathematica [C] time = 0.095932, size = 12, normalized size = 0.44

$$\frac{e^{\tanh^{-1}(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a + b*x]/(1 - a^2 - 2*a*b*x - b^2*x^2),x]
```

```
[Out] E^ArcTanh[a + b*x]/b
```

Maple [A] time = 0.032, size = 42, normalized size = 1.6

$$-\frac{(bx+a+1)^2(bx+a-1)}{b}\left(-b^2x^2-2\,xab-a^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/(-b^2*x^2-2*a*b*x-a^2+1),x)
```

```
[Out] -(b*x+a+1)^2*(b*x+a-1)/b/(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)
```

Maxima [B] time = 1.51165, size = 286, normalized size = 10.59

$$\frac{b^2\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}}{b^3x+ab^2+\sqrt{b^2}b}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}}{b^3x+ab^2-\sqrt{b^2}b}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}}{\sqrt{b^2}b^2x+a\sqrt{b^2}b+b^2}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}}{\sqrt{b^2}b^2x+a\sqrt{b^2}b-b^2}\right)}{2\sqrt{a^2b^2-(a^2-1)b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/(-b^2*x^2-2*a*b*x-a^2+1),x, algorit
hm="maxima")
```

```
[Out] 1/2*b^2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/(b^3*x + a*b^2 + sqrt(b^2)*b) -
 sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/(b^3*x + a*b^2 - sqrt(b^2)*b) - sqrt(-b
^2*x^2 - 2*a*b*x - a^2 + 1)/(sqrt(b^2)*b^2*x + a*sqrt(b^2)*b + b^2) - sqrt(
-b^2*x^2 - 2*a*b*x - a^2 + 1)/(sqrt(b^2)*b^2*x + a*sqrt(b^2)*b - b^2))/sqrt
(a^2*b^2 - (a^2 - 1)*b^2)
```

Fricas [A] time = 1.66461, size = 77, normalized size = 2.85

$$-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}}{b^2x+(a-1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/(-b^2*x^2-2*a*b*x-a^2+1),x, algorit
hm="fricas")
```

```
[Out] -sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/(b^2*x + (a - 1)*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{a\sqrt{-a^2-2abx-b^2x^2+1}+bx\sqrt{-a^2-2abx-b^2x^2+1}-\sqrt{-a^2-2abx-b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)/(-b**2*x**2-2*a*b*x-a**2+1),x)
```

```
[Out] -Integral(1/(a*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1) + b*x*sqrt(-a**2 - 2*a
*b*x - b**2*x**2 + 1) - sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)), x)
```

Giac [A] time = 1.20698, size = 54, normalized size = 2.

$$\frac{2}{\left(\frac{\sqrt{-(bx+a)^2+1}|b|+b}{b^2x+ab}-1\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/(-b^2*x^2-2*a*b*x-a^2+1),x, algorit
hm="giac")
```

```
[Out] 2/(((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 1)*abs(b))
```

3.873 $\int \frac{e^{\tanh^{-1}(a+bx)}}{x(1-a^2-2abx-b^2x^2)}\,dx$

Optimal. Leaf size=93

$$\frac{\sqrt{a+bx+1}}{(1-a)\sqrt{-a-bx+1}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)\sqrt{1-a^2}}$$

```
[Out] Sqrt[1 + a + b*x]/((1 - a)*Sqrt[1 - a - b*x]) - (2*ArcTanh[(Sqrt[1 - a]*Sqr
t[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 - a)*Sqrt[1 - a^2])
```

Rubi [A] time = 0.131745, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6164, 96, 93, 208}

$$\frac{\sqrt{a+bx+1}}{(1-a)\sqrt{-a-bx+1}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]/(x*(1 - a^2 - 2*a*b*x - b^2*x^2)),x]
```

```
[Out] Sqrt[1 + a + b*x]/((1 - a)*Sqrt[1 - a - b*x]) - (2*ArcTanh[(Sqrt[1 - a]*Sqr
t[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 - a)*Sqrt[1 - a^2])
```

Rule 6164

```
Int[E^(ArcTanh[(a_) + (b_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> Dist[(c/(1 - a^2))^p, Int[u*(1 - a - b*x)^(p - n
/2)*(1 + a + b*x)^(p + n/2), x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && E
qQ[b*d - 2*a*e, 0] && EqQ[b^2*c + e*(1 - a^2), 0] && (IntegerQ[p] || GtQ[c/
(1 - a^2), 0])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(a+bx)}}{x\left(1-a^2-2abx-b^2x^2\right)}\,dx = \int \frac{1}{x(1-a-bx)^{3/2}\sqrt{1+a+bx}}\,dx$$

$$= \frac{\sqrt{1+a+bx}}{(1-a)\sqrt{1-a-bx}} + \frac{\int \frac{1}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{1-a}$$

$$= \frac{\sqrt{1+a+bx}}{(1-a)\sqrt{1-a-bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx, x, \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\right)}{1-a}$$

$$= \frac{\sqrt{1+a+bx}}{(1-a)\sqrt{1-a-bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{(1-a)\sqrt{1-a^2}}$$

Mathematica [A] time = 0.242331, size = 118, normalized size = 1.27

$$-\frac{-\frac{\sqrt{-a^2-2abx-b^2x^2+1}}{a+bx-1}-\frac{\log\left(\sqrt{1-a^2}\sqrt{-a^2-2abx-b^2x^2+1}-a^2-abx+1\right)}{\sqrt{1-a^2}}+\frac{\log(x)}{\sqrt{1-a^2}}}{a-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]/(x*(1 - a^2 - 2*a*b*x - b^2*x^2)),x]
```

```
[Out] -((-(Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]/(-1 + a + b*x)) + Log[x]/Sqrt[1 - a^
2] - Log[1 - a^2 - a*b*x + Sqrt[1 - a^2]*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]]
/Sqrt[1 - a^2])/(-1 + a))
```

Maple [B] time = 0.037, size = 391, normalized size = 4.2

$$2\,{\frac {b \left(-2\,{b}^{2}x-2\,ab \right) }{ \left(-4\,{b}^{2} \left(-{a}^{2}+1 \right) -4\,{a}^{2}{b}^{2} \right) \sqrt {-{b}^{2}{x}^{2}-2\,xab-{a}^{2}+1}}}+{\frac {1}{-{a}^{2}+1}{\frac {1}{\sqrt {-{b}^{2}{x}^{2}-2\,xab-{a}^{2}+1}}}}+{\frac {xab}{-{a}^{2}+1}{\frac {1}{\sqrt {-{b}^{2}{x}^{2}-2\,xab-}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x/(-b^2*x^2-2*a*b*x-a^2+1),x)
```

```
[Out] 2*b*(-2*b^2*x-2*a*b)/(-4*b^2*(-a^2+1)-4*a^2*b^2)/(-b^2*x^2-2*a*b*x-a^2+1)^(
1/2)+1/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+a*b/(-a^2+1)/(-b^2*x^2-2*a*b
*x-a^2+1)^(1/2)*x+a^2/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/(-a^2+1)^(3
/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x
)+1/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a+a^2*b/(-a^2+1)/(-b^2*x^2-2*a*
b*x-a^2+1)^(1/2)*x+1/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^3-1/(-a^2+1)
^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
)/x)*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx+a+1}{\left(b^2x^2+2\,abx+a^2-1\right)\sqrt{-(bx+a)^2+1x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x/(-b^2*x^2-2*a*b*x-a^2+1),x, algor
ithm="maxima")
```

```
[Out] -integrate((b*x + a + 1)/((b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-(b*x + a)^2 +
 1)*x), x)
```

Fricas [A] time = 1.76028, size = 730, normalized size = 7.85

$$\left[-\frac{\sqrt{-a^2+1}(bx+a-1)\log\left(\frac{(2\,a^2-1)b^2x^2+2\,a^4+4\,(a^3-a)bx-2\,\sqrt{-b^2x^2-2\,abx-a^2+1}(abx+a^2-1)\sqrt{-a^2+1}-4\,a^2+2}{x^2}\right)-2\,\sqrt{-b^2x^2-2\,abx-a^2+}}{2\left(a^4-2\,a^3+\left(a^3-a^2-a+1\right)bx+2\,a-1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x/(-b^2*x^2-2*a*b*x-a^2+1),x, algor
ithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + 1)*(b*x + a - 1)*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a
^3 - a)*b*x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-
a^2 + 1) - 4*a^2 + 2)/x^2) - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1)
)/(a^4 - 2*a^3 + (a^3 - a^2 - a + 1)*b*x + 2*a - 1), -(sqrt(a^2 - 1)*(b*x +
 a - 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^
2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) - sqrt(-b^2
*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1))/(a^4 - 2*a^3 + (a^3 - a^2 - a + 1)*b*x
 + 2*a - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{ax\sqrt{-a^2-2abx-b^2x^2+1}+bx^2\sqrt{-a^2-2abx-b^2x^2+1}-x\sqrt{-a^2-2abx-b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)/x/(-b**2*x**2-2*a*b*x-a**2+1),x)
```

```
[Out] -Integral(1/(a*x*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1) + b*x**2*sqrt(-a**2
- 2*a*b*x - b**2*x**2 + 1) - x*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)), x)
```

Giac [A] time = 1.21274, size = 151, normalized size = 1.62

$$-\frac{2\,b\arctan\left(\frac{\frac{\left(\sqrt{-(bx+a)^2+1}|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}(a|b|-|b|)}-\frac{2\,b}{(a|b|-|b|)\left(\frac{\sqrt{-(bx+a)^2+1}|b|+b}{b^2x+ab}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x/(-b^2*x^2-2*a*b*x-a^2+1),x, algor
ithm="giac")
```

```
[Out] -2*b*arctan(((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)/sqrt(
a^2 - 1))/(sqrt(a^2 - 1)*(a*abs(b) - abs(b))) - 2*b/((a*abs(b) - abs(b))*((
sqrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 1))
```

3.874 $\int \frac{e^{\tanh^{-1}(a+bx)}}{x^2(1-a^2-2abx-b^2x^2)}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt{a+bx+1}}{(1-a^2)\,x\sqrt{-a-bx+1}}-\frac{2(2a+1)b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)^2(a+1)\sqrt{1-a^2}}+\frac{(a+2)b\sqrt{a+bx+1}}{(1-a)^2(a+1)\sqrt{-a-bx+1}}$$

```
[Out] ((2 + a)*b*Sqrt[1 + a + b*x])/((1 - a)^2*(1 + a)*Sqrt[1 - a - b*x]) - Sqrt[
1 + a + b*x]/((1 - a^2)*x*Sqrt[1 - a - b*x]) - (2*(1 + 2*a)*b*ArcTanh[(Sqrt
[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 - a)^2*(1
+ a)*Sqrt[1 - a^2])
```

Rubi [A] time = 0.159194, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6164, 103, 152, 12, 93, 208}

$$-\frac{\sqrt{a+bx+1}}{(1-a^2)\,x\sqrt{-a-bx+1}}-\frac{2(2a+1)b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)^2(a+1)\sqrt{1-a^2}}+\frac{(a+2)b\sqrt{a+bx+1}}{(1-a)^2(a+1)\sqrt{-a-bx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]/(x^2*(1 - a^2 - 2*a*b*x - b^2*x^2)),x]
```

```
[Out] ((2 + a)*b*Sqrt[1 + a + b*x])/((1 - a)^2*(1 + a)*Sqrt[1 - a - b*x]) - Sqrt[
1 + a + b*x]/((1 - a^2)*x*Sqrt[1 - a - b*x]) - (2*(1 + 2*a)*b*ArcTanh[(Sqrt
[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 - a)^2*(1
+ a)*Sqrt[1 - a^2])
```

Rule 6164

```
Int[E^(ArcTanh[(a_) + (b_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> Dist[(c/(1 - a^2))^p, Int[u*(1 - a - b*x)^(p - n
/2)*(1 + a + b*x)^(p + n/2), x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && E
qQ[b*d - 2*a*e, 0] && EqQ[b^2*c + e*(1 - a^2), 0] && (IntegerQ[p] || GtQ[c/
(1 - a^2), 0])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(a+bx)}}{x^2\left(1-a^2-2abx-b^2x^2\right)}\,dx=\int \frac{1}{x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}\,dx$$

$$=-\frac{\sqrt{1+a+bx}}{\left(1-a^2\right)x\sqrt{1-a-bx}}-\frac{\int \frac{-(1+2a)b-b^2x}{x(1-a-bx)^{3/2}\sqrt{1+a+bx}}\,dx}{1-a^2}$$

$$=\frac{(2+a)b\sqrt{1+a+bx}}{(1-a)^2(1+a)\sqrt{1-a-bx}}-\frac{\sqrt{1+a+bx}}{\left(1-a^2\right)x\sqrt{1-a-bx}}+\frac{\int \frac{(1+2a)b^2}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{(1-a)^2(1+a)b}$$

$$=\frac{(2+a)b\sqrt{1+a+bx}}{(1-a)^2(1+a)\sqrt{1-a-bx}}-\frac{\sqrt{1+a+bx}}{\left(1-a^2\right)x\sqrt{1-a-bx}}+\frac{((1+2a)b)\int \frac{1}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}}{(1-a)^2(1+a)}$$

$$=\frac{(2+a)b\sqrt{1+a+bx}}{(1-a)^2(1+a)\sqrt{1-a-bx}}-\frac{\sqrt{1+a+bx}}{\left(1-a^2\right)x\sqrt{1-a-bx}}+\frac{(2(1+2a)b)\,\mathrm{Subst}\left(\int \frac{1}{-1-a-(\cdots}\right.}{(1-a)^2(1-\cdots}$$

$$=\frac{(2+a)b\sqrt{1+a+bx}}{(1-a)^2(1+a)\sqrt{1-a-bx}}-\frac{\sqrt{1+a+bx}}{\left(1-a^2\right)x\sqrt{1-a-bx}}-\frac{2(1+2a)b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a\cdots}}{\sqrt{1+a}\sqrt{1-a\cdots}}\right.}{(1-a)^2(1+a)\sqrt{1-a^2}}$$

Mathematica [A] time = 0.298647, size = 149, normalized size = 0.99

$$-\frac{\sqrt{-a^2-2abx-b^2x^2+1}\left(\frac{b}{a+bx-1}+\frac{1}{ax+x}\right)+\frac{(2a+1)b\log\left(\sqrt{1-a^2}\sqrt{-a^2-2abx-b^2x^2+1}-a^2-abx+1\right)}{(a+1)\sqrt{1-a^2}}-\frac{(2a+1)b\log(x)}{(a+1)\sqrt{1-a^2}}}{(a-1)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]/(x^2*(1 - a^2 - 2*a*b*x - b^2*x^2)),x]
```

```
[Out] -((Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*((x + a*x)^(-1) + b/(-1 + a + b*x)) -
((1 + 2*a)*b*Log[x])/((1 + a)*Sqrt[1 - a^2]) + ((1 + 2*a)*b*Log[1 - a^2 - a
```

```
*b*x + Sqrt[1 - a^2]*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]])/((1 + a)*Sqrt[1 -
a^2]))/(-1 + a)^2)
```

Maple [B] time = 0.062, size = 671, normalized size = 4.5

$$-\frac{1}{\left(-a^2+1\right)x\sqrt{-b^2x^2-2xab-a^2+1}}+3\frac{ab}{\left(-a^2+1\right)^2\sqrt{-b^2x^2-2xab-a^2+1}}+3\frac{a^2b^2x}{\left(-a^2+1\right)^2\sqrt{-b^2x^2-2xab-a^2+1}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^2/(-b^2*x^2-2*a*b*x-a^2+1),x)
```

```
[Out] -1/(-a^2+1)/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3*a*b/(-a^2+1)^2/(-b^2*x^2-2*a
*b*x-a^2+1)^(1/2)+3*a^2*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+3*a
^3*b/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3*a*b/(-a^2+1)^(5/2)*ln((-2*
a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+2/(-a^2+1
)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x*b^2+2/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^
(1/2)*a*b-1/(-a^2+1)/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a+3*a^2*b/(-a^2+1)^2/
(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3*a^3*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1
)^(1/2)*x+3*a^4*b/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3*a^2*b/(-a^2+1
)^(5/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2
))/x)+3/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x*b^2*a+3/(-a^2+1)/(-b^2*x^
2-2*a*b*x-a^2+1)^(1/2)*a^2*b+b/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-b/(-
a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)
^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx+a+1}{\left(b^2x^2+2\,abx+a^2-1\right)\sqrt{-(bx+a)^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^2/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="maxima")
```

```
[Out] -integrate((b*x + a + 1)/((b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-(b*x + a)^2 +
 1)*x^2), x)
```

Fricas [A] time = 1.83045, size = 1021, normalized size = 6.81

$$\left[-\frac{\left((2\,a+1)b^2x^2+\left(2\,a^2-a-1\right)bx\right)\sqrt{-a^2+1}\log\left(\frac{\left(2\,a^2-1\right)b^2x^2+2\,a^4+4\left(a^3-a\right)bx+2\sqrt{-b^2x^2-2\,abx-a^2+1}\left(abx+a^2-1\right)\sqrt{-a^2+1}-4\,a^2+2}{x^2}\right)+2\,\sqrt{}}{2\left(\left(a^5-a^4-2\,a^3+2\,a^2+a-1\right)bx^2+\left(a^6-2\,a^5-a^4+4\,a^3-a^2-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^2/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="fricas")
```

[Out] [-1/2*(((2*a + 1)*b^2*x^2 + (2*a^2 - a - 1)*b*x)*sqrt(-a^2 + 1)*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*x + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1) - 4*a^2 + 2)/x^2) + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^3 + (a^3 + 2*a^2 - a - 2)*b*x - a^2 - a + 1))/((a^5 - a^4 - 2*a^3 + 2*a^2 + a - 1)*b*x^2 + (a^6 - 2*a^5 - a^4 + 4*a^3 - a^2 - 2*a + 1)*x), (((2*a + 1)*b^2*x^2 + (2*a^2 - a - 1)*b*x)*sqrt(a^2 - 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^3 + (a^3 + 2*a^2 - a - 2)*b*x - a^2 - a + 1))/((a^5 - a^4 - 2*a^3 + 2*a^2 + a - 1)*b*x^2 + (a^6 - 2*a^5 - a^4 + 4*a^3 - a^2 - 2*a + 1)*x)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{ax^2\sqrt{-a^2-2abx-b^2x^2+1}+bx^3\sqrt{-a^2-2abx-b^2x^2+1}-x^2\sqrt{-a^2-2abx-b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)/x**2/(-b**2*x**2-2*a*b*x-a**2+1), x)

[Out] -Integral(1/(a*x**2*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1) + b*x**3*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1) - x**2*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)), x)

Giac [B] time = 1.22364, size = 701, normalized size = 4.67

$$\frac{2\left(2\,ab^2+b^2\right)\arctan\left(\frac{\frac{\left(\sqrt{-(bx+a)^2+1}|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\left(a^3|b|-a^2|b|-a|b|+|b|\right)\sqrt{a^2-1}}+\frac{2\left(\frac{\left(\sqrt{-(bx+a)^2+1}|b|+b\right)^2a^3b^2}{\left(b^2x+ab\right)^2}+a^3b^2-\frac{2\left(\sqrt{-(bx+a)^2+1}|b|+b\right)a^2b^2}{b^2x+ab}+\frac{\left(\sqrt{-(bx+a)^2+1}|b|+b\right)}{\left(b^2x+ab\right)^2}\right.}{\left(a^4|b|-a^3|b|-a^2|b|+a|b|\right)\left(\frac{\left(\sqrt{-(bx+a)^2+1}|b|+b\right)a}{b^2x+ab}-\frac{\left(\sqrt{-(bx+a)^2+1}|b|\right.}{\left(b^2x+ab\right)^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^2/(-b^2*x^2-2*a*b*x-a^2+1),x, algorithm="giac")

[Out] 2*(2*a*b^2 + b^2)*arctan(((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/((a^3*abs(b) - a^2*abs(b) - a*abs(b) + abs(b))*sqrt(a^2 - 1)) + 2*((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^3*b^2/(b^2*x + a*b)^2 + a^3*b^2 - 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a^2*b^2/(b^2*x + a*b) + (sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^2*b^2/(b^2*x + a*b)^2 + a^2*b^2 - 3*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a*b^2/(b^2*x + a*b) + a*b^2 - (sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*b^2/(b^2*x + a*b) + (sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*b^2/(b^2*x + a*b)^2)/((a^4*abs(b) - a^3*abs(b) - a^2*abs(b) + a*abs(b))*((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - (sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a/(b^2*x + a*b)^2 + (sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a/(b^2*x + a*b)^3 - a + 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2/(b^2*x + a*b)^2))

3.875 $\int e^{n\tanh^{-1}(a+bx)}x^m\,dx$

Optimal. Leaf size=109

$$\frac{x^{m+1}(-a-bx+1)^{-n/2}(a+bx+1)^{n/2}\left(1-\frac{bx}{1-a}\right)^{n/2}\left(\frac{bx}{a+1}+1\right)^{-n/2}F_1\left(m+1;\frac{n}{2},-\frac{n}{2};m+2;\frac{bx}{1-a},-\frac{bx}{a+1}\right)}{m+1}$$

```
[Out] (x^(1 + m)*(1 + a + b*x)^(n/2)*(1 - (b*x)/(1 - a))^(n/2)*AppellF1[1 + m, n/
2, -n/2, 2 + m, (b*x)/(1 - a), -((b*x)/(1 + a))])/((1 + m)*(1 - a - b*x)^(n
/2)*(1 + (b*x)/(1 + a))^(n/2))
```

Rubi [A] time = 0.0759803, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6163, 135, 133}

$$\frac{x^{m+1}(-a-bx+1)^{-n/2}(a+bx+1)^{n/2}\left(1-\frac{bx}{1-a}\right)^{n/2}\left(\frac{bx}{a+1}+1\right)^{-n/2}F_1\left(m+1;\frac{n}{2},-\frac{n}{2};m+2;\frac{bx}{1-a},-\frac{bx}{a+1}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a + b*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(1 + a + b*x)^(n/2)*(1 - (b*x)/(1 - a))^(n/2)*AppellF1[1 + m, n/
2, -n/2, 2 + m, (b*x)/(1 - a), -((b*x)/(1 + a))])/((1 + m)*(1 - a - b*x)^(n
/2)*(1 + (b*x)/(1 + a))^(n/2))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(a+bx)}x^m\,dx &= \int x^m(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\,dx\\ &= \left((1-a-bx)^{-n/2}\left(1-\frac{bx}{1-a}\right)^{n/2}\right)\int x^m(1+a+bx)^{n/2}\left(1-\frac{bx}{1-a}\right)^{-n/2}dx\\ &= \left((1-a-bx)^{-n/2}(1+a+bx)^{n/2}\left(1-\frac{bx}{1-a}\right)^{n/2}\left(1+\frac{bx}{1+a}\right)^{-n/2}\right)\int x^m\left(1-\frac{bx}{1-a}\right)^{-n/2}\left(1+\frac{bx}{1+\ldots}\right.\\ &= \frac{x^{1+m}(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\left(1-\frac{bx}{1-a}\right)^{n/2}\left(1+\frac{bx}{1+a}\right)^{-n/2}F_1\left(1+m;\frac{n}{2},-\frac{n}{2};2+m;\frac{bx}{1-a},-\ldots\right.}{1+m}\end{aligned}$$

Mathematica [F] time = 0.790792, size = 0, normalized size = 0.

$$\int e^{n\tanh^{-1}(a+bx)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a + b*x])*x^m,x]
```

```
[Out] Integrate[E^(n*ArcTanh[a + b*x])*x^m, x]
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int e^{n\operatorname{Artanh}(bx+a)}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(b*x+a))*x^m,x)
```

```
[Out] int(exp(n*arctanh(b*x+a))*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m e^{n \operatorname{atanh}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(b*x+a))*x**m,x)
```

```
[Out] Integral(x**m*exp(n*atanh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

3.876 $\int e^{n\tanh^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=206

$$\frac{2^{\frac{n}{2}-2}\left(-36a^2n+24a^3+12a\left(n^2+2\right)-n\left(n^2+8\right)\right)(-a-bx+1)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(-a-b\right.}{3b^4(2-n)}$$

```
[Out] -(x^2*(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2))/(4*b^2) - ((1 - a
- b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2)*(6 + 18*a^2 - 10*a*n + n^2 - 2*b
*(6*a - n)*x))/(24*b^4) + (2^(-2 + n/2)*(24*a^3 - 36*a^2*n + 12*a*(2 + n^2)
 - n*(8 + n^2))*(1 - a - b*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2
- n/2, (1 - a - b*x)/2])/(3*b^4*(2 - n))
```

Rubi [A] time = 0.182662, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6163, 100, 147, 69}

$$\frac{2^{\frac{n}{2}-2}\left(-36a^2n+24a^3+12a\left(n^2+2\right)-n\left(n^2+8\right)\right)(-a-bx+1)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(-a-bx+1)\right)}{3b^4(2-n)}-\frac{(a+bx+}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a + b*x])*x^3,x]
```


```
[Out] -(x^2*(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2))/(4*b^2) - ((1 - a
- b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2)*(6 + 18*a^2 - 10*a*n + n^2 - 2*b
*(6*a - n)*x))/(24*b^4) + (2^(-2 + n/2)*(24*a^3 - 36*a^2*n + 12*a*(2 + n^2)
 - n*(8 + n^2))*(1 - a - b*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2
- n/2, (1 - a - b*x)/2])/(3*b^4*(2 - n))
```


Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
```

```
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tanh^{-1}(a+bx)}x^3\,dx = \int x^3(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\,dx$$
$$= -\frac{x^2(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{4b^2} - \frac{\int x(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\left(-2\left(1-a^2\right)+b(6a-n)x\right.}{4b^2}$$
$$= -\frac{x^2(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{4b^2} - \frac{(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}\left(6+18a^2-10an+n^2-2b(\right.}{24b^4}$$
$$= -\frac{x^2(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{4b^2} - \frac{(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}\left(6+18a^2-10an+n^2-2b(\right.}{24b^4}$$

Mathematica [A] time = 0.251091, size = 220, normalized size = 1.07

$$(-a-bx+1)^{1-\frac{n}{2}}\left(-2^{\frac{n}{2}+3}(n-6a)\text{Hypergeometric2F1}\left(-\frac{n}{2}-2,1-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(-a-bx+1)\right)-(a+1)2^{\frac{n}{2}+3}(5a-n+1)\text{H}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a + b*x])*x^3,x]
```

```
[Out] ((1 - a - b*x)^(1 - n/2)*(b^2*(-2 + n)*x^2*(1 + a + b*x)^(1 + n/2) - 2^(3 +
 n/2)*(-6*a + n)*Hypergeometric2F1[-2 - n/2, 1 - n/2, 2 - n/2, (1 - a - b*x
)/2] - 2^(3 + n/2)*(1 + a)*(1 + 5*a - n)*Hypergeometric2F1[-1 - n/2, 1 - n/
2, 2 - n/2, (1 - a - b*x)/2] + 2^(1 + n/2)*(1 + a)^2*(2 + 4*a - n)*Hypergeo
metric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a - b*x)/2]))/(4*b^4*(2 - n))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(bx+a)}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(b*x+a))*x^3,x)
```

```
[Out] int(exp(n*arctanh(b*x+a))*x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(\frac{bx + a + 1}{bx + a - 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3 \left(\frac{bx + a + 1}{bx + a - 1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 e^{n \operatorname{atanh}(a+bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(b*x+a))*x**3,x)
```

```
[Out] Integral(x**3*exp(n*atanh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(\frac{bx + a + 1}{bx + a - 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

3.877 $\int e^{n\tanh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=170

$$-\frac{2^{n/2}\left(6a^2-6an+n^2+2\right)(-a-bx+1)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(-a-bx+1)\right)}{3b^3(2-n)}+\frac{(4a-n)(a+bx+$$

```
[Out] ((4*a - n)*(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2))/(6*b^3) - (x*
(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2))/(3*b^2) - (2^(n/2)*(2 +
6*a^2 - 6*a*n + n^2)*(1 - a - b*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/
2, 2 - n/2, (1 - a - b*x)/2])/(3*b^3*(2 - n))
```

Rubi [A] time = 0.158488, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6163, 90, 80, 69}

$$-\frac{2^{n/2}\left(6a^2-6an+n^2+2\right)(-a-bx+1)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(-a-bx+1)\right)}{3b^3(2-n)}+\frac{(4a-n)(a+bx+1)^{\frac{n+2}{2}}(-a-bx+1}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a + b*x])*x^2,x]
```

```
[Out] ((4*a - n)*(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2))/(6*b^3) - (x*
(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2))/(3*b^2) - (2^(n/2)*(2 +
6*a^2 - 6*a*n + n^2)*(1 - a - b*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/
2, 2 - n/2, (1 - a - b*x)/2])/(3*b^3*(2 - n))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tanh^{-1}(a+bx)}x^2\,dx = \int x^2(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\,dx$$
$$= -\frac{x(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{3b^2} - \frac{\int (1-a-bx)^{-n/2}(1+a+bx)^{n/2}\left(-1+a^2+b(4a-n)x\right) d}{3b^2}$$
$$= \frac{(4a-n)(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{6b^3} - \frac{x(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{3b^2} + \frac{\left(2+6a^2-6an+\right.}{}$$
$$= \frac{(4a-n)(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{6b^3} - \frac{x(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{3b^2} - \frac{2^{n/2}\left(2+6a^2-6\right.}{}$$

Mathematica [A] time = 0.113065, size = 127, normalized size = 0.75

$$\frac{(-a-bx+1)^{1-\frac{n}{2}}\left(\frac{2^{\frac{n}{2}+1}\left(6a^2-6an+n^2+2\right)\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(-a-bx+1)\right)}{n-2}+(4a-n)(a+bx+1)^{\frac{n}{2}+1}-2bx(a+bx+1\right.}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a + b*x])*x^2,x]
```

```
[Out] ((1 - a - b*x)^(1 - n/2)*((4*a - n)*(1 + a + b*x)^(1 + n/2) - 2*b*x*(1 + a
+ b*x)^(1 + n/2) + (2^(1 + n/2)*(2 + 6*a^2 - 6*a*n + n^2)*Hypergeometric2F1
[1 - n/2, -n/2, 2 - n/2, (1 - a - b*x)/2])/(-2 + n)))/(6*b^3)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(bx+a)}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(b*x+a))*x^2,x)
```

```
[Out] int(exp(n*arctanh(b*x+a))*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2e^{n\,\text{atanh}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(b*x+a))*x**2,x)
```

```
[Out] Integral(x**2*exp(n*atanh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

3.878 $\int e^{n\tanh^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=114

$$\frac{2^{n/2}(2a-n)(-a-bx+1)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(-a-bx+1)\right)}{b^2(2-n)}-\frac{(-a-bx+1)^{1-\frac{n}{2}}(a+bx+1)}{2b^2}$$

```
[Out] -((1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2))/(2*b^2) + (2^(n/2)*(2*
a - n)*(1 - a - b*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1
 - a - b*x)/2])/(b^2*(2 - n))
```

Rubi [A] time = 0.0687108, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6163, 80, 69}

$$\frac{2^{n/2}(2a-n)(-a-bx+1)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(-a-bx+1)\right)}{b^2(2-n)}-\frac{(-a-bx+1)^{1-\frac{n}{2}}(a+bx+1)^{\frac{n+2}{2}}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a + b*x])*x,x]
```

```
[Out] -((1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2))/(2*b^2) + (2^(n/2)*(2*
a - n)*(1 - a - b*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1
 - a - b*x)/2])/(b^2*(2 - n))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tanh^{-1}(a+bx)}x\,dx=\int x(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\,dx$$
$$=-\frac{(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{2b^2}-\frac{(2a-n)\int(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\,dx}{2b}$$
$$=-\frac{(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{2b^2}+\frac{2^{n/2}(2a-n)(1-a-bx)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-a-bx)\right)}{b^2(2-n)}$$

Mathematica [A] time = 0.0403405, size = 96, normalized size = 0.84

$$\frac{(-a-bx+1)^{1-\frac{n}{2}}\left(\frac{b2^{\frac{n}{2}+1}(n-2a)\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(-a-bx+1)\right)}{n-2}-b(a+bx+1)^{\frac{n}{2}+1}\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a + b*x])*x,x]
```

```
[Out] ((1 - a - b*x)^(1 - n/2)*(-(b*(1 + a + b*x)^(1 + n/2)) + (2^(1 + n/2)*b*(-2
*a + n)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a - b*x)/2])/(-2 + n
)))/(2*b^3)
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int e^{n\operatorname{Artanh}(bx+a)}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(b*x+a))*x,x)
```

```
[Out] int(exp(n*arctanh(b*x+a))*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x,x, algorithm="fricas")
```

```
[Out] integral(x*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{n\,\mathrm{atanh}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(b*x+a))*x,x)
```

```
[Out] Integral(x*exp(n*atanh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x,x, algorithm="giac")
```

```
[Out] integrate(x*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

3.879 $\int e^{n \tanh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=71

$$-\frac{2^{\frac{n}{2}+1}(-a-bx+1)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(-a-bx+1)\right)}{b(2-n)}$$

```
[Out] -((2^(1 + n/2)*(1 - a - b*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 -
 n/2, (1 - a - b*x)/2])/(b*(2 - n)))
```

Rubi [A] time = 0.0141212, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6161, 69}

$$-\frac{2^{\frac{n}{2}+1}(-a-bx+1)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(-a-bx+1)\right)}{b(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a + b*x]),x]
```

```
[Out] -((2^(1 + n/2)*(1 - a - b*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 -
 n/2, (1 - a - b*x)/2])/(b*(2 - n)))
```

Rule 6161

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 + a*c
 + b*c*x)^(n/2)/(1 - a*c - b*c*x)^(n/2), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{n \tanh^{-1}(a+bx)}\,dx &= \int (1-a-bx)^{-n/2}(1+a+bx)^{n/2}\,dx\\ &= -\frac{2^{1+\frac{n}{2}}(1-a-bx)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-a-bx)\right)}{b(2-n)}\end{aligned}$$

Mathematica [A] time = 0.0406865, size = 50, normalized size = 0.7

$$\frac{4e^{(n+2)\tanh^{-1}(a+bx)}\text{Hypergeometric2F1}\left(2,\frac{n}{2}+1,\frac{n}{2}+2,-e^{2\tanh^{-1}(a+bx)}\right)}{b(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a + b*x]),x]
```

```
[Out] (4*E^((2 + n)*ArcTanh[a + b*x])*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E^(
2*ArcTanh[a + b*x])])/(b*(2 + n))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int e^{n \operatorname{Artanh}(bx+a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(b*x+a)),x)
```

```
[Out] int(exp(n*arctanh(b*x+a)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] integral(((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n \operatorname{atanh}(a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(b*x+a)),x)
```

```
[Out] Integral(exp(n*atanh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{bx + a + 1}{bx + a - 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

3.880 $\int \frac{e^{n\tanh^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=135

$$\frac{2(-a-bx+1)^{-n/2}(a+bx+1)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{(a+1)(-a-bx+1)}{(1-a)(a+bx+1)}\right)}{n}-\frac{2^{\frac{n}{2}+1}(-a-bx+1)^{-n/2}\text{Hyperge}}{}$$

```
[Out] (2*(1 + a + b*x)^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, ((1 + a)*(1 - a
- b*x))/((1 - a)*(1 + a + b*x))])/(n*(1 - a - b*x)^(n/2)) - (2^(1 + n/2)*Hy
pergeometric2F1[-n/2, -n/2, 1 - n/2, (1 - a - b*x)/2])/(n*(1 - a - b*x)^(n/
2))
```

Rubi [A] time = 0.0704511, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6163, 105, 69, 131}

$$\frac{2(-a-bx+1)^{-n/2}(a+bx+1)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{(a+1)(-a-bx+1)}{(1-a)(a+bx+1)}\right)}{n}-\frac{2^{\frac{n}{2}+1}(-a-bx+1)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(-a-b\right.}{n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a + b*x])/x,x]
```

```
[Out] (2*(1 + a + b*x)^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, ((1 + a)*(1 - a
- b*x))/((1 - a)*(1 + a + b*x))])/(n*(1 - a - b*x)^(n/2)) - (2^(1 + n/2)*Hy
pergeometric2F1[-n/2, -n/2, 1 - n/2, (1 - a - b*x)/2])/(n*(1 - a - b*x)^(n/
2))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
```

```
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(a+bx)}}{x}\,dx = \int \frac{(1-a-bx)^{-n/2}(1+a+bx)^{n/2}}{x}\,dx$$
$$= -\left((-1+a)\int \frac{(1-a-bx)^{-1-\frac{n}{2}}(1+a+bx)^{n/2}}{x}\,dx\right) - b\int (1-a-bx)^{-1-\frac{n}{2}}(1+a+bx)^{n/2}\,dx$$
$$= \frac{2(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{(1+a)(1-a-bx)}{(1-a)(1+a+bx)}\right)}{n} - \frac{2^{1+\frac{n}{2}}(1-a-bx)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};\right.}{n}$$

Mathematica [A] time = 0.0354214, size = 111, normalized size = 0.82

$$\frac{2(-a-bx+1)^{-n/2}\left((a+bx+1)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{(a+1)(a+bx-1)}{(a-1)(a+bx+1)}\right)-2^{n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2}\right.\right.}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a + b*x])/x,x]
```

```
[Out] (2*((1 + a + b*x)^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, ((1 + a)*(-1 +
a + b*x))/((-1 + a)*(1 + a + b*x))] - 2^(n/2)*Hypergeometric2F1[-n/2, -n/2,
 1 - n/2, (1 - a - b*x)/2]))/(n*(1 - a - b*x)^(n/2))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{e^{n\,\mathrm{Artanh}(bx+a)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(b*x+a))/x,x)
```

```
[Out] int(exp(n*arctanh(b*x+a))/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{atanh}(a+bx)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(b*x+a))/x,x)
```

```
[Out] Integral(exp(n*atanh(a + b*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x, x)
```

3.881 $\int \frac{e^{n\tanh^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=92

$$-\frac{4b(-a-bx+1)^{1-\frac{n}{2}}(a+bx+1)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{(a+1)(-a-bx+1)}{(1-a)(a+bx+1)}\right)}{(1-a)^2(2-n)}$$

```
[Out] (-4*b*(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((-2 + n)/2)*Hypergeometric2F1[
2, 1 - n/2, 2 - n/2, ((1 + a)*(1 - a - b*x))/((1 - a)*(1 + a + b*x))])/((1
- a)^2*(2 - n))
```

Rubi [A] time = 0.0415899, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 131}

$$-\frac{4b(-a-bx+1)^{1-\frac{n}{2}}(a+bx+1)^{\frac{n-2}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{(a+1)(-a-bx+1)}{(1-a)(a+bx+1)}\right)}{(1-a)^2(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a + b*x])/x^2,x]
```

```
[Out] (-4*b*(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((-2 + n)/2)*Hypergeometric2F1[
2, 1 - n/2, 2 - n/2, ((1 + a)*(1 - a - b*x))/((1 - a)*(1 + a + b*x))])/((1
- a)^2*(2 - n))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{(1-a-bx)^{-n/2}(1+a+bx)^{n/2}}{x^2}\,dx\\ &= -\frac{4b(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{(1+a)(1-a-bx)}{(1-a)(1+a+bx)}\right)}{(1-a)^2(2-n)}\end{aligned}$$

Mathematica [A] time = 0.0219383, size = 83, normalized size = 0.9

$$\frac{4b(-a-bx+1)^{1-\frac{n}{2}}(a+bx+1)^{\frac{n}{2}-1}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{(a+1)(a+bx-1)}{(a-1)(a+bx+1)}\right)}{(a-1)^2(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a + b*x])/x^2,x]
```

```
[Out] (4*b*(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^(-1 + n/2)*Hypergeometric2F1[2,
1 - n/2, 2 - n/2, ((1 + a)*(-1 + a + b*x))/((-1 + a)*(1 + a + b*x))])/((-1
+ a)^2*(-2 + n))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{e^{n\operatorname{Artanh}(bx+a)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(b*x+a))/x^2,x)
```

```
[Out] int(exp(n*arctanh(b*x+a))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x^2,x, algorithm="fricas")
```

```
[Out] integral(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n\operatorname{atanh}(a+bx)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(b*x+a))/x**2,x)
```

```
[Out] Integral(exp(n*atanh(a + b*x))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] integrate(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x^2, x)
```

3.882 $\int \frac{e^{n \tanh^{-1}(a+bx)}}{x^3} \, dx$

Optimal. Leaf size=152

$$-\frac{2b^2(2a+n)(a+bx+1)^{\frac{n-2}{2}}(-a-bx+1)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{(a+1)(-a-bx+1)}{(1-a)(a+bx+1)}\right)}{(1-a)^3(a+1)(2-n)}-\frac{(a+bx+1)^{\frac{n+2}{2}}(\cdots}{2(1-\cdots}$$

```
[Out] -((1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2))/(2*(1 - a^2)*x^2) - (2
*b^2*(2*a + n)*(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((-2 + n)/2)*Hypergeom
etric2F1[2, 1 - n/2, 2 - n/2, ((1 + a)*(1 - a - b*x))/((1 - a)*(1 + a + b*x
))])/((1 - a)^3*(1 + a)*(2 - n))
```

Rubi [A] time = 0.092375, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6163, 96, 131}

$$-\frac{(a+bx+1)^{\frac{n+2}{2}}(-a-bx+1)^{1-\frac{n}{2}}}{2\left(1-a^2\right)x^2}-\frac{2b^2(2a+n)(a+bx+1)^{\frac{n-2}{2}}(-a-bx+1)^{1-\frac{n}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{(a+1)(-a-bx+1)}{(1-a)(a+bx+1)}\right)}{(1-a)^3(a+1)(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a + b*x])/x^3,x]
```

```
[Out] -((1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2))/(2*(1 - a^2)*x^2) - (2
*b^2*(2*a + n)*(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((-2 + n)/2)*Hypergeom
etric2F1[2, 1 - n/2, 2 - n/2, ((1 + a)*(1 - a - b*x))/((1 - a)*(1 + a + b*x
))])/((1 - a)^3*(1 + a)*(2 - n))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(a+bx)}}{x^3}\,dx = \int \frac{(1-a-bx)^{-n/2}(1+a+bx)^{n/2}}{x^3}\,dx$$
$$= -\frac{(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{2\left(1-a^2\right)x^2} + \frac{(b(2a+n))\int \frac{(1-a-bx)^{-n/2}(1+a+bx)^{n/2}}{x^2}\,dx}{2\left(1-a^2\right)}$$
$$= -\frac{(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{2\left(1-a^2\right)x^2} - \frac{2b^2(2a+n)(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\right.}{(1-a)^3(1+a)(2-n)}$$

Mathematica [A] time = 0.055405, size = 123, normalized size = 0.81

$$\frac{(-a-bx+1)^{1-\frac{n}{2}}(a+bx+1)^{\frac{n}{2}-1}\left((a-1)^2(n-2)(a+bx+1)^2-4b^2x^2(2a+n)\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{(a+1}{(a-1}\right.\right.}{2(a-1)^3(a+1)(n-2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a + b*x])/x^3,x]
```

```
[Out] ((1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^(-1 + n/2)*((-1 + a)^2*(-2 + n)*(1 +
 a + b*x)^2 - 4*b^2*(2*a + n)*x^2*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, ((
1 + a)*(-1 + a + b*x))/((-1 + a)*(1 + a + b*x))]))/(2*(-1 + a)^3*(1 + a)*(-
2 + n)*x^2)
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{e^{n\operatorname{Artanh}(bx+a)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(b*x+a))/x^3,x)
```

```
[Out] int(exp(n*arctanh(b*x+a))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x^3,x, algorithm="fricas")
```

```
[Out] integral(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{atanh}(a+bx)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(b*x+a))/x**3,x)
```

```
[Out] Integral(exp(n*atanh(a + b*x))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x^3,x, algorithm="giac")
```

```
[Out] integrate(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x^3, x)
```

3.883 $\int e^{\tanh^{-1}(ax)} \left(c-a^2cx^2\right)^4 \, dx$

Optimal. Leaf size=127

$$-\frac{c^4 \left(1-a^2x^2\right)^{9/2}}{9a}+\frac{1}{8}c^4x \left(1-a^2x^2\right)^{7/2}+\frac{7}{48}c^4x \left(1-a^2x^2\right)^{5/2}+\frac{35}{192}c^4x \left(1-a^2x^2\right)^{3/2}+\frac{35}{128}c^4x\sqrt{1-a^2x^2}+\frac{35c^4 \sin^{-1}(ax)}{128a}$$

```
[Out] (35*c^4*x*Sqrt[1 - a^2*x^2])/128 + (35*c^4*x*(1 - a^2*x^2)^(3/2))/192 + (7*
c^4*x*(1 - a^2*x^2)^(5/2))/48 + (c^4*x*(1 - a^2*x^2)^(7/2))/8 - (c^4*(1 - a
^2*x^2)^(9/2))/(9*a) + (35*c^4*ArcSin[a*x])/(128*a)
```

Rubi [A] time = 0.0640731, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6138, 641, 195, 216}

$$-\frac{c^4 \left(1-a^2x^2\right)^{9/2}}{9a}+\frac{1}{8}c^4x \left(1-a^2x^2\right)^{7/2}+\frac{7}{48}c^4x \left(1-a^2x^2\right)^{5/2}+\frac{35}{192}c^4x \left(1-a^2x^2\right)^{3/2}+\frac{35}{128}c^4x\sqrt{1-a^2x^2}+\frac{35c^4 \sin^{-1}(ax)}{128a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a^2*c*x^2)^4,x]
```

```
[Out] (35*c^4*x*Sqrt[1 - a^2*x^2])/128 + (35*c^4*x*(1 - a^2*x^2)^(3/2))/192 + (7*
c^4*x*(1 - a^2*x^2)^(5/2))/48 + (c^4*x*(1 - a^2*x^2)^(7/2))/8 - (c^4*(1 - a
^2*x^2)^(9/2))/(9*a) + (35*c^4*ArcSin[a*x])/(128*a)
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4 dx = c^4\int (1+ax)\left(1-a^2x^2\right)^{7/2} dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a} + c^4\int \left(1-a^2x^2\right)^{7/2} dx$$
$$= \frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2} - \frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a} + \frac{1}{8}\left(7c^4\right)\int \left(1-a^2x^2\right)^{5/2} dx$$
$$= \frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2} + \frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2} - \frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a} + \frac{1}{48}\left(35c^4\right)\int \left(1-a^2x^2\right)^{3/}$$
$$= \frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2} + \frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2} - \frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a} + \frac{1}{6}$$
$$= \frac{35}{128}c^4x\sqrt{1-a^2x^2} + \frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2} + \frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2} -$$
$$= \frac{35}{128}c^4x\sqrt{1-a^2x^2} + \frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2} + \frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2} -$$

Mathematica [A] time = 0.144968, size = 107, normalized size = 0.84

$$-\frac{c^4\left(\sqrt{1-a^2x^2}\left(128a^8x^8+144a^7x^7-512a^6x^6-600a^5x^5+768a^4x^4+978a^3x^3-512a^2x^2-837ax+128\right)+630\sin^{-1}\right.}{1152a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a^2*c*x^2)^4,x]
```

```
[Out] -(c^4*(Sqrt[1 - a^2*x^2]*(128 - 837*a*x - 512*a^2*x^2 + 978*a^3*x^3 + 768*a
^4*x^4 - 600*a^5*x^5 - 512*a^6*x^6 + 144*a^7*x^7 + 128*a^8*x^8) + 630*ArcSi
n[Sqrt[1 - a*x]/Sqrt[2]]))/(1152*a)
```

Maple [B] time = 0.088, size = 229, normalized size = 1.8

$$-\frac{a^6c^4x^7}{8}\sqrt{-a^2x^2+1}+\frac{25\,a^4c^4x^5}{48}\sqrt{-a^2x^2+1}-\frac{163\,a^2c^4x^3}{192}\sqrt{-a^2x^2+1}-\frac{c^4a^7x^8}{9}\sqrt{-a^2x^2+1}+\frac{4\,c^4a^5x^6}{9}\sqrt{-a^2x^2+1}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/8*c^4*a^6*x^7*(-a^2*x^2+1)^(1/2)+25/48*c^4*a^4*x^5*(-a^2*x^2+1)^(1/2)-16
3/192*c^4*a^2*x^3*(-a^2*x^2+1)^(1/2)-1/9*c^4*a^7*x^8*(-a^2*x^2+1)^(1/2)+4/9
*c^4*a^5*x^6*(-a^2*x^2+1)^(1/2)-2/3*c^4*a^3*x^4*(-a^2*x^2+1)^(1/2)+4/9*c^4*
a*x^2*(-a^2*x^2+1)^(1/2)+35/128*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*
x^2+1)^(1/2))+93/128*c^4*x*(-a^2*x^2+1)^(1/2)-1/9*c^4*(-a^2*x^2+1)^(1/2)/a
```

Maxima [B] time = 1.47239, size = 296, normalized size = 2.33

$$-\frac{1}{9}\sqrt{-a^2x^2+1}a^7c^4x^8-\frac{1}{8}\sqrt{-a^2x^2+1}a^6c^4x^7+\frac{4}{9}\sqrt{-a^2x^2+1}a^5c^4x^6+\frac{25}{48}\sqrt{-a^2x^2+1}a^4c^4x^5-\frac{2}{3}\sqrt{-a^2x^2+1}a^3c^4x^4-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="maxima"
)
```

```
[Out] -1/9*sqrt(-a^2*x^2 + 1)*a^7*c^4*x^8 - 1/8*sqrt(-a^2*x^2 + 1)*a^6*c^4*x^7 +
4/9*sqrt(-a^2*x^2 + 1)*a^5*c^4*x^6 + 25/48*sqrt(-a^2*x^2 + 1)*a^4*c^4*x^5 -
 2/3*sqrt(-a^2*x^2 + 1)*a^3*c^4*x^4 - 163/192*sqrt(-a^2*x^2 + 1)*a^2*c^4*x^
3 + 4/9*sqrt(-a^2*x^2 + 1)*a*c^4*x^2 + 93/128*sqrt(-a^2*x^2 + 1)*c^4*x + 35
/128*c^4*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - 1/9*sqrt(-a^2*x^2 + 1)*c^4/a
```

Fricas [A] time = 1.58702, size = 312, normalized size = 2.46

$$-\frac{630\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(128\,a^8c^4x^8+144\,a^7c^4x^7-512\,a^6c^4x^6-600\,a^5c^4x^5+768\,a^4c^4x^4+978\,a^3c^4x^3-512\,a^2c^4\cdots\right.}{1152\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="fricas"
)
```

```
[Out] -1/1152*(630*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (128*a^8*c^4*x^8
+ 144*a^7*c^4*x^7 - 512*a^6*c^4*x^6 - 600*a^5*c^4*x^5 + 768*a^4*c^4*x^4 + 9
78*a^3*c^4*x^3 - 512*a^2*c^4*x^2 - 837*a*c^4*x + 128*c^4)*sqrt(-a^2*x^2 + 1
))/a
```

Sympy [A] time = 20.9683, size = 454, normalized size = 3.57

$$\begin{cases} -\frac{\frac{c^4(-a^2x^2+1)^{\frac{3}{2}}}{3}-c^4\left(\begin{cases}\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\operatorname{asin}(ax)}{2} & \text{for}\ ax>-1\wedge ax<1\end{cases}\right)+3c^4\left(\begin{cases}-\frac{ax(-2a^2x^2+1)\sqrt{-a^2x^2+1}}{8}+\frac{\operatorname{asin}(ax)}{8} & \text{for}\ ax>-1\wedge ax\cdots\end{cases}\right.}{\cdots} \\ c^4x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] Piecewise((-(c**4*(-a**2*x**2 + 1)**(3/2)/3 - c**4*Piecewise((a*x*sqrt(-a**
2*x**2 + 1)/2 + asin(a*x)/2, (a*x > -1) & (a*x < 1))) + 3*c**4*Piecewise((-
a*x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/8 + asin(a*x)/8, (a*x > -1) & (
a*x < 1))) + 3*c**4*Piecewise(((-a**2*x**2 + 1)**(5/2)/5 - (-a**2*x**2 + 1)
**(3/2)/3, (a*x > -1) & (a*x < 1))) - 3*c**4*Piecewise((-a**3*x**3*(-a**2*x
**2 + 1)**(3/2)/6 - a*x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/16 + asin(a
*x)/16, (a*x > -1) & (a*x < 1))) - 3*c**4*Piecewise((-(-a**2*x**2 + 1)**(7/
2)/7 + 2*(-a**2*x**2 + 1)**(5/2)/5 - (-a**2*x**2 + 1)**(3/2)/3, (a*x > -1)
& (a*x < 1))) + c**4*Piecewise((-a**3*x**3*(-a**2*x**2 + 1)**(3/2)/6 - a*x*
(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/32 - a*x*sqrt(-a**2*x**2 + 1)*(-16*
a**6*x**6 + 24*a**4*x**4 - 10*a**2*x**2 + 1)/128 + 5*asin(a*x)/128, (a*x >
-1) & (a*x < 1))) + c**4*Piecewise((((-a**2*x**2 + 1)**(9/2)/9 - 3*(-a**2*x*
*2 + 1)**(7/2)/7 + 3*(-a**2*x**2 + 1)**(5/2)/5 - (-a**2*x**2 + 1)**(3/2)/3,
 (a*x > -1) & (a*x < 1))))/a, Ne(a, 0)), (c**4*x, True))
```

Giac [A] time = 1.19874, size = 171, normalized size = 1.35

$$\frac{35\,c^4\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{128\,|a|}-\frac{1}{1152}\,\sqrt{-a^2x^2+1}\left(\frac{128\,c^4}{a}-\left(837\,c^4+2\left(256\,ac^4-\left(489\,a^2c^4+4\left(96\,a^3c^4-\left(75\,a^4c^4+2\left(32\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 35/128*c^4*arcsin(a*x)*sgn(a)/abs(a) - 1/1152*sqrt(-a^2*x^2 + 1)*(128*c^4/a
 - (837*c^4 + 2*(256*a*c^4 - (489*a^2*c^4 + 4*(96*a^3*c^4 - (75*a^4*c^4 + 2
*(32*a^5*c^4 - (8*a^7*c^4*x + 9*a^6*c^4)*x)*x)*x)*x)*x)*x)*x)
```

3.884 $\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=105

$$-\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}+\frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{5}{16}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\sin^{-1}(ax)}{16a}$$

```
[Out] (5*c^3*x*Sqrt[1 - a^2*x^2])/16 + (5*c^3*x*(1 - a^2*x^2)^(3/2))/24 + (c^3*x*
(1 - a^2*x^2)^(5/2))/6 - (c^3*(1 - a^2*x^2)^(7/2))/(7*a) + (5*c^3*ArcSin[a*
x])/(16*a)
```

Rubi [A] time = 0.0560546, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6138, 641, 195, 216}

$$-\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}+\frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{5}{16}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\sin^{-1}(ax)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a^2*c*x^2)^3,x]
```

```
[Out] (5*c^3*x*Sqrt[1 - a^2*x^2])/16 + (5*c^3*x*(1 - a^2*x^2)^(3/2))/24 + (c^3*x*
(1 - a^2*x^2)^(5/2))/6 - (c^3*(1 - a^2*x^2)^(7/2))/(7*a) + (5*c^3*ArcSin[a*
x])/(16*a)
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3 dx &= c^3\int (1+ax)\left(1-a^2x^2\right)^{5/2} dx \\
&= -\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+c^3\int \left(1-a^2x^2\right)^{5/2} dx \\
&= \frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}-\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{1}{6}\left(5c^3\right)\int \left(1-a^2x^2\right)^{3/2} dx \\
&= \frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}-\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{1}{8}\left(5c^3\right)\int \sqrt{1-a^2x^2}\,dx \\
&= \frac{5}{16}c^3x\sqrt{1-a^2x^2}+\frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}-\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{1}{16}\left(5\right. \\
&= \frac{5}{16}c^3x\sqrt{1-a^2x^2}+\frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}-\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{5c^3\,\mathrm{si}}{1}
\end{aligned}
$$

Mathematica [A] time = 0.119618, size = 91, normalized size = 0.87

$$
\frac{c^3\left(\sqrt{1-a^2x^2}\left(48a^6x^6+56a^5x^5-144a^4x^4-182a^3x^3+144a^2x^2+231ax-48\right)-210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{336a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*(Sqrt[1 - a^2*x^2]*(-48 + 231*a*x + 144*a^2*x^2 - 182*a^3*x^3 - 144*a^
4*x^4 + 56*a^5*x^5 + 48*a^6*x^6) - 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(336
*a)
```

Maple [B] time = 0.053, size = 183, normalized size = 1.7

$$
\frac{c^3a^5x^6}{7}\sqrt{-a^2x^2+1}-\frac{3\,c^3a^3x^4}{7}\sqrt{-a^2x^2+1}+\frac{3\,c^3ax^2}{7}\sqrt{-a^2x^2+1}-\frac{c^3}{7\,a}\sqrt{-a^2x^2+1}+\frac{a^4c^3x^5}{6}\sqrt{-a^2x^2+1}-\frac{13\,c^3a^2x^3}{24}\sqrt{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/7*c^3*a^5*x^6*(-a^2*x^2+1)^(1/2)-3/7*c^3*a^3*x^4*(-a^2*x^2+1)^(1/2)+3/7*c
^3*a*x^2*(-a^2*x^2+1)^(1/2)-1/7*c^3*(-a^2*x^2+1)^(1/2)/a+1/6*c^3*a^4*x^5*(-
a^2*x^2+1)^(1/2)-13/24*c^3*a^2*x^3*(-a^2*x^2+1)^(1/2)+11/16*c^3*x*(-a^2*x^2
+1)^(1/2)+5/16*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.45203, size = 234, normalized size = 2.23

$$
\frac{1}{7}\sqrt{-a^2x^2+1}a^5c^3x^6+\frac{1}{6}\sqrt{-a^2x^2+1}a^4c^3x^5-\frac{3}{7}\sqrt{-a^2x^2+1}a^3c^3x^4-\frac{13}{24}\sqrt{-a^2x^2+1}a^2c^3x^3+\frac{3}{7}\sqrt{-a^2x^2+1}ac^3x^2+\frac{11}{16}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] 1/7*sqrt(-a^2*x^2 + 1)*a^5*c^3*x^6 + 1/6*sqrt(-a^2*x^2 + 1)*a^4*c^3*x^5 - 3
/7*sqrt(-a^2*x^2 + 1)*a^3*c^3*x^4 - 13/24*sqrt(-a^2*x^2 + 1)*a^2*c^3*x^3 +
3/7*sqrt(-a^2*x^2 + 1)*a*c^3*x^2 + 11/16*sqrt(-a^2*x^2 + 1)*c^3*x + 5/16*c^
3*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - 1/7*sqrt(-a^2*x^2 + 1)*c^3/a
```

Fricas [A] time = 1.59597, size = 258, normalized size = 2.46

$$-\frac{210\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(48\,a^6c^3x^6+56\,a^5c^3x^5-144\,a^4c^3x^4-182\,a^3c^3x^3+144\,a^2c^3x^2+231\,ac^3x-48\,c^3\right)\sqrt{-a^2x^2}}{336\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/336*(210*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (48*a^6*c^3*x^6 +
56*a^5*c^3*x^5 - 144*a^4*c^3*x^4 - 182*a^3*c^3*x^3 + 144*a^2*c^3*x^2 + 231*
a*c^3*x - 48*c^3)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [A] time = 11.3747, size = 267, normalized size = 2.54

$$\begin{cases}\frac{-\frac{c^3(-a^2x^2+1)^{\frac{3}{2}}}{3}+c^3\left(\begin{cases}\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\operatorname{asin}(ax)}{2} & \text{for}\ ax>-1\wedge ax<1\end{cases}\right)-2c^3\left(\begin{cases}-\frac{ax(-2a^2x^2+1)\sqrt{-a^2x^2+1}}{8}+\frac{\operatorname{asin}(ax)}{8} & \text{for}\ ax>-1\wedge ax<\end{cases}\right.}{} \\ c^3x\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**3,x)
```

```
[Out] Piecewise(((-c**3*(-a**2*x**2 + 1)**(3/2)/3 + c**3*Piecewise((a*x*sqrt(-a**
2*x**2 + 1)/2 + asin(a*x)/2, (a*x > -1) & (a*x < 1))) - 2*c**3*Piecewise((-
a*x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/8 + asin(a*x)/8, (a*x > -1) & (
a*x < 1))) - 2*c**3*Piecewise(((-a**2*x**2 + 1)**(5/2)/5 - (-a**2*x**2 + 1)
**(3/2)/3, (a*x > -1) & (a*x < 1))) + c**3*Piecewise((-a**3*x**3*(-a**2*x**
2 + 1)**(3/2)/6 - a*x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/16 + asin(a*x
)/16, (a*x > -1) & (a*x < 1))) + c**3*Piecewise((-(-a**2*x**2 + 1)**(7/2)/7
 + 2*(-a**2*x**2 + 1)**(5/2)/5 - (-a**2*x**2 + 1)**(3/2)/3, (a*x > -1) & (a
*x < 1))))/a, Ne(a, 0)), (c**3*x, True))
```

Giac [A] time = 1.17427, size = 139, normalized size = 1.32

$$\frac{5\,c^3\arcsin(ax)\,\mathrm{sgn}(a)}{16\,|a|}-\frac{1}{336}\sqrt{-a^2x^2+1}\left(\frac{48\,c^3}{a}-\left(231\,c^3+2\left(72\,ac^3-\left(91\,a^2c^3+4\left(18\,a^3c^3-\left(6\,a^5c^3x+7\,a^4c^3\right)x\right)x\right)x\right)x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 5/16*c^3*arcsin(a*x)*sgn(a)/abs(a) - 1/336*sqrt(-a^2*x^2 + 1)*(48*c^3/a - (
231*c^3 + 2*(72*a*c^3 - (91*a^2*c^3 + 4*(18*a^3*c^3 - (6*a^5*c^3*x + 7*a^4*
c^3)*x)*x)*x)*x)*x)
```

3.885 $\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=83

$$-\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}+\frac{3}{8}c^2x\sqrt{1-a^2x^2}+\frac{3c^2\sin^{-1}(ax)}{8a}$$

```
[Out] (3*c^2*x*Sqrt[1 - a^2*x^2])/8 + (c^2*x*(1 - a^2*x^2)^(3/2))/4 - (c^2*(1 - a
^2*x^2)^(5/2))/(5*a) + (3*c^2*ArcSin[a*x])/(8*a)
```

Rubi [A] time = 0.0463788, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6138, 641, 195, 216}

$$-\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}+\frac{3}{8}c^2x\sqrt{1-a^2x^2}+\frac{3c^2\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a^2*c*x^2)^2,x]
```

```
[Out] (3*c^2*x*Sqrt[1 - a^2*x^2])/8 + (c^2*x*(1 - a^2*x^2)^(3/2))/4 - (c^2*(1 - a
^2*x^2)^(5/2))/(5*a) + (3*c^2*ArcSin[a*x])/(8*a)
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx &= c^2\int(1+ax)\left(1-a^2x^2\right)^{3/2}\,dx\\ &= -\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+c^2\int\left(1-a^2x^2\right)^{3/2}\,dx\\ &= \frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}-\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{4}\left(3c^2\right)\int\sqrt{1-a^2x^2}\,dx\\ &= \frac{3}{8}c^2x\sqrt{1-a^2x^2}+\frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}-\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{8}\left(3c^2\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{3}{8}c^2x\sqrt{1-a^2x^2}+\frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}-\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{3c^2\sin^{-1}(ax)}{8a}\end{aligned}$$

Mathematica [A] time = 0.0934339, size = 75, normalized size = 0.9

$$-\frac{c^2\left(\sqrt{1-a^2x^2}\left(8a^4x^4+10a^3x^3-16a^2x^2-25ax+8\right)+30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{40a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a^2*c*x^2)^2,x]
```

```
[Out] -(c^2*(Sqrt[1 - a^2*x^2]*(8 - 25*a*x - 16*a^2*x^2 + 10*a^3*x^3 + 8*a^4*x^4)
 + 30*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(40*a)
```

Maple [A] time = 0.04, size = 137, normalized size = 1.7

$$-\frac{c^2a^3x^4}{5}\sqrt{-a^2x^2+1}+\frac{2\,ac^2x^2}{5}\sqrt{-a^2x^2+1}-\frac{c^2}{5\,a}\sqrt{-a^2x^2+1}-\frac{a^2c^2x^3}{4}\sqrt{-a^2x^2+1}+\frac{5\,xc^2}{8}\sqrt{-a^2x^2+1}+\frac{3\,c^2}{8}\arctan\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/5*c^2*a^3*x^4*(-a^2*x^2+1)^(1/2)+2/5*c^2*a*x^2*(-a^2*x^2+1)^(1/2)-1/5*c^
2*(-a^2*x^2+1)^(1/2)/a-1/4*c^2*a^2*x^3*(-a^2*x^2+1)^(1/2)+5/8*c^2*x*(-a^2*x
^2+1)^(1/2)+3/8*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.44935, size = 171, normalized size = 2.06

$$-\frac{1}{5}\sqrt{-a^2x^2+1}a^3c^2x^4-\frac{1}{4}\sqrt{-a^2x^2+1}a^2c^2x^3+\frac{2}{5}\sqrt{-a^2x^2+1}ac^2x^2+\frac{5}{8}\sqrt{-a^2x^2+1}c^2x+\frac{3\,c^2\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\,\sqrt{a^2}}-\frac{\sqrt{-a^2x^2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] -1/5*sqrt(-a^2*x^2 + 1)*a^3*c^2*x^4 - 1/4*sqrt(-a^2*x^2 + 1)*a^2*c^2*x^3 +
2/5*sqrt(-a^2*x^2 + 1)*a*c^2*x^2 + 5/8*sqrt(-a^2*x^2 + 1)*c^2*x + 3/8*c^2*a
```

```
rcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - 1/5*sqrt(-a^2*x^2 + 1)*c^2/a
```

Fricas [A] time = 1.55262, size = 201, normalized size = 2.42

$$-\frac{30\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(8\,a^4c^2x^4+10\,a^3c^2x^3-16\,a^2c^2x^2-25\,ac^2x+8\,c^2\right)\sqrt{-a^2x^2+1}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/40*(30*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (8*a^4*c^2*x^4 + 10*
a^3*c^2*x^3 - 16*a^2*c^2*x^2 - 25*a*c^2*x + 8*c^2)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [A] time = 4.7066, size = 144, normalized size = 1.73

$$\begin{cases} -\frac{\frac{c^2(-a^2x^2+1)^{\frac{3}{2}}}{3}-c^2\left(\begin{cases}\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\operatorname{asin}(ax)}{2} & \text{for}\ ax>-1\wedge ax<1\end{cases}\right)+c^2\left(\begin{cases}-\frac{ax(-2a^2x^2+1)\sqrt{-a^2x^2+1}}{8}+\frac{\operatorname{asin}(ax)}{8} & \text{for}\ ax>-1\wedge ax<\end{cases}\right.}{a} \\ c^2x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((-(c**2*(-a**2*x**2 + 1)**(3/2)/3 - c**2*Piecewise((a*x*sqrt(-a**
2*x**2 + 1)/2 + asin(a*x)/2, (a*x > -1) & (a*x < 1))) + c**2*Piecewise((-a*
x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/8 + asin(a*x)/8, (a*x > -1) & (a*
x < 1))) + c**2*Piecewise(((-a**2*x**2 + 1)**(5/2)/5 - (-a**2*x**2 + 1)**(3
/2)/3, (a*x > -1) & (a*x < 1))))/a, Ne(a, 0)), (c**2*x, True))
```

Giac [A] time = 1.17372, size = 105, normalized size = 1.27

$$\frac{3\,c^2\arcsin(ax)\operatorname{sgn}(a)}{8\,|a|}+\frac{1}{40}\sqrt{-a^2x^2+1}\left(\left(25\,c^2+2\left(8\,ac^2-\left(4\,a^3c^2x+5\,a^2c^2\right)x\right)x\right)x-\frac{8\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 3/8*c^2*arcsin(a*x)*sgn(a)/abs(a) + 1/40*sqrt(-a^2*x^2 + 1)*((25*c^2 + 2*(8
*a*c^2 - (4*a^3*c^2*x + 5*a^2*c^2)*x)*x)*x - 8*c^2/a)
```

3.886 $\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=55

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}$$

```
[Out] (c*x*Sqrt[1 - a^2*x^2])/2 - (c*(1 - a^2*x^2)^(3/2))/(3*a) + (c*ArcSin[a*x])
/(2*a)
```

Rubi [A] time = 0.029047, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6138, 641, 195, 216}

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a^2*c*x^2),x]
```

```
[Out] (c*x*Sqrt[1 - a^2*x^2])/2 - (c*(1 - a^2*x^2)^(3/2))/(3*a) + (c*ArcSin[a*x])
/(2*a)
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)dx &= c\int(1+ax)\sqrt{1-a^2x^2}\,dx\\ &= -\frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+c\int\sqrt{1-a^2x^2}\,dx\\ &= \frac{1}{2}cx\sqrt{1-a^2x^2}-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c\int\frac{1}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{1}{2}cx\sqrt{1-a^2x^2}-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{c\sin^{-1}(ax)}{2a}\end{aligned}$$

Mathematica [A] time = 0.0687981, size = 57, normalized size = 1.04

$$\frac{c\left(\sqrt{1-a^2x^2}\left(2a^2x^2+3ax-2\right)-6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a^2*c*x^2),x]
```

```
[Out] (c*(Sqrt[1 - a^2*x^2]*(-2 + 3*a*x + 2*a^2*x^2) - 6*ArcSin[Sqrt[1 - a*x]/Sqr
t[2]]))/(6*a)
```

Maple [A] time = 0.036, size = 83, normalized size = 1.5

$$\frac{acx^2}{3}\sqrt{-a^2x^2+1}-\frac{c}{3\,a}\sqrt{-a^2x^2+1}+\frac{cx}{2}\sqrt{-a^2x^2+1}+\frac{c}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c),x)
```

```
[Out] 1/3*c*a*x^2*(-a^2*x^2+1)^(1/2)-1/3*c*(-a^2*x^2+1)^(1/2)/a+1/2*c*x*(-a^2*x^2
+1)^(1/2)+1/2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.43572, size = 99, normalized size = 1.8

$$\frac{1}{3}\sqrt{-a^2x^2+1}acx^2+\frac{1}{2}\sqrt{-a^2x^2+1}cx+\frac{c\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}}-\frac{\sqrt{-a^2x^2+1}c}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-a^2*x^2 + 1)*a*c*x^2 + 1/2*sqrt(-a^2*x^2 + 1)*c*x + 1/2*c*arcsin(
a^2*x/sqrt(a^2))/sqrt(a^2) - 1/3*sqrt(-a^2*x^2 + 1)*c/a
```

Fricas [A] time = 1.52435, size = 140, normalized size = 2.55

$$-\frac{6\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(2\,a^2cx^2+3\,acx-2\,c\right)\sqrt{-a^2x^2+1}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/6*(6*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (2*a^2*c*x^2 + 3*a*c*x -
 2*c)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [A] time = 2.54884, size = 53, normalized size = 0.96

$$\begin{cases}\frac{-\frac{c\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3}+c\left(\begin{cases}\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\operatorname{asin}(ax)}{2} & \text{for}\ ax>-1\wedge ax<1\end{cases}\right)}{a} & \text{for}\ a\neq 0\\ cx & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c),x)
```

```
[Out] Piecewise(((-c*(-a**2*x**2 + 1)**(3/2)/3 + c*Piecewise((a*x*sqrt(-a**2*x**2
 + 1)/2 + asin(a*x)/2, (a*x > -1) & (a*x < 1))))/a, Ne(a, 0)), (c*x, True))
```

Giac [A] time = 1.23489, size = 62, normalized size = 1.13

$$\frac{c\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{2\,|a|}+\frac{1}{6}\,\sqrt{-a^2x^2+1}\left((2\,acx+3\,c)x-\frac{2\,c}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*c*arcsin(a*x)*sgn(a)/abs(a) + 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c*x + 3*c)*x
 - 2*c/a)
```

3.887 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{c-a^2cx^2}\,dx$

Optimal. Leaf size=101

$$\frac{x^3(ax+1)}{a^2c\sqrt{1-a^2x^2}}+\frac{4x^2\sqrt{1-a^2x^2}}{3a^3c}+\frac{(9ax+16)\sqrt{1-a^2x^2}}{6a^5c}-\frac{3\sin^{-1}(ax)}{2a^5c}$$

```
[Out] (x^3*(1 + a*x))/(a^2*c*Sqrt[1 - a^2*x^2]) + (4*x^2*Sqrt[1 - a^2*x^2])/(3*a^
3*c) + ((16 + 9*a*x)*Sqrt[1 - a^2*x^2])/(6*a^5*c) - (3*ArcSin[a*x])/(2*a^5*
c)
```

Rubi [A] time = 0.132048, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6148, 819, 833, 780, 216}

$$\frac{x^3(ax+1)}{a^2c\sqrt{1-a^2x^2}}+\frac{4x^2\sqrt{1-a^2x^2}}{3a^3c}+\frac{(9ax+16)\sqrt{1-a^2x^2}}{6a^5c}-\frac{3\sin^{-1}(ax)}{2a^5c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2),x]
```

```
[Out] (x^3*(1 + a*x))/(a^2*c*Sqrt[1 - a^2*x^2]) + (4*x^2*Sqrt[1 - a^2*x^2])/(3*a^
3*c) + ((16 + 9*a*x)*Sqrt[1 - a^2*x^2])/(6*a^5*c) - (3*ArcSin[a*x])/(2*a^5*
c)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^4}{c-a^2cx^2}\,dx &= \frac{\int \frac{x^4(1+ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c}\\
&= \frac{x^3(1+ax)}{a^2c\sqrt{1-a^2x^2}} - \frac{\int \frac{x^2(3+4ax)}{\sqrt{1-a^2x^2}}\,dx}{a^2c}\\
&= \frac{x^3(1+ax)}{a^2c\sqrt{1-a^2x^2}} + \frac{4x^2\sqrt{1-a^2x^2}}{3a^3c} + \frac{\int \frac{x\left(-8a-9a^2x\right)}{\sqrt{1-a^2x^2}}\,dx}{3a^4c}\\
&= \frac{x^3(1+ax)}{a^2c\sqrt{1-a^2x^2}} + \frac{4x^2\sqrt{1-a^2x^2}}{3a^3c} + \frac{(16+9ax)\sqrt{1-a^2x^2}}{6a^5c} - \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^4c}\\
&= \frac{x^3(1+ax)}{a^2c\sqrt{1-a^2x^2}} + \frac{4x^2\sqrt{1-a^2x^2}}{3a^3c} + \frac{(16+9ax)\sqrt{1-a^2x^2}}{6a^5c} - \frac{3\sin^{-1}(ax)}{2a^5c}
\end{aligned}$$

Mathematica [A] time = 0.0473442, size = 74, normalized size = 0.73

$$-\frac{2a^4x^4+3a^3x^3+8a^2x^2+9\sqrt{1-a^2x^2}\sin^{-1}(ax)-9ax-16}{6a^5c\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2),x]
```

```
[Out] -(-16 - 9*a*x + 8*a^2*x^2 + 3*a^3*x^3 + 2*a^4*x^4 + 9*Sqrt[1 - a^2*x^2]*Arc
Sin[a*x])/(6*a^5*c*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.04, size = 143, normalized size = 1.4

$$\frac{x^2}{3\,a^3c}\sqrt{-a^2x^2+1}+\frac{5}{3\,a^5c}\sqrt{-a^2x^2+1}+\frac{x}{2\,a^4c}\sqrt{-a^2x^2+1}-\frac{3}{2\,a^4c}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{1}{ca^6}\sqrt{-a^2\left(x-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c),x)
```

```
[Out] 1/3*x^2*(-a^2*x^2+1)^(1/2)/a^3/c+5/3*(-a^2*x^2+1)^(1/2)/a^5/c+1/2*x*(-a^2*x
^2+1)^(1/2)/a^4/c-3/2/c/a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(
1/2))-1/c/a^6/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [B] time = 1.84586, size = 502, normalized size = 4.97

$$-\frac{a^2c\left(\frac{6\sqrt{-a^2x^2+1}c}{\sqrt{a^2c^2}a^6cx+a^6c^2}+\frac{6\sqrt{-a^2x^2+1}c}{\sqrt{a^2c^2}a^6cx-a^6c^2}-\frac{6\sqrt{-a^2x^2+1}}{a^7cx+\sqrt{a^2c^2}a^5}+\frac{6\sqrt{-a^2x^2+1}}{a^7cx-\sqrt{a^2c^2}a^5}-\frac{4\sqrt{a^2c^2}\sqrt{-a^2x^2+1}x^2}{a^5c^2}-\frac{6\sqrt{a^2c^2}\sqrt{-a^2x^2+1}x}{a^6c^2}+\frac{25\sqrt{a^2c^2}\sqrt{-a^2x^2+1}}{a^7c^2}-\cdots\right.}{12\sqrt{a^2c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -1/12*a^2*c*(6*sqrt(-a^2*x^2 + 1)*c/(sqrt(a^2*c^2)*a^6*c*x + a^6*c^2) + 6*s
qrt(-a^2*x^2 + 1)*c/(sqrt(a^2*c^2)*a^6*c*x - a^6*c^2) - 6*sqrt(-a^2*x^2 + 1
)/(a^7*c*x + sqrt(a^2*c^2)*a^5) + 6*sqrt(-a^2*x^2 + 1)/(a^7*c*x - sqrt(a^2*
c^2)*a^5) - 4*sqrt(a^2*c^2)*sqrt(-a^2*x^2 + 1)*x^2/(a^5*c^2) - 6*sqrt(a^2*c
^2)*sqrt(-a^2*x^2 + 1)*x/(a^6*c^2) + 25*sqrt(a^2*c^2)*sqrt(-a^2*x^2 + 1)/(a
^7*c^2) - 30*sqrt(a^2*c^2)*arcsin(x/(c*sqrt(1/(a^2*c^2))))/(a^8*c^3*sqrt(1/
(a^2*c^2))) - 45*(a^2*c^2)^(3/2)*sqrt(-a^2*x^2 + 1)/(a^9*c^4) + 48*(a^2*c^2
)^(3/2)*arcsin(x/(c*sqrt(1/(a^2*c^2))))/(a^10*c^5*sqrt(1/(a^2*c^2))))/sqrt(
a^2*c^2)
```

Fricas [A] time = 1.59177, size = 198, normalized size = 1.96

$$\frac{16\,ax+18\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^3x^3+a^2x^2+7\,ax-16\right)\sqrt{-a^2x^2+1}-16}{6\left(a^6cx-a^5c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] 1/6*(16*a*x + 18*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^3*
x^3 + a^2*x^2 + 7*a*x - 16)*sqrt(-a^2*x^2 + 1) - 16)/(a^6*c*x - a^5*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{x^4}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int\frac{ax^5}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(x**4/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)
 + Integral(a*x**5/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1))
, x))/c
```

Giac [A] time = 1.19979, size = 138, normalized size = 1.37

$$\frac{1}{6}\sqrt{-a^2x^2+1}\left(x\left(\frac{2x}{a^3c}+\frac{3}{a^4c}\right)+\frac{10}{a^5c}\right)-\frac{3\arcsin(ax)\operatorname{sgn}(a)}{2a^4c|a|}+\frac{2}{a^4c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] 1/6*sqrt(-a^2*x^2 + 1)*(x*(2*x/(a^3*c) + 3/(a^4*c)) + 10/(a^5*c)) - 3/2*arc
sin(a*x)*sgn(a)/(a^4*c*abs(a)) + 2/(a^4*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/
(a^2*x) - 1)*abs(a))
```

3.888 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{c-a^2cx^2} dx$

Optimal. Leaf size=74

$$\frac{x^2(ax+1)}{a^2c\sqrt{1-a^2x^2}}+\frac{(3ax+4)\sqrt{1-a^2x^2}}{2a^4c}-\frac{3\sin^{-1}(ax)}{2a^4c}$$

```
[Out] (x^2*(1 + a*x))/(a^2*c*Sqrt[1 - a^2*x^2]) + ((4 + 3*a*x)*Sqrt[1 - a^2*x^2])
/(2*a^4*c) - (3*ArcSin[a*x])/(2*a^4*c)
```

Rubi [A] time = 0.107834, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 819, 780, 216}

$$\frac{x^2(ax+1)}{a^2c\sqrt{1-a^2x^2}}+\frac{(3ax+4)\sqrt{1-a^2x^2}}{2a^4c}-\frac{3\sin^{-1}(ax)}{2a^4c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2),x]
```

```
[Out] (x^2*(1 + a*x))/(a^2*c*Sqrt[1 - a^2*x^2]) + ((4 + 3*a*x)*Sqrt[1 - a^2*x^2])
/(2*a^4*c) - (3*ArcSin[a*x])/(2*a^4*c)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^3}{c-a^2cx^2}\,dx &= \frac{\int \frac{x^3(1+ax)}{(1-a^2x^2)^{3/2}}\,dx}{c}\\
&= \frac{x^2(1+ax)}{a^2c\sqrt{1-a^2x^2}} - \frac{\int \frac{x(2+3ax)}{\sqrt{1-a^2x^2}}\,dx}{a^2c}\\
&= \frac{x^2(1+ax)}{a^2c\sqrt{1-a^2x^2}} + \frac{(4+3ax)\sqrt{1-a^2x^2}}{2a^4c} - \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^3c}\\
&= \frac{x^2(1+ax)}{a^2c\sqrt{1-a^2x^2}} + \frac{(4+3ax)\sqrt{1-a^2x^2}}{2a^4c} - \frac{3\sin^{-1}(ax)}{2a^4c}
\end{aligned}
$$

Mathematica [A] time = 0.0390005, size = 65, normalized size = 0.88

$$
-\frac{a^3x^3+2a^2x^2+3\sqrt{1-a^2x^2}\sin^{-1}(ax)-3ax-4}{2a^4c\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2),x]
```

```
[Out] -(-4 - 3*a*x + 2*a^2*x^2 + a^3*x^3 + 3*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(2*a^
4*c*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.039, size = 119, normalized size = 1.6

$$
\frac{x}{2\,a^3c}\sqrt{-a^2x^2+1}-\frac{3}{2\,a^3c}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{1}{a^4c}\sqrt{-a^2x^2+1}-\frac{1}{ca^5}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c),x)
```

```
[Out] 1/2*x*(-a^2*x^2+1)^(1/2)/a^3/c-3/2/c/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(
-a^2*x^2+1)^(1/2))+(-a^2*x^2+1)^(1/2)/a^4/c-1/c/a^5/(x-1/a)*(-a^2*(x-1/a)^2
-2*a*(x-1/a))^(1/2)
```

Maxima [B] time = 1.71213, size = 414, normalized size = 5.59

$$
-\frac{a^2c\left(\frac{\sqrt{-a^2x^2+1}c}{\sqrt{a^2c^2}a^5cx+a^5c^2}+\frac{\sqrt{-a^2x^2+1}c}{\sqrt{a^2c^2}a^5cx-a^5c^2}-\frac{\sqrt{-a^2x^2+1}}{a^6cx+\sqrt{a^2c^2}a^4}+\frac{\sqrt{-a^2x^2+1}}{a^6cx-\sqrt{a^2c^2}a^4}-\frac{\sqrt{a^2c^2}\sqrt{-a^2x^2+1}x}{a^5c^2}-\frac{2\sqrt{a^2c^2}\sqrt{-a^2x^2+1}}{a^6c^2}+\frac{\sqrt{a^2c^2}\arcsin\left(\frac{x}{c\sqrt{\frac{1}{a^2c^2}}}\right)}{a^7c^3\sqrt{\frac{1}{a^2c^2}}}\right.}{2\sqrt{a^2c^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -1/2*a^2*c*(sqrt(-a^2*x^2 + 1)*c/(sqrt(a^2*c^2)*a^5*c*x + a^5*c^2) + sqrt(-
a^2*x^2 + 1)*c/(sqrt(a^2*c^2)*a^5*c*x - a^5*c^2) - sqrt(-a^2*x^2 + 1)/(a^6*
c*x + sqrt(a^2*c^2)*a^4) + sqrt(-a^2*x^2 + 1)/(a^6*c*x - sqrt(a^2*c^2)*a^4)
 - sqrt(a^2*c^2)*sqrt(-a^2*x^2 + 1)*x/(a^5*c^2) - 2*sqrt(a^2*c^2)*sqrt(-a^2
*x^2 + 1)/(a^6*c^2) + sqrt(a^2*c^2)*arcsin(x/(c*sqrt(1/(a^2*c^2))))/(a^7*c^
3*sqrt(1/(a^2*c^2))) + 2*(a^2*c^2)^(3/2)*arcsin(x/(c*sqrt(1/(a^2*c^2))))/(a
^9*c^5*sqrt(1/(a^2*c^2))))/sqrt(a^2*c^2)
```

Fricas [A] time = 1.59515, size = 174, normalized size = 2.35

$$\frac{4\,ax+6\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(a^2x^2+ax-4\right)\sqrt{-a^2x^2+1}-4}{2\left(a^5cx-a^4c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] 1/2*(4*a*x + 6*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a^2*x^2
+ a*x - 4)*sqrt(-a^2*x^2 + 1) - 4)/(a^5*c*x - a^4*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^4}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(x**3/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)
 + Integral(a*x**4/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1))
, x))/c
```

Giac [A] time = 1.20777, size = 122, normalized size = 1.65

$$\frac{1}{2}\sqrt{-a^2x^2+1}\left(\frac{x}{a^3c}+\frac{2}{a^4c}\right)-\frac{3\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{2\,a^3c|a|}+\frac{2}{a^3c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*(x/(a^3*c) + 2/(a^4*c)) - 3/2*arcsin(a*x)*sgn(a)/(a^
3*c*abs(a)) + 2/(a^3*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)
)
```

3.889 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{c-a^2cx^2}\,dx$

Optimal. Leaf size=60

$$\frac{ax+1}{a^3c\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{a^3c}-\frac{\sin^{-1}(ax)}{a^3c}$$

```
[Out] (1 + a*x)/(a^3*c*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*x^2]/(a^3*c) - ArcSin[a*
x]/(a^3*c)
```

Rubi [A] time = 0.1074, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6148, 797, 641, 216, 637}

$$\frac{ax+1}{a^3c\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{a^3c}-\frac{\sin^{-1}(ax)}{a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2),x]
```

```
[Out] (1 + a*x)/(a^3*c*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*x^2]/(a^3*c) - ArcSin[a*
x]/(a^3*c)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^2}{c-a^2cx^2}\,dx = \frac{\int \frac{x^2(1+ax)}{(1-a^2x^2)^{3/2}}\,dx}{c}$$
$$= \frac{\int \frac{1+ax}{(1-a^2x^2)^{3/2}}\,dx}{a^2c} - \frac{\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx}{a^2c}$$
$$= \frac{1+ax}{a^3c\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}}{a^3c} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2c}$$
$$= \frac{1+ax}{a^3c\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}}{a^3c} - \frac{\sin^{-1}(ax)}{a^3c}$$

Mathematica [A] time = 0.0312228, size = 54, normalized size = 0.9

$$\frac{-a^2x^2-\sqrt{1-a^2x^2}\sin^{-1}(ax)+ax+2}{a^3c\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2),x]
```

```
[Out] (2 + a*x - a^2*x^2 - Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(a^3*c*Sqrt[1 - a^2*x^2
])
```

Maple [A] time = 0.036, size = 98, normalized size = 1.6

$$\frac{1}{a^3c}\sqrt{-a^2x^2+1}-\frac{1}{a^2c}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{1}{ca^4}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c),x)
```

```
[Out] (-a^2*x^2+1)^(1/2)/a^3/c-1/c/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2
+1)^(1/2))-1/c/a^4/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [B] time = 1.60072, size = 320, normalized size = 5.33

$$-\frac{a^2c\left(\frac{\sqrt{-a^2x^2+1}c}{\sqrt{a^2c^2}a^4cx+a^4c^2}+\frac{\sqrt{-a^2x^2+1}c}{\sqrt{a^2c^2}a^4cx-a^4c^2}-\frac{\sqrt{-a^2x^2+1}}{a^5cx+\sqrt{a^2c^2}a^3}+\frac{\sqrt{-a^2x^2+1}}{a^5cx-\sqrt{a^2c^2}a^3}-\frac{2\sqrt{a^2c^2}\sqrt{-a^2x^2+1}}{a^5c^2}+\frac{2\sqrt{a^2c^2}\arcsin\left(\frac{x}{c\sqrt{\frac{1}{a^2c^2}}}\right)}{a^6c^3\sqrt{\frac{1}{a^2c^2}}}\right)}{2\sqrt{a^2c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -1/2*a^2*c*(sqrt(-a^2*x^2 + 1)*c/(sqrt(a^2*c^2)*a^4*c*x + a^4*c^2) + sqrt(-
a^2*x^2 + 1)*c/(sqrt(a^2*c^2)*a^4*c*x - a^4*c^2) - sqrt(-a^2*x^2 + 1)/(a^5*
c*x + sqrt(a^2*c^2)*a^3) + sqrt(-a^2*x^2 + 1)/(a^5*c*x - sqrt(a^2*c^2)*a^3)
 - 2*sqrt(a^2*c^2)*sqrt(-a^2*x^2 + 1)/(a^5*c^2) + 2*sqrt(a^2*c^2)*arcsin(x/
(c*sqrt(1/(a^2*c^2))))/(a^6*c^3*sqrt(1/(a^2*c^2))))/sqrt(a^2*c^2)
```

Fricas [A] time = 1.57227, size = 155, normalized size = 2.58

$$\frac{2\,ax+2\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}(ax-2)-2}{a^4cx-a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] (2*a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2
 + 1)*(a*x - 2) - 2)/(a^4*c*x - a^3*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^3}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(x**2/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)
 + Integral(a*x**3/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1))
, x))/c
```

Giac [A] time = 1.24204, size = 105, normalized size = 1.75

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{a^2c|a|}+\frac{\sqrt{-a^2x^2+1}}{a^3c}+\frac{2}{a^2c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] -arcsin(a*x)*sgn(a)/(a^2*c*abs(a)) + sqrt(-a^2*x^2 + 1)/(a^3*c) + 2/(a^2*c*
((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

3.890 $\int \frac{e^{\tanh^{-1}(ax)}x}{c-a^2cx^2} dx$

Optimal. Leaf size=39

$$\frac{ax+1}{a^2c\sqrt{1-a^2x^2}}-\frac{\sin^{-1}(ax)}{a^2c}$$

```
[Out] (1 + a*x)/(a^2*c*Sqrt[1 - a^2*x^2]) - ArcSin[a*x]/(a^2*c)
```

Rubi [A] time = 0.0639509, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6148, 778, 216}

$$\frac{ax+1}{a^2c\sqrt{1-a^2x^2}}-\frac{\sin^{-1}(ax)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2),x]
```

```
[Out] (1 + a*x)/(a^2*c*Sqrt[1 - a^2*x^2]) - ArcSin[a*x]/(a^2*c)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x}{c-a^2cx^2} dx &= \frac{\int \frac{x(1+ax)}{(1-a^2x^2)^{3/2}} dx}{c} \\
&= \frac{1+ax}{a^2c\sqrt{1-a^2x^2}}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}} dx}{ac} \\
&= \frac{1+ax}{a^2c\sqrt{1-a^2x^2}}-\frac{\sin^{-1}(ax)}{a^2c}
\end{aligned}$$

Mathematica [A] time = 0.0293188, size = 45, normalized size = 1.15

$$\frac{\frac{ax}{\sqrt{1-a^2x^2}}+\frac{1}{\sqrt{1-a^2x^2}}-\sin^{-1}(ax)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2),x]
```

```
[Out] (1/Sqrt[1 - a^2*x^2] + (a*x)/Sqrt[1 - a^2*x^2] - ArcSin[a*x])/(a^2*c)
```

Maple [B] time = 0.036, size = 79, normalized size = 2.

$$-\frac{1}{ac}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{1}{a^3c}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c),x)
```

```
[Out] -1/c/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-1/c/a^3/(x-1/a)
*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [B] time = 1.5419, size = 281, normalized size = 7.21

$$-\frac{a^2c\left(\frac{\sqrt{-a^2x^2+1}c}{\sqrt{a^2c^2}a^3cx+a^3c^2}+\frac{\sqrt{-a^2x^2+1}c}{\sqrt{a^2c^2}a^3cx-a^3c^2}-\frac{\sqrt{-a^2x^2+1}}{a^4cx+\sqrt{a^2c^2}a^2}+\frac{\sqrt{-a^2x^2+1}}{a^4cx-\sqrt{a^2c^2}a^2}+\frac{2\sqrt{a^2c^2}\arcsin\left(\frac{x}{c\sqrt{\frac{1}{a^2c^2}}}\right)}{a^5c^3\sqrt{\frac{1}{a^2c^2}}}\right)}{2\sqrt{a^2c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c),x, algorithm="maxima"
)
```

```
[Out] -1/2*a^2*c*(sqrt(-a^2*x^2 + 1)*c/(sqrt(a^2*c^2)*a^3*c*x + a^3*c^2) + sqrt(-
a^2*x^2 + 1)*c/(sqrt(a^2*c^2)*a^3*c*x - a^3*c^2) - sqrt(-a^2*x^2 + 1)/(a^4*
c*x + sqrt(a^2*c^2)*a^2) + sqrt(-a^2*x^2 + 1)/(a^4*c*x - sqrt(a^2*c^2)*a^2)
 + 2*sqrt(a^2*c^2)*arcsin(x/(c*sqrt(1/(a^2*c^2))))/(a^5*c^3*sqrt(1/(a^2*c^2
))))/sqrt(a^2*c^2)
```

Fricas [A] time = 1.54584, size = 139, normalized size = 3.56

$$\frac{ax+2\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\sqrt{-a^2x^2+1}-1}{a^3cx-a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c),x, algorithm="fricas"
)
```

```
[Out] (a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 +
 1) - 1)/(a^3*c*x - a^2*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx + \int \frac{ax^2}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(x/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) +
Integral(a*x**2/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x
))/c
```

Giac [A] time = 1.20704, size = 80, normalized size = 2.05

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{ac|a|} + \frac{2}{ac\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(a*c*abs(a)) + 2/(a*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/
(a^2*x) - 1)*abs(a))
```

3.891 $\int \frac{e^{\tanh^{-1}(ax)}}{c-a^2cx^2} dx$

Optimal. Leaf size=13

$$\frac{e^{\tanh^{-1}(ax)}}{ac}$$

```
[Out] E^ArcTanh[a*x]/(a*c)
```

Rubi [A] time = 0.0287232, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {6137}

$$\frac{e^{\tanh^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a^2*c*x^2),x]
```

```
[Out] E^ArcTanh[a*x]/(a*c)
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{c-a^2cx^2} dx = \frac{e^{\tanh^{-1}(ax)}}{ac}$$

Mathematica [A] time = 0.0077084, size = 26, normalized size = 2.

$$\frac{\sqrt{ax+1}}{ac\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a^2*c*x^2),x]
```

```
[Out] Sqrt[1 + a*x]/(a*c*Sqrt[1 - a*x])
```

Maple [A] time = 0.03, size = 25, normalized size = 1.9

$$\frac{ax+1}{ac}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c),x)
```

```
[Out] (a*x+1)/(-a^2*x^2+1)^(1/2)/a/c
```

Maxima [B] time = 1.4996, size = 220, normalized size = 16.92

$$-\frac{a^2c\left(\frac{\sqrt{-a^2x^2+1}c}{\sqrt{a^2c^2}a^2cx+a^2c^2}+\frac{\sqrt{-a^2x^2+1}c}{\sqrt{a^2c^2}a^2cx-a^2c^2}-\frac{\sqrt{-a^2x^2+1}}{a^3cx+\sqrt{a^2c^2}a}+\frac{\sqrt{-a^2x^2+1}}{a^3cx-\sqrt{a^2c^2}a}\right)}{2\sqrt{a^2c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*c*(sqrt(-a^2*x^2 + 1)*c/(sqrt(a^2*c^2)*a^2*c*x + a^2*c^2) + sqrt(-
a^2*x^2 + 1)*c/(sqrt(a^2*c^2)*a^2*c*x - a^2*c^2) - sqrt(-a^2*x^2 + 1)/(a^3*
c*x + sqrt(a^2*c^2)*a) + sqrt(-a^2*x^2 + 1)/(a^3*c*x - sqrt(a^2*c^2)*a))/sq
rt(a^2*c^2)
```

Fricas [A] time = 1.53849, size = 65, normalized size = 5.

$$\frac{ax-\sqrt{-a^2x^2+1}-1}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] (a*x - sqrt(-a^2*x^2 + 1) - 1)/(a^2*c*x - a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(a*x/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)
+ Integral(1/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/
c
```

Giac [A] time = 1.18228, size = 50, normalized size = 3.85

$$\frac{2}{c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

3.892 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-a^2cx^2)}\,dx$

Optimal. Leaf size=44

$$\frac{ax+1}{c\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

```
[Out] (1 + a*x)/(c*Sqrt[1 - a^2*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c
```

Rubi [A] time = 0.0993619, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6148, 823, 12, 266, 63, 208}

$$\frac{ax+1}{c\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)),x]
```

```
[Out] (1 + a*x)/(c*Sqrt[1 - a^2*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)}\,dx &= \frac{\int \frac{1+ax}{x\left(1-a^2x^2\right)^{3/2}}\,dx}{c}\\
&= \frac{1+ax}{c\sqrt{1-a^2x^2}}+\frac{\int \frac{a^2}{x\sqrt{1-a^2x^2}}\,dx}{a^2c}\\
&= \frac{1+ax}{c\sqrt{1-a^2x^2}}+\frac{\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c}\\
&= \frac{1+ax}{c\sqrt{1-a^2x^2}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2c}\\
&= \frac{1+ax}{c\sqrt{1-a^2x^2}}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2c}\\
&= \frac{1+ax}{c\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}
\end{aligned}$$

Mathematica [A] time = 0.0213502, size = 55, normalized size = 1.25

$$-\frac{a^3(-x)-a^2}{a^2c\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)),x]
```

```
[Out] -((-a^2 - a^3*x)/(a^2*c*Sqrt[1 - a^2*x^2])) - ArcTanh[Sqrt[1 - a^2*x^2]]/c
```

Maple [A] time = 0.039, size = 60, normalized size = 1.4

$$-\frac{1}{c}\left(\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{1}{a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c),x)
```

```
[Out] -1/c*(arctanh(1/(-a^2*x^2+1)^(1/2))+1/a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a)
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax+1}{\left(a^2cx^2-c\right)\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c),x, algorithm="maxima"
)
```

```
[Out] -integrate((a*x + 1)/((a^2*c*x^2 - c)*sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [A] time = 1.56868, size = 119, normalized size = 2.7

$$\frac{ax+(ax-1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}-1}{acx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c),x, algorithm="fricas"
)
```

```
[Out] (a*x + (a*x - 1)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1) - 1)/
(a*c*x - c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{-a^2x^3\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(a/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) +
Integral(1/(-a**2*x**3*sqrt(-a**2*x**2 + 1) + x*sqrt(-a**2*x**2 + 1)), x))/
c
```

Giac [A] time = 1.22, size = 108, normalized size = 2.45

$$-\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{c|a|}+\frac{2a}{c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c*abs(a))
 + 2*a/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

3.893 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)}\,dx$

Optimal. Leaf size=70

$$\frac{ax+1}{cx\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-a^2x^2}}{cx}-\frac{a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

```
[Out] (1 + a*x)/(c*x*Sqrt[1 - a^2*x^2]) - (2*Sqrt[1 - a^2*x^2])/(c*x) - (a*ArcTan
h[Sqrt[1 - a^2*x^2]])/c
```

Rubi [A] time = 0.116746, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6148, 823, 807, 266, 63, 208}

$$\frac{ax+1}{cx\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-a^2x^2}}{cx}-\frac{a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)),x]
```

```
[Out] (1 + a*x)/(c*x*Sqrt[1 - a^2*x^2]) - (2*Sqrt[1 - a^2*x^2])/(c*x) - (a*ArcTan
h[Sqrt[1 - a^2*x^2]])/c
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)}\,dx &= \frac{\int \frac{1+ax}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{c}\\
&= \frac{1+ax}{cx\sqrt{1-a^2x^2}}+\frac{\int \frac{2a^2+a^3x}{x^2\sqrt{1-a^2x^2}}\,dx}{a^2c}\\
&= \frac{1+ax}{cx\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-a^2x^2}}{cx}+\frac{a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c}\\
&= \frac{1+ax}{cx\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-a^2x^2}}{cx}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2c}\\
&= \frac{1+ax}{cx\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-a^2x^2}}{cx}-\frac{\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{ac}\\
&= \frac{1+ax}{cx\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-a^2x^2}}{cx}-\frac{a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}
\end{aligned}
$$

Mathematica [A] time = 0.0259739, size = 67, normalized size = 0.96

$$
\frac{2a^2x^2-ax\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+ax-1}{cx\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)),x]
```

```
[Out] (-1 + a*x + 2*a^2*x^2 - a*x*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(
c*x*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.042, size = 75, normalized size = 1.1

$$
-\frac{1}{c}\left(\frac{1}{x}\sqrt{-a^2x^2+1}+a\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c),x)
```

```
[Out] -1/c*((-a^2*x^2+1)^(1/2)/x+a*arctanh(1/(-a^2*x^2+1)^(1/2))+1/(x-1/a)*(-a^2*
(x-1/a)^2-2*a*(x-1/a))^(1/2))
```

Maxima [A] time = 1.00868, size = 140, normalized size = 2.

$$-\frac{\frac{a^2\log\left(\sqrt{-a^2x^2+1}+1\right)}{c}-\frac{a^2\log\left(\sqrt{-a^2x^2+1}-1\right)}{c}-\frac{2\,a^2}{\sqrt{-a^2x^2+1}c}}{2\,a}+\frac{2\,a^2x^2-1}{\sqrt{ax+1}\sqrt{-ax+1}cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -1/2*(a^2*log(sqrt(-a^2*x^2 + 1) + 1)/c - a^2*log(sqrt(-a^2*x^2 + 1) - 1)/c
 - 2*a^2/(sqrt(-a^2*x^2 + 1)*c))/a + (2*a^2*x^2 - 1)/(sqrt(a*x + 1)*sqrt(-a
*x + 1)*c*x)
```

Fricas [A] time = 1.58153, size = 157, normalized size = 2.24

$$\frac{a^2x^2-ax+\left(a^2x^2-ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}(2\,ax-1)}{acx^2-cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] (a^2*x^2 - a*x + (a^2*x^2 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^
2*x^2 + 1)*(2*a*x - 1))/(a*c*x^2 - c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a}{-a^2x^3\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx+\int\frac{1}{-a^2x^4\sqrt{-a^2x^2+1}+x^2\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(a/(-a**2*x**3*sqrt(-a**2*x**2 + 1) + x*sqrt(-a**2*x**2 + 1)), x)
+ Integral(1/(-a**2*x**4*sqrt(-a**2*x**2 + 1) + x**2*sqrt(-a**2*x**2 + 1)),
 x))/c
```

Giac [B] time = 1.21332, size = 215, normalized size = 3.07

$$-\frac{a^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{c|a|}-\frac{\left(a^2-\frac{5\left(\sqrt{-a^2x^2+1}|a|+a\right)}{x}\right)a^2x}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}-\frac{\sqrt{-a^2x^2+1}|a|+a}{2cx|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] -a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c*abs(a
)) - 1/2*(a^2 - 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/x)*a^2*x/((sqrt(-a^2*x^2
+ 1)*abs(a) + a)*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) -
1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(c*x*abs(a))
```

3.894 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)}\,dx$

Optimal. Leaf size=99

$$-\frac{2a\sqrt{1-a^2x^2}}{cx}-\frac{3\sqrt{1-a^2x^2}}{2cx^2}+\frac{ax+1}{cx^2\sqrt{1-a^2x^2}}-\frac{3a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}$$

```
[Out] (1 + a*x)/(c*x^2*Sqrt[1 - a^2*x^2]) - (3*Sqrt[1 - a^2*x^2])/(2*c*x^2) - (2*
a*Sqrt[1 - a^2*x^2])/(c*x) - (3*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c)
```

Rubi [A] time = 0.138867, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 823, 835, 807, 266, 63, 208}

$$-\frac{2a\sqrt{1-a^2x^2}}{cx}-\frac{3\sqrt{1-a^2x^2}}{2cx^2}+\frac{ax+1}{cx^2\sqrt{1-a^2x^2}}-\frac{3a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)),x]
```

```
[Out] (1 + a*x)/(c*x^2*Sqrt[1 - a^2*x^2]) - (3*Sqrt[1 - a^2*x^2])/(2*c*x^2) - (2*
a*Sqrt[1 - a^2*x^2])/(c*x) - (3*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)}\,dx &= \frac{\int \frac{1+ax}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx}{c}\\
&= \frac{1+ax}{cx^2\sqrt{1-a^2x^2}}+\frac{\int \frac{3a^2+2a^3x}{x^3\sqrt{1-a^2x^2}}\,dx}{a^2c}\\
&= \frac{1+ax}{cx^2\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{2cx^2}-\frac{\int \frac{-4a^3-3a^4x}{x^2\sqrt{1-a^2x^2}}\,dx}{2a^2c}\\
&= \frac{1+ax}{cx^2\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{2cx^2}-\frac{2a\sqrt{1-a^2x^2}}{cx}+\frac{\left(3a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{2c}\\
&= \frac{1+ax}{cx^2\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{2cx^2}-\frac{2a\sqrt{1-a^2x^2}}{cx}+\frac{\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{4c}\\
&= \frac{1+ax}{cx^2\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{2cx^2}-\frac{2a\sqrt{1-a^2x^2}}{cx}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{2c}\\
&= \frac{1+ax}{cx^2\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{2cx^2}-\frac{2a\sqrt{1-a^2x^2}}{cx}-\frac{3a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}
\end{aligned}
$$

Mathematica [A] time = 0.0363257, size = 83, normalized size = 0.84

$$
-\frac{-4a^3x^3-3a^2x^2+3a^2x^2\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+2ax+1}{2cx^2\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)),x]
```

```
[Out] -(1 + 2*a*x - 3*a^2*x^2 - 4*a^3*x^3 + 3*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcTanh[S
qrt[1 - a^2*x^2]])/(2*c*x^2*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.041, size = 97, normalized size = 1.

$$-\frac{1}{c}\left(\frac{a}{x}\sqrt{-a^2x^2+1}+\frac{3a^2}{2}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+a\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}+\frac{1}{2x^2}\sqrt{-a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c),x)
```

```
[Out] -1/c*(a*(-a^2*x^2+1)^(1/2)/x+3/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))+a/(x-1/a
)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1/2*(-a^2*x^2+1)^(1/2)/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{ax+1}{\left(a^2cx^2-c\right)\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -integrate((a*x + 1)/((a^2*c*x^2 - c)*sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [A] time = 1.54449, size = 197, normalized size = 1.99

$$\frac{2a^3x^3-2a^2x^2+3\left(a^3x^3-a^2x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(4a^2x^2-ax-1\right)\sqrt{-a^2x^2+1}}{2\left(acx^3-cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*a^3*x^3 - 2*a^2*x^2 + 3*(a^3*x^3 - a^2*x^2)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) - (4*a^2*x^2 - a*x - 1)*sqrt(-a^2*x^2 + 1))/(a*c*x^3 - c*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a}{-a^2x^4\sqrt{-a^2x^2+1}+x^2\sqrt{-a^2x^2+1}}\,dx+\int\frac{1}{-a^2x^5\sqrt{-a^2x^2+1}+x^3\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(a/(-a**2*x**4*sqrt(-a**2*x**2 + 1) + x**2*sqrt(-a**2*x**2 + 1)),
x) + Integral(1/(-a**2*x**5*sqrt(-a**2*x**2 + 1) + x**3*sqrt(-a**2*x**2 + 1
)), x))/c
```

<u>Giac</u> [B] time = 1.21046, size = 302, normalized size = 3.05

$$-\frac{\left(a^3+\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)a}{x}-\frac{20\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{ax^2}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}-\frac{3a^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{2c|a|}-\frac{\frac{4\left(\sqrt{-a^2x^2+1}|a|+a\right)ac|a|}{x}+\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c|a|}{ax^2}}{8a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] -1/8*(a^3 + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 20*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^2/(a*x^2))*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c*((sqrt(
-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) - 3/2*a^3*log(1/2*abs(-2*sqr
t(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c*abs(a)) - 1/8*(4*(sqrt(-a^2*
x^2 + 1)*abs(a) + a)*a*c*abs(a)/x + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c*abs
(a)/(a*x^2))/(a^2*c^2)
```

3.895 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-a^2cx^2)}\,dx$

Optimal. Leaf size=128

$$-\frac{8a^2\sqrt{1-a^2x^2}}{3cx}-\frac{3a\sqrt{1-a^2x^2}}{2cx^2}-\frac{4\sqrt{1-a^2x^2}}{3cx^3}+\frac{ax+1}{cx^3\sqrt{1-a^2x^2}}-\frac{3a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}$$

```
[Out] (1 + a*x)/(c*x^3*Sqrt[1 - a^2*x^2]) - (4*Sqrt[1 - a^2*x^2])/(3*c*x^3) - (3*
a*Sqrt[1 - a^2*x^2])/(2*c*x^2) - (8*a^2*Sqrt[1 - a^2*x^2])/(3*c*x) - (3*a^3
*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c)
```

Rubi [A] time = 0.166154, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 823, 835, 807, 266, 63, 208}

$$-\frac{8a^2\sqrt{1-a^2x^2}}{3cx}-\frac{3a\sqrt{1-a^2x^2}}{2cx^2}-\frac{4\sqrt{1-a^2x^2}}{3cx^3}+\frac{ax+1}{cx^3\sqrt{1-a^2x^2}}-\frac{3a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*(c - a^2*c*x^2)),x]
```

```
[Out] (1 + a*x)/(c*x^3*Sqrt[1 - a^2*x^2]) - (4*Sqrt[1 - a^2*x^2])/(3*c*x^3) - (3*
a*Sqrt[1 - a^2*x^2])/(2*c*x^2) - (8*a^2*Sqrt[1 - a^2*x^2])/(3*c*x) - (3*a^3
*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^4\left(c-a^2cx^2\right)}\,dx &= \frac{\int \frac{1+ax}{x^4\left(1-a^2x^2\right)^{3/2}}\,dx}{c}\\
&= \frac{1+ax}{cx^3\sqrt{1-a^2x^2}}+\frac{\int \frac{4a^2+3a^3x}{x^4\sqrt{1-a^2x^2}}\,dx}{a^2c}\\
&= \frac{1+ax}{cx^3\sqrt{1-a^2x^2}}-\frac{4\sqrt{1-a^2x^2}}{3cx^3}-\frac{\int \frac{-9a^3-8a^4x}{x^3\sqrt{1-a^2x^2}}\,dx}{3a^2c}\\
&= \frac{1+ax}{cx^3\sqrt{1-a^2x^2}}-\frac{4\sqrt{1-a^2x^2}}{3cx^3}-\frac{3a\sqrt{1-a^2x^2}}{2cx^2}+\frac{\int \frac{16a^4+9a^5x}{x^2\sqrt{1-a^2x^2}}\,dx}{6a^2c}\\
&= \frac{1+ax}{cx^3\sqrt{1-a^2x^2}}-\frac{4\sqrt{1-a^2x^2}}{3cx^3}-\frac{3a\sqrt{1-a^2x^2}}{2cx^2}-\frac{8a^2\sqrt{1-a^2x^2}}{3cx}+\frac{\left(3a^3\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{2c}\\
&= \frac{1+ax}{cx^3\sqrt{1-a^2x^2}}-\frac{4\sqrt{1-a^2x^2}}{3cx^3}-\frac{3a\sqrt{1-a^2x^2}}{2cx^2}-\frac{8a^2\sqrt{1-a^2x^2}}{3cx}+\frac{\left(3a^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x\right.}{4c}\\
&= \frac{1+ax}{cx^3\sqrt{1-a^2x^2}}-\frac{4\sqrt{1-a^2x^2}}{3cx^3}-\frac{3a\sqrt{1-a^2x^2}}{2cx^2}-\frac{8a^2\sqrt{1-a^2x^2}}{3cx}-\frac{(3a)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-\right.}{2c}\\
&= \frac{1+ax}{cx^3\sqrt{1-a^2x^2}}-\frac{4\sqrt{1-a^2x^2}}{3cx^3}-\frac{3a\sqrt{1-a^2x^2}}{2cx^2}-\frac{8a^2\sqrt{1-a^2x^2}}{3cx}-\frac{3a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}
\end{aligned}
$$

Mathematica [A] time = 0.0366877, size = 91, normalized size = 0.71

$$-\frac{-16a^4x^4-9a^3x^3+8a^2x^2+9a^3x^3\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+3ax+2}{6cx^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*(c - a^2*c*x^2)),x]
```

```
[Out] -(2 + 3*a*x + 8*a^2*x^2 - 9*a^3*x^3 - 16*a^4*x^4 + 9*a^3*x^3*Sqrt[1 - a^2*x
^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(6*c*x^3*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.043, size = 140, normalized size = 1.1

$$-\frac{1}{c}\left(\frac{5a^2}{3x}\sqrt{-a^2x^2+1}+a^3\operatorname{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+a^2\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}-a\left(-\frac{1}{2x^2}\sqrt{-a^2x^2+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c),x)
```

```
[Out] -1/c*(5/3*a^2*(-a^2*x^2+1)^(1/2)/x+a^3*arctanh(1/(-a^2*x^2+1)^(1/2))+a^2/(x
-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-a*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2
*a^2*arctanh(1/(-a^2*x^2+1)^(1/2)))+1/3*(-a^2*x^2+1)^(1/2)/x^3)
```

Maxima [A] time = 1.01802, size = 200, normalized size = 1.56

$$-\frac{\frac{3a^4\log\left(\sqrt{-a^2x^2+1}+1\right)}{c}-\frac{3a^4\log\left(\sqrt{-a^2x^2+1}-1\right)}{c}+\frac{2\left(3\left(a^2x^2-1\right)a^4+2a^4\right)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}c-\sqrt{-a^2x^2+1}c}}{4a}+\frac{8a^4x^4-4a^2x^2-1}{3\sqrt{ax+1}\sqrt{-ax+1}cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -1/4*(3*a^4*log(sqrt(-a^2*x^2 + 1) + 1)/c - 3*a^4*log(sqrt(-a^2*x^2 + 1) -
1)/c + 2*(3*(a^2*x^2 - 1)*a^4 + 2*a^4)/((-a^2*x^2 + 1)^(3/2)*c - sqrt(-a^2*
x^2 + 1)*c))/a + 1/3*(8*a^4*x^4 - 4*a^2*x^2 - 1)/(sqrt(a*x + 1)*sqrt(-a*x +
 1)*c*x^3)
```

Fricas [A] time = 1.54098, size = 215, normalized size = 1.68

$$\frac{6a^4x^4-6a^3x^3+9\left(a^4x^4-a^3x^3\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(16a^3x^3-7a^2x^2-ax-2\right)\sqrt{-a^2x^2+1}}{6\left(acx^4-cx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] 1/6*(6*a^4*x^4 - 6*a^3*x^3 + 9*(a^4*x^4 - a^3*x^3)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) - (16*a^3*x^3 - 7*a^2*x^2 - a*x - 2)*sqrt(-a^2*x^2 + 1))/(a*c*x^4 -
 c*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{-a^2x^5\sqrt{-a^2x^2+1}+x^3\sqrt{-a^2x^2+1}}\,dx + \int \frac{1}{-a^2x^6\sqrt{-a^2x^2+1}+x^4\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**4/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(a/(-a**2*x**5*sqrt(-a**2*x**2 + 1) + x**3*sqrt(-a**2*x**2 + 1)),
x) + Integral(1/(-a**2*x**6*sqrt(-a**2*x**2 + 1) + x**4*sqrt(-a**2*x**2 + 1
)), x))/c
```

Giac [B] time = 1.18719, size = 382, normalized size = 2.98

$$-\frac{\left(a^4+\frac{2\left(\sqrt{-a^2x^2+1}|a|+a\right)a^2}{x}+\frac{18\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{x^2}-\frac{69\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^2x^3}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}-\frac{3a^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{2c|a|}-\frac{21\left(\sqrt{-a^2x^2+1}|a|+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] -1/24*(a^4 + 2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2/x + 18*(sqrt(-a^2*x^2 +
1)*abs(a) + a)^2/x^2 - 69*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^2*x^3))*a^6*
x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(
a^2*x) - 1)*abs(a)) - 3/2*a^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*
a)/(a^2*abs(x)))/(c*abs(a)) - 1/24*(21*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*
c^2/x + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^2/x^2 + (sqrt(-a^2*x^2 +
1)*abs(a) + a)^3*c^2/x^3)/(a^2*c^3*abs(a))
```

3.896 $\int \frac{e^{\tanh^{-1}(ax)}x^6}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=137

$$\frac{x^5(ax+1)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x^3(6ax+5)}{3a^4c^2\sqrt{1-a^2x^2}}-\frac{8x^2\sqrt{1-a^2x^2}}{3a^5c^2}-\frac{(15ax+32)\sqrt{1-a^2x^2}}{6a^7c^2}+\frac{5\sin^{-1}(ax)}{2a^7c^2}$$

```
[Out] (x^5*(1 + a*x))/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - (x^3*(5 + 6*a*x))/(3*a^4*
c^2*Sqrt[1 - a^2*x^2]) - (8*x^2*Sqrt[1 - a^2*x^2])/(3*a^5*c^2) - ((32 + 15*
a*x)*Sqrt[1 - a^2*x^2])/(6*a^7*c^2) + (5*ArcSin[a*x])/(2*a^7*c^2)
```

Rubi [A] time = 0.154315, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6148, 819, 833, 780, 216}

$$\frac{x^5(ax+1)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x^3(6ax+5)}{3a^4c^2\sqrt{1-a^2x^2}}-\frac{8x^2\sqrt{1-a^2x^2}}{3a^5c^2}-\frac{(15ax+32)\sqrt{1-a^2x^2}}{6a^7c^2}+\frac{5\sin^{-1}(ax)}{2a^7c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^6)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (x^5*(1 + a*x))/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - (x^3*(5 + 6*a*x))/(3*a^4*
c^2*Sqrt[1 - a^2*x^2]) - (8*x^2*Sqrt[1 - a^2*x^2])/(3*a^5*c^2) - ((32 + 15*
a*x)*Sqrt[1 - a^2*x^2])/(6*a^7*c^2) + (5*ArcSin[a*x])/(2*a^7*c^2)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^6}{\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{x^6(1+ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}\\
&= \frac{x^5(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{x^4(5+6ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}\\
&= \frac{x^5(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x^3(5+6ax)}{3a^4c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{x^2(15+24ax)}{\sqrt{1-a^2x^2}}\,dx}{3a^4c^2}\\
&= \frac{x^5(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x^3(5+6ax)}{3a^4c^2\sqrt{1-a^2x^2}} - \frac{8x^2\sqrt{1-a^2x^2}}{3a^5c^2} - \frac{\int \frac{x\left(-48a-45a^2x\right)}{\sqrt{1-a^2x^2}}\,dx}{9a^6c^2}\\
&= \frac{x^5(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x^3(5+6ax)}{3a^4c^2\sqrt{1-a^2x^2}} - \frac{8x^2\sqrt{1-a^2x^2}}{3a^5c^2} - \frac{(32+15ax)\sqrt{1-a^2x^2}}{6a^7c^2} + \frac{5\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^6c^2}\\
&= \frac{x^5(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x^3(5+6ax)}{3a^4c^2\sqrt{1-a^2x^2}} - \frac{8x^2\sqrt{1-a^2x^2}}{3a^5c^2} - \frac{(32+15ax)\sqrt{1-a^2x^2}}{6a^7c^2} + \frac{5\sin^{-1}(ax)}{2a^7c^2}
\end{aligned}$$

Mathematica [A] time = 0.0664661, size = 93, normalized size = 0.68

$$\frac{2a^5x^5+a^4x^4+11a^3x^3-31a^2x^2+15(ax-1)\sqrt{1-a^2x^2}\sin^{-1}(ax)-17ax+32}{6a^7c^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^6)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (32 - 17*a*x - 31*a^2*x^2 + 11*a^3*x^3 + a^4*x^4 + 2*a^5*x^5 + 15*(-1 + a*x
)*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(6*a^7*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.047, size = 225, normalized size = 1.6

$$-\frac{x^2}{3a^5c^2}\sqrt{-a^2x^2+1}-\frac{8}{3c^2a^7}\sqrt{-a^2x^2+1}-\frac{x}{2c^2a^6}\sqrt{-a^2x^2+1}+\frac{5}{2c^2a^6}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{1}{4c^2a^8\left(x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/3*x^2*(-a^2*x^2+1)^(1/2)/a^5/c^2-8/3/c^2/a^7*(-a^2*x^2+1)^(1/2)-1/2/c^2/
a^6*x*(-a^2*x^2+1)^(1/2)+5/2/c^2/a^6/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2
*x^2+1)^(1/2))-1/4/c^2/a^8/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1/6/c
^2/a^9/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+31/12/c^2/a^8/(x-1/a)*(
-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^6}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)*x^6/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [A] time = 1.64351, size = 343, normalized size = 2.5

$$-\frac{32\,a^3x^3-32\,a^2x^2-32\,ax+30\left(a^3x^3-a^2x^2-ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^5x^5+a^4x^4+11\,a^3x^3-31\,a^2x^2-17\,ax\right.}{6\left(a^{10}c^2x^3-a^9c^2x^2-a^8c^2x+a^7c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] -1/6*(32*a^3*x^3 - 32*a^2*x^2 - 32*a*x + 30*(a^3*x^3 - a^2*x^2 - a*x + 1)*a
rctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^5*x^5 + a^4*x^4 + 11*a^3*x^3 -
 31*a^2*x^2 - 17*a*x + 32)*sqrt(-a^2*x^2 + 1) + 32)/(a^10*c^2*x^3 - a^9*c^2
*x^2 - a^8*c^2*x + a^7*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^6}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^7}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**6/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(x**6/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**
2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**7/(a**4*x**4*sqrt(-a**2*
x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c
**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^6}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*x^6/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

3.897 $\int \frac{e^{\tanh^{-1}(ax)}x^5}{\left(c-a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=110

$$\frac{x^4(ax+1)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x^2(5ax+4)}{3a^4c^2\sqrt{1-a^2x^2}}-\frac{(15ax+16)\sqrt{1-a^2x^2}}{6a^6c^2}+\frac{5\sin^{-1}(ax)}{2a^6c^2}$$

```
[Out] (x^4*(1 + a*x))/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - (x^2*(4 + 5*a*x))/(3*a^4*
c^2*Sqrt[1 - a^2*x^2]) - ((16 + 15*a*x)*Sqrt[1 - a^2*x^2])/(6*a^6*c^2) + (5
*ArcSin[a*x])/(2*a^6*c^2)
```

Rubi [A] time = 0.135965, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 819, 780, 216}

$$\frac{x^4(ax+1)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x^2(5ax+4)}{3a^4c^2\sqrt{1-a^2x^2}}-\frac{(15ax+16)\sqrt{1-a^2x^2}}{6a^6c^2}+\frac{5\sin^{-1}(ax)}{2a^6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^5)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (x^4*(1 + a*x))/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - (x^2*(4 + 5*a*x))/(3*a^4*
c^2*Sqrt[1 - a^2*x^2]) - ((16 + 15*a*x)*Sqrt[1 - a^2*x^2])/(6*a^6*c^2) + (5
*ArcSin[a*x])/(2*a^6*c^2)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^5}{\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{x^5(1+ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}\\ &= \frac{x^4(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{\int \frac{x^3(4+5ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}\\ &= \frac{x^4(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x^2(4+5ax)}{3a^4c^2\sqrt{1-a^2x^2}}+\frac{\int \frac{x(8+15ax)}{\sqrt{1-a^2x^2}}\,dx}{3a^4c^2}\\ &= \frac{x^4(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x^2(4+5ax)}{3a^4c^2\sqrt{1-a^2x^2}}-\frac{(16+15ax)\sqrt{1-a^2x^2}}{6a^6c^2}+\frac{5\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^5c^2}\\ &= \frac{x^4(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x^2(4+5ax)}{3a^4c^2\sqrt{1-a^2x^2}}-\frac{(16+15ax)\sqrt{1-a^2x^2}}{6a^6c^2}+\frac{5\sin^{-1}(ax)}{2a^6c^2}\end{aligned}$$

Mathematica [A] time = 0.049769, size = 86, normalized size = 0.78

$$\frac{3a^4x^4+3a^3x^3-23a^2x^2+15(ax-1)\sqrt{1-a^2x^2}\sin^{-1}(ax)-ax+16}{6a^6c^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^5)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (16 - a*x - 23*a^2*x^2 + 3*a^3*x^3 + 3*a^4*x^4 + 15*(-1 + a*x)*Sqrt[1 - a^2
*x^2]*ArcSin[a*x])/(6*a^6*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.045, size = 202, normalized size = 1.8

$$-\frac{x}{2c^2a^5}\sqrt{-a^2x^2+1}+\frac{5}{2c^2a^5}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{1}{c^2a^6}\sqrt{-a^2x^2+1}+\frac{1}{4c^2a^7\left(x+a^{-1}\right)}\sqrt{-a^2\left(x+a^{-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/2/c^2/a^5*x*(-a^2*x^2+1)^(1/2)+5/2/c^2/a^5/(a^2)^(1/2)*arctan((a^2)^(1/2
)*x/(-a^2*x^2+1)^(1/2))-1/c^2/a^6*(-a^2*x^2+1)^(1/2)+1/4/c^2/a^7/(x+1/a)*(-
a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1/6/c^2/a^8/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*
(x-1/a))^(1/2)+25/12/c^2/a^7/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a\int \frac{x^6}{\left(a^4c^2x^4-2\,a^2c^2x^2+c^2\right)\sqrt{ax+1}\sqrt{-ax+1}}\,dx-\frac{\frac{3\sqrt{-a^2x^2+1}}{c^2}-\frac{6\,a^2x^2-5}{\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}}{3\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] a*integrate(x^6/((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(-a*
x + 1)), x) - 1/3*(3*sqrt(-a^2*x^2 + 1)/c^2 - (6*a^2*x^2 - 5)/((-a^2*x^2 +
1)^(3/2)*c^2))/a^6
```

Fricas [A] time = 1.56941, size = 323, normalized size = 2.94

$$-\frac{16\,a^3x^3-16\,a^2x^2-16\,ax+30\left(a^3x^3-a^2x^2-ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3\,a^4x^4+3\,a^3x^3-23\,a^2x^2-ax+16\right)\sqrt{-a^2}}{6\left(a^9c^2x^3-a^8c^2x^2-a^7c^2x+a^6c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] -1/6*(16*a^3*x^3 - 16*a^2*x^2 - 16*a*x + 30*(a^3*x^3 - a^2*x^2 - a*x + 1)*a
rctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (3*a^4*x^4 + 3*a^3*x^3 - 23*a^2*x^2
 - a*x + 16)*sqrt(-a^2*x^2 + 1) + 16)/(a^9*c^2*x^3 - a^8*c^2*x^2 - a^7*c^2*
x + a^6*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^5}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^6}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**5/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(x**5/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**
2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**6/(a**4*x**4*sqrt(-a**2*
x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c
**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^5}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*x^5/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

3.898 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=99

$$\frac{x^3(ax+1)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(4ax+3)}{3a^4c^2\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3a^5c^2}+\frac{\sin^{-1}(ax)}{a^5c^2}$$

```
[Out] (x^3*(1 + a*x))/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - (x*(3 + 4*a*x))/(3*a^4*c^
2*Sqrt[1 - a^2*x^2]) - (8*Sqrt[1 - a^2*x^2])/(3*a^5*c^2) + ArcSin[a*x]/(a^5
*c^2)
```

Rubi [A] time = 0.122405, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 819, 641, 216}

$$\frac{x^3(ax+1)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(4ax+3)}{3a^4c^2\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3a^5c^2}+\frac{\sin^{-1}(ax)}{a^5c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (x^3*(1 + a*x))/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - (x*(3 + 4*a*x))/(3*a^4*c^
2*Sqrt[1 - a^2*x^2]) - (8*Sqrt[1 - a^2*x^2])/(3*a^5*c^2) + ArcSin[a*x]/(a^5
*c^2)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{x^4(1+ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}$$
$$= \frac{x^3(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{x^2(3+4ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}$$
$$= \frac{x^3(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x(3+4ax)}{3a^4c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{3+8ax}{\sqrt{1-a^2x^2}}\,dx}{3a^4c^2}$$
$$= \frac{x^3(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x(3+4ax)}{3a^4c^2\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3a^5c^2} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^4c^2}$$
$$= \frac{x^3(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x(3+4ax)}{3a^4c^2\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3a^5c^2} + \frac{\sin^{-1}(ax)}{a^5c^2}$$

Mathematica [A] time = 0.0495607, size = 78, normalized size = 0.79

$$\frac{3a^3x^3-7a^2x^2+3(ax-1)\sqrt{1-a^2x^2}\sin^{-1}(ax)-5ax+8}{3a^5c^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (8 - 5*a*x - 7*a^2*x^2 + 3*a^3*x^3 + 3*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSin[
a*x])/(3*a^5*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.046, size = 180, normalized size = 1.8

$$-\frac{1}{a^5c^2}\sqrt{-a^2x^2+1}+\frac{1}{a^4c^2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{1}{4\,c^2a^6\left(x+a^{-1}\right)}\sqrt{-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)}+\frac{1}{6\,c^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a^5/c^2+1/c^2/a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^
2*x^2+1)^(1/2))-1/4/c^2/a^6/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1/6/
c^2/a^7/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+19/12/c^2/a^6/(x-1/a)*
(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^4}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)*x^4/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [A] time = 1.58879, size = 300, normalized size = 3.03

$$-\frac{8\,a^{3}x^{3}-8\,a^{2}x^{2}-8\,ax+6\left(a^{3}x^{3}-a^{2}x^{2}-ax+1\right)\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)+\left(3\,a^{3}x^{3}-7\,a^{2}x^{2}-5\,ax+8\right)\sqrt{-a^{2}x^{2}+1}+8}{3\left(a^{8}c^{2}x^{3}-a^{7}c^{2}x^{2}-a^{6}c^{2}x+a^{5}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] -1/3*(8*a^3*x^3 - 8*a^2*x^2 - 8*a*x + 6*(a^3*x^3 - a^2*x^2 - a*x + 1)*arcta
n((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (3*a^3*x^3 - 7*a^2*x^2 - 5*a*x + 8)*sqr
t(-a^2*x^2 + 1) + 8)/(a^8*c^2*x^3 - a^7*c^2*x^2 - a^6*c^2*x + a^5*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^{4}}{a^{4}x^{4}\sqrt{-a^{2}x^{2}+1}-2a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}+\sqrt{-a^{2}x^{2}+1}}\,dx+\int \frac{ax^{5}}{a^{4}x^{4}\sqrt{-a^{2}x^{2}+1}-2a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}+\sqrt{-a^{2}x^{2}+1}}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(x**4/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**
2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**5/(a**4*x**4*sqrt(-a**2*
x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c
**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^{4}}{\left(a^{2}cx^{2}-c\right)^{2}\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*x^4/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

3.899 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=74

$$\frac{x^2(ax+1)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{3ax+2}{3a^4c^2\sqrt{1-a^2x^2}}+\frac{\sin^{-1}(ax)}{a^4c^2}$$

```
[Out] (x^2*(1 + a*x))/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - (2 + 3*a*x)/(3*a^4*c^2*Sq
rt[1 - a^2*x^2]) + ArcSin[a*x]/(a^4*c^2)
```

Rubi [A] time = 0.103928, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 819, 778, 216}

$$\frac{x^2(ax+1)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{3ax+2}{3a^4c^2\sqrt{1-a^2x^2}}+\frac{\sin^{-1}(ax)}{a^4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (x^2*(1 + a*x))/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - (2 + 3*a*x)/(3*a^4*c^2*Sq
rt[1 - a^2*x^2]) + ArcSin[a*x]/(a^4*c^2)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^2}\,dx=\frac{\int \frac{x^3(1+ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}$$
$$=\frac{x^2(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{\int \frac{x(2+3ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}$$
$$=\frac{x^2(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{2+3ax}{3a^4c^2\sqrt{1-a^2x^2}}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3c^2}$$
$$=\frac{x^2(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{2+3ax}{3a^4c^2\sqrt{1-a^2x^2}}+\frac{\sin^{-1}(ax)}{a^4c^2}$$

Mathematica [A] time = 0.0395464, size = 69, normalized size = 0.93

$$\frac{-4a^2x^2+3(ax-1)\sqrt{1-a^2x^2}\sin^{-1}(ax)+ax+2}{3a^4c^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (2 + a*x - 4*a^2*x^2 + 3*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(3*a^4*c
^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.042, size = 160, normalized size = 2.2

$$\frac{1}{c^2a^3}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{1}{4\,c^2a^5\left(x+a^{-1}\right)}\sqrt{-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)}+\frac{1}{6\,c^2a^6}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/c^2/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/4/c^2/a^5/
(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1/6/c^2/a^6/(x-1/a)^2*(-a^2*(x-1
/a)^2-2*a*(x-1/a))^(1/2)+13/12/c^2/a^5/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a\int \frac{x^4}{\left(a^4c^2x^4-2\,a^2c^2x^2+c^2\right)\sqrt{ax+1}\sqrt{-ax+1}}\,dx+\frac{3\,a^2x^2-2}{3\left(-a^2x^2+1\right)^{\frac{3}{2}}a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] a*integrate(x^4/((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(-a*
x + 1)), x) + 1/3*(3*a^2*x^2 - 2)/((-a^2*x^2 + 1)^(3/2)*a^4*c^2)
```

Fricas [B] time = 1.59463, size = 281, normalized size = 3.8

$$-\frac{2\,a^3x^3-2\,a^2x^2-2\,ax+6\left(a^3x^3-a^2x^2-ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(4\,a^2x^2-ax-2\right)\sqrt{-a^2x^2+1}+2}{3\left(a^7c^2x^3-a^6c^2x^2-a^5c^2x+a^4c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] -1/3*(2*a^3*x^3 - 2*a^2*x^2 - 2*a*x + 6*(a^3*x^3 - a^2*x^2 - a*x + 1)*arcta
n((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (4*a^2*x^2 - a*x - 2)*sqrt(-a^2*x^2 + 1
) + 2)/(a^7*c^2*x^3 - a^6*c^2*x^2 - a^5*c^2*x + a^4*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^4}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(x**3/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**
2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**4/(a**4*x**4*sqrt(-a**2*
x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c
**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^3}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*x^3/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

3.900 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=57

$$\frac{x^2(ax+1)}{3ac^2\left(1-a^2x^2\right)^{3/2}}-\frac{2}{3a^3c^2\sqrt{1-a^2x^2}}$$

```
[Out] (x^2*(1 + a*x))/(3*a*c^2*(1 - a^2*x^2)^(3/2)) - 2/(3*a^3*c^2*Sqrt[1 - a^2*x
^2])
```

Rubi [A] time = 0.093813, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 796, 12, 261}

$$\frac{x^2(ax+1)}{3ac^2\left(1-a^2x^2\right)^{3/2}}-\frac{2}{3a^3c^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (x^2*(1 + a*x))/(3*a*c^2*(1 - a^2*x^2)^(3/2)) - 2/(3*a^3*c^2*Sqrt[1 - a^2*x
^2])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 796

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(x^2*(a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^2}\,dx = \frac{\int \frac{x^2(1+ax)}{(1-a^2x^2)^{5/2}}\,dx}{c^2}$$

$$= \frac{x^2(1+ax)}{3ac^2(1-a^2x^2)^{3/2}} - \frac{\int \frac{2ax}{(1-a^2x^2)^{3/2}}\,dx}{3a^2c^2}$$

$$= \frac{x^2(1+ax)}{3ac^2(1-a^2x^2)^{3/2}} - \frac{2\int \frac{x}{(1-a^2x^2)^{3/2}}\,dx}{3ac^2}$$

$$= \frac{x^2(1+ax)}{3ac^2(1-a^2x^2)^{3/2}} - \frac{2}{3a^3c^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0197114, size = 45, normalized size = 0.79

$$\frac{-a^2x^2-2ax+2}{3a^3c^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (2 - 2*a*x - a^2*x^2)/(3*a^3*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.029, size = 41, normalized size = 0.7

$$-\frac{a^2x^2+2\,ax-2}{(3\,ax-3)\,c^2a^3}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/3*(a^2*x^2+2*a*x-2)/(a*x-1)/c^2/(-a^2*x^2+1)^(1/2)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^2}{(a^2cx^2-c)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)*x^2/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [A] time = 1.62398, size = 182, normalized size = 3.19

$$-\frac{2\,a^{3}x^{3}-2\,a^{2}x^{2}-2\,ax-\left(a^{2}x^{2}+2\,ax-2\right)\sqrt{-a^{2}x^{2}+1}+2}{3\left(a^{6}c^{2}x^{3}-a^{5}c^{2}x^{2}-a^{4}c^{2}x+a^{3}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] -1/3*(2*a^3*x^3 - 2*a^2*x^2 - 2*a*x - (a^2*x^2 + 2*a*x - 2)*sqrt(-a^2*x^2 +
 1) + 2)/(a^6*c^2*x^3 - a^5*c^2*x^2 - a^4*c^2*x + a^3*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^{2}}{a^{4}x^{4}\sqrt{-a^{2}x^{2}+1}-2a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}+\sqrt{-a^{2}x^{2}+1}}\,dx+\int \frac{ax^{3}}{a^{4}x^{4}\sqrt{-a^{2}x^{2}+1}-2a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}+\sqrt{-a^{2}x^{2}+1}}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(x**2/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**
2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**3/(a**4*x**4*sqrt(-a**2*
x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c
**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^{2}}{\left(a^{2}cx^{2}-c\right)^{2}\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*x^2/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

3.901 $\int \frac{e^{\tanh^{-1}(ax)}x}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=55

$$\frac{ax+1}{3a^2c^2(1-a^2x^2)^{3/2}}-\frac{x}{3ac^2\sqrt{1-a^2x^2}}$$

```
[Out] (1 + a*x)/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - x/(3*a*c^2*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0637577, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6148, 778, 191}

$$\frac{ax+1}{3a^2c^2(1-a^2x^2)^{3/2}}-\frac{x}{3ac^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (1 + a*x)/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - x/(3*a*c^2*Sqrt[1 - a^2*x^2])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x}{(c-a^2cx^2)^2}\,dx &= \frac{\int \frac{x(1+ax)}{(1-a^2x^2)^{5/2}}\,dx}{c^2}\\
&= \frac{1+ax}{3a^2c^2(1-a^2x^2)^{3/2}}-\frac{\int \frac{1}{(1-a^2x^2)^{3/2}}\,dx}{3ac^2}\\
&= \frac{1+ax}{3a^2c^2(1-a^2x^2)^{3/2}}-\frac{x}{3ac^2\sqrt{1-a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0193405, size = 44, normalized size = 0.8

$$\frac{-a^2x^2+ax-1}{3a^2c^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-1 + a*x - a^2*x^2)/(3*a^2*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.03, size = 41, normalized size = 0.8

$$-\frac{a^2x^2-ax+1}{(3\,ax-3)\,c^2a^2}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/3*(a^2*x^2-a*x+1)/(a*x-1)/c^2/(-a^2*x^2+1)^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a\int\frac{x^2}{\left(a^4c^2x^4-2\,a^2c^2x^2+c^2\right)\sqrt{ax+1}\sqrt{-ax+1}}\,dx+\frac{1}{3\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^2,x, algorithm="maxim
a")
```

```
[Out] a*integrate(x^2/((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(-a*
x + 1)), x) + 1/3/((-a^2*x^2 + 1)^(3/2)*a^2*c^2)
```

Fricas [A] time = 1.52715, size = 170, normalized size = 3.09

$$\frac{a^3x^3-a^2x^2-ax+\left(a^2x^2-ax+1\right)\sqrt{-a^2x^2+1}+1}{3\left(a^5c^2x^3-a^4c^2x^2-a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^2,x, algorithm="frica
s")
```

```
[Out] 1/3*(a^3*x^3 - a^2*x^2 - a*x + (a^2*x^2 - a*x + 1)*sqrt(-a^2*x^2 + 1) + 1)/
(a^5*c^2*x^3 - a^4*c^2*x^2 - a^3*c^2*x + a^2*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^2}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(x/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 +
 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**2/(a**4*x**4*sqrt(-a**2*x**
2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^2,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)*x/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

3.902 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=52

$$\frac{2x}{3c^2\sqrt{1-a^2x^2}}+\frac{ax+1}{3ac^2\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] (1 + a*x)/(3*a*c^2*(1 - a^2*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0396439, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6138, 639, 191}

$$\frac{2x}{3c^2\sqrt{1-a^2x^2}}+\frac{ax+1}{3ac^2\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] (1 + a*x)/(3*a*c^2*(1 - a^2*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[1 - a^2*x^2])
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{1+ax}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}\\
&= \frac{1+ax}{3ac^2\left(1-a^2x^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^2}\\
&= \frac{1+ax}{3ac^2\left(1-a^2x^2\right)^{3/2}}+\frac{2x}{3c^2\sqrt{1-a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0131722, size = 45, normalized size = 0.87

$$\frac{2a^2x^2 - 2ax - 1}{3ac^2(ax - 1)\sqrt{1 - a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-1 - 2*a*x + 2*a^2*x^2)/(3*a*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.03, size = 42, normalized size = 0.8

$$\frac{2a^2x^2 - 2ax - 1}{(3ax - 3)c^2a}\frac{1}{\sqrt{-a^2x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/3*(2*a^2*x^2-2*a*x-1)/(a*x-1)/c^2/(-a^2*x^2+1)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax + 1}{\left(a^2cx^2 - c\right)^2\sqrt{-a^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [B] time = 1.52428, size = 173, normalized size = 3.33

$$\frac{a^3x^3 - a^2x^2 - ax - \left(2a^2x^2 - 2ax - 1\right)\sqrt{-a^2x^2 + 1} + 1}{3\left(a^4c^2x^3 - a^3c^2x^2 - a^2c^2x + ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] 1/3*(a^3*x^3 - a^2*x^2 - a*x - (2*a^2*x^2 - 2*a*x - 1)*sqrt(-a^2*x^2 + 1) +
 1)/(a^4*c^2*x^3 - a^3*c^2*x^2 - a^2*c^2*x + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(a*x/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2
 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**4*sqrt(-a**2*x**2 +
 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

3.903 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=74

$$\frac{ax+1}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{2ax+3}{3c^2\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

```
[Out] (1 + a*x)/(3*c^2*(1 - a^2*x^2)^(3/2)) + (3 + 2*a*x)/(3*c^2*Sqrt[1 - a^2*x^2
]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c^2
```

Rubi [A] time = 0.116499, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6148, 823, 12, 266, 63, 208}

$$\frac{ax+1}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{2ax+3}{3c^2\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)^2),x]
```

```
[Out] (1 + a*x)/(3*c^2*(1 - a^2*x^2)^(3/2)) + (3 + 2*a*x)/(3*c^2*Sqrt[1 - a^2*x^2
]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c^2
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{1+ax}{x\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}\\
&= \frac{1+ax}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{\int \frac{3a^2+2a^3x}{x\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}\\
&= \frac{1+ax}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{3+2ax}{3c^2\sqrt{1-a^2x^2}}+\frac{\int \frac{3a^4}{x\sqrt{1-a^2x^2}}\,dx}{3a^4c^2}\\
&= \frac{1+ax}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{3+2ax}{3c^2\sqrt{1-a^2x^2}}+\frac{\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c^2}\\
&= \frac{1+ax}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{3+2ax}{3c^2\sqrt{1-a^2x^2}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2c^2}\\
&= \frac{1+ax}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{3+2ax}{3c^2\sqrt{1-a^2x^2}}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2c^2}\\
&= \frac{1+ax}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{3+2ax}{3c^2\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}
\end{aligned}
$$

Mathematica [A] time = 0.0322058, size = 77, normalized size = 1.04

$$
\frac{2a^2x^2-3(ax-1)\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+ax-4}{3c^2(ax-1)\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)^2),x]
```

```
[Out] (-4 + a*x + 2*a^2*x^2 - 3*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2
*x^2]])/(3*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.042, size = 182, normalized size = 2.5

$$\frac{1}{c^2}\left(-\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{1}{4a\left(x+a^{-1}\right)}\sqrt{-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)}+\frac{1}{2a}\left(\frac{1}{3a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/c^2*(-arctanh(1/(-a^2*x^2+1)^(1/2))+1/4/a/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+
1/a))^(1/2)+1/2/a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(
x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))-3/4/a/(x-1/a)*(-a^2*(x-1/a)^2-2*
a*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^2,x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [A] time = 1.59626, size = 262, normalized size = 3.54

$$\frac{4\,a^3x^3-4\,a^2x^2-4\,ax+3\left(a^3x^3-a^2x^2-ax+1\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(2\,a^2x^2+ax-4\right)\sqrt{-a^2x^2+1}+4}{3\left(a^3c^2x^3-a^2c^2x^2-ac^2x+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^2,x, algorithm="frica
s")
```

```
[Out] 1/3*(4*a^3*x^3 - 4*a^2*x^2 - 4*a*x + 3*(a^3*x^3 - a^2*x^2 - a*x + 1)*log((s
qrt(-a^2*x^2 + 1) - 1)/x) - (2*a^2*x^2 + a*x - 4)*sqrt(-a^2*x^2 + 1) + 4)/(
a^3*c^2*x^3 - a^2*c^2*x^2 - a*c^2*x + c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^4x^5\sqrt{-a^2x^2+1}-2a^2x^3\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(a/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 +
 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**5*sqrt(-a**2*x**2 + 1
) - 2*a**2*x**3*sqrt(-a**2*x**2 + 1) + x*sqrt(-a**2*x**2 + 1)), x))/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^2,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)*x), x)
```

3.904 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=105

$$\frac{ax+1}{3c^2x\left(1-a^2x^2\right)^{3/2}}-\frac{8\sqrt{1-a^2x^2}}{3c^2x}+\frac{3ax+4}{3c^2x\sqrt{1-a^2x^2}}-\frac{a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

```
[Out] (1 + a*x)/(3*c^2*x*(1 - a^2*x^2)^(3/2)) + (4 + 3*a*x)/(3*c^2*x*Sqrt[1 - a^2
*x^2]) - (8*Sqrt[1 - a^2*x^2])/(3*c^2*x) - (a*ArcTanh[Sqrt[1 - a^2*x^2]])/c
^2
```

Rubi [A] time = 0.141686, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6148, 823, 807, 266, 63, 208}

$$\frac{ax+1}{3c^2x\left(1-a^2x^2\right)^{3/2}}-\frac{8\sqrt{1-a^2x^2}}{3c^2x}+\frac{3ax+4}{3c^2x\sqrt{1-a^2x^2}}-\frac{a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)^2),x]
```

```
[Out] (1 + a*x)/(3*c^2*x*(1 - a^2*x^2)^(3/2)) + (4 + 3*a*x)/(3*c^2*x*Sqrt[1 - a^2
*x^2]) - (8*Sqrt[1 - a^2*x^2])/(3*c^2*x) - (a*ArcTanh[Sqrt[1 - a^2*x^2]])/c
^2
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{1+ax}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}\\
&= \frac{1+ax}{3c^2x\left(1-a^2x^2\right)^{3/2}}+\frac{\int \frac{4a^2+3a^3x}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}\\
&= \frac{1+ax}{3c^2x\left(1-a^2x^2\right)^{3/2}}+\frac{4+3ax}{3c^2x\sqrt{1-a^2x^2}}+\frac{\int \frac{8a^4+3a^5x}{x^2\sqrt{1-a^2x^2}}\,dx}{3a^4c^2}\\
&= \frac{1+ax}{3c^2x\left(1-a^2x^2\right)^{3/2}}+\frac{4+3ax}{3c^2x\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3c^2x}+\frac{a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c^2}\\
&= \frac{1+ax}{3c^2x\left(1-a^2x^2\right)^{3/2}}+\frac{4+3ax}{3c^2x\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3c^2x}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2c^2}\\
&= \frac{1+ax}{3c^2x\left(1-a^2x^2\right)^{3/2}}+\frac{4+3ax}{3c^2x\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3c^2x}-\frac{\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{ac^2}\\
&= \frac{1+ax}{3c^2x\left(1-a^2x^2\right)^{3/2}}+\frac{4+3ax}{3c^2x\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3c^2x}-\frac{a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}
\end{aligned}
$$

Mathematica [A] time = 0.0417575, size = 91, normalized size = 0.87

$$
\frac{8a^3x^3-5a^2x^2-3ax(ax-1)\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-7ax+3}{3c^2x(ax-1)\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)^2),x]
```

```
[Out] (3 - 7*a*x - 5*a^2*x^2 + 8*a^3*x^3 - 3*a*x*(-1 + a*x)*Sqrt[1 - a^2*x^2]*Arc
Tanh[Sqrt[1 - a^2*x^2]])/(3*c^2*x*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.046, size = 150, normalized size = 1.4

$$\frac{1}{c^2}\left(-\frac{1}{x}\sqrt{-a^2x^2+1}-a\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{1}{4x+4a^{-1}}\sqrt{-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)}+\frac{1}{6a}\sqrt{-a^2\left(x-a^{-1}\right)^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/c^2*(-(-a^2*x^2+1)^(1/2)/x-a*arctanh(1/(-a^2*x^2+1)^(1/2))-1/4/(x+1/a)*(-
a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1/6/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a
))^(1/2)-17/12/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))
```

Maxima [A] time = 1.03892, size = 194, normalized size = 1.85

$$-\frac{\frac{3a^2\log\left(\sqrt{-a^2x^2+1}+1\right)}{c^2}-\frac{3a^2\log\left(\sqrt{-a^2x^2+1}-1\right)}{c^2}+\frac{2\left(3\left(a^2x^2-1\right)a^2-a^2\right)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}}{6a}+\frac{8a^4x^4-12a^2x^2+3}{3\left(a^2c^2x^3-c^2x\right)\sqrt{ax+1}\sqrt{-ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] -1/6*(3*a^2*log(sqrt(-a^2*x^2 + 1) + 1)/c^2 - 3*a^2*log(sqrt(-a^2*x^2 + 1)
- 1)/c^2 + 2*(3*(a^2*x^2 - 1)*a^2 - a^2)/((-a^2*x^2 + 1)^(3/2)*c^2))/a + 1/
3*(8*a^4*x^4 - 12*a^2*x^2 + 3)/((a^2*c^2*x^3 - c^2*x)*sqrt(a*x + 1)*sqrt(-a
*x + 1))
```

Fricas [A] time = 1.59098, size = 305, normalized size = 2.9

$$\frac{4a^4x^4-4a^3x^3-4a^2x^2+4ax+3\left(a^4x^4-a^3x^3-a^2x^2+ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(8a^3x^3-5a^2x^2-7ax+3\right)\sqrt{-a^2x^2+}}{3\left(a^3c^2x^4-a^2c^2x^3-ac^2x^2+c^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] 1/3*(4*a^4*x^4 - 4*a^3*x^3 - 4*a^2*x^2 + 4*a*x + 3*(a^4*x^4 - a^3*x^3 - a^2
*x^2 + a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (8*a^3*x^3 - 5*a^2*x^2 - 7*a*
x + 3)*sqrt(-a^2*x^2 + 1))/(a^3*c^2*x^4 - a^2*c^2*x^3 - a*c^2*x^2 + c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{a^4x^5\sqrt{-a^2x^2+1}-2a^2x^3\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^4x^6\sqrt{-a^2x^2+1}-2a^2x^4\sqrt{-a^2x^2+1}+x^2\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(a/(a**4*x**5*sqrt(-a**2*x**2 + 1) - 2*a**2*x**3*sqrt(-a**2*x**2 +
 1) + x*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**6*sqrt(-a**2*x**2 +
 1) - 2*a**2*x**4*sqrt(-a**2*x**2 + 1) + x**2*sqrt(-a**2*x**2 + 1)), x))/c*
*2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)*x^2), x)
```

3.905 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=134

$$-\frac{8a\sqrt{1-a^2x^2}}{3c^2x}-\frac{5\sqrt{1-a^2x^2}}{2c^2x^2}+\frac{4ax+5}{3c^2x^2\sqrt{1-a^2x^2}}+\frac{ax+1}{3c^2x^2\left(1-a^2x^2\right)^{3/2}}-\frac{5a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

```
[Out] (1 + a*x)/(3*c^2*x^2*(1 - a^2*x^2)^(3/2)) + (5 + 4*a*x)/(3*c^2*x^2*Sqrt[1 -
 a^2*x^2]) - (5*Sqrt[1 - a^2*x^2])/(2*c^2*x^2) - (8*a*Sqrt[1 - a^2*x^2])/(3
*c^2*x) - (5*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c^2)
```

Rubi [A] time = 0.159217, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 823, 835, 807, 266, 63, 208}

$$-\frac{8a\sqrt{1-a^2x^2}}{3c^2x}-\frac{5\sqrt{1-a^2x^2}}{2c^2x^2}+\frac{4ax+5}{3c^2x^2\sqrt{1-a^2x^2}}+\frac{ax+1}{3c^2x^2\left(1-a^2x^2\right)^{3/2}}-\frac{5a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)^2),x]
```


```
[Out] (1 + a*x)/(3*c^2*x^2*(1 - a^2*x^2)^(3/2)) + (5 + 4*a*x)/(3*c^2*x^2*Sqrt[1 -
 a^2*x^2]) - (5*Sqrt[1 - a^2*x^2])/(2*c^2*x^2) - (8*a*Sqrt[1 - a^2*x^2])/(3
*c^2*x) - (5*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c^2)
```


Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{1+ax}{x^3\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}$$

$$= \frac{1+ax}{3c^2x^2\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{5a^2+4a^3x}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}$$

$$= \frac{1+ax}{3c^2x^2\left(1-a^2x^2\right)^{3/2}} + \frac{5+4ax}{3c^2x^2\sqrt{1-a^2x^2}} + \frac{\int \frac{15a^4+8a^5x}{x^3\sqrt{1-a^2x^2}}\,dx}{3a^4c^2}$$

$$= \frac{1+ax}{3c^2x^2\left(1-a^2x^2\right)^{3/2}} + \frac{5+4ax}{3c^2x^2\sqrt{1-a^2x^2}} - \frac{5\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{\int \frac{-16a^5-15a^6x}{x^2\sqrt{1-a^2x^2}}\,dx}{6a^4c^2}$$

$$= \frac{1+ax}{3c^2x^2\left(1-a^2x^2\right)^{3/2}} + \frac{5+4ax}{3c^2x^2\sqrt{1-a^2x^2}} - \frac{5\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{8a\sqrt{1-a^2x^2}}{3c^2x} + \frac{\left(5a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{2c^2}$$

$$= \frac{1+ax}{3c^2x^2\left(1-a^2x^2\right)^{3/2}} + \frac{5+4ax}{3c^2x^2\sqrt{1-a^2x^2}} - \frac{5\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{8a\sqrt{1-a^2x^2}}{3c^2x} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2}}\right.}{4c^2}$$

$$= \frac{1+ax}{3c^2x^2\left(1-a^2x^2\right)^{3/2}} + \frac{5+4ax}{3c^2x^2\sqrt{1-a^2x^2}} - \frac{5\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{8a\sqrt{1-a^2x^2}}{3c^2x} - \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x\right.}{2c^2}$$

$$= \frac{1+ax}{3c^2x^2\left(1-a^2x^2\right)^{3/2}} + \frac{5+4ax}{3c^2x^2\sqrt{1-a^2x^2}} - \frac{5\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{8a\sqrt{1-a^2x^2}}{3c^2x} - \frac{5a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right.}{2c^2}$$

Mathematica [A] time = 0.0459144, size = 103, normalized size = 0.77

$$\frac{16a^4x^4-a^3x^3-23a^2x^2-15a^2x^2(ax-1)\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+3ax+3}{6c^2x^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)^2),x]
```

```
[Out] (3 + 3*a*x - 23*a^2*x^2 - a^3*x^3 + 16*a^4*x^4 - 15*a^2*x^2*(-1 + a*x)*Sqrt
[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(6*c^2*x^2*(-1 + a*x)*Sqrt[1 - a^
2*x^2])
```

Maple [A] time = 0.046, size = 214, normalized size = 1.6

$$\frac{1}{c^2}\left(-\frac{a}{x}\sqrt{-a^2x^2+1}-\frac{5\,a^2}{2}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{a}{4\,x+4\,a^{-1}}\sqrt{-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)}+\frac{a}{2}\left(\frac{1}{3\,a}\sqrt{-a^2\left(x-a\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/c^2*(-a*(-a^2*x^2+1)^(1/2)/x-5/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))+1/4*a/
(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1/2*a*(1/3/a/(x-1/a)^2*(-a^2*(x-
1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))-7
/4*a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/2*(-a^2*x^2+1)^(1/2)/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [A] time = 1.54513, size = 342, normalized size = 2.55

$$\frac{14\,a^5x^5-14\,a^4x^4-14\,a^3x^3+14\,a^2x^2+15\left(a^5x^5-a^4x^4-a^3x^3+a^2x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(16\,a^4x^4-a^3x^3-23\,a^2x^2+3\,a\right.}{6\left(a^3c^2x^5-a^2c^2x^4-ac^2x^3+c^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] 1/6*(14*a^5*x^5 - 14*a^4*x^4 - 14*a^3*x^3 + 14*a^2*x^2 + 15*(a^5*x^5 - a^4*
x^4 - a^3*x^3 + a^2*x^2)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (16*a^4*x^4 - a^
3*x^3 - 23*a^2*x^2 + 3*a*x + 3)*sqrt(-a^2*x^2 + 1))/(a^3*c^2*x^5 - a^2*c^2*
x^4 - a*c^2*x^3 + c^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{a^4x^6\sqrt{-a^2x^2+1}-2a^2x^4\sqrt{-a^2x^2+1}+x^2\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^4x^7\sqrt{-a^2x^2+1}-2a^2x^5\sqrt{-a^2x^2+1}+x^3\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(a/(a**4*x**6*sqrt(-a**2*x**2 + 1) - 2*a**2*x**4*sqrt(-a**2*x**2 +
 1) + x**2*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**7*sqrt(-a**2*x**
2 + 1) - 2*a**2*x**5*sqrt(-a**2*x**2 + 1) + x**3*sqrt(-a**2*x**2 + 1)), x))
/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)*x^3), x)
```

3.906 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=161

$$-\frac{16a^2\sqrt{1-a^2x^2}}{3c^2x}-\frac{5a\sqrt{1-a^2x^2}}{2c^2x^2}-\frac{8\sqrt{1-a^2x^2}}{3c^2x^3}+\frac{5ax+6}{3c^2x^3\sqrt{1-a^2x^2}}+\frac{ax+1}{3c^2x^3(1-a^2x^2)^{3/2}}-\frac{5a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

```
[Out] (1 + a*x)/(3*c^2*x^3*(1 - a^2*x^2)^(3/2)) + (6 + 5*a*x)/(3*c^2*x^3*Sqrt[1 -
 a^2*x^2]) - (8*Sqrt[1 - a^2*x^2])/(3*c^2*x^3) - (5*a*Sqrt[1 - a^2*x^2])/(2
*c^2*x^2) - (16*a^2*Sqrt[1 - a^2*x^2])/(3*c^2*x) - (5*a^3*ArcTanh[Sqrt[1 -
a^2*x^2]])/(2*c^2)
```

Rubi [A] time = 0.188517, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 823, 835, 807, 266, 63, 208}

$$-\frac{16a^2\sqrt{1-a^2x^2}}{3c^2x}-\frac{5a\sqrt{1-a^2x^2}}{2c^2x^2}-\frac{8\sqrt{1-a^2x^2}}{3c^2x^3}+\frac{5ax+6}{3c^2x^3\sqrt{1-a^2x^2}}+\frac{ax+1}{3c^2x^3(1-a^2x^2)^{3/2}}-\frac{5a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*(c - a^2*c*x^2)^2),x]
```

```
[Out] (1 + a*x)/(3*c^2*x^3*(1 - a^2*x^2)^(3/2)) + (6 + 5*a*x)/(3*c^2*x^3*Sqrt[1 -
 a^2*x^2]) - (8*Sqrt[1 - a^2*x^2])/(3*c^2*x^3) - (5*a*Sqrt[1 - a^2*x^2])/(2
*c^2*x^2) - (16*a^2*Sqrt[1 - a^2*x^2])/(3*c^2*x) - (5*a^3*ArcTanh[Sqrt[1 -
a^2*x^2]])/(2*c^2)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^4\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{1+ax}{x^4\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}$$

$$= \frac{1+ax}{3c^2x^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{6a^2+5a^3x}{x^4\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}$$

$$= \frac{1+ax}{3c^2x^3\left(1-a^2x^2\right)^{3/2}} + \frac{6+5ax}{3c^2x^3\sqrt{1-a^2x^2}} + \frac{\int \frac{24a^4+15a^5x}{x^4\sqrt{1-a^2x^2}}\,dx}{3a^4c^2}$$

$$= \frac{1+ax}{3c^2x^3\left(1-a^2x^2\right)^{3/2}} + \frac{6+5ax}{3c^2x^3\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{\int \frac{-45a^5-48a^6x}{x^3\sqrt{1-a^2x^2}}\,dx}{9a^4c^2}$$

$$= \frac{1+ax}{3c^2x^3\left(1-a^2x^2\right)^{3/2}} + \frac{6+5ax}{3c^2x^3\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{5a\sqrt{1-a^2x^2}}{2c^2x^2} + \frac{\int \frac{96a^6+45a^7x}{x^2\sqrt{1-a^2x^2}}\,dx}{18a^4c^2}$$

$$= \frac{1+ax}{3c^2x^3\left(1-a^2x^2\right)^{3/2}} + \frac{6+5ax}{3c^2x^3\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{5a\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{16a^2\sqrt{1-a^2x^2}}{3c^2x} + \frac{\left(5a^3\right)\int}{}$$

$$= \frac{1+ax}{3c^2x^3\left(1-a^2x^2\right)^{3/2}} + \frac{6+5ax}{3c^2x^3\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{5a\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{16a^2\sqrt{1-a^2x^2}}{3c^2x} + \frac{\left(5a^3\right)\text{S}}{}$$

$$= \frac{1+ax}{3c^2x^3\left(1-a^2x^2\right)^{3/2}} + \frac{6+5ax}{3c^2x^3\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{5a\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{16a^2\sqrt{1-a^2x^2}}{3c^2x} - \frac{(5a)\,\text{Sul}}{}$$

$$= \frac{1+ax}{3c^2x^3\left(1-a^2x^2\right)^{3/2}} + \frac{6+5ax}{3c^2x^3\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{5a\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{16a^2\sqrt{1-a^2x^2}}{3c^2x} - \frac{5a^3\,\text{tan}}{}$$

Mathematica [A] time = 0.0510961, size = 110, normalized size = 0.68

$$\frac{32a^5x^5-17a^4x^4-31a^3x^3+11a^2x^2-15a^3x^3(ax-1)\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+ax+2}{6c^2x^3(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*(c - a^2*c*x^2)^2),x]
```

```
[Out] (2 + a*x + 11*a^2*x^2 - 31*a^3*x^3 - 17*a^4*x^4 + 32*a^5*x^5 - 15*a^3*x^3*(
-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(6*c^2*x^3*(-1 + a*
x)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.046, size = 260, normalized size = 1.6

$$\frac{1}{c^2}\left(-\frac{8a^2}{3x}\sqrt{-a^2x^2+1}-2a^3\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{a^2}{4x+4a^{-1}}\sqrt{-a^2\left(x+a^{-1}\right)^2+2a\left(x+a^{-1}\right)}+\frac{a^2}{2}\left(\frac{1}{3a}\sqrt{-a^2\left(x-a\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/c^2*(-8/3*a^2*(-a^2*x^2+1)^(1/2)/x-2*a^3*arctanh(1/(-a^2*x^2+1)^(1/2))-1/
4*a^2/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1/2*a^2*(1/3/a/(x-1/a)^2*(
-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^
(1/2))-9/4*a^2/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+a*(-1/2*(-a^2*x^2
+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2)))-1/3*(-a^2*x^2+1)^(1/2)
/x^3)
```

Maxima [A] time = 1.03299, size = 258, normalized size = 1.6

$$-\frac{\frac{15\,a^4\log\left(\sqrt{-a^2x^2+1}+1\right)}{c^2}-\frac{15\,a^4\log\left(\sqrt{-a^2x^2+1}-1\right)}{c^2}-\frac{2\left(15\left(a^2x^2-1\right)^2a^4+10\left(a^2x^2-1\right)a^4-2\,a^4\right)}{\left(-a^2x^2+1\right)^{\frac{5}{2}}c^2-\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}}{12\,a}+\frac{16\,a^6x^6-24\,a^4x^4+6\,a^2x^2+1}{3\left(a^2c^2x^5-c^2x^3\right)\sqrt{ax+1}\sqrt{-ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] -1/12*(15*a^4*log(sqrt(-a^2*x^2 + 1) + 1)/c^2 - 15*a^4*log(sqrt(-a^2*x^2 +
1) - 1)/c^2 - 2*(15*(a^2*x^2 - 1)^2*a^4 + 10*(a^2*x^2 - 1)*a^4 - 2*a^4)/((-
a^2*x^2 + 1)^(5/2)*c^2 - (-a^2*x^2 + 1)^(3/2)*c^2))/a + 1/3*(16*a^6*x^6 - 2
4*a^4*x^4 + 6*a^2*x^2 + 1)/((a^2*c^2*x^5 - c^2*x^3)*sqrt(a*x + 1)*sqrt(-a*x
 + 1))
```

Fricas [A] time = 1.53575, size = 360, normalized size = 2.24

$$\frac{14\,a^6x^6-14\,a^5x^5-14\,a^4x^4+14\,a^3x^3+15\left(a^6x^6-a^5x^5-a^4x^4+a^3x^3\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(32\,a^5x^5-17\,a^4x^4-31\,a^3x^3\right.}{6\left(a^3c^2x^6-a^2c^2x^5-ac^2x^4+c^2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] 1/6*(14*a^6*x^6 - 14*a^5*x^5 - 14*a^4*x^4 + 14*a^3*x^3 + 15*(a^6*x^6 - a^5*
x^5 - a^4*x^4 + a^3*x^3)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (32*a^5*x^5 - 17
*a^4*x^4 - 31*a^3*x^3 + 11*a^2*x^2 + a*x + 2)*sqrt(-a^2*x^2 + 1))/(a^3*c^2*
x^6 - a^2*c^2*x^5 - a*c^2*x^4 + c^2*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{a^4x^7\sqrt{-a^2x^2+1}-2a^2x^5\sqrt{-a^2x^2+1}+x^3\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^4x^8\sqrt{-a^2x^2+1}-2a^2x^6\sqrt{-a^2x^2+1}+x^4\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**4/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(a/(a**4*x**7*sqrt(-a**2*x**2 + 1) - 2*a**2*x**5*sqrt(-a**2*x**2 +
 1) + x**3*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**8*sqrt(-a**2*x**
2 + 1) - 2*a**2*x**6*sqrt(-a**2*x**2 + 1) + x**4*sqrt(-a**2*x**2 + 1)), x))
/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)*x^4), x)
```

3.907 $\int \frac{e^{\tanh^{-1}(ax)}x^7}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=143

$$\frac{x^6(ax+1)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}}-\frac{x^4(7ax+6)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}}+\frac{x^2(35ax+24)}{15a^6c^3\sqrt{1-a^2x^2}}+\frac{(35ax+32)\sqrt{1-a^2x^2}}{10a^8c^3}-\frac{7\sin^{-1}(ax)}{2a^8c^3}$$

```
[Out] (x^6*(1 + a*x))/(5*a^2*c^3*(1 - a^2*x^2)^(5/2)) - (x^4*(6 + 7*a*x))/(15*a^4
*c^3*(1 - a^2*x^2)^(3/2)) + (x^2*(24 + 35*a*x))/(15*a^6*c^3*Sqrt[1 - a^2*x^
2]) + ((32 + 35*a*x)*Sqrt[1 - a^2*x^2])/(10*a^8*c^3) - (7*ArcSin[a*x])/(2*a
^8*c^3)
```

Rubi [A] time = 0.18099, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 819, 780, 216}

$$\frac{x^6(ax+1)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}}-\frac{x^4(7ax+6)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}}+\frac{x^2(35ax+24)}{15a^6c^3\sqrt{1-a^2x^2}}+\frac{(35ax+32)\sqrt{1-a^2x^2}}{10a^8c^3}-\frac{7\sin^{-1}(ax)}{2a^8c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^7)/(c - a^2*c*x^2)^3,x]
```


```
[Out] (x^6*(1 + a*x))/(5*a^2*c^3*(1 - a^2*x^2)^(5/2)) - (x^4*(6 + 7*a*x))/(15*a^4
*c^3*(1 - a^2*x^2)^(3/2)) + (x^2*(24 + 35*a*x))/(15*a^6*c^3*Sqrt[1 - a^2*x^
2]) + ((32 + 35*a*x)*Sqrt[1 - a^2*x^2])/(10*a^8*c^3) - (7*ArcSin[a*x])/(2*a
^8*c^3)
```


Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^7}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{x^7(1+ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$

$$= \frac{x^6(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{x^5(6+7ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5a^2c^3}$$

$$= \frac{x^6(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^4(6+7ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{x^3(24+35ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15a^4c^3}$$

$$= \frac{x^6(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^4(6+7ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{x^2(24+35ax)}{15a^6c^3\sqrt{1-a^2x^2}} - \frac{\int \frac{x(48+105ax)}{\sqrt{1-a^2x^2}}\,dx}{15a^6c^3}$$

$$= \frac{x^6(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^4(6+7ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{x^2(24+35ax)}{15a^6c^3\sqrt{1-a^2x^2}} + \frac{(32+35ax)\sqrt{1-a^2x^2}}{10a^8c^3} - \frac{7\int \frac{1}{\sqrt{1-a^2}}}{2a^7c^3}$$

$$= \frac{x^6(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^4(6+7ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{x^2(24+35ax)}{15a^6c^3\sqrt{1-a^2x^2}} + \frac{(32+35ax)\sqrt{1-a^2x^2}}{10a^8c^3} - \frac{7\sin^{-1}(a}{2a^8c^3}$$

Mathematica [A] time = 0.0766133, size = 116, normalized size = 0.81

$$\frac{-15a^6x^6-15a^5x^5+176a^4x^4+4a^3x^3-249a^2x^2-105(ax-1)^2(ax+1)\sqrt{1-a^2x^2}\sin^{-1}(ax)+9ax+96}{30a^8c^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^7)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (96 + 9*a*x - 249*a^2*x^2 + 4*a^3*x^3 + 176*a^4*x^4 - 15*a^5*x^5 - 15*a^6*x
^6 - 105*(-1 + a*x)^2*(1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(30*a^8*c^3*
(-1 + a*x)^2*(1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.053, size = 283, normalized size = 2.

$$\frac{x}{2c^3a^7}\sqrt{-a^2x^2+1} - \frac{7}{2c^3a^7}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} + \frac{1}{c^3a^8}\sqrt{-a^2x^2+1} - \frac{1}{20c^3a^{11}}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^7/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/2/c^3/a^7*x*(-a^2*x^2+1)^(1/2)-7/2/c^3/a^7/(a^2)^(1/2)*arctan((a^2)^(1/2)
*x/(-a^2*x^2+1)^(1/2))+1/c^3/a^8*(-a^2*x^2+1)^(1/2)-1/20/c^3/a^11/(x-1/a)^3
```

```
*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-7/15/c^3/a^10/(x-1/a)^2*(-a^2*(x-1/a)^2
-2*a*(x-1/a))^(1/2)-773/240/c^3/a^9/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1
/2)+1/24/c^3/a^10/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)-31/48/c^3/a^
9/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a\int \frac{x^8}{(a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3)\sqrt{ax+1}\sqrt{-ax+1}}\,dx+\frac{\frac{5\sqrt{-a^2x^2+1}}{c^3}+\frac{5\,a^2x^2+15\,(a^2x^2-1)^2-4}{(-a^2x^2+1)^{\frac{5}{2}}c^3}}{5\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^7/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -a*integrate(x^8/((a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(
a*x + 1)*sqrt(-a*x + 1)), x) + 1/5*(5*sqrt(-a^2*x^2 + 1)/c^3 + (5*a^2*x^2 +
 15*(a^2*x^2 - 1)^2 - 4)/((-a^2*x^2 + 1)^(5/2)*c^3))/a^8
```

Fricas [A] time = 1.69224, size = 485, normalized size = 3.39

$$\frac{96\,a^5x^5-96\,a^4x^4-192\,a^3x^3+192\,a^2x^2+96\,ax+210\,(a^5x^5-a^4x^4-2\,a^3x^3+2\,a^2x^2+ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+}{30\,(a^{13}c^3x^5-a^{12}c^3x^4-2\,a^{11}c^3x^3+2\,a^{10}c^3x^2+a^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^7/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] 1/30*(96*a^5*x^5 - 96*a^4*x^4 - 192*a^3*x^3 + 192*a^2*x^2 + 96*a*x + 210*(a
^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*arctan((sqrt(-a^2*x^2 +
 1) - 1)/(a*x)) + (15*a^6*x^6 + 15*a^5*x^5 - 176*a^4*x^4 - 4*a^3*x^3 + 249*
a^2*x^2 - 9*a*x - 96)*sqrt(-a^2*x^2 + 1) - 96)/(a^13*c^3*x^5 - a^12*c^3*x^4
 - 2*a^11*c^3*x^3 + 2*a^10*c^3*x^2 + a^9*c^3*x - a^8*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^7}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^8}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**7/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(x**7/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x*
*2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + In
tegral(a*x**8/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**
2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)x^7}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^7/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)*x^7/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

3.908 $\int \frac{e^{\tanh^{-1}(ax)}x^6}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=133

$$\frac{x^5(ax+1)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}}-\frac{x^3(6ax+5)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}}+\frac{x(8ax+5)}{5a^6c^3\sqrt{1-a^2x^2}}+\frac{16\sqrt{1-a^2x^2}}{5a^7c^3}-\frac{\sin^{-1}(ax)}{a^7c^3}$$

```
[Out] (x^5*(1 + a*x))/(5*a^2*c^3*(1 - a^2*x^2)^(5/2)) - (x^3*(5 + 6*a*x))/(15*a^4
*c^3*(1 - a^2*x^2)^(3/2)) + (x*(5 + 8*a*x))/(5*a^6*c^3*Sqrt[1 - a^2*x^2]) +
 (16*Sqrt[1 - a^2*x^2])/(5*a^7*c^3) - ArcSin[a*x]/(a^7*c^3)
```

Rubi [A] time = 0.154496, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 819, 641, 216}

$$\frac{x^5(ax+1)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}}-\frac{x^3(6ax+5)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}}+\frac{x(8ax+5)}{5a^6c^3\sqrt{1-a^2x^2}}+\frac{16\sqrt{1-a^2x^2}}{5a^7c^3}-\frac{\sin^{-1}(ax)}{a^7c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^6)/(c - a^2*c*x^2)^3,x]
```


```
[Out] (x^5*(1 + a*x))/(5*a^2*c^3*(1 - a^2*x^2)^(5/2)) - (x^3*(5 + 6*a*x))/(15*a^4
*c^3*(1 - a^2*x^2)^(3/2)) + (x*(5 + 8*a*x))/(5*a^6*c^3*Sqrt[1 - a^2*x^2]) +
 (16*Sqrt[1 - a^2*x^2])/(5*a^7*c^3) - ArcSin[a*x]/(a^7*c^3)
```


Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^6}{(c-a^2cx^2)^3}\,dx = \frac{\int \frac{x^6(1+ax)}{(1-a^2x^2)^{7/2}}\,dx}{c^3}$$
$$= \frac{x^5(1+ax)}{5a^2c^3(1-a^2x^2)^{5/2}} - \frac{\int \frac{x^4(5+6ax)}{(1-a^2x^2)^{5/2}}\,dx}{5a^2c^3}$$
$$= \frac{x^5(1+ax)}{5a^2c^3(1-a^2x^2)^{5/2}} - \frac{x^3(5+6ax)}{15a^4c^3(1-a^2x^2)^{3/2}} + \frac{\int \frac{x^2(15+24ax)}{(1-a^2x^2)^{3/2}}\,dx}{15a^4c^3}$$
$$= \frac{x^5(1+ax)}{5a^2c^3(1-a^2x^2)^{5/2}} - \frac{x^3(5+6ax)}{15a^4c^3(1-a^2x^2)^{3/2}} + \frac{x(5+8ax)}{5a^6c^3\sqrt{1-a^2x^2}} - \frac{\int \frac{15+48ax}{\sqrt{1-a^2x^2}}\,dx}{15a^6c^3}$$
$$= \frac{x^5(1+ax)}{5a^2c^3(1-a^2x^2)^{5/2}} - \frac{x^3(5+6ax)}{15a^4c^3(1-a^2x^2)^{3/2}} + \frac{x(5+8ax)}{5a^6c^3\sqrt{1-a^2x^2}} + \frac{16\sqrt{1-a^2x^2}}{5a^7c^3} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^6c^3}$$
$$= \frac{x^5(1+ax)}{5a^2c^3(1-a^2x^2)^{5/2}} - \frac{x^3(5+6ax)}{15a^4c^3(1-a^2x^2)^{3/2}} + \frac{x(5+8ax)}{5a^6c^3\sqrt{1-a^2x^2}} + \frac{16\sqrt{1-a^2x^2}}{5a^7c^3} - \frac{\sin^{-1}(ax)}{a^7c^3}$$

Mathematica [A] time = 0.0728295, size = 108, normalized size = 0.81

$$\frac{-15a^5x^5+38a^4x^4+52a^3x^3-87a^2x^2-15(ax-1)^2(ax+1)\sqrt{1-a^2x^2}\sin^{-1}(ax)-33ax+48}{15a^7c^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^6)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (48 - 33*a*x - 87*a^2*x^2 + 52*a^3*x^3 + 38*a^4*x^4 - 15*a^5*x^5 - 15*(-1 +
 a*x)^2*(1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(15*a^7*c^3*(-1 + a*x)^2*(
1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.05, size = 262, normalized size = 2.

$$\frac{1}{a^7c^3}\sqrt{-a^2x^2+1} - \frac{1}{a^6c^3}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} - \frac{1}{20\,c^3a^{10}}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3} - \frac{23}{60\,c^3a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^3,x)
```

```
[Out] (-a^2*x^2+1)^(1/2)/a^7/c^3-1/c^3/a^6/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2
*x^2+1)^(1/2))-1/20/c^3/a^10/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2
3/60/c^3/a^9/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-493/240/c^3/a^8/(
x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/24/c^3/a^9/(x+1/a)^2*(-a^2*(x+1
/a)^2+2*a*(x+1/a))^(1/2)+25/48/c^3/a^8/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)x^6}{(a^2cx^2-c)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)*x^6/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [A] time = 1.58447, size = 462, normalized size = 3.47

$$\frac{48\,a^5x^5-48\,a^4x^4-96\,a^3x^3+96\,a^2x^2+48\,ax+30\left(a^5x^5-a^4x^4-2\,a^3x^3+2\,a^2x^2+ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+(1}{15\left(a^{12}c^3x^5-a^{11}c^3x^4-2\,a^{10}c^3x^3+2\,a^9c^3x^2+a^8c^3x-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] 1/15*(48*a^5*x^5 - 48*a^4*x^4 - 96*a^3*x^3 + 96*a^2*x^2 + 48*a*x + 30*(a^5*
x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*arctan((sqrt(-a^2*x^2 + 1)
 - 1)/(a*x)) + (15*a^5*x^5 - 38*a^4*x^4 - 52*a^3*x^3 + 87*a^2*x^2 + 33*a*x
- 48)*sqrt(-a^2*x^2 + 1) - 48)/(a^12*c^3*x^5 - a^11*c^3*x^4 - 2*a^10*c^3*x^
3 + 2*a^9*c^3*x^2 + a^8*c^3*x - a^7*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^6}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^7}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**6/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(x**6/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x*
*2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + In
tegral(a*x**7/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**
2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)x^6}{(a^2cx^2-c)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)*x^6/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

3.909 $\int \frac{e^{\tanh^{-1}(ax)} x^5}{(c-a^2cx^2)^3} dx$

Optimal. Leaf size=108

$$\frac{x^4(ax+1)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}}-\frac{x^2(5ax+4)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}}+\frac{15ax+8}{15a^6c^3\sqrt{1-a^2x^2}}-\frac{\sin^{-1}(ax)}{a^6c^3}$$

```
[Out] (x^4*(1 + a*x))/(5*a^2*c^3*(1 - a^2*x^2)^(5/2)) - (x^2*(4 + 5*a*x))/(15*a^4
*c^3*(1 - a^2*x^2)^(3/2)) + (8 + 15*a*x)/(15*a^6*c^3*Sqrt[1 - a^2*x^2]) - A
rcSin[a*x]/(a^6*c^3)
```

Rubi [A] time = 0.129224, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 819, 778, 216}

$$\frac{x^4(ax+1)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}}-\frac{x^2(5ax+4)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}}+\frac{15ax+8}{15a^6c^3\sqrt{1-a^2x^2}}-\frac{\sin^{-1}(ax)}{a^6c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^5)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (x^4*(1 + a*x))/(5*a^2*c^3*(1 - a^2*x^2)^(5/2)) - (x^2*(4 + 5*a*x))/(15*a^4
*c^3*(1 - a^2*x^2)^(3/2)) + (8 + 15*a*x)/(15*a^6*c^3*Sqrt[1 - a^2*x^2]) - A
rcSin[a*x]/(a^6*c^3)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^5}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{x^5(1+ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$

$$= \frac{x^4(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{x^3(4+5ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5a^2c^3}$$

$$= \frac{x^4(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^2(4+5ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{x(8+15ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15a^4c^3}$$

$$= \frac{x^4(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^2(4+5ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{8+15ax}{15a^6c^3\sqrt{1-a^2x^2}} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^5c^3}$$

$$= \frac{x^4(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^2(4+5ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{8+15ax}{15a^6c^3\sqrt{1-a^2x^2}} - \frac{\sin^{-1}(ax)}{a^6c^3}$$

Mathematica [A] time = 0.0555427, size = 100, normalized size = 0.93

$$\frac{23a^4x^4-8a^3x^3-27a^2x^2-15(ax-1)^2(ax+1)\sqrt{1-a^2x^2}\sin^{-1}(ax)+7ax+8}{15a^6c^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^5)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (8 + 7*a*x - 27*a^2*x^2 - 8*a^3*x^3 + 23*a^4*x^4 - 15*(-1 + a*x)^2*(1 + a*x
)*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(15*a^6*c^3*(-1 + a*x)^2*(1 + a*x)*Sqrt[1
- a^2*x^2])
```

Maple [B] time = 0.049, size = 243, normalized size = 2.3

$$-\frac{1}{c^3a^5}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} - \frac{1}{20\,c^3a^9}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3} - \frac{3}{10\,c^3a^8}\sqrt{-a^2\left(x-a^{-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/c^3/a^5/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-1/20/c^3/a^
9/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-3/10/c^3/a^8/(x-1/a)^2*(-a^2
*(x-1/a)^2-2*a*(x-1/a))^(1/2)-91/80/c^3/a^7/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-
1/a))^(1/2)+1/24/c^3/a^8/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)-19/48
/c^3/a^7/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a\int \frac{x^6}{\left(a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3\right)\sqrt{ax+1}\sqrt{-ax+1}}\,dx+\frac{10\,a^2x^2+15\left(a^2x^2-1\right)^2-7}{15\left(-a^2x^2+1\right)^{\frac{5}{2}}a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -a*integrate(x^6/((a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(
a*x + 1)*sqrt(-a*x + 1)), x) + 1/15*(10*a^2*x^2 + 15*(a^2*x^2 - 1)^2 - 7)/(
(-a^2*x^2 + 1)^(5/2)*a^6*c^3)
```

Fricas [B] time = 1.67021, size = 433, normalized size = 4.01

$$\frac{8\,a^5x^5-8\,a^4x^4-16\,a^3x^3+16\,a^2x^2+8\,ax+30\left(a^5x^5-a^4x^4-2\,a^3x^3+2\,a^2x^2+ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(23\,a^4\right.}{15\left(a^{11}c^3x^5-a^{10}c^3x^4-2\,a^9c^3x^3+2\,a^8c^3x^2+a^7c^3x-a^6c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] 1/15*(8*a^5*x^5 - 8*a^4*x^4 - 16*a^3*x^3 + 16*a^2*x^2 + 8*a*x + 30*(a^5*x^5
 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) -
1)/(a*x)) - (23*a^4*x^4 - 8*a^3*x^3 - 27*a^2*x^2 + 7*a*x + 8)*sqrt(-a^2*x^2
 + 1) - 8)/(a^11*c^3*x^5 - a^10*c^3*x^4 - 2*a^9*c^3*x^3 + 2*a^8*c^3*x^2 + a
^7*c^3*x - a^6*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^5}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^6}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**5/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(x**5/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x*
*2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + In
tegral(a*x**6/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**
2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)x^5}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)*x^5/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

3.910 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=81

$$\frac{x^4(ax+1)}{5ac^3\left(1-a^2x^2\right)^{5/2}}+\frac{4}{5a^5c^3\sqrt{1-a^2x^2}}-\frac{4}{15a^5c^3\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] (x^4*(1 + a*x))/(5*a*c^3*(1 - a^2*x^2)^(5/2)) - 4/(15*a^5*c^3*(1 - a^2*x^2)
^(3/2)) + 4/(5*a^5*c^3*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.116196, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 805, 266, 43}

$$\frac{x^4(ax+1)}{5ac^3\left(1-a^2x^2\right)^{5/2}}+\frac{4}{5a^5c^3\sqrt{1-a^2x^2}}-\frac{4}{15a^5c^3\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (x^4*(1 + a*x))/(5*a*c^3*(1 - a^2*x^2)^(5/2)) - 4/(15*a^5*c^3*(1 - a^2*x^2)
^(3/2)) + 4/(5*a^5*c^3*Sqrt[1 - a^2*x^2])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{x^4(1+ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$

$$= \frac{x^4(1+ax)}{5ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{4\int \frac{x^3}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5ac^3}$$

$$= \frac{x^4(1+ax)}{5ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{2\,\text{Subst}\left(\int \frac{x}{\left(1-a^2x\right)^{5/2}}\,dx, x, x^2\right)}{5ac^3}$$

$$= \frac{x^4(1+ax)}{5ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{2\,\text{Subst}\left(\int \left(\frac{1}{a^2\left(1-a^2x\right)^{5/2}} - \frac{1}{a^2\left(1-a^2x\right)^{3/2}}\right)dx, x, x^2\right)}{5ac^3}$$

$$= \frac{x^4(1+ax)}{5ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{4}{15a^5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{4}{5a^5c^3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0263161, size = 68, normalized size = 0.84

$$\frac{3a^4x^4+12a^3x^3-12a^2x^2-8ax+8}{15a^5c^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (8 - 8*a*x - 12*a^2*x^2 + 12*a^3*x^3 + 3*a^4*x^4)/(15*a^5*c^3*(-1 + a*x)^2*
(1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.03, size = 58, normalized size = 0.7

$$-\frac{3\,x^4a^4+12\,x^3a^3-12\,a^2x^2-8\,ax+8}{(15\,ax-15)\,c^3a^5}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/15*(3*a^4*x^4+12*a^3*x^3-12*a^2*x^2-8*a*x+8)/(a*x-1)/c^3/(-a^2*x^2+1)^(3
/2)/a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)x^4}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)*x^4/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [B] time = 1.51358, size = 300, normalized size = 3.7

$$\frac{8 a^5x^5 - 8 a^4x^4 - 16 a^3x^3 + 16 a^2x^2 + 8 ax - \left(3 a^4x^4 + 12 a^3x^3 - 12 a^2x^2 - 8 ax + 8\right)\sqrt{-a^2x^2 + 1} - 8}{15 \left(a^{10}c^3x^5 - a^9c^3x^4 - 2 a^8c^3x^3 + 2 a^7c^3x^2 + a^6c^3x - a^5c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] 1/15*(8*a^5*x^5 - 8*a^4*x^4 - 16*a^3*x^3 + 16*a^2*x^2 + 8*a*x - (3*a^4*x^4
+ 12*a^3*x^3 - 12*a^2*x^2 - 8*a*x + 8)*sqrt(-a^2*x^2 + 1) - 8)/(a^10*c^3*x^
5 - a^9*c^3*x^4 - 2*a^8*c^3*x^3 + 2*a^7*c^3*x^2 + a^6*c^3*x - a^5*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^4}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\, dx + \int \frac{ax^5}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\, dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(x**4/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x*
*2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + In
tegral(a*x**5/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**
2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax + 1)x^4}{\left(a^2cx^2 - c\right)^3\sqrt{-a^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)*x^4/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

3.911 $\int \frac{e^{\tanh^{-1}(ax)} x^3}{\left(c-a^2cx^2\right)^3} \, dx$

Optimal. Leaf size=74

$$\frac{ax^5}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{1}{3a^4c^3\left(1-a^2x^2\right)^{3/2}}+\frac{1}{5a^4c^3\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] 1/(5*a^4*c^3*(1 - a^2*x^2)^(5/2)) + (a*x^5)/(5*c^3*(1 - a^2*x^2)^(5/2)) - 1
/(3*a^4*c^3*(1 - a^2*x^2)^(3/2))
```

Rubi [A] time = 0.105676, antiderivative size = 88, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 819, 778, 191}

$$\frac{x^2(ax+1)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}}+\frac{x}{5a^3c^3\sqrt{1-a^2x^2}}-\frac{3ax+2}{15a^4c^3\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (x^2*(1 + a*x))/(5*a^2*c^3*(1 - a^2*x^2)^(5/2)) - (2 + 3*a*x)/(15*a^4*c^3*(
1 - a^2*x^2)^(3/2)) + x/(5*a^3*c^3*Sqrt[1 - a^2*x^2])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{x^3(1+ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$

$$= \frac{x^2(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{x(2+3ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5a^2c^3}$$

$$= \frac{x^2(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{2+3ax}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{5a^3c^3}$$

$$= \frac{x^2(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{2+3ax}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{x}{5a^3c^3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0238963, size = 68, normalized size = 0.92

$$\frac{3a^4x^4-3a^3x^3+3a^2x^2+2ax-2}{15a^4c^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (-2 + 2*a*x + 3*a^2*x^2 - 3*a^3*x^3 + 3*a^4*x^4)/(15*a^4*c^3*(-1 + a*x)^2*(
1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.03, size = 58, normalized size = 0.8

$$-\frac{3\,x^4a^4-3\,x^3a^3+3\,a^2x^2+2\,ax-2}{(15\,ax-15)\,c^3a^4}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/15*(3*a^4*x^4-3*a^3*x^3+3*a^2*x^2+2*a*x-2)/(a*x-1)/c^3/(-a^2*x^2+1)^(3/2
)/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a\int \frac{x^4}{\left(a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3\right)\sqrt{ax+1}\sqrt{-ax+1}}\,dx + \frac{5\,a^2x^2-2}{15\left(-a^2x^2+1\right)^{\frac{5}{2}}a^4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -a*integrate(x^4/((a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(
a*x + 1)*sqrt(-a*x + 1)), x) + 1/15*(5*a^2*x^2 - 2)/((-a^2*x^2 + 1)^(5/2)*a
^4*c^3)
```

Fricas [B] time = 1.56795, size = 294, normalized size = 3.97

$$-\frac{2\,a^{5}x^{5}-2\,a^{4}x^{4}-4\,a^{3}x^{3}+4\,a^{2}x^{2}+2\,ax+\left(3\,a^{4}x^{4}-3\,a^{3}x^{3}+3\,a^{2}x^{2}+2\,ax-2\right)\sqrt{-a^{2}x^{2}+1}-2}{15\left(a^{9}c^{3}x^{5}-a^{8}c^{3}x^{4}-2\,a^{7}c^{3}x^{3}+2\,a^{6}c^{3}x^{2}+a^{5}c^{3}x-a^{4}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] -1/15*(2*a^5*x^5 - 2*a^4*x^4 - 4*a^3*x^3 + 4*a^2*x^2 + 2*a*x + (3*a^4*x^4 -
 3*a^3*x^3 + 3*a^2*x^2 + 2*a*x - 2)*sqrt(-a^2*x^2 + 1) - 2)/(a^9*c^3*x^5 -
a^8*c^3*x^4 - 2*a^7*c^3*x^3 + 2*a^6*c^3*x^2 + a^5*c^3*x - a^4*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^{3}}{-a^{6}x^{6}\sqrt{-a^{2}x^{2}+1}+3a^{4}x^{4}\sqrt{-a^{2}x^{2}+1}-3a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}+\sqrt{-a^{2}x^{2}+1}}\,dx+\int \frac{ax^{4}}{-a^{6}x^{6}\sqrt{-a^{2}x^{2}+1}+3a^{4}x^{4}\sqrt{-a^{2}x^{2}+1}-3a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}+\sqrt{-a^{2}x^{2}+1}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(x**3/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x*
*2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + In
tegral(a*x**4/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**
2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)x^{3}}{\left(a^{2}cx^{2}-c\right)^{3}\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)*x^3/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

3.912 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=88

$$\frac{x^2(ax+1)}{5ac^3\left(1-a^2x^2\right)^{5/2}}-\frac{2x}{15a^2c^3\sqrt{1-a^2x^2}}-\frac{2(1-ax)}{15a^3c^3\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] (x^2*(1 + a*x))/(5*a*c^3*(1 - a^2*x^2)^(5/2)) - (2*(1 - a*x))/(15*a^3*c^3*(
1 - a^2*x^2)^(3/2)) - (2*x)/(15*a^2*c^3*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.107533, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 796, 778, 191}

$$\frac{x^2(ax+1)}{5ac^3\left(1-a^2x^2\right)^{5/2}}-\frac{2x}{15a^2c^3\sqrt{1-a^2x^2}}-\frac{2(1-ax)}{15a^3c^3\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (x^2*(1 + a*x))/(5*a*c^3*(1 - a^2*x^2)^(5/2)) - (2*(1 - a*x))/(15*a^3*c^3*(
1 - a^2*x^2)^(3/2)) - (2*x)/(15*a^2*c^3*Sqrt[1 - a^2*x^2])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 796

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(x^2*(a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{x^2(1+ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$

$$= \frac{x^2(1+ax)}{5ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{x\left(2a-2a^2x\right)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5a^2c^3}$$

$$= \frac{x^2(1+ax)}{5ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{2(1-ax)}{15a^3c^3\left(1-a^2x^2\right)^{3/2}} - \frac{2\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15a^2c^3}$$

$$= \frac{x^2(1+ax)}{5ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{2(1-ax)}{15a^3c^3\left(1-a^2x^2\right)^{3/2}} - \frac{2x}{15a^2c^3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0263293, size = 68, normalized size = 0.77

$$\frac{-2a^4x^4+2a^3x^3+3a^2x^2+2ax-2}{15a^3c^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (-2 + 2*a*x + 3*a^2*x^2 + 2*a^3*x^3 - 2*a^4*x^4)/(15*a^3*c^3*(-1 + a*x)^2*(
1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.033, size = 58, normalized size = 0.7

$$\frac{2\,x^4a^4-2\,x^3a^3-3\,a^2x^2-2\,ax+2}{(15\,ax-15)\,c^3a^3}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/15*(2*a^4*x^4-2*a^3*x^3-3*a^2*x^2-2*a*x+2)/(a*x-1)/c^3/(-a^2*x^2+1)^(3/2)
/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)x^2}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)*x^2/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [A] time = 1.57908, size = 294, normalized size = 3.34

$$-\frac{2\,a^5x^5-2\,a^4x^4-4\,a^3x^3+4\,a^2x^2+2\,ax-\left(2\,a^4x^4-2\,a^3x^3-3\,a^2x^2-2\,ax+2\right)\sqrt{-a^2x^2+1}-2}{15\left(a^8c^3x^5-a^7c^3x^4-2\,a^6c^3x^3+2\,a^5c^3x^2+a^4c^3x-a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] -1/15*(2*a^5*x^5 - 2*a^4*x^4 - 4*a^3*x^3 + 4*a^2*x^2 + 2*a*x - (2*a^4*x^4 -
 2*a^3*x^3 - 3*a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1) - 2)/(a^8*c^3*x^5 -
a^7*c^3*x^4 - 2*a^6*c^3*x^3 + 2*a^5*c^3*x^2 + a^4*c^3*x - a^3*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^3}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(x**2/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x*
*2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + In
tegral(a*x**3/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**
2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)x^2}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)*x^2/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

3.913 $\int \frac{e^{\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=80

$$-\frac{2x}{15ac^3\sqrt{1-a^2x^2}}-\frac{x}{15ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{ax+1}{5a^2c^3\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] (1 + a*x)/(5*a^2*c^3*(1 - a^2*x^2)^(5/2)) - x/(15*a*c^3*(1 - a^2*x^2)^(3/2)
) - (2*x)/(15*a*c^3*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0690854, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {6148, 778, 192, 191}

$$-\frac{2x}{15ac^3\sqrt{1-a^2x^2}}-\frac{x}{15ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{ax+1}{5a^2c^3\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (1 + a*x)/(5*a^2*c^3*(1 - a^2*x^2)^(5/2)) - x/(15*a*c^3*(1 - a^2*x^2)^(3/2)
) - (2*x)/(15*a*c^3*Sqrt[1 - a^2*x^2])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x}{(c-a^2cx^2)^3}\,dx = \frac{\int \frac{x(1+ax)}{(1-a^2x^2)^{7/2}}\,dx}{c^3}$$
$$= \frac{1+ax}{5a^2c^3(1-a^2x^2)^{5/2}} - \frac{\int \frac{1}{(1-a^2x^2)^{5/2}}\,dx}{5ac^3}$$
$$= \frac{1+ax}{5a^2c^3(1-a^2x^2)^{5/2}} - \frac{x}{15ac^3(1-a^2x^2)^{3/2}} - \frac{2\int \frac{1}{(1-a^2x^2)^{3/2}}\,dx}{15ac^3}$$
$$= \frac{1+ax}{5a^2c^3(1-a^2x^2)^{5/2}} - \frac{x}{15ac^3(1-a^2x^2)^{3/2}} - \frac{2x}{15ac^3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0311734, size = 68, normalized size = 0.85

$$\frac{-2a^4x^4+2a^3x^3+3a^2x^2-3ax+3}{15a^2c^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (3 - 3*a*x + 3*a^2*x^2 + 2*a^3*x^3 - 2*a^4*x^4)/(15*a^2*c^3*(-1 + a*x)^2*(1
 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.032, size = 58, normalized size = 0.7

$$\frac{2x^4a^4-2x^3a^3-3a^2x^2+3ax-3}{(15ax-15)c^3a^2}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/15*(2*a^4*x^4-2*a^3*x^3-3*a^2*x^2+3*a*x-3)/(a*x-1)/c^3/(-a^2*x^2+1)^(3/2)
/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a\int \frac{x^2}{\left(a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3\right)\sqrt{ax+1}\sqrt{-ax+1}}\,dx + \frac{1}{5\left(-a^2x^2+1\right)^{\frac{5}{2}}a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^3,x, algorithm="maxim
a")
```

```
[Out] -a*integrate(x^2/((a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(
a*x + 1)*sqrt(-a*x + 1)), x) + 1/5/((-a^2*x^2 + 1)^(5/2)*a^2*c^3)
```

Fricas [B] time = 1.58584, size = 293, normalized size = 3.66

$$\frac{3\,a^5x^5 - 3\,a^4x^4 - 6\,a^3x^3 + 6\,a^2x^2 + 3\,ax + \left(2\,a^4x^4 - 2\,a^3x^3 - 3\,a^2x^2 + 3\,ax - 3\right)\sqrt{-a^2x^2 + 1} - 3}{15\left(a^7c^3x^5 - a^6c^3x^4 - 2\,a^5c^3x^3 + 2\,a^4c^3x^2 + a^3c^3x - a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^3,x, algorithm="frica
s")
```

```
[Out] 1/15*(3*a^5*x^5 - 3*a^4*x^4 - 6*a^3*x^3 + 6*a^2*x^2 + 3*a*x + (2*a^4*x^4 -
2*a^3*x^3 - 3*a^2*x^2 + 3*a*x - 3)*sqrt(-a^2*x^2 + 1) - 3)/(a^7*c^3*x^5 - a
^6*c^3*x^4 - 2*a^5*c^3*x^3 + 2*a^4*c^3*x^2 + a^3*c^3*x - a^2*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx + \int \frac{ax^2}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(x/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2
+ 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integ
ral(a*x**2/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 +
 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)x}{\left(a^2cx^2 - c\right)^3\sqrt{-a^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^3,x, algorithm="giac"
)
```

```
[Out] integrate(-(a*x + 1)*x/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

3.914 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=74

$$\frac{8x}{15c^3\sqrt{1-a^2x^2}}+\frac{4x}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{ax+1}{5ac^3\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] (1 + a*x)/(5*a*c^3*(1 - a^2*x^2)^(5/2)) + (4*x)/(15*c^3*(1 - a^2*x^2)^(3/2)
) + (8*x)/(15*c^3*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0453206, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6138, 639, 192, 191}

$$\frac{8x}{15c^3\sqrt{1-a^2x^2}}+\frac{4x}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{ax+1}{5ac^3\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a^2*c*x^2)^3,x]
```

```
[Out] (1 + a*x)/(5*a*c^3*(1 - a^2*x^2)^(5/2)) + (4*x)/(15*c^3*(1 - a^2*x^2)^(3/2)
) + (8*x)/(15*c^3*Sqrt[1 - a^2*x^2])
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx=\frac{\int \frac{1+ax}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$

$$=\frac{1+ax}{5ac^3\left(1-a^2x^2\right)^{5/2}}+\frac{4\int \frac{1}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^3}$$

$$=\frac{1+ax}{5ac^3\left(1-a^2x^2\right)^{5/2}}+\frac{4x}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{8\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^3}$$

$$=\frac{1+ax}{5ac^3\left(1-a^2x^2\right)^{5/2}}+\frac{4x}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{8x}{15c^3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.021533, size = 59, normalized size = 0.8

$$\frac{8a^4x^4-8a^3x^3-12a^2x^2+12ax+3}{15ac^3(1-ax)^{5/2}(ax+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a^2*c*x^2)^3,x]
```

```
[Out] (3 + 12*a*x - 12*a^2*x^2 - 8*a^3*x^3 + 8*a^4*x^4)/(15*a*c^3*(1 - a*x)^(5/2)
*(1 + a*x)^(3/2))
```

Maple [A] time = 0.033, size = 58, normalized size = 0.8

$$-{\frac{8\,{x}^{4}{a}^{4}-8\,{x}^{3}{a}^{3}-12\,{a}^{2}{x}^{2}+12\,ax+3}{ \left(15\,ax-15 \right) {c}^{3}a} \left(-{a}^{2}{x}^{2}+1 \right) ^{-{\frac{3}{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/15*(8*a^4*x^4-8*a^3*x^3-12*a^2*x^2+12*a*x+3)/(a*x-1)/c^3/(-a^2*x^2+1)^(3
/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax+1}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -integrate((a*x + 1)/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [B] time = 1.52217, size = 293, normalized size = 3.96

$$\frac{3\,a^5x^5 - 3\,a^4x^4 - 6\,a^3x^3 + 6\,a^2x^2 + 3\,ax - \left(8\,a^4x^4 - 8\,a^3x^3 - 12\,a^2x^2 + 12\,ax + 3\right)\sqrt{-a^2x^2 + 1} - 3}{15\left(a^6c^3x^5 - a^5c^3x^4 - 2\,a^4c^3x^3 + 2\,a^3c^3x^2 + a^2c^3x - ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/15*(3*a^5*x^5 - 3*a^4*x^4 - 6*a^3*x^3 + 6*a^2*x^2 + 3*a*x - (8*a^4*x^4 -
8*a^3*x^3 - 12*a^2*x^2 + 12*a*x + 3)*sqrt(-a^2*x^2 + 1) - 3)/(a^6*c^3*x^5 -
 a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^3*c^3*x^2 + a^2*c^3*x - a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx + \int \frac{1}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(a*x/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**
2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Int
egral(1/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 + 1)
 - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{ax + 1}{\left(a^2cx^2 - c\right)^3\sqrt{-a^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

3.915 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=101

$$\frac{ax+1}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{8ax+15}{15c^3\sqrt{1-a^2x^2}}+\frac{4ax+5}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

```
[Out] (1 + a*x)/(5*c^3*(1 - a^2*x^2)^(5/2)) + (5 + 4*a*x)/(15*c^3*(1 - a^2*x^2)^(
3/2)) + (15 + 8*a*x)/(15*c^3*Sqrt[1 - a^2*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]
]/c^3
```

Rubi [A] time = 0.139788, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6148, 823, 12, 266, 63, 208}

$$\frac{ax+1}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{8ax+15}{15c^3\sqrt{1-a^2x^2}}+\frac{4ax+5}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)^3),x]
```

```
[Out] (1 + a*x)/(5*c^3*(1 - a^2*x^2)^(5/2)) + (5 + 4*a*x)/(15*c^3*(1 - a^2*x^2)^(
3/2)) + (15 + 8*a*x)/(15*c^3*Sqrt[1 - a^2*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]
]/c^3
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^3}\,dx &= \frac{\int \frac{1+ax}{x\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}\\
&= \frac{1+ax}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{\int \frac{5a^2+4a^3x}{x\left(1-a^2x^2\right)^{5/2}}\,dx}{5a^2c^3}\\
&= \frac{1+ax}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{5+4ax}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{\int \frac{15a^4+8a^5x}{x\left(1-a^2x^2\right)^{3/2}}\,dx}{15a^4c^3}\\
&= \frac{1+ax}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{5+4ax}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{15+8ax}{15c^3\sqrt{1-a^2x^2}}+\frac{\int \frac{15a^6}{x\sqrt{1-a^2x^2}}\,dx}{15a^6c^3}\\
&= \frac{1+ax}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{5+4ax}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{15+8ax}{15c^3\sqrt{1-a^2x^2}}+\frac{\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c^3}\\
&= \frac{1+ax}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{5+4ax}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{15+8ax}{15c^3\sqrt{1-a^2x^2}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2c^3}\\
&= \frac{1+ax}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{5+4ax}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{15+8ax}{15c^3\sqrt{1-a^2x^2}}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2c^3}\\
&= \frac{1+ax}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{5+4ax}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{15+8ax}{15c^3\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}
\end{aligned}
$$

Mathematica [A] time = 0.0501983, size = 108, normalized size = 1.07

$$
\frac{8a^4x^4+7a^3x^3-27a^2x^2-15(ax-1)^2(ax+1)\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-8ax+23}{15c^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)^3),x]
```

```
[Out] (23 - 8*a*x - 27*a^2*x^2 + 7*a^3*x^3 + 8*a^4*x^4 - 15*(-1 + a*x)^2*(1 + a*x
)*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(15*c^3*(-1 + a*x)^2*(1 + a
*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.048, size = 384, normalized size = 3.8

$$-\frac{1}{c^3}\left(\frac{1}{4a^2}\left(\frac{1}{5a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}-\frac{2a}{5}\left(\frac{1}{3a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-2}-\frac{1}{3}\sqrt{-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/c^3*(1/4/a^2*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2/5*a*(
1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a
)^2-2*a*(x-1/a))^(1/2)))+1/8/a*(-1/3/a/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a
))^(1/2)-1/3/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))+arctanh(1/(-a^2*x^
2+1)^(1/2))-5/16/a/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)-1/2/a*(1/3/a/
(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*
a*(x-1/a))^(1/2))+11/16/a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax+1}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^3,x, algorithm="maxim
a")
```

```
[Out] -integrate((a*x + 1)/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [B] time = 1.67801, size = 419, normalized size = 4.15

$$\frac{23\,a^5x^5-23\,a^4x^4-46\,a^3x^3+46\,a^2x^2+23\,ax+15\left(a^5x^5-a^4x^4-2\,a^3x^3+2\,a^2x^2+ax-1\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(8\,a^4x^4+\right.}{15\left(a^5c^3x^5-a^4c^3x^4-2\,a^3c^3x^3+2\,a^2c^3x^2+ac^3x-c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^3,x, algorithm="frica
s")
```

```
[Out] 1/15*(23*a^5*x^5 - 23*a^4*x^4 - 46*a^3*x^3 + 46*a^2*x^2 + 23*a*x + 15*(a^5*
x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log((sqrt(-a^2*x^2 + 1) -
1)/x) - (8*a^4*x^4 + 7*a^3*x^3 - 27*a^2*x^2 - 8*a*x + 23)*sqrt(-a^2*x^2 + 1
) - 23)/(a^5*c^3*x^5 - a^4*c^3*x^4 - 2*a^3*c^3*x^3 + 2*a^2*c^3*x^2 + a*c^3*
x - c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{-a^6x^7\sqrt{-a^2x^2+1}+3a^4x^5\sqrt{-a^2x^2+1}-3a^2x^3\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(a/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2
+ 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integ
ral(1/(-a**6*x**7*sqrt(-a**2*x**2 + 1) + 3*a**4*x**5*sqrt(-a**2*x**2 + 1) -
 3*a**2*x**3*sqrt(-a**2*x**2 + 1) + x*sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{ax+1}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^3,x, algorithm="giac"
)
```

```
[Out] integrate(-(a*x + 1)/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)*x), x)
```

3.916 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=135

$$\frac{ax+1}{5c^3x\left(1-a^2x^2\right)^{5/2}}-\frac{16\sqrt{1-a^2x^2}}{5c^3x}+\frac{5ax+8}{5c^3x\sqrt{1-a^2x^2}}+\frac{5ax+6}{15c^3x\left(1-a^2x^2\right)^{3/2}}-\frac{a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

```
[Out] (1 + a*x)/(5*c^3*x*(1 - a^2*x^2)^(5/2)) + (6 + 5*a*x)/(15*c^3*x*(1 - a^2*x^
2)^(3/2)) + (8 + 5*a*x)/(5*c^3*x*Sqrt[1 - a^2*x^2]) - (16*Sqrt[1 - a^2*x^2]
)/(5*c^3*x) - (a*ArcTanh[Sqrt[1 - a^2*x^2]])/c^3
```

Rubi [A] time = 0.170999, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6148, 823, 807, 266, 63, 208}

$$\frac{ax+1}{5c^3x\left(1-a^2x^2\right)^{5/2}}-\frac{16\sqrt{1-a^2x^2}}{5c^3x}+\frac{5ax+8}{5c^3x\sqrt{1-a^2x^2}}+\frac{5ax+6}{15c^3x\left(1-a^2x^2\right)^{3/2}}-\frac{a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)^3),x]
```


```
[Out] (1 + a*x)/(5*c^3*x*(1 - a^2*x^2)^(5/2)) + (6 + 5*a*x)/(15*c^3*x*(1 - a^2*x^
2)^(3/2)) + (8 + 5*a*x)/(5*c^3*x*Sqrt[1 - a^2*x^2]) - (16*Sqrt[1 - a^2*x^2]
)/(5*c^3*x) - (a*ArcTanh[Sqrt[1 - a^2*x^2]])/c^3
```


Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^3}\,dx &= \frac{\int \frac{1+ax}{x^2\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3} \\
&= \frac{1+ax}{5c^3x\left(1-a^2x^2\right)^{5/2}}+\frac{\int \frac{6a^2+5a^3x}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx}{5a^2c^3} \\
&= \frac{1+ax}{5c^3x\left(1-a^2x^2\right)^{5/2}}+\frac{6+5ax}{15c^3x\left(1-a^2x^2\right)^{3/2}}+\frac{\int \frac{24a^4+15a^5x}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{15a^4c^3} \\
&= \frac{1+ax}{5c^3x\left(1-a^2x^2\right)^{5/2}}+\frac{6+5ax}{15c^3x\left(1-a^2x^2\right)^{3/2}}+\frac{8+5ax}{5c^3x\sqrt{1-a^2x^2}}+\frac{\int \frac{48a^6+15a^7x}{x^2\sqrt{1-a^2x^2}}\,dx}{15a^6c^3} \\
&= \frac{1+ax}{5c^3x\left(1-a^2x^2\right)^{5/2}}+\frac{6+5ax}{15c^3x\left(1-a^2x^2\right)^{3/2}}+\frac{8+5ax}{5c^3x\sqrt{1-a^2x^2}}-\frac{16\sqrt{1-a^2x^2}}{5c^3x}+\frac{a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c^3} \\
&= \frac{1+ax}{5c^3x\left(1-a^2x^2\right)^{5/2}}+\frac{6+5ax}{15c^3x\left(1-a^2x^2\right)^{3/2}}+\frac{8+5ax}{5c^3x\sqrt{1-a^2x^2}}-\frac{16\sqrt{1-a^2x^2}}{5c^3x}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a\ldots}}\ldots\right)}{2c^3} \\
&= \frac{1+ax}{5c^3x\left(1-a^2x^2\right)^{5/2}}+\frac{6+5ax}{15c^3x\left(1-a^2x^2\right)^{3/2}}+\frac{8+5ax}{5c^3x\sqrt{1-a^2x^2}}-\frac{16\sqrt{1-a^2x^2}}{5c^3x}-\frac{\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\ldots\right)}{ac\ldots} \\
&= \frac{1+ax}{5c^3x\left(1-a^2x^2\right)^{5/2}}+\frac{6+5ax}{15c^3x\left(1-a^2x^2\right)^{3/2}}+\frac{8+5ax}{5c^3x\sqrt{1-a^2x^2}}-\frac{16\sqrt{1-a^2x^2}}{5c^3x}-\frac{a\tanh^{-1}\left(\sqrt{1-a\ldots}\right)}{c^3}
\end{aligned}
$$

Mathematica [A] time = 0.0589028, size = 121, normalized size = 0.9

$$
\frac{48a^5x^5-33a^4x^4-87a^3x^3+52a^2x^2-15ax(ax-1)^2(ax+1)\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+38ax-15}{15c^3x(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)^3),x]
```

```
[Out] (-15 + 38*a*x + 52*a^2*x^2 - 87*a^3*x^3 - 33*a^4*x^4 + 48*a^5*x^5 - 15*a*x*
(-1 + a*x)^2*(1 + a*x)*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(15*c^
3*x*(-1 + a*x)^2*(1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.052, size = 314, normalized size = 2.3

$$-\frac{1}{c^3}\left(\frac{1}{x}\sqrt{-a^2x^2+1}+\frac{1}{4a}\left(\frac{1}{5a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}-\frac{2a}{5}\left(\frac{1}{3a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2a\left(x-a^{-1}\right)}\left(x-\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/c^3*((-a^2*x^2+1)^(1/2)/x+1/4/a*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-
1/a))^(1/2)-2/5*a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(
x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)))+1/24/a/(x+1/a)^2*(-a^2*(x+1/a)^
2+2*a*(x+1/a))^(1/2)+23/48/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+a*arc
tanh(1/(-a^2*x^2+1)^(1/2))-1/4/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/
2)+27/16/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))
```

Maxima [A] time = 1.05485, size = 240, normalized size = 1.78

$$-\frac{\frac{15\,a^2\log\left(\sqrt{-a^2x^2+1}+1\right)}{c^3}-\frac{15\,a^2\log\left(\sqrt{-a^2x^2+1}-1\right)}{c^3}-\frac{2\left(15\left(a^2x^2-1\right)^2a^2-5\left(a^2x^2-1\right)a^2+3\,a^2\right)}{\left(-a^2x^2+1\right)^{\frac{5}{2}}c^3}}{30\,a}+\frac{16\,a^6x^6-40\,a^4x^4+30\,a^2x^2-5}{5\left(a^4c^3x^5-2\,a^2c^3x^3+c^3x\right)\sqrt{ax+1}\sqrt{-ax+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -1/30*(15*a^2*log(sqrt(-a^2*x^2 + 1) + 1)/c^3 - 15*a^2*log(sqrt(-a^2*x^2 +
1) - 1)/c^3 - 2*(15*(a^2*x^2 - 1)^2*a^2 - 5*(a^2*x^2 - 1)*a^2 + 3*a^2)/((-a
^2*x^2 + 1)^(5/2)*c^3))/a + 1/5*(16*a^6*x^6 - 40*a^4*x^4 + 30*a^2*x^2 - 5)/
((a^4*c^3*x^5 - 2*a^2*c^3*x^3 + c^3*x)*sqrt(a*x + 1)*sqrt(-a*x + 1))
```

Fricas [A] time = 1.70131, size = 464, normalized size = 3.44

$$\frac{23\,a^6x^6-23\,a^5x^5-46\,a^4x^4+46\,a^3x^3+23\,a^2x^2-23\,ax+15\left(a^6x^6-a^5x^5-2\,a^4x^4+2\,a^3x^3+a^2x^2-ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right.}{15\left(a^5c^3x^6-a^4c^3x^5-2\,a^3c^3x^4+2\,a^2c^3x^3+ac^3x^2-c^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] 1/15*(23*a^6*x^6 - 23*a^5*x^5 - 46*a^4*x^4 + 46*a^3*x^3 + 23*a^2*x^2 - 23*a
*x + 15*(a^6*x^6 - a^5*x^5 - 2*a^4*x^4 + 2*a^3*x^3 + a^2*x^2 - a*x)*log((sq
rt(-a^2*x^2 + 1) - 1)/x) - (48*a^5*x^5 - 33*a^4*x^4 - 87*a^3*x^3 + 52*a^2*x
^2 + 38*a*x - 15)*sqrt(-a^2*x^2 + 1))/(a^5*c^3*x^6 - a^4*c^3*x^5 - 2*a^3*c^
3*x^4 + 2*a^2*c^3*x^3 + a*c^3*x^2 - c^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{-a^6x^7\sqrt{-a^2x^2+1}+3a^4x^5\sqrt{-a^2x^2+1}-3a^2x^3\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{-a^6x^8\sqrt{-a^2x^2+1}+3a^4x^6\sqrt{-a^2x^2+1}-3a^2x^4\sqrt{-a^2x^2+1}+x^2\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(a/(-a**6*x**7*sqrt(-a**2*x**2 + 1) + 3*a**4*x**5*sqrt(-a**2*x**2
+ 1) - 3*a**2*x**3*sqrt(-a**2*x**2 + 1) + x*sqrt(-a**2*x**2 + 1)), x) + Int
egral(1/(-a**6*x**8*sqrt(-a**2*x**2 + 1) + 3*a**4*x**6*sqrt(-a**2*x**2 + 1)
 - 3*a**2*x**4*sqrt(-a**2*x**2 + 1) + x**2*sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{ax+1}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)*x^2), x)
```

3.917 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=164

$$-\frac{16a\sqrt{1-a^2x^2}}{5c^3x}-\frac{7\sqrt{1-a^2x^2}}{2c^3x^2}+\frac{24ax+35}{15c^3x^2\sqrt{1-a^2x^2}}+\frac{6ax+7}{15c^3x^2\left(1-a^2x^2\right)^{3/2}}+\frac{ax+1}{5c^3x^2\left(1-a^2x^2\right)^{5/2}}-\frac{7a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^3}$$

```
[Out] (1 + a*x)/(5*c^3*x^2*(1 - a^2*x^2)^(5/2)) + (7 + 6*a*x)/(15*c^3*x^2*(1 - a^
2*x^2)^(3/2)) + (35 + 24*a*x)/(15*c^3*x^2*Sqrt[1 - a^2*x^2]) - (7*Sqrt[1 -
a^2*x^2])/(2*c^3*x^2) - (16*a*Sqrt[1 - a^2*x^2])/(5*c^3*x) - (7*a^2*ArcTanh
[Sqrt[1 - a^2*x^2]])/(2*c^3)
```

Rubi [A] time = 0.194332, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 823, 835, 807, 266, 63, 208}

$$-\frac{16a\sqrt{1-a^2x^2}}{5c^3x}-\frac{7\sqrt{1-a^2x^2}}{2c^3x^2}+\frac{24ax+35}{15c^3x^2\sqrt{1-a^2x^2}}+\frac{6ax+7}{15c^3x^2\left(1-a^2x^2\right)^{3/2}}+\frac{ax+1}{5c^3x^2\left(1-a^2x^2\right)^{5/2}}-\frac{7a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)^3),x]
```

```
[Out] (1 + a*x)/(5*c^3*x^2*(1 - a^2*x^2)^(5/2)) + (7 + 6*a*x)/(15*c^3*x^2*(1 - a^
2*x^2)^(3/2)) + (35 + 24*a*x)/(15*c^3*x^2*Sqrt[1 - a^2*x^2]) - (7*Sqrt[1 -
a^2*x^2])/(2*c^3*x^2) - (16*a*Sqrt[1 - a^2*x^2])/(5*c^3*x) - (7*a^2*ArcTanh
[Sqrt[1 - a^2*x^2]])/(2*c^3)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{1+ax}{x^3\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$

$$= \frac{1+ax}{5c^3x^2\left(1-a^2x^2\right)^{5/2}} + \frac{\int \frac{7a^2+6a^3x}{x^3\left(1-a^2x^2\right)^{5/2}}\,dx}{5a^2c^3}$$

$$= \frac{1+ax}{5c^3x^2\left(1-a^2x^2\right)^{5/2}} + \frac{7+6ax}{15c^3x^2\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{35a^4+24a^5x}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx}{15a^4c^3}$$

$$= \frac{1+ax}{5c^3x^2\left(1-a^2x^2\right)^{5/2}} + \frac{7+6ax}{15c^3x^2\left(1-a^2x^2\right)^{3/2}} + \frac{35+24ax}{15c^3x^2\sqrt{1-a^2x^2}} + \frac{\int \frac{105a^6+48a^7x}{x^3\sqrt{1-a^2x^2}}\,dx}{15a^6c^3}$$

$$= \frac{1+ax}{5c^3x^2\left(1-a^2x^2\right)^{5/2}} + \frac{7+6ax}{15c^3x^2\left(1-a^2x^2\right)^{3/2}} + \frac{35+24ax}{15c^3x^2\sqrt{1-a^2x^2}} - \frac{7\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{\int \frac{-96a^7-105a^8x}{x^2\sqrt{1-a^2x^2}}\,dx}{30a^6c^3}$$

$$= \frac{1+ax}{5c^3x^2\left(1-a^2x^2\right)^{5/2}} + \frac{7+6ax}{15c^3x^2\left(1-a^2x^2\right)^{3/2}} + \frac{35+24ax}{15c^3x^2\sqrt{1-a^2x^2}} - \frac{7\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{16a\sqrt{1-a^2x^2}}{5c^3x} + \cdots$$

$$= \frac{1+ax}{5c^3x^2\left(1-a^2x^2\right)^{5/2}} + \frac{7+6ax}{15c^3x^2\left(1-a^2x^2\right)^{3/2}} + \frac{35+24ax}{15c^3x^2\sqrt{1-a^2x^2}} - \frac{7\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{16a\sqrt{1-a^2x^2}}{5c^3x} + \cdots$$

$$= \frac{1+ax}{5c^3x^2\left(1-a^2x^2\right)^{5/2}} + \frac{7+6ax}{15c^3x^2\left(1-a^2x^2\right)^{3/2}} + \frac{35+24ax}{15c^3x^2\sqrt{1-a^2x^2}} - \frac{7\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{16a\sqrt{1-a^2x^2}}{5c^3x} - \cdots$$

$$= \frac{1+ax}{5c^3x^2\left(1-a^2x^2\right)^{5/2}} + \frac{7+6ax}{15c^3x^2\left(1-a^2x^2\right)^{3/2}} + \frac{35+24ax}{15c^3x^2\sqrt{1-a^2x^2}} - \frac{7\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{16a\sqrt{1-a^2x^2}}{5c^3x} - \cdots$$

Mathematica [A] time = 0.0671797, size = 133, normalized size = 0.81

$$\frac{96a^6x^6+9a^5x^5-249a^4x^4+4a^3x^3+176a^2x^2-105a^2x^2(ax-1)^2(ax+1)\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-15ax-15}{30c^3x^2(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)^3),x]
```

```
[Out] (-15 - 15*a*x + 176*a^2*x^2 + 4*a^3*x^3 - 249*a^4*x^4 + 9*a^5*x^5 + 96*a^6*
x^6 - 105*a^2*x^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 -
 a^2*x^2]])/(30*c^3*x^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.052, size = 326, normalized size = 2.

$$-\frac{1}{c^3}\left(\frac{a}{x}\sqrt{-a^2x^2+1}+\frac{1}{20\,a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-3}-\frac{11\,a}{10}\left(\frac{1}{3\,a}\sqrt{-a^2\left(x-a^{-1}\right)^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/c^3*(a*(-a^2*x^2+1)^(1/2)/x+1/20/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a)
)^(1/2)-11/10*a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-
1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))+1/8*a*(-1/3/a/(x+1/a)^2*(-a^2*(x+1
/a)^2+2*a*(x+1/a))^(1/2)-1/3/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))+7/
2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))-9/16*a/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1
/a))^(1/2)+39/16*a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1/2*(-a^2*x^2
+1)^(1/2)/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax+1}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [A] time = 1.62287, size = 505, normalized size = 3.08

$$\frac{116\,a^7x^7-116\,a^6x^6-232\,a^5x^5+232\,a^4x^4+116\,a^3x^3-116\,a^2x^2+105\left(a^7x^7-a^6x^6-2\,a^5x^5+2\,a^4x^4+a^3x^3-a^2x^2\right)1}{30\left(a^5c^3x^7-a^4c^3x^6-2\,a^3c^3x^5+2\,a^2c^3x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] 1/30*(116*a^7*x^7 - 116*a^6*x^6 - 232*a^5*x^5 + 232*a^4*x^4 + 116*a^3*x^3 -
 116*a^2*x^2 + 105*(a^7*x^7 - a^6*x^6 - 2*a^5*x^5 + 2*a^4*x^4 + a^3*x^3 - a
^2*x^2)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (96*a^6*x^6 + 9*a^5*x^5 - 249*a^4
*x^4 + 4*a^3*x^3 + 176*a^2*x^2 - 15*a*x - 15)*sqrt(-a^2*x^2 + 1))/(a^5*c^3*
x^7 - a^4*c^3*x^6 - 2*a^3*c^3*x^5 + 2*a^2*c^3*x^4 + a*c^3*x^3 - c^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{-a^6x^8\sqrt{-a^2x^2+1}+3a^4x^6\sqrt{-a^2x^2+1}-3a^2x^4\sqrt{-a^2x^2+1}+x^2\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{-a^6x^9\sqrt{-a^2x^2+1}+3a^4x^7\sqrt{-a^2x^2+1}-3a^2x^5\sqrt{-a^2x^2+1}+x^3\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(a/(-a**6*x**8*sqrt(-a**2*x**2 + 1) + 3*a**4*x**6*sqrt(-a**2*x**2
+ 1) - 3*a**2*x**4*sqrt(-a**2*x**2 + 1) + x**2*sqrt(-a**2*x**2 + 1)), x) +
```

```
Integral(1/(-a**6*x**9*sqrt(-a**2*x**2 + 1) + 3*a**4*x**7*sqrt(-a**2*x**2 +
 1) - 3*a**2*x**5*sqrt(-a**2*x**2 + 1) + x**3*sqrt(-a**2*x**2 + 1)), x))/c*
*3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{ax+1}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)*x^3), x)
```

3.918 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-a^2cx^2)^4} dx$

Optimal. Leaf size=96

$$\frac{16x}{35c^4\sqrt{1-a^2x^2}}+\frac{8x}{35c^4(1-a^2x^2)^{3/2}}+\frac{6x}{35c^4(1-a^2x^2)^{5/2}}+\frac{ax+1}{7ac^4(1-a^2x^2)^{7/2}}$$

```
[Out] (1 + a*x)/(7*a*c^4*(1 - a^2*x^2)^(7/2)) + (6*x)/(35*c^4*(1 - a^2*x^2)^(5/2)
) + (8*x)/(35*c^4*(1 - a^2*x^2)^(3/2)) + (16*x)/(35*c^4*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.053146, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6138, 639, 192, 191}

$$\frac{16x}{35c^4\sqrt{1-a^2x^2}}+\frac{8x}{35c^4(1-a^2x^2)^{3/2}}+\frac{6x}{35c^4(1-a^2x^2)^{5/2}}+\frac{ax+1}{7ac^4(1-a^2x^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a^2*c*x^2)^4,x]
```

```
[Out] (1 + a*x)/(7*a*c^4*(1 - a^2*x^2)^(7/2)) + (6*x)/(35*c^4*(1 - a^2*x^2)^(5/2)
) + (8*x)/(35*c^4*(1 - a^2*x^2)^(3/2)) + (16*x)/(35*c^4*Sqrt[1 - a^2*x^2])
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{\int \frac{1+ax}{(1-a^2x^2)^{9/2}}\,dx}{c^4}$$

$$= \frac{1+ax}{7ac^4(1-a^2x^2)^{7/2}} + \frac{6\int \frac{1}{(1-a^2x^2)^{7/2}}\,dx}{7c^4}$$

$$= \frac{1+ax}{7ac^4(1-a^2x^2)^{7/2}} + \frac{6x}{35c^4(1-a^2x^2)^{5/2}} + \frac{24\int \frac{1}{(1-a^2x^2)^{5/2}}\,dx}{35c^4}$$

$$= \frac{1+ax}{7ac^4(1-a^2x^2)^{7/2}} + \frac{6x}{35c^4(1-a^2x^2)^{5/2}} + \frac{8x}{35c^4(1-a^2x^2)^{3/2}} + \frac{16\int \frac{1}{(1-a^2x^2)^{3/2}}\,dx}{35c^4}$$

$$= \frac{1+ax}{7ac^4(1-a^2x^2)^{7/2}} + \frac{6x}{35c^4(1-a^2x^2)^{5/2}} + \frac{8x}{35c^4(1-a^2x^2)^{3/2}} + \frac{16x}{35c^4\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0332737, size = 75, normalized size = 0.78

$$\frac{-16a^6x^6+16a^5x^5+40a^4x^4-40a^3x^3-30a^2x^2+30ax+5}{35ac^4(1-ax)^{7/2}(ax+1)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a^2*c*x^2)^4,x]
```

```
[Out] (5 + 30*a*x - 30*a^2*x^2 - 40*a^3*x^3 + 40*a^4*x^4 + 16*a^5*x^5 - 16*a^6*x^
6)/(35*a*c^4*(1 - a*x)^(7/2)*(1 + a*x)^(5/2))
```

Maple [A] time = 0.032, size = 74, normalized size = 0.8

$$\frac{16\,x^6a^6-16\,x^5a^5-40\,x^4a^4+40\,x^3a^3+30\,a^2x^2-30\,ax-5}{(35\,ax-35)\,c^4a}\left(-a^2x^2+1\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] 1/35*(16*a^6*x^6-16*a^5*x^5-40*a^4*x^4+40*a^3*x^3+30*a^2*x^2-30*a*x-5)/(a*x
-1)/c^4/(-a^2*x^2+1)^(5/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{(a^2cx^2-c)^4\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^4*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [B] time = 1.78667, size = 412, normalized size = 4.29

$$\frac{5\,a^7x^7 - 5\,a^6x^6 - 15\,a^5x^5 + 15\,a^4x^4 + 15\,a^3x^3 - 15\,a^2x^2 - 5\,ax - \left(16\,a^6x^6 - 16\,a^5x^5 - 40\,a^4x^4 + 40\,a^3x^3 + 30\,a^2x^2 - 30}{35\left(a^8c^4x^7 - a^7c^4x^6 - 3\,a^6c^4x^5 + 3\,a^5c^4x^4 + 3\,a^4c^4x^3 - 3\,a^3c^4x^2 - a^2c^4x + ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas"
)
```

```
[Out] 1/35*(5*a^7*x^7 - 5*a^6*x^6 - 15*a^5*x^5 + 15*a^4*x^4 + 15*a^3*x^3 - 15*a^2
*x^2 - 5*a*x - (16*a^6*x^6 - 16*a^5*x^5 - 40*a^4*x^4 + 40*a^3*x^3 + 30*a^2*
x^2 - 30*a*x - 5)*sqrt(-a^2*x^2 + 1) + 5)/(a^8*c^4*x^7 - a^7*c^4*x^6 - 3*a^
6*c^4*x^5 + 3*a^5*c^4*x^4 + 3*a^4*c^4*x^3 - 3*a^3*c^4*x^2 - a^2*c^4*x + a*c
^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{a^8x^8\sqrt{-a^2x^2+1}-4a^6x^6\sqrt{-a^2x^2+1}+6a^4x^4\sqrt{-a^2x^2+1}-4a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx + \int \frac{1}{a^8x^8\sqrt{-a^2x^2+1}-4a^6x^6\sqrt{-a^2x^2+1}+6a^4x^4\sqrt{-a^2x^2+1}-4a^2x^2\sqrt{}}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (Integral(a*x/(a**8*x**8*sqrt(-a**2*x**2 + 1) - 4*a**6*x**6*sqrt(-a**2*x**2
 + 1) + 6*a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**2*x**2*sqrt(-a**2*x**2 + 1)
 + sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**8*x**8*sqrt(-a**2*x**2 + 1) -
 4*a**6*x**6*sqrt(-a**2*x**2 + 1) + 6*a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a*
*2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax + 1}{\left(a^2cx^2 - c\right)^4\sqrt{-a^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^4*sqrt(-a^2*x^2 + 1)), x)
```

3.919 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-a^2cx^2)^5} dx$

Optimal. Leaf size=118

$$\frac{128x}{315c^5\sqrt{1-a^2x^2}}+\frac{64x}{315c^5\left(1-a^2x^2\right)^{3/2}}+\frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}}+\frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}}+\frac{ax+1}{9ac^5\left(1-a^2x^2\right)^{9/2}}$$

```
[Out] (1 + a*x)/(9*a*c^5*(1 - a^2*x^2)^(9/2)) + (8*x)/(63*c^5*(1 - a^2*x^2)^(7/2)
) + (16*x)/(105*c^5*(1 - a^2*x^2)^(5/2)) + (64*x)/(315*c^5*(1 - a^2*x^2)^(3
/2)) + (128*x)/(315*c^5*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0641603, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6138, 639, 192, 191}

$$\frac{128x}{315c^5\sqrt{1-a^2x^2}}+\frac{64x}{315c^5\left(1-a^2x^2\right)^{3/2}}+\frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}}+\frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}}+\frac{ax+1}{9ac^5\left(1-a^2x^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a^2*c*x^2)^5,x]
```

```
[Out] (1 + a*x)/(9*a*c^5*(1 - a^2*x^2)^(9/2)) + (8*x)/(63*c^5*(1 - a^2*x^2)^(7/2)
) + (16*x)/(105*c^5*(1 - a^2*x^2)^(5/2)) + (64*x)/(315*c^5*(1 - a^2*x^2)^(3
/2)) + (128*x)/(315*c^5*Sqrt[1 - a^2*x^2])
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^5}\,dx = \frac{\int \frac{1+ax}{\left(1-a^2x^2\right)^{11/2}}\,dx}{c^5}$$

$$= \frac{1+ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8\int \frac{1}{\left(1-a^2x^2\right)^{9/2}}\,dx}{9c^5}$$

$$= \frac{1+ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} + \frac{16\int \frac{1}{\left(1-a^2x^2\right)^{7/2}}\,dx}{21c^5}$$

$$= \frac{1+ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} + \frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}} + \frac{64\int \frac{1}{\left(1-a^2x^2\right)^{5/2}}\,dx}{105c^5}$$

$$= \frac{1+ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} + \frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}} + \frac{64x}{315c^5\left(1-a^2x^2\right)^{3/2}} + \frac{128\int \frac{1}{(1-a^2}}{315}$$

$$= \frac{1+ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} + \frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}} + \frac{64x}{315c^5\left(1-a^2x^2\right)^{3/2}} + \frac{128x}{315c^5\sqrt{1-}}$$

Mathematica [A] time = 0.0314079, size = 91, normalized size = 0.77

$$\frac{128a^8x^8-128a^7x^7-448a^6x^6+448a^5x^5+560a^4x^4-560a^3x^3-280a^2x^2+280ax+35}{315ac^5(1-ax)^{9/2}(ax+1)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a^2*c*x^2)^5,x]
```

```
[Out] (35 + 280*a*x - 280*a^2*x^2 - 560*a^3*x^3 + 560*a^4*x^4 + 448*a^5*x^5 - 448
*a^6*x^6 - 128*a^7*x^7 + 128*a^8*x^8)/(315*a*c^5*(1 - a*x)^(9/2)*(1 + a*x)^
(7/2))
```

Maple [A] time = 0.032, size = 90, normalized size = 0.8

$$-\frac{128\,a^8x^8-128\,a^7x^7-448\,x^6a^6+448\,x^5a^5+560\,x^4a^4-560\,x^3a^3-280\,a^2x^2+280\,ax+35}{(315\,ax-315)\,c^5a}\left(-a^2x^2+1\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^5,x)
```

```
[Out] -1/315*(128*a^8*x^8-128*a^7*x^7-448*a^6*x^6+448*a^5*x^5+560*a^4*x^4-560*a^3
*x^3-280*a^2*x^2+280*a*x+35)/(a*x-1)/c^5/(-a^2*x^2+1)^(7/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax+1}{\left(a^2cx^2-c\right)^5\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^5,x, algorithm="maxima"
)
```

```
[Out] -integrate((a*x + 1)/((a^2*c*x^2 - c)^5*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [B] time = 2.02386, size = 554, normalized size = 4.69

$$\frac{35\,a^9x^9 - 35\,a^8x^8 - 140\,a^7x^7 + 140\,a^6x^6 + 210\,a^5x^5 - 210\,a^4x^4 - 140\,a^3x^3 + 140\,a^2x^2 + 35\,ax - \big(128\,a^8x^8 - 128\,a^7x^7 - 44}{315\,\big(a^{10}c^5x^9 - a^9c^5x^8 - 4\,a^8c^5x^7 + 4\,a^7c^5x^6 + 6\,a^6c^5x^5 - 6\,a^5c^5x^4 - 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^5,x, algorithm="fricas"
)
```

```
[Out] 1/315*(35*a^9*x^9 - 35*a^8*x^8 - 140*a^7*x^7 + 140*a^6*x^6 + 210*a^5*x^5 -
210*a^4*x^4 - 140*a^3*x^3 + 140*a^2*x^2 + 35*a*x - (128*a^8*x^8 - 128*a^7*x
^7 - 448*a^6*x^6 + 448*a^5*x^5 + 560*a^4*x^4 - 560*a^3*x^3 - 280*a^2*x^2 +
280*a*x + 35)*sqrt(-a^2*x^2 + 1) - 35)/(a^10*c^5*x^9 - a^9*c^5*x^8 - 4*a^8*
c^5*x^7 + 4*a^7*c^5*x^6 + 6*a^6*c^5*x^5 - 6*a^5*c^5*x^4 - 4*a^4*c^5*x^3 + 4
*a^3*c^5*x^2 + a^2*c^5*x - a*c^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{-a^{10}x^{10}\sqrt{-a^2x^2+1}+5a^8x^8\sqrt{-a^2x^2+1}-10a^6x^6\sqrt{-a^2x^2+1}+10a^4x^4\sqrt{-a^2x^2+1}-5a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx + \int \frac{}{-a^{10}x^{10}\sqrt{-a^2x^2+1}+5a^8x^8\sqrt{-a^2x^2+1}-10}}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**5,x)
```

```
[Out] (Integral(a*x/(-a**10*x**10*sqrt(-a**2*x**2 + 1) + 5*a**8*x**8*sqrt(-a**2*x
**2 + 1) - 10*a**6*x**6*sqrt(-a**2*x**2 + 1) + 10*a**4*x**4*sqrt(-a**2*x**2
 + 1) - 5*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Inte
gral(1/(-a**10*x**10*sqrt(-a**2*x**2 + 1) + 5*a**8*x**8*sqrt(-a**2*x**2 + 1
) - 10*a**6*x**6*sqrt(-a**2*x**2 + 1) + 10*a**4*x**4*sqrt(-a**2*x**2 + 1) -
 5*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{ax + 1}{\left(a^2cx^2 - c\right)^5\sqrt{-a^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^5,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)/((a^2*c*x^2 - c)^5*sqrt(-a^2*x^2 + 1)), x)
```

3.920 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{x^3}{3a^2}-\frac{x^2}{2a^3}-\frac{x}{a^4}-\frac{\log(1-ax)}{a^5}-\frac{x^4}{4a}$$

```
[Out] -(x/a^4) - x^2/(2*a^3) - x^3/(3*a^2) - x^4/(4*a) - Log[1 - a*x]/a^5
```

Rubi [A] time = 0.0999109, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 43}

$$-\frac{x^3}{3a^2}-\frac{x^2}{2a^3}-\frac{x}{a^4}-\frac{\log(1-ax)}{a^5}-\frac{x^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x/a^4) - x^2/(2*a^3) - x^3/(3*a^2) - x^4/(4*a) - Log[1 - a*x]/a^5
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^4}{\sqrt{1-a^2x^2}}\,dx &= \int \frac{x^4}{1-ax}\,dx\\ &= \int \left(-\frac{1}{a^4}-\frac{x}{a^3}-\frac{x^2}{a^2}-\frac{x^3}{a}-\frac{1}{a^4(-1+ax)}\right)dx\\ &= -\frac{x}{a^4}-\frac{x^2}{2a^3}-\frac{x^3}{3a^2}-\frac{x^4}{4a}-\frac{\log(1-ax)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.024949, size = 49, normalized size = 1.

$$-\frac{x^3}{3a^2}-\frac{x^2}{2a^3}-\frac{x}{a^4}-\frac{\log(1-ax)}{a^5}-\frac{x^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x/a^4) - x^2/(2*a^3) - x^3/(3*a^2) - x^4/(4*a) - Log[1 - a*x]/a^5
```

Maple [A] time = 0.029, size = 43, normalized size = 0.9

$$-\frac{x^4}{4\,a}-\frac{x^3}{3\,a^2}-\frac{x^2}{2\,a^3}-\frac{x}{a^4}-\frac{\ln\left(ax-1\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)*x^4,x)
```

```
[Out] -1/4*x^4/a-1/3*x^3/a^2-1/2*x^2/a^3-x/a^4-1/a^5*ln(a*x-1)
```

Maxima [A] time = 0.9511, size = 58, normalized size = 1.18

$$-\frac{3\,a^3x^4+4\,a^2x^3+6\,ax^2+12\,x}{12\,a^4}-\frac{\log\left(ax-1\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^4,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^3*x^4 + 4*a^2*x^3 + 6*a*x^2 + 12*x)/a^4 - log(a*x - 1)/a^5
```

Fricas [A] time = 1.45332, size = 100, normalized size = 2.04

$$-\frac{3\,a^4x^4+4\,a^3x^3+6\,a^2x^2+12\,ax+12\,\log\left(ax-1\right)}{12\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*x^4 + 4*a^3*x^3 + 6*a^2*x^2 + 12*a*x + 12*log(a*x - 1))/a^5
```

Sympy [A] time = 0.263434, size = 39, normalized size = 0.8

$$-\frac{x^4}{4a}-\frac{x^3}{3a^2}-\frac{x^2}{2a^3}-\frac{x}{a^4}-\frac{\log\left(ax-1\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)*x**4,x)
```

```
[Out] -x**4/(4*a) - x**3/(3*a**2) - x**2/(2*a**3) - x/a**4 - log(a*x - 1)/a**5
```

Giac [A] time = 1.21649, size = 59, normalized size = 1.2

$$-\frac{3\,a^3x^4+4\,a^2x^3+6\,ax^2+12\,x}{12\,a^4}-\frac{\log\left(|ax-1|\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^4,x, algorithm="giac")
```

```
[Out] -1/12*(3*a^3*x^4 + 4*a^2*x^3 + 6*a*x^2 + 12*x)/a^4 - log(abs(a*x - 1))/a^5
```

3.921 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=39

$$-\frac{x^2}{2a^2}-\frac{x}{a^3}-\frac{\log(1-ax)}{a^4}-\frac{x^3}{3a}$$

```
[Out] -(x/a^3) - x^2/(2*a^2) - x^3/(3*a) - Log[1 - a*x]/a^4
```

Rubi [A] time = 0.0960698, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 43}

$$-\frac{x^2}{2a^2}-\frac{x}{a^3}-\frac{\log(1-ax)}{a^4}-\frac{x^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x/a^3) - x^2/(2*a^2) - x^3/(3*a) - Log[1 - a*x]/a^4
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^3}{\sqrt{1-a^2x^2}}\,dx &= \int \frac{x^3}{1-ax}\,dx\\
&= \int \left(-\frac{1}{a^3}-\frac{x}{a^2}-\frac{x^2}{a}-\frac{1}{a^3(-1+ax)}\right)dx\\
&= -\frac{x}{a^3}-\frac{x^2}{2a^2}-\frac{x^3}{3a}-\frac{\log(1-ax)}{a^4}
\end{aligned}$$

Mathematica [A] time = 0.019688, size = 39, normalized size = 1.

$$-\frac{x^2}{2a^2}-\frac{x}{a^3}-\frac{\log(1-ax)}{a^4}-\frac{x^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x/a^3) - x^2/(2*a^2) - x^3/(3*a) - Log[1 - a*x]/a^4
```

Maple [A] time = 0.026, size = 35, normalized size = 0.9

$$-\frac{x^3}{3\,a}-\frac{x^2}{2\,a^2}-\frac{x}{a^3}-\frac{\ln\left(ax-1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)*x^3,x)
```

```
[Out] -1/3*x^3/a-1/2*x^2/a^2-x/a^3-1/a^4*ln(a*x-1)
```

Maxima [A] time = 0.951527, size = 47, normalized size = 1.21

$$-\frac{2\,a^2x^3+3\,ax^2+6\,x}{6\,a^3}-\frac{\log\left(ax-1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^2*x^3 + 3*a*x^2 + 6*x)/a^3 - log(a*x - 1)/a^4
```

Fricas [A] time = 1.48202, size = 80, normalized size = 2.05

$$-\frac{2\,a^3x^3+3\,a^2x^2+6\,ax+6\,\log\left(ax-1\right)}{6\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^3,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a^3*x^3 + 3*a^2*x^2 + 6*a*x + 6*log(a*x - 1))/a^4
```

Sympy [A] time = 0.25716, size = 31, normalized size = 0.79

$$-\frac{x^3}{3a}-\frac{x^2}{2a^2}-\frac{x}{a^3}-\frac{\log\left(ax-1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)*x**3,x)
```

```
[Out] -x**3/(3*a) - x**2/(2*a**2) - x/a**3 - log(a*x - 1)/a**4
```

Giac [A] time = 1.13088, size = 49, normalized size = 1.26

$$-\frac{2\,a^2x^3+3\,ax^2+6\,x}{6\,a^3}-\frac{\log\left(|ax-1|\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^3,x, algorithm="giac")
```

```
[Out] -1/6*(2*a^2*x^3 + 3*a*x^2 + 6*x)/a^3 - log(abs(a*x - 1))/a^4
```

3.922 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{x}{a^2}-\frac{\log(1-ax)}{a^3}-\frac{x^2}{2a}$$

```
[Out] -(x/a^2) - x^2/(2*a) - Log[1 - a*x]/a^3
```

Rubi [A] time = 0.0906503, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 43}

$$-\frac{x}{a^2}-\frac{\log(1-ax)}{a^3}-\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x/a^2) - x^2/(2*a) - Log[1 - a*x]/a^3
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^2}{\sqrt{1-a^2x^2}}\,dx &= \int \frac{x^2}{1-ax}\,dx\\ &= \int \left(-\frac{1}{a^2}-\frac{x}{a}-\frac{1}{a^2(-1+ax)}\right)dx\\ &= -\frac{x}{a^2}-\frac{x^2}{2a}-\frac{\log(1-ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.0164728, size = 29, normalized size = 1.

$$-\frac{x}{a^2}-\frac{\log(1-ax)}{a^3}-\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x/a^2) - x^2/(2*a) - Log[1 - a*x]/a^3
```

Maple [A] time = 0.026, size = 27, normalized size = 0.9

$$-\frac{x^2}{2\,a}-\frac{x}{a^2}-\frac{\ln(ax-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)*x^2,x)
```

```
[Out] -1/2*x^2/a-x/a^2-1/a^3*ln(a*x-1)
```

Maxima [A] time = 0.942624, size = 35, normalized size = 1.21

$$-\frac{ax^2+2\,x}{2\,a^2}-\frac{\log(ax-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*x^2 + 2*x)/a^2 - log(a*x - 1)/a^3
```

Fricas [A] time = 1.49826, size = 61, normalized size = 2.1

$$-\frac{a^2x^2+2\,ax+2\,\log(ax-1)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*x^2 + 2*a*x + 2*log(a*x - 1))/a^3
```

Sympy [A] time = 0.25169, size = 22, normalized size = 0.76

$$-\frac{x^2}{2a}-\frac{x}{a^2}-\frac{\log(ax-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)*x**2,x)
```

```
[Out] -x**2/(2*a) - x/a**2 - log(a*x - 1)/a**3
```

Giac [A] time = 1.15345, size = 36, normalized size = 1.24

$$-\frac{ax^2+2x}{2a^2}-\frac{\log(|ax-1|)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*x^2 + 2*x)/a^2 - log(abs(a*x - 1))/a^3
```

3.923 $\int \frac{e^{\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{\log(1-ax)}{a^2}-\frac{x}{a}$$

```
[Out] -(x/a) - Log[1 - a*x]/a^2
```

Rubi [A] time = 0.0622626, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6150, 43}

$$-\frac{\log(1-ax)}{a^2}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x/a) - Log[1 - a*x]/a^2
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx &= \int \frac{x}{1-ax}\,dx\\ &= \int \left(-\frac{1}{a}-\frac{1}{a(-1+ax)}\right)dx\\ &= -\frac{x}{a}-\frac{\log(1-ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0129541, size = 19, normalized size = 1.

$$-\frac{\log(1-ax)}{a^2}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x/a) - Log[1 - a*x]/a^2
```

Maple [A] time = 0.026, size = 19, normalized size = 1.

$$-\frac{x}{a} - \frac{\ln(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)*x,x)
```

```
[Out] -x/a-1/a^2*ln(a*x-1)
```

Maxima [A] time = 0.944947, size = 24, normalized size = 1.26

$$-\frac{x}{a} - \frac{\log(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x,x, algorithm="maxima")
```

```
[Out] -x/a - log(a*x - 1)/a^2
```

Fricas [A] time = 1.71153, size = 36, normalized size = 1.89

$$-\frac{ax + \log(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x,x, algorithm="fricas")
```

```
[Out] -(a*x + log(a*x - 1))/a^2
```

Sympy [A] time = 0.247795, size = 14, normalized size = 0.74

$$-\frac{x}{a} - \frac{\log(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)*x,x)
```

```
[Out] -x/a - log(a*x - 1)/a**2
```

Giac [A] time = 1.17948, size = 26, normalized size = 1.37

$$-\frac{x}{a} - \frac{\log(|ax-1|)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x,x, algorithm="giac")
```

```
[Out] -x/a - log(abs(a*x - 1))/a^2
```

3.924 $\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=12

$$-\frac{\log(1-ax)}{a}$$

[Out] -(Log[1 - a*x]/a)

Rubi [A] time = 0.0341307, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6140, 31}

$$-\frac{\log(1-ax)}{a}$$

Antiderivative was successfully verified.

[In] Int[E^ArcTanh[a*x]/Sqrt[1 - a^2*x^2],x]

[Out] -(Log[1 - a*x]/a)

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}} dx &= \int \frac{1}{1-ax} dx\\ &= -\frac{\log(1-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0047174, size = 12, normalized size = 1.

$$-\frac{\log(1-ax)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[E^ArcTanh[a*x]/Sqrt[1 - a^2*x^2],x]

[Out] -(Log[1 - a*x]/a)

Maple [A] time = 0.026, size = 12, normalized size = 1.

$$-\frac{\ln (ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1),x)
```

```
[Out] -1/a*ln(a*x-1)
```

Maxima [A] time = 0.962576, size = 15, normalized size = 1.25

$$-\frac{\log (ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -log(a*x - 1)/a
```

Fricas [A] time = 1.68061, size = 23, normalized size = 1.92

$$-\frac{\log (ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -log(a*x - 1)/a
```

Sympy [A] time = 0.062884, size = 8, normalized size = 0.67

$$-\frac{\log (ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1),x)
```

```
[Out] -log(a*x - 1)/a
```

Giac [A] time = 1.15186, size = 16, normalized size = 1.33

$$-\frac{\log (|ax-1|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -log(abs(a*x - 1))/a
```

3.925 $\int \frac{e^{\tanh^{-1}(ax)}}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=12

$$\log(x)-\log(1-ax)$$

```
[Out] Log[x] - Log[1 - a*x]
```

Rubi [A] time = 0.0795024, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6150, 36, 29, 31}

$$\log(x)-\log(1-ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Log[x] - Log[1 - a*x]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{x\sqrt{1-a^2x^2}}\,dx &= \int \frac{1}{x(1-ax)}\,dx\\ &= a\int \frac{1}{1-ax}\,dx+\int \frac{1}{x}\,dx\\ &= \log(x)-\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.006461, size = 12, normalized size = 1.

$$\log(x)-\log(1-ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Log[x] - Log[1 - a*x]
```

Maple [A] time = 0.032, size = 12, normalized size = 1.

$$\ln(x) - \ln(ax - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)/x,x)
```

```
[Out] ln(x)-ln(a*x-1)
```

Maxima [A] time = 0.95007, size = 15, normalized size = 1.25

$$-\log(ax - 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x,x, algorithm="maxima")
```

```
[Out] -log(a*x - 1) + log(x)
```

Fricas [A] time = 1.76315, size = 32, normalized size = 2.67

$$-\log(ax - 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x,x, algorithm="fricas")
```

```
[Out] -log(a*x - 1) + log(x)
```

Sympy [A] time = 0.111181, size = 8, normalized size = 0.67

$$\log(x) - \log\left(x - \frac{1}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)/x,x)
```

```
[Out] log(x) - log(x - 1/a)
```

Giac [A] time = 1.15031, size = 18, normalized size = 1.5

$$-\log(|ax-1|)+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x,x, algorithm="giac")
```

```
[Out] -log(abs(a*x - 1)) + log(abs(x))
```

3.926 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=20

$$a\log(x) - a\log(1-ax) - \frac{1}{x}$$

```
[Out] -x^(-1) + a*Log[x] - a*Log[1 - a*x]
```

Rubi [A] time = 0.0874084, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 44}

$$a\log(x) - a\log(1-ax) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -x^(-1) + a*Log[x] - a*Log[1 - a*x]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{x^2\sqrt{1-a^2x^2}}\,dx &= \int \frac{1}{x^2(1-ax)}\,dx\\ &= \int \left(\frac{1}{x^2}+\frac{a}{x}-\frac{a^2}{-1+ax}\right)dx\\ &= -\frac{1}{x}+a\log(x)-a\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.012582, size = 20, normalized size = 1.

$$a\log(x) - a\log(1-ax) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -x^(-1) + a*Log[x] - a*Log[1 - a*x]
```

Maple [A] time = 0.033, size = 20, normalized size = 1.

$$-x^{-1} + a\ln(x) - a\ln(ax - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)/x^2,x)
```

```
[Out] -1/x+a*ln(x)-a*ln(a*x-1)
```

Maxima [A] time = 0.942688, size = 26, normalized size = 1.3

$$-a\log(ax - 1) + a\log(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^2,x, algorithm="maxima")
```

```
[Out] -a*log(a*x - 1) + a*log(x) - 1/x
```

Fricas [A] time = 1.7383, size = 54, normalized size = 2.7

$$-\frac{ax\log(ax - 1) - ax\log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*x*log(a*x - 1) - a*x*log(x) + 1)/x
```

Sympy [A] time = 0.137859, size = 15, normalized size = 0.75

$$-a\left(-\log(x) + \log\left(x - \frac{1}{a}\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)/x**2,x)
```

```
[Out] -a*(-log(x) + log(x - 1/a)) - 1/x
```

Giac [A] time = 1.15307, size = 28, normalized size = 1.4

$$-a\log(|ax - 1|) + a\log(|x|) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^2,x, algorithm="giac")
```

```
[Out] -a*log(abs(a*x - 1)) + a*log(abs(x)) - 1/x
```

3.927 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=32

$$a^2\log(x)-a^2\log(1-ax)-\frac{a}{x}-\frac{1}{2x^2}$$

```
[Out] -1/(2*x^2) - a/x + a^2*Log[x] - a^2*Log[1 - a*x]
```

Rubi [A] time = 0.090373, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 44}

$$a^2\log(x)-a^2\log(1-ax)-\frac{a}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -1/(2*x^2) - a/x + a^2*Log[x] - a^2*Log[1 - a*x]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^3\sqrt{1-a^2x^2}}\,dx &= \int \frac{1}{x^3(1-ax)}\,dx\\
&= \int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{a^2}{x}-\frac{a^3}{-1+ax}\right)dx\\
&= -\frac{1}{2x^2}-\frac{a}{x}+a^2\log(x)-a^2\log(1-ax)
\end{aligned}$$

Mathematica [A] time = 0.0136628, size = 32, normalized size = 1.

$$a^2\log(x)-a^2\log(1-ax)-\frac{a}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -1/(2*x^2) - a/x + a^2*Log[x] - a^2*Log[1 - a*x]
```

Maple [A] time = 0.031, size = 30, normalized size = 0.9

$$-\frac{1}{2x^2}-\frac{a}{x}+a^2\ln(x)-a^2\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)/x^3,x)
```

```
[Out] -1/2/x^2-a/x+a^2*ln(x)-a^2*ln(a*x-1)
```

Maxima [A] time = 0.955309, size = 39, normalized size = 1.22

$$-a^2\log(ax-1)+a^2\log(x)-\frac{2ax+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^3,x, algorithm="maxima")
```

```
[Out] -a^2*log(a*x - 1) + a^2*log(x) - 1/2*(2*a*x + 1)/x^2
```

Fricas [A] time = 1.67278, size = 89, normalized size = 2.78

$$-\frac{2a^2x^2\log(ax-1)-2a^2x^2\log(x)+2ax+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*x^2*log(a*x - 1) - 2*a^2*x^2*log(x) + 2*a*x + 1)/x^2
```

Sympy [A] time = 0.311324, size = 26, normalized size = 0.81

$$-a^2\left(-\log(x)+\log\left(x-\frac{1}{a}\right)\right)-\frac{2ax+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)/x**3,x)
```

```
[Out] -a**2*(-log(x) + log(x - 1/a)) - (2*a*x + 1)/(2*x**2)
```

Giac [A] time = 1.1655, size = 42, normalized size = 1.31

$$-a^2\log(|ax-1|)+a^2\log(|x|)-\frac{2ax+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^3,x, algorithm="giac")
```

```
[Out] -a^2*log(abs(a*x - 1)) + a^2*log(abs(x)) - 1/2*(2*a*x + 1)/x^2
```

3.928 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=42

$$-\frac{a^2}{x}+a^3\log(x)-a^3\log(1-ax)-\frac{a}{2x^2}-\frac{1}{3x^3}$$

```
[Out] -1/(3*x^3) - a/(2*x^2) - a^2/x + a^3*Log[x] - a^3*Log[1 - a*x]
```

Rubi [A] time = 0.090742, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 44}

$$-\frac{a^2}{x}+a^3\log(x)-a^3\log(1-ax)-\frac{a}{2x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -1/(3*x^3) - a/(2*x^2) - a^2/x + a^3*Log[x] - a^3*Log[1 - a*x]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{x^4\sqrt{1-a^2x^2}}\,dx &= \int \frac{1}{x^4(1-ax)}\,dx\\ &= \int \left(\frac{1}{x^4}+\frac{a}{x^3}+\frac{a^2}{x^2}+\frac{a^3}{x}-\frac{a^4}{-1+ax}\right)dx\\ &= -\frac{1}{3x^3}-\frac{a}{2x^2}-\frac{a^2}{x}+a^3\log(x)-a^3\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.015382, size = 42, normalized size = 1.

$$-\frac{a^2}{x}+a^3\log(x)-a^3\log(1-ax)-\frac{a}{2x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -1/(3*x^3) - a/(2*x^2) - a^2/x + a^3*Log[x] - a^3*Log[1 - a*x]
```

Maple [A] time = 0.031, size = 38, normalized size = 0.9

$$-\frac{1}{3\,x^{3}}-\frac{a}{2\,x^{2}}-\frac{a^{2}}{x}+a^{3}\ln\left(x\right)-a^{3}\ln\left(ax-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)/x^4,x)
```

```
[Out] -1/3/x^3-1/2*a/x^2-a^2/x+a^3*ln(x)-a^3*ln(a*x-1)
```

Maxima [A] time = 0.957407, size = 50, normalized size = 1.19

$$-a^{3}\log\left(ax-1\right)+a^{3}\log\left(x\right)-\frac{6\,a^{2}x^{2}+3\,ax+2}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^4,x, algorithm="maxima")
```

```
[Out] -a^3*log(a*x - 1) + a^3*log(x) - 1/6*(6*a^2*x^2 + 3*a*x + 2)/x^3
```

Fricas [A] time = 1.85445, size = 105, normalized size = 2.5

$$-\frac{6\,a^{3}x^{3}\log\left(ax-1\right)-6\,a^{3}x^{3}\log\left(x\right)+6\,a^{2}x^{2}+3\,ax+2}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^3*x^3*log(a*x - 1) - 6*a^3*x^3*log(x) + 6*a^2*x^2 + 3*a*x + 2)/x^
3
```

Sympy [A] time = 0.340384, size = 34, normalized size = 0.81

$$-a^{3}\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)-\frac{6a^{2}x^{2}+3ax+2}{6x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)/x**4,x)
```

```
[Out] -a**3*(-log(x) + log(x - 1/a)) - (6*a**2*x**2 + 3*a*x + 2)/(6*x**3)
```

Giac [A] time = 1.13409, size = 53, normalized size = 1.26

$$-a^3\log(|ax-1|) + a^3\log(|x|) - \frac{6a^2x^2 + 3ax + 2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^4,x, algorithm="giac")
```

```
[Out] -a^3*log(abs(a*x - 1)) + a^3*log(abs(x)) - 1/6*(6*a^2*x^2 + 3*a*x + 2)/x^3
```

3.929 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{x^2}{2a^3}+\frac{x}{a^4}+\frac{1}{2a^5(1-ax)}+\frac{7\log(1-ax)}{4a^5}+\frac{\log(ax+1)}{4a^5}$$

```
[Out] x/a^4 + x^2/(2*a^3) + 1/(2*a^5*(1 - a*x)) + (7*Log[1 - a*x])/(4*a^5) + Log[
1 + a*x]/(4*a^5)
```

Rubi [A] time = 0.115073, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{x^2}{2a^3}+\frac{x}{a^4}+\frac{1}{2a^5(1-ax)}+\frac{7\log(1-ax)}{4a^5}+\frac{\log(ax+1)}{4a^5}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] x/a^4 + x^2/(2*a^3) + 1/(2*a^5*(1 - a*x)) + (7*Log[1 - a*x])/(4*a^5) + Log[
1 + a*x]/(4*a^5)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^4}{(1-a^2x^2)^{3/2}}\,dx &= \int \frac{x^4}{(1-ax)^2(1+ax)}\,dx\\ &= \int \left(\frac{1}{a^4}+\frac{x}{a^3}+\frac{1}{2a^4(-1+ax)^2}+\frac{7}{4a^4(-1+ax)}+\frac{1}{4a^4(1+ax)}\right)dx\\ &= \frac{x}{a^4}+\frac{x^2}{2a^3}+\frac{1}{2a^5(1-ax)}+\frac{7\log(1-ax)}{4a^5}+\frac{\log(1+ax)}{4a^5}\end{aligned}$$

Mathematica [A] time = 0.060876, size = 45, normalized size = 0.78

$$\frac{2\left(a^2x^2+2ax+\frac{1}{1-ax}\right)+7\log(1-ax)+\log(ax+1)}{4a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (2*(2*a*x + a^2*x^2 + (1 - a*x)^(-1)) + 7*Log[1 - a*x] + Log[1 + a*x])/(4*a
^5)
```

Maple [A] time = 0.035, size = 49, normalized size = 0.8

$$\frac{x^2}{2a^3}+\frac{x}{a^4}+\frac{\ln(ax+1)}{4a^5}-\frac{1}{2a^5(ax-1)}+\frac{7\ln(ax-1)}{4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2*x^4,x)
```

```
[Out] 1/2*x^2/a^3+x/a^4+1/4*ln(a*x+1)/a^5-1/2/a^5/(a*x-1)+7/4/a^5*ln(a*x-1)
```

Maxima [A] time = 0.94287, size = 70, normalized size = 1.21

$$-\frac{1}{2\left(a^6x-a^5\right)}+\frac{ax^2+2x}{2a^4}+\frac{\log(ax+1)}{4a^5}+\frac{7\log(ax-1)}{4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^4,x, algorithm="maxima")
```

```
[Out] -1/2/(a^6*x - a^5) + 1/2*(a*x^2 + 2*x)/a^4 + 1/4*log(a*x + 1)/a^5 + 7/4*log
(a*x - 1)/a^5
```

Fricas [A] time = 1.77871, size = 144, normalized size = 2.48

$$\frac{2a^3x^3+2a^2x^2-4ax+(ax-1)\log(ax+1)+7(ax-1)\log(ax-1)-2}{4\left(a^6x-a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^4,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^3*x^3 + 2*a^2*x^2 - 4*a*x + (a*x - 1)*log(a*x + 1) + 7*(a*x - 1)*l
og(a*x - 1) - 2)/(a^6*x - a^5)
```

Sympy [A] time = 0.426931, size = 48, normalized size = 0.83

$$-\frac{1}{2a^6x-2a^5}+\frac{x^2}{2a^3}+\frac{x}{a^4}+\frac{\frac{7\log\left(x-\frac{1}{a}\right)}{4}+\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2*x**4,x)
```

```
[Out] -1/(2*a**6*x - 2*a**5) + x**2/(2*a**3) + x/a**4 + (7*log(x - 1/a)/4 + log(x
 + 1/a)/4)/a**5
```

Giac [A] time = 1.16013, size = 76, normalized size = 1.31

$$\frac{\log\left(|ax+1|\right)}{4\,a^5} + \frac{7\,\log\left(|ax-1|\right)}{4\,a^5} + \frac{a^3x^2 + 2\,a^2x}{2\,a^6} - \frac{1}{2\,(ax-1)a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^4,x, algorithm="giac")
```

```
[Out] 1/4*log(abs(a*x + 1))/a^5 + 7/4*log(abs(a*x - 1))/a^5 + 1/2*(a^3*x^2 + 2*a^
2*x)/a^6 - 1/2/((a*x - 1)*a^5)
```

3.930 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{x}{a^3}+\frac{1}{2a^4(1-ax)}+\frac{5\log(1-ax)}{4a^4}-\frac{\log(ax+1)}{4a^4}$$

```
[Out] x/a^3 + 1/(2*a^4*(1 - a*x)) + (5*Log[1 - a*x])/(4*a^4) - Log[1 + a*x]/(4*a^
4)
```

Rubi [A] time = 0.113325, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{x}{a^3}+\frac{1}{2a^4(1-ax)}+\frac{5\log(1-ax)}{4a^4}-\frac{\log(ax+1)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] x/a^3 + 1/(2*a^4*(1 - a*x)) + (5*Log[1 - a*x])/(4*a^4) - Log[1 + a*x]/(4*a^
4)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^3}{(1-a^2x^2)^{3/2}}\,dx &= \int \frac{x^3}{(1-ax)^2(1+ax)}\,dx\\ &= \int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx\\ &= \frac{x}{a^3}+\frac{1}{2a^4(1-ax)}+\frac{5\log(1-ax)}{4a^4}-\frac{\log(1+ax)}{4a^4}\end{aligned}$$

Mathematica [A] time = 0.0306449, size = 39, normalized size = 0.81

$$\frac{4ax+\frac{2}{1-ax}+5\log(1-ax)-\log(ax+1)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (4*a*x + 2/(1 - a*x) + 5*Log[1 - a*x] - Log[1 + a*x])/(4*a^4)
```

Maple [A] time = 0.033, size = 41, normalized size = 0.9

$$\frac{x}{a^3}-\frac{\ln(ax+1)}{4a^4}-\frac{1}{2a^4(ax-1)}+\frac{5\ln(ax-1)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2*x^3,x)
```

```
[Out] x/a^3-1/4/a^4*ln(a*x+1)-1/2/a^4/(a*x-1)+5/4/a^4*ln(a*x-1)
```

Maxima [A] time = 0.948588, size = 58, normalized size = 1.21

$$-\frac{1}{2\left(a^5x-a^4\right)}+\frac{x}{a^3}-\frac{\log(ax+1)}{4a^4}+\frac{5\log(ax-1)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^3,x, algorithm="maxima")
```

```
[Out] -1/2/(a^5*x - a^4) + x/a^3 - 1/4*log(a*x + 1)/a^4 + 5/4*log(a*x - 1)/a^4
```

Fricas [A] time = 1.77264, size = 128, normalized size = 2.67

$$\frac{4a^2x^2-4ax-(ax-1)\log(ax+1)+5(ax-1)\log(ax-1)-2}{4\left(a^5x-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
- 2)/(a^5*x - a^4)
```

Sympy [A] time = 0.412554, size = 39, normalized size = 0.81

$$-\frac{1}{2a^5x-2a^4}+\frac{x}{a^3}+\frac{\frac{5\log\left(x-\frac{1}{a}\right)}{4}-\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2*x**3,x)
```

```
[Out] -1/(2*a**5*x - 2*a**4) + x/a**3 + (5*log(x - 1/a)/4 - log(x + 1/a)/4)/a**4
```

Giac [A] time = 1.15815, size = 57, normalized size = 1.19

$$\frac{x}{a^3} - \frac{\log(|ax+1|)}{4\,a^4} + \frac{5\,\log(|ax-1|)}{4\,a^4} - \frac{1}{2\,(ax-1)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^3,x, algorithm="giac")
```

```
[Out] x/a^3 - 1/4*log(abs(a*x + 1))/a^4 + 5/4*log(abs(a*x - 1))/a^4 - 1/2/((a*x -
 1)*a^4)
```

3.931 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=43

$$\frac{1}{2a^3(1-ax)}+\frac{3\log(1-ax)}{4a^3}+\frac{\log(ax+1)}{4a^3}$$

```
[Out] 1/(2*a^3*(1 - a*x)) + (3*Log[1 - a*x])/(4*a^3) + Log[1 + a*x]/(4*a^3)
```

Rubi [A] time = 0.113338, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{1}{2a^3(1-ax)}+\frac{3\log(1-ax)}{4a^3}+\frac{\log(ax+1)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] 1/(2*a^3*(1 - a*x)) + (3*Log[1 - a*x])/(4*a^3) + Log[1 + a*x]/(4*a^3)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{3/2}}\,dx &= \int \frac{x^2}{(1-ax)^2(1+ax)}\,dx\\ &= \int \left(\frac{1}{2a^2(-1+ax)^2}+\frac{3}{4a^2(-1+ax)}+\frac{1}{4a^2(1+ax)}\right)dx\\ &= \frac{1}{2a^3(1-ax)}+\frac{3\log(1-ax)}{4a^3}+\frac{\log(1+ax)}{4a^3}\end{aligned}$$

Mathematica [A] time = 0.0243129, size = 33, normalized size = 0.77

$$\frac{\frac{2}{1-ax}+3\log(1-ax)+\log(ax+1)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (2/(1 - a*x) + 3*Log[1 - a*x] + Log[1 + a*x])/(4*a^3)
```

Maple [A] time = 0.033, size = 36, normalized size = 0.8

$$\frac{\ln(ax+1)}{4a^3} - \frac{1}{2a^3(ax-1)} + \frac{3\ln(ax-1)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2*x^2,x)
```

```
[Out] 1/4*ln(a*x+1)/a^3-1/2/a^3/(a*x-1)+3/4/a^3*ln(a*x-1)
```

Maxima [A] time = 0.969361, size = 51, normalized size = 1.19

$$-\frac{1}{2\left(a^4x - a^3\right)} + \frac{\log(ax+1)}{4a^3} + \frac{3\log(ax-1)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^2,x, algorithm="maxima")
```

```
[Out] -1/2/(a^4*x - a^3) + 1/4*log(a*x + 1)/a^3 + 3/4*log(a*x - 1)/a^3
```

Fricas [A] time = 1.74665, size = 101, normalized size = 2.35

$$\frac{(ax-1)\log(ax+1) + 3(ax-1)\log(ax-1) - 2}{4\left(a^4x - a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/4*((a*x - 1)*log(a*x + 1) + 3*(a*x - 1)*log(a*x - 1) - 2)/(a^4*x - a^3)
```

Sympy [A] time = 0.385481, size = 34, normalized size = 0.79

$$-\frac{1}{2a^4x - 2a^3} + \frac{\frac{3\log\left(x - \frac{1}{a}\right)}{4} + \frac{\log\left(x + \frac{1}{a}\right)}{4}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2*x**2,x)
```

```
[Out] -1/(2*a**4*x - 2*a**3) + (3*log(x - 1/a)/4 + log(x + 1/a)/4)/a**3
```

Giac [A] time = 1.19562, size = 50, normalized size = 1.16

$$\frac{\log\left(|ax+1|\right)}{4\,a^3}+\frac{3\,\log\left(|ax-1|\right)}{4\,a^3}-\frac{1}{2\,(ax-1)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^2,x, algorithm="giac")
```

```
[Out] 1/4*log(abs(a*x + 1))/a^3 + 3/4*log(abs(a*x - 1))/a^3 - 1/2/((a*x - 1)*a^3)
```

3.932 $\int \frac{e^{\tanh^{-1}(ax)}x}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2a^2(1-ax)}-\frac{\tanh^{-1}(ax)}{2a^2}$$

```
[Out] 1/(2*a^2*(1 - a*x)) - ArcTanh[a*x]/(2*a^2)
```

Rubi [A] time = 0.0775094, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6150, 77, 207}

$$\frac{1}{2a^2(1-ax)}-\frac{\tanh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] 1/(2*a^2*(1 - a*x)) - ArcTanh[a*x]/(2*a^2)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x}{\left(1-a^2x^2\right)^{3/2}}\,dx = \int \frac{x}{(1-ax)^2(1+ax)}\,dx$$
$$= \int \left(\frac{1}{2a(-1+ax)^2} + \frac{1}{2a\left(-1+a^2x^2\right)}\right)dx$$
$$= \frac{1}{2a^2(1-ax)} + \frac{\int \frac{1}{-1+a^2x^2}\,dx}{2a}$$
$$= \frac{1}{2a^2(1-ax)} - \frac{\tanh^{-1}(ax)}{2a^2}$$

Mathematica [A] time = 0.0190793, size = 22, normalized size = 0.81

$$\frac{\frac{1}{1-ax} - \tanh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] ((1 - a*x)^(-1) - ArcTanh[a*x])/(2*a^2)
```

Maple [A] time = 0.035, size = 36, normalized size = 1.3

$$-\frac{\ln(ax+1)}{4a^2} - \frac{1}{2a^2(ax-1)} + \frac{\ln(ax-1)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2*x,x)
```

```
[Out] -1/4/a^2*ln(a*x+1)-1/2/a^2/(a*x-1)+1/4/a^2*ln(a*x-1)
```

Maxima [A] time = 0.946489, size = 51, normalized size = 1.89

$$-\frac{1}{2\left(a^3x-a^2\right)} - \frac{\log(ax+1)}{4a^2} + \frac{\log(ax-1)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x,x, algorithm="maxima")
```

```
[Out] -1/2/(a^3*x - a^2) - 1/4*log(a*x + 1)/a^2 + 1/4*log(a*x - 1)/a^2
```

Fricas [A] time = 1.70477, size = 100, normalized size = 3.7

$$-\frac{(ax-1)\log(ax+1) - (ax-1)\log(ax-1) + 2}{4\left(a^3x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x,x, algorithm="fricas")
```

```
[Out] -1/4*((a*x - 1)*log(a*x + 1) - (a*x - 1)*log(a*x - 1) + 2)/(a^3*x - a^2)
```

Sympy [A] time = 0.335607, size = 32, normalized size = 1.19

$$-\frac{1}{2a^3x - 2a^2} + \frac{\frac{\log\left(x-\frac{1}{a}\right)}{4} - \frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2*x,x)
```

```
[Out] -1/(2*a**3*x - 2*a**2) + (log(x - 1/a)/4 - log(x + 1/a)/4)/a**2
```

Giac [A] time = 1.21206, size = 50, normalized size = 1.85

$$-\frac{\log\left(|ax+1|\right)}{4\,a^2} + \frac{\log\left(|ax-1|\right)}{4\,a^2} - \frac{1}{2\,(ax-1)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x,x, algorithm="giac")
```

```
[Out] -1/4*log(abs(a*x + 1))/a^2 + 1/4*log(abs(a*x - 1))/a^2 - 1/2/((a*x - 1)*a^2
)
```

3.933 $\int \frac{e^{\tanh^{-1}(ax)}}{(1-a^2x^2)^{3/2}} \, dx$

Optimal. Leaf size=27

$$\frac{1}{2a(1-ax)}+\frac{\tanh^{-1}(ax)}{2a}$$

```
[Out] 1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a)
```

Rubi [A] time = 0.0484609, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6140, 44, 207}

$$\frac{1}{2a(1-ax)}+\frac{\tanh^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] 1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}} \, dx &= \int \frac{1}{(1-ax)^2(1+ax)} \, dx\\
&= \int \left(\frac{1}{2(-1+ax)^2}-\frac{1}{2\left(-1+a^2x^2\right)}\right) dx\\
&= \frac{1}{2a(1-ax)}-\frac{1}{2}\int \frac{1}{-1+a^2x^2} \, dx\\
&= \frac{1}{2a(1-ax)}+\frac{\tanh^{-1}(ax)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.015646, size = 20, normalized size = 0.74

$$\frac{\frac{1}{1-ax}+\tanh^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] ((1 - a*x)^(-1) + ArcTanh[a*x])/(2*a)
```

Maple [A] time = 0.033, size = 36, normalized size = 1.3

$$\frac{\ln(ax+1)}{4a}-\frac{1}{2a(ax-1)}-\frac{\ln(ax-1)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2,x)
```

```
[Out] 1/4*ln(a*x+1)/a-1/2/a/(a*x-1)-1/4/a*ln(a*x-1)
```

Maxima [A] time = 0.94299, size = 49, normalized size = 1.81

$$\frac{\log(ax+1)}{4a}-\frac{\log(ax-1)}{4a}-\frac{1}{2\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/4*log(a*x + 1)/a - 1/4*log(a*x - 1)/a - 1/2/(a^2*x - a)
```

Fricas [A] time = 1.69701, size = 96, normalized size = 3.56

$$\frac{(ax-1)\log(ax+1)-(ax-1)\log(ax-1)-2}{4\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((a*x - 1)*log(a*x + 1) - (a*x - 1)*log(a*x - 1) - 2)/(a^2*x - a)
```

Sympy [A] time = 0.352961, size = 29, normalized size = 1.07

$$-\frac{1}{2a^2x-2a}+\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{4}+\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2,x)
```

```
[Out] -1/(2*a**2*x - 2*a) + (-log(x - 1/a)/4 + log(x + 1/a)/4)/a
```

Giac [A] time = 1.16653, size = 50, normalized size = 1.85

$$\frac{\log\left(|ax+1|\right)}{4\,a} - \frac{\log\left(|ax-1|\right)}{4\,a} - \frac{1}{2\,(ax-1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/4*log(abs(a*x + 1))/a - 1/4*log(abs(a*x - 1))/a - 1/2/((a*x - 1)*a)
```

3.934 $\int \frac{e^{\tanh^{-1}(ax)}}{x(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2(1-ax)}-\frac{3}{4}\log(1-ax)-\frac{1}{4}\log(ax+1)+\log(x)$$

```
[Out] 1/(2*(1 - a*x)) + Log[x] - (3*Log[1 - a*x])/4 - Log[1 + a*x]/4
```

Rubi [A] time = 0.108868, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 72}

$$\frac{1}{2(1-ax)}-\frac{3}{4}\log(1-ax)-\frac{1}{4}\log(ax+1)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] 1/(2*(1 - a*x)) + Log[x] - (3*Log[1 - a*x])/4 - Log[1 + a*x]/4
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{3/2}}\,dx &= \int \frac{1}{x(1-ax)^2(1+ax)}\,dx\\
&= \int \left(\frac{1}{x}+\frac{a}{2(-1+ax)^2}-\frac{3a}{4(-1+ax)}-\frac{a}{4(1+ax)}\right)dx\\
&= \frac{1}{2(1-ax)}+\log(x)-\frac{3}{4}\log(1-ax)-\frac{1}{4}\log(1+ax)
\end{aligned}$$

Mathematica [A] time = 0.0308583, size = 32, normalized size = 0.89

$$\frac{1}{2-2ax}-\frac{3}{4}\log(1-ax)-\frac{1}{4}\log(ax+1)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (2 - 2*a*x)^(-1) + Log[x] - (3*Log[1 - a*x])/4 - Log[1 + a*x]/4
```

Maple [A] time = 0.036, size = 29, normalized size = 0.8

$$\ln(x) - \frac{\ln(ax+1)}{4} - \frac{1}{2\,ax-2} - \frac{3\,\ln(ax-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2/x,x)
```

```
[Out] ln(x)-1/4*ln(a*x+1)-1/2/(a*x-1)-3/4*ln(a*x-1)
```

Maxima [A] time = 0.944727, size = 38, normalized size = 1.06

$$-\frac{1}{2\,(ax-1)} - \frac{1}{4}\log(ax+1) - \frac{3}{4}\log(ax-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x,x, algorithm="maxima")
```

```
[Out] -1/2/(a*x - 1) - 1/4*log(a*x + 1) - 3/4*log(a*x - 1) + log(x)
```

Fricas [A] time = 1.83539, size = 126, normalized size = 3.5

$$-\frac{(ax-1)\log(ax+1) + 3\,(ax-1)\log(ax-1) - 4\,(ax-1)\log(x) + 2}{4\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x,x, algorithm="fricas")
```

```
[Out] -1/4*((a*x - 1)*log(a*x + 1) + 3*(a*x - 1)*log(a*x - 1) - 4*(a*x - 1)*log(x
) + 2)/(a*x - 1)
```

Sympy [A] time = 0.419124, size = 29, normalized size = 0.81

$$\log(x) - \frac{3\log\left(x - \frac{1}{a}\right)}{4} - \frac{\log\left(x + \frac{1}{a}\right)}{4} - \frac{1}{2ax-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2/x,x)
```

```
[Out] log(x) - 3*log(x - 1/a)/4 - log(x + 1/a)/4 - 1/(2*a*x - 2)
```

Giac [A] time = 1.13694, size = 42, normalized size = 1.17

$$-\frac{1}{2(ax-1)}-\frac{1}{4}\log(|ax+1|)-\frac{3}{4}\log(|ax-1|)+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x,x, algorithm="giac")
```

```
[Out] -1/2/(a*x - 1) - 1/4*log(abs(a*x + 1)) - 3/4*log(abs(a*x - 1)) + log(abs(x)
)
```

3.935 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{a}{2(1-ax)}+a\log(x)-\frac{5}{4}a\log(1-ax)+\frac{1}{4}a\log(ax+1)-\frac{1}{x}$$

```
[Out] -x^(-1) + a/(2*(1 - a*x)) + a*Log[x] - (5*a*Log[1 - a*x])/4 + (a*Log[1 + a*
x])/4
```

Rubi [A] time = 0.109848, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{a}{2(1-ax)}+a\log(x)-\frac{5}{4}a\log(1-ax)+\frac{1}{4}a\log(ax+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -x^(-1) + a/(2*(1 - a*x)) + a*Log[x] - (5*a*Log[1 - a*x])/4 + (a*Log[1 + a*
x])/4
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx &= \int \frac{1}{x^2(1-ax)^2(1+ax)}\,dx\\ &= \int \left(\frac{1}{x^2}+\frac{a}{x}+\frac{a^2}{2(-1+ax)^2}-\frac{5a^2}{4(-1+ax)}+\frac{a^2}{4(1+ax)}\right)dx\\ &= -\frac{1}{x}+\frac{a}{2(1-ax)}+a\log(x)-\frac{5}{4}a\log(1-ax)+\frac{1}{4}a\log(1+ax)\end{aligned}$$

Mathematica [A] time = 0.035168, size = 46, normalized size = 1.

$$\frac{a}{2(1-ax)}+a\log(x)-\frac{5}{4}a\log(1-ax)+\frac{1}{4}a\log(ax+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -x^(-1) + a/(2*(1 - a*x)) + a*Log[x] - (5*a*Log[1 - a*x])/4 + (a*Log[1 + a*
x])/4
```

Maple [A] time = 0.037, size = 39, normalized size = 0.9

$$-x^{-1}+a\ln\left(x\right)+\frac{a\ln\left(ax+1\right)}{4}-\frac{a}{2\,ax-2}-\frac{5\,a\ln\left(ax-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2/x^2,x)
```

```
[Out] -1/x+a*ln(x)+1/4*a*ln(a*x+1)-1/2*a/(a*x-1)-5/4*a*ln(a*x-1)
```

Maxima [A] time = 0.962485, size = 57, normalized size = 1.24

$$\frac{1}{4}\,a\log\left(ax+1\right)-\frac{5}{4}\,a\log\left(ax-1\right)+a\log\left(x\right)-\frac{3\,ax-2}{2\left(ax^{2}-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*a*log(a*x + 1) - 5/4*a*log(a*x - 1) + a*log(x) - 1/2*(3*a*x - 2)/(a*x^2
 - x)
```

Fricas [A] time = 1.73207, size = 163, normalized size = 3.54

$$-\frac{6\,ax-\left(a^{2}x^{2}-ax\right)\log\left(ax+1\right)+5\left(a^{2}x^{2}-ax\right)\log\left(ax-1\right)-4\left(a^{2}x^{2}-ax\right)\log\left(x\right)-4}{4\left(ax^{2}-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*x - (a^2*x^2 - a*x)*log(a*x + 1) + 5*(a^2*x^2 - a*x)*log(a*x - 1)
 - 4*(a^2*x^2 - a*x)*log(x) - 4)/(a*x^2 - x)
```

Sympy [A] time = 0.525264, size = 42, normalized size = 0.91

$$a\log\left(x\right)-\frac{5a\log\left(x-\frac{1}{a}\right)}{4}+\frac{a\log\left(x+\frac{1}{a}\right)}{4}-\frac{3ax-2}{2ax^{2}-2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2/x**2,x)
```

```
[Out] a*log(x) - 5*a*log(x - 1/a)/4 + a*log(x + 1/a)/4 - (3*a*x - 2)/(2*a*x**2 -
2*x)
```

Giac [A] time = 1.13994, size = 59, normalized size = 1.28

$$\frac{1}{4}\,a\log\left(|ax+1|\right)-\frac{5}{4}\,a\log\left(|ax-1|\right)+a\log\left(|x|\right)-\frac{3\,ax-2}{2\,(ax-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/4*a*log(abs(a*x + 1)) - 5/4*a*log(abs(a*x - 1)) + a*log(abs(x)) - 1/2*(3*
a*x - 2)/((a*x - 1)*x)
```

3.936 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{a^2}{2(1-ax)}+2a^2\log(x)-\frac{7}{4}a^2\log(1-ax)-\frac{1}{4}a^2\log(ax+1)-\frac{a}{x}-\frac{1}{2x^2}$$

```
[Out] -1/(2*x^2) - a/x + a^2/(2*(1 - a*x)) + 2*a^2*Log[x] - (7*a^2*Log[1 - a*x])/
4 - (a^2*Log[1 + a*x])/4
```

Rubi [A] time = 0.118239, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{a^2}{2(1-ax)}+2a^2\log(x)-\frac{7}{4}a^2\log(1-ax)-\frac{1}{4}a^2\log(ax+1)-\frac{a}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -1/(2*x^2) - a/x + a^2/(2*(1 - a*x)) + 2*a^2*Log[x] - (7*a^2*Log[1 - a*x])/
4 - (a^2*Log[1 + a*x])/4
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx &= \int \frac{1}{x^3(1-ax)^2(1+ax)}\,dx\\
&= \int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{2a^2}{x}+\frac{a^3}{2(-1+ax)^2}-\frac{7a^3}{4(-1+ax)}-\frac{a^3}{4(1+ax)}\right)dx\\
&= -\frac{1}{2x^2}-\frac{a}{x}+\frac{a^2}{2(1-ax)}+2a^2\log(x)-\frac{7}{4}a^2\log(1-ax)-\frac{1}{4}a^2\log(1+ax)
\end{aligned}$$

Mathematica [A] time = 0.0483971, size = 59, normalized size = 0.94

$$\frac{1}{4}\left(\frac{2a^2}{1-ax}+8a^2\log(x)-7a^2\log(1-ax)-a^2\log(ax+1)-\frac{4a}{x}-\frac{2}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-2/x^2 - (4*a)/x + (2*a^2)/(1 - a*x) + 8*a^2*Log[x] - 7*a^2*Log[1 - a*x] -
 a^2*Log[1 + a*x])/4
```

Maple [A] time = 0.039, size = 54, normalized size = 0.9

$$-\frac{1}{2\,x^{2}}-\frac{a}{x}+2\,a^{2}\ln \left(x\right)-\frac{a^{2}\ln \left(ax+1\right)}{4}-\frac{a^{2}}{2\,ax-2}-\frac{7\,a^{2}\ln \left(ax-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2/x^3,x)
```

```
[Out] -1/2/x^2-a/x+2*a^2*ln(x)-1/4*a^2*ln(a*x+1)-1/2*a^2/(a*x-1)-7/4*a^2*ln(a*x-1
)
```

Maxima [A] time = 0.954497, size = 80, normalized size = 1.27

$$-\frac{1}{4}\,a^{2}\log \left(ax+1\right)-\frac{7}{4}\,a^{2}\log \left(ax-1\right)+2\,a^{2}\log \left(x\right)-\frac{3\,a^{2}x^{2}-ax-1}{2\,\left(ax^{3}-x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*log(a*x + 1) - 7/4*a^2*log(a*x - 1) + 2*a^2*log(x) - 1/2*(3*a^2*x^
2 - a*x - 1)/(a*x^3 - x^2)
```

Fricas [A] time = 1.69433, size = 198, normalized size = 3.14

$$-\frac{6\,a^{2}x^{2}-2\,ax+\left(a^{3}x^{3}-a^{2}x^{2}\right)\log \left(ax+1\right)+7\,\left(a^{3}x^{3}-a^{2}x^{2}\right)\log \left(ax-1\right)-8\,\left(a^{3}x^{3}-a^{2}x^{2}\right)\log \left(x\right)-2}{4\,\left(ax^{3}-x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a^2*x^2 - 2*a*x + (a^3*x^3 - a^2*x^2)*log(a*x + 1) + 7*(a^3*x^3 - a
^2*x^2)*log(a*x - 1) - 8*(a^3*x^3 - a^2*x^2)*log(x) - 2)/(a*x^3 - x^2)
```

Sympy [A] time = 0.583806, size = 58, normalized size = 0.92

$$2a^{2}\log \left(x\right)-\frac{7a^{2}\log \left(x-\frac{1}{a}\right)}{4}-\frac{a^{2}\log \left(x+\frac{1}{a}\right)}{4}-\frac{3a^{2}x^{2}-ax-1}{2ax^{3}-2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2/x**3,x)
```

```
[Out] 2*a**2*log(x) - 7*a**2*log(x - 1/a)/4 - a**2*log(x + 1/a)/4 - (3*a**2*x**2
- a*x - 1)/(2*a*x**3 - 2*x**2)
```

Giac [A] time = 1.17738, size = 80, normalized size = 1.27

$$-\frac{1}{4}\,a^2\log\left(|ax+1|\right)-\frac{7}{4}\,a^2\log\left(|ax-1|\right)+2\,a^2\log\left(|x|\right)-\frac{3\,a^2x^2-ax-1}{2\,(ax-1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/4*a^2*log(abs(a*x + 1)) - 7/4*a^2*log(abs(a*x - 1)) + 2*a^2*log(abs(x))
- 1/2*(3*a^2*x^2 - a*x - 1)/((a*x - 1)*x^2)
```

3.937 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{a^3}{2(1-ax)}-\frac{2a^2}{x}+2a^3\log(x)-\frac{9}{4}a^3\log(1-ax)+\frac{1}{4}a^3\log(ax+1)-\frac{a}{2x^2}-\frac{1}{3x^3}$$

```
[Out] -1/(3*x^3) - a/(2*x^2) - (2*a^2)/x + a^3/(2*(1 - a*x)) + 2*a^3*Log[x] - (9*
a^3*Log[1 - a*x])/4 + (a^3*Log[1 + a*x])/4
```

Rubi [A] time = 0.121673, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{a^3}{2(1-ax)}-\frac{2a^2}{x}+2a^3\log(x)-\frac{9}{4}a^3\log(1-ax)+\frac{1}{4}a^3\log(ax+1)-\frac{a}{2x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -1/(3*x^3) - a/(2*x^2) - (2*a^2)/x + a^3/(2*(1 - a*x)) + 2*a^3*Log[x] - (9*
a^3*Log[1 - a*x])/4 + (a^3*Log[1 + a*x])/4
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{x^4\left(1-a^2x^2\right)^{3/2}}\,dx &= \int \frac{1}{x^4(1-ax)^2(1+ax)}\,dx\\ &= \int \left(\frac{1}{x^4}+\frac{a}{x^3}+\frac{2a^2}{x^2}+\frac{2a^3}{x}+\frac{a^4}{2(-1+ax)^2}-\frac{9a^4}{4(-1+ax)}+\frac{a^4}{4(1+ax)}\right)dx\\ &= -\frac{1}{3x^3}-\frac{a}{2x^2}-\frac{2a^2}{x}+\frac{a^3}{2(1-ax)}+2a^3\log(x)-\frac{9}{4}a^3\log(1-ax)+\frac{1}{4}a^3\log(1+ax)\end{aligned}$$

Mathematica [A] time = 0.0514541, size = 67, normalized size = 0.92

$$\frac{1}{12}\left(\frac{6a^3}{1-ax}-\frac{24a^2}{x}+24a^3\log(x)-27a^3\log(1-ax)+3a^3\log(ax+1)-\frac{6a}{x^2}-\frac{4}{x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-4/x^3 - (6*a)/x^2 - (24*a^2)/x + (6*a^3)/(1 - a*x) + 24*a^3*Log[x] - 27*a
^3*Log[1 - a*x] + 3*a^3*Log[1 + a*x])/12
```

Maple [A] time = 0.036, size = 62, normalized size = 0.9

$$-\frac{1}{3x^3}-\frac{a}{2x^2}-2\frac{a^2}{x}+2a^3\ln(x)+\frac{a^3\ln(ax+1)}{4}-\frac{a^3}{2ax-2}-\frac{9a^3\ln(ax-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2/x^4,x)
```

```
[Out] -1/3/x^3-1/2*a/x^2-2*a^2/x+2*a^3*ln(x)+1/4*a^3*ln(a*x+1)-1/2*a^3/(a*x-1)-9/
4*a^3*ln(a*x-1)
```

Maxima [A] time = 0.964068, size = 90, normalized size = 1.23

$$\frac{1}{4}a^3\log(ax+1)-\frac{9}{4}a^3\log(ax-1)+2a^3\log(x)-\frac{15a^3x^3-9a^2x^2-ax-2}{6\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*log(a*x + 1) - 9/4*a^3*log(a*x - 1) + 2*a^3*log(x) - 1/6*(15*a^3*x^
3 - 9*a^2*x^2 - a*x - 2)/(a*x^4 - x^3)
```

Fricas [A] time = 1.71931, size = 224, normalized size = 3.07

$$-\frac{30a^3x^3-18a^2x^2-2ax-3\left(a^4x^4-a^3x^3\right)\log(ax+1)+27\left(a^4x^4-a^3x^3\right)\log(ax-1)-24\left(a^4x^4-a^3x^3\right)\log(x)-4}{12\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(30*a^3*x^3 - 18*a^2*x^2 - 2*a*x - 3*(a^4*x^4 - a^3*x^3)*log(a*x + 1)
 + 27*(a^4*x^4 - a^3*x^3)*log(a*x - 1) - 24*(a^4*x^4 - a^3*x^3)*log(x) - 4)
/(a*x^4 - x^3)
```

Sympy [A] time = 0.631542, size = 66, normalized size = 0.9

$$2a^3\log(x)-\frac{9a^3\log\left(x-\frac{1}{a}\right)}{4}+\frac{a^3\log\left(x+\frac{1}{a}\right)}{4}-\frac{15a^3x^3-9a^2x^2-ax-2}{6ax^4-6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2/x**4,x)
```

```
[Out] 2*a**3*log(x) - 9*a**3*log(x - 1/a)/4 + a**3*log(x + 1/a)/4 - (15*a**3*x**3
 - 9*a**2*x**2 - a*x - 2)/(6*a*x**4 - 6*x**3)
```

Giac [A] time = 1.14705, size = 90, normalized size = 1.23

$$\frac{1}{4}a^3\log\left(|ax+1|\right)-\frac{9}{4}a^3\log\left(|ax-1|\right)+2\,a^3\log\left(|x|\right)-\frac{15\,a^3x^3-9\,a^2x^2-ax-2}{6\,(ax-1)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/4*a^3*log(abs(a*x + 1)) - 9/4*a^3*log(abs(a*x - 1)) + 2*a^3*log(abs(x)) -
 1/6*(15*a^3*x^3 - 9*a^2*x^2 - a*x - 2)/((a*x - 1)*x^3)
```

3.938 $\int \frac{e^{\tanh^{-1}(ax)}x^6}{\left(1-a^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{x^2}{2a^5}-\frac{x}{a^6}-\frac{5}{4a^7(1-ax)}-\frac{1}{8a^7(ax+1)}+\frac{1}{8a^7(1-ax)^2}-\frac{39\log(1-ax)}{16a^7}-\frac{9\log(ax+1)}{16a^7}$$

```
[Out] -(x/a^6) - x^2/(2*a^5) + 1/(8*a^7*(1 - a*x)^2) - 5/(4*a^7*(1 - a*x)) - 1/(8
*a^7*(1 + a*x)) - (39*Log[1 - a*x])/(16*a^7) - (9*Log[1 + a*x])/(16*a^7)
```

Rubi [A] time = 0.140105, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$-\frac{x^2}{2a^5}-\frac{x}{a^6}-\frac{5}{4a^7(1-ax)}-\frac{1}{8a^7(ax+1)}+\frac{1}{8a^7(1-ax)^2}-\frac{39\log(1-ax)}{16a^7}-\frac{9\log(ax+1)}{16a^7}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^6)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] -(x/a^6) - x^2/(2*a^5) + 1/(8*a^7*(1 - a*x)^2) - 5/(4*a^7*(1 - a*x)) - 1/(8
*a^7*(1 + a*x)) - (39*Log[1 - a*x])/(16*a^7) - (9*Log[1 + a*x])/(16*a^7)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^6}{\left(1-a^2x^2\right)^{5/2}}\,dx &= \int \frac{x^6}{(1-ax)^3(1+ax)^2}\,dx\\ &= \int \left(-\frac{1}{a^6}-\frac{x}{a^5}-\frac{1}{4a^6(-1+ax)^3}-\frac{5}{4a^6(-1+ax)^2}-\frac{39}{16a^6(-1+ax)}+\frac{1}{8a^6(1+ax)^2}-\frac{9}{16a^6(1+ax)}\right)dx\\ &= -\frac{x}{a^6}-\frac{x^2}{2a^5}+\frac{1}{8a^7(1-ax)^2}-\frac{5}{4a^7(1-ax)}-\frac{1}{8a^7(1+ax)}-\frac{39\log(1-ax)}{16a^7}-\frac{9\log(1+ax)}{16a^7}\end{aligned}$$

Mathematica [A] time = 0.0901663, size = 65, normalized size = 0.74

$$\frac{2\left(-4a^2x^2-8ax+\frac{10}{ax-1}-\frac{1}{ax+1}+\frac{1}{(ax-1)^2}\right)-39\log(1-ax)-9\log(ax+1)}{16a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^6)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (2*(-8*a*x - 4*a^2*x^2 + (-1 + a*x)^(-2) + 10/(-1 + a*x) - (1 + a*x)^(-1))
- 39*Log[1 - a*x] - 9*Log[1 + a*x])/(16*a^7)
```

Maple [A] time = 0.035, size = 74, normalized size = 0.8

$$-\frac{x^2}{2\,a^5}-\frac{x}{a^6}-\frac{1}{8\,a^7\,(ax+1)}-\frac{9\,\ln(ax+1)}{16\,a^7}+\frac{1}{8\,a^7\,(ax-1)^2}+\frac{5}{4\,a^7\,(ax-1)}-\frac{39\,\ln(ax-1)}{16\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3*x^6,x)
```

```
[Out] -1/2*x^2/a^5-x/a^6-1/8/a^7/(a*x+1)-9/16*ln(a*x+1)/a^7+1/8/a^7/(a*x-1)^2+5/4
/a^7/(a*x-1)-39/16/a^7*ln(a*x-1)
```

Maxima [A] time = 0.955406, size = 108, normalized size = 1.23

$$\frac{9\,a^2x^2+3\,ax-10}{8\left(a^{10}x^3-a^9x^2-a^8x+a^7\right)}-\frac{ax^2+2\,x}{2\,a^6}-\frac{9\,\log(ax+1)}{16\,a^7}-\frac{39\,\log(ax-1)}{16\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^6,x, algorithm="maxima")
```

```
[Out] 1/8*(9*a^2*x^2 + 3*a*x - 10)/(a^10*x^3 - a^9*x^2 - a^8*x + a^7) - 1/2*(a*x^
2 + 2*x)/a^6 - 9/16*log(a*x + 1)/a^7 - 39/16*log(a*x - 1)/a^7
```

Fricas [A] time = 1.77636, size = 271, normalized size = 3.08

$$-\frac{8\,a^5x^5+8\,a^4x^4-24\,a^3x^3-26\,a^2x^2+10\,ax+9\left(a^3x^3-a^2x^2-ax+1\right)\log(ax+1)+39\left(a^3x^3-a^2x^2-ax+1\right)\log(ax-}{16\left(a^{10}x^3-a^9x^2-a^8x+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^6,x, algorithm="fricas")
```

```
[Out] -1/16*(8*a^5*x^5 + 8*a^4*x^4 - 24*a^3*x^3 - 26*a^2*x^2 + 10*a*x + 9*(a^3*x^
3 - a^2*x^2 - a*x + 1)*log(a*x + 1) + 39*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(
a*x - 1) + 20)/(a^10*x^3 - a^9*x^2 - a^8*x + a^7)
```

Sympy [A] time = 0.62623, size = 80, normalized size = 0.91

$$\frac{9a^2x^2+3ax-10}{8a^{10}x^3-8a^9x^2-8a^8x+8a^7}-\frac{x^2}{2a^5}-\frac{x}{a^6}-\frac{3\left(\frac{13\log\left(x-\frac{1}{a}\right)}{16}+\frac{3\log\left(x+\frac{1}{a}\right)}{16}\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3*x**6,x)
```

```
[Out] (9*a**2*x**2 + 3*a*x - 10)/(8*a**10*x**3 - 8*a**9*x**2 - 8*a**8*x + 8*a**7)
 - x**2/(2*a**5) - x/a**6 - 3*(13*log(x - 1/a)/16 + 3*log(x + 1/a)/16)/a**7
```

Giac [A] time = 1.17428, size = 104, normalized size = 1.18

$$-\frac{9\log(|ax+1|)}{16\,a^7}-\frac{39\log(|ax-1|)}{16\,a^7}-\frac{a^5x^2+2\,a^4x}{2\,a^{10}}+\frac{9\,a^2x^2+3\,ax-10}{8\,(ax+1)(ax-1)^2a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^6,x, algorithm="giac")
```

```
[Out] -9/16*log(abs(a*x + 1))/a^7 - 39/16*log(abs(a*x - 1))/a^7 - 1/2*(a^5*x^2 +
2*a^4*x)/a^10 + 1/8*(9*a^2*x^2 + 3*a*x - 10)/((a*x + 1)*(a*x - 1)^2*a^7)
```

3.939 $\int \frac{e^{\tanh^{-1}(ax)}x^5}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{x}{a^5}-\frac{1}{a^6(1-ax)}+\frac{1}{8a^6(ax+1)}+\frac{1}{8a^6(1-ax)^2}-\frac{23\log(1-ax)}{16a^6}+\frac{7\log(ax+1)}{16a^6}$$

```
[Out] -(x/a^5) + 1/(8*a^6*(1 - a*x)^2) - 1/(a^6*(1 - a*x)) + 1/(8*a^6*(1 + a*x))
- (23*Log[1 - a*x])/(16*a^6) + (7*Log[1 + a*x])/(16*a^6)
```

Rubi [A] time = 0.126824, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$-\frac{x}{a^5}-\frac{1}{a^6(1-ax)}+\frac{1}{8a^6(ax+1)}+\frac{1}{8a^6(1-ax)^2}-\frac{23\log(1-ax)}{16a^6}+\frac{7\log(ax+1)}{16a^6}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^5)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] -(x/a^5) + 1/(8*a^6*(1 - a*x)^2) - 1/(a^6*(1 - a*x)) + 1/(8*a^6*(1 + a*x))
- (23*Log[1 - a*x])/(16*a^6) + (7*Log[1 + a*x])/(16*a^6)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^5}{\left(1-a^2x^2\right)^{5/2}}\,dx &= \int \frac{x^5}{(1-ax)^3(1+ax)^2}\,dx\\
&= \int \left(-\frac{1}{a^5}-\frac{1}{4a^5(-1+ax)^3}-\frac{1}{a^5(-1+ax)^2}-\frac{23}{16a^5(-1+ax)}-\frac{1}{8a^5(1+ax)^2}+\frac{7}{16a^5(1+ax)}\right)dx\\
&= -\frac{x}{a^5}+\frac{1}{8a^6(1-ax)^2}-\frac{1}{a^6(1-ax)}+\frac{1}{8a^6(1+ax)}-\frac{23\log(1-ax)}{16a^6}+\frac{7\log(1+ax)}{16a^6}
\end{aligned}$$

Mathematica [A] time = 0.0721872, size = 55, normalized size = 0.72

$$\frac{2\left(-8ax+\frac{8}{ax-1}+\frac{1}{ax+1}+\frac{1}{(ax-1)^2}\right)-23\log(1-ax)+7\log(ax+1)}{16a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^5)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (2*(-8*a*x + (-1 + a*x)^(-2) + 8/(-1 + a*x) + (1 + a*x)^(-1)) - 23*Log[1 -
a*x] + 7*Log[1 + a*x])/(16*a^6)
```

Maple [A] time = 0.036, size = 65, normalized size = 0.9

$$-\frac{x}{a^5}+\frac{1}{8\,a^6\,(ax+1)}+\frac{7\,\ln\,(ax+1)}{16\,a^6}-\frac{23\,\ln\,(ax-1)}{16\,a^6}+\frac{1}{8\,a^6\,(ax-1)^2}+\frac{1}{a^6\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3*x^5,x)
```

```
[Out] -x/a^5+1/8/a^6/(a*x+1)+7/16*ln(a*x+1)/a^6-23/16/a^6*ln(a*x-1)+1/8/a^6/(a*x-
1)^2+1/a^6/(a*x-1)
```

Maxima [A] time = 0.952949, size = 97, normalized size = 1.28

$$\frac{9\,a^2x^2-ax-6}{8\left(a^9x^3-a^8x^2-a^7x+a^6\right)}-\frac{x}{a^5}+\frac{7\,\log\,(ax+1)}{16\,a^6}-\frac{23\,\log\,(ax-1)}{16\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(9*a^2*x^2 - a*x - 6)/(a^9*x^3 - a^8*x^2 - a^7*x + a^6) - x/a^5 + 7/16*
log(a*x + 1)/a^6 - 23/16*log(a*x - 1)/a^6
```

Fricas [A] time = 1.78357, size = 255, normalized size = 3.36

$$-\frac{16\,a^4x^4-16\,a^3x^3-34\,a^2x^2+18\,ax-7\left(a^3x^3-a^2x^2-ax+1\right)\log\,(ax+1)+23\left(a^3x^3-a^2x^2-ax+1\right)\log\,(ax-1)+}{16\left(a^9x^3-a^8x^2-a^7x+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^4*x^4 - 16*a^3*x^3 - 34*a^2*x^2 + 18*a*x - 7*(a^3*x^3 - a^2*x^2
 - a*x + 1)*log(a*x + 1) + 23*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) +
12)/(a^9*x^3 - a^8*x^2 - a^7*x + a^6)
```

Sympy [A] time = 0.615176, size = 70, normalized size = 0.92

$$\frac{9a^2x^2-ax-6}{8a^9x^3-8a^8x^2-8a^7x+8a^6}-\frac{x}{a^5}-\frac{\frac{23\log\left(x-\frac{1}{a}\right)}{16}-\frac{7\log\left(x+\frac{1}{a}\right)}{16}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3*x**5,x)
```

```
[Out] (9*a**2*x**2 - a*x - 6)/(8*a**9*x**3 - 8*a**8*x**2 - 8*a**7*x + 8*a**6) - x
/a**5 - (23*log(x - 1/a)/16 - 7*log(x + 1/a)/16)/a**6
```

Giac [A] time = 1.2163, size = 86, normalized size = 1.13

$$-\frac{x}{a^5} + \frac{7 \log(|ax + 1|)}{16\, a^6} - \frac{23 \log(|ax - 1|)}{16\, a^6} + \frac{9\, a^2 x^2 - ax - 6}{8\,(ax + 1)(ax - 1)^2 a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^5,x, algorithm="giac")
```

```
[Out] -x/a^5 + 7/16*log(abs(a*x + 1))/a^6 - 23/16*log(abs(a*x - 1))/a^6 + 1/8*(9*
a^2*x^2 - a*x - 6)/((a*x + 1)*(a*x - 1)^2*a^6)
```

3.940 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=72

$$-\frac{3}{4a^5(1-ax)}-\frac{1}{8a^5(ax+1)}+\frac{1}{8a^5(1-ax)^2}-\frac{11\log(1-ax)}{16a^5}-\frac{5\log(ax+1)}{16a^5}$$

```
[Out] 1/(8*a^5*(1 - a*x)^2) - 3/(4*a^5*(1 - a*x)) - 1/(8*a^5*(1 + a*x)) - (11*Log
[1 - a*x])/(16*a^5) - (5*Log[1 + a*x])/(16*a^5)
```

Rubi [A] time = 0.124757, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$-\frac{3}{4a^5(1-ax)}-\frac{1}{8a^5(ax+1)}+\frac{1}{8a^5(1-ax)^2}-\frac{11\log(1-ax)}{16a^5}-\frac{5\log(ax+1)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] 1/(8*a^5*(1 - a*x)^2) - 3/(4*a^5*(1 - a*x)) - 1/(8*a^5*(1 + a*x)) - (11*Log
[1 - a*x])/(16*a^5) - (5*Log[1 + a*x])/(16*a^5)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^4}{(1-a^2x^2)^{5/2}}\,dx &= \int \frac{x^4}{(1-ax)^3(1+ax)^2}\,dx\\ &= \int \left(-\frac{1}{4a^4(-1+ax)^3}-\frac{3}{4a^4(-1+ax)^2}-\frac{11}{16a^4(-1+ax)}+\frac{1}{8a^4(1+ax)^2}-\frac{5}{16a^4(1+ax)}\right)dx\\ &= \frac{1}{8a^5(1-ax)^2}-\frac{3}{4a^5(1-ax)}-\frac{1}{8a^5(1+ax)}-\frac{11\log(1-ax)}{16a^5}-\frac{5\log(1+ax)}{16a^5}\end{aligned}$$

Mathematica [A] time = 0.057497, size = 55, normalized size = 0.76

$$\frac{\frac{2(5a^2x^2+3ax-6)}{(ax-1)^2(ax+1)}-11\log(1-ax)-5\log(ax+1)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] ((2*(-6 + 3*a*x + 5*a^2*x^2))/((-1 + a*x)^2*(1 + a*x)) - 11*Log[1 - a*x] -
5*Log[1 + a*x])/(16*a^5)
```

Maple [A] time = 0.036, size = 60, normalized size = 0.8

$$-\frac{1}{8 a^5 (ax+1)}-\frac{5 \ln (ax+1)}{16 a^5}+\frac{1}{8 a^5 (ax-1)^2}+\frac{3}{4 a^5 (ax-1)}-\frac{11 \ln (ax-1)}{16 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3*x^4,x)
```

```
[Out] -1/8/a^5/(a*x+1)-5/16*ln(a*x+1)/a^5+1/8/a^5/(a*x-1)^2+3/4/a^5/(a*x-1)-11/16
/a^5*ln(a*x-1)
```

Maxima [A] time = 0.949897, size = 89, normalized size = 1.24

$$\frac{5 a^2 x^2+3 ax-6}{8 \left(a^8 x^3-a^7 x^2-a^6 x+a^5\right)}-\frac{5 \log (ax+1)}{16 a^5}-\frac{11 \log (ax-1)}{16 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^4,x, algorithm="maxima")
```

```
[Out] 1/8*(5*a^2*x^2 + 3*a*x - 6)/(a^8*x^3 - a^7*x^2 - a^6*x + a^5) - 5/16*log(a*
x + 1)/a^5 - 11/16*log(a*x - 1)/a^5
```

Fricas [A] time = 2.06296, size = 217, normalized size = 3.01

$$\frac{10 a^2 x^2+6 ax-5 \left(a^3 x^3-a^2 x^2-ax+1\right) \log (ax+1)-11 \left(a^3 x^3-a^2 x^2-ax+1\right) \log (ax-1)-12}{16 \left(a^8 x^3-a^7 x^2-a^6 x+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^4,x, algorithm="fricas")
```

```
[Out] 1/16*(10*a^2*x^2 + 6*a*x - 5*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) - 1
1*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 12)/(a^8*x^3 - a^7*x^2 - a^6
*x + a^5)
```

Sympy [A] time = 0.548471, size = 66, normalized size = 0.92

$$\frac{5a^2x^2+3ax-6}{8a^8x^3-8a^7x^2-8a^6x+8a^5}-\frac{\frac{11\log\left(x-\frac{1}{a}\right)}{16}+\frac{5\log\left(x+\frac{1}{a}\right)}{16}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3*x**4,x)
```

```
[Out] (5*a**2*x**2 + 3*a*x - 6)/(8*a**8*x**3 - 8*a**7*x**2 - 8*a**6*x + 8*a**5) -
 (11*log(x - 1/a)/16 + 5*log(x + 1/a)/16)/a**5
```

Giac [A] time = 1.20655, size = 78, normalized size = 1.08

$$-\frac{5\,\log\left(|ax+1|\right)}{16\,a^{5}}-\frac{11\,\log\left(|ax-1|\right)}{16\,a^{5}}+\frac{5\,a^{2}x^{2}+3\,ax-6}{8\,(ax+1)(ax-1)^{2}a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^4,x, algorithm="giac")
```

```
[Out] -5/16*log(abs(a*x + 1))/a^5 - 11/16*log(abs(a*x - 1))/a^5 + 1/8*(5*a^2*x^2
+ 3*a*x - 6)/((a*x + 1)*(a*x - 1)^2*a^5)
```

3.941 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{2a^4(1-ax)}+\frac{1}{8a^4(ax+1)}+\frac{1}{8a^4(1-ax)^2}+\frac{3\tanh^{-1}(ax)}{8a^4}$$

```
[Out] 1/(8*a^4*(1 - a*x)^2) - 1/(2*a^4*(1 - a*x)) + 1/(8*a^4*(1 + a*x)) + (3*ArcT
anh[a*x])/(8*a^4)
```

Rubi [A] time = 0.123667, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6150, 88, 207}

$$-\frac{1}{2a^4(1-ax)}+\frac{1}{8a^4(ax+1)}+\frac{1}{8a^4(1-ax)^2}+\frac{3\tanh^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] 1/(8*a^4*(1 - a*x)^2) - 1/(2*a^4*(1 - a*x)) + 1/(8*a^4*(1 + a*x)) + (3*ArcT
anh[a*x])/(8*a^4)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{5/2}}\,dx &= \int \frac{x^3}{(1-ax)^3(1+ax)^2}\,dx \\
&= \int \left(-\frac{1}{4a^3(-1+ax)^3}-\frac{1}{2a^3(-1+ax)^2}-\frac{1}{8a^3(1+ax)^2}-\frac{3}{8a^3\left(-1+a^2x^2\right)}\right)dx \\
&= \frac{1}{8a^4(1-ax)^2}-\frac{1}{2a^4(1-ax)}+\frac{1}{8a^4(1+ax)}-\frac{3\int \frac{1}{-1+a^2x^2}\,dx}{8a^3} \\
&= \frac{1}{8a^4(1-ax)^2}-\frac{1}{2a^4(1-ax)}+\frac{1}{8a^4(1+ax)}+\frac{3\tanh^{-1}(ax)}{8a^4}
\end{aligned}
$$

Mathematica [A] time = 0.0319314, size = 53, normalized size = 0.95

$$
\frac{5a^2x^2-ax+3(ax-1)^2(ax+1)\tanh^{-1}(ax)-2}{8a^4(ax-1)^2(ax+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (-2 - a*x + 5*a^2*x^2 + 3*(-1 + a*x)^2*(1 + a*x)*ArcTanh[a*x])/(8*a^4*(-1 +
 a*x)^2*(1 + a*x))
```

Maple [A] time = 0.035, size = 60, normalized size = 1.1

$$
\frac{1}{8\,a^4\,(ax+1)}+\frac{3\,\ln(ax+1)}{16\,a^4}+\frac{1}{8\,a^4\,(ax-1)^2}+\frac{1}{2\,a^4\,(ax-1)}-\frac{3\,\ln(ax-1)}{16\,a^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3*x^3,x)
```

```
[Out] 1/8/a^4/(a*x+1)+3/16/a^4*ln(a*x+1)+1/8/a^4/(a*x-1)^2+1/2/a^4/(a*x-1)-3/16/a
^4*ln(a*x-1)
```

Maxima [A] time = 0.950193, size = 89, normalized size = 1.59

$$
\frac{5\,a^2x^2-ax-2}{8\left(a^7x^3-a^6x^2-a^5x+a^4\right)}+\frac{3\,\log(ax+1)}{16\,a^4}-\frac{3\,\log(ax-1)}{16\,a^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(5*a^2*x^2 - a*x - 2)/(a^7*x^3 - a^6*x^2 - a^5*x + a^4) + 3/16*log(a*x
+ 1)/a^4 - 3/16*log(a*x - 1)/a^4
```

Fricas [B] time = 2.3696, size = 215, normalized size = 3.84

$$\frac{10\,a^2x^2 - 2\,ax + 3\left(a^3x^3 - a^2x^2 - ax + 1\right)\log\left(ax + 1\right) - 3\left(a^3x^3 - a^2x^2 - ax + 1\right)\log\left(ax - 1\right) - 4}{16\left(a^7x^3 - a^6x^2 - a^5x + a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(10*a^2*x^2 - 2*a*x + 3*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) - 3
*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 4)/(a^7*x^3 - a^6*x^2 - a^5*x
 + a^4)
```

Sympy [A] time = 0.488704, size = 65, normalized size = 1.16

$$\frac{5a^2x^2 - ax - 2}{8a^7x^3 - 8a^6x^2 - 8a^5x + 8a^4} - \frac{\frac{3\log\left(x - \frac{1}{a}\right)}{16} - \frac{3\log\left(x + \frac{1}{a}\right)}{16}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3*x**3,x)
```

```
[Out] (5*a**2*x**2 - a*x - 2)/(8*a**7*x**3 - 8*a**6*x**2 - 8*a**5*x + 8*a**4) - (
3*log(x - 1/a)/16 - 3*log(x + 1/a)/16)/a**4
```

Giac [A] time = 1.22821, size = 78, normalized size = 1.39

$$\frac{3\,\log\left(|ax + 1|\right)}{16\,a^4} - \frac{3\,\log\left(|ax - 1|\right)}{16\,a^4} + \frac{5\,a^2x^2 - ax - 2}{8\,(ax + 1)(ax - 1)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^3,x, algorithm="giac")
```

```
[Out] 3/16*log(abs(a*x + 1))/a^4 - 3/16*log(abs(a*x - 1))/a^4 + 1/8*(5*a^2*x^2 -
a*x - 2)/((a*x + 1)*(a*x - 1)^2*a^4)
```

3.942 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{4a^3(1-ax)}-\frac{1}{8a^3(ax+1)}+\frac{1}{8a^3(1-ax)^2}-\frac{\tanh^{-1}(ax)}{8a^3}$$

```
[Out] 1/(8*a^3*(1 - a*x)^2) - 1/(4*a^3*(1 - a*x)) - 1/(8*a^3*(1 + a*x)) - ArcTanh
[a*x]/(8*a^3)
```

Rubi [A] time = 0.122589, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6150, 88, 207}

$$-\frac{1}{4a^3(1-ax)}-\frac{1}{8a^3(ax+1)}+\frac{1}{8a^3(1-ax)^2}-\frac{\tanh^{-1}(ax)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] 1/(8*a^3*(1 - a*x)^2) - 1/(4*a^3*(1 - a*x)) - 1/(8*a^3*(1 + a*x)) - ArcTanh
[a*x]/(8*a^3)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{5/2}}\,dx &= \int \frac{x^2}{(1-ax)^3(1+ax)^2}\,dx \\
&= \int \left(-\frac{1}{4a^2(-1+ax)^3}-\frac{1}{4a^2(-1+ax)^2}+\frac{1}{8a^2(1+ax)^2}+\frac{1}{8a^2\left(-1+a^2x^2\right)}\right)dx \\
&= \frac{1}{8a^3(1-ax)^2}-\frac{1}{4a^3(1-ax)}-\frac{1}{8a^3(1+ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{8a^2} \\
&= \frac{1}{8a^3(1-ax)^2}-\frac{1}{4a^3(1-ax)}-\frac{1}{8a^3(1+ax)}-\frac{\tanh^{-1}(ax)}{8a^3}
\end{aligned}
$$

Mathematica [A] time = 0.0302282, size = 52, normalized size = 0.93

$$
\frac{a^2x^2+3ax-(ax-1)^2(ax+1)\tanh^{-1}(ax)-2}{8a^3(ax-1)^2(ax+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (-2 + 3*a*x + a^2*x^2 - (-1 + a*x)^2*(1 + a*x)*ArcTanh[a*x])/(8*a^3*(-1 + a
*x)^2*(1 + a*x))
```

Maple [A] time = 0.035, size = 60, normalized size = 1.1

$$
-\frac{1}{8\,a^3\,(ax+1)}-\frac{\ln(ax+1)}{16\,a^3}+\frac{1}{8\,a^3\,(ax-1)^2}+\frac{1}{4\,a^3\,(ax-1)}+\frac{\ln(ax-1)}{16\,a^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3*x^2,x)
```

```
[Out] -1/8/a^3/(a*x+1)-1/16*ln(a*x+1)/a^3+1/8/a^3/(a*x-1)^2+1/4/a^3/(a*x-1)+1/16/
a^3*ln(a*x-1)
```

Maxima [A] time = 0.94536, size = 88, normalized size = 1.57

$$
\frac{a^2x^2+3\,ax-2}{8\left(a^6x^3-a^5x^2-a^4x+a^3\right)}-\frac{\log(ax+1)}{16\,a^3}+\frac{\log(ax-1)}{16\,a^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^2,x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*x^2 + 3*a*x - 2)/(a^6*x^3 - a^5*x^2 - a^4*x + a^3) - 1/16*log(a*x
 + 1)/a^3 + 1/16*log(a*x - 1)/a^3
```

Fricas [B] time = 2.33037, size = 208, normalized size = 3.71

$$\frac{2\,a^2x^2+6\,ax-\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)+\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax-1\right)-4}{16\left(a^6x^3-a^5x^2-a^4x+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^2,x, algorithm="fricas")
```

```
[Out] 1/16*(2*a^2*x^2 + 6*a*x - (a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) + (a^3
*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 4)/(a^6*x^3 - a^5*x^2 - a^4*x + a^
3)
```

Sympy [A] time = 0.47591, size = 61, normalized size = 1.09

$$\frac{a^2x^2+3ax-2}{8a^6x^3-8a^5x^2-8a^4x+8a^3}-\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{16}+\frac{\log\left(x+\frac{1}{a}\right)}{16}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3*x**2,x)
```

```
[Out] (a**2*x**2 + 3*a*x - 2)/(8*a**6*x**3 - 8*a**5*x**2 - 8*a**4*x + 8*a**3) - (
-log(x - 1/a)/16 + log(x + 1/a)/16)/a**3
```

Giac [A] time = 1.18126, size = 77, normalized size = 1.38

$$-\frac{\log\left(|ax+1|\right)}{16\,a^3}+\frac{\log\left(|ax-1|\right)}{16\,a^3}+\frac{a^2x^2+3\,ax-2}{8\,(ax+1)(ax-1)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^2,x, algorithm="giac")
```

```
[Out] -1/16*log(abs(a*x + 1))/a^3 + 1/16*log(abs(a*x - 1))/a^3 + 1/8*(a^2*x^2 + 3
*a*x - 2)/((a*x + 1)*(a*x - 1)^2*a^3)
```

3.943 $\int \frac{e^{\tanh^{-1}(ax)}x}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{1}{8a^2(ax+1)}+\frac{1}{8a^2(1-ax)^2}-\frac{\tanh^{-1}(ax)}{8a^2}$$

```
[Out] 1/(8*a^2*(1 - a*x)^2) + 1/(8*a^2*(1 + a*x)) - ArcTanh[a*x]/(8*a^2)
```

Rubi [A] time = 0.0841084, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6150, 77, 207}

$$\frac{1}{8a^2(ax+1)}+\frac{1}{8a^2(1-ax)^2}-\frac{\tanh^{-1}(ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] 1/(8*a^2*(1 - a*x)^2) + 1/(8*a^2*(1 + a*x)) - ArcTanh[a*x]/(8*a^2)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x}{\left(1-a^2x^2\right)^{5/2}}\,dx = \int \frac{x}{(1-ax)^3(1+ax)^2}\,dx$$
$$= \int \left(-\frac{1}{4a(-1+ax)^3} - \frac{1}{8a(1+ax)^2} + \frac{1}{8a\left(-1+a^2x^2\right)}\right)dx$$
$$= \frac{1}{8a^2(1-ax)^2} + \frac{1}{8a^2(1+ax)} + \frac{\int \frac{1}{-1+a^2x^2}\,dx}{8a}$$
$$= \frac{1}{8a^2(1-ax)^2} + \frac{1}{8a^2(1+ax)} - \frac{\tanh^{-1}(ax)}{8a^2}$$

Mathematica [A] time = 0.0276091, size = 28, normalized size = 0.68

$$\frac{\frac{1}{ax+1} + \frac{1}{(ax-1)^2} - \tanh^{-1}(ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] ((-1 + a*x)^(-2) + (1 + a*x)^(-1) - ArcTanh[a*x])/(8*a^2)
```

Maple [A] time = 0.036, size = 48, normalized size = 1.2

$$\frac{1}{8\,a^2\,(ax+1)} - \frac{\ln(ax+1)}{16\,a^2} + \frac{1}{8\,a^2\,(ax-1)^2} + \frac{\ln(ax-1)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3*x,x)
```

```
[Out] 1/8/a^2/(a*x+1)-1/16/a^2*ln(a*x+1)+1/8/a^2/(a*x-1)^2+1/16/a^2*ln(a*x-1)
```

Maxima [A] time = 0.967148, size = 88, normalized size = 2.15

$$\frac{a^2x^2 - ax + 2}{8\left(a^5x^3 - a^4x^2 - a^3x + a^2\right)} - \frac{\log(ax+1)}{16\,a^2} + \frac{\log(ax-1)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x,x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*x^2 - a*x + 2)/(a^5*x^3 - a^4*x^2 - a^3*x + a^2) - 1/16*log(a*x +
1)/a^2 + 1/16*log(a*x - 1)/a^2
```

Fricas [B] time = 2.13842, size = 208, normalized size = 5.07

$$\frac{2\,a^2x^2 - 2\,ax - \left(a^3x^3 - a^2x^2 - ax + 1\right)\log(ax+1) + \left(a^3x^3 - a^2x^2 - ax + 1\right)\log(ax-1) + 4}{16\left(a^5x^3 - a^4x^2 - a^3x + a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x,x, algorithm="fricas")
```

```
[Out] 1/16*(2*a^2*x^2 - 2*a*x - (a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) + (a^3
*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) + 4)/(a^5*x^3 - a^4*x^2 - a^3*x + a^
2)
```

Sympy [A] time = 0.479255, size = 60, normalized size = 1.46

$$\frac{a^2x^2 - ax + 2}{8a^5x^3 - 8a^4x^2 - 8a^3x + 8a^2} - \frac{-\frac{\log\left(x-\frac{1}{a}\right)}{16} + \frac{\log\left(x+\frac{1}{a}\right)}{16}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3*x,x)
```

```
[Out] (a**2*x**2 - a*x + 2)/(8*a**5*x**3 - 8*a**4*x**2 - 8*a**3*x + 8*a**2) - (-l
og(x - 1/a)/16 + log(x + 1/a)/16)/a**2
```

Giac [A] time = 1.20794, size = 77, normalized size = 1.88

$$-\frac{\log\left(|ax+1|\right)}{16\,a^2} + \frac{\log\left(|ax-1|\right)}{16\,a^2} + \frac{a^2x^2 - ax + 2}{8\,(ax+1)(ax-1)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x,x, algorithm="giac")
```

```
[Out] -1/16*log(abs(a*x + 1))/a^2 + 1/16*log(abs(a*x - 1))/a^2 + 1/8*(a^2*x^2 - a
*x + 2)/((a*x + 1)*(a*x - 1)^2*a^2)
```

3.944 $\int \frac{e^{\tanh^{-1}(ax)}}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=56

$$\frac{1}{4a(1-ax)}-\frac{1}{8a(ax+1)}+\frac{1}{8a(1-ax)^2}+\frac{3\tanh^{-1}(ax)}{8a}$$

```
[Out] 1/(8*a*(1 - a*x)^2) + 1/(4*a*(1 - a*x)) - 1/(8*a*(1 + a*x)) + (3*ArcTanh[a*
x])/(8*a)
```

Rubi [A] time = 0.0571836, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6140, 44, 207}

$$\frac{1}{4a(1-ax)}-\frac{1}{8a(ax+1)}+\frac{1}{8a(1-ax)^2}+\frac{3\tanh^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] 1/(8*a*(1 - a*x)^2) + 1/(4*a*(1 - a*x)) - 1/(8*a*(1 + a*x)) + (3*ArcTanh[a*
x])/(8*a)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx &= \int \frac{1}{(1-ax)^3(1+ax)^2}\,dx \\
&= \int \left(-\frac{1}{4(-1+ax)^3}+\frac{1}{4(-1+ax)^2}+\frac{1}{8(1+ax)^2}-\frac{3}{8\left(-1+a^2x^2\right)}\right)dx \\
&= \frac{1}{8a(1-ax)^2}+\frac{1}{4a(1-ax)}-\frac{1}{8a(1+ax)}-\frac{3}{8}\int \frac{1}{-1+a^2x^2}\,dx \\
&= \frac{1}{8a(1-ax)^2}+\frac{1}{4a(1-ax)}-\frac{1}{8a(1+ax)}+\frac{3\tanh^{-1}(ax)}{8a}
\end{aligned}
$$

Mathematica [A] time = 0.0247539, size = 53, normalized size = 0.95

$$
\frac{-3a^2x^2+3ax+3(ax-1)^2(ax+1)\tanh^{-1}(ax)+2}{8a(ax-1)^2(ax+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (2 + 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^2*(1 + a*x)*ArcTanh[a*x])/(8*a*(-1 +
a*x)^2*(1 + a*x))
```

Maple [A] time = 0.036, size = 60, normalized size = 1.1

$$
-\frac{1}{8\,a\,(ax+1)}+\frac{3\,\ln(ax+1)}{16\,a}+\frac{1}{8\,a\,(ax-1)^2}-\frac{1}{4\,a\,(ax-1)}-\frac{3\,\ln(ax-1)}{16\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3,x)
```

```
[Out] -1/8/a/(a*x+1)+3/16*ln(a*x+1)/a+1/8/a/(a*x-1)^2-1/4/a/(a*x-1)-3/16/a*ln(a*x
-1)
```

Maxima [A] time = 0.968295, size = 86, normalized size = 1.54

$$
-\frac{3\,a^2x^2-3\,ax-2}{8\left(a^4x^3-a^3x^2-a^2x+a\right)}+\frac{3\,\log(ax+1)}{16\,a}-\frac{3\,\log(ax-1)}{16\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*a^2*x^2 - 3*a*x - 2)/(a^4*x^3 - a^3*x^2 - a^2*x + a) + 3/16*log(a*x
 + 1)/a - 3/16*log(a*x - 1)/a
```

Fricas [B] time = 2.02067, size = 212, normalized size = 3.79

$$-\frac{6\,a^{2}x^{2}-6\,ax-3\left(a^{3}x^{3}-a^{2}x^{2}-ax+1\right)\log\left(ax+1\right)+3\left(a^{3}x^{3}-a^{2}x^{2}-ax+1\right)\log\left(ax-1\right)-4}{16\left(a^{4}x^{3}-a^{3}x^{2}-a^{2}x+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*a^2*x^2 - 6*a*x - 3*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) + 3
*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 4)/(a^4*x^3 - a^3*x^2 - a^2*x
 + a)
```

Sympy [A] time = 0.489683, size = 65, normalized size = 1.16

$$-\frac{3a^{2}x^{2}-3ax-2}{8a^{4}x^{3}-8a^{3}x^{2}-8a^{2}x+8a}-\frac{\frac{3\log\left(x-\frac{1}{a}\right)}{16}-\frac{3\log\left(x+\frac{1}{a}\right)}{16}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3,x)
```

```
[Out] -(3*a**2*x**2 - 3*a*x - 2)/(8*a**4*x**3 - 8*a**3*x**2 - 8*a**2*x + 8*a) - (
3*log(x - 1/a)/16 - 3*log(x + 1/a)/16)/a
```

Giac [A] time = 1.16864, size = 78, normalized size = 1.39

$$\frac{3\,\log\left(|ax+1|\right)}{16\,a}-\frac{3\,\log\left(|ax-1|\right)}{16\,a}-\frac{3\,a^{2}x^{2}-3\,ax-2}{8\,(ax+1)(ax-1)^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] 3/16*log(abs(a*x + 1))/a - 3/16*log(abs(a*x - 1))/a - 1/8*(3*a^2*x^2 - 3*a*
x - 2)/((a*x + 1)*(a*x - 1)^2*a)
```

3.945 $\int \frac{e^{\tanh^{-1}(ax)}}{x(1-a^2x^2)^{5/2}} \, dx$

Optimal. Leaf size=59

$$\frac{1}{2(1-ax)} + \frac{1}{8(ax+1)} + \frac{1}{8(1-ax)^2} - \frac{11}{16}\log(1-ax) - \frac{5}{16}\log(ax+1) + \log(x)$$

```
[Out] 1/(8*(1 - a*x)^2) + 1/(2*(1 - a*x)) + 1/(8*(1 + a*x)) + Log[x] - (11*Log[1
 - a*x])/16 - (5*Log[1 + a*x])/16
```

Rubi [A] time = 0.117907, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{1}{2(1-ax)} + \frac{1}{8(ax+1)} + \frac{1}{8(1-ax)^2} - \frac{11}{16}\log(1-ax) - \frac{5}{16}\log(ax+1) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(1 - a^2*x^2)^(5/2)),x]
```

```
[Out] 1/(8*(1 - a*x)^2) + 1/(2*(1 - a*x)) + 1/(8*(1 + a*x)) + Log[x] - (11*Log[1
 - a*x])/16 - (5*Log[1 + a*x])/16
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{5/2}} \, dx &= \int \frac{1}{x(1-ax)^3(1+ax)^2} \, dx \\
&= \int \left(\frac{1}{x} - \frac{a}{4(-1+ax)^3} + \frac{a}{2(-1+ax)^2} - \frac{11a}{16(-1+ax)} - \frac{a}{8(1+ax)^2} - \frac{5a}{16(1+ax)}\right) dx \\
&= \frac{1}{8(1-ax)^2} + \frac{1}{2(1-ax)} + \frac{1}{8(1+ax)} + \log(x) - \frac{11}{16}\log(1-ax) - \frac{5}{16}\log(1+ax)
\end{aligned}$$

Mathematica [A] time = 0.0452678, size = 54, normalized size = 0.92

$$\frac{1}{16}\left(\frac{8}{1-ax} + \frac{2}{ax+1} + \frac{2}{(ax-1)^2} - 11\log(1-ax) - 5\log(ax+1) + 16\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(1 - a^2*x^2)^(5/2)),x]
```

```
[Out] (8/(1 - a*x) + 2/(-1 + a*x)^2 + 2/(1 + a*x) + 16*Log[x] - 11*Log[1 - a*x] -
 5*Log[1 + a*x])/16
```

Maple [A] time = 0.038, size = 47, normalized size = 0.8

$$\ln(x) + \frac{1}{8\,ax+8} - \frac{5\ln(ax+1)}{16} + \frac{1}{8\,(ax-1)^2} - \frac{1}{2\,ax-2} - \frac{11\ln(ax-1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3/x,x)
```

```
[Out] ln(x)+1/8/(a*x+1)-5/16*ln(a*x+1)+1/8/(a*x-1)^2-1/2/(a*x-1)-11/16*ln(a*x-1)
```

Maxima [A] time = 0.958549, size = 77, normalized size = 1.31

$$-\frac{3\,a^2x^2+ax-6}{8\left(a^3x^3-a^2x^2-ax+1\right)} - \frac{5}{16}\log(ax+1) - \frac{11}{16}\log(ax-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x,x, algorithm="maxima")
```

```
[Out] -1/8*(3*a^2*x^2 + a*x - 6)/(a^3*x^3 - a^2*x^2 - a*x + 1) - 5/16*log(a*x + 1
) - 11/16*log(a*x - 1) + log(x)
```

Fricas [B] time = 2.13587, size = 269, normalized size = 4.56

$$-\frac{6\,a^2x^2+2\,ax+5\left(a^3x^3-a^2x^2-ax+1\right)\log(ax+1)+11\left(a^3x^3-a^2x^2-ax+1\right)\log(ax-1)-16\left(a^3x^3-a^2x^2-ax\right.}{16\left(a^3x^3-a^2x^2-ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x,x, algorithm="fricas")
```

```
[Out] -1/16*(6*a^2*x^2 + 2*a*x + 5*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) + 1
1*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 16*(a^3*x^3 - a^2*x^2 - a*x
+ 1)*log(x) - 12)/(a^3*x^3 - a^2*x^2 - a*x + 1)
```

Sympy [A] time = 0.590731, size = 60, normalized size = 1.02

$$-\frac{3a^2x^2+ax-6}{8a^3x^3-8a^2x^2-8ax+8} + \log(x) - \frac{11\log\left(x-\frac{1}{a}\right)}{16} - \frac{5\log\left(x+\frac{1}{a}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3/x,x)
```

```
[Out] -(3*a**2*x**2 + a*x - 6)/(8*a**3*x**3 - 8*a**2*x**2 - 8*a*x + 8) + log(x) -
 11*log(x - 1/a)/16 - 5*log(x + 1/a)/16
```

Giac [A] time = 1.16475, size = 69, normalized size = 1.17

$$-\frac{3\,a^2x^2+ax-6}{8\,(ax+1)(ax-1)^2}-\frac{5}{16}\,\log\left(|ax+1|\right)-\frac{11}{16}\,\log\left(|ax-1|\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x,x, algorithm="giac")
```

```
[Out] -1/8*(3*a^2*x^2 + a*x - 6)/((a*x + 1)*(a*x - 1)^2) - 5/16*log(abs(a*x + 1))
 - 11/16*log(abs(a*x - 1)) + log(abs(x))
```

3.946 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{3a}{4(1-ax)}-\frac{a}{8(ax+1)}+\frac{a}{8(1-ax)^2}+a\log(x)-\frac{23}{16}a\log(1-ax)+\frac{7}{16}a\log(ax+1)-\frac{1}{x}$$

```
[Out] -x^(-1) + a/(8*(1 - a*x)^2) + (3*a)/(4*(1 - a*x)) - a/(8*(1 + a*x)) + a*Log
[x] - (23*a*Log[1 - a*x])/16 + (7*a*Log[1 + a*x])/16
```

Rubi [A] time = 0.126846, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{3a}{4(1-ax)}-\frac{a}{8(ax+1)}+\frac{a}{8(1-ax)^2}+a\log(x)-\frac{23}{16}a\log(1-ax)+\frac{7}{16}a\log(ax+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^(5/2)),x]
```

```
[Out] -x^(-1) + a/(8*(1 - a*x)^2) + (3*a)/(4*(1 - a*x)) - a/(8*(1 + a*x)) + a*Log
[x] - (23*a*Log[1 - a*x])/16 + (7*a*Log[1 + a*x])/16
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx &= \int \frac{1}{x^2(1-ax)^3(1+ax)^2}\,dx\\
&= \int \left(\frac{1}{x^2}+\frac{a}{x}-\frac{a^2}{4(-1+ax)^3}+\frac{3a^2}{4(-1+ax)^2}-\frac{23a^2}{16(-1+ax)}+\frac{a^2}{8(1+ax)^2}+\frac{7a^2}{16(1+ax)}\right)dx\\
&= -\frac{1}{x}+\frac{a}{8(1-ax)^2}+\frac{3a}{4(1-ax)}-\frac{a}{8(1+ax)}+a\log(x)-\frac{23}{16}a\log(1-ax)+\frac{7}{16}a\log(1+ax)
\end{aligned}$$

Mathematica [A] time = 0.0601992, size = 65, normalized size = 0.92

$$\frac{1}{16}\left(\frac{12a}{1-ax}-\frac{2a}{ax+1}+\frac{2a}{(ax-1)^2}+16a\log(x)-23a\log(1-ax)+7a\log(ax+1)-\frac{16}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^(5/2)),x]
```

```
[Out] (-16/x + (12*a)/(1 - a*x) + (2*a)/(-1 + a*x)^2 - (2*a)/(1 + a*x) + 16*a*Log
[x] - 23*a*Log[1 - a*x] + 7*a*Log[1 + a*x])/16
```

Maple [A] time = 0.038, size = 59, normalized size = 0.8

$$-x^{-1}+a\ln(x)-\frac{a}{8\,ax+8}+\frac{7\,a\ln(ax+1)}{16}+\frac{a}{8\,(ax-1)^2}-\frac{3\,a}{4\,ax-4}-\frac{23\,a\ln(ax-1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3/x^2,x)
```

```
[Out] -1/x+a*ln(x)-1/8*a/(a*x+1)+7/16*a*ln(a*x+1)+1/8*a/(a*x-1)^2-3/4*a/(a*x-1)-2
3/16*a*ln(a*x-1)
```

Maxima [A] time = 0.95228, size = 97, normalized size = 1.37

$$\frac{7}{16}\,a\log(ax+1)-\frac{23}{16}\,a\log(ax-1)+a\log(x)-\frac{15\,a^3x^3-11\,a^2x^2-14\,ax+8}{8\left(a^3x^4-a^2x^3-ax^2+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^2,x, algorithm="maxima")
```

```
[Out] 7/16*a*log(a*x + 1) - 23/16*a*log(a*x - 1) + a*log(x) - 1/8*(15*a^3*x^3 - 1
1*a^2*x^2 - 14*a*x + 8)/(a^3*x^4 - a^2*x^3 - a*x^2 + x)
```

Fricas [B] time = 2.03365, size = 316, normalized size = 4.45

$$-\frac{30\,a^3x^3-22\,a^2x^2-28\,ax-7\left(a^4x^4-a^3x^3-a^2x^2+ax\right)\log(ax+1)+23\left(a^4x^4-a^3x^3-a^2x^2+ax\right)\log(ax-1)-16\,(a^4 \cdots}{16\left(a^3x^4-a^2x^3-ax^2+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/16*(30*a^3*x^3 - 22*a^2*x^2 - 28*a*x - 7*(a^4*x^4 - a^3*x^3 - a^2*x^2 +
a*x)*log(a*x + 1) + 23*(a^4*x^4 - a^3*x^3 - a^2*x^2 + a*x)*log(a*x - 1) - 1
6*(a^4*x^4 - a^3*x^3 - a^2*x^2 + a*x)*log(x) + 16)/(a^3*x^4 - a^2*x^3 - a*x
^2 + x)
```

Sympy [A] time = 0.772041, size = 78, normalized size = 1.1

$$a\log(x)-\frac{23a\log\left(x-\frac{1}{a}\right)}{16}+\frac{7a\log\left(x+\frac{1}{a}\right)}{16}-\frac{15a^3x^3-11a^2x^2-14ax+8}{8a^3x^4-8a^2x^3-8ax^2+8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3/x**2,x)
```

```
[Out] a*log(x) - 23*a*log(x - 1/a)/16 + 7*a*log(x + 1/a)/16 - (15*a**3*x**3 - 11*
a**2*x**2 - 14*a*x + 8)/(8*a**3*x**4 - 8*a**2*x**3 - 8*a*x**2 + 8*x)
```

Giac [A] time = 1.20685, size = 90, normalized size = 1.27

$$\frac{7}{16}\,a\log\left(|ax+1|\right)-\frac{23}{16}\,a\log\left(|ax-1|\right)+a\log\left(|x|\right)-\frac{15\,a^3x^3-11\,a^2x^2-14\,ax+8}{8\,(ax+1)(ax-1)^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^2,x, algorithm="giac")
```

```
[Out] 7/16*a*log(abs(a*x + 1)) - 23/16*a*log(abs(a*x - 1)) + a*log(abs(x)) - 1/8*
(15*a^3*x^3 - 11*a^2*x^2 - 14*a*x + 8)/((a*x + 1)*(a*x - 1)^2*x)
```

3.947 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=89

$$\frac{a^2}{1-ax}+\frac{a^2}{8(ax+1)}+\frac{a^2}{8(1-ax)^2}+3a^2\log(x)-\frac{39}{16}a^2\log(1-ax)-\frac{9}{16}a^2\log(ax+1)-\frac{a}{x}-\frac{1}{2x^2}$$

```
[Out] -1/(2*x^2) - a/x + a^2/(8*(1 - a*x)^2) + a^2/(1 - a*x) + a^2/(8*(1 + a*x))
+ 3*a^2*Log[x] - (39*a^2*Log[1 - a*x])/16 - (9*a^2*Log[1 + a*x])/16
```

Rubi [A] time = 0.13547, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{a^2}{1-ax}+\frac{a^2}{8(ax+1)}+\frac{a^2}{8(1-ax)^2}+3a^2\log(x)-\frac{39}{16}a^2\log(1-ax)-\frac{9}{16}a^2\log(ax+1)-\frac{a}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^(5/2)),x]
```

```
[Out] -1/(2*x^2) - a/x + a^2/(8*(1 - a*x)^2) + a^2/(1 - a*x) + a^2/(8*(1 + a*x))
+ 3*a^2*Log[x] - (39*a^2*Log[1 - a*x])/16 - (9*a^2*Log[1 + a*x])/16
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(1-a^2x^2\right)^{5/2}}\,dx &= \int \frac{1}{x^3(1-ax)^3(1+ax)^2}\,dx\\
&= \int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{3a^2}{x}-\frac{a^3}{4(-1+ax)^3}+\frac{a^3}{(-1+ax)^2}-\frac{39a^3}{16(-1+ax)}-\frac{a^3}{8(1+ax)^2}-\frac{9a^3}{16(1+ax)}\right)dx\\
&= -\frac{1}{2x^2}-\frac{a}{x}+\frac{a^2}{8(1-ax)^2}+\frac{a^2}{1-ax}+\frac{a^2}{8(1+ax)}+3a^2\log(x)-\frac{39}{16}a^2\log(1-ax)-\frac{9}{16}a^2\log(1+ax
\end{aligned}$$

Mathematica [A] time = 0.071539, size = 83, normalized size = 0.93

$$\frac{1}{16}\left(\frac{16a^2}{1-ax}+\frac{2a^2}{ax+1}+\frac{2a^2}{(ax-1)^2}+48a^2\log(x)-39a^2\log(1-ax)-9a^2\log(ax+1)-\frac{16a}{x}-\frac{8}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^(5/2)),x]
```

```
[Out] (-8/x^2 - (16*a)/x + (16*a^2)/(1 - a*x) + (2*a^2)/(-1 + a*x)^2 + (2*a^2)/(1
 + a*x) + 48*a^2*Log[x] - 39*a^2*Log[1 - a*x] - 9*a^2*Log[1 + a*x])/16
```

Maple [A] time = 0.041, size = 78, normalized size = 0.9

$$-\frac{1}{2x^{2}}-\frac{a}{x}+3a^{2}\ln(x)+\frac{a^{2}}{8ax+8}-\frac{9a^{2}\ln(ax+1)}{16}-\frac{a^{2}}{ax-1}+\frac{a^{2}}{8(ax-1)^{2}}-\frac{39a^{2}\ln(ax-1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3/x^3,x)
```

```
[Out] -1/2/x^2-a/x+3*a^2*ln(x)+1/8*a^2/(a*x+1)-9/16*a^2*ln(a*x+1)-a^2/(a*x-1)+1/8
*a^2/(a*x-1)^2-39/16*a^2*ln(a*x-1)
```

Maxima [A] time = 0.961134, size = 120, normalized size = 1.35

$$-\frac{9}{16}a^{2}\log(ax+1)-\frac{39}{16}a^{2}\log(ax-1)+3a^{2}\log(x)-\frac{15a^{4}x^{4}-3a^{3}x^{3}-22a^{2}x^{2}+4ax+4}{8\left(a^{3}x^{5}-a^{2}x^{4}-ax^{3}+x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^3,x, algorithm="maxima")
```

```
[Out] -9/16*a^2*log(a*x + 1) - 39/16*a^2*log(a*x - 1) + 3*a^2*log(x) - 1/8*(15*a^
4*x^4 - 3*a^3*x^3 - 22*a^2*x^2 + 4*a*x + 4)/(a^3*x^5 - a^2*x^4 - a*x^3 + x^
2)
```

Fricas [B] time = 2.14004, size = 348, normalized size = 3.91

$$-\frac{30a^{4}x^{4}-6a^{3}x^{3}-44a^{2}x^{2}+8ax+9\left(a^{5}x^{5}-a^{4}x^{4}-a^{3}x^{3}+a^{2}x^{2}\right)\log(ax+1)+39\left(a^{5}x^{5}-a^{4}x^{4}-a^{3}x^{3}+a^{2}x^{2}\right)\log(}{16\left(a^{3}x^{5}-a^{2}x^{4}-ax^{3}+x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(30*a^4*x^4 - 6*a^3*x^3 - 44*a^2*x^2 + 8*a*x + 9*(a^5*x^5 - a^4*x^4 -
 a^3*x^3 + a^2*x^2)*log(a*x + 1) + 39*(a^5*x^5 - a^4*x^4 - a^3*x^3 + a^2*x^
2)*log(a*x - 1) - 48*(a^5*x^5 - a^4*x^4 - a^3*x^3 + a^2*x^2)*log(x) + 8)/(a
^3*x^5 - a^2*x^4 - a*x^3 + x^2)
```

Sympy [A] time = 0.917268, size = 95, normalized size = 1.07

$$3a^{2}\log(x)-\frac{39a^{2}\log\left(x-\frac{1}{a}\right)}{16}-\frac{9a^{2}\log\left(x+\frac{1}{a}\right)}{16}-\frac{15a^{4}x^{4}-3a^{3}x^{3}-22a^{2}x^{2}+4ax+4}{8a^{3}x^{5}-8a^{2}x^{4}-8ax^{3}+8x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3/x**3,x)
```

```
[Out] 3*a**2*log(x) - 39*a**2*log(x - 1/a)/16 - 9*a**2*log(x + 1/a)/16 - (15*a**4
*x**4 - 3*a**3*x**3 - 22*a**2*x**2 + 4*a*x + 4)/(8*a**3*x**5 - 8*a**2*x**4
- 8*a*x**3 + 8*x**2)
```

Giac [A] time = 1.23689, size = 111, normalized size = 1.25

$$-\frac{9}{16}\,a^2\log\left(|ax+1|\right)-\frac{39}{16}\,a^2\log\left(|ax-1|\right)+3\,a^2\log\left(|x|\right)-\frac{15\,a^4x^4-3\,a^3x^3-22\,a^2x^2+4\,ax+4}{8\,(ax+1)(ax-1)^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^3,x, algorithm="giac")
```

```
[Out] -9/16*a^2*log(abs(a*x + 1)) - 39/16*a^2*log(abs(a*x - 1)) + 3*a^2*log(abs(x
)) - 1/8*(15*a^4*x^4 - 3*a^3*x^3 - 22*a^2*x^2 + 4*a*x + 4)/((a*x + 1)*(a*x
- 1)^2*x^2)
```

3.948 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4\left(1-a^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=102

$$\frac{5a^3}{4(1-ax)}-\frac{a^3}{8(ax+1)}+\frac{a^3}{8(1-ax)^2}-\frac{3a^2}{x}+3a^3\log(x)-\frac{59}{16}a^3\log(1-ax)+\frac{11}{16}a^3\log(ax+1)-\frac{a}{2x^2}-\frac{1}{3x^3}$$

```
[Out] -1/(3*x^3) - a/(2*x^2) - (3*a^2)/x + a^3/(8*(1 - a*x)^2) + (5*a^3)/(4*(1 -
a*x)) - a^3/(8*(1 + a*x)) + 3*a^3*Log[x] - (59*a^3*Log[1 - a*x])/16 + (11*a
^3*Log[1 + a*x])/16
```

Rubi [A] time = 0.145136, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{5a^3}{4(1-ax)}-\frac{a^3}{8(ax+1)}+\frac{a^3}{8(1-ax)^2}-\frac{3a^2}{x}+3a^3\log(x)-\frac{59}{16}a^3\log(1-ax)+\frac{11}{16}a^3\log(ax+1)-\frac{a}{2x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*(1 - a^2*x^2)^(5/2)),x]
```

```
[Out] -1/(3*x^3) - a/(2*x^2) - (3*a^2)/x + a^3/(8*(1 - a*x)^2) + (5*a^3)/(4*(1 -
a*x)) - a^3/(8*(1 + a*x)) + 3*a^3*Log[x] - (59*a^3*Log[1 - a*x])/16 + (11*a
^3*Log[1 + a*x])/16
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{x^4\left(1-a^2x^2\right)^{5/2}}\,dx &= \int \frac{1}{x^4(1-ax)^3(1+ax)^2}\,dx\\ &= \int \left(\frac{1}{x^4}+\frac{a}{x^3}+\frac{3a^2}{x^2}+\frac{3a^3}{x}-\frac{a^4}{4(-1+ax)^3}+\frac{5a^4}{4(-1+ax)^2}-\frac{59a^4}{16(-1+ax)}+\frac{a^4}{8(1+ax)^2}+\frac{11}{16(1}\right.\\ &= -\frac{1}{3x^3}-\frac{a}{2x^2}-\frac{3a^2}{x}+\frac{a^3}{8(1-ax)^2}+\frac{5a^3}{4(1-ax)}-\frac{a^3}{8(1+ax)}+3a^3\log(x)-\frac{59}{16}a^3\log(1-ax)+\frac{1}{1}\end{aligned}$$

Mathematica [A] time = 0.0830081, size = 91, normalized size = 0.89

$$\frac{1}{48}\left(\frac{60a^3}{1-ax}-\frac{6a^3}{ax+1}+\frac{6a^3}{(ax-1)^2}-\frac{144a^2}{x}+144a^3\log(x)-177a^3\log(1-ax)+33a^3\log(ax+1)-\frac{24a}{x^2}-\frac{16}{x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*(1 - a^2*x^2)^(5/2)),x]
```

```
[Out] (-16/x^3 - (24*a)/x^2 - (144*a^2)/x + (60*a^3)/(1 - a*x) + (6*a^3)/(-1 + a*
x)^2 - (6*a^3)/(1 + a*x) + 144*a^3*Log[x] - 177*a^3*Log[1 - a*x] + 33*a^3*L
og[1 + a*x])/48
```

Maple [A] time = 0.042, size = 86, normalized size = 0.8

$$-\frac{1}{3x^3}-\frac{a}{2x^2}-3\frac{a^2}{x}+3a^3\ln(x)-\frac{a^3}{8ax+8}+\frac{11a^3\ln(ax+1)}{16}+\frac{a^3}{8(ax-1)^2}-\frac{5a^3}{4ax-4}-\frac{59a^3\ln(ax-1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3/x^4,x)
```

```
[Out] -1/3/x^3-1/2*a/x^2-3*a^2/x+3*a^3*ln(x)-1/8*a^3/(a*x+1)+11/16*a^3*ln(a*x+1)+
1/8*a^3/(a*x-1)^2-5/4*a^3/(a*x-1)-59/16*a^3*ln(a*x-1)
```

Maxima [A] time = 0.963698, size = 131, normalized size = 1.28

$$\frac{11}{16}a^3\log(ax+1)-\frac{59}{16}a^3\log(ax-1)+3a^3\log(x)-\frac{105a^5x^5-69a^4x^4-106a^3x^3+52a^2x^2+4ax+8}{24\left(a^3x^6-a^2x^5-ax^4+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^4,x, algorithm="maxima")
```

```
[Out] 11/16*a^3*log(a*x + 1) - 59/16*a^3*log(a*x - 1) + 3*a^3*log(x) - 1/24*(105*
a^5*x^5 - 69*a^4*x^4 - 106*a^3*x^3 + 52*a^2*x^2 + 4*a*x + 8)/(a^3*x^6 - a^2
*x^5 - a*x^4 + x^3)
```

Fricas [B] time = 2.03898, size = 378, normalized size = 3.71

$$-\frac{210a^5x^5-138a^4x^4-212a^3x^3+104a^2x^2+8ax-33\left(a^6x^6-a^5x^5-a^4x^4+a^3x^3\right)\log(ax+1)+177\left(a^6x^6-a^5x^5-a^4x\right.}{48\left(a^3x^6-a^2x^5-ax^4+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(210*a^5*x^5 - 138*a^4*x^4 - 212*a^3*x^3 + 104*a^2*x^2 + 8*a*x - 33*(
a^6*x^6 - a^5*x^5 - a^4*x^4 + a^3*x^3)*log(a*x + 1) + 177*(a^6*x^6 - a^5*x^
5 - a^4*x^4 + a^3*x^3)*log(a*x - 1) - 144*(a^6*x^6 - a^5*x^5 - a^4*x^4 + a^
3*x^3)*log(x) + 16)/(a^3*x^6 - a^2*x^5 - a*x^4 + x^3)
```

Sympy [A] time = 1.17001, size = 104, normalized size = 1.02

$$3a^3\log(x) - \frac{59a^3\log\left(x - \frac{1}{a}\right)}{16} + \frac{11a^3\log\left(x + \frac{1}{a}\right)}{16} - \frac{105a^5x^5 - 69a^4x^4 - 106a^3x^3 + 52a^2x^2 + 4ax + 8}{24a^3x^6 - 24a^2x^5 - 24ax^4 + 24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3/x**4,x)
```

```
[Out] 3*a**3*log(x) - 59*a**3*log(x - 1/a)/16 + 11*a**3*log(x + 1/a)/16 - (105*a*
*5*x**5 - 69*a**4*x**4 - 106*a**3*x**3 + 52*a**2*x**2 + 4*a*x + 8)/(24*a**3
*x**6 - 24*a**2*x**5 - 24*a*x**4 + 24*x**3)
```

Giac [A] time = 1.17724, size = 122, normalized size = 1.2

$$\frac{11}{16}a^3\log(|ax+1|) - \frac{59}{16}a^3\log(|ax-1|) + 3a^3\log(|x|) - \frac{105a^5x^5 - 69a^4x^4 - 106a^3x^3 + 52a^2x^2 + 4ax + 8}{24(ax+1)(ax-1)^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^4,x, algorithm="giac")
```

```
[Out] 11/16*a^3*log(abs(a*x + 1)) - 59/16*a^3*log(abs(a*x - 1)) + 3*a^3*log(abs(x
)) - 1/24*(105*a^5*x^5 - 69*a^4*x^4 - 106*a^3*x^3 + 52*a^2*x^2 + 4*a*x + 8)
/((a*x + 1)*(a*x - 1)^2*x^3)
```

3.949 $\int e^{\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=74

$$\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}+\frac{x^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}$$

```
[Out] (x^3*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - a^2*x^2]) + (a*x^4*Sqrt[c - a^2*c*x^2
])/(4*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.184429, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 43}

$$\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}+\frac{x^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^3*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - a^2*x^2]) + (a*x^4*Sqrt[c - a^2*c*x^2
])/(4*Sqrt[1 - a^2*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{\tanh^{-1}(ax)}x^2\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}} \\ &= \frac{\sqrt{c-a^2cx^2}\int x^2(1+ax)\,dx}{\sqrt{1-a^2x^2}} \\ &= \frac{\sqrt{c-a^2cx^2}\int \left(x^2+ax^3\right)\,dx}{\sqrt{1-a^2x^2}} \\ &= \frac{x^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}+\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0231548, size = 42, normalized size = 0.57

$$\frac{x^3(3ax+4)\sqrt{c-a^2cx^2}}{12\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^3*(4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.027, size = 37, normalized size = 0.5

$$\frac{x^3\,(3\,ax+4)}{12}\sqrt{-a^2cx^2+c}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/12*x^3*(3*a*x+4)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02855, size = 107, normalized size = 1.45

$$-\frac{\sqrt{-a^2cx^2+c}\left(3\,ax^4+4\,x^3\right)\sqrt{-a^2x^2+1}}{12\,\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] -1/12*sqrt(-a^2*c*x^2 + c)*(3*a*x^4 + 4*x^3)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 -
1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x +
 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax+1)x^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^2/sqrt(-a^2*x^2 + 1), x)
```

3.950 $\int e^{\tanh^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=74

$$\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}+\frac{x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}$$

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/(2*Sqrt[1 - a^2*x^2]) + (a*x^3*Sqrt[c - a^2*c*x^2
])/(3*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.127794, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6153, 6150, 43}

$$\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}+\frac{x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/(2*Sqrt[1 - a^2*x^2]) + (a*x^3*Sqrt[c - a^2*c*x^2
])/(3*Sqrt[1 - a^2*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int x(1+ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(x+ax^2\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}} + \frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0192384, size = 42, normalized size = 0.57

$$\frac{x^2(2ax+3)\sqrt{c-a^2cx^2}}{6\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^2*(3 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(6*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.028, size = 37, normalized size = 0.5

$$\frac{x^2\,(2\,ax+3)}{6}\sqrt{-a^2cx^2+c}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/6*x^2*(2*a*x+3)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.22186, size = 105, normalized size = 1.42

$$-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\left(2\,ax^3+3\,x^2\right)}{6\,\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] -1/6*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(2*a*x^3 + 3*x^2)/(a^2*x^2 - 1
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax+1)x}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x/sqrt(-a^2*x^2 + 1), x)
```

3.951 $\int e^{\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=69

$$\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}+\frac{x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

```
[Out] (x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] + (a*x^2*Sqrt[c - a^2*c*x^2])/(2*
Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0720662, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6143, 6140}

$$\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}+\frac{x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] + (a*x^2*Sqrt[c - a^2*c*x^2])/(2*
Sqrt[1 - a^2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx&=\frac{\sqrt{c-a^2cx^2}\int e^{\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}\\&=\frac{\sqrt{c-a^2cx^2}\int(1+ax)\,dx}{\sqrt{1-a^2x^2}}\\&=\frac{x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0153475, size = 40, normalized size = 0.58

$$\frac{\left(\frac{ax^2}{2}+x\right)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((x + (a*x^2)/2)*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.025, size = 34, normalized size = 0.5

$$\frac{x(ax+2)}{2}\sqrt{-a^2cx^2+c}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*x*(a*x+2)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.32863, size = 100, normalized size = 1.45

$$-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\left(ax^2+2x\right)}{2\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x^2 + 2*x)/(a^2*x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c(ax-1)(ax+1)}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/sqrt(-a^2*x^2 + 1), x)
```

3.952 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=65

$$\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

```
[Out] (a*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] + (Sqrt[c - a^2*c*x^2]*Log[x])/
Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.175835, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 43}

$$\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] (a*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] + (Sqrt[c - a^2*c*x^2]*Log[x])/
Sqrt[1 - a^2*x^2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{1+ax}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(a+\frac{1}{x}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}} + \frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0185091, size = 36, normalized size = 0.55

$$\frac{\sqrt{c-a^2cx^2}(ax+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x + Log[x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.093, size = 48, normalized size = 0.7

$$\frac{-ax-\ln(x)}{a^2x^2-1}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] (-a*x-ln(x))*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.4956, size = 555, normalized size = 8.54

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\left(x^4-1\right)\sqrt{c}-c}{a^2x^4-x^2}\right)-2\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}(ax-a)}{2\left(a^2x^2-1\right)}, \frac{\left(a^2x^2-1\right)\sqrt{-c}\,\mathrm{arctan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="f
ricas")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 - sqrt(-a^2*
c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x^4 - 1)*sqrt(c) - c)/(a^2*x^4 - x^2)) - 2*s
qrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x - a))/(a^2*x^2 - 1), ((a^2*x^2
- 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x^2 + 1)*sqrt
(-c)/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) - sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^
2 + 1)*(a*x - a))/(a^2*x^2 - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\left(ax+1\right)}{x\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x*sqrt(-(a*x - 1)*(a*x + 1
))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/(sqrt(-a^2*x^2 + 1)*x), x)
```

3.953 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=68

$$\frac{a\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}$$

```
[Out] -(Sqrt[c - a^2*c*x^2]/(x*Sqrt[1 - a^2*x^2])) + (a*Sqrt[c - a^2*c*x^2]*Log[x
])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.178264, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 43}

$$\frac{a\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(x*Sqrt[1 - a^2*x^2])) + (a*Sqrt[c - a^2*c*x^2]*Log[x
])/Sqrt[1 - a^2*x^2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{1+ax}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}+\frac{a}{x}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}} + \frac{a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0191225, size = 40, normalized size = 0.59

$$\frac{\sqrt{c-a^2cx^2}\left(a\log(x)-\frac{1}{x}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-x^(-1) + a*Log[x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.089, size = 50, normalized size = 0.7

$$\frac{-a\ln(x)\,x+1}{x\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] (-a*ln(x)*x+1)*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.37657, size = 555, normalized size = 8.16

$$\left[\frac{\left(a^3x^3-ax\right)\sqrt{c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\left(x^4-1\right)\sqrt{c}-c}{a^2x^4-x^2}\right)-2\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}(x-1)}{2\left(a^2x^3-x\right)}, \frac{\left(a^3x^3-ax\right)\sqrt{-c}\,\mathrm{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"fricas")
```

```
[Out] [1/2*((a^3*x^3 - a*x)*sqrt(c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 - sqrt(-a^
2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x^4 - 1)*sqrt(c) - c)/(a^2*x^4 - x^2)) - 2
*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x - 1))/(a^2*x^3 - x), ((a^3*x^3
- a*x)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x^2 + 1)*sq
rt(-c)/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) - sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*
x^2 + 1)*(x - 1))/(a^2*x^3 - x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{x^2\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**2*sqrt(-(a*x - 1)*(a*x
+ 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

3.954 $\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=89

$$\frac{2c(ax+1)^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{c(ax+1)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

```
[Out] (2*c*(1 + a*x)^3*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) - (c*(1 + a*x
)^4*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0872342, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6143, 6140, 43}

$$\frac{2c(ax+1)^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{c(ax+1)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*c*(1 + a*x)^3*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) - (c*(1 + a*x
)^4*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \frac{\left(c\sqrt{c-a^2cx^2}\right)\int e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(c\sqrt{c-a^2cx^2}\right)\int (1-ax)(1+ax)^2\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(c\sqrt{c-a^2cx^2}\right)\int \left(2(1+ax)^2-(1+ax)^3\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{2c(1+ax)^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}} - \frac{c(1+ax)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0270392, size = 57, normalized size = 0.64

$$-\frac{cx\left(3a^3x^3+4a^2x^2-6ax-12\right)\sqrt{c-a^2cx^2}}{12\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -(c*x*Sqrt[c - a^2*c*x^2]*(-12 - 6*a*x + 4*a^2*x^2 + 3*a^3*x^3))/(12*Sqrt[1
 - a^2*x^2])
```

Maple [A] time = 0.032, size = 65, normalized size = 0.7

$$\frac{x\left(3\,x^3a^3+4\,a^2x^2-6\,ax-12\right)}{(12\,ax-12)\,(ax+1)}\left(-a^2cx^2+c\right)^{\frac{3}{2}}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/12*x*(3*a^3*x^3+4*a^2*x^2-6*a*x-12)*(-a^2*c*x^2+c)^(3/2)/(a*x-1)/(a*x+1)/
(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.03253, size = 178, normalized size = 2.

$$-\frac{1}{3}\,a^2c^{\frac{3}{2}}x^3+c^{\frac{3}{2}}x+\frac{1}{4}\left(\frac{2\,a^8c^4\log\left(x^2-\frac{1}{a^2}\right)}{\left(a^4c\right)^{\frac{5}{2}}}+\frac{2\,a^6c^3x^2}{\left(a^4c\right)^{\frac{3}{2}}}+\frac{a^4c^2x^4}{\sqrt{a^4c}}-2\,c^2\sqrt{\frac{1}{a^4c}}\log\left(x^2-\frac{1}{a^2}\right)-\frac{4\,\sqrt{a^4cx^4-2\,a^2cx^2+cc}}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] -1/3*a^2*c^(3/2)*x^3 + c^(3/2)*x + 1/4*(2*a^8*c^4*log(x^2 - 1/a^2)/(a^4*c)^
(5/2) + 2*a^6*c^3*x^2/(a^4*c)^(3/2) + a^4*c^2*x^4/sqrt(a^4*c) - 2*c^2*sqrt(
```

```
1/(a^4*c))*log(x^2 - 1/a^2) - 4*sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c)*c/a^2)*a
```

Fricas [A] time = 2.08864, size = 147, normalized size = 1.65

$$\frac{\left(3\,a^{3}cx^{4}+4\,a^{2}cx^{3}-6\,acx^{2}-12\,cx\right)\sqrt{-a^{2}cx^{2}+c}\sqrt{-a^{2}x^{2}+1}}{12\,\left(a^{2}x^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/12*(3*a^3*c*x^4 + 4*a^2*c*x^3 - 6*a*c*x^2 - 12*c*x)*sqrt(-a^2*c*x^2 + c)*
sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}\left(ax+1\right)}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)/sqrt(-a^2*x^2 + 1), x)
```

3.955 $\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=136

$$\frac{c^2(ax+1)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}-\frac{4c^2(ax+1)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{c^2(ax+1)^4\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}$$

```
[Out] (c^2*(1 + a*x)^4*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) - (4*c^2*(1 + a
*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) + (c^2*(1 + a*x)^6*Sqrt[
c - a^2*c*x^2])/(6*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0949734, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6143, 6140, 43}

$$\frac{c^2(ax+1)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}-\frac{4c^2(ax+1)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{c^2(ax+1)^4\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*(1 + a*x)^4*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) - (4*c^2*(1 + a
*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) + (c^2*(1 + a*x)^6*Sqrt[
c - a^2*c*x^2])/(6*a*Sqrt[1 - a^2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}dx = \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int (1-ax)^2(1+ax)^3\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int \left(4(1+ax)^3-4(1+ax)^4+(1+ax)^5\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{c^2(1+ax)^4\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4c^2(1+ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{c^2(1+ax)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0380002, size = 60, normalized size = 0.44

$$\frac{c^2(ax+1)^4\left(5a^2x^2-14ax+11\right)\sqrt{c-a^2cx^2}}{30a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*(1 + a*x)^4*(11 - 14*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(30*a*Sqrt[
1 - a^2*x^2])
```

Maple [A] time = 0.03, size = 81, normalized size = 0.6

$$\frac{x\left(5x^5a^5+6x^4a^4-15x^3a^3-20a^2x^2+15ax+30\right)}{30\,(ax+1)^2(ax-1)^2}\left(-a^2cx^2+c\right)^{\frac{5}{2}}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/30*x*(5*a^5*x^5+6*a^4*x^4-15*a^3*x^3-20*a^2*x^2+15*a*x+30)*(-a^2*c*x^2+c)
^(5/2)/(a*x-1)^2/(a*x+1)^2/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.01905, size = 240, normalized size = 1.76

$$\frac{1}{5}a^4c^{\frac{5}{2}}x^5-\frac{2}{3}a^2c^{\frac{5}{2}}x^3+c^{\frac{5}{2}}x+\frac{1}{6}\left(\frac{4a^8c^5\log\left(x^2-\frac{1}{a^2}\right)}{\left(a^4c\right)^{\frac{5}{2}}}+\frac{4a^6c^4x^2}{\left(a^4c\right)^{\frac{3}{2}}}+\frac{2a^4c^3x^4}{\sqrt{a^4c}}-\sqrt{a^4cx^4-2a^2cx^2+c}a^2c^2x^4-4c^3\sqrt{\frac{1}{a^4c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

```
[Out] 1/5*a^4*c^(5/2)*x^5 - 2/3*a^2*c^(5/2)*x^3 + c^(5/2)*x + 1/6*(4*a^8*c^5*log(
x^2 - 1/a^2)/(a^4*c)^(5/2) + 4*a^6*c^4*x^2/(a^4*c)^(3/2) + 2*a^4*c^3*x^4/sq
```

```
rt(a^4*c) - sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c)*a^2*c^2*x^4 - 4*c^3*sqrt(1/(a
^4*c))*log(x^2 - 1/a^2) - 7*sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c)*c^2/a^2)*a
```

Fricas [A] time = 2.12908, size = 207, normalized size = 1.52

$$-\frac{\left(5\,a^{5}c^{2}x^{6}+6\,a^{4}c^{2}x^{5}-15\,a^{3}c^{2}x^{4}-20\,a^{2}c^{2}x^{3}+15\,ac^{2}x^{2}+30\,c^{2}x\right)\sqrt{-a^{2}cx^{2}+c}\sqrt{-a^{2}x^{2}+1}}{30\left(a^{2}x^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

```
[Out] -1/30*(5*a^5*c^2*x^6 + 6*a^4*c^2*x^5 - 15*a^3*c^2*x^4 - 20*a^2*c^2*x^3 + 15
*a*c^2*x^2 + 30*c^2*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}\left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}\left(ax+1\right)}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)/sqrt(-a^2*x^2 + 1), x)
```

3.956 $\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=183

$$-\frac{c^3(ax+1)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{6c^3(ax+1)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{2c^3(ax+1)^6\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}+\frac{8c^3(ax+1)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}$$

```
[Out] (8*c^3*(1 + a*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) - (2*c^3*(1
 + a*x)^6*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) + (6*c^3*(1 + a*x)^7*S
qrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) - (c^3*(1 + a*x)^8*Sqrt[c - a^2
*c*x^2])/(8*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.105045, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6143, 6140, 43}

$$-\frac{c^3(ax+1)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{6c^3(ax+1)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{2c^3(ax+1)^6\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}+\frac{8c^3(ax+1)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (8*c^3*(1 + a*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) - (2*c^3*(1
 + a*x)^6*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) + (6*c^3*(1 + a*x)^7*S
qrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) - (c^3*(1 + a*x)^8*Sqrt[c - a^2
*c*x^2])/(8*a*Sqrt[1 - a^2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}dx=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{7/2}dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int(1-ax)^3(1+ax)^4\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int\left(8(1+ax)^4-12(1+ax)^5+6(1+ax)^6-(1+ax)^7\right)dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{8c^3(1+ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}-\frac{2c^3(1+ax)^6\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}+\frac{6c^3(1+ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{c^3(1+a}{8a}$$

Mathematica [A] time = 0.0463095, size = 68, normalized size = 0.37

$$-\frac{c^3(ax+1)^5\left(35a^3x^3-135a^2x^2+185ax-93\right)\sqrt{c-a^2cx^2}}{280a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -(c^3*(1 + a*x)^5*Sqrt[c - a^2*c*x^2]*(-93 + 185*a*x - 135*a^2*x^2 + 35*a^3
*x^3))/(280*a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.03, size = 97, normalized size = 0.5

$$\frac{x\left(35\,a^7x^7+40\,x^6a^6-140\,x^5a^5-168\,x^4a^4+210\,x^3a^3+280\,a^2x^2-140\,ax-280\right)}{280\,(ax+1)^3\,(ax-1)^3}\left(-a^2cx^2+c\right)^{\frac{7}{2}}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/280*x*(35*a^7*x^7+40*a^6*x^6-140*a^5*x^5-168*a^4*x^4+210*a^3*x^3+280*a^2*
x^2-140*a*x-280)*(-a^2*c*x^2+c)^(7/2)/(a*x-1)^3/(a*x+1)^3/(-a^2*x^2+1)^(1/2
)
```

Maxima [A] time = 1.02817, size = 297, normalized size = 1.62

$$-\frac{1}{7}a^6c^{\frac{7}{2}}x^7+\frac{3}{5}a^4c^{\frac{7}{2}}x^5-a^2c^{\frac{7}{2}}x^3+c^{\frac{7}{2}}x+\frac{1}{8}\left(\sqrt{a^4cx^4-2\,a^2cx^2+c}a^4c^3x^6+\frac{6\,a^8c^6\log\left(x^2-\frac{1}{a^2}\right)}{\left(a^4c\right)^{\frac{5}{2}}}+\frac{6\,a^6c^5x^2}{\left(a^4c\right)^{\frac{3}{2}}}+\frac{3\,a^4c^4x^4}{\sqrt{a^4c}}-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="max
ima")
```

```
[Out] -1/7*a^6*c^(7/2)*x^7 + 3/5*a^4*c^(7/2)*x^5 - a^2*c^(7/2)*x^3 + c^(7/2)*x +
1/8*(sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c)*a^4*c^3*x^6 + 6*a^8*c^6*log(x^2 - 1/
```

```
a^2)/(a^4*c)^(5/2) + 6*a^6*c^5*x^2/(a^4*c)^(3/2) + 3*a^4*c^4*x^4/sqrt(a^4*c
) - 3*sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c)*a^2*c^3*x^4 - 6*c^4*sqrt(1/(a^4*c))
*log(x^2 - 1/a^2) - 10*sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c)*c^3/a^2)*a
```

Fricas [A] time = 2.18842, size = 263, normalized size = 1.44

$$\frac{\left(35\,a^7c^3x^8+40\,a^6c^3x^7-140\,a^5c^3x^6-168\,a^4c^3x^5+210\,a^3c^3x^4+280\,a^2c^3x^3-140\,ac^3x^2-280\,c^3x\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{280\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fri
cas")
```

```
[Out] 1/280*(35*a^7*c^3*x^8 + 40*a^6*c^3*x^7 - 140*a^5*c^3*x^6 - 168*a^4*c^3*x^5
+ 210*a^3*c^3*x^4 + 280*a^2*c^3*x^3 - 140*a*c^3*x^2 - 280*c^3*x)*sqrt(-a^2*
c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}\left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(ax+1\right)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="gia
c")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*(a*x + 1)/sqrt(-a^2*x^2 + 1), x)
```

3.957 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=194

$$-\frac{x^4\sqrt{1-a^2x^2}}{4a\sqrt{c-a^2cx^2}}-\frac{x^3\sqrt{1-a^2x^2}}{3a^2\sqrt{c-a^2cx^2}}-\frac{x^2\sqrt{1-a^2x^2}}{2a^3\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}}{a^4\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^5\sqrt{c-a^2cx^2}}$$

```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a^4*Sqrt[c - a^2*c*x^2])) - (x^2*Sqrt[1 - a^2*x^2]
)/(2*a^3*Sqrt[c - a^2*c*x^2]) - (x^3*Sqrt[1 - a^2*x^2])/(3*a^2*Sqrt[c - a^2
*c*x^2]) - (x^4*Sqrt[1 - a^2*x^2])/(4*a*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^
2*x^2]*Log[1 - a*x])/(a^5*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.212941, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 43}

$$-\frac{x^4\sqrt{1-a^2x^2}}{4a\sqrt{c-a^2cx^2}}-\frac{x^3\sqrt{1-a^2x^2}}{3a^2\sqrt{c-a^2cx^2}}-\frac{x^2\sqrt{1-a^2x^2}}{2a^3\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}}{a^4\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^5\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/Sqrt[c - a^2*c*x^2],x]
```


```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a^4*Sqrt[c - a^2*c*x^2])) - (x^2*Sqrt[1 - a^2*x^2]
)/(2*a^3*Sqrt[c - a^2*c*x^2]) - (x^3*Sqrt[1 - a^2*x^2])/(3*a^2*Sqrt[c - a^2
*c*x^2]) - (x^4*Sqrt[1 - a^2*x^2])/(4*a*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^
2*x^2]*Log[1 - a*x])/(a^5*Sqrt[c - a^2*c*x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^4}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^4}{1-ax}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{a^4}-\frac{x}{a^3}-\frac{x^2}{a^2}-\frac{x^3}{a}-\frac{1}{a^4(-1+ax)}\right)dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{x\sqrt{1-a^2x^2}}{a^4\sqrt{c-a^2cx^2}}-\frac{x^2\sqrt{1-a^2x^2}}{2a^3\sqrt{c-a^2cx^2}}-\frac{x^3\sqrt{1-a^2x^2}}{3a^2\sqrt{c-a^2cx^2}}-\frac{x^4\sqrt{1-a^2x^2}}{4a\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^5\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0451362, size = 71, normalized size = 0.37

$$-\frac{\sqrt{1-a^2x^2}\left(ax\left(3a^3x^3+4a^2x^2+6ax+12\right)+12\log(1-ax)\right)}{12a^5\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]*(a*x*(12 + 6*a*x + 4*a^2*x^2 + 3*a^3*x^3) + 12*Log[1 -
a*x]))/(12*a^5*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.085, size = 83, normalized size = 0.4

$$\frac{3x^4a^4+4x^3a^3+6a^2x^2+12ax+12\ln(ax-1)}{\left(12a^2x^2-12\right)ca^5}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/12*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(3*x^4*a^4+4*x^3*a^3+6*a^2*x
^2+12*a*x+12*ln(a*x-1))/(a^2*x^2-1)/c/a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^4}{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^4/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [A] time = 2.46467, size = 828, normalized size = 4.27

$$\left[\frac{6\left(a^{2}x^{2}-1\right)\sqrt{c}\log\left(\frac{a^{6}cx^{6}-4a^{5}cx^{5}+5a^{4}cx^{4}-4a^{2}cx^{2}+4acx+\left(a^{4}x^{4}-4a^{3}x^{3}+6a^{2}x^{2}-4ax\right)\sqrt{-a^{2}cx^{2}+c}\sqrt{-a^{2}x^{2}+1}\sqrt{c}-2c}{a^{4}x^{4}-2a^{3}x^{3}+2ax-1}\right)+\left(3a^{4}x^{4}+4a^{3}x^{3}+6a^{2}\right.}{12\left(a^{7}cx^{2}-a^{5}c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/12*(6*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 -
 4*a^2*c*x^2 + 4*a*c*x + (a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^
2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x
 - 1)) + (3*a^4*x^4 + 4*a^3*x^3 + 6*a^2*x^2 + 12*a*x)*sqrt(-a^2*c*x^2 + c)*
sqrt(-a^2*x^2 + 1))/(a^7*c*x^2 - a^5*c), -1/12*(12*(a^2*x^2 - 1)*sqrt(-c)*a
rctan(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c
)/(a^4*c*x^4 - 2*a^3*c*x^3 - a^2*c*x^2 + 2*a*c*x)) - (3*a^4*x^4 + 4*a^3*x^3
 + 6*a^2*x^2 + 12*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^7*c*x^2
- a^5*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}(ax+1)}{\sqrt{-(ax-1)(ax+1)}\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x
+ 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^{4}}{\sqrt{-a^{2}cx^{2}+c}\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^4/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

3.958 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=155

$$-\frac{x^3\sqrt{1-a^2x^2}}{3a\sqrt{c-a^2cx^2}}-\frac{x^2\sqrt{1-a^2x^2}}{2a^2\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}}{a^3\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^4\sqrt{c-a^2cx^2}}$$

```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a^3*Sqrt[c - a^2*c*x^2])) - (x^2*Sqrt[1 - a^2*x^2]
)/(2*a^2*Sqrt[c - a^2*c*x^2]) - (x^3*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[c - a^2*c
*x^2]) - (Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(a^4*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.209687, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 43}

$$-\frac{x^3\sqrt{1-a^2x^2}}{3a\sqrt{c-a^2cx^2}}-\frac{x^2\sqrt{1-a^2x^2}}{2a^2\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}}{a^3\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^4\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a^3*Sqrt[c - a^2*c*x^2])) - (x^2*Sqrt[1 - a^2*x^2]
)/(2*a^2*Sqrt[c - a^2*c*x^2]) - (x^3*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[c - a^2*c
*x^2]) - (Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(a^4*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^3}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^3}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^3}{1-ax}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{a^3}-\frac{x}{a^2}-\frac{x^2}{a}-\frac{1}{a^3(-1+ax)}\right)dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{x\sqrt{1-a^2x^2}}{a^3\sqrt{c-a^2cx^2}}-\frac{x^2\sqrt{1-a^2x^2}}{2a^2\sqrt{c-a^2cx^2}}-\frac{x^3\sqrt{1-a^2x^2}}{3a\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^4\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0365916, size = 63, normalized size = 0.41

$$-\frac{\sqrt{1-a^2x^2}\left(ax\left(2a^2x^2+3ax+6\right)+6\log(1-ax)\right)}{6a^4\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]*(a*x*(6 + 3*a*x + 2*a^2*x^2) + 6*Log[1 - a*x]))/(6*a^4*
Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.085, size = 75, normalized size = 0.5

$$\frac{2\,x^3a^3+3\,a^2x^2+6\,ax+6\,\ln(ax-1)}{\left(6\,a^2x^2-6\right)ca^4}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/6*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3+3*a^2*x^2+6*a*x+6*
ln(a*x-1))/(a^2*x^2-1)/c/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{-\frac{1}{6}\,a\left(\frac{2\left(a^2x^3+3\,x\right)}{a^4}-\frac{3\,\log(ax+1)}{a^5}+\frac{3\,\log(ax-1)}{a^5}\right)}{\sqrt{c}}+\frac{\sqrt{\frac{1}{a^4c}}\log\left(x^2-\frac{1}{a^2}\right)}{2\,a^2}+\frac{\sqrt{a^4cx^4-2\,a^2cx^2+c}}{2\,a^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] a*integrate(-x^4/((a*x + 1)*(a*x - 1)), x)/sqrt(c) + 1/2*sqrt(1/(a^4*c))*lo
g(x^2 - 1/a^2)/a^2 + 1/2*sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c)/(a^4*c)
```

Fricas [A] time = 2.4471, size = 788, normalized size = 5.08

$$\left[\frac{3\left(a^2x^2-1\right)\sqrt{c}\log\left(\frac{a^6cx^6-4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2+4\,acx+\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-2\,c}{a^4x^4-2\,a^3x^3+2\,ax-1}\right)+\left(2\,a^3x^3+3\,a^2x^2+6\right.}{6\left(a^6cx^2-a^4c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(3*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 -
4*a^2*c*x^2 + 4*a*c*x + (a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^2
*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x
- 1)) + (2*a^3*x^3 + 3*a^2*x^2 + 6*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2
+ 1))/(a^6*c*x^2 - a^4*c), -1/6*(6*(a^2*x^2 - 1)*sqrt(-c)*arctan(sqrt(-a^2*
c*x^2 + c)*(a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 - 2
*a^3*c*x^3 - a^2*c*x^2 + 2*a*c*x)) - (2*a^3*x^3 + 3*a^2*x^2 + 6*a*x)*sqrt(-
a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^6*c*x^2 - a^4*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x
+ 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^3}{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^3/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

3.959 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=116

$$-\frac{x^2\sqrt{1-a^2x^2}}{2a\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}}{a^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^3\sqrt{c-a^2cx^2}}$$

```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a^2*Sqrt[c - a^2*c*x^2])) - (x^2*Sqrt[1 - a^2*x^2]
)/(2*a*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(a^3*Sqrt[c
- a^2*c*x^2])
```

Rubi [A] time = 0.201208, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 43}

$$-\frac{x^2\sqrt{1-a^2x^2}}{2a\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}}{a^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/Sqrt[c - a^2*c*x^2],x]
```


```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a^2*Sqrt[c - a^2*c*x^2])) - (x^2*Sqrt[1 - a^2*x^2]
)/(2*a*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(a^3*Sqrt[c
- a^2*c*x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^2}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^2}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{1-ax}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{a^2}-\frac{x}{a}-\frac{1}{a^2(-1+ax)}\right)dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{x\sqrt{1-a^2x^2}}{a^2\sqrt{c-a^2cx^2}}-\frac{x^2\sqrt{1-a^2x^2}}{2a\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^3\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0310341, size = 54, normalized size = 0.47

$$-\frac{\sqrt{1-a^2x^2}(ax(ax+2)+2\log(1-ax))}{2a^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]*(a*x*(2 + a*x) + 2*Log[1 - a*x]))/(2*a^3*Sqrt[c - a^2*c
*x^2])
```

Maple [A] time = 0.085, size = 66, normalized size = 0.6

$$\frac{a^2x^2+2\,ax+2\,\ln(ax-1)}{\left(2\,a^2x^2-2\right)ca^3}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a^2*x^2+2*a*x+2*ln(a*x-1))/(
a^2*x^2-1)/c/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^2}{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^2/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [A] time = 2.3834, size = 748, normalized size = 6.45

$$\left[\frac{\left(a^{2} x^{2}-1\right) \sqrt{c} \log \left(\frac{a^{6} c x^{6}-4 a^{5} c x^{5}+5 a^{4} c x^{4}-4 a^{2} c x^{2}+4 a c x+\left(a^{4} x^{4}-4 a^{3} x^{3}+6 a^{2} x^{2}-4 a x\right) \sqrt{-a^{2} c x^{2}+c} \sqrt{-a^{2} x^{2}+1} \sqrt{c}-2 c}{a^{4} x^{4}-2 a^{3} x^{3}+2 a x-1}\right)+\sqrt{-a^{2} c x^{2}+c}\left(a^{2} x^{2}+2 a x\right.}{2\left(a^{5} c x^{2}-a^{3} c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*
a^2*c*x^2 + 4*a*c*x + (a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^2*c
*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x -
1)) + sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 2*a*x)*sqrt(-a^2*x^2 + 1))/(a^5*c*x^2
 - a^3*c), -1/2*(2*(a^2*x^2 - 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*(a^2*
x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 - 2*a^3*c*x^3 - a^2
*c*x^2 + 2*a*c*x)) - sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 2*a*x)*sqrt(-a^2*x^2 +
 1))/(a^5*c*x^2 - a^3*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}(a x+1)}{\sqrt{-(a x-1)(a x+1)} \sqrt{-c(a x-1)(a x+1)}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x
+ 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a x+1) x^{2}}{\sqrt{-a^{2} c x^{2}+c} \sqrt{-a^{2} x^{2}+1}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^2/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

3.960 $\int \frac{e^{\tanh^{-1}(ax)}x}{\sqrt{c-a^2cx^2}} dx$

Optimal. Leaf size=77

$$-\frac{x\sqrt{1-a^2x^2}}{a\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^2\sqrt{c-a^2cx^2}}$$

```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a^2*c*x^2])) - (Sqrt[1 - a^2*x^2]*Log[1
 - a*x])/(a^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.134378, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6153, 6150, 43}

$$-\frac{x\sqrt{1-a^2x^2}}{a\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a^2*c*x^2])) - (Sqrt[1 - a^2*x^2]*Log[1
 - a*x])/(a^2*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x}{1-ax}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{a}-\frac{1}{a(-1+ax)}\right)dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{x\sqrt{1-a^2x^2}}{a\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^2\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.027004, size = 45, normalized size = 0.58

$$-\frac{\sqrt{1-a^2x^2}(ax+\log(1-ax))}{a^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(a*x + Log[1 - a*x]))/(a^2*Sqrt[c - a^2*c*x^2]))
```

Maple [A] time = 0.082, size = 55, normalized size = 0.7

$$\frac{ax+\ln(ax-1)}{c\left(a^2x^2-1\right)a^2}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*x+ln(a*x-1))/(a^2*x^2-1)/c/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{-\frac{1}{2}a\left(\frac{2x}{a^2}-\frac{\log(ax+1)}{a^3}+\frac{\log(ax-1)}{a^3}\right)}{\sqrt{c}}+\frac{1}{2}\sqrt{\frac{1}{a^4c}}\log\left(x^2-\frac{1}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] a*integrate(-x^2/((a*x + 1)*(a*x - 1)), x)/sqrt(c) + 1/2*sqrt(1/(a^4*c))*lo
g(x^2 - 1/a^2)
```

Fricas [B] time = 2.51605, size = 703, normalized size = 9.13

$$\left[\frac{2\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}ax+\left(a^2x^2-1\right)\sqrt{c}\log\left(\frac{a^6cx^6-4a^5cx^5+5a^4cx^4-4a^2cx^2+4acx+\left(a^4x^4-4a^3x^3+6a^2x^2-4ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{a^4x^4-2a^3x^3+2ax-1}\right)}{2\left(a^4cx^2-a^2c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/2*(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x + (a^2*x^2 - 1)*sqrt(c)
*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 + 4*a*c*x + (a^4*
x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1
)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)))/(a^4*c*x^2 - a^2*c), (
sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x - (a^2*x^2 - 1)*sqrt(-c)*arctan
(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^
4*c*x^4 - 2*a^3*c*x^3 - a^2*c*x^2 + 2*a*c*x)))/(a^4*c*x^2 - a^2*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,(ax+1)}{\sqrt{-(ax-1)(ax+1)}\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1
))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x}{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)*x/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

3.961 $\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=41

$$-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a\sqrt{c-a^2cx^2}}$$

```
[Out] -((Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(a*Sqrt[c - a^2*c*x^2]))
```

Rubi [A] time = 0.0711826, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6143, 6140, 31}

$$-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(a*Sqrt[c - a^2*c*x^2]))
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{1}{1-ax}\,dx}{\sqrt{c-a^2cx^2}}\\
&= -\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0123819, size = 41, normalized size = 1.

$$-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(a*Sqrt[c - a^2*c*x^2]))
```

Maple [A] time = 0.082, size = 51, normalized size = 1.2

$$\frac{\ln(ax-1)}{c\left(a^2x^2-1\right)a}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)/c/a*ln(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax+1}{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [B] time = 2.47969, size = 494, normalized size = 12.05

$$\left[\frac{\log\left(\frac{a^6cx^6-4a^5cx^5+5a^4cx^4-4a^2cx^2+4acx+\left(a^4x^4-4a^3x^3+6a^2x^2-4ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-2c}{a^4x^4-2a^3x^3+2ax-1}\right)}{2a\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-2ax+2\right)\sqrt{}}{a^4cx^4-2a^3cx^3-a^2cx^2}\right.}{ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/2*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 + 4*a*c*x + (
a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2
 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1))/(a*sqrt(c)), -sqrt(
-c)*arctan(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sq
```

```
rt(-c)/(a^4*c*x^4 - 2*a^3*c*x^3 - a^2*c*x^2 + 2*a*c*x))/(a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-(ax-1)(ax+1)}\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

3.962 $\int \frac{e^{\tanh^{-1}(ax)}}{x\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=71

$$\frac{\sqrt{1-a^2x^2}\log(x)}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] (Sqrt[1 - a^2*x^2]*Log[x])/Sqrt[c - a^2*c*x^2] - (Sqrt[1 - a^2*x^2]*Log[1 -
 a*x])/Sqrt[c - a^2*c*x^2]
```

Rubi [A] time = 0.181653, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6153, 6150, 36, 29, 31}

$$\frac{\sqrt{1-a^2x^2}\log(x)}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*Log[x])/Sqrt[c - a^2*c*x^2] - (Sqrt[1 - a^2*x^2]*Log[1 -
 a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x(1-ax)}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x}\,dx}{\sqrt{c-a^2cx^2}} + \frac{\left(a\sqrt{1-a^2x^2}\right)\int \frac{1}{1-ax}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\log(x)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0172875, size = 42, normalized size = 0.59

$$\frac{\sqrt{1-a^2x^2}(\log(x)-\log(1-ax))}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(Log[x] - Log[1 - a*x]))/Sqrt[c - a^2*c*x^2]
```

Maple [A] time = 0.087, size = 53, normalized size = 0.8

$$\frac{-\ln(x)+\ln(ax-1)}{c\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-ln(x)+ln(a*x-1))*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [A] time = 2.629, size = 662, normalized size = 9.32

$$\left[\frac{\log\left(-\frac{4\,a^5cx^5-\left(2\,a^6-4\,a^5+6\,a^4-4\,a^3+a^2\right)cx^6-\left(4\,a^4+4\,a^3-6\,a^2+4\,a-1\right)cx^4+5\,a^2cx^2-4\,acx+\left(4\,a^3x^3-\left(4\,a^3-6\,a^2+4\,a-1\right)x^4-6\,a^2x^2+4\,ax-1\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2}}{a^4x^6-2\,a^3x^5+2\,ax^3-x^2}\right.}{2\,\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/2*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6*a^4 - 4*a^3 + a^2)*c*x^6 - (4*a
^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2*c*x^2 - 4*a*c*x + (4*a^3*x^3 -
(4*a^3 - 6*a^2 + 4*a - 1)*x^4 - 6*a^2*x^2 + 4*a*x - 1)*sqrt(-a^2*c*x^2 + c)
*sqrt(-a^2*x^2 + 1)*sqrt(c) + c)/(a^4*x^6 - 2*a^3*x^5 + 2*a*x^3 - x^2))/sqr
t(c), -sqrt(-c)*arctan(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((2*a^2 - 2
*a + 1)*x^2 - 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^3 - (2*a^3 - a^2)*c*x^4 - (a^2
 - 2*a + 1)*c*x^2 - 2*a*c*x + c))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{x\sqrt{-(ax-1)(ax+1)}\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(x*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1
))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x), x)
```

3.963 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\log(x)}{\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] -(Sqrt[1 - a^2*x^2]/(x*Sqrt[c - a^2*c*x^2])) + (a*Sqrt[1 - a^2*x^2]*Log[x])
/Sqrt[c - a^2*c*x^2] - (a*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/Sqrt[c - a^2*c*x^
2]
```

Rubi [A] time = 0.190474, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 44}

$$-\frac{\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\log(x)}{\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*Sqrt[c - a^2*c*x^2]),x]
```


```
[Out] -(Sqrt[1 - a^2*x^2]/(x*Sqrt[c - a^2*c*x^2])) + (a*Sqrt[1 - a^2*x^2]*Log[x])
/Sqrt[c - a^2*c*x^2] - (a*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/Sqrt[c - a^2*c*x^
2]
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^2\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x^2\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x^2(1-ax)}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x^2}+\frac{a}{x}-\frac{a^2}{-1+ax}\right)dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}} + \frac{a\sqrt{1-a^2x^2}\log(x)}{\sqrt{c-a^2cx^2}} - \frac{a\sqrt{1-a^2x^2}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0296793, size = 50, normalized size = 0.47

$$\frac{\sqrt{1-a^2x^2}\left(a\log(x)-a\log(1-ax)-\frac{1}{x}\right)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-x^(-1) + a*Log[x] - a*Log[1 - a*x]))/Sqrt[c - a^2*c*x^
2]
```

Maple [A] time = 0.089, size = 62, normalized size = 0.6

$$\frac{-a\ln(x)\,x+\ln(ax-1)\,xa+1}{c\left(a^2x^2-1\right)x}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-a*ln(x)*x+ln(a*x-1)*x*a+1)*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/(a^2
*x^2-1)/c/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^2), x)
```

Fricas [A] time = 2.64452, size = 894, normalized size = 8.36

$$\left[\frac{\left(a^3x^3-ax\right)\sqrt{c}\log\left(-\frac{4\,a^5cx^5-\left(2\,a^6-4\,a^5+6\,a^4-4\,a^3+a^2\right)cx^6-\left(4\,a^4+4\,a^3-6\,a^2+4\,a-1\right)cx^4+5\,a^2cx^2-4\,acx+\left(4\,a^3x^3-\left(4\,a^3-6\,a^2+4\,a-1\right)x^4-6\,a^2x^2+4\,ax-1\right)\sqrt{}}{a^4x^6-2\,a^3x^5+2\,ax^3-x^2}\right.}{2\left(a^2cx^3-cx\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/2*((a^3*x^3 - a*x)*sqrt(c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6*a^4 -
4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2*c*x^2
- 4*a*c*x + (4*a^3*x^3 - (4*a^3 - 6*a^2 + 4*a - 1)*x^4 - 6*a^2*x^2 + 4*a*x
- 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) + c)/(a^4*x^6 - 2*a^3*
x^5 + 2*a*x^3 - x^2)) - 2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x - 1))/
(a^2*c*x^3 - c*x), -((a^3*x^3 - a*x)*sqrt(-c)*arctan(-sqrt(-a^2*c*x^2 + c)*
sqrt(-a^2*x^2 + 1)*((2*a^2 - 2*a + 1)*x^2 - 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^
3 - (2*a^3 - a^2)*c*x^4 - (a^2 - 2*a + 1)*c*x^2 - 2*a*c*x + c)) + sqrt(-a^2
*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x - 1))/(a^2*c*x^3 - c*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{x^2\sqrt{-\left(ax-1\right)\left(ax+1\right)}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x
+ 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^2), x)
```

3.964 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=148

$$-\frac{a\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}\log(x)}{\sqrt{c-a^2cx^2}}-\frac{a^2\sqrt{1-a^2x^2}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] -Sqrt[1 - a^2*x^2]/(2*x^2*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/(x*S
qrt[c - a^2*c*x^2]) + (a^2*Sqrt[1 - a^2*x^2]*Log[x])/Sqrt[c - a^2*c*x^2] -
(a^2*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Rubi [A] time = 0.19717, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 44}

$$-\frac{a\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}\log(x)}{\sqrt{c-a^2cx^2}}-\frac{a^2\sqrt{1-a^2x^2}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*Sqrt[c - a^2*c*x^2]),x]
```


```
[Out] -Sqrt[1 - a^2*x^2]/(2*x^2*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/(x*S
qrt[c - a^2*c*x^2]) + (a^2*Sqrt[1 - a^2*x^2]*Log[x])/Sqrt[c - a^2*c*x^2] -
(a^2*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x^3\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x^3(1-ax)}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{a^2}{x}-\frac{a^3}{-1+ax}\right)dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}} - \frac{a\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}} + \frac{a^2\sqrt{1-a^2x^2}\log(x)}{\sqrt{c-a^2cx^2}} - \frac{a^2\sqrt{1-a^2x^2}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0405171, size = 62, normalized size = 0.42

$$\frac{\sqrt{1-a^2x^2}\left(a^2\log(x)-a^2\log(1-ax)-\frac{a}{x}-\frac{1}{2x^2}\right)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-1/(2*x^2) - a/x + a^2*Log[x] - a^2*Log[1 - a*x]))/Sqrt
[c - a^2*c*x^2]
```

Maple [A] time = 0.099, size = 76, normalized size = 0.5

$$-\frac{2\,a^2\ln(x)\,x^2-2\,\ln(ax-1)\,a^2x^2-2\,ax-1}{\left(2\,a^2x^2-2\right)cx^2}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(2*a^2*ln(x)*x^2-2*ln(a*x-1)
*a^2*x^2-2*a*x-1)/(a^2*x^2-1)/c/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.70757, size = 969, normalized size = 6.55

$$\left[\frac{\left(a^4x^4-a^2x^2\right)\sqrt{c}\log\left(-\frac{4\,a^5cx^5-\left(2\,a^6-4\,a^5+6\,a^4-4\,a^3+a^2\right)cx^6-\left(4\,a^4+4\,a^3-6\,a^2+4\,a-1\right)cx^4+5\,a^2cx^2-4\,acx+\left(4\,a^3x^3-\left(4\,a^3-6\,a^2+4\,a-1\right)x^4-6\,a^2x^2+4\,a\right.}{a^4x^6-2\,a^3x^5+2\,ax^3-x^2}\right.}{2\left(a^2cx^4-cx^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/2*((a^4*x^4 - a^2*x^2)*sqrt(c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6*a^
4 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2*c*
x^2 - 4*a*c*x + (4*a^3*x^3 - (4*a^3 - 6*a^2 + 4*a - 1)*x^4 - 6*a^2*x^2 + 4*
a*x - 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) + c)/(a^4*x^6 - 2*
a^3*x^5 + 2*a*x^3 - x^2)) - sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((2*a +
 1)*x^2 - 2*a*x - 1))/(a^2*c*x^4 - c*x^2), -1/2*(2*(a^4*x^4 - a^2*x^2)*sqrt
(-c)*arctan(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((2*a^2 - 2*a + 1)*x^2
 - 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^3 - (2*a^3 - a^2)*c*x^4 - (a^2 - 2*a + 1)
*c*x^2 - 2*a*c*x + c)) + sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((2*a + 1)
*x^2 - 2*a*x - 1))/(a^2*c*x^4 - c*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{x^3\sqrt{-(ax-1)(ax+1)}\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x
 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^3), x)
```

3.965 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=187

$$-\frac{a^2\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{3x^3\sqrt{c-a^2cx^2}}+\frac{a^3\sqrt{1-a^2x^2}\log(x)}{\sqrt{c-a^2cx^2}}-\frac{a^3\sqrt{1-a^2x^2}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] -Sqrt[1 - a^2*x^2]/(3*x^3*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/(2*x
^2*Sqrt[c - a^2*c*x^2]) - (a^2*Sqrt[1 - a^2*x^2])/(x*Sqrt[c - a^2*c*x^2]) +
 (a^3*Sqrt[1 - a^2*x^2]*Log[x])/Sqrt[c - a^2*c*x^2] - (a^3*Sqrt[1 - a^2*x^2
]*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Rubi [A] time = 0.19943, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 44}

$$-\frac{a^2\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{3x^3\sqrt{c-a^2cx^2}}+\frac{a^3\sqrt{1-a^2x^2}\log(x)}{\sqrt{c-a^2cx^2}}-\frac{a^3\sqrt{1-a^2x^2}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(3*x^3*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/(2*x
^2*Sqrt[c - a^2*c*x^2]) - (a^2*Sqrt[1 - a^2*x^2])/(x*Sqrt[c - a^2*c*x^2]) +
 (a^3*Sqrt[1 - a^2*x^2]*Log[x])/Sqrt[c - a^2*c*x^2] - (a^3*Sqrt[1 - a^2*x^2
]*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^4\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x^4\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x^4(1-ax)}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x^4}+\frac{a}{x^3}+\frac{a^2}{x^2}+\frac{a^3}{x}-\frac{a^4}{-1+ax}\right)dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{\sqrt{1-a^2x^2}}{3x^3\sqrt{c-a^2cx^2}} - \frac{a\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}} - \frac{a^2\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}} + \frac{a^3\sqrt{1-a^2x^2}\log(x)}{\sqrt{c-a^2cx^2}} - \frac{a^3\sqrt{1-a^2x^2}\log(1-}{\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0403855, size = 72, normalized size = 0.39

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{a^2}{x}+a^3\log(x)-a^3\log(1-ax)-\frac{a}{2x^2}-\frac{1}{3x^3}\right)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-1/(3*x^3) - a/(2*x^2) - a^2/x + a^3*Log[x] - a^3*Log[1
 - a*x]))/Sqrt[c - a^2*c*x^2]
```

Maple [A] time = 0.092, size = 84, normalized size = 0.5

$$-\frac{6\,a^3\ln(x)\,x^3-6\,\ln(ax-1)\,x^3a^3-6\,a^2x^2-3\,ax-2}{\left(6\,a^2x^2-6\right)cx^3}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/6*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(6*a^3*ln(x)*x^3-6*ln(a*x-1)
*x^3*a^3-6*a^2*x^2-3*a*x-2)/(a^2*x^2-1)/c/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^4), x)
```

Fricas [A] time = 2.56926, size = 1026, normalized size = 5.49

$$\left[\frac{3\left(a^5x^5 - a^3x^3\right)\sqrt{c}\log\left(-\frac{4\,a^5cx^5-\left(2\,a^6-4\,a^5+6\,a^4-4\,a^3+a^2\right)cx^6-\left(4\,a^4+4\,a^3-6\,a^2+4\,a-1\right)cx^4+5\,a^2cx^2-4\,acx+\left(4\,a^3x^3-\left(4\,a^3-6\,a^2+4\,a-1\right)x^4-6\,a^2x^2+4\,ax\right.}{a^4x^6-2\,a^3x^5+2\,ax^3-x^2}\right.}{6\left(a^2cx^5 - cx^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(3*(a^5*x^5 - a^3*x^3)*sqrt(c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6*
a^4 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2*
c*x^2 - 4*a*c*x + (4*a^3*x^3 - (4*a^3 - 6*a^2 + 4*a - 1)*x^4 - 6*a^2*x^2 +
4*a*x - 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) + c)/(a^4*x^6 -
2*a^3*x^5 + 2*a*x^3 - x^2)) + sqrt(-a^2*c*x^2 + c)*(6*a^2*x^2 - (6*a^2 + 3*
a + 2)*x^3 + 3*a*x + 2)*sqrt(-a^2*x^2 + 1))/(a^2*c*x^5 - c*x^3), -1/6*(6*(a
^5*x^5 - a^3*x^3)*sqrt(-c)*arctan(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*
((2*a^2 - 2*a + 1)*x^2 - 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^3 - (2*a^3 - a^2)*c
*x^4 - (a^2 - 2*a + 1)*c*x^2 - 2*a*c*x + c)) - sqrt(-a^2*c*x^2 + c)*(6*a^2*
x^2 - (6*a^2 + 3*a + 2)*x^3 + 3*a*x + 2)*sqrt(-a^2*x^2 + 1))/(a^2*c*x^5 - c
*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{x^4\sqrt{-(ax-1)(ax+1)}\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**4/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(x**4*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x
+ 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^4), x)
```

3.966 $\int \frac{e^{\tanh^{-1}(ax)}x^5}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=261

$$\frac{x^3\sqrt{1-a^2x^2}}{3a^3c\sqrt{c-a^2cx^2}}+\frac{x^2\sqrt{1-a^2x^2}}{2a^4c\sqrt{c-a^2cx^2}}+\frac{2x\sqrt{1-a^2x^2}}{a^5c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{2a^6c(1-ax)\sqrt{c-a^2cx^2}}+\frac{9\sqrt{1-a^2x^2}\log(1-ax)}{4a^6c\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{4a^6c\sqrt{}}$$

```
[Out] (2*x*Sqrt[1 - a^2*x^2])/(a^5*c*Sqrt[c - a^2*c*x^2]) + (x^2*Sqrt[1 - a^2*x^2
])/(2*a^4*c*Sqrt[c - a^2*c*x^2]) + (x^3*Sqrt[1 - a^2*x^2])/(3*a^3*c*Sqrt[c
- a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(2*a^6*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) +
 (9*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*a^6*c*Sqrt[c - a^2*c*x^2]) - (Sqrt[1
 - a^2*x^2]*Log[1 + a*x])/(4*a^6*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.245591, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 88}

$$\frac{x^3\sqrt{1-a^2x^2}}{3a^3c\sqrt{c-a^2cx^2}}+\frac{x^2\sqrt{1-a^2x^2}}{2a^4c\sqrt{c-a^2cx^2}}+\frac{2x\sqrt{1-a^2x^2}}{a^5c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{2a^6c(1-ax)\sqrt{c-a^2cx^2}}+\frac{9\sqrt{1-a^2x^2}\log(1-ax)}{4a^6c\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{4a^6c\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^5)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*x*Sqrt[1 - a^2*x^2])/(a^5*c*Sqrt[c - a^2*c*x^2]) + (x^2*Sqrt[1 - a^2*x^2
])/(2*a^4*c*Sqrt[c - a^2*c*x^2]) + (x^3*Sqrt[1 - a^2*x^2])/(3*a^3*c*Sqrt[c
- a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(2*a^6*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) +
 (9*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*a^6*c*Sqrt[c - a^2*c*x^2]) - (Sqrt[1
 - a^2*x^2]*Log[1 + a*x])/(4*a^6*c*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^5}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^5}{(1-a^2x^2)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^5}{(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{2}{a^5}+\frac{x}{a^4}+\frac{x^2}{a^3}+\frac{1}{2a^5(-1+ax)^2}+\frac{9}{4a^5(-1+ax)}-\frac{1}{4a^5(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{2x\sqrt{1-a^2x^2}}{a^5c\sqrt{c-a^2cx^2}}+\frac{x^2\sqrt{1-a^2x^2}}{2a^4c\sqrt{c-a^2cx^2}}+\frac{x^3\sqrt{1-a^2x^2}}{3a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{2a^6c(1-ax)\sqrt{c-a^2cx^2}}+\frac{9\sqrt{1-a^2x^2}\log(}{4a^6c\sqrt{c-a^2}}$$

Mathematica [A] time = 0.0725329, size = 87, normalized size = 0.33

$$\frac{\sqrt{1-a^2x^2}\left(4a^3x^3+6a^2x^2+24ax+\frac{6}{1-ax}+27\log(1-ax)-3\log(ax+1)\right)}{12a^6c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^5)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(24*a*x + 6*a^2*x^2 + 4*a^3*x^3 + 6/(1 - a*x) + 27*Log[1
 - a*x] - 3*Log[1 + a*x]))/(12*a^6*c*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.098, size = 119, normalized size = 0.5

$$\frac{-4x^4a^4-2x^3a^3-18a^2x^2+3ax\ln(ax+1)-27\ln(ax-1)xa+24ax-3\ln(ax+1)+27\ln(ax-1)+6}{(12a^2x^2-12)c^2a^6(ax-1)}\sqrt{-a^2x^2+1}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/12*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(-4*x^4*a^4-2*x^3*a^3-18*a^2
*x^2+3*a*x*ln(a*x+1)-27*ln(a*x-1)*x*a+24*a*x-3*ln(a*x+1)+27*ln(a*x-1)+6)/(a
^2*x^2-1)/c^2/a^6/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a\int -\frac{x^6}{\left(a^2c^{\frac{3}{2}}x^2-c^{\frac{3}{2}}\right)(ax+1)(ax-1)}\,dx-\frac{1}{2\left(a^8c^{\frac{3}{2}}x^2-a^6c^{\frac{3}{2}}\right)}+\frac{\log\left(-a^2cx^2+c\right)}{a^6c^{\frac{3}{2}}}-\frac{\sqrt{a^4cx^4-2a^2cx^2+c}}{2a^6c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -a*integrate(-x^6/((a^2*c^(3/2)*x^2 - c^(3/2))*(a*x + 1)*(a*x - 1)), x) - 1
/2/(a^8*c^(3/2)*x^2 - a^6*c^(3/2)) + log(-a^2*c*x^2 + c)/(a^6*c^(3/2)) - 1/
2*sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c)/(a^6*c^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^5}{a^5c^2x^5-a^4c^2x^4-2\,a^3c^2x^3+2\,a^2c^2x^2+ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^5/(a^5*c^2*x^5 - a^4*c^
2*x^4 - 2*a^3*c^2*x^3 + 2*a^2*c^2*x^2 + a*c^2*x - c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**5/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**5*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)x^5}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^5/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.967 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=218

$$\frac{x^2\sqrt{1-a^2x^2}}{2a^3c\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-a^2x^2}}{a^4c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{2a^5c(1-ax)\sqrt{c-a^2cx^2}}+\frac{7\sqrt{1-a^2x^2}\log(1-ax)}{4a^5c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(ax+1)}{4a^5c\sqrt{c-a^2cx^2}}$$

```
[Out] (x*Sqrt[1 - a^2*x^2])/(a^4*c*Sqrt[c - a^2*c*x^2]) + (x^2*Sqrt[1 - a^2*x^2])
/(2*a^3*c*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(2*a^5*c*(1 - a*x)*Sqrt[
c - a^2*c*x^2]) + (7*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*a^5*c*Sqrt[c - a^2*
c*x^2]) + (Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(4*a^5*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.235765, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 88}

$$\frac{x^2\sqrt{1-a^2x^2}}{2a^3c\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-a^2x^2}}{a^4c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{2a^5c(1-ax)\sqrt{c-a^2cx^2}}+\frac{7\sqrt{1-a^2x^2}\log(1-ax)}{4a^5c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(ax+1)}{4a^5c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(a^4*c*Sqrt[c - a^2*c*x^2]) + (x^2*Sqrt[1 - a^2*x^2])
/(2*a^3*c*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(2*a^5*c*(1 - a*x)*Sqrt[
c - a^2*c*x^2]) + (7*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*a^5*c*Sqrt[c - a^2*
c*x^2]) + (Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(4*a^5*c*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^4}{(1-a^2x^2)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^4}{(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{a^4}+\frac{x}{a^3}+\frac{1}{2a^4(-1+ax)^2}+\frac{7}{4a^4(-1+ax)}+\frac{1}{4a^4(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{x\sqrt{1-a^2x^2}}{a^4c\sqrt{c-a^2cx^2}} + \frac{x^2\sqrt{1-a^2x^2}}{2a^3c\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-a^2x^2}}{2a^5c(1-ax)\sqrt{c-a^2cx^2}} + \frac{7\sqrt{1-a^2x^2}\log(1-ax)}{4a^5c\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-}}{4a}$$

Mathematica [A] time = 0.059801, size = 77, normalized size = 0.35

$$\frac{\sqrt{1-a^2x^2}\left(2\left(a^2x^2+2ax+\frac{1}{1-ax}\right)+7\log(1-ax)+\log(ax+1)\right)}{4a^5c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*(2*a*x + a^2*x^2 + (1 - a*x)^(-1)) + 7*Log[1 - a*x] +
 Log[1 + a*x]))/(4*a^5*c*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.101, size = 110, normalized size = 0.5

$$-\frac{2x^3a^3+2a^2x^2+ax\ln(ax+1)+7\ln(ax-1)xa-4ax-\ln(ax+1)-7\ln(ax-1)-2}{(4a^2x^2-4)c^2a^5(ax-1)}\sqrt{-a^2x^2+1}\sqrt{-c(a^2x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3+2*a^2*x^2+a*x*ln(
a*x+1)+7*ln(a*x-1)*x*a-4*a*x-ln(a*x+1)-7*ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a^5/(
a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^4}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^4/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^4}{a^5c^2x^5-a^4c^2x^4-2\,a^3c^2x^3+2\,a^2c^2x^2+ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^4/(a^5*c^2*x^5 - a^4*c^
2*x^4 - 2*a^3*c^2*x^3 + 2*a^2*c^2*x^2 + a*c^2*x - c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**4*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)x^4}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^4/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.968 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=176

$$\frac{x\sqrt{1-a^2x^2}}{a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{2a^4c(1-ax)\sqrt{c-a^2cx^2}}+\frac{5\sqrt{1-a^2x^2}\log(1-ax)}{4a^4c\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(ax+1)}{4a^4c\sqrt{c-a^2cx^2}}$$

```
[Out] (x*Sqrt[1 - a^2*x^2])/(a^3*c*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(2*a^
4*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (5*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*
a^4*c*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(4*a^4*c*Sqrt
[c - a^2*c*x^2])
```

Rubi [A] time = 0.231571, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 88}

$$\frac{x\sqrt{1-a^2x^2}}{a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{2a^4c(1-ax)\sqrt{c-a^2cx^2}}+\frac{5\sqrt{1-a^2x^2}\log(1-ax)}{4a^4c\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(ax+1)}{4a^4c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(a^3*c*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(2*a^
4*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (5*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*
a^4*c*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(4*a^4*c*Sqrt
[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^3}{(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{x\sqrt{1-a^2x^2}}{a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{2a^4c(1-ax)\sqrt{c-a^2cx^2}}+\frac{5\sqrt{1-a^2x^2}\log(1-ax)}{4a^4c\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1+ax)}{4a^4c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0524373, size = 71, normalized size = 0.4

$$\frac{\sqrt{1-a^2x^2}\left(4ax+\frac{2}{1-ax}+5\log(1-ax)-\log(ax+1)\right)}{4a^4c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(4*a*x + 2/(1 - a*x) + 5*Log[1 - a*x] - Log[1 + a*x]))/(
4*a^4*c*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.092, size = 102, normalized size = 0.6

$$\frac{-4\,a^2x^2+ax\ln(ax+1)-5\,\ln(ax-1)\,xa+4\,ax-\ln(ax+1)+5\,\ln(ax-1)+2}{\left(4\,a^2x^2-4\right)c^2a^4\,(ax-1)}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(-4*a^2*x^2+a*x*ln(a*x+1)-5*l
n(a*x-1)*x*a+4*a*x-ln(a*x+1)+5*ln(a*x-1)+2)/(a^2*x^2-1)/c^2/a^4/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a\int -\frac{x^4}{\left(a^2c^{\frac{3}{2}}x^2-c^{\frac{3}{2}}\right)(ax+1)(ax-1)}\,dx-\frac{1}{2\left(a^6c^{\frac{3}{2}}x^2-a^4c^{\frac{3}{2}}\right)}+\frac{\log\left(-a^2cx^2+c\right)}{2\,a^4c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -a*integrate(-x^4/((a^2*c^(3/2)*x^2 - c^(3/2))*(a*x + 1)*(a*x - 1)), x) - 1
/2/(a^6*c^(3/2)*x^2 - a^4*c^(3/2)) + 1/2*log(-a^2*c*x^2 + c)/(a^4*c^(3/2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^3}{a^5c^2x^5-a^4c^2x^4-2\,a^3c^2x^3+2\,a^2c^2x^2+ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^3/(a^5*c^2*x^5 - a^4*c^
2*x^4 - 2*a^3*c^2*x^3 + 2*a^2*c^2*x^2 + a*c^2*x - c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**3*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^3}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^3/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.969 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{1-a^2x^2}}{2a^3c(1-ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(ax+1)}{4a^3c\sqrt{c-a^2cx^2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(2*a^3*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2
*x^2]*Log[1 - a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*Log[
1 + a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.221601, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 88}

$$\frac{\sqrt{1-a^2x^2}}{2a^3c(1-ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(ax+1)}{4a^3c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a^3*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2
*x^2]*Log[1 - a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*Log[
1 + a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{2a^2(-1+ax)^2}+\frac{3}{4a^2(-1+ax)}+\frac{1}{4a^2(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}}{2a^3c(1-ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(1+ax)}{4a^3c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0405422, size = 76, normalized size = 0.55

$$\frac{\sqrt{1-a^2x^2}\left(\frac{1}{2a^3(1-ax)}+\frac{3\log(1-ax)}{4a^3}+\frac{\log(ax+1)}{4a^3}\right)}{c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1/(2*a^3*(1 - a*x)) + (3*Log[1 - a*x])/(4*a^3) + Log[1
 + a*x]/(4*a^3)))/(c*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.095, size = 90, normalized size = 0.7

$$-\frac{ax\ln(ax+1)+3\ln(ax-1)xa-\ln(ax+1)-3\ln(ax-1)-2}{\left(4a^2x^2-4\right)c^2a^3(ax-1)}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*x*ln(a*x+1)+3*ln(a*x-1)*x
*a-ln(a*x+1)-3*ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a^3/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^2}{a^5c^2x^5-a^4c^2x^4-2\,a^3c^2x^3+2\,a^2c^2x^2+ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^2/(a^5*c^2*x^5 - a^4*c^
2*x^4 - 2*a^3*c^2*x^3 + 2*a^2*c^2*x^2 + a*c^2*x - c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\,(ax+1)}{\sqrt{-\,(ax-1)\,(ax+1)}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)x^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.970 $\int \frac{e^{\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=91

$$\frac{\sqrt{1-a^2x^2}}{2a^2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2a^2c\sqrt{c-a^2cx^2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(2*a^2*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x
^2]*ArcTanh[a*x])/(2*a^2*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.151133, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6153, 6150, 77, 207}

$$\frac{\sqrt{1-a^2x^2}}{2a^2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2a^2c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2)^(3/2),x]
```


```
[Out] Sqrt[1 - a^2*x^2]/(2*a^2*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x
^2]*ArcTanh[a*x])/(2*a^2*c*Sqrt[c - a^2*c*x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x}{(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{2a(-1+ax)^2}+\frac{1}{2a\left(-1+a^2x^2\right)}\right)dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}}{2a^2c(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\int \frac{1}{-1+a^2x^2}\,dx}{2ac\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}}{2a^2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2a^2c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0348936, size = 60, normalized size = 0.66

$$\frac{\sqrt{1-a^2x^2}\left(\frac{1}{2a^2(1-ax)}-\frac{\tanh^{-1}(ax)}{2a^2}\right)}{c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1/(2*a^2*(1 - a*x)) - ArcTanh[a*x]/(2*a^2)))/(c*Sqrt[c
- a^2*c*x^2])
```

Maple [A] time = 0.092, size = 88, normalized size = 1.

$$\frac{ax\ln(ax+1)-\ln(ax-1)xa-\ln(ax+1)+\ln(ax-1)+2}{\left(4\,a^2x^2-4\right)c^2a^2\,(ax-1)}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*x*ln(a*x+1)-ln(a*x-1)*x*a-
ln(a*x+1)+ln(a*x-1)+2)/(a^2*x^2-1)/c^2/a^2/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a\int -\frac{x^2}{\left(a^2c^{\frac{3}{2}}x^2-c^{\frac{3}{2}}\right)(ax+1)(ax-1)}\,dx-\frac{1}{2\left(a^4c^{\frac{3}{2}}x^2-a^2c^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] -a*integrate(-x^2/((a^2*c^(3/2)*x^2 - c^(3/2))*(a*x + 1)*(a*x - 1)), x) - 1
/2/(a^4*c^(3/2)*x^2 - a^2*c^(3/2))
```

Fricas [A] time = 2.10398, size = 710, normalized size = 7.8

$$\left[\frac{4\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}ax+\left(a^3x^3-a^2x^2-ax+1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5a^4cx^4-5a^2cx^2+4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-c}{a^6x^6-3a^4x^4+3a^2x^2-1}\right)}{8\left(a^5c^2x^3-a^4c^2x^2-a^3c^2x+a^2c^2\right)}, \frac{2}{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] [1/8*(4*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x + (a^3*x^3 - a^2*x^2 -
a*x + 1)*sqrt(c)*log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 + 4*(a^3*x^3 +
 a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4
*x^4 + 3*a^2*x^2 - 1)))/(a^5*c^2*x^3 - a^4*c^2*x^2 - a^3*c^2*x + a^2*c^2),
1/4*(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x - (a^3*x^3 - a^2*x^2 - a
*x + 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqrt(-c
)*x/(a^4*c*x^4 - c)))/(a^5*c^2*x^3 - a^4*c^2*x^2 - a^3*c^2*x + a^2*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,(ax+1)}{\sqrt{-(ax-1)(ax+1)}\,(-c\,(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**
(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)*x/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.971 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{\sqrt{1-a^2x^2}}{2ac(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2ac\sqrt{c-a^2cx^2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(2*a*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2
]*ArcTanh[a*x])/(2*a*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.0928384, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6143, 6140, 44, 207}

$$\frac{\sqrt{1-a^2x^2}}{2ac(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2ac\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2
]*ArcTanh[a*x])/(2*a*c*Sqrt[c - a^2*c*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{2(-1+ax)^2}-\frac{1}{2\left(-1+a^2x^2\right)}\right)dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}}{2ac(1-ax)\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int \frac{1}{-1+a^2x^2}\,dx}{2c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}}{2ac(1-ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2ac\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0296628, size = 60, normalized size = 0.66

$$\frac{\sqrt{1-a^2x^2}\left(\frac{1}{2a(1-ax)}+\frac{\tanh^{-1}(ax)}{2a}\right)}{c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a)))/(c*Sqrt[c - a^
2*c*x^2])
```

Maple [A] time = 0.09, size = 88, normalized size = 1.

$$-\frac{ax\ln(ax+1)-\ln(ax-1)xa-\ln(ax+1)+\ln(ax-1)-2}{\left(4a^2x^2-4\right)c^2a(ax-1)}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*x*ln(a*x+1)-ln(a*x-1)*x*a
-ln(a*x+1)+ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [A] time = 2.01044, size = 705, normalized size = 7.75

$$\left[\frac{4\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}ax+\left(a^3x^3-a^2x^2-ax+1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5a^4cx^4-5a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-c}{a^6x^6-3a^4x^4+3a^2x^2-1}\right)}{8\left(a^4c^2x^3-a^3c^2x^2-a^2c^2x+ac^2\right)}, \ 2\sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/8*(4*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x + (a^3*x^3 - a^2*x^2 -
a*x + 1)*sqrt(c)*log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*x^3 +
 a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4
*x^4 + 3*a^2*x^2 - 1)))/(a^4*c^2*x^3 - a^3*c^2*x^2 - a^2*c^2*x + a*c^2), 1/
4*(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x + (a^3*x^3 - a^2*x^2 - a*x
 + 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqrt(-c)*
x/(a^4*c*x^4 - c)))/(a^4*c^2*x^3 - a^3*c^2*x^2 - a^2*c^2*x + a*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-(ax-1)(ax+1)}\left(-c(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3
/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.972 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=165

$$\frac{\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\log(1-ax)}{4c\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(ax+1)}{4c\sqrt{c-a^2cx^2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(2*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*
Log[x])/(c*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*c*S
qrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(4*c*Sqrt[c - a^2*c*
x^2])
```

Rubi [A] time = 0.219334, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 72}

$$\frac{\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\log(1-ax)}{4c\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(ax+1)}{4c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)^(3/2)),x]
```


```
[Out] Sqrt[1 - a^2*x^2]/(2*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*
Log[x])/(c*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*c*S
qrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(4*c*Sqrt[c - a^2*c*
x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x}+\frac{a}{2(-1+ax)^2}-\frac{3a}{4(-1+ax)}-\frac{a}{4(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\log(1-ax)}{4c\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1+ax)}{4c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0511237, size = 65, normalized size = 0.39

$$\frac{\sqrt{1-a^2x^2}\left(\frac{1}{2-2ax}-\frac{3}{4}\log(1-ax)-\frac{1}{4}\log(ax+1)+\log(x)\right)}{c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*((2 - 2*a*x)^(-1) + Log[x] - (3*Log[1 - a*x])/4 - Log[1
+ a*x]/4))/(c*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.095, size = 96, normalized size = 0.6

$$-\frac{4\,a\ln(x)\,x-ax\ln(ax+1)-3\,\ln(ax-1)\,xa-4\,\ln(x)+\ln(ax+1)+3\,\ln(ax-1)-2}{4\,c^2\,(ax-1)\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(4*a*ln(x)*x-a*x*ln(a*x+1)-3
*ln(a*x-1)*x*a-4*ln(x)+ln(a*x+1)+3*ln(a*x-1)-2)/c^2/(a*x-1)/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{a^5c^2x^6-a^4c^2x^5-2\,a^3c^2x^4+2\,a^2c^2x^3+ac^2x^2-c^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^5*c^2*x^6 - a^4*c^2*x^
5 - 2*a^3*c^2*x^4 + 2*a^2*c^2*x^3 + a*c^2*x^2 - c^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{x\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/(x*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**
(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)*x), x)
```

3.973 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=206

$$\frac{a\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{5a\sqrt{1-a^2x^2}\log(1-ax)}{4c\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\log(ax+1)}{4c\sqrt{c-a^2cx^2}}$$

```
[Out] -(Sqrt[1 - a^2*x^2]/(c*x*Sqrt[c - a^2*c*x^2])) + (a*Sqrt[1 - a^2*x^2])/(2*c
*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (a*Sqrt[1 - a^2*x^2]*Log[x])/(c*Sqrt[c -
a^2*c*x^2]) - (5*a*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*c*Sqrt[c - a^2*c*x^2]
) + (a*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(4*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.226395, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 88}

$$\frac{a\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{5a\sqrt{1-a^2x^2}\log(1-ax)}{4c\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\log(ax+1)}{4c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```


```
[Out] -(Sqrt[1 - a^2*x^2]/(c*x*Sqrt[c - a^2*c*x^2])) + (a*Sqrt[1 - a^2*x^2])/(2*c
*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (a*Sqrt[1 - a^2*x^2]*Log[x])/(c*Sqrt[c -
a^2*c*x^2]) - (5*a*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*c*Sqrt[c - a^2*c*x^2]
) + (a*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(4*c*Sqrt[c - a^2*c*x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x^2(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x^2}+\frac{a}{x}+\frac{a^2}{2(-1+ax)^2}-\frac{5a^2}{4(-1+ax)}+\frac{a^2}{4(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$
$$= -\frac{\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{5a\sqrt{1-a^2x^2}\log(1-ax)}{4c\sqrt{c-a^2cx^2}}+$$

Mathematica [A] time = 0.0584262, size = 76, normalized size = 0.37

$$\frac{\sqrt{1-a^2x^2}\left(\frac{2a}{1-ax}+4a\log(x)-5a\log(1-ax)+a\log(ax+1)-\frac{4}{x}\right)}{4c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-4/x + (2*a)/(1 - a*x) + 4*a*Log[x] - 5*a*Log[1 - a*x]
+ a*Log[1 + a*x]))/(4*c*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.096, size = 122, normalized size = 0.6

$$-\frac{4a^2\ln(x)x^2+\ln(ax+1)a^2x^2-5\ln(ax-1)a^2x^2-4a\ln(x)x-ax\ln(ax+1)+5\ln(ax-1)xa-6ax+4}{\left(4a^2x^2-4\right)c^2(ax-1)x}\sqrt{-a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(4*a^2*ln(x)*x^2+ln(a*x+1)*a
^2*x^2-5*ln(a*x-1)*a^2*x^2-4*a*ln(x)*x-a*x*ln(a*x+1)+5*ln(a*x-1)*x*a-6*a*x+
4)/(a^2*x^2-1)/c^2/(a*x-1)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{a^5c^2x^7-a^4c^2x^6-2\,a^3c^2x^5+2\,a^2c^2x^4+ac^2x^3-c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^5*c^2*x^7 - a^4*c^2*x^
6 - 2*a^3*c^2*x^5 + 2*a^2*c^2*x^4 + a*c^2*x^3 - c^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax+1}{x^2\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)*x^2), x)
```

3.974 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=255

$$\frac{a^2\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}}+\frac{2a^2\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{7a^2\sqrt{1-a^2x^2}\log(1-ax)}{4c\sqrt{c-a^2cx^2}}-\frac{a^2\sqrt{1-}}{4}$$

```
[Out] -Sqrt[1 - a^2*x^2]/(2*c*x^2*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/(c
*x*Sqrt[c - a^2*c*x^2]) + (a^2*Sqrt[1 - a^2*x^2])/(2*c*(1 - a*x)*Sqrt[c - a
^2*c*x^2]) + (2*a^2*Sqrt[1 - a^2*x^2]*Log[x])/(c*Sqrt[c - a^2*c*x^2]) - (7*
a^2*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*c*Sqrt[c - a^2*c*x^2]) - (a^2*Sqrt[1
 - a^2*x^2]*Log[1 + a*x])/(4*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.238413, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 88}

$$\frac{a^2\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}}+\frac{2a^2\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{7a^2\sqrt{1-a^2x^2}\log(1-ax)}{4c\sqrt{c-a^2cx^2}}-\frac{a^2\sqrt{1-}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```


```
[Out] -Sqrt[1 - a^2*x^2]/(2*c*x^2*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/(c
*x*Sqrt[c - a^2*c*x^2]) + (a^2*Sqrt[1 - a^2*x^2])/(2*c*(1 - a*x)*Sqrt[c - a
^2*c*x^2]) + (2*a^2*Sqrt[1 - a^2*x^2]*Log[x])/(c*Sqrt[c - a^2*c*x^2]) - (7*
a^2*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*c*Sqrt[c - a^2*c*x^2]) - (a^2*Sqrt[1
 - a^2*x^2]*Log[1 + a*x])/(4*c*Sqrt[c - a^2*c*x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x^3(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{2a^2}{x}+\frac{a^3}{2(-1+ax)^2}-\frac{7a^3}{4(-1+ax)}-\frac{a^3}{4(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$

$$= -\frac{\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}+\frac{2a^2\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{7a^2\sqrt{1-a^2}}{4c\sqrt{c}}$$

Mathematica [A] time = 0.0612631, size = 91, normalized size = 0.36

$$\frac{\sqrt{1-a^2x^2}\left(\frac{2a^2}{1-ax}+8a^2\log(x)-7a^2\log(1-ax)-a^2\log(ax+1)-\frac{4a}{x}-\frac{2}{x^2}\right)}{4c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2/x^2 - (4*a)/x + (2*a^2)/(1 - a*x) + 8*a^2*Log[x] - 7
*a^2*Log[1 - a*x] - a^2*Log[1 + a*x]))/(4*c*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.096, size = 142, normalized size = 0.6

$$-\frac{8a^3\ln(x)x^3-a^3x^3\ln(ax+1)-7\ln(ax-1)x^3a^3-8a^2\ln(x)x^2+\ln(ax+1)a^2x^2+7\ln(ax-1)a^2x^2-6a^2x^2+2ax}{\left(4a^2x^2-4\right)c^2(ax-1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(8*a^3*ln(x)*x^3-a^3*x^3*ln(
a*x+1)-7*ln(a*x-1)*x^3*a^3-8*a^2*ln(x)*x^2+ln(a*x+1)*a^2*x^2+7*ln(a*x-1)*a^
2*x^2-6*a^2*x^2+2*a*x+2)/(a^2*x^2-1)/c^2/(a*x-1)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{a^5c^2x^8-a^4c^2x^7-2\,a^3c^2x^6+2\,a^2c^2x^5+ac^2x^4-c^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^5*c^2*x^8 - a^4*c^2*x^
7 - 2*a^3*c^2*x^6 + 2*a^2*c^2*x^5 + a*c^2*x^4 - c^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax+1}{x^3\sqrt{-\left(ax-1\right)\left(ax+1\right)}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)*x^3), x)
```

3.975 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=297

$$\frac{a^3\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{2a^2\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{3cx^3\sqrt{c-a^2cx^2}}+\frac{2a^3\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{9a^3\sqrt{1-a^2x^2}\log(1}{4c\sqrt{c-a^2cx^2}}$$

```
[Out] -Sqrt[1 - a^2*x^2]/(3*c*x^3*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/(2
*c*x^2*Sqrt[c - a^2*c*x^2]) - (2*a^2*Sqrt[1 - a^2*x^2])/(c*x*Sqrt[c - a^2*c
*x^2]) + (a^3*Sqrt[1 - a^2*x^2])/(2*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (2*a
^3*Sqrt[1 - a^2*x^2]*Log[x])/(c*Sqrt[c - a^2*c*x^2]) - (9*a^3*Sqrt[1 - a^2*
x^2]*Log[1 - a*x])/(4*c*Sqrt[c - a^2*c*x^2]) + (a^3*Sqrt[1 - a^2*x^2]*Log[1
 + a*x])/(4*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.241217, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 88}

$$\frac{a^3\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{2a^2\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{3cx^3\sqrt{c-a^2cx^2}}+\frac{2a^3\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{9a^3\sqrt{1-a^2x^2}\log(1}{4c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(3*c*x^3*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/(2
*c*x^2*Sqrt[c - a^2*c*x^2]) - (2*a^2*Sqrt[1 - a^2*x^2])/(c*x*Sqrt[c - a^2*c
*x^2]) + (a^3*Sqrt[1 - a^2*x^2])/(2*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (2*a
^3*Sqrt[1 - a^2*x^2]*Log[x])/(c*Sqrt[c - a^2*c*x^2]) - (9*a^3*Sqrt[1 - a^2*
x^2]*Log[1 - a*x])/(4*c*Sqrt[c - a^2*c*x^2]) + (a^3*Sqrt[1 - a^2*x^2]*Log[1
 + a*x])/(4*c*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^4\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x^4\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x^4(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x^4}+\frac{a}{x^3}+\frac{2a^2}{x^2}+\frac{2a^3}{x}+\frac{a^4}{2(-1+ax)^2}-\frac{9a^4}{4(-1+ax)}+\frac{a^4}{4(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$
$$= -\frac{\sqrt{1-a^2x^2}}{3cx^3\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}}-\frac{2a^2\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}+\frac{a^3\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}+\frac{2a^3\sqrt{1-a^2}}{c\sqrt{c-a}}$$

Mathematica [A] time = 0.0727007, size = 99, normalized size = 0.33

$$\frac{\sqrt{1-a^2x^2}\left(\frac{6a^3}{1-ax}-\frac{24a^2}{x}+24a^3\log(x)-27a^3\log(1-ax)+3a^3\log(ax+1)-\frac{6a}{x^2}-\frac{4}{x^3}\right)}{12c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-4/x^3 - (6*a)/x^2 - (24*a^2)/x + (6*a^3)/(1 - a*x) + 2
4*a^3*Log[x] - 27*a^3*Log[1 - a*x] + 3*a^3*Log[1 + a*x]))/(12*c*Sqrt[c - a^
2*c*x^2])
```

Maple [A] time = 0.097, size = 151, normalized size = 0.5

$$-\frac{24\,a^4\ln(x)\,x^4+3\,\ln(ax+1)\,a^4x^4-27\,\ln(ax-1)\,a^4x^4-24\,a^3\ln(x)\,x^3-3\,a^3x^3\ln(ax+1)+27\,\ln(ax-1)\,x^3a^3-3}{\left(12\,a^2x^2-12\right)c^2x^3\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/12*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(24*a^4*ln(x)*x^4+3*ln(a*x+
1)*a^4*x^4-27*ln(a*x-1)*a^4*x^4-24*a^3*ln(x)*x^3-3*a^3*x^3*ln(a*x+1)+27*ln(
a*x-1)*x^3*a^3-30*x^3*a^3+18*a^2*x^2+2*a*x+4)/(a^2*x^2-1)/c^2/x^3/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{a^5c^2x^9-a^4c^2x^8-2\,a^3c^2x^7+2\,a^2c^2x^6+ac^2x^5-c^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^5*c^2*x^9 - a^4*c^2*x^
8 - 2*a^3*c^2*x^7 + 2*a^2*c^2*x^6 + a*c^2*x^5 - c^2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax+1}{x^4\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**4/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/(x**4*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)*x^4), x)
```

3.976 $\int \frac{e^{\tanh^{-1}(ax)}x^6}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=312

$$-\frac{x^2\sqrt{1-a^2x^2}}{2a^5c^2\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}}{a^6c^2\sqrt{c-a^2cx^2}}-\frac{5\sqrt{1-a^2x^2}}{4a^7c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^7c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^7c^2(1-ax)^2\sqrt{c-a^2cx^2}}$$

```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a^6*c^2*Sqrt[c - a^2*c*x^2])) - (x^2*Sqrt[1 - a^2*
x^2])/(2*a^5*c^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*a^7*c^2*(1 - a
*x)^2*Sqrt[c - a^2*c*x^2]) - (5*Sqrt[1 - a^2*x^2])/(4*a^7*c^2*(1 - a*x)*Sqr
t[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a^7*c^2*(1 + a*x)*Sqrt[c - a^2*c*x
^2]) - (39*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(16*a^7*c^2*Sqrt[c - a^2*c*x^2])
 - (9*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(16*a^7*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.254843, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 88}

$$-\frac{x^2\sqrt{1-a^2x^2}}{2a^5c^2\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}}{a^6c^2\sqrt{c-a^2cx^2}}-\frac{5\sqrt{1-a^2x^2}}{4a^7c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^7c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^7c^2(1-ax)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^6)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a^6*c^2*Sqrt[c - a^2*c*x^2])) - (x^2*Sqrt[1 - a^2*
x^2])/(2*a^5*c^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*a^7*c^2*(1 - a
*x)^2*Sqrt[c - a^2*c*x^2]) - (5*Sqrt[1 - a^2*x^2])/(4*a^7*c^2*(1 - a*x)*Sqr
t[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a^7*c^2*(1 + a*x)*Sqrt[c - a^2*c*x
^2]) - (39*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(16*a^7*c^2*Sqrt[c - a^2*c*x^2])
 - (9*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(16*a^7*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^6}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^6}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^6}{(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{a^6}-\frac{x}{a^5}-\frac{1}{4a^6(-1+ax)^3}-\frac{5}{4a^6(-1+ax)^2}-\frac{39}{16a^6(-1+ax)}+\frac{1}{8a^6(1+ax)^2}-\frac{9}{16a^6(1+ax)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= -\frac{x\sqrt{1-a^2x^2}}{a^6c^2\sqrt{c-a^2cx^2}}-\frac{x^2\sqrt{1-a^2x^2}}{2a^5c^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^7c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{5\sqrt{1-a^2x^2}}{4a^7c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{}{8a}$$

Mathematica [A] time = 0.117292, size = 97, normalized size = 0.31

$$\frac{\sqrt{1-a^2x^2}\left(2\left(-4a^2x^2-8ax+\frac{10}{ax-1}-\frac{1}{ax+1}+\frac{1}{(ax-1)^2}\right)-39\log(1-ax)-9\log(ax+1)\right)}{16a^7c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^6)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*(-8*a*x - 4*a^2*x^2 + (-1 + a*x)^(-2) + 10/(-1 + a*x)
 - (1 + a*x)^(-1)) - 39*Log[1 - a*x] - 9*Log[1 + a*x]))/(16*a^7*c^2*Sqrt[c
- a^2*c*x^2])
```

Maple [A] time = 0.096, size = 190, normalized size = 0.6

$$\frac{8x^5a^5+8x^4a^4+9a^3x^3\ln(ax+1)+39\ln(ax-1)x^3a^3-24x^3a^3-9\ln(ax+1)a^2x^2-39\ln(ax-1)a^2x^2-26a^2x^2-}{\left(16a^2x^2-16\right)c^3a^7(ax+1)(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(8*x^5*a^5+8*x^4*a^4+9*a^3*x
^3*ln(a*x+1)+39*ln(a*x-1)*x^3*a^3-24*x^3*a^3-9*ln(a*x+1)*a^2*x^2-39*ln(a*x-
1)*a^2*x^2-26*a^2*x^2-9*a*x*ln(a*x+1)-39*ln(a*x-1)*x*a+10*a*x+9*ln(a*x+1)+3
9*ln(a*x-1)+20)/(a^2*x^2-1)/c^3/a^7/(a*x+1)/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^6}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^6/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^6}{a^7c^3x^7-a^6c^3x^6-3\,a^5c^3x^5+3\,a^4c^3x^4+3\,a^3c^3x^3-3\,a^2c^3x^2-ac^3x+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^6/(a^7*c^3*x^7 - a^6*c^3
*x^6 - 3*a^5*c^3*x^5 + 3*a^4*c^3*x^4 + 3*a^3*c^3*x^3 - 3*a^2*c^3*x^2 - a*c^
3*x + c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}\,(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**6/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**6*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^6}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^6/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.977 $\int \frac{e^{\tanh^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=268

$$-\frac{x\sqrt{1-a^2x^2}}{a^5c^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{a^6c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^6c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^6c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{23\sqrt{1-a^2x^2}\log(1}{16a^6c^2\sqrt{c-a^2c}}$$

```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a^5*c^2*Sqrt[c - a^2*c*x^2])) + Sqrt[1 - a^2*x^2]/
(8*a^6*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(a^6*c^2*(1
 - a*x)*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*a^6*c^2*(1 + a*x)*Sqrt[
c - a^2*c*x^2]) - (23*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(16*a^6*c^2*Sqrt[c -
a^2*c*x^2]) + (7*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(16*a^6*c^2*Sqrt[c - a^2*c
*x^2])
```

Rubi [A] time = 0.246031, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 88}

$$-\frac{x\sqrt{1-a^2x^2}}{a^5c^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{a^6c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^6c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^6c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{23\sqrt{1-a^2x^2}\log(1}{16a^6c^2\sqrt{c-a^2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^5)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a^5*c^2*Sqrt[c - a^2*c*x^2])) + Sqrt[1 - a^2*x^2]/
(8*a^6*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(a^6*c^2*(1
 - a*x)*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*a^6*c^2*(1 + a*x)*Sqrt[
c - a^2*c*x^2]) - (23*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(16*a^6*c^2*Sqrt[c -
a^2*c*x^2]) + (7*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(16*a^6*c^2*Sqrt[c - a^2*c
*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^5}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^5}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^5}{(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{a^5}-\frac{1}{4a^5(-1+ax)^3}-\frac{1}{a^5(-1+ax)^2}-\frac{23}{16a^5(-1+ax)}-\frac{1}{8a^5(1+ax)^2}+\frac{7}{16a^5(1+ax)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= -\frac{x\sqrt{1-a^2x^2}}{a^5c^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^6c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{a^6c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^6c^2(1+ax)\sqrt{c-}}$$

Mathematica [A] time = 0.0984278, size = 87, normalized size = 0.32

$$\frac{\sqrt{1-a^2x^2}\left(2\left(-8ax+\frac{8}{ax-1}+\frac{1}{ax+1}+\frac{1}{(ax-1)^2}\right)-23\log(1-ax)+7\log(ax+1)\right)}{16a^6c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^5)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*(-8*a*x + (-1 + a*x)^(-2) + 8/(-1 + a*x) + (1 + a*x)^
(-1)) - 23*Log[1 - a*x] + 7*Log[1 + a*x]))/(16*a^6*c^2*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.095, size = 182, normalized size = 0.7

$$-\frac{-16x^4a^4+7a^3x^3\ln(ax+1)-23\ln(ax-1)x^3a^3+16x^3a^3-7\ln(ax+1)a^2x^2+23\ln(ax-1)a^2x^2+34a^2x^2-7}{\left(16a^2x^2-16\right)c^3a^6(ax+1)(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(-16*x^4*a^4+7*a^3*x^3*ln(a
*x+1)-23*ln(a*x-1)*x^3*a^3+16*x^3*a^3-7*ln(a*x+1)*a^2*x^2+23*ln(a*x-1)*a^2*
x^2+34*a^2*x^2-7*a*x*ln(a*x+1)+23*ln(a*x-1)*x*a-18*a*x+7*ln(a*x+1)-23*ln(a*
x-1)-12)/(a^2*x^2-1)/c^3/a^6/(a*x+1)/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a\int -\frac{x^6}{\left(a^4c^{\frac{5}{2}}x^4-2a^2c^{\frac{5}{2}}x^2+c^{\frac{5}{2}}\right)(ax+1)(ax-1)}\,dx+\frac{1}{4\left(a^{10}c^{\frac{5}{2}}x^4-2a^8c^{\frac{5}{2}}x^2+a^6c^{\frac{5}{2}}\right)}+\frac{1}{a^8c^{\frac{5}{2}}x^2-a^6c^{\frac{5}{2}}}-\frac{\log\left(-a^2cx^2+\right.}{2a^6c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] a*integrate(-x^6/((a^4*c^(5/2)*x^4 - 2*a^2*c^(5/2)*x^2 + c^(5/2))*(a*x + 1)
*(a*x - 1)), x) + 1/4/(a^10*c^(5/2)*x^4 - 2*a^8*c^(5/2)*x^2 + a^6*c^(5/2))
+ 1/(a^8*c^(5/2)*x^2 - a^6*c^(5/2)) - 1/2*log(-a^2*c*x^2 + c)/(a^6*c^(5/2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^5}{a^7c^3x^7-a^6c^3x^6-3\,a^5c^3x^5+3\,a^4c^3x^4+3\,a^3c^3x^3-3\,a^2c^3x^2-ac^3x+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^5/(a^7*c^3*x^7 - a^6*c^3
*x^6 - 3*a^5*c^3*x^5 + 3*a^4*c^3*x^4 + 3*a^3*c^3*x^3 - 3*a^2*c^3*x^2 - a*c^
3*x + c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5\,(ax+1)}{\sqrt{-\,(ax-1)\,(ax+1)}\,(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**5/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**5*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)x^5}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^5/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.978 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=232

$$-\frac{3\sqrt{1-a^2x^2}}{4a^5c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^5c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^5c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{11\sqrt{1-a^2x^2}\log(1-ax)}{16a^5c^2\sqrt{c-a^2cx^2}}-\frac{5\sqrt{1}}{16}$$

```
[Out] Sqrt[1 - a^2*x^2]/(8*a^5*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 -
 a^2*x^2])/(4*a^5*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8
*a^5*c^2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (11*Sqrt[1 - a^2*x^2]*Log[1 - a*x
])/(16*a^5*c^2*Sqrt[c - a^2*c*x^2]) - (5*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(1
6*a^5*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.237612, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 88}

$$-\frac{3\sqrt{1-a^2x^2}}{4a^5c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^5c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^5c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{11\sqrt{1-a^2x^2}\log(1-ax)}{16a^5c^2\sqrt{c-a^2cx^2}}-\frac{5\sqrt{1}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(8*a^5*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 -
 a^2*x^2])/(4*a^5*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8
*a^5*c^2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (11*Sqrt[1 - a^2*x^2]*Log[1 - a*x
])/(16*a^5*c^2*Sqrt[c - a^2*c*x^2]) - (5*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(1
6*a^5*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^4}{(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{4a^4(-1+ax)^3}-\frac{3}{4a^4(-1+ax)^2}-\frac{11}{16a^4(-1+ax)}+\frac{1}{8a^4(1+ax)^2}-\frac{5}{16a^4(1+ax)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}}{8a^5c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}}{4a^5c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^5c^2(1+ax)\sqrt{c-a^2cx^2}}-\frac{11\sqrt{1-a^2x^2}\,\mathrm{lo}}{16a^5c^2\sqrt{c-}}$$

Mathematica [A] time = 0.0908829, size = 87, normalized size = 0.38

$$\frac{\sqrt{1-a^2x^2}\left(\frac{2\left(5a^2x^2+3ax-6\right)}{(ax-1)^2(ax+1)}-11\log(1-ax)-5\log(ax+1)\right)}{16a^5c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*((2*(-6 + 3*a*x + 5*a^2*x^2))/((-1 + a*x)^2*(1 + a*x)) -
 11*Log[1 - a*x] - 5*Log[1 + a*x]))/(16*a^5*c^2*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.093, size = 166, normalized size = 0.7

$$\frac{5\,a^3x^3\ln(ax+1)+11\,\ln(ax-1)\,x^3a^3-5\,\ln(ax+1)\,a^2x^2-11\,\ln(ax-1)\,a^2x^2-10\,a^2x^2-5\,ax\ln(ax+1)-11\,\ln(ax-}{\left(16\,a^2x^2-16\right)c^3a^5\,(ax-1)^2\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(5*a^3*x^3*ln(a*x+1)+11*ln(a
*x-1)*x^3*a^3-5*ln(a*x+1)*a^2*x^2-11*ln(a*x-1)*a^2*x^2-10*a^2*x^2-5*a*x*ln(
a*x+1)-11*ln(a*x-1)*x*a-6*a*x+5*ln(a*x+1)+11*ln(a*x-1)+12)/(a^2*x^2-1)/c^3/
a^5/(a*x-1)^2/(a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^4}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^4/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^4}{a^7c^3x^7-a^6c^3x^6-3\,a^5c^3x^5+3\,a^4c^3x^4+3\,a^3c^3x^3-3\,a^2c^3x^2-ac^3x+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^4/(a^7*c^3*x^7 - a^6*c^3
*x^6 - 3*a^5*c^3*x^5 + 3*a^4*c^3*x^4 + 3*a^3*c^3*x^3 - 3*a^2*c^3*x^2 - a*c^
3*x + c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}\,(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^4}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^4/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.979 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{\sqrt{1-a^2x^2}}{2a^4c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^4c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^4c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^4c^2\sqrt{c-a^2cx^2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(8*a^4*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^
2*x^2]/(2*a^4*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*a^4
*c^2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8
*a^4*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.23539, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6153, 6150, 88, 207}

$$-\frac{\sqrt{1-a^2x^2}}{2a^4c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^4c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^4c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^4c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2)^(5/2),x]
```


```
[Out] Sqrt[1 - a^2*x^2]/(8*a^4*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^
2*x^2]/(2*a^4*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*a^4
*c^2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8
*a^4*c^2*Sqrt[c - a^2*c*x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^3}{(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{4a^3(-1+ax)^3}-\frac{1}{2a^3(-1+ax)^2}-\frac{1}{8a^3(1+ax)^2}-\frac{3}{8a^3\left(-1+a^2x^2\right)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}}{8a^4c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2a^4c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^4c^2(1+ax)\sqrt{c-a^2cx^2}}-\frac{\left(3\sqrt{1-a^2x\right.}}{8a^3c^2}$$

$$= \frac{\sqrt{1-a^2x^2}}{8a^4c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2a^4c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^4c^2(1+ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x}}{8a^4c^2\sqrt{}}$$

Mathematica [A] time = 0.0575734, size = 85, normalized size = 0.46

$$\frac{\sqrt{1-a^2x^2}\left(5a^2x^2-ax+3(ax-1)^2(ax+1)\tanh^{-1}(ax)-2\right)}{8a^4c^2(ax-1)^2(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2 - a*x + 5*a^2*x^2 + 3*(-1 + a*x)^2*(1 + a*x)*ArcTanh
[a*x]))/(8*a^4*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.099, size = 166, normalized size = 0.9

$$-\frac{3a^3x^3\ln(ax+1)-3\ln(ax-1)x^3a^3-3\ln(ax+1)a^2x^2+3\ln(ax-1)a^2x^2+10a^2x^2-3ax\ln(ax+1)+3\ln(ax}{\left(16a^2x^2-16\right)c^3a^4(ax-1)^2(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*x^3*ln(a*x+1)-3*ln(a
*x-1)*x^3*a^3-3*ln(a*x+1)*a^2*x^2+3*ln(a*x-1)*a^2*x^2+10*a^2*x^2-3*a*x*ln(a
*x+1)+3*ln(a*x-1)*x*a-2*a*x+3*ln(a*x+1)-3*ln(a*x-1)-4)/(a^2*x^2-1)/c^3/a^4/
(a*x-1)^2/(a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a\int -\frac{x^4}{\left(a^4c^{\frac{5}{2}}x^4-2a^2c^{\frac{5}{2}}x^2+c^{\frac{5}{2}}\right)(ax+1)(ax-1)}\,dx+\frac{1}{4\left(a^8c^{\frac{5}{2}}x^4-2a^6c^{\frac{5}{2}}x^2+a^4c^{\frac{5}{2}}\right)}+\frac{1}{2\left(a^6c^{\frac{5}{2}}x^2-a^4c^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] a*integrate(-x^4/((a^4*c^(5/2)*x^4 - 2*a^2*c^(5/2)*x^2 + c^(5/2))*(a*x + 1)
*(a*x - 1)), x) + 1/4/(a^8*c^(5/2)*x^4 - 2*a^6*c^(5/2)*x^2 + a^4*c^(5/2)) +
 1/2/(a^6*c^(5/2)*x^2 - a^4*c^(5/2))
```

Fricas [A] time = 2.05747, size = 945, normalized size = 5.14

$$\left[\frac{3\left(a^5x^5 - a^4x^4 - 2a^3x^3 + 2a^2x^2 + ax - 1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5a^4cx^4-5a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-c}{a^6x^6-3a^4x^4+3a^2x^2-1}\right) - 4\left(2a^3x^3 + 3a^2\right.}{32\left(a^9c^3x^5 - a^8c^3x^4 - 2a^7c^3x^3 + 2a^6c^3x^2 + a^5c^3x - a^4c^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] [1/32*(3*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*sqrt(c)*log(
-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*x^3 + a*x)*sqrt(-a^2*c*x^2
 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)
) - 4*(2*a^3*x^3 + 3*a^2*x^2 - 3*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 +
1))/(a^9*c^3*x^5 - a^8*c^3*x^4 - 2*a^7*c^3*x^3 + 2*a^6*c^3*x^2 + a^5*c^3*x
- a^4*c^3), 1/16*(3*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*s
qrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*
c*x^4 - c)) - 2*(2*a^3*x^3 + 3*a^2*x^2 - 3*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-
a^2*x^2 + 1))/(a^9*c^3*x^5 - a^8*c^3*x^4 - 2*a^7*c^3*x^3 + 2*a^6*c^3*x^2 +
a^5*c^3*x - a^4*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3(ax+1)}{\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**3*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^3}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^3/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.980 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{\sqrt{1-a^2x^2}}{4a^3c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^3c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^3c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^3c^2\sqrt{c-a^2cx^2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(8*a^3*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^
2*x^2]/(4*a^3*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a^3
*c^2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*a
^3*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.236701, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6153, 6150, 88, 207}

$$-\frac{\sqrt{1-a^2x^2}}{4a^3c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^3c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^3c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^3c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(8*a^3*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^
2*x^2]/(4*a^3*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a^3
*c^2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*a
^3*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{4a^2(-1+ax)^3}-\frac{1}{4a^2(-1+ax)^2}+\frac{1}{8a^2(1+ax)^2}+\frac{1}{8a^2\left(-1+a^2x^2\right)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}}{8a^3c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{4a^3c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^3c^2(1+ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^2c^2\sqrt{}}$$

$$= \frac{\sqrt{1-a^2x^2}}{8a^3c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{4a^3c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^3c^2(1+ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^3c^2\sqrt{c}}$$

Mathematica [A] time = 0.0571427, size = 84, normalized size = 0.46

$$\frac{\sqrt{1-a^2x^2}\left(a^2x^2+3ax-(ax-1)^2(ax+1)\tanh^{-1}(ax)-2\right)}{8a^3c^2(ax-1)^2(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2 + 3*a*x + a^2*x^2 - (-1 + a*x)^2*(1 + a*x)*ArcTanh[a
*x]))/(8*a^3*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.095, size = 161, normalized size = 0.9

$$\frac{a^3x^3\ln(ax+1)-\ln(ax-1)x^3a^3-\ln(ax+1)a^2x^2+\ln(ax-1)a^2x^2-2a^2x^2-ax\ln(ax+1)+\ln(ax-1)xa-6ax}{\left(16a^2x^2-16\right)c^3a^3(ax+1)(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a^3*x^3*ln(a*x+1)-ln(a*x-1)
*x^3*a^3-ln(a*x+1)*a^2*x^2+ln(a*x-1)*a^2*x^2-2*a^2*x^2-a*x*ln(a*x+1)+ln(a*x
-1)*x*a-6*a*x+ln(a*x+1)-ln(a*x-1)+4)/(a^2*x^2-1)/c^3/a^3/(a*x+1)/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^2}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^2/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [A] time = 2.06943, size = 930, normalized size = 5.05

$$\left[\frac{\left(a^5x^5 - a^4x^4 - 2a^3x^3 + 2a^2x^2 + ax - 1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5a^4cx^4-5a^2cx^2+4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-c}{a^6x^6-3a^4x^4+3a^2x^2-1}\right) - 4\left(2a^3x^3 - a^2x^2 + \right.}{32\left(a^8c^3x^5 - a^7c^3x^4 - 2a^6c^3x^3 + 2a^5c^3x^2 + a^4c^3x - a^3c^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] [1/32*((a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*sqrt(c)*log(-(
a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 + 4*(a^3*x^3 + a*x)*sqrt(-a^2*c*x^2 +
 c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1))
- 4*(2*a^3*x^3 - a^2*x^2 + a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a
^8*c^3*x^5 - a^7*c^3*x^4 - 2*a^6*c^3*x^3 + 2*a^5*c^3*x^2 + a^4*c^3*x - a^3*
c^3), -1/16*((a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*sqrt(-c)
*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*c*x^4 -
 c)) + 2*(2*a^3*x^3 - a^2*x^2 + a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1
))/(a^8*c^3*x^5 - a^7*c^3*x^4 - 2*a^6*c^3*x^3 + 2*a^5*c^3*x^2 + a^4*c^3*x -
 a^3*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2(ax+1)}{\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^2}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^2/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.981 $\int \frac{e^{\tanh^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{\sqrt{1-a^2x^2}}{8a^2c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^2c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^2c^2\sqrt{c-a^2cx^2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(8*a^2*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^
2*x^2]/(8*a^2*c^2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*ArcTa
nh[a*x])/(8*a^2*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.16511, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6153, 6150, 77, 207}

$$\frac{\sqrt{1-a^2x^2}}{8a^2c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^2c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^2c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(8*a^2*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^
2*x^2]/(8*a^2*c^2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*ArcTa
nh[a*x])/(8*a^2*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x}{(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{4a(-1+ax)^3}-\frac{1}{8a(1+ax)^2}+\frac{1}{8a\left(-1+a^2x^2\right)}\right)dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}}{8a^2c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^2c^2(1+ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\int \frac{1}{-1+a^2x^2}\,dx}{8ac^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}}{8a^2c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^2c^2(1+ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^2c^2\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0647348, size = 60, normalized size = 0.44

$$\frac{\sqrt{1-a^2x^2}\left(\frac{1}{ax+1}+\frac{1}{(ax-1)^2}-\tanh^{-1}(ax)\right)}{8a^2c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*((-1 + a*x)^(-2) + (1 + a*x)^(-1) - ArcTanh[a*x]))/(8*a^
2*c^2*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.095, size = 161, normalized size = 1.2

$$\frac{a^3x^3\ln(ax+1)-\ln(ax-1)x^3a^3-\ln(ax+1)a^2x^2+\ln(ax-1)a^2x^2-2a^2x^2-ax\ln(ax+1)+\ln(ax-1)xa+2ax}{\left(16a^2x^2-16\right)c^3a^2(ax+1)(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a^3*x^3*ln(a*x+1)-ln(a*x-1)
*x^3*a^3-ln(a*x+1)*a^2*x^2+ln(a*x-1)*a^2*x^2-2*a^2*x^2-a*x*ln(a*x+1)+ln(a*x
-1)*x*a+2*a*x+ln(a*x+1)-ln(a*x-1)-4)/(a^2*x^2-1)/c^3/a^2/(a*x+1)/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a\int -\frac{x^2}{\left(a^4c^{\frac{5}{2}}x^4-2a^2c^{\frac{5}{2}}x^2+c^{\frac{5}{2}}\right)(ax+1)(ax-1)}\,dx+\frac{1}{4\left(a^6c^{\frac{5}{2}}x^4-2a^4c^{\frac{5}{2}}x^2+a^2c^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="m
axima")
```

```
[Out] a*integrate(-x^2/((a^4*c^(5/2)*x^4 - 2*a^2*c^(5/2)*x^2 + c^(5/2))*(a*x + 1)
*(a*x - 1)), x) + 1/4/(a^6*c^(5/2)*x^4 - 2*a^4*c^(5/2)*x^2 + a^2*c^(5/2))
```

Fricas [A] time = 2.08522, size = 936, normalized size = 6.83

$$\left[\frac{\left(a^5x^5 - a^4x^4 - 2\,a^3x^3 + 2\,a^2x^2 + ax - 1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5\,a^4cx^4-5\,a^2cx^2+4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-c}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}\right) + 4\left(2\,a^3x^3 - 3\,a^2x^2\right.}{32\left(a^7c^3x^5 - a^6c^3x^4 - 2\,a^5c^3x^3 + 2\,a^4c^3x^2 + a^3c^3x - a^2c^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="f
ricas")
```

```
[Out] [1/32*((a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*sqrt(c)*log(-(
a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 + 4*(a^3*x^3 + a*x)*sqrt(-a^2*c*x^2 +
 c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1))
+ 4*(2*a^3*x^3 - 3*a^2*x^2 - a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/
(a^7*c^3*x^5 - a^6*c^3*x^4 - 2*a^5*c^3*x^3 + 2*a^4*c^3*x^2 + a^3*c^3*x - a^
2*c^3), -1/16*((a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*sqrt(-
c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*c*x^4
 - c)) - 2*(2*a^3*x^3 - 3*a^2*x^2 - a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2
 + 1))/(a^7*c^3*x^5 - a^6*c^3*x^4 - 2*a^5*c^3*x^3 + 2*a^4*c^3*x^2 + a^3*c^3
*x - a^2*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**
(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)*x/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.982 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=184

$$\frac{\sqrt{1-a^2x^2}}{4ac^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8ac^2\sqrt{c-a^2cx^2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(8*a*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*
x^2]/(4*a*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a*c^2*(
1 + a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*a*c^2
*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.112075, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6143, 6140, 44, 207}

$$\frac{\sqrt{1-a^2x^2}}{4ac^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8ac^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(8*a*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*
x^2]/(4*a*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a*c^2*(
1 + a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*a*c^2
*Sqrt[c - a^2*c*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{4(-1+ax)^3}+\frac{1}{4(-1+ax)^2}+\frac{1}{8(1+ax)^2}-\frac{3}{8\left(-1+a^2x^2\right)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{4ac^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(1+ax)\sqrt{c-a^2cx^2}}-\frac{\left(3\sqrt{1-a^2x^2}\right)}{8c^2\sqrt{c-}}$$
$$= \frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{4ac^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(1+ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\,\text{ta}}{8ac^2\sqrt{c-}}$$

Mathematica [A] time = 0.0496307, size = 85, normalized size = 0.46

$$\frac{\sqrt{1-a^2x^2}\left(-3a^2x^2+3ax+3(ax-1)^2(ax+1)\tanh^{-1}(ax)+2\right)}{8ac^2(ax-1)^2(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2 + 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^2*(1 + a*x)*ArcTan
h[a*x]))/(8*a*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.097, size = 166, normalized size = 0.9

$$-\frac{3a^3x^3\ln(ax+1)-3\ln(ax-1)x^3a^3-3\ln(ax+1)a^2x^2+3\ln(ax-1)a^2x^2-6a^2x^2-3ax\ln(ax+1)+3\ln(ax-}{\left(16a^2x^2-16\right)c^3a(ax+1)(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*x^3*ln(a*x+1)-3*ln(a
*x-1)*x^3*a^3-3*ln(a*x+1)*a^2*x^2+3*ln(a*x-1)*a^2*x^2-6*a^2*x^2-3*a*x*ln(a*
x+1)+3*ln(a*x-1)*x*a+6*a*x+3*ln(a*x+1)-3*ln(a*x-1)+4)/(a^2*x^2-1)/c^3/a/(a*
x+1)/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [A] time = 2.12643, size = 934, normalized size = 5.08

$$\left[\frac{3\left(a^5x^5 - a^4x^4 - 2a^3x^3 + 2a^2x^2 + ax - 1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5a^4cx^4-5a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-c}{a^6x^6-3a^4x^4+3a^2x^2-1}\right) + 4\left(2a^3x^3 + a^2x^2\right.}{32\left(a^6c^3x^5 - a^5c^3x^4 - 2a^4c^3x^3 + 2a^3c^3x^2 + a^2c^3x - ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/32*(3*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*sqrt(c)*log(
-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*x^3 + a*x)*sqrt(-a^2*c*x^2
 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)
) + 4*(2*a^3*x^3 + a^2*x^2 - 5*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)
)/(a^6*c^3*x^5 - a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^3*c^3*x^2 + a^2*c^3*x -
a*c^3), 1/16*(3*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*sqrt(
-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*c*x^
4 - c)) + 2*(2*a^3*x^3 + a^2*x^2 - 5*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^
2 + 1))/(a^6*c^3*x^5 - a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^3*c^3*x^2 + a^2*c^
3*x - a*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-(ax-1)(ax+1)}\left(-c(ax-1)(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(5
/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.983 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=252

$$\frac{\sqrt{1-a^2x^2}}{2c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(x)}{c^2\sqrt{c-a^2cx^2}}-\frac{11\sqrt{1-a^2x^2}\log(1-}{16c^2\sqrt{c-a^2cx^2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(8*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^
2]/(2*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*c^2*(1 + a*
x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*Log[x])/(c^2*Sqrt[c - a^2*c*x^
2]) - (11*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(16*c^2*Sqrt[c - a^2*c*x^2]) - (5
*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(16*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.243718, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 88}

$$\frac{\sqrt{1-a^2x^2}}{2c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(x)}{c^2\sqrt{c-a^2cx^2}}-\frac{11\sqrt{1-a^2x^2}\log(1-}{16c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)^(5/2)),x]
```



```
[Out] Sqrt[1 - a^2*x^2]/(8*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^
2]/(2*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*c^2*(1 + a*
x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*Log[x])/(c^2*Sqrt[c - a^2*c*x^
2]) - (11*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(16*c^2*Sqrt[c - a^2*c*x^2]) - (5
*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(16*c^2*Sqrt[c - a^2*c*x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x}-\frac{a}{4(-1+ax)^3}+\frac{a}{2(-1+ax)^2}-\frac{11a}{16(-1+ax)}-\frac{a}{8(1+ax)^2}-\frac{5a}{16(1+ax)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{2c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8c^2(1+ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(x)}{c^2\sqrt{c-a^2cx^2}}-$$

Mathematica [A] time = 0.0725986, size = 86, normalized size = 0.34

$$\frac{\sqrt{1-a^2x^2}\left(\frac{8}{1-ax}+\frac{2}{ax+1}+\frac{2}{(ax-1)^2}-11\log(1-ax)-5\log(ax+1)+16\log(x)\right)}{16c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(8/(1 - a*x) + 2/(-1 + a*x)^2 + 2/(1 + a*x) + 16*Log[x]
- 11*Log[1 - a*x] - 5*Log[1 + a*x]))/(16*c^2*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.093, size = 193, normalized size = 0.8

$$-\frac{16a^3\ln(x)x^3-5a^3x^3\ln(ax+1)-11\ln(ax-1)x^3a^3-16a^2\ln(x)x^2+5\ln(ax+1)a^2x^2+11\ln(ax-1)a^2x^2-6a^2x}{\left(16a^2x^2-16\right)c^3(ax-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(16*a^3*ln(x)*x^3-5*a^3*x^3
*ln(a*x+1)-11*ln(a*x-1)*x^3*a^3-16*a^2*ln(x)*x^2+5*ln(a*x+1)*a^2*x^2+11*ln(
a*x-1)*a^2*x^2-6*a^2*x^2-16*a*ln(x)*x+5*a*x*ln(a*x+1)+11*ln(a*x-1)*x*a-2*a*
x+16*ln(x)-5*ln(a*x+1)-11*ln(a*x-1)+12)/(a^2*x^2-1)/c^3/(a*x-1)^2/(a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{a^7c^3x^8-a^6c^3x^7-3\,a^5c^3x^6+3\,a^4c^3x^5+3\,a^3c^3x^4-3\,a^2c^3x^3-ac^3x^2+c^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^7*c^3*x^8 - a^6*c^3*x^7
 - 3*a^5*c^3*x^6 + 3*a^4*c^3*x^5 + 3*a^3*c^3*x^4 - 3*a^2*c^3*x^3 - a*c^3*x^
2 + c^3*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{x\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/(x*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**
(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)*x), x)
```

3.984 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=295

$$\frac{3a\sqrt{1-a^2x^2}}{4c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{8c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\log(x)}{c^2\sqrt{c-a^2cx^2}}-23a$$

```
[Out] -(Sqrt[1 - a^2*x^2]/(c^2*x*Sqrt[c - a^2*c*x^2])) + (a*Sqrt[1 - a^2*x^2])/(8
*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + (3*a*Sqrt[1 - a^2*x^2])/(4*c^2*(1 -
 a*x)*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/(8*c^2*(1 + a*x)*Sqrt[c
- a^2*c*x^2]) + (a*Sqrt[1 - a^2*x^2]*Log[x])/(c^2*Sqrt[c - a^2*c*x^2]) - (2
3*a*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(16*c^2*Sqrt[c - a^2*c*x^2]) + (7*a*Sqr
t[1 - a^2*x^2]*Log[1 + a*x])/(16*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.245048, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 88}

$$\frac{3a\sqrt{1-a^2x^2}}{4c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{8c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\log(x)}{c^2\sqrt{c-a^2cx^2}}-23a$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/(c^2*x*Sqrt[c - a^2*c*x^2])) + (a*Sqrt[1 - a^2*x^2])/(8
*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + (3*a*Sqrt[1 - a^2*x^2])/(4*c^2*(1 -
 a*x)*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/(8*c^2*(1 + a*x)*Sqrt[c
- a^2*c*x^2]) + (a*Sqrt[1 - a^2*x^2]*Log[x])/(c^2*Sqrt[c - a^2*c*x^2]) - (2
3*a*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(16*c^2*Sqrt[c - a^2*c*x^2]) + (7*a*Sqr
t[1 - a^2*x^2]*Log[1 + a*x])/(16*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx=\frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$=\frac{\sqrt{1-a^2x^2}\int \frac{1}{x^2(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$=\frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x^2}+\frac{a}{x}-\frac{a^2}{4(-1+ax)^3}+\frac{3a^2}{4(-1+ax)^2}-\frac{23a^2}{16(-1+ax)}+\frac{a^2}{8(1+ax)^2}+\frac{7a^2}{16(1+ax)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$
$$=-\frac{\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{3a\sqrt{1-a^2x^2}}{4c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{8c^2(1+ax)\sqrt{c-a^2cx}}$$

Mathematica [A] time = 0.0885629, size = 97, normalized size = 0.33

$$\frac{\sqrt{1-a^2x^2}\left(\frac{12a}{1-ax}-\frac{2a}{ax+1}+\frac{2a}{(ax-1)^2}+16a\log(x)-23a\log(1-ax)+7a\log(ax+1)-\frac{16}{x}\right)}{16c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-16/x + (12*a)/(1 - a*x) + (2*a)/(-1 + a*x)^2 - (2*a)/(
1 + a*x) + 16*a*Log[x] - 23*a*Log[1 - a*x] + 7*a*Log[1 + a*x]))/(16*c^2*Sqr
t[c - a^2*c*x^2])
```

Maple [A] time = 0.1, size = 222, normalized size = 0.8

$$-\frac{16\,a^4\ln(x)\,x^4+7\,\ln(ax+1)\,a^4x^4-23\,\ln(ax-1)\,a^4x^4-16\,a^3\ln(x)\,x^3-7\,a^3x^3\ln(ax+1)+23\,\ln(ax-1)\,x^3a^3-3}{(16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(16*a^4*ln(x)*x^4+7*ln(a*x+
1)*a^4*x^4-23*ln(a*x-1)*a^4*x^4-16*a^3*ln(x)*x^3-7*a^3*x^3*ln(a*x+1)+23*ln(
a*x-1)*x^3*a^3-30*x^3*a^3-16*a^2*ln(x)*x^2-7*ln(a*x+1)*a^2*x^2+23*ln(a*x-1)
*a^2*x^2+22*a^2*x^2+16*a*ln(x)*x+7*a*x*ln(a*x+1)-23*ln(a*x-1)*x*a+28*a*x-16
)/(a^2*x^2-1)/c^3/(a*x-1)^2/(a*x+1)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{a^7c^3x^9-a^6c^3x^8-3\,a^5c^3x^7+3\,a^4c^3x^6+3\,a^3c^3x^5-3\,a^2c^3x^4-ac^3x^3+c^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^7*c^3*x^9 - a^6*c^3*x^8
 - 3*a^5*c^3*x^7 + 3*a^4*c^3*x^6 + 3*a^3*c^3*x^5 - 3*a^2*c^3*x^4 - a*c^3*x^
3 + c^3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{x^2\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)*x^2), x)
```

3.985 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=345

$$\frac{a^2\sqrt{1-a^2x^2}}{c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}}{8c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2c^2x^2\sqrt{c-a^2cx^2}}+\frac{3a^2}{c}$$

```
[Out] -Sqrt[1 - a^2*x^2]/(2*c^2*x^2*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/
(c^2*x*Sqrt[c - a^2*c*x^2]) + (a^2*Sqrt[1 - a^2*x^2])/(8*c^2*(1 - a*x)^2*Sq
rt[c - a^2*c*x^2]) + (a^2*Sqrt[1 - a^2*x^2])/(c^2*(1 - a*x)*Sqrt[c - a^2*c*
x^2]) + (a^2*Sqrt[1 - a^2*x^2])/(8*c^2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (3*
a^2*Sqrt[1 - a^2*x^2]*Log[x])/(c^2*Sqrt[c - a^2*c*x^2]) - (39*a^2*Sqrt[1 -
a^2*x^2]*Log[1 - a*x])/(16*c^2*Sqrt[c - a^2*c*x^2]) - (9*a^2*Sqrt[1 - a^2*x
^2]*Log[1 + a*x])/(16*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.256693, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 88}

$$\frac{a^2\sqrt{1-a^2x^2}}{c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}}{8c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2c^2x^2\sqrt{c-a^2cx^2}}+\frac{3a^2}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)^(5/2)),x]
```


```
[Out] -Sqrt[1 - a^2*x^2]/(2*c^2*x^2*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/
(c^2*x*Sqrt[c - a^2*c*x^2]) + (a^2*Sqrt[1 - a^2*x^2])/(8*c^2*(1 - a*x)^2*Sq
rt[c - a^2*c*x^2]) + (a^2*Sqrt[1 - a^2*x^2])/(c^2*(1 - a*x)*Sqrt[c - a^2*c*
x^2]) + (a^2*Sqrt[1 - a^2*x^2])/(8*c^2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (3*
a^2*Sqrt[1 - a^2*x^2]*Log[x])/(c^2*Sqrt[c - a^2*c*x^2]) - (39*a^2*Sqrt[1 -
a^2*x^2]*Log[1 - a*x])/(16*c^2*Sqrt[c - a^2*c*x^2]) - (9*a^2*Sqrt[1 - a^2*x
^2]*Log[1 + a*x])/(16*c^2*Sqrt[c - a^2*c*x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x^3(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{3a^2}{x}-\frac{a^3}{4(-1+ax)^3}+\frac{a^3}{(-1+ax)^2}-\frac{39a^3}{16(-1+ax)}-\frac{a^3}{8(1+ax)^2}-\frac{9a^3}{16(1+ax)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= -\frac{\sqrt{1-a^2x^2}}{2c^2x^2\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}}{c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{a}{8c^2(1}$$

Mathematica [A] time = 0.124727, size = 115, normalized size = 0.33

$$\frac{\sqrt{1-a^2x^2}\left(\frac{16a^2}{1-ax}+\frac{2a^2}{ax+1}+\frac{2a^2}{(ax-1)^2}+48a^2\log(x)-39a^2\log(1-ax)-9a^2\log(ax+1)-\frac{16a}{x}-\frac{8}{x^2}\right)}{16c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-8/x^2 - (16*a)/x + (16*a^2)/(1 - a*x) + (2*a^2)/(-1 +
a*x)^2 + (2*a^2)/(1 + a*x) + 48*a^2*Log[x] - 39*a^2*Log[1 - a*x] - 9*a^2*Lo
g[1 + a*x]))/(16*c^2*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.105, size = 242, normalized size = 0.7

$$-\frac{48\,a^5\ln(x)\,x^5-9\,\ln(ax+1)\,x^5a^5-39\,\ln(ax-1)\,x^5a^5-48\,a^4\ln(x)\,x^4+9\,\ln(ax+1)\,a^4x^4+39\,\ln(ax-1)\,a^4x^4-30}{(16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(48*a^5*ln(x)*x^5-9*ln(a*x+
1)*x^5*a^5-39*ln(a*x-1)*x^5*a^5-48*a^4*ln(x)*x^4+9*ln(a*x+1)*a^4*x^4+39*ln(
a*x-1)*a^4*x^4-30*x^4*a^4-48*a^3*ln(x)*x^3+9*a^3*x^3*ln(a*x+1)+39*ln(a*x-1)
*x^3*a^3+6*x^3*a^3+48*a^2*ln(x)*x^2-9*ln(a*x+1)*a^2*x^2-39*ln(a*x-1)*a^2*x^
2+44*a^2*x^2-8*a*x-8)/(a^2*x^2-1)/c^3/(a*x-1)^2/(a*x+1)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{a^7c^3x^{10}-a^6c^3x^9-3\,a^5c^3x^8+3\,a^4c^3x^7+3\,a^3c^3x^6-3\,a^2c^3x^5-ac^3x^4+c^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^7*c^3*x^10 - a^6*c^3*x^
9 - 3*a^5*c^3*x^8 + 3*a^4*c^3*x^7 + 3*a^3*c^3*x^6 - 3*a^2*c^3*x^5 - a*c^3*x
^4 + c^3*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{x^3\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)*x^3), x)
```

3.986 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=277

$$\frac{3\sqrt{1-a^2x^2}}{16ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^3(ax+1)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{32ac^3(ax+1)^2\sqrt{c-a^2cx^2}}+\frac{}{24ac^3(1-}$$

```
[Out] Sqrt[1 - a^2*x^2]/(24*a*c^3*(1 - a*x)^3*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 -
a^2*x^2])/(32*a*c^3*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2*x^2]
)/(16*a*c^3*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(32*a*c^3*(1
 + a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a*c^3*(1 + a*x)*Sqrt[
c - a^2*c*x^2]) + (5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(16*a*c^3*Sqrt[c - a^2
*c*x^2])
```

Rubi [A] time = 0.132034, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6143, 6140, 44, 207}

$$\frac{3\sqrt{1-a^2x^2}}{16ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^3(ax+1)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{32ac^3(ax+1)^2\sqrt{c-a^2cx^2}}+\frac{}{24ac^3(1-}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```


```
[Out] Sqrt[1 - a^2*x^2]/(24*a*c^3*(1 - a*x)^3*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 -
a^2*x^2])/(32*a*c^3*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2*x^2]
)/(16*a*c^3*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(32*a*c^3*(1
 + a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a*c^3*(1 + a*x)*Sqrt[
c - a^2*c*x^2]) + (5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(16*a*c^3*Sqrt[c - a^2
*c*x^2])
```


Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^4(1+ax)^3}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{8(-1+ax)^4}-\frac{3}{16(-1+ax)^3}+\frac{3}{16(-1+ax)^2}+\frac{1}{16(1+ax)^3}+\frac{1}{8(1+ax)^2}-\frac{5}{16\left(-1+a^2x^2\right)}\right)dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}}{24ac^3(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{16ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{\cdots}}{32ac^3(1+\cdots}$$

$$= \frac{\sqrt{1-a^2x^2}}{24ac^3(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{16ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{\cdots}}{32ac^3(1+\cdots}$$

Mathematica [A] time = 0.0789628, size = 103, normalized size = 0.37

$$\frac{\sqrt{1-a^2x^2}\left(-15a^4x^4+15a^3x^3+25a^2x^2-25ax+15(ax-1)^3(ax+1)^2\tanh^{-1}(ax)-8\right)}{48ac^3(ax-1)^3(ax+1)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-8 - 25*a*x + 25*a^2*x^2 + 15*a^3*x^3 - 15*a^4*x^4 + 15
*(-1 + a*x)^3*(1 + a*x)^2*ArcTanh[a*x]))/(48*a*c^3*(-1 + a*x)^3*(1 + a*x)^2
*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.103, size = 238, normalized size = 0.9

$$-\frac{15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5-15\ln(ax+1)a^4x^4+15\ln(ax-1)a^4x^4-30x^4a^4-30a^3x^3\ln(ax+1)+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/96*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x+1)*x^5*a^5-15*ln
(a*x-1)*x^5*a^5-15*ln(a*x+1)*a^4*x^4+15*ln(a*x-1)*a^4*x^4-30*x^4*a^4-30*a^3
*x^3*ln(a*x+1)+30*ln(a*x-1)*x^3*a^3+30*x^3*a^3+30*ln(a*x+1)*a^2*x^2-30*ln(a
*x-1)*a^2*x^2+50*a^2*x^2+15*a*x*ln(a*x+1)-15*ln(a*x-1)*x*a-50*a*x-15*ln(a*x
+1)+15*ln(a*x-1)-16)/(a^2*x^2-1)/c^4/a/(a*x+1)^2/(a*x-1)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(7/2)*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [A] time = 2.13687, size = 1165, normalized size = 4.21

$$\left[\frac{15\left(a^7x^7-a^6x^6-3\,a^5x^5+3\,a^4x^4+3\,a^3x^3-3\,a^2x^2-ax+1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5\,a^4cx^4-5\,a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-c}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}\right)}{192\left(a^8c^4x^7-a^7c^4x^6-3\,a^6c^4x^5+3\,a^5c^4x^4+3\,a^4c^4x^3-3\,a^3c^4x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fri
cas")
```

```
[Out] [1/192*(15*(a^7*x^7 - a^6*x^6 - 3*a^5*x^5 + 3*a^4*x^4 + 3*a^3*x^3 - 3*a^2*x
^2 - a*x + 1)*sqrt(c)*log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*
x^3 + a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 -
3*a^4*x^4 + 3*a^2*x^2 - 1)) + 4*(8*a^5*x^5 + 7*a^4*x^4 - 31*a^3*x^3 - 9*a^2
*x^2 + 33*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^8*c^4*x^7 - a^7*
c^4*x^6 - 3*a^6*c^4*x^5 + 3*a^5*c^4*x^4 + 3*a^4*c^4*x^3 - 3*a^3*c^4*x^2 - a
^2*c^4*x + a*c^4), 1/96*(15*(a^7*x^7 - a^6*x^6 - 3*a^5*x^5 + 3*a^4*x^4 + 3*
a^3*x^3 - 3*a^2*x^2 - a*x + 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(
-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*c*x^4 - c)) + 2*(8*a^5*x^5 + 7*a^4*x^4 - 31
*a^3*x^3 - 9*a^2*x^2 + 33*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^
8*c^4*x^7 - a^7*c^4*x^6 - 3*a^6*c^4*x^5 + 3*a^5*c^4*x^4 + 3*a^4*c^4*x^3 - 3
*a^3*c^4*x^2 - a^2*c^4*x + a*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(7
/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(7/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.987 $\int e^{\tanh^{-1}(ax)} x^m \left(c - a^2 c x^2\right)^2 \, dx$

Optimal. Leaf size=80

$$\frac{c^2 x^{m+1} \text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{m+1}{2}, \frac{m+3}{2}, a^2 x^2\right)}{m+1} + \frac{a c^2 x^{m+2} \text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{m+2}{2}, \frac{m+4}{2}, a^2 x^2\right)}{m+2}$$

```
[Out] (c^2*x^(1 + m)*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 +
 m) + (a*c^2*x^(2 + m)*Hypergeometric2F1[-3/2, (2 + m)/2, (4 + m)/2, a^2*x^
2])/(2 + m)
```

Rubi [A] time = 0.095878, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6148, 808, 364}

$$\frac{c^2 x^{m+1} {}_2F_1\left(-\frac{3}{2}, \frac{m+1}{2}; \frac{m+3}{2}; a^2 x^2\right)}{m+1} + \frac{a c^2 x^{m+2} {}_2F_1\left(-\frac{3}{2}, \frac{m+2}{2}; \frac{m+4}{2}; a^2 x^2\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*x^(1 + m)*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 +
 m) + (a*c^2*x^(2 + m)*Hypergeometric2F1[-3/2, (2 + m)/2, (4 + m)/2, a^2*x^
2])/(2 + m)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^2\,dx=c^2\int x^m(1+ax)\left(1-a^2x^2\right)^{3/2}\,dx$$
$$=c^2\int x^m\left(1-a^2x^2\right)^{3/2}\,dx+\left(ac^2\right)\int x^{1+m}\left(1-a^2x^2\right)^{3/2}\,dx$$
$$=\frac{c^2x^{1+m}\,{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m}+\frac{ac^2x^{2+m}\,{}_2F_1\left(-\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}$$

Mathematica [A] time = 0.0358562, size = 82, normalized size = 1.02

$$c^2\left(\frac{x^{m+1}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{2},\frac{m+1}{2}+1,a^2x^2\right)}{m+1}+\frac{ax^{m+2}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+2}{2},\frac{m+2}{2}+1,a^2x^2\right)}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2)^2,x]
```

```
[Out] c^2*((x^(1 + m)*Hypergeometric2F1[-3/2, (1 + m)/2, 1 + (1 + m)/2, a^2*x^2])
/(1 + m) + (a*x^(2 + m)*Hypergeometric2F1[-3/2, (2 + m)/2, 1 + (2 + m)/2, a
^2*x^2])/(2 + m))
```

Maple [B] time = 0.323, size = 227, normalized size = 2.8

$$\frac{a^5c^2x^{6+m}}{6+m}{}_2F_1\left(\frac{1}{2},3+\frac{m}{2};4+\frac{m}{2};a^2x^2\right)-2\,\frac{a^3c^2x^{4+m}{}_2F_1(1/2,2+m/2;3+m/2;a^2x^2)}{4+m}+\frac{ac^2x^{2+m}}{2+m}{}_2F_1\left(\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^2,x)
```

```
[Out] a^5*c^2/(6+m)*x^(6+m)*hypergeom([1/2,3+1/2*m],[4+1/2*m],a^2*x^2)-2*a^3*c^2/
(4+m)*x^(4+m)*hypergeom([1/2,2+1/2*m],[3+1/2*m],a^2*x^2)+a*c^2/(2+m)*x^(2+m
)*hypergeom([1/2,1+1/2*m],[2+1/2*m],a^2*x^2)+c^2*a^4/(5+m)*x^(5+m)*hypergeo
m([1/2,5/2+1/2*m],[7/2+1/2*m],a^2*x^2)-2*c^2*a^2/(3+m)*x^(3+m)*hypergeom([1
/2,3/2+1/2*m],[5/2+1/2*m],a^2*x^2)+c^2/(1+m)*x^(1+m)*hypergeom([1/2,1/2+1/2
*m],[3/2+1/2*m],a^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2-c\right)^2(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] integrate((a^2*c*x^2 - c)^2*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^3c^2x^3 + a^2c^2x^2 - ac^2x - c^2\right)\sqrt{-a^2x^2 + 1}x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] integral(-(a^3*c^2*x^3 + a^2*c^2*x^2 - a*c^2*x - c^2)*sqrt(-a^2*x^2 + 1)*x^
m, x)
```

Sympy [C] time = 11.6798, size = 223, normalized size = 2.79

$$-\frac{a^3c^2x^4x^m\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle| a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{m}{2}+3\right)} - \frac{a^2c^2x^3x^m\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle| a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{ac^2x^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\right.}{2\Gamma\left(\frac{m}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(-a**2*c*x**2+c)**2,x)
```

```
[Out] -a**3*c**2*x**4*x**m*gamma(m/2 + 2)*hyper((-1/2, m/2 + 2), (m/2 + 3,), a**2
*x**2*exp_polar(2*I*pi))/(2*gamma(m/2 + 3)) - a**2*c**2*x**3*x**m*gamma(m/2
 + 3/2)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), a**2*x**2*exp_polar(2*I*pi))
/(2*gamma(m/2 + 5/2)) + a*c**2*x**2*x**m*gamma(m/2 + 1)*hyper((-1/2, m/2 +
1), (m/2 + 2,), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(m/2 + 2)) + c**2*x*x*
*m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), a**2*x**2*exp_po
lar(2*I*pi))/(2*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 - c\right)^2(ax + 1)x^m}{\sqrt{-a^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a^2*c*x^2 - c)^2*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

3.988 $\int e^{\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=76

$$\frac{cx^{m+1}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{m+1}+\frac{acx^{m+2}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{m+2}$$

```
[Out] (c*x^(1 + m)*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m
) + (a*c*x^(2 + m)*Hypergeometric2F1[-1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/
(2 + m)
```

Rubi [A] time = 0.0789355, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6148, 808, 364}

$$\frac{cx^{m+1}\,_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{acx^{m+2}\,_2F_1\left(-\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^(1 + m)*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m
) + (a*c*x^(2 + m)*Hypergeometric2F1[-1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/
(2 + m)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)dx = c\int x^m(1+ax)\sqrt{1-a^2x^2}\,dx$$
$$= c\int x^m\sqrt{1-a^2x^2}\,dx + (ac)\int x^{1+m}\sqrt{1-a^2x^2}\,dx$$
$$= \frac{cx^{1+m}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m} + \frac{acx^{2+m}\,{}_2F_1\left(-\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}$$

Mathematica [A] time = 0.0418694, size = 72, normalized size = 0.95

$$cx^{m+1}\left(\frac{ax\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2}+1,\frac{m}{2}+2,a^2x^2\right)}{m+2}+\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2),x]
```

```
[Out] c*x^(1 + m)*((a*x*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2, a^2*x^2])/(2 +
m) + Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a^2*x^2]/(1 + m))
```

Maple [B] time = 0.185, size = 143, normalized size = 1.9

$$-\frac{a^3cx^{4+m}}{4+m}{}_2\text{F}_1\left(\frac{1}{2},2+\frac{m}{2};\,3+\frac{m}{2};\,a^2x^2\right)+\frac{acx^{2+m}}{2+m}{}_2\text{F}_1\left(\frac{1}{2},1+\frac{m}{2};\,2+\frac{m}{2};\,a^2x^2\right)-\frac{a^2cx^{3+m}}{3+m}{}_2\text{F}_1\left(\frac{1}{2},\frac{3}{2}+\frac{m}{2};\,\frac{5}{2}+\frac{m}{2};\,a^2x^2\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c),x)
```

```
[Out] -a^3*c/(4+m)*x^(4+m)*hypergeom([1/2,2+1/2*m],[3+1/2*m],a^2*x^2)+a*c/(2+m)*x
^(2+m)*hypergeom([1/2,1+1/2*m],[2+1/2*m],a^2*x^2)-c*a^2/(3+m)*x^(3+m)*hyper
geom([1/2,3/2+1/2*m],[5/2+1/2*m],a^2*x^2)+c/(1+m)*x^(1+m)*hypergeom([1/2,1/
2+1/2*m],[3/2+1/2*m],a^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2cx^2-c\right)(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -integrate((a^2*c*x^2 - c)*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-a^2x^2+1}(acx+c)x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*c*x + c)*x^m, x)
```

Sympy [C] time = 4.91502, size = 104, normalized size = 1.37

$$\frac{acx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\\frac{m}{2}+2\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{cxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(-a**2*c*x**2+c),x)
```

```
[Out] a*c*x**2*x**m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), a**2*x**2*e
xp_polar(2*I*pi))/(2*gamma(m/2 + 2)) + c*x*x**m*gamma(m/2 + 1/2)*hyper((-1/
2, m/2 + 1/2), (m/2 + 3/2,), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(m/2 + 3/
2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(a^2cx^2-c\right)(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] integrate(-(a^2*c*x^2 - c)*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

3.989 $\int \frac{e^{\tanh^{-1}(ax)}x^m}{c-a^2cx^2} dx$

Optimal. Leaf size=80

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{c(m+1)}+\frac{ax^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{c(m+2)}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(c*(1 + m
)) + (a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(c
*(2 + m))
```

Rubi [A] time = 0.10641, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6148, 808, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{c(m+1)}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{c(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(c*(1 + m
)) + (a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(c
*(2 + m))
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^m}{c-a^2cx^2}\,dx = \frac{\int \frac{x^m(1+ax)}{(1-a^2x^2)^{3/2}}\,dx}{c}$$
$$= \frac{\int \frac{x^m}{(1-a^2x^2)^{3/2}}\,dx}{c} + \frac{a\int \frac{x^{1+m}}{(1-a^2x^2)^{3/2}}\,dx}{c}$$
$$= \frac{x^{1+m}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{c(1+m)} + \frac{ax^{2+m}\,{}_2F_1\left(\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{c(2+m)}$$

Mathematica [A] time = 0.0296975, size = 82, normalized size = 1.02

$$\frac{\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+1}{2}+1,a^2x^2\right)}{m+1}+\frac{ax^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+2}{2}+1,a^2x^2\right)}{m+2}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2),x]
```

```
[Out] ((x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, 1 + (1 + m)/2, a^2*x^2])/(1 +
 m) + (a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, 1 + (2 + m)/2, a^2*x^2
])/(2 + m))/c
```

Maple [F] time = 0.293, size = 0, normalized size = 0.

$$\int \frac{(ax+1)\,x^m}{-a^2cx^2+c}\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c),x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)x^m}{(a^2cx^2-c)\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -integrate((a*x + 1)*x^m/((a^2*c*x^2 - c)*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}x^m}{a^3cx^3-a^2cx^2-acx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m/(a^3*c*x^3 - a^2*c*x^2 - a*c*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{axx^m}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(x**m/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)
 + Integral(a*x*x**m/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1
)), x))/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)x^m}{\left(a^2cx^2-c\right)\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] integrate(-(a*x + 1)*x^m/((a^2*c*x^2 - c)*sqrt(-a^2*x^2 + 1)), x)
```

3.990 $\int \frac{e^{\tanh^{-1}(ax)}x^m}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=80

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{c^2(m+1)}+\frac{ax^{m+2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{c^2(m+2)}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(c^2*(1 +
 m)) + (a*x^(2 + m)*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/
(c^2*(2 + m))
```

Rubi [A] time = 0.105854, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6148, 808, 364}

$$\frac{x^{m+1}\,_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{c^2(m+1)}+\frac{ax^{m+2}\,_2F_1\left(\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{c^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(c^2*(1 +
 m)) + (a*x^(2 + m)*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/
(c^2*(2 + m))
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{x^m(1+ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}$$
$$= \frac{\int \frac{x^m}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2} + \frac{a\int \frac{x^{1+m}}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}$$
$$= \frac{x^{1+m}\,{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{c^2(1+m)} + \frac{ax^{2+m}\,{}_2F_1\left(\frac{5}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{c^2(2+m)}$$

Mathematica [A] time = 0.0308114, size = 82, normalized size = 1.02

$$\frac{\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+1}{2},\frac{m+1}{2}+1,a^2x^2\right)}{m+1}+\frac{ax^{m+2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+2}{2}+1,a^2x^2\right)}{m+2}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2)^2,x]
```

```
[Out] ((x^(1 + m)*Hypergeometric2F1[5/2, (1 + m)/2, 1 + (1 + m)/2, a^2*x^2])/(1 +
 m) + (a*x^(2 + m)*Hypergeometric2F1[5/2, (2 + m)/2, 1 + (2 + m)/2, a^2*x^2
])/(2 + m))/c^2
```

Maple [F] time = 0.291, size = 0, normalized size = 0.

$$\int \frac{(ax+1)\,x^m}{\left(-a^2cx^2+c\right)^2}\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^2,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^m}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)*x^m/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}x^m}{a^5c^2x^5-a^4c^2x^4-2\,a^3c^2x^3+2\,a^2c^2x^2+ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^m/(a^5*c^2*x^5 - a^4*c^2*x^4 - 2*a^3*c^2*x^3
 + 2*a^2*c^2*x^2 + a*c^2*x - c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{x^m}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int\frac{axx^m}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(x**m/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**
2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x*x**m/(a**4*x**4*sqrt(-a**
2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))
/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)x^m}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*x^m/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

3.991 $\int \frac{e^{\tanh^{-1}(ax)}x^m}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=80

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{c^3(m+1)}+\frac{ax^{m+2}\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{c^3(m+2)}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[7/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(c^3*(1 +
 m)) + (a*x^(2 + m)*Hypergeometric2F1[7/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/
(c^3*(2 + m))
```

Rubi [A] time = 0.109285, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6148, 808, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{c^3(m+1)}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{7}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{c^3(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2)^3,x]
```


```
[Out] (x^(1 + m)*Hypergeometric2F1[7/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(c^3*(1 +
 m)) + (a*x^(2 + m)*Hypergeometric2F1[7/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/
(c^3*(2 + m))
```


Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^m}{(c-a^2cx^2)^3}\,dx = \frac{\int \frac{x^m(1+ax)}{(1-a^2x^2)^{7/2}}\,dx}{c^3}$$
$$= \frac{\int \frac{x^m}{(1-a^2x^2)^{7/2}}\,dx}{c^3} + \frac{a\int \frac{x^{1+m}}{(1-a^2x^2)^{7/2}}\,dx}{c^3}$$
$$= \frac{x^{1+m}\,{}_2F_1\left(\frac{7}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{c^3(1+m)} + \frac{ax^{2+m}\,{}_2F_1\left(\frac{7}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{c^3(2+m)}$$

Mathematica [A] time = 0.0302, size = 82, normalized size = 1.02

$$\frac{\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{m+1}{2},\frac{m+1}{2}+1,a^2x^2\right)}{m+1}+\frac{ax^{m+2}\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{m+2}{2},\frac{m+2}{2}+1,a^2x^2\right)}{m+2}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2)^3,x]
```

```
[Out] ((x^(1 + m)*Hypergeometric2F1[7/2, (1 + m)/2, 1 + (1 + m)/2, a^2*x^2])/(1 +
 m) + (a*x^(2 + m)*Hypergeometric2F1[7/2, (2 + m)/2, 1 + (2 + m)/2, a^2*x^2
])/(2 + m))/c^3
```

Maple [F] time = 0.29, size = 0, normalized size = 0.

$$\int \frac{(ax+1)\,x^m}{(-a^2cx^2+c)^3}\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^3,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)x^m}{(a^2cx^2-c)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)*x^m/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}x^m}{a^7c^3x^7-a^6c^3x^6-3\,a^5c^3x^5+3\,a^4c^3x^4+3\,a^3c^3x^3-3\,a^2c^3x^2-ac^3x+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m/(a^7*c^3*x^7 - a^6*c^3*x^6 - 3*a^5*c^3*x^5
+ 3*a^4*c^3*x^4 + 3*a^3*c^3*x^3 - 3*a^2*c^3*x^2 - a*c^3*x + c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{axx^m}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(x**m/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x*
*2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + In
tegral(a*x*x**m/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x
**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c*
*3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)x^m}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)*x^m/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

3.992 $\int e^{\tanh^{-1}(ax)} x^m \left(1-a^2x^2\right)^{5/2} dx$

Optimal. Leaf size=82

$$-\frac{2a^2x^{m+3}}{m+3}-\frac{2a^3x^{m+4}}{m+4}+\frac{a^4x^{m+5}}{m+5}+\frac{a^5x^{m+6}}{m+6}+\frac{ax^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}$$

```
[Out] x^(1 + m)/(1 + m) + (a*x^(2 + m))/(2 + m) - (2*a^2*x^(3 + m))/(3 + m) - (2*
a^3*x^(4 + m))/(4 + m) + (a^4*x^(5 + m))/(5 + m) + (a^5*x^(6 + m))/(6 + m)
```

Rubi [A] time = 0.117699, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$-\frac{2a^2x^{m+3}}{m+3}-\frac{2a^3x^{m+4}}{m+4}+\frac{a^4x^{m+5}}{m+5}+\frac{a^5x^{m+6}}{m+6}+\frac{ax^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*(1 - a^2*x^2)^(5/2),x]
```

```
[Out] x^(1 + m)/(1 + m) + (a*x^(2 + m))/(2 + m) - (2*a^2*x^(3 + m))/(3 + m) - (2*
a^3*x^(4 + m))/(4 + m) + (a^4*x^(5 + m))/(5 + m) + (a^5*x^(6 + m))/(6 + m)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)} x^m \left(1-a^2x^2\right)^{5/2} dx &= \int x^m (1-ax)^2 (1+ax)^3 \, dx\\ &= \int \left(x^m+ax^{1+m}-2a^2x^{2+m}-2a^3x^{3+m}+a^4x^{4+m}+a^5x^{5+m}\right) dx\\ &= \frac{x^{1+m}}{1+m}+\frac{ax^{2+m}}{2+m}-\frac{2a^2x^{3+m}}{3+m}-\frac{2a^3x^{4+m}}{4+m}+\frac{a^4x^{5+m}}{5+m}+\frac{a^5x^{6+m}}{6+m}\end{aligned}$$

Mathematica [A] time = 0.0462876, size = 70, normalized size = 0.85

$$x^{m+1}\left(\frac{a^5x^5}{m+6}+\frac{a^4x^4}{m+5}-\frac{2a^3x^3}{m+4}-\frac{2a^2x^2}{m+3}+\frac{ax}{m+2}+\frac{1}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*(1 - a^2*x^2)^(5/2),x]
```

```
[Out] x^(1 + m)*((1 + m)^(-1) + (a*x)/(2 + m) - (2*a^2*x^2)/(3 + m) - (2*a^3*x^3)
/(4 + m) + (a^4*x^4)/(5 + m) + (a^5*x^5)/(6 + m))
```

Maple [B] time = 0.032, size = 338, normalized size = 4.1

$$x^{1+m}\left(a^5m^5x^5 + 15\,a^5m^4x^5 + 85\,a^5m^3x^5 + a^4m^5x^4 + 225\,a^5m^2x^5 + 16\,a^4m^4x^4 + 274\,a^5mx^5 + 95\,a^4m^3x^4 - 2\,a^3m^5x^3 + 12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)*(-a^2*x^2+1)^2*x^m,x)
```

```
[Out] x^(1+m)*(a^5*m^5*x^5+15*a^5*m^4*x^5+85*a^5*m^3*x^5+a^4*m^5*x^4+225*a^5*m^2*
x^5+16*a^4*m^4*x^4+274*a^5*m*x^5+95*a^4*m^3*x^4-2*a^3*m^5*x^3+120*a^5*x^5+2
60*a^4*m^2*x^4-34*a^3*m^4*x^3+324*a^4*m*x^4-214*a^3*m^3*x^3-2*a^2*m^5*x^2+1
44*a^4*x^4-614*a^3*m^2*x^3-36*a^2*m^4*x^2-792*a^3*m*x^3-242*a^2*m^3*x^2+a*m
^5*x-360*a^3*x^3-744*a^2*m^2*x^2+19*a*m^4*x-1016*a^2*m*x^2+137*a*m^3*x+m^5-
480*a^2*x^2+461*a*m^2*x+20*m^4+702*a*m*x+155*m^3+360*a*x+580*m^2+1044*m+720
)/(6+m)/(5+m)/(4+m)/(3+m)/(2+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^2*x^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.95738, size = 682, normalized size = 8.32

$$\left(\left(a^5m^5 + 15\,a^5m^4 + 85\,a^5m^3 + 225\,a^5m^2 + 274\,a^5m + 120\,a^5\right)x^6 + \left(a^4m^5 + 16\,a^4m^4 + 95\,a^4m^3 + 260\,a^4m^2 + 324\,a^4m + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^2*x^m,x, algorithm="fricas")
```

```
[Out] ((a^5*m^5 + 15*a^5*m^4 + 85*a^5*m^3 + 225*a^5*m^2 + 274*a^5*m + 120*a^5)*x^
6 + (a^4*m^5 + 16*a^4*m^4 + 95*a^4*m^3 + 260*a^4*m^2 + 324*a^4*m + 144*a^4)
*x^5 - 2*(a^3*m^5 + 17*a^3*m^4 + 107*a^3*m^3 + 307*a^3*m^2 + 396*a^3*m + 18
0*a^3)*x^4 - 2*(a^2*m^5 + 18*a^2*m^4 + 121*a^2*m^3 + 372*a^2*m^2 + 508*a^2*
m + 240*a^2)*x^3 + (a*m^5 + 19*a*m^4 + 137*a*m^3 + 461*a*m^2 + 702*a*m + 36
0*a)*x^2 + (m^5 + 20*m^4 + 155*m^3 + 580*m^2 + 1044*m + 720)*x)*x^m/(m^6 +
21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)
```

Sympy [A] time = 1.54547, size = 1760, normalized size = 21.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a**2*x**2+1)**2*x**m,x)
```

```
[Out] Piecewise((a**5*log(x) - a**4/x + a**3/x**2 + 2*a**2/(3*x**3) - a/(4*x**4)
- 1/(5*x**5), Eq(m, -6)), (a**5*x + a**4*log(x) + 2*a**3/x + a**2/x**2 - a/
(3*x**3) - 1/(4*x**4), Eq(m, -5)), (a**5*x**2/2 + a**4*x - 2*a**3*log(x) +
2*a**2/x - a/(2*x**2) - 1/(3*x**3), Eq(m, -4)), (a**5*x**3/3 + a**4*x**2/2
- 2*a**3*x - 2*a**2*log(x) - a/x - 1/(2*x**2), Eq(m, -3)), (a**5*x**4/4 + a
**4*x**3/3 - a**3*x**2 - 2*a**2*x + a*log(x) - 1/x, Eq(m, -2)), (a**5*x**5/
5 + a**4*x**4/4 - 2*a**3*x**3/3 - a**2*x**2 + a*x + log(x), Eq(m, -1)), (a*
*5*m**5*x**6*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*
m + 720) + 15*a**5*m**4*x**6*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1
624*m**2 + 1764*m + 720) + 85*a**5*m**3*x**6*x**m/(m**6 + 21*m**5 + 175*m**
4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 225*a**5*m**2*x**6*x**m/(m**6 +
21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 274*a**5*m*x**6
*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 1
20*a**5*x**6*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*
m + 720) + a**4*m**5*x**5*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624
*m**2 + 1764*m + 720) + 16*a**4*m**4*x**5*x**m/(m**6 + 21*m**5 + 175*m**4 +
 735*m**3 + 1624*m**2 + 1764*m + 720) + 95*a**4*m**3*x**5*x**m/(m**6 + 21*m
**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 260*a**4*m**2*x**5*
x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 32
4*a**4*m*x**5*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764
*m + 720) + 144*a**4*x**5*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624
*m**2 + 1764*m + 720) - 2*a**3*m**5*x**4*x**m/(m**6 + 21*m**5 + 175*m**4 +
735*m**3 + 1624*m**2 + 1764*m + 720) - 34*a**3*m**4*x**4*x**m/(m**6 + 21*m*
*5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) - 214*a**3*m**3*x**4*x
**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) - 614
*a**3*m**2*x**4*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 17
64*m + 720) - 792*a**3*m*x**4*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 +
1624*m**2 + 1764*m + 720) - 360*a**3*x**4*x**m/(m**6 + 21*m**5 + 175*m**4 +
 735*m**3 + 1624*m**2 + 1764*m + 720) - 2*a**2*m**5*x**3*x**m/(m**6 + 21*m*
*5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) - 36*a**2*m**4*x**3*x*
*m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) - 242*
a**2*m**3*x**3*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 176
4*m + 720) - 744*a**2*m**2*x**3*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3
+ 1624*m**2 + 1764*m + 720) - 1016*a**2*m*x**3*x**m/(m**6 + 21*m**5 + 175*m
**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) - 480*a**2*x**3*x**m/(m**6 + 21*
m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + a*m**5*x**2*x**m/(
m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 19*a*m**
4*x**2*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 72
0) + 137*a*m**3*x**2*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2
 + 1764*m + 720) + 461*a*m**2*x**2*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m*
*3 + 1624*m**2 + 1764*m + 720) + 702*a*m*x**2*x**m/(m**6 + 21*m**5 + 175*m*
*4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 360*a*x**2*x**m/(m**6 + 21*m**5
 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + m**5*x*x**m/(m**6 + 21
*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 20*m**4*x*x**m/(m
**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 155*m**3*
x*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) +
580*m**2*x*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m
+ 720) + 1044*m*x*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 +
1764*m + 720) + 720*x*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**
2 + 1764*m + 720), True))
```

Giac [B] time = 1.20047, size = 621, normalized size = 7.57

$$\frac{a^5m^5x^6x^m + 15\,a^5m^4x^6x^m + a^4m^5x^5x^m + 85\,a^5m^3x^6x^m + 16\,a^4m^4x^5x^m + 225\,a^5m^2x^6x^m - 2\,a^3m^5x^4x^m + 95\,a^4m^3x^5x^m + \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^2*x^m,x, algorithm="giac")
```

```
[Out] (a^5*m^5*x^6*x^m + 15*a^5*m^4*x^6*x^m + a^4*m^5*x^5*x^m + 85*a^5*m^3*x^6*x^
m + 16*a^4*m^4*x^5*x^m + 225*a^5*m^2*x^6*x^m - 2*a^3*m^5*x^4*x^m + 95*a^4*m
^3*x^5*x^m + 274*a^5*m*x^6*x^m - 34*a^3*m^4*x^4*x^m + 260*a^4*m^2*x^5*x^m +
 120*a^5*x^6*x^m - 2*a^2*m^5*x^3*x^m - 214*a^3*m^3*x^4*x^m + 324*a^4*m*x^5*
x^m - 36*a^2*m^4*x^3*x^m - 614*a^3*m^2*x^4*x^m + 144*a^4*x^5*x^m + a*m^5*x^
2*x^m - 242*a^2*m^3*x^3*x^m - 792*a^3*m*x^4*x^m + 19*a*m^4*x^2*x^m - 744*a^
2*m^2*x^3*x^m - 360*a^3*x^4*x^m + m^5*x*x^m + 137*a*m^3*x^2*x^m - 1016*a^2*
m*x^3*x^m + 20*m^4*x*x^m + 461*a*m^2*x^2*x^m - 480*a^2*x^3*x^m + 155*m^3*x*
x^m + 702*a*m*x^2*x^m + 580*m^2*x*x^m + 360*a*x^2*x^m + 1044*m*x*x^m + 720*
x*x^m)/(m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)
```

3.993 $\int e^{\tanh^{-1}(ax)} x^m \left(1-a^2x^2\right)^{3/2} dx$

Optimal. Leaf size=54

$$-\frac{a^2x^{m+3}}{m+3}-\frac{a^3x^{m+4}}{m+4}+\frac{ax^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}$$

```
[Out] x^(1 + m)/(1 + m) + (a*x^(2 + m))/(2 + m) - (a^2*x^(3 + m))/(3 + m) - (a^3*
x^(4 + m))/(4 + m)
```

Rubi [A] time = 0.0987892, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 75}

$$-\frac{a^2x^{m+3}}{m+3}-\frac{a^3x^{m+4}}{m+4}+\frac{ax^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*(1 - a^2*x^2)^(3/2),x]
```

```
[Out] x^(1 + m)/(1 + m) + (a*x^(2 + m))/(2 + m) - (a^2*x^(3 + m))/(3 + m) - (a^3*
x^(4 + m))/(4 + m)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)} x^m \left(1-a^2x^2\right)^{3/2} dx &= \int x^m(1-ax)(1+ax)^2 \, dx\\
&= \int \left(x^m+ax^{1+m}-a^2x^{2+m}-a^3x^{3+m}\right) dx\\
&= \frac{x^{1+m}}{1+m}+\frac{ax^{2+m}}{2+m}-\frac{a^2x^{3+m}}{3+m}-\frac{a^3x^{4+m}}{4+m}
\end{aligned}$$

Mathematica [A] time = 0.0937093, size = 54, normalized size = 1.

$$\frac{x^{m+1}\left((2m+5)\left(\frac{a^2x^2}{m+3}+\frac{2ax}{m+2}+\frac{1}{m+1}\right)-(ax+1)^3\right)}{m+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*(-(1 + a*x)^3 + (5 + 2*m)*((1 + m)^(-1) + (2*a*x)/(2 + m) + (a^2
*x^2)/(3 + m))))/(4 + m)
```

Maple [B] time = 0.031, size = 142, normalized size = 2.6

$$-\frac{x^{1+m}\left(a^3m^3x^3+6\,a^3m^2x^3+11\,a^3mx^3+a^2m^3x^2+6\,x^3a^3+7\,a^2m^2x^2+14\,a^2mx^2-am^3x+8\,a^2x^2-8\,am^2x-19\,amx-\right.}{(4+m)\,(3+m)\,(2+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)*(-a^2*x^2+1)*x^m,x)
```

```
[Out] -x^(1+m)*(a^3*m^3*x^3+6*a^3*m^2*x^3+11*a^3*m*x^3+a^2*m^3*x^2+6*a^3*x^3+7*a^
2*m^2*x^2+14*a^2*m*x^2-a*m^3*x+8*a^2*x^2-8*a*m^2*x-19*a*m*x-m^3-12*a*x-9*m^
2-26*m-24)/(4+m)/(3+m)/(2+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)*x^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.98979, size = 278, normalized size = 5.15

$$-\frac{\left(\left(a^3m^3+6\,a^3m^2+11\,a^3m+6\,a^3\right)x^4+\left(a^2m^3+7\,a^2m^2+14\,a^2m+8\,a^2\right)x^3-\left(am^3+8\,am^2+19\,am+12\,a\right)x^2-\left(m^3+9\right.\right.}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)*x^m,x, algorithm="fricas")
```

```
[Out] -((a^3*m^3 + 6*a^3*m^2 + 11*a^3*m + 6*a^3)*x^4 + (a^2*m^3 + 7*a^2*m^2 + 14*
a^2*m + 8*a^2)*x^3 - (a*m^3 + 8*a*m^2 + 19*a*m + 12*a)*x^2 - (m^3 + 9*m^2 +
 26*m + 24)*x)*x^m/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

Sympy [A] time = 0.808258, size = 585, normalized size = 10.83

$$\begin{cases} -a^3\log(x)+\frac{a^2}{x}-\frac{a}{2x^2}-\frac{1}{3x^3} \\ -a^3x-a^2\log(x)-\frac{a}{x}-\frac{1}{2x^2} \\ -\frac{a^3x^2}{2}-a^2x+a\log(x)-\frac{1}{x} \\ -\frac{a^3x^3}{3}-\frac{a^2x^2}{2}+ax+\log(x) \\ -\frac{a^3m^3x^4x^m}{m^4+10m^3+35m^2+50m+24}-\frac{6a^3m^2x^4x^m}{m^4+10m^3+35m^2+50m+24}-\frac{11a^3mx^4x^m}{m^4+10m^3+35m^2+50m+24}-\frac{6a^3x^4x^m}{m^4+10m^3+35m^2+50m+24}-\frac{a^2m^3x^3x^m}{m^4+10m^3+35m^2+50m+24}- \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a**2*x**2+1)*x**m,x)
```

```
[Out] Piecewise((-a**3*log(x) + a**2/x - a/(2*x**2) - 1/(3*x**3), Eq(m, -4)), (-a
**3*x - a**2*log(x) - a/x - 1/(2*x**2), Eq(m, -3)), (-a**3*x**2/2 - a**2*x
+ a*log(x) - 1/x, Eq(m, -2)), (-a**3*x**3/3 - a**2*x**2/2 + a*x + log(x), E
q(m, -1)), (-a**3*m**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) - 6
*a**3*m**2*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) - 11*a**3*m*x**
4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) - 6*a**3*x**4*x**m/(m**4 + 10
*m**3 + 35*m**2 + 50*m + 24) - a**2*m**3*x**3*x**m/(m**4 + 10*m**3 + 35*m**
2 + 50*m + 24) - 7*a**2*m**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 2
4) - 14*a**2*m*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) - 8*a**2*x*
*3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + a*m**3*x**2*x**m/(m**4 + 1
0*m**3 + 35*m**2 + 50*m + 24) + 8*a*m**2*x**2*x**m/(m**4 + 10*m**3 + 35*m**
2 + 50*m + 24) + 19*a*m*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
12*a*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + m**3*x*x**m/(m**4 +
 10*m**3 + 35*m**2 + 50*m + 24) + 9*m**2*x*x**m/(m**4 + 10*m**3 + 35*m**2 +
 50*m + 24) + 26*m*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*x*x**
m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24), True))
```

Giac [B] time = 1.17111, size = 266, normalized size = 4.93

$$-\frac{a^3m^3x^4x^m + 6\,a^3m^2x^4x^m + a^2m^3x^3x^m + 11\,a^3mx^4x^m + 7\,a^2m^2x^3x^m + 6\,a^3x^4x^m - am^3x^2x^m + 14\,a^2mx^3x^m - 8\,am^2x^2}{m^4 + 10\,m^3 + 35\,m^2 + 50\,m + 24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)*x^m,x, algorithm="giac")
```

```
[Out] -(a^3*m^3*x^4*x^m + 6*a^3*m^2*x^4*x^m + a^2*m^3*x^3*x^m + 11*a^3*m*x^4*x^m
+ 7*a^2*m^2*x^3*x^m + 6*a^3*x^4*x^m - a*m^3*x^2*x^m + 14*a^2*m*x^3*x^m - 8*
a*m^2*x^2*x^m + 8*a^2*x^3*x^m - m^3*x*x^m - 19*a*m*x^2*x^m - 9*m^2*x*x^m -
12*a*x^2*x^m - 26*m*x*x^m - 24*x*x^m)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

3.994 $\int e^{\tanh^{-1}(ax)} x^m \sqrt{1-a^2x^2}\,dx$

Optimal. Leaf size=24

$$\frac{ax^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}$$

```
[Out] x^(1 + m)/(1 + m) + (a*x^(2 + m))/(2 + m)
```

Rubi [A] time = 0.0799803, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 43}

$$\frac{ax^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*Sqrt[1 - a^2*x^2],x]
```

```
[Out] x^(1 + m)/(1 + m) + (a*x^(2 + m))/(2 + m)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x^m\sqrt{1-a^2x^2}\,dx&=\int x^m(1+ax)\,dx\\&=\int\left(x^m+ax^{1+m}\right)dx\\&=\frac{x^{1+m}}{1+m}+\frac{ax^{2+m}}{2+m}\end{aligned}$$

Mathematica [A] time = 0.0132547, size = 20, normalized size = 0.83

$$x^{m+1}\left(\frac{ax}{m+2}+\frac{1}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*Sqrt[1 - a^2*x^2],x]
```

```
[Out] x^(1 + m)*((1 + m)^(-1) + (a*x)/(2 + m))
```

Maple [A] time = 0.027, size = 27, normalized size = 1.1

$$\frac{x^{1+m} (amx + ax + m + 2)}{(2 + m) (1 + m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)*x^m,x)
```

```
[Out] x^(1+m)*(a*m*x+a*x+m+2)/(2+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*x^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13452, size = 66, normalized size = 2.75

$$\frac{\left((am + a)x^2 + (m + 2)x\right)x^m}{m^2 + 3\,m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*x^m,x, algorithm="fricas")
```

```
[Out] ((a*m + a)*x^2 + (m + 2)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [A] time = 0.277716, size = 82, normalized size = 3.42

$$\begin{cases} a\log(x) - \frac{1}{x} & \text{for}\ m = -2 \\ ax + \log(x) & \text{for}\ m = -1 \\ \frac{amx^2x^m}{m^2+3m+2} + \frac{ax^2x^m}{m^2+3m+2} + \frac{mxx^m}{m^2+3m+2} + \frac{2xx^m}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*x**m,x)
```

```
[Out] Piecewise((a*log(x) - 1/x, Eq(m, -2)), (a*x + log(x), Eq(m, -1)), (a*m*x**2
*x**m/(m**2 + 3*m + 2) + a*x**2*x**m/(m**2 + 3*m + 2) + m*x*x**m/(m**2 + 3*
m + 2) + 2*x*x**m/(m**2 + 3*m + 2), True))
```

Giac [A] time = 1.13644, size = 55, normalized size = 2.29

$$\frac{amx^2x^m + ax^2x^m + mxx^m + 2\,xx^m}{m^2 + 3\,m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*x^m,x, algorithm="giac")
```

```
[Out] (a*m*x^2*x^m + a*x^2*x^m + m*x*x^m + 2*x*x^m)/(m^2 + 3*m + 2)
```

3.995 $\int \frac{e^{\tanh^{-1}(ax)}x^m}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=22

$$\frac{x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{m+1}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(1 + m)
```

Rubi [A] time = 0.081731, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 64}

$$\frac{x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(1 + m)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^m}{\sqrt{1-a^2x^2}}\,dx &= \int \frac{x^m}{1-ax}\,dx\\ &= \frac{x^{1+m}\,{}_2F_1(1,1+m;2+m;ax)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.01085, size = 22, normalized size = 1.

$$\frac{x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(1 + m)
```

Maple [C] time = 0.193, size = 100, normalized size = 4.6

$$-\frac{1}{2a}\left(-a^{2}\right)^{-\frac{m}{2}}\left(-2\frac{x^{m}\left(-a^{2}\right)^{m/2}(-m-2)}{(2+m)\,m}-x^{m}\left(-a^{2}\right)^{\frac{m}{2}}\operatorname{LerchPhi}\left(a^{2}x^{2},1,\frac{m}{2}\right)\right)+\frac{x^{1+m}}{1+m}\left(\frac{1}{2}+\frac{m}{2}\right)\operatorname{LerchPhi}\left(a^{2}x^{2},1,\frac{1}{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)*x^m,x)
```

```
[Out] -1/2/a*(-a^2)^(-1/2*m)*(-2/(2+m)*x^m*(-a^2)^(1/2*m)*(-m-2)/m-x^m*(-a^2)^(1/
2*m)*LerchPhi(a^2*x^2,1,1/2*m))+1/(1+m)*x^(1+m)*(1/2+1/2*m)*LerchPhi(a^2*x^
2,1,1/2+1/2*m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)x^{m}}{a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^m,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)*x^m/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{x^{m}}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^m,x, algorithm="fricas")
```

```
[Out] integral(-x^m/(a*x - 1), x)
```

Sympy [B] time = 2.0484, size = 44, normalized size = 2.

$$\frac{mxx^{m}\Phi\left(ax,1,m+1\right)\Gamma\left(m+1\right)}{\Gamma\left(m+2\right)}+\frac{xx^{m}\Phi\left(ax,1,m+1\right)\Gamma\left(m+1\right)}{\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)*x**m,x)
```

```
[Out] m*x*x**m*lerchphi(a*x, 1, m + 1)*gamma(m + 1)/gamma(m + 2) + x*x**m*lerchph
i(a*x, 1, m + 1)*gamma(m + 1)/gamma(m + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^m,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)*x^m/(a^2*x^2 - 1), x)
```

3.996 $\int \frac{e^{\tanh^{-1}(ax)}x^m}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=70

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{m+1}+\frac{ax^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{m+2}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m) + (
a*x^(2 + m)*Hypergeometric2F1[2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2 + m)
```

Rubi [A] time = 0.10401, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6150, 82, 73, 364}

$$\frac{x^{m+1}\,_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,_2F_1\left(2,\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m) + (
a*x^(2 + m)*Hypergeometric2F1[2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2 + m)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 82

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^m}{\left(1-a^2x^2\right)^{3/2}}\,dx=\int \frac{x^m}{(1-ax)^2(1+ax)}\,dx$$
$$=a\int \frac{x^{1+m}}{(1-ax)^2(1+ax)^2}\,dx+\int \frac{x^m}{(1-ax)^2(1+ax)^2}\,dx$$
$$=a\int \frac{x^{1+m}}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{x^m}{\left(1-a^2x^2\right)^2}\,dx$$
$$=\frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m}+\frac{ax^{2+m}\,{}_2F_1\left(2,\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}$$

Mathematica [A] time = 0.0347338, size = 67, normalized size = 0.96

$$x^{m+1}\left(\frac{ax\text{Hypergeometric2F1}\left(2,\frac{m}{2}+1,\frac{m}{2}+2,a^2x^2\right)}{m+2}+\frac{\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] x^(1 + m)*((a*x*Hypergeometric2F1[2, 1 + m/2, 2 + m/2, a^2*x^2])/(2 + m) +
Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, a^2*x^2]/(1 + m))
```

Maple [C] time = 0.207, size = 177, normalized size = 2.5

$$-\frac{1}{2a}\left(-a^2\right)^{-\frac{m}{2}}\left(\frac{x^m(-m-2)}{(2+m)\left(-a^2x^2+1\right)}\left(-a^2\right)^{\frac{m}{2}}+\frac{x^m m}{2}\left(-a^2\right)^{\frac{m}{2}}\text{LerchPhi}\left(a^2x^2,1,\frac{m}{2}\right)\right)+\frac{1}{2}\left(-a^2\right)^{-\frac{1}{2}-\frac{m}{2}}\left(-2\frac{x^{1+m}\left(-a^2\right)^{1/\cdots}}{(1+m)\left(-\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2*x^m,x)
```

```
[Out] -1/2/a*(-a^2)^(-1/2*m)*(1/(2+m)*x^m*(-a^2)^(1/2*m)*(-m-2)/(-a^2*x^2+1)+1/2*
x^m*(-a^2)^(1/2*m)*m*LerchPhi(a^2*x^2,1,1/2*m))+1/2*(-a^2)^(-1/2-1/2*m)*(-2
/(1+m)*x^(1+m)*(-a^2)^(1/2+1/2*m)*(-1-m)/(-2*a^2*x^2+2)+2/(1+m)*x^(1+m)*(-a
^2)^(1/2+1/2*m)*(-1/4*m^2+1/4)*LerchPhi(a^2*x^2,1,1/2+1/2*m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^m}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^m,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*x^m/(a^2*x^2 - 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{a^3x^3 - a^2x^2 - ax + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m/(a^3*x^3 - a^2*x^2 - a*x + 1), x)
```

Sympy [C] time = 4.73726, size = 673, normalized size = 9.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2*x**m,x)
```

```
[Out] -a**2*m**2*x**3*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*ga
mma(m/2 + 1/2)/(8*a**2*x**2*gamma(m/2 + 3/2) - 8*gamma(m/2 + 3/2)) + a**2*x
**3*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/
2)/(8*a**2*x**2*gamma(m/2 + 3/2) - 8*gamma(m/2 + 3/2)) + a*(-a**2*m**2*x**4
*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1)*gamma(m/2 + 1)/(8*a
**2*x**2*gamma(m/2 + 2) - 8*gamma(m/2 + 2)) - 2*a**2*m*x**4*x**m*lerchphi(a
**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1)*gamma(m/2 + 1)/(8*a**2*x**2*gamma(m
/2 + 2) - 8*gamma(m/2 + 2)) + m**2*x**2*x**m*lerchphi(a**2*x**2*exp_polar(2
*I*pi), 1, m/2 + 1)*gamma(m/2 + 1)/(8*a**2*x**2*gamma(m/2 + 2) - 8*gamma(m/
2 + 2)) + 2*m*x**2*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1)*g
amma(m/2 + 1)/(8*a**2*x**2*gamma(m/2 + 2) - 8*gamma(m/2 + 2)) - 2*m*x**2*x*
*m*gamma(m/2 + 1)/(8*a**2*x**2*gamma(m/2 + 2) - 8*gamma(m/2 + 2)) - 4*x**2*
x**m*gamma(m/2 + 1)/(8*a**2*x**2*gamma(m/2 + 2) - 8*gamma(m/2 + 2))) + m**2
*x*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2
)/(8*a**2*x**2*gamma(m/2 + 3/2) - 8*gamma(m/2 + 3/2)) - 2*m*x*x**m*gamma(m/
2 + 1/2)/(8*a**2*x**2*gamma(m/2 + 3/2) - 8*gamma(m/2 + 3/2)) - x*x**m*lerch
phi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**2*x**
2*gamma(m/2 + 3/2) - 8*gamma(m/2 + 3/2)) - 2*x*x**m*gamma(m/2 + 1/2)/(8*a**
2*x**2*gamma(m/2 + 3/2) - 8*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^m}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^m,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*x^m/(a^2*x^2 - 1)^2, x)
```

3.997 $\int \frac{e^{\tanh^{-1}(ax)}x^m}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=70

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(3,\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{m+1}+\frac{ax^{m+2}\text{Hypergeometric2F1}\left(3,\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{m+2}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m) + (
a*x^(2 + m)*Hypergeometric2F1[3, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2 + m)
```

Rubi [A] time = 0.116348, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6150, 82, 73, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(3,\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m) + (
a*x^(2 + m)*Hypergeometric2F1[3, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2 + m)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 82

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^m}{\left(1-a^2x^2\right)^{5/2}}\,dx = \int \frac{x^m}{(1-ax)^3(1+ax)^2}\,dx$$
$$= a\int \frac{x^{1+m}}{(1-ax)^3(1+ax)^3}\,dx + \int \frac{x^m}{(1-ax)^3(1+ax)^3}\,dx$$
$$= a\int \frac{x^{1+m}}{\left(1-a^2x^2\right)^3}\,dx + \int \frac{x^m}{\left(1-a^2x^2\right)^3}\,dx$$
$$= \frac{x^{1+m}\,{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m} + \frac{ax^{2+m}\,{}_2F_1\left(3,\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}$$

Mathematica [A] time = 0.0333006, size = 67, normalized size = 0.96

$$x^{m+1}\left(\frac{ax\text{Hypergeometric2F1}\left(3,\frac{m}{2}+1,\frac{m}{2}+2,a^2x^2\right)}{m+2}+\frac{\text{Hypergeometric2F1}\left(3,\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] x^(1 + m)*((a*x*Hypergeometric2F1[3, 1 + m/2, 2 + m/2, a^2*x^2])/(2 + m) +
Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, a^2*x^2]/(1 + m))
```

Maple [C] time = 0.277, size = 224, normalized size = 3.2

$$\frac{1}{4}\left(-a^2\right)^{-\frac{1}{2}-\frac{m}{2}}\left(\frac{x^{1+m}\left(a^2m^2x^2-2a^2mx^2-3a^2x^2-m^2+4m+5\right)}{(2m+2)\left(-a^2x^2+1\right)^2}\left(-a^2\right)^{\frac{1}{2}+\frac{m}{2}}+4\frac{x^{1+m}\left(-a^2\right)^{1/2+m/2}\left(1/16m^3-3/16m^2-n\right.}{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3*x^m,x)
```

```
[Out] 1/4*(-a^2)^(-1/2-1/2*m)*(1/2/(1+m)*x^(1+m)*(-a^2)^(1/2+1/2*m)*(a^2*m^2*x^2-
2*a^2*m*x^2-3*a^2*x^2-m^2+4*m+5)/(-a^2*x^2+1)^2+4/(1+m)*x^(1+m)*(-a^2)^(1/2
+1/2*m)*(1/16*m^3-3/16*m^2-1/16*m+3/16)*LerchPhi(a^2*x^2,1,1/2+1/2*m))-1/4/
a*(-a^2)^(-1/2*m)*(-1/2*x^m*(-a^2)^(1/2*m)*(a^2*m*x^2-m+2)/(-a^2*x^2+1)^2-1
/4*x^m*(-a^2)^(1/2*m)*(-2+m)*m*LerchPhi(a^2*x^2,1,1/2*m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)x^m}{\left(a^2x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^m,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)*x^m/(a^2*x^2 - 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^m}{a^5x^5 - a^4x^4 - 2\,a^3x^3 + 2\,a^2x^2 + ax - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^m,x, algorithm="fricas")
```

```
[Out] integral(-x^m/(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1), x)
```

Sympy [C] time = 7.94969, size = 2152, normalized size = 30.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3*x**m,x)
```

```
[Out] a**4*m**3*x**5*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gam
ma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2
) + 32*gamma(m/2 + 3/2)) - 3*a**4*m**2*x**5*x**m*lerchphi(a**2*x**2*exp_pol
ar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2 + 3/2) -
 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) - a**4*m*x**5*x**m*le
rchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**4
*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2
)) + 3*a**4*x**5*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*g
amma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3
/2) + 32*gamma(m/2 + 3/2)) - 2*a**2*m**3*x**3*x**m*lerchphi(a**2*x**2*exp_p
olar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2 + 3/2)
 - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) + 6*a**2*m**2*x**3*
x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(
32*a**4*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/
2 + 3/2)) + 2*a**2*m**2*x**3*x**m*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2
+ 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) + 2*a**2*m*x*
*3*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2
)/(32*a**4*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma
(m/2 + 3/2)) - 4*a**2*m*x**3*x**m*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2
+ 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) - 6*a**2*x**3
*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/
(32*a**4*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m
/2 + 3/2)) - 6*a**2*x**3*x**m*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2 + 3/
2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) + a*(a**4*m**3*x*
*6*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1)*gamma(m/2 + 1)/(3
2*a**4*x**4*gamma(m/2 + 2) - 64*a**2*x**2*gamma(m/2 + 2) + 32*gamma(m/2 + 2
)) - 4*a**4*m*x**6*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1)*g
amma(m/2 + 1)/(32*a**4*x**4*gamma(m/2 + 2) - 64*a**2*x**2*gamma(m/2 + 2) +
32*gamma(m/2 + 2)) - 2*a**2*m**3*x**4*x**m*lerchphi(a**2*x**2*exp_polar(2*I
*pi), 1, m/2 + 1)*gamma(m/2 + 1)/(32*a**4*x**4*gamma(m/2 + 2) - 64*a**2*x**
2*gamma(m/2 + 2) + 32*gamma(m/2 + 2)) + 2*a**2*m**2*x**4*x**m*gamma(m/2 + 1
)/(32*a**4*x**4*gamma(m/2 + 2) - 64*a**2*x**2*gamma(m/2 + 2) + 32*gamma(m/2
 + 2)) + 8*a**2*m*x**4*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 +
```

```
1)*gamma(m/2 + 1)/(32*a**4*x**4*gamma(m/2 + 2) - 64*a**2*x**2*gamma(m/2 + 2
) + 32*gamma(m/2 + 2)) - 8*a**2*x**4*x**m*gamma(m/2 + 1)/(32*a**4*x**4*gamm
a(m/2 + 2) - 64*a**2*x**2*gamma(m/2 + 2) + 32*gamma(m/2 + 2)) + m**3*x**2*x
**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1)*gamma(m/2 + 1)/(32*a*
*4*x**4*gamma(m/2 + 2) - 64*a**2*x**2*gamma(m/2 + 2) + 32*gamma(m/2 + 2)) -
 2*m**2*x**2*x**m*gamma(m/2 + 1)/(32*a**4*x**4*gamma(m/2 + 2) - 64*a**2*x**
2*gamma(m/2 + 2) + 32*gamma(m/2 + 2)) - 4*m*x**2*x**m*lerchphi(a**2*x**2*ex
p_polar(2*I*pi), 1, m/2 + 1)*gamma(m/2 + 1)/(32*a**4*x**4*gamma(m/2 + 2) -
64*a**2*x**2*gamma(m/2 + 2) + 32*gamma(m/2 + 2)) + 4*m*x**2*x**m*gamma(m/2
+ 1)/(32*a**4*x**4*gamma(m/2 + 2) - 64*a**2*x**2*gamma(m/2 + 2) + 32*gamma(
m/2 + 2)) + 16*x**2*x**m*gamma(m/2 + 1)/(32*a**4*x**4*gamma(m/2 + 2) - 64*a
**2*x**2*gamma(m/2 + 2) + 32*gamma(m/2 + 2))) + m**3*x*x**m*lerchphi(a**2*x
**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m
/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) - 3*m**2*x
*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/
(32*a**4*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m
/2 + 3/2)) - 2*m**2*x*x**m*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2 + 3/2)
- 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) - m*x*x**m*lerchphi(
a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**4*x**4*g
amma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) + 8*
m*x*x**m*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gam
ma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) + 3*x*x**m*lerchphi(a**2*x**2*exp_pola
r(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2 + 3/2) -
64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) + 10*x*x**m*gamma(m/2
+ 1/2)/(32*a**4*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*
gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)x^m}{\left(a^2x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^m,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)*x^m/(a^2*x^2 - 1)^3, x)
```

3.998 $\int e^{\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=274

$$\frac{c^2x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}+\frac{ac^2x^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}-\frac{2a^2c^2x^{m+3}\sqrt{c-a^2cx^2}}{(m+3)\sqrt{1-a^2x^2}}-\frac{2a^3c^2x^{m+4}\sqrt{c-a^2cx^2}}{(m+4)\sqrt{1-a^2x^2}}+\frac{a^4c^2x^{m+5}\sqrt{c-a^2cx^2}}{(m+5)\sqrt{1-a^2x^2}}+\frac{a}{}$$

```
[Out] (c^2*x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) + (a*c^2*x^
(2 + m)*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2]) - (2*a^2*c^2*x^(3
+ m)*Sqrt[c - a^2*c*x^2])/((3 + m)*Sqrt[1 - a^2*x^2]) - (2*a^3*c^2*x^(4 + m
)*Sqrt[c - a^2*c*x^2])/((4 + m)*Sqrt[1 - a^2*x^2]) + (a^4*c^2*x^(5 + m)*Sqr
t[c - a^2*c*x^2])/((5 + m)*Sqrt[1 - a^2*x^2]) + (a^5*c^2*x^(6 + m)*Sqrt[c -
 a^2*c*x^2])/((6 + m)*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.229839, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 88}

$$\frac{c^2x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}+\frac{ac^2x^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}-\frac{2a^2c^2x^{m+3}\sqrt{c-a^2cx^2}}{(m+3)\sqrt{1-a^2x^2}}-\frac{2a^3c^2x^{m+4}\sqrt{c-a^2cx^2}}{(m+4)\sqrt{1-a^2x^2}}+\frac{a^4c^2x^{m+5}\sqrt{c-a^2cx^2}}{(m+5)\sqrt{1-a^2x^2}}+\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) + (a*c^2*x^
(2 + m)*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2]) - (2*a^2*c^2*x^(3
+ m)*Sqrt[c - a^2*c*x^2])/((3 + m)*Sqrt[1 - a^2*x^2]) - (2*a^3*c^2*x^(4 + m
)*Sqrt[c - a^2*c*x^2])/((4 + m)*Sqrt[1 - a^2*x^2]) + (a^4*c^2*x^(5 + m)*Sqr
t[c - a^2*c*x^2])/((5 + m)*Sqrt[1 - a^2*x^2]) + (a^5*c^2*x^(6 + m)*Sqrt[c -
 a^2*c*x^2])/((6 + m)*Sqrt[1 - a^2*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^{5/2}dx=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int e^{\tanh^{-1}(ax)}x^m\left(1-a^2x^2\right)^{5/2}dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int x^m(1-ax)^2(1+ax)^3\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int\left(x^m+ax^{1+m}-2a^2x^{2+m}-2a^3x^{3+m}+a^4x^{4+m}+a^5x^{5+m}\right)dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{c^2x^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}}+\frac{ac^2x^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}}-\frac{2a^2c^2x^{3+m}\sqrt{c-a^2cx^2}}{(3+m)\sqrt{1-a^2x^2}}-\frac{2a^3c^2x^{4+m}\sqrt{c-a}}{(4+m)\sqrt{1-a^2}}$$

Mathematica [A] time = 0.093042, size = 102, normalized size = 0.37

$$\frac{c^2x^{m+1}\sqrt{c-a^2cx^2}\left(\frac{a^5x^5}{m+6}+\frac{a^4x^4}{m+5}-\frac{2a^3x^3}{m+4}-\frac{2a^2x^2}{m+3}+\frac{ax}{m+2}+\frac{1}{m+1}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*x^(1 + m)*Sqrt[c - a^2*c*x^2]*((1 + m)^(-1) + (a*x)/(2 + m) - (2*a^2*x
^2)/(3 + m) - (2*a^3*x^3)/(4 + m) + (a^4*x^4)/(5 + m) + (a^5*x^5)/(6 + m)))
/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.033, size = 377, normalized size = 1.4

$$\frac{x^{1+m}\left(a^5m^5x^5+15\,a^5m^4x^5+85\,a^5m^3x^5+a^4m^5x^4+225\,a^5m^2x^5+16\,a^4m^4x^4+274\,a^5mx^5+95\,a^4m^3x^4-2\,a^3m^5x^3+12\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] x^(1+m)*(a^5*m^5*x^5+15*a^5*m^4*x^5+85*a^5*m^3*x^5+a^4*m^5*x^4+225*a^5*m^2*
x^5+16*a^4*m^4*x^4+274*a^5*m*x^5+95*a^4*m^3*x^4-2*a^3*m^5*x^3+120*a^5*x^5+2
60*a^4*m^2*x^4-34*a^3*m^4*x^3+324*a^4*m*x^4-214*a^3*m^3*x^3-2*a^2*m^5*x^2+1
44*a^4*x^4-614*a^3*m^2*x^3-36*a^2*m^4*x^2-792*a^3*m*x^3-242*a^2*m^3*x^2+a*m
^5*x-360*a^3*x^3-744*a^2*m^2*x^2+19*a*m^4*x-1016*a^2*m*x^2+137*a*m^3*x+m^5-
480*a^2*x^2+461*a*m^2*x+20*m^4+702*a*m*x+155*m^3+360*a*x+580*m^2+1044*m+720
)*(-a^2*c*x^2+c)^(5/2)/(6+m)/(5+m)/(4+m)/(3+m)/(2+m)/(1+m)/(a*x-1)^2/(a*x+1
)^2/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.01227, size = 194, normalized size = 0.71

$$\frac{\left(\left(m^2+6\,m+8\right)a^4c^{\frac{5}{2}}x^6-2\left(m^2+8\,m+12\right)a^2c^{\frac{5}{2}}x^4+\left(m^2+10\,m+24\right)c^{\frac{5}{2}}x^2\right)ax^m}{m^3+12\,m^2+44\,m+48}+\frac{\left(\left(m^2+4\,m+3\right)a^4c^{\frac{5}{2}}x^5-2\left(m^2+6\right.\right.}{m^3+9\,m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] ((m^2 + 6*m + 8)*a^4*c^(5/2)*x^6 - 2*(m^2 + 8*m + 12)*a^2*c^(5/2)*x^4 + (m^
2 + 10*m + 24)*c^(5/2)*x^2)*a*x^m/(m^3 + 12*m^2 + 44*m + 48) + ((m^2 + 4*m
+ 3)*a^4*c^(5/2)*x^5 - 2*(m^2 + 6*m + 5)*a^2*c^(5/2)*x^3 + (m^2 + 8*m + 15)
*c^(5/2)*x)*x^m/(m^3 + 9*m^2 + 23*m + 15)
```

Fricas [A] time = 2.26833, size = 1058, normalized size = 3.86

$$\frac{\left(\left(a^5c^2m^5 + 15\,a^5c^2m^4 + 85\,a^5c^2m^3 + 225\,a^5c^2m^2 + 274\,a^5c^2m + 120\,a^5c^2\right)x^6 + \left(a^4c^2m^5 + 16\,a^4c^2m^4 + 95\,a^4c^2m^3 + 26\cdots\right.\right.}{\ }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] ((a^5*c^2*m^5 + 15*a^5*c^2*m^4 + 85*a^5*c^2*m^3 + 225*a^5*c^2*m^2 + 274*a^5
*c^2*m + 120*a^5*c^2)*x^6 + (a^4*c^2*m^5 + 16*a^4*c^2*m^4 + 95*a^4*c^2*m^3
+ 260*a^4*c^2*m^2 + 324*a^4*c^2*m + 144*a^4*c^2)*x^5 - 2*(a^3*c^2*m^5 + 17*
a^3*c^2*m^4 + 107*a^3*c^2*m^3 + 307*a^3*c^2*m^2 + 396*a^3*c^2*m + 180*a^3*c
^2)*x^4 - 2*(a^2*c^2*m^5 + 18*a^2*c^2*m^4 + 121*a^2*c^2*m^3 + 372*a^2*c^2*m
^2 + 508*a^2*c^2*m + 240*a^2*c^2)*x^3 + (a*c^2*m^5 + 19*a*c^2*m^4 + 137*a*c
^2*m^3 + 461*a*c^2*m^2 + 702*a*c^2*m + 360*a*c^2)*x^2 + (c^2*m^5 + 20*c^2*m
^4 + 155*c^2*m^3 + 580*c^2*m^2 + 1044*c^2*m + 720*c^2)*x)*sqrt(-a^2*c*x^2 +
 c)*sqrt(-a^2*x^2 + 1)*x^m/(m^6 + 21*m^5 + 175*m^4 + 735*m^3 - (a^2*m^6 + 2
1*a^2*m^5 + 175*a^2*m^4 + 735*a^2*m^3 + 1624*a^2*m^2 + 1764*a^2*m + 720*a^2
)*x^2 + 1624*m^2 + 1764*m + 720)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2 + c\right)^{\frac{5}{2}}(ax + 1)x^m}{\sqrt{-a^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

3.999 $\int e^{\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=174

$$\frac{cx^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}+\frac{acx^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}-\frac{a^2cx^{m+3}\sqrt{c-a^2cx^2}}{(m+3)\sqrt{1-a^2x^2}}-\frac{a^3cx^{m+4}\sqrt{c-a^2cx^2}}{(m+4)\sqrt{1-a^2x^2}}$$

```
[Out] (c*x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) + (a*c*x^(2 +
 m)*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2]) - (a^2*c*x^(3 + m)*Sqr
t[c - a^2*c*x^2])/((3 + m)*Sqrt[1 - a^2*x^2]) - (a^3*c*x^(4 + m)*Sqrt[c - a
^2*c*x^2])/((4 + m)*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.2033, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 75}

$$\frac{cx^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}+\frac{acx^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}-\frac{a^2cx^{m+3}\sqrt{c-a^2cx^2}}{(m+3)\sqrt{1-a^2x^2}}-\frac{a^3cx^{m+4}\sqrt{c-a^2cx^2}}{(m+4)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) + (a*c*x^(2 +
 m)*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2]) - (a^2*c*x^(3 + m)*Sqr
t[c - a^2*c*x^2])/((3 + m)*Sqrt[1 - a^2*x^2]) - (a^3*c*x^(4 + m)*Sqrt[c - a
^2*c*x^2])/((4 + m)*Sqrt[1 - a^2*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^{3/2}dx=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int e^{\tanh^{-1}(ax)}x^m\left(1-a^2x^2\right)^{3/2}dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int x^m(1-ax)(1+ax)^2\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int\left(x^m+ax^{1+m}-a^2x^{2+m}-a^3x^{3+m}\right)dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{cx^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}}+\frac{acx^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}}-\frac{a^2cx^{3+m}\sqrt{c-a^2cx^2}}{(3+m)\sqrt{1-a^2x^2}}-\frac{a^3cx^{4+m}\sqrt{c-a^2c}}{(4+m)\sqrt{1-a^2x}}$$

Mathematica [A] time = 0.0903083, size = 84, normalized size = 0.48

$$\frac{cx^{m+1}\sqrt{c-a^2cx^2}\left((2m+5)\left(\frac{a^2x^2}{m+3}+\frac{2ax}{m+2}+\frac{1}{m+1}\right)-(ax+1)^3\right)}{(m+4)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*x^(1 + m)*Sqrt[c - a^2*c*x^2]*(-(1 + a*x)^3 + (5 + 2*m)*((1 + m)^(-1) +
(2*a*x)/(2 + m) + (a^2*x^2)/(3 + m))))/((4 + m)*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.03, size = 180, normalized size = 1.

$$\frac{x^{1+m}\left(a^3m^3x^3+6a^3m^2x^3+11a^3mx^3+a^2m^3x^2+6x^3a^3+7a^2m^2x^2+14a^2mx^2-am^3x+8a^2x^2-8am^2x-19amx\right.}{(ax+1)(ax-1)(4+m)(3+m)(2+m)(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] x^(1+m)*(a^3*m^3*x^3+6*a^3*m^2*x^3+11*a^3*m*x^3+a^2*m^3*x^2+6*a^3*x^3+7*a^2
*m^2*x^2+14*a^2*m*x^2-a*m^3*x+8*a^2*x^2-8*a*m^2*x-19*a*m*x-m^3-12*a*x-9*m^2
-26*m-24)*(-a^2*c*x^2+c)^(3/2)/(4+m)/(3+m)/(2+m)/(1+m)/(a*x-1)/(a*x+1)/(-a^
2*x^2+1)^(1/2)
```

Maxima [A] time = 1.00073, size = 108, normalized size = 0.62

$$-\frac{\left(a^2c^{\frac{3}{2}}(m+2)x^4-c^{\frac{3}{2}}(m+4)x^2\right)ax^m}{m^2+6m+8}-\frac{\left(a^2c^{\frac{3}{2}}(m+1)x^3-c^{\frac{3}{2}}(m+3)x\right)x^m}{m^2+4m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -(a^2*c^(3/2)*(m + 2)*x^4 - c^(3/2)*(m + 4)*x^2)*a*x^m/(m^2 + 6*m + 8) - (a
^2*c^(3/2)*(m + 1)*x^3 - c^(3/2)*(m + 3)*x)*x^m/(m^2 + 4*m + 3)
```

Fricas [A] time = 2.37073, size = 459, normalized size = 2.64

$$-\frac{\sqrt{-a^2cx^2+c}\left(\left(a^3cm^3+6\,a^3cm^2+11\,a^3cm+6\,a^3c\right)x^4+\left(a^2cm^3+7\,a^2cm^2+14\,a^2cm+8\,a^2c\right)x^3-\left(acm^3+8\,acm^2+19\right.}{m^4+10\,m^3-\left(a^2m^4+10\,a^2m^3+35\,a^2m^2+50\,a^2m+24\,a^2\right)x^2+35\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*((a^3*c*m^3 + 6*a^3*c*m^2 + 11*a^3*c*m + 6*a^3*c)*x^4
 + (a^2*c*m^3 + 7*a^2*c*m^2 + 14*a^2*c*m + 8*a^2*c)*x^3 - (a*c*m^3 + 8*a*c*
m^2 + 19*a*c*m + 12*a*c)*x^2 - (c*m^3 + 9*c*m^2 + 26*c*m + 24*c)*x)*sqrt(-a
^2*x^2 + 1)*x^m/(m^4 + 10*m^3 - (a^2*m^4 + 10*a^2*m^3 + 35*a^2*m^2 + 50*a^2
*m + 24*a^2)*x^2 + 35*m^2 + 50*m + 24)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

3.1000 $\int e^{\tanh^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=82

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}+\frac{ax^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}$$

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) + (a*x^(2 + m)*
Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.173754, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 43}

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}+\frac{ax^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) + (a*x^(2 + m)*
Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{\tanh^{-1}(ax)}x^m\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int x^m(1+ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(x^m+ax^{1+m}\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}} + \frac{ax^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0326556, size = 49, normalized size = 0.6

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}\left(\frac{ax}{m+2}+\frac{1}{m+1}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*((1 + m)^(-1) + (a*x)/(2 + m))*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2
*x^2]
```

Maple [A] time = 0.027, size = 52, normalized size = 0.6

$$\frac{x^{1+m}(amx+ax+m+2)}{(2+m)(1+m)}\sqrt{-a^2cx^2+c}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] x^(1+m)*(a*m*x+a*x+m+2)*(-a^2*c*x^2+c)^(1/2)/(2+m)/(1+m)/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.03725, size = 41, normalized size = 0.5

$$\frac{a\sqrt{c}x^2x^m}{m+2}+\frac{\sqrt{c}xx^m}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] a*sqrt(c)*x^2*x^m/(m + 2) + sqrt(c)*x*x^m/(m + 1)
```

Fricas [A] time = 2.17953, size = 167, normalized size = 2.04

$$-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\left((am+a)x^2+(m+2)x\right)x^m}{\left(a^2m^2+3\,a^2m+2\,a^2\right)x^2-m^2-3\,m-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((a*m + a)*x^2 + (m + 2)*x)*x^m/((
a^2*m^2 + 3*a^2*m + 2*a^2)*x^2 - m^2 - 3*m - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x +
 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

3.1001 $\int \frac{e^{\tanh^{-1}(ax)}x^m}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{1-a^2x^2}x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{(m+1)\sqrt{c-a^2cx^2}}$$

```
[Out] (x^(1 + m)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/((1 +
 m)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.183227, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 64}

$$\frac{\sqrt{1-a^2x^2}x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{(m+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/((1 +
 m)*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^m}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^m}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^m}{1-ax}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{x^{1+m}\sqrt{1-a^2x^2}\,{}_2F_1(1,1+m;2+m;ax)}{(1+m)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0249566, size = 51, normalized size = 1.

$$\frac{\sqrt{1-a^2x^2}x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{(m+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/((1 +
 m)*Sqrt[c - a^2*c*x^2])
```

Maple [F] time = 0.333, size = 0, normalized size = 0.

$$\int (ax+1)\,x^m \frac{1}{\sqrt{-a^2x^2+1}} \frac{1}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^m}{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^m/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^m}{a^3cx^3-a^2cx^2-acx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^m/(a^3*c*x^3 - a^2*c*x^2
 - a*c*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m (ax+1)}{\sqrt{-(ax-1)(ax+1)}\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x
+ 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^m}{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^m/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

3.1002 $\int \frac{e^{\tanh^{-1}(ax)}x^m}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{\sqrt{1-a^2x^2}x^{m+1}\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{c(m+1)\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}x^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{c(m+2)\sqrt{c-a^2cx^2}}$$

```
[Out] (x^(1 + m)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, a^2
*x^2])/(c*(1 + m)*Sqrt[c - a^2*c*x^2]) + (a*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hyp
ergeometric2F1[2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(c*(2 + m)*Sqrt[c - a^2*c
*x^2])
```

Rubi [A] time = 0.21553, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6153, 6150, 82, 73, 364}

$$\frac{\sqrt{1-a^2x^2}x^{m+1}\,_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{c(m+1)\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}x^{m+2}\,_2F_1\left(2,\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{c(m+2)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2)^(3/2),x]
```


```
[Out] (x^(1 + m)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, a^2
*x^2])/(c*(1 + m)*Sqrt[c - a^2*c*x^2]) + (a*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hyp
ergeometric2F1[2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(c*(2 + m)*Sqrt[c - a^2*c
*x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 82

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] && !RationalQ[
p] && !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^m}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x^m}{(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x^m}{(1-ax)^2(1+ax)^2}\,dx}{c\sqrt{c-a^2cx^2}}+\frac{\left(a\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{(1-ax)^2(1+ax)^2}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x^m}{\left(1-a^2x^2\right)^2}\,dx}{c\sqrt{c-a^2cx^2}}+\frac{\left(a\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\left(1-a^2x^2\right)^2}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{x^{1+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{c(1+m)\sqrt{c-a^2cx^2}}+\frac{ax^{2+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(2,\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{c(2+m)\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0416802, size = 107, normalized size = 0.8

$$
\frac{\sqrt{1-a^2x^2}\left(\frac{x^{m+1}\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+1}{2}+1,a^2x^2\right)}{m+1}+\frac{ax^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+2}{2}+1,a^2x^2\right)}{m+2}\right)}{c\sqrt{c-a^2cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*((x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/
2, a^2*x^2])/(1 + m) + (a*x^(2 + m)*Hypergeometric2F1[2, (2 + m)/2, 1 + (2
+ m)/2, a^2*x^2])/(2 + m)))/(c*Sqrt[c - a^2*c*x^2])
```

Maple [F] time = 0.328, size = 0, normalized size = 0.

$$
\int (ax+1)\,x^m\frac{1}{\sqrt{-a^2x^2+1}}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^m}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^m/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^m}{a^5c^2x^5-a^4c^2x^4-2\,a^3c^2x^3+2\,a^2c^2x^2+ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^m/(a^5*c^2*x^5 - a^4*c^
2*x^4 - 2*a^3*c^2*x^3 + 2*a^2*c^2*x^2 + a*c^2*x - c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**m*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^m}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^m/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.1003 $\int \frac{e^{\tanh^{-1}(ax)}x^m}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=134

$$\frac{\sqrt{1-a^2x^2}x^{m+1}\text{Hypergeometric2F1}\left(3,\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{c^2(m+1)\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}x^{m+2}\text{Hypergeometric2F1}\left(3,\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{c^2(m+2)\sqrt{c-a^2cx^2}}$$

```
[Out] (x^(1 + m)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, a^2
*x^2])/(c^2*(1 + m)*Sqrt[c - a^2*c*x^2]) + (a*x^(2 + m)*Sqrt[1 - a^2*x^2]*H
ypergeometric2F1[3, (2 + m)/2, (4 + m)/2, a^2*x^2])/(c^2*(2 + m)*Sqrt[c - a
^2*c*x^2])
```

Rubi [A] time = 0.211823, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6153, 6150, 82, 73, 364}

$$\frac{\sqrt{1-a^2x^2}x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{c^2(m+1)\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}x^{m+2}\,{}_2F_1\left(3,\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{c^2(m+2)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, a^2
*x^2])/(c^2*(1 + m)*Sqrt[c - a^2*c*x^2]) + (a*x^(2 + m)*Sqrt[1 - a^2*x^2]*H
ypergeometric2F1[3, (2 + m)/2, (4 + m)/2, a^2*x^2])/(c^2*(2 + m)*Sqrt[c - a
^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 82

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] && !RationalQ[
p] && !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^m}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x^m}{(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x^m}{(1-ax)^3(1+ax)^3}\,dx}{c^2\sqrt{c-a^2cx^2}}+\frac{\left(a\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{(1-ax)^3(1+ax)^3}\,dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x^m}{\left(1-a^2x^2\right)^3}\,dx}{c^2\sqrt{c-a^2cx^2}}+\frac{\left(a\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\left(1-a^2x^2\right)^3}\,dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{x^{1+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{c^2(1+m)\sqrt{c-a^2cx^2}}+\frac{ax^{2+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(3,\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{c^2(2+m)\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.040429, size = 107, normalized size = 0.8

$$\frac{\sqrt{1-a^2x^2}\left(\frac{x^{m+1}\text{Hypergeometric2F1}\left(3,\frac{m+1}{2},\frac{m+1}{2}+1,a^2x^2\right)}{m+1}+\frac{ax^{m+2}\text{Hypergeometric2F1}\left(3,\frac{m+2}{2},\frac{m+2}{2}+1,a^2x^2\right)}{m+2}\right)}{c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*((x^(1 + m)*Hypergeometric2F1[3, (1 + m)/2, 1 + (1 + m)/
2, a^2*x^2])/(1 + m) + (a*x^(2 + m)*Hypergeometric2F1[3, (2 + m)/2, 1 + (2
+ m)/2, a^2*x^2])/(2 + m)))/(c^2*Sqrt[c - a^2*c*x^2])
```

Maple [F] time = 0.344, size = 0, normalized size = 0.

$$\int (ax+1)\,x^m\frac{1}{\sqrt{-a^2x^2+1}}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^m}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^m/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^m}{a^7c^3x^7-a^6c^3x^6-3\,a^5c^3x^5+3\,a^4c^3x^4+3\,a^3c^3x^3-3\,a^2c^3x^2-ac^3x+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^m/(a^7*c^3*x^7 - a^6*c^3
*x^6 - 3*a^5*c^3*x^5 + 3*a^4*c^3*x^4 + 3*a^3*c^3*x^3 - 3*a^2*c^3*x^2 - a*c^
3*x + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^m}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^m/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.1004 $\int e^{\tanh^{-1}(ax)} x^m \left(c - a^2 c x^2\right)^p \, dx$

Optimal. Leaf size=136

$$\frac{x^{m+1} \left(1-a^2x^2\right)^{-p} \left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1}{2}-p,\frac{m+3}{2},a^2x^2\right)}{m+1}+\frac{ax^{m+2} \left(1-a^2x^2\right)^{-p} \left(c-a^2cx^2\right)^p \text{Hype}}{m}$$

```
[Out] (x^(1 + m)*(c - a^2*c*x^2)^p*Hypergeometric2F1[(1 + m)/2, 1/2 - p, (3 + m)/
2, a^2*x^2])/((1 + m)*(1 - a^2*x^2)^p) + (a*x^(2 + m)*(c - a^2*c*x^2)^p*Hyp
ergeometric2F1[(2 + m)/2, 1/2 - p, (4 + m)/2, a^2*x^2])/((2 + m)*(1 - a^2*x
^2)^p)
```

Rubi [A] time = 0.149491, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6153, 6148, 808, 364}

$$\frac{x^{m+1} \left(1-a^2x^2\right)^{-p} \left(c-a^2cx^2\right)^p {}_2F_1\left(\frac{m+1}{2},\frac{1}{2}-p;\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{ax^{m+2} \left(1-a^2x^2\right)^{-p} \left(c-a^2cx^2\right)^p {}_2F_1\left(\frac{m+2}{2},\frac{1}{2}-p;\frac{m+4}{2};a^2\right.}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2)^p,x]
```


```
[Out] (x^(1 + m)*(c - a^2*c*x^2)^p*Hypergeometric2F1[(1 + m)/2, 1/2 - p, (3 + m)/
2, a^2*x^2])/((1 + m)*(1 - a^2*x^2)^p) + (a*x^(2 + m)*(c - a^2*c*x^2)^p*Hyp
ergeometric2F1[(2 + m)/2, 1/2 - p, (4 + m)/2, a^2*x^2])/((2 + m)*(1 - a^2*x
^2)^p)
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)} x^m \left(c-a^2cx^2\right)^p \, dx = \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{\tanh^{-1}(ax)} x^m \left(1-a^2x^2\right)^p \, dx$$
$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^m(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p} \, dx$$
$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^m\left(1-a^2x^2\right)^{-\frac{1}{2}+p} \, dx + \left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int$$
$$= \frac{x^{1+m}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p {}_2F_1\left(\frac{1+m}{2},\frac{1}{2}-p;\frac{3+m}{2};a^2x^2\right)}{1+m} + \frac{ax^{2+m}\left(1-a^2x^2\right)^{-p}\left(c-a\right.}{}$$

Mathematica [A] time = 0.041569, size = 114, normalized size = 0.84

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1}{2}-p,\frac{m+1}{2}+1,a^2x^2\right)}{m+1}+\frac{ax^{m+2}\text{Hypergeometric2F1}\left(\frac{m+2}{2},\frac{1}{2}\right.}{m+2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((c - a^2*c*x^2)^p*((x^(1 + m)*Hypergeometric2F1[(1 + m)/2, 1/2 - p, 1 + (1
 + m)/2, a^2*x^2])/(1 + m) + (a*x^(2 + m)*Hypergeometric2F1[(2 + m)/2, 1/2
- p, 1 + (2 + m)/2, a^2*x^2])/(2 + m)))/(1 - a^2*x^2)^p
```

Maple [F] time = 0.329, size = 0, normalized size = 0.

$$\int (ax+1)\, x^m \left(-a^2cx^2+c\right)^p \frac{1}{\sqrt{-a^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p x^m}{\sqrt{-a^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^p,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p*x^m/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x^m}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^p,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^m/(a*x - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^p,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p*x^m/sqrt(-a^2*x^2 + 1), x)
```

3.1005 $\int e^{\tanh^{-1}(ax)} x^3 \left(1-a^2x^2\right)^p \, dx$

Optimal. Leaf size=85

$$\frac{1}{5}ax^5\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-p,\frac{7}{2},a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^4(2p+1)}+\frac{\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^4(2p+3)}$$

```
[Out] -((1 - a^2*x^2)^(1/2 + p)/(a^4*(1 + 2*p))) + (1 - a^2*x^2)^(3/2 + p)/(a^4*(
3 + 2*p)) + (a*x^5*Hypergeometric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/5
```

Rubi [A] time = 0.113711, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6148, 764, 266, 43, 364}

$$\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^4(2p+1)}+\frac{\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^4(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^3*(1 - a^2*x^2)^p,x]
```

```
[Out] -((1 - a^2*x^2)^(1/2 + p)/(a^4*(1 + 2*p))) + (1 - a^2*x^2)^(3/2 + p)/(a^4*(
3 + 2*p)) + (a*x^5*Hypergeometric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/5
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^3\left(1-a^2x^2\right)^p\,dx=\int x^3(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx$$
$$=a\int x^4\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx+\int x^3\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx$$
$$=\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{2}\text{Subst}\left(\int x\left(1-a^2x\right)^{-\frac{1}{2}+p}\,dx,x,x^2\right)$$
$$=\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{2}\text{Subst}\left(\int\left(\frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{a^2}-\frac{\left(1-a^2x\right)^{\frac{1}{2}+p}}{a^2}\right)dx,x,x^2\right)$$
$$=-\frac{\left(1-a^2x^2\right)^{\frac{1}{2}+p}}{a^4(1+2p)}+\frac{\left(1-a^2x^2\right)^{\frac{3}{2}+p}}{a^4(3+2p)}+\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)$$

Mathematica [A] time = 0.0695191, size = 77, normalized size = 0.91

$$\frac{1}{5}ax^5\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-p,\frac{7}{2},a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}\left(a^2(2p+1)x^2+2\right)}{a^4\left(4p^2+8p+3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^3*(1 - a^2*x^2)^p,x]
```

```
[Out] -(((1 - a^2*x^2)^(1/2 + p)*(2 + a^2*(1 + 2*p)*x^2))/(a^4*(3 + 8*p + 4*p^2))
) + (a*x^5*Hypergeometric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/5
```

Maple [A] time = 0.357, size = 47, normalized size = 0.6

$$\frac{ax^5}{5}{}_2\text{F}_1(\frac{5}{2},\frac{1}{2}-p;\ \frac{7}{2};\ a^2x^2)+\frac{x^4}{4}{}_2\text{F}_1(2,\frac{1}{2}-p;\ 3;\ a^2x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*x^2+1)^p,x)
```

```
[Out] 1/5*a*x^5*hypergeom([5/2,1/2-p],[7/2],a^2*x^2)+1/4*x^4*hypergeom([2,1/2-p],
[3],a^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a\int\frac{x^4e^{\left(p\log(ax+1)+p\log(-ax+1)\right)}}{\sqrt{ax+1}\sqrt{-ax+1}}\,dx+\frac{\left(a^4\left(2p+1\right)x^4-a^2\left(2p-1\right)x^2-2\right)\left(-a^2x^2+1\right)^p}{\sqrt{-a^2x^2+1}\left(4p^2+8p+3\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*x^2+1)^p,x, algorithm="maxim
a")
```

```
[Out] a*integrate(x^4*e^(p*log(a*x + 1) + p*log(-a*x + 1))/(sqrt(a*x + 1)*sqrt(-a
*x + 1)), x) + (a^4*(2*p + 1)*x^4 - a^2*(2*p - 1)*x^2 - 2)*(-a^2*x^2 + 1)^p
/(sqrt(-a^2*x^2 + 1)*(4*p^2 + 8*p + 3)*a^4)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x^3}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*x^2+1)^p,x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x^3/(a*x - 1), x)
```

Sympy [C] time = 29.6525, size = 258, normalized size = 3.04

$$-\frac{aa^{2p}x^5x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{5}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p+\frac{5}{2}\\ p+1,p+\frac{7}{2}\end{matrix}\middle| a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}x^5x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{5}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-p-\frac{5}{2}\\ \frac{1}{2},-p-\frac{3}{2}\end{matrix}\middle| \frac{1}{a^2x^2}\right)}{2\sqrt{\pi}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3*(-a**2*x**2+1)**p,x)
```

```
[Out] -a*a**(2*p)*x**5*x**(2*p)*exp(I*pi*p)*gamma(-p - 5/2)*gamma(p + 1/2)*hyper(
(1/2, 1, p + 5/2), (p + 1, p + 7/2), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(p
i)*gamma(-p - 3/2)*gamma(p + 1)) - a*a**(2*p)*x**5*x**(2*p)*exp(I*pi*p)*gam
ma(-p - 5/2)*gamma(p + 1/2)*hyper((1, -p, -p - 5/2), (1/2, -p - 3/2), 1/(a*
*2*x**2))/(2*sqrt(pi)*gamma(-p - 3/2)*gamma(p + 1)) - meijerg(((-p - 1, 1),
 (-1,)), ((-p - 3/2, -p - 1), (0,)), exp_polar(-I*pi)/(a**2*x**2))*gamma(p
+ 1/2)/(2*pi*a**4) - meijerg(((-1, -p - 2, 1), ()), ((-p - 2,), (-p - 3/2,
0)), exp_polar(-I*pi)/(a**2*x**2))*gamma(p + 1/2)/(2*a**4*gamma(-p)*gamma(p
 + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^p x^3}{\sqrt{-a^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*x^2+1)^p,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^p*x^3/sqrt(-a^2*x^2 + 1), x)
```

3.1006 $\int e^{\tanh^{-1}(ax)}x^2\left(1-a^2x^2\right)^p\,dx$

Optimal. Leaf size=84

$$\frac{1}{3}x^3\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-p,\frac{5}{2},a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^3(2p+1)}+\frac{\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^3(2p+3)}$$

```
[Out] -((1 - a^2*x^2)^(1/2 + p)/(a^3*(1 + 2*p))) + (1 - a^2*x^2)^(3/2 + p)/(a^3*(
3 + 2*p)) + (x^3*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/3
```

Rubi [A] time = 0.115129, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6148, 764, 364, 266, 43}

$$\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^3(2p+1)}+\frac{\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^3(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2*(1 - a^2*x^2)^p,x]
```


```
[Out] -((1 - a^2*x^2)^(1/2 + p)/(a^3*(1 + 2*p))) + (1 - a^2*x^2)^(3/2 + p)/(a^3*(
3 + 2*p)) + (x^3*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/3
```


Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x^2\left(1-a^2x^2\right)^p\,dx &= \int x^2(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\ &= a\int x^3\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx+\int x^2\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\ &= \frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{1}{2}a\,\text{Subst}\left(\int x\left(1-a^2x\right)^{-\frac{1}{2}+p}\,dx,x,x^2\right)\\ &= \frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{1}{2}a\,\text{Subst}\left(\int\left(\frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{a^2}-\frac{\left(1-a^2x\right)^{\frac{1}{2}+p}}{a^2}\right)dx,x,x^2\right)\\ &= -\frac{\left(1-a^2x^2\right)^{\frac{1}{2}+p}}{a^3(1+2p)}+\frac{\left(1-a^2x^2\right)^{\frac{3}{2}+p}}{a^3(3+2p)}+\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0483701, size = 76, normalized size = 0.9

$$\frac{1}{3}x^3\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-p,\frac{5}{2},a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}\left(a^2(2p+1)x^2+2\right)}{a^3\left(4p^2+8p+3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^2*(1 - a^2*x^2)^p,x]
```

```
[Out] -(((1 - a^2*x^2)^(1/2 + p)*(2 + a^2*(1 + 2*p)*x^2))/(a^3*(3 + 8*p + 4*p^2))
) + (x^3*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/3
```

Maple [A] time = 0.322, size = 47, normalized size = 0.6

$$\frac{x^4a}{4}{}_2F_1(2,\frac{1}{2}-p;\,3;\,a^2x^2)+\frac{x^3}{3}{}_2F_1(\frac{3}{2},\frac{1}{2}-p;\,\frac{5}{2};\,a^2x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*x^2+1)^p,x)
```

```
[Out] 1/4*a*x^4*hypergeom([2,1/2-p],[3],a^2*x^2)+1/3*x^3*hypergeom([3/2,1/2-p],[5
/2],a^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(ax+1)\left(-a^2x^2+1\right)^{p-\frac{1}{2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*x^2+1)^p,x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^(p - 1/2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x^2}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*x^2+1)^p,x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x^2/(a*x - 1), x)
```

Sympy [C] time = 19.2566, size = 255, normalized size = 3.04

$$-\frac{a^{2p}x^3x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p+\frac{3}{2}\\ p+1,p+\frac{5}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+1\right)}-\frac{a^{2p}x^3x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-p-\frac{3}{2}\\ \frac{1}{2},-p-\frac{1}{2}\end{matrix}\middle|\frac{1}{a^2x^2}\right)}{2\sqrt{\pi}\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(-a**2*x**2+1)**p,x)
```

```
[Out] -a**(2*p)*x**3*x**(2*p)*exp(I*pi*p)*gamma(-p - 3/2)*gamma(p + 1/2)*hyper((1
/2, 1, p + 3/2), (p + 1, p + 5/2), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)
*gamma(-p - 1/2)*gamma(p + 1)) - a**(2*p)*x**3*x**(2*p)*exp(I*pi*p)*gamma(-
p - 3/2)*gamma(p + 1/2)*hyper((1, -p, -p - 3/2), (1/2, -p - 1/2), 1/(a**2*x
**2))/(2*sqrt(pi)*gamma(-p - 1/2)*gamma(p + 1)) - meijerg(((-p - 1, 1), (-1
,)), ((-p - 3/2, -p - 1), (0,)), exp_polar(-I*pi)/(a**2*x**2))*gamma(p + 1/
2)/(2*pi*a**3) - meijerg(((-1, -p - 2, 1), ()), ((-p - 2,), (-p - 3/2, 0)),
 exp_polar(-I*pi)/(a**2*x**2))*gamma(p + 1/2)/(2*a**3*gamma(-p)*gamma(p + 1
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^p x^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*x^2+1)^p,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^p*x^2/sqrt(-a^2*x^2 + 1), x)
```

3.1007 $\int e^{\tanh^{-1}(ax)}x\left(1-a^2x^2\right)^p\,dx$

Optimal. Leaf size=58

$$\frac{1}{3}ax^3\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-p,\frac{5}{2},a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^2(2p+1)}$$

```
[Out] -((1 - a^2*x^2)^(1/2 + p)/(a^2*(1 + 2*p))) + (a*x^3*Hypergeometric2F1[3/2,
1/2 - p, 5/2, a^2*x^2])/3
```

Rubi [A] time = 0.0613426, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6148, 764, 261, 364}

$$\frac{1}{3}ax^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x*(1 - a^2*x^2)^p,x]
```

```
[Out] -((1 - a^2*x^2)^(1/2 + p)/(a^2*(1 + 2*p))) + (a*x^3*Hypergeometric2F1[3/2,
1/2 - p, 5/2, a^2*x^2])/3
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x\left(1-a^2x^2\right)^p\,dx=\int x(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx$$
$$=a\int x^2\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx+\int x\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx$$
$$=-\frac{\left(1-a^2x^2\right)^{\frac{1}{2}+p}}{a^2(1+2p)}+\frac{1}{3}ax^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)$$

Mathematica [A] time = 0.0238248, size = 60, normalized size = 1.03

$$\frac{1}{3}ax^3\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-p,\frac{5}{2},a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{2a^2\left(p+\frac{1}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x*(1 - a^2*x^2)^p,x]
```

```
[Out] -(1 - a^2*x^2)^(1/2 + p)/(2*a^2*(1/2 + p)) + (a*x^3*Hypergeometric2F1[3/2,
1/2 - p, 5/2, a^2*x^2])/3
```

Maple [A] time = 0.342, size = 47, normalized size = 0.8

$$\frac{x^3a}{3}{}_2\text{F}_1(\frac{3}{2},\frac{1}{2}-p;\ \frac{5}{2};\ a^2x^2)+\frac{x^2}{2}{}_2\text{F}_1(1,\frac{1}{2}-p;\ 2;\ a^2x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*x^2+1)^p,x)
```

```
[Out] 1/3*a*x^3*hypergeom([3/2,1/2-p],[5/2],a^2*x^2)+1/2*x^2*hypergeom([1,1/2-p],
[2],a^2*x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*x^2+1)^p,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*x^2+1)^p,x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x/(a*x - 1), x)
```

Sympy [C] time = 13.3856, size = 301, normalized size = 5.19

$$-\frac{aa^{2p}x^3x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p+\frac{3}{2}\\p+1,p+\frac{5}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}x^3x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-p-\frac{3}{2}\\\frac{1}{2},-p-\frac{1}{2}\end{matrix}\middle|\frac{1}{a^2x^2}\right)}{2\sqrt{\pi}\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(-a**2*x**2+1)**p,x)
```

```
[Out] -a*a**(2*p)*x**3*x**(2*p)*exp(I*pi*p)*gamma(-p - 3/2)*gamma(p + 1/2)*hyper(
(1/2, 1, p + 3/2), (p + 1, p + 5/2), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(p
i)*gamma(-p - 1/2)*gamma(p + 1)) - a*a**(2*p)*x**3*x**(2*p)*exp(I*pi*p)*gam
ma(-p - 3/2)*gamma(p + 1/2)*hyper((1, -p, -p - 3/2), (1/2, -p - 1/2), 1/(a*
*2*x**2))/(2*sqrt(pi)*gamma(-p - 1/2)*gamma(p + 1)) - a**(2*p)*x**2*x**(2*p
)*exp(I*pi*p)*gamma(-p - 1)*gamma(p + 1/2)*hyper((1/2, 1), (p + 2,), a**2*x
**2*exp_polar(2*I*pi))/(2*sqrt(pi)*gamma(-p)*gamma(p + 1)) - a**(2*p)*x**2*
x**(2*p)*exp(I*pi*p)*gamma(-p - 1)*gamma(p + 1/2)*hyper((1, -p - 1), (1/2,)
, 1/(a**2*x**2))/(2*sqrt(pi)*gamma(-p)*gamma(p + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^p x}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*x^2+1)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^p*x/sqrt(-a^2*x^2 + 1), x)
```

3.1008 $\int e^{\tanh^{-1}(ax)} \left(1-a^2x^2\right)^p \, dx$

Optimal. Leaf size=59

$$-\frac{2^{p+\frac{3}{2}}(1-ax)^{p+\frac{1}{2}}\text{Hypergeometric2F1}\left(-p-\frac{1}{2},p+\frac{1}{2},p+\frac{3}{2},\frac{1}{2}(1-ax)\right)}{a(2p+1)}$$

```
[Out] -((2^(3/2 + p)*(1 - a*x)^(1/2 + p)*Hypergeometric2F1[-1/2 - p, 1/2 + p, 3/2
 + p, (1 - a*x)/2])/(a*(1 + 2*p)))
```

Rubi [A] time = 0.0388424, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6140, 69}

$$-\frac{2^{p+\frac{3}{2}}(1-ax)^{p+\frac{1}{2}}\,{}_2F_1\left(-p-\frac{1}{2},p+\frac{1}{2};p+\frac{3}{2};\frac{1}{2}(1-ax)\right)}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(1 - a^2*x^2)^p,x]
```

```
[Out] -((2^(3/2 + p)*(1 - a*x)^(1/2 + p)*Hypergeometric2F1[-1/2 - p, 1/2 + p, 3/2
 + p, (1 - a*x)/2])/(a*(1 + 2*p)))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx &= \int (1-ax)^{-\frac{1}{2}+p}(1+ax)^{\frac{1}{2}+p}\,dx\\ &= -\frac{2^{\frac{3}{2}+p}(1-ax)^{\frac{1}{2}+p}\,{}_2F_1\left(-\frac{1}{2}-p,\frac{1}{2}+p;\frac{3}{2}+p;\frac{1}{2}(1-ax)\right)}{a(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.0138504, size = 59, normalized size = 1.

$$-\frac{2^{p+\frac{1}{2}}(1-ax)^{p+\frac{1}{2}}\text{Hypergeometric2F1}\left(-p-\frac{1}{2},p+\frac{1}{2},p+\frac{3}{2},\frac{1}{2}(1-ax)\right)}{a\left(p+\frac{1}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(1 - a^2*x^2)^p,x]
```

```
[Out] -((2^(1/2 + p)*(1 - a*x)^(1/2 + p)*Hypergeometric2F1[-1/2 - p, 1/2 + p, 3/2
 + p, (1 - a*x)/2])/(a*(1/2 + p)))
```

Maple [A] time = 0.321, size = 44, normalized size = 0.8

$$\frac{ax^2}{2}{}_2F_1(1,\frac{1}{2}-p;\,2;\,a^2x^2)+x{}_2F_1(\frac{1}{2},\frac{1}{2}-p;\,\frac{3}{2};\,a^2x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p,x)
```

```
[Out] 1/2*a*x^2*hypergeom([1,1/2-p],[2],a^2*x^2)+x*hypergeom([1/2,1/2-p],[3/2],a^
2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ax+1)\left(-a^2x^2+1\right)^{p-\frac{1}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^(p - 1/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/(a*x - 1), x)
```

Sympy [C] time = 10.8415, size = 292, normalized size = 4.95

$$-\frac{aa^{2p}x^2x^{2p}e^{i\pi p}\Gamma\left(-p-1\right)\Gamma\left(p+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},1\\ p+2\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\Gamma\left(-p\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}x^2x^{2p}e^{i\pi p}\Gamma\left(-p-1\right)\Gamma\left(p+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}1,-p-1\\ \frac{1}{2}\end{matrix}\middle|\frac{1}{a^2x^2}\right)}{2\sqrt{\pi}\Gamma\left(-p\right)\Gamma\left(p+1\right)}-\frac{a^{2p}xx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*x**2+1)**p,x)
```

```
[Out] -a*a**(2*p)*x**2*x**(2*p)*exp(I*pi*p)*gamma(-p - 1)*gamma(p + 1/2)*hyper((1
/2, 1), (p + 2,), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*gamma(-p)*gamma(
p + 1)) - a*a**(2*p)*x**2*x**(2*p)*exp(I*pi*p)*gamma(-p - 1)*gamma(p + 1/2)
*hyper((1, -p - 1), (1/2,), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(-p)*gamma(p +
1)) - a**(2*p)*x*x**(2*p)*exp(I*pi*p)*gamma(-p - 1/2)*gamma(p + 1/2)*hyper(
(1/2, 1, p + 1/2), (p + 1, p + 3/2), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(p
i)*gamma(1/2 - p)*gamma(p + 1)) - a**(2*p)*x*x**(2*p)*exp(I*pi*p)*gamma(-p
- 1/2)*gamma(p + 1/2)*hyper((1, -p, -p - 1/2), (1/2, 1/2 - p), 1/(a**2*x**2
))/(2*sqrt(pi)*gamma(1/2 - p)*gamma(p + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^p/sqrt(-a^2*x^2 + 1), x)
```

3.1009 $\int \frac{e^{\tanh^{-1}(ax)}(1-a^2x^2)^p}{x}\,dx$

Optimal. Leaf size=72

$$ax\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-p,\frac{3}{2},a^2x^2\right)-\frac{(1-a^2x^2)^{p+\frac{1}{2}}\text{ Hypergeometric2F1}\left(1,p+\frac{1}{2},p+\frac{3}{2},1-a^2x^2\right)}{2p+1}$$

```
[Out] a*x*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2] - ((1 - a^2*x^2)^(1/2 + p
)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rubi [A] time = 0.0927399, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6148, 764, 266, 65, 245}

$$ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{(1-a^2x^2)^{p+\frac{1}{2}}\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(1 - a^2*x^2)^p)/x,x]
```

```
[Out] a*x*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2] - ((1 - a^2*x^2)^(1/2 + p
)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x}\,dx &= \int \frac{(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx\\
&= a\int \left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx + \int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx\\
&= ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{x}\,dx,x,x^2\right)\\
&= ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right) - \frac{\left(1-a^2x^2\right)^{\frac{1}{2}+p}\,{}_2F_1\left(1,\frac{1}{2}+p;\frac{3}{2}+p;1-a^2x^2\right)}{1+2p}
\end{aligned}$$

Mathematica [A] time = 0.0251872, size = 74, normalized size = 1.03

$$ax\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-p,\frac{3}{2},a^2x^2\right) - \frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}\text{Hypergeometric2F1}\left(1,p+\frac{1}{2},p+\frac{3}{2},1-a^2x^2\right)}{2\left(p+\frac{1}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(1 - a^2*x^2)^p)/x,x]
```

```
[Out] a*x*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2] - ((1 - a^2*x^2)^(1/2 + p
)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(2*(1/2 + p))
```

Maple [A] time = 0.343, size = 90, normalized size = 1.3

$$ax{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\,\frac{3}{2};\,a^2x^2\right) + \frac{1}{2}\left(\left(\Psi\left(\frac{1}{2}-p\right)+\gamma+2\,\ln(x)+\ln\left(-a^2\right)\right)\Gamma\left(\frac{1}{2}-p\right)+\Gamma\left(\frac{3}{2}-p\right)a^2x^2{}_3F_2(1,1,\frac{3}{2}-p;\,2,2;$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x,x)
```

```
[Out] a*x*hypergeom([1/2,1/2-p],[3/2],a^2*x^2)+1/2*((Psi(1/2-p)+gamma+2*ln(x)+ln(
-a^2))*GAMMA(1/2-p)+GAMMA(3/2-p)*a^2*x^2*hypergeom([1,1,3/2-p],[2,2],a^2*x^
2))/GAMMA(1/2-p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^{p-\frac{1}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^(p - 1/2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{ax^2-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x,x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/(a*x^2 - x), x)
```

Sympy [C] time = 14.5145, size = 286, normalized size = 3.97

$$-\frac{aa^{2p}xx^{2p}e^{i\pi p}\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p+\frac{1}{2}\\ p+1,p+\frac{3}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\Gamma\left(\frac{1}{2}-p\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}xx^{2p}e^{i\pi p}\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-p-\frac{1}{2}\\ \frac{1}{2},\frac{1}{2}-p\end{matrix}\middle|\frac{1}{a^2x^2}\right)}{2\sqrt{\pi}\Gamma\left(\frac{1}{2}-p\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*x**2+1)**p/x,x)
```

```
[Out] -a*a**(2*p)*x*x**(2*p)*exp(I*pi*p)*gamma(-p - 1/2)*gamma(p + 1/2)*hyper((1/
2, 1, p + 1/2), (p + 1, p + 3/2), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*
gamma(1/2 - p)*gamma(p + 1)) - a*a**(2*p)*x*x**(2*p)*exp(I*pi*p)*gamma(-p -
 1/2)*gamma(p + 1/2)*hyper((1, -p, -p - 1/2), (1/2, 1/2 - p), 1/(a**2*x**2)
)/(2*sqrt(pi)*gamma(1/2 - p)*gamma(p + 1)) - a**(2*p)*x**(2*p)*exp(I*pi*p)*
gamma(-p)*gamma(p + 1/2)*hyper((1/2, 1, p), (p + 1, p + 1), a**2*x**2*exp_p
olar(2*I*pi))/(2*sqrt(pi)*gamma(1 - p)*gamma(p + 1)) - a**(2*p)*x**(2*p)*ex
p(I*pi*p)*gamma(-p)*gamma(p + 1/2)*hyper((1, -p, -p), (1/2, 1 - p), 1/(a**2
*x**2))/(2*sqrt(pi)*gamma(1 - p)*gamma(p + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^p}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^p/(sqrt(-a^2*x^2 + 1)*x), x)
```

3.1010 $\int \frac{e^{\tanh^{-1}(ax)}(1-a^2x^2)^p}{x^2}\,dx$

Optimal. Leaf size=75

$$-\frac{a\left(1-a^2x^2\right)^{p+\frac{1}{2}}\text{Hypergeometric2F1}\left(1,p+\frac{1}{2},p+\frac{3}{2},1-a^2x^2\right)}{2p+1}-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-p,\frac{1}{2},a^2x^2\right)}{x}$$

```
[Out] -(Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2]/x) - (a*(1 - a^2*x^2)^(1/2
 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rubi [A] time = 0.100196, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6148, 764, 364, 266, 65}

$$-\frac{a\left(1-a^2x^2\right)^{p+\frac{1}{2}}\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(1 - a^2*x^2)^p)/x^2,x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2]/x) - (a*(1 - a^2*x^2)^(1/2
 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
```

```
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^2}\,dx &= \int \frac{(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx\\ &= a\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx+\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx\\ &= -\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{x}\,dx,x,x^2\right)\\ &= -\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}-\frac{a\left(1-a^2x^2\right)^{\frac{1}{2}+p}\,{}_2F_1\left(1,\frac{1}{2}+p;\frac{3}{2}+p;1-a^2x^2\right)}{1+2p}\end{aligned}$$

Mathematica [A] time = 0.026374, size = 77, normalized size = 1.03

$$-\frac{a\left(1-a^2x^2\right)^{p+\frac{1}{2}}\text{Hypergeometric2F1}\left(1,p+\frac{1}{2},p+\frac{3}{2},1-a^2x^2\right)}{2\left(p+\frac{1}{2}\right)}-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-p,\frac{1}{2},a^2x^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(1 - a^2*x^2)^p)/x^2,x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2]/x) - (a*(1 - a^2*x^2)^(1/2
 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(2*(1/2 + p))
```

Maple [A] time = 0.331, size = 93, normalized size = 1.2

$$\frac{a}{2}\left(\left(\Psi\left(\frac{1}{2}-p\right)+\gamma+2\ln(x)+\ln\left(-a^2\right)\right)\Gamma\left(\frac{1}{2}-p\right)+\Gamma\left(\frac{3}{2}-p\right)a^2x^2{}_3F_2(1,1,\frac{3}{2}-p;\,2,2;\,a^2x^2)\right)\left(\Gamma\left(\frac{1}{2}-p\right)\right)^{-1}-\frac{1}{x}{}_2F_1(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x^2,x)
```

```
[Out] 1/2*a*((Psi(1/2-p)+gamma+2*ln(x)+ln(-a^2))*GAMMA(1/2-p)+GAMMA(3/2-p)*a^2*x^
2*hypergeom([1,1,3/2-p],[2,2],a^2*x^2))/GAMMA(1/2-p)-hypergeom([-1/2,1/2-p]
,[1/2],a^2*x^2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^{p-\frac{1}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^(p - 1/2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{ax^3-x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x^2,x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/(a*x^3 - x^2), x)
```

Sympy [C] time = 15.5307, size = 280, normalized size = 3.73

$$-\frac{aa^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p\\p+1,p+1\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\Gamma\left(1-p\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-p\\\frac{1}{2},1-p\end{matrix}\middle|\frac{1}{a^2x^2}\right)}{2\sqrt{\pi}\Gamma\left(1-p\right)\Gamma\left(p+1\right)}-\frac{a^{2p}x^{2p}e^{i\pi}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*x**2+1)**p/x**2,x)
```

```
[Out] -a*a**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*gamma(p + 1/2)*hyper((1/2, 1, p)
, (p + 1, p + 1), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*gamma(1 - p)*gam
ma(p + 1)) - a*a**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*gamma(p + 1/2)*hyper
((1, -p, -p), (1/2, 1 - p), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(1 - p)*gamma(p
 + 1)) - a**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1/2 - p)*gamma(p + 1/2)*hyper(
(1/2, 1, p - 1/2), (p + 1/2, p + 1), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(p
i)*x*gamma(3/2 - p)*gamma(p + 1)) - a**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1/2
 - p)*gamma(p + 1/2)*hyper((1, -p, 1/2 - p), (1/2, 3/2 - p), 1/(a**2*x**2))
/(2*sqrt(pi)*x*gamma(3/2 - p)*gamma(p + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^p}{\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x^2,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^p/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

3.1011 $\int \frac{e^{\tanh^{-1}(ax)}(1-a^2x^2)^p}{x^3}\,dx$

Optimal. Leaf size=78

$$-\frac{a^2\left(1-a^2x^2\right)^{p+\frac{1}{2}}\text{Hypergeometric2F1}\left(2,p+\frac{1}{2},p+\frac{3}{2},1-a^2x^2\right)}{2p+1}-\frac{a\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-p,\frac{1}{2},a^2x^2\right)}{x}$$

```
[Out] -((a*Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2])/x) - (a^2*(1 - a^2*x^2
)^(1/2 + p)*Hypergeometric2F1[2, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rubi [A] time = 0.0988352, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6148, 764, 266, 65, 364}

$$-\frac{a^2\left(1-a^2x^2\right)^{p+\frac{1}{2}}{}_2F_1\left(2,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-\frac{a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(1 - a^2*x^2)^p)/x^3,x]
```

```
[Out] -((a*Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2])/x) - (a^2*(1 - a^2*x^2
)^(1/2 + p)*Hypergeometric2F1[2, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^3}\,dx &= \int \frac{(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^3}\,dx\\ &= a\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx+\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^3}\,dx\\ &= -\frac{a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}+\frac{1}{2}\text{Subst}\left(\int \frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{x^2}\,dx,x,x^2\right)\\ &= -\frac{a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}-\frac{a^2\left(1-a^2x^2\right)^{\frac{1}{2}+p}{}_2F_1\left(2,\frac{1}{2}+p;\frac{3}{2}+p;1-a^2x^2\right)}{1+2p}\end{aligned}$$

Mathematica [A] time = 0.0218515, size = 80, normalized size = 1.03

$$-\frac{a^2\left(1-a^2x^2\right)^{p+\frac{1}{2}}\text{Hypergeometric2F1}\left(2,p+\frac{1}{2},p+\frac{3}{2},1-a^2x^2\right)}{2\left(p+\frac{1}{2}\right)}-\frac{a\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-p,\frac{1}{2},a^2x^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(1 - a^2*x^2)^p)/x^3,x]
```

```
[Out] -((a*Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2])/x) - (a^2*(1 - a^2*x^2
)^(1/2 + p)*Hypergeometric2F1[2, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(2*(1/2 +
p))
```

Maple [A] time = 0.339, size = 112, normalized size = 1.4

$$-\frac{a}{x}{}_2\text{F}_1(-\frac{1}{2},\frac{1}{2}-p;\,\frac{1}{2};\,a^2x^2)-\frac{a^2}{2}\left(\frac{1}{a^2x^2}\Gamma\left(\frac{1}{2}-p\right)-\left(\Psi\left(\frac{3}{2}-p\right)+\gamma-1+2\,\ln(x)+\ln\left(-a^2\right)\right)\Gamma\left(\frac{3}{2}-p\right)-\frac{a^2x^2}{2}\Gamma\left(\frac{5}{2}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x^3,x)
```

```
[Out] -a*hypergeom([-1/2,1/2-p],[1/2],a^2*x^2)/x-1/2*a^2*(GAMMA(1/2-p)/x^2/a^2-(P
si(3/2-p)+gamma-1+2*ln(x)+ln(-a^2))*GAMMA(3/2-p)-1/2*GAMMA(5/2-p)*a^2*x^2*h
ypergeom([1,1,5/2-p],[2,3],a^2*x^2))/GAMMA(1/2-p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^{p-\frac{1}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^(p - 1/2)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{ax^4-x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x^3,x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/(a*x^4 - x^3), x)
```

Sympy [C] time = 23.7387, size = 287, normalized size = 3.68

$$-\frac{aa^{2p}x^{2p}e^{i\pi p}\Gamma\left(\frac{1}{2}-p\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p-\frac{1}{2}\\p+\frac{1}{2},p+1\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}x\Gamma\left(\frac{3}{2}-p\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}x^{2p}e^{i\pi p}\Gamma\left(\frac{1}{2}-p\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,\frac{1}{2}-p\\\frac{1}{2},\frac{3}{2}-p\end{matrix}\middle|\frac{1}{a^2x^2}\right)}{2\sqrt{\pi}x\Gamma\left(\frac{3}{2}-p\right)\Gamma\left(p+1\right)}-\frac{a^{2p}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*x**2+1)**p/x**3,x)
```

```
[Out] -a*a**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1/2 - p)*gamma(p + 1/2)*hyper((1/2,
1, p - 1/2), (p + 1/2, p + 1), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*x*g
amma(3/2 - p)*gamma(p + 1)) - a*a**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1/2 - p
)*gamma(p + 1/2)*hyper((1, -p, 1/2 - p), (1/2, 3/2 - p), 1/(a**2*x**2))/(2*
sqrt(pi)*x*gamma(3/2 - p)*gamma(p + 1)) - a**(2*p)*x**(2*p)*exp(I*pi*p)*gam
ma(1 - p)*gamma(p + 1/2)*hyper((1/2, 1, p - 1), (p, p + 1), a**2*x**2*exp_p
olar(2*I*pi))/(2*sqrt(pi)*x**2*gamma(2 - p)*gamma(p + 1)) - a**(2*p)*x**(2*
p)*exp(I*pi*p)*gamma(1 - p)*gamma(p + 1/2)*hyper((1, -p, 1 - p), (1/2, 2 -
p), 1/(a**2*x**2))/(2*sqrt(pi)*x**2*gamma(2 - p)*gamma(p + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\left(-a^2x^2+1\right)^p}{\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x^3,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^p/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

3.1012 $\int e^{\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=134

$$\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-p,\frac{7}{2},a^2x^2\right)+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^4(2p+3)}-\frac{\sqrt{1-a^2x^2}\left(c\cdot\right.}{a^4(2p+}$$

```
[Out] -((Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^4*(1 + 2*p))) + ((1 - a^2*x^2)^(
3/2)*(c - a^2*c*x^2)^p)/(a^4*(3 + 2*p)) + (a*x^5*(c - a^2*c*x^2)^p*Hypergeo
metric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/(5*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.180644, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6153, 6148, 764, 266, 43, 364}

$$\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^4(2p+3)}-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^4(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^3*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^4*(1 + 2*p))) + ((1 - a^2*x^2)^(
3/2)*(c - a^2*c*x^2)^p)/(a^4*(3 + 2*p)) + (a*x^5*(c - a^2*c*x^2)^p*Hypergeo
metric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/(5*(1 - a^2*x^2)^p)
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{\tanh^{-1}(ax)}x^3\left(1-a^2x^2\right)^p\,dx\\ &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^3(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\ &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^3\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx+\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x\\ &= \frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\mathrm{S}\\ &= \frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\mathrm{S}\\ &= -\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^4(1+2p)}+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^4(3+2p)}+\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\end{aligned}$$

Mathematica [A] time = 0.0866559, size = 105, normalized size = 0.78

$$\frac{\left(c-a^2cx^2\right)^p\left(a^5x^5\left(1-a^2x^2\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-p,\frac{7}{2},a^2x^2\right)-\frac{5\sqrt{1-a^2x^2}\left(a^2(2p+1)x^2+2\right)}{4p^2+8p+3}\right)}{5a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^3*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((c - a^2*c*x^2)^p*((-5*Sqrt[1 - a^2*x^2]*(2 + a^2*(1 + 2*p)*x^2))/(3 + 8*p
 + 4*p^2) + (a^5*x^5*Hypergeometric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/(1 - a^
2*x^2)^p))/(5*a^4)
```

Maple [F] time = 0.322, size = 0, normalized size = 0.

$$\int(ax+1)\,x^3\left(-a^2cx^2+c\right)^p\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$ac^p \int \frac{x^4 e^{\left(p \log(ax+1) + p \log(-ax+1)\right)}}{\sqrt{ax+1}\sqrt{-ax+1}}\,dx + \frac{\left(a^4 c^p (2p+1)x^4 - a^2 c^p (2p-1)x^2 - 2c^p\right)\left(-a^2x^2+1\right)^p}{\sqrt{-a^2x^2+1}\left(4p^2+8p+3\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*c*x^2+c)^p,x, algorithm="max
ima")
```

```
[Out] a*c^p*integrate(x^4*e^(p*log(a*x + 1) + p*log(-a*x + 1))/(sqrt(a*x + 1)*sqr
t(-a*x + 1)), x) + (a^4*c^p*(2*p + 1)*x^4 - a^2*c^p*(2*p - 1)*x^2 - 2*c^p)*
(-a^2*x^2 + 1)^p/(sqrt(-a^2*x^2 + 1)*(4*p^2 + 8*p + 3)*a^4)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x^3}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*c*x^2+c)^p,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^3/(a*x - 1), x)
```

Sympy [C] time = 30.0238, size = 272, normalized size = 2.03

$$-\frac{aa^{2p}c^px^5x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{5}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p+\frac{5}{2}\\p+1,p+\frac{7}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+1\right)} - \frac{aa^{2p}c^px^5x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{5}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-p\\\frac{1}{2},-p-\end{matrix}\right.}{2\sqrt{\pi}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3*(-a**2*c*x**2+c)**p,x)
```

```
[Out] -a*a**(2*p)*c**p*x**5*x**(2*p)*exp(I*pi*p)*gamma(-p - 5/2)*gamma(p + 1/2)*h
yper((1/2, 1, p + 5/2), (p + 1, p + 7/2), a**2*x**2*exp_polar(2*I*pi))/(2*s
qrt(pi)*gamma(-p - 3/2)*gamma(p + 1)) - a*a**(2*p)*c**p*x**5*x**(2*p)*exp(I
*pi*p)*gamma(-p - 5/2)*gamma(p + 1/2)*hyper((1, -p, -p - 5/2), (1/2, -p - 3
/2), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(-p - 3/2)*gamma(p + 1)) - c**p*meijer
g(((-p - 1, 1), (-1,)), ((-p - 3/2, -p - 1), (0,)), exp_polar(-I*pi)/(a**2*
x**2))*gamma(p + 1/2)/(2*pi*a**4) - c**p*meijerg(((-1, -p - 2, 1), ()), ((-
p - 2,), (-p - 3/2, 0)), exp_polar(-I*pi)/(a**2*x**2))*gamma(p + 1/2)/(2*a*
*4*gamma(-p)*gamma(p + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p x^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*c*x^2+c)^p,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p*x^3/sqrt(-a^2*x^2 + 1), x)
```

3.1013 $\int e^{\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=133

$$\frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-p,\frac{5}{2},a^2x^2\right)+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^3(2p+3)}-\frac{\sqrt{1-a^2x^2}\left(c-\right.}{a^3(2p+}$$

```
[Out] -((Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^3*(1 + 2*p))) + ((1 - a^2*x^2)^(
3/2)*(c - a^2*c*x^2)^p)/(a^3*(3 + 2*p)) + (x^3*(c - a^2*c*x^2)^p*Hypergeome
tric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/(3*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.18136, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6153, 6148, 764, 364, 266, 43}

$$\frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^3(2p+3)}-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^3*(1 + 2*p))) + ((1 - a^2*x^2)^(
3/2)*(c - a^2*c*x^2)^p)/(a^3*(3 + 2*p)) + (x^3*(c - a^2*c*x^2)^p*Hypergeome
tric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/(3*(1 - a^2*x^2)^p)
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{\tanh^{-1}(ax)}x^2\left(1-a^2x^2\right)^p\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^2(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^2\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx+\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x\\
&= \frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{1}{2}\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\\
&= \frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{1}{2}\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\\
&= -\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^3(1+2p)}+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^3(3+2p)}+\frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\ {}_2
\end{aligned}$$

Mathematica [A] time = 0.0837518, size = 102, normalized size = 0.77

$$\frac{1}{3}\left(c-a^2cx^2\right)^p\left(x^3\left(1-a^2x^2\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-p,\frac{5}{2},a^2x^2\right)-\frac{3\sqrt{1-a^2x^2}\left(a^2(2p+1)x^2+2\right)}{a^3\left(4p^2+8p+3\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^2*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((c - a^2*c*x^2)^p*((-3*Sqrt[1 - a^2*x^2]*(2 + a^2*(1 + 2*p)*x^2))/(a^3*(3
+ 8*p + 4*p^2)) + (x^3*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/(1 -
a^2*x^2)^p))/3
```

Maple [F] time = 0.318, size = 0, normalized size = 0.

$$\int (ax+1)\,x^2\left(-a^2cx^2+c\right)^p\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p x^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^p,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p*x^2/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x^2}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^p,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^2/(a*x - 1), x)
```

Sympy [C] time = 20.0578, size = 269, normalized size = 2.02

$$-\frac{a^{2p}c^p x^3 x^{2p} e^{i\pi p}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p+\frac{3}{2}\\ p+1,p+\frac{5}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+1\right)} - \frac{a^{2p}c^p x^3 x^{2p} e^{i\pi p}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-p-\\ \frac{1}{2},-p-\frac{1}{2}\end{matrix}\right.}{2\sqrt{\pi}\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(-a**2*c*x**2+c)**p,x)
```

```
[Out] -a**(2*p)*c**p*x**3*x**(2*p)*exp(I*pi*p)*gamma(-p - 3/2)*gamma(p + 1/2)*hyp
er((1/2, 1, p + 3/2), (p + 1, p + 5/2), a**2*x**2*exp_polar(2*I*pi))/(2*sqr
t(pi)*gamma(-p - 1/2)*gamma(p + 1)) - a**(2*p)*c**p*x**3*x**(2*p)*exp(I*pi*
p)*gamma(-p - 3/2)*gamma(p + 1/2)*hyper((1, -p, -p - 3/2), (1/2, -p - 1/2),
 1/(a**2*x**2))/(2*sqrt(pi)*gamma(-p - 1/2)*gamma(p + 1)) - c**p*meijerg(((
-p - 1, 1), (-1,)), ((-p - 3/2, -p - 1), (0,)), exp_polar(-I*pi)/(a**2*x**2
))*gamma(p + 1/2)/(2*pi*a**3) - c**p*meijerg(((-1, -p - 2, 1), ()), ((-p -
2,), (-p - 3/2, 0)), exp_polar(-I*pi)/(a**2*x**2))*gamma(p + 1/2)/(2*a**3*g
amma(-p)*gamma(p + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p x^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^p,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p*x^2/sqrt(-a^2*x^2 + 1), x)
```

3.1014 $\int e^{\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=96

$$\frac{1}{3}ax^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-p,\frac{5}{2},a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^2(2p+1)}$$

```
[Out] -((Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^2*(1 + 2*p))) + (a*x^3*(c - a^2*
c*x^2)^p*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/(3*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.109345, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6153, 6148, 764, 261, 364}

$$\frac{1}{3}ax^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x*(c - a^2*c*x^2)^p,x]
```


```
[Out] -((Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^2*(1 + 2*p))) + (a*x^3*(c - a^2*
c*x^2)^p*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/(3*(1 - a^2*x^2)^p)
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{\tanh^{-1}(ax)}x\left(1-a^2x^2\right)^p\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx+\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^2\left(\right.$$
$$=-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^2(1+2p)}+\frac{1}{3}ax^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)$$

Mathematica [A] time = 0.0343258, size = 88, normalized size = 0.92

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{1}{3}ax^3\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-p,\frac{5}{2},a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{2a^2\left(p+\frac{1}{2}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((c - a^2*c*x^2)^p*(-(1 - a^2*x^2)^(1/2 + p)/(2*a^2*(1/2 + p)) + (a*x^3*Hyp
ergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/3))/(1 - a^2*x^2)^p
```

Maple [F] time = 0.313, size = 0, normalized size = 0.

$$\int (ax+1)\,x\left(-a^2cx^2+c\right)^p\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$ac^p\int\frac{x^2e^{\left(p\log(ax+1)+p\log(-ax+1)\right)}}{\sqrt{ax+1}\sqrt{-ax+1}}\,dx+\frac{\left(a^2c^px^2-c^p\right)\left(-a^2x^2+1\right)^p}{\sqrt{-a^2x^2+1}a^2\left(2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^p,x, algorithm="maxim
a")
```

```
[Out] a*c^p*integrate(x^2*e^(p*log(a*x + 1) + p*log(-a*x + 1))/(sqrt(a*x + 1)*sqr
t(-a*x + 1)), x) + (a^2*c^p*x^2 - c^p)*(-a^2*x^2 + 1)^p/(sqrt(-a^2*x^2 + 1)
```

```
*a^2*(2*p + 1))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^p,x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x/(a*x - 1), x)
```

Sympy [C] time = 13.5348, size = 314, normalized size = 3.27

$$-\frac{aa^{2p}c^p x^3 x^{2p} e^{i\pi p}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p+\frac{3}{2}\\ p+1,p+\frac{5}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}c^p x^3 x^{2p} e^{i\pi p}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-p\\ \frac{1}{2},-p-\end{matrix}\right.}{2\sqrt{\pi}\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(-a**2*c*x**2+c)**p,x)
```

```
[Out] -a*a**(2*p)*c**p*x**3*x**(2*p)*exp(I*pi*p)*gamma(-p - 3/2)*gamma(p + 1/2)*h
yper((1/2, 1, p + 3/2), (p + 1, p + 5/2), a**2*x**2*exp_polar(2*I*pi))/(2*s
qrt(pi)*gamma(-p - 1/2)*gamma(p + 1)) - a*a**(2*p)*c**p*x**3*x**(2*p)*exp(I
*pi*p)*gamma(-p - 3/2)*gamma(p + 1/2)*hyper((1, -p, -p - 3/2), (1/2, -p - 1
/2), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(-p - 1/2)*gamma(p + 1)) - a**(2*p)*c*
*p*x**2*x**(2*p)*exp(I*pi*p)*gamma(-p - 1)*gamma(p + 1/2)*hyper((1/2, 1), (
p + 2,), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*gamma(-p)*gamma(p + 1)) -
 a**(2*p)*c**p*x**2*x**(2*p)*exp(I*pi*p)*gamma(-p - 1)*gamma(p + 1/2)*hyper
((1, -p - 1), (1/2,), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(-p)*gamma(p + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p x}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^p,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p*x/sqrt(-a^2*x^2 + 1), x)
```

3.1015 $\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=86

$$-\frac{2^{p+\frac{3}{2}}(1-ax)^{p+\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(-p-\frac{1}{2},p+\frac{1}{2},p+\frac{3}{2},\frac{1}{2}(1-ax)\right)}{a(2p+1)}$$

```
[Out] -((2^(3/2 + p)*(1 - a*x)^(1/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2
 - p, 1/2 + p, 3/2 + p, (1 - a*x)/2])/(a*(1 + 2*p)*(1 - a^2*x^2)^p))
```

Rubi [A] time = 0.0644312, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6143, 6140, 69}

$$-\frac{2^{p+\frac{3}{2}}(1-ax)^{p+\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-\frac{1}{2},p+\frac{1}{2};p+\frac{3}{2};\frac{1}{2}(1-ax)\right)}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a^2*c*x^2)^p,x]
```



```
[Out] -((2^(3/2 + p)*(1 - a*x)^(1/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2
 - p, 1/2 + p, 3/2 + p, (1 - a*x)/2])/(a*(1 + 2*p)*(1 - a^2*x^2)^p))
```


Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int (1-ax)^{-\frac{1}{2}+p}(1+ax)^{\frac{1}{2}+p}\,dx\\
&= -\frac{2^{\frac{3}{2}+p}(1-ax)^{\frac{1}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2}-p,\frac{1}{2}+p;\frac{3}{2}+p;\frac{1}{2}(1-ax)\right)}{a(1+2p)}
\end{aligned}$$

Mathematica [A] time = 0.0173702, size = 86, normalized size = 1.

$$-\frac{2^{p+\frac{1}{2}}(1-ax)^{p+\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-p-\frac{1}{2},p+\frac{1}{2},p+\frac{3}{2},\frac{1}{2}(1-ax)\right)}{a\left(p+\frac{1}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((2^(1/2 + p)*(1 - a*x)^(1/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2
 - p, 1/2 + p, 3/2 + p, (1 - a*x)/2])/(a*(1/2 + p)*(1 - a^2*x^2)^p))
```

Maple [F] time = 0.322, size = 0, normalized size = 0.

$$\int (ax+1)\left(-a^2cx^2+c\right)^p\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

Sympy [C] time = 10.8214, size = 306, normalized size = 3.56

$$-\frac{aa^{2p}c^px^2x^{2p}e^{i\pi p}\Gamma\left(-p-1\right)\Gamma\left(p+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},1\\p+2\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\Gamma\left(-p\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}c^px^2x^{2p}e^{i\pi p}\Gamma\left(-p-1\right)\Gamma\left(p+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}1,-p-1\\\frac{1}{2}\end{matrix}\middle|\frac{1}{a^2x^2}\right)}{2\sqrt{\pi}\Gamma\left(-p\right)\Gamma\left(p+1\right)}-\dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**p,x)
```

```
[Out] -a*a**(2*p)*c**p*x**2*x**(2*p)*exp(I*pi*p)*gamma(-p - 1)*gamma(p + 1/2)*hyp
er((1/2, 1), (p + 2,), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*gamma(-p)*g
amma(p + 1)) - a*a**(2*p)*c**p*x**2*x**(2*p)*exp(I*pi*p)*gamma(-p - 1)*gamm
a(p + 1/2)*hyper((1, -p - 1), (1/2,), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(-p)*
gamma(p + 1)) - a**(2*p)*c**p*x*x**(2*p)*exp(I*pi*p)*gamma(-p - 1/2)*gamma(
p + 1/2)*hyper((1/2, 1, p + 1/2), (p + 1, p + 3/2), a**2*x**2*exp_polar(2*I
*pi))/(2*sqrt(pi)*gamma(1/2 - p)*gamma(p + 1)) - a**(2*p)*c**p*x*x**(2*p)*e
xp(I*pi*p)*gamma(-p - 1/2)*gamma(p + 1/2)*hyper((1, -p, -p - 1/2), (1/2, 1/
2 - p), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(1/2 - p)*gamma(p + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/sqrt(-a^2*x^2 + 1), x)
```

3.1016 $\int \frac{e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x}\,dx$

Optimal. Leaf size=110

$$ax\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-p,\frac{3}{2},a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}}{2p+1}$$

```
[Out] (a*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2])/(1 -
a^2*x^2)^p - (Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p*Hypergeometric2F1[1, 1/2
+ p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rubi [A] time = 0.159684, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6153, 6148, 764, 266, 65, 245}

$$ax\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p {}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p {}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a^2*c*x^2)^p)/x,x]
```

```
[Out] (a*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2])/(1 -
a^2*x^2)^p - (Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p*Hypergeometric2F1[1, 1/2
+ p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x}\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x}\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx+\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \left(1-a\right.\\
&= ax\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\text{Subs}\\
&= ax\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,\frac{1}{2}+\right.}{1+2p}
\end{aligned}$$

Mathematica [A] time = 0.0292093, size = 102, normalized size = 0.93

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(ax\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-p,\frac{3}{2},a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}\text{Hypergeometric2F1}\left(1,p+\frac{1}{2},\right.}{2\left(p+\frac{1}{2}\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a^2*c*x^2)^p)/x,x]
```

```
[Out] ((c - a^2*c*x^2)^p*(a*x*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2] - ((1
 - a^2*x^2)^(1/2 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/
(2*(1/2 + p))))/(1 - a^2*x^2)^p
```

Maple [F] time = 0.315, size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{x}\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x,x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{ax^2-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x,x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x^2 - x), x)
```

Sympy [C] time = 15.6866, size = 299, normalized size = 2.72

$$-\frac{aa^{2p}c^pxx^{2p}e^{i\pi p}\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p+\frac{1}{2}\\p+1,p+\frac{3}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\Gamma\left(\frac{1}{2}-p\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}c^pxx^{2p}e^{i\pi p}\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-p-\\\frac{1}{2},\frac{1}{2}-p\end{matrix}\right.}{2\sqrt{\pi}\Gamma\left(\frac{1}{2}-p\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**p/x,x)
```

```
[Out] -a*a**(2*p)*c**p*x*x**(2*p)*exp(I*pi*p)*gamma(-p - 1/2)*gamma(p + 1/2)*hype
r((1/2, 1, p + 1/2), (p + 1, p + 3/2), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt
(pi)*gamma(1/2 - p)*gamma(p + 1)) - a*a**(2*p)*c**p*x*x**(2*p)*exp(I*pi*p)*
gamma(-p - 1/2)*gamma(p + 1/2)*hyper((1, -p, -p - 1/2), (1/2, 1/2 - p), 1/(
a**2*x**2))/(2*sqrt(pi)*gamma(1/2 - p)*gamma(p + 1)) - a**(2*p)*c**p*x**(2*
p)*exp(I*pi*p)*gamma(-p)*gamma(p + 1/2)*hyper((1/2, 1, p), (p + 1, p + 1),
a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*gamma(1 - p)*gamma(p + 1)) - a**(2
*p)*c**p*x**(2*p)*exp(I*pi*p)*gamma(-p)*gamma(p + 1/2)*hyper((1, -p, -p), (
1/2, 1 - p), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(1 - p)*gamma(p + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(sqrt(-a^2*x^2 + 1)*x), x)
```

3.1017 $\int \frac{e^{\tanh^{-1}(ax)}(c-a^2cx^2)^p}{x^2}\,dx$

Optimal. Leaf size=113

$$-\frac{(1-a^2x^2)^{-p}\,(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-p,\frac{1}{2},a^2x^2\right)}{x}-\frac{a\sqrt{1-a^2x^2}\,(c-a^2cx^2)^p\,\text{Hypergeometric2F}}{2p+1}$$

```
[Out] -(((c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2])/(x*(1
- a^2*x^2)^p)) - (a*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p*Hypergeometric2F1[1
, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rubi [A] time = 0.166599, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6153, 6148, 764, 364, 266, 65}

$$-\frac{(1-a^2x^2)^{-p}\,(c-a^2cx^2)^p\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}-\frac{a\sqrt{1-a^2x^2}\,(c-a^2cx^2)^p\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a^2*c*x^2)^p)/x^2,x]
```

```
[Out] -(((c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2])/(x*(1
- a^2*x^2)^p)) - (a*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p*Hypergeometric2F1[1
, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x^2}\,dx = \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^2}\,dx$$
$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx$$
$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx + \left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1-a$$
$$= -\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p {}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x} + \frac{1}{2}\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\text{Sub}$$
$$= -\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p {}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x} - \frac{a\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p {}_2F_1\left(1,\frac{1}{2}+\right.}{1+2p}$$

Mathematica [A] time = 0.0361945, size = 105, normalized size = 0.93

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(-\frac{a\left(1-a^2x^2\right)^{p+\frac{1}{2}}\text{Hypergeometric2F1}\left(1,p+\frac{1}{2},p+\frac{3}{2},1-a^2x^2\right)}{2\left(p+\frac{1}{2}\right)}-\frac{\text{Hypergeometric2F1}\left(-\right.}{x}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a^2*c*x^2)^p)/x^2,x]
```

```
[Out] ((c - a^2*c*x^2)^p*(-(Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2]/x) - (
a*(1 - a^2*x^2)^(1/2 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^
2])/(2*(1/2 + p))))/(1 - a^2*x^2)^p
```

Maple [F] time = 0.328, size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{x^2}\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^2,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^2,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{ax^3-x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^2,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x^3 - x^2), x)
```

Sympy [C] time = 16.23, size = 294, normalized size = 2.6

$$-\frac{aa^{2p}c^px^{2p}e^{i\pi p}\Gamma\left(-p\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p\\p+1,p+1\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\Gamma\left(1-p\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}c^px^{2p}e^{i\pi p}\Gamma\left(-p\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-p\\\frac{1}{2},1-p\end{matrix}\middle|\frac{1}{a^2x^2}\right)}{2\sqrt{\pi}\Gamma\left(1-p\right)\Gamma\left(p+1\right)}-\frac{a^{2p}c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**p/x**2,x)
```

```
[Out] -a*a**(2*p)*c**p*x**(2*p)*exp(I*pi*p)*gamma(-p)*gamma(p + 1/2)*hyper((1/2,
1, p), (p + 1, p + 1), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*gamma(1 - p
)*gamma(p + 1)) - a*a**(2*p)*c**p*x**(2*p)*exp(I*pi*p)*gamma(-p)*gamma(p +
1/2)*hyper((1, -p, -p), (1/2, 1 - p), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(1 -
p)*gamma(p + 1)) - a**(2*p)*c**p*x**(2*p)*exp(I*pi*p)*gamma(1/2 - p)*gamma(
p + 1/2)*hyper((1/2, 1, p - 1/2), (p + 1/2, p + 1), a**2*x**2*exp_polar(2*I
*pi))/(2*sqrt(pi)*x*gamma(3/2 - p)*gamma(p + 1)) - a**(2*p)*c**p*x**(2*p)*e
xp(I*pi*p)*gamma(1/2 - p)*gamma(p + 1/2)*hyper((1, -p, 1/2 - p), (1/2, 3/2
- p), 1/(a**2*x**2))/(2*sqrt(pi)*x*gamma(3/2 - p)*gamma(p + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

3.1018 $\int \frac{e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x^3}\,dx$

Optimal. Leaf size=116

$$-\frac{a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-p,\frac{1}{2},a^2x^2\right)}{x}-\frac{a^2\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p \text{Hypergeometric}}{2p+1}$$

```
[Out] -((a*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2])/(x*(
1 - a^2*x^2)^p)) - (a^2*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p*Hypergeometric2
F1[2, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rubi [A] time = 0.160676, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6153, 6148, 764, 266, 65, 364}

$$-\frac{a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p {}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}-\frac{a^2\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p {}_2F_1\left(2,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a^2*c*x^2)^p)/x^3,x]
```

```
[Out] -((a*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2])/(x*(
1 - a^2*x^2)^p)) - (a^2*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p*Hypergeometric2
F1[2, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x^3}\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^3}\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^3}\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^3}\,dx+\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1-a
\\
&= -\frac{a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\text{Sub}
\\
&= -\frac{a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}-\frac{a^2\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(2,\frac{1}{2}\right.}{1+2p}
\end{aligned}$$

Mathematica [A] time = 0.0217302, size = 108, normalized size = 0.93

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(-\frac{a^2\left(1-a^2x^2\right)^{p+\frac{1}{2}}\text{Hypergeometric2F1}\left(2,p+\frac{1}{2},p+\frac{3}{2},1-a^2x^2\right)}{2\left(p+\frac{1}{2}\right)}-\frac{a\text{Hypergeometric2F1}}{x}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a^2*c*x^2)^p)/x^3,x]
```

```
[Out] ((c - a^2*c*x^2)^p*(-((a*Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2])/x)
 - (a^2*(1 - a^2*x^2)^(1/2 + p)*Hypergeometric2F1[2, 1/2 + p, 3/2 + p, 1 -
a^2*x^2])/(2*(1/2 + p))))/(1 - a^2*x^2)^p
```

Maple [F] time = 0.319, size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{x^3}\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^3,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^3,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{ax^4-x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^3,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x^4 - x^3), x)
```

Sympy [C] time = 24.6242, size = 301, normalized size = 2.59

$$-\frac{aa^{2p}c^px^{2p}e^{i\pi p}\Gamma\left(\frac{1}{2}-p\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2}, 1, p-\frac{1}{2}\\ p+\frac{1}{2}, p+1\end{matrix}\middle| a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}x\Gamma\left(\frac{3}{2}-p\right)\Gamma\left(p+1\right)} - \frac{aa^{2p}c^px^{2p}e^{i\pi p}\Gamma\left(\frac{1}{2}-p\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1, -p, \frac{1}{2}-p\\ \frac{1}{2}, \frac{3}{2}-p\end{matrix}\middle| \frac{1}{a^2x^2}\right)}{2\sqrt{\pi}x\Gamma\left(\frac{3}{2}-p\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**p/x**3,x)
```

```
[Out] -a*a**(2*p)*c**p*x**(2*p)*exp(I*pi*p)*gamma(1/2 - p)*gamma(p + 1/2)*hyper((
1/2, 1, p - 1/2), (p + 1/2, p + 1), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi
)*x*gamma(3/2 - p)*gamma(p + 1)) - a*a**(2*p)*c**p*x**(2*p)*exp(I*pi*p)*gam
ma(1/2 - p)*gamma(p + 1/2)*hyper((1, -p, 1/2 - p), (1/2, 3/2 - p), 1/(a**2*
x**2))/(2*sqrt(pi)*x*gamma(3/2 - p)*gamma(p + 1)) - a**(2*p)*c**p*x**(2*p)*
exp(I*pi*p)*gamma(1 - p)*gamma(p + 1/2)*hyper((1/2, 1, p - 1), (p, p + 1),
a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*x**2*gamma(2 - p)*gamma(p + 1)) -
a**(2*p)*c**p*x**(2*p)*exp(I*pi*p)*gamma(1 - p)*gamma(p + 1/2)*hyper((1, -p
, 1 - p), (1/2, 2 - p), 1/(a**2*x**2))/(2*sqrt(pi)*x**2*gamma(2 - p)*gamma(
p + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^3,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

3.1019 $\int e^{2\tanh^{-1}(ax)}x^4\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=29

$$\frac{1}{7}a^2cx^7+\frac{1}{3}acx^6+\frac{cx^5}{5}$$

```
[Out] (c*x^5)/5 + (a*c*x^6)/3 + (a^2*c*x^7)/7
```

Rubi [A] time = 0.0546501, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 43}

$$\frac{1}{7}a^2cx^7+\frac{1}{3}acx^6+\frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^4*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^5)/5 + (a*c*x^6)/3 + (a^2*c*x^7)/7
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^4\left(c-a^2cx^2\right)\,dx&=c\int x^4(1+ax)^2\,dx\\&=c\int\left(x^4+2ax^5+a^2x^6\right)\,dx\\&=\frac{cx^5}{5}+\frac{1}{3}acx^6+\frac{1}{7}a^2cx^7\end{aligned}$$

Mathematica [A] time = 0.0167055, size = 22, normalized size = 0.76

$$\frac{1}{105}cx^5\left(15a^2x^2+35ax+21\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^4*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^5*(21 + 35*a*x + 15*a^2*x^2))/105
```

Maple [A] time = 0.026, size = 23, normalized size = 0.8

$$c\left(\frac{a^2x^7}{7}+\frac{x^6a}{3}+\frac{x^5}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c),x)
```

```
[Out] c*(1/7*a^2*x^7+1/3*x^6*a+1/5*x^5)
```

Maxima [A] time = 0.946904, size = 31, normalized size = 1.07

$$\frac{1}{7}\,a^2cx^7+\frac{1}{3}\,acx^6+\frac{1}{5}\,cx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/7*a^2*c*x^7 + 1/3*a*c*x^6 + 1/5*c*x^5
```

Fricas [A] time = 1.86128, size = 55, normalized size = 1.9

$$\frac{1}{7}\,a^2cx^7+\frac{1}{3}\,acx^6+\frac{1}{5}\,cx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/7*a^2*c*x^7 + 1/3*a*c*x^6 + 1/5*c*x^5
```

Sympy [A] time = 0.077603, size = 24, normalized size = 0.83

$$\frac{a^2cx^7}{7}+\frac{acx^6}{3}+\frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**4*(-a**2*c*x**2+c),x)
```

```
[Out] a**2*c*x**7/7 + a*c*x**6/3 + c*x**5/5
```

Giac [A] time = 1.18084, size = 31, normalized size = 1.07

$$\frac{1}{7}\,a^2cx^7+\frac{1}{3}\,acx^6+\frac{1}{5}\,cx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/7*a^2*c*x^7 + 1/3*a*c*x^6 + 1/5*c*x^5
```

3.1020 $\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=29

$$\frac{1}{6}a^2cx^6+\frac{2}{5}acx^5+\frac{cx^4}{4}$$

```
[Out] (c*x^4)/4 + (2*a*c*x^5)/5 + (a^2*c*x^6)/6
```

Rubi [A] time = 0.0546217, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 43}

$$\frac{1}{6}a^2cx^6+\frac{2}{5}acx^5+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^4)/4 + (2*a*c*x^5)/5 + (a^2*c*x^6)/6
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)\,dx&=c\int x^3(1+ax)^2\,dx\\&=c\int\left(x^3+2ax^4+a^2x^5\right)\,dx\\&=\frac{cx^4}{4}+\frac{2}{5}acx^5+\frac{1}{6}a^2cx^6\end{aligned}$$

Mathematica [A] time = 0.0165648, size = 22, normalized size = 0.76

$$\frac{1}{60}cx^4\left(10a^2x^2+24ax+15\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^4*(15 + 24*a*x + 10*a^2*x^2))/60
```

Maple [A] time = 0.026, size = 23, normalized size = 0.8

$$c\left(\frac{x^6a^2}{6}+\frac{2\,ax^5}{5}+\frac{x^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c),x)
```

```
[Out] c*(1/6*x^6*a^2+2/5*a*x^5+1/4*x^4)
```

Maxima [A] time = 0.934618, size = 31, normalized size = 1.07

$$\frac{1}{6}\,a^2cx^6+\frac{2}{5}\,acx^5+\frac{1}{4}\,cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/6*a^2*c*x^6 + 2/5*a*c*x^5 + 1/4*c*x^4
```

Fricas [A] time = 1.92134, size = 55, normalized size = 1.9

$$\frac{1}{6}\,a^2cx^6+\frac{2}{5}\,acx^5+\frac{1}{4}\,cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/6*a^2*c*x^6 + 2/5*a*c*x^5 + 1/4*c*x^4
```

Sympy [A] time = 0.077675, size = 26, normalized size = 0.9

$$\frac{a^2cx^6}{6}+\frac{2acx^5}{5}+\frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(-a**2*c*x**2+c),x)
```

```
[Out] a**2*c*x**6/6 + 2*a*c*x**5/5 + c*x**4/4
```

Giac [A] time = 1.16305, size = 31, normalized size = 1.07

$$\frac{1}{6}\,a^2cx^6+\frac{2}{5}\,acx^5+\frac{1}{4}\,cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/6*a^2*c*x^6 + 2/5*a*c*x^5 + 1/4*c*x^4
```

3.1021 $\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)dx$

Optimal. Leaf size=29

$$\frac{1}{5}a^2cx^5+\frac{1}{2}acx^4+\frac{cx^3}{3}$$

```
[Out] (c*x^3)/3 + (a*c*x^4)/2 + (a^2*c*x^5)/5
```

Rubi [A] time = 0.0534224, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 43}

$$\frac{1}{5}a^2cx^5+\frac{1}{2}acx^4+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^3)/3 + (a*c*x^4)/2 + (a^2*c*x^5)/5
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)dx&=c\int x^2(1+ax)^2\,dx\\&=c\int\left(x^2+2ax^3+a^2x^4\right)dx\\&=\frac{cx^3}{3}+\frac{1}{2}acx^4+\frac{1}{5}a^2cx^5\end{aligned}$$

Mathematica [A] time = 0.014817, size = 22, normalized size = 0.76

$$\frac{1}{30}cx^3\left(6a^2x^2+15ax+10\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^3*(10 + 15*a*x + 6*a^2*x^2))/30
```

Maple [A] time = 0.024, size = 23, normalized size = 0.8

$$c\left(\frac{x^5a^2}{5}+\frac{x^4a}{2}+\frac{x^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c),x)
```

```
[Out] c*(1/5*x^5*a^2+1/2*x^4*a+1/3*x^3)
```

Maxima [A] time = 0.939426, size = 31, normalized size = 1.07

$$\frac{1}{5}\,a^2cx^5+\frac{1}{2}\,acx^4+\frac{1}{3}\,cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/5*a^2*c*x^5 + 1/2*a*c*x^4 + 1/3*c*x^3
```

Fricas [A] time = 1.94123, size = 55, normalized size = 1.9

$$\frac{1}{5}\,a^2cx^5+\frac{1}{2}\,acx^4+\frac{1}{3}\,cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/5*a^2*c*x^5 + 1/2*a*c*x^4 + 1/3*c*x^3
```

Sympy [A] time = 0.077207, size = 24, normalized size = 0.83

$$\frac{a^2cx^5}{5}+\frac{acx^4}{2}+\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(-a**2*c*x**2+c),x)
```

```
[Out] a**2*c*x**5/5 + a*c*x**4/2 + c*x**3/3
```

Giac [A] time = 1.12715, size = 31, normalized size = 1.07

$$\frac{1}{5}\,a^2cx^5+\frac{1}{2}\,acx^4+\frac{1}{3}\,cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/5*a^2*c*x^5 + 1/2*a*c*x^4 + 1/3*c*x^3
```

3.1022 $\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=29

$$\frac{1}{4}a^2cx^4+\frac{2}{3}acx^3+\frac{cx^2}{2}$$

[Out] (c*x^2)/2 + (2*a*c*x^3)/3 + (a^2*c*x^4)/4

Rubi [A] time = 0.040211, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6150, 43}

$$\frac{1}{4}a^2cx^4+\frac{2}{3}acx^3+\frac{cx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2),x]

[Out] (c*x^2)/2 + (2*a*c*x^3)/3 + (a^2*c*x^4)/4

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)\,dx&=c\int x(1+ax)^2\,dx\\&=c\int\left(x+2ax^2+a^2x^3\right)\,dx\\&=\frac{cx^2}{2}+\frac{2}{3}acx^3+\frac{1}{4}a^2cx^4\end{aligned}$$

Mathematica [A] time = 0.014375, size = 22, normalized size = 0.76

$$\frac{1}{12}cx^2\left(3a^2x^2+8ax+6\right)$$

Antiderivative was successfully verified.

[In] Integrate[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2),x]

```
[Out] (c*x^2*(6 + 8*a*x + 3*a^2*x^2))/12
```

Maple [A] time = 0.025, size = 23, normalized size = 0.8

$$c\left(\frac{a^2x^4}{4}+\frac{2x^3a}{3}+\frac{x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c),x)
```

```
[Out] c*(1/4*a^2*x^4+2/3*x^3*a+1/2*x^2)
```

Maxima [A] time = 0.954782, size = 31, normalized size = 1.07

$$\frac{1}{4}a^2cx^4+\frac{2}{3}acx^3+\frac{1}{2}cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c*x^4 + 2/3*a*c*x^3 + 1/2*c*x^2
```

Fricas [A] time = 1.91731, size = 55, normalized size = 1.9

$$\frac{1}{4}a^2cx^4+\frac{2}{3}acx^3+\frac{1}{2}cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/4*a^2*c*x^4 + 2/3*a*c*x^3 + 1/2*c*x^2
```

Sympy [A] time = 0.076318, size = 26, normalized size = 0.9

$$\frac{a^2cx^4}{4}+\frac{2acx^3}{3}+\frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(-a**2*c*x**2+c),x)
```

```
[Out] a**2*c*x**4/4 + 2*a*c*x**3/3 + c*x**2/2
```

Giac [A] time = 1.13828, size = 31, normalized size = 1.07

$$\frac{1}{4}a^2cx^4+\frac{2}{3}acx^3+\frac{1}{2}cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/4*a^2*c*x^4 + 2/3*a*c*x^3 + 1/2*c*x^2
```

3.1023 $\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=15

$$\frac{c(ax+1)^3}{3a}$$

```
[Out] (c*(1 + a*x)^3)/(3*a)
```

Rubi [A] time = 0.0190191, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6140, 32}

$$\frac{c(ax+1)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] (c*(1 + a*x)^3)/(3*a)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx &= c\int(1+ax)^2\,dx\\ &= \frac{c(1+ax)^3}{3a}\end{aligned}$$

Mathematica [A] time = 0.0119897, size = 19, normalized size = 1.27

$$c\left(\frac{a^2x^3}{3}+ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] c*(x + a*x^2 + (a^2*x^3)/3)
```

Maple [A] time = 0.025, size = 14, normalized size = 0.9

$$\frac{c\,(ax+1)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c),x)
```

```
[Out] 1/3*c*(a*x+1)^3/a
```

Maxima [A] time = 0.941747, size = 26, normalized size = 1.73

$$\frac{1}{3}\,a^2cx^3 + acx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c*x^3 + a*c*x^2 + c*x
```

Fricas [A] time = 1.83929, size = 42, normalized size = 2.8

$$\frac{1}{3}\,a^2cx^3 + acx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/3*a^2*c*x^3 + a*c*x^2 + c*x
```

Sympy [A] time = 0.076215, size = 19, normalized size = 1.27

$$\frac{a^2cx^3}{3} + acx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c),x)
```

```
[Out] a**2*c*x**3/3 + a*c*x**2 + c*x
```

Giac [A] time = 1.14134, size = 26, normalized size = 1.73

$$\frac{1}{3}\,a^2cx^3 + acx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c),x, algorithm="giac")

[Out] 1/3*a^2*c*x^3 + a*c*x^2 + c*x
```

3.1024 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)}{x} dx$

Optimal. Leaf size=21

$$\frac{1}{2}a^2cx^2 + 2acx + c\log(x)$$

```
[Out] 2*a*c*x + (a^2*c*x^2)/2 + c*Log[x]
```

Rubi [A] time = 0.0493203, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 43}

$$\frac{1}{2}a^2cx^2 + 2acx + c\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2))/x,x]
```

```
[Out] 2*a*c*x + (a^2*c*x^2)/2 + c*Log[x]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x} dx &= c\int \frac{(1+ax)^2}{x} dx \\
&= c\int \left(2a + \frac{1}{x} + a^2x\right) dx \\
&= 2acx + \frac{1}{2}a^2cx^2 + c\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0147296, size = 19, normalized size = 0.9

$$c\left(\frac{a^2x^2}{2} + 2ax + \log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2))/x,x]
```

```
[Out] c*(2*a*x + (a^2*x^2)/2 + Log[x])
```

Maple [A] time = 0.026, size = 20, normalized size = 1.

$$2\,acx+\frac{a^{2}c x^{2}}{2}+c\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x,x)
```

```
[Out] 2*a*c*x+1/2*a^2*c*x^2+c*ln(x)
```

Maxima [A] time = 0.944807, size = 26, normalized size = 1.24

$$\frac{1}{2}\,a^{2} c x^{2}+2\,acx+c\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c*x^2 + 2*a*c*x + c*log(x)
```

Fricas [A] time = 2.01512, size = 49, normalized size = 2.33

$$\frac{1}{2}\,a^{2} c x^{2}+2\,acx+c\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*a^2*c*x^2 + 2*a*c*x + c*log(x)
```

Sympy [A] time = 0.10553, size = 20, normalized size = 0.95

$$\frac{a^{2} c x^{2}}{2}+2acx+c\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)/x,x)
```

```
[Out] a**2*c*x**2/2 + 2*a*c*x + c*log(x)
```

Giac [A] time = 1.16146, size = 27, normalized size = 1.29

$$\frac{1}{2}\,a^{2} c x^{2}+2\,acx+c\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*a^2*c*x^2 + 2*a*c*x + c*log(abs(x))
```

3.1025 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)}{x^2}\,dx$

Optimal. Leaf size=19

$$a^2cx + 2ac\log(x) - \frac{c}{x}$$

```
[Out] -(c/x) + a^2*c*x + 2*a*c*Log[x]
```

Rubi [A] time = 0.0494398, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 43}

$$a^2cx + 2ac\log(x) - \frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2))/x^2,x]
```

```
[Out] -(c/x) + a^2*c*x + 2*a*c*Log[x]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^2}\,dx &= c\int \frac{(1+ax)^2}{x^2}\,dx\\
&= c\int \left(a^2+\frac{1}{x^2}+\frac{2a}{x}\right)dx\\
&= -\frac{c}{x}+a^2cx+2ac\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0125687, size = 18, normalized size = 0.95

$$c\left(a^2x + 2a\log(x) - \frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2))/x^2,x]
```

```
[Out] c*(-x^(-1) + a^2*x + 2*a*Log[x])
```

Maple [A] time = 0.032, size = 20, normalized size = 1.1

$$-\frac{c}{x}+cxa^{2}+2\,ac\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^2,x)
```

```
[Out] -c/x+c*x*a^2+2*a*c*ln(x)
```

Maxima [A] time = 0.94102, size = 26, normalized size = 1.37

$$a^{2}cx+2\,ac\log\left(x\right)-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^2,x, algorithm="maxima")
```

```
[Out] a^2*c*x + 2*a*c*log(x) - c/x
```

Fricas [A] time = 1.71466, size = 49, normalized size = 2.58

$$\frac{a^{2}cx^{2}+2\,acx\log\left(x\right)-c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^2,x, algorithm="fricas")
```

```
[Out] (a^2*c*x^2 + 2*a*c*x*log(x) - c)/x
```

Sympy [A] time = 0.277468, size = 17, normalized size = 0.89

$$a^{2}cx+2ac\log\left(x\right)-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)/x**2,x)
```

```
[Out] a**2*c*x + 2*a*c*log(x) - c/x
```

Giac [A] time = 1.15575, size = 27, normalized size = 1.42

$$a^{2}cx+2\,ac\log\left(|x|\right)-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^2,x, algorithm="giac")
```

```
[Out] a^2*c*x + 2*a*c*log(abs(x)) - c/x
```

3.1026 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)}{x^3}\,dx$

Optimal. Leaf size=23

$$a^2c\log(x)-\frac{2ac}{x}-\frac{c}{2x^2}$$

```
[Out] -c/(2*x^2) - (2*a*c)/x + a^2*c*Log[x]
```

Rubi [A] time = 0.0521579, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 43}

$$a^2c\log(x)-\frac{2ac}{x}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2))/x^3,x]
```

```
[Out] -c/(2*x^2) - (2*a*c)/x + a^2*c*Log[x]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^3}\,dx &= c\int \frac{(1+ax)^2}{x^3}\,dx\\ &= c\int \left(\frac{1}{x^3}+\frac{2a}{x^2}+\frac{a^2}{x}\right)dx\\ &= -\frac{c}{2x^2}-\frac{2ac}{x}+a^2c\log(x)\end{aligned}$$

Mathematica [A] time = 0.0166373, size = 22, normalized size = 0.96

$$c\left(a^2\log(x)-\frac{2a}{x}-\frac{1}{2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2))/x^3,x]
```

```
[Out] c*(-1/(2*x^2) - (2*a)/x + a^2*Log[x])
```

Maple [A] time = 0.032, size = 22, normalized size = 1.

$$-\frac{c}{2x^2} - 2\frac{ac}{x} + a^2c\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^3,x)
```

```
[Out] -1/2*c/x^2-2*a*c/x+a^2*c*ln(x)
```

Maxima [A] time = 0.952203, size = 27, normalized size = 1.17

$$a^2c\log(x) - \frac{4\,acx + c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^3,x, algorithm="maxima")
```

```
[Out] a^2*c*log(x) - 1/2*(4*a*c*x + c)/x^2
```

Fricas [A] time = 1.71111, size = 59, normalized size = 2.57

$$\frac{2\,a^2cx^2\log(x) - 4\,acx - c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*c*x^2*log(x) - 4*a*c*x - c)/x^2
```

Sympy [A] time = 0.321967, size = 20, normalized size = 0.87

$$a^2c\log(x) - \frac{4acx + c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)/x**3,x)
```

```
[Out] a**2*c*log(x) - (4*a*c*x + c)/(2*x**2)
```

Giac [A] time = 1.13781, size = 28, normalized size = 1.22

$$a^2c\log(|x|) - \frac{4\,acx + c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^3,x, algorithm="giac")
```

```
[Out] a^2*c*log(abs(x)) - 1/2*(4*a*c*x + c)/x^2
```

3.1027 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)}{x^4}\,dx$

Optimal. Leaf size=15

$$-\frac{c(ax+1)^3}{3x^3}$$

```
[Out] -(c*(1 + a*x)^3)/(3*x^3)
```

Rubi [A] time = 0.0448639, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 37}

$$-\frac{c(ax+1)^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2))/x^4,x]
```

```
[Out] -(c*(1 + a*x)^3)/(3*x^3)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^4}\,dx &= c\int \frac{(1+ax)^2}{x^4}\,dx\\ &= -\frac{c(1+ax)^3}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.0087305, size = 15, normalized size = 1.

$$-\frac{c(ax+1)^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2))/x^4,x]
```

```
[Out] -(c*(1 + a*x)^3)/(3*x^3)
```

Maple [A] time = 0.03, size = 23, normalized size = 1.5

$$c\left(-\frac{a^2}{x}-\frac{a}{x^2}-\frac{1}{3\,x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^4,x)
```

```
[Out] c*(-a^2/x-a/x^2-1/3/x^3)
```

Maxima [A] time = 0.940828, size = 28, normalized size = 1.87

$$-\frac{3\,a^2cx^2+3\,acx+c}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a^2*c*x^2 + 3*a*c*x + c)/x^3
```

Fricas [A] time = 1.66385, size = 51, normalized size = 3.4

$$-\frac{3\,a^2cx^2+3\,acx+c}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*c*x^2 + 3*a*c*x + c)/x^3
```

Sympy [A] time = 0.330752, size = 24, normalized size = 1.6

$$-\frac{3a^2cx^2+3acx+c}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)/x**4,x)
```

```
[Out] -(3*a**2*c*x**2 + 3*a*c*x + c)/(3*x**3)
```

Giac [A] time = 1.13783, size = 28, normalized size = 1.87

$$-\frac{3\,a^{2}cx^{2}+3\,acx+c}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*a^2*c*x^2 + 3*a*c*x + c)/x^3
```

3.1028 $\int e^{2\tanh^{-1}(ax)}x^4\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=48

$$-\frac{1}{9}a^4c^2x^9-\frac{1}{4}a^3c^2x^8+\frac{1}{3}ac^2x^6+\frac{c^2x^5}{5}$$

```
[Out] (c^2*x^5)/5 + (a*c^2*x^6)/3 - (a^3*c^2*x^8)/4 - (a^4*c^2*x^9)/9
```

Rubi [A] time = 0.0881659, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 75}

$$-\frac{1}{9}a^4c^2x^9-\frac{1}{4}a^3c^2x^8+\frac{1}{3}ac^2x^6+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^4*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*x^5)/5 + (a*c^2*x^6)/3 - (a^3*c^2*x^8)/4 - (a^4*c^2*x^9)/9
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^4\left(c-a^2cx^2\right)^2\,dx&=c^2\int x^4(1-ax)(1+ax)^3\,dx\\&=c^2\int\left(x^4+2ax^5-2a^3x^7-a^4x^8\right)\,dx\\&=\frac{c^2x^5}{5}+\frac{1}{3}ac^2x^6-\frac{1}{4}a^3c^2x^8-\frac{1}{9}a^4c^2x^9\end{aligned}$$

Mathematica [A] time = 0.024491, size = 32, normalized size = 0.67

$$-\frac{1}{180}c^2x^5\left(20a^4x^4+45a^3x^3-60ax-36\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^4*(c - a^2*c*x^2)^2,x]
```

```
[Out] -(c^2*x^5*(-36 - 60*a*x + 45*a^3*x^3 + 20*a^4*x^4))/180
```

Maple [A] time = 0.024, size = 33, normalized size = 0.7

$$c^2\left(-\frac{a^4x^9}{9}-\frac{a^3x^8}{4}+\frac{x^6a}{3}+\frac{x^5}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c)^2,x)
```

```
[Out] c^2*(-1/9*a^4*x^9-1/4*a^3*x^8+1/3*x^6*a+1/5*x^5)
```

Maxima [A] time = 0.940892, size = 54, normalized size = 1.12

$$-\frac{1}{9}a^4c^2x^9-\frac{1}{4}a^3c^2x^8+\frac{1}{3}ac^2x^6+\frac{1}{5}c^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] -1/9*a^4*c^2*x^9 - 1/4*a^3*c^2*x^8 + 1/3*a*c^2*x^6 + 1/5*c^2*x^5
```

Fricas [A] time = 1.66844, size = 89, normalized size = 1.85

$$-\frac{1}{9}a^4c^2x^9-\frac{1}{4}a^3c^2x^8+\frac{1}{3}ac^2x^6+\frac{1}{5}c^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/9*a^4*c^2*x^9 - 1/4*a^3*c^2*x^8 + 1/3*a*c^2*x^6 + 1/5*c^2*x^5
```

Sympy [A] time = 0.08837, size = 41, normalized size = 0.85

$$-\frac{a^4c^2x^9}{9}-\frac{a^3c^2x^8}{4}+\frac{ac^2x^6}{3}+\frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**4*(-a**2*c*x**2+c)**2,x)
```

```
[Out] -a**4*c**2*x**9/9 - a**3*c**2*x**8/4 + a*c**2*x**6/3 + c**2*x**5/5
```

Giac [A] time = 1.14486, size = 54, normalized size = 1.12

$$-\frac{1}{9}a^4c^2x^9 - \frac{1}{4}a^3c^2x^8 + \frac{1}{3}ac^2x^6 + \frac{1}{5}c^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/9*a^4*c^2*x^9 - 1/4*a^3*c^2*x^8 + 1/3*a*c^2*x^6 + 1/5*c^2*x^5
```

3.1029 $\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=48

$$-\frac{1}{8}a^4c^2x^8-\frac{2}{7}a^3c^2x^7+\frac{2}{5}ac^2x^5+\frac{c^2x^4}{4}$$

```
[Out] (c^2*x^4)/4 + (2*a*c^2*x^5)/5 - (2*a^3*c^2*x^7)/7 - (a^4*c^2*x^8)/8
```

Rubi [A] time = 0.0892372, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 75}

$$-\frac{1}{8}a^4c^2x^8-\frac{2}{7}a^3c^2x^7+\frac{2}{5}ac^2x^5+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*x^4)/4 + (2*a*c^2*x^5)/5 - (2*a^3*c^2*x^7)/7 - (a^4*c^2*x^8)/8
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^2\,dx&=c^2\int x^3(1-ax)(1+ax)^3\,dx\\&=c^2\int\left(x^3+2ax^4-2a^3x^6-a^4x^7\right)dx\\&=\frac{c^2x^4}{4}+\frac{2}{5}ac^2x^5-\frac{2}{7}a^3c^2x^7-\frac{1}{8}a^4c^2x^8\end{aligned}$$

Mathematica [A] time = 0.0222726, size = 40, normalized size = 0.83

$$c^2\left(-\frac{1}{8}a^4x^8-\frac{2a^3x^7}{7}+\frac{2ax^5}{5}+\frac{x^4}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^2,x]
```

```
[Out] c^2*(x^4/4 + (2*a*x^5)/5 - (2*a^3*x^7)/7 - (a^4*x^8)/8)
```

Maple [A] time = 0.023, size = 33, normalized size = 0.7

$$c^{2}\left(-\frac{a^{4}x^{8}}{8}-\frac{2\,x^{7}a^{3}}{7}+\frac{2\,a x^{5}}{5}+\frac{x^{4}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^2,x)
```

```
[Out] c^2*(-1/8*a^4*x^8-2/7*x^7*a^3+2/5*a*x^5+1/4*x^4)
```

Maxima [A] time = 0.944045, size = 54, normalized size = 1.12

$$-\frac{1}{8}\,a^{4}c^{2}x^{8}-\frac{2}{7}\,a^{3}c^{2}x^{7}+\frac{2}{5}\,a c^{2}x^{5}+\frac{1}{4}\,c^{2}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] -1/8*a^4*c^2*x^8 - 2/7*a^3*c^2*x^7 + 2/5*a*c^2*x^5 + 1/4*c^2*x^4
```

Fricas [A] time = 1.63187, size = 89, normalized size = 1.85

$$-\frac{1}{8}\,a^{4}c^{2}x^{8}-\frac{2}{7}\,a^{3}c^{2}x^{7}+\frac{2}{5}\,a c^{2}x^{5}+\frac{1}{4}\,c^{2}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/8*a^4*c^2*x^8 - 2/7*a^3*c^2*x^7 + 2/5*a*c^2*x^5 + 1/4*c^2*x^4
```

Sympy [A] time = 0.088677, size = 44, normalized size = 0.92

$$-\frac{a^{4}c^{2}x^{8}}{8}-\frac{2a^{3}c^{2}x^{7}}{7}+\frac{2ac^{2}x^{5}}{5}+\frac{c^{2}x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(-a**2*c*x**2+c)**2,x)
```

```
[Out] -a**4*c**2*x**8/8 - 2*a**3*c**2*x**7/7 + 2*a*c**2*x**5/5 + c**2*x**4/4
```

Giac [A] time = 1.1107, size = 54, normalized size = 1.12

$$-\frac{1}{8}a^4c^2x^8 - \frac{2}{7}a^3c^2x^7 + \frac{2}{5}ac^2x^5 + \frac{1}{4}c^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*a^4*c^2*x^8 - 2/7*a^3*c^2*x^7 + 2/5*a*c^2*x^5 + 1/4*c^2*x^4
```

3.1030 $\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=48

$$-\frac{1}{7}a^4c^2x^7-\frac{1}{3}a^3c^2x^6+\frac{1}{2}ac^2x^4+\frac{c^2x^3}{3}$$

```
[Out] (c^2*x^3)/3 + (a*c^2*x^4)/2 - (a^3*c^2*x^6)/3 - (a^4*c^2*x^7)/7
```

Rubi [A] time = 0.0896065, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 75}

$$-\frac{1}{7}a^4c^2x^7-\frac{1}{3}a^3c^2x^6+\frac{1}{2}ac^2x^4+\frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*x^3)/3 + (a*c^2*x^4)/2 - (a^3*c^2*x^6)/3 - (a^4*c^2*x^7)/7
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^2\,dx &= c^2\int x^2(1-ax)(1+ax)^3\,dx\\ &= c^2\int\left(x^2+2ax^3-2a^3x^5-a^4x^6\right)dx\\ &= \frac{c^2x^3}{3}+\frac{1}{2}ac^2x^4-\frac{1}{3}a^3c^2x^6-\frac{1}{7}a^4c^2x^7\end{aligned}$$

Mathematica [A] time = 0.0210124, size = 32, normalized size = 0.67

$$-\frac{1}{42}c^2x^3\left(6a^4x^4+14a^3x^3-21ax-14\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^2,x]
```

```
[Out] -(c^2*x^3*(-14 - 21*a*x + 14*a^3*x^3 + 6*a^4*x^4))/42
```

Maple [A] time = 0.025, size = 33, normalized size = 0.7

$$c^{2}\left(-\frac{x^{7}a^{4}}{7}-\frac{x^{6}a^{3}}{3}+\frac{x^{4}a}{2}+\frac{x^{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^2,x)
```

```
[Out] c^2*(-1/7*x^7*a^4-1/3*x^6*a^3+1/2*x^4*a+1/3*x^3)
```

Maxima [A] time = 0.944307, size = 54, normalized size = 1.12

$$-\frac{1}{7}a^{4}c^{2}x^{7}-\frac{1}{3}a^{3}c^{2}x^{6}+\frac{1}{2}ac^{2}x^{4}+\frac{1}{3}c^{2}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] -1/7*a^4*c^2*x^7 - 1/3*a^3*c^2*x^6 + 1/2*a*c^2*x^4 + 1/3*c^2*x^3
```

Fricas [A] time = 1.68789, size = 89, normalized size = 1.85

$$-\frac{1}{7}a^{4}c^{2}x^{7}-\frac{1}{3}a^{3}c^{2}x^{6}+\frac{1}{2}ac^{2}x^{4}+\frac{1}{3}c^{2}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/7*a^4*c^2*x^7 - 1/3*a^3*c^2*x^6 + 1/2*a*c^2*x^4 + 1/3*c^2*x^3
```

Sympy [A] time = 0.08737, size = 41, normalized size = 0.85

$$-\frac{a^{4}c^{2}x^{7}}{7}-\frac{a^{3}c^{2}x^{6}}{3}+\frac{ac^{2}x^{4}}{2}+\frac{c^{2}x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(-a**2*c*x**2+c)**2,x)
```

```
[Out] -a**4*c**2*x**7/7 - a**3*c**2*x**6/3 + a*c**2*x**4/2 + c**2*x**3/3
```

Giac [A] time = 1.12429, size = 54, normalized size = 1.12

$$-\frac{1}{7}a^4c^2x^7 - \frac{1}{3}a^3c^2x^6 + \frac{1}{2}ac^2x^4 + \frac{1}{3}c^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/7*a^4*c^2*x^7 - 1/3*a^3*c^2*x^6 + 1/2*a*c^2*x^4 + 1/3*c^2*x^3
```

3.1031 $\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=48

$$-\frac{1}{6}a^4c^2x^6-\frac{2}{5}a^3c^2x^5+\frac{2}{3}ac^2x^3+\frac{c^2x^2}{2}$$

```
[Out] (c^2*x^2)/2 + (2*a*c^2*x^3)/3 - (2*a^3*c^2*x^5)/5 - (a^4*c^2*x^6)/6
```

Rubi [A] time = 0.066772, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 75}

$$-\frac{1}{6}a^4c^2x^6-\frac{2}{5}a^3c^2x^5+\frac{2}{3}ac^2x^3+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*x^2)/2 + (2*a*c^2*x^3)/3 - (2*a^3*c^2*x^5)/5 - (a^4*c^2*x^6)/6
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^2\,dx&=c^2\int x(1-ax)(1+ax)^3\,dx\\&=c^2\int\left(x+2ax^2-2a^3x^4-a^4x^5\right)dx\\&=\frac{c^2x^2}{2}+\frac{2}{3}ac^2x^3-\frac{2}{5}a^3c^2x^5-\frac{1}{6}a^4c^2x^6\end{aligned}$$

Mathematica [A] time = 0.0182033, size = 32, normalized size = 0.67

$$-\frac{1}{30}c^2x^2\left(5a^4x^4+12a^3x^3-20ax-15\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2)^2,x]
```

```
[Out] -(c^2*x^2*(-15 - 20*a*x + 12*a^3*x^3 + 5*a^4*x^4))/30
```

Maple [A] time = 0.024, size = 33, normalized size = 0.7

$$c^{2}\left(-\frac{x^{6}a^{4}}{6}-\frac{2\,x^{5}a^{3}}{5}+\frac{2\,x^{3}a}{3}+\frac{x^{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^2,x)
```

```
[Out] c^2*(-1/6*x^6*a^4-2/5*x^5*a^3+2/3*x^3*a+1/2*x^2)
```

Maxima [A] time = 0.948752, size = 54, normalized size = 1.12

$$-\frac{1}{6}\,a^{4}c^{2}x^{6}-\frac{2}{5}\,a^{3}c^{2}x^{5}+\frac{2}{3}\,ac^{2}x^{3}+\frac{1}{2}\,c^{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/6*a^4*c^2*x^6 - 2/5*a^3*c^2*x^5 + 2/3*a*c^2*x^3 + 1/2*c^2*x^2
```

Fricas [A] time = 1.69476, size = 89, normalized size = 1.85

$$-\frac{1}{6}\,a^{4}c^{2}x^{6}-\frac{2}{5}\,a^{3}c^{2}x^{5}+\frac{2}{3}\,ac^{2}x^{3}+\frac{1}{2}\,c^{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*a^4*c^2*x^6 - 2/5*a^3*c^2*x^5 + 2/3*a*c^2*x^3 + 1/2*c^2*x^2
```

Sympy [A] time = 0.087026, size = 44, normalized size = 0.92

$$-\frac{a^{4}c^{2}x^{6}}{6}-\frac{2a^{3}c^{2}x^{5}}{5}+\frac{2ac^{2}x^{3}}{3}+\frac{c^{2}x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(-a**2*c*x**2+c)**2,x)
```

```
[Out] -a**4*c**2*x**6/6 - 2*a**3*c**2*x**5/5 + 2*a*c**2*x**3/3 + c**2*x**2/2
```

Giac [A] time = 1.14133, size = 54, normalized size = 1.12

$$-\frac{1}{6}\,a^{4}c^{2}x^{6}-\frac{2}{5}\,a^{3}c^{2}x^{5}+\frac{2}{3}\,ac^{2}x^{3}+\frac{1}{2}\,c^{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/6*a^4*c^2*x^6 - 2/5*a^3*c^2*x^5 + 2/3*a*c^2*x^3 + 1/2*c^2*x^2
```

3.1032 $\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=35

$$\frac{c^2(ax+1)^4}{2a}-\frac{c^2(ax+1)^5}{5a}$$

```
[Out] (c^2*(1 + a*x)^4)/(2*a) - (c^2*(1 + a*x)^5)/(5*a)
```

Rubi [A] time = 0.0370518, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$\frac{c^2(ax+1)^4}{2a}-\frac{c^2(ax+1)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(1 + a*x)^4)/(2*a) - (c^2*(1 + a*x)^5)/(5*a)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx &= c^2\int (1-ax)(1+ax)^3\,dx\\ &= c^2\int \left(2(1+ax)^3-(1+ax)^4\right)dx\\ &= \frac{c^2(1+ax)^4}{2a}-\frac{c^2(1+ax)^5}{5a}\end{aligned}$$

Mathematica [A] time = 0.0130379, size = 23, normalized size = 0.66

$$-\frac{c^2(ax+1)^4(2ax-3)}{10a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] -(c^2*(1 + a*x)^4*(-3 + 2*a*x))/(10*a)
```

Maple [A] time = 0.023, size = 28, normalized size = 0.8

$$c^2\left(-\frac{x^5a^4}{5}-\frac{x^4a^3}{2}+ax^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2,x)
```

```
[Out] c^2*(-1/5*x^5*a^4-1/2*x^4*a^3+a*x^2+x)
```

Maxima [A] time = 0.950569, size = 49, normalized size = 1.4

$$-\frac{1}{5}a^4c^2x^5-\frac{1}{2}a^3c^2x^4+ac^2x^2+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 + c^2*x
```

Fricas [A] time = 1.69535, size = 76, normalized size = 2.17

$$-\frac{1}{5}a^4c^2x^5-\frac{1}{2}a^3c^2x^4+ac^2x^2+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 + c^2*x
```

Sympy [A] time = 0.085515, size = 36, normalized size = 1.03

$$-\frac{a^4c^2x^5}{5}-\frac{a^3c^2x^4}{2}+ac^2x^2+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**2,x)
```

```
[Out] -a**4*c**2*x**5/5 - a**3*c**2*x**4/2 + a*c**2*x**2 + c**2*x
```

Giac [A] time = 1.15739, size = 49, normalized size = 1.4

$$-\frac{1}{5}a^4c^2x^5-\frac{1}{2}a^3c^2x^4+ac^2x^2+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 + c^2*x
```

3.1033 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x} dx$

Optimal. Leaf size=40

$$-\frac{1}{4}a^4c^2x^4-\frac{2}{3}a^3c^2x^3+2ac^2x+c^2\log(x)$$

[Out] 2*a*c^2*x - (2*a^3*c^2*x^3)/3 - (a^4*c^2*x^4)/4 + c^2*Log[x]

Rubi [A] time = 0.0746111, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 75}

$$-\frac{1}{4}a^4c^2x^4-\frac{2}{3}a^3c^2x^3+2ac^2x+c^2\log(x)$$

Antiderivative was successfully verified.

[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x,x]

[Out] 2*a*c^2*x - (2*a^3*c^2*x^3)/3 - (a^4*c^2*x^4)/4 + c^2*Log[x]

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x} dx &= c^2\int \frac{(1-ax)(1+ax)^3}{x} dx\\
&= c^2\int \left(2a+\frac{1}{x}-2a^3x^2-a^4x^3\right) dx\\
&= 2ac^2x-\frac{2}{3}a^3c^2x^3-\frac{1}{4}a^4c^2x^4+c^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0150586, size = 33, normalized size = 0.82

$$-\frac{1}{12}c^2\left(3a^4x^4+8a^3x^3-24ax-12\log(x)+3\right)$$

Antiderivative was successfully verified.

[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x,x]

```
[Out] -(c^2*(3 - 24*a*x + 8*a^3*x^3 + 3*a^4*x^4 - 12*Log[x]))/12
```

Maple [A] time = 0.026, size = 37, normalized size = 0.9

$$2\,ac^{2}x-\frac{2\,a^{3}c^{2}x^{3}}{3}-\frac{a^{4}c^{2}x^{4}}{4}+c^{2}\ln \left(x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x,x)
```

```
[Out] 2*a*c^2*x-2/3*a^3*c^2*x^3-1/4*a^4*c^2*x^4+c^2*ln(x)
```

Maxima [A] time = 0.944819, size = 49, normalized size = 1.22

$$-\frac{1}{4}\,a^{4}c^{2}x^{4}-\frac{2}{3}\,a^{3}c^{2}x^{3}+2\,ac^{2}x+c^{2}\log \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x,x, algorithm="maxima")
```

```
[Out] -1/4*a^4*c^2*x^4 - 2/3*a^3*c^2*x^3 + 2*a*c^2*x + c^2*log(x)
```

Fricas [A] time = 1.77666, size = 82, normalized size = 2.05

$$-\frac{1}{4}\,a^{4}c^{2}x^{4}-\frac{2}{3}\,a^{3}c^{2}x^{3}+2\,ac^{2}x+c^{2}\log \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x,x, algorithm="fricas")
```

```
[Out] -1/4*a^4*c^2*x^4 - 2/3*a^3*c^2*x^3 + 2*a*c^2*x + c^2*log(x)
```

Sympy [A] time = 0.291865, size = 39, normalized size = 0.98

$$-\frac{a^{4}c^{2}x^{4}}{4}-\frac{2a^{3}c^{2}x^{3}}{3}+2ac^{2}x+c^{2}\log \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**2/x,x)
```

```
[Out] -a**4*c**2*x**4/4 - 2*a**3*c**2*x**3/3 + 2*a*c**2*x + c**2*log(x)
```

Giac [A] time = 1.12561, size = 50, normalized size = 1.25

$$-\frac{1}{4}\,a^{4}c^{2}x^{4}-\frac{2}{3}\,a^{3}c^{2}x^{3}+2\,ac^{2}x+c^{2}\log \left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x,x, algorithm="giac")
```

```
[Out] -1/4*a^4*c^2*x^4 - 2/3*a^3*c^2*x^3 + 2*a*c^2*x + c^2*log(abs(x))
```

3.1034 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^2}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{3}a^4c^2x^3-a^3c^2x^2+2ac^2\log(x)-\frac{c^2}{x}$$

```
[Out] -(c^2/x) - a^3*c^2*x^2 - (a^4*c^2*x^3)/3 + 2*a*c^2*Log[x]
```

Rubi [A] time = 0.0818491, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 75}

$$-\frac{1}{3}a^4c^2x^3-a^3c^2x^2+2ac^2\log(x)-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^2,x]
```

```
[Out] -(c^2/x) - a^3*c^2*x^2 - (a^4*c^2*x^3)/3 + 2*a*c^2*Log[x]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^2}\,dx &= c^2\int \frac{(1-ax)(1+ax)^3}{x^2}\,dx\\
&= c^2\int \left(\frac{1}{x^2}+\frac{2a}{x}-2a^3x-a^4x^2\right)dx\\
&= -\frac{c^2}{x}-a^3c^2x^2-\frac{1}{3}a^4c^2x^3+2ac^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0132127, size = 41, normalized size = 1.

$$-\frac{1}{3}a^4c^2x^3-a^3c^2x^2+2ac^2\log(x)-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^2,x]
```

```
[Out] -(c^2/x) - a^3*c^2*x^2 - (a^4*c^2*x^3)/3 + 2*a*c^2*Log[x]
```

Maple [A] time = 0.033, size = 40, normalized size = 1.

$$-\frac{c^{2}}{x}-a^{3}c^{2}x^{2}-\frac{a^{4}c^{2}x^{3}}{3}+2\,ac^{2}\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^2,x)
```

```
[Out] -c^2/x-a^3*c^2*x^2-1/3*a^4*c^2*x^3+2*a*c^2*ln(x)
```

Maxima [A] time = 0.949476, size = 53, normalized size = 1.29

$$-\frac{1}{3}\,a^{4}c^{2}x^{3}-a^{3}c^{2}x^{2}+2\,ac^{2}\log\left(x\right)-\frac{c^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^2,x, algorithm="maxima"
)
```

```
[Out] -1/3*a^4*c^2*x^3 - a^3*c^2*x^2 + 2*a*c^2*log(x) - c^2/x
```

Fricas [A] time = 1.67054, size = 88, normalized size = 2.15

$$-\frac{a^{4}c^{2}x^{4}+3\,a^{3}c^{2}x^{3}-6\,ac^{2}x\log\left(x\right)+3\,c^{2}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^2,x, algorithm="fricas"
)
```

```
[Out] -1/3*(a^4*c^2*x^4 + 3*a^3*c^2*x^3 - 6*a*c^2*x*log(x) + 3*c^2)/x
```

Sympy [A] time = 0.306966, size = 36, normalized size = 0.88

$$-\frac{a^{4}c^{2}x^{3}}{3}-a^{3}c^{2}x^{2}+2ac^{2}\log\left(x\right)-\frac{c^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**2/x**2,x)
```

```
[Out] -a**4*c**2*x**3/3 - a**3*c**2*x**2 + 2*a*c**2*log(x) - c**2/x
```

Giac [A] time = 1.13346, size = 54, normalized size = 1.32

$$-\frac{1}{3}a^4c^2x^3 - a^3c^2x^2 + 2ac^2\log(|x|) - \frac{c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^2,x, algorithm="giac")
```

```
[Out] -1/3*a^4*c^2*x^3 - a^3*c^2*x^2 + 2*a*c^2*log(abs(x)) - c^2/x
```

3.1035 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^3}\,dx$

Optimal. Leaf size=17

$$-\frac{c^2(ax+1)^4}{2x^2}$$

```
[Out] -(c^2*(1 + a*x)^4)/(2*x^2)
```

Rubi [A] time = 0.0692598, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 74}

$$-\frac{c^2(ax+1)^4}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^3,x]
```

```
[Out] -(c^2*(1 + a*x)^4)/(2*x^2)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^3}\,dx &= c^2\int \frac{(1-ax)(1+ax)^3}{x^3}\,dx\\ &= -\frac{c^2(1+ax)^4}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0148526, size = 17, normalized size = 1.

$$-\frac{c^2(ax+1)^4}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^3,x]
```

```
[Out] -(c^2*(1 + a*x)^4)/(2*x^2)
```

Maple [A] time = 0.031, size = 31, normalized size = 1.8

$$c^2\left(-\frac{x^2a^4}{2}-2\,xa^3-2\,\frac{a}{x}-\frac{1}{2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^3,x)
```

```
[Out] c^2*(-1/2*x^2*a^4-2*x*a^3-2*a/x-1/2/x^2)
```

Maxima [B] time = 0.948993, size = 50, normalized size = 2.94

$$-\frac{1}{2}\,a^4c^2x^2-2\,a^3c^2x-\frac{4\,ac^2x+c^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^3,x, algorithm="maxima"
)
```

```
[Out] -1/2*a^4*c^2*x^2 - 2*a^3*c^2*x - 1/2*(4*a*c^2*x + c^2)/x^2
```

Fricas [B] time = 1.64385, size = 78, normalized size = 4.59

$$-\frac{a^4c^2x^4+4\,a^3c^2x^3+4\,ac^2x+c^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^3,x, algorithm="fricas"
)
```

```
[Out] -1/2*(a^4*c^2*x^4 + 4*a^3*c^2*x^3 + 4*a*c^2*x + c^2)/x^2
```

Sympy [B] time = 0.315029, size = 39, normalized size = 2.29

$$-\frac{a^4c^2x^2}{2}-2a^3c^2x-\frac{4ac^2x+c^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**2/x**3,x)
```

```
[Out] -a**4*c**2*x**2/2 - 2*a**3*c**2*x - (4*a*c**2*x + c**2)/(2*x**2)
```

Giac [B] time = 1.11806, size = 50, normalized size = 2.94

$$-\frac{1}{2}a^4c^2x^2 - 2a^3c^2x - \frac{4ac^2x + c^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*a^4*c^2*x^2 - 2*a^3*c^2*x - 1/2*(4*a*c^2*x + c^2)/x^2
```

3.1036 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^4}\,dx$

Optimal. Leaf size=39

$$a^4\left(-c^2\right)x-2a^3c^2\log(x)-\frac{ac^2}{x^2}-\frac{c^2}{3x^3}$$

```
[Out] -c^2/(3*x^3) - (a*c^2)/x^2 - a^4*c^2*x - 2*a^3*c^2*Log[x]
```

Rubi [A] time = 0.0802476, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 75}

$$a^4\left(-c^2\right)x-2a^3c^2\log(x)-\frac{ac^2}{x^2}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^4,x]
```

```
[Out] -c^2/(3*x^3) - (a*c^2)/x^2 - a^4*c^2*x - 2*a^3*c^2*Log[x]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^4}\,dx &= c^2\int \frac{(1-ax)(1+ax)^3}{x^4}\,dx\\
&= c^2\int \left(-a^4+\frac{1}{x^4}+\frac{2a}{x^3}-\frac{2a^3}{x}\right)dx\\
&= -\frac{c^2}{3x^3}-\frac{ac^2}{x^2}-a^4c^2x-2a^3c^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0150206, size = 39, normalized size = 1.

$$a^4\left(-c^2\right)x-2a^3c^2\log(x)-\frac{ac^2}{x^2}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^4,x]

[Out] -c^2/(3*x^3) - (a*c^2)/x^2 - a^4*c^2*x - 2*a^3*c^2*Log[x]
```

Maple [A] time = 0.031, size = 38, normalized size = 1.

$$-\frac{c^2}{3\,x^3}-\frac{ac^2}{x^2}-a^4c^2x-2\,a^3c^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^4,x)

[Out] -1/3*c^2/x^3-a*c^2/x^2-a^4*c^2*x-2*a^3*c^2*ln(x)
```

Maxima [A] time = 0.948565, size = 49, normalized size = 1.26

$$-a^4c^2x-2\,a^3c^2\log(x)-\frac{3\,ac^2x+c^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^4,x, algorithm="maxima"
)

[Out] -a^4*c^2*x - 2*a^3*c^2*log(x) - 1/3*(3*a*c^2*x + c^2)/x^3
```

Fricas [A] time = 1.67061, size = 90, normalized size = 2.31

$$-\frac{3\,a^4c^2x^4+6\,a^3c^2x^3\log(x)+3\,ac^2x+c^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^4,x, algorithm="fricas"
)

[Out] -1/3*(3*a^4*c^2*x^4 + 6*a^3*c^2*x^3*log(x) + 3*a*c^2*x + c^2)/x^3
```

Sympy [A] time = 0.35641, size = 37, normalized size = 0.95

$$-a^4c^2x-2a^3c^2\log(x)-\frac{3ac^2x+c^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**2/x**4,x)

[Out] -a**4*c**2*x - 2*a**3*c**2*log(x) - (3*a*c**2*x + c**2)/(3*x**3)
```

Giac [A] time = 1.13319, size = 50, normalized size = 1.28

$$-a^4c^2x - 2\,a^3c^2\log\left(|x|\right) - \frac{3\,ac^2x + c^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^4,x, algorithm="giac")
```

```
[Out] -a^4*c^2*x - 2*a^3*c^2*log(abs(x)) - 1/3*(3*a*c^2*x + c^2)/x^3
```

3.1037 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^5}\,dx$

Optimal. Leaf size=43

$$\frac{2a^3c^2}{x}+a^4\left(-c^2\right)\log(x)-\frac{2ac^2}{3x^3}-\frac{c^2}{4x^4}$$

```
[Out] -c^2/(4*x^4) - (2*a*c^2)/(3*x^3) + (2*a^3*c^2)/x - a^4*c^2*Log[x]
```

Rubi [A] time = 0.0791424, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 75}

$$\frac{2a^3c^2}{x}+a^4\left(-c^2\right)\log(x)-\frac{2ac^2}{3x^3}-\frac{c^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^5,x]
```

```
[Out] -c^2/(4*x^4) - (2*a*c^2)/(3*x^3) + (2*a^3*c^2)/x - a^4*c^2*Log[x]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^5}\,dx &= c^2\int \frac{(1-ax)(1+ax)^3}{x^5}\,dx\\ &= c^2\int \left(\frac{1}{x^5}+\frac{2a}{x^4}-\frac{2a^3}{x^2}-\frac{a^4}{x}\right)dx\\ &= -\frac{c^2}{4x^4}-\frac{2ac^2}{3x^3}+\frac{2a^3c^2}{x}-a^4c^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.0173615, size = 35, normalized size = 0.81

$$c^2\left(\frac{2a^3}{x}+a^4(-\log(x))-\frac{2a}{3x^3}-\frac{1}{4x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^5,x]
```

```
[Out] c^2*(-1/(4*x^4) - (2*a)/(3*x^3) + (2*a^3)/x - a^4*Log[x])
```

Maple [A] time = 0.03, size = 40, normalized size = 0.9

$$-\frac{c^2}{4\,x^4}-\frac{2\,ac^2}{3\,x^3}+2\,\frac{a^3c^2}{x}-a^4c^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^5,x)
```

```
[Out] -1/4*c^2/x^4-2/3*a*c^2/x^3+2*a^3*c^2/x-a^4*c^2*ln(x)
```

Maxima [A] time = 0.954989, size = 54, normalized size = 1.26

$$-a^4c^2\log(x)+\frac{24\,a^3c^2x^3-8\,ac^2x-3\,c^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^5,x, algorithm="maxima"
)
```

```
[Out] -a^4*c^2*log(x) + 1/12*(24*a^3*c^2*x^3 - 8*a*c^2*x - 3*c^2)/x^4
```

Fricas [A] time = 1.66952, size = 97, normalized size = 2.26

$$-\frac{12\,a^4c^2x^4\log(x)-24\,a^3c^2x^3+8\,ac^2x+3\,c^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^5,x, algorithm="fricas"
)
```

```
[Out] -1/12*(12*a^4*c^2*x^4*log(x) - 24*a^3*c^2*x^3 + 8*a*c^2*x + 3*c^2)/x^4
```

Sympy [A] time = 0.396636, size = 39, normalized size = 0.91

$$-a^4c^2\log(x)+\frac{24a^3c^2x^3-8ac^2x-3c^2}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**2/x**5,x)
```

```
[Out] -a**4*c**2*log(x) + (24*a**3*c**2*x**3 - 8*a*c**2*x - 3*c**2)/(12*x**4)
```

Giac [A] time = 1.12034, size = 55, normalized size = 1.28

$$-a^4c^2\log(|x|) + \frac{24\,a^3c^2x^3 - 8\,ac^2x - 3\,c^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^5,x, algorithm="giac")
```

```
[Out] -a^4*c^2*log(abs(x)) + 1/12*(24*a^3*c^2*x^3 - 8*a*c^2*x - 3*c^2)/x^4
```

3.1038 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^6} dx$

Optimal. Leaf size=42

$$\frac{a^3c^2}{x^2}+\frac{a^4c^2}{x}-\frac{ac^2}{2x^4}-\frac{c^2}{5x^5}$$

```
[Out] -c^2/(5*x^5) - (a*c^2)/(2*x^4) + (a^3*c^2)/x^2 + (a^4*c^2)/x
```

Rubi [A] time = 0.080575, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 75}

$$\frac{a^3c^2}{x^2}+\frac{a^4c^2}{x}-\frac{ac^2}{2x^4}-\frac{c^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^6,x]
```

```
[Out] -c^2/(5*x^5) - (a*c^2)/(2*x^4) + (a^3*c^2)/x^2 + (a^4*c^2)/x
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^6} dx &= c^2\int \frac{(1-ax)(1+ax)^3}{x^6} dx\\ &= c^2\int \left(\frac{1}{x^6}+\frac{2a}{x^5}-\frac{2a^3}{x^3}-\frac{a^4}{x^2}\right) dx\\ &= -\frac{c^2}{5x^5}-\frac{ac^2}{2x^4}+\frac{a^3c^2}{x^2}+\frac{a^4c^2}{x}\end{aligned}$$

Mathematica [A] time = 0.010401, size = 23, normalized size = 0.55

$$\frac{c^2(ax+1)^4(3ax-2)}{10x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^6,x]
```

```
[Out] (c^2*(1 + a*x)^4*(-2 + 3*a*x))/(10*x^5)
```

Maple [A] time = 0.032, size = 31, normalized size = 0.7

$$c^2\left(-\frac{a}{2x^4}+\frac{a^4}{x}-\frac{1}{5x^5}+\frac{a^3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^6,x)
```

```
[Out] c^2*(-1/2*a/x^4+a^4/x-1/5/x^5+a^3/x^2)
```

Maxima [A] time = 0.95437, size = 54, normalized size = 1.29

$$\frac{10\,a^4c^2x^4+10\,a^3c^2x^3-5\,ac^2x-2\,c^2}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^6,x, algorithm="maxima"
)
```

```
[Out] 1/10*(10*a^4*c^2*x^4 + 10*a^3*c^2*x^3 - 5*a*c^2*x - 2*c^2)/x^5
```

Fricas [A] time = 1.6516, size = 86, normalized size = 2.05

$$\frac{10\,a^4c^2x^4+10\,a^3c^2x^3-5\,ac^2x-2\,c^2}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^6,x, algorithm="fricas"
)
```

```
[Out] 1/10*(10*a^4*c^2*x^4 + 10*a^3*c^2*x^3 - 5*a*c^2*x - 2*c^2)/x^5
```

Sympy [A] time = 0.412481, size = 41, normalized size = 0.98

$$\frac{10a^4c^2x^4+10a^3c^2x^3-5ac^2x-2c^2}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**2/x**6,x)
```

```
[Out] (10*a**4*c**2*x**4 + 10*a**3*c**2*x**3 - 5*a*c**2*x - 2*c**2)/(10*x**5)
```

Giac [A] time = 1.11447, size = 54, normalized size = 1.29

$$\frac{10\,a^4c^2x^4 + 10\,a^3c^2x^3 - 5\,ac^2x - 2\,c^2}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/10*(10*a^4*c^2*x^4 + 10*a^3*c^2*x^3 - 5*a*c^2*x - 2*c^2)/x^5
```

3.1039 $\int e^{2\tanh^{-1}(ax)}x^4\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=87

$$\frac{1}{11}a^6c^3x^{11}+\frac{1}{5}a^5c^3x^{10}-\frac{1}{9}a^4c^3x^9-\frac{1}{2}a^3c^3x^8-\frac{1}{7}a^2c^3x^7+\frac{1}{3}ac^3x^6+\frac{c^3x^5}{5}$$

```
[Out] (c^3*x^5)/5 + (a*c^3*x^6)/3 - (a^2*c^3*x^7)/7 - (a^3*c^3*x^8)/2 - (a^4*c^3*
x^9)/9 + (a^5*c^3*x^10)/5 + (a^6*c^3*x^11)/11
```

Rubi [A] time = 0.10185, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$\frac{1}{11}a^6c^3x^{11}+\frac{1}{5}a^5c^3x^{10}-\frac{1}{9}a^4c^3x^9-\frac{1}{2}a^3c^3x^8-\frac{1}{7}a^2c^3x^7+\frac{1}{3}ac^3x^6+\frac{c^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^4*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*x^5)/5 + (a*c^3*x^6)/3 - (a^2*c^3*x^7)/7 - (a^3*c^3*x^8)/2 - (a^4*c^3*
x^9)/9 + (a^5*c^3*x^10)/5 + (a^6*c^3*x^11)/11
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^4\left(c-a^2cx^2\right)^3\,dx&=c^3\int x^4(1-ax)^2(1+ax)^4\,dx\\&=c^3\int\left(x^4+2ax^5-a^2x^6-4a^3x^7-a^4x^8+2a^5x^9+a^6x^{10}\right)dx\\&=\frac{c^3x^5}{5}+\frac{1}{3}ac^3x^6-\frac{1}{7}a^2c^3x^7-\frac{1}{2}a^3c^3x^8-\frac{1}{9}a^4c^3x^9+\frac{1}{5}a^5c^3x^{10}+\frac{1}{11}a^6c^3x^{11}\end{aligned}$$

Mathematica [A] time = 0.037536, size = 70, normalized size = 0.8

$$c^3\left(\frac{a^6x^{11}}{11}+\frac{a^5x^{10}}{5}-\frac{a^4x^9}{9}-\frac{a^3x^8}{2}-\frac{a^2x^7}{7}+\frac{ax^6}{3}+\frac{x^5}{5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^4*(c - a^2*c*x^2)^3,x]
```

```
[Out] c^3*(x^5/5 + (a*x^6)/3 - (a^2*x^7)/7 - (a^3*x^8)/2 - (a^4*x^9)/9 + (a^5*x^1
0)/5 + (a^6*x^11)/11)
```

Maple [A] time = 0.026, size = 57, normalized size = 0.7

$$c^{3}\left(\frac{a^{6}x^{11}}{11}+\frac{a^{5}x^{10}}{5}-\frac{a^{4}x^{9}}{9}-\frac{a^{3}x^{8}}{2}-\frac{a^{2}x^{7}}{7}+\frac{x^{6}a}{3}+\frac{x^{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*(1/11*a^6*x^11+1/5*a^5*x^10-1/9*a^4*x^9-1/2*a^3*x^8-1/7*a^2*x^7+1/3*x^6
*a+1/5*x^5)
```

Maxima [A] time = 0.955658, size = 99, normalized size = 1.14

$$\frac{1}{11}a^{6}c^{3}x^{11}+\frac{1}{5}a^{5}c^{3}x^{10}-\frac{1}{9}a^{4}c^{3}x^{9}-\frac{1}{2}a^{3}c^{3}x^{8}-\frac{1}{7}a^{2}c^{3}x^{7}+\frac{1}{3}ac^{3}x^{6}+\frac{1}{5}c^{3}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] 1/11*a^6*c^3*x^11 + 1/5*a^5*c^3*x^10 - 1/9*a^4*c^3*x^9 - 1/2*a^3*c^3*x^8 -
1/7*a^2*c^3*x^7 + 1/3*a*c^3*x^6 + 1/5*c^3*x^5
```

Fricas [A] time = 1.6361, size = 165, normalized size = 1.9

$$\frac{1}{11}a^{6}c^{3}x^{11}+\frac{1}{5}a^{5}c^{3}x^{10}-\frac{1}{9}a^{4}c^{3}x^{9}-\frac{1}{2}a^{3}c^{3}x^{8}-\frac{1}{7}a^{2}c^{3}x^{7}+\frac{1}{3}ac^{3}x^{6}+\frac{1}{5}c^{3}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/11*a^6*c^3*x^11 + 1/5*a^5*c^3*x^10 - 1/9*a^4*c^3*x^9 - 1/2*a^3*c^3*x^8 -
1/7*a^2*c^3*x^7 + 1/3*a*c^3*x^6 + 1/5*c^3*x^5
```

Sympy [A] time = 0.09861, size = 76, normalized size = 0.87

$$\frac{a^{6}c^{3}x^{11}}{11}+\frac{a^{5}c^{3}x^{10}}{5}-\frac{a^{4}c^{3}x^{9}}{9}-\frac{a^{3}c^{3}x^{8}}{2}-\frac{a^{2}c^{3}x^{7}}{7}+\frac{ac^{3}x^{6}}{3}+\frac{c^{3}x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**4*(-a**2*c*x**2+c)**3,x)
```

```
[Out] a**6*c**3*x**11/11 + a**5*c**3*x**10/5 - a**4*c**3*x**9/9 - a**3*c**3*x**8/
2 - a**2*c**3*x**7/7 + a*c**3*x**6/3 + c**3*x**5/5
```

Giac [A] time = 1.14489, size = 99, normalized size = 1.14

$$\frac{1}{11}a^6c^3x^{11} + \frac{1}{5}a^5c^3x^{10} - \frac{1}{9}a^4c^3x^9 - \frac{1}{2}a^3c^3x^8 - \frac{1}{7}a^2c^3x^7 + \frac{1}{3}ac^3x^6 + \frac{1}{5}c^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/11*a^6*c^3*x^11 + 1/5*a^5*c^3*x^10 - 1/9*a^4*c^3*x^9 - 1/2*a^3*c^3*x^8 -
1/7*a^2*c^3*x^7 + 1/3*a*c^3*x^6 + 1/5*c^3*x^5
```

3.1040 $\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=87

$$\frac{1}{10}a^6c^3x^{10}+\frac{2}{9}a^5c^3x^9-\frac{1}{8}a^4c^3x^8-\frac{4}{7}a^3c^3x^7-\frac{1}{6}a^2c^3x^6+\frac{2}{5}ac^3x^5+\frac{c^3x^4}{4}$$

```
[Out] (c^3*x^4)/4 + (2*a*c^3*x^5)/5 - (a^2*c^3*x^6)/6 - (4*a^3*c^3*x^7)/7 - (a^4*
c^3*x^8)/8 + (2*a^5*c^3*x^9)/9 + (a^6*c^3*x^10)/10
```

Rubi [A] time = 0.0989541, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$\frac{1}{10}a^6c^3x^{10}+\frac{2}{9}a^5c^3x^9-\frac{1}{8}a^4c^3x^8-\frac{4}{7}a^3c^3x^7-\frac{1}{6}a^2c^3x^6+\frac{2}{5}ac^3x^5+\frac{c^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*x^4)/4 + (2*a*c^3*x^5)/5 - (a^2*c^3*x^6)/6 - (4*a^3*c^3*x^7)/7 - (a^4*
c^3*x^8)/8 + (2*a^5*c^3*x^9)/9 + (a^6*c^3*x^10)/10
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^3\,dx&=c^3\int x^3(1-ax)^2(1+ax)^4\,dx\\&=c^3\int\left(x^3+2ax^4-a^2x^5-4a^3x^6-a^4x^7+2a^5x^8+a^6x^9\right)dx\\&=\frac{c^3x^4}{4}+\frac{2}{5}ac^3x^5-\frac{1}{6}a^2c^3x^6-\frac{4}{7}a^3c^3x^7-\frac{1}{8}a^4c^3x^8+\frac{2}{9}a^5c^3x^9+\frac{1}{10}a^6c^3x^{10}\end{aligned}$$

Mathematica [A] time = 0.0334374, size = 70, normalized size = 0.8

$$c^3\left(\frac{a^6x^{10}}{10}+\frac{2a^5x^9}{9}-\frac{a^4x^8}{8}-\frac{4a^3x^7}{7}-\frac{a^2x^6}{6}+\frac{2ax^5}{5}+\frac{x^4}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^3,x]
```

```
[Out] c^3*(x^4/4 + (2*a*x^5)/5 - (a^2*x^6)/6 - (4*a^3*x^7)/7 - (a^4*x^8)/8 + (2*a
^5*x^9)/9 + (a^6*x^10)/10)
```

Maple [A] time = 0.026, size = 57, normalized size = 0.7

$$c^{3}\left(\frac{a^{6}x^{10}}{10}+\frac{2\,a^{5}x^{9}}{9}-\frac{a^{4}x^{8}}{8}-\frac{4\,x^{7}a^{3}}{7}-\frac{x^{6}a^{2}}{6}+\frac{2\,ax^{5}}{5}+\frac{x^{4}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*(1/10*a^6*x^10+2/9*a^5*x^9-1/8*a^4*x^8-4/7*x^7*a^3-1/6*x^6*a^2+2/5*a*x^
5+1/4*x^4)
```

Maxima [A] time = 0.942435, size = 99, normalized size = 1.14

$$\frac{1}{10}\,a^{6}c^{3}x^{10}+\frac{2}{9}\,a^{5}c^{3}x^{9}-\frac{1}{8}\,a^{4}c^{3}x^{8}-\frac{4}{7}\,a^{3}c^{3}x^{7}-\frac{1}{6}\,a^{2}c^{3}x^{6}+\frac{2}{5}\,ac^{3}x^{5}+\frac{1}{4}\,c^{3}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] 1/10*a^6*c^3*x^10 + 2/9*a^5*c^3*x^9 - 1/8*a^4*c^3*x^8 - 4/7*a^3*c^3*x^7 - 1
/6*a^2*c^3*x^6 + 2/5*a*c^3*x^5 + 1/4*c^3*x^4
```

Fricas [A] time = 1.63667, size = 163, normalized size = 1.87

$$\frac{1}{10}\,a^{6}c^{3}x^{10}+\frac{2}{9}\,a^{5}c^{3}x^{9}-\frac{1}{8}\,a^{4}c^{3}x^{8}-\frac{4}{7}\,a^{3}c^{3}x^{7}-\frac{1}{6}\,a^{2}c^{3}x^{6}+\frac{2}{5}\,ac^{3}x^{5}+\frac{1}{4}\,c^{3}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/10*a^6*c^3*x^10 + 2/9*a^5*c^3*x^9 - 1/8*a^4*c^3*x^8 - 4/7*a^3*c^3*x^7 - 1
/6*a^2*c^3*x^6 + 2/5*a*c^3*x^5 + 1/4*c^3*x^4
```

Sympy [A] time = 0.096982, size = 82, normalized size = 0.94

$$\frac{a^{6}c^{3}x^{10}}{10}+\frac{2a^{5}c^{3}x^{9}}{9}-\frac{a^{4}c^{3}x^{8}}{8}-\frac{4a^{3}c^{3}x^{7}}{7}-\frac{a^{2}c^{3}x^{6}}{6}+\frac{2ac^{3}x^{5}}{5}+\frac{c^{3}x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(-a**2*c*x**2+c)**3,x)
```

```
[Out] a**6*c**3*x**10/10 + 2*a**5*c**3*x**9/9 - a**4*c**3*x**8/8 - 4*a**3*c**3*x*
*7/7 - a**2*c**3*x**6/6 + 2*a*c**3*x**5/5 + c**3*x**4/4
```

Giac [A] time = 1.12946, size = 99, normalized size = 1.14

$$\frac{1}{10}a^6c^3x^{10} + \frac{2}{9}a^5c^3x^9 - \frac{1}{8}a^4c^3x^8 - \frac{4}{7}a^3c^3x^7 - \frac{1}{6}a^2c^3x^6 + \frac{2}{5}ac^3x^5 + \frac{1}{4}c^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/10*a^6*c^3*x^10 + 2/9*a^5*c^3*x^9 - 1/8*a^4*c^3*x^8 - 4/7*a^3*c^3*x^7 - 1
/6*a^2*c^3*x^6 + 2/5*a*c^3*x^5 + 1/4*c^3*x^4
```

3.1041 $\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=84

$$\frac{c^3(ax+1)^9}{9a^3}-\frac{3c^3(ax+1)^8}{4a^3}+\frac{13c^3(ax+1)^7}{7a^3}-\frac{2c^3(ax+1)^6}{a^3}+\frac{4c^3(ax+1)^5}{5a^3}$$

```
[Out] (4*c^3*(1 + a*x)^5)/(5*a^3) - (2*c^3*(1 + a*x)^6)/a^3 + (13*c^3*(1 + a*x)^7
)/(7*a^3) - (3*c^3*(1 + a*x)^8)/(4*a^3) + (c^3*(1 + a*x)^9)/(9*a^3)
```

Rubi [A] time = 0.10281, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$\frac{c^3(ax+1)^9}{9a^3}-\frac{3c^3(ax+1)^8}{4a^3}+\frac{13c^3(ax+1)^7}{7a^3}-\frac{2c^3(ax+1)^6}{a^3}+\frac{4c^3(ax+1)^5}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^3,x]
```

```
[Out] (4*c^3*(1 + a*x)^5)/(5*a^3) - (2*c^3*(1 + a*x)^6)/a^3 + (13*c^3*(1 + a*x)^7
)/(7*a^3) - (3*c^3*(1 + a*x)^8)/(4*a^3) + (c^3*(1 + a*x)^9)/(9*a^3)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^3\,dx&=c^3\int x^2(1-ax)^2(1+ax)^4\,dx\\&=c^3\int\left(\frac{4(1+ax)^4}{a^2}-\frac{12(1+ax)^5}{a^2}+\frac{13(1+ax)^6}{a^2}-\frac{6(1+ax)^7}{a^2}+\frac{(1+ax)^8}{a^2}\right)dx\\&=\frac{4c^3(1+ax)^5}{5a^3}-\frac{2c^3(1+ax)^6}{a^3}+\frac{13c^3(1+ax)^7}{7a^3}-\frac{3c^3(1+ax)^8}{4a^3}+\frac{c^3(1+ax)^9}{9a^3}\end{aligned}$$

Mathematica [A] time = 0.0317254, size = 70, normalized size = 0.83

$$c^3\left(\frac{a^6x^9}{9}+\frac{a^5x^8}{4}-\frac{a^4x^7}{7}-\frac{2a^3x^6}{3}-\frac{a^2x^5}{5}+\frac{ax^4}{2}+\frac{x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^3,x]
```

```
[Out] c^3*(x^3/3 + (a*x^4)/2 - (a^2*x^5)/5 - (2*a^3*x^6)/3 - (a^4*x^7)/7 + (a^5*x
^8)/4 + (a^6*x^9)/9)
```

Maple [A] time = 0.026, size = 57, normalized size = 0.7

$$c^{3}\left(\frac{x^{9}a^{6}}{9}+\frac{a^{5}x^{8}}{4}-\frac{x^{7}a^{4}}{7}-\frac{2\,x^{6}a^{3}}{3}-\frac{x^{5}a^{2}}{5}+\frac{x^{4}a}{2}+\frac{x^{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*(1/9*x^9*a^6+1/4*a^5*x^8-1/7*x^7*a^4-2/3*x^6*a^3-1/5*x^5*a^2+1/2*x^4*a+
1/3*x^3)
```

Maxima [A] time = 0.945745, size = 99, normalized size = 1.18

$$\frac{1}{9}\,a^{6}c^{3}x^{9}+\frac{1}{4}\,a^{5}c^{3}x^{8}-\frac{1}{7}\,a^{4}c^{3}x^{7}-\frac{2}{3}\,a^{3}c^{3}x^{6}-\frac{1}{5}\,a^{2}c^{3}x^{5}+\frac{1}{2}\,ac^{3}x^{4}+\frac{1}{3}\,c^{3}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] 1/9*a^6*c^3*x^9 + 1/4*a^5*c^3*x^8 - 1/7*a^4*c^3*x^7 - 2/3*a^3*c^3*x^6 - 1/5
*a^2*c^3*x^5 + 1/2*a*c^3*x^4 + 1/3*c^3*x^3
```

Fricas [A] time = 1.61706, size = 161, normalized size = 1.92

$$\frac{1}{9}\,a^{6}c^{3}x^{9}+\frac{1}{4}\,a^{5}c^{3}x^{8}-\frac{1}{7}\,a^{4}c^{3}x^{7}-\frac{2}{3}\,a^{3}c^{3}x^{6}-\frac{1}{5}\,a^{2}c^{3}x^{5}+\frac{1}{2}\,ac^{3}x^{4}+\frac{1}{3}\,c^{3}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/9*a^6*c^3*x^9 + 1/4*a^5*c^3*x^8 - 1/7*a^4*c^3*x^7 - 2/3*a^3*c^3*x^6 - 1/5
*a^2*c^3*x^5 + 1/2*a*c^3*x^4 + 1/3*c^3*x^3
```

Sympy [A] time = 0.097526, size = 78, normalized size = 0.93

$$\frac{a^{6}c^{3}x^{9}}{9}+\frac{a^{5}c^{3}x^{8}}{4}-\frac{a^{4}c^{3}x^{7}}{7}-\frac{2a^{3}c^{3}x^{6}}{3}-\frac{a^{2}c^{3}x^{5}}{5}+\frac{ac^{3}x^{4}}{2}+\frac{c^{3}x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(-a**2*c*x**2+c)**3,x)
```

```
[Out] a**6*c**3*x**9/9 + a**5*c**3*x**8/4 - a**4*c**3*x**7/7 - 2*a**3*c**3*x**6/3
 - a**2*c**3*x**5/5 + a*c**3*x**4/2 + c**3*x**3/3
```

Giac [A] time = 1.1377, size = 99, normalized size = 1.18

$$\frac{1}{9}\,a^6c^3x^9+\frac{1}{4}\,a^5c^3x^8-\frac{1}{7}\,a^4c^3x^7-\frac{2}{3}\,a^3c^3x^6-\frac{1}{5}\,a^2c^3x^5+\frac{1}{2}\,ac^3x^4+\frac{1}{3}\,c^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/9*a^6*c^3*x^9 + 1/4*a^5*c^3*x^8 - 1/7*a^4*c^3*x^7 - 2/3*a^3*c^3*x^6 - 1/5
*a^2*c^3*x^5 + 1/2*a*c^3*x^4 + 1/3*c^3*x^3
```

3.1042 $\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=69

$$\frac{c^3(ax+1)^8}{8a^2}-\frac{5c^3(ax+1)^7}{7a^2}+\frac{4c^3(ax+1)^6}{3a^2}-\frac{4c^3(ax+1)^5}{5a^2}$$

```
[Out] (-4*c^3*(1 + a*x)^5)/(5*a^2) + (4*c^3*(1 + a*x)^6)/(3*a^2) - (5*c^3*(1 + a*
x)^7)/(7*a^2) + (c^3*(1 + a*x)^8)/(8*a^2)
```

Rubi [A] time = 0.0799643, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 77}

$$\frac{c^3(ax+1)^8}{8a^2}-\frac{5c^3(ax+1)^7}{7a^2}+\frac{4c^3(ax+1)^6}{3a^2}-\frac{4c^3(ax+1)^5}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2)^3,x]
```

```
[Out] (-4*c^3*(1 + a*x)^5)/(5*a^2) + (4*c^3*(1 + a*x)^6)/(3*a^2) - (5*c^3*(1 + a*
x)^7)/(7*a^2) + (c^3*(1 + a*x)^8)/(8*a^2)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^3\,dx &= c^3\int x(1-ax)^2(1+ax)^4\,dx\\ &= c^3\int\left(-\frac{4(1+ax)^4}{a}+\frac{8(1+ax)^5}{a}-\frac{5(1+ax)^6}{a}+\frac{(1+ax)^7}{a}\right)dx\\ &= -\frac{4c^3(1+ax)^5}{5a^2}+\frac{4c^3(1+ax)^6}{3a^2}-\frac{5c^3(1+ax)^7}{7a^2}+\frac{c^3(1+ax)^8}{8a^2}\end{aligned}$$

Mathematica [A] time = 0.0297239, size = 70, normalized size = 1.01

$$c^3\left(\frac{a^6x^8}{8}+\frac{2a^5x^7}{7}-\frac{a^4x^6}{6}-\frac{4a^3x^5}{5}-\frac{a^2x^4}{4}+\frac{2ax^3}{3}+\frac{x^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2)^3,x]
```

```
[Out] c^3*(x^2/2 + (2*a*x^3)/3 - (a^2*x^4)/4 - (4*a^3*x^5)/5 - (a^4*x^6)/6 + (2*a
^5*x^7)/7 + (a^6*x^8)/8)
```

Maple [A] time = 0.024, size = 57, normalized size = 0.8

$$c^{3}\left(\frac{x^{8}a^{6}}{8}+\frac{2\,a^{5}x^{7}}{7}-\frac{x^{6}a^{4}}{6}-\frac{4\,x^{5}a^{3}}{5}-\frac{a^{2}x^{4}}{4}+\frac{2\,x^{3}a}{3}+\frac{x^{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*(1/8*x^8*a^6+2/7*a^5*x^7-1/6*x^6*a^4-4/5*x^5*a^3-1/4*a^2*x^4+2/3*x^3*a+
1/2*x^2)
```

Maxima [A] time = 0.961356, size = 99, normalized size = 1.43

$$\frac{1}{8}\,a^{6}c^{3}x^{8}+\frac{2}{7}\,a^{5}c^{3}x^{7}-\frac{1}{6}\,a^{4}c^{3}x^{6}-\frac{4}{5}\,a^{3}c^{3}x^{5}-\frac{1}{4}\,a^{2}c^{3}x^{4}+\frac{2}{3}\,ac^{3}x^{3}+\frac{1}{2}\,c^{3}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^6*c^3*x^8 + 2/7*a^5*c^3*x^7 - 1/6*a^4*c^3*x^6 - 4/5*a^3*c^3*x^5 - 1/4
*a^2*c^3*x^4 + 2/3*a*c^3*x^3 + 1/2*c^3*x^2
```

Fricas [A] time = 1.71556, size = 161, normalized size = 2.33

$$\frac{1}{8}\,a^{6}c^{3}x^{8}+\frac{2}{7}\,a^{5}c^{3}x^{7}-\frac{1}{6}\,a^{4}c^{3}x^{6}-\frac{4}{5}\,a^{3}c^{3}x^{5}-\frac{1}{4}\,a^{2}c^{3}x^{4}+\frac{2}{3}\,ac^{3}x^{3}+\frac{1}{2}\,c^{3}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/8*a^6*c^3*x^8 + 2/7*a^5*c^3*x^7 - 1/6*a^4*c^3*x^6 - 4/5*a^3*c^3*x^5 - 1/4
*a^2*c^3*x^4 + 2/3*a*c^3*x^3 + 1/2*c^3*x^2
```

Sympy [A] time = 0.096283, size = 82, normalized size = 1.19

$$\frac{a^{6}c^{3}x^{8}}{8}+\frac{2a^{5}c^{3}x^{7}}{7}-\frac{a^{4}c^{3}x^{6}}{6}-\frac{4a^{3}c^{3}x^{5}}{5}-\frac{a^{2}c^{3}x^{4}}{4}+\frac{2ac^{3}x^{3}}{3}+\frac{c^{3}x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(-a**2*c*x**2+c)**3,x)
```

```
[Out] a**6*c**3*x**8/8 + 2*a**5*c**3*x**7/7 - a**4*c**3*x**6/6 - 4*a**3*c**3*x**5
/5 - a**2*c**3*x**4/4 + 2*a*c**3*x**3/3 + c**3*x**2/2
```

Giac [A] time = 1.13904, size = 99, normalized size = 1.43

$$\frac{1}{8}a^6c^3x^8 + \frac{2}{7}a^5c^3x^7 - \frac{1}{6}a^4c^3x^6 - \frac{4}{5}a^3c^3x^5 - \frac{1}{4}a^2c^3x^4 + \frac{2}{3}ac^3x^3 + \frac{1}{2}c^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*a^6*c^3*x^8 + 2/7*a^5*c^3*x^7 - 1/6*a^4*c^3*x^6 - 4/5*a^3*c^3*x^5 - 1/4
*a^2*c^3*x^4 + 2/3*a*c^3*x^3 + 1/2*c^3*x^2
```

3.1043 $\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=52

$$\frac{c^3(ax+1)^7}{7a}-\frac{2c^3(ax+1)^6}{3a}+\frac{4c^3(ax+1)^5}{5a}$$

```
[Out] (4*c^3*(1 + a*x)^5)/(5*a) - (2*c^3*(1 + a*x)^6)/(3*a) + (c^3*(1 + a*x)^7)/(
7*a)
```

Rubi [A] time = 0.0499648, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$\frac{c^3(ax+1)^7}{7a}-\frac{2c^3(ax+1)^6}{3a}+\frac{4c^3(ax+1)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (4*c^3*(1 + a*x)^5)/(5*a) - (2*c^3*(1 + a*x)^6)/(3*a) + (c^3*(1 + a*x)^7)/(
7*a)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx&=c^3\int(1-ax)^2(1+ax)^4\,dx\\&=c^3\int\left(4(1+ax)^4-4(1+ax)^5+(1+ax)^6\right)dx\\&=\frac{4c^3(1+ax)^5}{5a}-\frac{2c^3(1+ax)^6}{3a}+\frac{c^3(1+ax)^7}{7a}\end{aligned}$$

Mathematica [A] time = 0.0211414, size = 31, normalized size = 0.6

$$\frac{c^3(ax+1)^5\left(15a^2x^2-40ax+29\right)}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*(1 + a*x)^5*(29 - 40*a*x + 15*a^2*x^2))/(105*a)
```

Maple [A] time = 0.025, size = 52, normalized size = 1.

$$c^{3}\left(\frac{x^{7}a^{6}}{7}+\frac{x^{6}a^{5}}{3}-\frac{x^{5}a^{4}}{5}-x^{4}a^{3}-\frac{x^{3}a^{2}}{3}+ax^{2}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*(1/7*x^7*a^6+1/3*x^6*a^5-1/5*x^5*a^4-x^4*a^3-1/3*x^3*a^2+a*x^2+x)
```

Maxima [A] time = 0.963406, size = 93, normalized size = 1.79

$$\frac{1}{7}a^{6}c^{3}x^{7}+\frac{1}{3}a^{5}c^{3}x^{6}-\frac{1}{5}a^{4}c^{3}x^{5}-a^{3}c^{3}x^{4}-\frac{1}{3}a^{2}c^{3}x^{3}+ac^{3}x^{2}+c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 - 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 - 1/3*a^2
*c^3*x^3 + a*c^3*x^2 + c^3*x
```

Fricas [A] time = 1.68886, size = 142, normalized size = 2.73

$$\frac{1}{7}a^{6}c^{3}x^{7}+\frac{1}{3}a^{5}c^{3}x^{6}-\frac{1}{5}a^{4}c^{3}x^{5}-a^{3}c^{3}x^{4}-\frac{1}{3}a^{2}c^{3}x^{3}+ac^{3}x^{2}+c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 - 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 - 1/3*a^2
*c^3*x^3 + a*c^3*x^2 + c^3*x
```

Sympy [A] time = 0.108122, size = 70, normalized size = 1.35

$$\frac{a^{6}c^{3}x^{7}}{7}+\frac{a^{5}c^{3}x^{6}}{3}-\frac{a^{4}c^{3}x^{5}}{5}-a^{3}c^{3}x^{4}-\frac{a^{2}c^{3}x^{3}}{3}+ac^{3}x^{2}+c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**3,x)
```

```
[Out] a**6*c**3*x**7/7 + a**5*c**3*x**6/3 - a**4*c**3*x**5/5 - a**3*c**3*x**4 - a
**2*c**3*x**3/3 + a*c**3*x**2 + c**3*x
```

Giac [A] time = 1.12947, size = 93, normalized size = 1.79

$$\frac{1}{7}a^6c^3x^7 + \frac{1}{3}a^5c^3x^6 - \frac{1}{5}a^4c^3x^5 - a^3c^3x^4 - \frac{1}{3}a^2c^3x^3 + ac^3x^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 - 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 - 1/3*a^2
*c^3*x^3 + a*c^3*x^2 + c^3*x
```

3.1044 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3}{x}\,dx$

Optimal. Leaf size=79

$$\frac{1}{6}a^6c^3x^6+\frac{2}{5}a^5c^3x^5-\frac{1}{4}a^4c^3x^4-\frac{4}{3}a^3c^3x^3-\frac{1}{2}a^2c^3x^2+2ac^3x+c^3\log(x)$$

```
[Out] 2*a*c^3*x - (a^2*c^3*x^2)/2 - (4*a^3*c^3*x^3)/3 - (a^4*c^3*x^4)/4 + (2*a^5*
c^3*x^5)/5 + (a^6*c^3*x^6)/6 + c^3*Log[x]
```

Rubi [A] time = 0.0856803, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$\frac{1}{6}a^6c^3x^6+\frac{2}{5}a^5c^3x^5-\frac{1}{4}a^4c^3x^4-\frac{4}{3}a^3c^3x^3-\frac{1}{2}a^2c^3x^2+2ac^3x+c^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3)/x,x]
```

```
[Out] 2*a*c^3*x - (a^2*c^3*x^2)/2 - (4*a^3*c^3*x^3)/3 - (a^4*c^3*x^4)/4 + (2*a^5*
c^3*x^5)/5 + (a^6*c^3*x^6)/6 + c^3*Log[x]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3}{x}\,dx &= c^3\int \frac{(1-ax)^2(1+ax)^4}{x}\,dx\\ &= c^3\int \left(2a+\frac{1}{x}-a^2x-4a^3x^2-a^4x^3+2a^5x^4+a^6x^5\right)dx\\ &= 2ac^3x-\frac{1}{2}a^2c^3x^2-\frac{4}{3}a^3c^3x^3-\frac{1}{4}a^4c^3x^4+\frac{2}{5}a^5c^3x^5+\frac{1}{6}a^6c^3x^6+c^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.0290283, size = 51, normalized size = 0.65

$$c^3\left(\frac{1}{60}ax\left(10a^5x^5+24a^4x^4-15a^3x^3-80a^2x^2-30ax+120\right)+\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3)/x,x]
```

```
[Out] c^3*((a*x*(120 - 30*a*x - 80*a^2*x^2 - 15*a^3*x^3 + 24*a^4*x^4 + 10*a^5*x^5
))/60 + Log[x])
```

Maple [A] time = 0.026, size = 70, normalized size = 0.9

$$2\,ac^3x-\frac{a^2c^3x^2}{2}-\frac{4\,a^3c^3x^3}{3}-\frac{a^4c^3x^4}{4}+\frac{2\,a^5c^3x^5}{5}+\frac{a^6c^3x^6}{6}+c^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x,x)
```

```
[Out] 2*a*c^3*x-1/2*a^2*c^3*x^2-4/3*a^3*c^3*x^3-1/4*a^4*c^3*x^4+2/5*a^5*c^3*x^5+1
/6*a^6*c^3*x^6+c^3*ln(x)
```

Maxima [A] time = 0.961181, size = 93, normalized size = 1.18

$$\frac{1}{6}\,a^6c^3x^6+\frac{2}{5}\,a^5c^3x^5-\frac{1}{4}\,a^4c^3x^4-\frac{4}{3}\,a^3c^3x^3-\frac{1}{2}\,a^2c^3x^2+2\,ac^3x+c^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x,x, algorithm="maxima")
```

```
[Out] 1/6*a^6*c^3*x^6 + 2/5*a^5*c^3*x^5 - 1/4*a^4*c^3*x^4 - 4/3*a^3*c^3*x^3 - 1/2
*a^2*c^3*x^2 + 2*a*c^3*x + c^3*log(x)
```

Fricas [A] time = 1.9625, size = 154, normalized size = 1.95

$$\frac{1}{6}\,a^6c^3x^6+\frac{2}{5}\,a^5c^3x^5-\frac{1}{4}\,a^4c^3x^4-\frac{4}{3}\,a^3c^3x^3-\frac{1}{2}\,a^2c^3x^2+2\,ac^3x+c^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x,x, algorithm="fricas")
```

```
[Out] 1/6*a^6*c^3*x^6 + 2/5*a^5*c^3*x^5 - 1/4*a^4*c^3*x^4 - 4/3*a^3*c^3*x^3 - 1/2
*a^2*c^3*x^2 + 2*a*c^3*x + c^3*log(x)
```

Sympy [A] time = 0.32037, size = 76, normalized size = 0.96

$$\frac{a^6c^3x^6}{6}+\frac{2a^5c^3x^5}{5}-\frac{a^4c^3x^4}{4}-\frac{4a^3c^3x^3}{3}-\frac{a^2c^3x^2}{2}+2ac^3x+c^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**3/x,x)
```

```
[Out] a**6*c**3*x**6/6 + 2*a**5*c**3*x**5/5 - a**4*c**3*x**4/4 - 4*a**3*c**3*x**3
/3 - a**2*c**3*x**2/2 + 2*a*c**3*x + c**3*log(x)
```

Giac [A] time = 1.18015, size = 95, normalized size = 1.2

$$\frac{1}{6}a^6c^3x^6 + \frac{2}{5}a^5c^3x^5 - \frac{1}{4}a^4c^3x^4 - \frac{4}{3}a^3c^3x^3 - \frac{1}{2}a^2c^3x^2 + 2\,ac^3x + c^3\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x,x, algorithm="giac")
```

```
[Out] 1/6*a^6*c^3*x^6 + 2/5*a^5*c^3*x^5 - 1/4*a^4*c^3*x^4 - 4/3*a^3*c^3*x^3 - 1/2
*a^2*c^3*x^2 + 2*a*c^3*x + c^3*log(abs(x))
```

3.1045 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^3}{x^2}\,dx$

Optimal. Leaf size=76

$$\frac{1}{5}a^6c^3x^5+\frac{1}{2}a^5c^3x^4-\frac{1}{3}a^4c^3x^3-2a^3c^3x^2-a^2c^3x+2ac^3\log(x)-\frac{c^3}{x}$$

```
[Out] -(c^3/x) - a^2*c^3*x - 2*a^3*c^3*x^2 - (a^4*c^3*x^3)/3 + (a^5*c^3*x^4)/2 +
(a^6*c^3*x^5)/5 + 2*a*c^3*Log[x]
```

Rubi [A] time = 0.0908948, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$\frac{1}{5}a^6c^3x^5+\frac{1}{2}a^5c^3x^4-\frac{1}{3}a^4c^3x^3-2a^3c^3x^2-a^2c^3x+2ac^3\log(x)-\frac{c^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3)/x^2,x]
```

```
[Out] -(c^3/x) - a^2*c^3*x - 2*a^3*c^3*x^2 - (a^4*c^3*x^3)/3 + (a^5*c^3*x^4)/2 +
(a^6*c^3*x^5)/5 + 2*a*c^3*Log[x]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3}{x^2}\,dx &= c^3\int \frac{(1-ax)^2(1+ax)^4}{x^2}\,dx\\ &= c^3\int \left(-a^2+\frac{1}{x^2}+\frac{2a}{x}-4a^3x-a^4x^2+2a^5x^3+a^6x^4\right)dx\\ &= -\frac{c^3}{x}-a^2c^3x-2a^3c^3x^2-\frac{1}{3}a^4c^3x^3+\frac{1}{2}a^5c^3x^4+\frac{1}{5}a^6c^3x^5+2ac^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.025573, size = 59, normalized size = 0.78

$$c^3\left(\frac{a^6x^5}{5}+\frac{a^5x^4}{2}-\frac{a^4x^3}{3}-2a^3x^2-a^2x+2a\log(x)-\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3)/x^2,x]
```

```
[Out] c^3*(-x^(-1) - a^2*x - 2*a^3*x^2 - (a^4*x^3)/3 + (a^5*x^4)/2 + (a^6*x^5)/5
+ 2*a*Log[x])
```

Maple [A] time = 0.031, size = 71, normalized size = 0.9

$$-\frac{c^3}{x} - a^2c^3x - 2\,a^3c^3x^2 - \frac{a^4c^3x^3}{3} + \frac{a^5c^3x^4}{2} + \frac{a^6c^3x^5}{5} + 2\,ac^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^2,x)
```

```
[Out] -c^3/x-a^2*c^3*x-2*a^3*c^3*x^2-1/3*a^4*c^3*x^3+1/2*a^5*c^3*x^4+1/5*a^6*c^3*
x^5+2*a*c^3*ln(x)
```

Maxima [A] time = 0.956938, size = 95, normalized size = 1.25

$$\frac{1}{5}\,a^6c^3x^5 + \frac{1}{2}\,a^5c^3x^4 - \frac{1}{3}\,a^4c^3x^3 - 2\,a^3c^3x^2 - a^2c^3x + 2\,ac^3\log(x) - \frac{c^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^2,x, algorithm="maxima"
)
```

```
[Out] 1/5*a^6*c^3*x^5 + 1/2*a^5*c^3*x^4 - 1/3*a^4*c^3*x^3 - 2*a^3*c^3*x^2 - a^2*c
^3*x + 2*a*c^3*log(x) - c^3/x
```

Fricas [A] time = 1.93189, size = 163, normalized size = 2.14

$$\frac{6\,a^6c^3x^6 + 15\,a^5c^3x^5 - 10\,a^4c^3x^4 - 60\,a^3c^3x^3 - 30\,a^2c^3x^2 + 60\,ac^3x\log(x) - 30\,c^3}{30\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^2,x, algorithm="fricas"
)
```

```
[Out] 1/30*(6*a^6*c^3*x^6 + 15*a^5*c^3*x^5 - 10*a^4*c^3*x^4 - 60*a^3*c^3*x^3 - 30
*a^2*c^3*x^2 + 60*a*c^3*x*log(x) - 30*c^3)/x
```

Sympy [A] time = 0.343045, size = 70, normalized size = 0.92

$$\frac{a^6c^3x^5}{5} + \frac{a^5c^3x^4}{2} - \frac{a^4c^3x^3}{3} - 2a^3c^3x^2 - a^2c^3x + 2ac^3\log(x) - \frac{c^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**3/x**2,x)
```

```
[Out] a**6*c**3*x**5/5 + a**5*c**3*x**4/2 - a**4*c**3*x**3/3 - 2*a**3*c**3*x**2 -
 a**2*c**3*x + 2*a*c**3*log(x) - c**3/x
```

Giac [A] time = 1.14991, size = 96, normalized size = 1.26

$$\frac{1}{5}a^6c^3x^5 + \frac{1}{2}a^5c^3x^4 - \frac{1}{3}a^4c^3x^3 - 2a^3c^3x^2 - a^2c^3x + 2ac^3\log(|x|) - \frac{c^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/5*a^6*c^3*x^5 + 1/2*a^5*c^3*x^4 - 1/3*a^4*c^3*x^3 - 2*a^3*c^3*x^2 - a^2*c
^3*x + 2*a*c^3*log(abs(x)) - c^3/x
```

3.1046 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^3}{x^3}\,dx$

Optimal. Leaf size=78

$$\frac{1}{4}a^6c^3x^4+\frac{2}{3}a^5c^3x^3-\frac{1}{2}a^4c^3x^2-4a^3c^3x-a^2c^3\log(x)-\frac{2ac^3}{x}-\frac{c^3}{2x^2}$$

```
[Out] -c^3/(2*x^2) - (2*a*c^3)/x - 4*a^3*c^3*x - (a^4*c^3*x^2)/2 + (2*a^5*c^3*x^3
)/3 + (a^6*c^3*x^4)/4 - a^2*c^3*Log[x]
```

Rubi [A] time = 0.095062, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$\frac{1}{4}a^6c^3x^4+\frac{2}{3}a^5c^3x^3-\frac{1}{2}a^4c^3x^2-4a^3c^3x-a^2c^3\log(x)-\frac{2ac^3}{x}-\frac{c^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3)/x^3,x]
```

```
[Out] -c^3/(2*x^2) - (2*a*c^3)/x - 4*a^3*c^3*x - (a^4*c^3*x^2)/2 + (2*a^5*c^3*x^3
)/3 + (a^6*c^3*x^4)/4 - a^2*c^3*Log[x]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3}{x^3}\,dx &= c^3\int \frac{(1-ax)^2(1+ax)^4}{x^3}\,dx\\ &= c^3\int \left(-4a^3+\frac{1}{x^3}+\frac{2a}{x^2}-\frac{a^2}{x}-a^4x+2a^5x^2+a^6x^3\right)dx\\ &= -\frac{c^3}{2x^2}-\frac{2ac^3}{x}-4a^3c^3x-\frac{1}{2}a^4c^3x^2+\frac{2}{3}a^5c^3x^3+\frac{1}{4}a^6c^3x^4-a^2c^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.0239966, size = 58, normalized size = 0.74

$$\frac{c^3\left(3a^6x^6+8a^5x^5-6a^4x^4-48a^3x^3-12a^2x^2\log(x)-24ax-6\right)}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3)/x^3,x]
```

```
[Out] (c^3*(-6 - 24*a*x - 48*a^3*x^3 - 6*a^4*x^4 + 8*a^5*x^5 + 3*a^6*x^6 - 12*a^2
*x^2*Log[x]))/(12*x^2)
```

Maple [A] time = 0.033, size = 71, normalized size = 0.9

$$-\frac{c^3}{2\,x^2}-2\,\frac{ac^3}{x}-4\,a^3c^3x-\frac{a^4c^3x^2}{2}+\frac{2\,a^5c^3x^3}{3}+\frac{a^6c^3x^4}{4}-a^2c^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^3,x)
```

```
[Out] -1/2*c^3/x^2-2*a*c^3/x-4*a^3*c^3*x-1/2*a^4*c^3*x^2+2/3*a^5*c^3*x^3+1/4*a^6*
c^3*x^4-a^2*c^3*ln(x)
```

Maxima [A] time = 0.965557, size = 93, normalized size = 1.19

$$\frac{1}{4}\,a^6c^3x^4+\frac{2}{3}\,a^5c^3x^3-\frac{1}{2}\,a^4c^3x^2-4\,a^3c^3x-a^2c^3\log(x)-\frac{4\,ac^3x+c^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^3,x, algorithm="maxima"
)
```

```
[Out] 1/4*a^6*c^3*x^4 + 2/3*a^5*c^3*x^3 - 1/2*a^4*c^3*x^2 - 4*a^3*c^3*x - a^2*c^3
*log(x) - 1/2*(4*a*c^3*x + c^3)/x^2
```

Fricas [A] time = 2.01215, size = 162, normalized size = 2.08

$$\frac{3\,a^6c^3x^6+8\,a^5c^3x^5-6\,a^4c^3x^4-48\,a^3c^3x^3-12\,a^2c^3x^2\log(x)-24\,ac^3x-6\,c^3}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^3,x, algorithm="fricas"
)
```

```
[Out] 1/12*(3*a^6*c^3*x^6 + 8*a^5*c^3*x^5 - 6*a^4*c^3*x^4 - 48*a^3*c^3*x^3 - 12*a
^2*c^3*x^2*log(x) - 24*a*c^3*x - 6*c^3)/x^2
```

Sympy [A] time = 0.370847, size = 73, normalized size = 0.94

$$\frac{a^6c^3x^4}{4}+\frac{2a^5c^3x^3}{3}-\frac{a^4c^3x^2}{2}-4a^3c^3x-a^2c^3\log(x)-\frac{4ac^3x+c^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**3/x**3,x)
```

```
[Out] a**6*c**3*x**4/4 + 2*a**5*c**3*x**3/3 - a**4*c**3*x**2/2 - 4*a**3*c**3*x -
a**2*c**3*log(x) - (4*a*c**3*x + c**3)/(2*x**2)
```

Giac [A] time = 1.16721, size = 95, normalized size = 1.22

$$\frac{1}{4}a^6c^3x^4 + \frac{2}{3}a^5c^3x^3 - \frac{1}{2}a^4c^3x^2 - 4a^3c^3x - a^2c^3\log(|x|) - \frac{4ac^3x + c^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/4*a^6*c^3*x^4 + 2/3*a^5*c^3*x^3 - 1/2*a^4*c^3*x^2 - 4*a^3*c^3*x - a^2*c^3
*log(abs(x)) - 1/2*(4*a*c^3*x + c^3)/x^2
```

3.1047 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^3}{x^4}\,dx$

Optimal. Leaf size=72

$$\frac{1}{3}a^6c^3x^3+a^5c^3x^2-a^4c^3x+\frac{a^2c^3}{x}-4a^3c^3\log(x)-\frac{ac^3}{x^2}-\frac{c^3}{3x^3}$$

```
[Out] -c^3/(3*x^3) - (a*c^3)/x^2 + (a^2*c^3)/x - a^4*c^3*x + a^5*c^3*x^2 + (a^6*c
^3*x^3)/3 - 4*a^3*c^3*Log[x]
```

Rubi [A] time = 0.0929093, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$\frac{1}{3}a^6c^3x^3+a^5c^3x^2-a^4c^3x+\frac{a^2c^3}{x}-4a^3c^3\log(x)-\frac{ac^3}{x^2}-\frac{c^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3)/x^4,x]
```

```
[Out] -c^3/(3*x^3) - (a*c^3)/x^2 + (a^2*c^3)/x - a^4*c^3*x + a^5*c^3*x^2 + (a^6*c
^3*x^3)/3 - 4*a^3*c^3*Log[x]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3}{x^4}\,dx &= c^3\int \frac{(1-ax)^2(1+ax)^4}{x^4}\,dx\\ &= c^3\int \left(-a^4+\frac{1}{x^4}+\frac{2a}{x^3}-\frac{a^2}{x^2}-\frac{4a^3}{x}+2a^5x+a^6x^2\right)dx\\ &= -\frac{c^3}{3x^3}-\frac{ac^3}{x^2}+\frac{a^2c^3}{x}-a^4c^3x+a^5c^3x^2+\frac{1}{3}a^6c^3x^3-4a^3c^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.0293488, size = 55, normalized size = 0.76

$$c^3\left(\frac{a^6x^3}{3}+a^5x^2-a^4x+\frac{a^2}{x}-4a^3\log(x)-\frac{a}{x^2}-\frac{1}{3x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3)/x^4,x]
```

```
[Out] c^3*(-1/(3*x^3) - a/x^2 + a^2/x - a^4*x + a^5*x^2 + (a^6*x^3)/3 - 4*a^3*Log
[x])
```

Maple [A] time = 0.031, size = 69, normalized size = 1.

$$-\frac{c^3}{3\,x^3}-\frac{ac^3}{x^2}+\frac{a^2c^3}{x}-a^4c^3x+a^5c^3x^2+\frac{a^6c^3x^3}{3}-4\,a^3c^3\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^4,x)
```

```
[Out] -1/3*c^3/x^3-a*c^3/x^2+a^2*c^3/x-a^4*c^3*x+a^5*c^3*x^2+1/3*a^6*c^3*x^3-4*a^
3*c^3*ln(x)
```

Maxima [A] time = 0.942724, size = 95, normalized size = 1.32

$$\frac{1}{3}\,a^6c^3x^3+a^5c^3x^2-a^4c^3x-4\,a^3c^3\log\left(x\right)+\frac{3\,a^2c^3x^2-3\,ac^3x-c^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^4,x, algorithm="maxima"
)
```

```
[Out] 1/3*a^6*c^3*x^3 + a^5*c^3*x^2 - a^4*c^3*x - 4*a^3*c^3*log(x) + 1/3*(3*a^2*c
^3*x^2 - 3*a*c^3*x - c^3)/x^3
```

Fricas [A] time = 2.03493, size = 153, normalized size = 2.12

$$\frac{a^6c^3x^6+3\,a^5c^3x^5-3\,a^4c^3x^4-12\,a^3c^3x^3\log\left(x\right)+3\,a^2c^3x^2-3\,ac^3x-c^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^4,x, algorithm="fricas"
)
```

```
[Out] 1/3*(a^6*c^3*x^6 + 3*a^5*c^3*x^5 - 3*a^4*c^3*x^4 - 12*a^3*c^3*x^3*log(x) +
3*a^2*c^3*x^2 - 3*a*c^3*x - c^3)/x^3
```

Sympy [A] time = 0.409236, size = 70, normalized size = 0.97

$$\frac{a^6c^3x^3}{3}+a^5c^3x^2-a^4c^3x-4a^3c^3\log\left(x\right)+\frac{3a^2c^3x^2-3ac^3x-c^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**3/x**4,x)
```

```
[Out] a**6*c**3*x**3/3 + a**5*c**3*x**2 - a**4*c**3*x - 4*a**3*c**3*log(x) + (3*a
**2*c**3*x**2 - 3*a*c**3*x - c**3)/(3*x**3)
```

Giac [A] time = 1.15008, size = 96, normalized size = 1.33

$$\frac{1}{3}a^6c^3x^3 + a^5c^3x^2 - a^4c^3x - 4a^3c^3\log(|x|) + \frac{3a^2c^3x^2 - 3ac^3x - c^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/3*a^6*c^3*x^3 + a^5*c^3*x^2 - a^4*c^3*x - 4*a^3*c^3*log(abs(x)) + 1/3*(3*
a^2*c^3*x^2 - 3*a*c^3*x - c^3)/x^3
```

3.1048 $\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=69

$$-\frac{c^4(ax+1)^9}{9a}+\frac{3c^4(ax+1)^8}{4a}-\frac{12c^4(ax+1)^7}{7a}+\frac{4c^4(ax+1)^6}{3a}$$

```
[Out] (4*c^4*(1 + a*x)^6)/(3*a) - (12*c^4*(1 + a*x)^7)/(7*a) + (3*c^4*(1 + a*x)^8
)/(4*a) - (c^4*(1 + a*x)^9)/(9*a)
```

Rubi [A] time = 0.056424, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$-\frac{c^4(ax+1)^9}{9a}+\frac{3c^4(ax+1)^8}{4a}-\frac{12c^4(ax+1)^7}{7a}+\frac{4c^4(ax+1)^6}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (4*c^4*(1 + a*x)^6)/(3*a) - (12*c^4*(1 + a*x)^7)/(7*a) + (3*c^4*(1 + a*x)^8
)/(4*a) - (c^4*(1 + a*x)^9)/(9*a)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx &= c^4\int(1-ax)^3(1+ax)^5\,dx\\ &= c^4\int\left(8(1+ax)^5-12(1+ax)^6+6(1+ax)^7-(1+ax)^8\right)dx\\ &= \frac{4c^4(1+ax)^6}{3a}-\frac{12c^4(1+ax)^7}{7a}+\frac{3c^4(1+ax)^8}{4a}-\frac{c^4(1+ax)^9}{9a}\end{aligned}$$

Mathematica [A] time = 0.024709, size = 39, normalized size = 0.57

$$-\frac{c^4(ax+1)^6\left(28a^3x^3-105a^2x^2+138ax-65\right)}{252a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] -(c^4*(1 + a*x)^6*(-65 + 138*a*x - 105*a^2*x^2 + 28*a^3*x^3))/(252*a)
```

Maple [A] time = 0.024, size = 59, normalized size = 0.9

$$c^{4}\left(-\frac{x^{9}a^{8}}{9}-\frac{a^{7}x^{8}}{4}+\frac{2\,x^{7}a^{6}}{7}+x^{6}a^{5}-\frac{3\,x^{4}a^{3}}{2}-\frac{2\,x^{3}a^{2}}{3}+ax^{2}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^4,x)
```

```
[Out] c^4*(-1/9*x^9*a^8-1/4*a^7*x^8+2/7*x^7*a^6+x^6*a^5-3/2*x^4*a^3-2/3*x^3*a^2+a
*x^2+x)
```

Maxima [A] time = 0.944406, size = 107, normalized size = 1.55

$$-\frac{1}{9}\,a^{8}c^{4}x^{9}-\frac{1}{4}\,a^{7}c^{4}x^{8}+\frac{2}{7}\,a^{6}c^{4}x^{7}+a^{5}c^{4}x^{6}-\frac{3}{2}\,a^{3}c^{4}x^{4}-\frac{2}{3}\,a^{2}c^{4}x^{3}+ac^{4}x^{2}+c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] -1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 + 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^
3*c^4*x^4 - 2/3*a^2*c^4*x^3 + a*c^4*x^2 + c^4*x
```

Fricas [A] time = 1.92478, size = 167, normalized size = 2.42

$$-\frac{1}{9}\,a^{8}c^{4}x^{9}-\frac{1}{4}\,a^{7}c^{4}x^{8}+\frac{2}{7}\,a^{6}c^{4}x^{7}+a^{5}c^{4}x^{6}-\frac{3}{2}\,a^{3}c^{4}x^{4}-\frac{2}{3}\,a^{2}c^{4}x^{3}+ac^{4}x^{2}+c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 + 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^
3*c^4*x^4 - 2/3*a^2*c^4*x^3 + a*c^4*x^2 + c^4*x
```

Sympy [A] time = 0.111312, size = 87, normalized size = 1.26

$$-\frac{a^{8}c^{4}x^{9}}{9}-\frac{a^{7}c^{4}x^{8}}{4}+\frac{2a^{6}c^{4}x^{7}}{7}+a^{5}c^{4}x^{6}-\frac{3a^{3}c^{4}x^{4}}{2}-\frac{2a^{2}c^{4}x^{3}}{3}+ac^{4}x^{2}+c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] -a**8*c**4*x**9/9 - a**7*c**4*x**8/4 + 2*a**6*c**4*x**7/7 + a**5*c**4*x**6
- 3*a**3*c**4*x**4/2 - 2*a**2*c**4*x**3/3 + a*c**4*x**2 + c**4*x
```

Giac [A] time = 1.18944, size = 107, normalized size = 1.55

$$-\frac{1}{9}a^8c^4x^9 - \frac{1}{4}a^7c^4x^8 + \frac{2}{7}a^6c^4x^7 + a^5c^4x^6 - \frac{3}{2}a^3c^4x^4 - \frac{2}{3}a^2c^4x^3 + ac^4x^2 + c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 + 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^
3*c^4*x^4 - 2/3*a^2*c^4*x^3 + a*c^4*x^2 + c^4*x
```

3.1049 $\int \frac{e^{2\tanh^{-1}(ax)}x^4}{c-a^2cx^2}\,dx$

Optimal. Leaf size=63

$$\frac{x^3}{3a^2c}+\frac{x^2}{a^3c}+\frac{3x}{a^4c}+\frac{1}{a^5c(1-ax)}+\frac{4\log(1-ax)}{a^5c}$$

```
[Out] (3*x)/(a^4*c) + x^2/(a^3*c) + x^3/(3*a^2*c) + 1/(a^5*c*(1 - a*x)) + (4*Log[
1 - a*x])/(a^5*c)
```

Rubi [A] time = 0.111975, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 43}

$$\frac{x^3}{3a^2c}+\frac{x^2}{a^3c}+\frac{3x}{a^4c}+\frac{1}{a^5c(1-ax)}+\frac{4\log(1-ax)}{a^5c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2),x]
```

```
[Out] (3*x)/(a^4*c) + x^2/(a^3*c) + x^3/(3*a^2*c) + 1/(a^5*c*(1 - a*x)) + (4*Log[
1 - a*x])/(a^5*c)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x^4}{c-a^2cx^2}\,dx &= \frac{\int \frac{x^4}{(1-ax)^2}\,dx}{c}\\
&= \frac{\int \left(\frac{3}{a^4}+\frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{1}{a^4(-1+ax)^2}+\frac{4}{a^4(-1+ax)}\right)dx}{c}\\
&= \frac{3x}{a^4c}+\frac{x^2}{a^3c}+\frac{x^3}{3a^2c}+\frac{1}{a^5c(1-ax)}+\frac{4\log(1-ax)}{a^5c}
\end{aligned}$$

Mathematica [A] time = 0.037359, size = 49, normalized size = 0.78

$$\frac{a^3x^3+3a^2x^2+9ax+\frac{3}{1-ax}+12\log(1-ax)}{3a^5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2),x]
```

```
[Out] (9*a*x + 3*a^2*x^2 + a^3*x^3 + 3/(1 - a*x) + 12*Log[1 - a*x])/(3*a^5*c)
```

Maple [A] time = 0.034, size = 61, normalized size = 1.

$$\frac{x^3}{3\,a^2c}+\frac{x^2}{a^3c}+3\,\frac{x}{a^4c}-\frac{1}{ca^5\,(ax-1)}+4\,\frac{\ln(ax-1)}{ca^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c),x)
```

```
[Out] 1/3*x^3/a^2/c+x^2/a^3/c+3*x/a^4/c-1/c/a^5/(a*x-1)+4/c/a^5*ln(a*x-1)
```

Maxima [A] time = 0.959523, size = 77, normalized size = 1.22

$$-\frac{1}{a^6cx-a^5c}+\frac{a^2x^3+3\,ax^2+9\,x}{3\,a^4c}+\frac{4\,\log(ax-1)}{a^5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/(a^6*c*x - a^5*c) + 1/3*(a^2*x^3 + 3*a*x^2 + 9*x)/(a^4*c) + 4*log(a*x -
1)/(a^5*c)
```

Fricas [A] time = 1.94782, size = 131, normalized size = 2.08

$$\frac{a^4x^4+2\,a^3x^3+6\,a^2x^2-9\,ax+12\,(ax-1)\log(ax-1)-3}{3\left(a^6cx-a^5c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/3*(a^4*x^4 + 2*a^3*x^3 + 6*a^2*x^2 - 9*a*x + 12*(a*x - 1)*log(a*x - 1) -
3)/(a^6*c*x - a^5*c)
```

Sympy [A] time = 0.357478, size = 53, normalized size = 0.84

$$-\frac{1}{a^6cx-a^5c}+\frac{x^3}{3a^2c}+\frac{x^2}{a^3c}+\frac{3x}{a^4c}+\frac{4\log(ax-1)}{a^5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**4/(-a**2*c*x**2+c),x)
```

```
[Out] -1/(a**6*c*x - a**5*c) + x**3/(3*a**2*c) + x**2/(a**3*c) + 3*x/(a**4*c) + 4
*log(a*x - 1)/(a**5*c)
```

Giac [A] time = 1.18937, size = 95, normalized size = 1.51

$$\frac{4\,\log\left(|ax-1|\right)}{a^5c}-\frac{1}{(ax-1)a^5c}+\frac{a^4c^2x^3+3\,a^3c^2x^2+9\,a^2c^2x}{3\,a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 4*log(abs(a*x - 1))/(a^5*c) - 1/((a*x - 1)*a^5*c) + 1/3*(a^4*c^2*x^3 + 3*a^
3*c^2*x^2 + 9*a^2*c^2*x)/(a^6*c^3)
```

3.1050 $\int \frac{e^{2\tanh^{-1}(ax)}x^3}{c-a^2cx^2}\,dx$

Optimal. Leaf size=53

$$\frac{x^2}{2a^2c}+\frac{2x}{a^3c}+\frac{1}{a^4c(1-ax)}+\frac{3\log(1-ax)}{a^4c}$$

```
[Out] (2*x)/(a^3*c) + x^2/(2*a^2*c) + 1/(a^4*c*(1 - a*x)) + (3*Log[1 - a*x])/(a^4
*c)
```

Rubi [A] time = 0.10409, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 43}

$$\frac{x^2}{2a^2c}+\frac{2x}{a^3c}+\frac{1}{a^4c(1-ax)}+\frac{3\log(1-ax)}{a^4c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2),x]
```

```
[Out] (2*x)/(a^3*c) + x^2/(2*a^2*c) + 1/(a^4*c*(1 - a*x)) + (3*Log[1 - a*x])/(a^4
*c)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x^3}{c-a^2cx^2}\,dx &= \frac{\int \frac{x^3}{(1-ax)^2}\,dx}{c}\\
&= \frac{\int \left(\frac{2}{a^3}+\frac{x}{a^2}+\frac{1}{a^3(-1+ax)^2}+\frac{3}{a^3(-1+ax)}\right)dx}{c}\\
&= \frac{2x}{a^3c}+\frac{x^2}{2a^2c}+\frac{1}{a^4c(1-ax)}+\frac{3\log(1-ax)}{a^4c}
\end{aligned}$$

Mathematica [A] time = 0.0299233, size = 41, normalized size = 0.77

$$\frac{a^2x^2+4ax+\frac{2}{1-ax}+6\log(1-ax)}{2a^4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2),x]
```

```
[Out] (4*a*x + a^2*x^2 + 2/(1 - a*x) + 6*Log[1 - a*x])/(2*a^4*c)
```

Maple [A] time = 0.034, size = 51, normalized size = 1.

$$\frac{x^2}{2\,a^2c}+2\,\frac{x}{a^3c}-\frac{1}{ca^4\,(ax-1)}+3\,\frac{\ln(ax-1)}{ca^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c),x)
```

```
[Out] 1/2*x^2/a^2/c+2*x/a^3/c-1/c/a^4/(a*x-1)+3/c/a^4*ln(a*x-1)
```

Maxima [A] time = 0.965163, size = 66, normalized size = 1.25

$$-\frac{1}{a^5cx-a^4c}+\frac{ax^2+4\,x}{2\,a^3c}+\frac{3\,\log(ax-1)}{a^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/(a^5*c*x - a^4*c) + 1/2*(a*x^2 + 4*x)/(a^3*c) + 3*log(a*x - 1)/(a^4*c)
```

Fricas [A] time = 1.90143, size = 113, normalized size = 2.13

$$\frac{a^3x^3+3\,a^2x^2-4\,ax+6\,(ax-1)\log(ax-1)-2}{2\left(a^5cx-a^4c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*x^3 + 3*a^2*x^2 - 4*a*x + 6*(a*x - 1)*log(a*x - 1) - 2)/(a^5*c*x -
a^4*c)
```

Sympy [A] time = 0.35384, size = 44, normalized size = 0.83

$$-\frac{1}{a^5cx-a^4c}+\frac{x^2}{2a^2c}+\frac{2x}{a^3c}+\frac{3\log(ax-1)}{a^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3/(-a**2*c*x**2+c),x)
```

```
[Out] -1/(a**5*c*x - a**4*c) + x**2/(2*a**2*c) + 2*x/(a**3*c) + 3*log(a*x - 1)/(a
**4*c)
```

Giac [A] time = 1.16419, size = 72, normalized size = 1.36

$$\frac{3\,\log\left(|ax-1|\right)}{a^4c}+\frac{a^2cx^2+4\,acx}{2\,a^4c^2}-\frac{1}{(ax-1)a^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 3*log(abs(a*x - 1))/(a^4*c) + 1/2*(a^2*c*x^2 + 4*a*c*x)/(a^4*c^2) - 1/((a*x
 - 1)*a^4*c)
```

3.1051 $\int \frac{e^{2\tanh^{-1}(ax)}x^2}{c-a^2cx^2}\,dx$

Optimal. Leaf size=39

$$\frac{x}{a^2c}+\frac{1}{a^3c(1-ax)}+\frac{2\log(1-ax)}{a^3c}$$

```
[Out] x/(a^2*c) + 1/(a^3*c*(1 - a*x)) + (2*Log[1 - a*x])/(a^3*c)
```

Rubi [A] time = 0.0955794, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 43}

$$\frac{x}{a^2c}+\frac{1}{a^3c(1-ax)}+\frac{2\log(1-ax)}{a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2),x]
```

```
[Out] x/(a^2*c) + 1/(a^3*c*(1 - a*x)) + (2*Log[1 - a*x])/(a^3*c)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}x^2}{c-a^2cx^2}\,dx &= \frac{\int \frac{x^2}{(1-ax)^2}\,dx}{c}\\ &= \frac{\int \left(\frac{1}{a^2}+\frac{1}{a^2(-1+ax)^2}+\frac{2}{a^2(-1+ax)}\right)dx}{c}\\ &= \frac{x}{a^2c}+\frac{1}{a^3c(1-ax)}+\frac{2\log(1-ax)}{a^3c}\end{aligned}$$

Mathematica [A] time = 0.024749, size = 28, normalized size = 0.72

$$\frac{ax+\frac{1}{1-ax}+2\log(1-ax)}{a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2),x]
```

```
[Out] (a*x + (1 - a*x)^(-1) + 2*Log[1 - a*x])/(a^3*c)
```

Maple [A] time = 0.035, size = 39, normalized size = 1.

$$\frac{x}{a^2c} - \frac{1}{a^3c\,(ax-1)} + 2\,\frac{\ln(ax-1)}{a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c),x)
```

```
[Out] x/a^2/c-1/c/a^3/(a*x-1)+2/c/a^3*ln(a*x-1)
```

Maxima [A] time = 0.953028, size = 54, normalized size = 1.38

$$-\frac{1}{a^4cx - a^3c} + \frac{x}{a^2c} + \frac{2\,\log(ax-1)}{a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/(a^4*c*x - a^3*c) + x/(a^2*c) + 2*log(a*x - 1)/(a^3*c)
```

Fricas [A] time = 2.02149, size = 89, normalized size = 2.28

$$\frac{a^2x^2 - ax + 2\,(ax-1)\log(ax-1) - 1}{a^4cx - a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] (a^2*x^2 - a*x + 2*(a*x - 1)*log(a*x - 1) - 1)/(a^4*c*x - a^3*c)
```

Sympy [A] time = 0.330464, size = 32, normalized size = 0.82

$$-\frac{1}{a^4cx - a^3c} + \frac{x}{a^2c} + \frac{2\log(ax-1)}{a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2/(-a**2*c*x**2+c),x)
```

```
[Out] -1/(a**4*c*x - a**3*c) + x/(a**2*c) + 2*log(a*x - 1)/(a**3*c)
```

Giac [A] time = 1.13432, size = 53, normalized size = 1.36

$$\frac{x}{a^2c} + \frac{2 \log\left(|ax - 1|\right)}{a^3c} - \frac{1}{(ax - 1)a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] x/(a^2*c) + 2*log(abs(a*x - 1))/(a^3*c) - 1/((a*x - 1)*a^3*c)
```

3.1052 $\int \frac{e^{2\tanh^{-1}(ax)}x}{c-a^2cx^2}\,dx$

Optimal. Leaf size=30

$$\frac{1}{a^2c(1-ax)}+\frac{\log(1-ax)}{a^2c}$$

```
[Out] 1/(a^2*c*(1 - a*x)) + Log[1 - a*x]/(a^2*c)
```

Rubi [A] time = 0.06539, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 43}

$$\frac{1}{a^2c(1-ax)}+\frac{\log(1-ax)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x)/(c - a^2*c*x^2),x]
```

```
[Out] 1/(a^2*c*(1 - a*x)) + Log[1 - a*x]/(a^2*c)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x}{c-a^2cx^2}\,dx &= \frac{\int \frac{x}{(1-ax)^2}\,dx}{c}\\
&= \frac{\int \left(\frac{1}{a(-1+ax)^2}+\frac{1}{a(-1+ax)}\right)dx}{c}\\
&= \frac{1}{a^2c(1-ax)}+\frac{\log(1-ax)}{a^2c}
\end{aligned}$$

Mathematica [A] time = 0.0188966, size = 23, normalized size = 0.77

$$\frac{\frac{1}{1-ax}+\log(1-ax)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x)/(c - a^2*c*x^2),x]
```

```
[Out] ((1 - a*x)^(-1) + Log[1 - a*x])/(a^2*c)
```

Maple [A] time = 0.03, size = 30, normalized size = 1.

$$-\frac{1}{a^2c\,(ax-1)}+\frac{\ln(ax-1)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c),x)
```

```
[Out] -1/c/a^2/(a*x-1)+1/c/a^2*ln(a*x-1)
```

Maxima [A] time = 0.953644, size = 42, normalized size = 1.4

$$-\frac{1}{a^3cx-a^2c}+\frac{\log(ax-1)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/(a^3*c*x - a^2*c) + log(a*x - 1)/(a^2*c)
```

Fricas [A] time = 1.92952, size = 65, normalized size = 2.17

$$\frac{(ax-1)\log(ax-1)-1}{a^3cx-a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] ((a*x - 1)*log(a*x - 1) - 1)/(a^3*c*x - a^2*c)
```

Sympy [A] time = 0.302677, size = 24, normalized size = 0.8

$$-\frac{1}{a^3cx-a^2c}+\frac{\log(ax-1)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x/(-a**2*c*x**2+c),x)
```

```
[Out] -1/(a**3*c*x - a**2*c) + log(a*x - 1)/(a**2*c)
```

Giac [A] time = 1.13642, size = 41, normalized size = 1.37

$$\frac{\log\left(|ax-1|\right)}{a^2c} - \frac{1}{(ax-1)a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] log(abs(a*x - 1))/(a^2*c) - 1/((a*x - 1)*a^2*c)
```

3.1053 $\int \frac{e^{2\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=15

$$\frac{1}{ac(1-ax)}$$

```
[Out] 1/(a*c*(1 - a*x))
```

Rubi [A] time = 0.0331008, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 32}

$$\frac{1}{ac(1-ax)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] 1/(a*c*(1 - a*x))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx &= \frac{\int \frac{1}{(1-ax)^2}\,dx}{c}\\ &= \frac{1}{ac(1-ax)}\end{aligned}$$

Mathematica [C] time = 0.0121428, size = 18, normalized size = 1.2

$$\frac{e^{2\tanh^{-1}(ax)}}{2ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] E^(2*ArcTanh[a*x])/(2*a*c)
```

Maple [A] time = 0.026, size = 16, normalized size = 1.1

$$-\frac{1}{ac\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c),x)
```

```
[Out] -1/c/a/(a*x-1)
```

Maxima [A] time = 0.966117, size = 20, normalized size = 1.33

$$-\frac{1}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/(a^2*c*x - a*c)
```

Fricas [A] time = 1.97788, size = 27, normalized size = 1.8

$$-\frac{1}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/(a^2*c*x - a*c)
```

Sympy [A] time = 0.297726, size = 12, normalized size = 0.8

$$-\frac{1}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a**2*c*x**2+c),x)
```

```
[Out] -1/(a**2*c*x - a*c)
```

Giac [A] time = 1.14754, size = 20, normalized size = 1.33

$$-\frac{1}{(ax-1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -1/((a*x - 1)*a*c)
```

3.1054 $\int \frac{e^{2\tanh^{-1}(ax)}}{x(c-a^2cx^2)}\,dx$

Optimal. Leaf size=31

$$\frac{1}{c(1-ax)}-\frac{\log(1-ax)}{c}+\frac{\log(x)}{c}$$

```
[Out] 1/(c*(1 - a*x)) + Log[x]/c - Log[1 - a*x]/c
```

Rubi [A] time = 0.0866635, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 44}

$$\frac{1}{c(1-ax)}-\frac{\log(1-ax)}{c}+\frac{\log(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x*(c - a^2*c*x^2)),x]
```

```
[Out] 1/(c*(1 - a*x)) + Log[x]/c - Log[1 - a*x]/c
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)}\,dx &= \frac{\int \frac{1}{x(1-ax)^2}\,dx}{c}\\
&= \frac{\int \left(\frac{1}{x}+\frac{a}{(-1+ax)^2}-\frac{a}{-1+ax}\right)dx}{c}\\
&= \frac{1}{c(1-ax)}+\frac{\log(x)}{c}-\frac{\log(1-ax)}{c}
\end{aligned}$$

Mathematica [A] time = 0.0198567, size = 24, normalized size = 0.77

$$\frac{\frac{1}{1-ax}-\log(1-ax)+\log(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x*(c - a^2*c*x^2)),x]
```

```
[Out] ((1 - a*x)^(-1) + Log[x] - Log[1 - a*x])/c
```

Maple [A] time = 0.031, size = 31, normalized size = 1.

$$\frac{\ln(x)}{c} - \frac{1}{c(ax-1)} - \frac{\ln(ax-1)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c),x)
```

```
[Out] ln(x)/c-1/c/(a*x-1)-1/c*ln(a*x-1)
```

Maxima [A] time = 0.958175, size = 41, normalized size = 1.32

$$-\frac{\log(ax-1)}{c} + \frac{\log(x)}{c} - \frac{1}{acx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -log(a*x - 1)/c + log(x)/c - 1/(a*c*x - c)
```

Fricas [A] time = 2.03213, size = 84, normalized size = 2.71

$$-\frac{(ax-1)\log(ax-1) - (ax-1)\log(x) + 1}{acx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -((a*x - 1)*log(a*x - 1) - (a*x - 1)*log(x) + 1)/(a*c*x - c)
```

Sympy [A] time = 0.364284, size = 19, normalized size = 0.61

$$-\frac{1}{acx-c} + \frac{\log(x) - \log\left(x - \frac{1}{a}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x/(-a**2*c*x**2+c),x)
```

```
[Out] -1/(a*c*x - c) + (log(x) - log(x - 1/a))/c
```

Giac [A] time = 1.20644, size = 43, normalized size = 1.39

$$-\frac{\log\left(|ax-1|\right)}{c}+\frac{\log\left(|x|\right)}{c}-\frac{1}{(ax-1)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -log(abs(a*x - 1))/c + log(abs(x))/c - 1/((a*x - 1)*c)
```

3.1055 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)}\,dx$

Optimal. Leaf size=43

$$\frac{a}{c(1-ax)}+\frac{2a\log(x)}{c}-\frac{2a\log(1-ax)}{c}-\frac{1}{cx}$$

```
[Out] -(1/(c*x)) + a/(c*(1 - a*x)) + (2*a*Log[x])/c - (2*a*Log[1 - a*x])/c
```

Rubi [A] time = 0.0938533, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 44}

$$\frac{a}{c(1-ax)}+\frac{2a\log(x)}{c}-\frac{2a\log(1-ax)}{c}-\frac{1}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)),x]
```

```
[Out] -(1/(c*x)) + a/(c*(1 - a*x)) + (2*a*Log[x])/c - (2*a*Log[1 - a*x])/c
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)}\,dx &= \frac{\int \frac{1}{x^2(1-ax)^2}\,dx}{c}\\
&= \frac{\int \left(\frac{1}{x^2}+\frac{2a}{x}+\frac{a^2}{(-1+ax)^2}-\frac{2a^2}{-1+ax}\right)dx}{c}\\
&= -\frac{1}{cx}+\frac{a}{c(1-ax)}+\frac{2a\log(x)}{c}-\frac{2a\log(1-ax)}{c}
\end{aligned}$$

Mathematica [A] time = 0.0301129, size = 35, normalized size = 0.81

$$\frac{\frac{a}{1-ax}+2a\log(x)-2a\log(1-ax)-\frac{1}{x}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)),x]
```

```
[Out] (-x^(-1) + a/(1 - a*x) + 2*a*Log[x] - 2*a*Log[1 - a*x])/c
```

Maple [A] time = 0.033, size = 43, normalized size = 1.

$$-\frac{1}{cx}+2\,\frac{a\ln\left(x\right)}{c}-\frac{a}{c\left(ax-1\right)}-2\,\frac{a\ln\left(ax-1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c),x)
```

```
[Out] -1/c/x+2*a*ln(x)/c-1/c*a/(a*x-1)-2/c*a*ln(a*x-1)
```

Maxima [A] time = 0.966991, size = 57, normalized size = 1.33

$$-\frac{2\,a\log\left(ax-1\right)}{c}+\frac{2\,a\log\left(x\right)}{c}-\frac{2\,ax-1}{acx^{2}-cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -2*a*log(a*x - 1)/c + 2*a*log(x)/c - (2*a*x - 1)/(a*c*x^2 - c*x)
```

Fricas [A] time = 2.00823, size = 122, normalized size = 2.84

$$-\frac{2\,ax+2\,\left(a^{2}x^{2}-ax\right)\log\left(ax-1\right)-2\,\left(a^{2}x^{2}-ax\right)\log\left(x\right)-1}{acx^{2}-cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -(2*a*x + 2*(a^2*x^2 - a*x)*log(a*x - 1) - 2*(a^2*x^2 - a*x)*log(x) - 1)/(a
*c*x^2 - c*x)
```

Sympy [A] time = 0.411368, size = 31, normalized size = 0.72

$$\frac{2a\left(\log\left(x\right)-\log\left(x-\frac{1}{a}\right)\right)}{c}-\frac{2ax-1}{acx^{2}-cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**2/(-a**2*c*x**2+c),x)
```

```
[Out] 2*a*(log(x) - log(x - 1/a))/c - (2*a*x - 1)/(a*c*x**2 - c*x)
```

Giac [A] time = 1.13968, size = 61, normalized size = 1.42

$$-\frac{2\,a\log\left(|ax-1|\right)}{c}+\frac{2\,a\log\left(|x|\right)}{c}-\frac{2\,ax-1}{\left(ax^2-x\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -2*a*log(abs(a*x - 1))/c + 2*a*log(abs(x))/c - (2*a*x - 1)/((a*x^2 - x)*c)
```

3.1056 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)}\,dx$

Optimal. Leaf size=60

$$\frac{a^2}{c(1-ax)}+\frac{3a^2\log(x)}{c}-\frac{3a^2\log(1-ax)}{c}-\frac{2a}{cx}-\frac{1}{2cx^2}$$

```
[Out] -1/(2*c*x^2) - (2*a)/(c*x) + a^2/(c*(1 - a*x)) + (3*a^2*Log[x])/c - (3*a^2*
Log[1 - a*x])/c
```

Rubi [A] time = 0.101499, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 44}

$$\frac{a^2}{c(1-ax)}+\frac{3a^2\log(x)}{c}-\frac{3a^2\log(1-ax)}{c}-\frac{2a}{cx}-\frac{1}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)),x]
```

```
[Out] -1/(2*c*x^2) - (2*a)/(c*x) + a^2/(c*(1 - a*x)) + (3*a^2*Log[x])/c - (3*a^2*
Log[1 - a*x])/c
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)}\,dx &= \frac{\int \frac{1}{x^3(1-ax)^2}\,dx}{c}\\ &= \frac{\int \left(\frac{1}{x^3}+\frac{2a}{x^2}+\frac{3a^2}{x}+\frac{a^3}{(-1+ax)^2}-\frac{3a^3}{-1+ax}\right)dx}{c}\\ &= -\frac{1}{2cx^2}-\frac{2a}{cx}+\frac{a^2}{c(1-ax)}+\frac{3a^2\log(x)}{c}-\frac{3a^2\log(1-ax)}{c}\end{aligned}$$

Mathematica [A] time = 0.0477582, size = 52, normalized size = 0.87

$$\frac{\frac{-6a^2x^2+3ax+1}{x^2(ax-1)}+6a^2\log(x)-6a^2\log(1-ax)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)),x]
```

```
[Out] ((1 + 3*a*x - 6*a^2*x^2)/(x^2*(-1 + a*x)) + 6*a^2*Log[x] - 6*a^2*Log[1 - a*
x])/(2*c)
```

Maple [A] time = 0.035, size = 58, normalized size = 1.

$$-\frac{1}{2\,c{x}^{2}}-2\,{\frac{a}{cx}}+3\,{\frac{{a}^{2}\ln \left(x \right) }{c}}-{\frac{{a}^{2}}{c \left(ax-1 \right) }}-3\,{\frac{{a}^{2}\ln \left(ax-1 \right) }{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c),x)
```

```
[Out] -1/2/c/x^2-2*a/c/x+3*a^2*ln(x)/c-1/c*a^2/(a*x-1)-3/c*a^2*ln(a*x-1)
```

Maxima [A] time = 0.956291, size = 76, normalized size = 1.27

$$-\frac{3\,a^{2}\log \left(a x - 1\right)}{c} + \frac{3\,a^{2}\log \left(x\right)}{c} - \frac{6\,a^{2} x^{2} - 3\,a x - 1}{2\,{\left(a c x^{3} - c x^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -3*a^2*log(a*x - 1)/c + 3*a^2*log(x)/c - 1/2*(6*a^2*x^2 - 3*a*x - 1)/(a*c*x
^3 - c*x^2)
```

Fricas [A] time = 2.03348, size = 157, normalized size = 2.62

$$-\frac{6\,a^{2} x^{2} - 3\,a x + 6\,{\left(a^{3} x^{3} - a^{2} x^{2}\right)}\log \left(a x - 1\right) - 6\,{\left(a^{3} x^{3} - a^{2} x^{2}\right)}\log \left(x\right) - 1}{2\,{\left(a c x^{3} - c x^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(6*a^2*x^2 - 3*a*x + 6*(a^3*x^3 - a^2*x^2)*log(a*x - 1) - 6*(a^3*x^3 -
 a^2*x^2)*log(x) - 1)/(a*c*x^3 - c*x^2)
```

Sympy [A] time = 0.444763, size = 46, normalized size = 0.77

$$\frac{3a^{2}\left(\log{\left(x \right)} - \log{\left(x - \frac{1}{a} \right)}\right)}{c} - \frac{6a^{2}x^{2} - 3ax - 1}{2acx^{3} - 2cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**3/(-a**2*c*x**2+c),x)
```

```
[Out] 3*a**2*(log(x) - log(x - 1/a))/c - (6*a**2*x**2 - 3*a*x - 1)/(2*a*c*x**3 -
2*c*x**2)
```

Giac [A] time = 1.12318, size = 76, normalized size = 1.27

$$-\frac{3\,a^2\log\left(|ax-1|\right)}{c}+\frac{3\,a^2\log\left(|x|\right)}{c}-\frac{6\,a^2x^2-3\,ax-1}{2\,(ax-1)cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -3*a^2*log(abs(a*x - 1))/c + 3*a^2*log(abs(x))/c - 1/2*(6*a^2*x^2 - 3*a*x -
 1)/((a*x - 1)*c*x^2)
```

3.1057 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^4(c-a^2cx^2)}\,dx$

Optimal. Leaf size=71

$$\frac{a^3}{c(1-ax)}-\frac{3a^2}{cx}+\frac{4a^3\log(x)}{c}-\frac{4a^3\log(1-ax)}{c}-\frac{a}{cx^2}-\frac{1}{3cx^3}$$

```
[Out] -1/(3*c*x^3) - a/(c*x^2) - (3*a^2)/(c*x) + a^3/(c*(1 - a*x)) + (4*a^3*Log[x
])/c - (4*a^3*Log[1 - a*x])/c
```

Rubi [A] time = 0.109847, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 44}

$$\frac{a^3}{c(1-ax)}-\frac{3a^2}{cx}+\frac{4a^3\log(x)}{c}-\frac{4a^3\log(1-ax)}{c}-\frac{a}{cx^2}-\frac{1}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^4*(c - a^2*c*x^2)),x]
```

```
[Out] -1/(3*c*x^3) - a/(c*x^2) - (3*a^2)/(c*x) + a^3/(c*(1 - a*x)) + (4*a^3*Log[x
])/c - (4*a^3*Log[1 - a*x])/c
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x^4\left(c-a^2cx^2\right)}\,dx &= \frac{\int \frac{1}{x^4(1-ax)^2}\,dx}{c}\\
&= \frac{\int \left(\frac{1}{x^4}+\frac{2a}{x^3}+\frac{3a^2}{x^2}+\frac{4a^3}{x}+\frac{a^4}{(-1+ax)^2}-\frac{4a^4}{-1+ax}\right)dx}{c}\\
&= -\frac{1}{3cx^3}-\frac{a}{cx^2}-\frac{3a^2}{cx}+\frac{a^3}{c(1-ax)}+\frac{4a^3\log(x)}{c}-\frac{4a^3\log(1-ax)}{c}
\end{aligned}$$

Mathematica [A] time = 0.0393452, size = 71, normalized size = 1.

$$\frac{a^3}{c(1-ax)}-\frac{3a^2}{cx}+\frac{4a^3\log(x)}{c}-\frac{4a^3\log(1-ax)}{c}-\frac{a}{cx^2}-\frac{1}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^4*(c - a^2*c*x^2)),x]
```

```
[Out] -1/(3*c*x^3) - a/(c*x^2) - (3*a^2)/(c*x) + a^3/(c*(1 - a*x)) + (4*a^3*Log[x
])/c - (4*a^3*Log[1 - a*x])/c
```

Maple [A] time = 0.034, size = 69, normalized size = 1.

$$-\frac{1}{3cx^3}-\frac{a}{cx^2}-3\frac{a^2}{cx}+4\frac{a^3\ln(x)}{c}-\frac{a^3}{c(ax-1)}-4\frac{a^3\ln(ax-1)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c),x)
```

```
[Out] -1/3/c/x^3-a/c/x^2-3*a^2/c/x+4*a^3*ln(x)/c-1/c*a^3/(a*x-1)-4/c*a^3*ln(a*x-1
)
```

Maxima [A] time = 0.94526, size = 86, normalized size = 1.21

$$-\frac{4a^3\log(ax-1)}{c}+\frac{4a^3\log(x)}{c}-\frac{12a^3x^3-6a^2x^2-2ax-1}{3\left(acx^4-cx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -4*a^3*log(a*x - 1)/c + 4*a^3*log(x)/c - 1/3*(12*a^3*x^3 - 6*a^2*x^2 - 2*a*
x - 1)/(a*c*x^4 - c*x^3)
```

Fricas [A] time = 2.04781, size = 177, normalized size = 2.49

$$-\frac{12a^3x^3-6a^2x^2-2ax+12\left(a^4x^4-a^3x^3\right)\log(ax-1)-12\left(a^4x^4-a^3x^3\right)\log(x)-1}{3\left(acx^4-cx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/3*(12*a^3*x^3 - 6*a^2*x^2 - 2*a*x + 12*(a^4*x^4 - a^3*x^3)*log(a*x - 1)
- 12*(a^4*x^4 - a^3*x^3)*log(x) - 1)/(a*c*x^4 - c*x^3)
```

Sympy [A] time = 0.492618, size = 54, normalized size = 0.76

$$\frac{4a^3\left(\log(x)-\log\left(x-\frac{1}{a}\right)\right)}{c}-\frac{12a^3x^3-6a^2x^2-2ax-1}{3acx^4-3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**4/(-a**2*c*x**2+c),x)
```

```
[Out] 4*a**3*(log(x) - log(x - 1/a))/c - (12*a**3*x**3 - 6*a**2*x**2 - 2*a*x - 1)
/(3*a*c*x**4 - 3*c*x**3)
```

Giac [A] time = 1.15145, size = 86, normalized size = 1.21

$$-\frac{4\,a^3\log\left(|ax-1|\right)}{c}+\frac{4\,a^3\log\left(|x|\right)}{c}-\frac{12\,a^3x^3-6\,a^2x^2-2\,ax-1}{3\,(ax-1)cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -4*a^3*log(abs(a*x - 1))/c + 4*a^3*log(abs(x))/c - 1/3*(12*a^3*x^3 - 6*a^2*
x^2 - 2*a*x - 1)/((a*x - 1)*c*x^3)
```

3.1058 $\int \frac{e^{2\tanh^{-1}(ax)}x^4}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=79

$$-\frac{x}{a^4c^2}-\frac{7}{4a^5c^2(1-ax)}+\frac{1}{4a^5c^2(1-ax)^2}-\frac{17\log(1-ax)}{8a^5c^2}+\frac{\log(ax+1)}{8a^5c^2}$$

```
[Out] -(x/(a^4*c^2)) + 1/(4*a^5*c^2*(1 - a*x)^2) - 7/(4*a^5*c^2*(1 - a*x)) - (17*
Log[1 - a*x])/(8*a^5*c^2) + Log[1 + a*x]/(8*a^5*c^2)
```

Rubi [A] time = 0.118692, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$-\frac{x}{a^4c^2}-\frac{7}{4a^5c^2(1-ax)}+\frac{1}{4a^5c^2(1-ax)^2}-\frac{17\log(1-ax)}{8a^5c^2}+\frac{\log(ax+1)}{8a^5c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(x/(a^4*c^2)) + 1/(4*a^5*c^2*(1 - a*x)^2) - 7/(4*a^5*c^2*(1 - a*x)) - (17*
Log[1 - a*x])/(8*a^5*c^2) + Log[1 + a*x]/(8*a^5*c^2)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}x^4}{(c-a^2cx^2)^2}\,dx &= \frac{\int \frac{x^4}{(1-ax)^3(1+ax)}\,dx}{c^2}\\ &= \frac{\int \left(-\frac{1}{a^4}-\frac{1}{2a^4(-1+ax)^3}-\frac{7}{4a^4(-1+ax)^2}-\frac{17}{8a^4(-1+ax)}+\frac{1}{8a^4(1+ax)}\right)dx}{c^2}\\ &= -\frac{x}{a^4c^2}+\frac{1}{4a^5c^2(1-ax)^2}-\frac{7}{4a^5c^2(1-ax)}-\frac{17\log(1-ax)}{8a^5c^2}+\frac{\log(1+ax)}{8a^5c^2}\end{aligned}$$

Mathematica [A] time = 0.0445352, size = 68, normalized size = 0.86

$$\frac{-\frac{x}{a^4}-\frac{7}{4a^5(1-ax)}+\frac{1}{4a^5(1-ax)^2}-\frac{17\log(1-ax)}{8a^5}+\frac{\log(ax+1)}{8a^5}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-(x/a^4) + 1/(4*a^5*(1 - a*x)^2) - 7/(4*a^5*(1 - a*x)) - (17*Log[1 - a*x])
/(8*a^5) + Log[1 + a*x]/(8*a^5))/c^2
```

Maple [A] time = 0.036, size = 69, normalized size = 0.9

$$-\frac{x}{a^4c^2}+\frac{\ln(ax+1)}{8a^5c^2}+\frac{7}{4a^5c^2(ax-1)}-\frac{17\ln(ax-1)}{8a^5c^2}+\frac{1}{4a^5c^2(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -x/a^4/c^2+1/8*ln(a*x+1)/a^5/c^2+7/4/c^2/a^5/(a*x-1)-17/8/c^2/a^5*ln(a*x-1)
+1/4/c^2/a^5/(a*x-1)^2
```

Maxima [A] time = 0.956094, size = 101, normalized size = 1.28

$$\frac{7ax-6}{4\left(a^7c^2x^2-2a^6c^2x+a^5c^2\right)}-\frac{x}{a^4c^2}+\frac{\log(ax+1)}{8a^5c^2}-\frac{17\log(ax-1)}{8a^5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] 1/4*(7*a*x - 6)/(a^7*c^2*x^2 - 2*a^6*c^2*x + a^5*c^2) - x/(a^4*c^2) + 1/8*l
og(a*x + 1)/(a^5*c^2) - 17/8*log(a*x - 1)/(a^5*c^2)
```

Fricas [A] time = 2.04113, size = 215, normalized size = 2.72

$$-\frac{8a^3x^3-16a^2x^2-6ax-\left(a^2x^2-2ax+1\right)\log(ax+1)+17\left(a^2x^2-2ax+1\right)\log(ax-1)+12}{8\left(a^7c^2x^2-2a^6c^2x+a^5c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
 17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^7*c^2*x^2 - 2*a^6*c^2*x + a
^5*c^2)
```

Sympy [A] time = 0.594508, size = 70, normalized size = 0.89

$$\frac{7ax-6}{4a^7c^2x^2-8a^6c^2x+4a^5c^2}-\frac{x}{a^4c^2}-\frac{\frac{17\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**4/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (7*a*x - 6)/(4*a**7*c**2*x**2 - 8*a**6*c**2*x + 4*a**5*c**2) - x/(a**4*c**2
) - (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**2)
```

Giac [A] time = 1.16385, size = 82, normalized size = 1.04

$$-\frac{x}{a^4c^2}+\frac{\log(|ax+1|)}{8\,a^5c^2}-\frac{17\,\log(|ax-1|)}{8\,a^5c^2}+\frac{7\,ax-6}{4\,(ax-1)^2a^5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -x/(a^4*c^2) + 1/8*log(abs(a*x + 1))/(a^5*c^2) - 17/8*log(abs(a*x - 1))/(a^
5*c^2) + 1/4*(7*a*x - 6)/((a*x - 1)^2*a^5*c^2)
```

3.1059 $\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=70

$$-\frac{5}{4a^4c^2(1-ax)}+\frac{1}{4a^4c^2(1-ax)^2}-\frac{7\log(1-ax)}{8a^4c^2}-\frac{\log(ax+1)}{8a^4c^2}$$

```
[Out] 1/(4*a^4*c^2*(1 - a*x)^2) - 5/(4*a^4*c^2*(1 - a*x)) - (7*Log[1 - a*x])/(8*a
^4*c^2) - Log[1 + a*x]/(8*a^4*c^2)
```

Rubi [A] time = 0.106701, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$-\frac{5}{4a^4c^2(1-ax)}+\frac{1}{4a^4c^2(1-ax)^2}-\frac{7\log(1-ax)}{8a^4c^2}-\frac{\log(ax+1)}{8a^4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^2,x]
```

```
[Out] 1/(4*a^4*c^2*(1 - a*x)^2) - 5/(4*a^4*c^2*(1 - a*x)) - (7*Log[1 - a*x])/(8*a
^4*c^2) - Log[1 + a*x]/(8*a^4*c^2)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^2}\,dx &= \frac{\int \frac{x^3}{(1-ax)^3(1+ax)}\,dx}{c^2}\\
&= \frac{\int \left(-\frac{1}{2a^3(-1+ax)^3}-\frac{5}{4a^3(-1+ax)^2}-\frac{7}{8a^3(-1+ax)}-\frac{1}{8a^3(1+ax)}\right)dx}{c^2}\\
&= \frac{1}{4a^4c^2(1-ax)^2}-\frac{5}{4a^4c^2(1-ax)}-\frac{7\log(1-ax)}{8a^4c^2}-\frac{\log(1+ax)}{8a^4c^2}
\end{aligned}$$

Mathematica [A] time = 0.0358648, size = 53, normalized size = 0.76

$$-\frac{-10ax+7(ax-1)^2\log(1-ax)+(ax-1)^2\log(ax+1)+8}{8a^4c^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(8 - 10*a*x + 7*(-1 + a*x)^2*Log[1 - a*x] + (-1 + a*x)^2*Log[1 + a*x])/(8*
a^4*c^2*(-1 + a*x)^2)
```

Maple [A] time = 0.033, size = 60, normalized size = 0.9

$$-\frac{\ln(ax+1)}{8a^4c^2}+\frac{1}{4a^4c^2(ax-1)^2}+\frac{5}{4a^4c^2(ax-1)}-\frac{7\ln(ax-1)}{8a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/8*ln(a*x+1)/a^4/c^2+1/4/c^2/a^4/(a*x-1)^2+5/4/c^2/a^4/(a*x-1)-7/8/c^2/a^
4*ln(a*x-1)
```

Maxima [A] time = 0.958823, size = 89, normalized size = 1.27

$$\frac{5ax-4}{4\left(a^6c^2x^2-2a^5c^2x+a^4c^2\right)}-\frac{\log(ax+1)}{8a^4c^2}-\frac{7\log(ax-1)}{8a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] 1/4*(5*a*x - 4)/(a^6*c^2*x^2 - 2*a^5*c^2*x + a^4*c^2) - 1/8*log(a*x + 1)/(a
^4*c^2) - 7/8*log(a*x - 1)/(a^4*c^2)
```

Fricas [A] time = 2.22161, size = 178, normalized size = 2.54

$$\frac{10ax-\left(a^2x^2-2ax+1\right)\log(ax+1)-7\left(a^2x^2-2ax+1\right)\log(ax-1)-8}{8\left(a^6c^2x^2-2a^5c^2x+a^4c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] 1/8*(10*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) - 7*(a^2*x^2 - 2*a*x + 1)*
log(a*x - 1) - 8)/(a^6*c^2*x^2 - 2*a^5*c^2*x + a^4*c^2)
```

Sympy [A] time = 0.537136, size = 61, normalized size = 0.87

$$\frac{5ax-4}{4a^6c^2x^2-8a^5c^2x+4a^4c^2}-\frac{\frac{7\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (5*a*x - 4)/(4*a**6*c**2*x**2 - 8*a**5*c**2*x + 4*a**4*c**2) - (7*log(x - 1
/a)/8 + log(x + 1/a)/8)/(a**4*c**2)
```

Giac [A] time = 1.141, size = 70, normalized size = 1.

$$-\frac{\log\left(|ax+1|\right)}{8\,a^4c^2}-\frac{7\,\log\left(|ax-1|\right)}{8\,a^4c^2}+\frac{5\,ax-4}{4\,(ax-1)^2a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a^4*c^2) - 7/8*log(abs(a*x - 1))/(a^4*c^2) + 1/4*(5
*a*x - 4)/((a*x - 1)^2*a^4*c^2)
```

3.1060 $\int \frac{e^{2\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=51

$$-\frac{3}{4a^3c^2(1-ax)}+\frac{1}{4a^3c^2(1-ax)^2}+\frac{\tanh^{-1}(ax)}{4a^3c^2}$$

```
[Out] 1/(4*a^3*c^2*(1 - a*x)^2) - 3/(4*a^3*c^2*(1 - a*x)) + ArcTanh[a*x]/(4*a^3*c
^2)
```

Rubi [A] time = 0.105425, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6150, 88, 207}

$$-\frac{3}{4a^3c^2(1-ax)}+\frac{1}{4a^3c^2(1-ax)^2}+\frac{\tanh^{-1}(ax)}{4a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^2,x]
```

```
[Out] 1/(4*a^3*c^2*(1 - a*x)^2) - 3/(4*a^3*c^2*(1 - a*x)) + ArcTanh[a*x]/(4*a^3*c
^2)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^2}\,dx = \frac{\int \frac{x^2}{(1-ax)^3(1+ax)}\,dx}{c^2}$$
$$= \frac{\int \left(-\frac{1}{2a^2(-1+ax)^3} - \frac{3}{4a^2(-1+ax)^2} - \frac{1}{4a^2(-1+a^2x^2)}\right)dx}{c^2}$$
$$= \frac{1}{4a^3c^2(1-ax)^2} - \frac{3}{4a^3c^2(1-ax)} - \frac{\int \frac{1}{-1+a^2x^2}\,dx}{4a^2c^2}$$
$$= \frac{1}{4a^3c^2(1-ax)^2} - \frac{3}{4a^3c^2(1-ax)} + \frac{\tanh^{-1}(ax)}{4a^3c^2}$$

Mathematica [A] time = 0.026789, size = 35, normalized size = 0.69

$$\frac{3ax + (ax-1)^2\tanh^{-1}(ax) - 2}{4a^3c^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-2 + 3*a*x + (-1 + a*x)^2*ArcTanh[a*x])/(4*a^3*c^2*(-1 + a*x)^2)
```

Maple [A] time = 0.032, size = 60, normalized size = 1.2

$$\frac{\ln(ax+1)}{8c^2a^3} + \frac{1}{4c^2a^3(ax-1)^2} + \frac{3}{4c^2a^3(ax-1)} - \frac{\ln(ax-1)}{8c^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/8/c^2/a^3*ln(a*x+1)+1/4/c^2/a^3/(a*x-1)^2+3/4/c^2/a^3/(a*x-1)-1/8/c^2/a^3
*ln(a*x-1)
```

Maxima [A] time = 0.947161, size = 89, normalized size = 1.75

$$\frac{3ax-2}{4(a^5c^2x^2 - 2a^4c^2x + a^3c^2)} + \frac{\log(ax+1)}{8a^3c^2} - \frac{\log(ax-1)}{8a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] 1/4*(3*a*x - 2)/(a^5*c^2*x^2 - 2*a^4*c^2*x + a^3*c^2) + 1/8*log(a*x + 1)/(a
^3*c^2) - 1/8*log(a*x - 1)/(a^3*c^2)
```

Fricas [A] time = 2.32425, size = 174, normalized size = 3.41

$$\frac{6\,ax + \left(a^2x^2 - 2\,ax + 1\right)\log\left(ax + 1\right) - \left(a^2x^2 - 2\,ax + 1\right)\log\left(ax - 1\right) - 4}{8\left(a^5c^2x^2 - 2\,a^4c^2x + a^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] 1/8*(6*a*x + (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) - (a^2*x^2 - 2*a*x + 1)*log
(a*x - 1) - 4)/(a^5*c^2*x^2 - 2*a^4*c^2*x + a^3*c^2)
```

Sympy [A] time = 0.468091, size = 60, normalized size = 1.18

$$\frac{3ax - 2}{4a^5c^2x^2 - 8a^4c^2x + 4a^3c^2} - \frac{\frac{\log\left(x - \frac{1}{a}\right)}{8} - \frac{\log\left(x + \frac{1}{a}\right)}{8}}{a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (3*a*x - 2)/(4*a**5*c**2*x**2 - 8*a**4*c**2*x + 4*a**3*c**2) - (log(x - 1/a
)/8 - log(x + 1/a)/8)/(a**3*c**2)
```

Giac [A] time = 1.1554, size = 70, normalized size = 1.37

$$\frac{\log\left(|ax + 1|\right)}{8\,a^3c^2} - \frac{\log\left(|ax - 1|\right)}{8\,a^3c^2} + \frac{3\,ax - 2}{4\left(ax - 1\right)^2a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*log(abs(a*x + 1))/(a^3*c^2) - 1/8*log(abs(a*x - 1))/(a^3*c^2) + 1/4*(3*
a*x - 2)/((a*x - 1)^2*a^3*c^2)
```

3.1061 $\int \frac{e^{2\tanh^{-1}(ax)}x}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{4a^2c^2(1-ax)}+\frac{1}{4a^2c^2(1-ax)^2}-\frac{\tanh^{-1}(ax)}{4a^2c^2}$$

```
[Out] 1/(4*a^2*c^2*(1 - a*x)^2) - 1/(4*a^2*c^2*(1 - a*x)) - ArcTanh[a*x]/(4*a^2*c
^2)
```

Rubi [A] time = 0.0795022, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6150, 77, 207}

$$-\frac{1}{4a^2c^2(1-ax)}+\frac{1}{4a^2c^2(1-ax)^2}-\frac{\tanh^{-1}(ax)}{4a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^2,x]
```

```
[Out] 1/(4*a^2*c^2*(1 - a*x)^2) - 1/(4*a^2*c^2*(1 - a*x)) - ArcTanh[a*x]/(4*a^2*c
^2)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x}{(c-a^2cx^2)^2}\,dx = \frac{\int \frac{x}{(1-ax)^3(1+ax)}\,dx}{c^2}$$
$$= \frac{\int \left(-\frac{1}{2a(-1+ax)^3} - \frac{1}{4a(-1+ax)^2} + \frac{1}{4a(-1+a^2x^2)}\right)dx}{c^2}$$
$$= \frac{1}{4a^2c^2(1-ax)^2} - \frac{1}{4a^2c^2(1-ax)} + \frac{\int \frac{1}{-1+a^2x^2}\,dx}{4ac^2}$$
$$= \frac{1}{4a^2c^2(1-ax)^2} - \frac{1}{4a^2c^2(1-ax)} - \frac{\tanh^{-1}(ax)}{4a^2c^2}$$

Mathematica [A] time = 0.0247767, size = 34, normalized size = 0.67

$$\frac{ax-(ax-1)^2\tanh^{-1}(ax)}{4a^2c^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (a*x - (-1 + a*x)^2*ArcTanh[a*x])/(4*a^2*c^2*(-1 + a*x)^2)
```

Maple [A] time = 0.033, size = 60, normalized size = 1.2

$$-\frac{\ln(ax+1)}{8a^2c^2} + \frac{1}{4a^2c^2(ax-1)^2} + \frac{1}{4a^2c^2(ax-1)} + \frac{\ln(ax-1)}{8a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/8/c^2/a^2*ln(a*x+1)+1/4/c^2/a^2/(a*x-1)^2+1/4/c^2/a^2/(a*x-1)+1/8/c^2/a^
2*ln(a*x-1)
```

Maxima [A] time = 0.962135, size = 80, normalized size = 1.57

$$\frac{x}{4(a^3c^2x^2-2a^2c^2x+ac^2)} - \frac{\log(ax+1)}{8a^2c^2} + \frac{\log(ax-1)}{8a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) - 1/8*log(a*x + 1)/(a^2*c^2) + 1/
8*log(a*x - 1)/(a^2*c^2)
```

Fricas [A] time = 2.32214, size = 169, normalized size = 3.31

$$\frac{2\,ax - \left(a^2x^2 - 2\,ax + 1\right)\log\left(ax + 1\right) + \left(a^2x^2 - 2\,ax + 1\right)\log\left(ax - 1\right)}{8\left(a^4c^2x^2 - 2\,a^3c^2x + a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*log
(a*x - 1))/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2)
```

Sympy [A] time = 0.449353, size = 53, normalized size = 1.04

$$\frac{x}{4a^3c^2x^2 - 8a^2c^2x + 4ac^2} - \frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8} + \frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x/(-a**2*c*x**2+c)**2,x)
```

```
[Out] x/(4*a**3*c**2*x**2 - 8*a**2*c**2*x + 4*a*c**2) - (-log(x - 1/a)/8 + log(x
+ 1/a)/8)/(a**2*c**2)
```

Giac [A] time = 1.14136, size = 63, normalized size = 1.24

$$-\frac{\log\left(|ax + 1|\right)}{8\,a^2c^2} + \frac{\log\left(|ax - 1|\right)}{8\,a^2c^2} + \frac{x}{4\,(ax - 1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a^2*c^2) + 1/8*log(abs(a*x - 1))/(a^2*c^2) + 1/4*x/
((a*x - 1)^2*a*c^2)
```

3.1062 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=51

$$\frac{1}{4ac^2(1-ax)}+\frac{1}{4ac^2(1-ax)^2}+\frac{\tanh^{-1}(ax)}{4ac^2}$$

```
[Out] 1/(4*a*c^2*(1 - a*x)^2) + 1/(4*a*c^2*(1 - a*x)) + ArcTanh[a*x]/(4*a*c^2)
```

Rubi [A] time = 0.052897, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6140, 44, 207}

$$\frac{1}{4ac^2(1-ax)}+\frac{1}{4ac^2(1-ax)^2}+\frac{\tanh^{-1}(ax)}{4ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] 1/(4*a*c^2*(1 - a*x)^2) + 1/(4*a*c^2*(1 - a*x)) + ArcTanh[a*x]/(4*a*c^2)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{1}{(1-ax)^3(1+ax)}\,dx}{c^2}\\
&= \frac{\int \left(-\frac{1}{2(-1+ax)^3}+\frac{1}{4(-1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^2}\\
&= \frac{1}{4ac^2(1-ax)^2}+\frac{1}{4ac^2(1-ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^2}\\
&= \frac{1}{4ac^2(1-ax)^2}+\frac{1}{4ac^2(1-ax)}+\frac{\tanh^{-1}(ax)}{4ac^2}
\end{aligned}$$

Mathematica [A] time = 0.0194994, size = 35, normalized size = 0.69

$$\frac{-ax+(ax-1)^2\tanh^{-1}(ax)+2}{4ac^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] (2 - a*x + (-1 + a*x)^2*ArcTanh[a*x])/(4*a*c^2*(-1 + a*x)^2)
```

Maple [A] time = 0.032, size = 60, normalized size = 1.2

$$\frac{\ln(ax+1)}{8\,ac^2}+\frac{1}{4\,ac^2\,(ax-1)^2}-\frac{1}{4\,ac^2\,(ax-1)}-\frac{\ln(ax-1)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/8*ln(a*x+1)/a/c^2+1/4/c^2/a/(a*x-1)^2-1/4/c^2/a/(a*x-1)-1/8/c^2/a*ln(a*x-
1)
```

Maxima [A] time = 0.972146, size = 85, normalized size = 1.67

$$-\frac{ax-2}{4\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}+\frac{\log(ax+1)}{8\,ac^2}-\frac{\log(ax-1)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a*x - 2)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) + 1/8*log(a*x + 1)/(a*c^
2) - 1/8*log(a*x - 1)/(a*c^2)
```

Fricas [A] time = 2.28199, size = 173, normalized size = 3.39

$$-\frac{2\,ax-\left(a^2x^2-2\,ax+1\right)\log(ax+1)+\left(a^2x^2-2\,ax+1\right)\log(ax-1)-4}{8\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*lo
g(a*x - 1) - 4)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [A] time = 0.474745, size = 56, normalized size = 1.1

$$-\frac{ax-2}{4a^3c^2x^2-8a^2c^2x+4ac^2}-\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -(a*x - 2)/(4*a**3*c**2*x**2 - 8*a**2*c**2*x + 4*a*c**2) - (log(x - 1/a)/8
- log(x + 1/a)/8)/(a*c**2)
```

Giac [A] time = 1.14465, size = 69, normalized size = 1.35

$$\frac{\log\left(|ax+1|\right)}{8\,ac^2}-\frac{\log\left(|ax-1|\right)}{8\,ac^2}-\frac{ax-2}{4\,(ax-1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*log(abs(a*x + 1))/(a*c^2) - 1/8*log(abs(a*x - 1))/(a*c^2) - 1/4*(a*x -
2)/((a*x - 1)^2*a*c^2)
```

3.1063 $\int \frac{e^{2\tanh^{-1}(ax)}}{x(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=64

$$\frac{3}{4c^2(1-ax)}+\frac{1}{4c^2(1-ax)^2}-\frac{7\log(1-ax)}{8c^2}-\frac{\log(ax+1)}{8c^2}+\frac{\log(x)}{c^2}$$

```
[Out] 1/(4*c^2*(1 - a*x)^2) + 3/(4*c^2*(1 - a*x)) + Log[x]/c^2 - (7*Log[1 - a*x])
/(8*c^2) - Log[1 + a*x]/(8*c^2)
```

Rubi [A] time = 0.0992315, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 72}

$$\frac{3}{4c^2(1-ax)}+\frac{1}{4c^2(1-ax)^2}-\frac{7\log(1-ax)}{8c^2}-\frac{\log(ax+1)}{8c^2}+\frac{\log(x)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^2),x]
```

```
[Out] 1/(4*c^2*(1 - a*x)^2) + 3/(4*c^2*(1 - a*x)) + Log[x]/c^2 - (7*Log[1 - a*x])
/(8*c^2) - Log[1 + a*x]/(8*c^2)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{1}{x(1-ax)^3(1+ax)}\,dx}{c^2}\\
&= \frac{\int \left(\frac{1}{x}-\frac{a}{2(-1+ax)^3}+\frac{3a}{4(-1+ax)^2}-\frac{7a}{8(-1+ax)}-\frac{a}{8(1+ax)}\right)dx}{c^2}\\
&= \frac{1}{4c^2(1-ax)^2}+\frac{3}{4c^2(1-ax)}+\frac{\log(x)}{c^2}-\frac{7\log(1-ax)}{8c^2}-\frac{\log(1+ax)}{8c^2}
\end{aligned}$$

Mathematica [A] time = 0.0381694, size = 48, normalized size = 0.75

$$\frac{\frac{6}{1-ax}+\frac{2}{(ax-1)^2}-7\log(1-ax)-\log(ax+1)+8\log(x)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^2),x]
```

```
[Out] (6/(1 - a*x) + 2/(-1 + a*x)^2 + 8*Log[x] - 7*Log[1 - a*x] - Log[1 + a*x])/(
8*c^2)
```

Maple [A] time = 0.038, size = 54, normalized size = 0.8

$$\frac{\ln(x)}{c^2} - \frac{\ln(ax+1)}{8c^2} + \frac{1}{4c^2(ax-1)^2} - \frac{3}{4c^2(ax-1)} - \frac{7\ln(ax-1)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^2,x)
```

```
[Out] ln(x)/c^2-1/8*ln(a*x+1)/c^2+1/4/c^2/(a*x-1)^2-3/4/c^2/(a*x-1)-7/8/c^2*ln(a*
x-1)
```

Maxima [A] time = 0.956038, size = 81, normalized size = 1.27

$$-\frac{3\,ax-4}{4\left(a^2c^2x^2-2\,ac^2x+c^2\right)} - \frac{\log(ax+1)}{8\,c^2} - \frac{7\,\log(ax-1)}{8\,c^2} + \frac{\log(x)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(3*a*x - 4)/(a^2*c^2*x^2 - 2*a*c^2*x + c^2) - 1/8*log(a*x + 1)/c^2 - 7
/8*log(a*x - 1)/c^2 + log(x)/c^2
```

Fricas [A] time = 2.40164, size = 215, normalized size = 3.36

$$-\frac{6\,ax+\left(a^2x^2-2\,ax+1\right)\log(ax+1)+7\left(a^2x^2-2\,ax+1\right)\log(ax-1)-8\left(a^2x^2-2\,ax+1\right)\log(x)-8}{8\left(a^2c^2x^2-2\,ac^2x+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(6*a*x + (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + 7*(a^2*x^2 - 2*a*x + 1)*
log(a*x - 1) - 8*(a^2*x^2 - 2*a*x + 1)*log(x) - 8)/(a^2*c^2*x^2 - 2*a*c^2*x
 + c^2)
```

Sympy [A] time = 0.66909, size = 58, normalized size = 0.91

$$-\frac{3ax-4}{4a^2c^2x^2-8ac^2x+4c^2} - \frac{-\log(x)+\frac{7\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -(3*a*x - 4)/(4*a**2*c**2*x**2 - 8*a*c**2*x + 4*c**2) - (-log(x) + 7*log(x
- 1/a)/8 + log(x + 1/a)/8)/c**2
```

Giac [A] time = 1.1363, size = 68, normalized size = 1.06

$$-\frac{\log\left(|ax+1|\right)}{8\,c^2}-\frac{7\,\log\left(|ax-1|\right)}{8\,c^2}+\frac{\log\left(|x|\right)}{c^2}-\frac{3\,ax-4}{4\,(ax-1)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/c^2 - 7/8*log(abs(a*x - 1))/c^2 + log(abs(x))/c^2 -
1/4*(3*a*x - 4)/((a*x - 1)^2*c^2)
```

3.1064 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=78

$$\frac{5a}{4c^2(1-ax)}+\frac{a}{4c^2(1-ax)^2}+\frac{2a\log(x)}{c^2}-\frac{17a\log(1-ax)}{8c^2}+\frac{a\log(ax+1)}{8c^2}-\frac{1}{c^2x}$$

```
[Out] -(1/(c^2*x)) + a/(4*c^2*(1 - a*x)^2) + (5*a)/(4*c^2*(1 - a*x)) + (2*a*Log[x
])/c^2 - (17*a*Log[1 - a*x])/(8*c^2) + (a*Log[1 + a*x])/(8*c^2)
```

Rubi [A] time = 0.111547, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$\frac{5a}{4c^2(1-ax)}+\frac{a}{4c^2(1-ax)^2}+\frac{2a\log(x)}{c^2}-\frac{17a\log(1-ax)}{8c^2}+\frac{a\log(ax+1)}{8c^2}-\frac{1}{c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^2),x]
```

```
[Out] -(1/(c^2*x)) + a/(4*c^2*(1 - a*x)^2) + (5*a)/(4*c^2*(1 - a*x)) + (2*a*Log[x
])/c^2 - (17*a*Log[1 - a*x])/(8*c^2) + (a*Log[1 + a*x])/(8*c^2)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{1}{x^2(1-ax)^3(1+ax)}\,dx}{c^2}\\ &= \frac{\int \left(\frac{1}{x^2}+\frac{2a}{x}-\frac{a^2}{2(-1+ax)^3}+\frac{5a^2}{4(-1+ax)^2}-\frac{17a^2}{8(-1+ax)}+\frac{a^2}{8(1+ax)}\right)dx}{c^2}\\ &= -\frac{1}{c^2x}+\frac{a}{4c^2(1-ax)^2}+\frac{5a}{4c^2(1-ax)}+\frac{2a\log(x)}{c^2}-\frac{17a\log(1-ax)}{8c^2}+\frac{a\log(1+ax)}{8c^2}\end{aligned}$$

Mathematica [A] time = 0.0509784, size = 57, normalized size = 0.73

$$\frac{\frac{10a}{1-ax}+\frac{2a}{(ax-1)^2}+16a\log(x)-17a\log(1-ax)+a\log(ax+1)-\frac{8}{x}}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^2),x]
```

```
[Out] (-8/x + (10*a)/(1 - a*x) + (2*a)/(-1 + a*x)^2 + 16*a*Log[x] - 17*a*Log[1 -
a*x] + a*Log[1 + a*x])/(8*c^2)
```

Maple [A] time = 0.039, size = 68, normalized size = 0.9

$$-\frac{1}{xc^2}+2\,\frac{a\ln(x)}{c^2}+\frac{a\ln(ax+1)}{8\,c^2}+\frac{a}{4\,c^2\,(ax-1)^2}-\frac{5\,a}{4\,c^2\,(ax-1)}-\frac{17\,a\ln(ax-1)}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/x/c^2+2*a*ln(x)/c^2+1/8*a*ln(a*x+1)/c^2+1/4/c^2*a/(a*x-1)^2-5/4/c^2*a/(a
*x-1)-17/8/c^2*a*ln(a*x-1)
```

Maxima [A] time = 0.955182, size = 103, normalized size = 1.32

$$-\frac{9\,a^2x^2-14\,ax+4}{4\left(a^2c^2x^3-2\,ac^2x^2+c^2x\right)}+\frac{a\log(ax+1)}{8\,c^2}-\frac{17\,a\log(ax-1)}{8\,c^2}+\frac{2\,a\log(x)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] -1/4*(9*a^2*x^2 - 14*a*x + 4)/(a^2*c^2*x^3 - 2*a*c^2*x^2 + c^2*x) + 1/8*a*l
og(a*x + 1)/c^2 - 17/8*a*log(a*x - 1)/c^2 + 2*a*log(x)/c^2
```

Fricas [A] time = 2.30388, size = 266, normalized size = 3.41

$$-\frac{18\,a^2x^2-28\,ax-\left(a^3x^3-2\,a^2x^2+ax\right)\log(ax+1)+17\left(a^3x^3-2\,a^2x^2+ax\right)\log(ax-1)-16\left(a^3x^3-2\,a^2x^2+ax\right)\mathrm{lo}}{8\left(a^2c^2x^3-2\,ac^2x^2+c^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/8*(18*a^2*x^2 - 28*a*x - (a^3*x^3 - 2*a^2*x^2 + a*x)*log(a*x + 1) + 17*(
a^3*x^3 - 2*a^2*x^2 + a*x)*log(a*x - 1) - 16*(a^3*x^3 - 2*a^2*x^2 + a*x)*lo
g(x) + 8)/(a^2*c^2*x^3 - 2*a*c^2*x^2 + c^2*x)
```

Sympy [A] time = 0.795853, size = 76, normalized size = 0.97

$$-\frac{9a^2x^2-14ax+4}{4a^2c^2x^3-8ac^2x^2+4c^2x}-\frac{-2a\log(x)+\frac{17a\log\left(x-\frac{1}{a}\right)}{8}-\frac{a\log\left(x+\frac{1}{a}\right)}{8}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -(9*a**2*x**2 - 14*a*x + 4)/(4*a**2*c**2*x**3 - 8*a*c**2*x**2 + 4*c**2*x) -
 (-2*a*log(x) + 17*a*log(x - 1/a)/8 - a*log(x + 1/a)/8)/c**2
```

Giac [A] time = 1.14039, size = 88, normalized size = 1.13

$$\frac{a\log\left(|ax+1|\right)}{8\,c^2}-\frac{17\,a\log\left(|ax-1|\right)}{8\,c^2}+\frac{2\,a\log\left(|x|\right)}{c^2}-\frac{9\,a^2x^2-14\,ax+4}{4\,(ax-1)^2c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*a*log(abs(a*x + 1))/c^2 - 17/8*a*log(abs(a*x - 1))/c^2 + 2*a*log(abs(x)
)/c^2 - 1/4*(9*a^2*x^2 - 14*a*x + 4)/((a*x - 1)^2*c^2*x)
```

3.1065 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=99

$$\frac{7a^2}{4c^2(1-ax)}+\frac{a^2}{4c^2(1-ax)^2}+\frac{4a^2\log(x)}{c^2}-\frac{31a^2\log(1-ax)}{8c^2}-\frac{a^2\log(ax+1)}{8c^2}-\frac{2a}{c^2x}-\frac{1}{2c^2x^2}$$

```
[Out] -1/(2*c^2*x^2) - (2*a)/(c^2*x) + a^2/(4*c^2*(1 - a*x)^2) + (7*a^2)/(4*c^2*(
1 - a*x)) + (4*a^2*Log[x])/c^2 - (31*a^2*Log[1 - a*x])/(8*c^2) - (a^2*Log[1
 + a*x])/(8*c^2)
```

Rubi [A] time = 0.122351, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$\frac{7a^2}{4c^2(1-ax)}+\frac{a^2}{4c^2(1-ax)^2}+\frac{4a^2\log(x)}{c^2}-\frac{31a^2\log(1-ax)}{8c^2}-\frac{a^2\log(ax+1)}{8c^2}-\frac{2a}{c^2x}-\frac{1}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^2),x]
```

```
[Out] -1/(2*c^2*x^2) - (2*a)/(c^2*x) + a^2/(4*c^2*(1 - a*x)^2) + (7*a^2)/(4*c^2*(
1 - a*x)) + (4*a^2*Log[x])/c^2 - (31*a^2*Log[1 - a*x])/(8*c^2) - (a^2*Log[1
 + a*x])/(8*c^2)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{1}{x^3(1-ax)^3(1+ax)}\,dx}{c^2}\\ &= \frac{\int \left(\frac{1}{x^3}+\frac{2a}{x^2}+\frac{4a^2}{x}-\frac{a^3}{2(-1+ax)^3}+\frac{7a^3}{4(-1+ax)^2}-\frac{31a^3}{8(-1+ax)}-\frac{a^3}{8(1+ax)}\right)dx}{c^2}\\ &= -\frac{1}{2c^2x^2}-\frac{2a}{c^2x}+\frac{a^2}{4c^2(1-ax)^2}+\frac{7a^2}{4c^2(1-ax)}+\frac{4a^2\log(x)}{c^2}-\frac{31a^2\log(1-ax)}{8c^2}-\frac{a^2\log(1+ax)}{8c^2}\end{aligned}$$

Mathematica [A] time = 0.0957903, size = 72, normalized size = 0.73

$$-\frac{\frac{14a^2}{ax-1}-\frac{2a^2}{(ax-1)^2}-32a^2\log(x)+31a^2\log(1-ax)+a^2\log(ax+1)+\frac{16a}{x}+\frac{4}{x^2}}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^2),x]
```

```
[Out] -(4/x^2 + (16*a)/x - (2*a^2)/(-1 + a*x)^2 + (14*a^2)/(-1 + a*x) - 32*a^2*Lo
g[x] + 31*a^2*Log[1 - a*x] + a^2*Log[1 + a*x])/(8*c^2)
```

Maple [A] time = 0.039, size = 87, normalized size = 0.9

$$-\frac{1}{2\,c^2x^2}-2\,\frac{a}{xc^2}+4\,\frac{a^2\ln(x)}{c^2}-\frac{a^2\ln(ax+1)}{8\,c^2}+\frac{a^2}{4\,c^2\,(ax-1)^2}-\frac{7\,a^2}{4\,c^2\,(ax-1)}-\frac{31\,a^2\ln(ax-1)}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/2/c^2/x^2-2*a/x/c^2+4*a^2*ln(x)/c^2-1/8*a^2*ln(a*x+1)/c^2+1/4/c^2*a^2/(a
*x-1)^2-7/4/c^2*a^2/(a*x-1)-31/8/c^2*a^2*ln(a*x-1)
```

Maxima [A] time = 0.95721, size = 124, normalized size = 1.25

$$-\frac{a^2\log(ax+1)}{8\,c^2}-\frac{31\,a^2\log(ax-1)}{8\,c^2}+\frac{4\,a^2\log(x)}{c^2}-\frac{15\,a^3x^3-22\,a^2x^2+4\,ax+2}{4\left(a^2c^2x^4-2\,ac^2x^3+c^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] -1/8*a^2*log(a*x + 1)/c^2 - 31/8*a^2*log(a*x - 1)/c^2 + 4*a^2*log(x)/c^2 -
1/4*(15*a^3*x^3 - 22*a^2*x^2 + 4*a*x + 2)/(a^2*c^2*x^4 - 2*a*c^2*x^3 + c^2*
x^2)
```

Fricas [A] time = 2.34618, size = 301, normalized size = 3.04

$$-\frac{30\,a^3x^3-44\,a^2x^2+8\,ax+\left(a^4x^4-2\,a^3x^3+a^2x^2\right)\log(ax+1)+31\left(a^4x^4-2\,a^3x^3+a^2x^2\right)\log(ax-1)-32\left(a^4x^4-2\,a^3\right.}{8\left(a^2c^2x^4-2\,ac^2x^3+c^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/8*(30*a^3*x^3 - 44*a^2*x^2 + 8*a*x + (a^4*x^4 - 2*a^3*x^3 + a^2*x^2)*log
(a*x + 1) + 31*(a^4*x^4 - 2*a^3*x^3 + a^2*x^2)*log(a*x - 1) - 32*(a^4*x^4 -
```

```
2*a^3*x^3 + a^2*x^2)*log(x) + 4)/(a^2*c^2*x^4 - 2*a*c^2*x^3 + c^2*x^2)
```

Sympy [A] time = 0.884441, size = 92, normalized size = 0.93

$$-\frac{15a^3x^3-22a^2x^2+4ax+2}{4a^2c^2x^4-8ac^2x^3+4c^2x^2}-\frac{-4a^2\log(x)+\frac{31a^2\log\left(x-\frac{1}{a}\right)}{8}+\frac{a^2\log\left(x+\frac{1}{a}\right)}{8}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**3/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -(15*a**3*x**3 - 22*a**2*x**2 + 4*a*x + 2)/(4*a**2*c**2*x**4 - 8*a*c**2*x**
3 + 4*c**2*x**2) - (-4*a**2*log(x) + 31*a**2*log(x - 1/a)/8 + a**2*log(x +
1/a)/8)/c**2
```

Giac [A] time = 1.13579, size = 107, normalized size = 1.08

$$-\frac{a^2\log(|ax+1|)}{8\,c^2}-\frac{31\,a^2\log(|ax-1|)}{8\,c^2}+\frac{4\,a^2\log(|x|)}{c^2}-\frac{15\,a^3x^3-22\,a^2x^2+4\,ax+2}{4\,(ax-1)^2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*a^2*log(abs(a*x + 1))/c^2 - 31/8*a^2*log(abs(a*x - 1))/c^2 + 4*a^2*log
(abs(x))/c^2 - 1/4*(15*a^3*x^3 - 22*a^2*x^2 + 4*a*x + 2)/((a*x - 1)^2*c^2*x
^2)
```

3.1066 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^4(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=110

$$\frac{9a^3}{4c^2(1-ax)}+\frac{a^3}{4c^2(1-ax)^2}-\frac{4a^2}{c^2x}+\frac{6a^3\log(x)}{c^2}-\frac{49a^3\log(1-ax)}{8c^2}+\frac{a^3\log(ax+1)}{8c^2}-\frac{a}{c^2x^2}-\frac{1}{3c^2x^3}$$

```
[Out] -1/(3*c^2*x^3) - a/(c^2*x^2) - (4*a^2)/(c^2*x) + a^3/(4*c^2*(1 - a*x)^2) +
(9*a^3)/(4*c^2*(1 - a*x)) + (6*a^3*Log[x])/c^2 - (49*a^3*Log[1 - a*x])/(8*c
^2) + (a^3*Log[1 + a*x])/(8*c^2)
```

Rubi [A] time = 0.133771, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$\frac{9a^3}{4c^2(1-ax)}+\frac{a^3}{4c^2(1-ax)^2}-\frac{4a^2}{c^2x}+\frac{6a^3\log(x)}{c^2}-\frac{49a^3\log(1-ax)}{8c^2}+\frac{a^3\log(ax+1)}{8c^2}-\frac{a}{c^2x^2}-\frac{1}{3c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^4*(c - a^2*c*x^2)^2),x]
```

```
[Out] -1/(3*c^2*x^3) - a/(c^2*x^2) - (4*a^2)/(c^2*x) + a^3/(4*c^2*(1 - a*x)^2) +
(9*a^3)/(4*c^2*(1 - a*x)) + (6*a^3*Log[x])/c^2 - (49*a^3*Log[1 - a*x])/(8*c
^2) + (a^3*Log[1 + a*x])/(8*c^2)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{x^4\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{1}{x^4(1-ax)^3(1+ax)}\,dx}{c^2}\\ &= \frac{\int \left(\frac{1}{x^4}+\frac{2a}{x^3}+\frac{4a^2}{x^2}+\frac{6a^3}{x}-\frac{a^4}{2(-1+ax)^3}+\frac{9a^4}{4(-1+ax)^2}-\frac{49a^4}{8(-1+ax)}+\frac{a^4}{8(1+ax)}\right)dx}{c^2}\\ &= -\frac{1}{3c^2x^3}-\frac{a}{c^2x^2}-\frac{4a^2}{c^2x}+\frac{a^3}{4c^2(1-ax)^2}+\frac{9a^3}{4c^2(1-ax)}+\frac{6a^3\log(x)}{c^2}-\frac{49a^3\log(1-ax)}{8c^2}+\frac{a^3\log(1+}{8c^2}\end{aligned}$$

Mathematica [A] time = 0.0786887, size = 87, normalized size = 0.79

$$\frac{\frac{9a^3}{4-4ax}+\frac{a^3}{4(ax-1)^2}-\frac{4a^2}{x}+6a^3\log(x)-\frac{49}{8}a^3\log(1-ax)+\frac{1}{8}a^3\log(ax+1)-\frac{a}{x^2}-\frac{1}{3x^3}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^4*(c - a^2*c*x^2)^2),x]
```

```
[Out] (-1/(3*x^3) - a/x^2 - (4*a^2)/x + (9*a^3)/(4 - 4*a*x) + a^3/(4*(-1 + a*x)^2
) + 6*a^3*Log[x] - (49*a^3*Log[1 - a*x])/8 + (a^3*Log[1 + a*x])/8)/c^2
```

Maple [A] time = 0.038, size = 98, normalized size = 0.9

$$-\frac{1}{3\,c^2x^3}-\frac{a}{c^2x^2}-4\,\frac{a^2}{xc^2}+6\,\frac{a^3\ln(x)}{c^2}+\frac{a^3\ln(ax+1)}{8\,c^2}+\frac{a^3}{4\,c^2(ax-1)^2}-\frac{9\,a^3}{4\,c^2(ax-1)}-\frac{49\,a^3\ln(ax-1)}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/3/c^2/x^3-a/c^2/x^2-4*a^2/x/c^2+6*a^3*ln(x)/c^2+1/8*a^3*ln(a*x+1)/c^2+1/
4/c^2*a^3/(a*x-1)^2-9/4/c^2*a^3/(a*x-1)-49/8/c^2*a^3*ln(a*x-1)
```

Maxima [A] time = 0.96654, size = 135, normalized size = 1.23

$$\frac{a^3\log(ax+1)}{8\,c^2}-\frac{49\,a^3\log(ax-1)}{8\,c^2}+\frac{6\,a^3\log(x)}{c^2}-\frac{75\,a^4x^4-114\,a^3x^3+28\,a^2x^2+4\,ax+4}{12\left(a^2c^2x^5-2\,ac^2x^4+c^2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] 1/8*a^3*log(a*x + 1)/c^2 - 49/8*a^3*log(a*x - 1)/c^2 + 6*a^3*log(x)/c^2 - 1
/12*(75*a^4*x^4 - 114*a^3*x^3 + 28*a^2*x^2 + 4*a*x + 4)/(a^2*c^2*x^5 - 2*a*
c^2*x^4 + c^2*x^3)
```

Fricas [A] time = 2.35848, size = 328, normalized size = 2.98

$$-\frac{150\,a^4x^4-228\,a^3x^3+56\,a^2x^2+8\,ax-3\left(a^5x^5-2\,a^4x^4+a^3x^3\right)\log(ax+1)+147\left(a^5x^5-2\,a^4x^4+a^3x^3\right)\log(ax-1}{24\left(a^2c^2x^5-2\,ac^2x^4+c^2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/24*(150*a^4*x^4 - 228*a^3*x^3 + 56*a^2*x^2 + 8*a*x - 3*(a^5*x^5 - 2*a^4*
x^4 + a^3*x^3)*log(a*x + 1) + 147*(a^5*x^5 - 2*a^4*x^4 + a^3*x^3)*log(a*x -
```

```
 1) - 144*(a^5*x^5 - 2*a^4*x^4 + a^3*x^3)*log(x) + 8)/(a^2*c^2*x^5 - 2*a*c^
2*x^4 + c^2*x^3)
```

Sympy [A] time = 1.00502, size = 100, normalized size = 0.91

$$-\frac{75a^4x^4 - 114a^3x^3 + 28a^2x^2 + 4ax + 4}{12a^2c^2x^5 - 24ac^2x^4 + 12c^2x^3} - \frac{-6a^3\log(x) + \frac{49a^3\log\left(x-\frac{1}{a}\right)}{8} - \frac{a^3\log\left(x+\frac{1}{a}\right)}{8}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**4/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -(75*a**4*x**4 - 114*a**3*x**3 + 28*a**2*x**2 + 4*a*x + 4)/(12*a**2*c**2*x*
*5 - 24*a*c**2*x**4 + 12*c**2*x**3) - (-6*a**3*log(x) + 49*a**3*log(x - 1/a
)/8 - a**3*log(x + 1/a)/8)/c**2
```

Giac [A] time = 1.12835, size = 117, normalized size = 1.06

$$\frac{a^3\log(|ax+1|)}{8c^2} - \frac{49a^3\log(|ax-1|)}{8c^2} + \frac{6a^3\log(|x|)}{c^2} - \frac{75a^4x^4 - 114a^3x^3 + 28a^2x^2 + 4ax + 4}{12(ax-1)^2c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*a^3*log(abs(a*x + 1))/c^2 - 49/8*a^3*log(abs(a*x - 1))/c^2 + 6*a^3*log(
abs(x))/c^2 - 1/12*(75*a^4*x^4 - 114*a^3*x^3 + 28*a^2*x^2 + 4*a*x + 4)/((a*
x - 1)^2*c^2*x^3)
```

3.1067 $\int \frac{e^{2\tanh^{-1}(ax)}x^5}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=105

$$\frac{23}{16a^6c^3(1-ax)}+\frac{1}{16a^6c^3(ax+1)}-\frac{1}{2a^6c^3(1-ax)^2}+\frac{1}{12a^6c^3(1-ax)^3}+\frac{13\log(1-ax)}{16a^6c^3}+\frac{3\log(ax+1)}{16a^6c^3}$$

```
[Out] 1/(12*a^6*c^3*(1 - a*x)^3) - 1/(2*a^6*c^3*(1 - a*x)^2) + 23/(16*a^6*c^3*(1
- a*x)) + 1/(16*a^6*c^3*(1 + a*x)) + (13*Log[1 - a*x])/(16*a^6*c^3) + (3*Lo
g[1 + a*x])/(16*a^6*c^3)
```

Rubi [A] time = 0.133259, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$\frac{23}{16a^6c^3(1-ax)}+\frac{1}{16a^6c^3(ax+1)}-\frac{1}{2a^6c^3(1-ax)^2}+\frac{1}{12a^6c^3(1-ax)^3}+\frac{13\log(1-ax)}{16a^6c^3}+\frac{3\log(ax+1)}{16a^6c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^5)/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(12*a^6*c^3*(1 - a*x)^3) - 1/(2*a^6*c^3*(1 - a*x)^2) + 23/(16*a^6*c^3*(1
- a*x)) + 1/(16*a^6*c^3*(1 + a*x)) + (13*Log[1 - a*x])/(16*a^6*c^3) + (3*Lo
g[1 + a*x])/(16*a^6*c^3)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x^5}{(c-a^2cx^2)^3}\,dx &= \frac{\int \frac{x^5}{(1-ax)^4(1+ax)^2}\,dx}{c^3}\\
&= \frac{\int \left(\frac{1}{4a^5(-1+ax)^4}+\frac{1}{a^5(-1+ax)^3}+\frac{23}{16a^5(-1+ax)^2}+\frac{13}{16a^5(-1+ax)}-\frac{1}{16a^5(1+ax)^2}+\frac{3}{16a^5(1+ax)}\right)dx}{c^3}\\
&= \frac{1}{12a^6c^3(1-ax)^3}-\frac{1}{2a^6c^3(1-ax)^2}+\frac{23}{16a^6c^3(1-ax)}+\frac{1}{16a^6c^3(1+ax)}+\frac{13\log(1-ax)}{16a^6c^3}+\frac{3\log(
\end{aligned}$$

Mathematica [A] time = 0.053934, size = 87, normalized size = 0.83

$$\frac{-66a^3x^3+36a^2x^2+74ax+39(ax-1)^3(ax+1)\log(1-ax)+9(ax-1)^3(ax+1)\log(ax+1)-52}{48a^6c^3(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^5)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (-52 + 74*a*x + 36*a^2*x^2 - 66*a^3*x^3 + 39*(-1 + a*x)^3*(1 + a*x)*Log[1 -
 a*x] + 9*(-1 + a*x)^3*(1 + a*x)*Log[1 + a*x])/(48*a^6*c^3*(-1 + a*x)^3*(1
+ a*x))
```

Maple [A] time = 0.039, size = 90, normalized size = 0.9

$$\frac{1}{16\,a^6c^3\,(ax+1)}+\frac{3\,\ln(ax+1)}{16\,a^6c^3}-\frac{1}{2\,a^6c^3\,(ax-1)^2}-\frac{1}{12\,a^6c^3\,(ax-1)^3}-\frac{23}{16\,a^6c^3\,(ax-1)}+\frac{13\,\ln(ax-1)}{16\,a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^5/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/16/a^6/c^3/(a*x+1)+3/16*ln(a*x+1)/a^6/c^3-1/2/c^3/a^6/(a*x-1)^2-1/12/c^3/
a^6/(a*x-1)^3-23/16/c^3/a^6/(a*x-1)+13/16/c^3/a^6*ln(a*x-1)
```

Maxima [A] time = 0.959164, size = 127, normalized size = 1.21

$$-\frac{33\,a^3x^3-18\,a^2x^2-37\,ax+26}{24\,\left(a^{10}c^3x^4-2\,a^9c^3x^3+2\,a^7c^3x-a^6c^3\right)}+\frac{3\,\log(ax+1)}{16\,a^6c^3}+\frac{13\,\log(ax-1)}{16\,a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^5/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -1/24*(33*a^3*x^3 - 18*a^2*x^2 - 37*a*x + 26)/(a^10*c^3*x^4 - 2*a^9*c^3*x^3
 + 2*a^7*c^3*x - a^6*c^3) + 3/16*log(a*x + 1)/(a^6*c^3) + 13/16*log(a*x - 1
)/(a^6*c^3)
```

Fricas [A] time = 2.37255, size = 277, normalized size = 2.64

$$-\frac{66\,a^3x^3-36\,a^2x^2-74\,ax-9\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log(ax+1)-39\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log(ax-1)+52}{48\left(a^{10}c^3x^4-2\,a^9c^3x^3+2\,a^7c^3x-a^6c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^5/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/48*(66*a^3*x^3 - 36*a^2*x^2 - 74*a*x - 9*(a^4*x^4 - 2*a^3*x^3 + 2*a*x -
1)*log(a*x + 1) - 39*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) + 52)/(
```

```
a^10*c^3*x^4 - 2*a^9*c^3*x^3 + 2*a^7*c^3*x - a^6*c^3)
```

Sympy [A] time = 0.786879, size = 92, normalized size = 0.88

$$-\frac{33a^3x^3 - 18a^2x^2 - 37ax + 26}{24a^{10}c^3x^4 - 48a^9c^3x^3 + 48a^7c^3x - 24a^6c^3} + \frac{\frac{13\log\left(x-\frac{1}{a}\right)}{16} + \frac{3\log\left(x+\frac{1}{a}\right)}{16}}{a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**5/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(33*a**3*x**3 - 18*a**2*x**2 - 37*a*x + 26)/(24*a**10*c**3*x**4 - 48*a**9*
c**3*x**3 + 48*a**7*c**3*x - 24*a**6*c**3) + (13*log(x - 1/a)/16 + 3*log(x
+ 1/a)/16)/(a**6*c**3)
```

Giac [A] time = 1.15004, size = 101, normalized size = 0.96

$$\frac{3\log\left(|ax+1|\right)}{16\,a^6c^3} + \frac{13\log\left(|ax-1|\right)}{16\,a^6c^3} - \frac{33\,a^3x^3 - 18\,a^2x^2 - 37\,ax + 26}{24\,(ax+1)(ax-1)^3a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^5/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 3/16*log(abs(a*x + 1))/(a^6*c^3) + 13/16*log(abs(a*x - 1))/(a^6*c^3) - 1/24
*(33*a^3*x^3 - 18*a^2*x^2 - 37*a*x + 26)/((a*x + 1)*(a*x - 1)^3*a^6*c^3)
```

3.1068 $\int \frac{e^{2\tanh^{-1}(ax)}x^4}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=86

$$\frac{11}{16a^5c^3(1-ax)}-\frac{1}{16a^5c^3(ax+1)}-\frac{3}{8a^5c^3(1-ax)^2}+\frac{1}{12a^5c^3(1-ax)^3}-\frac{\tanh^{-1}(ax)}{4a^5c^3}$$

```
[Out] 1/(12*a^5*c^3*(1 - a*x)^3) - 3/(8*a^5*c^3*(1 - a*x)^2) + 11/(16*a^5*c^3*(1
- a*x)) - 1/(16*a^5*c^3*(1 + a*x)) - ArcTanh[a*x]/(4*a^5*c^3)
```

Rubi [A] time = 0.137553, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6150, 88, 207}

$$\frac{11}{16a^5c^3(1-ax)}-\frac{1}{16a^5c^3(ax+1)}-\frac{3}{8a^5c^3(1-ax)^2}+\frac{1}{12a^5c^3(1-ax)^3}-\frac{\tanh^{-1}(ax)}{4a^5c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(12*a^5*c^3*(1 - a*x)^3) - 3/(8*a^5*c^3*(1 - a*x)^2) + 11/(16*a^5*c^3*(1
- a*x)) - 1/(16*a^5*c^3*(1 + a*x)) - ArcTanh[a*x]/(4*a^5*c^3)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^4}{(c-a^2cx^2)^3}\,dx = \frac{\int \frac{x^4}{(1-ax)^4(1+ax)^2}\,dx}{c^3}$$
$$= \frac{\int \left(\frac{1}{4a^4(-1+ax)^4} + \frac{3}{4a^4(-1+ax)^3} + \frac{11}{16a^4(-1+ax)^2} + \frac{1}{16a^4(1+ax)^2} + \frac{1}{4a^4(-1+a^2x^2)}\right)dx}{c^3}$$
$$= \frac{1}{12a^5c^3(1-ax)^3} - \frac{3}{8a^5c^3(1-ax)^2} + \frac{11}{16a^5c^3(1-ax)} - \frac{1}{16a^5c^3(1+ax)} + \frac{\int \frac{1}{-1+a^2x^2}\,dx}{4a^4c^3}$$
$$= \frac{1}{12a^5c^3(1-ax)^3} - \frac{3}{8a^5c^3(1-ax)^2} + \frac{11}{16a^5c^3(1-ax)} - \frac{1}{16a^5c^3(1+ax)} - \frac{\tanh^{-1}(ax)}{4a^5c^3}$$

Mathematica [A] time = 0.0411151, size = 64, normalized size = 0.74

$$\frac{-9a^3x^3 + 6a^2x^2 + 5ax - 3(ax-1)^3(ax+1)\tanh^{-1}(ax) - 4}{12a^5c^3(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (-4 + 5*a*x + 6*a^2*x^2 - 9*a^3*x^3 - 3*(-1 + a*x)^3*(1 + a*x)*ArcTanh[a*x]
)/(12*a^5*c^3*(-1 + a*x)^3*(1 + a*x))
```

Maple [A] time = 0.036, size = 90, normalized size = 1.1

$$-\frac{1}{16\,a^5c^3\,(ax+1)} - \frac{\ln(ax+1)}{8\,a^5c^3} - \frac{1}{12\,a^5c^3\,(ax-1)^3} - \frac{3}{8\,a^5c^3\,(ax-1)^2} - \frac{11}{16\,a^5c^3\,(ax-1)} + \frac{\ln(ax-1)}{8\,a^5c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/16/a^5/c^3/(a*x+1)-1/8/c^3/a^5*ln(a*x+1)-1/12/c^3/a^5/(a*x-1)^3-3/8/c^3/
a^5/(a*x-1)^2-11/16/c^3/a^5/(a*x-1)+1/8/c^3/a^5*ln(a*x-1)
```

Maxima [A] time = 0.969717, size = 127, normalized size = 1.48

$$-\frac{9\,a^3x^3 - 6\,a^2x^2 - 5\,ax + 4}{12\left(a^9c^3x^4 - 2\,a^8c^3x^3 + 2\,a^6c^3x - a^5c^3\right)} - \frac{\log(ax+1)}{8\,a^5c^3} + \frac{\log(ax-1)}{8\,a^5c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -1/12*(9*a^3*x^3 - 6*a^2*x^2 - 5*a*x + 4)/(a^9*c^3*x^4 - 2*a^8*c^3*x^3 + 2*
a^6*c^3*x - a^5*c^3) - 1/8*log(a*x + 1)/(a^5*c^3) + 1/8*log(a*x - 1)/(a^5*c
^3)
```

Fricas [A] time = 2.38175, size = 273, normalized size = 3.17

$$-\frac{18\,a^3x^3-12\,a^2x^2-10\,ax+3\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(ax+1\right)-3\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(ax-1\right)+8}{24\left(a^9c^3x^4-2\,a^8c^3x^3+2\,a^6c^3x-a^5c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/24*(18*a^3*x^3 - 12*a^2*x^2 - 10*a*x + 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x -
1)*log(a*x + 1) - 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) + 8)/(a^
9*c^3*x^4 - 2*a^8*c^3*x^3 + 2*a^6*c^3*x - a^5*c^3)
```

Sympy [A] time = 0.685253, size = 88, normalized size = 1.02

$$-\frac{9a^3x^3-6a^2x^2-5ax+4}{12a^9c^3x^4-24a^8c^3x^3+24a^6c^3x-12a^5c^3}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**4/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(9*a**3*x**3 - 6*a**2*x**2 - 5*a*x + 4)/(12*a**9*c**3*x**4 - 24*a**8*c**3*
x**3 + 24*a**6*c**3*x - 12*a**5*c**3) + (log(x - 1/a)/8 - log(x + 1/a)/8)/(
a**5*c**3)
```

Giac [A] time = 1.13778, size = 101, normalized size = 1.17

$$-\frac{\log\left(|ax+1|\right)}{8\,a^5c^3}+\frac{\log\left(|ax-1|\right)}{8\,a^5c^3}-\frac{9\,a^3x^3-6\,a^2x^2-5\,ax+4}{12\,(ax+1)(ax-1)^3a^5c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a^5*c^3) + 1/8*log(abs(a*x - 1))/(a^5*c^3) - 1/12*(
9*a^3*x^3 - 6*a^2*x^2 - 5*a*x + 4)/((a*x + 1)*(a*x - 1)^3*a^5*c^3)
```

3.1069 $\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=86

$$\frac{3}{16a^4c^3(1-ax)}+\frac{1}{16a^4c^3(ax+1)}-\frac{1}{4a^4c^3(1-ax)^2}+\frac{1}{12a^4c^3(1-ax)^3}+\frac{\tanh^{-1}(ax)}{8a^4c^3}$$

```
[Out] 1/(12*a^4*c^3*(1 - a*x)^3) - 1/(4*a^4*c^3*(1 - a*x)^2) + 3/(16*a^4*c^3*(1 -
 a*x)) + 1/(16*a^4*c^3*(1 + a*x)) + ArcTanh[a*x]/(8*a^4*c^3)
```

Rubi [A] time = 0.12337, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6150, 88, 207}

$$\frac{3}{16a^4c^3(1-ax)}+\frac{1}{16a^4c^3(ax+1)}-\frac{1}{4a^4c^3(1-ax)^2}+\frac{1}{12a^4c^3(1-ax)^3}+\frac{\tanh^{-1}(ax)}{8a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(12*a^4*c^3*(1 - a*x)^3) - 1/(4*a^4*c^3*(1 - a*x)^2) + 3/(16*a^4*c^3*(1 -
 a*x)) + 1/(16*a^4*c^3*(1 + a*x)) + ArcTanh[a*x]/(8*a^4*c^3)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{x^3}{(1-ax)^4(1+ax)^2}\,dx}{c^3}$$
$$= \frac{\int \left(\frac{1}{4a^3(-1+ax)^4}+\frac{1}{2a^3(-1+ax)^3}+\frac{3}{16a^3(-1+ax)^2}-\frac{1}{16a^3(1+ax)^2}-\frac{1}{8a^3\left(-1+a^2x^2\right)}\right)dx}{c^3}$$
$$= \frac{1}{12a^4c^3(1-ax)^3}-\frac{1}{4a^4c^3(1-ax)^2}+\frac{3}{16a^4c^3(1-ax)}+\frac{1}{16a^4c^3(1+ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{8a^3c^3}$$
$$= \frac{1}{12a^4c^3(1-ax)^3}-\frac{1}{4a^4c^3(1-ax)^2}+\frac{3}{16a^4c^3(1-ax)}+\frac{1}{16a^4c^3(1+ax)}+\frac{\tanh^{-1}(ax)}{8a^4c^3}$$

Mathematica [A] time = 0.0423958, size = 64, normalized size = 0.74

$$\frac{-3a^3x^3-6a^2x^2+7ax+3(ax-1)^3(ax+1)\tanh^{-1}(ax)-2}{24a^4c^3(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (-2 + 7*a*x - 6*a^2*x^2 - 3*a^3*x^3 + 3*(-1 + a*x)^3*(1 + a*x)*ArcTanh[a*x]
)/(24*a^4*c^3*(-1 + a*x)^3*(1 + a*x))
```

Maple [A] time = 0.036, size = 90, normalized size = 1.1

$$\frac{1}{16\,a^4c^3\,(ax+1)}+\frac{\ln(ax+1)}{16\,a^4c^3}-\frac{1}{12\,a^4c^3\,(ax-1)^3}-\frac{1}{4\,a^4c^3\,(ax-1)^2}-\frac{3}{16\,a^4c^3\,(ax-1)}-\frac{\ln(ax-1)}{16\,a^4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/16/a^4/c^3/(a*x+1)+1/16/c^3/a^4*ln(a*x+1)-1/12/c^3/a^4/(a*x-1)^3-1/4/c^3/
a^4/(a*x-1)^2-3/16/c^3/a^4/(a*x-1)-1/16/c^3/a^4*ln(a*x-1)
```

Maxima [A] time = 0.965609, size = 127, normalized size = 1.48

$$-\frac{3\,a^3x^3+6\,a^2x^2-7\,ax+2}{24\left(a^8c^3x^4-2\,a^7c^3x^3+2\,a^5c^3x-a^4c^3\right)}+\frac{\log(ax+1)}{16\,a^4c^3}-\frac{\log(ax-1)}{16\,a^4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -1/24*(3*a^3*x^3 + 6*a^2*x^2 - 7*a*x + 2)/(a^8*c^3*x^4 - 2*a^7*c^3*x^3 + 2*
a^5*c^3*x - a^4*c^3) + 1/16*log(a*x + 1)/(a^4*c^3) - 1/16*log(a*x - 1)/(a^4
*c^3)
```

Fricas [A] time = 2.25539, size = 271, normalized size = 3.15

$$-\frac{6\,a^{3}x^{3}+12\,a^{2}x^{2}-14\,ax-3\left(a^{4}x^{4}-2\,a^{3}x^{3}+2\,ax-1\right)\log\left(ax+1\right)+3\left(a^{4}x^{4}-2\,a^{3}x^{3}+2\,ax-1\right)\log\left(ax-1\right)+4}{48\left(a^{8}c^{3}x^{4}-2\,a^{7}c^{3}x^{3}+2\,a^{5}c^{3}x-a^{4}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/48*(6*a^3*x^3 + 12*a^2*x^2 - 14*a*x - 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1
)*log(a*x + 1) + 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) + 4)/(a^8
*c^3*x^4 - 2*a^7*c^3*x^3 + 2*a^5*c^3*x - a^4*c^3)
```

Sympy [A] time = 0.669184, size = 88, normalized size = 1.02

$$-\frac{3a^{3}x^{3}+6a^{2}x^{2}-7ax+2}{24a^{8}c^{3}x^{4}-48a^{7}c^{3}x^{3}+48a^{5}c^{3}x-24a^{4}c^{3}}+\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{16}+\frac{\log\left(x+\frac{1}{a}\right)}{16}}{a^{4}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(3*a**3*x**3 + 6*a**2*x**2 - 7*a*x + 2)/(24*a**8*c**3*x**4 - 48*a**7*c**3*
x**3 + 48*a**5*c**3*x - 24*a**4*c**3) + (-log(x - 1/a)/16 + log(x + 1/a)/16
)/(a**4*c**3)
```

Giac [A] time = 1.16654, size = 101, normalized size = 1.17

$$\frac{\log\left(|ax+1|\right)}{16\,a^{4}c^{3}}-\frac{\log\left(|ax-1|\right)}{16\,a^{4}c^{3}}-\frac{3\,a^{3}x^{3}+6\,a^{2}x^{2}-7\,ax+2}{24\,(ax+1)(ax-1)^{3}a^{4}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*log(abs(a*x + 1))/(a^4*c^3) - 1/16*log(abs(a*x - 1))/(a^4*c^3) - 1/24*
(3*a^3*x^3 + 6*a^2*x^2 - 7*a*x + 2)/((a*x + 1)*(a*x - 1)^3*a^4*c^3)
```

3.1070 $\int \frac{e^{2\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=31

$$-\frac{1-2ax}{6a^3c^3(1-ax)^3(ax+1)}$$

```
[Out] -(1 - 2*a*x)/(6*a^3*c^3*(1 - a*x)^3*(1 + a*x))
```

Rubi [A] time = 0.0889472, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 81}

$$-\frac{1-2ax}{6a^3c^3(1-ax)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^3,x]
```

```
[Out] -(1 - 2*a*x)/(6*a^3*c^3*(1 - a*x)^3*(1 + a*x))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^3}\,dx &= \frac{\int \frac{x^2}{(1-ax)^4(1+ax)^2}\,dx}{c^3}\\ &= -\frac{1-2ax}{6a^3c^3(1-ax)^3(1+ax)}\end{aligned}$$

Mathematica [A] time = 0.0288219, size = 30, normalized size = 0.97

$$\frac{1-2ax}{6a^3c^3(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (1 - 2*a*x)/(6*a^3*c^3*(-1 + a*x)^3*(1 + a*x))
```

Maple [A] time = 0.034, size = 54, normalized size = 1.7

$$\frac{1}{c^3}\left(-\frac{1}{16\,a^3\,(ax+1)}-\frac{1}{12\,a^3\,(ax-1)^3}-\frac{1}{8\,a^3\,(ax-1)^2}+\frac{1}{16\,a^3\,(ax-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/c^3*(-1/16/a^3/(a*x+1)-1/12/a^3/(a*x-1)^3-1/8/a^3/(a*x-1)^2+1/16/a^3/(a*x
-1))
```

Maxima [A] time = 0.968847, size = 66, normalized size = 2.13

$$-\frac{2\,ax-1}{6\left(a^7c^3x^4-2\,a^6c^3x^3+2\,a^4c^3x-a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -1/6*(2*a*x - 1)/(a^7*c^3*x^4 - 2*a^6*c^3*x^3 + 2*a^4*c^3*x - a^3*c^3)
```

Fricas [A] time = 2.21707, size = 97, normalized size = 3.13

$$-\frac{2\,ax-1}{6\left(a^7c^3x^4-2\,a^6c^3x^3+2\,a^4c^3x-a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/6*(2*a*x - 1)/(a^7*c^3*x^4 - 2*a^6*c^3*x^3 + 2*a^4*c^3*x - a^3*c^3)
```

Sympy [A] time = 0.580728, size = 49, normalized size = 1.58

$$-\frac{2ax-1}{6a^7c^3x^4-12a^6c^3x^3+12a^4c^3x-6a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(2*a*x - 1)/(6*a**7*c**3*x**4 - 12*a**6*c**3*x**3 + 12*a**4*c**3*x - 6*a**
3*c**3)
```

Giac [A] time = 1.13938, size = 61, normalized size = 1.97

$$-\frac{1}{16\,(ax+1)a^3c^3}+\frac{3\,a^2x^2-12\,ax+5}{48\,(ax-1)^3a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/16/((a*x + 1)*a^3*c^3) + 1/48*(3*a^2*x^2 - 12*a*x + 5)/((a*x - 1)^3*a^3*
c^3)
```

3.1071 $\int \frac{e^{2\tanh^{-1}(ax)}x}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=68

$$-\frac{1}{16a^2c^3(1-ax)}+\frac{1}{16a^2c^3(ax+1)}+\frac{1}{12a^2c^3(1-ax)^3}-\frac{\tanh^{-1}(ax)}{8a^2c^3}$$

```
[Out] 1/(12*a^2*c^3*(1 - a*x)^3) - 1/(16*a^2*c^3*(1 - a*x)) + 1/(16*a^2*c^3*(1 +
a*x)) - ArcTanh[a*x]/(8*a^2*c^3)
```

Rubi [A] time = 0.0880824, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6150, 77, 207}

$$-\frac{1}{16a^2c^3(1-ax)}+\frac{1}{16a^2c^3(ax+1)}+\frac{1}{12a^2c^3(1-ax)^3}-\frac{\tanh^{-1}(ax)}{8a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(12*a^2*c^3*(1 - a*x)^3) - 1/(16*a^2*c^3*(1 - a*x)) + 1/(16*a^2*c^3*(1 +
a*x)) - ArcTanh[a*x]/(8*a^2*c^3)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x}{(c-a^2cx^2)^3}\,dx = \frac{\int \frac{x}{(1-ax)^4(1+ax)^2}\,dx}{c^3}$$
$$= \frac{\int \left(\frac{1}{4a(-1+ax)^4} - \frac{1}{16a(-1+ax)^2} - \frac{1}{16a(1+ax)^2} + \frac{1}{8a(-1+a^2x^2)}\right)dx}{c^3}$$
$$= \frac{1}{12a^2c^3(1-ax)^3} - \frac{1}{16a^2c^3(1-ax)} + \frac{1}{16a^2c^3(1+ax)} + \frac{\int \frac{1}{-1+a^2x^2}\,dx}{8ac^3}$$
$$= \frac{1}{12a^2c^3(1-ax)^3} - \frac{1}{16a^2c^3(1-ax)} + \frac{1}{16a^2c^3(1+ax)} - \frac{\tanh^{-1}(ax)}{8a^2c^3}$$

Mathematica [A] time = 0.0339208, size = 60, normalized size = 0.88

$$\frac{-\frac{1}{16a^2(1-ax)} + \frac{1}{16a^2(ax+1)} + \frac{1}{12a^2(1-ax)^3} - \frac{\tanh^{-1}(ax)}{8a^2}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (1/(12*a^2*(1 - a*x)^3) - 1/(16*a^2*(1 - a*x)) + 1/(16*a^2*(1 + a*x)) - Arc
Tanh[a*x]/(8*a^2))/c^3
```

Maple [A] time = 0.036, size = 75, normalized size = 1.1

$$\frac{1}{16\,a^2c^3\,(ax+1)} - \frac{\ln(ax+1)}{16\,a^2c^3} - \frac{1}{12\,a^2c^3\,(ax-1)^3} + \frac{1}{16\,a^2c^3\,(ax-1)} + \frac{\ln(ax-1)}{16\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/16/a^2/c^3/(a*x+1)-1/16/c^3/a^2*ln(a*x+1)-1/12/c^3/a^2/(a*x-1)^3+1/16/c^3
/a^2/(a*x-1)+1/16/c^3/a^2*ln(a*x-1)
```

Maxima [A] time = 0.967842, size = 126, normalized size = 1.85

$$\frac{3\,a^3x^3 - 6\,a^2x^2 + ax - 2}{24\left(a^6c^3x^4 - 2\,a^5c^3x^3 + 2\,a^3c^3x - a^2c^3\right)} - \frac{\log(ax+1)}{16\,a^2c^3} + \frac{\log(ax-1)}{16\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(3*a^3*x^3 - 6*a^2*x^2 + a*x - 2)/(a^6*c^3*x^4 - 2*a^5*c^3*x^3 + 2*a^3
*c^3*x - a^2*c^3) - 1/16*log(a*x + 1)/(a^2*c^3) + 1/16*log(a*x - 1)/(a^2*c^
3)
```

Fricas [B] time = 2.31226, size = 269, normalized size = 3.96

$$\frac{6\,a^{3}x^{3}-12\,a^{2}x^{2}+2\,ax-3\left(a^{4}x^{4}-2\,a^{3}x^{3}+2\,ax-1\right)\log\left(ax+1\right)+3\left(a^{4}x^{4}-2\,a^{3}x^{3}+2\,ax-1\right)\log\left(ax-1\right)-4}{48\left(a^{6}c^{3}x^{4}-2\,a^{5}c^{3}x^{3}+2\,a^{3}c^{3}x-a^{2}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(6*a^3*x^3 - 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*
log(a*x + 1) + 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) - 4)/(a^6*c
^3*x^4 - 2*a^5*c^3*x^3 + 2*a^3*c^3*x - a^2*c^3)
```

Sympy [A] time = 0.640087, size = 87, normalized size = 1.28

$$\frac{3a^{3}x^{3}-6a^{2}x^{2}+ax-2}{24a^{6}c^{3}x^{4}-48a^{5}c^{3}x^{3}+48a^{3}c^{3}x-24a^{2}c^{3}}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{16}-\frac{\log\left(x+\frac{1}{a}\right)}{16}}{a^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (3*a**3*x**3 - 6*a**2*x**2 + a*x - 2)/(24*a**6*c**3*x**4 - 48*a**5*c**3*x**
3 + 48*a**3*c**3*x - 24*a**2*c**3) + (log(x - 1/a)/16 - log(x + 1/a)/16)/(a
**2*c**3)
```

Giac [A] time = 1.1233, size = 100, normalized size = 1.47

$$-\frac{\log\left(|ax+1|\right)}{16\,a^{2}c^{3}}+\frac{\log\left(|ax-1|\right)}{16\,a^{2}c^{3}}+\frac{3\,a^{3}x^{3}-6\,a^{2}x^{2}+ax-2}{24\,(ax+1)(ax-1)^{3}a^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/16*log(abs(a*x + 1))/(a^2*c^3) + 1/16*log(abs(a*x - 1))/(a^2*c^3) + 1/24
*(3*a^3*x^3 - 6*a^2*x^2 + a*x - 2)/((a*x + 1)*(a*x - 1)^3*a^2*c^3)
```

3.1072 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=86

$$\frac{3}{16ac^3(1-ax)}-\frac{1}{16ac^3(ax+1)}+\frac{1}{8ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{\tanh^{-1}(ax)}{4ac^3}$$

```
[Out] 1/(12*a*c^3*(1 - a*x)^3) + 1/(8*a*c^3*(1 - a*x)^2) + 3/(16*a*c^3*(1 - a*x))
 - 1/(16*a*c^3*(1 + a*x)) + ArcTanh[a*x]/(4*a*c^3)
```

Rubi [A] time = 0.0676645, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6140, 44, 207}

$$\frac{3}{16ac^3(1-ax)}-\frac{1}{16ac^3(ax+1)}+\frac{1}{8ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{\tanh^{-1}(ax)}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(12*a*c^3*(1 - a*x)^3) + 1/(8*a*c^3*(1 - a*x)^2) + 3/(16*a*c^3*(1 - a*x))
 - 1/(16*a*c^3*(1 + a*x)) + ArcTanh[a*x]/(4*a*c^3)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{1}{(1-ax)^4(1+ax)^2}\,dx}{c^3}$$

$$= \frac{\int \left(\frac{1}{4(-1+ax)^4} - \frac{1}{4(-1+ax)^3} + \frac{3}{16(-1+ax)^2} + \frac{1}{16(1+ax)^2} - \frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^3}$$

$$= \frac{1}{12ac^3(1-ax)^3} + \frac{1}{8ac^3(1-ax)^2} + \frac{3}{16ac^3(1-ax)} - \frac{1}{16ac^3(1+ax)} - \frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^3}$$

$$= \frac{1}{12ac^3(1-ax)^3} + \frac{1}{8ac^3(1-ax)^2} + \frac{3}{16ac^3(1-ax)} - \frac{1}{16ac^3(1+ax)} + \frac{\tanh^{-1}(ax)}{4ac^3}$$

Mathematica [A] time = 0.0313864, size = 63, normalized size = 0.73

$$-\frac{3a^3x^3 - 6a^2x^2 + ax - 3(ax-1)^3(ax+1)\tanh^{-1}(ax) + 4}{12ac^3(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] -(4 + a*x - 6*a^2*x^2 + 3*a^3*x^3 - 3*(-1 + a*x)^3*(1 + a*x)*ArcTanh[a*x])/
(12*a*c^3*(-1 + a*x)^3*(1 + a*x))
```

Maple [A] time = 0.036, size = 90, normalized size = 1.1

$$-\frac{1}{16\,ac^3\,(ax+1)} + \frac{\ln(ax+1)}{8\,ac^3} - \frac{1}{12\,ac^3\,(ax-1)^3} + \frac{1}{8\,ac^3\,(ax-1)^2} - \frac{3}{16\,ac^3\,(ax-1)} - \frac{\ln(ax-1)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/16/a/c^3/(a*x+1)+1/8*ln(a*x+1)/a/c^3-1/12/c^3/a/(a*x-1)^3+1/8/c^3/a/(a*x
-1)^2-3/16/c^3/a/(a*x-1)-1/8/c^3/a*ln(a*x-1)
```

Maxima [A] time = 0.963124, size = 123, normalized size = 1.43

$$-\frac{3\,a^3x^3 - 6\,a^2x^2 + ax + 4}{12\left(a^5c^3x^4 - 2\,a^4c^3x^3 + 2\,a^2c^3x - ac^3\right)} + \frac{\log(ax+1)}{8\,ac^3} - \frac{\log(ax-1)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^3*x^3 - 6*a^2*x^2 + a*x + 4)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^
2*c^3*x - a*c^3) + 1/8*log(a*x + 1)/(a*c^3) - 1/8*log(a*x - 1)/(a*c^3)
```

Fricas [A] time = 2.11872, size = 267, normalized size = 3.1

$$-\frac{6\,a^3x^3 - 12\,a^2x^2 + 2\,ax - 3\left(a^4x^4 - 2\,a^3x^3 + 2\,ax - 1\right)\log\left(ax+1\right) + 3\left(a^4x^4 - 2\,a^3x^3 + 2\,ax - 1\right)\log\left(ax-1\right) + 8}{24\left(a^5c^3x^4 - 2\,a^4c^3x^3 + 2\,a^2c^3x - ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^3*x^3 - 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)
*log(a*x + 1) + 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) + 8)/(a^5*
c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [A] time = 0.657277, size = 83, normalized size = 0.97

$$-\frac{3a^3x^3 - 6a^2x^2 + ax + 4}{12a^5c^3x^4 - 24a^4c^3x^3 + 24a^2c^3x - 12ac^3} + \frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8} + \frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(3*a**3*x**3 - 6*a**2*x**2 + a*x + 4)/(12*a**5*c**3*x**4 - 24*a**4*c**3*x*
*3 + 24*a**2*c**3*x - 12*a*c**3) + (-log(x - 1/a)/8 + log(x + 1/a)/8)/(a*c*
*3)
```

Giac [A] time = 1.13387, size = 100, normalized size = 1.16

$$\frac{\log\left(|ax+1|\right)}{8\,ac^3} - \frac{\log\left(|ax-1|\right)}{8\,ac^3} - \frac{3\,a^3x^3 - 6\,a^2x^2 + ax + 4}{12\,(ax+1)(ax-1)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*log(abs(a*x + 1))/(a*c^3) - 1/8*log(abs(a*x - 1))/(a*c^3) - 1/12*(3*a^3
*x^3 - 6*a^2*x^2 + a*x + 4)/((a*x + 1)*(a*x - 1)^3*a*c^3)
```

3.1073 $\int \frac{e^{2\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=93

$$\frac{11}{16c^3(1-ax)}+\frac{1}{16c^3(ax+1)}+\frac{1}{4c^3(1-ax)^2}+\frac{1}{12c^3(1-ax)^3}-\frac{13\log(1-ax)}{16c^3}-\frac{3\log(ax+1)}{16c^3}+\frac{\log(x)}{c^3}$$

```
[Out] 1/(12*c^3*(1 - a*x)^3) + 1/(4*c^3*(1 - a*x)^2) + 11/(16*c^3*(1 - a*x)) + 1/
(16*c^3*(1 + a*x)) + Log[x]/c^3 - (13*Log[1 - a*x])/(16*c^3) - (3*Log[1 + a
*x])/(16*c^3)
```

Rubi [A] time = 0.118672, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$\frac{11}{16c^3(1-ax)}+\frac{1}{16c^3(ax+1)}+\frac{1}{4c^3(1-ax)^2}+\frac{1}{12c^3(1-ax)^3}-\frac{13\log(1-ax)}{16c^3}-\frac{3\log(ax+1)}{16c^3}+\frac{\log(x)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^3),x]
```

```
[Out] 1/(12*c^3*(1 - a*x)^3) + 1/(4*c^3*(1 - a*x)^2) + 11/(16*c^3*(1 - a*x)) + 1/
(16*c^3*(1 + a*x)) + Log[x]/c^3 - (13*Log[1 - a*x])/(16*c^3) - (3*Log[1 + a
*x])/(16*c^3)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^3}\,dx &= \frac{\int \frac{1}{x(1-ax)^4(1+ax)^2}\,dx}{c^3}\\ &= \frac{\int \left(\frac{1}{x}+\frac{a}{4(-1+ax)^4}-\frac{a}{2(-1+ax)^3}+\frac{11a}{16(-1+ax)^2}-\frac{13a}{16(-1+ax)}-\frac{a}{16(1+ax)^2}-\frac{3a}{16(1+ax)}\right)dx}{c^3}\\ &= \frac{1}{12c^3(1-ax)^3}+\frac{1}{4c^3(1-ax)^2}+\frac{11}{16c^3(1-ax)}+\frac{1}{16c^3(1+ax)}+\frac{\log(x)}{c^3}-\frac{13\log(1-ax)}{16c^3}-\frac{3\log}{}\end{aligned}$$

Mathematica [A] time = 0.0670385, size = 66, normalized size = 0.71

$$\frac{\frac{33}{1-ax}+\frac{3}{ax+1}+\frac{12}{(ax-1)^2}-\frac{4}{(ax-1)^3}-39\log(1-ax)-9\log(ax+1)+48\log(x)}{48c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^3),x]
```

```
[Out] (33/(1 - a*x) - 4/(-1 + a*x)^3 + 12/(-1 + a*x)^2 + 3/(1 + a*x) + 48*Log[x]
- 39*Log[1 - a*x] - 9*Log[1 + a*x])/(48*c^3)
```

Maple [A] time = 0.039, size = 78, normalized size = 0.8

$$\frac{\ln(x)}{c^3}+\frac{1}{16c^3(ax+1)}-\frac{3\ln(ax+1)}{16c^3}-\frac{1}{12c^3(ax-1)^3}+\frac{1}{4c^3(ax-1)^2}-\frac{11}{16c^3(ax-1)}-\frac{13\ln(ax-1)}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^3,x)
```

```
[Out] ln(x)/c^3+1/16/c^3/(a*x+1)-3/16*ln(a*x+1)/c^3-1/12/c^3/(a*x-1)^3+1/4/c^3/(a
*x-1)^2-11/16/c^3/(a*x-1)-13/16/c^3*ln(a*x-1)
```

Maxima [A] time = 0.979564, size = 120, normalized size = 1.29

$$-\frac{15a^3x^3-18a^2x^2-19ax+26}{24\left(a^4c^3x^4-2a^3c^3x^3+2ac^3x-c^3\right)}-\frac{3\log(ax+1)}{16c^3}-\frac{13\log(ax-1)}{16c^3}+\frac{\log(x)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(15*a^3*x^3 - 18*a^2*x^2 - 19*a*x + 26)/(a^4*c^3*x^4 - 2*a^3*c^3*x^3
+ 2*a*c^3*x - c^3) - 3/16*log(a*x + 1)/c^3 - 13/16*log(a*x - 1)/c^3 + log(x
)/c^3
```

Fricas [A] time = 2.38349, size = 329, normalized size = 3.54

$$-\frac{30a^3x^3-36a^2x^2-38ax+9\left(a^4x^4-2a^3x^3+2ax-1\right)\log(ax+1)+39\left(a^4x^4-2a^3x^3+2ax-1\right)\log(ax-1)-48\left(a^4x^4-2a^3x^3+2ax-1\right)\log(x)+52}{48\left(a^4c^3x^4-2a^3c^3x^3+2ac^3x-c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(30*a^3*x^3 - 36*a^2*x^2 - 38*a*x + 9*(a^4*x^4 - 2*a^3*x^3 + 2*a*x -
1)*log(a*x + 1) + 39*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) - 48*(a
^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(x) + 52)/(a^4*c^3*x^4 - 2*a^3*c^3*x^3 +
 2*a*c^3*x - c^3)
```

Sympy [A] time = 0.924615, size = 87, normalized size = 0.94

$$-\frac{15a^3x^3-18a^2x^2-19ax+26}{24a^4c^3x^4-48a^3c^3x^3+48ac^3x-24c^3}+\frac{\log(x)-\frac{13\log\left(x-\frac{1}{a}\right)}{16}-\frac{3\log\left(x+\frac{1}{a}\right)}{16}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(15*a**3*x**3 - 18*a**2*x**2 - 19*a*x + 26)/(24*a**4*c**3*x**4 - 48*a**3*c
**3*x**3 + 48*a*c**3*x - 24*c**3) + (log(x) - 13*log(x - 1/a)/16 - 3*log(x
+ 1/a)/16)/c**3
```

Giac [A] time = 1.13504, size = 99, normalized size = 1.06

$$-\frac{3\log(|ax+1|)}{16c^3}-\frac{13\log(|ax-1|)}{16c^3}+\frac{\log(|x|)}{c^3}-\frac{15a^3x^3-18a^2x^2-19ax+26}{24(ax+1)(ax-1)^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -3/16*log(abs(a*x + 1))/c^3 - 13/16*log(abs(a*x - 1))/c^3 + log(abs(x))/c^3
 - 1/24*(15*a^3*x^3 - 18*a^2*x^2 - 19*a*x + 26)/((a*x + 1)*(a*x - 1)^3*c^3)
```

3.1074 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=109

$$\frac{23a}{16c^3(1-ax)}-\frac{a}{16c^3(ax+1)}+\frac{3a}{8c^3(1-ax)^2}+\frac{a}{12c^3(1-ax)^3}+\frac{2a\log(x)}{c^3}-\frac{9a\log(1-ax)}{4c^3}+\frac{a\log(ax+1)}{4c^3}-\frac{1}{c^3x}$$

```
[Out] -(1/(c^3*x)) + a/(12*c^3*(1 - a*x)^3) + (3*a)/(8*c^3*(1 - a*x)^2) + (23*a)/
(16*c^3*(1 - a*x)) - a/(16*c^3*(1 + a*x)) + (2*a*Log[x])/c^3 - (9*a*Log[1 -
 a*x])/(4*c^3) + (a*Log[1 + a*x])/(4*c^3)
```

Rubi [A] time = 0.135269, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$\frac{23a}{16c^3(1-ax)}-\frac{a}{16c^3(ax+1)}+\frac{3a}{8c^3(1-ax)^2}+\frac{a}{12c^3(1-ax)^3}+\frac{2a\log(x)}{c^3}-\frac{9a\log(1-ax)}{4c^3}+\frac{a\log(ax+1)}{4c^3}-\frac{1}{c^3x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^3),x]
```

```
[Out] -(1/(c^3*x)) + a/(12*c^3*(1 - a*x)^3) + (3*a)/(8*c^3*(1 - a*x)^2) + (23*a)/
(16*c^3*(1 - a*x)) - a/(16*c^3*(1 + a*x)) + (2*a*Log[x])/c^3 - (9*a*Log[1 -
 a*x])/(4*c^3) + (a*Log[1 + a*x])/(4*c^3)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^3}\,dx &= \frac{\int \frac{1}{x^2(1-ax)^4(1+ax)^2}\,dx}{c^3}\\
&= \frac{\int \left(\frac{1}{x^2}+\frac{2a}{x}+\frac{a^2}{4(-1+ax)^4}-\frac{3a^2}{4(-1+ax)^3}+\frac{23a^2}{16(-1+ax)^2}-\frac{9a^2}{4(-1+ax)}+\frac{a^2}{16(1+ax)^2}+\frac{a^2}{4(1+ax)}\right)dx}{c^3}\\
&= -\frac{1}{c^3x}+\frac{a}{12c^3(1-ax)^3}+\frac{3a}{8c^3(1-ax)^2}+\frac{23a}{16c^3(1-ax)}-\frac{a}{16c^3(1+ax)}+\frac{2a\log(x)}{c^3}-\frac{9a\log(1-ax}{4c^3}
\end{aligned}$$

Mathematica [A] time = 0.0831823, size = 78, normalized size = 0.72

$$\frac{\frac{69a}{1-ax}-\frac{3a}{ax+1}+\frac{18a}{(ax-1)^2}-\frac{4a}{(ax-1)^3}+96a\log(x)-108a\log(1-ax)+12a\log(ax+1)-\frac{48}{x}}{48c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^3),x]
```

```
[Out] (-48/x + (69*a)/(1 - a*x) - (4*a)/(-1 + a*x)^3 + (18*a)/(-1 + a*x)^2 - (3*a
)/(1 + a*x) + 96*a*Log[x] - 108*a*Log[1 - a*x] + 12*a*Log[1 + a*x])/(48*c^3
)
```

Maple [A] time = 0.04, size = 94, normalized size = 0.9

$$-\frac{1}{c^3x}+2\frac{a\ln(x)}{c^3}-\frac{a}{16c^3(ax+1)}+\frac{a\ln(ax+1)}{4c^3}-\frac{a}{12c^3(ax-1)^3}+\frac{3a}{8c^3(ax-1)^2}-\frac{23a}{16c^3(ax-1)}-\frac{9a\ln(ax-1)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/c^3/x+2*a*ln(x)/c^3-1/16*a/c^3/(a*x+1)+1/4*a*ln(a*x+1)/c^3-1/12/c^3*a/(a
*x-1)^3+3/8/c^3*a/(a*x-1)^2-23/16/c^3*a/(a*x-1)-9/4/c^3*a*ln(a*x-1)
```

Maxima [A] time = 0.991831, size = 140, normalized size = 1.28

$$-\frac{15a^4x^4-24a^3x^3-7a^2x^2+23ax-6}{6\left(a^4c^3x^5-2a^3c^3x^4+2ac^3x^2-c^3x\right)}+\frac{a\log(ax+1)}{4c^3}-\frac{9a\log(ax-1)}{4c^3}+\frac{2a\log(x)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -1/6*(15*a^4*x^4 - 24*a^3*x^3 - 7*a^2*x^2 + 23*a*x - 6)/(a^4*c^3*x^5 - 2*a^
3*c^3*x^4 + 2*a*c^3*x^2 - c^3*x) + 1/4*a*log(a*x + 1)/c^3 - 9/4*a*log(a*x -
 1)/c^3 + 2*a*log(x)/c^3
```

Fricas [A] time = 2.66167, size = 377, normalized size = 3.46

$$-\frac{30a^4x^4-48a^3x^3-14a^2x^2+46ax-3\left(a^5x^5-2a^4x^4+2a^2x^2-ax\right)\log(ax+1)+27\left(a^5x^5-2a^4x^4+2a^2x^2-ax\right)}{12\left(a^4c^3x^5-2a^3c^3x^4+2ac^3x^2-c^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/12*(30*a^4*x^4 - 48*a^3*x^3 - 14*a^2*x^2 + 46*a*x - 3*(a^5*x^5 - 2*a^4*x
^4 + 2*a^2*x^2 - a*x)*log(a*x + 1) + 27*(a^5*x^5 - 2*a^4*x^4 + 2*a^2*x^2 -
```

```
a*x)*log(a*x - 1) - 24*(a^5*x^5 - 2*a^4*x^4 + 2*a^2*x^2 - a*x)*log(x) - 12)
/(a^4*c^3*x^5 - 2*a^3*c^3*x^4 + 2*a*c^3*x^2 - c^3*x)
```

Sympy [A] time = 1.12012, size = 104, normalized size = 0.95

$$-\frac{15a^4x^4 - 24a^3x^3 - 7a^2x^2 + 23ax - 6}{6a^4c^3x^5 - 12a^3c^3x^4 + 12ac^3x^2 - 6c^3x} + \frac{2a\log(x) - \frac{9a\log\left(x-\frac{1}{a}\right)}{4} + \frac{a\log\left(x+\frac{1}{a}\right)}{4}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(15*a**4*x**4 - 24*a**3*x**3 - 7*a**2*x**2 + 23*a*x - 6)/(6*a**4*c**3*x**5
 - 12*a**3*c**3*x**4 + 12*a*c**3*x**2 - 6*c**3*x) + (2*a*log(x) - 9*a*log(x
 - 1/a)/4 + a*log(x + 1/a)/4)/c**3
```

Giac [A] time = 1.15346, size = 119, normalized size = 1.09

$$\frac{a\log(|ax+1|)}{4c^3} - \frac{9a\log(|ax-1|)}{4c^3} + \frac{2a\log(|x|)}{c^3} - \frac{15a^4x^4 - 24a^3x^3 - 7a^2x^2 + 23ax - 6}{6(ax+1)(ax-1)^3c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*a*log(abs(a*x + 1))/c^3 - 9/4*a*log(abs(a*x - 1))/c^3 + 2*a*log(abs(x))
/c^3 - 1/6*(15*a^4*x^4 - 24*a^3*x^3 - 7*a^2*x^2 + 23*a*x - 6)/((a*x + 1)*(a
*x - 1)^3*c^3*x)
```

3.1075 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=134

$$\frac{39a^2}{16c^3(1-ax)}+\frac{a^2}{16c^3(ax+1)}+\frac{a^2}{2c^3(1-ax)^2}+\frac{a^2}{12c^3(1-ax)^3}+\frac{5a^2\log(x)}{c^3}-\frac{75a^2\log(1-ax)}{16c^3}-\frac{5a^2\log(ax+1)}{16c^3}-\frac{2a}{c^3x}$$

```
[Out] -1/(2*c^3*x^2) - (2*a)/(c^3*x) + a^2/(12*c^3*(1 - a*x)^3) + a^2/(2*c^3*(1 -
 a*x)^2) + (39*a^2)/(16*c^3*(1 - a*x)) + a^2/(16*c^3*(1 + a*x)) + (5*a^2*Lo
g[x])/c^3 - (75*a^2*Log[1 - a*x])/(16*c^3) - (5*a^2*Log[1 + a*x])/(16*c^3)
```

Rubi [A] time = 0.151327, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$\frac{39a^2}{16c^3(1-ax)}+\frac{a^2}{16c^3(ax+1)}+\frac{a^2}{2c^3(1-ax)^2}+\frac{a^2}{12c^3(1-ax)^3}+\frac{5a^2\log(x)}{c^3}-\frac{75a^2\log(1-ax)}{16c^3}-\frac{5a^2\log(ax+1)}{16c^3}-\frac{2a}{c^3x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^3),x]
```


```
[Out] -1/(2*c^3*x^2) - (2*a)/(c^3*x) + a^2/(12*c^3*(1 - a*x)^3) + a^2/(2*c^3*(1 -
 a*x)^2) + (39*a^2)/(16*c^3*(1 - a*x)) + a^2/(16*c^3*(1 + a*x)) + (5*a^2*Lo
g[x])/c^3 - (75*a^2*Log[1 - a*x])/(16*c^3) - (5*a^2*Log[1 + a*x])/(16*c^3)
```


Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^3}\,dx &= \frac{\int \frac{1}{x^3(1-ax)^4(1+ax)^2}\,dx}{c^3}\\
&= \frac{\int \left(\frac{1}{x^3}+\frac{2a}{x^2}+\frac{5a^2}{x}+\frac{a^3}{4(-1+ax)^4}-\frac{a^3}{(-1+ax)^3}+\frac{39a^3}{16(-1+ax)^2}-\frac{75a^3}{16(-1+ax)}-\frac{a^3}{16(1+ax)^2}-\frac{5a^3}{16(1+ax)}\right)dx}{c^3}\\
&= -\frac{1}{2c^3x^2}-\frac{2a}{c^3x}+\frac{a^2}{12c^3(1-ax)^3}+\frac{a^2}{2c^3(1-ax)^2}+\frac{39a^2}{16c^3(1-ax)}+\frac{a^2}{16c^3(1+ax)}+\frac{5a^2\log(x)}{c^3}-
\end{aligned}$$

Mathematica [A] time = 0.108706, size = 98, normalized size = 0.73

$$\frac{\frac{117a^2}{1-ax}+\frac{3a^2}{ax+1}+\frac{24a^2}{(ax-1)^2}-\frac{4a^2}{(ax-1)^3}+240a^2\log(x)-225a^2\log(1-ax)-15a^2\log(ax+1)-\frac{96a}{x}-\frac{24}{x^2}}{48c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^3),x]
```

```
[Out] (-24/x^2 - (96*a)/x + (117*a^2)/(1 - a*x) - (4*a^2)/(-1 + a*x)^3 + (24*a^2)
/(-1 + a*x)^2 + (3*a^2)/(1 + a*x) + 240*a^2*Log[x] - 225*a^2*Log[1 - a*x] -
 15*a^2*Log[1 + a*x])/(48*c^3)
```

Maple [A] time = 0.042, size = 117, normalized size = 0.9

$$-\frac{1}{2c^3x^2}-2\frac{a}{c^3x}+5\frac{a^2\ln(x)}{c^3}+\frac{a^2}{16c^3(ax+1)}-\frac{5a^2\ln(ax+1)}{16c^3}-\frac{a^2}{12c^3(ax-1)^3}+\frac{a^2}{2c^3(ax-1)^2}-\frac{39a^2}{16c^3(ax-1)}-\frac{75a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/2/c^3/x^2-2*a/c^3/x+5*a^2*ln(x)/c^3+1/16*a^2/c^3/(a*x+1)-5/16*a^2*ln(a*x
+1)/c^3-1/12/c^3*a^2/(a*x-1)^3+1/2/c^3*a^2/(a*x-1)^2-39/16/c^3*a^2/(a*x-1)-
75/16/c^3*a^2*ln(a*x-1)
```

Maxima [A] time = 0.968231, size = 162, normalized size = 1.21

$$-\frac{105a^5x^5-150a^4x^4-85a^3x^3+170a^2x^2-24ax-12}{24\left(a^4c^3x^6-2a^3c^3x^5+2ac^3x^3-c^3x^2\right)}-\frac{5a^2\log(ax+1)}{16c^3}-\frac{75a^2\log(ax-1)}{16c^3}+\frac{5a^2\log(x)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -1/24*(105*a^5*x^5 - 150*a^4*x^4 - 85*a^3*x^3 + 170*a^2*x^2 - 24*a*x - 12)/
(a^4*c^3*x^6 - 2*a^3*c^3*x^5 + 2*a*c^3*x^3 - c^3*x^2) - 5/16*a^2*log(a*x +
1)/c^3 - 75/16*a^2*log(a*x - 1)/c^3 + 5*a^2*log(x)/c^3
```

Fricas [A] time = 3.10859, size = 423, normalized size = 3.16

$$-\frac{210a^5x^5-300a^4x^4-170a^3x^3+340a^2x^2-48ax+15\left(a^6x^6-2a^5x^5+2a^3x^3-a^2x^2\right)\log(ax+1)+225\left(a^6x^6-2a^5x\right.}{48\left(a^4c^3x^6-2a^3c^3x^5+2ac^3x^3-c^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/48*(210*a^5*x^5 - 300*a^4*x^4 - 170*a^3*x^3 + 340*a^2*x^2 - 48*a*x + 15*
(a^6*x^6 - 2*a^5*x^5 + 2*a^3*x^3 - a^2*x^2)*log(a*x + 1) + 225*(a^6*x^6 - 2
*a^5*x^5 + 2*a^3*x^3 - a^2*x^2)*log(a*x - 1) - 240*(a^6*x^6 - 2*a^5*x^5 + 2
*a^3*x^3 - a^2*x^2)*log(x) - 24)/(a^4*c^3*x^6 - 2*a^3*c^3*x^5 + 2*a*c^3*x^3
 - c^3*x^2)
```

Sympy [A] time = 1.43152, size = 121, normalized size = 0.9

$$-\frac{105a^5x^5-150a^4x^4-85a^3x^3+170a^2x^2-24ax-12}{24a^4c^3x^6-48a^3c^3x^5+48ac^3x^3-24c^3x^2}+\frac{5a^2\log(x)-\frac{75a^2\log\left(x-\frac{1}{a}\right)}{16}-\frac{5a^2\log\left(x+\frac{1}{a}\right)}{16}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**3/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(105*a**5*x**5 - 150*a**4*x**4 - 85*a**3*x**3 + 170*a**2*x**2 - 24*a*x - 1
2)/(24*a**4*c**3*x**6 - 48*a**3*c**3*x**5 + 48*a*c**3*x**3 - 24*c**3*x**2)
+ (5*a**2*log(x) - 75*a**2*log(x - 1/a)/16 - 5*a**2*log(x + 1/a)/16)/c**3
```

Giac [A] time = 1.14353, size = 138, normalized size = 1.03

$$-\frac{5\,a^2\log(|ax+1|)}{16\,c^3}-\frac{75\,a^2\log(|ax-1|)}{16\,c^3}+\frac{5\,a^2\log(|x|)}{c^3}-\frac{105\,a^5x^5-150\,a^4x^4-85\,a^3x^3+170\,a^2x^2-24\,ax-12}{24\,(ax+1)(ax-1)^3c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -5/16*a^2*log(abs(a*x + 1))/c^3 - 75/16*a^2*log(abs(a*x - 1))/c^3 + 5*a^2*l
og(abs(x))/c^3 - 1/24*(105*a^5*x^5 - 150*a^4*x^4 - 85*a^3*x^3 + 170*a^2*x^2
 - 24*a*x - 12)/((a*x + 1)*(a*x - 1)^3*c^3*x^2)
```

3.1076 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=121

$$\frac{5}{32ac^4(1-ax)}-\frac{5}{64ac^4(ax+1)}+\frac{3}{32ac^4(1-ax)^2}-\frac{1}{64ac^4(ax+1)^2}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{32ac^4(1-ax)^4}+\frac{15\tanh^{-1}(ax)}{64ac^4}$$

```
[Out] 1/(32*a*c^4*(1 - a*x)^4) + 1/(16*a*c^4*(1 - a*x)^3) + 3/(32*a*c^4*(1 - a*x)
^2) + 5/(32*a*c^4*(1 - a*x)) - 1/(64*a*c^4*(1 + a*x)^2) - 5/(64*a*c^4*(1 +
a*x)) + (15*ArcTanh[a*x])/(64*a*c^4)
```

Rubi [A] time = 0.0910009, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6140, 44, 207}

$$\frac{5}{32ac^4(1-ax)}-\frac{5}{64ac^4(ax+1)}+\frac{3}{32ac^4(1-ax)^2}-\frac{1}{64ac^4(ax+1)^2}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{32ac^4(1-ax)^4}+\frac{15\tanh^{-1}(ax)}{64ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^4,x]
```


```
[Out] 1/(32*a*c^4*(1 - a*x)^4) + 1/(16*a*c^4*(1 - a*x)^3) + 3/(32*a*c^4*(1 - a*x)
^2) + 5/(32*a*c^4*(1 - a*x)) - 1/(64*a*c^4*(1 + a*x)^2) - 5/(64*a*c^4*(1 +
a*x)) + (15*ArcTanh[a*x])/(64*a*c^4)
```


Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \frac{\int \frac{1}{(1-ax)^5(1+ax)^3}\,dx}{c^4}$$
$$= \frac{\int \left(-\frac{1}{8(-1+ax)^5}+\frac{3}{16(-1+ax)^4}-\frac{3}{16(-1+ax)^3}+\frac{5}{32(-1+ax)^2}+\frac{1}{32(1+ax)^3}+\frac{5}{64(1+ax)^2}-\frac{15}{64\left(-1+a^2x^2\right)}\right)dx}{c^4}$$
$$= \frac{1}{32ac^4(1-ax)^4}+\frac{1}{16ac^4(1-ax)^3}+\frac{3}{32ac^4(1-ax)^2}+\frac{5}{32ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)^2}-\frac{5}{64ac^4(1}$$
$$= \frac{1}{32ac^4(1-ax)^4}+\frac{1}{16ac^4(1-ax)^3}+\frac{3}{32ac^4(1-ax)^2}+\frac{5}{32ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)^2}-\frac{5}{64ac^4(1}$$

Mathematica [A] time = 0.0599402, size = 82, normalized size = 0.68

$$\frac{-15a^5x^5+30a^4x^4+10a^3x^3-50a^2x^2+17ax+15(ax-1)^4(ax+1)^2\tanh^{-1}(ax)+16}{64ac^4(ax-1)^4(ax+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (16 + 17*a*x - 50*a^2*x^2 + 10*a^3*x^3 + 30*a^4*x^4 - 15*a^5*x^5 + 15*(-1 +
 a*x)^4*(1 + a*x)^2*ArcTanh[a*x])/(64*a*c^4*(-1 + a*x)^4*(1 + a*x)^2)
```

Maple [A] time = 0.036, size = 120, normalized size = 1.

$$-\frac{1}{64\,ac^4\,(ax+1)^2}-\frac{5}{64\,ac^4\,(ax+1)}+\frac{15\,\ln(ax+1)}{128\,ac^4}+\frac{1}{32\,ac^4\,(ax-1)^4}-\frac{1}{16\,ac^4\,(ax-1)^3}+\frac{3}{32\,ac^4\,(ax-1)^2}-\frac{}{32\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/64/a/c^4/(a*x+1)^2-5/64/a/c^4/(a*x+1)+15/128*ln(a*x+1)/a/c^4+1/32/c^4/a/
(a*x-1)^4-1/16/c^4/a/(a*x-1)^3+3/32/c^4/a/(a*x-1)^2-5/32/c^4/a/(a*x-1)-15/1
28/c^4/a*ln(a*x-1)
```

Maxima [A] time = 0.982536, size = 189, normalized size = 1.56

$$-\frac{15\,a^5x^5-30\,a^4x^4-10\,a^3x^3+50\,a^2x^2-17\,ax-16}{64\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4\right)}+\frac{15\,\log(ax+1)}{128\,ac^4}-\frac{15\,\log(ax-1)}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] -1/64*(15*a^5*x^5 - 30*a^4*x^4 - 10*a^3*x^3 + 50*a^2*x^2 - 17*a*x - 16)/(a^
7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a
^2*c^4*x + a*c^4) + 15/128*log(a*x + 1)/(a*c^4) - 15/128*log(a*x - 1)/(a*c^
4)
```

Fricas [B] time = 2.77373, size = 458, normalized size = 3.79

$$-\frac{30\,a^{5}x^{5}-60\,a^{4}x^{4}-20\,a^{3}x^{3}+100\,a^{2}x^{2}-34\,ax-15\left(a^{6}x^{6}-2\,a^{5}x^{5}-a^{4}x^{4}+4\,a^{3}x^{3}-a^{2}x^{2}-2\,ax+1\right)\log\left(ax+1\right)+15\,(}{128\left(a^{7}c^{4}x^{6}-2\,a^{6}c^{4}x^{5}-a^{5}c^{4}x^{4}+4\,a^{4}c^{4}x^{3}-a^{3}c^{4}x^{2}-2\,a^{2}c^{4}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/128*(30*a^5*x^5 - 60*a^4*x^4 - 20*a^3*x^3 + 100*a^2*x^2 - 34*a*x - 15*(a
^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x + 1
) + 15*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*lo
g(a*x - 1) - 32)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3
 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [A] time = 0.980046, size = 143, normalized size = 1.18

$$-\frac{15a^{5}x^{5}-30a^{4}x^{4}-10a^{3}x^{3}+50a^{2}x^{2}-17ax-16}{64a^{7}c^{4}x^{6}-128a^{6}c^{4}x^{5}-64a^{5}c^{4}x^{4}+256a^{4}c^{4}x^{3}-64a^{3}c^{4}x^{2}-128a^{2}c^{4}x+64ac^{4}}-\frac{\frac{15\log\left(x-\frac{1}{a}\right)}{128}-\frac{15\log\left(x+\frac{1}{a}\right)}{128}}{ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] -(15*a**5*x**5 - 30*a**4*x**4 - 10*a**3*x**3 + 50*a**2*x**2 - 17*a*x - 16)/
(64*a**7*c**4*x**6 - 128*a**6*c**4*x**5 - 64*a**5*c**4*x**4 + 256*a**4*c**4
*x**3 - 64*a**3*c**4*x**2 - 128*a**2*c**4*x + 64*a*c**4) - (15*log(x - 1/a)
/128 - 15*log(x + 1/a)/128)/(a*c**4)
```

Giac [A] time = 1.16133, size = 123, normalized size = 1.02

$$\frac{15\,\log\left(|ax+1|\right)}{128\,ac^{4}}-\frac{15\,\log\left(|ax-1|\right)}{128\,ac^{4}}-\frac{15\,a^{5}x^{5}-30\,a^{4}x^{4}-10\,a^{3}x^{3}+50\,a^{2}x^{2}-17\,ax-16}{64\,(ax+1)^{2}(ax-1)^{4}ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 15/128*log(abs(a*x + 1))/(a*c^4) - 15/128*log(abs(a*x - 1))/(a*c^4) - 1/64*
(15*a^5*x^5 - 30*a^4*x^4 - 10*a^3*x^3 + 50*a^2*x^2 - 17*a*x - 16)/((a*x + 1
)^2*(a*x - 1)^4*a*c^4)
```

3.1077 $\int e^{2\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=137

$$-\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{3(5ax+8)\sqrt{c-a^2cx^2}}{20a^4}+\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

```
[Out] (-3*x^2*Sqrt[c - a^2*c*x^2])/(5*a^2) - (x^3*Sqrt[c - a^2*c*x^2])/(2*a) - (x
^4*Sqrt[c - a^2*c*x^2])/5 - (3*(8 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(20*a^4) +
(3*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(4*a^4)
```

Rubi [A] time = 0.329491, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6151, 1809, 833, 780, 217, 203}

$$-\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{3(5ax+8)\sqrt{c-a^2cx^2}}{20a^4}+\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```


```
[Out] (-3*x^2*Sqrt[c - a^2*c*x^2])/(5*a^2) - (x^3*Sqrt[c - a^2*c*x^2])/(2*a) - (x
^4*Sqrt[c - a^2*c*x^2])/5 - (3*(8 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(20*a^4) +
(3*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(4*a^4)
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{2\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx &= c\int \frac{x^3(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{\int \frac{x^3\left(-9a^2c-10a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{5a^2}\\
&= -\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{\int \frac{x^2\left(30a^3c^2+36a^4c^2x\right)}{\sqrt{c-a^2cx^2}}\,dx}{20a^4c}\\
&= -\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{\int \frac{x\left(-72a^4c^3-90a^5c^3x\right)}{\sqrt{c-a^2cx^2}}\,dx}{60a^6c^2}\\
&= -\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4}+\frac{(3c)\int \frac{}{\sqrt{c-}}}{4a}\\
&= -\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4}+\frac{(3c)\,\mathrm{Subs}}{}\\
&= -\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4}+\frac{3\sqrt{c}\tan^{-}}{4}
\end{aligned}
$$

Mathematica [A] time = 0.142663, size = 96, normalized size = 0.7

$$
-\frac{\left(4a^4x^4+10a^3x^3+12a^2x^2+15ax+24\right)\sqrt{c-a^2cx^2}+15\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{20a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]*(24 + 15*a*x + 12*a^2*x^2 + 10*a^3*x^3 + 4*a^4*x^4) +
 15*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(20
*a^4)
```

Maple [A] time = 0.043, size = 210, normalized size = 1.5

$$\frac{x^2}{5a^2c}\left(-a^2cx^2+c\right)^{\frac{3}{2}}+\frac{4}{5ca^4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}+\frac{x}{2a^3c}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{5x}{4a^3}\sqrt{-a^2cx^2+c}-\frac{5c}{4a^3}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/5*x^2*(-a^2*c*x^2+c)^(3/2)/a^2/c+4/5/c/a^4*(-a^2*c*x^2+c)^(3/2)+1/2/a^3*x
*(-a^2*c*x^2+c)^(3/2)/c-5/4/a^3*x*(-a^2*c*x^2+c)^(1/2)-5/4/a^3*c/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/a^4*(-c*a^2*(x-1/a)^2-2*
a*c*(x-1/a))^(1/2)+2/a^3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-
1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.8889, size = 433, normalized size = 3.16

$$\left[-\frac{2\left(4a^4x^4+10a^3x^3+12a^2x^2+15ax+24\right)\sqrt{-a^2cx^2+c}-15\sqrt{-c}\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{40a^4},-\frac{\left(4a^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [-1/40*(2*(4*a^4*x^4 + 10*a^3*x^3 + 12*a^2*x^2 + 15*a*x + 24)*sqrt(-a^2*c*x
^2 + c) - 15*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
 - c))/a^4, -1/20*((4*a^4*x^4 + 10*a^3*x^3 + 12*a^2*x^2 + 15*a*x + 24)*sqrt
(-a^2*c*x^2 + c) + 15*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*
c*x^2 - c)))/a^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^3\sqrt{-a^2cx^2+c}}{ax-1}\,dx-\int\frac{ax^4\sqrt{-a^2cx^2+c}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(x**3*sqrt(-a**2*c*x**2 + c)/(a*x - 1), x) - Integral(a*x**4*sqrt(
-a**2*c*x**2 + c)/(a*x - 1), x)
```

Giac [A] time = 1.18249, size = 124, normalized size = 0.91

$$-\frac{1}{20}\sqrt{-a^2cx^2+c}\left(\left(2\left(\left(2x+\frac{5}{a}\right)x+\frac{6}{a^2}\right)x+\frac{15}{a^3}\right)x+\frac{24}{a^4}\right)-\frac{3c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{4a^3\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] -1/20*sqrt(-a^2*c*x^2 + c)*((2*((2*x + 5/a)*x + 6/a^2)*x + 15/a^3)*x + 24/a
^4) - 3/4*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^3*sqrt(-c)*
abs(a))
```

3.1078 $\int e^{2\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=112

$$-\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{2x^2\sqrt{c-a^2cx^2}}{3a}-\frac{(21ax+32)\sqrt{c-a^2cx^2}}{24a^3}+\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

```
[Out] (-2*x^2*Sqrt[c - a^2*c*x^2])/(3*a) - (x^3*Sqrt[c - a^2*c*x^2])/4 - ((32 + 2
1*a*x)*Sqrt[c - a^2*c*x^2])/(24*a^3) + (7*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(8*a^3)
```

Rubi [A] time = 0.287331, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6151, 1809, 833, 780, 217, 203}

$$-\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{2x^2\sqrt{c-a^2cx^2}}{3a}-\frac{(21ax+32)\sqrt{c-a^2cx^2}}{24a^3}+\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```


```
[Out] (-2*x^2*Sqrt[c - a^2*c*x^2])/(3*a) - (x^3*Sqrt[c - a^2*c*x^2])/4 - ((32 + 2
1*a*x)*Sqrt[c - a^2*c*x^2])/(24*a^3) + (7*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(8*a^3)
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx &= c\int \frac{x^2(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{\int \frac{x^2\left(-7a^2c-8a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{4a^2}\\
&= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a}-\frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{\int \frac{x\left(16a^3c^2+21a^4c^2x\right)}{\sqrt{c-a^2cx^2}}\,dx}{12a^4c}\\
&= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a}-\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3}+\frac{(7c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{8a^2}\\
&= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a}-\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3}+\frac{(7c)\,\mathrm{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,\right.}{8a^2}\\
&= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a}-\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3}+\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}
\end{aligned}$$

Mathematica [A] time = 0.10731, size = 88, normalized size = 0.79

$$-\frac{\left(6a^3x^3+16a^2x^2+21ax+32\right)\sqrt{c-a^2cx^2}+21\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]*(32 + 21*a*x + 16*a^2*x^2 + 6*a^3*x^3) + 21*Sqrt[c]*A
rcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(24*a^3)
```

Maple [B] time = 0.041, size = 186, normalized size = 1.7

$$\frac{x}{4\,a^2c}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{9\,x}{8\,a^2}\sqrt{-a^2cx^2+c}-\frac{9\,c}{8\,a^2}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}+\frac{2}{3\,a^3c}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-2\,\frac{1}{a^3}\sqrt{-ca^2\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/4*x*(-a^2*c*x^2+c)^(3/2)/a^2/c-9/8/a^2*x*(-a^2*c*x^2+c)^(1/2)-9/8/a^2*c/(
a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/3/a^3*(-a^2*c*x
^2+c)^(3/2)/c-2/a^3*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)+2/a^2*c/(a^2*c)^
(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.23165, size = 398, normalized size = 3.55

$$\left[-\frac{2\left(6a^3x^3+16a^2x^2+21ax+32\right)\sqrt{-a^2cx^2+c}-21\sqrt{-c}\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{48a^3}, -\frac{\left(6a^3x^3+16a^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [-1/48*(2*(6*a^3*x^3 + 16*a^2*x^2 + 21*a*x + 32)*sqrt(-a^2*c*x^2 + c) - 21*
sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^3, -
1/24*((6*a^3*x^3 + 16*a^2*x^2 + 21*a*x + 32)*sqrt(-a^2*c*x^2 + c) + 21*sqrt
(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2\sqrt{-a^2cx^2+c}}{ax-1}\,dx-\int \frac{ax^3\sqrt{-a^2cx^2+c}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(x**2*sqrt(-a**2*c*x**2 + c)/(a*x - 1), x) - Integral(a*x**3*sqrt(
-a**2*c*x**2 + c)/(a*x - 1), x)
```

Giac [A] time = 1.17479, size = 113, normalized size = 1.01

$$-\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3x+\frac{8}{a}\right)x+\frac{21}{a^2}\right)x+\frac{32}{a^3}\right)-\frac{7c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8a^2\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] -1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*x + 8/a)*x + 21/a^2)*x + 32/a^3) - 7/8*c*
log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^2*sqrt(-c)*abs(a))
```

3.1079 $\int e^{2\tanh^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=84

$$-\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(3ax+5)\sqrt{c-a^2cx^2}}{3a^2}+\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

```
[Out] -(x^2*Sqrt[c - a^2*c*x^2])/3 - ((5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2) +
(Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/a^2
```

Rubi [A] time = 0.182244, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6151, 1809, 780, 217, 203}

$$-\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(3ax+5)\sqrt{c-a^2cx^2}}{3a^2}+\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(x^2*Sqrt[c - a^2*c*x^2])/3 - ((5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2) +
(Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/a^2
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= c\int \frac{x(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\\ &= -\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{\int \frac{x\left(-5a^2c-6a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{3a^2}\\ &= -\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{c\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a}\\ &= -\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{c\,\mathrm{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)}{a}\\ &= -\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0897938, size = 79, normalized size = 0.94

$$-\frac{\left(a^2x^2+3ax+5\right)\sqrt{c-a^2cx^2}+3\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((5 + 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2] + 3*Sqrt[c]*ArcTan[(a*x*Sqrt[c
- a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^2)
```

Maple [B] time = 0.038, size = 164, normalized size = 2.

$$\frac{1}{3\,a^2c}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{x}{a}\sqrt{-a^2cx^2+c}-\frac{c}{a}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}-2\,\frac{1}{a^2}\sqrt{-ca^2\left(x-a^{-1}\right)^2-2\,ac\left(x-a^{-1}\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(-a^2*c*x^2+c)^(3/2)/a^2/c-x/a*(-a^2*c*x^2+c)^(1/2)-1/a*c/(a^2*c)^(1/2)
*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/a^2*(-c*a^2*(x-1/a)^2-2*a*c
*(x-1/a))^(1/2)+2/a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^
2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.86243, size = 347, normalized size = 4.13

$$\left[-\frac{2\sqrt{-a^2cx^2+c}\left(a^2x^2+3\,ax+5\right)-3\sqrt{-c}\log\left(2\,a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{6\,a^2},-\frac{\sqrt{-a^2cx^2+c}\left(a^2x^2+3\,ax+5\right)}{3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/6*(2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 3*a*x + 5) - 3*sqrt(-c)*log(2*a^2*
c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^2, -1/3*(sqrt(-a^2*c*x^
2 + c)*(a^2*x^2 + 3*a*x + 5) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt
(c)*x/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x\sqrt{-a^2cx^2+c}}{ax-1}\,dx-\int\frac{ax^2\sqrt{-a^2cx^2+c}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(x*sqrt(-a**2*c*x**2 + c)/(a*x - 1), x) - Integral(a*x**2*sqrt(-a*
*2*c*x**2 + c)/(a*x - 1), x)
```

Giac [A] time = 1.17876, size = 99, normalized size = 1.18

$$-\frac{1}{3}\sqrt{-a^2cx^2+c}\left(\left(x+\frac{3}{a}\right)x+\frac{5}{a^2}\right)-\frac{c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{a\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] -1/3*sqrt(-a^2*c*x^2 + c)*((x + 3/a)*x + 5/a^2) - c*log(abs(-sqrt(-a^2*c)*x
 + sqrt(-a^2*c*x^2 + c)))/(a*sqrt(-c)*abs(a))
```

3.1080 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=86

$$-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(2*a) - ((1 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a) + (3
*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rubi [A] time = 0.0784366, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6141, 671, 641, 217, 203}

$$-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(2*a) - ((1 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a) + (3
*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= c\int \frac{(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}(3c)\int \frac{1+ax}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}(3c)\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.0506045, size = 76, normalized size = 0.88

$$
-\frac{\sqrt{c-a^2cx^2}\left(\sqrt{1-a^2x^2}(ax+4)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]
/Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.035, size = 134, normalized size = 1.6

$$
-\frac{x}{2}\sqrt{-a^2cx^2+c}-\frac{c}{2}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}-2\frac{1}{a}\sqrt{-ca^2\left(x-a^{-1}\right)^2-2\,ac\left(x-a^{-1}\right)}+2\frac{c}{\sqrt{a^2c}}\arctan\left(\sqrt{\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/2*x*(-a^2*c*x^2+c)^(1/2)-1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^
2*c*x^2+c)^(1/2))-2/a*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)+2*c/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.74273, size = 309, normalized size = 3.59

$$\left[-\frac{2\sqrt{-a^{2}cx^{2}+c}(ax+4)-3\sqrt{-c}\log\left(2a^{2}cx^{2}+2\sqrt{-a^{2}cx^{2}+c}a\sqrt{-c}x-c\right)}{4a}, -\frac{\sqrt{-a^{2}cx^{2}+c}(ax+4)+3\sqrt{c}\arctan\left(\frac{\sqrt{-a^{2}c}}{a^{2}c}\right.}{2a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x + 4) - 3*sqrt(-c)*log(2*a^2*c*x^2 + 2*sq
rt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, -1/2*(sqrt(-a^2*c*x^2 + c)*(a*x +
4) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{-a^{2}cx^{2}+c}}{ax-1}\,dx-\int\frac{ax\sqrt{-a^{2}cx^{2}+c}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(sqrt(-a**2*c*x**2 + c)/(a*x - 1), x) - Integral(a*x*sqrt(-a**2*c*
x**2 + c)/(a*x - 1), x)
```

Giac [A] time = 1.1505, size = 84, normalized size = 0.98

$$-\frac{1}{2}\sqrt{-a^{2}cx^{2}+c}\left(x+\frac{4}{a}\right)-\frac{3c\log\left(\left|-\sqrt{-a^{2}c}x+\sqrt{-a^{2}cx^{2}+c}\right|\right)}{2\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*c*x^2 + c)*(x + 4/a) - 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(
-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

3.1081 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=78

$$-\sqrt{c-a^2cx^2}+2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -Sqrt[c - a^2*c*x^2] + 2*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]
- Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.24485, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6151, 1809, 844, 217, 203, 266, 63, 208}

$$-\sqrt{c-a^2cx^2}+2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] -Sqrt[c - a^2*c*x^2] + 2*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]
- Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= c\int \frac{(1+ax)^2}{x\sqrt{c-a^2cx^2}}\,dx\\
&= -\sqrt{c-a^2cx^2}-\frac{\int \frac{-a^2c-2a^3cx}{x\sqrt{c-a^2cx^2}}\,dx}{a^2}\\
&= -\sqrt{c-a^2cx^2}+c\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx+(2ac)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\sqrt{c-a^2cx^2}+\frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)+(2ac)\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{}{\sqrt{c}}\right.\\
&= -\sqrt{c-a^2cx^2}+2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)}{a^2}\\
&= -\sqrt{c-a^2cx^2}+2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0775226, size = 99, normalized size = 1.27

$$
-\sqrt{c-a^2cx^2}-\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)-2\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)+\sqrt{c}\log(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] -Sqrt[c - a^2*c*x^2] - 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*
(-1 + a^2*x^2))] + Sqrt[c]*Log[x] - Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*
x^2]]
```

Maple [A] time = 0.04, size = 128, normalized size = 1.6

$$\sqrt{-a^2cx^2+c}-\sqrt{c}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right)-2\sqrt{-ca^2\left(x-a^{-1}\right)^2-2ac\left(x-a^{-1}\right)}+2\frac{ac}{\sqrt{a^2c}}\arctan\left(\sqrt{a^2c}x\frac{}{\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] (-a^2*c*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-2*(
-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)+2*a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1
/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{-a^2cx^2+c}(ax+1)^2}{\left(a^2x^2-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^2/((a^2*x^2 - 1)*x), x)
```

Fricas [A] time = 2.71875, size = 444, normalized size = 5.69

$$\left[-2\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+\frac{1}{2}\sqrt{c}\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)-\sqrt{-a^2cx^2+c},-\sqrt{-c}\arctan\left(\frac{\sqrt{-}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="frica
s")
```

```
[Out] [-2*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2*
sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) - sqrt
(-a^2*c*x^2 + c), -sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2
 - c)) + sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c
) - sqrt(-a^2*c*x^2 + c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{-a^2cx^2+c}}{ax^2-x}\,dx-\int\frac{ax\sqrt{-a^2cx^2+c}}{ax^2-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] -Integral(sqrt(-a**2*c*x**2 + c)/(a*x**2 - x), x) - Integral(a*x*sqrt(-a**2
*c*x**2 + c)/(a*x**2 - x), x)
```

Giac [A] time = 1.17322, size = 131, normalized size = 1.68

$$\frac{2\,c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{2\,a\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}-\sqrt{-a^2cx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac"
)
```

```
[Out] 2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) + 2*
a*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) - sqrt(-
a^2*c*x^2 + c)
```

3.1082 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=82

$$-\frac{\sqrt{c-a^2cx^2}}{x}+a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -(Sqrt[c - a^2*c*x^2]/x) + a*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^
2]] - 2*a*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.247375, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6151, 1807, 844, 217, 203, 266, 63, 208}

$$-\frac{\sqrt{c-a^2cx^2}}{x}+a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/x) + a*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^
2]] - 2*a*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= c\int \frac{(1+ax)^2}{x^2\sqrt{c-a^2cx^2}}\,dx \\
&= -\frac{\sqrt{c-a^2cx^2}}{x} - \int \frac{-2ac-a^2cx}{x\sqrt{c-a^2cx^2}}\,dx \\
&= -\frac{\sqrt{c-a^2cx^2}}{x} + (2ac)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx \\
&= -\frac{\sqrt{c-a^2cx^2}}{x} + (ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) + \left(a^2c\right)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, -\right. \\
&= -\frac{\sqrt{c-a^2cx^2}}{x} + a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)}{a} \\
&= -\frac{\sqrt{c-a^2cx^2}}{x} + a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - 2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.087087, size = 106, normalized size = 1.29

$$-\frac{\sqrt{c-a^2cx^2}}{x} - 2a\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) - a\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right) + 2a\sqrt{c}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/x) - a*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt
[c]*(-1 + a^2*x^2))] + 2*a*Sqrt[c]*Log[x] - 2*a*Sqrt[c]*Log[c + Sqrt[c]*Sqr
t[c - a^2*c*x^2]]
```

Maple [B] time = 0.042, size = 211, normalized size = 2.6

$$-\frac{1}{cx}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-a^2x\sqrt{-a^2cx^2+c}-a^2c\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}-2\ln\left(\frac{2c+2\sqrt{c}\sqrt{-a^2cx^2+c}}{x}\right)\sqrt{c}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] -1/c/x*(-a^2*c*x^2+c)^(3/2)-a^2*x*(-a^2*c*x^2+c)^(1/2)-a^2*c/(a^2*c)^(1/2)*
arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2*ln((2*c+2*c^(1/2)*(-a^2*c*x^
2+c)^(1/2))/x)*c^(1/2)*a+2*(-a^2*c*x^2+c)^(1/2)*a-2*a*(-c*a^2*(x-1/a)^2-2*a
*c*(x-1/a))^(1/2)+2*a^2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1
/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.72641, size = 479, normalized size = 5.84

$$\left[-\frac{a\sqrt{c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)-a\sqrt{c}x\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+\sqrt{-a^2cx^2+c}}{x},-\frac{4a\sqrt{-c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] [-(a*sqrt(c)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - a
*sqrt(c)*x*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + s
qrt(-a^2*c*x^2 + c))/x, -1/2*(4*a*sqrt(-c)*x*arctan(sqrt(-a^2*c*x^2 + c)*sq
rt(-c)/(a^2*c*x^2 - c)) - a*sqrt(-c)*x*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2
+ c)*a*sqrt(-c)*x - c) + 2*sqrt(-a^2*c*x^2 + c))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{-a^2cx^2+c}}{ax^3-x^2}\,dx-\int\frac{ax\sqrt{-a^2cx^2+c}}{ax^3-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] -Integral(sqrt(-a**2*c*x**2 + c)/(a*x**3 - x**2), x) - Integral(a*x*sqrt(-a
**2*c*x**2 + c)/(a*x**3 - x**2), x)
```

Giac [A] time = 1.17811, size = 180, normalized size = 2.2

$$\frac{4\,ac\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{a^2\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}+\frac{2\,a^2\sqrt{-c}c}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="gia
c")
```

```
[Out] 4*a*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) +
a^2*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) + 2*a^
2*sqrt(-c)*c/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*abs(a))
```

3.1083 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=78

$$-\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -Sqrt[c - a^2*c*x^2]/(2*x^2) - (2*a*Sqrt[c - a^2*c*x^2])/x - (3*a^2*Sqrt[c]
*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rubi [A] time = 0.246493, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6151, 1807, 807, 266, 63, 208}

$$-\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(2*x^2) - (2*a*Sqrt[c - a^2*c*x^2])/x - (3*a^2*Sqrt[c]
*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx &= c\int \frac{(1+ax)^2}{x^3\sqrt{c-a^2cx^2}}\,dx \\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{1}{2}\int \frac{-4ac-3a^2cx}{x^2\sqrt{c-a^2cx^2}}\,dx \\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} + \frac{1}{2}\left(3a^2c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx \\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} + \frac{1}{4}\left(3a^2c\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} - \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right) \\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} - \frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.101987, size = 79, normalized size = 1.01

$$-\frac{(4ax+1)\sqrt{c-a^2cx^2}}{2x^2} - \frac{3}{2}a^2\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) + \frac{3}{2}a^2\sqrt{c}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] -((1 + 4*a*x)*Sqrt[c - a^2*c*x^2])/(2*x^2) + (3*a^2*Sqrt[c]*Log[x])/2 - (3*
a^2*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/2
```

Maple [B] time = 0.045, size = 239, normalized size = 3.1

$$-2\,\frac{a\left(-a^2cx^2+c\right)^{3/2}}{cx} - 2\,a^3x\sqrt{-a^2cx^2+c} - 2\,\frac{a^3c}{\sqrt{a^2c}}\arctan\left(\frac{\sqrt{a^2c}x}{\sqrt{-a^2cx^2+c}}\right) - \frac{3\,a^2}{2}\sqrt{c}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right) + \frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x)
```

```
[Out] -2*a/c/x*(-a^2*c*x^2+c)^(3/2)-2*a^3*x*(-a^2*c*x^2+c)^(1/2)-2*a^3*c/(a^2*c)^
(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-3/2*c^(1/2)*ln((2*c+2*c^
```

```
(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a^2+3/2*(-a^2*c*x^2+c)^(1/2)*a^2-2*a^2*(-c*a
^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)+2*a^3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2
)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))-1/2/c/x^2*(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-a^2cx^2+c}(ax+1)^2}{(a^2x^2-1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^2/((a^2*x^2 - 1)*x^3), x)
```

Fricas [A] time = 2.58519, size = 339, normalized size = 4.35

$$\left[\frac{3\,a^2\sqrt{c}x^2\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-2\,\sqrt{-a^2cx^2+c}(4\,ax+1)}{4\,x^2},-\frac{3\,a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)+\sqrt{-a^2cx^2+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fri
cas")
```

```
[Out] [1/4*(3*a^2*sqrt(c)*x^2*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) - 2*sqrt(-a^2*c*x^2 + c)*(4*a*x + 1))/x^2, -1/2*(3*a^2*sqrt(-c)*x
^2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*c*x^2
+ c)*(4*a*x + 1))/x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-a^2cx^2+c}}{ax^4-x^3}\,dx-\int \frac{ax\sqrt{-a^2cx^2+c}}{ax^4-x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(1/2)/x**3,x)
```

```
[Out] -Integral(sqrt(-a**2*c*x**2 + c)/(a*x**4 - x**3), x) - Integral(a*x*sqrt(-a
**2*c*x**2 + c)/(a*x**4 - x**3), x)
```

Giac [B] time = 1.14897, size = 271, normalized size = 3.47

$$\frac{3\,a^2c\arctan\left(-\frac{\sqrt{-a^2}cx-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\left(\sqrt{-a^2}cx-\sqrt{-a^2cx^2+c}\right)^3a^2c-4\left(\sqrt{-a^2}cx-\sqrt{-a^2cx^2+c}\right)^2a\sqrt{-c}c|a|+\left(\sqrt{-a^2}cx\right.}{\left(\left(\sqrt{-a^2}cx-\sqrt{-a^2cx^2+c}\right)^2-c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="gia
c")
```

```
[Out] 3*a^2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
- ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^2*c - 4*(sqrt(-a^2*c)*x - sq
rt(-a^2*c*x^2 + c))^2*a*sqrt(-c)*c*abs(a) + (sqrt(-a^2*c)*x - sqrt(-a^2*c*x
^2 + c))*a^2*c^2 + 4*a*sqrt(-c)*c^2*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*
x^2 + c))^2 - c)^2
```

3.1084 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=101

$$-\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\left(-\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -Sqrt[c - a^2*c*x^2]/(3*x^3) - (a*Sqrt[c - a^2*c*x^2])/x^2 - (5*a^2*Sqrt[c
- a^2*c*x^2])/(3*x) - a^3*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.271994, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1807, 835, 807, 266, 63, 208}

$$-\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\left(-\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(3*x^3) - (a*Sqrt[c - a^2*c*x^2])/x^2 - (5*a^2*Sqrt[c
- a^2*c*x^2])/(3*x) - a^3*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
```

```
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx &= c\int \frac{(1+ax)^2}{x^4\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{1}{3}\int \frac{-6ac-5a^2cx}{x^3\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{\int \frac{10a^2c^2+6a^3c^2x}{x^2\sqrt{c-a^2cx^2}}\,dx}{6c}\\
&= -\frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+\left(a^3c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+\frac{1}{2}\left(a^3c\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-a\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)\\
&= -\frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.114498, size = 82, normalized size = 0.81

$$-\frac{\left(5a^2x^2+3ax+1\right)\sqrt{c-a^2cx^2}}{3x^3}-a^3\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+a^3\sqrt{c}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] -((1 + 3*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*x^3) + a^3*Sqrt[c]*Log[x]
 - a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

Maple [B] time = 0.049, size = 262, normalized size = 2.6

$$-2\frac{a^2\left(-a^2cx^2+c\right)^{3/2}}{cx}-2a^4x\sqrt{-a^2cx^2+c}-2\frac{a^4c}{\sqrt{a^2c}}\arctan\left(\frac{\sqrt{a^2c}x}{\sqrt{-a^2cx^2+c}}\right)-\sqrt{c}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right)a^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x)
```

```
[Out] -2*a^2/c/x*(-a^2*c*x^2+c)^(3/2)-2*a^4*x*(-a^2*c*x^2+c)^(1/2)-2*a^4*c/(a^2*c
)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-c^(1/2)*ln((2*c+2*c^(1
/2)*(-a^2*c*x^2+c)^(1/2))/x)*a^3+(-a^2*c*x^2+c)^(1/2)*a^3-2*a^3*(-c*a^2*(x-
1/a)^2-2*a*c*(x-1/a))^(1/2)+2*a^4*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-
c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))-a/c/x^2*(-a^2*c*x^2+c)^(3/2)-1/3/c/x^
3*(-a^2*c*x^2+c)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.65801, size = 371, normalized size = 3.67

$$\left[\frac{3a^3\sqrt{c}x^3\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)-2\sqrt{-a^2cx^2+c}\left(5a^2x^2+3ax+1\right)}{6x^3},-\frac{3a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)+\sqrt{-}}{3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fri
cas")
```

```
[Out] [1/6*(3*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) - 2*sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 + 3*a*x + 1))/x^3, -1/3*(3*a^
3*sqrt(-c)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt
(-a^2*c*x^2 + c)*(5*a^2*x^2 + 3*a*x + 1))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{-a^2cx^2+c}}{ax^5-x^4}\,dx-\int\frac{ax\sqrt{-a^2cx^2+c}}{ax^5-x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(1/2)/x**4,x)
```

```
[Out] -Integral(sqrt(-a**2*c*x**2 + c)/(a*x**5 - x**4), x) - Integral(a*x*sqrt(-a
**2*c*x**2 + c)/(a*x**5 - x**4), x)
```

Giac [B] time = 1.16775, size = 338, normalized size = 3.35

$$\frac{2\,a^3c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{2\left(3\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5 a^3c - 3\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^4 a^2\sqrt{-c}c|a| + 12\left(\sqrt{-\right.\right.}{3\left(\left(\sqrt{-a^2c}x-\sqrt{-a\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="gia
c")
```

```
[Out] 2*a^3*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
- 2/3*(3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c - 3*(sqrt(-a^2*c)*
x - sqrt(-a^2*c*x^2 + c))^4*a^2*sqrt(-c)*c*abs(a) + 12*(sqrt(-a^2*c)*x - sq
rt(-a^2*c*x^2 + c))^2*a^2*sqrt(-c)*c^2*abs(a) - 3*(sqrt(-a^2*c)*x - sqrt(-a
^2*c*x^2 + c))*a^3*c^3 - 5*a^2*sqrt(-c)*c^3*abs(a))/((sqrt(-a^2*c)*x - sqrt
(-a^2*c*x^2 + c))^2 - c)^3
```

3.1085 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=130

$$-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -Sqrt[c - a^2*c*x^2]/(4*x^4) - (2*a*Sqrt[c - a^2*c*x^2])/(3*x^3) - (7*a^2*S
qrt[c - a^2*c*x^2])/(8*x^2) - (4*a^3*Sqrt[c - a^2*c*x^2])/(3*x) - (7*a^4*Sq
rt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```

Rubi [A] time = 0.309271, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1807, 835, 807, 266, 63, 208}

$$-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```


```
[Out] -Sqrt[c - a^2*c*x^2]/(4*x^4) - (2*a*Sqrt[c - a^2*c*x^2])/(3*x^3) - (7*a^2*S
qrt[c - a^2*c*x^2])/(8*x^2) - (4*a^3*Sqrt[c - a^2*c*x^2])/(3*x) - (7*a^4*Sq
rt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx &= c\int \frac{(1+ax)^2}{x^5\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{1}{4}\int \frac{-8ac-7a^2cx}{x^4\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{\int \frac{21a^2c^2+16a^3c^2x}{x^3\sqrt{c-a^2cx^2}}\,dx}{12c}\\
&= -\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{\int \frac{-32a^3c^3-21a^4c^3x}{x^2\sqrt{c-a^2cx^2}}\,dx}{24c^2}\\
&= -\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{1}{8}\left(7a^4c\right)\int \frac{1}{x\sqrt{c-a^2c}}\\
&= -\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{1}{16}\left(7a^4c\right)\text{Subst}\left(\int\right.\\
&= -\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}-\frac{1}{8}\left(7a^2\right)\text{Subst}\left(\int \frac{}{\frac{1}{a^2}}\right.\\
&= -\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}-\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-}}{\sqrt{}}\right.
\end{aligned}
$$

Mathematica [A] time = 0.136727, size = 95, normalized size = 0.73

$$
-\frac{\left(32a^3x^3+21a^2x^2+16ax+6\right)\sqrt{c-a^2cx^2}}{24x^4}-\frac{7}{8}a^4\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+\frac{7}{8}a^4\sqrt{c}\log(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]*(6 + 16*a*x + 21*a^2*x^2 + 32*a^3*x^3))/(24*x^4) + (7
*a^4*Sqrt[c]*Log[x])/8 - (7*a^4*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]
])/8
```

Maple [B] time = 0.05, size = 287, normalized size = 2.2

$$-\frac{1}{4\,c x^4}\left(-a^2 c x^2+c\right)^{\frac{3}{2}}-\frac{9\,a^2}{8\,c x^2}\left(-a^2 c x^2+c\right)^{\frac{3}{2}}-\frac{7\,a^4}{8}\sqrt{c}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{-a^2 c x^2+c}\right)\right)+\frac{7\,a^4}{8}\sqrt{-a^2 c x^2+c}-2\,\frac{a^3\left(-a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x)
```

```
[Out] -1/4/c/x^4*(-a^2*c*x^2+c)^(3/2)-9/8*a^2/c/x^2*(-a^2*c*x^2+c)^(3/2)-7/8*a^4*
c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)+7/8*a^4*(-a^2*c*x^2+c)^(
1/2)-2*a^3/c/x*(-a^2*c*x^2+c)^(3/2)-2*a^5*x*(-a^2*c*x^2+c)^(1/2)-2*a^5*c/(a
^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2*a^4*(-c*a^2*(x-1
/a)^2-2*a*c*(x-1/a))^(1/2)+2*a^5*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c
*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))-2/3*a/c/x^3*(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-a^2 c x^2+c}(ax+1)^2}{\left(a^2x^2-1\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^2/((a^2*x^2 - 1)*x^5), x)
```

Fricas [A] time = 2.62499, size = 417, normalized size = 3.21

$$\left[\frac{21\,a^4\sqrt{c}x^4\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-2\left(32\,a^3x^3+21\,a^2x^2+16\,ax+6\right)\sqrt{-a^2cx^2+c}}{48\,x^4},-\frac{21\,a^4\sqrt{-c}x^4\arctan\left(\frac{\sqrt{-a^2c}}{a^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="fri
cas")
```

```
[Out] [1/48*(21*a^4*sqrt(c)*x^4*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) - 2*(32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqrt(-a^2*c*x^2 + c)
)/x^4, -1/24*(21*a^4*sqrt(-c)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2
*c*x^2 - c)) + (32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqrt(-a^2*c*x^2 + c))
/x^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-a^2cx^2+c}}{ax^6-x^5}\,dx - \int \frac{ax\sqrt{-a^2cx^2+c}}{ax^6-x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(1/2)/x**5,x)
```

```
[Out] -Integral(sqrt(-a**2*c*x**2 + c)/(a*x**6 - x**5), x) - Integral(a*x*sqrt(-a
**2*c*x**2 + c)/(a*x**6 - x**5), x)
```

Giac [B] time = 1.18367, size = 437, normalized size = 3.36

$$\frac{7\,a^4c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\,\sqrt{-c}} - \frac{21\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^7a^4c - 45\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5a^4c^2 + 96\left(\sqrt{-a^2c}x-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="gia
c")
```

```
[Out] 7/4*a^4*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c
) - 1/12*(21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^4*c - 45*(sqrt(-a^
2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^2 + 96*(sqrt(-a^2*c)*x - sqrt(-a^2*c
*x^2 + c))^4*a^3*sqrt(-c)*c^2*abs(a) - 45*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2
 + c))^3*a^4*c^3 - 128*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*sqrt(-
c)*c^3*abs(a) + 21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^4*c^4 + 32*a^3
*sqrt(-c)*c^4*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^4
```

3.1086 $\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=161

$$\frac{c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^4}-\frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2}-\frac{x^3\left(c-a^2cx^2\right)^{3/2}}{3a}-\frac{11x^2\left(c-a^2cx^2\right)^{3/2}}{35a^2}+\frac{cx\sqrt{c-a^2cx^2}}{8a^3}-\frac{(105ax+88)\left(c\right.}{420a^4}$$

```
[Out] (c*x*Sqrt[c - a^2*c*x^2])/(8*a^3) - (11*x^2*(c - a^2*c*x^2)^(3/2))/(35*a^2)
 - (x^3*(c - a^2*c*x^2)^(3/2))/(3*a) - (x^4*(c - a^2*c*x^2)^(3/2))/7 - ((88
 + 105*a*x)*(c - a^2*c*x^2)^(3/2))/(420*a^4) + (c^(3/2)*ArcTan[(a*Sqrt[c]*x
)/Sqrt[c - a^2*c*x^2]])/(8*a^4)
```

Rubi [A] time = 0.335875, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1809, 833, 780, 195, 217, 203}

$$\frac{c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^4}-\frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2}-\frac{x^3\left(c-a^2cx^2\right)^{3/2}}{3a}-\frac{11x^2\left(c-a^2cx^2\right)^{3/2}}{35a^2}+\frac{cx\sqrt{c-a^2cx^2}}{8a^3}-\frac{(105ax+88)\left(c\right.}{420a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^(3/2),x]
```


```
[Out] (c*x*Sqrt[c - a^2*c*x^2])/(8*a^3) - (11*x^2*(c - a^2*c*x^2)^(3/2))/(35*a^2)
 - (x^3*(c - a^2*c*x^2)^(3/2))/(3*a) - (x^4*(c - a^2*c*x^2)^(3/2))/7 - ((88
 + 105*a*x)*(c - a^2*c*x^2)^(3/2))/(420*a^4) + (c^(3/2)*ArcTan[(a*Sqrt[c]*x
)/Sqrt[c - a^2*c*x^2]])/(8*a^4)
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^{3/2}\,dx &= c\int x^3(1+ax)^2\sqrt{c-a^2cx^2}\,dx\\
&= -\frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2}-\frac{\int x^3\left(-11a^2c-14a^3cx\right)\sqrt{c-a^2cx^2}\,dx}{7a^2}\\
&= -\frac{x^3\left(c-a^2cx^2\right)^{3/2}}{3a}-\frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2}+\frac{\int x^2\left(42a^3c^2+66a^4c^2x\right)\sqrt{c-a^2cx^2}\,dx}{42a^4c}\\
&= -\frac{11x^2\left(c-a^2cx^2\right)^{3/2}}{35a^2}-\frac{x^3\left(c-a^2cx^2\right)^{3/2}}{3a}-\frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2}-\frac{\int x\left(-132a^4c^3-210a\right.}{210a}\\
&= -\frac{11x^2\left(c-a^2cx^2\right)^{3/2}}{35a^2}-\frac{x^3\left(c-a^2cx^2\right)^{3/2}}{3a}-\frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2}-\frac{(88+105ax)\left(c-a^2c\right.}{420a^4}\\
&= \frac{cx\sqrt{c-a^2cx^2}}{8a^3}-\frac{11x^2\left(c-a^2cx^2\right)^{3/2}}{35a^2}-\frac{x^3\left(c-a^2cx^2\right)^{3/2}}{3a}-\frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2}-\frac{(88+}{}\\
&= \frac{cx\sqrt{c-a^2cx^2}}{8a^3}-\frac{11x^2\left(c-a^2cx^2\right)^{3/2}}{35a^2}-\frac{x^3\left(c-a^2cx^2\right)^{3/2}}{3a}-\frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2}-\frac{(88+}{}\\
&= \frac{cx\sqrt{c-a^2cx^2}}{8a^3}-\frac{11x^2\left(c-a^2cx^2\right)^{3/2}}{35a^2}-\frac{x^3\left(c-a^2cx^2\right)^{3/2}}{3a}-\frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2}-\frac{(88+}{}
\end{aligned}$$

Mathematica [A] time = 0.136877, size = 113, normalized size = 0.7

$$\frac{c\left(120a^6x^6+280a^5x^5+144a^4x^4-70a^3x^3-88a^2x^2-105ax-176\right)\sqrt{c-a^2cx^2}-105c^{3/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{840a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-176 - 105*a*x - 88*a^2*x^2 - 70*a^3*x^3 + 144*a^4*
x^4 + 280*a^5*x^5 + 120*a^6*x^6) - 105*c^(3/2)*ArcTan[(a*x*Sqrt[c - a^2*c*x
^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(840*a^4)
```

Maple [B] time = 0.046, size = 268, normalized size = 1.7

$$\frac{x^2}{7a^2c}\left(-a^2cx^2+c\right)^{\frac{5}{2}}+\frac{16}{35ca^4}\left(-a^2cx^2+c\right)^{\frac{5}{2}}+\frac{x}{3a^3c}\left(-a^2cx^2+c\right)^{\frac{5}{2}}-\frac{7x}{12a^3}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{7cx}{8a^3}\sqrt{-a^2cx^2+c}-\frac{7c^2}{8a^3}\text{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/7*x^2*(-a^2*c*x^2+c)^(5/2)/a^2/c+16/35/c/a^4*(-a^2*c*x^2+c)^(5/2)+1/3/a^3
*x*(-a^2*c*x^2+c)^(5/2)/c-7/12/a^3*x*(-a^2*c*x^2+c)^(3/2)-7/8*c*x*(-a^2*c*x
^2+c)^(1/2)/a^3-7/8/a^3*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^
2+c)^(1/2))-2/3/a^4*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)+1/a^3*c*(-c*a^2*
(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x+1/a^3*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/
2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.75058, size = 566, normalized size = 3.52

$$\left[\frac{105\sqrt{-c}c\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+2\left(120a^6cx^6+280a^5cx^5+144a^4cx^4-70a^3cx^3-88a^2cx^2-10\right.}{1680a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/1680*(105*sqrt(-c)*c*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)
*x - c) + 2*(120*a^6*c*x^6 + 280*a^5*c*x^5 + 144*a^4*c*x^4 - 70*a^3*c*x^3 -
 88*a^2*c*x^2 - 105*a*c*x - 176*c)*sqrt(-a^2*c*x^2 + c))/a^4, -1/840*(105*c
^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - (120*a^6*
c*x^6 + 280*a^5*c*x^5 + 144*a^4*c*x^4 - 70*a^3*c*x^3 - 88*a^2*c*x^2 - 105*a
```

```
*c*x - 176*c)*sqrt(-a^2*c*x^2 + c))/a^4]
```

Sympy [A] time = 11.9152, size = 420, normalized size = 2.61

$$a^2c\left(\begin{cases}\frac{x^6\sqrt{-a^2cx^2+c}}{7}-\frac{x^4\sqrt{-a^2cx^2+c}}{35a^2}-\frac{4x^2\sqrt{-a^2cx^2+c}}{105a^4}-\frac{8\sqrt{-a^2cx^2+c}}{105a^6} & \text{for } a\neq 0\\ \frac{\sqrt{c}x^6}{6} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{ia^2\sqrt{c}x^7}{6\sqrt{a^2x^2-1}}-\frac{5i\sqrt{c}x^5}{24\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}x^3}{48a^2\sqrt{a^2x^2-1}}+\frac{}{16a} \\ -\frac{a^2\sqrt{c}x^7}{6\sqrt{-a^2x^2+1}}+\frac{5\sqrt{c}x^5}{24\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}x^3}{48a^2\sqrt{-a^2x^2+1}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] a**2*c*Piecewise((x**6*sqrt(-a**2*c*x**2 + c)/7 - x**4*sqrt(-a**2*c*x**2 +
c)/(35*a**2) - 4*x**2*sqrt(-a**2*c*x**2 + c)/(105*a**4) - 8*sqrt(-a**2*c*x*
*2 + c)/(105*a**6), Ne(a, 0)), (sqrt(c)*x**6/6, True)) + 2*a*c*Piecewise((I
*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*x**5/(24*sqrt(a**2
*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(1
6*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(16*a**5), Abs(a**2*x**2
) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c)*x**5/(24*s
qrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - sqrt(c
)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(16*a**5), True)) +
c*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5 - x**2*sqrt(-a**2*c*x**2 + c)/(1
5*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4), Ne(a, 0)), (sqrt(c)*x**4/4, T
rue))
```

Giac [A] time = 1.17976, size = 158, normalized size = 0.98

$$\frac{1}{840}\sqrt{-a^2cx^2+c}\left(\left(2\left(\left(4\left(5\left(3a^2cx+7ac\right)x+18c\right)x-\frac{35c}{a}\right)x-\frac{44c}{a^2}\right)x-\frac{105c}{a^3}\right)x-\frac{176c}{a^4}\right)-\frac{c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2c}\right.\right.}{8a^3\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] 1/840*sqrt(-a^2*c*x^2 + c)*((2*((4*(5*(3*a^2*c*x + 7*a*c)*x + 18*c)*x - 35*
c/a)*x - 44*c/a^2)*x - 105*c/a^3)*x - 176*c/a^4) - 1/8*c^2*log(abs(-sqrt(-a
^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^3*sqrt(-c)*abs(a))
```

3.1087 $\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=136

$$\frac{3c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a^3}-\frac{1}{6}x^3\left(c-a^2cx^2\right)^{3/2}-\frac{2x^2\left(c-a^2cx^2\right)^{3/2}}{5a}+\frac{3cx\sqrt{c-a^2cx^2}}{16a^2}-\frac{(45ax+32)\left(c-a^2cx^2\right)^{3/2}}{120a^3}$$

```
[Out] (3*c*x*Sqrt[c - a^2*c*x^2])/(16*a^2) - (2*x^2*(c - a^2*c*x^2)^(3/2))/(5*a)
- (x^3*(c - a^2*c*x^2)^(3/2))/6 - ((32 + 45*a*x)*(c - a^2*c*x^2)^(3/2))/(12
0*a^3) + (3*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a^3)
```

Rubi [A] time = 0.308353, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1809, 833, 780, 195, 217, 203}

$$\frac{3c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a^3}-\frac{1}{6}x^3\left(c-a^2cx^2\right)^{3/2}-\frac{2x^2\left(c-a^2cx^2\right)^{3/2}}{5a}+\frac{3cx\sqrt{c-a^2cx^2}}{16a^2}-\frac{(45ax+32)\left(c-a^2cx^2\right)^{3/2}}{120a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (3*c*x*Sqrt[c - a^2*c*x^2])/(16*a^2) - (2*x^2*(c - a^2*c*x^2)^(3/2))/(5*a)
- (x^3*(c - a^2*c*x^2)^(3/2))/6 - ((32 + 45*a*x)*(c - a^2*c*x^2)^(3/2))/(12
0*a^3) + (3*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a^3)
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^{3/2}\,dx &= c\int x^2(1+ax)^2\sqrt{c-a^2cx^2}\,dx\\
&= -\frac{1}{6}x^3\left(c-a^2cx^2\right)^{3/2}-\frac{\int x^2\left(-9a^2c-12a^3cx\right)\sqrt{c-a^2cx^2}\,dx}{6a^2}\\
&= -\frac{2x^2\left(c-a^2cx^2\right)^{3/2}}{5a}-\frac{1}{6}x^3\left(c-a^2cx^2\right)^{3/2}+\frac{\int x\left(24a^3c^2+45a^4c^2x\right)\sqrt{c-a^2cx^2}\,dx}{30a^4c}\\
&= -\frac{2x^2\left(c-a^2cx^2\right)^{3/2}}{5a}-\frac{1}{6}x^3\left(c-a^2cx^2\right)^{3/2}-\frac{(32+45ax)\left(c-a^2cx^2\right)^{3/2}}{120a^3}+\frac{(3c)\int\sqrt{c-}}{8a^2}\\
&= \frac{3cx\sqrt{c-a^2cx^2}}{16a^2}-\frac{2x^2\left(c-a^2cx^2\right)^{3/2}}{5a}-\frac{1}{6}x^3\left(c-a^2cx^2\right)^{3/2}-\frac{(32+45ax)\left(c-a^2cx^2\right)^{3/2}}{120a^3}\\
&= \frac{3cx\sqrt{c-a^2cx^2}}{16a^2}-\frac{2x^2\left(c-a^2cx^2\right)^{3/2}}{5a}-\frac{1}{6}x^3\left(c-a^2cx^2\right)^{3/2}-\frac{(32+45ax)\left(c-a^2cx^2\right)^{3/2}}{120a^3}\\
&= \frac{3cx\sqrt{c-a^2cx^2}}{16a^2}-\frac{2x^2\left(c-a^2cx^2\right)^{3/2}}{5a}-\frac{1}{6}x^3\left(c-a^2cx^2\right)^{3/2}-\frac{(32+45ax)\left(c-a^2cx^2\right)^{3/2}}{120a^3}
\end{aligned}
$$

Mathematica [A] time = 0.118549, size = 105, normalized size = 0.77

$$
\frac{c\left(40a^5x^5+96a^4x^4+50a^3x^3-32a^2x^2-45ax-64\right)\sqrt{c-a^2cx^2}-45c^{3/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{240a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-64 - 45*a*x - 32*a^2*x^2 + 50*a^3*x^3 + 96*a^4*x^4
 + 40*a^5*x^5) - 45*c^(3/2)*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 +
```

```
a^2*x^2))])/(240*a^3)
```

Maple [B] time = 0.04, size = 244, normalized size = 1.8

$$\frac{x}{6a^2c}\left(-a^2cx^2+c\right)^{\frac{5}{2}}-\frac{13x}{24a^2}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{13cx}{16a^2}\sqrt{-a^2cx^2+c}-\frac{13c^2}{16a^2}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}+\frac{2}{5a^3c}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/6*x*(-a^2*c*x^2+c)^(5/2)/a^2/c-13/24/a^2*x*(-a^2*c*x^2+c)^(3/2)-13/16*c*x
*(-a^2*c*x^2+c)^(1/2)/a^2-13/16/a^2*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*
x/(-a^2*c*x^2+c)^(1/2))+2/5/a^3*(-a^2*c*x^2+c)^(5/2)/c-2/3/a^3*(-c*a^2*(x-1
/a)^2-2*a*c*(x-1/a))^(3/2)+1/a^2*c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x
+1/a^2*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-
1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.66154, size = 508, normalized size = 3.74

$$\left[\frac{45\sqrt{-c}c\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+2\left(40a^5cx^5+96a^4cx^4+50a^3cx^3-32a^2cx^2-45acx-64c\right)\sqrt{-}}{480a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/480*(45*sqrt(-c)*c*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
 - c) + 2*(40*a^5*c*x^5 + 96*a^4*c*x^4 + 50*a^3*c*x^3 - 32*a^2*c*x^2 - 45*a
*c*x - 64*c)*sqrt(-a^2*c*x^2 + c))/a^3, -1/240*(45*c^(3/2)*arctan(sqrt(-a^2
*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - (40*a^5*c*x^5 + 96*a^4*c*x^4 + 5
0*a^3*c*x^3 - 32*a^2*c*x^2 - 45*a*c*x - 64*c)*sqrt(-a^2*c*x^2 + c))/a^3]
```

Sympy [C] time = 13.3175, size = 515, normalized size = 3.79

$$a^2c\left(\begin{cases} \frac{ia^2\sqrt{c}x^7}{6\sqrt{a^2x^2-1}} - \frac{5i\sqrt{c}x^5}{24\sqrt{a^2x^2-1}} - \frac{i\sqrt{c}x^3}{48a^2\sqrt{a^2x^2-1}} + \frac{i\sqrt{c}x}{16a^4\sqrt{a^2x^2-1}} - \frac{i\sqrt{c}\operatorname{acosh}(ax)}{16a^5} & \text{for } |a^2x^2| > 1 \\ -\frac{a^2\sqrt{c}x^7}{6\sqrt{-a^2x^2+1}} + \frac{5\sqrt{c}x^5}{24\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}x^3}{48a^2\sqrt{-a^2x^2+1}} - \frac{\sqrt{c}x}{16a^4\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\operatorname{asin}(ax)}{16a^5} & \text{otherwise} \end{cases}\right) + 2ac\left(\begin{cases} \frac{x^4\sqrt{-a^2cx^2+c}}{5} - \frac{x^2\sqrt{-a^2cx}}{15a^2} \\ \frac{\sqrt{c}x^4}{4} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] a**2*c*Piecewise((I*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)
*x**5/(24*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2 - 1
)) + I*sqrt(c)*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(16*a
**5), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5
*sqrt(c)*x**5/(24*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a**2*
x**2 + 1)) - sqrt(c)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(
16*a**5), True)) + 2*a*c*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5 - x**2*sq
rt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4), Ne(a,
0)), (sqrt(c)*x**4/4, True)) + c*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**
2*x**2 - 1)) - 3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a*
*2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1
), (-a**2*sqrt(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a
**2*x**2 + 1)) - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x
)/(8*a**3), True))
```

Giac [A] time = 1.16868, size = 144, normalized size = 1.06

$$\frac{1}{240}\sqrt{-a^2cx^2+c}\left(\left(2\left(\left(4\left(5a^2cx+12ac\right)x+25c\right)x-\frac{16c}{a}\right)x-\frac{45c}{a^2}\right)x-\frac{64c}{a^3}\right)-\frac{3c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{16a^2\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] 1/240*sqrt(-a^2*c*x^2 + c)*((2*((4*(5*a^2*c*x + 12*a*c)*x + 25*c)*x - 16*c/
a)*x - 45*c/a^2)*x - 64*c/a^3) - 3/16*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a
^2*c*x^2 + c)))/(a^2*sqrt(-c)*abs(a))
```

3.1088 $\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=111

$$\frac{c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^2}-\frac{1}{5}x^2\left(c-a^2cx^2\right)^{3/2}+\frac{cx\sqrt{c-a^2cx^2}}{4a}-\frac{(15ax+14)\left(c-a^2cx^2\right)^{3/2}}{30a^2}$$

```
[Out] (c*x*Sqrt[c - a^2*c*x^2])/(4*a) - (x^2*(c - a^2*c*x^2)^(3/2))/5 - ((14 + 15
*a*x)*(c - a^2*c*x^2)^(3/2))/(30*a^2) + (c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(4*a^2)
```

Rubi [A] time = 0.194993, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6151, 1809, 780, 195, 217, 203}

$$\frac{c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^2}-\frac{1}{5}x^2\left(c-a^2cx^2\right)^{3/2}+\frac{cx\sqrt{c-a^2cx^2}}{4a}-\frac{(15ax+14)\left(c-a^2cx^2\right)^{3/2}}{30a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2)^(3/2),x]
```


```
[Out] (c*x*Sqrt[c - a^2*c*x^2])/(4*a) - (x^2*(c - a^2*c*x^2)^(3/2))/5 - ((14 + 15
*a*x)*(c - a^2*c*x^2)^(3/2))/(30*a^2) + (c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(4*a^2)
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^{3/2}\,dx &= c\int x(1+ax)^2\sqrt{c-a^2cx^2}\,dx\\
&= -\frac{1}{5}x^2\left(c-a^2cx^2\right)^{3/2}-\frac{\int x\left(-7a^2c-10a^3cx\right)\sqrt{c-a^2cx^2}\,dx}{5a^2}\\
&= -\frac{1}{5}x^2\left(c-a^2cx^2\right)^{3/2}-\frac{(14+15ax)\left(c-a^2cx^2\right)^{3/2}}{30a^2}+\frac{c\int\sqrt{c-a^2cx^2}\,dx}{2a}\\
&= \frac{cx\sqrt{c-a^2cx^2}}{4a}-\frac{1}{5}x^2\left(c-a^2cx^2\right)^{3/2}-\frac{(14+15ax)\left(c-a^2cx^2\right)^{3/2}}{30a^2}+\frac{c^2\int\frac{1}{\sqrt{c-a^2cx^2}}\,dx}{4a}\\
&= \frac{cx\sqrt{c-a^2cx^2}}{4a}-\frac{1}{5}x^2\left(c-a^2cx^2\right)^{3/2}-\frac{(14+15ax)\left(c-a^2cx^2\right)^{3/2}}{30a^2}+\frac{c^2\,\text{Subst}\left(\int\frac{1}{1+a^2cx^2}\right.}{4a}\\
&= \frac{cx\sqrt{c-a^2cx^2}}{4a}-\frac{1}{5}x^2\left(c-a^2cx^2\right)^{3/2}-\frac{(14+15ax)\left(c-a^2cx^2\right)^{3/2}}{30a^2}+\frac{c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right.}{4a^2}\end{aligned}$$

Mathematica [A] time = 0.0991286, size = 97, normalized size = 0.87

$$\frac{c\left(12a^4x^4+30a^3x^3+16a^2x^2-15ax-28\right)\sqrt{c-a^2cx^2}-15c^{3/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{60a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-28 - 15*a*x + 16*a^2*x^2 + 30*a^3*x^3 + 12*a^4*x^4
) - 15*c^(3/2)*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/
(60*a^2)
```

Maple [B] time = 0.037, size = 222, normalized size = 2.

$$\frac{1}{5a^2c}\left(-a^2cx^2+c\right)^{\frac{5}{2}}-\frac{x}{2a}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{3cx}{4a}\sqrt{-a^2cx^2+c}-\frac{3c^2}{4a}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}-\frac{2}{3a^2}\left(-ca^2\left(x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/5*(-a^2*c*x^2+c)^(5/2)/a^2/c-1/2*x/a*(-a^2*c*x^2+c)^(3/2)-3/4*c*x*(-a^2*c
*x^2+c)^(1/2)/a-3/4/a*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+
c)^(1/2))-2/3/a^2*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)+1/a*c*(-c*a^2*(x-1
/a)^2-2*a*c*(x-1/a))^(1/2)*x+1/a*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(
-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.853, size = 466, normalized size = 4.2

$$\left[\frac{15\,\sqrt{-c}c\log\left(2\,a^{2}cx^{2}+2\,\sqrt{-a^{2}cx^{2}+c}a\sqrt{-c}x-c\right)+2\left(12\,a^{4}cx^{4}+30\,a^{3}cx^{3}+16\,a^{2}cx^{2}-15\,acx-28\,c\right)\sqrt{-a^{2}cx^{2}+c}}{120\,a^{2}},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/120*(15*sqrt(-c)*c*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
 - c) + 2*(12*a^4*c*x^4 + 30*a^3*c*x^3 + 16*a^2*c*x^2 - 15*a*c*x - 28*c)*sq
rt(-a^2*c*x^2 + c))/a^2, -1/60*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sq
rt(c)*x/(a^2*c*x^2 - c)) - (12*a^4*c*x^4 + 30*a^3*c*x^3 + 16*a^2*c*x^2 - 15
*a*c*x - 28*c)*sqrt(-a^2*c*x^2 + c))/a^2]
```

Sympy [A] time = 8.7533, size = 306, normalized size = 2.76

$$a^{2}c\left(\begin{cases}\frac{x^{4}\sqrt{-a^{2}cx^{2}+c}}{5}-\frac{x^{2}\sqrt{-a^{2}cx^{2}+c}}{15a^{2}}-\frac{2\sqrt{-a^{2}cx^{2}+c}}{15a^{4}} & \text{for}\: a\neq 0\\\frac{\sqrt{c}x^{4}}{4} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{ia^{2}\sqrt{c}x^{5}}{4\sqrt{a^{2}x^{2}-1}}-\frac{3i\sqrt{c}x^{3}}{8\sqrt{a^{2}x^{2}-1}}+\frac{i\sqrt{c}x}{8a^{2}\sqrt{a^{2}x^{2}-1}}-\frac{i\sqrt{c}\operatorname{acosh}(ax)}{8a^{3}}\\-\frac{a^{2}\sqrt{c}x^{5}}{4\sqrt{-a^{2}x^{2}+1}}+\frac{3\sqrt{c}x^{3}}{8\sqrt{-a^{2}x^{2}+1}}-\frac{\sqrt{c}x}{8a^{2}\sqrt{-a^{2}x^{2}+1}}+\frac{\sqrt{c}\operatorname{asin}(ax)}{8a^{3}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] a**2*c*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5 - x**2*sqrt(-a**2*c*x**2 +
c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4), Ne(a, 0)), (sqrt(c)*x**4
/4, True)) + 2*a*c*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) -
```

```
3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x
**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt
(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1))
 - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), Tr
ue)) + c*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c
*x**2 + c)**(3/2)/(3*a**2*c), True))
```

Giac [A] time = 1.16434, size = 132, normalized size = 1.19

$$\frac{1}{60}\sqrt{-a^2cx^2+c}\left(\left(2\left(3\left(2a^2cx+5ac\right)x+8c\right)x-\frac{15c}{a}\right)x-\frac{28c}{a^2}\right)-\frac{c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{4a\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] 1/60*sqrt(-a^2*c*x^2 + c)*((2*(3*(2*a^2*c*x + 5*a*c)*x + 8*c)*x - 15*c/a)*x
 - 28*c/a^2) - 1/4*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a*
sqrt(-c)*abs(a))
```

3.1089 $\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=107

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}+\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

```
[Out] (5*c*x*Sqrt[c - a^2*c*x^2])/8 - (5*(c - a^2*c*x^2)^(3/2))/(12*a) - ((1 + a*
x)*(c - a^2*c*x^2)^(3/2))/(4*a) + (5*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
a^2*c*x^2]])/(8*a)
```

Rubi [A] time = 0.0855183, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6141, 671, 641, 195, 217, 203}

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}+\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2),x]
```


```
[Out] (5*c*x*Sqrt[c - a^2*c*x^2])/8 - (5*(c - a^2*c*x^2)^(3/2))/(12*a) - ((1 + a*
x)*(c - a^2*c*x^2)^(3/2))/(4*a) + (5*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
a^2*c*x^2]])/(8*a)
```


Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}dx &= c\int (1+ax)^2\sqrt{c-a^2cx^2}\,dx \\
&= -\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{1}{4}(5c)\int (1+ax)\sqrt{c-a^2cx^2}\,dx \\
&= -\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{1}{4}(5c)\int \sqrt{c-a^2cx^2}\,dx \\
&= \frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{1}{8}\left(5c^2\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{1}{8}\left(5c^2\right)\mathrm{Subst}\left(\int \frac{1}{1+a^2 \right. \\
&= \frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}
\end{aligned}$$

Mathematica [A] time = 0.0899084, size = 117, normalized size = 1.09

$$-\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(6a^4x^4+10a^3x^3-7a^2x^2-25ax+16\right)+30\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -(c*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(16 - 25*a*x - 7*a^2*x^2 + 10*a^3*x^
3 + 6*a^4*x^4) + 30*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(24*a*Sqr
t[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.036, size = 186, normalized size = 1.7

$$-\frac{x}{4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{3\,cx}{8}\sqrt{-a^2cx^2+c}-\frac{3\,c^2}{8}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}-\frac{2}{3\,a}\left(-ca^2\left(x-a^{-1}\right)^2-2\,ac\left(x-a^{-1}\right)\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*x*(-a^2*c*x^2+c)^(3/2)-3/8*c*x*(-a^2*c*x^2+c)^(1/2)-3/8*c^2/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/3/a*(-c*a^2*(x-1/a)^2-2*
```

```
a*c*(x-1/a))^(3/2)+c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x+c^2/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.68867, size = 413, normalized size = 3.86

$$\left[\frac{15\sqrt{-c}c\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+2\left(6a^3cx^3+16a^2cx^2+9acx-16c\right)\sqrt{-a^2cx^2+c}}{48a}, -\frac{15c^{\frac{3}{2}}\arctan\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/48*(15*sqrt(-c)*c*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c) + 2*(6*a^3*c*x^3 + 16*a^2*c*x^2 + 9*a*c*x - 16*c)*sqrt(-a^2*c*x^2 + c)
)/a, -1/24*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 -
 c)) - (6*a^3*c*x^3 + 16*a^2*c*x^2 + 9*a*c*x - 16*c)*sqrt(-a^2*c*x^2 + c))/
a]
```

Sympy [C] time = 9.04011, size = 340, normalized size = 3.18

$$a^2c\left(\begin{cases}\frac{ia^2\sqrt{c}x^5}{4\sqrt{a^2x^2-1}}-\frac{3i\sqrt{c}x^3}{8\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}x}{8a^2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\operatorname{acosh}(ax)}{8a^3} & \text{for } \left|a^2x^2\right|>1\\ -\frac{a^2\sqrt{c}x^5}{4\sqrt{-a^2x^2+1}}+\frac{3\sqrt{c}x^3}{8\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\operatorname{asin}(ax)}{8a^3} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}0 & \text{for } c=0\\ \frac{\sqrt{c}x^2}{2} & \text{for } a^2=0\\ -\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3a^2c} & \text{otherwise}\end{cases}\right)+c\left(\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] a**2*c*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*sqrt(c)
*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x**2 - 1)) -
I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**5/(4*
sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1)) - sqrt(c)*x
/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), True)) + 2*a*c
*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 +
c)**(3/2)/(3*a**2*c), True)) + c*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**
2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(
```

```
2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin
(a*x)/(2*a), True))
```

Giac [A] time = 1.13948, size = 115, normalized size = 1.07

$$\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3a^2cx+8ac\right)x+9c\right)x-\frac{16c}{a}\right)-\frac{5c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*a^2*c*x + 8*a*c)*x + 9*c)*x - 16*c/a) - 5/
8*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

3.1090 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=101

$$c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+c(ax+1)\sqrt{c-a^2cx^2}-\frac{1}{3}\left(c-a^2cx^2\right)^{3/2}$$

```
[Out] c*(1 + a*x)*Sqrt[c - a^2*c*x^2] - (c - a^2*c*x^2)^(3/2)/3 + c^(3/2)*ArcTan[
(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] - c^(3/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/Sq
rt[c]]
```

Rubi [A] time = 0.286373, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6151, 1809, 815, 844, 217, 203, 266, 63, 208}

$$c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+c(ax+1)\sqrt{c-a^2cx^2}-\frac{1}{3}\left(c-a^2cx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x,x]
```

```
[Out] c*(1 + a*x)*Sqrt[c - a^2*c*x^2] - (c - a^2*c*x^2)^(3/2)/3 + c^(3/2)*ArcTan[
(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] - c^(3/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/Sq
rt[c]]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x}\,dx = c\int \frac{(1+ax)^2\sqrt{c-a^2cx^2}}{x}\,dx$$

$$= -\frac{1}{3}\left(c-a^2cx^2\right)^{3/2} - \frac{\int \frac{\left(-3a^2c-6a^3cx\right)\sqrt{c-a^2cx^2}}{x}\,dx}{3a^2}$$

$$= c(1+ax)\sqrt{c-a^2cx^2} - \frac{1}{3}\left(c-a^2cx^2\right)^{3/2} + \frac{\int \frac{6a^4c^3+6a^5c^3x}{x\sqrt{c-a^2cx^2}}\,dx}{6a^4c}$$

$$= c(1+ax)\sqrt{c-a^2cx^2} - \frac{1}{3}\left(c-a^2cx^2\right)^{3/2} + c^2\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx + \left(ac^2\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,d$$

$$= c(1+ax)\sqrt{c-a^2cx^2} - \frac{1}{3}\left(c-a^2cx^2\right)^{3/2} + \frac{1}{2}c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) + \left(ac^2\right)\text{S}$$

$$= c(1+ax)\sqrt{c-a^2cx^2} - \frac{1}{3}\left(c-a^2cx^2\right)^{3/2} + c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\right.}{a^2}$$

$$= c(1+ax)\sqrt{c-a^2cx^2} - \frac{1}{3}\left(c-a^2cx^2\right)^{3/2} + c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-}}{\sqrt{}}\right.$$

Mathematica [A] time = 0.108017, size = 115, normalized size = 1.14

$$-c^{3/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+c^{3/2}\left(-\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)\right)+\frac{1}{3}c\left(a^2x^2+3ax+2\right)\sqrt{c-a^2cx^2}+c^{3/2}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x,x]
```

```
[Out] (c*(2 + 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2])/3 - c^(3/2)*ArcTan[(a*x*Sqrt[
c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))] + c^(3/2)*Log[x] - c^(3/2)*Log[c
+ Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

Maple [B] time = 0.041, size = 179, normalized size = 1.8

$$\frac{1}{3}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-c^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right)+\sqrt{-a^2cx^2+c}c-\frac{2}{3}\left(-ca^2\left(x-a^{-1}\right)^2-2ac\left(x-a^{-1}\right)\right)^{\frac{3}{2}}+ac\sqrt{\cdot}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x,x)
```

```
[Out] 1/3*(-a^2*c*x^2+c)^(3/2)-c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
+(-a^2*c*x^2+c)^(1/2)*c-2/3*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)+a*c*(-c*
a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x+a*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/
2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x,x, algorithm="maxim
a")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x), x)
```

Fricas [A] time = 2.77963, size = 539, normalized size = 5.34

$$\left[-c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-a^2cx^2+ca}\sqrt{c}x}{a^2cx^2-c}\right)+\frac{1}{2}c^{\frac{3}{2}}\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+\frac{1}{3}\left(a^2cx^2+3acx+2c\right)\sqrt{-a^2cx^2+\cdot}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x,x, algorithm="frica
s")
```

```
[Out] [-c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2*c^
(3/2)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + 1/3*(a
^2*c*x^2 + 3*a*c*x + 2*c)*sqrt(-a^2*c*x^2 + c), -sqrt(-c)*c*arctan(sqrt(-a^
2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 1/2*sqrt(-c)*c*log(2*a^2*c*x^2 + 2
*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) + 1/3*(a^2*c*x^2 + 3*a*c*x + 2*c)*s
qrt(-a^2*c*x^2 + c)]
```

Sympy [C] time = 8.74049, size = 267, normalized size = 2.64

$$a^2c\left(\begin{cases}0 & \text{for } c=0\\ \frac{\sqrt{c}x^2}{2} & \text{for } a^2=0\\ -\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3a^2c} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{ia^2\sqrt{c}x^3}{2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}x}{2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\operatorname{acosh}(ax)}{2a} & \text{for } \left|a^2x^2\right|>1\\ \frac{\sqrt{c}x\sqrt{-a^2x^2+1}}{2}+\frac{\sqrt{c}\operatorname{asin}(ax)}{2a} & \text{otherwise}\end{cases}\right)+c\left(\begin{cases}i\sqrt{c}\sqrt{a^2x^2-1}-\sqrt{c}\,\mathrm{l} \\ \sqrt{c}\sqrt{-a^2x^2+1}+\frac{\sqrt{c}\,\mathrm{l}}{}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2)/x,x)
```

```
[Out] a**2*c*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x
**2 + c)**(3/2)/(3*a**2*c), True)) + 2*a*c*Piecewise((I*a**2*sqrt(c)*x**3/(
2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*ac
osh(a*x)/(2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sq
rt(c)*asin(a*x)/(2*a), True)) + c*Piecewise((I*sqrt(c)*sqrt(a**2*x**2 - 1)
- sqrt(c)*log(a*x) + sqrt(c)*log(a**2*x**2)/2 + I*sqrt(c)*asin(1/(a*x)), Ab
s(a**2*x**2) > 1), (sqrt(c)*sqrt(-a**2*x**2 + 1) + sqrt(c)*log(a**2*x**2)/2
 - sqrt(c)*log(sqrt(-a**2*x**2 + 1) + 1), True))
```

Giac [A] time = 1.17446, size = 157, normalized size = 1.55

$$\frac{2\,c^2\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{a\sqrt{-c}c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}+\frac{1}{3}\sqrt{-a^2cx^2+c}\left(\left(a^2cx+3\,ac\right)x+2\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x,x, algorithm="giac"
)
```

```
[Out] 2*c^2*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) +
a*sqrt(-c)*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) + 1/3*
sqrt(-a^2*c*x^2 + c)*((a^2*c*x + 3*a*c)*x + 2*c)
```

3.1091 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^{3/2}}{x^2} dx$

Optimal. Leaf size=112

$$-\frac{1}{2}ac^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-2ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{1}{2}ac(4-ax)\sqrt{c-a^2cx^2}-\frac{(c-a^2cx^2)^{3/2}}{x}$$

```
[Out] (a*c*(4 - a*x)*Sqrt[c - a^2*c*x^2])/2 - (c - a^2*c*x^2)^(3/2)/x - (a*c^(3/2
)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/2 - 2*a*c^(3/2)*ArcTanh[Sqrt[c
 - a^2*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.285476, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6151, 1807, 815, 844, 217, 203, 266, 63, 208}

$$-\frac{1}{2}ac^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-2ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{1}{2}ac(4-ax)\sqrt{c-a^2cx^2}-\frac{(c-a^2cx^2)^{3/2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^2,x]
```


```
[Out] (a*c*(4 - a*x)*Sqrt[c - a^2*c*x^2])/2 - (c - a^2*c*x^2)^(3/2)/x - (a*c^(3/2
)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/2 - 2*a*c^(3/2)*ArcTanh[Sqrt[c
 - a^2*c*x^2]/Sqrt[c]]
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^2}dx = c\int \frac{(1+ax)^2\sqrt{c-a^2cx^2}}{x^2}dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{x} - \int \frac{\left(-2ac+a^2cx\right)\sqrt{c-a^2cx^2}}{x}dx$$
$$= \frac{1}{2}ac(4-ax)\sqrt{c-a^2cx^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{x} + \frac{\int \frac{4a^3c^3-a^4c^3x}{x\sqrt{c-a^2cx^2}}dx}{2a^2c}$$
$$= \frac{1}{2}ac(4-ax)\sqrt{c-a^2cx^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{x} + \left(2ac^2\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}dx - \frac{1}{2}\left(a^2c^2\right)\int$$
$$= \frac{1}{2}ac(4-ax)\sqrt{c-a^2cx^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{x} + \left(ac^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}dx, x, x^2\right) -$$
$$= \frac{1}{2}ac(4-ax)\sqrt{c-a^2cx^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{x} - \frac{1}{2}ac^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - \frac{(2c)\,\text{Subst}}{}$$
$$= \frac{1}{2}ac(4-ax)\sqrt{c-a^2cx^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{x} - \frac{1}{2}ac^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - 2ac^{3/2}\tanh$$

Mathematica [A] time = 0.15094, size = 124, normalized size = 1.11

$$-2ac^{3/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+\frac{1}{2}ac^{3/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)+\frac{c\left(a^2x^2+4ax-2\right)\sqrt{c-a^2cx^2}}{2x}+2ac^{3/2}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^2,x]
```

```
[Out] (c*(-2 + 4*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2])/(2*x) + (a*c^(3/2)*ArcTan[(a
*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/2 + 2*a*c^(3/2)*Log[x] -
 2*a*c^(3/2)*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

Maple [B] time = 0.042, size = 286, normalized size = 2.6

$$-\frac{1}{cx}\left(-a^2cx^2+c\right)^{\frac{5}{2}}-a^2x\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{3\,cxa^2}{2}\sqrt{-a^2cx^2+c}-\frac{3\,a^2c^2}{2}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}+\frac{2\,a}{3}\left(-a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^2,x)
```

```
[Out] -1/c/x*(-a^2*c*x^2+c)^(5/2)-a^2*x*(-a^2*c*x^2+c)^(3/2)-3/2*a^2*c*x*(-a^2*c*
x^2+c)^(1/2)-3/2*a^2*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c
)^(1/2))+2/3*a*(-a^2*c*x^2+c)^(3/2)-2*a*c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x
^2+c)^(1/2))/x)+2*a*(-a^2*c*x^2+c)^(1/2)*c-2/3*a*(-c*a^2*(x-1/a)^2-2*a*c*(x
-1/a))^(3/2)+a^2*c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x+a^2*c^2/(a^2*c)
^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^2,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x^2), x)
```

Fricas [A] time = 2.76932, size = 567, normalized size = 5.06

$$\left[\frac{ac^{\frac{3}{2}}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+2\,ac^{\frac{3}{2}}x\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+\left(a^2cx^2+4\,acx-2\,c\right)\sqrt{-a^2cx^2+c}}{2\,x}, -\frac{8\,a\sqrt{-c}cx\,\mathrm{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] [1/2*(a*c^(3/2)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c))
+ 2*a*c^(3/2)*x*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2
) + (a^2*c*x^2 + 4*a*c*x - 2*c)*sqrt(-a^2*c*x^2 + c))/x, -1/4*(8*a*sqrt(-c)
*c*x*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - a*sqrt(-c)*c*x
*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) - 2*(a^2*c*x^2
+ 4*a*c*x - 2*c)*sqrt(-a^2*c*x^2 + c))/x]
```

Sympy [C] time = 8.59299, size = 350, normalized size = 3.12

$$a^2c\left(\begin{cases}\frac{ia^2\sqrt{c}x^3}{2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}x}{2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\operatorname{acosh}(ax)}{2a} & \text{for } \left|a^2x^2\right|>1\\ \frac{\sqrt{c}x\sqrt{-a^2x^2+1}}{2}+\frac{\sqrt{c}\operatorname{asin}(ax)}{2a} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}i\sqrt{c}\sqrt{a^2x^2-1}-\sqrt{c}\log(ax)+\frac{\sqrt{c}\log\left(a^2x^2\right)}{2}+i\sqrt{c}\operatorname{asin}\left(\right. \\ \sqrt{c}\sqrt{-a^2x^2+1}+\frac{\sqrt{c}\log\left(a^2x^2\right)}{2}-\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right.\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2)/x**2,x)
```

```
[Out] a**2*c*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x
/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1),
 (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin(a*x)/(2*a), True)) + 2*a*
c*Piecewise((I*sqrt(c)*sqrt(a**2*x**2 - 1) - sqrt(c)*log(a*x) + sqrt(c)*log
(a**2*x**2)/2 + I*sqrt(c)*asin(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(c)*sqrt
(-a**2*x**2 + 1) + sqrt(c)*log(a**2*x**2)/2 - sqrt(c)*log(sqrt(-a**2*x**2 +
 1) + 1), True)) + c*Piecewise((-I*a**2*sqrt(c)*x/sqrt(a**2*x**2 - 1) + I*a
*sqrt(c)*acosh(a*x) + I*sqrt(c)/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2) > 1
), (a**2*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - a*sqrt(c)*asin(a*x) - sqrt(c)/(x*
sqrt(-a**2*x**2 + 1)), True))
```

Giac [A] time = 1.1971, size = 223, normalized size = 1.99

$$\frac{4\,ac^2\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{a^2\sqrt{-c}c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\,|a|}+\frac{2\,a^2\sqrt{-c}c^2}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a|}+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^2,x, algorithm="gia
c")
```

```
[Out] 4*a*c^2*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
- 1/2*a^2*sqrt(-c)*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a
) + 2*a^2*sqrt(-c)*c^2/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*abs
(a)) + 1/2*sqrt(-a^2*c*x^2 + c)*(a^2*c*x + 4*a*c)
```

3.1092 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=121

$$-2a^2c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-\frac{1}{2}a^2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{ac(4-ax)\sqrt{c-a^2cx^2}}{2x}-\frac{\left(c-a^2cx^2\right)^{3/2}}{2x^2}$$

```
[Out] -(a*c*(4 - a*x)*Sqrt[c - a^2*c*x^2])/(2*x) - (c - a^2*c*x^2)^(3/2)/(2*x^2)
- 2*a^2*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] - (a^2*c^(3/2)*Ar
cTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rubi [A] time = 0.28923, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6151, 1807, 813, 844, 217, 203, 266, 63, 208}

$$-2a^2c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-\frac{1}{2}a^2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{ac(4-ax)\sqrt{c-a^2cx^2}}{2x}-\frac{\left(c-a^2cx^2\right)^{3/2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^3,x]
```

```
[Out] -(a*c*(4 - a*x)*Sqrt[c - a^2*c*x^2])/(2*x) - (c - a^2*c*x^2)^(3/2)/(2*x^2)
- 2*a^2*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] - (a^2*c^(3/2)*Ar
cTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^3}\,dx = c\int \frac{(1+ax)^2\sqrt{c-a^2cx^2}}{x^3}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{2x^2} - \frac{1}{2}\int \frac{\left(-4ac-a^2cx\right)\sqrt{c-a^2cx^2}}{x^2}\,dx$$
$$= -\frac{ac(4-ax)\sqrt{c-a^2cx^2}}{2x} - \frac{\left(c-a^2cx^2\right)^{3/2}}{2x^2} + \frac{1}{4}\int \frac{2a^2c^2-8a^3c^2x}{x\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{ac(4-ax)\sqrt{c-a^2cx^2}}{2x} - \frac{\left(c-a^2cx^2\right)^{3/2}}{2x^2} + \frac{1}{2}\left(a^2c^2\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx - \left(2a^3c^2\right)\int \frac{}{\sqrt{c}}$$
$$= -\frac{ac(4-ax)\sqrt{c-a^2cx^2}}{2x} - \frac{\left(c-a^2cx^2\right)^{3/2}}{2x^2} + \frac{1}{4}\left(a^2c^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right) -$$
$$= -\frac{ac(4-ax)\sqrt{c-a^2cx^2}}{2x} - \frac{\left(c-a^2cx^2\right)^{3/2}}{2x^2} - 2a^2c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - \frac{1}{2}c\,\text{Subst}\left(\int\right.$$
$$= -\frac{ac(4-ax)\sqrt{c-a^2cx^2}}{2x} - \frac{\left(c-a^2cx^2\right)^{3/2}}{2x^2} - 2a^2c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - \frac{1}{2}a^2c^{3/2}\tanh^{-}$$

Mathematica [A] time = 0.208506, size = 129, normalized size = 1.07

$$\frac{1}{2}c\left(\frac{\left(2a^2x^2-4ax-1\right)\sqrt{c-a^2cx^2}}{x^2} - a^2\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) + 4a^2\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right) + a^2\sqrt{c}\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^3,x]
```

```
[Out] (c*(((-1 - 4*a*x + 2*a^2*x^2)*Sqrt[c - a^2*c*x^2])/x^2 + 4*a^2*Sqrt[c]*ArcT
an[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))] + a^2*Sqrt[c]*Log[x]
 - a^2*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]))/2
```

Maple [B] time = 0.046, size = 316, normalized size = 2.6

$$-2\,\frac{a\left(-a^2cx^2+c\right)^{5/2}}{cx} - 2\,a^3x\left(-a^2cx^2+c\right)^{3/2} - 3\,a^3cx\sqrt{-a^2cx^2+c} - 3\,\frac{a^3c^2}{\sqrt{a^2c}}\arctan\left(\frac{\sqrt{a^2c}x}{\sqrt{-a^2cx^2+c}}\right) + \frac{a^2}{6}\left(-a^2cx^2+c\right)^{\frac{3}{2}} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^3,x)
```

```
[Out] -2*a/c/x*(-a^2*c*x^2+c)^(5/2)-2*a^3*x*(-a^2*c*x^2+c)^(3/2)-3*a^3*c*x*(-a^2*
c*x^2+c)^(1/2)-3*a^3*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c
)^(1/2))+1/6*a^2*(-a^2*c*x^2+c)^(3/2)-1/2*a^2*c^(3/2)*ln((2*c+2*c^(1/2)*(-a
^2*c*x^2+c)^(1/2))/x)+1/2*a^2*(-a^2*c*x^2+c)^(1/2)*c-2/3*a^2*(-c*a^2*(x-1/a
)^2-2*a*c*(x-1/a))^(3/2)+a^3*c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x+a^3
*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^
(1/2))-1/2/c/x^2*(-a^2*c*x^2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^3,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x^3), x)
```

Fricas [A] time = 2.6843, size = 594, normalized size = 4.91

$$\left[\frac{8\,a^2c^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+a^2c^{\frac{3}{2}}x^2\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\left(2\,a^2cx^2-4\,acx-c\right)\sqrt{-a^2cx^2+c}}{4\,x^2},-\frac{a^2\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^3,x, algorithm="fri
cas")
```

```
[Out] [1/4*(8*a^2*c^(3/2)*x^2*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2
- c)) + a^2*c^(3/2)*x^2*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) + 2*(2*a^2*c*x^2 - 4*a*c*x - c)*sqrt(-a^2*c*x^2 + c))/x^2, -1/2*(
a^2*sqrt(-c)*c*x^2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) -
2*a^2*sqrt(-c)*c*x^2*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c) - (2*a^2*c*x^2 - 4*a*c*x - c)*sqrt(-a^2*c*x^2 + c))/x^2]
```

Sympy [C] time = 7.4378, size = 366, normalized size = 3.02

$$a^2c\left(\begin{cases} i\sqrt{c}\sqrt{a^2x^2-1}-\sqrt{c}\log(ax)+\frac{\sqrt{c}\log\left(a^2x^2\right)}{2}+i\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right) & \text{for } \left|a^2x^2\right|>1 \\ \sqrt{c}\sqrt{-a^2x^2+1}+\frac{\sqrt{c}\log\left(a^2x^2\right)}{2}-\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right) & \text{otherwise} \end{cases}\right)+2ac\left(\begin{cases} -\frac{ia^2\sqrt{c}x}{\sqrt{a^2x^2-1}}+ia\sqrt{c}\operatorname{acosh}(ax) \\ \frac{a^2\sqrt{c}x}{\sqrt{-a^2x^2+1}}-a\sqrt{c}\operatorname{asin}(ax)- \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2)/x**3,x)
```

```
[Out] a**2*c*Piecewise((I*sqrt(c)*sqrt(a**2*x**2 - 1) - sqrt(c)*log(a*x) + sqrt(c
)*log(a**2*x**2)/2 + I*sqrt(c)*asin(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(c)
*sqrt(-a**2*x**2 + 1) + sqrt(c)*log(a**2*x**2)/2 - sqrt(c)*log(sqrt(-a**2*x
**2 + 1) + 1), True)) + 2*a*c*Piecewise((-I*a**2*sqrt(c)*x/sqrt(a**2*x**2 -
 1) + I*a*sqrt(c)*acosh(a*x) + I*sqrt(c)/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*
x**2) > 1), (a**2*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - a*sqrt(c)*asin(a*x) - sq
rt(c)/(x*sqrt(-a**2*x**2 + 1)), True)) + c*Piecewise((a**2*sqrt(c)*acosh(1/
(a*x))/2 + a*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - sqrt(c)/(2*a*x**3*sqr
t(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*a**2*sqrt(c)*asin(1/(a*x
```

```
))/2 - I*a*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))
```

Giac [B] time = 1.18399, size = 370, normalized size = 3.06

$$\frac{a^2c^2\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{2\,a^3\sqrt{-c}c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}+\sqrt{-a^2cx^2+c}a^2c-\frac{\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+\dots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^3,x, algorithm="gia
c")
```

```
[Out] a^2*c^2*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
- 2*a^3*sqrt(-c)*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a)
+ sqrt(-a^2*c*x^2 + c)*a^2*c - ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a
^2*c^2*abs(a) - 4*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*sqrt(-c)*c^
2 + (sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^2*c^3*abs(a) + 4*a^3*sqrt(-c)
*c^3)/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^2*abs(a))
```

3.1093 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=115

$$a^3\left(-c^{3/2}\right)\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+a^3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{ac(ax+1)\sqrt{c-a^2cx^2}}{x^2}-\frac{\left(c-a^2cx^2\right)^{3/2}}{3x^3}$$

```
[Out] -((a*c*(1 + a*x)*Sqrt[c - a^2*c*x^2])/x^2) - (c - a^2*c*x^2)^(3/2)/(3*x^3)
- a^3*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] + a^3*c^(3/2)*ArcTa
nh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.2824, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6151, 1807, 811, 844, 217, 203, 266, 63, 208}

$$a^3\left(-c^{3/2}\right)\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+a^3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{ac(ax+1)\sqrt{c-a^2cx^2}}{x^2}-\frac{\left(c-a^2cx^2\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^4,x]
```

```
[Out] -((a*c*(1 + a*x)*Sqrt[c - a^2*c*x^2])/x^2) - (c - a^2*c*x^2)^(3/2)/(3*x^3)
- a^3*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] + a^3*c^(3/2)*ArcTa
nh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^4}\,dx = c\int \frac{(1+ax)^2\sqrt{c-a^2cx^2}}{x^4}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{3x^3} - \frac{1}{3}\int \frac{\left(-6ac-3a^2cx\right)\sqrt{c-a^2cx^2}}{x^3}\,dx$$
$$= -\frac{ac(1+ax)\sqrt{c-a^2cx^2}}{x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{3x^3} + \frac{\int \frac{-12a^3c^3-12a^4c^3x}{x\sqrt{c-a^2cx^2}}\,dx}{12c}$$
$$= -\frac{ac(1+ax)\sqrt{c-a^2cx^2}}{x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{3x^3} - \left(a^3c^2\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx - \left(a^4c^2\right)\int \frac{}{\sqrt{c}}$$
$$= -\frac{ac(1+ax)\sqrt{c-a^2cx^2}}{x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{3x^3} - \frac{1}{2}\left(a^3c^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right)$$
$$= -\frac{ac(1+ax)\sqrt{c-a^2cx^2}}{x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{3x^3} - a^3c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) + (ac)\,\text{Subst}$$
$$= -\frac{ac(1+ax)\sqrt{c-a^2cx^2}}{x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{3x^3} - a^3c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) + a^3c^{3/2}\tanh$$

Mathematica [A] time = 0.15585, size = 127, normalized size = 1.1

$$a^3c^{3/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) + a^3c^{3/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right) - a^3c^{3/2}\log(x) - \frac{c\left(2a^2x^2+3ax+1\right)\sqrt{c-a^2cx^2}}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^4,x]
```

```
[Out] -(c*(1 + 3*a*x + 2*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*x^3) + a^3*c^(3/2)*ArcT
an[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))] - a^3*c^(3/2)*Log[x]
 + a^3*c^(3/2)*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

Maple [B] time = 0.049, size = 339, normalized size = 3.

$$-\frac{4a^2}{3cx}\left(-a^2cx^2+c\right)^{\frac{5}{2}} - \frac{4a^4x}{3}\left(-a^2cx^2+c\right)^{\frac{3}{2}} - 2a^4cx\sqrt{-a^2cx^2+c} - 2\frac{a^4c^2}{\sqrt{a^2c}}\arctan\left(\frac{\sqrt{a^2c}x}{\sqrt{-a^2cx^2+c}}\right) - \frac{a^3}{3}\left(-a^2cx^2+c\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^4,x)
```

```
[Out] -4/3*a^2/c/x*(-a^2*c*x^2+c)^(5/2)-4/3*a^4*x*(-a^2*c*x^2+c)^(3/2)-2*a^4*c*x*
(-a^2*c*x^2+c)^(1/2)-2*a^4*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c
*x^2+c)^(1/2))-1/3*a^3*(-a^2*c*x^2+c)^(3/2)+a^3*c^(3/2)*ln((2*c+2*c^(1/2)*(
-a^2*c*x^2+c)^(1/2))/x)-a^3*(-a^2*c*x^2+c)^(1/2)*c-2/3*a^3*(-c*a^2*(x-1/a)^
2-2*a*c*(x-1/a))^(3/2)+a^4*c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x+a^4*c
^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1
/2))-a/c/x^2*(-a^2*c*x^2+c)^(5/2)-1/3/c/x^3*(-a^2*c*x^2+c)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^4,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.83923, size = 601, normalized size = 5.23

$$\left[\frac{6\,a^3c^{\frac{3}{2}}x^3\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+3\,a^3c^{\frac{3}{2}}x^3\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-2\left(2\,a^2cx^2+3\,acx+c\right)\sqrt{-a^2cx^2+c}}{6\,x^3}, \frac{6\,a^3\sqrt{-c}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^4,x, algorithm="fri
cas")
```

```
[Out] [1/6*(6*a^3*c^(3/2)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2
- c)) + 3*a^3*c^(3/2)*x^3*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) - 2*(2*a^2*c*x^2 + 3*a*c*x + c)*sqrt(-a^2*c*x^2 + c))/x^3, 1/6*
(6*a^3*sqrt(-c)*c*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c))
 + 3*a^3*sqrt(-c)*c*x^3*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)
*x - c) - 2*(2*a^2*c*x^2 + 3*a*c*x + c)*sqrt(-a^2*c*x^2 + c))/x^3]
```

Sympy [C] time = 10.0809, size = 359, normalized size = 3.12

$$a^2c\left(\begin{cases}-\frac{ia^2\sqrt{c}x}{\sqrt{a^2x^2-1}}+ia\sqrt{c}\operatorname{acosh}(ax)+\frac{i\sqrt{c}}{x\sqrt{a^2x^2-1}} & \text{for } \left|a^2x^2\right|>1\\ \frac{a^2\sqrt{c}x}{\sqrt{-a^2x^2+1}}-a\sqrt{c}\operatorname{asin}(ax)-\frac{\sqrt{c}}{x\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{a^2\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}+\frac{a\sqrt{c}}{2x\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{\sqrt{c}}{2ax^3\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for}\cdots\\ -\frac{ia^2\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{2x} & \text{oth}\cdots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2)/x**4,x)
```

```
[Out] a**2*c*Piecewise((-I*a**2*sqrt(c)*x/sqrt(a**2*x**2 - 1) + I*a*sqrt(c)*acosh
(a*x) + I*sqrt(c)/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2) > 1), (a**2*sqrt(
c)*x/sqrt(-a**2*x**2 + 1) - a*sqrt(c)*asin(a*x) - sqrt(c)/(x*sqrt(-a**2*x**
2 + 1)), True)) + 2*a*c*Piecewise((a**2*sqrt(c)*acosh(1/(a*x))/2 + a*sqrt(c
)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - sqrt(c)/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2
))), 1/Abs(a**2*x**2) > 1), (-I*a**2*sqrt(c)*asin(1/(a*x))/2 - I*a*sqrt(c)*
sqrt(1 - 1/(a**2*x**2))/(2*x), True)) + c*Piecewise((a**3*sqrt(c)*sqrt(-1 +
 1/(a**2*x**2))/3 - a*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/(3*x**2), 1/Abs(a**2
*x**2) > 1), (I*a**3*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/3 - I*a*sqrt(c)*sqrt(1
 - 1/(a**2*x**2))/(3*x**2), True))
```

Giac [B] time = 1.18994, size = 350, normalized size = 3.04

$$-\frac{2\,a^{3}c^{2}\arctan\left(-\frac{\sqrt{-a^{2}c}x-\sqrt{-a^{2}cx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{a^{4}\sqrt{-c}c\log\left(\left|-\sqrt{-a^{2}c}x+\sqrt{-a^{2}cx^{2}+c}\right|\right)}{|a|}-\frac{2\left(3\left(\sqrt{-a^{2}c}x-\sqrt{-a^{2}cx^{2}+c}\right)^{5}a^{3}c^{2}|}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^4,x, algorithm="gia
c")
```

```
[Out] -2*a^3*c^2*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-
c) - a^4*sqrt(-c)*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a)
 - 2/3*(3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c^2*abs(a) + 6*(sqr
t(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^4*sqrt(-c)*c^3 - 3*(sqrt(-a^2*c)*x
- sqrt(-a^2*c*x^2 + c))*a^3*c^4*abs(a) - 2*a^4*sqrt(-c)*c^4)/(((sqrt(-a^2*c
)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^3*abs(a))
```

3.1094 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=106

$$\frac{5}{8}a^4c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{5a^2c\sqrt{c-a^2cx^2}}{8x^2}-\frac{2a\left(c-a^2cx^2\right)^{3/2}}{3x^3}-\frac{\left(c-a^2cx^2\right)^{3/2}}{4x^4}$$

```
[Out] (-5*a^2*c*Sqrt[c - a^2*c*x^2])/(8*x^2) - (c - a^2*c*x^2)^(3/2)/(4*x^4) - (2
*a*(c - a^2*c*x^2)^(3/2))/(3*x^3) + (5*a^4*c^(3/2)*ArcTanh[Sqrt[c - a^2*c*x
^2]/Sqrt[c]])/8
```

Rubi [A] time = 0.256783, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1807, 807, 266, 47, 63, 208}

$$\frac{5}{8}a^4c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{5a^2c\sqrt{c-a^2cx^2}}{8x^2}-\frac{2a\left(c-a^2cx^2\right)^{3/2}}{3x^3}-\frac{\left(c-a^2cx^2\right)^{3/2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^5,x]
```


```
[Out] (-5*a^2*c*Sqrt[c - a^2*c*x^2])/(8*x^2) - (c - a^2*c*x^2)^(3/2)/(4*x^4) - (2
*a*(c - a^2*c*x^2)^(3/2))/(3*x^3) + (5*a^4*c^(3/2)*ArcTanh[Sqrt[c - a^2*c*x
^2]/Sqrt[c]])/8
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^5}\,dx &= c\int \frac{(1+ax)^2\sqrt{c-a^2cx^2}}{x^5}\,dx\\
&= -\frac{\left(c-a^2cx^2\right)^{3/2}}{4x^4}-\frac{1}{4}\int \frac{\left(-8ac-5a^2cx\right)\sqrt{c-a^2cx^2}}{x^4}\,dx\\
&= -\frac{\left(c-a^2cx^2\right)^{3/2}}{4x^4}-\frac{2a\left(c-a^2cx^2\right)^{3/2}}{3x^3}+\frac{1}{4}\left(5a^2c\right)\int \frac{\sqrt{c-a^2cx^2}}{x^3}\,dx\\
&= -\frac{\left(c-a^2cx^2\right)^{3/2}}{4x^4}-\frac{2a\left(c-a^2cx^2\right)^{3/2}}{3x^3}+\frac{1}{8}\left(5a^2c\right)\text{Subst}\left(\int \frac{\sqrt{c-a^2cx}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{5a^2c\sqrt{c-a^2cx^2}}{8x^2}-\frac{\left(c-a^2cx^2\right)^{3/2}}{4x^4}-\frac{2a\left(c-a^2cx^2\right)^{3/2}}{3x^3}-\frac{1}{16}\left(5a^4c^2\right)\text{Subst}\left(\int \frac{}{x\sqrt{c}}\right.\\
&= -\frac{5a^2c\sqrt{c-a^2cx^2}}{8x^2}-\frac{\left(c-a^2cx^2\right)^{3/2}}{4x^4}-\frac{2a\left(c-a^2cx^2\right)^{3/2}}{3x^3}+\frac{1}{8}\left(5a^2c\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x}{a^2}}\right.\\
&= -\frac{5a^2c\sqrt{c-a^2cx^2}}{8x^2}-\frac{\left(c-a^2cx^2\right)^{3/2}}{4x^4}-\frac{2a\left(c-a^2cx^2\right)^{3/2}}{3x^3}+\frac{5}{8}a^4c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx}}{\sqrt{c}}\right.
\end{aligned}$$

Mathematica [A] time = 0.149083, size = 96, normalized size = 0.91

$$\frac{5}{8}a^4c^{3/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)-\frac{5}{8}a^4c^{3/2}\log(x)+\frac{c\left(16a^3x^3-9a^2x^2-16ax-6\right)\sqrt{c-a^2cx^2}}{24x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^5,x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-6 - 16*a*x - 9*a^2*x^2 + 16*a^3*x^3))/(24*x^4) - (
5*a^4*c^(3/2)*Log[x])/8 + (5*a^4*c^(3/2)*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2
```

```
]])/8
```

Maple [B] time = 0.054, size = 364, normalized size = 3.4

$$-\frac{1}{4cx^4}\left(-a^2cx^2+c\right)^{\frac{5}{2}}-\frac{7a^2}{8cx^2}\left(-a^2cx^2+c\right)^{\frac{5}{2}}-\frac{5a^4}{24}\left(-a^2cx^2+c\right)^{\frac{3}{2}}+\frac{5a^4}{8}c^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right)-\frac{5a^4c}{8}\sqrt{-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^5,x)
```

```
[Out] -1/4/c/x^4*(-a^2*c*x^2+c)^(5/2)-7/8*a^2/c/x^2*(-a^2*c*x^2+c)^(5/2)-5/24*a^4
*(-a^2*c*x^2+c)^(3/2)+5/8*a^4*c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2
))/x)-5/8*a^4*(-a^2*c*x^2+c)^(1/2)*c-2/3*a^3/c/x*(-a^2*c*x^2+c)^(5/2)-2/3*a
^5*x*(-a^2*c*x^2+c)^(3/2)-a^5*c*x*(-a^2*c*x^2+c)^(1/2)-a^5*c^2/(a^2*c)^(1/2
)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/3*a^4*(-c*a^2*(x-1/a)^2-2*
a*c*(x-1/a))^(3/2)+a^5*c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x+a^5*c^2/(
a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
-2/3*a/c/x^3*(-a^2*c*x^2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^5,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x^5), x)
```

Fricas [A] time = 2.70584, size = 437, normalized size = 4.12

$$\left[\frac{15a^4c^{\frac{3}{2}}x^4\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+2\left(16a^3cx^3-9a^2cx^2-16acx-6c\right)\sqrt{-a^2cx^2+c}}{48x^4},\frac{15a^4\sqrt{-c}cx^4\arctan\left(\frac{\sqrt{-a^2cx^2}}{a^2cx}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^5,x, algorithm="fri
cas")
```

```
[Out] [1/48*(15*a^4*c^(3/2)*x^4*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) + 2*(16*a^3*c*x^3 - 9*a^2*c*x^2 - 16*a*c*x - 6*c)*sqrt(-a^2*c*x
^2 + c))/x^4, 1/24*(15*a^4*sqrt(-c)*c*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(
-c)/(a^2*c*x^2 - c)) + (16*a^3*c*x^3 - 9*a^2*c*x^2 - 16*a*c*x - 6*c)*sqrt(-
a^2*c*x^2 + c))/x^4]
```

Sympy [C] time = 8.46438, size = 447, normalized size = 4.22

$$a^2c\left(\begin{cases}\frac{a^2\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}+\frac{a\sqrt{c}}{2x\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{\sqrt{c}}{2ax^3\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{ia^2\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{a^3\sqrt{c}\sqrt{-1+\frac{1}{a^2x^2}}}{3}-\frac{a\sqrt{c}\sqrt{-1+\frac{1}{a^2x^2}}}{3x^2} & \text{for } \frac{1}{|a^2x^2|}>1\\ \frac{ia^3\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{3}-\frac{ia\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} & \text{otherwise}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2)/x**5,x)
```

```
[Out] a**2*c*Piecewise((a**2*sqrt(c)*acosh(1/(a*x))/2 + a*sqrt(c)/(2*x*sqrt(-1 +
1/(a**2*x**2))) - sqrt(c)/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x
**2) > 1), (-I*a**2*sqrt(c)*asin(1/(a*x))/2 - I*a*sqrt(c)*sqrt(1 - 1/(a**2*
x**2))/(2*x), True)) + 2*a*c*Piecewise((a**3*sqrt(c)*sqrt(-1 + 1/(a**2*x**2
))/3 - a*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/(3*x**2), 1/Abs(a**2*x**2) > 1),
(I*a**3*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/3 - I*a*sqrt(c)*sqrt(1 - 1/(a**2*x*
*2))/(3*x**2), True)) + c*Piecewise((a**4*sqrt(c)*acosh(1/(a*x))/8 - a**3*s
qrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*a*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a*
*2*x**2))) - sqrt(c)/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2)
> 1), (-I*a**4*sqrt(c)*asin(1/(a*x))/8 + I*a**3*sqrt(c)/(8*x*sqrt(1 - 1/(a*
*2*x**2))) - 3*I*a*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I*sqrt(c)/(4*
a*x**5*sqrt(1 - 1/(a**2*x**2))), True))
```

Giac [B] time = 1.17848, size = 501, normalized size = 4.73

$$-\frac{5\,a^4c^2\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}}-\frac{9\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^7a^4c^2+48\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^6a^3\sqrt{-c}c^2|a|-3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^5,x, algorithm="gia
c")
```

```
[Out] -5/4*a^4*c^2*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt
(-c) - 1/12*(9*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^4*c^2 + 48*(sqrt
(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^6*a^3*sqrt(-c)*c^2*abs(a) - 33*(sqrt(-a^
2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^3 - 48*(sqrt(-a^2*c)*x - sqrt(-a^2*c
*x^2 + c))^4*a^3*sqrt(-c)*c^3*abs(a) - 33*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2
 + c))^3*a^4*c^4 + 16*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*sqrt(-c
)*c^4*abs(a) + 9*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^4*c^5 - 16*a^3*s
qrt(-c)*c^5*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^4
```

3.1095 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=131

$$\frac{1}{4}a^5c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{a^3c\sqrt{c-a^2cx^2}}{4x^2}-\frac{7a^2\left(c-a^2cx^2\right)^{3/2}}{15x^3}-\frac{a\left(c-a^2cx^2\right)^{3/2}}{2x^4}-\frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5}$$

```
[Out] -(a^3*c*Sqrt[c - a^2*c*x^2])/(4*x^2) - (c - a^2*c*x^2)^(3/2)/(5*x^5) - (a*(
c - a^2*c*x^2)^(3/2))/(2*x^4) - (7*a^2*(c - a^2*c*x^2)^(3/2))/(15*x^3) + (a
^5*c^(3/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/4
```

Rubi [A] time = 0.29272, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6151, 1807, 835, 807, 266, 47, 63, 208}

$$\frac{1}{4}a^5c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{a^3c\sqrt{c-a^2cx^2}}{4x^2}-\frac{7a^2\left(c-a^2cx^2\right)^{3/2}}{15x^3}-\frac{a\left(c-a^2cx^2\right)^{3/2}}{2x^4}-\frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^6,x]
```

```
[Out] -(a^3*c*Sqrt[c - a^2*c*x^2])/(4*x^2) - (c - a^2*c*x^2)^(3/2)/(5*x^5) - (a*(
c - a^2*c*x^2)^(3/2))/(2*x^4) - (7*a^2*(c - a^2*c*x^2)^(3/2))/(15*x^3) + (a
^5*c^(3/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/4
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^6}\,dx = c\int \frac{(1+ax)^2\sqrt{c-a^2cx^2}}{x^6}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{1}{5}\int \frac{\left(-10ac-7a^2cx\right)\sqrt{c-a^2cx^2}}{x^5}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{2x^4} + \frac{\int \frac{\left(28a^2c^2+10a^3c^2x\right)\sqrt{c-a^2cx^2}}{x^4}dx}{20c}$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{2x^4} - \frac{7a^2\left(c-a^2cx^2\right)^{3/2}}{15x^3} + \frac{1}{2}\left(a^3c\right)\int \frac{\sqrt{c-a^2cx^2}}{x^3}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{2x^4} - \frac{7a^2\left(c-a^2cx^2\right)^{3/2}}{15x^3} + \frac{1}{4}\left(a^3c\right)\text{Subst}\left(\int \frac{\sqrt{c-a^2cx}}{x^2}\right.$$
$$= -\frac{a^3c\sqrt{c-a^2cx^2}}{4x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{2x^4} - \frac{7a^2\left(c-a^2cx^2\right)^{3/2}}{15x^3} - \frac{1}{8}\left(a^5c^2\right)\text{Su}$$
$$= -\frac{a^3c\sqrt{c-a^2cx^2}}{4x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{2x^4} - \frac{7a^2\left(c-a^2cx^2\right)^{3/2}}{15x^3} + \frac{1}{4}\left(a^3c\right)\text{Sub}$$
$$= -\frac{a^3c\sqrt{c-a^2cx^2}}{4x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{2x^4} - \frac{7a^2\left(c-a^2cx^2\right)^{3/2}}{15x^3} + \frac{1}{4}a^5c^{3/2}\tan$$

Mathematica [A] time = 0.149663, size = 104, normalized size = 0.79

$$\frac{1}{4}a^5c^{3/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) - \frac{1}{4}a^5c^{3/2}\log(x) + \frac{c\left(28a^4x^4+15a^3x^3-16a^2x^2-30ax-12\right)\sqrt{c-a^2cx^2}}{60x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^6,x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-12 - 30*a*x - 16*a^2*x^2 + 15*a^3*x^3 + 28*a^4*x^4
))/(60*x^5) - (a^5*c^(3/2)*Log[x])/4 + (a^5*c^(3/2)*Log[c + Sqrt[c]*Sqrt[c
- a^2*c*x^2]])/4
```

Maple [B] time = 0.063, size = 388, normalized size = 3.

$$-\frac{2a^6x}{3}\left(-a^2cx^2+c\right)^{\frac{3}{2}} + \frac{a^5}{4}c^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right) - \frac{a^5c}{4}\sqrt{-a^2cx^2+c} - \frac{1}{5cx^5}\left(-a^2cx^2+c\right)^{\frac{5}{2}} - \frac{a^5}{12}\left(-a^2cx^2+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^6,x)
```

```
[Out] -2/3*a^6*x*(-a^2*c*x^2+c)^(3/2)+1/4*a^5*c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x
^2+c)^(1/2))/x)-1/4*a^5*(-a^2*c*x^2+c)^(1/2)*c-1/5/c/x^5*(-a^2*c*x^2+c)^(5/
2)-1/12*a^5*(-a^2*c*x^2+c)^(3/2)+a^6*c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/
2)*x+a^6*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(
x-1/a))^(1/2))-1/2*a/c/x^4*(-a^2*c*x^2+c)^(5/2)-3/4*a^3/c/x^2*(-a^2*c*x^2+c
)^(5/2)-2/3*a^2/c/x^3*(-a^2*c*x^2+c)^(5/2)-2/3*a^4/c/x*(-a^2*c*x^2+c)^(5/2)
-a^6*c*x*(-a^2*c*x^2+c)^(1/2)-a^6*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/
```

```
(-a^2*c*x^2+c)^(1/2))-2/3*a^5*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^6,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.7732, size = 485, normalized size = 3.7

$$\left[\frac{15\,a^5c^{\frac{3}{2}}x^5\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\left(28\,a^4cx^4+15\,a^3cx^3-16\,a^2cx^2-30\,acx-12\,c\right)\sqrt{-a^2cx^2+c}}{120\,x^5},\ \frac{15\,a^5\sqrt{-c}cx}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^6,x, algorithm="fri
cas")
```

```
[Out] [1/120*(15*a^5*c^(3/2)*x^5*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
 - 2*c)/x^2) + 2*(28*a^4*c*x^4 + 15*a^3*c*x^3 - 16*a^2*c*x^2 - 30*a*c*x - 1
2*c)*sqrt(-a^2*c*x^2 + c))/x^5, 1/60*(15*a^5*sqrt(-c)*c*x^5*arctan(sqrt(-a^
2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (28*a^4*c*x^4 + 15*a^3*c*x^3 - 16*
a^2*c*x^2 - 30*a*c*x - 12*c)*sqrt(-a^2*c*x^2 + c))/x^5]
```

Sympy [C] time = 14.5692, size = 484, normalized size = 3.69

$$a^2c\left(\begin{cases}\frac{a^3\sqrt{c}\sqrt{-1+\frac{1}{a^2x^2}}}{3}-\frac{a\sqrt{c}\sqrt{-1+\frac{1}{a^2x^2}}}{3x^2} & \text{for}\ \frac{1}{|a^2x^2|}>1\\ \frac{ia^3\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{3}-\frac{ia\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{a^4\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{8}-\frac{a^3\sqrt{c}}{8x\sqrt{-1+\frac{1}{a^2x^2}}}+\frac{3a\sqrt{c}}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{\sqrt{c}}{4ax^5\sqrt{-1+\frac{1}{a^2x^2}}}\\ -\frac{ia^4\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{8}+\frac{ia^3\sqrt{c}}{8x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3ia\sqrt{c}}{8x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i\sqrt{c}}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2)/x**6,x)
```

```
[Out] a**2*c*Piecewise((a**3*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(c)*sqrt(
-1 + 1/(a**2*x**2))/(3*x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(c)*sqrt(1
 - 1/(a**2*x**2))/3 - I*a*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True))
+ 2*a*c*Piecewise((a**4*sqrt(c)*acosh(1/(a*x))/8 - a**3*sqrt(c)/(8*x*sqrt(-
1 + 1/(a**2*x**2))) + 3*a*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - sqrt(
c)/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2)))), 1/Abs(a**2*x**2) > 1), (-I*a**4*sqr
t(c)*asin(1/(a*x))/8 + I*a**3*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2))) - 3*I*a
*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I*sqrt(c)/(4*a*x**5*sqrt(1 - 1/
```

```
(a**2*x**2))), True)) + c*Piecewise((2*I*a**4*sqrt(c)*sqrt(a**2*x**2 - 1)/(
15*x) + I*a**2*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x**3) - I*sqrt(c)*sqrt(a**2*
x**2 - 1)/(5*x**5), Abs(a**2*x**2) > 1), (2*a**4*sqrt(c)*sqrt(-a**2*x**2 +
1)/(15*x) + a**2*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3) - sqrt(c)*sqrt(-a**
2*x**2 + 1)/(5*x**5), True))
```

Giac [B] time = 1.16269, size = 559, normalized size = 4.27

$$-\frac{a^5c^2\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{2\sqrt{-c}}+\frac{15\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^9a^5c^2-60\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^8a^4\sqrt{-c}c^2|a|+90\left(\sqrt{\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^6,x, algorithm="gia
c")
```

```
[Out] -1/2*a^5*c^2*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt
(-c) + 1/30*(15*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^9*a^5*c^2 - 60*(sqr
t(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^8*a^4*sqrt(-c)*c^2*abs(a) + 90*(sqrt(-a
^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^5*c^3 + 240*(sqrt(-a^2*c)*x - sqrt(-a^2
*c*x^2 + c))^6*a^4*sqrt(-c)*c^3*abs(a) - 40*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x
^2 + c))^4*a^4*sqrt(-c)*c^4*abs(a) - 90*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 +
 c))^3*a^5*c^5 + 80*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^4*sqrt(-c)*
c^5*abs(a) - 15*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^5*c^6 - 28*a^4*sq
rt(-c)*c^6*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^5
```

3.1096 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=156

$$\frac{3}{16}a^6c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{3a^4c\sqrt{c-a^2cx^2}}{16x^2}-\frac{4a^3\left(c-a^2cx^2\right)^{3/2}}{15x^3}-\frac{3a^2\left(c-a^2cx^2\right)^{3/2}}{8x^4}-\frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5}-\frac{(c-a}{}$$

```
[Out] (-3*a^4*c*Sqrt[c - a^2*c*x^2])/(16*x^2) - (c - a^2*c*x^2)^(3/2)/(6*x^6) - (
2*a*(c - a^2*c*x^2)^(3/2))/(5*x^5) - (3*a^2*(c - a^2*c*x^2)^(3/2))/(8*x^4)
- (4*a^3*(c - a^2*c*x^2)^(3/2))/(15*x^3) + (3*a^6*c^(3/2)*ArcTanh[Sqrt[c -
a^2*c*x^2]/Sqrt[c]])/16
```

Rubi [A] time = 0.330117, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6151, 1807, 835, 807, 266, 47, 63, 208}

$$\frac{3}{16}a^6c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{3a^4c\sqrt{c-a^2cx^2}}{16x^2}-\frac{4a^3\left(c-a^2cx^2\right)^{3/2}}{15x^3}-\frac{3a^2\left(c-a^2cx^2\right)^{3/2}}{8x^4}-\frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5}-\frac{(c-a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^7,x]
```


```
[Out] (-3*a^4*c*Sqrt[c - a^2*c*x^2])/(16*x^2) - (c - a^2*c*x^2)^(3/2)/(6*x^6) - (
2*a*(c - a^2*c*x^2)^(3/2))/(5*x^5) - (3*a^2*(c - a^2*c*x^2)^(3/2))/(8*x^4)
- (4*a^3*(c - a^2*c*x^2)^(3/2))/(15*x^3) + (3*a^6*c^(3/2)*ArcTanh[Sqrt[c -
a^2*c*x^2]/Sqrt[c]])/16
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^7}\,dx = c\int \frac{(1+ax)^2\sqrt{c-a^2cx^2}}{x^7}\,dx$$

$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6} - \frac{1}{6}\int \frac{\left(-12ac-9a^2cx\right)\sqrt{c-a^2cx^2}}{x^6}\,dx$$

$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5} + \frac{\int \frac{\left(45a^2c^2+24a^3c^2x\right)\sqrt{c-a^2cx^2}}{x^5}\,dx}{30c}$$

$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{3a^2\left(c-a^2cx^2\right)^{3/2}}{8x^4} - \frac{\int \frac{\left(-96a^3c^3-45a^4c^3x\right)\sqrt{c-a^2cx^2}}{x^4}}{120c^2}$$

$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{3a^2\left(c-a^2cx^2\right)^{3/2}}{8x^4} - \frac{4a^3\left(c-a^2cx^2\right)^{3/2}}{15x^3} + \frac{1}{8}(3$$

$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{3a^2\left(c-a^2cx^2\right)^{3/2}}{8x^4} - \frac{4a^3\left(c-a^2cx^2\right)^{3/2}}{15x^3} + \frac{1}{16}($$

$$= -\frac{3a^4c\sqrt{c-a^2cx^2}}{16x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{3a^2\left(c-a^2cx^2\right)^{3/2}}{8x^4} - \frac{4a^3(c}{}$$

$$= -\frac{3a^4c\sqrt{c-a^2cx^2}}{16x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{3a^2\left(c-a^2cx^2\right)^{3/2}}{8x^4} - \frac{4a^3(c}{}$$

$$= -\frac{3a^4c\sqrt{c-a^2cx^2}}{16x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{3a^2\left(c-a^2cx^2\right)^{3/2}}{8x^4} - \frac{4a^3(c}{}$$

Mathematica [A] time = 0.231218, size = 109, normalized size = 0.7

$$\frac{1}{240}c\left(\frac{\left(64a^5x^5+45a^4x^4+32a^3x^3-50a^2x^2-96ax-40\right)\sqrt{c-a^2cx^2}}{x^6}+45a^6\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)-45a^6\sqrt{c}\log\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^7,x]
```

```
[Out] (c*((Sqrt[c - a^2*c*x^2]*(-40 - 96*a*x - 50*a^2*x^2 + 32*a^3*x^3 + 45*a^4*x
^4 + 64*a^5*x^5))/x^6 - 45*a^6*Sqrt[c]*Log[x] + 45*a^6*Sqrt[c]*Log[c + Sqrt
[c]*Sqrt[c - a^2*c*x^2]]))/240
```

Maple [B] time = 0.074, size = 412, normalized size = 2.6

$$-\frac{2a^6}{3}\left(-ca^2\left(x-a^{-1}\right)^2-2ac\left(x-a^{-1}\right)\right)^{\frac{3}{2}}-\frac{35a^4}{48cx^2}\left(-a^2cx^2+c\right)^{\frac{5}{2}}-\frac{2a^7x}{3}\left(-a^2cx^2+c\right)^{\frac{3}{2}}+\frac{3a^6}{16}c^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^7,x)
```

```
[Out] -2/3*a^6*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)-35/48*a^4/c/x^2*(-a^2*c*x^2
+c)^(5/2)-2/3*a^7*x*(-a^2*c*x^2+c)^(3/2)+3/16*a^6*c^(3/2)*ln((2*c+2*c^(1/2)
*(-a^2*c*x^2+c)^(1/2))/x)-3/16*a^6*(-a^2*c*x^2+c)^(1/2)*c-1/6/c/x^6*(-a^2*c
*x^2+c)^(5/2)-2/3*a^3/c/x^3*(-a^2*c*x^2+c)^(5/2)-13/24*a^2/c/x^4*(-a^2*c*x^
```

```
2+c)^(5/2)+a^7*c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x+a^7*c^2/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))-2/3*a^5
/c/x*(-a^2*c*x^2+c)^(5/2)-a^7*c*x*(-a^2*c*x^2+c)^(1/2)-a^7*c^2/(a^2*c)^(1/2
)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/5*a/c/x^5*(-a^2*c*x^2+c)^(
5/2)-1/16*a^6*(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^7,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x^7), x)
```

Fricas [A] time = 2.84828, size = 527, normalized size = 3.38

$$\left[\frac{45\,a^6c^{\frac{3}{2}}x^6\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\left(64\,a^5cx^5+45\,a^4cx^4+32\,a^3cx^3-50\,a^2cx^2-96\,acx-40\,c\right)\sqrt{-a^2cx^2+c}}{480\,x^6},\ 45\,\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^7,x, algorithm="fri
cas")
```

```
[Out] [1/480*(45*a^6*c^(3/2)*x^6*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
 - 2*c)/x^2) + 2*(64*a^5*c*x^5 + 45*a^4*c*x^4 + 32*a^3*c*x^3 - 50*a^2*c*x^2
 - 96*a*c*x - 40*c)*sqrt(-a^2*c*x^2 + c))/x^6, 1/240*(45*a^6*sqrt(-c)*c*x^6
*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (64*a^5*c*x^5 + 45
*a^4*c*x^4 + 32*a^3*c*x^3 - 50*a^2*c*x^2 - 96*a*c*x - 40*c)*sqrt(-a^2*c*x^2
 + c))/x^6]
```

Sympy [C] time = 12.5351, size = 636, normalized size = 4.08

$$a^2c\left(\begin{cases}\frac{a^4\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{8}-\frac{a^3\sqrt{c}}{8x\sqrt{-1+\frac{1}{a^2x^2}}}+\frac{3a\sqrt{c}}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{\sqrt{c}}{4ax^5\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{ia^4\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{8}+\frac{ia^3\sqrt{c}}{8x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3ia\sqrt{c}}{8x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i\sqrt{c}}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{2ia^4\sqrt{c}\sqrt{a^2x^2-1}}{15x}+\frac{ia^2\sqrt{c}\sqrt{a^2x^2-1}}{15x^3}-\ldots\\ \frac{2a^4\sqrt{c}\sqrt{-a^2x^2+1}}{15x}+\frac{a^2\sqrt{c}\sqrt{-a^2x^2+1}}{15x^3}-\ldots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2)/x**7,x)
```

```
[Out] a**2*c*Piecewise((a**4*sqrt(c)*acosh(1/(a*x))/8 - a**3*sqrt(c)/(8*x*sqrt(-1
 + 1/(a**2*x**2))) + 3*a*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - sqrt(c
```

```
)/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*a**4*sqrt
(c)*asin(1/(a*x))/8 + I*a**3*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2))) - 3*I*a*
sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I*sqrt(c)/(4*a*x**5*sqrt(1 - 1/(
a**2*x**2))), True)) + 2*a*c*Piecewise((2*I*a**4*sqrt(c)*sqrt(a**2*x**2 - 1
)/(15*x) + I*a**2*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x**3) - I*sqrt(c)*sqrt(a*
*2*x**2 - 1)/(5*x**5), Abs(a**2*x**2) > 1), (2*a**4*sqrt(c)*sqrt(-a**2*x**2
 + 1)/(15*x) + a**2*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3) - sqrt(c)*sqrt(-
a**2*x**2 + 1)/(5*x**5), True)) + c*Piecewise((a**6*sqrt(c)*acosh(1/(a*x))/
16 - a**5*sqrt(c)/(16*x*sqrt(-1 + 1/(a**2*x**2))) + a**3*sqrt(c)/(48*x**3*s
qrt(-1 + 1/(a**2*x**2))) + 5*a*sqrt(c)/(24*x**5*sqrt(-1 + 1/(a**2*x**2))) -
 sqrt(c)/(6*a*x**7*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*a*
*6*sqrt(c)*asin(1/(a*x))/16 + I*a**5*sqrt(c)/(16*x*sqrt(1 - 1/(a**2*x**2)))
 - I*a**3*sqrt(c)/(48*x**3*sqrt(1 - 1/(a**2*x**2))) - 5*I*a*sqrt(c)/(24*x**
5*sqrt(1 - 1/(a**2*x**2))) + I*sqrt(c)/(6*a*x**7*sqrt(1 - 1/(a**2*x**2))),
True))
```

Giac [B] time = 1.18805, size = 598, normalized size = 3.83

$$-\frac{3\,a^{6}c^{2}\arctan\left(-\frac{\sqrt{-a^{2}c}x-\sqrt{-a^{2}cx^{2}+c}}{\sqrt{-c}}\right)}{8\sqrt{-c}}+\frac{45\left(\sqrt{-a^{2}c}x-\sqrt{-a^{2}cx^{2}+c}\right)^{11}a^{6}c^{2}+65\left(\sqrt{-a^{2}c}x-\sqrt{-a^{2}cx^{2}+c}\right)^{9}a^{6}c^{3}+960\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^7,x, algorithm="gia
c")
```

```
[Out] -3/8*a^6*c^2*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt
(-c) + 1/120*(45*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^11*a^6*c^2 + 65*(s
qrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^9*a^6*c^3 + 960*(sqrt(-a^2*c)*x - sqr
t(-a^2*c*x^2 + c))^8*a^5*sqrt(-c)*c^3*abs(a) - 750*(sqrt(-a^2*c)*x - sqrt(-
a^2*c*x^2 + c))^7*a^6*c^4 - 640*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^6*a
^5*sqrt(-c)*c^4*abs(a) - 750*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^6*
c^5 + 65*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^6*c^6 - 384*(sqrt(-a^2
*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^5*sqrt(-c)*c^6*abs(a) + 45*(sqrt(-a^2*c)*
x - sqrt(-a^2*c*x^2 + c))*a^6*c^7 + 64*a^5*sqrt(-c)*c^7*abs(a))/((sqrt(-a^2
*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^6
```

3.1097 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=181

$$\frac{1}{8}a^7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{a^5c\sqrt{c-a^2cx^2}}{8x^2}-\frac{22a^4\left(c-a^2cx^2\right)^{3/2}}{105x^3}-\frac{a^3\left(c-a^2cx^2\right)^{3/2}}{4x^4}-\frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5}-\frac{a\left(c-a^2c\right.}{3x^6}$$

```
[Out] -(a^5*c*Sqrt[c - a^2*c*x^2])/(8*x^2) - (c - a^2*c*x^2)^(3/2)/(7*x^7) - (a*(
c - a^2*c*x^2)^(3/2))/(3*x^6) - (11*a^2*(c - a^2*c*x^2)^(3/2))/(35*x^5) - (
a^3*(c - a^2*c*x^2)^(3/2))/(4*x^4) - (22*a^4*(c - a^2*c*x^2)^(3/2))/(105*x^
3) + (a^7*c^(3/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```

Rubi [A] time = 0.371403, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6151, 1807, 835, 807, 266, 47, 63, 208}

$$\frac{1}{8}a^7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{a^5c\sqrt{c-a^2cx^2}}{8x^2}-\frac{22a^4\left(c-a^2cx^2\right)^{3/2}}{105x^3}-\frac{a^3\left(c-a^2cx^2\right)^{3/2}}{4x^4}-\frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5}-\frac{a\left(c-a^2c\right.}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^8,x]
```



```
[Out] -(a^5*c*Sqrt[c - a^2*c*x^2])/(8*x^2) - (c - a^2*c*x^2)^(3/2)/(7*x^7) - (a*(
c - a^2*c*x^2)^(3/2))/(3*x^6) - (11*a^2*(c - a^2*c*x^2)^(3/2))/(35*x^5) - (
a^3*(c - a^2*c*x^2)^(3/2))/(4*x^4) - (22*a^4*(c - a^2*c*x^2)^(3/2))/(105*x^
3) + (a^7*c^(3/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^8}dx = c\int \frac{(1+ax)^2\sqrt{c-a^2cx^2}}{x^8}dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{1}{7}\int \frac{\left(-14ac-11a^2cx\right)\sqrt{c-a^2cx^2}}{x^7}dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6} + \frac{\int \frac{\left(66a^2c^2+42a^3c^2x\right)\sqrt{c-a^2cx^2}}{x^6}dx}{42c}$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6} - \frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5} - \frac{\int \frac{\left(-210a^3c^3-132a^4c^3x\right)\sqrt{c-a^2cx^2}}{x^5}d}{210c^2}$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6} - \frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5} - \frac{a^3\left(c-a^2cx^2\right)^{3/2}}{4x^4} + \int \frac{(528a^4c^4}{}$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6} - \frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5} - \frac{a^3\left(c-a^2cx^2\right)^{3/2}}{4x^4} - \frac{22a^4(c-}{10}$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6} - \frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5} - \frac{a^3\left(c-a^2cx^2\right)^{3/2}}{4x^4} - \frac{22a^4(c-}{10}$$
$$= -\frac{a^5c\sqrt{c-a^2cx^2}}{8x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6} - \frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5} - \frac{a^3(c-a^2c}{4x^4}$$
$$= -\frac{a^5c\sqrt{c-a^2cx^2}}{8x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6} - \frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5} - \frac{a^3(c-a^2c}{4x^4}$$
$$= -\frac{a^5c\sqrt{c-a^2cx^2}}{8x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6} - \frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5} - \frac{a^3(c-a^2c}{4x^4}$$

Mathematica [A] time = 0.188234, size = 120, normalized size = 0.66

$$\frac{1}{8}a^7c^{3/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) - \frac{1}{8}a^7c^{3/2}\log(x) + \frac{c\left(176a^6x^6+105a^5x^5+88a^4x^4+70a^3x^3-144a^2x^2-280ax-120\right)\sqrt{}}{840x^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^8,x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-120 - 280*a*x - 144*a^2*x^2 + 70*a^3*x^3 + 88*a^4*
x^4 + 105*a^5*x^5 + 176*a^6*x^6))/(840*x^7) - (a^7*c^(3/2)*Log[x])/8 + (a^7
*c^(3/2)*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/8
```

Maple [B] time = 0.093, size = 436, normalized size = 2.4

$$-\frac{a^7}{24}\left(-a^2cx^2+c\right)^{\frac{3}{2}} - \frac{1}{7\,cx^7}\left(-a^2cx^2+c\right)^{\frac{5}{2}} - \frac{a}{3\,cx^6}\left(-a^2cx^2+c\right)^{\frac{5}{2}} - \frac{2\,a^8x}{3}\left(-a^2cx^2+c\right)^{\frac{3}{2}} + \frac{a^7}{8}c^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{-a^2cx^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^8,x)
```

```
[Out] -1/24*a^7*(-a^2*c*x^2+c)^(3/2)-1/7/c/x^7*(-a^2*c*x^2+c)^(5/2)-1/3*a/c/x^6*(
-a^2*c*x^2+c)^(5/2)-2/3*a^8*x*(-a^2*c*x^2+c)^(3/2)+1/8*a^7*c^(3/2)*ln((2*c+
```

```
2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-1/8*a^7*(-a^2*c*x^2+c)^(1/2)*c+a^8*c*(-c
*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x+a^8*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^
(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))-2/3*a^6/c/x*(-a^2*c*x^2+c)^
(5/2)-a^8*c*x*(-a^2*c*x^2+c)^(1/2)-a^8*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/
2)*x/(-a^2*c*x^2+c)^(1/2))-7/12*a^3/c/x^4*(-a^2*c*x^2+c)^(5/2)-17/24*a^5/c/
x^2*(-a^2*c*x^2+c)^(5/2)-2/3*a^4/c/x^3*(-a^2*c*x^2+c)^(5/2)-16/35*a^2/c/x^5
*(-a^2*c*x^2+c)^(5/2)-2/3*a^7*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^8,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.78042, size = 585, normalized size = 3.23

$$\left[\frac{105\,a^{7}c^{\frac{3}{2}}x^{7}\log\left(-\frac{a^{2}cx^{2}-2\,\sqrt{-a^{2}cx^{2}+c}\sqrt{c}-2\,c}{x^{2}}\right)+2\left(176\,a^{6}cx^{6}+105\,a^{5}cx^{5}+88\,a^{4}cx^{4}+70\,a^{3}cx^{3}-144\,a^{2}cx^{2}-280\,acx-12\right.}{1680\,x^{7}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^8,x, algorithm="fri
cas")
```

```
[Out] [1/1680*(105*a^7*c^(3/2)*x^7*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(
c) - 2*c)/x^2) + 2*(176*a^6*c*x^6 + 105*a^5*c*x^5 + 88*a^4*c*x^4 + 70*a^3*c
*x^3 - 144*a^2*c*x^2 - 280*a*c*x - 120*c)*sqrt(-a^2*c*x^2 + c))/x^7, 1/840*
(105*a^7*sqrt(-c)*c*x^7*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c
)) + (176*a^6*c*x^6 + 105*a^5*c*x^5 + 88*a^4*c*x^4 + 70*a^3*c*x^3 - 144*a^2
*c*x^2 - 280*a*c*x - 120*c)*sqrt(-a^2*c*x^2 + c))/x^7]
```

Sympy [C] time = 20.0616, size = 660, normalized size = 3.65

$$a^{2}c\left(\begin{cases}\frac{2ia^{4}\sqrt{c}\sqrt{a^{2}x^{2}-1}}{15x}+\frac{ia^{2}\sqrt{c}\sqrt{a^{2}x^{2}-1}}{15x^{3}}-\frac{i\sqrt{c}\sqrt{a^{2}x^{2}-1}}{5x^{5}} & \text{for } \left|a^{2}x^{2}\right|>1\\ \frac{2a^{4}\sqrt{c}\sqrt{-a^{2}x^{2}+1}}{15x}+\frac{a^{2}\sqrt{c}\sqrt{-a^{2}x^{2}+1}}{15x^{3}}-\frac{\sqrt{c}\sqrt{-a^{2}x^{2}+1}}{5x^{5}} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{a^{6}\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{16}-\frac{a^{5}\sqrt{c}}{16x\sqrt{-1+\frac{1}{a^{2}x^{2}}}}+\frac{a^{3}\sqrt{c}}{48x^{3}\sqrt{-1+\frac{1}{a^{2}x^{2}}}} \\ -\frac{ia^{6}\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{16}+\frac{ia^{5}\sqrt{c}}{16x\sqrt{1-\frac{1}{a^{2}x^{2}}}}-\frac{ia^{3}\sqrt{c}}{48x^{3}\sqrt{1-\frac{1}{a^{2}x^{2}}}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2)/x**8,x)
```

```
[Out] a**2*c*Piecewise((2*I*a**4*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x) + I*a**2*sqrt
(c)*sqrt(a**2*x**2 - 1)/(15*x**3) - I*sqrt(c)*sqrt(a**2*x**2 - 1)/(5*x**5),
```

```
 Abs(a**2*x**2) > 1), (2*a**4*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x) + a**2*sq
rt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3) - sqrt(c)*sqrt(-a**2*x**2 + 1)/(5*x**5
), True)) + 2*a*c*Piecewise((a**6*sqrt(c)*acosh(1/(a*x))/16 - a**5*sqrt(c)/
(16*x*sqrt(-1 + 1/(a**2*x**2))) + a**3*sqrt(c)/(48*x**3*sqrt(-1 + 1/(a**2*x
**2))) + 5*a*sqrt(c)/(24*x**5*sqrt(-1 + 1/(a**2*x**2))) - sqrt(c)/(6*a*x**7
*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*a**6*sqrt(c)*asin(1/
(a*x))/16 + I*a**5*sqrt(c)/(16*x*sqrt(1 - 1/(a**2*x**2))) - I*a**3*sqrt(c)/
(48*x**3*sqrt(1 - 1/(a**2*x**2))) - 5*I*a*sqrt(c)/(24*x**5*sqrt(1 - 1/(a**2
*x**2))) + I*sqrt(c)/(6*a*x**7*sqrt(1 - 1/(a**2*x**2))), True)) + c*Piecewi
se((8*a**7*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/105 + 4*a**5*sqrt(c)*sqrt(-1 +
1/(a**2*x**2))/(105*x**2) + a**3*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/(35*x**4)
 - a*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/(7*x**6), 1/Abs(a**2*x**2) > 1), (8*I
*a**7*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/105 + 4*I*a**5*sqrt(c)*sqrt(1 - 1/(a*
*2*x**2))/(105*x**2) + I*a**3*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(35*x**4) - I
*a*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(7*x**6), True))
```

Giac [B] time = 1.19264, size = 714, normalized size = 3.94

$$-\frac{a^7c^2\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}}+\frac{105\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^{13}a^7c^2-700\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^{11}a^7c^3+1680\left(\sqrt{\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^8,x, algorithm="gia
c")
```

```
[Out] -1/4*a^7*c^2*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt
(-c) + 1/420*(105*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^13*a^7*c^2 - 700*
(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^11*a^7*c^3 + 1680*(sqrt(-a^2*c)*x -
 sqrt(-a^2*c*x^2 + c))^10*a^6*sqrt(-c)*c^3*abs(a) - 3395*(sqrt(-a^2*c)*x -
sqrt(-a^2*c*x^2 + c))^9*a^7*c^4 - 7280*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 +
c))^8*a^6*sqrt(-c)*c^4*abs(a) - 1120*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c)
)^6*a^6*sqrt(-c)*c^5*abs(a) + 3395*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^
5*a^7*c^6 - 2016*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^4*a^6*sqrt(-c)*c^6
*abs(a) + 700*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^7*c^7 + 1232*(sqr
t(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^6*sqrt(-c)*c^7*abs(a) - 105*(sqrt(-
a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^7*c^8 - 176*a^6*sqrt(-c)*c^8*abs(a))/((s
qrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^7
```

3.1098 $\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=187

$$\frac{3c^2x\sqrt{c-a^2cx^2}}{64a^3}+\frac{3c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{64a^4}-\frac{1}{9}x^4\left(c-a^2cx^2\right)^{5/2}-\frac{x^3\left(c-a^2cx^2\right)^{5/2}}{4a}-\frac{13x^2\left(c-a^2cx^2\right)^{5/2}}{63a^2}+\frac{cx\left(c-a^2cx\right.}{32a^3}$$

```
[Out] (3*c^2*x*Sqrt[c - a^2*c*x^2])/(64*a^3) + (c*x*(c - a^2*c*x^2)^(3/2))/(32*a^
3) - (13*x^2*(c - a^2*c*x^2)^(5/2))/(63*a^2) - (x^3*(c - a^2*c*x^2)^(5/2))/
(4*a) - (x^4*(c - a^2*c*x^2)^(5/2))/9 - ((208 + 315*a*x)*(c - a^2*c*x^2)^(5
/2))/(2520*a^4) + (3*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(64
*a^4)
```

Rubi [A] time = 0.37002, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1809, 833, 780, 195, 217, 203}

$$\frac{3c^2x\sqrt{c-a^2cx^2}}{64a^3}+\frac{3c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{64a^4}-\frac{1}{9}x^4\left(c-a^2cx^2\right)^{5/2}-\frac{x^3\left(c-a^2cx^2\right)^{5/2}}{4a}-\frac{13x^2\left(c-a^2cx^2\right)^{5/2}}{63a^2}+\frac{cx\left(c-a^2cx\right.}{32a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (3*c^2*x*Sqrt[c - a^2*c*x^2])/(64*a^3) + (c*x*(c - a^2*c*x^2)^(3/2))/(32*a^
3) - (13*x^2*(c - a^2*c*x^2)^(5/2))/(63*a^2) - (x^3*(c - a^2*c*x^2)^(5/2))/
(4*a) - (x^4*(c - a^2*c*x^2)^(5/2))/9 - ((208 + 315*a*x)*(c - a^2*c*x^2)^(5
/2))/(2520*a^4) + (3*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(64
*a^4)
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^{5/2}\,dx = c\int x^3(1+ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx$$

$$= -\frac{1}{9}x^4\left(c-a^2cx^2\right)^{5/2} - \frac{\int x^3\left(-13a^2c-18a^3cx\right)\left(c-a^2cx^2\right)^{3/2}\,dx}{9a^2}$$

$$= -\frac{x^3\left(c-a^2cx^2\right)^{5/2}}{4a} - \frac{1}{9}x^4\left(c-a^2cx^2\right)^{5/2} + \frac{\int x^2\left(54a^3c^2+104a^4c^2x\right)\left(c-a^2cx^2\right)^{3/2}\,dx}{72a^4c}$$

$$= -\frac{13x^2\left(c-a^2cx^2\right)^{5/2}}{63a^2} - \frac{x^3\left(c-a^2cx^2\right)^{5/2}}{4a} - \frac{1}{9}x^4\left(c-a^2cx^2\right)^{5/2} - \frac{\int x\left(-208a^4c^3-378\right.}{50}$$

$$= -\frac{13x^2\left(c-a^2cx^2\right)^{5/2}}{63a^2} - \frac{x^3\left(c-a^2cx^2\right)^{5/2}}{4a} - \frac{1}{9}x^4\left(c-a^2cx^2\right)^{5/2} - \frac{(208+315ax)\left(c-a^2\right.}{2520a^4}$$

$$= \frac{cx\left(c-a^2cx^2\right)^{3/2}}{32a^3} - \frac{13x^2\left(c-a^2cx^2\right)^{5/2}}{63a^2} - \frac{x^3\left(c-a^2cx^2\right)^{5/2}}{4a} - \frac{1}{9}x^4\left(c-a^2cx^2\right)^{5/2} - (20$$

$$= \frac{3c^2x\sqrt{c-a^2cx^2}}{64a^3} + \frac{cx\left(c-a^2cx^2\right)^{3/2}}{32a^3} - \frac{13x^2\left(c-a^2cx^2\right)^{5/2}}{63a^2} - \frac{x^3\left(c-a^2cx^2\right)^{5/2}}{4a} - \frac{1}{9}x^4($$

$$= \frac{3c^2x\sqrt{c-a^2cx^2}}{64a^3} + \frac{cx\left(c-a^2cx^2\right)^{3/2}}{32a^3} - \frac{13x^2\left(c-a^2cx^2\right)^{5/2}}{63a^2} - \frac{x^3\left(c-a^2cx^2\right)^{5/2}}{4a} - \frac{1}{9}x^4($$

$$= \frac{3c^2x\sqrt{c-a^2cx^2}}{64a^3} + \frac{cx\left(c-a^2cx^2\right)^{3/2}}{32a^3} - \frac{13x^2\left(c-a^2cx^2\right)^{5/2}}{63a^2} - \frac{x^3\left(c-a^2cx^2\right)^{5/2}}{4a} - \frac{1}{9}x^4($$

Mathematica [A] time = 0.214483, size = 131, normalized size = 0.7

$$-\frac{c^2\left(\left(2240a^8x^8+5040a^7x^7-320a^6x^6-7560a^5x^5-4416a^4x^4+630a^3x^3+832a^2x^2+945ax+1664\right)\sqrt{c-a^2cx^2}+9\right.}{20160a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(c^2*(Sqrt[c - a^2*c*x^2]*(1664 + 945*a*x + 832*a^2*x^2 + 630*a^3*x^3 - 44
16*a^4*x^4 - 7560*a^5*x^5 - 320*a^6*x^6 + 5040*a^7*x^7 + 2240*a^8*x^8) + 94
5*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))]))/(201
60*a^4)
```

Maple [B] time = 0.046, size = 330, normalized size = 1.8

$$\frac{x^2}{9a^2c}\left(-a^2cx^2+c\right)^{\frac{7}{2}}+\frac{20}{63ca^4}\left(-a^2cx^2+c\right)^{\frac{7}{2}}+\frac{x}{4a^3c}\left(-a^2cx^2+c\right)^{\frac{7}{2}}-\frac{3x}{8a^3}\left(-a^2cx^2+c\right)^{\frac{5}{2}}-\frac{15cx}{32a^3}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{45x}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/9*x^2*(-a^2*c*x^2+c)^(7/2)/a^2/c+20/63/c/a^4*(-a^2*c*x^2+c)^(7/2)+1/4/a^3
*x*(-a^2*c*x^2+c)^(7/2)/c-3/8/a^3*x*(-a^2*c*x^2+c)^(5/2)-15/32*c*x*(-a^2*c*
x^2+c)^(3/2)/a^3-45/64*c^2*x*(-a^2*c*x^2+c)^(1/2)/a^3-45/64/a^3*c^3/(a^2*c)
^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/5/a^4*(-c*a^2*(x-1/a)
^2-2*a*c*(x-1/a))^(5/2)+1/2/a^3*c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)*x+
3/4/a^3*c^2*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x+3/4/a^3*c^3/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.88005, size = 726, normalized size = 3.88

$$\left[\frac{945\sqrt{-c}c^2\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\left(2240a^8c^2x^8+5040a^7c^2x^7-320a^6c^2x^6-7560a^5c^2x^5-44\right.}{40320a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/40320*(945*sqrt(-c)*c^2*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(
-c)*x - c) - 2*(2240*a^8*c^2*x^8 + 5040*a^7*c^2*x^7 - 320*a^6*c^2*x^6 - 756
0*a^5*c^2*x^5 - 4416*a^4*c^2*x^4 + 630*a^3*c^2*x^3 + 832*a^2*c^2*x^2 + 945*
a*c^2*x + 1664*c^2)*sqrt(-a^2*c*x^2 + c))/a^4, -1/20160*(945*c^(5/2)*arctan
(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (2240*a^8*c^2*x^8 + 50
40*a^7*c^2*x^7 - 320*a^6*c^2*x^6 - 7560*a^5*c^2*x^5 - 4416*a^4*c^2*x^4 + 63
0*a^3*c^2*x^3 + 832*a^2*c^2*x^2 + 945*a*c^2*x + 1664*c^2)*sqrt(-a^2*c*x^2 +
 c))/a^4]
```

Sympy [A] time = 25.4706, size = 763, normalized size = 4.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] -a**4*c**2*Piecewise((x**8*sqrt(-a**2*c*x**2 + c)/9 - x**6*sqrt(-a**2*c*x**
2 + c)/(63*a**2) - 2*x**4*sqrt(-a**2*c*x**2 + c)/(105*a**4) - 8*x**2*sqrt(-
a**2*c*x**2 + c)/(315*a**6) - 16*sqrt(-a**2*c*x**2 + c)/(315*a**8), Ne(a, 0
)), (sqrt(c)*x**8/8, True)) - 2*a**3*c**2*Piecewise((I*a**2*sqrt(c)*x**9/(8
*sqrt(a**2*x**2 - 1)) - 7*I*sqrt(c)*x**7/(48*sqrt(a**2*x**2 - 1)) - I*sqrt(
c)*x**5/(192*a**2*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*x**3/(384*a**4*sqrt(a*
*2*x**2 - 1)) + 5*I*sqrt(c)*x/(128*a**6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*
acosh(a*x)/(128*a**7), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**9/(8*sqrt(-a*
*2*x**2 + 1)) + 7*sqrt(c)*x**7/(48*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**5/(19
2*a**2*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c)*x**3/(384*a**4*sqrt(-a**2*x**2 + 1
)) - 5*sqrt(c)*x/(128*a**6*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c)*asin(a*x)/(128
*a**7), True)) + 2*a*c**2*Piecewise((I*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2
- 1)) - 5*I*sqrt(c)*x**5/(24*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2
*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(
c)*acosh(a*x)/(16*a**5), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-
a**2*x**2 + 1)) + 5*sqrt(c)*x**5/(24*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(
48*a**2*sqrt(-a**2*x**2 + 1)) - sqrt(c)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) +
sqrt(c)*asin(a*x)/(16*a**5), True)) + c**2*Piecewise((x**4*sqrt(-a**2*c*x**
2 + c)/5 - x**2*sqrt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)
/(15*a**4), Ne(a, 0)), (sqrt(c)*x**4/4, True))
```

Giac [A] time = 1.18195, size = 209, normalized size = 1.12

$$\frac{1}{20160}\sqrt{-a^2cx^2+c}\left(\left(2\left(\left(4\left(552\,c^2+5\left(189\,ac^2+2\left(4\,a^2c^2-7\left(4\,a^4c^2x+9\,a^3c^2\right)x\right)x\right)x\right)x-\frac{315\,c^2}{a}\right)x-\frac{416\,c^2}{a^2}\right)x-\frac{945\,c^2}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

```
[Out] 1/20160*sqrt(-a^2*c*x^2 + c)*((2*((4*(552*c^2 + 5*(189*a*c^2 + 2*(4*a^2*c^2
 - 7*(4*a^4*c^2*x + 9*a^3*c^2)*x)*x)*x)*x - 315*c^2/a)*x - 416*c^2/a^2)*x -
```

```
 945*c^2/a^3)*x - 1664*c^2/a^4) - 3/64*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(a^3*sqrt(-c)*abs(a))
```

3.1099 $\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=162

$$\frac{11c^2x\sqrt{c-a^2cx^2}}{128a^2}+\frac{11c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{128a^3}-\frac{1}{8}x^3\left(c-a^2cx^2\right)^{5/2}-\frac{2x^2\left(c-a^2cx^2\right)^{5/2}}{7a}+\frac{11cx\left(c-a^2cx^2\right)^{3/2}}{192a^2}-\frac{(385ax+19}{16}$$

```
[Out] (11*c^2*x*Sqrt[c - a^2*c*x^2])/(128*a^2) + (11*c*x*(c - a^2*c*x^2)^(3/2))/(
192*a^2) - (2*x^2*(c - a^2*c*x^2)^(5/2))/(7*a) - (x^3*(c - a^2*c*x^2)^(5/2)
)/8 - ((192 + 385*a*x)*(c - a^2*c*x^2)^(5/2))/(1680*a^3) + (11*c^(5/2)*ArcT
an[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(128*a^3)
```

Rubi [A] time = 0.325294, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1809, 833, 780, 195, 217, 203}

$$\frac{11c^2x\sqrt{c-a^2cx^2}}{128a^2}+\frac{11c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{128a^3}-\frac{1}{8}x^3\left(c-a^2cx^2\right)^{5/2}-\frac{2x^2\left(c-a^2cx^2\right)^{5/2}}{7a}+\frac{11cx\left(c-a^2cx^2\right)^{3/2}}{192a^2}-\frac{(385ax+19}{16}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (11*c^2*x*Sqrt[c - a^2*c*x^2])/(128*a^2) + (11*c*x*(c - a^2*c*x^2)^(3/2))/(
192*a^2) - (2*x^2*(c - a^2*c*x^2)^(5/2))/(7*a) - (x^3*(c - a^2*c*x^2)^(5/2)
)/8 - ((192 + 385*a*x)*(c - a^2*c*x^2)^(5/2))/(1680*a^3) + (11*c^(5/2)*ArcT
an[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(128*a^3)
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^{5/2}dx = c\int x^2(1+ax)^2\left(c-a^2cx^2\right)^{3/2}dx$$

$$= -\frac{1}{8}x^3\left(c-a^2cx^2\right)^{5/2} - \frac{\int x^2\left(-11a^2c-16a^3cx\right)\left(c-a^2cx^2\right)^{3/2}dx}{8a^2}$$

$$= -\frac{2x^2\left(c-a^2cx^2\right)^{5/2}}{7a} - \frac{1}{8}x^3\left(c-a^2cx^2\right)^{5/2} + \frac{\int x\left(32a^3c^2+77a^4c^2x\right)\left(c-a^2cx^2\right)^{3/2}}{56a^4c}$$

$$= -\frac{2x^2\left(c-a^2cx^2\right)^{5/2}}{7a} - \frac{1}{8}x^3\left(c-a^2cx^2\right)^{5/2} - \frac{(192+385ax)\left(c-a^2cx^2\right)^{5/2}}{1680a^3} + \frac{(11c)\int}{}$$

$$= \frac{11cx\left(c-a^2cx^2\right)^{3/2}}{192a^2} - \frac{2x^2\left(c-a^2cx^2\right)^{5/2}}{7a} - \frac{1}{8}x^3\left(c-a^2cx^2\right)^{5/2} - \frac{(192+385ax)\left(c-\right.}{1680a^3}$$

$$= \frac{11c^2x\sqrt{c-a^2cx^2}}{128a^2} + \frac{11cx\left(c-a^2cx^2\right)^{3/2}}{192a^2} - \frac{2x^2\left(c-a^2cx^2\right)^{5/2}}{7a} - \frac{1}{8}x^3\left(c-a^2cx^2\right)^{5/2} -$$

$$= \frac{11c^2x\sqrt{c-a^2cx^2}}{128a^2} + \frac{11cx\left(c-a^2cx^2\right)^{3/2}}{192a^2} - \frac{2x^2\left(c-a^2cx^2\right)^{5/2}}{7a} - \frac{1}{8}x^3\left(c-a^2cx^2\right)^{5/2} -$$

$$= \frac{11c^2x\sqrt{c-a^2cx^2}}{128a^2} + \frac{11cx\left(c-a^2cx^2\right)^{3/2}}{192a^2} - \frac{2x^2\left(c-a^2cx^2\right)^{5/2}}{7a} - \frac{1}{8}x^3\left(c-a^2cx^2\right)^{5/2} -$$

Mathematica [A] time = 0.173982, size = 123, normalized size = 0.76

$$-\frac{c^2\left(\left(1680a^7x^7+3840a^6x^6-280a^5x^5-6144a^4x^4-3710a^3x^3+768a^2x^2+1155ax+1536\right)\sqrt{c-a^2cx^2}+1155\sqrt{c}\tan\right.}{13440a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(c^2*(Sqrt[c - a^2*c*x^2]*(1536 + 1155*a*x + 768*a^2*x^2 - 3710*a^3*x^3 -
6144*a^4*x^4 - 280*a^5*x^5 + 3840*a^6*x^6 + 1680*a^7*x^7) + 1155*Sqrt[c]*Ar
cTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))]))/(13440*a^3)
```

Maple [B] time = 0.04, size = 306, normalized size = 1.9

$$\frac{x}{8a^2c}\left(-a^2cx^2+c\right)^{\frac{7}{2}}-\frac{17x}{48a^2}\left(-a^2cx^2+c\right)^{\frac{5}{2}}-\frac{85cx}{192a^2}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{85xc^2}{128a^2}\sqrt{-a^2cx^2+c}-\frac{85c^3}{128a^2}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/8*x*(-a^2*c*x^2+c)^(7/2)/a^2/c-17/48/a^2*x*(-a^2*c*x^2+c)^(5/2)-85/192*c*
x*(-a^2*c*x^2+c)^(3/2)/a^2-85/128*c^2*x*(-a^2*c*x^2+c)^(1/2)/a^2-85/128/a^2
*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/7/a^3*(-a
^2*c*x^2+c)^(7/2)/c-2/5/a^3*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(5/2)+1/2/a^2*
c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)*x+3/4/a^2*c^2*(-c*a^2*(x-1/a)^2-2*
a*c*(x-1/a))^(1/2)*x+3/4/a^2*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a
^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.8533, size = 683, normalized size = 4.22

$$\left[\frac{1155\sqrt{-c}c^2\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\left(1680a^7c^2x^7+3840a^6c^2x^6-280a^5c^2x^5-6144a^4c^2x^4-3710\right.}{26880a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/26880*(1155*sqrt(-c)*c^2*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt
(-c)*x - c) - 2*(1680*a^7*c^2*x^7 + 3840*a^6*c^2*x^6 - 280*a^5*c^2*x^5 - 61
44*a^4*c^2*x^4 - 3710*a^3*c^2*x^3 + 768*a^2*c^2*x^2 + 1155*a*c^2*x + 1536*c
^2)*sqrt(-a^2*c*x^2 + c))/a^3, -1/13440*(1155*c^(5/2)*arctan(sqrt(-a^2*c*x^
```

```
2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (1680*a^7*c^2*x^7 + 3840*a^6*c^2*x^6
- 280*a^5*c^2*x^5 - 6144*a^4*c^2*x^4 - 3710*a^3*c^2*x^3 + 768*a^2*c^2*x^2 +
 1155*a*c^2*x + 1536*c^2)*sqrt(-a^2*c*x^2 + c))/a^3]
```

Sympy [C] time = 20.3507, size = 687, normalized size = 4.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] -a**4*c**2*Piecewise((I*a**2*sqrt(c)*x**9/(8*sqrt(a**2*x**2 - 1)) - 7*I*sqr
t(c)*x**7/(48*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**5/(192*a**2*sqrt(a**2*x**
2 - 1)) - 5*I*sqrt(c)*x**3/(384*a**4*sqrt(a**2*x**2 - 1)) + 5*I*sqrt(c)*x/(
128*a**6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*acosh(a*x)/(128*a**7), Abs(a**2
*x**2) > 1), (-a**2*sqrt(c)*x**9/(8*sqrt(-a**2*x**2 + 1)) + 7*sqrt(c)*x**7/
(48*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**5/(192*a**2*sqrt(-a**2*x**2 + 1)) +
5*sqrt(c)*x**3/(384*a**4*sqrt(-a**2*x**2 + 1)) - 5*sqrt(c)*x/(128*a**6*sqrt
(-a**2*x**2 + 1)) + 5*sqrt(c)*asin(a*x)/(128*a**7), True)) - 2*a**3*c**2*Pi
ecewise((x**6*sqrt(-a**2*c*x**2 + c)/7 - x**4*sqrt(-a**2*c*x**2 + c)/(35*a*
*2) - 4*x**2*sqrt(-a**2*c*x**2 + c)/(105*a**4) - 8*sqrt(-a**2*c*x**2 + c)/(
105*a**6), Ne(a, 0)), (sqrt(c)*x**6/6, True)) + 2*a*c**2*Piecewise((x**4*sq
rt(-a**2*c*x**2 + c)/5 - x**2*sqrt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a*
*2*c*x**2 + c)/(15*a**4), Ne(a, 0)), (sqrt(c)*x**4/4, True)) + c**2*Piecewi
se((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*sqrt(c)*x**3/(8*sqrt(
a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acos
h(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**5/(4*sqrt(-a**2*x**
2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1)) - sqrt(c)*x/(8*a**2*sqrt(
-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), True))
```

Giac [A] time = 1.20321, size = 193, normalized size = 1.19

$$\frac{1}{13440}\sqrt{-a^2cx^2+c}\left(\left(2\left(\left(1855\,c^2+4\left(768\,ac^2+5\left(7\,a^2c^2-6\left(7\,a^4c^2x+16\,a^3c^2\right)x\right)x\right)x\right)x-\frac{384\,c^2}{a}\right)x-\frac{1155\,c^2}{a^2}\right)x-\frac{15}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

```
[Out] 1/13440*sqrt(-a^2*c*x^2 + c)*((2*((1855*c^2 + 4*(768*a*c^2 + 5*(7*a^2*c^2 -
 6*(7*a^4*c^2*x + 16*a^3*c^2)*x)*x)*x)*x - 384*c^2/a)*x - 1155*c^2/a^2)*x -
 1536*c^2/a^3) - 11/128*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c))
)/(a^2*sqrt(-c)*abs(a))
```

3.1100 $\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=137

$$\frac{c^2x\sqrt{c-a^2cx^2}}{8a}+\frac{c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^2}+\frac{cx\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{1}{7}x^2\left(c-a^2cx^2\right)^{5/2}-\frac{(35ax+27)\left(c-a^2cx^2\right)^{5/2}}{105a^2}$$

```
[Out] (c^2*x*Sqrt[c - a^2*c*x^2])/(8*a) + (c*x*(c - a^2*c*x^2)^(3/2))/(12*a) - (x
^2*(c - a^2*c*x^2)^(5/2))/7 - ((27 + 35*a*x)*(c - a^2*c*x^2)^(5/2))/(105*a^
2) + (c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(8*a^2)
```

Rubi [A] time = 0.21575, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6151, 1809, 780, 195, 217, 203}

$$\frac{c^2x\sqrt{c-a^2cx^2}}{8a}+\frac{c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^2}+\frac{cx\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{1}{7}x^2\left(c-a^2cx^2\right)^{5/2}-\frac{(35ax+27)\left(c-a^2cx^2\right)^{5/2}}{105a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2)^(5/2),x]
```


```
[Out] (c^2*x*Sqrt[c - a^2*c*x^2])/(8*a) + (c*x*(c - a^2*c*x^2)^(3/2))/(12*a) - (x
^2*(c - a^2*c*x^2)^(5/2))/7 - ((27 + 35*a*x)*(c - a^2*c*x^2)^(5/2))/(105*a^
2) + (c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(8*a^2)
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^{5/2}\,dx = c\int x(1+ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$= -\frac{1}{7}x^2\left(c-a^2cx^2\right)^{5/2} - \frac{\int x\left(-9a^2c-14a^3cx\right)\left(c-a^2cx^2\right)^{3/2}\,dx}{7a^2}$$
$$= -\frac{1}{7}x^2\left(c-a^2cx^2\right)^{5/2} - \frac{(27+35ax)\left(c-a^2cx^2\right)^{5/2}}{105a^2} + \frac{c\int\left(c-a^2cx^2\right)^{3/2}\,dx}{3a}$$
$$= \frac{cx\left(c-a^2cx^2\right)^{3/2}}{12a} - \frac{1}{7}x^2\left(c-a^2cx^2\right)^{5/2} - \frac{(27+35ax)\left(c-a^2cx^2\right)^{5/2}}{105a^2} + \frac{c^2\int\sqrt{c-a^2c}}{4a}$$
$$= \frac{c^2x\sqrt{c-a^2cx^2}}{8a} + \frac{cx\left(c-a^2cx^2\right)^{3/2}}{12a} - \frac{1}{7}x^2\left(c-a^2cx^2\right)^{5/2} - \frac{(27+35ax)\left(c-a^2cx^2\right)^{5}}{105a^2}$$
$$= \frac{c^2x\sqrt{c-a^2cx^2}}{8a} + \frac{cx\left(c-a^2cx^2\right)^{3/2}}{12a} - \frac{1}{7}x^2\left(c-a^2cx^2\right)^{5/2} - \frac{(27+35ax)\left(c-a^2cx^2\right)^{5}}{105a^2}$$
$$= \frac{c^2x\sqrt{c-a^2cx^2}}{8a} + \frac{cx\left(c-a^2cx^2\right)^{3/2}}{12a} - \frac{1}{7}x^2\left(c-a^2cx^2\right)^{5/2} - \frac{(27+35ax)\left(c-a^2cx^2\right)^{5}}{105a^2}$$

Mathematica [A] time = 0.150981, size = 115, normalized size = 0.84

$$-\frac{c^2\left(\left(120a^6x^6+280a^5x^5-24a^4x^4-490a^3x^3-312a^2x^2+105ax+216\right)\sqrt{c-a^2cx^2}+105\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)\right)}{840a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(c^2*(Sqrt[c - a^2*c*x^2]*(216 + 105*a*x - 312*a^2*x^2 - 490*a^3*x^3 - 24*
a^4*x^4 + 280*a^5*x^5 + 120*a^6*x^6) + 105*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2
*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))]))/(840*a^2)
```

Maple [B] time = 0.039, size = 284, normalized size = 2.1

$$\frac{1}{7a^2c}\left(-a^2cx^2+c\right)^{\frac{7}{2}} - \frac{x}{3a}\left(-a^2cx^2+c\right)^{\frac{5}{2}} - \frac{5cx}{12a}\left(-a^2cx^2+c\right)^{\frac{3}{2}} - \frac{5xc^2}{8a}\sqrt{-a^2cx^2+c} - \frac{5c^3}{8a}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/7*(-a^2*c*x^2+c)^(7/2)/a^2/c-1/3*x/a*(-a^2*c*x^2+c)^(5/2)-5/12*c*x*(-a^2*
c*x^2+c)^(3/2)/a-5/8*c^2*x*(-a^2*c*x^2+c)^(1/2)/a-5/8/a*c^3/(a^2*c)^(1/2)*a
rctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/5/a^2*(-c*a^2*(x-1/a)^2-2*a*c
*(x-1/a))^(5/2)+1/2/a*c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)*x+3/4/a*c^2*
(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x+3/4/a*c^3/(a^2*c)^(1/2)*arctan((a^
2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.85389, size = 609, normalized size = 4.45

$$\left[\frac{105\sqrt{-c}c^{2}\log\left(2a^{2}cx^{2}+2\sqrt{-a^{2}cx^{2}+c}a\sqrt{-c}x-c\right)-2\left(120a^{6}c^{2}x^{6}+280a^{5}c^{2}x^{5}-24a^{4}c^{2}x^{4}-490a^{3}c^{2}x^{3}-312a^{2}c^{2}x^{2}\right.}{1680a^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/1680*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) - 2*(120*a^6*c^2*x^6 + 280*a^5*c^2*x^5 - 24*a^4*c^2*x^4 - 490*a^3
*c^2*x^3 - 312*a^2*c^2*x^2 + 105*a*c^2*x + 216*c^2)*sqrt(-a^2*c*x^2 + c))/a
^2, -1/840*(105*c^(5/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2
- c)) + (120*a^6*c^2*x^6 + 280*a^5*c^2*x^5 - 24*a^4*c^2*x^4 - 490*a^3*c^2*x
^3 - 312*a^2*c^2*x^2 + 105*a*c^2*x + 216*c^2)*sqrt(-a^2*c*x^2 + c))/a^2]
```

Sympy [A] time = 18.4704, size = 586, normalized size = 4.28

$$-a^{4}c^{2}\left(\begin{cases}\frac{x^{6}\sqrt{-a^{2}cx^{2}+c}}{7}-\frac{x^{4}\sqrt{-a^{2}cx^{2}+c}}{35a^{2}}-\frac{4x^{2}\sqrt{-a^{2}cx^{2}+c}}{105a^{4}}-\frac{8\sqrt{-a^{2}cx^{2}+c}}{105a^{6}} & \text{for}\ a\neq 0\\ \frac{\sqrt{cx^{6}}}{6} & \text{otherwise}\end{cases}\right)-2a^{3}c^{2}\left(\begin{cases}\frac{ia^{2}\sqrt{cx^{7}}}{6\sqrt{a^{2}x^{2}-1}}-\frac{5i\sqrt{cx^{5}}}{24\sqrt{a^{2}x^{2}-1}}-\frac{i\sqrt{cx^{3}}}{48a^{2}\sqrt{a^{2}x^{2}-1}}-\\ -\frac{a^{2}\sqrt{cx^{7}}}{6\sqrt{-a^{2}x^{2}+1}}+\frac{5\sqrt{cx^{5}}}{24\sqrt{-a^{2}x^{2}+1}}+\frac{\sqrt{cx^{3}}}{48a^{2}\sqrt{-a^{2}\ldots}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] -a**4*c**2*Piecewise((x**6*sqrt(-a**2*c*x**2 + c)/7 - x**4*sqrt(-a**2*c*x**
2 + c)/(35*a**2) - 4*x**2*sqrt(-a**2*c*x**2 + c)/(105*a**4) - 8*sqrt(-a**2*
c*x**2 + c)/(105*a**6), Ne(a, 0)), (sqrt(c)*x**6/6, True)) - 2*a**3*c**2*Pi
ecewise((I*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*x**5/(24
*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2 - 1)) + I*sq
rt(c)*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(16*a**5), Abs
(a**2*x**2) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c)*
x**5/(24*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a**2*x**2 + 1)
) - sqrt(c)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(16*a**5),
 True)) + 2*a*c**2*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) -
 3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x
**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt
(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1))
 - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), Tr
ue)) + c**2*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**
2*c*x**2 + c)**(3/2)/(3*a**2*c), True))
```

Giac [A] time = 1.18258, size = 177, normalized size = 1.29

$$\frac{1}{840}\sqrt{-a^2cx^2+c}\left(\left(2\left(156\,c^2+\left(245\,ac^2+4\left(3\,a^2c^2-5\left(3\,a^4c^2x+7\,a^3c^2\right)x\right)x\right)x\right)x-\frac{105\,c^2}{a}\right)x-\frac{216\,c^2}{a^2}\right)-\frac{c^3\log\left(\left|-\sqrt{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac"
)
```

```
[Out] 1/840*sqrt(-a^2*c*x^2 + c)*((2*(156*c^2 + (245*a*c^2 + 4*(3*a^2*c^2 - 5*(3*
a^4*c^2*x + 7*a^3*c^2)*x)*x)*x)*x - 105*c^2/a)*x - 216*c^2/a^2) - 1/8*c^3*l
og(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a*sqrt(-c)*abs(a))
```

3.1101 $\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=130

$$\frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{(ax+1)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

```
[Out] (7*c^2*x*Sqrt[c - a^2*c*x^2])/16 + (7*c*x*(c - a^2*c*x^2)^(3/2))/24 - (7*(c
 - a^2*c*x^2)^(5/2))/(30*a) - ((1 + a*x)*(c - a^2*c*x^2)^(5/2))/(6*a) + (7*
c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a)
```

Rubi [A] time = 0.0983591, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6141, 671, 641, 195, 217, 203}

$$\frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{(ax+1)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (7*c^2*x*Sqrt[c - a^2*c*x^2])/16 + (7*c*x*(c - a^2*c*x^2)^(3/2))/24 - (7*(c
 - a^2*c*x^2)^(5/2))/(30*a) - ((1 + a*x)*(c - a^2*c*x^2)^(5/2))/(6*a) + (7*
c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a)
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}dx = c\int(1+ax)^2\left(c-a^2cx^2\right)^{3/2}dx$$
$$= -\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{1}{6}(7c)\int(1+ax)\left(c-a^2cx^2\right)^{3/2}dx$$
$$= -\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{1}{6}(7c)\int\left(c-a^2cx^2\right)^{3/2}dx$$
$$= \frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{1}{8}\left(7c^2\right)\int\sqrt{c-a^2}$$
$$= \frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}$$
$$= \frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}$$
$$= \frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}$$

Mathematica [A] time = 0.135569, size = 135, normalized size = 1.04

$$\frac{c^2\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(40a^6x^6+56a^5x^5-106a^4x^4-182a^3x^3+57a^2x^2+231ax-96\right)-210\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{240a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(-96 + 231*a*x + 57*a^2*x^2 - 182*a
^3*x^3 - 106*a^4*x^4 + 56*a^5*x^5 + 40*a^6*x^6) - 210*Sqrt[1 - a*x]*ArcSin[
Sqrt[1 - a*x]/Sqrt[2]]))/(240*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.036, size = 242, normalized size = 1.9

$$-\frac{x}{6}\left(-a^2cx^2+c\right)^{\frac{5}{2}}-\frac{5\,cx}{24}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{5\,xc^2}{16}\sqrt{-a^2cx^2+c}-\frac{5\,c^3}{16}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}-\frac{2}{5\,a}\left(-ca^2\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/6*x*(-a^2*c*x^2+c)^(5/2)-5/24*c*x*(-a^2*c*x^2+c)^(3/2)-5/16*c^2*x*(-a^2*
c*x^2+c)^(1/2)-5/16*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)
^(1/2))-2/5/a*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(5/2)+1/2*c*(-c*a^2*(x-1/a)^
2-2*a*c*(x-1/a))^(3/2)*x+3/4*c^2*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x+3
/4*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a)
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.81666, size = 545, normalized size = 4.19

$$\left[\frac{105\sqrt{-c}c^2\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\left(40a^5c^2x^5+96a^4c^2x^4-10a^3c^2x^3-192a^2c^2x^2-135ac^2x+96\right.}{480a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas"
)
```

```
[Out] [1/480*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c
)*x - c) - 2*(40*a^5*c^2*x^5 + 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 192*a^2*c^
2*x^2 - 135*a*c^2*x + 96*c^2)*sqrt(-a^2*c*x^2 + c))/a, -1/240*(105*c^(5/2)*
arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (40*a^5*c^2*x^5
+ 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 192*a^2*c^2*x^2 - 135*a*c^2*x + 96*c^2)
*sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [C] time = 14.3509, size = 478, normalized size = 3.68

$$-a^4c^2\left(\begin{cases}\frac{ia^2\sqrt{c}x^7}{6\sqrt{a^2x^2-1}}-\frac{5i\sqrt{c}x^5}{24\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}x^3}{48a^2\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}x}{16a^4\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\operatorname{acosh}(ax)}{16a^5} & \text{for } |a^2x^2|>1\\ -\frac{a^2\sqrt{c}x^7}{6\sqrt{-a^2x^2+1}}+\frac{5\sqrt{c}x^5}{24\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}x^3}{48a^2\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}x}{16a^4\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\operatorname{asin}(ax)}{16a^5} & \text{otherwise}\end{cases}\right)-2a^3c^2\left(\begin{cases}\frac{x^4\sqrt{-a^2cx^2+c}}{5}-\frac{x^2\sqrt{}}{} \\ \frac{\sqrt{c}x^4}{4}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] -a**4*c**2*Piecewise((I*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqr
t(c)*x**5/(24*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2
 - 1)) + I*sqrt(c)*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(
```

```
16*a**5), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1))
 + 5*sqrt(c)*x**5/(24*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a
**2*x**2 + 1)) - sqrt(c)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*
x)/(16*a**5), True)) - 2*a**3*c**2*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5
 - x**2*sqrt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**
4), Ne(a, 0)), (sqrt(c)*x**4/4, True)) + 2*a*c**2*Piecewise((0, Eq(c, 0)),
(sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True
)) + c**2*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c
)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) >
1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin(a*x)/(2*a), True))
```

Giac [A] time = 1.14759, size = 157, normalized size = 1.21

$$-\frac{7\,c^3\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{16\sqrt{-c}|a|}+\frac{1}{240}\sqrt{-a^2cx^2+c}\left(\left(135\,c^2+2\left(96\,ac^2+\left(5\,a^2c^2-4\left(5\,a^4c^2x+12\,a^3c^2\right)x\right)x\right)x\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -7/16*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a)
) + 1/240*sqrt(-a^2*c*x^2 + c)*((135*c^2 + 2*(96*a*c^2 + (5*a^2*c^2 - 4*(5*
a^4*c^2*x + 12*a^3*c^2)*x)*x)*x)*x - 96*c^2/a)
```

3.1102 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^{5/2}}{x}\,dx$

Optimal. Leaf size=136

$$\frac{1}{4}c^2(3ax+4)\sqrt{c-a^2cx^2}+\frac{3}{4}c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{1}{6}c(3ax+2)\left(c-a^2cx^2\right)^{3/2}-\frac{1}{5}\left(c-a^2\right.$$

```
[Out] (c^2*(4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/4 + (c*(2 + 3*a*x)*(c - a^2*c*x^2)^(3
/2))/6 - (c - a^2*c*x^2)^(5/2)/5 + (3*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
 a^2*c*x^2]])/4 - c^(5/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.333872, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6151, 1809, 815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{4}c^2(3ax+4)\sqrt{c-a^2cx^2}+\frac{3}{4}c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{1}{6}c(3ax+2)\left(c-a^2cx^2\right)^{3/2}-\frac{1}{5}\left(c-a^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x,x]
```

```
[Out] (c^2*(4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/4 + (c*(2 + 3*a*x)*(c - a^2*c*x^2)^(3
/2))/6 - (c - a^2*c*x^2)^(5/2)/5 + (3*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
 a^2*c*x^2]])/4 - c^(5/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x}\,dx = c\int \frac{(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}{x}\,dx$$

$$= -\frac{1}{5}\left(c-a^2cx^2\right)^{5/2} - \frac{\int \frac{\left(-5a^2c-10a^3cx\right)\left(c-a^2cx^2\right)^{3/2}}{x}dx}{5a^2}$$

$$= \frac{1}{6}c(2+3ax)\left(c-a^2cx^2\right)^{3/2} - \frac{1}{5}\left(c-a^2cx^2\right)^{5/2} + \frac{\int \frac{\left(20a^4c^3+30a^5c^3x\right)\sqrt{c-a^2cx^2}}{x}dx}{20a^4c}$$

$$= \frac{1}{4}c^2(4+3ax)\sqrt{c-a^2cx^2} + \frac{1}{6}c(2+3ax)\left(c-a^2cx^2\right)^{3/2} - \frac{1}{5}\left(c-a^2cx^2\right)^{5/2} - \frac{\int \frac{-40a^6c^5-30a}{x\sqrt{c-a^2cx}}}{40a^6c^2}$$

$$= \frac{1}{4}c^2(4+3ax)\sqrt{c-a^2cx^2} + \frac{1}{6}c(2+3ax)\left(c-a^2cx^2\right)^{3/2} - \frac{1}{5}\left(c-a^2cx^2\right)^{5/2} + c^3\int \frac{1}{x\sqrt{c-}}$$

$$= \frac{1}{4}c^2(4+3ax)\sqrt{c-a^2cx^2} + \frac{1}{6}c(2+3ax)\left(c-a^2cx^2\right)^{3/2} - \frac{1}{5}\left(c-a^2cx^2\right)^{5/2} + \frac{1}{2}c^3\,\text{Subst}\Big($$

$$= \frac{1}{4}c^2(4+3ax)\sqrt{c-a^2cx^2} + \frac{1}{6}c(2+3ax)\left(c-a^2cx^2\right)^{3/2} - \frac{1}{5}\left(c-a^2cx^2\right)^{5/2} + \frac{3}{4}c^{5/2}\tan^{-1}\Big($$

$$= \frac{1}{4}c^2(4+3ax)\sqrt{c-a^2cx^2} + \frac{1}{6}c(2+3ax)\left(c-a^2cx^2\right)^{3/2} - \frac{1}{5}\left(c-a^2cx^2\right)^{5/2} + \frac{3}{4}c^{5/2}\tan^{-1}\Big($$

Mathematica [A] time = 0.147828, size = 136, normalized size = 1.

$$-\frac{1}{60}c^2\left(12a^4x^4+30a^3x^3-4a^2x^2-75ax-68\right)\sqrt{c-a^2cx^2} - c^{5/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) - \frac{3}{4}c^{5/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right) +$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x,x]
```

```
[Out] -(c^2*Sqrt[c - a^2*c*x^2]*(-68 - 75*a*x - 4*a^2*x^2 + 30*a^3*x^3 + 12*a^4*x
^4))/60 - (3*c^(5/2)*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^
2))])/4 + c^(5/2)*Log[x] - c^(5/2)*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

Maple [B] time = 0.042, size = 235, normalized size = 1.7

$$\frac{1}{5}\left(-a^2cx^2+c\right)^{\frac{5}{2}} + \frac{c}{3}\left(-a^2cx^2+c\right)^{\frac{3}{2}} - c^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right) + \sqrt{-a^2cx^2+c}\,c^2 - \frac{2}{5}\left(-ca^2\left(x-a^{-1}\right)^2 - 2ac\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x,x)
```

```
[Out] 1/5*(-a^2*c*x^2+c)^(5/2)+1/3*c*(-a^2*c*x^2+c)^(3/2)-c^(5/2)*ln((2*c+2*c^(1/
2)*(-a^2*c*x^2+c)^(1/2))/x)+(-a^2*c*x^2+c)^(1/2)*c^2-2/5*(-c*a^2*(x-1/a)^2-
2*a*c*(x-1/a))^(5/2)+1/2*a*c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)*x+3/4*a
*c^2*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x+3/4*a*c^3/(a^2*c)^(1/2)*arcta
n((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x,x, algorithm="maxim
a")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x), x)
```

Fricas [A] time = 3.31672, size = 671, normalized size = 4.93

$$\left[-\frac{3}{4}c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+\frac{1}{2}c^{\frac{5}{2}}\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)-\frac{1}{60}\left(12a^4c^2x^4+30a^3c^2x^3-4a^2c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x,x, algorithm="frica
s")
```

```
[Out] [-3/4*c^(5/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/
2*c^(5/2)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) - 1/
60*(12*a^4*c^2*x^4 + 30*a^3*c^2*x^3 - 4*a^2*c^2*x^2 - 75*a*c^2*x - 68*c^2)*
sqrt(-a^2*c*x^2 + c), -sqrt(-c)*c^2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a
^2*c*x^2 - c)) + 3/8*sqrt(-c)*c^2*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*
a*sqrt(-c)*x - c) - 1/60*(12*a^4*c^2*x^4 + 30*a^3*c^2*x^3 - 4*a^2*c^2*x^2 -
 75*a*c^2*x - 68*c^2)*sqrt(-a^2*c*x^2 + c)]
```

Sympy [C] time = 21.5904, size = 508, normalized size = 3.74

$$-a^4c^2\left(\begin{cases}\frac{x^4\sqrt{-a^2cx^2+c}}{5}-\frac{x^2\sqrt{-a^2cx^2+c}}{15a^2}-\frac{2\sqrt{-a^2cx^2+c}}{15a^4} & \text{for } a\neq 0\\ \frac{\sqrt{c}x^4}{4} & \text{otherwise}\end{cases}\right)-2a^3c^2\left(\begin{cases}\frac{ia^2\sqrt{c}x^5}{4\sqrt{a^2x^2-1}}-\frac{3i\sqrt{c}x^3}{8\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}x}{8a^2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\operatorname{acosh}(ax)}{8a^3}\\ -\frac{a^2\sqrt{c}x^5}{4\sqrt{-a^2x^2+1}}+\frac{3\sqrt{c}x^3}{8\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\operatorname{asin}}{8a^3}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(5/2)/x,x)
```

```
[Out] -a**4*c**2*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5 - x**2*sqrt(-a**2*c*x**
2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4), Ne(a, 0)), (sqrt(c)*
x**4/4, True)) - 2*a**3*c**2*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x*
*2 - 1)) - 3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*s
qrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (
-a**2*sqrt(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*
x**2 + 1)) - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8
*a**3), True)) + 2*a*c**2*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**2*x**2
- 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(2*a), A
bs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin(a*x)/(
2*a), True)) + c**2*Piecewise((I*sqrt(c)*sqrt(a**2*x**2 - 1) - sqrt(c)*log(
a*x) + sqrt(c)*log(a**2*x**2)/2 + I*sqrt(c)*asin(1/(a*x)), Abs(a**2*x**2) >
```

```
1), (sqrt(c)*sqrt(-a**2*x**2 + 1) + sqrt(c)*log(a**2*x**2)/2 - sqrt(c)*log
(sqrt(-a**2*x**2 + 1) + 1), True))
```

Giac [A] time = 1.15838, size = 203, normalized size = 1.49

$$\frac{2\,c^3\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{3\,a\sqrt{-c}c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{4\,|a|}+\frac{1}{60}\sqrt{-a^2cx^2+c}\left(68\,c^2+\left(75\,ac^2+2\left(2\,a^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x,x, algorithm="giac"
)
```

```
[Out] 2*c^3*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) +
3/4*a*sqrt(-c)*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a)
+ 1/60*sqrt(-a^2*c*x^2 + c)*(68*c^2 + (75*a*c^2 + 2*(2*a^2*c^2 - 3*(2*a^4*c
^2*x + 5*a^3*c^2)*x)*x)*x)
```

3.1103 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=141

$$\frac{1}{8}ac^2(16-9ax)\sqrt{c-a^2cx^2}-\frac{9}{8}ac^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-2ac^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{1}{12}ac(8-9ax)\left(c-a^2cx^2\right)^{3/2}$$

```
[Out] (a*c^2*(16 - 9*a*x)*Sqrt[c - a^2*c*x^2])/8 + (a*c*(8 - 9*a*x)*(c - a^2*c*x^
2)^(3/2))/12 - (c - a^2*c*x^2)^(5/2)/x - (9*a*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/
Sqrt[c - a^2*c*x^2]])/8 - 2*a*c^(5/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.334867, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6151, 1807, 815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{8}ac^2(16-9ax)\sqrt{c-a^2cx^2}-\frac{9}{8}ac^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-2ac^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{1}{12}ac(8-9ax)\left(c-a^2cx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x^2,x]
```


```
[Out] (a*c^2*(16 - 9*a*x)*Sqrt[c - a^2*c*x^2])/8 + (a*c*(8 - 9*a*x)*(c - a^2*c*x^
2)^(3/2))/12 - (c - a^2*c*x^2)^(5/2)/x - (9*a*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/
Sqrt[c - a^2*c*x^2]])/8 - 2*a*c^(5/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x^2}\,dx = c\int \frac{(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}{x^2}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{5/2}}{x} - \int \frac{\left(-2ac+3a^2cx\right)\left(c-a^2cx^2\right)^{3/2}}{x}\,dx$$
$$= \frac{1}{12}ac(8-9ax)\left(c-a^2cx^2\right)^{3/2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{x} + \frac{\int \frac{\left(8a^3c^3-9a^4c^3x\right)\sqrt{c-a^2cx^2}}{x}\,dx}{4a^2c}$$
$$= \frac{1}{8}ac^2(16-9ax)\sqrt{c-a^2cx^2} + \frac{1}{12}ac(8-9ax)\left(c-a^2cx^2\right)^{3/2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{x} - \frac{\int \frac{-16a}{x\sqrt{}}}{}$$
$$= \frac{1}{8}ac^2(16-9ax)\sqrt{c-a^2cx^2} + \frac{1}{12}ac(8-9ax)\left(c-a^2cx^2\right)^{3/2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{x} + \left(2ac^3\right)$$
$$= \frac{1}{8}ac^2(16-9ax)\sqrt{c-a^2cx^2} + \frac{1}{12}ac(8-9ax)\left(c-a^2cx^2\right)^{3/2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{x} + \left(ac^3\right)$$
$$= \frac{1}{8}ac^2(16-9ax)\sqrt{c-a^2cx^2} + \frac{1}{12}ac(8-9ax)\left(c-a^2cx^2\right)^{3/2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{x} - \frac{9}{8}ac^{5/2}$$
$$= \frac{1}{8}ac^2(16-9ax)\sqrt{c-a^2cx^2} + \frac{1}{12}ac(8-9ax)\left(c-a^2cx^2\right)^{3/2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{x} - \frac{9}{8}ac^{5/2}$$

Mathematica [A] time = 0.232064, size = 143, normalized size = 1.01

$$-\frac{c^2\left(6a^4x^4+16a^3x^3-3a^2x^2-64ax+24\right)\sqrt{c-a^2cx^2}}{24x} - 2ac^{5/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) + \frac{9}{8}ac^{5/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x^2,x]
```

```
[Out] -(c^2*Sqrt[c - a^2*c*x^2]*(24 - 64*a*x - 3*a^2*x^2 + 16*a^3*x^3 + 6*a^4*x^4
))/(24*x) + (9*a*c^(5/2)*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^
2*x^2))])/8 + 2*a*c^(5/2)*Log[x] - 2*a*c^(5/2)*Log[c + Sqrt[c]*Sqrt[c - a^2
*c*x^2]]
```

Maple [B] time = 0.045, size = 367, normalized size = 2.6

$$-\frac{1}{cx}\left(-a^2cx^2+c\right)^{\frac{7}{2}} - a^2x\left(-a^2cx^2+c\right)^{\frac{5}{2}} - \frac{5\,cxa^2}{4}\left(-a^2cx^2+c\right)^{\frac{3}{2}} - \frac{15\,a^2c^2x}{8}\sqrt{-a^2cx^2+c} - \frac{15\,a^2c^3}{8}\arctan\left(x\sqrt{a^2c}\frac{}{\sqrt{-a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^2,x)
```

```
[Out] -1/c/x*(-a^2*c*x^2+c)^(7/2)-a^2*x*(-a^2*c*x^2+c)^(5/2)-5/4*a^2*c*x*(-a^2*c*
x^2+c)^(3/2)-15/8*a^2*c^2*x*(-a^2*c*x^2+c)^(1/2)-15/8*a^2*c^3/(a^2*c)^(1/2)
*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/5*a*(-a^2*c*x^2+c)^(5/2)+2/
3*a*c*(-a^2*c*x^2+c)^(3/2)-2*a*c^(5/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/
2))/x)+2*a*(-a^2*c*x^2+c)^(1/2)*c^2-2/5*a*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^
```

```
(5/2)+1/2*a^2*c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)*x+3/4*a^2*c^2*(-c*a^
2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x+3/4*a^2*c^3/(a^2*c)^(1/2)*arctan((a^2*c)
^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^2,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x^2), x)
```

Fricas [A] time = 2.79737, size = 702, normalized size = 4.98

$$\left[\frac{27\,ac^{\frac{5}{2}}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+24\,ac^{\frac{5}{2}}x\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-\left(6\,a^4c^2x^4+16\,a^3c^2x^3-3\,a^2c^2x^2-64\,ac^2x+24\right.}{24\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] [1/24*(27*a*c^(5/2)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 -
c)) + 24*a*c^(5/2)*x*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c
)/x^2) - (6*a^4*c^2*x^4 + 16*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 64*a*c^2*x + 24*
c^2)*sqrt(-a^2*c*x^2 + c))/x, -1/48*(96*a*sqrt(-c)*c^2*x*arctan(sqrt(-a^2*c
*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - 27*a*sqrt(-c)*c^2*x*log(2*a^2*c*x^2 -
 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) + 2*(6*a^4*c^2*x^4 + 16*a^3*c^2*x
^3 - 3*a^2*c^2*x^2 - 64*a*c^2*x + 24*c^2)*sqrt(-a^2*c*x^2 + c))/x]
```

Sympy [C] time = 13.3135, size = 483, normalized size = 3.43

$$-a^4c^2\left(\begin{cases}\frac{ia^2\sqrt{c}x^5}{4\sqrt{a^2x^2-1}}-\frac{3i\sqrt{c}x^3}{8\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}x}{8a^2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\operatorname{acosh}(ax)}{8a^3} & \text{for } \left|a^2x^2\right|>1\\ -\frac{a^2\sqrt{c}x^5}{4\sqrt{-a^2x^2+1}}+\frac{3\sqrt{c}x^3}{8\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\operatorname{asin}(ax)}{8a^3} & \text{otherwise}\end{cases}\right)-2a^3c^2\left(\begin{cases}0 & \text{for } c=0\\ \frac{\sqrt{c}x^2}{2} & \text{for } a^2=0\\ -\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3a^2c} & \text{otherwise}\end{cases}\right)+2ac$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(5/2)/x**2,x)
```

```
[Out] -a**4*c**2*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*sqr
t(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x**2 - 1)
) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**5
```

```
/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1)) - sqrt(
c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), True)) - 2
*a**3*c**2*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2
*c*x**2 + c)**(3/2)/(3*a**2*c), True)) + 2*a*c**2*Piecewise((I*sqrt(c)*sqrt
(a**2*x**2 - 1) - sqrt(c)*log(a*x) + sqrt(c)*log(a**2*x**2)/2 + I*sqrt(c)*a
sin(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(c)*sqrt(-a**2*x**2 + 1) + sqrt(c)*
log(a**2*x**2)/2 - sqrt(c)*log(sqrt(-a**2*x**2 + 1) + 1), True)) + c**2*Pie
cewise((-I*a**2*sqrt(c)*x/sqrt(a**2*x**2 - 1) + I*a*sqrt(c)*acosh(a*x) + I*
sqrt(c)/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2) > 1), (a**2*sqrt(c)*x/sqrt(
-a**2*x**2 + 1) - a*sqrt(c)*asin(a*x) - sqrt(c)/(x*sqrt(-a**2*x**2 + 1)), T
rue))
```

Giac [A] time = 1.20294, size = 263, normalized size = 1.87

$$\frac{4\,ac^3\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{9\,a^2\sqrt{-c}c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8\,|a|}+\frac{2\,a^2\sqrt{-c}c^3}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^2,x, algorithm="gia
c")
```

```
[Out] 4*a*c^3*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
- 9/8*a^2*sqrt(-c)*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs
(a) + 2*a^2*sqrt(-c)*c^3/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*a
bs(a)) + 1/24*sqrt(-a^2*c*x^2 + c)*(64*a*c^2 + (3*a^2*c^2 - 2*(3*a^4*c^2*x
+ 8*a^3*c^2)*x)*x)
```

3.1104 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=151

$$-\frac{1}{2}a^2c^2(6ax+1)\sqrt{c-a^2cx^2}-3a^2c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\frac{1}{2}a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{ac(ax+12)\left(c-a^2cx^2\right)^{3/2}}{6x}-$$

```
[Out] -(a^2*c^2*(1 + 6*a*x)*Sqrt[c - a^2*c*x^2])/2 - (a*c*(12 + a*x)*(c - a^2*c*x
^2)^(3/2))/(6*x) - (c - a^2*c*x^2)^(5/2)/(2*x^2) - 3*a^2*c^(5/2)*ArcTan[(a*
Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] + (a^2*c^(5/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/
Sqrt[c]])/2
```

Rubi [A] time = 0.331636, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {6151, 1807, 813, 815, 844, 217, 203, 266, 63, 208}

$$-\frac{1}{2}a^2c^2(6ax+1)\sqrt{c-a^2cx^2}-3a^2c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\frac{1}{2}a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{ac(ax+12)\left(c-a^2cx^2\right)^{3/2}}{6x}-$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x^3,x]
```

```
[Out] -(a^2*c^2*(1 + 6*a*x)*Sqrt[c - a^2*c*x^2])/2 - (a*c*(12 + a*x)*(c - a^2*c*x
^2)^(3/2))/(6*x) - (c - a^2*c*x^2)^(5/2)/(2*x^2) - 3*a^2*c^(5/2)*ArcTan[(a*
Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] + (a^2*c^(5/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/
Sqrt[c]])/2
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
```

```
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x^3}\,dx = c\int \frac{(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}{x^3}\,dx$$

$$= -\frac{\left(c-a^2cx^2\right)^{5/2}}{2x^2} - \frac{1}{2}\int \frac{\left(-4ac+a^2cx\right)\left(c-a^2cx^2\right)^{3/2}}{x^2}\,dx$$

$$= -\frac{ac(12+ax)\left(c-a^2cx^2\right)^{3/2}}{6x} - \frac{\left(c-a^2cx^2\right)^{5/2}}{2x^2} + \frac{1}{4}\int \frac{\left(-2a^2c^2-24a^3c^2x\right)\sqrt{c-a^2cx^2}}{x}\,dx$$

$$= -\frac{1}{2}a^2c^2(1+6ax)\sqrt{c-a^2cx^2} - \frac{ac(12+ax)\left(c-a^2cx^2\right)^{3/2}}{6x} - \frac{\left(c-a^2cx^2\right)^{5/2}}{2x^2} - \frac{\int \frac{4a^4c^4+24a}{x\sqrt{c-a^2c}}}{8a^2c}$$

$$= -\frac{1}{2}a^2c^2(1+6ax)\sqrt{c-a^2cx^2} - \frac{ac(12+ax)\left(c-a^2cx^2\right)^{3/2}}{6x} - \frac{\left(c-a^2cx^2\right)^{5/2}}{2x^2} - \frac{1}{2}\left(a^2c^3\right)\int$$

$$= -\frac{1}{2}a^2c^2(1+6ax)\sqrt{c-a^2cx^2} - \frac{ac(12+ax)\left(c-a^2cx^2\right)^{3/2}}{6x} - \frac{\left(c-a^2cx^2\right)^{5/2}}{2x^2} - \frac{1}{4}\left(a^2c^3\right)\text{Su}$$

$$= -\frac{1}{2}a^2c^2(1+6ax)\sqrt{c-a^2cx^2} - \frac{ac(12+ax)\left(c-a^2cx^2\right)^{3/2}}{6x} - \frac{\left(c-a^2cx^2\right)^{5/2}}{2x^2} - 3a^2c^{5/2}\tan$$

$$= -\frac{1}{2}a^2c^2(1+6ax)\sqrt{c-a^2cx^2} - \frac{ac(12+ax)\left(c-a^2cx^2\right)^{3/2}}{6x} - \frac{\left(c-a^2cx^2\right)^{5/2}}{2x^2} - 3a^2c^{5/2}\tan$$

Mathematica [A] time = 0.260316, size = 151, normalized size = 1.

$$-\frac{c^2\left(2a^4x^4+6a^3x^3-2a^2x^2+12ax+3\right)\sqrt{c-a^2cx^2}}{6x^2} + \frac{1}{2}a^2c^{5/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) + 3a^2c^{5/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right) -$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x^3,x]
```

```
[Out] -(c^2*Sqrt[c - a^2*c*x^2]*(3 + 12*a*x - 2*a^2*x^2 + 6*a^3*x^3 + 2*a^4*x^4))
/(6*x^2) + 3*a^2*c^(5/2)*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^
2*x^2))] - (a^2*c^(5/2)*Log[x])/2 + (a^2*c^(5/2)*Log[c + Sqrt[c]*Sqrt[c - a
^2*c*x^2]])/2
```

Maple [B] time = 0.048, size = 399, normalized size = 2.6

$$-2\,\frac{a\left(-a^2cx^2+c\right)^{7/2}}{cx} - 2\,a^3x\left(-a^2cx^2+c\right)^{5/2} - \frac{5\,a^3cx}{2}\left(-a^2cx^2+c\right)^{\frac{3}{2}} - \frac{15\,a^3c^2x}{4}\sqrt{-a^2cx^2+c} - \frac{15\,a^3c^3}{4}\arctan\left(x\sqrt{a^2c}\frac{}{\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^3,x)
```

```
[Out] -2*a/c/x*(-a^2*c*x^2+c)^(7/2)-2*a^3*x*(-a^2*c*x^2+c)^(5/2)-5/2*a^3*c*x*(-a^
2*c*x^2+c)^(3/2)-15/4*a^3*c^2*x*(-a^2*c*x^2+c)^(1/2)-15/4*a^3*c^3/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-1/10*a^2*(-a^2*c*x^2+c)^(
5/2)-1/6*a^2*c*(-a^2*c*x^2+c)^(3/2)+1/2*a^2*c^(5/2)*ln((2*c+2*c^(1/2)*(-a^2
*c*x^2+c)^(1/2))/x)-1/2*a^2*(-a^2*c*x^2+c)^(1/2)*c^2-2/5*a^2*(-c*a^2*(x-1/a
)^2-2*a*c*(x-1/a))^(5/2)+1/2*a^3*c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)*x
```

```
+3/4*a^3*c^2*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x+3/4*a^3*c^3/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))-1/2/c/x
^2*(-a^2*c*x^2+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^3,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x^3), x)
```

Fricas [A] time = 2.96939, size = 717, normalized size = 4.75

$$\left[\frac{36\,a^2c^{\frac{5}{2}}x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+3\,a^2c^{\frac{5}{2}}x^2\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-2\left(2\,a^4c^2x^4+6\,a^3c^2x^3-2\,a^2c^2x^2+12\,ac^2\right.}{12\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^3,x, algorithm="fri
cas")
```

```
[Out] [1/12*(36*a^2*c^(5/2)*x^2*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^
2 - c)) + 3*a^2*c^(5/2)*x^2*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c
) - 2*c)/x^2) - 2*(2*a^4*c^2*x^4 + 6*a^3*c^2*x^3 - 2*a^2*c^2*x^2 + 12*a*c^2
*x + 3*c^2)*sqrt(-a^2*c*x^2 + c))/x^2, 1/6*(3*a^2*sqrt(-c)*c^2*x^2*arctan(s
qrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 9*a^2*sqrt(-c)*c^2*x^2*log(
2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) - (2*a^4*c^2*x^4 + 6
*a^3*c^2*x^3 - 2*a^2*c^2*x^2 + 12*a*c^2*x + 3*c^2)*sqrt(-a^2*c*x^2 + c))/x^
2]
```

Sympy [C] time = 10.2947, size = 401, normalized size = 2.66

$$-a^4c^2\left(\begin{cases}0 & \text{for } c=0\\ \frac{\sqrt{c}x^2}{2} & \text{for } a^2=0\\ -\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3a^2c} & \text{otherwise}\end{cases}\right)-2a^3c^2\left(\begin{cases}\frac{ia^2\sqrt{c}x^3}{2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}x}{2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\operatorname{acosh}(ax)}{2a} & \text{for } \left|a^2x^2\right|>1\\ \frac{\sqrt{c}x\sqrt{-a^2x^2+1}}{2}+\frac{\sqrt{c}\operatorname{asin}(ax)}{2a} & \text{otherwise}\end{cases}\right)+2ac^2\left(\begin{cases}-\frac{ia^2\sqrt{c}x}{\sqrt{a^2x^2-1}}\\ \frac{a^2\sqrt{c}x}{\sqrt{-a^2x^2+1}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(5/2)/x**3,x)
```

```
[Out] -a**4*c**2*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2
*c*x**2 + c)**(3/2)/(3*a**2*c), True)) - 2*a**3*c**2*Piecewise((I*a**2*sqrt
```

```
(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I*
sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2 +
 1)/2 + sqrt(c)*asin(a*x)/(2*a), True)) + 2*a*c**2*Piecewise((-I*a**2*sqrt(
c)*x/sqrt(a**2*x**2 - 1) + I*a*sqrt(c)*acosh(a*x) + I*sqrt(c)/(x*sqrt(a**2*
x**2 - 1)), Abs(a**2*x**2) > 1), (a**2*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - a*s
qrt(c)*asin(a*x) - sqrt(c)/(x*sqrt(-a**2*x**2 + 1)), True)) + c**2*Piecewis
e((a**2*sqrt(c)*acosh(1/(a*x))/2 + a*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2)))
 - sqrt(c)/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*
a**2*sqrt(c)*asin(1/(a*x))/2 - I*a*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), T
rue))
```

Giac [B] time = 1.18431, size = 408, normalized size = 2.7

$$-\frac{a^2c^3\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{3\,a^3\sqrt{-c}c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}+\frac{1}{3}\sqrt{-a^2cx^2+c}\left(a^2c^2-\left(a^4c^2x+3\,a^3c^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^3,x, algorithm="gia
c")
```

```
[Out] -a^2*c^3*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
 - 3*a^3*sqrt(-c)*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(
a) + 1/3*sqrt(-a^2*c*x^2 + c)*(a^2*c^2 - (a^4*c^2*x + 3*a^3*c^2)*x) - ((sqr
t(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^2*c^3*abs(a) - 4*(sqrt(-a^2*c)*x -
sqrt(-a^2*c*x^2 + c))^2*a^3*sqrt(-c)*c^3 + (sqrt(-a^2*c)*x - sqrt(-a^2*c*x^
2 + c))*a^2*c^4*abs(a) + 4*a^3*sqrt(-c)*c^4)/(((sqrt(-a^2*c)*x - sqrt(-a^2*
c*x^2 + c))^2 - c)^2*abs(a))
```

3.1105 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=155

$$-\frac{a^2c^2(6ax+1)\sqrt{c-a^2cx^2}}{2x}-\frac{1}{2}a^3c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+3a^3c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{ac(6-ax)\left(c-a^2cx^2\right)^{3/2}}{6x^2}-$$

```
[Out] -(a^2*c^2*(1 + 6*a*x)*Sqrt[c - a^2*c*x^2])/(2*x) - (a*c*(6 - a*x)*(c - a^2*
c*x^2)^(3/2))/(6*x^2) - (c - a^2*c*x^2)^(5/2)/(3*x^3) - (a^3*c^(5/2)*ArcTan
[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/2 + 3*a^3*c^(5/2)*ArcTanh[Sqrt[c - a^2
*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.34214, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6151, 1807, 813, 844, 217, 203, 266, 63, 208}

$$-\frac{a^2c^2(6ax+1)\sqrt{c-a^2cx^2}}{2x}-\frac{1}{2}a^3c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+3a^3c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{ac(6-ax)\left(c-a^2cx^2\right)^{3/2}}{6x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x^4,x]
```


```
[Out] -(a^2*c^2*(1 + 6*a*x)*Sqrt[c - a^2*c*x^2])/(2*x) - (a*c*(6 - a*x)*(c - a^2*
c*x^2)^(3/2))/(6*x^2) - (c - a^2*c*x^2)^(5/2)/(3*x^3) - (a^3*c^(5/2)*ArcTan
[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/2 + 3*a^3*c^(5/2)*ArcTanh[Sqrt[c - a^2
*c*x^2]/Sqrt[c]]
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
```

```
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x^4}\,dx = c\int \frac{(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}{x^4}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{5/2}}{3x^3} - \frac{1}{3}\int \frac{\left(-6ac-a^2cx\right)\left(c-a^2cx^2\right)^{3/2}}{x^3}\,dx$$
$$= -\frac{ac(6-ax)\left(c-a^2cx^2\right)^{3/2}}{6x^2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{3x^3} + \frac{1}{8}\int \frac{\left(4a^2c^2-24a^3c^2x\right)\sqrt{c-a^2cx^2}}{x^2}\,dx$$
$$= -\frac{a^2c^2(1+6ax)\sqrt{c-a^2cx^2}}{2x} - \frac{ac(6-ax)\left(c-a^2cx^2\right)^{3/2}}{6x^2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{3x^3} - \frac{1}{16}\int \frac{48a}{x}$$
$$= -\frac{a^2c^2(1+6ax)\sqrt{c-a^2cx^2}}{2x} - \frac{ac(6-ax)\left(c-a^2cx^2\right)^{3/2}}{6x^2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{3x^3} - \left(3a^3c^3\right)\int$$
$$= -\frac{a^2c^2(1+6ax)\sqrt{c-a^2cx^2}}{2x} - \frac{ac(6-ax)\left(c-a^2cx^2\right)^{3/2}}{6x^2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{3x^3} - \frac{1}{2}\left(3a^3c^3\right)$$
$$= -\frac{a^2c^2(1+6ax)\sqrt{c-a^2cx^2}}{2x} - \frac{ac(6-ax)\left(c-a^2cx^2\right)^{3/2}}{6x^2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{3x^3} - \frac{1}{2}a^3c^{5/2}\tan$$
$$= -\frac{a^2c^2(1+6ax)\sqrt{c-a^2cx^2}}{2x} - \frac{ac(6-ax)\left(c-a^2cx^2\right)^{3/2}}{6x^2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{3x^3} - \frac{1}{2}a^3c^{5/2}\tan$$

Mathematica [A] time = 0.263262, size = 149, normalized size = 0.96

$$-\frac{c^2\left(3a^4x^4+12a^3x^3-2a^2x^2+6ax+2\right)\sqrt{c-a^2cx^2}}{6x^3} + 3a^3c^{5/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) + \frac{1}{2}a^3c^{5/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x^4,x]
```

```
[Out] -(c^2*Sqrt[c - a^2*c*x^2]*(2 + 6*a*x - 2*a^2*x^2 + 12*a^3*x^3 + 3*a^4*x^4))
/(6*x^3) + (a^3*c^(5/2)*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2
*x^2))])/2 - 3*a^3*c^(5/2)*Log[x] + 3*a^3*c^(5/2)*Log[c + Sqrt[c]*Sqrt[c -
a^2*c*x^2]]
```

Maple [B] time = 0.05, size = 423, normalized size = 2.7

$$-\frac{2a^2}{3cx}\left(-a^2cx^2+c\right)^{\frac{7}{2}} - \frac{2a^4x}{3}\left(-a^2cx^2+c\right)^{\frac{5}{2}} - \frac{5a^4cx}{6}\left(-a^2cx^2+c\right)^{\frac{3}{2}} - \frac{5a^4c^2x}{4}\sqrt{-a^2cx^2+c} - \frac{5a^4c^3}{4}\arctan\left(x\sqrt{a^2c}\frac{}{\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^4,x)
```

```
[Out] -2/3*a^2/c/x*(-a^2*c*x^2+c)^(7/2)-2/3*a^4*x*(-a^2*c*x^2+c)^(5/2)-5/6*a^4*c*
x*(-a^2*c*x^2+c)^(3/2)-5/4*a^4*c^2*x*(-a^2*c*x^2+c)^(1/2)-5/4*a^4*c^3/(a^2*
c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-3/5*a^3*(-a^2*c*x^2+c
)^(5/2)-a^3*c*(-a^2*c*x^2+c)^(3/2)+3*a^3*c^(5/2)*ln((2*c+2*c^(1/2)*(-a^2*c*
x^2+c)^(1/2))/x)-3*a^3*(-a^2*c*x^2+c)^(1/2)*c^2-2/5*a^3*(-c*a^2*(x-1/a)^2-2
*a*c*(x-1/a))^(5/2)+1/2*a^4*c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)*x+3/4*
```

```
a^4*c^2*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x+3/4*a^4*c^3/(a^2*c)^(1/2)*
arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))-a/c/x^2*(-a^
2*c*x^2+c)^(7/2)-1/3/c/x^3*(-a^2*c*x^2+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^4,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x^4), x)
```

Fricas [A] time = 2.7942, size = 717, normalized size = 4.63

$$\left[\frac{3\,a^3c^{\frac{5}{2}}x^3\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+9\,a^3c^{\frac{5}{2}}x^3\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-\left(3\,a^4c^2x^4+12\,a^3c^2x^3-2\,a^2c^2x^2+6\,ac^2x+2\right.}{6\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^4,x, algorithm="fri
cas")
```

```
[Out] [1/6*(3*a^3*c^(5/2)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2
- c)) + 9*a^3*c^(5/2)*x^3*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) - (3*a^4*c^2*x^4 + 12*a^3*c^2*x^3 - 2*a^2*c^2*x^2 + 6*a*c^2*x +
 2*c^2)*sqrt(-a^2*c*x^2 + c))/x^3, 1/12*(36*a^3*sqrt(-c)*c^2*x^3*arctan(sqr
t(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 3*a^3*sqrt(-c)*c^2*x^3*log(2*
a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) - 2*(3*a^4*c^2*x^4 + 1
2*a^3*c^2*x^3 - 2*a^2*c^2*x^2 + 6*a*c^2*x + 2*c^2)*sqrt(-a^2*c*x^2 + c))/x^
3]
```

Sympy [C] time = 14.1235, size = 478, normalized size = 3.08

$$-a^4c^2\left(\begin{cases}\frac{ia^2\sqrt{c}x^3}{2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}x}{2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\operatorname{acosh}(ax)}{2a} & \text{for}\ \left|a^2x^2\right|>1\\ \frac{\sqrt{c}x\sqrt{-a^2x^2+1}}{2}+\frac{\sqrt{c}\operatorname{asin}(ax)}{2a} & \text{otherwise}\end{cases}\right)-2a^3c^2\left(\begin{cases}i\sqrt{c}\sqrt{a^2x^2-1}-\sqrt{c}\log(ax)+\frac{\sqrt{c}\log\left(a^2x^2\right)}{2}+i\sqrt{c}\,\text{as}\\ \sqrt{c}\sqrt{-a^2x^2+1}+\frac{\sqrt{c}\log\left(a^2x^2\right)}{2}-\sqrt{c}\log\left(\sqrt{-a^2x^2+\right.\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(5/2)/x**4,x)
```

```
[Out] -a**4*c**2*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(
c)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) >
```

```
 1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin(a*x)/(2*a), True)) -
2*a**3*c**2*Piecewise((I*sqrt(c)*sqrt(a**2*x**2 - 1) - sqrt(c)*log(a*x) + s
qrt(c)*log(a**2*x**2)/2 + I*sqrt(c)*asin(1/(a*x)), Abs(a**2*x**2) > 1), (sq
rt(c)*sqrt(-a**2*x**2 + 1) + sqrt(c)*log(a**2*x**2)/2 - sqrt(c)*log(sqrt(-a
**2*x**2 + 1) + 1), True)) + 2*a*c**2*Piecewise((a**2*sqrt(c)*acosh(1/(a*x)
)/2 + a*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - sqrt(c)/(2*a*x**3*sqrt(-1
+ 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*a**2*sqrt(c)*asin(1/(a*x))/2
- I*a*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True)) + c**2*Piecewise((a**3*
sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/(3*
x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/3 - I
*a*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True))
```

Giac [B] time = 1.20414, size = 394, normalized size = 2.54

$$-\frac{6\,a^3c^3\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{a^4\sqrt{-c}c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\,|a|}-\frac{1}{2}\left(a^4c^2x+4\,a^3c^2\right)\sqrt{-a^2cx^2+c}-\cdots 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^4,x, algorithm="gia
c")
```

```
[Out] -6*a^3*c^3*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-
c) - 1/2*a^4*sqrt(-c)*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/
abs(a) - 1/2*(a^4*c^2*x + 4*a^3*c^2)*sqrt(-a^2*c*x^2 + c) - 2/3*(3*(sqrt(-a
^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c^3*abs(a) + 3*(sqrt(-a^2*c)*x - sqrt
(-a^2*c*x^2 + c))^4*a^4*sqrt(-c)*c^3 - 3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2
+ c))*a^3*c^5*abs(a) + a^4*sqrt(-c)*c^5)/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^
2 + c))^2 - c)^3*abs(a))
```

3.1106 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=155

$$\frac{a^3c^2(16-9ax)\sqrt{c-a^2cx^2}}{8x}+2a^4c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\frac{9}{8}a^4c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{ac(9ax+16)\left(c-a^2cx^2\right)^{3/2}}{24x^3}-$$

```
[Out] (a^3*c^2*(16 - 9*a*x)*Sqrt[c - a^2*c*x^2])/(8*x) - (a*c*(16 + 9*a*x)*(c - a
^2*c*x^2)^(3/2))/(24*x^3) - (c - a^2*c*x^2)^(5/2)/(4*x^4) + 2*a^4*c^(5/2)*A
rcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] + (9*a^4*c^(5/2)*ArcTanh[Sqrt[c -
a^2*c*x^2]/Sqrt[c]])/8
```

Rubi [A] time = 0.333742, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {6151, 1807, 811, 813, 844, 217, 203, 266, 63, 208}

$$\frac{a^3c^2(16-9ax)\sqrt{c-a^2cx^2}}{8x}+2a^4c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\frac{9}{8}a^4c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{ac(9ax+16)\left(c-a^2cx^2\right)^{3/2}}{24x^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x^5,x]
```


```
[Out] (a^3*c^2*(16 - 9*a*x)*Sqrt[c - a^2*c*x^2])/(8*x) - (a*c*(16 + 9*a*x)*(c - a
^2*c*x^2)^(3/2))/(24*x^3) - (c - a^2*c*x^2)^(5/2)/(4*x^4) + 2*a^4*c^(5/2)*A
rcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] + (9*a^4*c^(5/2)*ArcTanh[Sqrt[c -
a^2*c*x^2]/Sqrt[c]])/8
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
```

```
&& !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x^5}dx = c\int \frac{(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}{x^5}dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{5/2}}{4x^4} - \frac{1}{4}\int \frac{\left(-8ac-3a^2cx\right)\left(c-a^2cx^2\right)^{3/2}}{x^4}dx$$
$$= -\frac{ac(16+9ax)\left(c-a^2cx^2\right)^{3/2}}{24x^3} - \frac{\left(c-a^2cx^2\right)^{5/2}}{4x^4} + \frac{\int \frac{\left(-32a^3c^3-18a^4c^3x\right)\sqrt{c-a^2cx^2}}{x^2}dx}{16c}$$
$$= \frac{a^3c^2(16-9ax)\sqrt{c-a^2cx^2}}{8x} - \frac{ac(16+9ax)\left(c-a^2cx^2\right)^{3/2}}{24x^3} - \frac{\left(c-a^2cx^2\right)^{5/2}}{4x^4} - \frac{\int \frac{36a^4c^4-64a}{x\sqrt{c-a^2c}}}{32c}$$
$$= \frac{a^3c^2(16-9ax)\sqrt{c-a^2cx^2}}{8x} - \frac{ac(16+9ax)\left(c-a^2cx^2\right)^{3/2}}{24x^3} - \frac{\left(c-a^2cx^2\right)^{5/2}}{4x^4} - \frac{1}{8}\left(9a^4c^3\right)\int$$
$$= \frac{a^3c^2(16-9ax)\sqrt{c-a^2cx^2}}{8x} - \frac{ac(16+9ax)\left(c-a^2cx^2\right)^{3/2}}{24x^3} - \frac{\left(c-a^2cx^2\right)^{5/2}}{4x^4} - \frac{1}{16}\left(9a^4c^3\right)$$
$$= \frac{a^3c^2(16-9ax)\sqrt{c-a^2cx^2}}{8x} - \frac{ac(16+9ax)\left(c-a^2cx^2\right)^{3/2}}{24x^3} - \frac{\left(c-a^2cx^2\right)^{5/2}}{4x^4} + 2a^4c^{5/2}\tan$$
$$= \frac{a^3c^2(16-9ax)\sqrt{c-a^2cx^2}}{8x} - \frac{ac(16+9ax)\left(c-a^2cx^2\right)^{3/2}}{24x^3} - \frac{\left(c-a^2cx^2\right)^{5/2}}{4x^4} + 2a^4c^{5/2}\tan$$

Mathematica [A] time = 0.261657, size = 151, normalized size = 0.97

$$-\frac{c^2\left(24a^4x^4-64a^3x^3-3a^2x^2+16ax+6\right)\sqrt{c-a^2cx^2}}{24x^4} + \frac{9}{8}a^4c^{5/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) - 2a^4c^{5/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x^5,x]
```

```
[Out] -(c^2*Sqrt[c - a^2*c*x^2]*(6 + 16*a*x - 3*a^2*x^2 - 64*a^3*x^3 + 24*a^4*x^4
))/(24*x^4) - 2*a^4*c^(5/2)*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 +
 a^2*x^2))] - (9*a^4*c^(5/2)*Log[x])/8 + (9*a^4*c^(5/2)*Log[c + Sqrt[c]*Sqr
t[c - a^2*c*x^2]])/8
```

Maple [B] time = 0.061, size = 447, normalized size = 2.9

$$\frac{2a^5x}{3}\left(-a^2cx^2+c\right)^{\frac{5}{2}} - \frac{9a^4}{40}\left(-a^2cx^2+c\right)^{\frac{5}{2}} - \frac{2a^4}{5}\left(-ca^2\left(x-a^{-1}\right)^2 - 2ac\left(x-a^{-1}\right)\right)^{\frac{5}{2}} - \frac{3a^4c}{8}\left(-a^2cx^2+c\right)^{\frac{3}{2}} + \frac{9a^4}{8}c^{\frac{5}{2}}\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^5,x)
```

```
[Out] 2/3*a^5*x*(-a^2*c*x^2+c)^(5/2)-9/40*a^4*(-a^2*c*x^2+c)^(5/2)-2/5*a^4*(-c*a^
2*(x-1/a)^2-2*a*c*(x-1/a))^(5/2)-3/8*a^4*c*(-a^2*c*x^2+c)^(3/2)+9/8*a^4*c^(
5/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-9/8*a^4*(-a^2*c*x^2+c)^(1/2
)*c^2-1/4/c/x^4*(-a^2*c*x^2+c)^(7/2)-2/3*a/c/x^3*(-a^2*c*x^2+c)^(7/2)+1/2*a
^5*c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)*x+3/4*a^5*c^2*(-c*a^2*(x-1/a)^2
-2*a*c*(x-1/a))^(1/2)*x+3/4*a^5*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-
```

```
c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))+2/3*a^3/c/x*(-a^2*c*x^2+c)^(7/2)+5/6*
a^5*c*x*(-a^2*c*x^2+c)^(3/2)+5/4*a^5*c^2*x*(-a^2*c*x^2+c)^(1/2)+5/4*a^5*c^3
/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-5/8*a^2/c/x^2*(
-a^2*c*x^2+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^5,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x^5), x)
```

Fricas [A] time = 2.84422, size = 729, normalized size = 4.7

$$\left[-\frac{96\,a^4c^{\frac{5}{2}}x^4\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)-27\,a^4c^{\frac{5}{2}}x^4\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+2\left(24\,a^4c^2x^4-64\,a^3c^2x^3-3\,a^2c^2x^2+\right.}{48\,x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^5,x, algorithm="fri
cas")
```

```
[Out] [-1/48*(96*a^4*c^(5/2)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x
^2 - c)) - 27*a^4*c^(5/2)*x^4*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt
(c) - 2*c)/x^2) + 2*(24*a^4*c^2*x^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a
*c^2*x + 6*c^2)*sqrt(-a^2*c*x^2 + c))/x^4, 1/24*(27*a^4*sqrt(-c)*c^2*x^4*ar
ctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 24*a^4*sqrt(-c)*c^2*x
^4*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) - (24*a^4*c^2
*x^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt(-a^2*c*x^2
 + c))/x^4]
```

Sympy [C] time = 11.2951, size = 575, normalized size = 3.71

$$-a^4c^2\left(\begin{cases} i\sqrt{c}\sqrt{a^2x^2-1}-\sqrt{c}\log(ax)+\frac{\sqrt{c}\log\left(a^2x^2\right)}{2}+i\sqrt{c}\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{for } \left|a^2x^2\right|>1 \\ \sqrt{c}\sqrt{-a^2x^2+1}+\frac{\sqrt{c}\log\left(a^2x^2\right)}{2}-\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right) & \text{otherwise} \end{cases}\right)-2a^3c^2\left(\begin{cases} -\frac{ia^2\sqrt{c}x}{\sqrt{a^2x^2-1}}+ia\sqrt{c}\,\mathrm{acosh} \\ \frac{a^2\sqrt{c}x}{\sqrt{-a^2x^2+1}}-a\sqrt{c}\,\mathrm{asin}\,(a \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(5/2)/x**5,x)
```

```
[Out] -a**4*c**2*Piecewise((I*sqrt(c)*sqrt(a**2*x**2 - 1) - sqrt(c)*log(a*x) + sq
rt(c)*log(a**2*x**2)/2 + I*sqrt(c)*asin(1/(a*x)), Abs(a**2*x**2) > 1), (sqr
t(c)*sqrt(-a**2*x**2 + 1) + sqrt(c)*log(a**2*x**2)/2 - sqrt(c)*log(sqrt(-a*
*2*x**2 + 1) + 1), True)) - 2*a**3*c**2*Piecewise((-I*a**2*sqrt(c)*x/sqrt(a
**2*x**2 - 1) + I*a*sqrt(c)*acosh(a*x) + I*sqrt(c)/(x*sqrt(a**2*x**2 - 1)),
 Abs(a**2*x**2) > 1), (a**2*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - a*sqrt(c)*asin
(a*x) - sqrt(c)/(x*sqrt(-a**2*x**2 + 1)), True)) + 2*a*c**2*Piecewise((a**3
*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/(3
*x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/3 -
I*a*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True)) + c**2*Piecewise((a**4
*sqrt(c)*acosh(1/(a*x))/8 - a**3*sqrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3
*a*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - sqrt(c)/(4*a*x**5*sqrt(-1 +
1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*a**4*sqrt(c)*asin(1/(a*x))/8 +
I*a**3*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2))) - 3*I*a*sqrt(c)/(8*x**3*sqrt(1
 - 1/(a**2*x**2))) + I*sqrt(c)/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True))
```

Giac [B] time = 1.20622, size = 594, normalized size = 3.83

$$-\frac{9\,a^4c^3\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\,\sqrt{-c}}+\frac{2\,a^5\sqrt{-c}c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}-\sqrt{-a^2cx^2+c}a^4c^2+\frac{3\left(\sqrt{-a^2c}x-\sqrt{-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^5,x, algorithm="gia
c")
```

```
[Out] -9/4*a^4*c^3*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt
(-c) + 2*a^5*sqrt(-c)*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/
abs(a) - sqrt(-a^2*c*x^2 + c)*a^4*c^2 + 1/12*(3*(sqrt(-a^2*c)*x - sqrt(-a^2
*c*x^2 + c))^7*a^4*c^3*abs(a) - 96*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^
6*a^5*sqrt(-c)*c^3 + 21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^4*a
bs(a) + 192*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^4*a^5*sqrt(-c)*c^4 + 21
*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^4*c^5*abs(a) - 160*(sqrt(-a^2*
c)*x - sqrt(-a^2*c*x^2 + c))^2*a^5*sqrt(-c)*c^5 + 3*(sqrt(-a^2*c)*x - sqrt(
-a^2*c*x^2 + c))*a^4*c^6*abs(a) + 64*a^5*sqrt(-c)*c^6)/(((sqrt(-a^2*c)*x -
sqrt(-a^2*c*x^2 + c))^2 - c)^4*abs(a))
```

3.1107 $\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=153

$$\frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{45c^{7/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{128a}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}-\frac{(ax+1)\left(c-a^2cx^2\right)^{7/2}}{8a}-$$

```
[Out] (45*c^3*x*Sqrt[c - a^2*c*x^2])/128 + (15*c^2*x*(c - a^2*c*x^2)^(3/2))/64 +
(3*c*x*(c - a^2*c*x^2)^(5/2))/16 - (9*(c - a^2*c*x^2)^(7/2))/(56*a) - ((1 +
 a*x)*(c - a^2*c*x^2)^(7/2))/(8*a) + (45*c^(7/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(128*a)
```

Rubi [A] time = 0.109702, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6141, 671, 641, 195, 217, 203}

$$\frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{45c^{7/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{128a}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}-\frac{(ax+1)\left(c-a^2cx^2\right)^{7/2}}{8a}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (45*c^3*x*Sqrt[c - a^2*c*x^2])/128 + (15*c^2*x*(c - a^2*c*x^2)^(3/2))/64 +
(3*c*x*(c - a^2*c*x^2)^(5/2))/16 - (9*(c - a^2*c*x^2)^(7/2))/(56*a) - ((1 +
 a*x)*(c - a^2*c*x^2)^(7/2))/(8*a) + (45*c^(7/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(128*a)
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}dx &= c\int(1+ax)^2\left(c-a^2cx^2\right)^{5/2}dx\\ &= -\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}+\frac{1}{8}(9c)\int(1+ax)\left(c-a^2cx^2\right)^{5/2}dx\\ &= -\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}+\frac{1}{8}(9c)\int\left(c-a^2cx^2\right)^{5/2}dx\\ &= \frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}-\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}+\frac{1}{16}\left(15c^2\right)\int\left(c-a^2c\right.\\ &= \frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}-\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}\\ &= \frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}-\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}-\frac{(1}{}\\ &= \frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}-\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}-\frac{(1}{}\\ &= \frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}-\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}-\frac{(1}{}\end{aligned}$$

Mathematica [A] time = 0.132572, size = 151, normalized size = 0.99

$$-\frac{c^3\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(112a^8x^8+144a^7x^7-424a^6x^6-600a^5x^5+558a^4x^4+978a^3x^3-187a^2x^2-837ax+256\right)+63\right.}{896a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -(c^3*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(256 - 837*a*x - 187*a^2*x^2 + 978
*a^3*x^3 + 558*a^4*x^4 - 600*a^5*x^5 - 424*a^6*x^6 + 144*a^7*x^7 + 112*a^8*
x^8) + 630*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(896*a*Sqrt[1 - a*
x]*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.037, size = 296, normalized size = 1.9

$$-\frac{x}{8}\left(-a^2cx^2+c\right)^{\frac{7}{2}}-\frac{7\,cx}{48}\left(-a^2cx^2+c\right)^{\frac{5}{2}}-\frac{35\,xc^2}{192}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{35\,c^3x}{128}\sqrt{-a^2cx^2+c}-\frac{35\,c^4}{128}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/8*x*(-a^2*c*x^2+c)^(7/2)-7/48*c*x*(-a^2*c*x^2+c)^(5/2)-35/192*c^2*x*(-a^
2*c*x^2+c)^(3/2)-35/128*c^3*x*(-a^2*c*x^2+c)^(1/2)-35/128*c^4/(a^2*c)^(1/2)
*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/7/a*(-c*a^2*(x-1/a)^2-2*a*c
*(x-1/a))^(7/2)+1/3*c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(5/2)*x+5/12*c^2*(-c
*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)*x+5/8*c^3*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/
a))^(1/2)*x+5/8*c^4/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-
2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.92892, size = 655, normalized size = 4.28

$$\left[\frac{315\sqrt{-c}c^3\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+2\left(112a^7c^3x^7+256a^6c^3x^6-168a^5c^3x^5-768a^4c^3x^4-210a^3c\right.}{1792a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas"
)
```

```
[Out] [1/1792*(315*sqrt(-c)*c^3*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) + 2*(112*a^7*c^3*x^7 + 256*a^6*c^3*x^6 - 168*a^5*c^3*x^5 - 768*a^
4*c^3*x^4 - 210*a^3*c^3*x^3 + 768*a^2*c^3*x^2 + 581*a*c^3*x - 256*c^3)*sqrt
(-a^2*c*x^2 + c))/a, -1/896*(315*c^(7/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt
(c)*x/(a^2*c*x^2 - c)) - (112*a^7*c^3*x^7 + 256*a^6*c^3*x^6 - 168*a^5*c^3*x
^5 - 768*a^4*c^3*x^4 - 210*a^3*c^3*x^3 + 768*a^2*c^3*x^2 + 581*a*c^3*x - 25
6*c^3)*sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [C] time = 28.1027, size = 1091, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] a**6*c**3*Piecewise((I*a**2*sqrt(c)*x**9/(8*sqrt(a**2*x**2 - 1)) - 7*I*sqrt
(c)*x**7/(48*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**5/(192*a**2*sqrt(a**2*x**2
 - 1)) - 5*I*sqrt(c)*x**3/(384*a**4*sqrt(a**2*x**2 - 1)) + 5*I*sqrt(c)*x/(1
28*a**6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*acosh(a*x)/(128*a**7), Abs(a**2*
x**2) > 1), (-a**2*sqrt(c)*x**9/(8*sqrt(-a**2*x**2 + 1)) + 7*sqrt(c)*x**7/(
48*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**5/(192*a**2*sqrt(-a**2*x**2 + 1)) + 5
*sqrt(c)*x**3/(384*a**4*sqrt(-a**2*x**2 + 1)) - 5*sqrt(c)*x/(128*a**6*sqrt(
-a**2*x**2 + 1)) + 5*sqrt(c)*asin(a*x)/(128*a**7), True)) + 2*a**5*c**3*Pie
cewise((x**6*sqrt(-a**2*c*x**2 + c)/7 - x**4*sqrt(-a**2*c*x**2 + c)/(35*a**
2) - 4*x**2*sqrt(-a**2*c*x**2 + c)/(105*a**4) - 8*sqrt(-a**2*c*x**2 + c)/(1
05*a**6), Ne(a, 0)), (sqrt(c)*x**6/6, True)) - a**4*c**3*Piecewise((I*a**2*
sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*x**5/(24*sqrt(a**2*x**2
 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(16*a**4
*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(16*a**5), Abs(a**2*x**2) > 1)
, (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c)*x**5/(24*sqrt(-a
**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - sqrt(c)*x/(1
6*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(16*a**5), True)) - 4*a**3
*c**3*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5 - x**2*sqrt(-a**2*c*x**2 + c
)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4), Ne(a, 0)), (sqrt(c)*x**4/
4, True)) - a**2*c**3*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)
) - 3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**
2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*s
qrt(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 +
1)) - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3),
 True)) + 2*a*c**3*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)),
(-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True)) + c**3*Piecewise((I*a**2*sqr
t(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I
*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2
+ 1)/2 + sqrt(c)*asin(a*x)/(2*a), True))
```

Giac [A] time = 1.20269, size = 190, normalized size = 1.24

$$-\frac{45\,c^4\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{128\,\sqrt{-c}|a|}-\frac{1}{896}\,\sqrt{-a^2cx^2+c}\left(\frac{256\,c^3}{a}-\left(581\,c^3+2\left(384\,ac^3-\left(105\,a^2c^3+4\left(96\,a^3c^3+\left(21\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] -45/128*c^4*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(
a)) - 1/896*sqrt(-a^2*c*x^2 + c)*(256*c^3/a - (581*c^3 + 2*(384*a*c^3 - (10
5*a^2*c^3 + 4*(96*a^3*c^3 + (21*a^4*c^3 - 2*(7*a^6*c^3*x + 16*a^5*c^3)*x)*x
)*x)*x)*x)*x)
```

3.1108 $\int \frac{e^{2\tanh^{-1}(ax)}x^3}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=137

$$\frac{x^2\sqrt{c-a^2cx^2}}{3a^2c}+\frac{x\sqrt{c-a^2cx^2}}{a^3c}+\frac{11\sqrt{c-a^2cx^2}}{3a^4c}+\frac{(ax+1)^2}{a^4\sqrt{c-a^2cx^2}}-\frac{3\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^4\sqrt{c}}$$

```
[Out] (1 + a*x)^2/(a^4*Sqrt[c - a^2*c*x^2]) + (11*Sqrt[c - a^2*c*x^2])/(3*a^4*c)
+ (x*Sqrt[c - a^2*c*x^2])/(a^3*c) + (x^2*Sqrt[c - a^2*c*x^2])/(3*a^2*c) - (
3*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(a^4*Sqrt[c])
```

Rubi [A] time = 0.355805, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6151, 1635, 1815, 641, 217, 203}

$$\frac{x^2\sqrt{c-a^2cx^2}}{3a^2c}+\frac{x\sqrt{c-a^2cx^2}}{a^3c}+\frac{11\sqrt{c-a^2cx^2}}{3a^4c}+\frac{(ax+1)^2}{a^4\sqrt{c-a^2cx^2}}-\frac{3\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^3)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (1 + a*x)^2/(a^4*Sqrt[c - a^2*c*x^2]) + (11*Sqrt[c - a^2*c*x^2])/(3*a^4*c)
+ (x*Sqrt[c - a^2*c*x^2])/(a^3*c) + (x^2*Sqrt[c - a^2*c*x^2])/(3*a^2*c) - (
3*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(a^4*Sqrt[c])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x^3}{\sqrt{c-a^2cx^2}}\,dx &= c\int \frac{x^3(1+ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{(1+ax)^2}{a^4\sqrt{c-a^2cx^2}} - \int \frac{(1+ax)\left(\frac{2}{a^3}+\frac{x}{a^2}+\frac{x^2}{a}\right)}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{(1+ax)^2}{a^4\sqrt{c-a^2cx^2}} + \frac{x^2\sqrt{c-a^2cx^2}}{3a^2c} + \frac{\int \frac{-\frac{6c}{a}-11cx-6acx^2}{\sqrt{c-a^2cx^2}}\,dx}{3a^2c}\\
&= \frac{(1+ax)^2}{a^4\sqrt{c-a^2cx^2}} + \frac{x\sqrt{c-a^2cx^2}}{a^3c} + \frac{x^2\sqrt{c-a^2cx^2}}{3a^2c} - \frac{\int \frac{18ac^2+22a^2c^2x}{\sqrt{c-a^2cx^2}}\,dx}{6a^4c^2}\\
&= \frac{(1+ax)^2}{a^4\sqrt{c-a^2cx^2}} + \frac{11\sqrt{c-a^2cx^2}}{3a^4c} + \frac{x\sqrt{c-a^2cx^2}}{a^3c} + \frac{x^2\sqrt{c-a^2cx^2}}{3a^2c} - \frac{3\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a^3}\\
&= \frac{(1+ax)^2}{a^4\sqrt{c-a^2cx^2}} + \frac{11\sqrt{c-a^2cx^2}}{3a^4c} + \frac{x\sqrt{c-a^2cx^2}}{a^3c} + \frac{x^2\sqrt{c-a^2cx^2}}{3a^2c} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)}{a^3}\\
&= \frac{(1+ax)^2}{a^4\sqrt{c-a^2cx^2}} + \frac{11\sqrt{c-a^2cx^2}}{3a^4c} + \frac{x\sqrt{c-a^2cx^2}}{a^3c} + \frac{x^2\sqrt{c-a^2cx^2}}{3a^2c} - \frac{3\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^4\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.171213, size = 97, normalized size = 0.71

$$
\frac{\frac{\left(a^3x^3+2a^2x^2+5ax-14\right)\sqrt{c-a^2cx^2}}{ax-1}+9\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{3a^4c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^3)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((Sqrt[c - a^2*c*x^2]*(-14 + 5*a*x + 2*a^2*x^2 + a^3*x^3))/(-1 + a*x) + 9*S
qrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^4*c
)
```

Maple [A] time = 0.041, size = 149, normalized size = 1.1

$$
\frac{x^2}{3\,a^2c}\sqrt{-a^2cx^2+c}+\frac{8}{3\,a^4c}\sqrt{-a^2cx^2+c}+\frac{x}{a^3c}\sqrt{-a^2cx^2+c}-3\,\frac{1}{a^3\sqrt{a^2c}}\arctan\left(\frac{\sqrt{a^2c}x}{\sqrt{-a^2cx^2+c}}\right)-2\,\frac{1}{a^5c}\sqrt{-ca^2\left(x-a^{-1}\right)^2 -
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/3*x^2*(-a^2*c*x^2+c)^(1/2)/a^2/c+8/3*(-a^2*c*x^2+c)^(1/2)/a^4/c+x*(-a^2*c
*x^2+c)^(1/2)/a^3/c-3/a^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+
c)^(1/2))-2/a^5/c/(x-1/a)*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.95172, size = 446, normalized size = 3.26

$$\left[-\frac{9\,(ax-1)\sqrt{-c}\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\left(a^3x^3+2\,a^2x^2+5\,ax-14\right)\sqrt{-a^2cx^2+c}}{6\left(a^5cx-a^4c\right)}, \frac{9\,(ax-1)\sqrt{c}\,\mathrm{a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [-1/6*(9*(a*x - 1)*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt
(-c)*x - c) - 2*(a^3*x^3 + 2*a^2*x^2 + 5*a*x - 14)*sqrt(-a^2*c*x^2 + c))/(a
^5*c*x - a^4*c), 1/3*(9*(a*x - 1)*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqr
t(c)*x/(a^2*c*x^2 - c)) + (a^3*x^3 + 2*a^2*x^2 + 5*a*x - 14)*sqrt(-a^2*c*x^
2 + c))/(a^5*c*x - a^4*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^3}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx-\int\frac{ax^4}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(x**3/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x) -
Integral(a*x**4/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] undef
```

3.1109 $\int \frac{e^{2\tanh^{-1}(ax)}x^2}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=93

$$\frac{(ax+1)^2}{a^3\sqrt{c-a^2cx^2}}+\frac{(ax+6)\sqrt{c-a^2cx^2}}{2a^3c}-\frac{5\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a^3\sqrt{c}}$$

```
[Out] (1 + a*x)^2/(a^3*Sqrt[c - a^2*c*x^2]) + ((6 + a*x)*Sqrt[c - a^2*c*x^2])/(2*
a^3*c) - (5*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a^3*Sqrt[c])
```

Rubi [A] time = 0.248951, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1635, 780, 217, 203}

$$\frac{(ax+1)^2}{a^3\sqrt{c-a^2cx^2}}+\frac{(ax+6)\sqrt{c-a^2cx^2}}{2a^3c}-\frac{5\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a^3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (1 + a*x)^2/(a^3*Sqrt[c - a^2*c*x^2]) + ((6 + a*x)*Sqrt[c - a^2*c*x^2])/(2*
a^3*c) - (5*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a^3*Sqrt[c])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x^2}{\sqrt{c-a^2cx^2}}\,dx &= c\int \frac{x^2(1+ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx \\
&= \frac{(1+ax)^2}{a^3\sqrt{c-a^2cx^2}} - \int \frac{\left(\frac{2}{a^2}+\frac{x}{a}\right)(1+ax)}{\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{(1+ax)^2}{a^3\sqrt{c-a^2cx^2}} + \frac{(6+ax)\sqrt{c-a^2cx^2}}{2a^3c} - \frac{5\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{2a^2} \\
&= \frac{(1+ax)^2}{a^3\sqrt{c-a^2cx^2}} + \frac{(6+ax)\sqrt{c-a^2cx^2}}{2a^3c} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)}{2a^2} \\
&= \frac{(1+ax)^2}{a^3\sqrt{c-a^2cx^2}} + \frac{(6+ax)\sqrt{c-a^2cx^2}}{2a^3c} - \frac{5\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a^3\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.102573, size = 94, normalized size = 1.01

$$\frac{\left(a^2x^2+3ax-8\right)\sqrt{c-a^2cx^2}+5\sqrt{c}(ax-1)\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{2a^3c(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((-8 + 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2] + 5*Sqrt[c]*(-1 + a*x)*ArcTan[(
a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(2*a^3*c*(-1 + a*x))
```

Maple [A] time = 0.039, size = 126, normalized size = 1.4

$$\frac{x}{2\,a^2c}\sqrt{-a^2cx^2+c} - \frac{5}{2\,a^2}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}} + 2\,\frac{\sqrt{-a^2cx^2+c}}{a^3c} - 2\,\frac{1}{a^4c}\sqrt{-ca^2\left(x-a^{-1}\right)^2 - 2\,ac\left(x-a^{-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*x/a^2/c*(-a^2*c*x^2+c)^(1/2)-5/2/a^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)
*x/(-a^2*c*x^2+c)^(1/2))+2/a^3/c*(-a^2*c*x^2+c)^(1/2)-2/a^4/c/(x-1/a)*(-c*a
^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.82969, size = 410, normalized size = 4.41

$$\left[-\frac{5\,(ax-1)\sqrt{-c}\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\,\sqrt{-a^2cx^2+c}\left(a^2x^2+3\,ax-8\right)}{4\left(a^4cx-a^3c\right)},\frac{5\,(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-}\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [-1/4*(5*(a*x - 1)*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt
(-c)*x - c) - 2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 3*a*x - 8))/(a^4*c*x - a^3*
c), 1/2*(5*(a*x - 1)*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c
*x^2 - c)) + sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 3*a*x - 8))/(a^4*c*x - a^3*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx-\int\frac{ax^3}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(x**2/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x) -
Integral(a*x**3/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] undef
```

3.1110 $\int \frac{e^{2\tanh^{-1}(ax)}x}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=84

$$\frac{(ax+1)^2}{a^2\sqrt{c-a^2cx^2}}+\frac{2\sqrt{c-a^2cx^2}}{a^2c}-\frac{2\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2\sqrt{c}}$$

```
[Out] (1 + a*x)^2/(a^2*Sqrt[c - a^2*c*x^2]) + (2*Sqrt[c - a^2*c*x^2])/(a^2*c) - (
2*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(a^2*Sqrt[c])
```

Rubi [A] time = 0.119463, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6151, 789, 641, 217, 203}

$$\frac{(ax+1)^2}{a^2\sqrt{c-a^2cx^2}}+\frac{2\sqrt{c-a^2cx^2}}{a^2c}-\frac{2\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (1 + a*x)^2/(a^2*Sqrt[c - a^2*c*x^2]) + (2*Sqrt[c - a^2*c*x^2])/(a^2*c) - (
2*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(a^2*Sqrt[c])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x}{\sqrt{c-a^2cx^2}}\,dx &= c\int \frac{x(1+ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{(1+ax)^2}{a^2\sqrt{c-a^2cx^2}} - \frac{2\int \frac{1+ax}{\sqrt{c-a^2cx^2}}\,dx}{a}\\
&= \frac{(1+ax)^2}{a^2\sqrt{c-a^2cx^2}} + \frac{2\sqrt{c-a^2cx^2}}{a^2c} - \frac{2\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a}\\
&= \frac{(1+ax)^2}{a^2\sqrt{c-a^2cx^2}} + \frac{2\sqrt{c-a^2cx^2}}{a^2c} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)}{a}\\
&= \frac{(1+ax)^2}{a^2\sqrt{c-a^2cx^2}} + \frac{2\sqrt{c-a^2cx^2}}{a^2c} - \frac{2\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.108018, size = 78, normalized size = 0.93

$$\frac{\frac{(ax-3)\sqrt{c-a^2cx^2}}{ax-1} + 2\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (((-3 + a*x)*Sqrt[c - a^2*c*x^2])/(-1 + a*x) + 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c
 - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(a^2*c)
```

Maple [A] time = 0.036, size = 103, normalized size = 1.2

$$\frac{1}{a^2c}\sqrt{-a^2cx^2+c} - 2\,\frac{1}{a\sqrt{a^2c}}\arctan\left(\frac{\sqrt{a^2c}x}{\sqrt{-a^2cx^2+c}}\right) - 2\,\frac{1}{a^3c}\sqrt{-ca^2\left(x-a^{-1}\right)^2 - 2\,ac\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-a^2*c*x^2+c)^(1/2)/a^2/c-2/a/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c
*x^2+c)^(1/2))-2/a^3/c/(x-1/a)*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.8286, size = 362, normalized size = 4.31

$$\left[-\frac{(ax-1)\sqrt{-c}\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-\sqrt{-a^2cx^2+c}(ax-3)}{a^3cx-a^2c}, \frac{2\,(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+\sqrt{}}{a^3cx-a^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] [-((a*x - 1)*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
 - c) - sqrt(-a^2*c*x^2 + c)*(a*x - 3))/(a^3*c*x - a^2*c), (2*(a*x - 1)*sqr
t(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + sqrt(-a^2*c
*x^2 + c)*(a*x - 3))/(a^3*c*x - a^2*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx-\int \frac{ax^2}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(x/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x) - Int
egral(a*x**2/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] undef
```

3.1111 $\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=60

$$\frac{2(ax+1)}{a\sqrt{c-a^2cx^2}}-\frac{\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] (2*(1 + a*x))/(a*Sqrt[c - a^2*c*x^2]) - ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c
*x^2]]/(a*Sqrt[c])
```

Rubi [A] time = 0.0660823, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6141, 653, 217, 203}

$$\frac{2(ax+1)}{a\sqrt{c-a^2cx^2}}-\frac{\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*(1 + a*x))/(a*Sqrt[c - a^2*c*x^2]) - ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c
*x^2]]/(a*Sqrt[c])
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\ &= \frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}-\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\ &= \frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}-\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\ &= \frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}-\frac{\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0332282, size = 82, normalized size = 1.37

$$\frac{2\sqrt{1-a^2x^2}\left(\sqrt{ax+1}+\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-ax}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2]*(Sqrt[1 + a*x] + Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sq
rt[2]]))/(a*Sqrt[1 - a*x]*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.034, size = 80, normalized size = 1.3

$$-\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}-2\frac{1}{a^2c}\sqrt{-ca^2\left(x-a^{-1}\right)^2-2ac\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/a^2/c/(x-1/
a)*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.79638, size = 338, normalized size = 5.63

$$\left[-\frac{(ax-1)\sqrt{-c}\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+ca}\sqrt{-c}x-c\right)+4\,\sqrt{-a^2cx^2+c}}{2\,\left(a^2cx-ac\right)}, \frac{(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+ca}\sqrt{c}x}{a^2cx^2-c}\right)-2\,\sqrt{-a}}{a^2cx-ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/2*((a*x - 1)*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) + 4*sqrt(-a^2*c*x^2 + c))/(a^2*c*x - a*c), ((a*x - 1)*sqrt(c)*arc
tan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*c*x^2 +
 c))/(a^2*c*x - a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{ax}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx-\int\frac{1}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x) - I
ntegral(1/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] undef
```

3.1112 $\int \frac{e^{2\tanh^{-1}(ax)}}{x\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=52

$$\frac{2(ax+1)}{\sqrt{c-a^2cx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] (2*(1 + a*x))/Sqrt[c - a^2*c*x^2] - ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]/Sq
rt[c]
```

Rubi [A] time = 0.22813, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1805, 266, 63, 208}

$$\frac{2(ax+1)}{\sqrt{c-a^2cx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x*Sqrt[c - a^2*c*x^2]),x]
```


```
[Out] (2*(1 + a*x))/Sqrt[c - a^2*c*x^2] - ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]/Sq
rt[c]
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{x\sqrt{c-a^2cx^2}}\,dx &= c\int \frac{(1+ax)^2}{x\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{2(1+ax)}{\sqrt{c-a^2cx^2}}+\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{2(1+ax)}{\sqrt{c-a^2cx^2}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)\\
&= \frac{2(1+ax)}{\sqrt{c-a^2cx^2}}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)}{a^2c}\\
&= \frac{2(1+ax)}{\sqrt{c-a^2cx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.11062, size = 66, normalized size = 1.27

$$\frac{2\sqrt{c-a^2cx^2}}{c-acx}-\frac{\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)}{\sqrt{c}}+\frac{\log(x)}{\sqrt{c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*Sqrt[c - a^2*c*x^2])/(c - a*c*x) + Log[x]/Sqrt[c] - Log[c + Sqrt[c]*Sqrt
[c - a^2*c*x^2]]/Sqrt[c]
```

Maple [A] time = 0.037, size = 80, normalized size = 1.5

$$-\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right)\frac{1}{\sqrt{c}}-2\,\frac{1}{ac}\sqrt{-ca^2\left(x-a^{-1}\right)^2-2\,ac\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-2/a/c/(x-1/a)*(-c*a^2
*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2}{\sqrt{-a^2cx^2+c}(a^2x^2-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate((a*x + 1)^2/(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)*x), x)
```

Fricas [A] time = 2.77843, size = 325, normalized size = 6.25

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)-4\sqrt{-a^2cx^2+c}}{2(acx-c)}, -\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)+2\sqrt{-a^2cx^2+c}}{acx-c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*((a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) - 4*sqrt(-a^2*c*x^2 + c))/(a*c*x - c), -((a*x - 1)*sqrt(-c)*arcta
n(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 2*sqrt(-a^2*c*x^2 + c))/
(a*c*x - c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{ax^2\sqrt{-a^2cx^2+c}-x\sqrt{-a^2cx^2+c}}\,dx-\int \frac{1}{ax^2\sqrt{-a^2cx^2+c}-x\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x**2*sqrt(-a**2*c*x**2 + c) - x*sqrt(-a**2*c*x**2 + c)), x
) - Integral(1/(a*x**2*sqrt(-a**2*c*x**2 + c) - x*sqrt(-a**2*c*x**2 + c)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] sage2
```

3.1113 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^2\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=77

$$\frac{2a(ax+1)}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{cx}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] (2*a*(1 + a*x))/Sqrt[c - a^2*c*x^2] - Sqrt[c - a^2*c*x^2]/(c*x) - (2*a*ArcT
anh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c]
```

Rubi [A] time = 0.249583, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6151, 1805, 807, 266, 63, 208}

$$\frac{2a(ax+1)}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{cx}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^2*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*a*(1 + a*x))/Sqrt[c - a^2*c*x^2] - Sqrt[c - a^2*c*x^2]/(c*x) - (2*a*ArcT
anh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x^2\sqrt{c-a^2cx^2}}\,dx &= c\int \frac{(1+ax)^2}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{2a(1+ax)}{\sqrt{c-a^2cx^2}} - \int \frac{-1-2ax}{x^2\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{2a(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{cx} + (2a)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{2a(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{cx} + a\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right)\\
&= \frac{2a(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{cx} - \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)}{ac}\\
&= \frac{2a(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{cx} - \frac{2a\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.133278, size = 78, normalized size = 1.01

$$\frac{(1-3ax)\sqrt{c-a^2cx^2}}{cx(ax-1)} - \frac{2a\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)}{\sqrt{c}} + \frac{2a\log(x)}{\sqrt{c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^2*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] ((1 - 3*a*x)*Sqrt[c - a^2*c*x^2])/(c*x*(-1 + a*x)) + (2*a*Log[x])/Sqrt[c] -
 (2*a*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/Sqrt[c]
```

Maple [A] time = 0.04, size = 99, normalized size = 1.3

$$-\frac{1}{cx}\sqrt{-a^2cx^2+c} - 2\frac{a}{\sqrt{c}}\ln\left(\frac{2c+2\sqrt{c}\sqrt{-a^2cx^2+c}}{x}\right) - 2\frac{1}{c}\sqrt{-ca^2\left(x-a^{-1}\right)^2-2ac\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -(-a^2*c*x^2+c)^(1/2)/c/x-2*a/c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2
))/x)-2/c/(x-1/a)*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2}{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)^2/(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)*x^2), x)
```

Fricas [A] time = 2.75176, size = 377, normalized size = 4.9

$$\left[\frac{\left(a^2x^2-ax\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)-\sqrt{-a^2cx^2+c}(3ax-1)}{acx^2-cx}, -\frac{2\left(a^2x^2-ax\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)+\sqrt{\cdot}}{acx^2-cx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [((a^2*x^2 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) - sqrt(-a^2*c*x^2 + c)*(3*a*x - 1))/(a*c*x^2 - c*x), -(2*(a^2*x
^2 - a*x)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) +
sqrt(-a^2*c*x^2 + c)*(3*a*x - 1))/(a*c*x^2 - c*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{ax^3\sqrt{-a^2cx^2+c}-x^2\sqrt{-a^2cx^2+c}}\,dx-\int \frac{1}{ax^3\sqrt{-a^2cx^2+c}-x^2\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**2/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x**3*sqrt(-a**2*c*x**2 + c) - x**2*sqrt(-a**2*c*x**2 + c))
, x) - Integral(1/(a*x**3*sqrt(-a**2*c*x**2 + c) - x**2*sqrt(-a**2*c*x**2 +
 c)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] undef
```

3.1114 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^3\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=109

$$\frac{2a^2(ax+1)}{\sqrt{c-a^2cx^2}}-\frac{2a\sqrt{c-a^2cx^2}}{cx}-\frac{\sqrt{c-a^2cx^2}}{2cx^2}-\frac{5a^2\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

```
[Out] (2*a^2*(1 + a*x))/Sqrt[c - a^2*c*x^2] - Sqrt[c - a^2*c*x^2]/(2*c*x^2) - (2*
a*Sqrt[c - a^2*c*x^2])/(c*x) - (5*a^2*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])
/(2*Sqrt[c])
```

Rubi [A] time = 0.323369, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1805, 1807, 807, 266, 63, 208}

$$\frac{2a^2(ax+1)}{\sqrt{c-a^2cx^2}}-\frac{2a\sqrt{c-a^2cx^2}}{cx}-\frac{\sqrt{c-a^2cx^2}}{2cx^2}-\frac{5a^2\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^3*Sqrt[c - a^2*c*x^2]),x]
```


```
[Out] (2*a^2*(1 + a*x))/Sqrt[c - a^2*c*x^2] - Sqrt[c - a^2*c*x^2]/(2*c*x^2) - (2*
a*Sqrt[c - a^2*c*x^2])/(c*x) - (5*a^2*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])
/(2*Sqrt[c])
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x^3\sqrt{c-a^2cx^2}}\,dx &= c\int \frac{(1+ax)^2}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{2a^2(1+ax)}{\sqrt{c-a^2cx^2}}-\int \frac{-1-2ax-2a^2x^2}{x^3\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{2a^2(1+ax)}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{2cx^2}+\frac{\int \frac{4ac+5a^2cx}{x^2\sqrt{c-a^2cx^2}}\,dx}{2c}\\
&= \frac{2a^2(1+ax)}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{2cx^2}-\frac{2a\sqrt{c-a^2cx^2}}{cx}+\frac{1}{2}\left(5a^2\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{2a^2(1+ax)}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{2cx^2}-\frac{2a\sqrt{c-a^2cx^2}}{cx}+\frac{1}{4}\left(5a^2\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)\\
&= \frac{2a^2(1+ax)}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{2cx^2}-\frac{2a\sqrt{c-a^2cx^2}}{cx}-\frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)}{2c}\\
&= \frac{2a^2(1+ax)}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{2cx^2}-\frac{2a\sqrt{c-a^2cx^2}}{cx}-\frac{5a^2\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.161434, size = 94, normalized size = 0.86

$$\frac{\frac{\left(-8a^2x^2+3ax+1\right)\sqrt{c-a^2cx^2}}{x^2(ax-1)}-5a^2\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+5a^2\sqrt{c}\log(x)}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^3*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (((1 + 3*a*x - 8*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(x^2*(-1 + a*x)) + 5*a^2*Sqr
t[c]*Log[x] - 5*a^2*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/(2*c)
```

Maple [A] time = 0.042, size = 124, normalized size = 1.1

$$-2\,\frac{\sqrt{-a^2cx^2+ca}}{cx}-\frac{5\,a^2}{2}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right)\frac{1}{\sqrt{c}}-2\,\frac{a}{c}\sqrt{-ca^2\left(x-a^{-1}\right)^2-2\,ac\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}-\frac{1}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -2*a*(-a^2*c*x^2+c)^(1/2)/c/x-5/2*a^2/c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2
+c)^(1/2))/x)-2*a/c/(x-1/a)*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)-1/2*(-a^
2*c*x^2+c)^(1/2)/c/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(ax+1)^2}{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)^2/(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)*x^3), x)
```

Fricas [A] time = 2.69927, size = 441, normalized size = 4.05

$$\left[\frac{5\left(a^3x^3-a^2x^2\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-2\sqrt{-a^2cx^2+c}\left(8\,a^2x^2-3\,ax-1\right)}{4\left(acx^3-cx^2\right)},-\frac{5\left(a^3x^3-a^2x^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*(5*(a^3*x^3 - a^2*x^2)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c
)*sqrt(c) - 2*c)/x^2) - 2*sqrt(-a^2*c*x^2 + c)*(8*a^2*x^2 - 3*a*x - 1))/(a*
c*x^3 - c*x^2), -1/2*(5*(a^3*x^3 - a^2*x^2)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2
 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*c*x^2 + c)*(8*a^2*x^2 - 3*a*x -
 1))/(a*c*x^3 - c*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{ax^4\sqrt{-a^2cx^2+c}-x^3\sqrt{-a^2cx^2+c}}\,dx-\int \frac{1}{ax^4\sqrt{-a^2cx^2+c}-x^3\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**3/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x**4*sqrt(-a**2*c*x**2 + c) - x**3*sqrt(-a**2*c*x**2 + c))
, x) - Integral(1/(a*x**4*sqrt(-a**2*c*x**2 + c) - x**3*sqrt(-a**2*c*x**2 +
 c)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] undef
```

3.1115 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^4\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=135

$$\frac{2a^3(ax+1)}{\sqrt{c-a^2cx^2}}-\frac{8a^2\sqrt{c-a^2cx^2}}{3cx}-\frac{a\sqrt{c-a^2cx^2}}{cx^2}-\frac{\sqrt{c-a^2cx^2}}{3cx^3}-\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] (2*a^3*(1 + a*x))/Sqrt[c - a^2*c*x^2] - Sqrt[c - a^2*c*x^2]/(3*c*x^3) - (a*
Sqrt[c - a^2*c*x^2])/(c*x^2) - (8*a^2*Sqrt[c - a^2*c*x^2])/(3*c*x) - (3*a^3
*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c]
```

Rubi [A] time = 0.406531, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1805, 1807, 807, 266, 63, 208}

$$\frac{2a^3(ax+1)}{\sqrt{c-a^2cx^2}}-\frac{8a^2\sqrt{c-a^2cx^2}}{3cx}-\frac{a\sqrt{c-a^2cx^2}}{cx^2}-\frac{\sqrt{c-a^2cx^2}}{3cx^3}-\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^4*Sqrt[c - a^2*c*x^2]),x]
```


```
[Out] (2*a^3*(1 + a*x))/Sqrt[c - a^2*c*x^2] - Sqrt[c - a^2*c*x^2]/(3*c*x^3) - (a*
Sqrt[c - a^2*c*x^2])/(c*x^2) - (8*a^2*Sqrt[c - a^2*c*x^2])/(3*c*x) - (3*a^3
*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c]
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x^4\sqrt{c-a^2cx^2}}\,dx &= c\int \frac{(1+ax)^2}{x^4\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{2a^3(1+ax)}{\sqrt{c-a^2cx^2}} - \int \frac{-1-2ax-2a^2x^2-2a^3x^3}{x^4\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{2a^3(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{3cx^3} + \frac{\int \frac{6ac+8a^2cx+6a^3cx^2}{x^3\sqrt{c-a^2cx^2}}\,dx}{3c}\\
&= \frac{2a^3(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{3cx^3} - \frac{a\sqrt{c-a^2cx^2}}{cx^2} - \frac{\int \frac{-16a^2c^2-18a^3c^2x}{x^2\sqrt{c-a^2cx^2}}\,dx}{6c^2}\\
&= \frac{2a^3(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{3cx^3} - \frac{a\sqrt{c-a^2cx^2}}{cx^2} - \frac{8a^2\sqrt{c-a^2cx^2}}{3cx} + \left(3a^3\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{2a^3(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{3cx^3} - \frac{a\sqrt{c-a^2cx^2}}{cx^2} - \frac{8a^2\sqrt{c-a^2cx^2}}{3cx} + \frac{1}{2}\left(3a^3\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, \right.\\
&= \frac{2a^3(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{3cx^3} - \frac{a\sqrt{c-a^2cx^2}}{cx^2} - \frac{8a^2\sqrt{c-a^2cx^2}}{3cx} - \frac{(3a)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2c\right.}{c}\\
&= \frac{2a^3(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{3cx^3} - \frac{a\sqrt{c-a^2cx^2}}{cx^2} - \frac{8a^2\sqrt{c-a^2cx^2}}{3cx} - \frac{3a^3\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.165517, size = 101, normalized size = 0.75

$$
\frac{\left(-14a^3x^3+5a^2x^2+2ax+1\right)\sqrt{c-a^2cx^2}}{3cx^3(ax-1)} - \frac{3a^3\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)}{\sqrt{c}} + \frac{3a^3\log(x)}{\sqrt{c}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^4*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(1 + 2*a*x + 5*a^2*x^2 - 14*a^3*x^3))/(3*c*x^3*(-1 + a
*x)) + (3*a^3*Log[x])/Sqrt[c] - (3*a^3*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
)/Sqrt[c]
```

Maple [A] time = 0.045, size = 150, normalized size = 1.1

$$-\frac{8a^2}{3cx}\sqrt{-a^2cx^2+c}-3\frac{a^3}{\sqrt{c}}\ln\left(\frac{2c+2\sqrt{c}\sqrt{-a^2cx^2+c}}{x}\right)-2\frac{a^2}{c}\sqrt{-ca^2\left(x-a^{-1}\right)^2-2ac\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}-\frac{a}{cx^2}\sqrt{-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -8/3*a^2*(-a^2*c*x^2+c)^(1/2)/c/x-3*a^3/c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x
^2+c)^(1/2))/x)-2*a^2/c/(x-1/a)*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)-a*(-
a^2*c*x^2+c)^(1/2)/c/x^2-1/3*(-a^2*c*x^2+c)^(1/2)/c/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2}{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)^2/(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)*x^4), x)
```

Fricas [A] time = 2.86264, size = 477, normalized size = 3.53

$$\left[\frac{9\left(a^4x^4-a^3x^3\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)-2\left(14a^3x^3-5a^2x^2-2ax-1\right)\sqrt{-a^2cx^2+c}}{6\left(acx^4-cx^3\right)},-\frac{9\left(a^4x^4-a^3x^3\right)\sqrt{-c}\,a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/6*(9*(a^4*x^4 - a^3*x^3)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c
)*sqrt(c) - 2*c)/x^2) - 2*(14*a^3*x^3 - 5*a^2*x^2 - 2*a*x - 1)*sqrt(-a^2*c*
x^2 + c))/(a*c*x^4 - c*x^3), -1/3*(9*(a^4*x^4 - a^3*x^3)*sqrt(-c)*arctan(sq
rt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (14*a^3*x^3 - 5*a^2*x^2 - 2*
a*x - 1)*sqrt(-a^2*c*x^2 + c))/(a*c*x^4 - c*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{ax^5\sqrt{-a^2cx^2+c}-x^4\sqrt{-a^2cx^2+c}}\,dx-\int \frac{1}{ax^5\sqrt{-a^2cx^2+c}-x^4\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**4/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x**5*sqrt(-a**2*c*x**2 + c) - x**4*sqrt(-a**2*c*x**2 + c))
, x) - Integral(1/(a*x**5*sqrt(-a**2*c*x**2 + c) - x**4*sqrt(-a**2*c*x**2 +
 c)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] undef
```

3.1116 $\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{\sqrt{c-a^2cx^2}}{a^4c^2}+\frac{2\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^4c^{3/2}}+\frac{(ax+1)^2}{3a^4\left(c-a^2cx^2\right)^{3/2}}-\frac{8(ax+1)}{3a^4c\sqrt{c-a^2cx^2}}$$

```
[Out] (1 + a*x)^2/(3*a^4*(c - a^2*c*x^2)^(3/2)) - (8*(1 + a*x))/(3*a^4*c*Sqrt[c -
 a^2*c*x^2]) - Sqrt[c - a^2*c*x^2]/(a^4*c^2) + (2*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(a^4*c^(3/2))
```

Rubi [A] time = 0.320685, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1635, 641, 217, 203}

$$-\frac{\sqrt{c-a^2cx^2}}{a^4c^2}+\frac{2\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^4c^{3/2}}+\frac{(ax+1)^2}{3a^4\left(c-a^2cx^2\right)^{3/2}}-\frac{8(ax+1)}{3a^4c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (1 + a*x)^2/(3*a^4*(c - a^2*c*x^2)^(3/2)) - (8*(1 + a*x))/(3*a^4*c*Sqrt[c -
 a^2*c*x^2]) - Sqrt[c - a^2*c*x^2]/(a^4*c^2) + (2*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(a^4*c^(3/2))
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= c\int \frac{x^3(1+ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx\\
&= \frac{(1+ax)^2}{3a^4\left(c-a^2cx^2\right)^{3/2}}-\frac{1}{3}\int \frac{(1+ax)\left(\frac{2}{a^3}+\frac{3x}{a^2}+\frac{3x^2}{a}\right)}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{(1+ax)^2}{3a^4\left(c-a^2cx^2\right)^{3/2}}-\frac{8(1+ax)}{3a^4c\sqrt{c-a^2cx^2}}+\frac{\int \frac{\frac{6}{a^3}+\frac{3x}{a^2}}{\sqrt{c-a^2cx^2}}\,dx}{3c}\\
&= \frac{(1+ax)^2}{3a^4\left(c-a^2cx^2\right)^{3/2}}-\frac{8(1+ax)}{3a^4c\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{a^4c^2}+\frac{2\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a^3c}\\
&= \frac{(1+ax)^2}{3a^4\left(c-a^2cx^2\right)^{3/2}}-\frac{8(1+ax)}{3a^4c\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{a^4c^2}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)}{a^3c}\\
&= \frac{(1+ax)^2}{3a^4\left(c-a^2cx^2\right)^{3/2}}-\frac{8(1+ax)}{3a^4c\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{a^4c^2}+\frac{2\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^4c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.166011, size = 90, normalized size = 0.77

$$
\frac{\frac{\left(-3a^2x^2+14ax-10\right)\sqrt{c-a^2cx^2}}{(ax-1)^2}-6\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{3a^4c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (((-10 + 14*a*x - 3*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(-1 + a*x)^2 - 6*Sqrt[c]*
ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^4*c^2)
```

Maple [A] time = 0.04, size = 190, normalized size = 1.6

$$
\frac{x^2}{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}-4\,\frac{1}{a^4\sqrt{-a^2cx^2+c}c}-4\,\frac{x}{a^3\sqrt{-a^2cx^2+c}c}+2\,\frac{1}{a^3c\sqrt{a^2c}}\arctan\left(\frac{\sqrt{a^2c}x}{\sqrt{-a^2cx^2+c}}\right)-\frac{2}{3\,a^5c}\left(x-a^{-1}\right)^{-1}\frac{}{\sqrt{-ca}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] x^2/a^2/c/(-a^2*c*x^2+c)^(1/2)-4/c/a^4/(-a^2*c*x^2+c)^(1/2)-4/a^3/c*x/(-a^2
*c*x^2+c)^(1/2)+2/a^3/c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)
```

```
^(1/2))-2/3/a^5/c/(x-1/a)/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)+4/3/a^3/c/
(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.79853, size = 506, normalized size = 4.32

$$\left[-\frac{3\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+\sqrt{-a^2cx^2+c}\left(3\,a^2x^2-14\,ax+10\right)}{3\left(a^6c^2x^2-2\,a^5c^2x+a^4c^2\right)},-\frac{6\left(a^2x^2-2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] [-1/3*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2
 + c)*a*sqrt(-c)*x - c) + sqrt(-a^2*c*x^2 + c)*(3*a^2*x^2 - 14*a*x + 10))/(
a^6*c^2*x^2 - 2*a^5*c^2*x + a^4*c^2), -1/3*(6*(a^2*x^2 - 2*a*x + 1)*sqrt(c)
*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + sqrt(-a^2*c*x^2
 + c)*(3*a^2*x^2 - 14*a*x + 10))/(a^6*c^2*x^2 - 2*a^5*c^2*x + a^4*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^3}{-a^3cx^3\sqrt{-a^2cx^2+c}+a^2cx^2\sqrt{-a^2cx^2+c}+acx\sqrt{-a^2cx^2+c}-c\sqrt{-a^2cx^2+c}}\,dx-\int\frac{}{-a^3cx^3\sqrt{-a^2cx^2+c}+a^2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(x**3/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt(-a**
2*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c)), x
) - Integral(a*x**4/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt
(-a**2*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c
)), x)
```

Giac [A] time = 1.21464, size = 89, normalized size = 0.76

$$\frac{2\,\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{a^3c^2|a|}-\frac{\sqrt{-a^2cx^2+c}}{a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] 2*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^3*c^2*abs(a)
) - sqrt(-a^2*c*x^2 + c)/(a^4*c^2)
```

3.1117 $\int \frac{e^{2\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^3c^{3/2}}+\frac{(ax+1)^2}{3a^3(c-a^2cx^2)^{3/2}}-\frac{5(ax+1)}{3a^3c\sqrt{c-a^2cx^2}}$$

```
[Out] (1 + a*x)^2/(3*a^3*(c - a^2*c*x^2)^(3/2)) - (5*(1 + a*x))/(3*a^3*c*Sqrt[c -
 a^2*c*x^2]) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]/(a^3*c^(3/2))
```

Rubi [A] time = 0.262683, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1635, 778, 217, 203}

$$\frac{\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^3c^{3/2}}+\frac{(ax+1)^2}{3a^3(c-a^2cx^2)^{3/2}}-\frac{5(ax+1)}{3a^3c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (1 + a*x)^2/(3*a^3*(c - a^2*c*x^2)^(3/2)) - (5*(1 + a*x))/(3*a^3*c*Sqrt[c -
 a^2*c*x^2]) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]/(a^3*c^(3/2))
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= c\int \frac{x^2(1+ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx \\
&= \frac{(1+ax)^2}{3a^3\left(c-a^2cx^2\right)^{3/2}} - \frac{1}{3}\int \frac{\left(\frac{2}{a^2}+\frac{3x}{a}\right)(1+ax)}{\left(c-a^2cx^2\right)^{3/2}}\,dx \\
&= \frac{(1+ax)^2}{3a^3\left(c-a^2cx^2\right)^{3/2}} - \frac{5(1+ax)}{3a^3c\sqrt{c-a^2cx^2}} + \frac{\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a^2c} \\
&= \frac{(1+ax)^2}{3a^3\left(c-a^2cx^2\right)^{3/2}} - \frac{5(1+ax)}{3a^3c\sqrt{c-a^2cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)}{a^2c} \\
&= \frac{(1+ax)^2}{3a^3\left(c-a^2cx^2\right)^{3/2}} - \frac{5(1+ax)}{3a^3c\sqrt{c-a^2cx^2}} + \frac{\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^3c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.116325, size = 82, normalized size = 0.88

$$\frac{\frac{(5ax-4)\sqrt{c-a^2cx^2}}{(ax-1)^2} - 3\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{3a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (((-4 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(-1 + a*x)^2 - 3*Sqrt[c]*ArcTan[(a*x*Sq
rt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^3*c^2)
```

Maple [B] time = 0.039, size = 166, normalized size = 1.8

$$-3\frac{x}{a^2\sqrt{-a^2cx^2+c}c} + \frac{1}{a^2c}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}} - 2\frac{1}{a^3\sqrt{-a^2cx^2+c}c} - \frac{2}{3a^4c}\left(x-a^{-1}\right)^{-1}\frac{1}{\sqrt{-ca^2\left(x-a^{-1}\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -3*x/a^2/c/(-a^2*c*x^2+c)^(1/2)+1/a^2/c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*
x/(-a^2*c*x^2+c)^(1/2))-2/a^3/c/(-a^2*c*x^2+c)^(1/2)-2/3/a^4/c/(x-1/a)/(-c*
a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)+4/3/a^2/c/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a
))^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.75023, size = 471, normalized size = 5.06

$$\left[-\frac{3\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\,\sqrt{-a^2cx^2+c}(5\,ax-4)}{6\left(a^5c^2x^2-2\,a^4c^2x+a^3c^2\right)},-\frac{3\left(a^2x^2-2\,ax+1\right)\sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] [-1/6*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2
 + c)*a*sqrt(-c)*x - c) - 2*sqrt(-a^2*c*x^2 + c)*(5*a*x - 4))/(a^5*c^2*x^2
- 2*a^4*c^2*x + a^3*c^2), -1/3*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*arctan(sqrt
(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - sqrt(-a^2*c*x^2 + c)*(5*a*x
 - 4))/(a^5*c^2*x^2 - 2*a^4*c^2*x + a^3*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2}{-a^3cx^3\sqrt{-a^2cx^2+c}+a^2cx^2\sqrt{-a^2cx^2+c}+acx\sqrt{-a^2cx^2+c}-c\sqrt{-a^2cx^2+c}}\,dx-\int\frac{}{-a^3cx^3\sqrt{-a^2cx^2+c}+a^2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(x**2/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt(-a**
2*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c)), x
) - Integral(a*x**3/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt
(-a**2*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c
)), x)
```

Giac [A] time = 1.21104, size = 58, normalized size = 0.62

$$\frac{\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{a^2c^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^2*c^2*abs(a))
```

3.1118 $\int \frac{e^{2\tanh^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{(ax+1)^2}{3a^2\left(c-a^2cx^2\right)^{3/2}}-\frac{2(ax+1)}{3a^2c\sqrt{c-a^2cx^2}}$$

```
[Out] (1 + a*x)^2/(3*a^2*(c - a^2*c*x^2)^(3/2)) - (2*(1 + a*x))/(3*a^2*c*Sqrt[c -
 a^2*c*x^2])
```

Rubi [A] time = 0.117417, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6151, 789, 637}

$$\frac{(ax+1)^2}{3a^2\left(c-a^2cx^2\right)^{3/2}}-\frac{2(ax+1)}{3a^2c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (1 + a*x)^2/(3*a^2*(c - a^2*c*x^2)^(3/2)) - (2*(1 + a*x))/(3*a^2*c*Sqrt[c -
 a^2*c*x^2])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{3/2}}\,dx = c\int \frac{x(1+ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx$$
$$= \frac{(1+ax)^2}{3a^2\left(c-a^2cx^2\right)^{3/2}} - \frac{2\int \frac{1+ax}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{3a}$$
$$= \frac{(1+ax)^2}{3a^2\left(c-a^2cx^2\right)^{3/2}} - \frac{2(1+ax)}{3a^2c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0560338, size = 38, normalized size = 0.63

$$\frac{(2ax-1)\sqrt{c-a^2cx^2}}{3a^2c^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((-1 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2*c^2*(-1 + a*x)^2)
```

Maple [A] time = 0.032, size = 32, normalized size = 0.5

$$\frac{(2\,ax-1)\,(ax+1)^2}{3\,a^2}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/3*(2*a*x-1)*(a*x+1)^2/(-a^2*c*x^2+c)^(3/2)/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.65501, size = 103, normalized size = 1.72

$$\frac{\sqrt{-a^2cx^2+c}(2\,ax-1)}{3\left(a^4c^2x^2-2\,a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*(2*a*x - 1)/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-a^3cx^3\sqrt{-a^2cx^2+c}+a^2cx^2\sqrt{-a^2cx^2+c}+acx\sqrt{-a^2cx^2+c}-c\sqrt{-a^2cx^2+c}}\,dx-\int \frac{}{-a^3cx^3\sqrt{-a^2cx^2+c}+a^2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(x/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt(-a**2*c
*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c)), x) -
 Integral(a*x**2/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt(-a
**2*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c)),
 x)
```

Giac [B] time = 1.20668, size = 158, normalized size = 2.63

$$-\frac{\left(ac+3\sqrt{-a^2c}\sqrt{c}\right)\operatorname{sgn}(x)}{3\left(a^3c^{\frac{5}{2}}-\sqrt{-a^2c}a^2c^2\right)}-\frac{2\left(a\sqrt{c}+3\sqrt{-a^2c+\frac{c}{x^2}}-\frac{3\sqrt{c}}{x}\right)}{3\left(a\sqrt{c}+\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^3\sqrt{c}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] -1/3*(a*c + 3*sqrt(-a^2*c)*sqrt(c))*sgn(x)/(a^3*c^(5/2) - sqrt(-a^2*c)*a^2*
c^2) - 2/3*(a*sqrt(c) + 3*sqrt(-a^2*c + c/x^2) - 3*sqrt(c)/x)/((a*sqrt(c) +
 sqrt(-a^2*c + c/x^2) - sqrt(c)/x)^3*sqrt(c)*sgn(x))
```

3.1119 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{x}{3c\sqrt{c-a^2cx^2}}+\frac{2(ax+1)}{3a\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] (2*(1 + a*x))/(3*a*(c - a^2*c*x^2)^(3/2)) + x/(3*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.0636296, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6141, 653, 191}

$$\frac{x}{3c\sqrt{c-a^2cx^2}}+\frac{2(ax+1)}{3a\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*(1 + a*x))/(3*a*(c - a^2*c*x^2)^(3/2)) + x/(3*c*Sqrt[c - a^2*c*x^2])
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx\\
&= \frac{2(1+ax)}{3a\left(c-a^2cx^2\right)^{3/2}}+\frac{1}{3}\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{2(1+ax)}{3a\left(c-a^2cx^2\right)^{3/2}}+\frac{x}{3c\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0300728, size = 63, normalized size = 1.24

$$-\frac{(ax-2)\sqrt{ax+1}\sqrt{1-a^2x^2}}{3ac(1-ax)^{3/2}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((-2 + a*x)*Sqrt[1 + a*x]*Sqrt[1 - a^2*x^2])/(3*a*c*(1 - a*x)^(3/2)*Sqrt[c
 - a^2*c*x^2])
```

Maple [A] time = 0.031, size = 31, normalized size = 0.6

$$-\frac{(ax-2)(ax+1)^2}{3a}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/3*(a*x-2)*(a*x+1)^2/(-a^2*c*x^2+c)^(3/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.03509, size = 99, normalized size = 1.94

$$-\frac{\sqrt{-a^2cx^2+c}(ax-2)}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/3*sqrt(-a^2*c*x^2 + c)*(a*x - 2)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{ax}{-a^3cx^3\sqrt{-a^2cx^2+c}+a^2cx^2\sqrt{-a^2cx^2+c}+acx\sqrt{-a^2cx^2+c}-c\sqrt{-a^2cx^2+c}}\,dx-\int\frac{}{-a^3cx^3\sqrt{-a^2cx^2+c}+a^2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(a*x/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt(-a**2
*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c)), x)
 - Integral(1/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt(-a**2
*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c)), x)
```

Giac [B] time = 1.188, size = 200, normalized size = 3.92

$$-\frac{\left(ac-3\sqrt{-a^2c}\sqrt{c}\right)\operatorname{sgn}(x)}{3\left(a^2c^{\frac{5}{2}}-\sqrt{-a^2c}ac^2\right)}+\frac{2\left(2a^2c+3a\sqrt{c}\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)+3\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^2\right)}{3\left(a\sqrt{c}+\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^3c\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(a*c - 3*sqrt(-a^2*c)*sqrt(c))*sgn(x)/(a^2*c^(5/2) - sqrt(-a^2*c)*a*c^
2) + 2/3*(2*a^2*c + 3*a*sqrt(c)*(sqrt(-a^2*c + c/x^2) - sqrt(c)/x) + 3*(sqr
t(-a^2*c + c/x^2) - sqrt(c)/x)^2)/((a*sqrt(c) + sqrt(-a^2*c + c/x^2) - sqrt
(c)/x)^3*c*sgn(x))
```

3.1120 $\int \frac{e^{2\tanh^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{2(ax+1)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{4ax+3}{3c\sqrt{c-a^2cx^2}}$$

```
[Out] (2*(1 + a*x))/(3*(c - a^2*c*x^2)^(3/2)) + (3 + 4*a*x)/(3*c*Sqrt[c - a^2*c*x
^2]) - ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]/c^(3/2)
```

Rubi [A] time = 0.268081, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1805, 823, 12, 266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{2(ax+1)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{4ax+3}{3c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (2*(1 + a*x))/(3*(c - a^2*c*x^2)^(3/2)) + (3 + 4*a*x)/(3*c*Sqrt[c - a^2*c*x
^2]) - ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]/c^(3/2)
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx &= c\int \frac{(1+ax)^2}{x\left(c-a^2cx^2\right)^{5/2}}\,dx\\
&= \frac{2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}-\frac{1}{3}\int \frac{-3-4ax}{x\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{3+4ax}{3c\sqrt{c-a^2cx^2}}-\frac{\int -\frac{3a^2c^2}{x\sqrt{c-a^2cx^2}}\,dx}{3a^2c^3}\\
&= \frac{2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{3+4ax}{3c\sqrt{c-a^2cx^2}}+\frac{\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx}{c}\\
&= \frac{2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{3+4ax}{3c\sqrt{c-a^2cx^2}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)}{2c}\\
&= \frac{2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{3+4ax}{3c\sqrt{c-a^2cx^2}}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)}{a^2c^2}\\
&= \frac{2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{3+4ax}{3c\sqrt{c-a^2cx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.111037, size = 75, normalized size = 0.91

$$
\frac{(5-4ax)\sqrt{c-a^2cx^2}}{3c^2(ax-1)^2}-\frac{\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)}{c^{3/2}}+\frac{\log(x)}{c^{3/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((5 - 4*a*x)*Sqrt[c - a^2*c*x^2])/(3*c^2*(-1 + a*x)^2) + Log[x]/c^(3/2) - L
og[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]/c^(3/2)
```

Maple [B] time = 0.041, size = 152, normalized size = 1.9

$$\frac{1}{c}\frac{1}{\sqrt{-a^2cx^2+c}}-\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right)c^{-\frac{3}{2}}-\frac{2}{3ac}\left(x-a^{-1}\right)^{-1}\frac{1}{\sqrt{-ca^2\left(x-a^{-1}\right)^2-2ac\left(x-a^{-1}\right)}}-\frac{2}{3ac^2}(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/c/(-a^2*c*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/
x)-2/3/a/c/(x-1/a)/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)-2/3/a/c^2*(-2*a^2
*c*(x-1/a)-2*a*c)/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(ax+1)^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -integrate((a*x + 1)^2/((-a^2*c*x^2 + c)^(3/2)*(a^2*x^2 - 1)*x), x)
```

Fricas [A] time = 2.95345, size = 452, normalized size = 5.51

$$\left[\frac{3\left(a^2x^2-2ax+1\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)-2\sqrt{-a^2cx^2+c}(4ax-5)}{6\left(a^2c^2x^2-2ac^2x+c^2\right)},-\frac{3\left(a^2x^2-2ax+1\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2}}{a}\right.}{3\left(a^2c^2x^2-2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/6*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 +
 c)*sqrt(c) - 2*c)/x^2) - 2*sqrt(-a^2*c*x^2 + c)*(4*a*x - 5))/(a^2*c^2*x^2
- 2*a*c^2*x + c^2), -1/3*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(sqrt(-a^2
*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*c*x^2 + c)*(4*a*x - 5))/(
a^2*c^2*x^2 - 2*a*c^2*x + c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{-a^3cx^4\sqrt{-a^2cx^2+c}+a^2cx^3\sqrt{-a^2cx^2+c}+acx^2\sqrt{-a^2cx^2+c}-cx\sqrt{-a^2cx^2+c}}\,dx-\int \frac{}{-a^3cx^4\sqrt{-a^2cx^2+c}+a^2cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(a*x/(-a**3*c*x**4*sqrt(-a**2*c*x**2 + c) + a**2*c*x**3*sqrt(-a**2
*c*x**2 + c) + a*c*x**2*sqrt(-a**2*c*x**2 + c) - c*x*sqrt(-a**2*c*x**2 + c)
), x) - Integral(1/(-a**3*c*x**4*sqrt(-a**2*c*x**2 + c) + a**2*c*x**3*sqrt(
-a**2*c*x**2 + c) + a*c*x**2*sqrt(-a**2*c*x**2 + c) - c*x*sqrt(-a**2*c*x**2
 + c)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.1121 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=108

$$-\frac{\sqrt{c-a^2cx^2}}{c^2x}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{2a(ax+1)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{a(7ax+6)}{3c\sqrt{c-a^2cx^2}}$$

```
[Out] (2*a*(1 + a*x))/(3*(c - a^2*c*x^2)^(3/2)) + (a*(6 + 7*a*x))/(3*c*Sqrt[c - a
^2*c*x^2]) - Sqrt[c - a^2*c*x^2]/(c^2*x) - (2*a*ArcTanh[Sqrt[c - a^2*c*x^2]
/Sqrt[c]])/c^(3/2)
```

Rubi [A] time = 0.337047, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6151, 1805, 807, 266, 63, 208}

$$-\frac{\sqrt{c-a^2cx^2}}{c^2x}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{2a(ax+1)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{a(7ax+6)}{3c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```


```
[Out] (2*a*(1 + a*x))/(3*(c - a^2*c*x^2)^(3/2)) + (a*(6 + 7*a*x))/(3*c*Sqrt[c - a
^2*c*x^2]) - Sqrt[c - a^2*c*x^2]/(c^2*x) - (2*a*ArcTanh[Sqrt[c - a^2*c*x^2]
/Sqrt[c]])/c^(3/2)
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx &= c\int \frac{(1+ax)^2}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx\\
&= \frac{2a(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}-\frac{1}{3}\int \frac{-3-6ax-4a^2x^2}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{2a(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{a(6+7ax)}{3c\sqrt{c-a^2cx^2}}+\frac{\int \frac{3+6ax}{x^2\sqrt{c-a^2cx^2}}\,dx}{3c}\\
&= \frac{2a(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{a(6+7ax)}{3c\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{c^2x}+\frac{(2a)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx}{c}\\
&= \frac{2a(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{a(6+7ax)}{3c\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{c^2x}+\frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)}{c}\\
&= \frac{2a(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{a(6+7ax)}{3c\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{c^2x}-\frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)}{ac^2}\\
&= \frac{2a(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{a(6+7ax)}{3c\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{c^2x}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.130704, size = 89, normalized size = 0.82

$$\frac{\left(-10a^2x^2+14ax-3\right)\sqrt{c-a^2cx^2}}{3c^2x(ax-1)^2}-\frac{2a\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)}{c^{3/2}}+\frac{2a\log(x)}{c^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((-3 + 14*a*x - 10*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*c^2*x*(-1 + a*x)^2) + (
2*a*Log[x])/c^(3/2) - (2*a*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/c^(3/2)
```

Maple [A] time = 0.042, size = 178, normalized size = 1.7

$$-\frac{1}{cx}\frac{1}{\sqrt{-a^2cx^2+c}}+2\frac{a^2x}{\sqrt{-a^2cx^2+c}c}+2\frac{a}{\sqrt{-a^2cx^2+c}c}-2\frac{a}{c^{3/2}}\ln\left(\frac{2c+2\sqrt{c}\sqrt{-a^2cx^2+c}}{x}\right)-\frac{2}{3c}\left(x-a^{-1}\right)^{-1}\frac{}{\sqrt{-ca^2(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/c/x/(-a^2*c*x^2+c)^(1/2)+2*a^2/c*x/(-a^2*c*x^2+c)^(1/2)+2*a/c/(-a^2*c*x^
2+c)^(1/2)-2*a/c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-2/3/c/(x-
1/a)/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)+4/3*a^2/c/(-c*a^2*(x-1/a)^2-2*a
*c*(x-1/a))^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(ax+1)^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)^2/((-a^2*c*x^2 + c)^(3/2)*(a^2*x^2 - 1)*x^2), x)
```

Fricas [A] time = 3.19552, size = 514, normalized size = 4.76

$$\left[\frac{3\left(a^3x^3-2a^2x^2+ax\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)-\sqrt{-a^2cx^2+c}\left(10a^2x^2-14ax+3\right)}{3\left(a^2c^2x^3-2ac^2x^2+c^2x\right)},-\frac{6\left(a^3x^3-2a^2x^2+ax\right)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/3*(3*(a^3*x^3 - 2*a^2*x^2 + a*x)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c
*x^2 + c)*sqrt(c) - 2*c)/x^2) - sqrt(-a^2*c*x^2 + c)*(10*a^2*x^2 - 14*a*x +
 3))/(a^2*c^2*x^3 - 2*a*c^2*x^2 + c^2*x), -1/3*(6*(a^3*x^3 - 2*a^2*x^2 + a*
x)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a
^2*c*x^2 + c)*(10*a^2*x^2 - 14*a*x + 3))/(a^2*c^2*x^3 - 2*a*c^2*x^2 + c^2*x
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{-a^3cx^5\sqrt{-a^2cx^2+c}+a^2cx^4\sqrt{-a^2cx^2+c}+acx^3\sqrt{-a^2cx^2+c}-cx^2\sqrt{-a^2cx^2+c}}\,dx-\int \frac{}{-a^3cx^5\sqrt{-a^2cx^2+c}+a^2cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(a*x/(-a**3*c*x**5*sqrt(-a**2*c*x**2 + c) + a**2*c*x**4*sqrt(-a**2
*c*x**2 + c) + a*c*x**3*sqrt(-a**2*c*x**2 + c) - c*x**2*sqrt(-a**2*c*x**2 +
 c)), x) - Integral(1/(-a**3*c*x**5*sqrt(-a**2*c*x**2 + c) + a**2*c*x**4*sq
rt(-a**2*c*x**2 + c) + a*c*x**3*sqrt(-a**2*c*x**2 + c) - c*x**2*sqrt(-a**2*
c*x**2 + c)), x)
```

Giac [A] time = 1.22582, size = 135, normalized size = 1.25

$$-2\,a^2\sqrt{-c}c\left(\frac{2\,|a|\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{a^3c^3}-\frac{1}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)a^2c^2}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] -2*a^2*sqrt(-c)*c*(2*abs(a)*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))
/sqrt(-c))/(a^3*c^3) - 1/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*a
^2*c^2))*abs(a)
```

3.1122 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{2a\sqrt{c-a^2cx^2}}{c^2x}-\frac{\sqrt{c-a^2cx^2}}{2c^2x^2}-\frac{7a^2\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{a^2(10ax+9)}{3c\sqrt{c-a^2cx^2}}+\frac{2a^2(ax+1)}{3\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] (2*a^2*(1 + a*x))/(3*(c - a^2*c*x^2)^(3/2)) + (a^2*(9 + 10*a*x))/(3*c*Sqrt[
c - a^2*c*x^2]) - Sqrt[c - a^2*c*x^2]/(2*c^2*x^2) - (2*a*Sqrt[c - a^2*c*x^2
])/(c^2*x) - (7*a^2*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/(2*c^(3/2))
```

Rubi [A] time = 0.419906, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1805, 1807, 807, 266, 63, 208}

$$-\frac{2a\sqrt{c-a^2cx^2}}{c^2x}-\frac{\sqrt{c-a^2cx^2}}{2c^2x^2}-\frac{7a^2\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{a^2(10ax+9)}{3c\sqrt{c-a^2cx^2}}+\frac{2a^2(ax+1)}{3\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```



```
[Out] (2*a^2*(1 + a*x))/(3*(c - a^2*c*x^2)^(3/2)) + (a^2*(9 + 10*a*x))/(3*c*Sqrt[
c - a^2*c*x^2]) - Sqrt[c - a^2*c*x^2]/(2*c^2*x^2) - (2*a*Sqrt[c - a^2*c*x^2
])/(c^2*x) - (7*a^2*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/(2*c^(3/2))
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx &= c\int \frac{(1+ax)^2}{x^3\left(c-a^2cx^2\right)^{5/2}}\,dx\\
&= \frac{2a^2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}-\frac{1}{3}\int \frac{-3-6ax-6a^2x^2-4a^3x^3}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{2a^2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2(9+10ax)}{3c\sqrt{c-a^2cx^2}}+\frac{\int \frac{3+6ax+9a^2x^2}{x^3\sqrt{c-a^2cx^2}}\,dx}{3c}\\
&= \frac{2a^2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2(9+10ax)}{3c\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{2c^2x^2}-\frac{\int \frac{-12ac-21a^2cx}{x^2\sqrt{c-a^2cx^2}}\,dx}{6c^2}\\
&= \frac{2a^2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2(9+10ax)}{3c\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{2c^2x^2}-\frac{2a\sqrt{c-a^2cx^2}}{c^2x}+\frac{\left(7a^2\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx}{2c}\\
&= \frac{2a^2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2(9+10ax)}{3c\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{2c^2x^2}-\frac{2a\sqrt{c-a^2cx^2}}{c^2x}+\frac{\left(7a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x\right.}{4c}\\
&= \frac{2a^2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2(9+10ax)}{3c\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{2c^2x^2}-\frac{2a\sqrt{c-a^2cx^2}}{c^2x}-\frac{7\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-\right.}{2c^2}\\
&= \frac{2a^2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2(9+10ax)}{3c\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{2c^2x^2}-\frac{2a\sqrt{c-a^2cx^2}}{c^2x}-\frac{7a^2\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{2c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.144843, size = 105, normalized size = 0.74

$$-\frac{(32a^3x^3-43a^2x^2+6ax+3)\sqrt{c-a^2cx^2}}{6c^2x^2(ax-1)^2}-\frac{7a^2\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)}{2c^{3/2}}+\frac{7a^2\log(x)}{2c^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]*(3 + 6*a*x - 43*a^2*x^2 + 32*a^3*x^3))/(6*c^2*x^2*(-1
 + a*x)^2) + (7*a^2*Log[x])/(2*c^(3/2)) - (7*a^2*Log[c + Sqrt[c]*Sqrt[c - a
^2*c*x^2]])/(2*c^(3/2))
```

Maple [A] time = 0.043, size = 205, normalized size = 1.4

$$-2\,\frac{a}{cx\sqrt{-a^2cx^2+c}}+4\,\frac{xa^3}{\sqrt{-a^2cx^2+cc}}+\frac{7\,a^2}{2\,c}\frac{1}{\sqrt{-a^2cx^2+c}}-\frac{7\,a^2}{2}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right)c^{-\frac{3}{2}}-\frac{2\,a}{3\,c}\left(x-a^{-1}\right)^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -2*a/c/x/(-a^2*c*x^2+c)^(1/2)+4*a^3/c*x/(-a^2*c*x^2+c)^(1/2)+7/2*a^2/c/(-a^
2*c*x^2+c)^(1/2)-7/2*a^2/c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
-2/3*a/c/(x-1/a)/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)+4/3*a^3/c/(-c*a^2*(
x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x-1/2/c/x^2/(-a^2*c*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(ax+1)^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)^2/((-a^2*c*x^2 + c)^(3/2)*(a^2*x^2 - 1)*x^3), x)
```

Fricas [A] time = 3.84559, size = 570, normalized size = 4.01

$$\left[\frac{21\left(a^4x^4-2\,a^3x^3+a^2x^2\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-2\left(32\,a^3x^3-43\,a^2x^2+6\,ax+3\right)\sqrt{-a^2cx^2+c}}{12\left(a^2c^2x^4-2\,ac^2x^3+c^2x^2\right)},-\frac{21\left(a^4x^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/12*(21*(a^4*x^4 - 2*a^3*x^3 + a^2*x^2)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(
-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) - 2*(32*a^3*x^3 - 43*a^2*x^2 + 6*a*x +
3)*sqrt(-a^2*c*x^2 + c))/(a^2*c^2*x^4 - 2*a*c^2*x^3 + c^2*x^2), -1/6*(21*(a
^4*x^4 - 2*a^3*x^3 + a^2*x^2)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)
/(a^2*c*x^2 - c)) + (32*a^3*x^3 - 43*a^2*x^2 + 6*a*x + 3)*sqrt(-a^2*c*x^2 +
 c))/(a^2*c^2*x^4 - 2*a*c^2*x^3 + c^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{-a^3cx^6\sqrt{-a^2cx^2+c}+a^2cx^5\sqrt{-a^2cx^2+c}+acx^4\sqrt{-a^2cx^2+c}-cx^3\sqrt{-a^2cx^2+c}}\,dx-\int \frac{}{-a^3cx^6\sqrt{-a^2cx^2+c}+a^2cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(a*x/(-a**3*c*x**6*sqrt(-a**2*c*x**2 + c) + a**2*c*x**5*sqrt(-a**2
*c*x**2 + c) + a*c*x**4*sqrt(-a**2*c*x**2 + c) - c*x**3*sqrt(-a**2*c*x**2 +
 c)), x) - Integral(1/(-a**3*c*x**6*sqrt(-a**2*c*x**2 + c) + a**2*c*x**5*sq
rt(-a**2*c*x**2 + c) + a*c*x**4*sqrt(-a**2*c*x**2 + c) - c*x**3*sqrt(-a**2*
c*x**2 + c)), x)
```

Giac [A] time = 1.24404, size = 275, normalized size = 1.94

$$a^4c^2\left(\frac{7\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^3}-\frac{\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^3a-4\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2\sqrt{-c}|a|+\left(\sqrt{-a^2c}x-\sqrt{-}\right.}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)^2a^3c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] a^4*c^2*(7*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/(a^2*s
qrt(-c)*c^3) - ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a - 4*(sqrt(-a^2*
c)*x - sqrt(-a^2*c*x^2 + c))^2*sqrt(-c)*abs(a) + (sqrt(-a^2*c)*x - sqrt(-a^
2*c*x^2 + c))*a*c + 4*sqrt(-c)*c*abs(a))/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^
2 + c))^2 - c)^2*a^3*c^3))
```

3.1123 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{2x}{5c^2\sqrt{c-a^2cx^2}}+\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}+\frac{2(ax+1)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] (2*(1 + a*x))/(5*a*(c - a^2*c*x^2)^(5/2)) + x/(5*c*(c - a^2*c*x^2)^(3/2)) +
 (2*x)/(5*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.0715208, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6141, 653, 192, 191}

$$\frac{2x}{5c^2\sqrt{c-a^2cx^2}}+\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}+\frac{2(ax+1)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (2*(1 + a*x))/(5*a*(c - a^2*c*x^2)^(5/2)) + x/(5*c*(c - a^2*c*x^2)^(3/2)) +
 (2*x)/(5*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{7/2}}\,dx\\
&= \frac{2(1+ax)}{5a\left(c-a^2cx^2\right)^{5/2}}+\frac{3}{5}\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx\\
&= \frac{2(1+ax)}{5a\left(c-a^2cx^2\right)^{5/2}}+\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{5c}\\
&= \frac{2(1+ax)}{5a\left(c-a^2cx^2\right)^{5/2}}+\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}+\frac{2x}{5c^2\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0357158, size = 53, normalized size = 0.72

$$
\frac{2a^3x^3-4a^2x^2+ax+2}{5ac^2(ax-1)^2\sqrt{c-a^2cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (2 + a*x - 4*a^2*x^2 + 2*a^3*x^3)/(5*a*c^2*(-1 + a*x)^2*Sqrt[c - a^2*c*x^2]
)
```

Maple [A] time = 0.03, size = 47, normalized size = 0.6

$$
\frac{\left(2\,x^3a^3-4\,a^2x^2+ax+2\right)(ax+1)^2}{5\,a}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/5*(2*a^3*x^3-4*a^2*x^2+a*x+2)*(a*x+1)^2/(-a^2*c*x^2+c)^(5/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.68227, size = 153, normalized size = 2.07

$$-\frac{\left(2\,a^{3}x^{3}-4\,a^{2}x^{2}+ax+2\right)\sqrt{-a^{2}cx^{2}+c}}{5\,\left(a^{5}c^{3}x^{4}-2\,a^{4}c^{3}x^{3}+2\,a^{2}c^{3}x-ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/5*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 2)*sqrt(-a^2*c*x^2 + c)/(a^5*c^3*x^4 -
2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{ax}{a^{5}c^{2}x^{5}\sqrt{-a^{2}cx^{2}+c}-a^{4}c^{2}x^{4}\sqrt{-a^{2}cx^{2}+c}-2a^{3}c^{2}x^{3}\sqrt{-a^{2}cx^{2}+c}+2a^{2}c^{2}x^{2}\sqrt{-a^{2}cx^{2}+c}+ac^{2}x\sqrt{-a^{2}cx^{2}+c}-c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] -Integral(a*x/(a**5*c**2*x**5*sqrt(-a**2*c*x**2 + c) - a**4*c**2*x**4*sqrt(
-a**2*c*x**2 + c) - 2*a**3*c**2*x**3*sqrt(-a**2*c*x**2 + c) + 2*a**2*c**2*x
**2*sqrt(-a**2*c*x**2 + c) + a*c**2*x*sqrt(-a**2*c*x**2 + c) - c**2*sqrt(-a
**2*c*x**2 + c)), x) - Integral(1/(a**5*c**2*x**5*sqrt(-a**2*c*x**2 + c) -
a**4*c**2*x**4*sqrt(-a**2*c*x**2 + c) - 2*a**3*c**2*x**3*sqrt(-a**2*c*x**2
+ c) + 2*a**2*c**2*x**2*sqrt(-a**2*c*x**2 + c) + a*c**2*x*sqrt(-a**2*c*x**2
 + c) - c**2*sqrt(-a**2*c*x**2 + c)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{(ax+1)^{2}}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}\left(a^{2}x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2/((-a^2*c*x^2 + c)^(5/2)*(a^2*x^2 - 1)), x)
```

3.1124 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=97

$$\frac{8x}{21c^3\sqrt{c-a^2cx^2}}+\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}+\frac{2(ax+1)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

```
[Out] (2*(1 + a*x))/(7*a*(c - a^2*c*x^2)^(7/2)) + x/(7*c*(c - a^2*c*x^2)^(5/2)) +
 (4*x)/(21*c^2*(c - a^2*c*x^2)^(3/2)) + (8*x)/(21*c^3*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.0800799, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6141, 653, 192, 191}

$$\frac{8x}{21c^3\sqrt{c-a^2cx^2}}+\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}+\frac{2(ax+1)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (2*(1 + a*x))/(7*a*(c - a^2*c*x^2)^(7/2)) + x/(7*c*(c - a^2*c*x^2)^(5/2)) +
 (4*x)/(21*c^2*(c - a^2*c*x^2)^(3/2)) + (8*x)/(21*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{9/2}}\,dx$$
$$= \frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}} + \frac{5}{7}\int \frac{1}{\left(c-a^2cx^2\right)^{7/2}}\,dx$$
$$= \frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}} + \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} + \frac{4\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{7c}$$
$$= \frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}} + \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} + \frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}} + \frac{8\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{21c^2}$$
$$= \frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}} + \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} + \frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}} + \frac{8x}{21c^3\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0498029, size = 96, normalized size = 0.99

$$-\frac{\sqrt{1-a^2x^2}\left(8a^5x^5-16a^4x^4-4a^3x^3+24a^2x^2-9ax-6\right)}{21ac^3(1-ax)^{7/2}(ax+1)^{3/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]*(-6 - 9*a*x + 24*a^2*x^2 - 4*a^3*x^3 - 16*a^4*x^4 + 8*a
^5*x^5))/(21*a*c^3*(1 - a*x)^(7/2)*(1 + a*x)^(3/2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.031, size = 64, normalized size = 0.7

$$-\frac{\left(8\,x^5a^5-16\,x^4a^4-4\,x^3a^3+24\,a^2x^2-9\,ax-6\right)(ax+1)^2}{21\,a}\left(-a^2cx^2+c\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/21*(8*a^5*x^5-16*a^4*x^4-4*a^3*x^3+24*a^2*x^2-9*a*x-6)*(a*x+1)^2/(-a^2*c
*x^2+c)^(7/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.55198, size = 251, normalized size = 2.59

$$-\frac{\left(8\,a^{5}x^{5}-16\,a^{4}x^{4}-4\,a^{3}x^{3}+24\,a^{2}x^{2}-9\,ax-6\right)\sqrt{-a^{2}cx^{2}+c}}{21\left(a^{7}c^{4}x^{6}-2\,a^{6}c^{4}x^{5}-a^{5}c^{4}x^{4}+4\,a^{4}c^{4}x^{3}-a^{3}c^{4}x^{2}-2\,a^{2}c^{4}x+ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas"
)
```

```
[Out] -1/21*(8*a^5*x^5 - 16*a^4*x^4 - 4*a^3*x^3 + 24*a^2*x^2 - 9*a*x - 6)*sqrt(-a
^2*c*x^2 + c)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 -
a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{ax}{-a^{7}c^{3}x^{7}\sqrt{-a^{2}cx^{2}+c}+a^{6}c^{3}x^{6}\sqrt{-a^{2}cx^{2}+c}+3a^{5}c^{3}x^{5}\sqrt{-a^{2}cx^{2}+c}-3a^{4}c^{3}x^{4}\sqrt{-a^{2}cx^{2}+c}-3a^{3}c^{3}x^{3}\sqrt{-a^{2}cx^{2}+c}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] -Integral(a*x/(-a**7*c**3*x**7*sqrt(-a**2*c*x**2 + c) + a**6*c**3*x**6*sqrt
(-a**2*c*x**2 + c) + 3*a**5*c**3*x**5*sqrt(-a**2*c*x**2 + c) - 3*a**4*c**3*
x**4*sqrt(-a**2*c*x**2 + c) - 3*a**3*c**3*x**3*sqrt(-a**2*c*x**2 + c) + 3*a
**2*c**3*x**2*sqrt(-a**2*c*x**2 + c) + a*c**3*x*sqrt(-a**2*c*x**2 + c) - c*
*3*sqrt(-a**2*c*x**2 + c)), x) - Integral(1/(-a**7*c**3*x**7*sqrt(-a**2*c*x
**2 + c) + a**6*c**3*x**6*sqrt(-a**2*c*x**2 + c) + 3*a**5*c**3*x**5*sqrt(-a
**2*c*x**2 + c) - 3*a**4*c**3*x**4*sqrt(-a**2*c*x**2 + c) - 3*a**3*c**3*x**
3*sqrt(-a**2*c*x**2 + c) + 3*a**2*c**3*x**2*sqrt(-a**2*c*x**2 + c) + a*c**3
*x*sqrt(-a**2*c*x**2 + c) - c**3*sqrt(-a**2*c*x**2 + c)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{(ax+1)^{2}}{\left(-a^{2}cx^{2}+c\right)^{\frac{7}{2}}\left(a^{2}x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2/((-a^2*c*x^2 + c)^(7/2)*(a^2*x^2 - 1)), x)
```

3.1125 $\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=120

$$-\frac{a^2c^3x^{m+3}}{m+3}-\frac{4a^3c^3x^{m+4}}{m+4}-\frac{a^4c^3x^{m+5}}{m+5}+\frac{2a^5c^3x^{m+6}}{m+6}+\frac{a^6c^3x^{m+7}}{m+7}+\frac{2ac^3x^{m+2}}{m+2}+\frac{c^3x^{m+1}}{m+1}$$

```
[Out] (c^3*x^(1 + m))/(1 + m) + (2*a*c^3*x^(2 + m))/(2 + m) - (a^2*c^3*x^(3 + m))
/(3 + m) - (4*a^3*c^3*x^(4 + m))/(4 + m) - (a^4*c^3*x^(5 + m))/(5 + m) + (2
*a^5*c^3*x^(6 + m))/(6 + m) + (a^6*c^3*x^(7 + m))/(7 + m)
```

Rubi [A] time = 0.1127, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 88}

$$-\frac{a^2c^3x^{m+3}}{m+3}-\frac{4a^3c^3x^{m+4}}{m+4}-\frac{a^4c^3x^{m+5}}{m+5}+\frac{2a^5c^3x^{m+6}}{m+6}+\frac{a^6c^3x^{m+7}}{m+7}+\frac{2ac^3x^{m+2}}{m+2}+\frac{c^3x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*x^(1 + m))/(1 + m) + (2*a*c^3*x^(2 + m))/(2 + m) - (a^2*c^3*x^(3 + m))
/(3 + m) - (4*a^3*c^3*x^(4 + m))/(4 + m) - (a^4*c^3*x^(5 + m))/(5 + m) + (2
*a^5*c^3*x^(6 + m))/(6 + m) + (a^6*c^3*x^(7 + m))/(7 + m)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^3\,dx&=c^3\int x^m(1-ax)^2(1+ax)^4\,dx\\&=c^3\int\left(x^m+2ax^{1+m}-a^2x^{2+m}-4a^3x^{3+m}-a^4x^{4+m}+2a^5x^{5+m}+a^6x^{6+m}\right)dx\\&=\frac{c^3x^{1+m}}{1+m}+\frac{2ac^3x^{2+m}}{2+m}-\frac{a^2c^3x^{3+m}}{3+m}-\frac{4a^3c^3x^{4+m}}{4+m}-\frac{a^4c^3x^{5+m}}{5+m}+\frac{2a^5c^3x^{6+m}}{6+m}+\frac{a^6c^3x^{7+m}}{7+m}\end{aligned}$$

Mathematica [A] time = 0.0638301, size = 88, normalized size = 0.73

$$c^3x^{m+1}\left(\frac{a^6x^6}{m+7}+\frac{2a^5x^5}{m+6}-\frac{a^4x^4}{m+5}-\frac{4a^3x^3}{m+4}-\frac{a^2x^2}{m+3}+\frac{2ax}{m+2}+\frac{1}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^3,x]
```

```
[Out] c^3*x^(1 + m)*((1 + m)^(-1) + (2*a*x)/(2 + m) - (a^2*x^2)/(3 + m) - (4*a^3*
x^3)/(4 + m) - (a^4*x^4)/(5 + m) + (2*a^5*x^5)/(6 + m) + (a^6*x^6)/(7 + m))
```

Maple [B] time = 0.031, size = 476, normalized size = 4.

$$\frac{c^3x^{1+m}\left(a^6m^6x^6+21\,a^6m^5x^6+175\,a^6m^4x^6+2\,a^5m^6x^5+735\,a^6m^3x^6+44\,a^5m^5x^5+1624\,a^6m^2x^6+380\,a^5m^4x^5-a^4m^6x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*x^(1+m)*(a^6*m^6*x^6+21*a^6*m^5*x^6+175*a^6*m^4*x^6+2*a^5*m^6*x^5+735*a
^6*m^3*x^6+44*a^5*m^5*x^5+1624*a^6*m^2*x^6+380*a^5*m^4*x^5-a^4*m^6*x^4+1764
*a^6*m*x^6+1640*a^5*m^3*x^5-23*a^4*m^5*x^4+720*a^6*x^6+3698*a^5*m^2*x^5-207
*a^4*m^4*x^4-4*a^3*m^6*x^3+4076*a^5*m*x^5-925*a^4*m^3*x^4-96*a^3*m^5*x^3+16
80*a^5*x^5-2144*a^4*m^2*x^4-904*a^3*m^4*x^3-a^2*m^6*x^2-2412*a^4*m*x^4-4224
*a^3*m^3*x^3-25*a^2*m^5*x^2-1008*a^4*x^4-10180*a^3*m^2*x^3-247*a^2*m^4*x^2+
2*a*m^6*x-11808*a^3*m*x^3-1219*a^2*m^3*x^2+52*a*m^5*x-5040*a^3*x^3-3112*a^2
*m^2*x^2+540*a*m^4*x+m^6-3796*a^2*m*x^2+2840*a*m^3*x+27*m^5-1680*a^2*x^2+78
58*a*m^2*x+295*m^4+10548*a*m*x+1665*m^3+5040*a*x+5104*m^2+8028*m+5040)/(7+m
)/(6+m)/(5+m)/(4+m)/(3+m)/(2+m)/(1+m)
```

Maxima [B] time = 1.14391, size = 410, normalized size = 3.42

$$\frac{\left(\left(m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720\right)a^6c^3x^7+2\left(m^6+22\,m^5+190\,m^4+820\,m^3+1849\,m^2+20\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] ((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*a^6*c^3*x^7 +
 2*(m^6 + 22*m^5 + 190*m^4 + 820*m^3 + 1849*m^2 + 2038*m + 840)*a^5*c^3*x^6
 - (m^6 + 23*m^5 + 207*m^4 + 925*m^3 + 2144*m^2 + 2412*m + 1008)*a^4*c^3*x^
5 - 4*(m^6 + 24*m^5 + 226*m^4 + 1056*m^3 + 2545*m^2 + 2952*m + 1260)*a^3*c^
3*x^4 - (m^6 + 25*m^5 + 247*m^4 + 1219*m^3 + 3112*m^2 + 3796*m + 1680)*a^2*
c^3*x^3 + 2*(m^6 + 26*m^5 + 270*m^4 + 1420*m^3 + 3929*m^2 + 5274*m + 2520)*
a*c^3*x^2 + (m^6 + 27*m^5 + 295*m^4 + 1665*m^3 + 5104*m^2 + 8028*m + 5040)*
c^3*x)*x^m/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 1306
8*m + 5040)
```

Fricas [B] time = 3.26204, size = 1233, normalized size = 10.28

$$\frac{\left(\left(a^6c^3m^6+21\,a^6c^3m^5+175\,a^6c^3m^4+735\,a^6c^3m^3+1624\,a^6c^3m^2+1764\,a^6c^3m+720\,a^6c^3\right)x^7+2\left(a^5c^3m^6+22\,a^5c^3m^5\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] ((a^6*c^3*m^6 + 21*a^6*c^3*m^5 + 175*a^6*c^3*m^4 + 735*a^6*c^3*m^3 + 1624*a
^6*c^3*m^2 + 1764*a^6*c^3*m + 720*a^6*c^3)*x^7 + 2*(a^5*c^3*m^6 + 22*a^5*c^
3*m^5 + 190*a^5*c^3*m^4 + 820*a^5*c^3*m^3 + 1849*a^5*c^3*m^2 + 2038*a^5*c^3
*m + 840*a^5*c^3)*x^6 - (a^4*c^3*m^6 + 23*a^4*c^3*m^5 + 207*a^4*c^3*m^4 + 9
25*a^4*c^3*m^3 + 2144*a^4*c^3*m^2 + 2412*a^4*c^3*m + 1008*a^4*c^3)*x^5 - 4*
(a^3*c^3*m^6 + 24*a^3*c^3*m^5 + 226*a^3*c^3*m^4 + 1056*a^3*c^3*m^3 + 2545*a
^3*c^3*m^2 + 2952*a^3*c^3*m + 1260*a^3*c^3)*x^4 - (a^2*c^3*m^6 + 25*a^2*c^3
*m^5 + 247*a^2*c^3*m^4 + 1219*a^2*c^3*m^3 + 3112*a^2*c^3*m^2 + 3796*a^2*c^3
*m + 1680*a^2*c^3)*x^3 + 2*(a*c^3*m^6 + 26*a*c^3*m^5 + 270*a*c^3*m^4 + 1420
*a*c^3*m^3 + 3929*a*c^3*m^2 + 5274*a*c^3*m + 2520*a*c^3)*x^2 + (c^3*m^6 + 2
7*c^3*m^5 + 295*c^3*m^4 + 1665*c^3*m^3 + 5104*c^3*m^2 + 8028*c^3*m + 5040*c
^3)*x)*x^m/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 1306
8*m + 5040)
```

Sympy [A] time = 4.08118, size = 3009, normalized size = 25.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m*(-a**2*c*x**2+c)**3,x)
```

```
[Out] Piecewise((a**6*c**3*log(x) - 2*a**5*c**3/x + a**4*c**3/(2*x**2) + 4*a**3*c
**3/(3*x**3) + a**2*c**3/(4*x**4) - 2*a*c**3/(5*x**5) - c**3/(6*x**6), Eq(m
, -7)), (a**6*c**3*x + 2*a**5*c**3*log(x) + a**4*c**3/x + 2*a**3*c**3/x**2
+ a**2*c**3/(3*x**3) - a*c**3/(2*x**4) - c**3/(5*x**5), Eq(m, -6)), (a**6*c
**3*x**2/2 + 2*a**5*c**3*x - a**4*c**3*log(x) + 4*a**3*c**3/x + a**2*c**3/(
2*x**2) - 2*a*c**3/(3*x**3) - c**3/(4*x**4), Eq(m, -5)), (a**6*c**3*x**3/3
+ a**5*c**3*x**2 - a**4*c**3*x - 4*a**3*c**3*log(x) + a**2*c**3/x - a*c**3/
x**2 - c**3/(3*x**3), Eq(m, -4)), (a**6*c**3*x**4/4 + 2*a**5*c**3*x**3/3 -
a**4*c**3*x**2/2 - 4*a**3*c**3*x - a**2*c**3*log(x) - 2*a*c**3/x - c**3/(2*
x**2), Eq(m, -3)), (a**6*c**3*x**5/5 + a**5*c**3*x**4/2 - a**4*c**3*x**3/3
- 2*a**3*c**3*x**2 - a**2*c**3*x + 2*a*c**3*log(x) - c**3/x, Eq(m, -2)), (a
**6*c**3*x**6/6 + 2*a**5*c**3*x**5/5 - a**4*c**3*x**4/4 - 4*a**3*c**3*x**3/
3 - a**2*c**3*x**2/2 + 2*a*c**3*x + c**3*log(x), Eq(m, -1)), (a**6*c**3*m**
6*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
 + 13068*m + 5040) + 21*a**6*c**3*m**5*x**7*x**m/(m**7 + 28*m**6 + 322*m**5
 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 175*a**6*c**3*m**
4*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
 + 13068*m + 5040) + 735*a**6*c**3*m**3*x**7*x**m/(m**7 + 28*m**6 + 322*m**
5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1624*a**6*c**3*m
**2*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m*
*2 + 13068*m + 5040) + 1764*a**6*c**3*m*x**7*x**m/(m**7 + 28*m**6 + 322*m**
5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 720*a**6*c**3*x*
*7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 1
3068*m + 5040) + 2*a**5*c**3*m**6*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 19
60*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 44*a**5*c**3*m**5*x**6
*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 130
68*m + 5040) + 380*a**5*c**3*m**4*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 19
60*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1640*a**5*c**3*m**3*x*
*6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 1
3068*m + 5040) + 3698*a**5*c**3*m**2*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 +
 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 4076*a**5*c**3*m*x*
*6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 1
```

```
3068*m + 5040) + 1680*a**5*c**3*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960
*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) - a**4*c**3*m**6*x**5*x**m
/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
+ 5040) - 23*a**4*c**3*m**5*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**
4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) - 207*a**4*c**3*m**4*x**5*x**m
/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
+ 5040) - 925*a**4*c**3*m**3*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m*
*4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) - 2144*a**4*c**3*m**2*x**5*x*
*m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*
m + 5040) - 2412*a**4*c**3*m*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m*
*4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) - 1008*a**4*c**3*x**5*x**m/(m
**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5
040) - 4*a**3*c**3*m**6*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
6769*m**3 + 13132*m**2 + 13068*m + 5040) - 96*a**3*c**3*m**5*x**4*x**m/(m**
7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 504
0) - 904*a**3*c**3*m**4*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
6769*m**3 + 13132*m**2 + 13068*m + 5040) - 4224*a**3*c**3*m**3*x**4*x**m/(m
**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5
040) - 10180*a**3*c**3*m**2*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**
4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) - 11808*a**3*c**3*m*x**4*x**m/
(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
 5040) - 5040*a**3*c**3*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
6769*m**3 + 13132*m**2 + 13068*m + 5040) - a**2*c**3*m**6*x**3*x**m/(m**7 +
 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040)
- 25*a**2*c**3*m**5*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769
*m**3 + 13132*m**2 + 13068*m + 5040) - 247*a**2*c**3*m**4*x**3*x**m/(m**7 +
 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040)
- 1219*a**2*c**3*m**3*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 67
69*m**3 + 13132*m**2 + 13068*m + 5040) - 3112*a**2*c**3*m**2*x**3*x**m/(m**
7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 504
0) - 3796*a**2*c**3*m*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 67
69*m**3 + 13132*m**2 + 13068*m + 5040) - 1680*a**2*c**3*x**3*x**m/(m**7 + 2
8*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) +
2*a*c**3*m**6*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3
+ 13132*m**2 + 13068*m + 5040) + 52*a*c**3*m**5*x**2*x**m/(m**7 + 28*m**6 +
 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 540*a*c*
*3*m**4*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 1313
2*m**2 + 13068*m + 5040) + 2840*a*c**3*m**3*x**2*x**m/(m**7 + 28*m**6 + 322
*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 7858*a*c**3*
m**2*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m
**2 + 13068*m + 5040) + 10548*a*c**3*m*x**2*x**m/(m**7 + 28*m**6 + 322*m**5
 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 5040*a*c**3*x**2*
x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 1306
8*m + 5040) + c**3*m**6*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 676
9*m**3 + 13132*m**2 + 13068*m + 5040) + 27*c**3*m**5*x*x**m/(m**7 + 28*m**6
 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 295*c*
*3*m**4*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m
**2 + 13068*m + 5040) + 1665*c**3*m**3*x*x**m/(m**7 + 28*m**6 + 322*m**5 +
1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 5104*c**3*m**2*x*x**
m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
 + 5040) + 8028*c**3*m*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769
*m**3 + 13132*m**2 + 13068*m + 5040) + 5040*c**3*x*x**m/(m**7 + 28*m**6 + 3
22*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2-c\right)^3(ax+1)^2x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 - c)^3*(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

3.1126 $\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=67

$$-\frac{2a^3c^2x^{m+4}}{m+4}-\frac{a^4c^2x^{m+5}}{m+5}+\frac{2ac^2x^{m+2}}{m+2}+\frac{c^2x^{m+1}}{m+1}$$

```
[Out] (c^2*x^(1 + m))/(1 + m) + (2*a*c^2*x^(2 + m))/(2 + m) - (2*a^3*c^2*x^(4 + m
))/(4 + m) - (a^4*c^2*x^(5 + m))/(5 + m)
```

Rubi [A] time = 0.0872407, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 75}

$$-\frac{2a^3c^2x^{m+4}}{m+4}-\frac{a^4c^2x^{m+5}}{m+5}+\frac{2ac^2x^{m+2}}{m+2}+\frac{c^2x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*x^(1 + m))/(1 + m) + (2*a*c^2*x^(2 + m))/(2 + m) - (2*a^3*c^2*x^(4 + m
))/(4 + m) - (a^4*c^2*x^(5 + m))/(5 + m)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^2\,dx&=c^2\int x^m(1-ax)(1+ax)^3\,dx\\&=c^2\int\left(x^m+2ax^{1+m}-2a^3x^{3+m}-a^4x^{4+m}\right)dx\\&=\frac{c^2x^{1+m}}{1+m}+\frac{2ac^2x^{2+m}}{2+m}-\frac{2a^3c^2x^{4+m}}{4+m}-\frac{a^4c^2x^{5+m}}{5+m}\end{aligned}$$

Mathematica [A] time = 0.104508, size = 69, normalized size = 1.03

$$\frac{c^2x^{m+1}\left(2(m+3)\left(\frac{a^3x^3}{m+4}+\frac{3a^2x^2}{m+3}+\frac{3ax}{m+2}+\frac{1}{m+1}\right)-(ax+1)^4\right)}{m+5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*x^(1 + m)*(-(1 + a*x)^4 + 2*(3 + m)*((1 + m)^(-1) + (3*a*x)/(2 + m) +
(3*a^2*x^2)/(3 + m) + (a^3*x^3)/(4 + m))))/(5 + m)
```

Maple [B] time = 0.029, size = 146, normalized size = 2.2

$$-\frac{c^2x^{1+m}\left(a^4m^3x^4+7\,a^4m^2x^4+14\,a^4mx^4+2\,a^3m^3x^3+8\,x^4a^4+16\,a^3m^2x^3+34\,a^3mx^3+20\,x^3a^3-2\,am^3x-20\,am^2x\right.}{(5+m)\,(4+m)\,(2+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^2,x)
```

```
[Out] -c^2*x^(1+m)*(a^4*m^3*x^4+7*a^4*m^2*x^4+14*a^4*m*x^4+2*a^3*m^3*x^3+8*a^4*x^
4+16*a^3*m^2*x^3+34*a^3*m*x^3+20*a^3*x^3-2*a*m^3*x-20*a*m^2*x-58*a*m*x-m^3-
40*a*x-11*m^2-38*m-40)/(5+m)/(4+m)/(2+m)/(1+m)
```

Maxima [A] time = 1.13279, size = 154, normalized size = 2.3

$$-\frac{\left(\left(m^3+7\,m^2+14\,m+8\right)a^4c^2x^5+2\left(m^3+8\,m^2+17\,m+10\right)a^3c^2x^4-2\left(m^3+10\,m^2+29\,m+20\right)ac^2x^2-\left(m^3+11\,m\right.\right.}{m^4+12\,m^3+49\,m^2+78\,m+40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] -((m^3 + 7*m^2 + 14*m + 8)*a^4*c^2*x^5 + 2*(m^3 + 8*m^2 + 17*m + 10)*a^3*c^
2*x^4 - 2*(m^3 + 10*m^2 + 29*m + 20)*a*c^2*x^2 - (m^3 + 11*m^2 + 38*m + 40)
*c^2*x)*x^m/(m^4 + 12*m^3 + 49*m^2 + 78*m + 40)
```

Fricas [B] time = 3.04994, size = 374, normalized size = 5.58

$$-\frac{\left(\left(a^4c^2m^3+7\,a^4c^2m^2+14\,a^4c^2m+8\,a^4c^2\right)x^5+2\left(a^3c^2m^3+8\,a^3c^2m^2+17\,a^3c^2m+10\,a^3c^2\right)x^4-2\left(ac^2m^3+10\,ac^2m^2\right.\right.}{m^4+12\,m^3+49\,m^2+78\,m+40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] -((a^4*c^2*m^3 + 7*a^4*c^2*m^2 + 14*a^4*c^2*m + 8*a^4*c^2)*x^5 + 2*(a^3*c^2
*m^3 + 8*a^3*c^2*m^2 + 17*a^3*c^2*m + 10*a^3*c^2)*x^4 - 2*(a*c^2*m^3 + 10*a
*c^2*m^2 + 29*a*c^2*m + 20*a*c^2)*x^2 - (c^2*m^3 + 11*c^2*m^2 + 38*c^2*m +
40*c^2)*x)*x^m/(m^4 + 12*m^3 + 49*m^2 + 78*m + 40)
```

Sympy [A] time = 2.19458, size = 706, normalized size = 10.54

$$\begin{cases} -a^4c^2\log(x) + \frac{2a^3c^2}{x} - \frac{2ac^2}{3x^3} - \frac{c^2}{4x^4} \\ -a^4c^2x - 2a^3c^2\log(x) - \frac{ac^2}{x^2} - \frac{c^2}{3x^3} \\ -\frac{a^4c^2x^3}{3} - a^3c^2x^2 + 2ac^2\log(x) - \frac{c^2}{x} \\ -\frac{a^4c^2x^4}{4} - \frac{2a^3c^2x^3}{3} + 2ac^2x + c^2\log(x) \\ -\frac{a^4c^2m^3x^5x^m}{m^4+12m^3+49m^2+78m+40} - \frac{7a^4c^2m^2x^5x^m}{m^4+12m^3+49m^2+78m+40} - \frac{14a^4c^2mx^5x^m}{m^4+12m^3+49m^2+78m+40} - \frac{8a^4c^2x^5x^m}{m^4+12m^3+49m^2+78m+40} - \frac{2a^3c^2m^3x^4x^m}{m^4+12m^3+49m^2+78m+40} - \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m*(-a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((-a**4*c**2*log(x) + 2*a**3*c**2/x - 2*a*c**2/(3*x**3) - c**2/(4*
x**4), Eq(m, -5)), (-a**4*c**2*x - 2*a**3*c**2*log(x) - a*c**2/x**2 - c**2/
(3*x**3), Eq(m, -4)), (-a**4*c**2*x**3/3 - a**3*c**2*x**2 + 2*a*c**2*log(x)
 - c**2/x, Eq(m, -2)), (-a**4*c**2*x**4/4 - 2*a**3*c**2*x**3/3 + 2*a*c**2*x
 + c**2*log(x), Eq(m, -1)), (-a**4*c**2*m**3*x**5*x**m/(m**4 + 12*m**3 + 49
*m**2 + 78*m + 40) - 7*a**4*c**2*m**2*x**5*x**m/(m**4 + 12*m**3 + 49*m**2 +
 78*m + 40) - 14*a**4*c**2*m*x**5*x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 4
0) - 8*a**4*c**2*x**5*x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 40) - 2*a**3*
c**2*m**3*x**4*x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 40) - 16*a**3*c**2*m
**2*x**4*x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 40) - 34*a**3*c**2*m*x**4*
x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 40) - 20*a**3*c**2*x**4*x**m/(m**4
+ 12*m**3 + 49*m**2 + 78*m + 40) + 2*a*c**2*m**3*x**2*x**m/(m**4 + 12*m**3
+ 49*m**2 + 78*m + 40) + 20*a*c**2*m**2*x**2*x**m/(m**4 + 12*m**3 + 49*m**2
 + 78*m + 40) + 58*a*c**2*m*x**2*x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 40
) + 40*a*c**2*x**2*x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 40) + c**2*m**3*
x*x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 40) + 11*c**2*m**2*x*x**m/(m**4 +
 12*m**3 + 49*m**2 + 78*m + 40) + 38*c**2*m*x*x**m/(m**4 + 12*m**3 + 49*m**
2 + 78*m + 40) + 40*c**2*x*x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 40), Tru
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(a^2cx^2 - c\right)^2 (ax + 1)^2 x^m}{a^2x^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)^2*(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

3.1127 $\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=42

$$\frac{a^2cx^{m+3}}{m+3}+\frac{2acx^{m+2}}{m+2}+\frac{cx^{m+1}}{m+1}$$

```
[Out] (c*x^(1 + m))/(1 + m) + (2*a*c*x^(2 + m))/(2 + m) + (a^2*c*x^(3 + m))/(3 +
m)
```

Rubi [A] time = 0.0628355, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 43}

$$\frac{a^2cx^{m+3}}{m+3}+\frac{2acx^{m+2}}{m+2}+\frac{cx^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^(1 + m))/(1 + m) + (2*a*c*x^(2 + m))/(2 + m) + (a^2*c*x^(3 + m))/(3 +
m)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)\,dx&=c\int x^m(1+ax)^2\,dx\\&=c\int\left(x^m+2ax^{1+m}+a^2x^{2+m}\right)dx\\&=\frac{cx^{1+m}}{1+m}+\frac{2acx^{2+m}}{2+m}+\frac{a^2cx^{3+m}}{3+m}\end{aligned}$$

Mathematica [A] time = 0.038799, size = 34, normalized size = 0.81

$$cx^{m+1}\left(\frac{a^2x^2}{m+3}+\frac{2ax}{m+2}+\frac{1}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2),x]
```

```
[Out] c*x^(1 + m)*((1 + m)^(-1) + (2*a*x)/(2 + m) + (a^2*x^2)/(3 + m))
```

Maple [A] time = 0.028, size = 74, normalized size = 1.8

$$\frac{cx^{1+m}\left(a^2m^2x^2+3\,a^2mx^2+2\,a^2x^2+2\,am^2x+8\,amx+6\,ax+m^2+5\,m+6\right)}{(3+m)\,(2+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c),x)
```

```
[Out] c*x^(1+m)*(a^2*m^2*x^2+3*a^2*m*x^2+2*a^2*x^2+2*a*m^2*x+8*a*m*x+6*a*x+m^2+5*
m+6)/(3+m)/(2+m)/(1+m)
```

Maxima [A] time = 1.119, size = 84, normalized size = 2.

$$\frac{\left(\left(m^2+3\,m+2\right)a^2cx^3+2\left(m^2+4\,m+3\right)acx^2+\left(m^2+5\,m+6\right)cx\right)x^m}{m^3+6\,m^2+11\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] ((m^2 + 3*m + 2)*a^2*c*x^3 + 2*(m^2 + 4*m + 3)*a*c*x^2 + (m^2 + 5*m + 6)*c*
x)*x^m/(m^3 + 6*m^2 + 11*m + 6)
```

Fricas [A] time = 2.82858, size = 178, normalized size = 4.24

$$\frac{\left(\left(a^2cm^2+3\,a^2cm+2\,a^2c\right)x^3+2\left(acm^2+4\,acm+3\,ac\right)x^2+\left(cm^2+5\,cm+6\,c\right)x\right)x^m}{m^3+6\,m^2+11\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] ((a^2*c*m^2 + 3*a^2*c*m + 2*a^2*c)*x^3 + 2*(a*c*m^2 + 4*a*c*m + 3*a*c)*x^2
+ (c*m^2 + 5*c*m + 6*c)*x)*x^m/(m^3 + 6*m^2 + 11*m + 6)
```

Sympy [A] time = 1.29439, size = 299, normalized size = 7.12

$$\begin{cases} a^2c\log(x)-\frac{2ac}{x}-\frac{c}{2x^2} \\ a^2cx+2ac\log(x)-\frac{c}{x} \\ \frac{a^2cx^2}{2}+2acx+c\log(x) \\ \frac{a^2cm^2x^3x^m}{m^3+6m^2+11m+6}+\frac{3a^2cmx^3x^m}{m^3+6m^2+11m+6}+\frac{2a^2cx^3x^m}{m^3+6m^2+11m+6}+\frac{2acm^2x^2x^m}{m^3+6m^2+11m+6}+\frac{8acmx^2x^m}{m^3+6m^2+11m+6}+\frac{6acx^2x^m}{m^3+6m^2+11m+6}+\frac{cm^2xx^m}{m^3+6m^2+11m+6}+\frac{}{m^3+} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m*(-a**2*c*x**2+c),x)
```

```
[Out] Piecewise((a**2*c*log(x) - 2*a*c/x - c/(2*x**2), Eq(m, -3)), (a**2*c*x + 2*
a*c*log(x) - c/x, Eq(m, -2)), (a**2*c*x**2/2 + 2*a*c*x + c*log(x), Eq(m, -1
)), (a**2*c*m**2*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6) + 3*a**2*c*m*x**3*x**
m/(m**3 + 6*m**2 + 11*m + 6) + 2*a**2*c*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6
) + 2*a*c*m**2*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + 8*a*c*m*x**2*x**m/(m*
*3 + 6*m**2 + 11*m + 6) + 6*a*c*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + c*m*
*2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 5*c*m*x*x**m/(m**3 + 6*m**2 + 11*m +
 6) + 6*c*x*x**m/(m**3 + 6*m**2 + 11*m + 6), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 - c\right)(ax+1)^2x^m}{a^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 - c)*(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

3.1128 $\int \frac{e^{2\tanh^{-1}(ax)}x^m}{c-a^2cx^2}\,dx$

Optimal. Leaf size=25

$$\frac{x^{m+1}\text{Hypergeometric2F1}(2,m+1,m+2,ax)}{c(m+1)}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, a*x])/(c*(1 + m))
```

Rubi [A] time = 0.0812267, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 64}

$$\frac{x^{m+1}\,{}_2F_1(2,m+1;m+2;ax)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, a*x])/(c*(1 + m))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}x^m}{c-a^2cx^2}\,dx &= \frac{\int \frac{x^m}{(1-ax)^2}\,dx}{c}\\ &= \frac{x^{1+m}\,{}_2F_1(2,1+m;2+m;ax)}{c(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0128715, size = 25, normalized size = 1.

$$\frac{x^{m+1}\text{Hypergeometric2F1}(2,m+1,m+2,ax)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, a*x])/(c*(1 + m))
```

Maple [F] time = 0.372, size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2 x^m}{(-a^2x^2+1)(-a^2cx^2+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c),x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2 x^m}{(a^2cx^2-c)(a^2x^2-1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^2*x^m/((a^2*c*x^2 - c)*(a^2*x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{a^2cx^2 - 2\,acx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m/(a^2*c*x^2 - 2*a*c*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m}{a^2x^2-2ax+1} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m/(-a**2*c*x**2+c),x)
```

```
[Out] Integral(x**m/(a**2*x**2 - 2*a*x + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2 x^m}{(a^2cx^2-c)(a^2x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^2*x^m/((a^2*c*x^2 - c)*(a^2*x^2 - 1)), x)
```

3.1129 $\int \frac{e^{2\tanh^{-1}(ax)}x^m}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=113

$$\frac{(2m^2-4m+1)\,x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{8c^2(m+1)}+\frac{x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-ax)}{8c^2(m+1)}$$

```
[Out] x^(1 + m)/(4*c^2*(1 - a*x)^2) + ((2 - m)*x^(1 + m))/(4*c^2*(1 - a*x)) + (x^
(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/(8*c^2*(1 + m)) + ((1 -
 4*m + 2*m^2)*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(8*c^2*(1
+ m))
```

Rubi [A] time = 0.178045, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6150, 103, 151, 156, 64}

$$\frac{(2m^2-4m+1)\,x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{8c^2(m+1)}+\frac{x^{m+1}\,{}_2F_1(1,m+1;m+2;-ax)}{8c^2(m+1)}+\frac{(2-m)x^{m+1}}{4c^2(1-ax)}+\frac{x^{m+1}}{4c^2(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^2,x]
```

```
[Out] x^(1 + m)/(4*c^2*(1 - a*x)^2) + ((2 - m)*x^(1 + m))/(4*c^2*(1 - a*x)) + (x^
(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/(8*c^2*(1 + m)) + ((1 -
 4*m + 2*m^2)*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(8*c^2*(1
+ m))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{x^m}{(1-ax)^3(1+ax)}\,dx}{c^2}\\
&= \frac{x^{1+m}}{4c^2(1-ax)^2} - \frac{\int \frac{x^m\left(-a(3-m)-a^2(1-m)x\right)}{(1-ax)^2(1+ax)}\,dx}{4ac^2}\\
&= \frac{x^{1+m}}{4c^2(1-ax)^2} + \frac{(2-m)x^{1+m}}{4c^2(1-ax)} + \frac{\int \frac{x^m\left(2a^2(1-m)^2-2a^3(2-m)mx\right)}{(1-ax)(1+ax)}\,dx}{8a^2c^2}\\
&= \frac{x^{1+m}}{4c^2(1-ax)^2} + \frac{(2-m)x^{1+m}}{4c^2(1-ax)} + \frac{\int \frac{x^m}{1+ax}\,dx}{8c^2} + \frac{\left(1-4m+2m^2\right)\int \frac{x^m}{1-ax}\,dx}{8c^2}\\
&= \frac{x^{1+m}}{4c^2(1-ax)^2} + \frac{(2-m)x^{1+m}}{4c^2(1-ax)} + \frac{x^{1+m}\,{}_2F_1(1,1+m;2+m;-ax)}{8c^2(1+m)} + \frac{\left(1-4m+2m^2\right)x^{1+m}\,{}_2F_1(1,1+r}{8c^2(1+m)}
\end{aligned}
$$

Mathematica [A] time = 0.0680812, size = 92, normalized size = 0.81

$$
\frac{x^{m+1}\left(\left(2m^2-4m+1\right)(ax-1)^2\text{Hypergeometric2F1}(1,m+1,m+2,ax)+(ax-1)^2\text{Hypergeometric2F1}(1,m+1,m}{8c^2(m+1)(ax-1)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (x^(1 + m)*(2*(1 + m)*(3 - 2*a*x + m*(-1 + a*x)) + (-1 + a*x)^2*Hypergeomet
ric2F1[1, 1 + m, 2 + m, -(a*x)] + (1 - 4*m + 2*m^2)*(-1 + a*x)^2*Hypergeome
tric2F1[1, 1 + m, 2 + m, a*x]))/(8*c^2*(1 + m)*(-1 + a*x)^2)
```

Maple [F] time = 0.418, size = 0, normalized size = 0.

$$
\int \frac{(ax+1)^2 x^m}{\left(-a^2x^2+1\right)\left(-a^2cx^2+c\right)^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^2,x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2 x^m}{\left(a^2cx^2-c\right)^2 \left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] -integrate((a*x + 1)^2*x^m/((a^2*c*x^2 - c)^2*(a^2*x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^m}{a^4c^2x^4-2\,a^3c^2x^3+2\,ac^2x-c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] integral(-x^m/(a^4*c^2*x^4 - 2*a^3*c^2*x^3 + 2*a*c^2*x - c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^m}{a^4x^4-2a^3x^3+2ax-1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -Integral(x**m/(a**4*x**4 - 2*a**3*x**3 + 2*a*x - 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)^2 x^m}{\left(a^2cx^2-c\right)^2 \left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2*x^m/((a^2*c*x^2 - c)^2*(a^2*x^2 - 1)), x)
```

3.1130 $\int \frac{e^{2\tanh^{-1}(ax)}x^m}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=203

$$\frac{(2-m)\left(2m^2-8m+3\right)x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{48c^3(m+1)}+\frac{(2-m)x^{m+1}\text{Hypergeometric2F1}(1,m+1,m}{16c^3(m+1)}$$

```
[Out] -((2 - m)*(4 - m)*x^(1 + m))/(24*c^3*(1 + a*x)) + x^(1 + m)/(6*c^3*(1 - a*x
)^3*(1 + a*x)) + ((4 - m)*x^(1 + m))/(12*c^3*(1 - a*x)^2*(1 + a*x)) + ((7 -
 2*m)*(2 - m)*x^(1 + m))/(24*c^3*(1 - a*x)*(1 + a*x)) + ((2 - m)*x^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/(16*c^3*(1 + m)) + ((2 - m)*(3
- 8*m + 2*m^2)*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(48*c^3*(
1 + m))
```

Rubi [A] time = 0.348774, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6150, 103, 151, 156, 64}

$$\frac{(2-m)\left(2m^2-8m+3\right)x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{48c^3(m+1)}+\frac{(2-m)x^{m+1}\,{}_2F_1(1,m+1;m+2;-ax)}{16c^3(m+1)}-\frac{(2-m)(4-m)x^{m+1}}{24c^3(ax+1)}+$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^3,x]
```


```
[Out] -((2 - m)*(4 - m)*x^(1 + m))/(24*c^3*(1 + a*x)) + x^(1 + m)/(6*c^3*(1 - a*x
)^3*(1 + a*x)) + ((4 - m)*x^(1 + m))/(12*c^3*(1 - a*x)^2*(1 + a*x)) + ((7 -
 2*m)*(2 - m)*x^(1 + m))/(24*c^3*(1 - a*x)*(1 + a*x)) + ((2 - m)*x^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/(16*c^3*(1 + m)) + ((2 - m)*(3
- 8*m + 2*m^2)*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(48*c^3*(
1 + m))
```


Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{x^m}{(1-ax)^4(1+ax)^2}\,dx}{c^3}$$

$$= \frac{x^{1+m}}{6c^3(1-ax)^3(1+ax)} - \frac{\int \frac{x^m\left(-a(5-m)-a^2(3-m)x\right)}{(1-ax)^3(1+ax)^2}\,dx}{6ac^3}$$

$$= \frac{x^{1+m}}{6c^3(1-ax)^3(1+ax)} + \frac{(4-m)x^{1+m}}{12c^3(1-ax)^2(1+ax)} + \frac{\int \frac{x^m\left(2a^2(2-m)(3-m)+2a^3(2-m)(4-m)x\right)}{(1-ax)^2(1+ax)^2}\,dx}{24a^2c^3}$$

$$= \frac{x^{1+m}}{6c^3(1-ax)^3(1+ax)} + \frac{(4-m)x^{1+m}}{12c^3(1-ax)^2(1+ax)} + \frac{(7-2m)(2-m)x^{1+m}}{24c^3(1-ax)(1+ax)} - \frac{\int \frac{x^m\left(2a^3(2-m)\left(1+7m-2m^2\right.\right.}{(1-ax)}}{48}$$

$$= -\frac{(2-m)(4-m)x^{1+m}}{24c^3(1+ax)} + \frac{x^{1+m}}{6c^3(1-ax)^3(1+ax)} + \frac{(4-m)x^{1+m}}{12c^3(1-ax)^2(1+ax)} + \frac{(7-2m)(2-m)x^{1+m}}{24c^3(1-ax)(1+ax)}$$

$$= -\frac{(2-m)(4-m)x^{1+m}}{24c^3(1+ax)} + \frac{x^{1+m}}{6c^3(1-ax)^3(1+ax)} + \frac{(4-m)x^{1+m}}{12c^3(1-ax)^2(1+ax)} + \frac{(7-2m)(2-m)x^{1+m}}{24c^3(1-ax)(1+ax)}$$

$$= -\frac{(2-m)(4-m)x^{1+m}}{24c^3(1+ax)} + \frac{x^{1+m}}{6c^3(1-ax)^3(1+ax)} + \frac{(4-m)x^{1+m}}{12c^3(1-ax)^2(1+ax)} + \frac{(7-2m)(2-m)x^{1+m}}{24c^3(1-ax)(1+ax)}$$

Mathematica [A] time = 0.149856, size = 194, normalized size = 0.96

$$x^{m+1}\left(-\left(2m^3-12m^2+19m-6\right)(ax+1)(ax-1)^3\text{Hypergeometric2F1}(1,m+1,m+2,ax)-3(m-2)(ax+1)(ax-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (x^(1 + m)*(-2*(m^3*(-1 + a*x)^2*(1 + a*x) + m^2*(-6 + 5*a*x + 6*a^2*x^2 -
5*a^3*x^3) + m*(11 - 6*a*x - 3*a^2*x^2 + 2*a^3*x^3) + 2*(9 - 6*a*x - 5*a^2*
x^2 + 4*a^3*x^3)) - 3*(-2 + m)*(-1 + a*x)^3*(1 + a*x)*Hypergeometric2F1[1,
1 + m, 2 + m, -(a*x)] - (-6 + 19*m - 12*m^2 + 2*m^3)*(-1 + a*x)^3*(1 + a*x)
*Hypergeometric2F1[1, 1 + m, 2 + m, a*x]))/(48*c^3*(1 + m)*(-1 + a*x)^3*(1
```

```
+ a*x))
```

Maple [F] time = 0.403, size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2 x^m}{\left(-a^2x^2+1\right)\left(-a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^3,x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2 x^m}{\left(a^2cx^2-c\right)^3\left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)^2*x^m/((a^2*c*x^2 - c)^3*(a^2*x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{a^6c^3x^6-2\,a^5c^3x^5-a^4c^3x^4+4\,a^3c^3x^3-a^2c^3x^2-2\,ac^3x+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] integral(x^m/(a^6*c^3*x^6 - 2*a^5*c^3*x^5 - a^4*c^3*x^4 + 4*a^3*c^3*x^3 - a
^2*c^3*x^2 - 2*a*c^3*x + c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m}{a^6x^6-2a^5x^5-a^4x^4+4a^3x^3-a^2x^2-2ax+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m/(-a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**m/(a**6*x**6 - 2*a**5*x**5 - a**4*x**4 + 4*a**3*x**3 - a**2*x**
2 - 2*a*x + 1), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2 x^m}{(a^2cx^2-c)^3(a^2x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^2*x^m/((a^2*c*x^2 - c)^3*(a^2*x^2 - 1)), x)
```

3.1131 $\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=176

$$\frac{c^2(2m+7)x^{m+1}\sqrt{c-a^2cx^2}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{(m+1)(m+6)\sqrt{1-a^2x^2}}+\frac{2ac^2x^{m+2}\sqrt{c-a^2cx^2}\text{Hypergeometric2F1}\left(-\frac{3}{2}\right.}{(m+2)\sqrt{1-a^2x^2}}$$

```
[Out] -((x^(1 + m)*(c - a^2*c*x^2)^(5/2))/(6 + m)) + (c^2*(7 + 2*m)*x^(1 + m)*Sqr
t[c - a^2*c*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((
1 + m)*(6 + m)*Sqrt[1 - a^2*x^2]) + (2*a*c^2*x^(2 + m)*Sqrt[c - a^2*c*x^2]*
Hypergeometric2F1[-3/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[1 - a
^2*x^2])
```

Rubi [A] time = 0.314157, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1809, 808, 365, 364}

$$\frac{c^2(2m+7)x^{m+1}\sqrt{c-a^2cx^2}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+6)\sqrt{1-a^2x^2}}+\frac{2ac^2x^{m+2}\sqrt{c-a^2cx^2}\,{}_2F_1\left(-\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{1-a^2x^2}}-\frac{x^{m+1}\left(c-a^2cx^2\right)^5}{m+6}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((x^(1 + m)*(c - a^2*c*x^2)^(5/2))/(6 + m)) + (c^2*(7 + 2*m)*x^(1 + m)*Sqr
t[c - a^2*c*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((
1 + m)*(6 + m)*Sqrt[1 - a^2*x^2]) + (2*a*c^2*x^(2 + m)*Sqrt[c - a^2*c*x^2]*
Hypergeometric2F1[-3/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[1 - a
^2*x^2])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^{5/2}\,dx = c\int x^m(1+ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$= -\frac{x^{1+m}\left(c-a^2cx^2\right)^{5/2}}{6+m} - \frac{\int x^m\left(-a^2c(7+2m)-2a^3c(6+m)x\right)\left(c-a^2cx^2\right)^{3/2}\,dx}{a^2(6+m)}$$
$$= -\frac{x^{1+m}\left(c-a^2cx^2\right)^{5/2}}{6+m} + (2ac)\int x^{1+m}\left(c-a^2cx^2\right)^{3/2}\,dx + \frac{(c(7+2m))\int x^m\left(c-a\right.}{6+m}$$
$$= -\frac{x^{1+m}\left(c-a^2cx^2\right)^{5/2}}{6+m} + \frac{\left(2ac^2\sqrt{c-a^2cx^2}\right)\int x^{1+m}\left(1-a^2x^2\right)^{3/2}\,dx}{\sqrt{1-a^2x^2}} + \frac{\left(c^2(7+2m)\right.}{}$$
$$= -\frac{x^{1+m}\left(c-a^2cx^2\right)^{5/2}}{6+m} + \frac{c^2(7+2m)x^{1+m}\sqrt{c-a^2cx^2}\,{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)(6+m)\sqrt{1-a^2x^2}} + \frac{2}{}$$

Mathematica [A] time = 0.220947, size = 180, normalized size = 1.02

$$\frac{c^2x^{m+1}\sqrt{c-a^2cx^2}\left(-\frac{a^4x^4\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+5}{2},\frac{m+7}{2},a^2x^2\right)}{m+5} - \frac{2a^3x^3\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2}+2,\frac{m}{2}+3,a^2x^2\right)}{m+4} + \frac{2ax\text{Hypergeometric2}}{m}\right.}{\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*x^(1 + m)*Sqrt[c - a^2*c*x^2]*((2*a*x*Hypergeometric2F1[-1/2, 1 + m/2,
 2 + m/2, a^2*x^2])/(2 + m) - (2*a^3*x^3*Hypergeometric2F1[-1/2, 2 + m/2, 3
 + m/2, a^2*x^2])/(4 + m) + Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a
^2*x^2]/(1 + m) - (a^4*x^4*Hypergeometric2F1[-1/2, (5 + m)/2, (7 + m)/2, a^
2*x^2])/(5 + m)))/Sqrt[1 - a^2*x^2]
```

Maple [F] time = 0.402, size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2x^m}{-a^2x^2+1}\left(-a^2cx^2+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(5/2),x)
```

[Out] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(5/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)^2x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^4c^2x^4+2\,a^3c^2x^3-2\,ac^2x-c^2\right)\sqrt{-a^2cx^2+c}\,x^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(-(a^4*c^2*x^4 + 2*a^3*c^2*x^3 - 2*a*c^2*x - c^2)*sqrt(-a^2*c*x^2 +
 c)*x^m, x)
```

Sympy [C] time = 88.0958, size = 226, normalized size = 1.28

$$-\frac{a^4c^{\frac{5}{2}}x^5x^m\Gamma\left(\frac{m}{2}+\frac{5}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{5}{2}\\ \frac{m}{2}+\frac{7}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}-\frac{a^3c^{\frac{5}{2}}x^4x^m\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{\Gamma\left(\frac{m}{2}+3\right)}+\frac{ac^{\frac{5}{2}}x^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\right.}{\Gamma\left(\frac{m}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] -a**4*c**(5/2)*x**5*x**m*gamma(m/2 + 5/2)*hyper((-1/2, m/2 + 5/2), (m/2 + 7
/2,), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(m/2 + 7/2)) - a**3*c**(5/2)*x**
4*x**m*gamma(m/2 + 2)*hyper((-1/2, m/2 + 2), (m/2 + 3,), a**2*x**2*exp_pola
r(2*I*pi))/gamma(m/2 + 3) + a*c**(5/2)*x**2*x**m*gamma(m/2 + 1)*hyper((-1/2
, m/2 + 1), (m/2 + 2,), a**2*x**2*exp_polar(2*I*pi))/gamma(m/2 + 2) + c**(5
/2)*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), a**2*x**
2*exp_polar(2*I*pi))/(2*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)^2x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(-(-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

3.1132 $\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=172

$$\frac{c(2m+5)x^{m+1}\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{(m+1)(m+4)\sqrt{1-a^2x^2}}+\frac{2acx^{m+2}\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\right.}{(m+2)\sqrt{1-a^2x^2}}$$

```
[Out] -((x^(1 + m)*(c - a^2*c*x^2)^(3/2))/(4 + m)) + (c*(5 + 2*m)*x^(1 + m)*Sqrt[
c - a^2*c*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1
+ m)*(4 + m)*Sqrt[1 - a^2*x^2]) + (2*a*c*x^(2 + m)*Sqrt[c - a^2*c*x^2]*Hype
rgeometric2F1[-1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[1 - a^2*x
^2])
```

Rubi [A] time = 0.299937, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1809, 808, 365, 364}

$$\frac{c(2m+5)x^{m+1}\sqrt{c-a^2cx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+4)\sqrt{1-a^2x^2}}+\frac{2acx^{m+2}\sqrt{c-a^2cx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{1-a^2x^2}}-\frac{x^{m+1}\left(c-a^2cx^2\right)^{3/2}}{m+4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((x^(1 + m)*(c - a^2*c*x^2)^(3/2))/(4 + m)) + (c*(5 + 2*m)*x^(1 + m)*Sqrt[
c - a^2*c*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1
+ m)*(4 + m)*Sqrt[1 - a^2*x^2]) + (2*a*c*x^(2 + m)*Sqrt[c - a^2*c*x^2]*Hype
rgeometric2F1[-1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[1 - a^2*x
^2])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^{3/2}dx = c\int x^m(1+ax)^2\sqrt{c-a^2cx^2}\,dx$$
$$= -\frac{x^{1+m}\left(c-a^2cx^2\right)^{3/2}}{4+m} - \frac{\int x^m\left(-a^2c(5+2m)-2a^3c(4+m)x\right)\sqrt{c-a^2cx^2}\,dx}{a^2(4+m)}$$
$$= -\frac{x^{1+m}\left(c-a^2cx^2\right)^{3/2}}{4+m} + (2ac)\int x^{1+m}\sqrt{c-a^2cx^2}\,dx + \frac{(c(5+2m))\int x^m\sqrt{c-a^2cx}$$
$$= -\frac{x^{1+m}\left(c-a^2cx^2\right)^{3/2}}{4+m} + \frac{\left(2ac\sqrt{c-a^2cx^2}\right)\int x^{1+m}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}} + \frac{\left(c(5+2m)\sqrt{c-}\right.}{(4+}$$
$$= -\frac{x^{1+m}\left(c-a^2cx^2\right)^{3/2}}{4+m} + \frac{c(5+2m)x^{1+m}\sqrt{c-a^2cx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)(4+m)\sqrt{1-a^2x^2}} + \frac{2a}{}$$

Mathematica [A] time = 0.153125, size = 158, normalized size = 0.92

$$\frac{cx^{m+1}\sqrt{c-a^2cx^2}\left(2a\left(m^2+4m+3\right)x\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2}+1,\frac{m}{2}+2,a^2x^2\right)+(m+2)\left(a^2(m+1)x^2\text{Hyperg}\right.\right.}{(m+1)(m+2)(m+3)\sqrt{1-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*x^(1 + m)*Sqrt[c - a^2*c*x^2]*(2*a*(3 + 4*m + m^2)*x*Hypergeometric2F1[-
1/2, 1 + m/2, 2 + m/2, a^2*x^2] + (2 + m)*((3 + m)*Hypergeometric2F1[-1/2,
(1 + m)/2, (3 + m)/2, a^2*x^2] + a^2*(1 + m)*x^2*Hypergeometric2F1[-1/2, (3
 + m)/2, (5 + m)/2, a^2*x^2])))/((1 + m)*(2 + m)*(3 + m)*Sqrt[1 - a^2*x^2])
```

Maple [F] time = 0.398, size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2x^m}{-a^2x^2+1}\left(-a^2cx^2+c\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^2x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2cx^2+2\,acx+c\right)\sqrt{-a^2cx^2+c}x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral((a^2*c*x^2 + 2*a*c*x + c)*sqrt(-a^2*c*x^2 + c)*x^m, x)
```

Sympy [C] time = 17.9426, size = 172, normalized size = 1.

$$\frac{a^2c^{\frac{3}{2}}x^3x^m\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{ac^{\frac{3}{2}}x^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{\Gamma\left(\frac{m}{2}+2\right)}+\frac{c^{\frac{3}{2}}xx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\\ \frac{m}{2}\end{matrix}\right.}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] a**2*c**(3/2)*x**3*x**m*gamma(m/2 + 3/2)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/
2,), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(m/2 + 5/2)) + a*c**(3/2)*x**2*x*
*m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), a**2*x**2*exp_polar(2*
I*pi))/gamma(m/2 + 2) + c**(3/2)*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 +
 1/2), (m/2 + 3/2,), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^2x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(-(-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

3.1133 $\int e^{2\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=172

$$\frac{c(2m+3)\sqrt{1-a^2x^2}x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}+\frac{2ac\sqrt{1-a^2x^2}x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\right.}{(m+2)\sqrt{c-a^2cx^2}}$$

```
[Out] -((x^(1 + m)*Sqrt[c - a^2*c*x^2])/(2 + m)) + (c*(3 + 2*m)*x^(1 + m)*Sqrt[1
- a^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*
(2 + m)*Sqrt[c - a^2*c*x^2]) + (2*a*c*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeom
etric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.29139, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1809, 808, 365, 364}

$$\frac{c(2m+3)\sqrt{1-a^2x^2}x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}+\frac{2ac\sqrt{1-a^2x^2}x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((x^(1 + m)*Sqrt[c - a^2*c*x^2])/(2 + m)) + (c*(3 + 2*m)*x^(1 + m)*Sqrt[1
- a^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*
(2 + m)*Sqrt[c - a^2*c*x^2]) + (2*a*c*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeom
etric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= c\int\frac{x^m(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\\ &= -\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-\frac{\int\frac{x^m\left(-a^2c(3+2m)-2a^3c(2+m)x\right)}{\sqrt{c-a^2cx^2}}\,dx}{a^2(2+m)}\\ &= -\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}+(2ac)\int\frac{x^{1+m}}{\sqrt{c-a^2cx^2}}\,dx+\frac{(c(3+2m))\int\frac{x^m}{\sqrt{c-a^2cx^2}}\,dx}{2+m}\\ &= -\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}+\frac{\left(2ac\sqrt{1-a^2x^2}\right)\int\frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}+\frac{\left(c(3+2m)\sqrt{1-a^2x^2}\right)\int\frac{x^m}{\sqrt{1-a^2x^2}}}{(2+m)\sqrt{c-a^2cx^2}}\\ &= -\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}+\frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}}+\frac{2acx^{2+m}\sqrt{1}}{}\end{aligned}$$

Mathematica [C] time = 0.222297, size = 130, normalized size = 0.76

$$\frac{x^{m+1}\left(-\frac{\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-ax}\sqrt{-c(ax+1)}F_1\left(m+1;\frac{1}{2},-\frac{1}{2};m+2;ax,-ax\right)}{\sqrt{ax-1}\sqrt{ax+1}}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*((-2*Sqrt[1 - a*x]*Sqrt[-(c*(1 + a*x))]*AppellF1[1 + m, 1/2, -1/
2, 2 + m, a*x, -(a*x)])/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (Sqrt[c - a^2*c*x^
2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/Sqrt[1 - a^2*x^2
]))/(1 + m)
```

Maple [F] time = 0.395, size = 0, normalized size = 0.

$$\int\frac{(ax+1)^2x^m}{-a^2x^2+1}\sqrt{-a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-a^2cx^2+c}(ax+1)^2x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}(ax+1)x^m}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/(a*x - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^m\sqrt{-a^2cx^2+c}}{ax-1}\,dx-\int \frac{axx^m\sqrt{-a^2cx^2+c}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(x**m*sqrt(-a**2*c*x**2 + c)/(a*x - 1), x) - Integral(a*x*x**m*sqr
t(-a**2*c*x**2 + c)/(a*x - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-a^2cx^2+c}(ax+1)^2x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(-sqrt(-a^2*c*x^2 + c)*(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

3.1134 $\int \frac{e^{2\tanh^{-1}(ax)}x^m}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=169

$$-\frac{(2m+1)\sqrt{1-a^2x^2}x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{(m+1)\sqrt{c-a^2cx^2}}-\frac{2a(m+1)\sqrt{1-a^2x^2}x^{m+2}\text{Hypergeometric2F1}}{(m+2)\sqrt{c-a^2cx^2}}$$

```
[Out] (2*x^(1 + m)*(1 + a*x))/Sqrt[c - a^2*c*x^2] - ((1 + 2*m)*x^(1 + m)*Sqrt[1 -
 a^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*S
qrt[c - a^2*c*x^2]) - (2*a*(1 + m)*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeometr
ic2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.293024, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1806, 808, 365, 364}

$$-\frac{(2m+1)\sqrt{1-a^2x^2}x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)\sqrt{c-a^2cx^2}}-\frac{2a(m+1)\sqrt{1-a^2x^2}x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}+\frac{2(ax+1)x^{m+1}}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^m)/Sqrt[c - a^2*c*x^2],x]
```


```
[Out] (2*x^(1 + m)*(1 + a*x))/Sqrt[c - a^2*c*x^2] - ((1 + 2*m)*x^(1 + m)*Sqrt[1 -
 a^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*S
qrt[c - a^2*c*x^2]) - (2*a*(1 + m)*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeometr
ic2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x^m}{\sqrt{c-a^2cx^2}}\,dx &= c\int \frac{x^m(1+ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx \\
&= \frac{2x^{1+m}(1+ax)}{\sqrt{c-a^2cx^2}} - \int \frac{x^m(1+2m+2a(1+m)x)}{\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{2x^{1+m}(1+ax)}{\sqrt{c-a^2cx^2}} - (2a(1+m))\int \frac{x^{1+m}}{\sqrt{c-a^2cx^2}}\,dx - (1+2m)\int \frac{x^m}{\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{2x^{1+m}(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\left(2a(1+m)\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}} - \frac{\left((1+2m)\sqrt{1-a^2x^2}\right)\int \frac{x^m}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}} \\
&= \frac{2x^{1+m}(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{(1+2m)x^{1+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)\sqrt{c-a^2cx^2}} - \frac{2a(1+m)x^{2+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(\right.}{(2+m)\sqrt{c-a^2}}
\end{aligned}$$

Mathematica [C] time = 0.114016, size = 66, normalized size = 0.39

$$\frac{\sqrt{1-a^2x^2}x^{m+1}F_1\left(m+1;\frac{3}{2},-\frac{1}{2};m+2;ax,-ax\right)}{(m+1)\sqrt{ax-1}\sqrt{-c(ax+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^m)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 - a^2*x^2]*AppellF1[1 + m, 3/2, -1/2, 2 + m, a*x, -(a*x)]
)/((1 + m)*Sqrt[-1 + a*x]*Sqrt[-(c*(1 + a*x))])
```

Maple [F] time = 0.397, size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2x^m}{-a^2x^2+1}\frac{1}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2 x^m}{\sqrt{-a^2cx^2+c}\,(a^2x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)^2*x^m/(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,x^m}{a^2cx^2-2\,acx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x^m/(a^2*c*x^2 - 2*a*c*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^m}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx-\int \frac{axx^m}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(x**m/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x) -
Integral(a*x*x**m/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)^2 x^m}{\sqrt{-a^2cx^2+c}\,(a^2x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)^2*x^m/(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)), x)
```

3.1135 $\int \frac{e^{2\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=183

$$\frac{(1-2m)\sqrt{1-a^2x^2}x^{m+1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{3c(m+1)\sqrt{c-a^2cx^2}}+\frac{2a(1-m)\sqrt{1-a^2x^2}x^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\right.}{3c(m+2)\sqrt{c-a^2cx^2}}$$

```
[Out] (2*x^(1 + m)*(1 + a*x))/(3*(c - a^2*c*x^2)^(3/2)) + ((1 - 2*m)*x^(1 + m)*Sq
rt[1 - a^2*x^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(3*c
*(1 + m)*Sqrt[c - a^2*c*x^2]) + (2*a*(1 - m)*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hy
pergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(3*c*(2 + m)*Sqrt[c -
a^2*c*x^2])
```

Rubi [A] time = 0.319657, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1806, 808, 365, 364}

$$\frac{(1-2m)\sqrt{1-a^2x^2}x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{3c(m+1)\sqrt{c-a^2cx^2}}+\frac{2a(1-m)\sqrt{1-a^2x^2}x^{m+2}\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{3c(m+2)\sqrt{c-a^2cx^2}}+\frac{2(ax+1)x^{m+1}}{3\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^(3/2),x]
```


```
[Out] (2*x^(1 + m)*(1 + a*x))/(3*(c - a^2*c*x^2)^(3/2)) + ((1 - 2*m)*x^(1 + m)*Sq
rt[1 - a^2*x^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(3*c
*(1 + m)*Sqrt[c - a^2*c*x^2]) + (2*a*(1 - m)*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hy
pergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(3*c*(2 + m)*Sqrt[c -
a^2*c*x^2])
```


Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] && !GtQ[m, 0]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
```

```
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= c\int \frac{x^m(1+ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx \\
&= \frac{2x^{1+m}(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} - \frac{1}{3}\int \frac{x^m(-1+2m-2a(1-m)x)}{\left(c-a^2cx^2\right)^{3/2}}\,dx \\
&= \frac{2x^{1+m}(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{1}{3}(2a(1-m))\int \frac{x^{1+m}}{\left(c-a^2cx^2\right)^{3/2}}\,dx - \frac{1}{3}(-1+2m)\int \frac{x^m}{\left(c-a^2cx^2\right)^{3/2}}\,dx \\
&= \frac{2x^{1+m}(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(2a(1-m)\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c\sqrt{c-a^2cx^2}} - \frac{\left((-1+2m)\sqrt{1-a^2x^2}\right)\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}}\,d}{3c\sqrt{c-a^2cx^2}} \\
&= \frac{2x^{1+m}(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{(1-2m)x^{1+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{3c(1+m)\sqrt{c-a^2cx^2}} + \frac{2a(1-m)x^{2+m}\sqrt{1-a^2x^2}}{3c(2+m)\sqrt{}}
\end{aligned}$$

Mathematica [C] time = 0.241863, size = 173, normalized size = 0.95

$$\frac{x^{m+1}\left(\sqrt{ax-1}\sqrt{ax+1}\sqrt{c-acx}F_1\left(m+1;\frac{1}{2},-\frac{1}{2};m+2;-ax,ax\right)+(ax-1)\sqrt{-c(ax+1)}\left(F_1\left(m+1;\frac{1}{2},-\frac{1}{2};m+2;ax,-\right.\right.\right.}{8c^2(m+1)\sqrt{-(ax-1)^2}\sqrt{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[c - a*c*x]*AppellF1[1 + m, 1/
2, -1/2, 2 + m, -(a*x), a*x] + (-1 + a*x)*Sqrt[-(c*(1 + a*x))]*(AppellF1[1
+ m, 1/2, -1/2, 2 + m, a*x, -(a*x)] + 2*AppellF1[1 + m, 3/2, -1/2, 2 + m, a
*x, -(a*x)] + 4*AppellF1[1 + m, 5/2, -1/2, 2 + m, a*x, -(a*x)])))/(8*c^2*(1
 + m)*Sqrt[-(-1 + a*x)^2]*Sqrt[1 + a*x])
```

Maple [F] time = 0.401, size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2 x^m}{-a^2x^2+1}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2 x^m}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)^2*x^m/((-a^2*c*x^2 + c)^(3/2)*(a^2*x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,x^m}{a^4c^2x^4-2\,a^3c^2x^3+2\,ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*x^m/(a^4*c^2*x^4 - 2*a^3*c^2*x^3 + 2*a*c^2*x
 - c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^m}{-a^3cx^3\sqrt{-a^2cx^2+c}+a^2cx^2\sqrt{-a^2cx^2+c}+acx\sqrt{-a^2cx^2+c}-c\sqrt{-a^2cx^2+c}}\,dx-\int \frac{}{-a^3cx^3\sqrt{-a^2cx^2+c}+a^2cx^2\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(x**m/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt(-a**
2*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c)), x
) - Integral(a*x*x**m/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sq
rt(-a**2*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 +
 c)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)^2 x^m}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)^2*x^m/((-a^2*c*x^2 + c)^(3/2)*(a^2*x^2 - 1)), x)
```

3.1136 $\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=55

$$-\frac{2^{p+1}(ax+1)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-p-1,p,p+1,\frac{1}{2}(1-ax)\right)}{ap}$$

```
[Out] -((2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 - a*x
)/2])/(a*p*(1 + a*x)^p))
```

Rubi [A] time = 0.0629494, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6141, 678, 69}

$$-\frac{2^{p+1}(ax+1)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-1,p;p+1;\frac{1}{2}(1-ax)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 - a*x
)/2])/(a*p*(1 + a*x)^p))
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p dx = c\int (1+ax)^2\left(c-a^2cx^2\right)^{-1+p} dx$$
$$= \left(c(1+ax)^{-p}(c-acx)^{-p}\left(c-a^2cx^2\right)^p\right)\int (1+ax)^{1+p}(c-acx)^{-1+p} dx$$
$$= -\frac{2^{1+p}(1+ax)^{-p}\left(c-a^2cx^2\right)^p {}_2F_1\left(-1-p,p;1+p;\frac{1}{2}(1-ax)\right)}{ap}$$

Mathematica [A] time = 0.0178815, size = 68, normalized size = 1.24

$$-\frac{2^{p+1}(1-ax)^p\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-p-1,p,p+1,\frac{1}{2}(1-ax)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((2^(1 + p)*(1 - a*x)^p*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 +
 p, (1 - a*x)/2])/(a*p*(1 - a^2*x^2)^p))
```

Maple [F] time = 0.423, size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2\left(-a^2cx^2+c\right)^p}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^2\left(-a^2cx^2+c\right)^p}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ax+1)\left(-a^2cx^2+c\right)^p}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral(-(a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

Sympy [C] time = 10.1833, size = 653, normalized size = 11.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**p,x)
```

```
[Out] -a*Piecewise((0**p*x/a - 0**p*log(1/(a**2*x**2))/(2*a**2) + 0**p*log(-1 + 1
/(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 + c**p*x**2*gamma(p)*gamm
a(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma
(-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(
-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), 1/(a**2*x**2))/(2*a*gamma(1/
2 - p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a - 0**p*log(1/(a**2*x
**2))/(2*a**2) + 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atanh(1/(a*x))
/a**2 + c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x
**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2
*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,
), 1/(a**2*x**2))/(2*a*gamma(1/2 - p)*gamma(p + 1)), True)) - Piecewise((0*
*p*log(a**2*x**2 - 1)/(2*a) - 0**p*acoth(a*x)/a + a*c**p*x**2*gamma(p)*gamm
a(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma
(-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/
2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3
/2 - p)*gamma(p + 1)), Abs(a**2*x**2) > 1), (0**p*log(-a**2*x**2 + 1)/(2*a)
 - 0**p*atanh(a*x)/a + a*c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p
), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*
p)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 -
p), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3/2 - p)*gamma(p + 1)), True
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)^2\left(-a^2cx^2+c\right)^p}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 1), x)
```

3.1137 $\int e^{3\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=136

$$-\frac{1}{6}acx^5\sqrt{1-a^2x^2}-\frac{3}{5}cx^4\sqrt{1-a^2x^2}-\frac{23cx^3\sqrt{1-a^2x^2}}{24a}-\frac{17cx^2\sqrt{1-a^2x^2}}{15a^2}-\frac{c(345ax+544)\sqrt{1-a^2x^2}}{240a^4}+\frac{23c\sin^{-1}(ax}{16a^4}$$

```
[Out] (-17*c*x^2*Sqrt[1 - a^2*x^2])/(15*a^2) - (23*c*x^3*Sqrt[1 - a^2*x^2])/(24*a
) - (3*c*x^4*Sqrt[1 - a^2*x^2])/5 - (a*c*x^5*Sqrt[1 - a^2*x^2])/6 - (c*(544
 + 345*a*x)*Sqrt[1 - a^2*x^2])/(240*a^4) + (23*c*ArcSin[a*x])/(16*a^4)
```

Rubi [A] time = 0.27617, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6148, 1809, 833, 780, 216}

$$-\frac{1}{6}acx^5\sqrt{1-a^2x^2}-\frac{3}{5}cx^4\sqrt{1-a^2x^2}-\frac{23cx^3\sqrt{1-a^2x^2}}{24a}-\frac{17cx^2\sqrt{1-a^2x^2}}{15a^2}-\frac{c(345ax+544)\sqrt{1-a^2x^2}}{240a^4}+\frac{23c\sin^{-1}(ax}{16a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^3*(c - a^2*c*x^2),x]
```

```
[Out] (-17*c*x^2*Sqrt[1 - a^2*x^2])/(15*a^2) - (23*c*x^3*Sqrt[1 - a^2*x^2])/(24*a
) - (3*c*x^4*Sqrt[1 - a^2*x^2])/5 - (a*c*x^5*Sqrt[1 - a^2*x^2])/6 - (c*(544
 + 345*a*x)*Sqrt[1 - a^2*x^2])/(240*a^4) + (23*c*ArcSin[a*x])/(16*a^4)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)dx = c\int \frac{x^3(1+ax)^3}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{1}{6}acx^5\sqrt{1-a^2x^2} - \frac{c\int \frac{x^3\left(-6a^2-23a^3x-18a^4x^2\right)}{\sqrt{1-a^2x^2}}\,dx}{6a^2}$$
$$= -\frac{3}{5}cx^4\sqrt{1-a^2x^2} - \frac{1}{6}acx^5\sqrt{1-a^2x^2} + \frac{c\int \frac{x^3\left(102a^4+115a^5x\right)}{\sqrt{1-a^2x^2}}\,dx}{30a^4}$$
$$= -\frac{23cx^3\sqrt{1-a^2x^2}}{24a} - \frac{3}{5}cx^4\sqrt{1-a^2x^2} - \frac{1}{6}acx^5\sqrt{1-a^2x^2} - \frac{c\int \frac{x^2\left(-345a^5-408a^6x\right)}{\sqrt{1-a^2x^2}}\,dx}{120a^6}$$
$$= -\frac{17cx^2\sqrt{1-a^2x^2}}{15a^2} - \frac{23cx^3\sqrt{1-a^2x^2}}{24a} - \frac{3}{5}cx^4\sqrt{1-a^2x^2} - \frac{1}{6}acx^5\sqrt{1-a^2x^2} + \frac{c\int \frac{x(816a^6\cdots}{\sqrt{1-\cdots}}}{36\cdots}$$
$$= -\frac{17cx^2\sqrt{1-a^2x^2}}{15a^2} - \frac{23cx^3\sqrt{1-a^2x^2}}{24a} - \frac{3}{5}cx^4\sqrt{1-a^2x^2} - \frac{1}{6}acx^5\sqrt{1-a^2x^2} - \frac{c(544+34\cdots}{2\cdots}$$
$$= -\frac{17cx^2\sqrt{1-a^2x^2}}{15a^2} - \frac{23cx^3\sqrt{1-a^2x^2}}{24a} - \frac{3}{5}cx^4\sqrt{1-a^2x^2} - \frac{1}{6}acx^5\sqrt{1-a^2x^2} - \frac{c(544+34\cdots}{2\cdots}$$

Mathematica [A] time = 0.107107, size = 70, normalized size = 0.51

$$\frac{345c\sin^{-1}(ax) - c\sqrt{1-a^2x^2}\left(40a^5x^5+144a^4x^4+230a^3x^3+272a^2x^2+345ax+544\right)}{240a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^3*(c - a^2*c*x^2),x]
```

```
[Out] (-(c*Sqrt[1 - a^2*x^2]*(544 + 345*a*x + 272*a^2*x^2 + 230*a^3*x^3 + 144*a^4
*x^4 + 40*a^5*x^5)) + 345*c*ArcSin[a*x])/(240*a^4)
```

Maple [A] time = 0.083, size = 191, normalized size = 1.4

$$\frac{a^3cx^7}{6}\frac{1}{\sqrt{-a^2x^2+1}} + \frac{19\,acx^5}{24}\frac{1}{\sqrt{-a^2x^2+1}} + \frac{23\,cx^3}{48\,a}\frac{1}{\sqrt{-a^2x^2+1}} - \frac{23\,cx}{16\,a^3}\frac{1}{\sqrt{-a^2x^2+1}} + \frac{23\,c}{16\,a^3}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right) \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c),x)
```

```
[Out] 1/6*c*a^3*x^7/(-a^2*x^2+1)^(1/2)+19/24*c*a*x^5/(-a^2*x^2+1)^(1/2)+23/48*c/a
*x^3/(-a^2*x^2+1)^(1/2)-23/16*c/a^3*x/(-a^2*x^2+1)^(1/2)+23/16*c/a^3/(a^2)^
```

```
(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+3/5*c*a^2*x^6/(-a^2*x^2+1)^(
1/2)+8/15*c*x^4/(-a^2*x^2+1)^(1/2)+17/15*c*x^2/a^2/(-a^2*x^2+1)^(1/2)-34/15
*c/a^4/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.45072, size = 244, normalized size = 1.79

$$\frac{a^3cx^7}{6\sqrt{-a^2x^2+1}} + \frac{3a^2cx^6}{5\sqrt{-a^2x^2+1}} + \frac{19acx^5}{24\sqrt{-a^2x^2+1}} + \frac{8cx^4}{15\sqrt{-a^2x^2+1}} + \frac{23cx^3}{48\sqrt{-a^2x^2+1}a} + \frac{17cx^2}{15\sqrt{-a^2x^2+1}a^2} - \frac{23c}{16\sqrt{-a^2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c),x, algorithm="max
ima")
```

```
[Out] 1/6*a^3*c*x^7/sqrt(-a^2*x^2 + 1) + 3/5*a^2*c*x^6/sqrt(-a^2*x^2 + 1) + 19/24
*a*c*x^5/sqrt(-a^2*x^2 + 1) + 8/15*c*x^4/sqrt(-a^2*x^2 + 1) + 23/48*c*x^3/(
sqrt(-a^2*x^2 + 1)*a) + 17/15*c*x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 23/16*c*x/(s
qrt(-a^2*x^2 + 1)*a^3) + 23/16*c*arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^3) -
34/15*c/(sqrt(-a^2*x^2 + 1)*a^4)
```

Fricas [A] time = 2.7007, size = 220, normalized size = 1.62

$$-\frac{690\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(40\,a^5cx^5 + 144\,a^4cx^4 + 230\,a^3cx^3 + 272\,a^2cx^2 + 345\,acx + 544\,c\right)\sqrt{-a^2x^2+1}}{240\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c),x, algorithm="fri
cas")
```

```
[Out] -1/240*(690*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (40*a^5*c*x^5 + 144*
a^4*c*x^4 + 230*a^3*c*x^3 + 272*a^2*c*x^2 + 345*a*c*x + 544*c)*sqrt(-a^2*x^
2 + 1))/a^4
```

Sympy [A] time = 20.1568, size = 483, normalized size = 3.55

$$a^3c\left(\begin{cases} -\frac{ix^7}{6\sqrt{a^2x^2-1}} - \frac{ix^5}{24a^2\sqrt{a^2x^2-1}} - \frac{5ix^3}{48a^4\sqrt{a^2x^2-1}} + \frac{5ix}{16a^6\sqrt{a^2x^2-1}} - \frac{5i\operatorname{acosh}(ax)}{16a^7} & \text{for } \left|a^2x^2\right| > 1 \\ \frac{x^7}{6\sqrt{-a^2x^2+1}} + \frac{x^5}{24a^2\sqrt{-a^2x^2+1}} + \frac{5x^3}{48a^4\sqrt{-a^2x^2+1}} - \frac{5x}{16a^6\sqrt{-a^2x^2+1}} + \frac{5\operatorname{asin}(ax)}{16a^7} & \text{otherwise} \end{cases}\right) + 3a^2c\left(\begin{cases} -\frac{x^4\sqrt{-a^2x^2+1}}{5a^2} - \frac{4x^2\sqrt{}}{} \\ \frac{x^6}{6} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**3*(-a**2*c*x**2+c),x)
```

```
[Out] a**3*c*Piecewise((-I*x**7/(6*sqrt(a**2*x**2 - 1)) - I*x**5/(24*a**2*sqrt(a*
*2*x**2 - 1)) - 5*I*x**3/(48*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(16*a**6*sqr
t(a**2*x**2 - 1)) - 5*I*acosh(a*x)/(16*a**7), Abs(a**2*x**2) > 1), (x**7/(6
*sqrt(-a**2*x**2 + 1)) + x**5/(24*a**2*sqrt(-a**2*x**2 + 1)) + 5*x**3/(48*a
**4*sqrt(-a**2*x**2 + 1)) - 5*x/(16*a**6*sqrt(-a**2*x**2 + 1)) + 5*asin(a*x
)/(16*a**7), True)) + 3*a**2*c*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/(5*a**
```

```
2) - 4*x**2*sqrt(-a**2*x**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(15*a**
6), Ne(a, 0)), (x**6/6, True)) + 3*a*c*Piecewise((-I*x**5/(4*sqrt(a**2*x**2
 - 1)) - I*x**3/(8*a**2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2
 - 1)) - 3*I*acosh(a*x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*
x**2 + 1)) + x**3/(8*a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x*
*2 + 1)) + 3*asin(a*x)/(8*a**5), True)) + c*Piecewise((-x**2*sqrt(-a**2*x**
2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True
))
```

Giac [A] time = 1.16678, size = 105, normalized size = 0.77

$$-\frac{1}{240}\sqrt{-a^2x^2+1}\left(\left(2\left(\left(4\left(5\,acx+18\,c\right)x+\frac{115\,c}{a}\right)x+\frac{136\,c}{a^2}\right)x+\frac{345\,c}{a^3}\right)x+\frac{544\,c}{a^4}\right)+\frac{23\,c\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{16\,a^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c),x, algorithm="gia
c")
```

```
[Out] -1/240*sqrt(-a^2*x^2 + 1)*((2*((4*(5*a*c*x + 18*c)*x + 115*c/a)*x + 136*c/a
^2)*x + 345*c/a^3)*x + 544*c/a^4) + 23/16*c*arcsin(a*x)*sgn(a)/(a^3*abs(a))
```

3.1138 $\int e^{3\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)dx$

Optimal. Leaf size=111

$$-\frac{1}{5}acx^4\sqrt{1-a^2x^2}-\frac{3}{4}cx^3\sqrt{1-a^2x^2}-\frac{19cx^2\sqrt{1-a^2x^2}}{15a}-\frac{c(195ax+304)\sqrt{1-a^2x^2}}{120a^3}+\frac{13c\sin^{-1}(ax)}{8a^3}$$

```
[Out] (-19*c*x^2*Sqrt[1 - a^2*x^2])/(15*a) - (3*c*x^3*Sqrt[1 - a^2*x^2])/4 - (a*c
*x^4*Sqrt[1 - a^2*x^2])/5 - (c*(304 + 195*a*x)*Sqrt[1 - a^2*x^2])/(120*a^3)
 + (13*c*ArcSin[a*x])/(8*a^3)
```

Rubi [A] time = 0.24661, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6148, 1809, 833, 780, 216}

$$-\frac{1}{5}acx^4\sqrt{1-a^2x^2}-\frac{3}{4}cx^3\sqrt{1-a^2x^2}-\frac{19cx^2\sqrt{1-a^2x^2}}{15a}-\frac{c(195ax+304)\sqrt{1-a^2x^2}}{120a^3}+\frac{13c\sin^{-1}(ax)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^2*(c - a^2*c*x^2),x]
```

```
[Out] (-19*c*x^2*Sqrt[1 - a^2*x^2])/(15*a) - (3*c*x^3*Sqrt[1 - a^2*x^2])/4 - (a*c
*x^4*Sqrt[1 - a^2*x^2])/5 - (c*(304 + 195*a*x)*Sqrt[1 - a^2*x^2])/(120*a^3)
 + (13*c*ArcSin[a*x])/(8*a^3)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{3\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)dx &= c\int\frac{x^2(1+ax)^3}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{1}{5}acx^4\sqrt{1-a^2x^2}-\frac{c\int\frac{x^2\left(-5a^2-19a^3x-15a^4x^2\right)}{\sqrt{1-a^2x^2}}\,dx}{5a^2}\\
&= -\frac{3}{4}cx^3\sqrt{1-a^2x^2}-\frac{1}{5}acx^4\sqrt{1-a^2x^2}+\frac{c\int\frac{x^2\left(65a^4+76a^5x\right)}{\sqrt{1-a^2x^2}}\,dx}{20a^4}\\
&= -\frac{19cx^2\sqrt{1-a^2x^2}}{15a}-\frac{3}{4}cx^3\sqrt{1-a^2x^2}-\frac{1}{5}acx^4\sqrt{1-a^2x^2}-\frac{c\int\frac{x\left(-152a^5-195a^6x\right)}{\sqrt{1-a^2x^2}}\,dx}{60a^6}\\
&= -\frac{19cx^2\sqrt{1-a^2x^2}}{15a}-\frac{3}{4}cx^3\sqrt{1-a^2x^2}-\frac{1}{5}acx^4\sqrt{1-a^2x^2}-\frac{c(304+195ax)\sqrt{1-a^2x^2}}{120a^3}+\ldots\\
&= -\frac{19cx^2\sqrt{1-a^2x^2}}{15a}-\frac{3}{4}cx^3\sqrt{1-a^2x^2}-\frac{1}{5}acx^4\sqrt{1-a^2x^2}-\frac{c(304+195ax)\sqrt{1-a^2x^2}}{120a^3}+\ldots
\end{aligned}$$

Mathematica [A] time = 0.0861077, size = 62, normalized size = 0.56

$$\frac{195c\sin^{-1}(ax)-c\sqrt{1-a^2x^2}\left(24a^4x^4+90a^3x^3+152a^2x^2+195ax+304\right)}{120a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^2*(c - a^2*c*x^2),x]
```

```
[Out] (-(c*Sqrt[1 - a^2*x^2]*(304 + 195*a*x + 152*a^2*x^2 + 90*a^3*x^3 + 24*a^4*x
^4)) + 195*c*ArcSin[a*x])/(120*a^3)
```

Maple [A] time = 0.065, size = 170, normalized size = 1.5

$$\frac{a^3cx^6}{5}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{16\,acx^4}{15}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{19\,cx^2}{15\,a}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{38\,c}{15\,a^3}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{3\,a^2cx^5}{4}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{7\,cx^3}{8}\frac{1}{\sqrt{-a^2x^2\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c),x)
```

```
[Out] 1/5*c*a^3*x^6/(-a^2*x^2+1)^(1/2)+16/15*c*a*x^4/(-a^2*x^2+1)^(1/2)+19/15*c/a
*x^2/(-a^2*x^2+1)^(1/2)-38/15*c/a^3/(-a^2*x^2+1)^(1/2)+3/4*c*a^2*x^5/(-a^2*
x^2+1)^(1/2)+7/8*c*x^3/(-a^2*x^2+1)^(1/2)-13/8*c*x/a^2/(-a^2*x^2+1)^(1/2)+1
3/8*c/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.45622, size = 216, normalized size = 1.95

$$\frac{a^3cx^6}{5\sqrt{-a^2x^2+1}}+\frac{3\,a^2cx^5}{4\sqrt{-a^2x^2+1}}+\frac{16\,acx^4}{15\sqrt{-a^2x^2+1}}+\frac{7\,cx^3}{8\sqrt{-a^2x^2+1}}+\frac{19\,cx^2}{15\sqrt{-a^2x^2+1}a}-\frac{13\,cx}{8\sqrt{-a^2x^2+1}a^2}+\frac{13\,c\arcsin(}{8\sqrt{a^2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c),x, algorithm="max
ima")
```

```
[Out] 1/5*a^3*c*x^6/sqrt(-a^2*x^2 + 1) + 3/4*a^2*c*x^5/sqrt(-a^2*x^2 + 1) + 16/15
*a*c*x^4/sqrt(-a^2*x^2 + 1) + 7/8*c*x^3/sqrt(-a^2*x^2 + 1) + 19/15*c*x^2/(s
qrt(-a^2*x^2 + 1)*a) - 13/8*c*x/(sqrt(-a^2*x^2 + 1)*a^2) + 13/8*c*arcsin(a^
2*x/sqrt(a^2))/(sqrt(a^2)*a^2) - 38/15*c/(sqrt(-a^2*x^2 + 1)*a^3)
```

Fricas [A] time = 2.67267, size = 197, normalized size = 1.77

$$-\frac{390\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(24\,a^4cx^4+90\,a^3cx^3+152\,a^2cx^2+195\,acx+304\,c\right)\sqrt{-a^2x^2+1}}{120\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c),x, algorithm="fri
cas")
```

```
[Out] -1/120*(390*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (24*a^4*c*x^4 + 90*a
^3*c*x^3 + 152*a^2*c*x^2 + 195*a*c*x + 304*c)*sqrt(-a^2*x^2 + 1))/a^3
```

Sympy [C] time = 16.7749, size = 371, normalized size = 3.34

$$a^3c\left(\begin{cases}-\frac{x^4\sqrt{-a^2x^2+1}}{5a^2}-\frac{4x^2\sqrt{-a^2x^2+1}}{15a^4}-\frac{8\sqrt{-a^2x^2+1}}{15a^6} & \text{for } a\neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)+3a^2c\left(\begin{cases}-\frac{ix^5}{4\sqrt{a^2x^2-1}}-\frac{ix^3}{8a^2\sqrt{a^2x^2-1}}+\frac{3ix}{8a^4\sqrt{a^2x^2-1}}-\frac{3i\operatorname{acosh}(ax)}{8a^5}\\ \frac{x^5}{4\sqrt{-a^2x^2+1}}+\frac{x^3}{8a^2\sqrt{-a^2x^2+1}}-\frac{3x}{8a^4\sqrt{-a^2x^2+1}}+\frac{3\operatorname{asin}(ax)}{8a^5}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2*(-a**2*c*x**2+c),x)
```

```
[Out] a**3*c*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/(5*a**2) - 4*x**2*sqrt(-a**2*x
**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(15*a**6), Ne(a, 0)), (x**6/6,
True)) + 3*a**2*c*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a*
*2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*
x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*
a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x
)/(8*a**5), True)) + 3*a*c*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) -
 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) + c*Piecewise(
(-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2)
> 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + as
in(a*x)/(2*a**3), True))
```

Giac [A] time = 1.177, size = 93, normalized size = 0.84

$$-\frac{1}{120}\sqrt{-a^2x^2+1}\left(\left(2\left(3\,(4\,acx+15\,c)x+\frac{76\,c}{a}\right)x+\frac{195\,c}{a^2}\right)x+\frac{304\,c}{a^3}\right)+\frac{13\,c\arcsin{(ax)}\operatorname{sgn}{(a)}}{8\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c),x, algorithm="gia
c")
```

```
[Out] -1/120*sqrt(-a^2*x^2 + 1)*((2*(3*(4*a*c*x + 15*c)*x + 76*c/a)*x + 195*c/a^2
)*x + 304*c/a^3) + 13/8*c*arcsin(a*x)*sgn(a)/(a^2*abs(a))
```

3.1139 $\int e^{3\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=120

$$-\frac{c\sqrt{1-a^2x^2}(ax+1)^3}{4a^2}-\frac{c\sqrt{1-a^2x^2}(ax+1)^2}{4a^2}-\frac{5c\sqrt{1-a^2x^2}(ax+1)}{8a^2}-\frac{15c\sqrt{1-a^2x^2}}{8a^2}+\frac{15c\sin^{-1}(ax)}{8a^2}$$

```
[Out] (-15*c*Sqrt[1 - a^2*x^2])/(8*a^2) - (5*c*(1 + a*x)*Sqrt[1 - a^2*x^2])/(8*a^
2) - (c*(1 + a*x)^2*Sqrt[1 - a^2*x^2])/(4*a^2) - (c*(1 + a*x)^3*Sqrt[1 - a^
2*x^2])/(4*a^2) + (15*c*ArcSin[a*x])/(8*a^2)
```

Rubi [A] time = 0.095905, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6148, 795, 671, 641, 216}

$$-\frac{c\sqrt{1-a^2x^2}(ax+1)^3}{4a^2}-\frac{c\sqrt{1-a^2x^2}(ax+1)^2}{4a^2}-\frac{5c\sqrt{1-a^2x^2}(ax+1)}{8a^2}-\frac{15c\sqrt{1-a^2x^2}}{8a^2}+\frac{15c\sin^{-1}(ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x*(c - a^2*c*x^2),x]
```

```
[Out] (-15*c*Sqrt[1 - a^2*x^2])/(8*a^2) - (5*c*(1 + a*x)*Sqrt[1 - a^2*x^2])/(8*a^
2) - (c*(1 + a*x)^2*Sqrt[1 - a^2*x^2])/(4*a^2) - (c*(1 + a*x)^3*Sqrt[1 - a^
2*x^2])/(4*a^2) + (15*c*ArcSin[a*x])/(8*a^2)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)dx = c\int \frac{x(1+ax)^3}{\sqrt{1-a^2x^2}}dx$$
$$= -\frac{c(1+ax)^3\sqrt{1-a^2x^2}}{4a^2} + \frac{(3c)\int \frac{(1+ax)^3}{\sqrt{1-a^2x^2}}dx}{4a}$$
$$= -\frac{c(1+ax)^2\sqrt{1-a^2x^2}}{4a^2} - \frac{c(1+ax)^3\sqrt{1-a^2x^2}}{4a^2} + \frac{(5c)\int \frac{(1+ax)^2}{\sqrt{1-a^2x^2}}dx}{4a}$$
$$= -\frac{5c(1+ax)\sqrt{1-a^2x^2}}{8a^2} - \frac{c(1+ax)^2\sqrt{1-a^2x^2}}{4a^2} - \frac{c(1+ax)^3\sqrt{1-a^2x^2}}{4a^2} + \frac{(15c)\int \frac{1+ax}{\sqrt{1-a^2x^2}}d}{8a}$$
$$= -\frac{15c\sqrt{1-a^2x^2}}{8a^2} - \frac{5c(1+ax)\sqrt{1-a^2x^2}}{8a^2} - \frac{c(1+ax)^2\sqrt{1-a^2x^2}}{4a^2} - \frac{c(1+ax)^3\sqrt{1-a^2x^2}}{4a^2} +$$
$$= -\frac{15c\sqrt{1-a^2x^2}}{8a^2} - \frac{5c(1+ax)\sqrt{1-a^2x^2}}{8a^2} - \frac{c(1+ax)^2\sqrt{1-a^2x^2}}{4a^2} - \frac{c(1+ax)^3\sqrt{1-a^2x^2}}{4a^2} +$$

Mathematica [A] time = 0.0726654, size = 54, normalized size = 0.45

$$\frac{15c\sin^{-1}(ax) - c\sqrt{1-a^2x^2}\left(2a^3x^3+8a^2x^2+15ax+24\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x*(c - a^2*c*x^2),x]
```

```
[Out] (-(c*Sqrt[1 - a^2*x^2]*(24 + 15*a*x + 8*a^2*x^2 + 2*a^3*x^3)) + 15*c*ArcSin
[a*x])/(8*a^2)
```

Maple [A] time = 0.052, size = 148, normalized size = 1.2

$$\frac{a^3cx^5}{4}\frac{1}{\sqrt{-a^2x^2+1}} + \frac{13\,acx^3}{8}\frac{1}{\sqrt{-a^2x^2+1}} - \frac{15\,cx}{8\,a}\frac{1}{\sqrt{-a^2x^2+1}} + \frac{15\,c}{8\,a}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} + a^2cx^4\frac{1}{\sqrt{-a^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c),x)
```

```
[Out] 1/4*c*a^3*x^5/(-a^2*x^2+1)^(1/2)+13/8*c*a*x^3/(-a^2*x^2+1)^(1/2)-15/8*c/a*x
/(-a^2*x^2+1)^(1/2)+15/8*c/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^
(1/2))+c*a^2*x^4/(-a^2*x^2+1)^(1/2)+2*c*x^2/(-a^2*x^2+1)^(1/2)-3*c/a^2/(-a^
2*x^2+1)^(1/2)
```

Maxima [A] time = 1.43889, size = 186, normalized size = 1.55

$$\frac{a^3cx^5}{4\sqrt{-a^2x^2+1}} + \frac{a^2cx^4}{\sqrt{-a^2x^2+1}} + \frac{13\,acx^3}{8\sqrt{-a^2x^2+1}} + \frac{2\,cx^2}{\sqrt{-a^2x^2+1}} - \frac{15\,cx}{8\sqrt{-a^2x^2+1}a} + \frac{15\,c\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\sqrt{a^2}a} - \frac{3\,c}{\sqrt{-a^2x^2+1}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] 1/4*a^3*c*x^5/sqrt(-a^2*x^2 + 1) + a^2*c*x^4/sqrt(-a^2*x^2 + 1) + 13/8*a*c*
x^3/sqrt(-a^2*x^2 + 1) + 2*c*x^2/sqrt(-a^2*x^2 + 1) - 15/8*c*x/(sqrt(-a^2*x
^2 + 1)*a) + 15/8*c*arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a) - 3*c/(sqrt(-a^2*
x^2 + 1)*a^2)
```

Fricas [A] time = 2.58929, size = 166, normalized size = 1.38

$$-\frac{30\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^3cx^3+8\,a^2cx^2+15\,acx+24\,c\right)\sqrt{-a^2x^2+1}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] -1/8*(30*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^3*c*x^3 + 8*a^2*c*
x^2 + 15*a*c*x + 24*c)*sqrt(-a^2*x^2 + 1))/a^2
```

Sympy [A] time = 14.0537, size = 326, normalized size = 2.72

$$a^3c\left(\begin{cases}-\frac{ix^5}{4\sqrt{a^2x^2-1}}-\frac{ix^3}{8a^2\sqrt{a^2x^2-1}}+\frac{3ix}{8a^4\sqrt{a^2x^2-1}}-\frac{3i\operatorname{acosh}(ax)}{8a^5} & \text{for } \left|a^2x^2\right|>1\\ \frac{x^5}{4\sqrt{-a^2x^2+1}}+\frac{x^3}{8a^2\sqrt{-a^2x^2+1}}-\frac{3x}{8a^4\sqrt{-a^2x^2+1}}+\frac{3\operatorname{asin}(ax)}{8a^5} & \text{otherwise}\end{cases}\right)+3a^2c\left(\begin{cases}-\frac{x^2\sqrt{-a^2x^2+1}}{3a^2}-\frac{2\sqrt{-a^2x^2+1}}{3a^4} & \text{for } a\neq\\ \frac{x^4}{4} & \text{otherwi}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x*(-a**2*c*x**2+c),x)
```

```
[Out] a**3*c*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(a**
2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a**5)
, Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqrt(-
a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**5),
 True)) + 3*a**2*c*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(
-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) + 3*a*c*Piecewise((-I*
x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1)
, (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a
*x)/(2*a**3), True)) + c*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2
 + 1)/a**2, True))
```

Giac [A] time = 1.16988, size = 78, normalized size = 0.65

$$-\frac{1}{8}\sqrt{-a^2x^2+1}\left(\left(2\,(acx+4\,c)x+\frac{15\,c}{a}\right)x+\frac{24\,c}{a^2}\right)+\frac{15\,c\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{8\,a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] -1/8*sqrt(-a^2*x^2 + 1)*((2*(a*c*x + 4*c)*x + 15*c/a)*x + 24*c/a^2) + 15/8*
c*arcsin(a*x)*sgn(a)/(a*abs(a))
```

3.1140 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=91

$$-\frac{c\sqrt{1-a^2x^2}(ax+1)^2}{3a}-\frac{5c\sqrt{1-a^2x^2}(ax+1)}{6a}-\frac{5c\sqrt{1-a^2x^2}}{2a}+\frac{5c\sin^{-1}(ax)}{2a}$$

```
[Out] (-5*c*Sqrt[1 - a^2*x^2])/(2*a) - (5*c*(1 + a*x)*Sqrt[1 - a^2*x^2])/(6*a) -
(c*(1 + a*x)^2*Sqrt[1 - a^2*x^2])/(3*a) + (5*c*ArcSin[a*x])/(2*a)
```

Rubi [A] time = 0.0568851, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6138, 671, 641, 216}

$$-\frac{c\sqrt{1-a^2x^2}(ax+1)^2}{3a}-\frac{5c\sqrt{1-a^2x^2}(ax+1)}{6a}-\frac{5c\sqrt{1-a^2x^2}}{2a}+\frac{5c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] (-5*c*Sqrt[1 - a^2*x^2])/(2*a) - (5*c*(1 + a*x)*Sqrt[1 - a^2*x^2])/(6*a) -
(c*(1 + a*x)^2*Sqrt[1 - a^2*x^2])/(3*a) + (5*c*ArcSin[a*x])/(2*a)
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = c\int \frac{(1+ax)^3}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c(1+ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{1}{3}(5c)\int \frac{(1+ax)^2}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{5c(1+ax)\sqrt{1-a^2x^2}}{6a}-\frac{c(1+ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{1}{2}(5c)\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{5c\sqrt{1-a^2x^2}}{2a}-\frac{5c(1+ax)\sqrt{1-a^2x^2}}{6a}-\frac{c(1+ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{1}{2}(5c)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{5c\sqrt{1-a^2x^2}}{2a}-\frac{5c(1+ax)\sqrt{1-a^2x^2}}{6a}-\frac{c(1+ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{5c\sin^{-1}(ax)}{2a}$$

Mathematica [A] time = 0.0476712, size = 57, normalized size = 0.63

$$-\frac{c\left(\sqrt{1-a^2x^2}\left(2a^2x^2+9ax+22\right)+30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -(c*(Sqrt[1 - a^2*x^2]*(22 + 9*a*x + 2*a^2*x^2) + 30*ArcSin[Sqrt[1 - a*x]/S
qrt[2]]))/(6*a)
```

Maple [A] time = 0.049, size = 125, normalized size = 1.4

$$\frac{a^3cx^4}{3}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{10\,acx^2}{3}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{11\,c}{3\,a}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{3\,a^2cx^3}{2}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{3\,cx}{2}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{5\,c}{2}\arctan\left(x\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c),x)
```

```
[Out] 1/3*c*a^3*x^4/(-a^2*x^2+1)^(1/2)+10/3*c*a*x^2/(-a^2*x^2+1)^(1/2)-11/3*c/a/(
-a^2*x^2+1)^(1/2)+3/2*c*a^2*x^3/(-a^2*x^2+1)^(1/2)-3/2*c*x/(-a^2*x^2+1)^(1/
2)+5/2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.46553, size = 155, normalized size = 1.7

$$\frac{a^3cx^4}{3\sqrt{-a^2x^2+1}}+\frac{3\,a^2cx^3}{2\sqrt{-a^2x^2+1}}+\frac{10\,acx^2}{3\sqrt{-a^2x^2+1}}-\frac{3\,cx}{2\sqrt{-a^2x^2+1}}+\frac{5\,c\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}}-\frac{11\,c}{3\sqrt{-a^2x^2+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c),x, algorithm="maxima"
)
```

```
[Out] 1/3*a^3*c*x^4/sqrt(-a^2*x^2 + 1) + 3/2*a^2*c*x^3/sqrt(-a^2*x^2 + 1) + 10/3*
a*c*x^2/sqrt(-a^2*x^2 + 1) - 3/2*c*x/sqrt(-a^2*x^2 + 1) + 5/2*c*arcsin(a^2*
```

```
x/sqrt(a^2))/sqrt(a^2) - 11/3*c/(sqrt(-a^2*x^2 + 1)*a)
```

Fricas [A] time = 2.63829, size = 143, normalized size = 1.57

$$-\frac{30\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^2cx^2+9\,acx+22\,c\right)\sqrt{-a^2x^2+1}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c),x, algorithm="fricas"
)
```

```
[Out] -1/6*(30*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^2*c*x^2 + 9*a*c*x
+ 22*c)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [A] time = 10.9856, size = 218, normalized size = 2.4

$$a^3c\left(\begin{cases}-\frac{x^2\sqrt{-a^2x^2+1}}{3a^2}-\frac{2\sqrt{-a^2x^2+1}}{3a^4} & \text{for } a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)+3a^2c\left(\begin{cases}-\frac{ix\sqrt{a^2x^2-1}}{2a^2}-\frac{i\operatorname{acosh}(ax)}{2a^3} & \text{for } \left|a^2x^2\right|>1\\ \frac{x^3}{2\sqrt{-a^2x^2+1}}-\frac{x}{2a^2\sqrt{-a^2x^2+1}}+\frac{\operatorname{asin}(ax)}{2a^3} & \text{otherwise}\end{cases}\right)+3ac\left(\begin{cases}\frac{x^2}{2}\\ -\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c),x)
```

```
[Out] a**3*c*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 +
 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) + 3*a**2*c*Piecewise((-I*x*sqrt(a*
*2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1), (x**3/(
2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(2*a*
*3), True)) + 3*a*c*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)
/a**2, True)) + c*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (
sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0))
```

Giac [A] time = 1.2289, size = 62, normalized size = 0.68

$$\frac{5\,c\arcsin(ax)\operatorname{sgn}(a)}{2\,|a|}-\frac{1}{6}\sqrt{-a^2x^2+1}\left((2\,acx+9\,c)x+\frac{22\,c}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 5/2*c*arcsin(a*x)*sgn(a)/abs(a) - 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c*x + 9*c)*x
 + 22*c/a)
```

3.1141 $\int \frac{e^{3\tanh^{-1}(ax)}(c-a^2cx^2)}{x} dx$

Optimal. Leaf size=66

$$-\frac{1}{2}acx\sqrt{1-a^2x^2}-3c\sqrt{1-a^2x^2}-c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{7}{2}c\sin^{-1}(ax)$$

```
[Out] -3*c*Sqrt[1 - a^2*x^2] - (a*c*x*Sqrt[1 - a^2*x^2])/2 + (7*c*ArcSin[a*x])/2
- c*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.19502, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 1809, 844, 216, 266, 63, 208}

$$-\frac{1}{2}acx\sqrt{1-a^2x^2}-3c\sqrt{1-a^2x^2}-c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{7}{2}c\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x,x]
```

```
[Out] -3*c*Sqrt[1 - a^2*x^2] - (a*c*x*Sqrt[1 - a^2*x^2])/2 + (7*c*ArcSin[a*x])/2
- c*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x}\,dx &= c\int \frac{(1+ax)^3}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{1}{2}acx\sqrt{1-a^2x^2}-\frac{c\int \frac{-2a^2-7a^3x-6a^4x^2}{x\sqrt{1-a^2x^2}}\,dx}{2a^2}\\
&= -3c\sqrt{1-a^2x^2}-\frac{1}{2}acx\sqrt{1-a^2x^2}+\frac{c\int \frac{2a^4+7a^5x}{x\sqrt{1-a^2x^2}}\,dx}{2a^4}\\
&= -3c\sqrt{1-a^2x^2}-\frac{1}{2}acx\sqrt{1-a^2x^2}+c\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx+\frac{1}{2}(7ac)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= -3c\sqrt{1-a^2x^2}-\frac{1}{2}acx\sqrt{1-a^2x^2}+\frac{7}{2}c\sin^{-1}(ax)+\frac{1}{2}c\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= -3c\sqrt{1-a^2x^2}-\frac{1}{2}acx\sqrt{1-a^2x^2}+\frac{7}{2}c\sin^{-1}(ax)-\frac{c\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2}\\
&= -3c\sqrt{1-a^2x^2}-\frac{1}{2}acx\sqrt{1-a^2x^2}+\frac{7}{2}c\sin^{-1}(ax)-c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0448069, size = 49, normalized size = 0.74

$$
-\frac{1}{2}c\left(\sqrt{1-a^2x^2}(ax+6)+2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-7\sin^{-1}(ax)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x,x]
```

```
[Out] -(c*((6 + a*x)*Sqrt[1 - a^2*x^2] - 7*ArcSin[a*x] + 2*ArcTanh[Sqrt[1 - a^2*x
^2]]))/2
```

Maple [B] time = 0.043, size = 121, normalized size = 1.8

$$
\frac{a^3cx^3}{2}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{acx}{2}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{7\,ac}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+3\frac{a^2cx^2}{\sqrt{-a^2x^2+1}}-3\frac{c}{\sqrt{-a^2x^2+1}}-c\mathrm{Art}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x,x)
```

```
[Out] 1/2*c*a^3*x^3/(-a^2*x^2+1)^(1/2)-1/2*c*a*x/(-a^2*x^2+1)^(1/2)+7/2*c*a/(a^2)
^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+3*c*a^2*x^2/(-a^2*x^2+1)^(1
/2)-3*c/(-a^2*x^2+1)^(1/2)-c*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [B] time = 1.47956, size = 167, normalized size = 2.53

$$\frac{a^3cx^3}{2\sqrt{-a^2x^2+1}}+\frac{3\,a^2cx^2}{\sqrt{-a^2x^2+1}}-\frac{acx}{2\sqrt{-a^2x^2+1}}+\frac{7\,ac\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}}-c\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{3\,c}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x,x, algorithm="maxim
a")
```

```
[Out] 1/2*a^3*c*x^3/sqrt(-a^2*x^2 + 1) + 3*a^2*c*x^2/sqrt(-a^2*x^2 + 1) - 1/2*a*c
*x/sqrt(-a^2*x^2 + 1) + 7/2*a*c*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - c*log(2
*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 3*c/sqrt(-a^2*x^2 + 1)
```

Fricas [A] time = 2.64576, size = 162, normalized size = 2.45

$$-7\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+c\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\frac{1}{2}\sqrt{-a^2x^2+1}(acx+6\,c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x,x, algorithm="frica
s")
```

```
[Out] -7*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + c*log((sqrt(-a^2*x^2 + 1) - 1
)/x) - 1/2*sqrt(-a^2*x^2 + 1)*(a*c*x + 6*c)
```

Sympy [C] time = 10.2942, size = 197, normalized size = 2.98

$$a^3c\left(\begin{cases}-\frac{ix\sqrt{a^2x^2-1}}{2a^2}-\frac{i\operatorname{acosh}(ax)}{2a^3} & \text{for } |a^2x^2|>1\\ \frac{x^3}{2\sqrt{-a^2x^2+1}}-\frac{x}{2a^2\sqrt{-a^2x^2+1}}+\frac{\operatorname{asin}(ax)}{2a^3} & \text{otherwise}\end{cases}\right)+3a^2c\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)+3ac\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right)\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right.\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)/x,x)
```

```
[Out] a**3*c*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3)
, Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**
2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) + 3*a**2*c*Piecewise((x**2/2, Eq(
a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) + 3*a*c*Piecewise((sqrt(a**(
-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a*
*2 < 0)) + c*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(
```

```
a*x)), True))
```

Giac [A] time = 1.18742, size = 103, normalized size = 1.56

$$\frac{7\,ac\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{2\,|a|}-\frac{ac\log\left(\frac{\left|-2\,\sqrt{-a^{2}x^{2}+1}|a|-2\,a\right|}{2\,a^{2}|x|}\right)}{|a|}-\frac{1}{2}\,\sqrt{-a^{2}x^{2}+1}(acx+6\,c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x,x, algorithm="giac"
)
```

```
[Out] 7/2*a*c*arcsin(a*x)*sgn(a)/abs(a) - a*c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*a
bs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/2*sqrt(-a^2*x^2 + 1)*(a*c*x + 6*c)
```

3.1142 $\int \frac{e^{3\tanh^{-1}(ax)}(c-a^2cx^2)}{x^2}\,dx$

Optimal. Leaf size=66

$$-ac\sqrt{1-a^2x^2}-\frac{c\sqrt{1-a^2x^2}}{x}-3ac\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+3ac\sin^{-1}(ax)$$

```
[Out] -(a*c*Sqrt[1 - a^2*x^2]) - (c*Sqrt[1 - a^2*x^2])/x + 3*a*c*ArcSin[a*x] - 3*
a*c*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.191489, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6148, 1807, 1809, 844, 216, 266, 63, 208}

$$-ac\sqrt{1-a^2x^2}-\frac{c\sqrt{1-a^2x^2}}{x}-3ac\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+3ac\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^2,x]
```

```
[Out] -(a*c*Sqrt[1 - a^2*x^2]) - (c*Sqrt[1 - a^2*x^2])/x + 3*a*c*ArcSin[a*x] - 3*
a*c*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^2}\,dx &= c\int \frac{(1+ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c\sqrt{1-a^2x^2}}{x} - c\int \frac{-3a-3a^2x-a^3x^2}{x\sqrt{1-a^2x^2}}\,dx\\
&= -ac\sqrt{1-a^2x^2} - \frac{c\sqrt{1-a^2x^2}}{x} + \frac{c\int \frac{3a^3+3a^4x}{x\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= -ac\sqrt{1-a^2x^2} - \frac{c\sqrt{1-a^2x^2}}{x} + (3ac)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx + \left(3a^2c\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= -ac\sqrt{1-a^2x^2} - \frac{c\sqrt{1-a^2x^2}}{x} + 3ac\sin^{-1}(ax) + \frac{1}{2}(3ac)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)\\
&= -ac\sqrt{1-a^2x^2} - \frac{c\sqrt{1-a^2x^2}}{x} + 3ac\sin^{-1}(ax) - \frac{(3c)\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a}\\
&= -ac\sqrt{1-a^2x^2} - \frac{c\sqrt{1-a^2x^2}}{x} + 3ac\sin^{-1}(ax) - 3ac\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0631801, size = 52, normalized size = 0.79

$$
c\left(-\frac{\sqrt{1-a^2x^2}(ax+1)}{x} - 3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) + 3a\sin^{-1}(ax)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^2,x]
```

```
[Out] c*(-(((1 + a*x)*Sqrt[1 - a^2*x^2])/x) + 3*a*ArcSin[a*x] - 3*a*ArcTanh[Sqrt[
1 - a^2*x^2]])
```

Maple [B] time = 0.043, size = 122, normalized size = 1.9

$$a^3cx^2\frac{1}{\sqrt{-a^2x^2+1}} - ac\frac{1}{\sqrt{-a^2x^2+1}} + cxa^2\frac{1}{\sqrt{-a^2x^2+1}} + 3\frac{a^2c}{\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right) - \frac{c}{x}\frac{1}{\sqrt{-a^2x^2+1}} - 3\,ca\mathrm{Artanh}\left(\frac{}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^2,x)
```

```
[Out] c*a^3*x^2/(-a^2*x^2+1)^(1/2)-c*a/(-a^2*x^2+1)^(1/2)+c*a^2*x/(-a^2*x^2+1)^(1
/2)+3*c*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-c/x/(-a^2*
x^2+1)^(1/2)-3*c*a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [B] time = 1.45713, size = 169, normalized size = 2.56

$$\frac{a^3cx^2}{\sqrt{-a^2x^2+1}} + \frac{a^2cx}{\sqrt{-a^2x^2+1}} + \frac{3\,a^2c\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}} - 3\,ac\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{ac}{\sqrt{-a^2x^2+1}} - \frac{c}{\sqrt{-a^2x^2+1}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^2,x, algorithm="max
ima")
```

```
[Out] a^3*c*x^2/sqrt(-a^2*x^2 + 1) + a^2*c*x/sqrt(-a^2*x^2 + 1) + 3*a^2*c*arcsin(
a^2*x/sqrt(a^2))/sqrt(a^2) - 3*a*c*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(
x)) - a*c/sqrt(-a^2*x^2 + 1) - c/(sqrt(-a^2*x^2 + 1)*x)
```

Fricas [A] time = 2.61367, size = 184, normalized size = 2.79

$$-\frac{6\,acx\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - 3\,acx\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) + acx + \sqrt{-a^2x^2+1}(acx+c)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^2,x, algorithm="fri
cas")
```

```
[Out] -(6*a*c*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 3*a*c*x*log((sqrt(-a^2*x
^2 + 1) - 1)/x) + a*c*x + sqrt(-a^2*x^2 + 1)*(a*c*x + c))/x
```

Sympy [C] time = 9.7716, size = 150, normalized size = 2.27

$$a^3c\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right) + 3a^2c\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right) + 3ac\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right) + c\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)/x**2,x)
```

```
[Out] a**3*c*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
 + 3*a**2*c*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-
1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + 3*a*c*Piecewise((-acosh(1/(a*x))
, 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True)) + c*Piecewise((-I*sqrt(a*
*2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))
```

Giac [B] time = 1.15961, size = 177, normalized size = 2.68

$$\frac{a^{4}cx}{2\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)|a|}+\frac{3\,a^{2}c\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{|a|}-\frac{3\,a^{2}c\log\left(\frac{\left|-2\,\sqrt{-a^{2}x^{2}+1}|a|-2\,a\right|}{2\,a^{2}|x|}\right)}{|a|}-\sqrt{-a^{2}x^{2}+1}ac-\frac{\left(\sqrt{-a^{2}x^{2}+1}|a|\right.}{2\,x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^2,x, algorithm="gia
c")
```

```
[Out] 1/2*a^4*c*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) + 3*a^2*c*arcsin(a*x)*
sgn(a)/abs(a) - 3*a^2*c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^
2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*a*c - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a)
 + a)*c/(x*abs(a))
```

3.1143 $\int \frac{e^{3\tanh^{-1}(ax)}(c-a^2cx^2)}{x^3}\,dx$

Optimal. Leaf size=76

$$-\frac{3ac\sqrt{1-a^2x^2}}{x}-\frac{c\sqrt{1-a^2x^2}}{2x^2}-\frac{7}{2}a^2c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+a^2c\sin^{-1}(ax)$$

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(2*x^2) - (3*a*c*Sqrt[1 - a^2*x^2])/x + a^2*c*ArcSin
[a*x] - (7*a^2*c*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rubi [A] time = 0.194184, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 1807, 844, 216, 266, 63, 208}

$$-\frac{3ac\sqrt{1-a^2x^2}}{x}-\frac{c\sqrt{1-a^2x^2}}{2x^2}-\frac{7}{2}a^2c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+a^2c\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^3,x]
```

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(2*x^2) - (3*a*c*Sqrt[1 - a^2*x^2])/x + a^2*c*ArcSin
[a*x] - (7*a^2*c*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^3}\,dx &= c\int \frac{(1+ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2}-\frac{1}{2}c\int \frac{-6a-7a^2x-2a^3x^2}{x^2\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2}-\frac{3ac\sqrt{1-a^2x^2}}{x}+\frac{1}{2}c\int \frac{7a^2+2a^3x}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2}-\frac{3ac\sqrt{1-a^2x^2}}{x}+\frac{1}{2}\left(7a^2c\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx+\left(a^3c\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2}-\frac{3ac\sqrt{1-a^2x^2}}{x}+a^2c\sin^{-1}(ax)+\frac{1}{4}\left(7a^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,\right.\\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2}-\frac{3ac\sqrt{1-a^2x^2}}{x}+a^2c\sin^{-1}(ax)-\frac{1}{2}(7c)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a}\right.\\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2}-\frac{3ac\sqrt{1-a^2x^2}}{x}+a^2c\sin^{-1}(ax)-\frac{7}{2}a^2c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0741761, size = 60, normalized size = 0.79

$$\frac{1}{2}c\left(-\frac{(6ax+1)\sqrt{1-a^2x^2}}{x^2}-7a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+2a^2\sin^{-1}(ax)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^3,x]
```

```
[Out] (c*(-(((1 + 6*a*x)*Sqrt[1 - a^2*x^2])/x^2) + 2*a^2*ArcSin[a*x] - 7*a^2*ArcT
anh[Sqrt[1 - a^2*x^2]]))/2
```

Maple [A] time = 0.045, size = 125, normalized size = 1.6

$$3\frac{a^3cx}{\sqrt{-a^2x^2+1}}+a^3c\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{a^2c}{2}\frac{1}{\sqrt{-a^2x^2+1}}-3\frac{ac}{x\sqrt{-a^2x^2+1}}-\frac{7a^2c}{2}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^3,x)
```

```
[Out] 3*c*a^3*x/(-a^2*x^2+1)^(1/2)+c*a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x
^2+1)^(1/2))+1/2*c*a^2/(-a^2*x^2+1)^(1/2)-3*c*a/x/(-a^2*x^2+1)^(1/2)-7/2*c*
a^2*arctanh(1/(-a^2*x^2+1)^(1/2))-1/2*c/x^2/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.46518, size = 173, normalized size = 2.28

$$\frac{3\,a^3cx}{\sqrt{-a^2x^2+1}}+\frac{a^3c\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}-\frac{7}{2}\,a^2c\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{a^2c}{2\sqrt{-a^2x^2+1}}-\frac{3\,ac}{\sqrt{-a^2x^2+1}x}-\frac{c}{2\sqrt{-a^2x^2+1}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^3,x, algorithm="max
ima")
```

```
[Out] 3*a^3*c*x/sqrt(-a^2*x^2 + 1) + a^3*c*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - 7/
2*a^2*c*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 1/2*a^2*c/sqrt(-a^2*x
^2 + 1) - 3*a*c/(sqrt(-a^2*x^2 + 1)*x) - 1/2*c/(sqrt(-a^2*x^2 + 1)*x^2)
```

Fricas [A] time = 2.68276, size = 194, normalized size = 2.55

$$-\frac{4\,a^2cx^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-7\,a^2cx^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}(6\,acx+c)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^3,x, algorithm="fri
cas")
```

```
[Out] -1/2*(4*a^2*c*x^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 7*a^2*c*x^2*log(
(sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1)*(6*a*c*x + c))/x^2
```

Sympy [C] time = 7.5597, size = 223, normalized size = 2.93

$$a^3c\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for}\: a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\: a^2<0\end{cases}\right)+3a^2c\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\: \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)+3ac\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\: |a^2x^2|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)/x**3,x)
```

```
[Out] a**3*c*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**
2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + 3*a**2*c*Piecewise((-acosh(1/(a*x)),
1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True)) + 3*a*c*Piecewise((-I*sqrt(
a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True)) + c
```

```
*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Ab
s(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2
))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))
```

Giac [B] time = 1.18294, size = 242, normalized size = 3.18

$$\frac{a^3c\arcsin(ax)\operatorname{sgn}(a)}{|a|}+\frac{\left(a^3c+\frac{12\left(\sqrt{-a^2x^2+1}|a|+a\right)ac}{x}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}|a|+a\right)^2|a|}-\frac{7a^3c\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{2|a|}-\frac{\frac{12\left(\sqrt{-a^2x^2+1}|a|+a\right)ac|a|}{x}+\left(\sqrt{-a^2\cdots}\right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^3,x, algorithm="gia
c")
```

```
[Out] a^3*c*arcsin(a*x)*sgn(a)/abs(a) + 1/8*(a^3*c + 12*(sqrt(-a^2*x^2 + 1)*abs(a
) + a)*a*c/x)*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)) - 7/2*a^3*
c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/
8*(12*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a*c*abs(a)/x + (sqrt(-a^2*x^2 + 1)*ab
s(a) + a)^2*c*abs(a)/(a*x^2))/a^2
```

3.1144 $\int \frac{e^{3\tanh^{-1}(ax)}(c-a^2cx^2)}{x^4}\,dx$

Optimal. Leaf size=94

$$-\frac{11a^2c\sqrt{1-a^2x^2}}{3x}-\frac{3ac\sqrt{1-a^2x^2}}{2x^2}-\frac{c\sqrt{1-a^2x^2}}{3x^3}-\frac{5}{2}a^3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(3*x^3) - (3*a*c*Sqrt[1 - a^2*x^2])/(2*x^2) - (11*a^
2*c*Sqrt[1 - a^2*x^2])/(3*x) - (5*a^3*c*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rubi [A] time = 0.20036, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6148, 1807, 807, 266, 63, 208}

$$-\frac{11a^2c\sqrt{1-a^2x^2}}{3x}-\frac{3ac\sqrt{1-a^2x^2}}{2x^2}-\frac{c\sqrt{1-a^2x^2}}{3x^3}-\frac{5}{2}a^3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^4,x]
```


```
[Out] -(c*Sqrt[1 - a^2*x^2])/(3*x^3) - (3*a*c*Sqrt[1 - a^2*x^2])/(2*x^2) - (11*a^
2*c*Sqrt[1 - a^2*x^2])/(3*x) - (5*a^3*c*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```


Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^4}\,dx &= c\int \frac{(1+ax)^3}{x^4\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c\sqrt{1-a^2x^2}}{3x^3}-\frac{1}{3}c\int \frac{-9a-11a^2x-3a^3x^2}{x^3\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c\sqrt{1-a^2x^2}}{3x^3}-\frac{3ac\sqrt{1-a^2x^2}}{2x^2}+\frac{1}{6}c\int \frac{22a^2+15a^3x}{x^2\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c\sqrt{1-a^2x^2}}{3x^3}-\frac{3ac\sqrt{1-a^2x^2}}{2x^2}-\frac{11a^2c\sqrt{1-a^2x^2}}{3x}+\frac{1}{2}\left(5a^3c\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c\sqrt{1-a^2x^2}}{3x^3}-\frac{3ac\sqrt{1-a^2x^2}}{2x^2}-\frac{11a^2c\sqrt{1-a^2x^2}}{3x}+\frac{1}{4}\left(5a^3c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,\right.\\
&= -\frac{c\sqrt{1-a^2x^2}}{3x^3}-\frac{3ac\sqrt{1-a^2x^2}}{2x^2}-\frac{11a^2c\sqrt{1-a^2x^2}}{3x}-\frac{1}{2}(5ac)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1}\right.\\
&= -\frac{c\sqrt{1-a^2x^2}}{3x^3}-\frac{3ac\sqrt{1-a^2x^2}}{2x^2}-\frac{11a^2c\sqrt{1-a^2x^2}}{3x}-\frac{5}{2}a^3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0615224, size = 60, normalized size = 0.64

$$-\frac{c\sqrt{1-a^2x^2}\left(22a^2x^2+9ax+2\right)}{6x^3}-\frac{5}{2}a^3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^4,x]
```

```
[Out] -(c*Sqrt[1 - a^2*x^2]*(2 + 9*a*x + 22*a^2*x^2))/(6*x^3) - (5*a^3*c*ArcTanh[
Sqrt[1 - a^2*x^2]])/2
```

Maple [B] time = 0.042, size = 184, normalized size = 2.

$$-c\left(a^3\frac{1}{\sqrt{-a^2x^2+1}}+3\frac{a^4x}{\sqrt{-a^2x^2+1}}-\frac{10a^2}{3}\left(-\frac{1}{x}\frac{1}{\sqrt{-a^2x^2+1}}+2\frac{a^2x}{\sqrt{-a^2x^2+1}}\right)+2a^3\left(\frac{1}{\sqrt{-a^2x^2+1}}-\text{Artanh}\left(\frac{1}{\sqrt{-a^2x}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^4,x)
```

```
[Out] -c*(a^3/(-a^2*x^2+1)^(1/2)+3*a^4*x/(-a^2*x^2+1)^(1/2)-10/3*a^2*(-1/x/(-a^2*
x^2+1)^(1/2)+2*a^2*x/(-a^2*x^2+1)^(1/2))+2*a^3*(1/(-a^2*x^2+1)^(1/2)-arctan
h(1/(-a^2*x^2+1)^(1/2)))-3*a*(-1/2/x^2/(-a^2*x^2+1)^(1/2)+3/2*a^2*(1/(-a^2*
x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2))))+1/3/x^3/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 0.975372, size = 173, normalized size = 1.84

$$\frac{11\,a^4cx}{3\,\sqrt{-a^2x^2+1}} - \frac{5}{2}\,a^3c\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{3\,a^3c}{2\,\sqrt{-a^2x^2+1}} - \frac{10\,a^2c}{3\,\sqrt{-a^2x^2+1}x} - \frac{3\,ac}{2\,\sqrt{-a^2x^2+1}x^2} - \frac{c}{3\,\sqrt{-a^2x^2+1}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^4,x, algorithm="max
ima")
```

```
[Out] 11/3*a^4*c*x/sqrt(-a^2*x^2 + 1) - 5/2*a^3*c*log(2*sqrt(-a^2*x^2 + 1)/abs(x)
 + 2/abs(x)) + 3/2*a^3*c/sqrt(-a^2*x^2 + 1) - 10/3*a^2*c/(sqrt(-a^2*x^2 + 1
)*x) - 3/2*a*c/(sqrt(-a^2*x^2 + 1)*x^2) - 1/3*c/(sqrt(-a^2*x^2 + 1)*x^3)
```

Fricas [A] time = 2.6251, size = 146, normalized size = 1.55

$$\frac{15\,a^3cx^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \left(22\,a^2cx^2 + 9\,acx + 2\,c\right)\sqrt{-a^2x^2+1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^4,x, algorithm="fri
cas")
```

```
[Out] 1/6*(15*a^3*c*x^3*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (22*a^2*c*x^2 + 9*a*c*x
 + 2*c)*sqrt(-a^2*x^2 + 1))/x^3
```

Sympy [C] time = 17.3745, size = 267, normalized size = 2.84

$$a^3c\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise} \end{cases}\right) + 3a^2c\left(\begin{cases} -\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\ |a^2x^2| > 1 \\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise} \end{cases}\right) + 3ac\left(\begin{cases} -\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2} - \frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} \\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2} - \frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}} + \frac{i}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)/x**4,x)
```

```
[Out] a**3*c*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)),
 True)) + 3*a**2*c*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1)
, (-sqrt(-a**2*x**2 + 1)/x, True)) + 3*a*c*Piecewise((-a**2*acosh(1/(a*x))/
2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1
/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**
2*x**2))), True)) + c*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sq
```

```
rt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 +
 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), True))
```

Giac [B] time = 1.16526, size = 294, normalized size = 3.13

$$\frac{\left(a^4c+\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)a^2c}{x}+\frac{45\left(\sqrt{-a^2x^2+1}|a|+a\right)^2c}{x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|}-\frac{5a^4c\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{2|a|}-\frac{\frac{45\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4c}{x}+\frac{9\left(\sqrt{-a^2x^2+1}|a\right.}{x^2}}{24a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^4,x, algorithm="gia
c")
```

```
[Out] 1/24*(a^4*c + 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2*c/x + 45*(sqrt(-a^2*x^2
 + 1)*abs(a) + a)^2*c/x^2)*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*abs(a
)) - 5/2*a^4*c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x))
)/abs(a) - 1/24*(45*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c/x + 9*(sqrt(-a^2*
x^2 + 1)*abs(a) + a)^2*a^2*c/x^2 + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c/x^3)
/(a^2*abs(a))
```

3.1145 $\int \frac{e^{3\tanh^{-1}(ax)}(c-a^2cx^2)}{x^5}\,dx$

Optimal. Leaf size=115

$$-\frac{3a^3c\sqrt{1-a^2x^2}}{x}-\frac{15a^2c\sqrt{1-a^2x^2}}{8x^2}-\frac{ac\sqrt{1-a^2x^2}}{x^3}-\frac{c\sqrt{1-a^2x^2}}{4x^4}-\frac{15}{8}a^4c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(4*x^4) - (a*c*Sqrt[1 - a^2*x^2])/x^3 - (15*a^2*c*Sq
rt[1 - a^2*x^2])/(8*x^2) - (3*a^3*c*Sqrt[1 - a^2*x^2])/x - (15*a^4*c*ArcTan
h[Sqrt[1 - a^2*x^2]])/8
```

Rubi [A] time = 0.223379, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 1807, 835, 807, 266, 63, 208}

$$-\frac{3a^3c\sqrt{1-a^2x^2}}{x}-\frac{15a^2c\sqrt{1-a^2x^2}}{8x^2}-\frac{ac\sqrt{1-a^2x^2}}{x^3}-\frac{c\sqrt{1-a^2x^2}}{4x^4}-\frac{15}{8}a^4c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^5,x]
```

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(4*x^4) - (a*c*Sqrt[1 - a^2*x^2])/x^3 - (15*a^2*c*Sq
rt[1 - a^2*x^2])/(8*x^2) - (3*a^3*c*Sqrt[1 - a^2*x^2])/x - (15*a^4*c*ArcTan
h[Sqrt[1 - a^2*x^2]])/8
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^5}\,dx = c\int \frac{(1+ax)^3}{x^5\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c\sqrt{1-a^2x^2}}{4x^4}-\frac{1}{4}c\int \frac{-12a-15a^2x-4a^3x^2}{x^4\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c\sqrt{1-a^2x^2}}{4x^4}-\frac{ac\sqrt{1-a^2x^2}}{x^3}+\frac{1}{12}c\int \frac{45a^2+36a^3x}{x^3\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c\sqrt{1-a^2x^2}}{4x^4}-\frac{ac\sqrt{1-a^2x^2}}{x^3}-\frac{15a^2c\sqrt{1-a^2x^2}}{8x^2}-\frac{1}{24}c\int \frac{-72a^3-45a^4x}{x^2\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c\sqrt{1-a^2x^2}}{4x^4}-\frac{ac\sqrt{1-a^2x^2}}{x^3}-\frac{15a^2c\sqrt{1-a^2x^2}}{8x^2}-\frac{3a^3c\sqrt{1-a^2x^2}}{x}+\frac{1}{8}\left(15a^4c\right)\int \frac{}{x\sqrt{1}}$$
$$= -\frac{c\sqrt{1-a^2x^2}}{4x^4}-\frac{ac\sqrt{1-a^2x^2}}{x^3}-\frac{15a^2c\sqrt{1-a^2x^2}}{8x^2}-\frac{3a^3c\sqrt{1-a^2x^2}}{x}+\frac{1}{16}\left(15a^4c\right)\text{Subst}$$
$$= -\frac{c\sqrt{1-a^2x^2}}{4x^4}-\frac{ac\sqrt{1-a^2x^2}}{x^3}-\frac{15a^2c\sqrt{1-a^2x^2}}{8x^2}-\frac{3a^3c\sqrt{1-a^2x^2}}{x}-\frac{1}{8}\left(15a^2c\right)\text{Subst}\Big($$
$$= -\frac{c\sqrt{1-a^2x^2}}{4x^4}-\frac{ac\sqrt{1-a^2x^2}}{x^3}-\frac{15a^2c\sqrt{1-a^2x^2}}{8x^2}-\frac{3a^3c\sqrt{1-a^2x^2}}{x}-\frac{15}{8}a^4c\tanh^{-1}\Big(\sqrt{1}$$

Mathematica [C] time = 0.110184, size = 97, normalized size = 0.84

$$\frac{1}{2}ac\Bigg(-2a^3\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(6a^2x^2+3ax+2\right)}{x^3}-3a^3\tanh^{-1}\Big(\sqrt{1-a^2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^5,x]
```

```
[Out] (a*c*(-((Sqrt[1 - a^2*x^2]*(2 + 3*a*x + 6*a^2*x^2))/x^3) - 3*a^3*ArcTanh[Sq
rt[1 - a^2*x^2]] - 2*a^3*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1
 - a^2*x^2]))/2
```

Maple [B] time = 0.044, size = 231, normalized size = 2.

$$-c\left(a^5x\frac{1}{\sqrt{-a^2x^2+1}}+\frac{1}{4x^4}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{13a^2}{4}\left(-\frac{1}{2x^2}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{3a^2}{2}\left(\frac{1}{\sqrt{-a^2x^2+1}}-\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)\right)+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^5,x)
```

```
[Out] -c*(a^5*x/(-a^2*x^2+1)^(1/2)+1/4/x^4/(-a^2*x^2+1)^(1/2)-13/4*a^2*(-1/2/x^2/
(-a^2*x^2+1)^(1/2)+3/2*a^2*(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/
2))))+2*a^3*(-1/x/(-a^2*x^2+1)^(1/2)+2*a^2*x/(-a^2*x^2+1)^(1/2))+3*a^4*(1/(
-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2)))-3*a*(-1/3/x^3/(-a^2*x^2+1)
^(1/2)+4/3*a^2*(-1/x/(-a^2*x^2+1)^(1/2)+2*a^2*x/(-a^2*x^2+1)^(1/2))))
```

Maxima [A] time = 0.96264, size = 201, normalized size = 1.75

$$\frac{3a^5cx}{\sqrt{-a^2x^2+1}}-\frac{15}{8}a^4c\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{15a^4c}{8\sqrt{-a^2x^2+1}}-\frac{2a^3c}{\sqrt{-a^2x^2+1}x}-\frac{13a^2c}{8\sqrt{-a^2x^2+1}x^2}-\frac{ac}{\sqrt{-a^2x^2+1}x^3}-\frac{}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^5,x, algorithm="max
ima")
```

```
[Out] 3*a^5*c*x/sqrt(-a^2*x^2 + 1) - 15/8*a^4*c*log(2*sqrt(-a^2*x^2 + 1)/abs(x) +
 2/abs(x)) + 15/8*a^4*c/sqrt(-a^2*x^2 + 1) - 2*a^3*c/(sqrt(-a^2*x^2 + 1)*x)
 - 13/8*a^2*c/(sqrt(-a^2*x^2 + 1)*x^2) - a*c/(sqrt(-a^2*x^2 + 1)*x^3) - 1/4
*c/(sqrt(-a^2*x^2 + 1)*x^4)
```

Fricas [A] time = 2.58346, size = 166, normalized size = 1.44

$$\frac{15a^4cx^4\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(24a^3cx^3+15a^2cx^2+8acx+2c\right)\sqrt{-a^2x^2+1}}{8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^5,x, algorithm="fri
cas")
```

```
[Out] 1/8*(15*a^4*c*x^4*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (24*a^3*c*x^3 + 15*a^2*
c*x^2 + 8*a*c*x + 2*c)*sqrt(-a^2*x^2 + 1))/x^4
```

Sympy [C] time = 10.2415, size = 411, normalized size = 3.57

$$a^3c\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for } |a^2x^2|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)+3a^2c\left(\begin{cases}-\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} & \text{for } \frac{1}{|a^2x^2|}>1\\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)+3ac\left(\begin{cases}-\frac{2ia^2\sqrt{a^2x^2-}}{3x}\\ -\frac{2a^2\sqrt{-a^2x^2}}{3x}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)/x**5,x)
```

```
[Out] a**3*c*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a*
*2*x**2 + 1)/x, True)) + 3*a**2*c*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqr
t(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2
 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2)))
, True)) + 3*a*c*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a*
*2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(
3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), True)) + c*Piecewise((-3*a**4*acosh(1
/(a*x))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(
a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1)
, (3*I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/
(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), Tr
ue))
```

Giac [B] time = 1.16481, size = 378, normalized size = 3.29

$$\frac{\left(a^5c+\frac{8\left(\sqrt{-a^2x^2+1}|a|+a\right)a^3c}{x}+\frac{32\left(\sqrt{-a^2x^2+1}|a|+a\right)^2ac}{x^2}+\frac{104\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c}{ax^3}\right)a^8x^4}{64\left(\sqrt{-a^2x^2+1}|a|+a\right)^4|a|}-\frac{15\,a^5c\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{8\,|a|}-\frac{104\left(\sqrt{-a^2x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^5,x, algorithm="gia
c")
```

```
[Out] 1/64*(a^5*c + 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^3*c/x + 32*(sqrt(-a^2*x^2
 + 1)*abs(a) + a)^2*a*c/x^2 + 104*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c/(a*x^
3))*a^8*x^4/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*abs(a)) - 15/8*a^5*c*log(1/2
*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/64*(104*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^5*c*abs(a)/x + 32*(sqrt(-a^2*x^2 + 1)*abs(
a) + a)^2*a^3*c*abs(a)/x^2 + 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a*c*abs(a)
/x^3 + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c*abs(a)/(a*x^4))/a^4
```

3.1146 $\int \frac{e^{3\tanh^{-1}(ax)}(c-a^2cx^2)}{x^6} dx$

Optimal. Leaf size=144

$$-\frac{38a^4c\sqrt{1-a^2x^2}}{15x}-\frac{13a^3c\sqrt{1-a^2x^2}}{8x^2}-\frac{19a^2c\sqrt{1-a^2x^2}}{15x^3}-\frac{3ac\sqrt{1-a^2x^2}}{4x^4}-\frac{c\sqrt{1-a^2x^2}}{5x^5}-\frac{13}{8}a^5c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(5*x^5) - (3*a*c*Sqrt[1 - a^2*x^2])/(4*x^4) - (19*a^
2*c*Sqrt[1 - a^2*x^2])/(15*x^3) - (13*a^3*c*Sqrt[1 - a^2*x^2])/(8*x^2) - (3
8*a^4*c*Sqrt[1 - a^2*x^2])/(15*x) - (13*a^5*c*ArcTanh[Sqrt[1 - a^2*x^2]])/8
```

Rubi [A] time = 0.25023, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 1807, 835, 807, 266, 63, 208}

$$-\frac{38a^4c\sqrt{1-a^2x^2}}{15x}-\frac{13a^3c\sqrt{1-a^2x^2}}{8x^2}-\frac{19a^2c\sqrt{1-a^2x^2}}{15x^3}-\frac{3ac\sqrt{1-a^2x^2}}{4x^4}-\frac{c\sqrt{1-a^2x^2}}{5x^5}-\frac{13}{8}a^5c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^6,x]
```

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(5*x^5) - (3*a*c*Sqrt[1 - a^2*x^2])/(4*x^4) - (19*a^
2*c*Sqrt[1 - a^2*x^2])/(15*x^3) - (13*a^3*c*Sqrt[1 - a^2*x^2])/(8*x^2) - (3
8*a^4*c*Sqrt[1 - a^2*x^2])/(15*x) - (13*a^5*c*ArcTanh[Sqrt[1 - a^2*x^2]])/8
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^6}\,dx &= c\int \frac{(1+ax)^3}{x^6\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{c\sqrt{1-a^2x^2}}{5x^5}-\frac{1}{5}c\int \frac{-15a-19a^2x-5a^3x^2}{x^5\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{c\sqrt{1-a^2x^2}}{5x^5}-\frac{3ac\sqrt{1-a^2x^2}}{4x^4}+\frac{1}{20}c\int \frac{76a^2+65a^3x}{x^4\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{c\sqrt{1-a^2x^2}}{5x^5}-\frac{3ac\sqrt{1-a^2x^2}}{4x^4}-\frac{19a^2c\sqrt{1-a^2x^2}}{15x^3}-\frac{1}{60}c\int \frac{-195a^3-152a^4x}{x^3\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{c\sqrt{1-a^2x^2}}{5x^5}-\frac{3ac\sqrt{1-a^2x^2}}{4x^4}-\frac{19a^2c\sqrt{1-a^2x^2}}{15x^3}-\frac{13a^3c\sqrt{1-a^2x^2}}{8x^2}+\frac{1}{120}c\int \frac{304a^4 \cdots}{x^2\sqrt{1 \cdots}} \\
&= -\frac{c\sqrt{1-a^2x^2}}{5x^5}-\frac{3ac\sqrt{1-a^2x^2}}{4x^4}-\frac{19a^2c\sqrt{1-a^2x^2}}{15x^3}-\frac{13a^3c\sqrt{1-a^2x^2}}{8x^2}-\frac{38a^4c\sqrt{1-a^2x^2}}{15x} \cdots \\
&= -\frac{c\sqrt{1-a^2x^2}}{5x^5}-\frac{3ac\sqrt{1-a^2x^2}}{4x^4}-\frac{19a^2c\sqrt{1-a^2x^2}}{15x^3}-\frac{13a^3c\sqrt{1-a^2x^2}}{8x^2}-\frac{38a^4c\sqrt{1-a^2x^2}}{15x} \cdots \\
&= -\frac{c\sqrt{1-a^2x^2}}{5x^5}-\frac{3ac\sqrt{1-a^2x^2}}{4x^4}-\frac{19a^2c\sqrt{1-a^2x^2}}{15x^3}-\frac{13a^3c\sqrt{1-a^2x^2}}{8x^2}-\frac{38a^4c\sqrt{1-a^2x^2}}{15x} \cdots \\
&= -\frac{c\sqrt{1-a^2x^2}}{5x^5}-\frac{3ac\sqrt{1-a^2x^2}}{4x^4}-\frac{19a^2c\sqrt{1-a^2x^2}}{15x^3}-\frac{13a^3c\sqrt{1-a^2x^2}}{8x^2}-\frac{38a^4c\sqrt{1-a^2x^2}}{15x} \cdots
\end{aligned}$$

Mathematica [C] time = 0.121111, size = 110, normalized size = 0.76

$$-3a^5c\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-a^2x^2\right)-\frac{c\sqrt{1-a^2x^2}\left(76a^4x^4+15a^3x^3+38a^2x^2+6\right)}{30x^5}-\frac{1}{2}a^5c\tanh \cdots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^6,x]
```

```
[Out] -(c*Sqrt[1 - a^2*x^2]*(6 + 38*a^2*x^2 + 15*a^3*x^3 + 76*a^4*x^4))/(30*x^5)
- (a^5*c*ArcTanh[Sqrt[1 - a^2*x^2]])/2 - 3*a^5*c*Sqrt[1 - a^2*x^2]*Hypergeo
metric2F1[1/2, 3, 3/2, 1 - a^2*x^2]
```

Maple [B] time = 0.049, size = 292, normalized size = 2.

$$-c\left(-3\,a\left(-1/4\,\frac{1}{x^4\sqrt{-a^2x^2+1}}+5/4\,a^2\left(-1/2\,\frac{1}{x^2\sqrt{-a^2x^2+1}}+3/2\,a^2\left(\frac{1}{\sqrt{-a^2x^2+1}}-\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)\right)\right)+3\,a^4\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^6,x)
```

```
[Out] -c*(-3*a*(-1/4/x^4/(-a^2*x^2+1)^(1/2)+5/4*a^2*(-1/2/x^2/(-a^2*x^2+1)^(1/2)+
3/2*a^2*(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2)))))+3*a^4*(-1/x/
(-a^2*x^2+1)^(1/2)+2*a^2*x/(-a^2*x^2+1)^(1/2))+1/5/x^5/(-a^2*x^2+1)^(1/2)-1
6/5*a^2*(-1/3/x^3/(-a^2*x^2+1)^(1/2)+4/3*a^2*(-1/x/(-a^2*x^2+1)^(1/2)+2*a^2
*x/(-a^2*x^2+1)^(1/2)))+a^5*(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1
/2)))+2*a^3*(-1/2/x^2/(-a^2*x^2+1)^(1/2)+3/2*a^2*(1/(-a^2*x^2+1)^(1/2)-arct
anh(1/(-a^2*x^2+1)^(1/2)))))
```

Maxima [A] time = 0.97974, size = 230, normalized size = 1.6

$$\frac{38\,a^6cx}{15\,\sqrt{-a^2x^2+1}}-\frac{13}{8}\,a^5c\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{13\,a^5c}{8\,\sqrt{-a^2x^2+1}}-\frac{19\,a^4c}{15\,\sqrt{-a^2x^2+1}x}-\frac{7\,a^3c}{8\,\sqrt{-a^2x^2+1}x^2}-\frac{16\,a^2c}{15\,\sqrt{-a^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^6,x, algorithm="max
ima")
```

```
[Out] 38/15*a^6*c*x/sqrt(-a^2*x^2 + 1) - 13/8*a^5*c*log(2*sqrt(-a^2*x^2 + 1)/abs(
x) + 2/abs(x)) + 13/8*a^5*c/sqrt(-a^2*x^2 + 1) - 19/15*a^4*c/(sqrt(-a^2*x^2
 + 1)*x) - 7/8*a^3*c/(sqrt(-a^2*x^2 + 1)*x^2) - 16/15*a^2*c/(sqrt(-a^2*x^2
+ 1)*x^3) - 3/4*a*c/(sqrt(-a^2*x^2 + 1)*x^4) - 1/5*c/(sqrt(-a^2*x^2 + 1)*x^
5)
```

Fricas [A] time = 2.61866, size = 197, normalized size = 1.37

$$\frac{195\,a^5cx^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(304\,a^4cx^4+195\,a^3cx^3+152\,a^2cx^2+90\,acx+24\,c\right)\sqrt{-a^2x^2+1}}{120\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^6,x, algorithm="fri
cas")
```

```
[Out] 1/120*(195*a^5*c*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (304*a^4*c*x^4 + 195
*a^3*c*x^3 + 152*a^2*c*x^2 + 90*a*c*x + 24*c)*sqrt(-a^2*x^2 + 1))/x^5
```

Sympy [C] time = 31.3626, size = 518, normalized size = 3.6

$$a^3c\left(\begin{cases}-\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} & \text{for } \frac{1}{|a^2x^2|}>1\\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)+3a^2c\left(\begin{cases}-\frac{2ia^2\sqrt{a^2x^2-1}}{3x}-\frac{i\sqrt{a^2x^2-1}}{3x^3} & \text{for } |a^2x^2|>1\\ -\frac{2a^2\sqrt{-a^2x^2+1}}{3x}-\frac{\sqrt{-a^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)+3ac$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)/x**6,x)
```

```
[Out] a**3*c*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x)
, 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**
2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True)) + 3*a**2*c*Piecewi
se((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**3), A
bs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 +
 1)/(3*x**3), True)) + 3*a*c*Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*a**3/(
8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - 1/(4*
a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*asin(1/(
a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1 - 1/(
a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True)) + c*Piecewise((
-8*a**5*sqrt(-1 + 1/(a**2*x**2))/15 - 4*a**3*sqrt(-1 + 1/(a**2*x**2))/(15*x
**2) - a*sqrt(-1 + 1/(a**2*x**2))/(5*x**4), 1/Abs(a**2*x**2) > 1), (-8*I*a*
*5*sqrt(1 - 1/(a**2*x**2))/15 - 4*I*a**3*sqrt(1 - 1/(a**2*x**2))/(15*x**2)
- I*a*sqrt(1 - 1/(a**2*x**2))/(5*x**4), True))
```

Giac [B] time = 1.16176, size = 448, normalized size = 3.11

$$\frac{\left(6a^6c+\frac{45\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4c}{x}+\frac{170\left(\sqrt{-a^2x^2+1}|a|+a\right)^2a^2c}{x^2}+\frac{480\left(\sqrt{-a^2x^2+1}|a|+a\right)^3c}{x^3}+\frac{1380\left(\sqrt{-a^2x^2+1}|a|+a\right)^4c}{a^2x^4}\right)a^{10}x^5}{960\left(\sqrt{-a^2x^2+1}|a|+a\right)^5|a|}-\frac{13a^6c\log\left(\frac{|-2}{8|}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^6,x, algorithm="gia
c")
```

```
[Out] 1/960*(6*a^6*c + 45*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c/x + 170*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)^2*a^2*c/x^2 + 480*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*
c/x^3 + 1380*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c/(a^2*x^4))*a^10*x^5/((sqrt
(-a^2*x^2 + 1)*abs(a) + a)^5*abs(a)) - 13/8*a^6*c*log(1/2*abs(-2*sqrt(-a^2*
x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/960*(1380*(sqrt(-a^2*x^2 +
1)*abs(a) + a)*a^8*c/x + 480*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^6*c/x^2 +
170*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a^4*c/x^3 + 45*(sqrt(-a^2*x^2 + 1)*ab
s(a) + a)^4*a^2*c/x^4 + 6*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c/x^5)/(a^4*abs
(a))
```

3.1147 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=121

$$-\frac{c^2(ax+1)^2\left(1-a^2x^2\right)^{3/2}}{5a}-\frac{7c^2(ax+1)\left(1-a^2x^2\right)^{3/2}}{20a}-\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7}{8}c^2x\sqrt{1-a^2x^2}+\frac{7c^2\sin^{-1}(ax)}{8a}$$

```
[Out] (7*c^2*x*Sqrt[1 - a^2*x^2])/8 - (7*c^2*(1 - a^2*x^2)^(3/2))/(12*a) - (7*c^2
*(1 + a*x)*(1 - a^2*x^2)^(3/2))/(20*a) - (c^2*(1 + a*x)^2*(1 - a^2*x^2)^(3/
2))/(5*a) + (7*c^2*ArcSin[a*x])/(8*a)
```

Rubi [A] time = 0.0715503, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6138, 671, 641, 195, 216}

$$-\frac{c^2(ax+1)^2\left(1-a^2x^2\right)^{3/2}}{5a}-\frac{7c^2(ax+1)\left(1-a^2x^2\right)^{3/2}}{20a}-\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7}{8}c^2x\sqrt{1-a^2x^2}+\frac{7c^2\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (7*c^2*x*Sqrt[1 - a^2*x^2])/8 - (7*c^2*(1 - a^2*x^2)^(3/2))/(12*a) - (7*c^2
*(1 + a*x)*(1 - a^2*x^2)^(3/2))/(20*a) - (c^2*(1 + a*x)^2*(1 - a^2*x^2)^(3/
2))/(5*a) + (7*c^2*ArcSin[a*x])/(8*a)
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = c^2\int(1+ax)^3\sqrt{1-a^2x^2}\,dx$$
$$= -\frac{c^2(1+ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{5}\left(7c^2\right)\int(1+ax)^2\sqrt{1-a^2x^2}\,dx$$
$$= -\frac{7c^2(1+ax)\left(1-a^2x^2\right)^{3/2}}{20a}-\frac{c^2(1+ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{4}\left(7c^2\right)\int(1+ax)\sqrt{1-a^2x^2}$$
$$= -\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}-\frac{7c^2(1+ax)\left(1-a^2x^2\right)^{3/2}}{20a}-\frac{c^2(1+ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{4}\left(7c^2\right)\int$$
$$= \frac{7}{8}c^2x\sqrt{1-a^2x^2}-\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}-\frac{7c^2(1+ax)\left(1-a^2x^2\right)^{3/2}}{20a}-\frac{c^2(1+ax)^2\left(1-a^2x^2\right)}{5a}$$
$$= \frac{7}{8}c^2x\sqrt{1-a^2x^2}-\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}-\frac{7c^2(1+ax)\left(1-a^2x^2\right)^{3/2}}{20a}-\frac{c^2(1+ax)^2\left(1-a^2x^2\right)}{5a}$$

Mathematica [A] time = 0.0990525, size = 75, normalized size = 0.62

$$\frac{c^2\left(\sqrt{1-a^2x^2}\left(24a^4x^4+90a^3x^3+112a^2x^2+15ax-136\right)-210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(Sqrt[1 - a^2*x^2]*(-136 + 15*a*x + 112*a^2*x^2 + 90*a^3*x^3 + 24*a^4*
x^4) - 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(120*a)
```

Maple [A] time = 0.066, size = 183, normalized size = 1.5

$$-\frac{c^2a^5x^6}{5}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{11\,c^2a^3x^4}{15}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{31\,ac^2x^2}{15}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{17\,c^2}{15\,a}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{3\,a^4c^2x^5}{4}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/5*c^2*a^5*x^6/(-a^2*x^2+1)^(1/2)-11/15*c^2*a^3*x^4/(-a^2*x^2+1)^(1/2)+31
/15*c^2*a*x^2/(-a^2*x^2+1)^(1/2)-17/15*c^2/a/(-a^2*x^2+1)^(1/2)-3/4*c^2*a^4
*x^5/(-a^2*x^2+1)^(1/2)+5/8*c^2*a^2*x^3/(-a^2*x^2+1)^(1/2)+1/8*c^2*x/(-a^2*
x^2+1)^(1/2)+7/8*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.46709, size = 234, normalized size = 1.93

$$-\frac{a^5c^2x^6}{5\sqrt{-a^2x^2+1}}-\frac{3\,a^4c^2x^5}{4\sqrt{-a^2x^2+1}}-\frac{11\,a^3c^2x^4}{15\sqrt{-a^2x^2+1}}+\frac{5\,a^2c^2x^3}{8\sqrt{-a^2x^2+1}}+\frac{31\,ac^2x^2}{15\sqrt{-a^2x^2+1}}+\frac{c^2x}{8\sqrt{-a^2x^2+1}}+\frac{7\,c^2\arcsin\left(\frac{a}{\sqrt{}}\right)}{8\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="maxim
a")
```

```
[Out] -1/5*a^5*c^2*x^6/sqrt(-a^2*x^2 + 1) - 3/4*a^4*c^2*x^5/sqrt(-a^2*x^2 + 1) -
11/15*a^3*c^2*x^4/sqrt(-a^2*x^2 + 1) + 5/8*a^2*c^2*x^3/sqrt(-a^2*x^2 + 1) +
 31/15*a*c^2*x^2/sqrt(-a^2*x^2 + 1) + 1/8*c^2*x/sqrt(-a^2*x^2 + 1) + 7/8*c^
2*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - 17/15*c^2/(sqrt(-a^2*x^2 + 1)*a)
```

Fricas [A] time = 2.62267, size = 209, normalized size = 1.73

$$-\frac{210\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(24\,a^4c^2x^4+90\,a^3c^2x^3+112\,a^2c^2x^2+15\,ac^2x-136\,c^2\right)\sqrt{-a^2x^2+1}}{120\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="frica
s")
```

```
[Out] -1/120*(210*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (24*a^4*c^2*x^4 +
90*a^3*c^2*x^3 + 112*a^2*c^2*x^2 + 15*a*c^2*x - 136*c^2)*sqrt(-a^2*x^2 + 1)
)/a
```

Sympy [C] time = 16.9266, size = 340, normalized size = 2.81

$$a^3c^2\left(\begin{cases}\frac{x^4\sqrt{-a^2x^2+1}}{5}-\frac{x^2\sqrt{-a^2x^2+1}}{15a^2}-\frac{2\sqrt{-a^2x^2+1}}{15a^4} & \text{for } a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)+3a^2c^2\left(\begin{cases}\frac{ia^2x^5}{4\sqrt{a^2x^2-1}}-\frac{3ix^3}{8\sqrt{a^2x^2-1}}+\frac{ix}{8a^2\sqrt{a^2x^2-1}}-\frac{i\operatorname{acosh}(ax)}{8a^3} & \text{fo}\\ -\frac{a^2x^5}{4\sqrt{-a^2x^2+1}}+\frac{3x^3}{8\sqrt{-a^2x^2+1}}-\frac{x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\operatorname{asin}(ax)}{8a^3} & \text{ot}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**2,x)
```

```
[Out] a**3*c**2*Piecewise((x**4*sqrt(-a**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 + 1
)/(15*a**2) - 2*sqrt(-a**2*x**2 + 1)/(15*a**4), Ne(a, 0)), (x**4/4, True))
+ 3*a**2*c**2*Piecewise((I*a**2*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8*
sqrt(a**2*x**2 - 1)) + I*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*a
**3), Abs(a**2*x**2) > 1), (-a**2*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(8
*sqrt(-a**2*x**2 + 1)) - x/(8*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a**
3), True)) + 3*a*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**
(3/2)/(3*a**2), True)) + c**2*Piecewise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)
) - I*x/(2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1),
(x*sqrt(-a**2*x**2 + 1)/2 + asin(a*x)/(2*a), True))
```

Giac [A] time = 1.14445, size = 105, normalized size = 0.87

$$\frac{7\,c^2\arcsin(ax)\operatorname{sgn}(a)}{8\,|a|}+\frac{1}{120}\,\sqrt{-a^2x^2+1}\left(\left(15\,c^2+2\left(56\,ac^2+3\left(4\,a^3c^2x+15\,a^2c^2\right)x\right)x\right)x-\frac{136\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="giac"
)
```

```
[Out] 7/8*c^2*arcsin(a*x)*sgn(a)/abs(a) + 1/120*sqrt(-a^2*x^2 + 1)*((15*c^2 + 2*(
56*a*c^2 + 3*(4*a^3*c^2*x + 15*a^2*c^2)*x)*x)*x - 136*c^2/a)
```

3.1148 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=143

$$-\frac{c^3(ax+1)^2\left(1-a^2x^2\right)^{5/2}}{7a}-\frac{3c^3(ax+1)\left(1-a^2x^2\right)^{5/2}}{14a}-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a}+\frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{9}{16}c^3x\sqrt{1-a^2x^2}+\frac{9c^3\,\mathrm{s}}{1}$$

```
[Out] (9*c^3*x*Sqrt[1 - a^2*x^2])/16 + (3*c^3*x*(1 - a^2*x^2)^(3/2))/8 - (3*c^3*(
1 - a^2*x^2)^(5/2))/(10*a) - (3*c^3*(1 + a*x)*(1 - a^2*x^2)^(5/2))/(14*a) -
 (c^3*(1 + a*x)^2*(1 - a^2*x^2)^(5/2))/(7*a) + (9*c^3*ArcSin[a*x])/(16*a)
```

Rubi [A] time = 0.081671, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6138, 671, 641, 195, 216}

$$-\frac{c^3(ax+1)^2\left(1-a^2x^2\right)^{5/2}}{7a}-\frac{3c^3(ax+1)\left(1-a^2x^2\right)^{5/2}}{14a}-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a}+\frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{9}{16}c^3x\sqrt{1-a^2x^2}+\frac{9c^3\,\mathrm{s}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (9*c^3*x*Sqrt[1 - a^2*x^2])/16 + (3*c^3*x*(1 - a^2*x^2)^(3/2))/8 - (3*c^3*(
1 - a^2*x^2)^(5/2))/(10*a) - (3*c^3*(1 + a*x)*(1 - a^2*x^2)^(5/2))/(14*a) -
 (c^3*(1 + a*x)^2*(1 - a^2*x^2)^(5/2))/(7*a) + (9*c^3*ArcSin[a*x])/(16*a)
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=c^3\int(1+ax)^3\left(1-a^2x^2\right)^{3/2}\,dx$$
$$=-\frac{c^3(1+ax)^2\left(1-a^2x^2\right)^{5/2}}{7a}+\frac{1}{7}\left(9c^3\right)\int(1+ax)^2\left(1-a^2x^2\right)^{3/2}\,dx$$
$$=-\frac{3c^3(1+ax)\left(1-a^2x^2\right)^{5/2}}{14a}-\frac{c^3(1+ax)^2\left(1-a^2x^2\right)^{5/2}}{7a}+\frac{1}{2}\left(3c^3\right)\int(1+ax)\left(1-a^2x^2\right)$$
$$=-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a}-\frac{3c^3(1+ax)\left(1-a^2x^2\right)^{5/2}}{14a}-\frac{c^3(1+ax)^2\left(1-a^2x^2\right)^{5/2}}{7a}+\frac{1}{2}\left(3c^3\right)\int$$
$$=\frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2}-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a}-\frac{3c^3(1+ax)\left(1-a^2x^2\right)^{5/2}}{14a}-\frac{c^3(1+ax)^2\left(1-a\right.}{7a}$$
$$=\frac{9}{16}c^3x\sqrt{1-a^2x^2}+\frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2}-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a}-\frac{3c^3(1+ax)\left(1-a^2x^2\right)^{5/2}}{14a}$$
$$=\frac{9}{16}c^3x\sqrt{1-a^2x^2}+\frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2}-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a}-\frac{3c^3(1+ax)\left(1-a^2x^2\right)^{5/2}}{14a}$$

Mathematica [A] time = 0.11944, size = 91, normalized size = 0.64

$$-\frac{c^3\left(\sqrt{1-a^2x^2}\left(80a^6x^6+280a^5x^5+208a^4x^4-350a^3x^3-656a^2x^2-245ax+368\right)+630\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{560a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -(c^3*(Sqrt[1 - a^2*x^2]*(368 - 245*a*x - 656*a^2*x^2 - 350*a^3*x^3 + 208*a
^4*x^4 + 280*a^5*x^5 + 80*a^6*x^6) + 630*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(5
60*a)
```

Maple [A] time = 0.104, size = 229, normalized size = 1.6

$$\frac{c^3a^7x^8}{7}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{8\,c^3a^5x^6}{35}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{54\,c^3a^3x^4}{35}\frac{1}{\sqrt{-a^2x^2+1}}+\frac{64\,c^3ax^2}{35}\frac{1}{\sqrt{-a^2x^2+1}}-\frac{23\,c^3}{35\,a}\frac{1}{\sqrt{-a^2x^2+1}}+a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/7*c^3*a^7*x^8/(-a^2*x^2+1)^(1/2)+8/35*c^3*a^5*x^6/(-a^2*x^2+1)^(1/2)-54/3
5*c^3*a^3*x^4/(-a^2*x^2+1)^(1/2)+64/35*c^3*a*x^2/(-a^2*x^2+1)^(1/2)-23/35*c
^3/a/(-a^2*x^2+1)^(1/2)+1/2*c^3*a^6*x^7/(-a^2*x^2+1)^(1/2)-9/8*c^3*a^4*x^5/
(-a^2*x^2+1)^(1/2)+3/16*c^3*a^2*x^3/(-a^2*x^2+1)^(1/2)+7/16*c^3*x/(-a^2*x^2
+1)^(1/2)+9/16*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.46271, size = 296, normalized size = 2.07

$$\frac{a^7c^3x^8}{7\sqrt{-a^2x^2+1}}+\frac{a^6c^3x^7}{2\sqrt{-a^2x^2+1}}+\frac{8a^5c^3x^6}{35\sqrt{-a^2x^2+1}}-\frac{9a^4c^3x^5}{8\sqrt{-a^2x^2+1}}-\frac{54a^3c^3x^4}{35\sqrt{-a^2x^2+1}}+\frac{3a^2c^3x^3}{16\sqrt{-a^2x^2+1}}+\frac{64ac^3x^2}{35\sqrt{-a^2x^2+1}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="maxim
a")
```

```
[Out] 1/7*a^7*c^3*x^8/sqrt(-a^2*x^2 + 1) + 1/2*a^6*c^3*x^7/sqrt(-a^2*x^2 + 1) + 8
/35*a^5*c^3*x^6/sqrt(-a^2*x^2 + 1) - 9/8*a^4*c^3*x^5/sqrt(-a^2*x^2 + 1) - 5
4/35*a^3*c^3*x^4/sqrt(-a^2*x^2 + 1) + 3/16*a^2*c^3*x^3/sqrt(-a^2*x^2 + 1) +
 64/35*a*c^3*x^2/sqrt(-a^2*x^2 + 1) + 7/16*c^3*x/sqrt(-a^2*x^2 + 1) + 9/16*
c^3*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - 23/35*c^3/(sqrt(-a^2*x^2 + 1)*a)
```

Fricas [A] time = 2.65055, size = 261, normalized size = 1.83

$$-\frac{630\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(80\,a^6c^3x^6+280\,a^5c^3x^5+208\,a^4c^3x^4-350\,a^3c^3x^3-656\,a^2c^3x^2-245\,ac^3x+368\,c^3\right)\sqrt{-a}}{560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="frica
s")
```

```
[Out] -1/560*(630*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (80*a^6*c^3*x^6 +
280*a^5*c^3*x^5 + 208*a^4*c^3*x^4 - 350*a^3*c^3*x^3 - 656*a^2*c^3*x^2 - 245
*a*c^3*x + 368*c^3)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [C] time = 22.7718, size = 632, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -a**5*c**3*Piecewise((x**6*sqrt(-a**2*x**2 + 1)/7 - x**4*sqrt(-a**2*x**2 +
1)/(35*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)/(105*a**4) - 8*sqrt(-a**2*x**2 +
 1)/(105*a**6), Ne(a, 0)), (x**6/6, True)) - 3*a**4*c**3*Piecewise((I*a**2*
x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*x**5/(24*sqrt(a**2*x**2 - 1)) - I*x**3/(
48*a**2*sqrt(a**2*x**2 - 1)) + I*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*acosh(
a*x)/(16*a**5), Abs(a**2*x**2) > 1), (-a**2*x**7/(6*sqrt(-a**2*x**2 + 1)) +
 5*x**5/(24*sqrt(-a**2*x**2 + 1)) + x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - x
/(16*a**4*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(16*a**5), True)) - 2*a**3*c**3
*Piecewise((x**4*sqrt(-a**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 + 1)/(15*a**
2) - 2*sqrt(-a**2*x**2 + 1)/(15*a**4), Ne(a, 0)), (x**4/4, True)) + 2*a**2*
c**3*Piecewise((I*a**2*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8*sqrt(a**2
*x**2 - 1)) + I*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*a**3), Abs
(a**2*x**2) > 1), (-a**2*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(8*sqrt(-a*
```

```
*2*x**2 + 1)) - x/(8*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a**3), True)
) + 3*a*c**3*Piecewise((x**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2)/(3*
a**2), True)) + c**3*Piecewise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)) - I*x/(
2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (x*sqrt(-
a**2*x**2 + 1)/2 + asin(a*x)/(2*a), True))
```

Giac [A] time = 1.17549, size = 138, normalized size = 0.97

$$\frac{9\,c^3\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{16\,|a|} - \frac{1}{560}\,\sqrt{-a^2x^2+1}\left(\frac{368\,c^3}{a} - \left(245\,c^3 + 2\left(328\,ac^3 + \left(175\,a^2c^3 - 4\left(26\,a^3c^3 + 5\left(2\,a^5c^3x + 7\,a^4c^3\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="giac"
)
```

```
[Out] 9/16*c^3*arcsin(a*x)*sgn(a)/abs(a) - 1/560*sqrt(-a^2*x^2 + 1)*(368*c^3/a -
(245*c^3 + 2*(328*a*c^3 + (175*a^2*c^3 - 4*(26*a^3*c^3 + 5*(2*a^5*c^3*x + 7
*a^4*c^3)*x)*x)*x)*x)*x)
```

3.1149 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=165

$$-\frac{c^4(ax+1)^2\left(1-a^2x^2\right)^{7/2}}{9a}-\frac{11c^4(ax+1)\left(1-a^2x^2\right)^{7/2}}{72a}-\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/2}$$

```
[Out] (55*c^4*x*Sqrt[1 - a^2*x^2])/128 + (55*c^4*x*(1 - a^2*x^2)^(3/2))/192 + (11
*c^4*x*(1 - a^2*x^2)^(5/2))/48 - (11*c^4*(1 - a^2*x^2)^(7/2))/(56*a) - (11*
c^4*(1 + a*x)*(1 - a^2*x^2)^(7/2))/(72*a) - (c^4*(1 + a*x)^2*(1 - a^2*x^2)^
(7/2))/(9*a) + (55*c^4*ArcSin[a*x])/(128*a)
```

Rubi [A] time = 0.0903055, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6138, 671, 641, 195, 216}

$$-\frac{c^4(ax+1)^2\left(1-a^2x^2\right)^{7/2}}{9a}-\frac{11c^4(ax+1)\left(1-a^2x^2\right)^{7/2}}{72a}-\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (55*c^4*x*Sqrt[1 - a^2*x^2])/128 + (55*c^4*x*(1 - a^2*x^2)^(3/2))/192 + (11
*c^4*x*(1 - a^2*x^2)^(5/2))/48 - (11*c^4*(1 - a^2*x^2)^(7/2))/(56*a) - (11*
c^4*(1 + a*x)*(1 - a^2*x^2)^(7/2))/(72*a) - (c^4*(1 + a*x)^2*(1 - a^2*x^2)^
(7/2))/(9*a) + (55*c^4*ArcSin[a*x])/(128*a)
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4 dx = c^4\int (1+ax)^3\left(1-a^2x^2\right)^{5/2} dx$$
$$= -\frac{c^4(1+ax)^2\left(1-a^2x^2\right)^{7/2}}{9a} + \frac{1}{9}\left(11c^4\right)\int (1+ax)^2\left(1-a^2x^2\right)^{5/2} dx$$
$$= -\frac{11c^4(1+ax)\left(1-a^2x^2\right)^{7/2}}{72a} - \frac{c^4(1+ax)^2\left(1-a^2x^2\right)^{7/2}}{9a} + \frac{1}{8}\left(11c^4\right)\int (1+ax)\left(1-a^2x\right.$$
$$= -\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a} - \frac{11c^4(1+ax)\left(1-a^2x^2\right)^{7/2}}{72a} - \frac{c^4(1+ax)^2\left(1-a^2x^2\right)^{7/2}}{9a} + \frac{1}{8}\left(11c^4\right.$$
$$= \frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2} - \frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a} - \frac{11c^4(1+ax)\left(1-a^2x^2\right)^{7/2}}{72a} - \frac{c^4(1+ax)^2\left(1\right.}{9a}$$
$$= \frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2} - \frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a} - \frac{11c^4(1+ax)\left(1-a^2\right.}{72a}$$
$$= \frac{55}{128}c^4x\sqrt{1-a^2x^2} + \frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2} - \frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a} -$$
$$= \frac{55}{128}c^4x\sqrt{1-a^2x^2} + \frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2} - \frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a} -$$

Mathematica [A] time = 0.146806, size = 107, normalized size = 0.65

$$\frac{c^4\left(\sqrt{1-a^2x^2}\left(896a^8x^8+3024a^7x^7+1024a^6x^6-7224a^5x^5-8448a^4x^4+3066a^3x^3+10240a^2x^2+4599ax-3712\right)-\right.}{8064a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(Sqrt[1 - a^2*x^2]*(-3712 + 4599*a*x + 10240*a^2*x^2 + 3066*a^3*x^3 -
8448*a^4*x^4 - 7224*a^5*x^5 + 1024*a^6*x^6 + 3024*a^7*x^7 + 896*a^8*x^8) -
6930*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(8064*a)
```

Maple [A] time = 0.24, size = 275, normalized size = 1.7

$$-\frac{3\,c^4a^8x^9}{8}\frac{1}{\sqrt{-a^2x^2+1}} + \frac{61\,a^6c^4x^7}{48}\frac{1}{\sqrt{-a^2x^2+1}} - \frac{245\,a^4c^4x^5}{192}\frac{1}{\sqrt{-a^2x^2+1}} - \frac{73\,a^2c^4x^3}{384}\frac{1}{\sqrt{-a^2x^2+1}} - \frac{c^4a^9x^{10}}{9}\frac{1}{\sqrt{-a^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^4,x)
```

```
[Out] -3/8*c^4*a^8*x^9/(-a^2*x^2+1)^(1/2)+61/48*c^4*a^6*x^7/(-a^2*x^2+1)^(1/2)-24
5/192*c^4*a^4*x^5/(-a^2*x^2+1)^(1/2)-73/384*c^4*a^2*x^3/(-a^2*x^2+1)^(1/2)-
```

```
1/9*c^4*a^9*x^10/(-a^2*x^2+1)^(1/2)-1/63*c^4*a^7*x^8/(-a^2*x^2+1)^(1/2)+74/
63*c^4*a^5*x^6/(-a^2*x^2+1)^(1/2)-146/63*c^4*a^3*x^4/(-a^2*x^2+1)^(1/2)+109
/63*c^4*a*x^2/(-a^2*x^2+1)^(1/2)-29/63*c^4/a/(-a^2*x^2+1)^(1/2)+55/128*c^4/
(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+73/128*c^4*x/(-a^2*x^2
+1)^(1/2)
```

Maxima [A] time = 1.47877, size = 358, normalized size = 2.17

$$-\frac{a^9c^4x^{10}}{9\sqrt{-a^2x^2+1}}-\frac{3\,a^8c^4x^9}{8\sqrt{-a^2x^2+1}}-\frac{a^7c^4x^8}{63\sqrt{-a^2x^2+1}}+\frac{61\,a^6c^4x^7}{48\sqrt{-a^2x^2+1}}+\frac{74\,a^5c^4x^6}{63\sqrt{-a^2x^2+1}}-\frac{245\,a^4c^4x^5}{192\sqrt{-a^2x^2+1}}-\frac{146\,a^3c^4x^4}{63\sqrt{-a^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="maxim
a")
```

```
[Out] -1/9*a^9*c^4*x^10/sqrt(-a^2*x^2 + 1) - 3/8*a^8*c^4*x^9/sqrt(-a^2*x^2 + 1) -
 1/63*a^7*c^4*x^8/sqrt(-a^2*x^2 + 1) + 61/48*a^6*c^4*x^7/sqrt(-a^2*x^2 + 1)
 + 74/63*a^5*c^4*x^6/sqrt(-a^2*x^2 + 1) - 245/192*a^4*c^4*x^5/sqrt(-a^2*x^2
 + 1) - 146/63*a^3*c^4*x^4/sqrt(-a^2*x^2 + 1) - 73/384*a^2*c^4*x^3/sqrt(-a^
2*x^2 + 1) + 109/63*a*c^4*x^2/sqrt(-a^2*x^2 + 1) + 73/128*c^4*x/sqrt(-a^2*x
^2 + 1) + 55/128*c^4*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - 29/63*c^4/(sqrt(-a
^2*x^2 + 1)*a)
```

Fricas [A] time = 2.7294, size = 325, normalized size = 1.97

$$-\frac{6930\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(896\,a^8c^4x^8+3024\,a^7c^4x^7+1024\,a^6c^4x^6-7224\,a^5c^4x^5-8448\,a^4c^4x^4+3066\,a^3c^4x^3+1\right.}{8064\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="frica
s")
```

```
[Out] -1/8064*(6930*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (896*a^8*c^4*x^8
 + 3024*a^7*c^4*x^7 + 1024*a^6*c^4*x^6 - 7224*a^5*c^4*x^5 - 8448*a^4*c^4*x^
4 + 3066*a^3*c^4*x^3 + 10240*a^2*c^4*x^2 + 4599*a*c^4*x - 3712*c^4)*sqrt(-a
^2*x^2 + 1))/a
```

Sympy [C] time = 39.6861, size = 996, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] a**7*c**4*Piecewise((x**8*sqrt(-a**2*x**2 + 1)/9 - x**6*sqrt(-a**2*x**2 + 1
)/(63*a**2) - 2*x**4*sqrt(-a**2*x**2 + 1)/(105*a**4) - 8*x**2*sqrt(-a**2*x*
*2 + 1)/(315*a**6) - 16*sqrt(-a**2*x**2 + 1)/(315*a**8), Ne(a, 0)), (x**8/8
```

```
, True)) + 3*a**6*c**4*Piecewise((I*a**2*x**9/(8*sqrt(a**2*x**2 - 1)) - 7*I
*x**7/(48*sqrt(a**2*x**2 - 1)) - I*x**5/(192*a**2*sqrt(a**2*x**2 - 1)) - 5*
I*x**3/(384*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(128*a**6*sqrt(a**2*x**2 - 1)
) - 5*I*acosh(a*x)/(128*a**7), Abs(a**2*x**2) > 1), (-a**2*x**9/(8*sqrt(-a*
*2*x**2 + 1)) + 7*x**7/(48*sqrt(-a**2*x**2 + 1)) + x**5/(192*a**2*sqrt(-a**
2*x**2 + 1)) + 5*x**3/(384*a**4*sqrt(-a**2*x**2 + 1)) - 5*x/(128*a**6*sqrt(
-a**2*x**2 + 1)) + 5*asin(a*x)/(128*a**7), True)) + a**5*c**4*Piecewise((x*
*6*sqrt(-a**2*x**2 + 1)/7 - x**4*sqrt(-a**2*x**2 + 1)/(35*a**2) - 4*x**2*sq
rt(-a**2*x**2 + 1)/(105*a**4) - 8*sqrt(-a**2*x**2 + 1)/(105*a**6), Ne(a, 0)
), (x**6/6, True)) - 5*a**4*c**4*Piecewise((I*a**2*x**7/(6*sqrt(a**2*x**2 -
 1)) - 5*I*x**5/(24*sqrt(a**2*x**2 - 1)) - I*x**3/(48*a**2*sqrt(a**2*x**2 -
 1)) + I*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(16*a**5), Abs(a**2
*x**2) > 1), (-a**2*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5*x**5/(24*sqrt(-a**2*x
**2 + 1)) + x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - x/(16*a**4*sqrt(-a**2*x**
2 + 1)) + asin(a*x)/(16*a**5), True)) - 5*a**3*c**4*Piecewise((x**4*sqrt(-a
**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 + 1)/(15*a**2) - 2*sqrt(-a**2*x**2 +
 1)/(15*a**4), Ne(a, 0)), (x**4/4, True)) + a**2*c**4*Piecewise((I*a**2*x**
5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8*sqrt(a**2*x**2 - 1)) + I*x/(8*a**2*
sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*x
**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(8*sqrt(-a**2*x**2 + 1)) - x/(8*a**2*
sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a**3), True)) + 3*a*c**4*Piecewise((x*
*2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2)/(3*a**2), True)) + c**4*Piece
wise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)) - I*x/(2*sqrt(a**2*x**2 - 1)) - I
*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (x*sqrt(-a**2*x**2 + 1)/2 + asin(a*
x)/(2*a), True))
```

Giac [A] time = 1.18579, size = 169, normalized size = 1.02

$$\frac{55\,c^4\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{128\,|a|} - \frac{1}{8064}\,\sqrt{-a^2x^2+1}\left(\frac{3712\,c^4}{a} - \left(4599\,c^4 + 2\left(5120\,ac^4 + \left(1533\,a^2c^4 - 4\left(1056\,a^3c^4 + \left(903\,a^4c\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="giac"
)
```

```
[Out] 55/128*c^4*arcsin(a*x)*sgn(a)/abs(a) - 1/8064*sqrt(-a^2*x^2 + 1)*(3712*c^4/
a - (4599*c^4 + 2*(5120*a*c^4 + (1533*a^2*c^4 - 4*(1056*a^3*c^4 + (903*a^4*
c^4 - 2*(64*a^5*c^4 + 7*(8*a^7*c^4*x + 27*a^6*c^4)*x)*x)*x)*x)*x)*x)*x)
```

3.1150 $\int \frac{e^{3\tanh^{-1}(ax)}x^2}{c-a^2cx^2}\,dx$

Optimal. Leaf size=95

$$\frac{(ax+1)^3}{3a^3c\left(1-a^2x^2\right)^{3/2}}-\frac{2(ax+1)^2}{a^3c\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{a^3c}+\frac{3\sin^{-1}(ax)}{a^3c}$$

```
[Out] (1 + a*x)^3/(3*a^3*c*(1 - a^2*x^2)^(3/2)) - (2*(1 + a*x)^2)/(a^3*c*Sqrt[1 -
 a^2*x^2]) - (3*Sqrt[1 - a^2*x^2])/(a^3*c) + (3*ArcSin[a*x])/(a^3*c)
```

Rubi [A] time = 0.186649, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6148, 1635, 21, 669, 641, 216}

$$\frac{(ax+1)^3}{3a^3c\left(1-a^2x^2\right)^{3/2}}-\frac{2(ax+1)^2}{a^3c\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{a^3c}+\frac{3\sin^{-1}(ax)}{a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2),x]
```

```
[Out] (1 + a*x)^3/(3*a^3*c*(1 - a^2*x^2)^(3/2)) - (2*(1 + a*x)^2)/(a^3*c*Sqrt[1 -
 a^2*x^2]) - (3*Sqrt[1 - a^2*x^2])/(a^3*c) + (3*ArcSin[a*x])/(a^3*c)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
```

```
tegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}x^2}{c-a^2cx^2}\,dx &= \frac{\int \frac{x^2(1+ax)^3}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c} \\
&= \frac{(1+ax)^3}{3a^3c\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{\left(\frac{3}{a^2}+\frac{3x}{a}\right)(1+ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c} \\
&= \frac{(1+ax)^3}{3a^3c\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{(1+ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a^2c} \\
&= \frac{(1+ax)^3}{3a^3c\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^2}{a^3c\sqrt{1-a^2x^2}} + \frac{3\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx}{a^2c} \\
&= \frac{(1+ax)^3}{3a^3c\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^2}{a^3c\sqrt{1-a^2x^2}} - \frac{3\sqrt{1-a^2x^2}}{a^3c} + \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2c} \\
&= \frac{(1+ax)^3}{3a^3c\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^2}{a^3c\sqrt{1-a^2x^2}} - \frac{3\sqrt{1-a^2x^2}}{a^3c} + \frac{3\sin^{-1}(ax)}{a^3c}
\end{aligned}$$

Mathematica [A] time = 0.0850362, size = 78, normalized size = 0.82

$$\frac{3a^3x^3-16a^2x^2+9(ax-1)\sqrt{1-a^2x^2}\sin^{-1}(ax)-5ax+14}{3a^3c(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2),x]
```

```
[Out] (14 - 5*a*x - 16*a^2*x^2 + 3*a^3*x^3 + 9*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSi
n[a*x])/(3*a^3*c*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.04, size = 178, normalized size = 1.9

$$\frac{x^2}{ac}\frac{1}{\sqrt{-a^2x^2+1}}-6\frac{1}{a^3c\sqrt{-a^2x^2+1}}-7\frac{x}{a^2c\sqrt{-a^2x^2+1}}+3\frac{1}{a^2c\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)-\frac{4}{3\,ca^4}\left(x-a^{-1}\right)^{-1}\frac{}{\sqrt{-a^2(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2/(-a^2*c*x^2+c),x)
```

```
[Out] 1/c*x^2/a/(-a^2*x^2+1)^(1/2)-6/c/a^3/(-a^2*x^2+1)^(1/2)-7/c*x/a^2/(-a^2*x^2
+1)^(1/2)+3/c/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-4/3/
c/a^4/(x-1/a)/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+8/3/c/a^2/(-a^2*(x-1/a)^2-
2*a*(x-1/a))^(1/2)*x
```

Maxima [B] time = 1.92079, size = 782, normalized size = 8.23

$$a^2c\left(\frac{\left(a^2c^2\right)^{\frac{3}{2}}}{\sqrt{a^2c^2}\sqrt{-a^2x^2+1}a^7c^3x+\sqrt{-a^2x^2+1}a^7c^4}-\frac{\left(a^2c^2\right)^{\frac{3}{2}}}{\sqrt{a^2c^2}\sqrt{-a^2x^2+1}a^7c^3x-\sqrt{-a^2x^2+1}a^7c^4}-\frac{4c}{\sqrt{a^2c^2}\sqrt{-a^2x^2+1}a^4cx+\sqrt{-a^2x^2+1}a^4c^2}-\frac{4c}{\sqrt{a^2c^2}\sqrt{-a^2x^2+1}a^4cx-\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2/(-a^2*c*x^2+c),x, algorithm="max
ima")
```

```
[Out] 1/6*a^2*c*((a^2*c^2)^(3/2)/(sqrt(a^2*c^2)*sqrt(-a^2*x^2 + 1)*a^7*c^3*x + sq
rt(-a^2*x^2 + 1)*a^7*c^4) - (a^2*c^2)^(3/2)/(sqrt(a^2*c^2)*sqrt(-a^2*x^2 +
1)*a^7*c^3*x - sqrt(-a^2*x^2 + 1)*a^7*c^4) - 4*c/(sqrt(a^2*c^2)*sqrt(-a^2*x
^2 + 1)*a^4*c*x + sqrt(-a^2*x^2 + 1)*a^4*c^2) - 4*c/(sqrt(a^2*c^2)*sqrt(-a^
2*x^2 + 1)*a^4*c*x - sqrt(-a^2*x^2 + 1)*a^4*c^2) + 3*sqrt(a^2*c^2)/(sqrt(a^
2*c^2)*sqrt(-a^2*x^2 + 1)*a^5*c*x + sqrt(-a^2*x^2 + 1)*a^5*c^2) - 3*sqrt(a^
2*c^2)/(sqrt(a^2*c^2)*sqrt(-a^2*x^2 + 1)*a^5*c*x - sqrt(-a^2*x^2 + 1)*a^5*c
^2) + 16*x/(sqrt(a^2*c^2)*sqrt(-a^2*x^2 + 1)*a^2) + 6*sqrt(a^2*c^2)*x^2/(sq
rt(-a^2*x^2 + 1)*a^3*c^2) - 16/(sqrt(a^2*c^2)*sqrt(-a^2*x^2 + 1)*a^3) - 42*
sqrt(a^2*c^2)*x/(sqrt(-a^2*x^2 + 1)*a^4*c^2) - 21*sqrt(a^2*c^2)/(sqrt(-a^2*
x^2 + 1)*a^5*c^2) + 18*sqrt(a^2*c^2)*arcsin(x/(c*sqrt(1/(a^2*c^2))))/(a^6*c
^3*sqrt(1/(a^2*c^2))) + (a^2*c^2)^(3/2)/(sqrt(-a^2*x^2 + 1)*a^7*c^4))/sqrt(
a^2*c^2)
```

Fricas [A] time = 2.58919, size = 240, normalized size = 2.53

$$-\frac{14\,a^2x^2-28\,ax+18\left(a^2x^2-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3\,a^2x^2-19\,ax+14\right)\sqrt{-a^2x^2+1}+14}{3\left(a^5cx^2-2\,a^4cx+a^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2/(-a^2*c*x^2+c),x, algorithm="fri
cas")
```

```
[Out] -1/3*(14*a^2*x^2 - 28*a*x + 18*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2
 + 1) - 1)/(a*x)) + (3*a^2*x^2 - 19*a*x + 14)*sqrt(-a^2*x^2 + 1) + 14)/(a^5*
c*x^2 - 2*a^4*c*x + a^3*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx + \int \frac{3ax^3}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx + \int \frac{3a^2x^4}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(x**2/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**
2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(3*a*x**3/(a**4*x**4*sqrt(-a**
2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)
+ Integral(3*a**2*x**4/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-
a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**5/(a**4*x**4*
sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 +
 1)), x))/c
```

Giac [A] time = 1.17991, size = 180, normalized size = 1.89

$$\frac{3\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{a^2c|a|} - \frac{\sqrt{-a^2x^2+1}}{a^3c} + \frac{2\left(\frac{24\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x} - \frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2} - 11\right)}{3\,a^2c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2/(-a^2*c*x^2+c),x, algorithm="gia
c")
```

```
[Out] 3*arcsin(a*x)*sgn(a)/(a^2*c*abs(a)) - sqrt(-a^2*x^2 + 1)/(a^3*c) + 2/3*(24*
(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)
^2/(a^4*x^2) - 11)/(a^2*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^3*a
bs(a))
```

3.1151 $\int \frac{e^{3\tanh^{-1}(ax)}x}{c-a^2cx^2}\,dx$

Optimal. Leaf size=70

$$\frac{(ax+1)^3}{3a^2c\left(1-a^2x^2\right)^{3/2}}-\frac{2(ax+1)}{a^2c\sqrt{1-a^2x^2}}+\frac{\sin^{-1}(ax)}{a^2c}$$

```
[Out] (1 + a*x)^3/(3*a^2*c*(1 - a^2*x^2)^(3/2)) - (2*(1 + a*x))/(a^2*c*Sqrt[1 - a
^2*x^2]) + ArcSin[a*x]/(a^2*c)
```

Rubi [A] time = 0.0860757, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 789, 653, 216}

$$\frac{(ax+1)^3}{3a^2c\left(1-a^2x^2\right)^{3/2}}-\frac{2(ax+1)}{a^2c\sqrt{1-a^2x^2}}+\frac{\sin^{-1}(ax)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*x)/(c - a^2*c*x^2),x]
```

```
[Out] (1 + a*x)^3/(3*a^2*c*(1 - a^2*x^2)^(3/2)) - (2*(1 + a*x))/(a^2*c*Sqrt[1 - a
^2*x^2]) + ArcSin[a*x]/(a^2*c)
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] && !IntegerQ[p] && LtQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}x}{c-a^2cx^2}\,dx = \frac{\int \frac{x(1+ax)^3}{(1-a^2x^2)^{5/2}}\,dx}{c}$$
$$= \frac{(1+ax)^3}{3a^2c\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{(1+ax)^2}{(1-a^2x^2)^{3/2}}\,dx}{ac}$$
$$= \frac{(1+ax)^3}{3a^2c\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)}{a^2c\sqrt{1-a^2x^2}} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{ac}$$
$$= \frac{(1+ax)^3}{3a^2c\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)}{a^2c\sqrt{1-a^2x^2}} + \frac{\sin^{-1}(ax)}{a^2c}$$

Mathematica [A] time = 0.0617958, size = 70, normalized size = 1.

$$\frac{-7a^2x^2+3(ax-1)\sqrt{1-a^2x^2}\sin^{-1}(ax)-2ax+5}{3a^2c(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*x)/(c - a^2*c*x^2),x]
```

```
[Out] (5 - 2*a*x - 7*a^2*x^2 + 3*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(3*a^2
*c*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.038, size = 155, normalized size = 2.2

$$-5\frac{x}{ac\sqrt{-a^2x^2+1}} + \frac{1}{ac}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} - 3\frac{1}{a^2c\sqrt{-a^2x^2+1}} - \frac{4}{3a^3c}\left(x-a^{-1}\right)^{-1}\frac{1}{\sqrt{-a^2\left(x-a^{-1}\right)^2-2a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x/(-a^2*c*x^2+c),x)
```

```
[Out] -5/c*x/a/(-a^2*x^2+1)^(1/2)+1/c/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^
2+1)^(1/2))-3/c/a^2/(-a^2*x^2+1)^(1/2)-4/3/c/a^3/(x-1/a)/(-a^2*(x-1/a)^2-2*
a*(x-1/a))^(1/2)+8/3/c/a/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x
```

Maxima [B] time = 1.75188, size = 572, normalized size = 8.17

$$-\frac{a^2c\left(\frac{2c}{\sqrt{a^2c^2}\sqrt{-a^2x^2+1}a^3cx+\sqrt{-a^2x^2+1}a^3c^2} + \frac{2c}{\sqrt{a^2c^2}\sqrt{-a^2x^2+1}a^3cx-\sqrt{-a^2x^2+1}a^3c^2} - \frac{2\sqrt{a^2c^2}}{\sqrt{a^2c^2}\sqrt{-a^2x^2+1}a^4cx+\sqrt{-a^2x^2+1}a^4c^2} + \frac{2\sqrt{a^2c}}{\sqrt{a^2c^2}\sqrt{-a^2x^2+1}a^4cx}\right.}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -1/3*a^2*c*(2*c/(sqrt(a^2*c^2)*sqrt(-a^2*x^2 + 1)*a^3*c*x + sqrt(-a^2*x^2 +
 1)*a^3*c^2) + 2*c/(sqrt(a^2*c^2)*sqrt(-a^2*x^2 + 1)*a^3*c*x - sqrt(-a^2*x^
2 + 1)*a^3*c^2) - 2*sqrt(a^2*c^2)/(sqrt(a^2*c^2)*sqrt(-a^2*x^2 + 1)*a^4*c*x
 + sqrt(-a^2*x^2 + 1)*a^4*c^2) + 2*sqrt(a^2*c^2)/(sqrt(a^2*c^2)*sqrt(-a^2*x
^2 + 1)*a^4*c*x - sqrt(-a^2*x^2 + 1)*a^4*c^2) - 8*x/(sqrt(a^2*c^2)*sqrt(-a^
2*x^2 + 1)*a) + 8/(sqrt(a^2*c^2)*sqrt(-a^2*x^2 + 1)*a^2) + 12*sqrt(a^2*c^2)
*x/(sqrt(-a^2*x^2 + 1)*a^3*c^2) + sqrt(a^2*c^2)/(sqrt(-a^2*x^2 + 1)*a^4*c^2
) + 3*(a^2*c^2)^(3/2)*x/(sqrt(-a^2*x^2 + 1)*a^5*c^4) - 3*sqrt(a^2*c^2)*arcs
in(x/(c*sqrt(1/(a^2*c^2))))/(a^5*c^3*sqrt(1/(a^2*c^2))))/sqrt(a^2*c^2)
```

Fricas [A] time = 2.62036, size = 217, normalized size = 3.1

$$-\frac{5\,a^2x^2-10\,ax+6\left(a^2x^2-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\sqrt{-a^2x^2+1}(7\,ax-5)+5}{3\left(a^4cx^2-2\,a^3cx+a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] -1/3*(5*a^2*x^2 - 10*a*x + 6*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2 +
1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*(7*a*x - 5) + 5)/(a^4*c*x^2 - 2*a^3*c*x
 + a^2*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{3ax^2}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{3a^2x^3}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+}}$$

$$c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(x/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 +
 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(3*a*x**2/(a**4*x**4*sqrt(-a**2*x
**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + I
ntegral(3*a**2*x**3/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**
2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**4/(a**4*x**4*sqr
t(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)
), x))/c
```

Giac [A] time = 1.19037, size = 151, normalized size = 2.16

$$\frac{\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{ac|a|}+\frac{2\left(\frac{12\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}-\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}-5\right)}{3\,ac\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] arcsin(a*x)*sgn(a)/(a*c*abs(a)) + 2/3*(12*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(
a^2*x) - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^4*x^2) - 5)/(a*c*((sqrt(-a^
2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^3*abs(a))
```

3.1152 $\int \frac{e^{3\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{e^{3\tanh^{-1}(ax)}}{3ac}$$

```
[Out] E^(3*ArcTanh[a*x])/(3*a*c)
```

Rubi [A] time = 0.0305452, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6137}

$$\frac{e^{3\tanh^{-1}(ax)}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2),x]

[Out] E^(3*ArcTanh[a*x])/(3*a*c)
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{3\tanh^{-1}(ax)}}{3ac}$$

Mathematica [A] time = 0.0111311, size = 29, normalized size = 1.61

$$\frac{(ax+1)^{3/2}}{3ac(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2),x]

[Out] (1 + a*x)^(3/2)/(3*a*c*(1 - a*x)^(3/2))
```

Maple [A] time = 0.029, size = 28, normalized size = 1.6

$$\frac{(ax+1)^3}{3\,ac}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c),x)
```

```
[Out] 1/3*(a*x+1)^3/a/c/(-a^2*x^2+1)^(3/2)
```

Maxima [B] time = 1.67419, size = 680, normalized size = 37.78

$$a^2c\left(\frac{\left(a^2c^2\right)^{\frac{3}{2}}}{\sqrt{a^2c^2}\sqrt{-a^2x^2+1}a^5c^3x+\sqrt{-a^2x^2+1}a^5c^4} - \frac{\left(a^2c^2\right)^{\frac{3}{2}}}{\sqrt{a^2c^2}\sqrt{-a^2x^2+1}a^5c^3x-\sqrt{-a^2x^2+1}a^5c^4} - \frac{4c}{\sqrt{a^2c^2}\sqrt{-a^2x^2+1}a^2cx+\sqrt{-a^2x^2+1}a^2c^2} - \frac{4c}{\sqrt{a^2c^2}\sqrt{-a^2x^2+1}a^2cx\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c),x, algorithm="maxima"
)
```

```
[Out] 1/6*a^2*c*((a^2*c^2)^(3/2)/(sqrt(a^2*c^2)*sqrt(-a^2*x^2 + 1)*a^5*c^3*x + sq
rt(-a^2*x^2 + 1)*a^5*c^4) - (a^2*c^2)^(3/2)/(sqrt(a^2*c^2)*sqrt(-a^2*x^2 +
1)*a^5*c^3*x - sqrt(-a^2*x^2 + 1)*a^5*c^4) - 4*c/(sqrt(a^2*c^2)*sqrt(-a^2*x
^2 + 1)*a^2*c*x + sqrt(-a^2*x^2 + 1)*a^2*c^2) - 4*c/(sqrt(a^2*c^2)*sqrt(-a^
2*x^2 + 1)*a^2*c*x - sqrt(-a^2*x^2 + 1)*a^2*c^2) + 16*x/(sqrt(a^2*c^2)*sqrt
(-a^2*x^2 + 1)) + 3*sqrt(a^2*c^2)/(sqrt(a^2*c^2)*sqrt(-a^2*x^2 + 1)*a^3*c*x
 + sqrt(-a^2*x^2 + 1)*a^3*c^2) - 3*sqrt(a^2*c^2)/(sqrt(a^2*c^2)*sqrt(-a^2*x
^2 + 1)*a^3*c*x - sqrt(-a^2*x^2 + 1)*a^3*c^2) - 16/(sqrt(a^2*c^2)*sqrt(-a^2
*x^2 + 1)*a) - 18*sqrt(a^2*c^2)*x/(sqrt(-a^2*x^2 + 1)*a^2*c^2) + 6*sqrt(a^2
*c^2)/(sqrt(-a^2*x^2 + 1)*a^3*c^2) + 4*(a^2*c^2)^(3/2)/(sqrt(-a^2*x^2 + 1)*
a^5*c^4))/sqrt(a^2*c^2)
```

Fricas [B] time = 2.526, size = 119, normalized size = 6.61

$$\frac{a^2x^2 - 2\,ax + \sqrt{-a^2x^2+1}(ax+1) + 1}{3\left(a^3cx^2 - 2\,a^2cx + ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c),x, algorithm="fricas"
)
```

```
[Out] 1/3*(a^2*x^2 - 2*a*x + sqrt(-a^2*x^2 + 1)*(a*x + 1) + 1)/(a^3*c*x^2 - 2*a^2
*c*x + a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{3ax}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx + \int \frac{3a^2x^2}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx + \int \frac{a^3x^3}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2\cdots}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(3*a*x/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x*
*2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(3*a**2*x**2/(a**4*x**4*sqrt(
-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)),
 x) + Integral(a**3*x**3/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt
(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**4*sqrt(-
a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)),
x))/c
```

Giac [B] time = 1.16098, size = 89, normalized size = 4.94

$$\frac{2\left(\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+1\right)}{3\,c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^4*x^2) + 1)/(c*((sqrt(-a^2*x^2
+ 1)*abs(a) + a)/(a^2*x) - 1)^3*abs(a))
```

3.1153 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=97

$$\frac{2\sqrt{1-a^2x^2}}{15ac^2(1-ax)}+\frac{2\sqrt{1-a^2x^2}}{15ac^2(1-ax)^2}+\frac{\sqrt{1-a^2x^2}}{5ac^2(1-ax)^3}$$

```
[Out] Sqrt[1 - a^2*x^2]/(5*a*c^2*(1 - a*x)^3) + (2*Sqrt[1 - a^2*x^2])/(15*a*c^2*(
1 - a*x)^2) + (2*Sqrt[1 - a^2*x^2])/(15*a*c^2*(1 - a*x))
```

Rubi [A] time = 0.0742634, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6138, 655, 659, 651}

$$\frac{2\sqrt{1-a^2x^2}}{15ac^2(1-ax)}+\frac{2\sqrt{1-a^2x^2}}{15ac^2(1-ax)^2}+\frac{\sqrt{1-a^2x^2}}{5ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(5*a*c^2*(1 - a*x)^3) + (2*Sqrt[1 - a^2*x^2])/(15*a*c^2*(
1 - a*x)^2) + (2*Sqrt[1 - a^2*x^2])/(15*a*c^2*(1 - a*x))
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{(1+ax)^3}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^2}$$
$$= \frac{\int \frac{1}{(1-ax)^3\sqrt{1-a^2x^2}}\,dx}{c^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{5ac^2(1-ax)^3} + \frac{2\int \frac{1}{(1-ax)^2\sqrt{1-a^2x^2}}\,dx}{5c^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{5ac^2(1-ax)^3} + \frac{2\sqrt{1-a^2x^2}}{15ac^2(1-ax)^2} + \frac{2\int \frac{1}{(1-ax)\sqrt{1-a^2x^2}}\,dx}{15c^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{5ac^2(1-ax)^3} + \frac{2\sqrt{1-a^2x^2}}{15ac^2(1-ax)^2} + \frac{2\sqrt{1-a^2x^2}}{15ac^2(1-ax)}$$

Mathematica [A] time = 0.0179653, size = 43, normalized size = 0.44

$$\frac{\sqrt{ax+1}\left(2a^2x^2-6ax+7\right)}{15ac^2(1-ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] (Sqrt[1 + a*x]*(7 - 6*a*x + 2*a^2*x^2))/(15*a*c^2*(1 - a*x)^(5/2))
```

Maple [A] time = 0.03, size = 49, normalized size = 0.5

$$-\frac{\left(2\,a^2x^2-6\,ax+7\right)(ax+1)^2}{(15\,ax-15)\,c^2a}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/15*(2*a^2*x^2-6*a*x+7)*(a*x+1)^2/(a*x-1)/c^2/(-a^2*x^2+1)^(3/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(a^2cx^2-c\right)^2\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)^3/((a^2*c*x^2 - c)^2*(-a^2*x^2 + 1)^(3/2)), x)
```

Fricas [A] time = 2.59292, size = 190, normalized size = 1.96

$$\frac{7 a^3x^3 - 21 a^2x^2 + 21 ax - \left(2 a^2x^2 - 6 ax + 7\right)\sqrt{-a^2x^2 + 1} - 7}{15 \left(a^4c^2x^3 - 3 a^3c^2x^2 + 3 a^2c^2x - ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="frica
s")
```

```
[Out] 1/15*(7*a^3*x^3 - 21*a^2*x^2 + 21*a*x - (2*a^2*x^2 - 6*a*x + 7)*sqrt(-a^2*x
^2 + 1) - 7)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{3ax}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx + \int \frac{3a^2x^2}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx + \int}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(3*a*x/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x
**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + I
ntegral(3*a**2*x**2/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a*
*2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)
 + Integral(a**3*x**3/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-
a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)),
x) + Integral(1/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x
**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c*
*2
```

Giac [A] time = 1.19143, size = 196, normalized size = 2.02

$$-\frac{2\left(\frac{20\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x} - \frac{40\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2} + \frac{30\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3} - \frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4} - 7\right)}{15\, c^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)^5 |a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac"
)
```

```
[Out] -2/15*(20*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 40*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^2/(a^4*x^2) + 30*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) -
15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 7)/(c^2*((sqrt(-a^2*x^2 +
1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

3.1154 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=119

$$\frac{8x}{35c^3\sqrt{1-a^2x^2}}+\frac{4}{35ac^3(1-ax)\sqrt{1-a^2x^2}}+\frac{4}{35ac^3(1-ax)^2\sqrt{1-a^2x^2}}+\frac{1}{7ac^3(1-ax)^3\sqrt{1-a^2x^2}}$$

```
[Out] (8*x)/(35*c^3*Sqrt[1 - a^2*x^2]) + 1/(7*a*c^3*(1 - a*x)^3*Sqrt[1 - a^2*x^2]
) + 4/(35*a*c^3*(1 - a*x)^2*Sqrt[1 - a^2*x^2]) + 4/(35*a*c^3*(1 - a*x)*Sqrt
[1 - a^2*x^2])
```

Rubi [A] time = 0.0885734, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6138, 655, 659, 191}

$$\frac{8x}{35c^3\sqrt{1-a^2x^2}}+\frac{4}{35ac^3(1-ax)\sqrt{1-a^2x^2}}+\frac{4}{35ac^3(1-ax)^2\sqrt{1-a^2x^2}}+\frac{1}{7ac^3(1-ax)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (8*x)/(35*c^3*Sqrt[1 - a^2*x^2]) + 1/(7*a*c^3*(1 - a*x)^3*Sqrt[1 - a^2*x^2]
) + 4/(35*a*c^3*(1 - a*x)^2*Sqrt[1 - a^2*x^2]) + 4/(35*a*c^3*(1 - a*x)*Sqrt
[1 - a^2*x^2])
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{\int \frac{(1+ax)^3}{(1-a^2x^2)^{9/2}}\,dx}{c^3}$$
$$= \frac{\int \frac{1}{(1-ax)^3(1-a^2x^2)^{3/2}}\,dx}{c^3}$$
$$= \frac{1}{7ac^3(1-ax)^3\sqrt{1-a^2x^2}} + \frac{4\int \frac{1}{(1-ax)^2(1-a^2x^2)^{3/2}}\,dx}{7c^3}$$
$$= \frac{1}{7ac^3(1-ax)^3\sqrt{1-a^2x^2}} + \frac{4}{35ac^3(1-ax)^2\sqrt{1-a^2x^2}} + \frac{12\int \frac{1}{(1-ax)(1-a^2x^2)^{3/2}}\,dx}{35c^3}$$
$$= \frac{1}{7ac^3(1-ax)^3\sqrt{1-a^2x^2}} + \frac{4}{35ac^3(1-ax)^2\sqrt{1-a^2x^2}} + \frac{4}{35ac^3(1-ax)\sqrt{1-a^2x^2}} + \frac{8\int \frac{1}{(1-a^2x^2)^{3/2}}\,d}{35c^3}$$
$$= \frac{8x}{35c^3\sqrt{1-a^2x^2}} + \frac{1}{7ac^3(1-ax)^3\sqrt{1-a^2x^2}} + \frac{4}{35ac^3(1-ax)^2\sqrt{1-a^2x^2}} + \frac{4}{35ac^3(1-ax)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0208613, size = 61, normalized size = 0.51

$$\frac{8a^4x^4-24a^3x^3+20a^2x^2+4ax-13}{35ac^3(ax-1)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (-13 + 4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4)/(35*a*c^3*(-1 + a*x)^3*
Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.033, size = 63, normalized size = 0.5

$$-\frac{(8x^4a^4-24x^3a^3+20a^2x^2+4ax-13)(ax+1)}{35c^3(ax-1)^2a}(-a^2x^2+1)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/35*(8*a^4*x^4-24*a^3*x^3+20*a^2*x^2+4*a*x-13)*(a*x+1)/(a*x-1)^2/c^3/(-a^
2*x^2+1)^(3/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ax+1)^3}{(a^2cx^2-c)^3(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxim
a")
```

```
[Out] -integrate((a*x + 1)^3/((a^2*c*x^2 - c)^3*(-a^2*x^2 + 1)^(3/2)), x)
```

Fricas [A] time = 2.6794, size = 308, normalized size = 2.59

$$\frac{13\,a^5x^5-39\,a^4x^4+26\,a^3x^3+26\,a^2x^2-39\,ax-\left(8\,a^4x^4-24\,a^3x^3+20\,a^2x^2+4\,ax-13\right)\sqrt{-a^2x^2+1}+13}{35\left(a^6c^3x^5-3\,a^5c^3x^4+2\,a^4c^3x^3+2\,a^3c^3x^2-3\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="frica
s")
```

```
[Out] 1/35*(13*a^5*x^5 - 39*a^4*x^4 + 26*a^3*x^3 + 26*a^2*x^2 - 39*a*x - (8*a^4*x
^4 - 24*a^3*x^3 + 20*a^2*x^2 + 4*a*x - 13)*sqrt(-a^2*x^2 + 1) + 13)/(a^6*c^
3*x^5 - 3*a^5*c^3*x^4 + 2*a^4*c^3*x^3 + 2*a^3*c^3*x^2 - 3*a^2*c^3*x + a*c^3
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{3ax}{a^8x^8\sqrt{-a^2x^2+1}-4a^6x^6\sqrt{-a^2x^2+1}+6a^4x^4\sqrt{-a^2x^2+1}-4a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int\frac{3a^2x^2}{a^8x^8\sqrt{-a^2x^2+1}-4a^6x^6\sqrt{-a^2x^2+1}+6a^4x^4\sqrt{-a^2x^2+1}-4a^2x^2\sqrt{-a^2\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(3*a*x/(a**8*x**8*sqrt(-a**2*x**2 + 1) - 4*a**6*x**6*sqrt(-a**2*x*
*2 + 1) + 6*a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**2*x**2*sqrt(-a**2*x**2 +
1) + sqrt(-a**2*x**2 + 1)), x) + Integral(3*a**2*x**2/(a**8*x**8*sqrt(-a**2
*x**2 + 1) - 4*a**6*x**6*sqrt(-a**2*x**2 + 1) + 6*a**4*x**4*sqrt(-a**2*x**2
 + 1) - 4*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Inte
gral(a**3*x**3/(a**8*x**8*sqrt(-a**2*x**2 + 1) - 4*a**6*x**6*sqrt(-a**2*x**
2 + 1) + 6*a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**2*x**2*sqrt(-a**2*x**2 + 1
) + sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**8*x**8*sqrt(-a**2*x**2 + 1)
- 4*a**6*x**6*sqrt(-a**2*x**2 + 1) + 6*a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a
**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ax+1)^3}{\left(a^2cx^2-c\right)^3\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac"
)
```

```
[Out] integrate(-(a*x + 1)^3/((a^2*c*x^2 - c)^3*(-a^2*x^2 + 1)^(3/2)), x)
```

3.1155 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=141

$$\frac{16x}{63c^4\sqrt{1-a^2x^2}}+\frac{8x}{63c^4\left(1-a^2x^2\right)^{3/2}}+\frac{2}{21ac^4(1-ax)\left(1-a^2x^2\right)^{3/2}}+\frac{2}{21ac^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}+\frac{1}{9ac^4(1-ax)^3\left(1-\right.}$$

```
[Out] (8*x)/(63*c^4*(1 - a^2*x^2)^(3/2)) + 1/(9*a*c^4*(1 - a*x)^3*(1 - a^2*x^2)^(
3/2)) + 2/(21*a*c^4*(1 - a*x)^2*(1 - a^2*x^2)^(3/2)) + 2/(21*a*c^4*(1 - a*x
)*(1 - a^2*x^2)^(3/2)) + (16*x)/(63*c^4*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0907069, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6138, 655, 659, 192, 191}

$$\frac{16x}{63c^4\sqrt{1-a^2x^2}}+\frac{8x}{63c^4\left(1-a^2x^2\right)^{3/2}}+\frac{2}{21ac^4(1-ax)\left(1-a^2x^2\right)^{3/2}}+\frac{2}{21ac^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}+\frac{1}{9ac^4(1-ax)^3\left(1-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (8*x)/(63*c^4*(1 - a^2*x^2)^(3/2)) + 1/(9*a*c^4*(1 - a*x)^3*(1 - a^2*x^2)^(
3/2)) + 2/(21*a*c^4*(1 - a*x)^2*(1 - a^2*x^2)^(3/2)) + 2/(21*a*c^4*(1 - a*x
)*(1 - a^2*x^2)^(3/2)) + (16*x)/(63*c^4*Sqrt[1 - a^2*x^2])
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \frac{\int \frac{(1+ax)^3}{\left(1-a^2x^2\right)^{11/2}}\,dx}{c^4}$$

$$= \frac{\int \frac{1}{(1-ax)^3\left(1-a^2x^2\right)^{5/2}}\,dx}{c^4}$$

$$= \frac{1}{9ac^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}} + \frac{2\int \frac{1}{(1-ax)^2\left(1-a^2x^2\right)^{5/2}}\,dx}{3c^4}$$

$$= \frac{1}{9ac^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}} + \frac{2}{21ac^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}} + \frac{10\int \frac{1}{(1-ax)\left(1-a^2x^2\right)^{5/2}}\,dx}{21c^4}$$

$$= \frac{1}{9ac^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}} + \frac{2}{21ac^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}} + \frac{2}{21ac^4(1-ax)\left(1-a^2x^2\right)^{3/2}} + \frac{8\int \frac{1}{\left(1-a^2x^2\right.}}{21c}$$

$$= \frac{8x}{63c^4\left(1-a^2x^2\right)^{3/2}} + \frac{1}{9ac^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}} + \frac{2}{21ac^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}} + \frac{2}{21ac^4(1-ax)\left(1-\right.}$$

$$= \frac{8x}{63c^4\left(1-a^2x^2\right)^{3/2}} + \frac{1}{9ac^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}} + \frac{2}{21ac^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}} + \frac{2}{21ac^4(1-ax)\left(1-\right.}$$

Mathematica [A] time = 0.0279528, size = 75, normalized size = 0.53

$$\frac{16a^6x^6-48a^5x^5+24a^4x^4+56a^3x^3-66a^2x^2+6ax+19}{63ac^4(1-ax)^{9/2}(ax+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (19 + 6*a*x - 66*a^2*x^2 + 56*a^3*x^3 + 24*a^4*x^4 - 48*a^5*x^5 + 16*a^6*x^
6)/(63*a*c^4*(1 - a*x)^(9/2)*(1 + a*x)^(3/2))
```

Maple [A] time = 0.033, size = 74, normalized size = 0.5

$$-\frac{16\,x^6a^6-48\,x^5a^5+24\,x^4a^4+56\,x^3a^3-66\,a^2x^2+6\,ax+19}{63\,c^4\,(ax-1)^3\,a}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/63*(16*a^6*x^6-48*a^5*x^5+24*a^4*x^4+56*a^3*x^3-66*a^2*x^2+6*a*x+19)/(a*
x-1)^3/c^4/(-a^2*x^2+1)^(3/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(a^2cx^2-c\right)^4\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)^3/((a^2*c*x^2 - c)^4*(-a^2*x^2 + 1)^(3/2)), x)
```

Fricas [A] time = 2.98434, size = 417, normalized size = 2.96

$$\frac{19\,a^7x^7-57\,a^6x^6+19\,a^5x^5+95\,a^4x^4-95\,a^3x^3-19\,a^2x^2+57\,ax-\left(16\,a^6x^6-48\,a^5x^5+24\,a^4x^4+56\,a^3x^3-66\,a^2x^2\right.}{63\left(a^8c^4x^7-3\,a^7c^4x^6+a^6c^4x^5+5\,a^5c^4x^4-5\,a^4c^4x^3-a^3c^4x^2+3\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="frica
s")
```

```
[Out] 1/63*(19*a^7*x^7 - 57*a^6*x^6 + 19*a^5*x^5 + 95*a^4*x^4 - 95*a^3*x^3 - 19*a
^2*x^2 + 57*a*x - (16*a^6*x^6 - 48*a^5*x^5 + 24*a^4*x^4 + 56*a^3*x^3 - 66*a
^2*x^2 + 6*a*x + 19)*sqrt(-a^2*x^2 + 1) - 19)/(a^8*c^4*x^7 - 3*a^7*c^4*x^6
+ a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^4*c^4*x^3 - a^3*c^4*x^2 + 3*a^2*c^4*x -
 a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3ax}{-a^{10}x^{10}\sqrt{-a^2x^2+1}+5a^8x^8\sqrt{-a^2x^2+1}-10a^6x^6\sqrt{-a^2x^2+1}+10a^4x^4\sqrt{-a^2x^2+1}-5a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{}{-a^{10}x^{10}\sqrt{-a^2x^2+1}+5a^8x^8\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (Integral(3*a*x/(-a**10*x**10*sqrt(-a**2*x**2 + 1) + 5*a**8*x**8*sqrt(-a**2
*x**2 + 1) - 10*a**6*x**6*sqrt(-a**2*x**2 + 1) + 10*a**4*x**4*sqrt(-a**2*x*
*2 + 1) - 5*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + In
tegral(3*a**2*x**2/(-a**10*x**10*sqrt(-a**2*x**2 + 1) + 5*a**8*x**8*sqrt(-a
**2*x**2 + 1) - 10*a**6*x**6*sqrt(-a**2*x**2 + 1) + 10*a**4*x**4*sqrt(-a**2
*x**2 + 1) - 5*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) +
 Integral(a**3*x**3/(-a**10*x**10*sqrt(-a**2*x**2 + 1) + 5*a**8*x**8*sqrt(-
a**2*x**2 + 1) - 10*a**6*x**6*sqrt(-a**2*x**2 + 1) + 10*a**4*x**4*sqrt(-a**
2*x**2 + 1) - 5*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)
+ Integral(1/(-a**10*x**10*sqrt(-a**2*x**2 + 1) + 5*a**8*x**8*sqrt(-a**2*x*
```

```
*2 + 1) - 10*a**6*x**6*sqrt(-a**2*x**2 + 1) + 10*a**4*x**4*sqrt(-a**2*x**2
+ 1) - 5*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(a^2cx^2-c\right)^4\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)^3/((a^2*c*x^2 - c)^4*(-a^2*x^2 + 1)^(3/2)), x)
```

3.1156 $\int e^{3\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=226

$$-\frac{ax^5\sqrt{c-a^2cx^2}}{5\sqrt{1-a^2x^2}}-\frac{3x^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}-\frac{4x^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{2x^2\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^4\sqrt{1-a^2x^2}}$$

```
[Out] (-4*x*Sqrt[c - a^2*c*x^2])/(a^3*Sqrt[1 - a^2*x^2]) - (2*x^2*Sqrt[c - a^2*c*
x^2])/(a^2*Sqrt[1 - a^2*x^2]) - (4*x^3*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a
^2*x^2]) - (3*x^4*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - a^2*x^2]) - (a*x^5*Sqrt[
c - a^2*c*x^2])/(5*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x]
)/(a^4*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.221166, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{ax^5\sqrt{c-a^2cx^2}}{5\sqrt{1-a^2x^2}}-\frac{3x^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}-\frac{4x^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{2x^2\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```


```
[Out] (-4*x*Sqrt[c - a^2*c*x^2])/(a^3*Sqrt[1 - a^2*x^2]) - (2*x^2*Sqrt[c - a^2*c*
x^2])/(a^2*Sqrt[1 - a^2*x^2]) - (4*x^3*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a
^2*x^2]) - (3*x^4*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - a^2*x^2]) - (a*x^5*Sqrt[
c - a^2*c*x^2])/(5*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x]
)/(a^4*Sqrt[1 - a^2*x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{3\tanh^{-1}(ax)}x^3\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x^3(1+ax)^2}{1-ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4}{a^3}-\frac{4x}{a^2}-\frac{4x^2}{a}-3x^3-ax^4-\frac{4}{a^3(-1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-a^2x^2}}-\frac{2x^2\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}-\frac{4x^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{3x^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}-\frac{ax^5\sqrt{c-a^2cx^2}}{5\sqrt{1-a^2x^2}}-$$

Mathematica [A] time = 0.0522266, size = 79, normalized size = 0.35

$$-\frac{\sqrt{c-a^2cx^2}\left(ax\left(12a^4x^4+45a^3x^3+80a^2x^2+120ax+240\right)+240\log(1-ax)\right)}{60a^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]*(a*x*(240 + 120*a*x + 80*a^2*x^2 + 45*a^3*x^3 + 12*a^
4*x^4) + 240*Log[1 - a*x]))/(60*a^4*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.089, size = 88, normalized size = 0.4

$$\frac{12\,x^5a^5+45\,x^4a^4+80\,x^3a^3+120\,a^2x^2+240\,ax+240\,\ln(ax-1)}{\left(60\,a^2x^2-60\right)a^4}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/60*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(12*x^5*a^5+45*x^4*a^4+80*x^
3*a^3+120*a^2*x^2+240*a*x+240*ln(a*x-1))/(a^2*x^2-1)/a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorith
m="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 3.0555, size = 869, normalized size = 3.85

$$\left[\frac{120\left(a^2x^2-1\right)\sqrt{c}\log\left(\frac{a^6cx^6-4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2+4\,acx+\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-2\,c}{a^4x^4-2\,a^3x^3+2\,ax-1}\right)+\left(12\,a^5x^5+45\,a^4x\right.}{60\left(a^6x^2-a^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/60*(120*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4
 - 4*a^2*c*x^2 + 4*a*c*x + (a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-
a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a
*x - 1)) + (12*a^5*x^5 + 45*a^4*x^4 + 80*a^3*x^3 + 120*a^2*x^2 + 240*a*x)*s
qrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^6*x^2 - a^4), -1/60*(240*(a^2*x^
2 - 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 2*a*x + 2)*sqrt(-a^2
*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 - 2*a^3*c*x^3 - a^2*c*x^2 + 2*a*c*x)) - (12*a
^5*x^5 + 45*a^4*x^4 + 80*a^3*x^3 + 120*a^2*x^2 + 240*a*x)*sqrt(-a^2*c*x^2 +
 c)*sqrt(-a^2*x^2 + 1))/(a^6*x^2 - a^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)^3}{(-\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**3*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x +
1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax+1)^3x^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3*x^3/(-a^2*x^2 + 1)^(3/2), x)
```

3.1157 $\int e^{3\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=185

$$-\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}-\frac{x^3\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{2x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^3\sqrt{1-a^2x^2}}$$

```
[Out] (-4*x*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - a^2*x^2]) - (2*x^2*Sqrt[c - a^2*c*
x^2])/(a*Sqrt[1 - a^2*x^2]) - (x^3*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] -
 (a*x^4*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c - a^2*c*x^2]
*Log[1 - a*x])/(a^3*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.210795, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}-\frac{x^3\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{2x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```


```
[Out] (-4*x*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - a^2*x^2]) - (2*x^2*Sqrt[c - a^2*c*
x^2])/(a*Sqrt[1 - a^2*x^2]) - (x^3*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] -
 (a*x^4*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c - a^2*c*x^2]
*Log[1 - a*x])/(a^3*Sqrt[1 - a^2*x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{3\tanh^{-1}(ax)}x^2\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x^2(1+ax)^2}{1-ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4}{a^2}-\frac{4x}{a}-3x^2-ax^3-\frac{4}{a^2(-1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}-\frac{2x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{x^3\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\,\mathrm{lo}}{a^3\sqrt{1-a}}$$

Mathematica [A] time = 0.0418343, size = 70, normalized size = 0.38

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{4x}{a^2}-\frac{4\log(1-ax)}{a^3}-\frac{ax^4}{4}-\frac{2x^2}{a}-x^3\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((-4*x)/a^2 - (2*x^2)/a - x^3 - (a*x^4)/4 - (4*Log[1 -
 a*x])/a^3))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.087, size = 79, normalized size = 0.4

$$\frac{x^4a^4+4x^3a^3+8a^2x^2+16ax+16\ln(ax-1)}{(4a^2x^2-4)a^3}\sqrt{-a^2x^2+1}\sqrt{-c(a^2x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(x^4*a^4+4*x^3*a^3+8*a^2*x^2+
16*a*x+16*ln(a*x-1))/(a^2*x^2-1)/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorith
m="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.94976, size = 809, normalized size = 4.37

$$\left[\frac{8\left(a^2x^2-1\right)\sqrt{c}\log\left(\frac{a^6cx^6-4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2+4\,acx+\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-2\,c}{a^4x^4-2\,a^3x^3+2\,ax-1}\right)+\left(a^4x^4+4\,a^3x^3+8\,a^2x^2\right.}{4\left(a^5x^2-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/4*(8*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 -
4*a^2*c*x^2 + 4*a*c*x + (a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^2
*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x
- 1)) + (a^4*x^4 + 4*a^3*x^3 + 8*a^2*x^2 + 16*a*x)*sqrt(-a^2*c*x^2 + c)*sqr
t(-a^2*x^2 + 1))/(a^5*x^2 - a^3), -1/4*(16*(a^2*x^2 - 1)*sqrt(-c)*arctan(sq
rt(-a^2*c*x^2 + c)*(a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c
*x^4 - 2*a^3*c*x^3 - a^2*c*x^2 + 2*a*c*x)) - (a^4*x^4 + 4*a^3*x^3 + 8*a^2*x
^2 + 16*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^5*x^2 - a^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)^3}{(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x +
1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax+1)^3x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3*x^2/(-a^2*x^2 + 1)^(3/2), x)
```

3.1158 $\int e^{3\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=150

$$-\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2\sqrt{1-a^2x^2}}$$

```
[Out] (-4*x*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) - (3*x^2*Sqrt[c - a^2*c*x^
2])/(2*Sqrt[1 - a^2*x^2]) - (a*x^3*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - a^2*x^2
]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^2*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.151583, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 77}

$$-\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-4*x*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) - (3*x^2*Sqrt[c - a^2*c*x^
2])/(2*Sqrt[1 - a^2*x^2]) - (a*x^3*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - a^2*x^2
]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^2*Sqrt[1 - a^2*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{3\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x(1+ax)^2}{1-ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4}{a}-3x-ax^2-\frac{4}{a(-1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4x\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0378538, size = 64, normalized size = 0.43

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(1-ax)}{a^2}-\frac{ax^3}{3}-\frac{4x}{a}-\frac{3x^2}{2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((-4*x)/a - (3*x^2)/2 - (a*x^3)/3 - (4*Log[1 - a*x])/a
^2))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.085, size = 72, normalized size = 0.5

$$\frac{2x^3a^3+9a^2x^2+24ax+24\ln(ax-1)}{(6a^2x^2-6)a^2}\sqrt{-a^2x^2+1}\sqrt{-c(a^2x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/6*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3+9*a^2*x^2+24*a*x+2
4*ln(a*x-1))/(a^2*x^2-1)/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.94047, size = 783, normalized size = 5.22

$$\left[\frac{12\left(a^2x^2-1\right)\sqrt{c}\log\left(\frac{a^6cx^6-4a^5cx^5+5a^4cx^4-4a^2cx^2+4acx+\left(a^4x^4-4a^3x^3+6a^2x^2-4ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-2c}{a^4x^4-2a^3x^3+2ax-1}\right)+\left(2a^3x^3+9a^2x^2+\right.}{6\left(a^4x^2-a^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(12*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 -
 4*a^2*c*x^2 + 4*a*c*x + (a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^
2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x
 - 1)) + (2*a^3*x^3 + 9*a^2*x^2 + 24*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^
2 + 1))/(a^4*x^2 - a^2), -1/6*(24*(a^2*x^2 - 1)*sqrt(-c)*arctan(sqrt(-a^2*c
*x^2 + c)*(a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 - 2*
a^3*c*x^3 - a^2*c*x^2 + 2*a*c*x)) - (2*a^3*x^3 + 9*a^2*x^2 + 24*a*x)*sqrt(-
a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^4*x^2 - a^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\left(ax+1\right)^3}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))
**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(ax+1\right)^3x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3*x/(-a^2*x^2 + 1)^(3/2), x)
```

3.1159 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=111

$$-\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{3x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a\sqrt{1-a^2x^2}}$$

```
[Out] (-3*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] - (a*x^2*Sqrt[c - a^2*c*x^2])/
(2*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a*Sqrt[1 - a^
2*x^2])
```

Rubi [A] time = 0.0845421, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$-\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{3x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-3*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] - (a*x^2*Sqrt[c - a^2*c*x^2])/
(2*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a*Sqrt[1 - a^
2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{1-ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-3-ax+\frac{4}{1-ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{3x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}} - \frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}} - \frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0259945, size = 54, normalized size = 0.49

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}-3x\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a))/Sqrt[1 - a^2*
x^2]
```

Maple [A] time = 0.085, size = 63, normalized size = 0.6

$$\frac{a^2x^2+6\,ax+8\,\ln(ax-1)}{\left(2\,a^2x^2-2\right)a}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a^2*x^2+6*a*x+8*ln(a*x-1))/(
a^2*x^2-1)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.91532, size = 734, normalized size = 6.61

$$\left[\frac{4\left(a^2x^2-1\right)\sqrt{c}\log\left(\frac{a^6cx^6-4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2+4\,acx+\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-2\,c}{a^4x^4-2\,a^3x^3+2\,ax-1}\right)+\sqrt{-a^2cx^2+c}\left(a^2x^2+6\right.}{2\left(a^3x^2-a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/2*(4*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 -
4*a^2*c*x^2 + 4*a*c*x + (a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^2
*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x
- 1)) + sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 6*a*x)*sqrt(-a^2*x^2 + 1))/(a^3*x^2
 - a), -1/2*(8*(a^2*x^2 - 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*(a^2*x^2
- 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 - 2*a^3*c*x^3 - a^2*c*x
^2 + 2*a*c*x)) - sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 6*a*x)*sqrt(-a^2*x^2 + 1))
/(a^3*x^2 - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\left(ax+1\right)^3}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**
(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(ax+1\right)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3/(-a^2*x^2 + 1)^(3/2), x)
```

3.1160 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=104

$$-\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -((a*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2]) + (Sqrt[c - a^2*c*x^2]*Log[x
])/Sqrt[1 - a^2*x^2] - (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/Sqrt[1 - a^2*x^
2]
```

Rubi [A] time = 0.193995, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 72}

$$-\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] -((a*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2]) + (Sqrt[c - a^2*c*x^2]*Log[x
])/Sqrt[1 - a^2*x^2] - (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/Sqrt[1 - a^2*x^
2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-a+\frac{1}{x}-\frac{4a}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}} + \frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}} - \frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.030042, size = 46, normalized size = 0.44

$$\frac{\sqrt{c-a^2cx^2}(-ax-4\log(1-ax)+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-(a*x) + Log[x] - 4*Log[1 - a*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.096, size = 55, normalized size = 0.5

$$\frac{ax-\ln(x)+4\ln(ax-1)}{a^2x^2-1}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] (a*x-ln(x)+4*ln(a*x-1))*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/(a^2*x^2-
1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}(ax+1)}{a^2x^3-2\,ax^2+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x + 1)/(a^2*x^3 - 2*a*x
^2 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)^3}{x\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x), x)
```

3.1161 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=108

$$-\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}+\frac{3a\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -(Sqrt[c - a^2*c*x^2]/(x*Sqrt[1 - a^2*x^2])) + (3*a*Sqrt[c - a^2*c*x^2]*Log
[x])/Sqrt[1 - a^2*x^2] - (4*a*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/Sqrt[1 - a^
2*x^2]
```

Rubi [A] time = 0.200332, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}+\frac{3a\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(x*Sqrt[1 - a^2*x^2])) + (3*a*Sqrt[c - a^2*c*x^2]*Log
[x])/Sqrt[1 - a^2*x^2] - (4*a*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/Sqrt[1 - a^
2*x^2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^2(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}+\frac{3a}{x}-\frac{4a^2}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}} + \frac{3a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}} - \frac{4a\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0336291, size = 51, normalized size = 0.47

$$\frac{\sqrt{c-a^2cx^2}\left(3a\log(x)-4a\log(1-ax)-\frac{1}{x}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-x^(-1) + 3*a*Log[x] - 4*a*Log[1 - a*x]))/Sqrt[1 - a^
2*x^2]
```

Maple [A] time = 0.089, size = 60, normalized size = 0.6

$$\frac{-3\,a\ln(x)\,x+4\,\ln(ax-1)\,xa+1}{x\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] (-3*a*ln(x)*x+4*ln(a*x-1)*x*a+1)*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/
(a^2*x^2-1)/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}(ax+1)}{a^2x^4-2\,ax^3+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorith
m="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x + 1)/(a^2*x^4 - 2*a*x
^3 + x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)^3}{x^2\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(x**2*(-(a*x - 1)*(a*x +
 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2cx^2+c}(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

3.1162 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=149

$$-\frac{3a\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}+\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -Sqrt[c - a^2*c*x^2]/(2*x^2*Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[c - a^2*c*x^2])/
(x*Sqrt[1 - a^2*x^2]) + (4*a^2*Sqrt[c - a^2*c*x^2]*Log[x])/Sqrt[1 - a^2*x^2
] - (4*a^2*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.206386, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{3a\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}+\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(2*x^2*Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[c - a^2*c*x^2])/
(x*Sqrt[1 - a^2*x^2]) + (4*a^2*Sqrt[c - a^2*c*x^2]*Log[x])/Sqrt[1 - a^2*x^2
] - (4*a^2*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/Sqrt[1 - a^2*x^2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^3}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^3(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^3}+\frac{3a}{x^2}+\frac{4a^2}{x}-\frac{4a^3}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}} - \frac{3a\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}} + \frac{4a^2\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}} - \frac{4a^2\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0424801, size = 63, normalized size = 0.42

$$\frac{\sqrt{c-a^2cx^2}\left(4a^2\log(x)-4a^2\log(1-ax)-\frac{3a}{x}-\frac{1}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/(2*x^2) - (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1 - a
*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.089, size = 73, normalized size = 0.5

$$-\frac{8\,a^2\ln(x)\,x^2-8\,\ln(ax-1)\,a^2x^2-6\,ax-1}{\left(2\,a^2x^2-2\right)x^2}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(8*a^2*ln(x)*x^2-8*ln(a*x-1)
*a^2*x^2-6*a*x-1)/(a^2*x^2-1)/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.20687, size = 961, normalized size = 6.45

$$\left[\frac{4\left(a^4x^4-a^2x^2\right)\sqrt{c}\log\left(-\frac{4a^5cx^5-\left(2a^6-4a^5+6a^4-4a^3+a^2\right)cx^6-\left(4a^4+4a^3-6a^2+4a-1\right)cx^4+5a^2cx^2-4acx+\left(4a^3x^3-\left(4a^3-6a^2+4a-1\right)x^4-6a^2x^2+\cdots\right.}{a^4x^6-2a^3x^5+2ax^3-x^2}\right.}{2\left(a^2x^4-x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorith
m="fricas")
```

```
[Out] [1/2*(4*(a^4*x^4 - a^2*x^2)*sqrt(c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6*
a^4 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2*
c*x^2 - 4*a*c*x + (4*a^3*x^3 - (4*a^3 - 6*a^2 + 4*a - 1)*x^4 - 6*a^2*x^2 +
4*a*x - 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) + c)/(a^4*x^6 -
2*a^3*x^5 + 2*a*x^3 - x^2)) - sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((6*a
 + 1)*x^2 - 6*a*x - 1))/(a^2*x^4 - x^2), -1/2*(8*(a^4*x^4 - a^2*x^2)*sqrt(-
c)*arctan(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((2*a^2 - 2*a + 1)*x^2 -
 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^3 - (2*a^3 - a^2)*c*x^4 - (a^2 - 2*a + 1)*c
*x^2 - 2*a*c*x + c)) + sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((6*a + 1)*x
^2 - 6*a*x - 1))/(a^2*x^4 - x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\left(ax+1\right)^3}{x^3\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(x**3*(-(a*x - 1)*(a*x +
 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(ax+1\right)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

3.1163 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=188

$$-\frac{4a^2\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{3a\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{3x^3\sqrt{1-a^2x^2}}+\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^3\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -Sqrt[c - a^2*c*x^2]/(3*x^3*Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[c - a^2*c*x^2])/
(2*x^2*Sqrt[1 - a^2*x^2]) - (4*a^2*Sqrt[c - a^2*c*x^2])/(x*Sqrt[1 - a^2*x^2
]) + (4*a^3*Sqrt[c - a^2*c*x^2]*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^3*Sqrt[c -
 a^2*c*x^2]*Log[1 - a*x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.209612, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{4a^2\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{3a\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{3x^3\sqrt{1-a^2x^2}}+\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^3\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(3*x^3*Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[c - a^2*c*x^2])/
(2*x^2*Sqrt[1 - a^2*x^2]) - (4*a^2*Sqrt[c - a^2*c*x^2])/(x*Sqrt[1 - a^2*x^2
]) + (4*a^3*Sqrt[c - a^2*c*x^2]*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^3*Sqrt[c -
 a^2*c*x^2]*Log[1 - a*x])/Sqrt[1 - a^2*x^2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^4}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^4(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^4}+\frac{3a}{x^3}+\frac{4a^2}{x^2}+\frac{4a^3}{x}-\frac{4a^4}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{3x^3\sqrt{1-a^2x^2}} - \frac{3a\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}} - \frac{4a^2\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}} + \frac{4a^3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}} - \frac{4a^3\sqrt{c-}}{}$$

Mathematica [A] time = 0.0457004, size = 73, normalized size = 0.39

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{4a^2}{x}+4a^3\log(x)-4a^3\log(1-ax)-\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/(3*x^3) - (3*a)/(2*x^2) - (4*a^2)/x + 4*a^3*Log[x]
 - 4*a^3*Log[1 - a*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.091, size = 81, normalized size = 0.4

$$-\frac{24\,a^3\ln(x)\,x^3-24\,\ln(ax-1)\,x^3a^3-24\,a^2x^2-9\,ax-2}{\left(6\,a^2x^2-6\right)x^3}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x)
```

```
[Out] -1/6*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(24*a^3*ln(x)*x^3-24*ln(a*x-
1)*x^3*a^3-24*a^2*x^2-9*a*x-2)/(a^2*x^2-1)/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.14665, size = 1023, normalized size = 5.44

$$\left[\frac{12\left(a^5x^5-a^3x^3\right)\sqrt{c}\log\left(-\frac{4a^5cx^5-\left(2a^6-4a^5+6a^4-4a^3+a^2\right)cx^6-\left(4a^4+4a^3-6a^2+4a-1\right)cx^4+5a^2cx^2-4acx+\left(4a^3x^3-\left(4a^3-6a^2+4a-1\right)x^4-6a^2x^2+4a\right.}{a^4x^6-2a^3x^5+2ax^3-x^2}\right.}{6\left(a^2x^5-x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorith
m="fricas")
```

```
[Out] [1/6*(12*(a^5*x^5 - a^3*x^3)*sqrt(c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6
*a^4 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2
*c*x^2 - 4*a*c*x + (4*a^3*x^3 - (4*a^3 - 6*a^2 + 4*a - 1)*x^4 - 6*a^2*x^2 +
 4*a*x - 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) + c)/(a^4*x^6 -
 2*a^3*x^5 + 2*a*x^3 - x^2)) + sqrt(-a^2*c*x^2 + c)*(24*a^2*x^2 - (24*a^2 +
 9*a + 2)*x^3 + 9*a*x + 2)*sqrt(-a^2*x^2 + 1))/(a^2*x^5 - x^3), -1/6*(24*(a
^5*x^5 - a^3*x^3)*sqrt(-c)*arctan(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*
((2*a^2 - 2*a + 1)*x^2 - 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^3 - (2*a^3 - a^2)*c
*x^4 - (a^2 - 2*a + 1)*c*x^2 - 2*a*c*x + c)) - sqrt(-a^2*c*x^2 + c)*(24*a^2
*x^2 - (24*a^2 + 9*a + 2)*x^3 + 9*a*x + 2)*sqrt(-a^2*x^2 + 1))/(a^2*x^5 - x
^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\left(ax+1\right)^3}{x^4\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(x**4*(-(a*x - 1)*(a*x +
 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2cx^2+c}\left(ax+1\right)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x^4), x)
```

3.1164 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=223

$$-\frac{4a^3\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{4x^4\sqrt{1-a^2x^2}}+\frac{4a^4\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^4\sqrt{c-a^2cx^2}\log(1-a}{\sqrt{1-a^2x^2}}$$

```
[Out] -Sqrt[c - a^2*c*x^2]/(4*x^4*Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - a^2*c*x^2])/(x
^3*Sqrt[1 - a^2*x^2]) - (2*a^2*Sqrt[c - a^2*c*x^2])/(x^2*Sqrt[1 - a^2*x^2])
 - (4*a^3*Sqrt[c - a^2*c*x^2])/(x*Sqrt[1 - a^2*x^2]) + (4*a^4*Sqrt[c - a^2*
c*x^2]*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])
/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.220376, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{4a^3\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{4x^4\sqrt{1-a^2x^2}}+\frac{4a^4\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^4\sqrt{c-a^2cx^2}\log(1-a}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(4*x^4*Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - a^2*c*x^2])/(x
^3*Sqrt[1 - a^2*x^2]) - (2*a^2*Sqrt[c - a^2*c*x^2])/(x^2*Sqrt[1 - a^2*x^2])
 - (4*a^3*Sqrt[c - a^2*c*x^2])/(x*Sqrt[1 - a^2*x^2]) + (4*a^4*Sqrt[c - a^2*
c*x^2]*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])
/Sqrt[1 - a^2*x^2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^5}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^5(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^5}+\frac{3a}{x^4}+\frac{4a^2}{x^3}+\frac{4a^3}{x^2}+\frac{4a^4}{x}-\frac{4a^5}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4\sqrt{1-a^2x^2}} - \frac{a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-a^2x^2}} - \frac{2a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-a^2x^2}} - \frac{4a^3\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}} + \frac{4a^4\sqrt{c-a^2cx^2}\log(x}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0534722, size = 79, normalized size = 0.35

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{2a^2}{x^2}-\frac{4a^3}{x}+4a^4\log(x)-4a^4\log(1-ax)-\frac{a}{x^3}-\frac{1}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/(4*x^4) - a/x^3 - (2*a^2)/x^2 - (4*a^3)/x + 4*a^4*
Log[x] - 4*a^4*Log[1 - a*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.093, size = 89, normalized size = 0.4

$$-\frac{16\,a^4\ln(x)\,x^4-16\,\ln(ax-1)\,a^4x^4-16\,x^3a^3-8\,a^2x^2-4\,ax-1}{\left(4\,a^2x^2-4\right)x^4}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(16*a^4*ln(x)*x^4-16*ln(a*x-
1)*a^4*x^4-16*x^3*a^3-8*a^2*x^2-4*a*x-1)/(a^2*x^2-1)/x^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.29985, size = 1076, normalized size = 4.83

$$\left[\frac{8\left(a^6x^6-a^4x^4\right)\sqrt{c}\log\left(-\frac{4a^5cx^5-\left(2a^6-4a^5+6a^4-4a^3+a^2\right)cx^6-\left(4a^4+4a^3-6a^2+4a-1\right)cx^4+5a^2cx^2-4acx+\left(4a^3x^3-\left(4a^3-6a^2+4a-1\right)x^4-6a^2x^2+\cdots\right.}{a^4x^6-2a^3x^5+2ax^3-x^2}\right.}{4\left(a^2x^6-\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorith
m="fricas")
```

```
[Out] [1/4*(8*(a^6*x^6 - a^4*x^4)*sqrt(c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6*
a^4 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2*
c*x^2 - 4*a*c*x + (4*a^3*x^3 - (4*a^3 - 6*a^2 + 4*a - 1)*x^4 - 6*a^2*x^2 +
4*a*x - 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) + c)/(a^4*x^6 -
2*a^3*x^5 + 2*a*x^3 - x^2)) + (16*a^3*x^3 - (16*a^3 + 8*a^2 + 4*a + 1)*x^4
+ 8*a^2*x^2 + 4*a*x + 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^2*x^6
- x^4), -1/4*(16*(a^6*x^6 - a^4*x^4)*sqrt(-c)*arctan(-sqrt(-a^2*c*x^2 + c)*
sqrt(-a^2*x^2 + 1)*((2*a^2 - 2*a + 1)*x^2 - 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^
3 - (2*a^3 - a^2)*c*x^4 - (a^2 - 2*a + 1)*c*x^2 - 2*a*c*x + c)) - (16*a^3*x
^3 - (16*a^3 + 8*a^2 + 4*a + 1)*x^4 + 8*a^2*x^2 + 4*a*x + 1)*sqrt(-a^2*c*x^
2 + c)*sqrt(-a^2*x^2 + 1))/(a^2*x^6 - x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)^3}{x^5\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(x**5*(-(a*x - 1)*(a*x +
 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x^5), x)
```

3.1165 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=44

$$\frac{c(ax+1)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

```
[Out] (c*(1 + a*x)^4*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0824788, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 32}

$$\frac{c(ax+1)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*(1 + a*x)^4*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx &= \frac{\left(c\sqrt{c-a^2cx^2}\right)\int e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\left(c\sqrt{c-a^2cx^2}\right)\int (1+ax)^3\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{c(1+ax)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0283183, size = 58, normalized size = 1.32

$$\frac{c\left(\frac{a^3x^4}{4}+a^2x^3+\frac{3ax^2}{2}+x\right)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(x + (3*a*x^2)/2 + a^2*x^3 + (a^3*x^4)/4))/Sqrt[1 -
a^2*x^2]
```

Maple [A] time = 0.027, size = 50, normalized size = 1.1

$$\frac{x\left(x^3a^3+4\,a^2x^2+6\,ax+4\right)}{4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4*x*(a^3*x^3+4*a^2*x^2+6*a*x+4)*(-a^2*c*x^2+c)^(3/2)/(-a^2*x^2+1)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.59729, size = 143, normalized size = 3.25

$$-\frac{\left(a^3cx^4+4\,a^2cx^3+6\,acx^2+4\,cx\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{4\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] -1/4*(a^3*c*x^4 + 4*a^2*c*x^3 + 6*a*c*x^2 + 4*c*x)*sqrt(-a^2*c*x^2 + c)*sqr
t(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c(ax-1)(ax+1))^{\frac{3}{2}}(ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1)
)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^3/(-a^2*x^2 + 1)^(3/2), x)
```

3.1166 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=93

$$\frac{2c^2(ax+1)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}-\frac{c^2(ax+1)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}$$

```
[Out] (2*c^2*(1 + a*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) - (c^2*(1 +
 a*x)^6*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0911275, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$\frac{2c^2(ax+1)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}-\frac{c^2(ax+1)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (2*c^2*(1 + a*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) - (c^2*(1 +
 a*x)^6*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - a^2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}dx=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int(1-ax)(1+ax)^4dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int\left(2(1+ax)^4-(1+ax)^5\right)dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{2c^2(1+ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}-\frac{c^2(1+ax)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0379155, size = 52, normalized size = 0.56

$$-\frac{c^2(ax+1)^5(5ax-7)\sqrt{c-a^2cx^2}}{30a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(c^2*(1 + a*x)^5*(-7 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(30*a*Sqrt[1 - a^2*x^2]
)
```

Maple [A] time = 0.028, size = 81, normalized size = 0.9

$$\frac{x\left(5x^5a^5+18x^4a^4+15x^3a^3-20a^2x^2-45ax-30\right)}{(30ax-30)(ax+1)}\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/30*x*(5*a^5*x^5+18*a^4*x^4+15*a^3*x^3-20*a^2*x^2-45*a*x-30)*(-a^2*c*x^2+c
)^(5/2)/(a*x-1)/(a*x+1)/(-a^2*x^2+1)^(3/2)
```

Maxima [B] time = 1.105, size = 320, normalized size = 3.44

$$-\frac{1}{3}a^2c^{\frac{5}{2}}x^3+\frac{1}{12}\left(\frac{2a^4c^3x^8}{\sqrt{a^4cx^4-2a^2cx^2+c}}-\frac{5a^2c^3x^6}{\sqrt{a^4cx^4-2a^2cx^2+c}}+\frac{3c^3x^4}{\sqrt{a^4cx^4-2a^2cx^2+c}}\right)a^3+c^{\frac{5}{2}}x-\frac{1}{5}\left(3a^2c^{\frac{5}{2}}x^5-5c^{\frac{5}{2}}x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="m
axima")
```

```
[Out] -1/3*a^2*c^(5/2)*x^3 + 1/12*(2*a^4*c^3*x^8/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c
) - 5*a^2*c^3*x^6/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) + 3*c^3*x^4/sqrt(a^4*c*
x^4 - 2*a^2*c*x^2 + c))*a^3 + c^(5/2)*x - 1/5*(3*a^2*c^(5/2)*x^5 - 5*c^(5/2
)*x^3)*a^2 + 3/4*(a^4*c^3*x^6/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) - 3*a^2*c^3
```

```
*x^4/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) + 2*c^3/(sqrt(a^4*c*x^4 - 2*a^2*c*x^
2 + c)*a^2))*a
```

Fricas [A] time = 2.51615, size = 207, normalized size = 2.23

$$\frac{\left(5\,a^5c^2x^6+18\,a^4c^2x^5+15\,a^3c^2x^4-20\,a^2c^2x^3-45\,ac^2x^2-30\,c^2x\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{30\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/30*(5*a^5*c^2*x^6 + 18*a^4*c^2*x^5 + 15*a^3*c^2*x^4 - 20*a^2*c^2*x^3 - 45
*a*c^2*x^2 - 30*c^2*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}\left(ax+1\right)^{3}}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1)
)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(ax+1\right)^{3}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^3/(-a^2*x^2 + 1)^(3/2), x)
```

3.1167 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=139

$$\frac{c^3(ax+1)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}-\frac{4c^3(ax+1)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{2c^3(ax+1)^6\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}$$

```
[Out] (2*c^3*(1 + a*x)^6*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) - (4*c^3*(1
 + a*x)^7*Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) + (c^3*(1 + a*x)^8*S
qrt[c - a^2*c*x^2])/(8*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.103955, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$\frac{c^3(ax+1)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}-\frac{4c^3(ax+1)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{2c^3(ax+1)^6\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (2*c^3*(1 + a*x)^6*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) - (4*c^3*(1
 + a*x)^7*Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) + (c^3*(1 + a*x)^8*S
qrt[c - a^2*c*x^2])/(8*a*Sqrt[1 - a^2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}dx=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{7/2}dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int(1-ax)^2(1+ax)^5dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int\left(4(1+ax)^5-4(1+ax)^6+(1+ax)^7\right)dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{2c^3(1+ax)^6\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{4c^3(1+ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{c^3(1+ax)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0545946, size = 60, normalized size = 0.43

$$\frac{c^3(ax+1)^6\left(21a^2x^2-54ax+37\right)\sqrt{c-a^2cx^2}}{168a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (c^3*(1 + a*x)^6*(37 - 54*a*x + 21*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(168*a*Sqr
t[1 - a^2*x^2])
```

Maple [A] time = 0.031, size = 97, normalized size = 0.7

$$\frac{x\left(21\,a^7x^7+72\,x^6a^6+28\,x^5a^5-168\,x^4a^4-210\,x^3a^3+56\,a^2x^2+252\,ax+168\right)}{168\,(ax+1)^2\,(ax-1)^2}\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/168*x*(21*a^7*x^7+72*a^6*x^6+28*a^5*x^5-168*a^4*x^4-210*a^3*x^3+56*a^2*x^
2+252*a*x+168)*(-a^2*c*x^2+c)^(7/2)/(a*x-1)^2/(a*x+1)^2/(-a^2*x^2+1)^(3/2)
```

Maxima [B] time = 1.11643, size = 436, normalized size = 3.14

$$\frac{1}{5}\,a^4c^{\frac{7}{2}}x^5-\frac{2}{3}\,a^2c^{\frac{7}{2}}x^3+c^{\frac{7}{2}}x-\frac{1}{24}\left(\frac{3\,a^6c^4x^{10}}{\sqrt{a^4cx^4-2\,a^2cx^2+c}}-\frac{11\,a^4c^4x^8}{\sqrt{a^4cx^4-2\,a^2cx^2+c}}+\frac{14\,a^2c^4x^6}{\sqrt{a^4cx^4-2\,a^2cx^2+c}}-\frac{6\,c^4}{\sqrt{a^4cx^4-2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="m
axima")
```

```
[Out] 1/5*a^4*c^(7/2)*x^5 - 2/3*a^2*c^(7/2)*x^3 + c^(7/2)*x - 1/24*(3*a^6*c^4*x^1
0/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) - 11*a^4*c^4*x^8/sqrt(a^4*c*x^4 - 2*a^2
*c*x^2 + c) + 14*a^2*c^4*x^6/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) - 6*c^4*x^4/
sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c))*a^3 + 1/35*(15*a^4*c^(7/2)*x^7 - 42*a^2*
```

```
c^(7/2)*x^5 + 35*c^(7/2)*x^3)*a^2 - 1/2*(a^6*c^4*x^8/sqrt(a^4*c*x^4 - 2*a^2
*c*x^2 + c) - 4*a^4*c^4*x^6/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) + 6*a^2*c^4*x
^4/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) - 3*c^4/(sqrt(a^4*c*x^4 - 2*a^2*c*x^2
+ c)*a^2))*a
```

Fricas [A] time = 2.53354, size = 262, normalized size = 1.88

$$-\frac{\left(21\,a^7c^3x^8 + 72\,a^6c^3x^7 + 28\,a^5c^3x^6 - 168\,a^4c^3x^5 - 210\,a^3c^3x^4 + 56\,a^2c^3x^3 + 252\,ac^3x^2 + 168\,c^3x\right)\sqrt{-a^2cx^2 + c}\sqrt{-a^2x^2 +}}{168\left(a^2x^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="f
ricas")
```

```
[Out] -1/168*(21*a^7*c^3*x^8 + 72*a^6*c^3*x^7 + 28*a^5*c^3*x^6 - 168*a^4*c^3*x^5
- 210*a^3*c^3*x^4 + 56*a^2*c^3*x^3 + 252*a*c^3*x^2 + 168*c^3*x)*sqrt(-a^2*c
*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2 + c\right)^{\frac{7}{2}}(ax + 1)^3}{\left(-a^2x^2 + 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="g
iac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*(a*x + 1)^3/(-a^2*x^2 + 1)^(3/2), x)
```

3.1168 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

Optimal. Leaf size=185

$$-\frac{c^4(ax+1)^{10}\sqrt{c-a^2cx^2}}{10a\sqrt{1-a^2x^2}}+\frac{2c^4(ax+1)^9\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{3c^4(ax+1)^8\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}}+\frac{8c^4(ax+1)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}$$

```
[Out] (8*c^4*(1 + a*x)^7*Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) - (3*c^4*(1
 + a*x)^8*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - a^2*x^2]) + (2*c^4*(1 + a*x)^9
*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) - (c^4*(1 + a*x)^10*Sqrt[c -
a^2*c*x^2])/(10*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.115192, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$-\frac{c^4(ax+1)^{10}\sqrt{c-a^2cx^2}}{10a\sqrt{1-a^2x^2}}+\frac{2c^4(ax+1)^9\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{3c^4(ax+1)^8\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}}+\frac{8c^4(ax+1)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (8*c^4*(1 + a*x)^7*Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) - (3*c^4*(1
 + a*x)^8*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - a^2*x^2]) + (2*c^4*(1 + a*x)^9
*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) - (c^4*(1 + a*x)^10*Sqrt[c -
a^2*c*x^2])/(10*a*Sqrt[1 - a^2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}dx = \frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{9/2}dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int(1-ax)^3(1+ax)^6dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int\left(8(1+ax)^6-12(1+ax)^7+6(1+ax)^8-(1+ax)^9\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{8c^4(1+ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}} - \frac{3c^4(1+ax)^8\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} + \frac{2c^4(1+ax)^9\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}} - \frac{c^4(1+}{10}$$

Mathematica [A] time = 0.0528237, size = 68, normalized size = 0.37

$$-\frac{c^4(ax+1)^7\left(21a^3x^3-77a^2x^2+98ax-44\right)\sqrt{c-a^2cx^2}}{210a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] -(c^4*(1 + a*x)^7*Sqrt[c - a^2*c*x^2]*(-44 + 98*a*x - 77*a^2*x^2 + 21*a^3*x
^3))/(210*a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.03, size = 97, normalized size = 0.5

$$\frac{x\left(21\,a^9x^9+70\,a^8x^8-240\,x^6a^6-210\,x^5a^5+252\,x^4a^4+420\,x^3a^3-315\,ax-210\right)}{210\,(ax+1)^3\,(ax-1)^3}\left(-a^2cx^2+c\right)^{\frac{9}{2}}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(9/2),x)
```

```
[Out] 1/210*x*(21*a^9*x^9+70*a^8*x^8-240*a^6*x^6-210*a^5*x^5+252*a^4*x^4+420*a^3*
x^3-315*a*x-210)*(-a^2*c*x^2+c)^(9/2)/(a*x-1)^3/(a*x+1)^3/(-a^2*x^2+1)^(3/2
)
```

Maxima [B] time = 1.1484, size = 552, normalized size = 2.98

$$-\frac{1}{7}\,a^6c^{\frac{9}{2}}x^7+\frac{3}{5}\,a^4c^{\frac{9}{2}}x^5-a^2c^{\frac{9}{2}}x^3+c^{\frac{9}{2}}x+\frac{1}{40}\left(\frac{4\,a^8c^5x^{12}}{\sqrt{a^4cx^4-2\,a^2cx^2+c}}-\frac{19\,a^6c^5x^{10}}{\sqrt{a^4cx^4-2\,a^2cx^2+c}}+\frac{35\,a^4c^5x^8}{\sqrt{a^4cx^4-2\,a^2cx^2+c}}-\frac{}{\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="m
axima")
```

```
[Out] -1/7*a^6*c^(9/2)*x^7 + 3/5*a^4*c^(9/2)*x^5 - a^2*c^(9/2)*x^3 + c^(9/2)*x +
1/40*(4*a^8*c^5*x^12/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) - 19*a^6*c^5*x^10/sq
rt(a^4*c*x^4 - 2*a^2*c*x^2 + c) + 35*a^4*c^5*x^8/sqrt(a^4*c*x^4 - 2*a^2*c*x
```

```
^2 + c) - 30*a^2*c^5*x^6/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) + 10*c^5*x^4/sqr
t(a^4*c*x^4 - 2*a^2*c*x^2 + c))*a^3 - 1/105*(35*a^6*c^(9/2)*x^9 - 135*a^4*c
^(9/2)*x^7 + 189*a^2*c^(9/2)*x^5 - 105*c^(9/2)*x^3)*a^2 + 3/8*(a^8*c^5*x^10
/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) - 5*a^6*c^5*x^8/sqrt(a^4*c*x^4 - 2*a^2*c
*x^2 + c) + 10*a^4*c^5*x^6/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) - 10*a^2*c^5*x
^4/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) + 4*c^5/(sqrt(a^4*c*x^4 - 2*a^2*c*x^2
+ c)*a^2))*a
```

Fricas [A] time = 2.5971, size = 265, normalized size = 1.43

$$\frac{\left(21\,a^9c^4x^{10}+70\,a^8c^4x^9-240\,a^6c^4x^7-210\,a^5c^4x^6+252\,a^4c^4x^5+420\,a^3c^4x^4-315\,ac^4x^2-210\,c^4x\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2}}{210\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="f
ricas")
```

```
[Out] 1/210*(21*a^9*c^4*x^10 + 70*a^8*c^4*x^9 - 240*a^6*c^4*x^7 - 210*a^5*c^4*x^6
 + 252*a^4*c^4*x^5 + 420*a^3*c^4*x^4 - 315*a*c^4*x^2 - 210*c^4*x)*sqrt(-a^2
*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="g
iac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*(a*x + 1)^3/(-a^2*x^2 + 1)^(3/2), x)
```

3.1169 $\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=83

$$\frac{2\sqrt{1-a^2x^2}}{a(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a\sqrt{c-a^2cx^2}}$$

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2
]*Log[1 - a*x])/(a*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.0885614, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$\frac{2\sqrt{1-a^2x^2}}{a(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/Sqrt[c - a^2*c*x^2],x]
```


```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2
]*Log[1 - a*x])/(a*Sqrt[c - a^2*c*x^2])
```


Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1+ax}{(1-ax)^2}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{2}{(-1+ax)^2}+\frac{1}{-1+ax}\right)dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{2\sqrt{1-a^2x^2}}{a(1-ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-a^2x^2}\log(1-ax)}{a\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0331972, size = 51, normalized size = 0.61

$$\frac{\sqrt{1-a^2x^2}\left(\frac{2}{1-ax}+\log(1-ax)\right)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2/(1 - a*x) + Log[1 - a*x]))/(a*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.087, size = 70, normalized size = 0.8

$$\frac{-\ln(ax-1)xa+\ln(ax-1)+2}{c\left(a^2x^2-1\right)a(ax-1)}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-ln(a*x-1)*x*a+ln(a*x-1)+2)*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/(a^2
*x^2-1)/c/a/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)^3/(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)), x)
```

Fricas [B] time = 2.95509, size = 803, normalized size = 9.67

$$\left[\frac{4\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}ax+\left(a^3x^3-a^2x^2-ax+1\right)\sqrt{c}\log\left(\frac{a^6cx^6-4a^5cx^5+5a^4cx^4-4a^2cx^2+4acx-\left(a^4x^4-4a^3x^3+6a^2x^2-4ax\right)\sqrt{-a^2cx^2}}{a^4x^4-2a^3x^3+2ax-1}\right.}{2\left(a^4cx^3-a^3cx^2-a^2cx+ac\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/2*(4*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x + (a^3*x^3 - a^2*x^2 -
a*x + 1)*sqrt(c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 +
 4*a*c*x - (a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^2*c*x^2 + c)*s
qrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)))/(a^4*c
*x^3 - a^3*c*x^2 - a^2*c*x + a*c), (2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 +
1)*a*x + (a^3*x^3 - a^2*x^2 - a*x + 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)
*(a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 - 2*a^3*c*x^3
 - a^2*c*x^2 + 2*a*c*x)))/(a^4*c*x^3 - a^3*c*x^2 - a^2*c*x + a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x
 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3/(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)), x)
```

3.1170 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{1-a^2x^2}}{2ac(1-ax)^2\sqrt{c-a^2cx^2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(2*a*c*(1 - a*x)^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.084061, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 32}

$$\frac{\sqrt{1-a^2x^2}}{2ac(1-ax)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^(3/2),x]

[Out] Sqrt[1 - a^2*x^2]/(2*a*c*(1 - a*x)^2*Sqrt[c - a^2*c*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{3\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^3}\,dx}{c\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}}{2ac(1-ax)^2\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0398864, size = 53, normalized size = 1.13

$$-\frac{\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{2ac^2(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]*Sqrt[c - a^2*c*x^2])/(2*a*c^2*(-1 + a*x)^3*(1 + a*x))
```

Maple [A] time = 0.027, size = 43, normalized size = 0.9

$$-\frac{(ax-1)(ax+1)^3}{2a}\left(-a^2x^2+1\right)^{-\frac{3}{2}}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*(a*x-1)/a*(a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)^3}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*c*x^2 + c)^(3/2)*(-a^2*x^2 + 1)^(3/2)), x)
```

Fricas [A] time = 2.56247, size = 144, normalized size = 3.06

$$\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\left(ax^2-2x\right)}{2\left(a^4c^2x^4-2a^3c^2x^3+2ac^2x-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x^2 - 2*x)/(a^4*c^2*x^4 - 2*
a^3*c^2*x^3 + 2*a*c^2*x - c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(a*x - 1)*(a*x + 1
))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*c*x^2 + c)^(3/2)*(-a^2*x^2 + 1)^(3/2)), x)
```

3.1171 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=185

$$\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{6ac^2(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8ac^2\sqrt{c-a^2cx^2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(6*a*c^2*(1 - a*x)^3*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*
x^2]/(8*a*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*a*c^2
*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*a*c^2
*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.11643, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6143, 6140, 44, 207}

$$\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{6ac^2(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8ac^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^(5/2),x]
```


```
[Out] Sqrt[1 - a^2*x^2]/(6*a*c^2*(1 - a*x)^3*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*
x^2]/(8*a*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*a*c^2
*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*a*c^2
*Sqrt[c - a^2*c*x^2])
```


Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{3\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^4(1+ax)}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{2(-1+ax)^4}-\frac{1}{4(-1+ax)^3}+\frac{1}{8(-1+ax)^2}-\frac{1}{8\left(-1+a^2x^2\right)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}}{6ac^2(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\int}{8c^2\sqrt{c-}}$$
$$= \frac{\sqrt{1-a^2x^2}}{6ac^2(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\,\mathrm{ta}}{8ac^2\sqrt{c-}}$$

Mathematica [A] time = 0.0515388, size = 73, normalized size = 0.39

$$\frac{\sqrt{1-a^2x^2}\left(-3a^2x^2+9ax+3(ax-1)^3\tanh^{-1}(ax)-10\right)}{24ac^2(ax-1)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-10 + 9*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^3*ArcTanh[a*x]))
/(24*a*c^2*(-1 + a*x)^3*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.095, size = 159, normalized size = 0.9

$$-\frac{3\,a^3x^3\ln(ax+1)-3\,\ln(ax-1)\,x^3a^3-9\,\ln(ax+1)\,a^2x^2+9\,\ln(ax-1)\,a^2x^2-6\,a^2x^2+9\,ax\ln(ax+1)-9\,\ln(ax-}{\left(48\,a^2x^2-48\right)c^3a\,(ax-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/48*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*x^3*ln(a*x+1)-3*ln(a
*x-1)*x^3*a^3-9*ln(a*x+1)*a^2*x^2+9*ln(a*x-1)*a^2*x^2-6*a^2*x^2+9*a*x*ln(a*
x+1)-9*ln(a*x-1)*x*a+18*a*x-3*ln(a*x+1)+3*ln(a*x-1)-20)/(a^2*x^2-1)/c^3/a/(
a*x-1)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*c*x^2 + c)^(5/2)*(-a^2*x^2 + 1)^(3/2)), x)
```

Fricas [A] time = 3.04767, size = 969, normalized size = 5.24

$$\left[\frac{3\left(a^5x^5-3\,a^4x^4+2\,a^3x^3+2\,a^2x^2-3\,ax+1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5\,a^4cx^4-5\,a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-c}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}\right)+4\left(10\,a^3x^3-\right.}{96\left(a^6c^3x^5-3\,a^5c^3x^4+2\,a^4c^3x^3+2\,a^3c^3x^2-3\,a^2c^3x+ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="f
ricas")
```

```
[Out] [1/96*(3*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 - 3*a*x + 1)*sqrt(c)*
log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*x^3 + a*x)*sqrt(-a^2*c
*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
- 1)) + 4*(10*a^3*x^3 - 27*a^2*x^2 + 21*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2
*x^2 + 1))/(a^6*c^3*x^5 - 3*a^5*c^3*x^4 + 2*a^4*c^3*x^3 + 2*a^3*c^3*x^2 - 3
*a^2*c^3*x + a*c^3), 1/48*(3*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 -
 3*a*x + 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqr
t(-c)*x/(a^4*c*x^4 - c)) + 2*(10*a^3*x^3 - 27*a^2*x^2 + 21*a*x)*sqrt(-a^2*c
*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^6*c^3*x^5 - 3*a^5*c^3*x^4 + 2*a^4*c^3*x^3
+ 2*a^3*c^3*x^2 - 3*a^2*c^3*x + a*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}\,(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(a*x - 1)*(a*x + 1
))**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*c*x^2 + c)^(5/2)*(-a^2*x^2 + 1)^(3/2)), x)
```

3.1172 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=278

$$\frac{\sqrt{1-a^2x^2}}{8ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{32ac^3(ax+1)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{12ac^3(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{}{16ac^3}$$

```
[Out] Sqrt[1 - a^2*x^2]/(16*a*c^3*(1 - a*x)^4*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2
*x^2]/(12*a*c^3*(1 - a*x)^3*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2*x^2])/(3
2*a*c^3*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*a*c^3*(1 -
a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(32*a*c^3*(1 + a*x)*Sqrt[c -
a^2*c*x^2]) + (5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(32*a*c^3*Sqrt[c - a^2*c*x
^2])
```

Rubi [A] time = 0.138623, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6143, 6140, 44, 207}

$$\frac{\sqrt{1-a^2x^2}}{8ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{32ac^3(ax+1)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{12ac^3(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{}{16ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(16*a*c^3*(1 - a*x)^4*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2
*x^2]/(12*a*c^3*(1 - a*x)^3*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2*x^2])/(3
2*a*c^3*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*a*c^3*(1 -
a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(32*a*c^3*(1 + a*x)*Sqrt[c -
a^2*c*x^2]) + (5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(32*a*c^3*Sqrt[c - a^2*c*x
^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{3\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^5(1+ax)^2}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{4(-1+ax)^5}+\frac{1}{4(-1+ax)^4}-\frac{3}{16(-1+ax)^3}+\frac{1}{8(-1+ax)^2}+\frac{1}{32(1+ax)^2}-\frac{5}{32\left(-1+a^2x^2\right)}\right)dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}}{16ac^3(1-ax)^4\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{12ac^3(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-}}{8ac^3(1-ax}$$

$$= \frac{\sqrt{1-a^2x^2}}{16ac^3(1-ax)^4\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{12ac^3(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-}}{8ac^3(1-ax}$$

Mathematica [A] time = 0.0768633, size = 101, normalized size = 0.36

$$\frac{\sqrt{1-a^2x^2}\left(-15a^4x^4+45a^3x^3-35a^2x^2-15ax+15(ax-1)^4(ax+1)\tanh^{-1}(ax)+32\right)}{96ac^3(ax-1)^4(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(32 - 15*a*x - 35*a^2*x^2 + 45*a^3*x^3 - 15*a^4*x^4 + 15
*(-1 + a*x)^4*(1 + a*x)*ArcTanh[a*x]))/(96*a*c^3*(-1 + a*x)^4*(1 + a*x)*Sqr
t[c - a^2*c*x^2])
```

Maple [A] time = 0.099, size = 238, normalized size = 0.9

$$-\frac{15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5-45\ln(ax+1)a^4x^4+45\ln(ax-1)a^4x^4-30x^4a^4+30a^3x^3\ln(ax+1)-30}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/192*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x+1)*x^5*a^5-15*l
n(a*x-1)*x^5*a^5-45*ln(a*x+1)*a^4*x^4+45*ln(a*x-1)*a^4*x^4-30*x^4*a^4+30*a^
3*x^3*ln(a*x+1)-30*ln(a*x-1)*x^3*a^3+90*x^3*a^3+30*ln(a*x+1)*a^2*x^2-30*ln(
a*x-1)*a^2*x^2-70*a^2*x^2-45*a*x*ln(a*x+1)+45*ln(a*x-1)*x*a-30*a*x+15*ln(a*
x+1)-15*ln(a*x-1)+64)/(a^2*x^2-1)/c^4/a/(a*x+1)/(a*x-1)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*c*x^2 + c)^(7/2)*(-a^2*x^2 + 1)^(3/2)), x)
```

Fricas [A] time = 2.93026, size = 1175, normalized size = 4.23

$$\left[\frac{15\left(a^7x^7-3a^6x^6+a^5x^5+5a^4x^4-5a^3x^3-a^2x^2+3ax-1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5a^4cx^4-5a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}}{a^6x^6-3a^4x^4+3a^2x^2-1}\right.}{384\left(a^8c^4x^7-3a^7c^4x^6+a^6c^4x^5+5a^5c^4x^4-5a^4c^4x^3-a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="f
ricas")
```

```
[Out] [1/384*(15*(a^7*x^7 - 3*a^6*x^6 + a^5*x^5 + 5*a^4*x^4 - 5*a^3*x^3 - a^2*x^2
 + 3*a*x - 1)*sqrt(c)*log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*
x^3 + a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 -
3*a^4*x^4 + 3*a^2*x^2 - 1)) + 4*(32*a^5*x^5 - 81*a^4*x^4 + 19*a^3*x^3 + 99*
a^2*x^2 - 81*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^8*c^4*x^7 - 3
*a^7*c^4*x^6 + a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^4*c^4*x^3 - a^3*c^4*x^2 +
3*a^2*c^4*x - a*c^4), 1/192*(15*(a^7*x^7 - 3*a^6*x^6 + a^5*x^5 + 5*a^4*x^4
- 5*a^3*x^3 - a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*s
qrt(-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*c*x^4 - c)) + 2*(32*a^5*x^5 - 81*a^4*x^
4 + 19*a^3*x^3 + 99*a^2*x^2 - 81*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 +
1))/(a^8*c^4*x^7 - 3*a^7*c^4*x^6 + a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^4*c^4*
x^3 - a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*c*x^2 + c)^(7/2)*(-a^2*x^2 + 1)^(3/2)), x)
```

3.1173 $\int e^{3\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=136

$$\frac{4x^{m+1}\sqrt{c-a^2cx^2}\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{(m+1)\sqrt{1-a^2x^2}}-\frac{3x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}-\frac{ax^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}$$

```
[Out] (-3*x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) - (a*x^(2 +
m)*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2]) + (4*x^(1 + m)*Sqrt[c -
 a^2*c*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/((1 + m)*Sqrt[1 - a^2*
x^2])
```

Rubi [A] time = 0.206928, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6153, 6150, 88, 64}

$$\frac{4x^{m+1}\sqrt{c-a^2cx^2}\,{}_2F_1(1,m+1;m+2;ax)}{(m+1)\sqrt{1-a^2x^2}}-\frac{3x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}-\frac{ax^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```


```
[Out] (-3*x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) - (a*x^(2 +
m)*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2]) + (4*x^(1 + m)*Sqrt[c -
 a^2*c*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/((1 + m)*Sqrt[1 - a^2*
x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
```

```
&& !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0]))
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{3\tanh^{-1}(ax)}x^m\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{x^m(1+ax)^2}{1-ax}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-3x^m-ax^{1+m}+\frac{4x^m}{1-ax}\right)dx}{\sqrt{1-a^2x^2}}\\
&= -\frac{3x^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}}-\frac{ax^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}}+\frac{\left(4\sqrt{c-a^2cx^2}\right)\int \frac{x^m}{1-ax}\,dx}{\sqrt{1-a^2x^2}}\\
&= -\frac{3x^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}}-\frac{ax^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}}+\frac{4x^{1+m}\sqrt{c-a^2cx^2}\,{}_2F_1(1,1+m;2+m;ax)}{(1+m)\sqrt{1-a^2x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0497853, size = 74, normalized size = 0.54

$$
-\frac{x^{m+1}\sqrt{c-a^2cx^2}(-4(m+2)\text{Hypergeometric2F1}(1,m+1,m+2,ax)+m(ax+3)+ax+6)}{(m+1)(m+2)\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]

[Out] -((x^(1 + m)*Sqrt[c - a^2*c*x^2]*(6 + a*x + m*(3 + a*x) - 4*(2 + m)*Hyperge
ometric2F1[1, 1 + m, 2 + m, a*x]))/((1 + m)*(2 + m)*Sqrt[1 - a^2*x^2]))
```

Maple [F] time = 0.421, size = 0, normalized size = 0.

$$
\int (ax+1)^3 x^m\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{-\frac{3}{2}}dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)

[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{-a^2cx^2+c}(ax+1)^3x^m}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3*x^m/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}(ax+1)x^m}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x + 1)*x^m/(a^2*x^2 - 2
*a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^m\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)^3}{(-\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x +
1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2cx^2+c}(ax+1)^3x^m}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3*x^m/(-a^2*x^2 + 1)^(3/2), x)
```

3.1174 $\int e^{3\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=251

$$\frac{(4m+2p+3)x^{m+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{3}{2}-p,\frac{m+3}{2},a^2x^2\right)}{(m+1)(m+2p)}+\frac{a(4m+6p+5)x^{m+2}\left(1-a\right.}{}$$

```
[Out] (-3*x^(1 + m)*(c - a^2*c*x^2)^p)/((m + 2*p)*Sqrt[1 - a^2*x^2]) - (a*x^(2 +
m)*(c - a^2*c*x^2)^p)/((1 + m + 2*p)*Sqrt[1 - a^2*x^2]) + ((3 + 4*m + 2*p)*
x^(1 + m)*(c - a^2*c*x^2)^p*Hypergeometric2F1[(1 + m)/2, 3/2 - p, (3 + m)/2
, a^2*x^2])/((1 + m)*(m + 2*p)*(1 - a^2*x^2)^p) + (a*(5 + 4*m + 6*p)*x^(2 +
 m)*(c - a^2*c*x^2)^p*Hypergeometric2F1[(2 + m)/2, 3/2 - p, (4 + m)/2, a^2*
x^2])/((2 + m)*(1 + m + 2*p)*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.416018, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6153, 6148, 1809, 808, 364}

$$\frac{(4m+2p+3)x^{m+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{m+1}{2},\frac{3}{2}-p;\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2p)}+\frac{a(4m+6p+5)x^{m+2}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{(m+2)(m+2p+}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^p,x]
```


```
[Out] (-3*x^(1 + m)*(c - a^2*c*x^2)^p)/((m + 2*p)*Sqrt[1 - a^2*x^2]) - (a*x^(2 +
m)*(c - a^2*c*x^2)^p)/((1 + m + 2*p)*Sqrt[1 - a^2*x^2]) + ((3 + 4*m + 2*p)*
x^(1 + m)*(c - a^2*c*x^2)^p*Hypergeometric2F1[(1 + m)/2, 3/2 - p, (3 + m)/2
, a^2*x^2])/((1 + m)*(m + 2*p)*(1 - a^2*x^2)^p) + (a*(5 + 4*m + 6*p)*x^(2 +
 m)*(c - a^2*c*x^2)^p*Hypergeometric2F1[(2 + m)/2, 3/2 - p, (4 + m)/2, a^2*
x^2])/((2 + m)*(1 + m + 2*p)*(1 - a^2*x^2)^p)
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
```

```
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{3\tanh^{-1}(ax)}x^m\left(1-a^2x^2\right)^p\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^m(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\,dx$$
$$=-\frac{ax^{2+m}\left(c-a^2cx^2\right)^p}{(1+m+2p)\sqrt{1-a^2x^2}}-\frac{\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^m\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\left(-a^2(1\right.}{a^2(1+}$$
$$=-\frac{3x^{1+m}\left(c-a^2cx^2\right)^p}{(m+2p)\sqrt{1-a^2x^2}}-\frac{ax^{2+m}\left(c-a^2cx^2\right)^p}{(1+m+2p)\sqrt{1-a^2x^2}}+\frac{\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^m}{}$$
$$=-\frac{3x^{1+m}\left(c-a^2cx^2\right)^p}{(m+2p)\sqrt{1-a^2x^2}}-\frac{ax^{2+m}\left(c-a^2cx^2\right)^p}{(1+m+2p)\sqrt{1-a^2x^2}}+\frac{\left((3+4m+2p)\left(1-a^2x^2\right)^{-p}\left(c-\right.\right.}{m}$$
$$=-\frac{3x^{1+m}\left(c-a^2cx^2\right)^p}{(m+2p)\sqrt{1-a^2x^2}}-\frac{ax^{2+m}\left(c-a^2cx^2\right)^p}{(1+m+2p)\sqrt{1-a^2x^2}}+\frac{(3+4m+2p)x^{1+m}\left(1-a^2x^2\right)^{-p}}{(1+}$$

Mathematica [A] time = 0.166521, size = 186, normalized size = 0.74

$$x^{m+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{3}{2}-p,\frac{m+3}{2},a^2x^2\right)}{m+1}+ax\left(\frac{3\text{Hypergeometric2F1}\left(\frac{m+2}{2},\frac{3}{2}\right.}{m+2}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^p,x]
```

```
[Out] (x^(1 + m)*(c - a^2*c*x^2)^p*(Hypergeometric2F1[(1 + m)/2, 3/2 - p, (3 + m)
/2, a^2*x^2]/(1 + m) + a*x*((3*Hypergeometric2F1[(2 + m)/2, 3/2 - p, (4 + m
)/2, a^2*x^2])/(2 + m) + a*x*((3*Hypergeometric2F1[(3 + m)/2, 3/2 - p, (5 +
 m)/2, a^2*x^2])/(3 + m) + (a*x*Hypergeometric2F1[(4 + m)/2, 3/2 - p, (6 +
m)/2, a^2*x^2])/(4 + m)))))/(1 - a^2*x^2)^p
```

Maple [F] time = 0.406, size = 0, normalized size = 0.

$$\int (ax+1)^3x^m\left(-a^2cx^2+c\right)^p\left(-a^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3(-a^2cx^2+c)^p x^m}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^p,x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p*x^m/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}(ax+1)(-a^2cx^2+c)^p x^m}{a^2x^2-2\,ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^p,x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*(-a^2*c*x^2 + c)^p*x^m/(a^2*x^2 - 2*a
*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**m*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3(-a^2cx^2+c)^p x^m}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^p,x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p*x^m/(-a^2*x^2 + 1)^(3/2), x)
```

3.1175 $\int e^{3\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=224

$$\frac{a(6p+17)x^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(\frac{5}{2},\frac{3}{2}-p,\frac{7}{2},a^2x^2\right)}{10(p+2)}-\frac{3\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^4(2p+3)}+\frac{7\sqrt{1-a}}{a}$$

```
[Out] (4*(c - a^2*c*x^2)^p)/(a^4*(1 - 2*p)*Sqrt[1 - a^2*x^2]) - (a*x^5*(c - a^2*c
*x^2)^p)/(2*(2 + p)*Sqrt[1 - a^2*x^2]) + (7*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^
2)^p)/(a^4*(1 + 2*p)) - (3*(1 - a^2*x^2)^(3/2)*(c - a^2*c*x^2)^p)/(a^4*(3 +
 2*p)) + (a*(17 + 6*p)*x^5*(c - a^2*c*x^2)^p*Hypergeometric2F1[5/2, 3/2 - p
, 7/2, a^2*x^2])/(10*(2 + p)*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.355352, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {6153, 6148, 1652, 446, 77, 459, 364}

$$\frac{a(6p+17)x^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{3}{2}-p;\frac{7}{2};a^2x^2\right)}{10(p+2)}-\frac{3\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^4(2p+3)}+\frac{7\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^4(2p+1)}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^p,x]
```

```
[Out] (4*(c - a^2*c*x^2)^p)/(a^4*(1 - 2*p)*Sqrt[1 - a^2*x^2]) - (a*x^5*(c - a^2*c
*x^2)^p)/(2*(2 + p)*Sqrt[1 - a^2*x^2]) + (7*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^
2)^p)/(a^4*(1 + 2*p)) - (3*(1 - a^2*x^2)^(3/2)*(c - a^2*c*x^2)^p)/(a^4*(3 +
 2*p)) + (a*(17 + 6*p)*x^5*(c - a^2*c*x^2)^p*Hypergeometric2F1[5/2, 3/2 - p
, 7/2, a^2*x^2])/(10*(2 + p)*(1 - a^2*x^2)^p)
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{3\tanh^{-1}(ax)}x^3\left(1-a^2x^2\right)^p\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^3(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\left(1+3a^2x^2\right)dx+\left(\left(1-a^2x^2\right)^{-p}\left(c\cdot\right.\right.$$
$$=-\frac{ax^5\left(c-a^2cx^2\right)^p}{2(2+p)\sqrt{1-a^2x^2}}+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\mathrm{Subst}\left(\int x\left(1-a^2x\right)^{-\frac{3}{2}+p}\left(1\cdot\right.\right.$$
$$=-\frac{ax^5\left(c-a^2cx^2\right)^p}{2(2+p)\sqrt{1-a^2x^2}}+\frac{a(17+6p)x^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{3}{2}-p;\frac{7}{2};a^2x^2\right.}{10(2+p)}$$
$$=\frac{4\left(c-a^2cx^2\right)^p}{a^4(1-2p)\sqrt{1-a^2x^2}}-\frac{ax^5\left(c-a^2cx^2\right)^p}{2(2+p)\sqrt{1-a^2x^2}}+\frac{7\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^4(1+2p)}-\frac{3\left(1-a^2x^2\right.}{a^4}$$

Mathematica [A] time = 0.202111, size = 176, normalized size = 0.79

$$\frac{\left(c-a^2cx^2\right)^p\left(21a^5x^5\left(1-a^2x^2\right)^{-p}\mathrm{Hypergeometric2F1}\left(\frac{5}{2},\frac{3}{2}-p,\frac{7}{2},a^2x^2\right)+5a^7x^7\left(1-a^2x^2\right)^{-p}\mathrm{Hypergeometric2F}\right.}{35a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((c - a^2*c*x^2)^p*(140/((1 - 2*p)*Sqrt[1 - a^2*x^2]) + (245*Sqrt[1 - a^2*x
^2])/(1 + 2*p) - (105*(1 - a^2*x^2)^(3/2))/(3 + 2*p) + (21*a^5*x^5*Hypergeo
metric2F1[5/2, 3/2 - p, 7/2, a^2*x^2])/(1 - a^2*x^2)^p + (5*a^7*x^7*Hyperge
ometric2F1[7/2, 3/2 - p, 9/2, a^2*x^2])/(1 - a^2*x^2)^p))/(35*a^4)
```

Maple [F] time = 0.411, size = 0, normalized size = 0.

$$\int (ax+1)^3 x^3 \left(-a^2cx^2+c\right)^p \left(-a^2x^2+1\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(a^2c^p\left(2p-1\right)x^2+2c^p\right)\left(-a^2x^2+1\right)^p}{\sqrt{-a^2x^2+1}\left(4p^2-1\right)a^4}-\int \frac{\left(a^3c^px^6+3a^2c^px^5+3ac^px^4\right)e^{\left(p\log(ax+1)+p\log(-ax+1)\right)}}{\left(a^2x^2-1\right)\sqrt{ax+1}\sqrt{-ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^p,x, algorithm="m
axima")
```

```
[Out] -(a^2*c^p*(2*p - 1)*x^2 + 2*c^p)*(-a^2*x^2 + 1)^p/(sqrt(-a^2*x^2 + 1)*(4*p^
2 - 1)*a^4) - integrate((a^3*c^p*x^6 + 3*a^2*c^p*x^5 + 3*a*c^p*x^4)*e^(p*lo
g(a*x + 1) + p*log(-a*x + 1))/((a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ax^4+x^3\right)\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{a^2x^2-2ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^p,x, algorithm="f
ricas")
```

```
[Out] integral((a*x^4 + x^3)*sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 2*a
*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\left(-c\left(ax-1\right)\left(ax+1\right)\right)^p\left(ax+1\right)^3}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**3*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral(x**3*(-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1
))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3 \left(-a^2cx^2+c\right)^p x^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^p,x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p*x^3/(-a^2*x^2 + 1)^(3/2), x)
```

3.1176 $\int e^{3\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=222

$$\frac{(2p+11)x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3}{2}-p,\frac{5}{2},a^2x^2\right)}{6(p+1)}-\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^3(2p+3)}+\frac{5\sqrt{1-a^2x^2}}{a^3(2}$$

```
[Out] (4*(c - a^2*c*x^2)^p)/(a^3*(1 - 2*p)*Sqrt[1 - a^2*x^2]) - (3*x^3*(c - a^2*c
*x^2)^p)/(2*(1 + p)*Sqrt[1 - a^2*x^2]) + (5*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^
2)^p)/(a^3*(1 + 2*p)) - ((1 - a^2*x^2)^(3/2)*(c - a^2*c*x^2)^p)/(a^3*(3 + 2
*p)) + ((11 + 2*p)*x^3*(c - a^2*c*x^2)^p*Hypergeometric2F1[3/2, 3/2 - p, 5/
2, a^2*x^2])/(6*(1 + p)*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.339021, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {6153, 6148, 1652, 459, 364, 446, 77}

$$\frac{(2p+11)x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{3}{2}-p;\frac{5}{2};a^2x^2\right)}{6(p+1)}-\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^3(2p+3)}+\frac{5\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^3(2p+1)}-\frac{3}{2(}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^p,x]
```

```
[Out] (4*(c - a^2*c*x^2)^p)/(a^3*(1 - 2*p)*Sqrt[1 - a^2*x^2]) - (3*x^3*(c - a^2*c
*x^2)^p)/(2*(1 + p)*Sqrt[1 - a^2*x^2]) + (5*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^
2)^p)/(a^3*(1 + 2*p)) - ((1 - a^2*x^2)^(3/2)*(c - a^2*c*x^2)^p)/(a^3*(3 + 2
*p)) + ((11 + 2*p)*x^3*(c - a^2*c*x^2)^p*Hypergeometric2F1[3/2, 3/2 - p, 5/
2, a^2*x^2])/(6*(1 + p)*(1 - a^2*x^2)^p)
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int e^{3\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{3\tanh^{-1}(ax)}x^2\left(1-a^2x^2\right)^p\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^2(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^2\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\left(1+3a^2x^2\right)dx+\left(\left(1-a^2x^2\right)^{-p}\left(c\right.\right.\\
&= -\frac{3x^3\left(c-a^2cx^2\right)^p}{2(1+p)\sqrt{1-a^2x^2}}+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\mathrm{Subst}\left(\int x\left(1-a^2x\right)^{-\frac{3}{2}+p}\left(3a\right.\right.\\
&= -\frac{3x^3\left(c-a^2cx^2\right)^p}{2(1+p)\sqrt{1-a^2x^2}}+\frac{(11+2p)x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p{}_2F_1\left(\frac{3}{2},\frac{3}{2}-p;\frac{5}{2};a^2x^2\right)}{6(1+p)}\\
&= \frac{4\left(c-a^2cx^2\right)^p}{a^3(1-2p)\sqrt{1-a^2x^2}}-\frac{3x^3\left(c-a^2cx^2\right)^p}{2(1+p)\sqrt{1-a^2x^2}}+\frac{5\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^3(1+2p)}-\frac{\left(1-a^2x^2\right)^{\cdots}}{a^3(\cdots}
\end{aligned}$$

Mathematica [A] time = 0.230338, size = 179, normalized size = 0.81

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{1}{3}x^3\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3}{2}-p,\frac{5}{2},a^2x^2\right)+\frac{3}{5}a^2x^5\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{3}{2}-p,\frac{7}{2},\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((c - a^2*c*x^2)^p*(((1 - a^2*x^2)^(-1/2 + p)*(-26 + 13*a^2*x^2 + a^4*x^4 -
 4*a^2*p^2*x^2*(3 + a^2*x^2) - 4*p*(3 + 5*a^2*x^2)))/(a^3*(-1 + 2*p)*(1 + 2
*p)*(3 + 2*p)) + (x^3*Hypergeometric2F1[3/2, 3/2 - p, 5/2, a^2*x^2])/3 + (3
*a^2*x^5*Hypergeometric2F1[5/2, 3/2 - p, 7/2, a^2*x^2])/5))/(1 - a^2*x^2)^p
```

Maple [F] time = 0.404, size = 0, normalized size = 0.

$$\int (ax+1)^3 x^2 \left(-a^2cx^2+c\right)^p \left(-a^2x^2+1\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(-a^2cx^2+c\right)^p x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^p,x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p*x^2/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(ax^3+x^2\right)\left(-a^2cx^2+c\right)^p}{a^2x^2-2\,ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^p,x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x^3 + x^2)*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 2*a
*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(-c\left(ax-1\right)\left(ax+1\right)\right)^p\left(ax+1\right)^3}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral(x**2*(-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1
))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3(-a^2cx^2+c)^p x^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^p,x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p*x^2/(-a^2*x^2 + 1)^(3/2), x)
```

3.1177 $\int e^{3\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=138

$$\frac{3\,2^{p+\frac{3}{2}}(1-ax)^{p-\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(-p-\frac{3}{2},p-\frac{1}{2},p+\frac{1}{2},\frac{1}{2}(1-ax)\right)}{a^2\left(-2p^2-p+1\right)}-\frac{(ax+1)^3\left(c-a^2cx\right.}{2a^2(p+1)\sqrt{1-a^2}}$$

```
[Out] -((1 + a*x)^3*(c - a^2*c*x^2)^p)/(2*a^2*(1 + p)*Sqrt[1 - a^2*x^2]) + (3*2^(
3/2 + p)*(1 - a*x)^(-1/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-3/2 - p,
 -1/2 + p, 1/2 + p, (1 - a*x)/2])/(a^2*(1 - p - 2*p^2)*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.169421, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6153, 6148, 795, 676, 69}

$$\frac{3\,2^{p+\frac{3}{2}}(1-ax)^{p-\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-\frac{3}{2},p-\frac{1}{2};p+\frac{1}{2};\frac{1}{2}(1-ax)\right)}{a^2\left(-2p^2-p+1\right)}-\frac{(ax+1)^3\left(c-a^2cx^2\right)^p}{2a^2(p+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((1 + a*x)^3*(c - a^2*c*x^2)^p)/(2*a^2*(1 + p)*Sqrt[1 - a^2*x^2]) + (3*2^(
3/2 + p)*(1 - a*x)^(-1/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-3/2 - p,
 -1/2 + p, 1/2 + p, (1 - a*x)/2])/(a^2*(1 - p - 2*p^2)*(1 - a^2*x^2)^p)
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 676

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(a^(p + 1)*d^(m - 1)*((d - e*x)/d)^(p + 1))/(a/d + (c*x)/e)^(p + 1), Int[(1
 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d, e, m}, x]
```

```
&& EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&
GtQ[a, 0] &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{3\tanh^{-1}(ax)}x\left(1-a^2x^2\right)^p\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\,dx\\
&= -\frac{(1+ax)^3\left(c-a^2cx^2\right)^p}{2a^2(1+p)\sqrt{1-a^2x^2}}+\frac{\left(3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\,d}{2a(1+p)}\\
&= -\frac{(1+ax)^3\left(c-a^2cx^2\right)^p}{2a^2(1+p)\sqrt{1-a^2x^2}}+\frac{\left(3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1-ax)^{-\frac{3}{2}+p}(1+ax)^{\frac{3}{2}+p}\,dx}{2a(1+p)}\\
&= -\frac{(1+ax)^3\left(c-a^2cx^2\right)^p}{2a^2(1+p)\sqrt{1-a^2x^2}}+\frac{3\,2^{\frac{3}{2}+p}(1-ax)^{-\frac{1}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{3}{2}-p\right.}{a^2(1-2p)(1+p)}
\end{aligned}
$$

Mathematica [A] time = 0.362921, size = 134, normalized size = 0.97

$$
\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(ax^3\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3}{2}-p,\frac{5}{2},a^2x^2\right)+\frac{1}{5}a^3x^5\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{3}{2}-p,\frac{7}{2},\right.\right.
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((c - a^2*c*x^2)^p*(((1 - a^2*x^2)^(-1/2 + p)*(4/(1 - 2*p) + (3 - 3*a^2*x^2
)/(1 + 2*p)))/a^2 + a*x^3*Hypergeometric2F1[3/2, 3/2 - p, 5/2, a^2*x^2] + (
a^3*x^5*Hypergeometric2F1[5/2, 3/2 - p, 7/2, a^2*x^2])/5))/(1 - a^2*x^2)^p
```

Maple [F] time = 0.401, size = 0, normalized size = 0.

$$
\int(ax+1)^3x\left(-a^2cx^2+c\right)^p\left(-a^2x^2+1\right)^{-\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(-a^2x^2+1\right)^p c^p}{\sqrt{-a^2x^2+1}a^2\left(2p-1\right)}-\int \frac{\left(a^3c^px^4+3\,a^2c^px^3+3\,ac^px^2\right)e^{\left(p\log(ax+1)+p\log(-ax+1)\right)}}{\left(a^2x^2-1\right)\sqrt{ax+1}\sqrt{-ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^p,x, algorithm="max
ima")
```

```
[Out] -(-a^2*x^2 + 1)^p*c^p/(sqrt(-a^2*x^2 + 1)*a^2*(2*p - 1)) - integrate((a^3*c
^p*x^4 + 3*a^2*c^p*x^3 + 3*a*c^p*x^2)*e^(p*log(a*x + 1) + p*log(-a*x + 1))/
((a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(ax^2+x\right)\left(-a^2cx^2+c\right)^p}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^p,x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x^2 + x)*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 2*a*x
 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(-c\left(ax-1\right)\left(ax+1\right)\right)^p\left(ax+1\right)^3}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral(x*(-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))*
*(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax+1\right)^3\left(-a^2cx^2+c\right)^p x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^p,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p*x/(-a^2*x^2 + 1)^(3/2), x)
```

3.1178 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=85

$$\frac{2^{p+\frac{5}{2}}(1-ax)^{p-\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(-p-\frac{3}{2},p-\frac{1}{2},p+\frac{1}{2},\frac{1}{2}(1-ax)\right)}{a(1-2p)}$$

```
[Out] (2^(5/2 + p)*(1 - a*x)^(-1/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-3/2
- p, -1/2 + p, 1/2 + p, (1 - a*x)/2])/(a*(1 - 2*p)*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.0740719, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6143, 6140, 69}

$$\frac{2^{p+\frac{5}{2}}(1-ax)^{p-\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-\frac{3}{2},p-\frac{1}{2};p+\frac{1}{2};\frac{1}{2}(1-ax)\right)}{a(1-2p)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```



```
[Out] (2^(5/2 + p)*(1 - a*x)^(-1/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-3/2
- p, -1/2 + p, 1/2 + p, (1 - a*x)/2])/(a*(1 - 2*p)*(1 - a^2*x^2)^p)
```


Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx\\ &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int (1-ax)^{-\frac{3}{2}+p}(1+ax)^{\frac{3}{2}+p}\,dx\\ &= \frac{2^{\frac{5}{2}+p}(1-ax)^{-\frac{1}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{3}{2}-p,-\frac{1}{2}+p;\frac{1}{2}+p;\frac{1}{2}(1-ax)\right)}{a(1-2p)}\end{aligned}$$

Mathematica [A] time = 0.0291076, size = 83, normalized size = 0.98

$$\frac{2^{p+\frac{5}{2}}(1-ax)^{p-\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-p-\frac{3}{2},p-\frac{1}{2},p+\frac{1}{2},\frac{1}{2}(1-ax)\right)}{a-2ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (2^(5/2 + p)*(1 - a*x)^(-1/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-3/2
- p, -1/2 + p, 1/2 + p, (1 - a*x)/2])/((a - 2*a*p)*(1 - a^2*x^2)^p)
```

Maple [F] time = 0.395, size = 0, normalized size = 0.

$$\int (ax+1)^3\left(-a^2cx^2+c\right)^p\left(-a^2x^2+1\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(-a^2cx^2+c\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}(ax+1)\left(-a^2cx^2+c\right)^p}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 2*a*x +
 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{p}\left(ax+1\right)^{3}}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(
3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax+1\right)^{3}\left(-a^{2}cx^{2}+c\right)^{p}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p/(-a^2*x^2 + 1)^(3/2), x)
```

3.1179 $\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x}\,dx$

Optimal. Leaf size=193

$$\frac{a(6p+1)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}-p,\frac{3}{2},a^2x^2\right)}{2p}-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p \text{Hypergeome}}{2p+}$$

```
[Out] (4*(c - a^2*c*x^2)^p)/((1 - 2*p)*Sqrt[1 - a^2*x^2]) - (a*x*(c - a^2*c*x^2)^
p)/(2*p*Sqrt[1 - a^2*x^2]) + (a*(1 + 6*p)*x*(c - a^2*c*x^2)^p*Hypergeometri
c2F1[1/2, 3/2 - p, 3/2, a^2*x^2])/(2*p*(1 - a^2*x^2)^p) - (Sqrt[1 - a^2*x^2
]*(c - a^2*c*x^2)^p*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1
 + 2*p)
```

Rubi [A] time = 0.291826, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {6153, 6148, 1652, 446, 79, 65, 388, 245}

$$\frac{a(6p+1)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\, {}_2F_1\left(\frac{1}{2},\frac{3}{2}-p;\frac{3}{2};a^2x^2\right)}{2p}-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p\, {}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^p)/x,x]
```

```
[Out] (4*(c - a^2*c*x^2)^p)/((1 - 2*p)*Sqrt[1 - a^2*x^2]) - (a*x*(c - a^2*c*x^2)^
p)/(2*p*Sqrt[1 - a^2*x^2]) + (a*(1 + 6*p)*x*(c - a^2*c*x^2)^p*Hypergeometri
c2F1[1/2, 3/2 - p, 3/2, a^2*x^2])/(2*p*(1 - a^2*x^2)^p) - (Sqrt[1 - a^2*x^2
]*(c - a^2*c*x^2)^p*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1
 + 2*p)
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x}\,dx = \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x}\,dx$$

$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}}{x}\,dx$$

$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\left(1+3a^2x^2\right)}{x}\,dx + \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)\right.$$

$$= -\frac{ax\left(c-a^2cx^2\right)^p}{2p\sqrt{1-a^2x^2}} + \frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\mathrm{Subst}\left(\int \frac{\left(1-a^2x\right)^{-\frac{3}{2}+p}\left(1+3a^2x\right)}{x}\,dx\right.$$

$$= \frac{4\left(c-a^2cx^2\right)^p}{(1-2p)\sqrt{1-a^2x^2}} - \frac{ax\left(c-a^2cx^2\right)^p}{2p\sqrt{1-a^2x^2}} + \frac{a(1+6p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-\right.}{2p}$$

$$= \frac{4\left(c-a^2cx^2\right)^p}{(1-2p)\sqrt{1-a^2x^2}} - \frac{ax\left(c-a^2cx^2\right)^p}{2p\sqrt{1-a^2x^2}} + \frac{a(1+6p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-\right.}{2p}$$

Mathematica [A] time = 0.223485, size = 159, normalized size = 0.82

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(-\frac{\left(1-a^2x^2\right)^{p-\frac{1}{2}}\text{Hypergeometric2F1}\left(1,p-\frac{1}{2},p+\frac{1}{2},1-a^2x^2\right)}{2\left(p-\frac{1}{2}\right)}+3ax\text{Hypergeometric2F}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^p)/x,x]
```

```
[Out] ((c - a^2*c*x^2)^p*((3*(1 - a^2*x^2)^(-1/2 + p))/(1 - 2*p) + 3*a*x*Hypergeo
metric2F1[1/2, 3/2 - p, 3/2, a^2*x^2] - ((1 - a^2*x^2)^(-1/2 + p)*Hypergeom
etric2F1[1, -1/2 + p, 1/2 + p, 1 - a^2*x^2])/(2*(-1/2 + p)) + (a^3*x^3*Hype
rgeometric2F1[3/2, 3/2 - p, 5/2, a^2*x^2])/3))/(1 - a^2*x^2)^p
```

Maple [F] time = 0.396, size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(-a^2cx^2+c\right)^p}{x}\left(-a^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(-a^2cx^2+c\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}(ax+1)\left(-a^2cx^2+c\right)^p}{a^2x^3-2\,ax^2+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x,x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^3 - 2*a*x^2
 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c(ax-1)(ax+1))^p (ax+1)^3}{x(-(ax-1)(ax+1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**p/x,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1))
**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3(-a^2cx^2+c)^p}{(-a^2x^2+1)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p/((-a^2*x^2 + 1)^(3/2)*x), x)
```

3.1180 $\int \frac{e^{3\tanh^{-1}(ax)}(c-a^2cx^2)^p}{x^2}\,dx$

Optimal. Leaf size=187

$$a^2(5-2p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}-p,\frac{3}{2},a^2x^2\right)-\frac{3a\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p \text{Hyperge}}{2p}$$

```
[Out] (4*a*(c - a^2*c*x^2)^p)/((1 - 2*p)*Sqrt[1 - a^2*x^2]) - (c - a^2*c*x^2)^p/(
x*Sqrt[1 - a^2*x^2]) + (a^2*(5 - 2*p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1
[1/2, 3/2 - p, 3/2, a^2*x^2])/(1 - a^2*x^2)^p - (3*a*Sqrt[1 - a^2*x^2]*(c -
 a^2*c*x^2)^p*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p
)
```

Rubi [A] time = 0.331396, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {6153, 6148, 1807, 1652, 446, 79, 65, 12, 245}

$$a^2(5-2p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p {}_2F_1\left(\frac{1}{2},\frac{3}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{3a\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p {}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^p)/x^2,x]
```

```
[Out] (4*a*(c - a^2*c*x^2)^p)/((1 - 2*p)*Sqrt[1 - a^2*x^2]) - (c - a^2*c*x^2)^p/(
x*Sqrt[1 - a^2*x^2]) + (a^2*(5 - 2*p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1
[1/2, 3/2 - p, 3/2, a^2*x^2])/(1 - a^2*x^2)^p - (3*a*Sqrt[1 - a^2*x^2]*(c -
 a^2*c*x^2)^p*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p
)
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x^2}\,dx = \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^2}\,dx$$

$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}}{x^2}\,dx$$

$$= -\frac{\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} - \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\left(-3a-a^2(5-2p)x-\right.}{x}$$

$$= -\frac{\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} + \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int a^2(5-2p)\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\,dx - \left(\left(1-\right.\right.$$

$$= -\frac{\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} - \frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int \frac{\left(1-a^2x\right)^{-\frac{3}{2}+p}\left(-3a-a^3x\right)}{x}\right.$$

$$= \frac{4a\left(c-a^2cx^2\right)^p}{(1-2p)\sqrt{1-a^2x^2}} - \frac{\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} + a^2(5-2p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}\right.$$

$$= \frac{4a\left(c-a^2cx^2\right)^p}{(1-2p)\sqrt{1-a^2x^2}} - \frac{\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} + a^2(5-2p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}\right.$$

Mathematica [A] time = 0.250108, size = 133, normalized size = 0.71

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(a\left(\frac{\left(1-a^2x^2\right)^{p-\frac{1}{2}}\left(3\text{Hypergeometric2F1}\left(1,p-\frac{1}{2},p+\frac{1}{2},1-a^2x^2\right)+1\right)}{1-2p}+3ax\text{Hypergeon}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^p)/x^2,x]
```

```
[Out] ((c - a^2*c*x^2)^p*(-(Hypergeometric2F1[-1/2, 3/2 - p, 1/2, a^2*x^2]/x) + a
*(3*a*x*Hypergeometric2F1[1/2, 3/2 - p, 3/2, a^2*x^2] + ((1 - a^2*x^2)^(-1/
2 + p)*(1 + 3*Hypergeometric2F1[1, -1/2 + p, 1/2 + p, 1 - a^2*x^2]))/(1 - 2
*p))))/(1 - a^2*x^2)^p
```

Maple [F] time = 0.396, size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3\left(-a^2cx^2+c\right)^p}{x^2}\left(-a^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^2,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3(-a^2cx^2+c)^p}{(-a^2x^2+1)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^2,x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}(ax+1)(-a^2cx^2+c)^p}{a^2x^4-2\,ax^3+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^2,x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^4 - 2*a*x^3
 + x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c\,(ax-1)\,(ax+1))^p\,(ax+1)^3}{x^2\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**p/x**2,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**3/(x**2*(-(a*x - 1)*(a*x +
1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3(-a^2cx^2+c)^p}{(-a^2x^2+1)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^2,x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

3.1181 $\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x^3}\,dx$

Optimal. Leaf size=194

$$a^3(7-6p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}-p,\frac{3}{2},a^2x^2\right)+\frac{a^2(9-2p)\left(c-a^2cx^2\right)^p \text{Hypergeon}}{2(1-2p)}$$

```
[Out] -(c - a^2*c*x^2)^p/(2*x^2*Sqrt[1 - a^2*x^2]) - (3*a*(c - a^2*c*x^2)^p)/(x*S
qrt[1 - a^2*x^2]) + (a^3*(7 - 6*p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/
2, 3/2 - p, 3/2, a^2*x^2])/(1 - a^2*x^2)^p + (a^2*(9 - 2*p)*(c - a^2*c*x^2)
^p*Hypergeometric2F1[1, -1/2 + p, 1/2 + p, 1 - a^2*x^2])/(2*(1 - 2*p)*Sqrt[
1 - a^2*x^2])
```

Rubi [A] time = 0.367373, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {6153, 6148, 1807, 764, 266, 65, 245}

$$a^3(7-6p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-p;\frac{3}{2};a^2x^2\right)+\frac{a^2(9-2p)\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,p-\frac{1}{2};p+\frac{1}{2};1-a^2x^2\right)}{2(1-2p)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^p)/x^3,x]
```

```
[Out] -(c - a^2*c*x^2)^p/(2*x^2*Sqrt[1 - a^2*x^2]) - (3*a*(c - a^2*c*x^2)^p)/(x*S
qrt[1 - a^2*x^2]) + (a^3*(7 - 6*p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/
2, 3/2 - p, 3/2, a^2*x^2])/(1 - a^2*x^2)^p + (a^2*(9 - 2*p)*(c - a^2*c*x^2)
^p*Hypergeometric2F1[1, -1/2 + p, 1/2 + p, 1 - a^2*x^2])/(2*(1 - 2*p)*Sqrt[
1 - a^2*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x^3}\,dx = \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^3}\,dx$$

$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}}{x^3}\,dx$$

$$= -\frac{\left(c-a^2cx^2\right)^p}{2x^2\sqrt{1-a^2x^2}} - \frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\left(-6a-a^2(9-2p)x-\right.}{x^2}$$

$$= -\frac{\left(c-a^2cx^2\right)^p}{2x^2\sqrt{1-a^2x^2}} - \frac{3a\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} + \frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(a^2(9-2p)+2a^3(\right.}{}$$

$$= -\frac{\left(c-a^2cx^2\right)^p}{2x^2\sqrt{1-a^2x^2}} - \frac{3a\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} + \left(a^3(7-6p)\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \left(1-a^2x^2\right)$$

$$= -\frac{\left(c-a^2cx^2\right)^p}{2x^2\sqrt{1-a^2x^2}} - \frac{3a\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} + a^3(7-6p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-p;\right.$$

$$= -\frac{\left(c-a^2cx^2\right)^p}{2x^2\sqrt{1-a^2x^2}} - \frac{3a\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} + a^3(7-6p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-p;\right.$$

Mathematica [A] time = 0.288748, size = 154, normalized size = 0.79

$$a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(a\left(\frac{\left(1-a^2x^2\right)^{p-\frac{1}{2}}\left(3\text{Hypergeometric2F1}\left(1,p-\frac{1}{2},p+\frac{1}{2},1-a^2x^2\right)+\text{Hypergeometric2F1}\right.}{1-2p}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^p)/x^3,x]
```

```
[Out] (a*(c - a^2*c*x^2)^p*((-3*Hypergeometric2F1[-1/2, 3/2 - p, 1/2, a^2*x^2])/x
 + a*(a*x*Hypergeometric2F1[1/2, 3/2 - p, 3/2, a^2*x^2] + ((1 - a^2*x^2)^(-
1/2 + p)*(3*Hypergeometric2F1[1, -1/2 + p, 1/2 + p, 1 - a^2*x^2] + Hypergeo
metric2F1[2, -1/2 + p, 1/2 + p, 1 - a^2*x^2]))/(1 - 2*p))))/(1 - a^2*x^2)^p
```

Maple [F] time = 0.395, size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3 \left(-a^2cx^2+c\right)^p}{x^3} \left(-a^2x^2+1\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^3,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3 \left(-a^2cx^2+c\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^3,x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}(ax+1)\left(-a^2cx^2+c\right)^p}{a^2x^5-2\,ax^4+x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^3,x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^5 - 2*a*x^4
 + x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c\,(ax-1)\,(ax+1))^p\,(ax+1)^3}{x^3\,(-\,(ax-1)\,(ax+1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**p/x**3,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**3/(x**3*(-(a*x - 1)*(a*x +
1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3(-a^2cx^2+c)^p}{(-a^2x^2+1)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^3,x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

3.1182 $\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx$

Optimal. Leaf size=66

$$-\frac{c^5(ax+1)^{11}}{11a}+\frac{3c^5(ax+1)^{10}}{5a}-\frac{4c^5(ax+1)^9}{3a}+\frac{c^5(ax+1)^8}{a}$$

```
[Out] (c^5*(1 + a*x)^8)/a - (4*c^5*(1 + a*x)^9)/(3*a) + (3*c^5*(1 + a*x)^10)/(5*a
) - (c^5*(1 + a*x)^11)/(11*a)
```

Rubi [A] time = 0.0643129, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$-\frac{c^5(ax+1)^{11}}{11a}+\frac{3c^5(ax+1)^{10}}{5a}-\frac{4c^5(ax+1)^9}{3a}+\frac{c^5(ax+1)^8}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] (c^5*(1 + a*x)^8)/a - (4*c^5*(1 + a*x)^9)/(3*a) + (3*c^5*(1 + a*x)^10)/(5*a
) - (c^5*(1 + a*x)^11)/(11*a)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx &= c^5\int(1-ax)^3(1+ax)^7\,dx\\ &= c^5\int\left(8(1+ax)^7-12(1+ax)^8+6(1+ax)^9-(1+ax)^{10}\right)dx\\ &= \frac{c^5(1+ax)^8}{a}-\frac{4c^5(1+ax)^9}{3a}+\frac{3c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}\end{aligned}$$

Mathematica [A] time = 0.0322227, size = 39, normalized size = 0.59

$$-\frac{c^5(ax+1)^8\left(15a^3x^3-54a^2x^2+67ax-29\right)}{165a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] -(c^5*(1 + a*x)^8*(-29 + 67*a*x - 54*a^2*x^2 + 15*a^3*x^3))/(165*a)
```

Maple [A] time = 0.026, size = 75, normalized size = 1.1

$$c^{5}\left(-\frac{a^{10}x^{11}}{11}-\frac{2\,a^{9}x^{10}}{5}-\frac{x^{9}a^{8}}{3}+a^{7}x^{8}+2\,x^{7}a^{6}-\frac{14\,x^{5}a^{4}}{5}-2\,x^{4}a^{3}+x^{3}a^{2}+2\,ax^{2}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^5,x)
```

```
[Out] c^5*(-1/11*a^10*x^11-2/5*a^9*x^10-1/3*x^9*a^8+a^7*x^8+2*x^7*a^6-14/5*x^5*a^
4-2*x^4*a^3+x^3*a^2+2*a*x^2+x)
```

Maxima [A] time = 0.970398, size = 136, normalized size = 2.06

$$-\frac{1}{11}\,a^{10}c^{5}x^{11}-\frac{2}{5}\,a^{9}c^{5}x^{10}-\frac{1}{3}\,a^{8}c^{5}x^{9}+a^{7}c^{5}x^{8}+2\,a^{6}c^{5}x^{7}-\frac{14}{5}\,a^{4}c^{5}x^{5}-2\,a^{3}c^{5}x^{4}+a^{2}c^{5}x^{3}+2\,ac^{5}x^{2}+c^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] -1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2*
a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^2
+ c^5*x
```

Fricas [A] time = 2.26109, size = 215, normalized size = 3.26

$$-\frac{1}{11}\,a^{10}c^{5}x^{11}-\frac{2}{5}\,a^{9}c^{5}x^{10}-\frac{1}{3}\,a^{8}c^{5}x^{9}+a^{7}c^{5}x^{8}+2\,a^{6}c^{5}x^{7}-\frac{14}{5}\,a^{4}c^{5}x^{5}-2\,a^{3}c^{5}x^{4}+a^{2}c^{5}x^{3}+2\,ac^{5}x^{2}+c^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] -1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2*
a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^2
+ c^5*x
```

Sympy [B] time = 0.122364, size = 109, normalized size = 1.65

$$-\frac{a^{10}c^{5}x^{11}}{11}-\frac{2a^{9}c^{5}x^{10}}{5}-\frac{a^{8}c^{5}x^{9}}{3}+a^{7}c^{5}x^{8}+2a^{6}c^{5}x^{7}-\frac{14a^{4}c^{5}x^{5}}{5}-2a^{3}c^{5}x^{4}+a^{2}c^{5}x^{3}+2ac^{5}x^{2}+c^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a**2*c*x**2+c)**5,x)
```

```
[Out] -a**10*c**5*x**11/11 - 2*a**9*c**5*x**10/5 - a**8*c**5*x**9/3 + a**7*c**5*x
**8 + 2*a**6*c**5*x**7 - 14*a**4*c**5*x**5/5 - 2*a**3*c**5*x**4 + a**2*c**5
```

```
*x**3 + 2*a*c**5*x**2 + c**5*x
```

Giac [A] time = 1.14567, size = 136, normalized size = 2.06

$$-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{2}{5}\,a^9c^5x^{10}-\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8+2\,a^6c^5x^7-\frac{14}{5}\,a^4c^5x^5-2\,a^3c^5x^4+a^2c^5x^3+2\,ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2*
a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^2
+ c^5*x
```

3.1183 $\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=52

$$\frac{c^4(ax+1)^9}{9a}-\frac{c^4(ax+1)^8}{2a}+\frac{4c^4(ax+1)^7}{7a}$$

```
[Out] (4*c^4*(1 + a*x)^7)/(7*a) - (c^4*(1 + a*x)^8)/(2*a) + (c^4*(1 + a*x)^9)/(9*
a)
```

Rubi [A] time = 0.0565974, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$\frac{c^4(ax+1)^9}{9a}-\frac{c^4(ax+1)^8}{2a}+\frac{4c^4(ax+1)^7}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (4*c^4*(1 + a*x)^7)/(7*a) - (c^4*(1 + a*x)^8)/(2*a) + (c^4*(1 + a*x)^9)/(9*
a)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx &= c^4\int(1-ax)^2(1+ax)^6\,dx\\ &= c^4\int\left(4(1+ax)^6-4(1+ax)^7+(1+ax)^8\right)dx\\ &= \frac{4c^4(1+ax)^7}{7a}-\frac{c^4(1+ax)^8}{2a}+\frac{c^4(1+ax)^9}{9a}\end{aligned}$$

Mathematica [A] time = 0.0251382, size = 31, normalized size = 0.6

$$\frac{c^4(ax+1)^7\left(14a^2x^2-35ax+23\right)}{126a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(1 + a*x)^7*(23 - 35*a*x + 14*a^2*x^2))/(126*a)
```

Maple [A] time = 0.026, size = 69, normalized size = 1.3

$$c^4\left(\frac{x^9a^8}{9}+\frac{a^7x^8}{2}+\frac{4\,x^7a^6}{7}-\frac{2\,x^6a^5}{3}-2\,x^5a^4-x^4a^3+\frac{4\,x^3a^2}{3}+2\,ax^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^4,x)
```

```
[Out] c^4*(1/9*x^9*a^8+1/2*a^7*x^8+4/7*x^7*a^6-2/3*x^6*a^5-2*x^5*a^4-x^4*a^3+4/3*
x^3*a^2+2*a*x^2+x)
```

Maxima [A] time = 0.954703, size = 124, normalized size = 2.38

$$\frac{1}{9}\,a^8c^4x^9+\frac{1}{2}\,a^7c^4x^8+\frac{4}{7}\,a^6c^4x^7-\frac{2}{3}\,a^5c^4x^6-2\,a^4c^4x^5-a^3c^4x^4+\frac{4}{3}\,a^2c^4x^3+2\,ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*a
^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x
```

Fricas [A] time = 2.25395, size = 190, normalized size = 3.65

$$\frac{1}{9}\,a^8c^4x^9+\frac{1}{2}\,a^7c^4x^8+\frac{4}{7}\,a^6c^4x^7-\frac{2}{3}\,a^5c^4x^6-2\,a^4c^4x^5-a^3c^4x^4+\frac{4}{3}\,a^2c^4x^3+2\,ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*a
^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x
```

Sympy [B] time = 0.117172, size = 100, normalized size = 1.92

$$\frac{a^8c^4x^9}{9}+\frac{a^7c^4x^8}{2}+\frac{4a^6c^4x^7}{7}-\frac{2a^5c^4x^6}{3}-2a^4c^4x^5-a^3c^4x^4+\frac{4a^2c^4x^3}{3}+2ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a**2*c*x**2+c)**4,x)
```

```
[Out] a**8*c**4*x**9/9 + a**7*c**4*x**8/2 + 4*a**6*c**4*x**7/7 - 2*a**5*c**4*x**6
/3 - 2*a**4*c**4*x**5 - a**3*c**4*x**4 + 4*a**2*c**4*x**3/3 + 2*a*c**4*x**2
```

```
+ c**4*x
```

Giac [A] time = 1.1567, size = 124, normalized size = 2.38

$$\frac{1}{9}a^8c^4x^9 + \frac{1}{2}a^7c^4x^8 + \frac{4}{7}a^6c^4x^7 - \frac{2}{3}a^5c^4x^6 - 2a^4c^4x^5 - a^3c^4x^4 + \frac{4}{3}a^2c^4x^3 + 2ac^4x^2 + c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*a
^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x
```

3.1184 $\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=35

$$\frac{c^3(ax+1)^6}{3a}-\frac{c^3(ax+1)^7}{7a}$$

```
[Out] (c^3*(1 + a*x)^6)/(3*a) - (c^3*(1 + a*x)^7)/(7*a)
```

Rubi [A] time = 0.0392315, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$\frac{c^3(ax+1)^6}{3a}-\frac{c^3(ax+1)^7}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*(1 + a*x)^6)/(3*a) - (c^3*(1 + a*x)^7)/(7*a)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx &= c^3\int(1-ax)(1+ax)^5\,dx\\
&= c^3\int\left(2(1+ax)^5-(1+ax)^6\right)dx\\
&= \frac{c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}
\end{aligned}$$

Mathematica [A] time = 0.0186514, size = 23, normalized size = 0.66

$$-\frac{c^3(ax+1)^6(3ax-4)}{21a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -(c^3*(1 + a*x)^6*(-4 + 3*a*x))/(21*a)
```

Maple [A] time = 0.024, size = 45, normalized size = 1.3

$$c^3\left(-\frac{x^7a^6}{7}-\frac{2\,x^6a^5}{3}-x^5a^4+\frac{5\,x^3a^2}{3}+2\,ax^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*(-1/7*x^7*a^6-2/3*x^6*a^5-x^5*a^4+5/3*x^3*a^2+2*a*x^2+x)
```

Maxima [A] time = 0.978482, size = 80, normalized size = 2.29

$$-\frac{1}{7}\,a^6c^3x^7-\frac{2}{3}\,a^5c^3x^6-a^4c^3x^5+\frac{5}{3}\,a^2c^3x^3+2\,ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c^
3*x^2 + c^3*x
```

Fricas [A] time = 2.17988, size = 122, normalized size = 3.49

$$-\frac{1}{7}\,a^6c^3x^7-\frac{2}{3}\,a^5c^3x^6-a^4c^3x^5+\frac{5}{3}\,a^2c^3x^3+2\,ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c^
3*x^2 + c^3*x
```

Sympy [B] time = 0.10525, size = 63, normalized size = 1.8

$$-\frac{a^6c^3x^7}{7}-\frac{2a^5c^3x^6}{3}-a^4c^3x^5+\frac{5a^2c^3x^3}{3}+2ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -a**6*c**3*x**7/7 - 2*a**5*c**3*x**6/3 - a**4*c**3*x**5 + 5*a**2*c**3*x**3/
3 + 2*a*c**3*x**2 + c**3*x
```

Giac [A] time = 1.13631, size = 80, normalized size = 2.29

$$-\frac{1}{7}a^6c^3x^7 - \frac{2}{3}a^5c^3x^6 - a^4c^3x^5 + \frac{5}{3}a^2c^3x^3 + 2ac^3x^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c^
3*x^2 + c^3*x
```

3.1185 $\int e^{4\tanh^{-1}(ax)} \left(c-a^2cx^2\right)^2 \, dx$

Optimal. Leaf size=17

$$\frac{c^2(ax+1)^5}{5a}$$

```
[Out] (c^2*(1 + a*x)^5)/(5*a)
```

Rubi [A] time = 0.0303655, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 32}

$$\frac{c^2(ax+1)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(1 + a*x)^5)/(5*a)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)} \left(c-a^2cx^2\right)^2 \, dx &= c^2 \int (1+ax)^4 \, dx\\ &= \frac{c^2(1+ax)^5}{5a}\end{aligned}$$

Mathematica [B] time = 0.0177917, size = 37, normalized size = 2.18

$$c^2\left(\frac{a^4x^5}{5}+a^3x^4+2a^2x^3+2ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] c^2*(x + 2*a*x^2 + 2*a^2*x^3 + a^3*x^4 + (a^4*x^5)/5)
```

Maple [A] time = 0.026, size = 16, normalized size = 0.9

$$\frac{c^2 (ax+1)^5}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/5*c^2*(a*x+1)^5/a
```

Maxima [B] time = 0.973064, size = 63, normalized size = 3.71

$$\frac{1}{5} a^4 c^2 x^5 + a^3 c^2 x^4 + 2 a^2 c^2 x^3 + 2 a c^2 x^2 + c^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x
```

Fricas [B] time = 2.29438, size = 93, normalized size = 5.47

$$\frac{1}{5} a^4 c^2 x^5 + a^3 c^2 x^4 + 2 a^2 c^2 x^3 + 2 a c^2 x^2 + c^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x
```

Sympy [B] time = 0.099785, size = 48, normalized size = 2.82

$$\frac{a^4 c^2 x^5}{5} + a^3 c^2 x^4 + 2a^2 c^2 x^3 + 2ac^2 x^2 + c^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a**2*c*x**2+c)**2,x)
```

```
[Out] a**4*c**2*x**5/5 + a**3*c**2*x**4 + 2*a**2*c**2*x**3 + 2*a*c**2*x**2 + c**2
*x
```

Giac [B] time = 1.14501, size = 63, normalized size = 3.71

$$\frac{1}{5} a^4 c^2 x^5 + a^3 c^2 x^4 + 2 a^2 c^2 x^3 + 2 a c^2 x^2 + c^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x
```

3.1186 $\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=36

$$-\frac{1}{3}a^2cx^3-2acx^2-\frac{8c\log(1-ax)}{a}-7cx$$

```
[Out] -7*c*x - 2*a*c*x^2 - (a^2*c*x^3)/3 - (8*c*Log[1 - a*x])/a
```

Rubi [A] time = 0.0321805, antiderivative size = 46, normalized size of antiderivative = 1.28, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6140, 43}

$$-\frac{c(ax+1)^3}{3a}-\frac{c(ax+1)^2}{a}-\frac{8c\log(1-ax)}{a}-4cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -4*c*x - (c*(1 + a*x)^2)/a - (c*(1 + a*x)^3)/(3*a) - (8*c*Log[1 - a*x])/a
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx &= c\int \frac{(1+ax)^3}{1-ax}\,dx\\
&= c\int \left(-4+\frac{8}{1-ax}-2(1+ax)-(1+ax)^2\right)\,dx\\
&= -4cx-\frac{c(1+ax)^2}{a}-\frac{c(1+ax)^3}{3a}-\frac{8c\log(1-ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0115599, size = 36, normalized size = 1.

$$-\frac{1}{3}a^2cx^3-2acx^2-\frac{8c\log(1-ax)}{a}-7cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -7*c*x - 2*a*c*x^2 - (a^2*c*x^3)/3 - (8*c*Log[1 - a*x])/a
```

Maple [A] time = 0.026, size = 34, normalized size = 0.9

$$-\frac{a^2cx^3}{3} - 2\,acx^2 - 7\,cx - 8\,\frac{c\ln(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c),x)
```

```
[Out] -1/3*a^2*c*x^3-2*a*c*x^2-7*c*x-8*c/a*ln(a*x-1)
```

Maxima [A] time = 0.965084, size = 45, normalized size = 1.25

$$-\frac{1}{3}\,a^2cx^3 - 2\,acx^2 - 7\,cx - \frac{8\,c\log(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/3*a^2*c*x^3 - 2*a*c*x^2 - 7*c*x - 8*c*log(a*x - 1)/a
```

Fricas [A] time = 2.29866, size = 88, normalized size = 2.44

$$-\frac{a^3cx^3 + 6\,a^2cx^2 + 21\,acx + 24\,c\log(ax-1)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/3*(a^3*c*x^3 + 6*a^2*c*x^2 + 21*a*c*x + 24*c*log(a*x - 1))/a
```

Sympy [A] time = 0.30259, size = 36, normalized size = 1.

$$-\frac{a^2cx^3}{3} - 2acx^2 - 7cx - \frac{8c\log(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a**2*c*x**2+c),x)
```

```
[Out] -a**2*c*x**3/3 - 2*a*c*x**2 - 7*c*x - 8*c*log(a*x - 1)/a
```

Giac [A] time = 1.12206, size = 59, normalized size = 1.64

$$-\frac{8\,c\log(|ax-1|)}{a} - \frac{a^5cx^3 + 6\,a^4cx^2 + 21\,a^3cx}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -8*c*log(abs(a*x - 1))/a - 1/3*(a^5*c*x^3 + 6*a^4*c*x^2 + 21*a^3*c*x)/a^3
```

3.1187 $\int \frac{e^{4\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=13

$$\frac{x}{c(1-ax)^2}$$

```
[Out] x/(c*(1 - a*x)^2)
```

Rubi [A] time = 0.0339631, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 34}

$$\frac{x}{c(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] x/(c*(1 - a*x)^2)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx &= \frac{\int \frac{1+ax}{(1-ax)^3}\,dx}{c}\\ &= \frac{x}{c(1-ax)^2}\end{aligned}$$

Mathematica [A] time = 0.0079842, size = 25, normalized size = 1.92

$$\frac{(ax+1)^2}{4ac(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] (1 + a*x)^2/(4*a*c*(1 - a*x)^2)
```

Maple [B] time = 0.03, size = 28, normalized size = 2.2

$$\frac{1}{c}\left(\frac{1}{a\left(ax-1\right)^{2}}+\frac{1}{a\left(ax-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c),x)
```

```
[Out] 1/c*(1/a/(a*x-1)^2+1/a/(a*x-1))
```

Maxima [A] time = 0.967232, size = 26, normalized size = 2.

$$\frac{x}{a^{2}cx^{2}-2\,acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] x/(a^2*c*x^2 - 2*a*c*x + c)
```

Fricas [A] time = 2.2211, size = 39, normalized size = 3.

$$\frac{x}{a^{2}cx^{2}-2\,acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] x/(a^2*c*x^2 - 2*a*c*x + c)
```

Sympy [B] time = 0.352385, size = 17, normalized size = 1.31

$$\frac{x}{a^{2}cx^{2}-2acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(-a**2*c*x**2+c),x)
```

```
[Out] x/(a**2*c*x**2 - 2*a*c*x + c)
```

Giac [A] time = 1.16115, size = 16, normalized size = 1.23

$$\frac{x}{\left(ax-1\right)^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] x/((a*x - 1)^2*c)
```

3.1188 $\int \frac{e^{4\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=18

$$\frac{1}{3ac^2(1-ax)^3}$$

```
[Out] 1/(3*a*c^2*(1 - a*x)^3)
```

Rubi [A] time = 0.035483, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 32}

$$\frac{1}{3ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - a^2*c*x^2)^2,x]

[Out] 1/(3*a*c^2*(1 - a*x)^3)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx &= \frac{\int \frac{1}{(1-ax)^4}\,dx}{c^2}\\ &= \frac{1}{3ac^2(1-ax)^3}\end{aligned}$$

Mathematica [A] time = 0.0160205, size = 17, normalized size = 0.94

$$-\frac{1}{3ac^2(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - a^2*c*x^2)^2,x]

[Out] -1/(3*a*c^2*(-1 + a*x)^3)
```

Maple [A] time = 0.026, size = 16, normalized size = 0.9

$$-\frac{1}{3\,ac^{2}\left(ax-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/3/c^2/a/(a*x-1)^3
```

Maxima [B] time = 0.955198, size = 55, normalized size = 3.06

$$-\frac{1}{3\left(a^{4}c^{2}x^{3}-3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x-ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Fricas [B] time = 2.17685, size = 78, normalized size = 4.33

$$-\frac{1}{3\left(a^{4}c^{2}x^{3}-3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x-ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/3/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [B] time = 0.401703, size = 42, normalized size = 2.33

$$-\frac{1}{3a^{4}c^{2}x^{3}-9a^{3}c^{2}x^{2}+9a^{2}c^{2}x-3ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -1/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2*c**2*x - 3*a*c**2)
```

Giac [A] time = 1.17256, size = 20, normalized size = 1.11

$$-\frac{1}{3\left(ax-1\right)^{3}ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/3/((a*x - 1)^3*a*c^2)
```

3.1189 $\int \frac{e^{4\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=87

$$\frac{1}{16ac^3(1-ax)}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{8ac^3(1-ax)^4}+\frac{\tanh^{-1}(ax)}{16ac^3}$$

```
[Out] 1/(8*a*c^3*(1 - a*x)^4) + 1/(12*a*c^3*(1 - a*x)^3) + 1/(16*a*c^3*(1 - a*x)^
2) + 1/(16*a*c^3*(1 - a*x)) + ArcTanh[a*x]/(16*a*c^3)
```

Rubi [A] time = 0.0677188, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6140, 44, 207}

$$\frac{1}{16ac^3(1-ax)}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{8ac^3(1-ax)^4}+\frac{\tanh^{-1}(ax)}{16ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(8*a*c^3*(1 - a*x)^4) + 1/(12*a*c^3*(1 - a*x)^3) + 1/(16*a*c^3*(1 - a*x)^
2) + 1/(16*a*c^3*(1 - a*x)) + ArcTanh[a*x]/(16*a*c^3)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{1}{(1-ax)^5(1+ax)}\,dx}{c^3}$$

$$= \frac{\int \left(-\frac{1}{2(-1+ax)^5} + \frac{1}{4(-1+ax)^4} - \frac{1}{8(-1+ax)^3} + \frac{1}{16(-1+ax)^2} - \frac{1}{16\left(-1+a^2x^2\right)}\right) dx}{c^3}$$

$$= \frac{1}{8ac^3(1-ax)^4} + \frac{1}{12ac^3(1-ax)^3} + \frac{1}{16ac^3(1-ax)^2} + \frac{1}{16ac^3(1-ax)} - \frac{\int \frac{1}{-1+a^2x^2}\,dx}{16c^3}$$

$$= \frac{1}{8ac^3(1-ax)^4} + \frac{1}{12ac^3(1-ax)^3} + \frac{1}{16ac^3(1-ax)^2} + \frac{1}{16ac^3(1-ax)} + \frac{\tanh^{-1}(ax)}{16ac^3}$$

Mathematica [A] time = 0.0340216, size = 52, normalized size = 0.6

$$\frac{-3a^3x^3 + 12a^2x^2 - 19ax + 3(ax-1)^4\tanh^{-1}(ax) + 16}{48ac^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (16 - 19*a*x + 12*a^2*x^2 - 3*a^3*x^3 + 3*(-1 + a*x)^4*ArcTanh[a*x])/(48*a*
c^3*(-1 + a*x)^4)
```

Maple [A] time = 0.033, size = 90, normalized size = 1.

$$\frac{\ln(ax+1)}{32\,ac^3} + \frac{1}{8\,ac^3\,(ax-1)^4} - \frac{1}{12\,ac^3\,(ax-1)^3} + \frac{1}{16\,ac^3\,(ax-1)^2} - \frac{1}{16\,ac^3\,(ax-1)} - \frac{\ln(ax-1)}{32\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/32*ln(a*x+1)/a/c^3+1/8/c^3/a/(a*x-1)^4-1/12/c^3/a/(a*x-1)^3+1/16/c^3/a/(a
*x-1)^2-1/16/c^3/a/(a*x-1)-1/32/c^3/a*ln(a*x-1)
```

Maxima [A] time = 0.97302, size = 138, normalized size = 1.59

$$-\frac{3\,a^3x^3 - 12\,a^2x^2 + 19\,ax - 16}{48\left(a^5c^3x^4 - 4\,a^4c^3x^3 + 6\,a^3c^3x^2 - 4\,a^2c^3x + ac^3\right)} + \frac{\log(ax+1)}{32\,ac^3} - \frac{\log(ax-1)}{32\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(3*a^3*x^3 - 12*a^2*x^2 + 19*a*x - 16)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 +
 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + 1/32*log(a*x + 1)/(a*c^3) - 1/32*lo
g(a*x - 1)/(a*c^3)
```

Fricas [A] time = 2.20007, size = 324, normalized size = 3.72

$$-\frac{6\,a^3x^3-24\,a^2x^2+38\,ax-3\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(ax+1\right)+3\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(\right.}{96\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/96*(6*a^3*x^3 - 24*a^2*x^2 + 38*a*x - 3*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2
 - 4*a*x + 1)*log(a*x + 1) + 3*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1
)*log(a*x - 1) - 32)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c
^3*x + a*c^3)
```

Sympy [A] time = 0.687613, size = 99, normalized size = 1.14

$$-\frac{3a^3x^3-12a^2x^2+19ax-16}{48a^5c^3x^4-192a^4c^3x^3+288a^3c^3x^2-192a^2c^3x+48ac^3}-\frac{\frac{\log\left(x-\frac{1}{a}\right)}{32}-\frac{\log\left(x+\frac{1}{a}\right)}{32}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(3*a**3*x**3 - 12*a**2*x**2 + 19*a*x - 16)/(48*a**5*c**3*x**4 - 192*a**4*c
**3*x**3 + 288*a**3*c**3*x**2 - 192*a**2*c**3*x + 48*a*c**3) - (log(x - 1/a
)/32 - log(x + 1/a)/32)/(a*c**3)
```

Giac [A] time = 1.17042, size = 92, normalized size = 1.06

$$\frac{\log\left(|ax+1|\right)}{32\,ac^3}-\frac{\log\left(|ax-1|\right)}{32\,ac^3}-\frac{3\,a^3x^3-12\,a^2x^2+19\,ax-16}{48\,(ax-1)^4ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*log(abs(a*x + 1))/(a*c^3) - 1/32*log(abs(a*x - 1))/(a*c^3) - 1/48*(3*a
^3*x^3 - 12*a^2*x^2 + 19*a*x - 16)/((a*x - 1)^4*a*c^3)
```

3.1190 $\int \frac{e^{4\tanh^{-1}(ax)}}{(c-a^2cx^2)^4} dx$

Optimal. Leaf size=122

$$\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}+\frac{1}{16ac^4(1-ax)^2}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{3\tanh^{-1}(ax)}{32ac^4}$$

```
[Out] 1/(20*a*c^4*(1 - a*x)^5) + 1/(16*a*c^4*(1 - a*x)^4) + 1/(16*a*c^4*(1 - a*x)
^3) + 1/(16*a*c^4*(1 - a*x)^2) + 5/(64*a*c^4*(1 - a*x)) - 1/(64*a*c^4*(1 +
a*x)) + (3*ArcTanh[a*x])/(32*a*c^4)
```

Rubi [A] time = 0.0893913, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6140, 44, 207}

$$\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}+\frac{1}{16ac^4(1-ax)^2}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{3\tanh^{-1}(ax)}{32ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] 1/(20*a*c^4*(1 - a*x)^5) + 1/(16*a*c^4*(1 - a*x)^4) + 1/(16*a*c^4*(1 - a*x)
^3) + 1/(16*a*c^4*(1 - a*x)^2) + 5/(64*a*c^4*(1 - a*x)) - 1/(64*a*c^4*(1 +
a*x)) + (3*ArcTanh[a*x])/(32*a*c^4)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{\int \frac{1}{(1-ax)^6(1+ax)^2}\,dx}{c^4}$$

$$= \frac{\int \left(\frac{1}{4(-1+ax)^6} - \frac{1}{4(-1+ax)^5} + \frac{3}{16(-1+ax)^4} - \frac{1}{8(-1+ax)^3} + \frac{5}{64(-1+ax)^2} + \frac{1}{64(1+ax)^2} - \frac{3}{32(-1+a^2x^2)}\right)dx}{c^4}$$

$$= \frac{1}{20ac^4(1-ax)^5} + \frac{1}{16ac^4(1-ax)^4} + \frac{1}{16ac^4(1-ax)^3} + \frac{1}{16ac^4(1-ax)^2} + \frac{5}{64ac^4(1-ax)} - \frac{1}{64ac^4(1+a}$$

$$= \frac{1}{20ac^4(1-ax)^5} + \frac{1}{16ac^4(1-ax)^4} + \frac{1}{16ac^4(1-ax)^3} + \frac{1}{16ac^4(1-ax)^2} + \frac{5}{64ac^4(1-ax)} - \frac{1}{64ac^4(1+a}$$

Mathematica [A] time = 0.0545091, size = 80, normalized size = 0.66

$$\frac{-15a^5x^5 + 60a^4x^4 - 80a^3x^3 + 20a^2x^2 + 47ax + 15(ax-1)^5(ax+1)\tanh^{-1}(ax) - 48}{160ac^4(ax-1)^5(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (-48 + 47*a*x + 20*a^2*x^2 - 80*a^3*x^3 + 60*a^4*x^4 - 15*a^5*x^5 + 15*(-1
+ a*x)^5*(1 + a*x)*ArcTanh[a*x])/(160*a*c^4*(-1 + a*x)^5*(1 + a*x))
```

Maple [A] time = 0.037, size = 120, normalized size = 1.

$$-\frac{1}{64\,ac^4\,(ax+1)} + \frac{3\ln(ax+1)}{64\,ac^4} - \frac{1}{20\,ac^4\,(ax-1)^5} + \frac{1}{16\,ac^4\,(ax-1)^4} - \frac{1}{16\,ac^4\,(ax-1)^3} + \frac{1}{16\,ac^4\,(ax-1)^2} - \frac{5}{64\,ac^4\,(ax-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/64/a/c^4/(a*x+1)+3/64*ln(a*x+1)/a/c^4-1/20/c^4/a/(a*x-1)^5+1/16/c^4/a/(a
*x-1)^4-1/16/c^4/a/(a*x-1)^3+1/16/c^4/a/(a*x-1)^2-5/64/c^4/a/(a*x-1)-3/64/c
^4/a*ln(a*x-1)
```

Maxima [A] time = 0.997916, size = 176, normalized size = 1.44

$$-\frac{15\,a^5x^5 - 60\,a^4x^4 + 80\,a^3x^3 - 20\,a^2x^2 - 47\,ax + 48}{160\left(a^7c^4x^6 - 4\,a^6c^4x^5 + 5\,a^5c^4x^4 - 5\,a^3c^4x^2 + 4\,a^2c^4x - ac^4\right)} + \frac{3\log(ax+1)}{64\,ac^4} - \frac{3\log(ax-1)}{64\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] -1/160*(15*a^5*x^5 - 60*a^4*x^4 + 80*a^3*x^3 - 20*a^2*x^2 - 47*a*x + 48)/(a
^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x -
a*c^4) + 3/64*log(a*x + 1)/(a*c^4) - 3/64*log(a*x - 1)/(a*c^4)
```

Fricas [A] time = 2.41364, size = 421, normalized size = 3.45

$$-\frac{30\,a^5x^5-120\,a^4x^4+160\,a^3x^3-40\,a^2x^2-94\,ax-15\left(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2+4\,ax-1\right)\log\left(ax+1\right)+15\left(a\right.}{320\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/320*(30*a^5*x^5 - 120*a^4*x^4 + 160*a^3*x^3 - 40*a^2*x^2 - 94*a*x - 15*(
a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x + 1) + 15*
(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x - 1) + 96
)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*
x - a*c^4)
```

Sympy [A] time = 0.974087, size = 129, normalized size = 1.06

$$-\frac{15a^5x^5-60a^4x^4+80a^3x^3-20a^2x^2-47ax+48}{160a^7c^4x^6-640a^6c^4x^5+800a^5c^4x^4-800a^3c^4x^2+640a^2c^4x-160ac^4}+\frac{-\frac{3\log\left(x-\frac{1}{a}\right)}{64}+\frac{3\log\left(x+\frac{1}{a}\right)}{64}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(-a**2*c*x**2+c)**4,x)
```

```
[Out] -(15*a**5*x**5 - 60*a**4*x**4 + 80*a**3*x**3 - 20*a**2*x**2 - 47*a*x + 48)/
(160*a**7*c**4*x**6 - 640*a**6*c**4*x**5 + 800*a**5*c**4*x**4 - 800*a**3*c*
*4*x**2 + 640*a**2*c**4*x - 160*a*c**4) + (-3*log(x - 1/a)/64 + 3*log(x + 1
/a)/64)/(a*c**4)
```

Giac [A] time = 1.14617, size = 123, normalized size = 1.01

$$\frac{3\,\log\left(|ax+1|\right)}{64\,ac^4}-\frac{3\,\log\left(|ax-1|\right)}{64\,ac^4}-\frac{15\,a^5x^5-60\,a^4x^4+80\,a^3x^3-20\,a^2x^2-47\,ax+48}{160\,(ax+1)(ax-1)^5ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 3/64*log(abs(a*x + 1))/(a*c^4) - 3/64*log(abs(a*x - 1))/(a*c^4) - 1/160*(15
*a^5*x^5 - 60*a^4*x^4 + 80*a^3*x^3 - 20*a^2*x^2 - 47*a*x + 48)/((a*x + 1)*(
a*x - 1)^5*a*c^4)
```

3.1191 $\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=63

$$\frac{c2^{p+2}(ax+1)^{1-p}\left(c-a^2cx^2\right)^{p-1}\text{Hypergeometric2F1}\left(-p-2,p-1,p,\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

```
[Out] (2^(2 + p)*c*(1 + a*x)^(1 - p)*(c - a^2*c*x^2)^(-1 + p)*Hypergeometric2F1[-
2 - p, -1 + p, p, (1 - a*x)/2])/(a*(1 - p))
```

Rubi [A] time = 0.0797909, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6141, 678, 69}

$$\frac{c2^{p+2}(ax+1)^{1-p}\left(c-a^2cx^2\right)^{p-1}\,{}_2F_1\left(-p-2,p-1;p;\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (2^(2 + p)*c*(1 + a*x)^(1 - p)*(c - a^2*c*x^2)^(-1 + p)*Hypergeometric2F1[-
2 - p, -1 + p, p, (1 - a*x)/2])/(a*(1 - p))
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p dx = c^2\int (1+ax)^4\left(c-a^2cx^2\right)^{-2+p} dx$$
$$= \left(c^2(1+ax)^{1-p}(c-acx)^{1-p}\left(c-a^2cx^2\right)^{-1+p}\right)\int (1+ax)^{2+p}(c-acx)^{-2+p} dx$$
$$= \frac{2^{2+p}c(1+ax)^{1-p}\left(c-a^2cx^2\right)^{-1+p}\,{}_2F_1\left(-2-p,-1+p;p;\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

Mathematica [A] time = 0.0201876, size = 72, normalized size = 1.14

$$-\frac{2^{p+2}(1-ax)^{p-1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-p-2,p-1,p,\frac{1}{2}(1-ax)\right)}{a(p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((2^(2 + p)*(1 - a*x)^(-1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-2 - p,
 -1 + p, p, (1 - a*x)/2])/(a*(-1 + p)*(1 - a^2*x^2)^p))
```

Maple [F] time = 0.434, size = 0, normalized size = 0.

$$\int \frac{(ax+1)^4\left(-a^2cx^2+c\right)^p}{\left(-a^2x^2+1\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^4\left(-a^2cx^2+c\right)^p}{\left(a^2x^2-1\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^4*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2x^2+2\,ax+1\right)\left(-a^2cx^2+c\right)^p}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c(ax-1)(ax+1))^p (ax+1)^2}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**2/(a*x - 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^4\left(-a^2cx^2+c\right)^p}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^4*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 1)^2, x)
```

3.1192 $\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=127

$$\frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a}+\frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2}+\frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{35}{128}c^4x\sqrt{1-a^2x^2}+\frac{35c^4\sin^{-1}(a$$

```
[Out] (35*c^4*x*Sqrt[1 - a^2*x^2])/128 + (35*c^4*x*(1 - a^2*x^2)^(3/2))/192 + (7*
c^4*x*(1 - a^2*x^2)^(5/2))/48 + (c^4*x*(1 - a^2*x^2)^(7/2))/8 + (c^4*(1 - a
^2*x^2)^(9/2))/(9*a) + (35*c^4*ArcSin[a*x])/(128*a)
```

Rubi [A] time = 0.0682698, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6139, 641, 195, 216}

$$\frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a}+\frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2}+\frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{35}{128}c^4x\sqrt{1-a^2x^2}+\frac{35c^4\sin^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^4/E^ArcTanh[a*x],x]
```

```
[Out] (35*c^4*x*Sqrt[1 - a^2*x^2])/128 + (35*c^4*x*(1 - a^2*x^2)^(3/2))/192 + (7*
c^4*x*(1 - a^2*x^2)^(5/2))/48 + (c^4*x*(1 - a^2*x^2)^(7/2))/8 + (c^4*(1 - a
^2*x^2)^(9/2))/(9*a) + (35*c^4*ArcSin[a*x])/(128*a)
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] && !Int
egerQ[p - n/2]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = c^4\int (1-ax)\left(1-a^2x^2\right)^{7/2}\,dx$$
$$= \frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a} + c^4\int \left(1-a^2x^2\right)^{7/2}\,dx$$
$$= \frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2} + \frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a} + \frac{1}{8}\left(7c^4\right)\int \left(1-a^2x^2\right)^{5/2}\,dx$$
$$= \frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2} + \frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2} + \frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a} + \frac{1}{48}\left(35c^4\right)\int \left(1-a^2x^2\right)^{3/2}$$
$$= \frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2} + \frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2} + \frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a} + \frac{1}{64}$$
$$= \frac{35}{128}c^4x\sqrt{1-a^2x^2} + \frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2} + \frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2} + \frac{c}{\ }$$
$$= \frac{35}{128}c^4x\sqrt{1-a^2x^2} + \frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2} + \frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2} + \frac{c}{\ }$$

Mathematica [A] time = 0.144339, size = 107, normalized size = 0.84

$$\frac{c^4\left(\sqrt{1-a^2x^2}\left(128a^8x^8-144a^7x^7-512a^6x^6+600a^5x^5+768a^4x^4-978a^3x^3-512a^2x^2+837ax+128\right)-630\sin^{-1}\left(\frac{\sqrt{1}}{\sqrt{}}\right.\right.}{1152a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^4/E^ArcTanh[a*x],x]
```

```
[Out] (c^4*(Sqrt[1 - a^2*x^2]*(128 + 837*a*x - 512*a^2*x^2 - 978*a^3*x^3 + 768*a^
4*x^4 + 600*a^5*x^5 - 512*a^6*x^6 - 144*a^7*x^7 + 128*a^8*x^8) - 630*ArcSin
[Sqrt[1 - a*x]/Sqrt[2]]))/(1152*a)
```

Maple [A] time = 0.07, size = 201, normalized size = 1.6

$$-\frac{c^4a^5x^6}{9}\left(-a^2x^2+1\right)^{\frac{3}{2}} + \frac{c^4a^3x^4}{3}\left(-a^2x^2+1\right)^{\frac{3}{2}} - \frac{c^4ax^2}{3}\left(-a^2x^2+1\right)^{\frac{3}{2}} + \frac{c^4}{9a}\left(-a^2x^2+1\right)^{\frac{3}{2}} + \frac{a^4c^4x^5}{8}\left(-a^2x^2+1\right)^{\frac{3}{2}} - \frac{19a^2c^4}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/9*c^4*a^5*x^6*(-a^2*x^2+1)^(3/2)+1/3*c^4*a^3*x^4*(-a^2*x^2+1)^(3/2)-1/3*
c^4*a*x^2*(-a^2*x^2+1)^(3/2)+1/9*c^4*(-a^2*x^2+1)^(3/2)/a+1/8*c^4*a^4*x^5*(
-a^2*x^2+1)^(3/2)-19/48*c^4*a^2*x^3*(-a^2*x^2+1)^(3/2)+29/64*c^4*x*(-a^2*x^
2+1)^(3/2)+35/128*c^4*x*(-a^2*x^2+1)^(1/2)+35/128*c^4/(a^2)^(1/2)*arctan((a
^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.48355, size = 246, normalized size = 1.94

$$-\frac{1}{9}\left(-a^2x^2+1\right)^{\frac{3}{2}}a^5c^4x^6 + \frac{1}{8}\left(-a^2x^2+1\right)^{\frac{3}{2}}a^4c^4x^5 + \frac{1}{3}\left(-a^2x^2+1\right)^{\frac{3}{2}}a^3c^4x^4 - \frac{19}{48}\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2c^4x^3 - \frac{1}{3}\left(-a^2x^2+1\right)^{\frac{3}{2}}ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/9*(-a^2*x^2 + 1)^(3/2)*a^5*c^4*x^6 + 1/8*(-a^2*x^2 + 1)^(3/2)*a^4*c^4*x^
5 + 1/3*(-a^2*x^2 + 1)^(3/2)*a^3*c^4*x^4 - 19/48*(-a^2*x^2 + 1)^(3/2)*a^2*c
^4*x^3 - 1/3*(-a^2*x^2 + 1)^(3/2)*a*c^4*x^2 + 29/64*(-a^2*x^2 + 1)^(3/2)*c^
4*x + 35/128*sqrt(-a^2*x^2 + 1)*c^4*x + 1/9*(-a^2*x^2 + 1)^(3/2)*c^4/a + 35
/128*c^4*arcsin(a*x)/a
```

Fricas [A] time = 2.22347, size = 312, normalized size = 2.46

$$-\frac{630\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(128\,a^8c^4x^8-144\,a^7c^4x^7-512\,a^6c^4x^6+600\,a^5c^4x^5+768\,a^4c^4x^4-978\,a^3c^4x^3-512\,a^2}{1152\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/1152*(630*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (128*a^8*c^4*x^8
- 144*a^7*c^4*x^7 - 512*a^6*c^4*x^6 + 600*a^5*c^4*x^5 + 768*a^4*c^4*x^4 - 9
78*a^3*c^4*x^3 - 512*a^2*c^4*x^2 + 837*a*c^4*x + 128*c^4)*sqrt(-a^2*x^2 + 1
))/a
```

Sympy [C] time = 20.0465, size = 996, normalized size = 7.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**4/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] a**7*c**4*Piecewise((x**8*sqrt(-a**2*x**2 + 1)/9 - x**6*sqrt(-a**2*x**2 + 1
)/(63*a**2) - 2*x**4*sqrt(-a**2*x**2 + 1)/(105*a**4) - 8*x**2*sqrt(-a**2*x*
*2 + 1)/(315*a**6) - 16*sqrt(-a**2*x**2 + 1)/(315*a**8), Ne(a, 0)), (x**8/8
, True)) - a**6*c**4*Piecewise((I*a**2*x**9/(8*sqrt(a**2*x**2 - 1)) - 7*I*x
**7/(48*sqrt(a**2*x**2 - 1)) - I*x**5/(192*a**2*sqrt(a**2*x**2 - 1)) - 5*I*
x**3/(384*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(128*a**6*sqrt(a**2*x**2 - 1))
- 5*I*acosh(a*x)/(128*a**7), Abs(a**2*x**2) > 1), (-a**2*x**9/(8*sqrt(-a**2
*x**2 + 1)) + 7*x**7/(48*sqrt(-a**2*x**2 + 1)) + x**5/(192*a**2*sqrt(-a**2*
x**2 + 1)) + 5*x**3/(384*a**4*sqrt(-a**2*x**2 + 1)) - 5*x/(128*a**6*sqrt(-a
**2*x**2 + 1)) + 5*asin(a*x)/(128*a**7), True)) - 3*a**5*c**4*Piecewise((x*
*6*sqrt(-a**2*x**2 + 1)/7 - x**4*sqrt(-a**2*x**2 + 1)/(35*a**2) - 4*x**2*sq
rt(-a**2*x**2 + 1)/(105*a**4) - 8*sqrt(-a**2*x**2 + 1)/(105*a**6), Ne(a, 0)
), (x**6/6, True)) + 3*a**4*c**4*Piecewise((I*a**2*x**7/(6*sqrt(a**2*x**2 -
 1)) - 5*I*x**5/(24*sqrt(a**2*x**2 - 1)) - I*x**3/(48*a**2*sqrt(a**2*x**2 -
 1)) + I*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(16*a**5), Abs(a**2
*x**2) > 1), (-a**2*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5*x**5/(24*sqrt(-a**2*x
**2 + 1)) + x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - x/(16*a**4*sqrt(-a**2*x**
2 + 1)) + asin(a*x)/(16*a**5), True)) + 3*a**3*c**4*Piecewise((x**4*sqrt(-a
**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 + 1)/(15*a**2) - 2*sqrt(-a**2*x**2 +
```

```
 1)/(15*a**4), Ne(a, 0)), (x**4/4, True)) - 3*a**2*c**4*Piecewise((I*a**2*x
**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8*sqrt(a**2*x**2 - 1)) + I*x/(8*a**
2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2
*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(8*sqrt(-a**2*x**2 + 1)) - x/(8*a**
2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a**3), True)) - a*c**4*Piecewise((x*
*2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2)/(3*a**2), True)) + c**4*Piece
wise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)) - I*x/(2*sqrt(a**2*x**2 - 1)) - I
*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (x*sqrt(-a**2*x**2 + 1)/2 + asin(a*
x)/(2*a), True))
```

Giac [A] time = 1.19541, size = 167, normalized size = 1.31

$$\frac{35\,c^4\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{128\,|a|}+\frac{1}{1152}\,\sqrt{-a^2x^2+1}\left(\frac{128\,c^4}{a}+\left(837\,c^4-2\left(256\,ac^4+\left(489\,a^2c^4-4\left(96\,a^3c^4+\left(75\,a^4c^4-2\left(32\,a^5\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 35/128*c^4*arcsin(a*x)*sgn(a)/abs(a) + 1/1152*sqrt(-a^2*x^2 + 1)*(128*c^4/a
 + (837*c^4 - 2*(256*a*c^4 + (489*a^2*c^4 - 4*(96*a^3*c^4 + (75*a^4*c^4 - 2
*(32*a^5*c^4 - (8*a^7*c^4*x - 9*a^6*c^4)*x)*x)*x)*x)*x)*x)*x)
```

3.1193 $\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=105

$$\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}+\frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{5}{16}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\sin^{-1}(ax)}{16a}$$

```
[Out] (5*c^3*x*Sqrt[1 - a^2*x^2])/16 + (5*c^3*x*(1 - a^2*x^2)^(3/2))/24 + (c^3*x*
(1 - a^2*x^2)^(5/2))/6 + (c^3*(1 - a^2*x^2)^(7/2))/(7*a) + (5*c^3*ArcSin[a*
x])/(16*a)
```

Rubi [A] time = 0.0590107, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6139, 641, 195, 216}

$$\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}+\frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{5}{16}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\sin^{-1}(ax)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/E^ArcTanh[a*x],x]
```

```
[Out] (5*c^3*x*Sqrt[1 - a^2*x^2])/16 + (5*c^3*x*(1 - a^2*x^2)^(3/2))/24 + (c^3*x*
(1 - a^2*x^2)^(5/2))/6 + (c^3*(1 - a^2*x^2)^(7/2))/(7*a) + (5*c^3*ArcSin[a*
x])/(16*a)
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx &= c^3\int (1-ax)\left(1-a^2x^2\right)^{5/2}\,dx\\
&= \frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+c^3\int \left(1-a^2x^2\right)^{5/2}\,dx\\
&= \frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}+\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{1}{6}\left(5c^3\right)\int \left(1-a^2x^2\right)^{3/2}\,dx\\
&= \frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}+\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{1}{8}\left(5c^3\right)\int \sqrt{1-a^2x^2}\,dx\\
&= \frac{5}{16}c^3x\sqrt{1-a^2x^2}+\frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}+\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{1}{16}\left(5c^3\right.\\
&= \frac{5}{16}c^3x\sqrt{1-a^2x^2}+\frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}+\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{5c^3\sin}{16}
\end{aligned}
$$

Mathematica [A] time = 0.121655, size = 91, normalized size = 0.87

$$
-\frac{c^3\left(\sqrt{1-a^2x^2}\left(48a^6x^6-56a^5x^5-144a^4x^4+182a^3x^3+144a^2x^2-231ax-48\right)+210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{336a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^3/E^ArcTanh[a*x],x]
```

```
[Out] -(c^3*(Sqrt[1 - a^2*x^2]*(-48 - 231*a*x + 144*a^2*x^2 + 182*a^3*x^3 - 144*a
^4*x^4 - 56*a^5*x^5 + 48*a^6*x^6) + 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(33
6*a)
```

Maple [A] time = 0.046, size = 155, normalized size = 1.5

$$
\frac{c^3a^3x^4}{7}\left(-a^2x^2+1\right)^{\frac{3}{2}}-\frac{2\,c^3ax^2}{7}\left(-a^2x^2+1\right)^{\frac{3}{2}}+\frac{c^3}{7\,a}\left(-a^2x^2+1\right)^{\frac{3}{2}}-\frac{c^3a^2x^3}{6}\left(-a^2x^2+1\right)^{\frac{3}{2}}+\frac{3\,c^3x}{8}\left(-a^2x^2+1\right)^{\frac{3}{2}}+\frac{5\,c^3x}{16}\sqrt{-}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/7*c^3*a^3*x^4*(-a^2*x^2+1)^(3/2)-2/7*c^3*a*x^2*(-a^2*x^2+1)^(3/2)+1/7*c^3
*(-a^2*x^2+1)^(3/2)/a-1/6*c^3*a^2*x^3*(-a^2*x^2+1)^(3/2)+3/8*c^3*x*(-a^2*x^
2+1)^(3/2)+5/16*c^3*x*(-a^2*x^2+1)^(1/2)+5/16*c^3/(a^2)^(1/2)*arctan((a^2)^
(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.50055, size = 184, normalized size = 1.75

$$
\frac{1}{7}\left(-a^2x^2+1\right)^{\frac{3}{2}}a^3c^3x^4-\frac{1}{6}\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2c^3x^3-\frac{2}{7}\left(-a^2x^2+1\right)^{\frac{3}{2}}ac^3x^2+\frac{3}{8}\left(-a^2x^2+1\right)^{\frac{3}{2}}c^3x+\frac{5}{16}\sqrt{-a^2x^2+1}c^3x+\frac{\left(-a^2\right.}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/7*(-a^2*x^2 + 1)^(3/2)*a^3*c^3*x^4 - 1/6*(-a^2*x^2 + 1)^(3/2)*a^2*c^3*x^3
 - 2/7*(-a^2*x^2 + 1)^(3/2)*a*c^3*x^2 + 3/8*(-a^2*x^2 + 1)^(3/2)*c^3*x + 5/
16*sqrt(-a^2*x^2 + 1)*c^3*x + 1/7*(-a^2*x^2 + 1)^(3/2)*c^3/a + 5/16*c^3*arc
sin(a*x)/a
```

Fricas [A] time = 2.42659, size = 258, normalized size = 2.46

$$-\frac{210\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(48\,a^6c^3x^6-56\,a^5c^3x^5-144\,a^4c^3x^4+182\,a^3c^3x^3+144\,a^2c^3x^2-231\,ac^3x-48\,c^3\right)\sqrt{-a}}{336\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/336*(210*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (48*a^6*c^3*x^6 -
56*a^5*c^3*x^5 - 144*a^4*c^3*x^4 + 182*a^3*c^3*x^3 + 144*a^2*c^3*x^2 - 231*
a*c^3*x - 48*c^3)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [C] time = 11.5812, size = 629, normalized size = 5.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -a**5*c**3*Piecewise((x**6*sqrt(-a**2*x**2 + 1)/7 - x**4*sqrt(-a**2*x**2 +
1)/(35*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)/(105*a**4) - 8*sqrt(-a**2*x**2 +
 1)/(105*a**6), Ne(a, 0)), (x**6/6, True)) + a**4*c**3*Piecewise((I*a**2*x*
*7/(6*sqrt(a**2*x**2 - 1)) - 5*I*x**5/(24*sqrt(a**2*x**2 - 1)) - I*x**3/(48
*a**2*sqrt(a**2*x**2 - 1)) + I*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*acosh(a*
x)/(16*a**5), Abs(a**2*x**2) > 1), (-a**2*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5
*x**5/(24*sqrt(-a**2*x**2 + 1)) + x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - x/(
16*a**4*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(16*a**5), True)) + 2*a**3*c**3*P
iecewise((x**4*sqrt(-a**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 + 1)/(15*a**2)
 - 2*sqrt(-a**2*x**2 + 1)/(15*a**4), Ne(a, 0)), (x**4/4, True)) - 2*a**2*c*
*3*Piecewise((I*a**2*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8*sqrt(a**2*x
**2 - 1)) + I*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*a**3), Abs(a
**2*x**2) > 1), (-a**2*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(8*sqrt(-a**2
*x**2 + 1)) - x/(8*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a**3), True))
- a*c**3*Piecewise((x**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2)/(3*a**2
), True)) + c**3*Piecewise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)) - I*x/(2*sq
rt(a**2*x**2 - 1)) - I*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (x*sqrt(-a**2
*x**2 + 1)/2 + asin(a*x)/(2*a), True))
```

Giac [A] time = 1.22509, size = 136, normalized size = 1.3

$$\frac{5\,c^3\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{16\,|a|}+\frac{1}{336}\,\sqrt{-a^2x^2+1}\left(\frac{48\,c^3}{a}+\left(231\,c^3-2\left(72\,ac^3+\left(91\,a^2c^3-4\left(18\,a^3c^3-\left(6\,a^5c^3x-7\,a^4c^3\right)x\right)x\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 5/16*c^3*arcsin(a*x)*sgn(a)/abs(a) + 1/336*sqrt(-a^2*x^2 + 1)*(48*c^3/a + (
231*c^3 - 2*(72*a*c^3 + (91*a^2*c^3 - 4*(18*a^3*c^3 - (6*a^5*c^3*x - 7*a^4*
c^3)*x)*x)*x)*x)*x)
```

3.1194 $\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=83

$$\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}+\frac{3}{8}c^2x\sqrt{1-a^2x^2}+\frac{3c^2\sin^{-1}(ax)}{8a}$$

```
[Out] (3*c^2*x*Sqrt[1 - a^2*x^2])/8 + (c^2*x*(1 - a^2*x^2)^(3/2))/4 + (c^2*(1 - a
^2*x^2)^(5/2))/(5*a) + (3*c^2*ArcSin[a*x])/(8*a)
```

Rubi [A] time = 0.0507057, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6139, 641, 195, 216}

$$\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}+\frac{3}{8}c^2x\sqrt{1-a^2x^2}+\frac{3c^2\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/E^ArcTanh[a*x],x]
```

```
[Out] (3*c^2*x*Sqrt[1 - a^2*x^2])/8 + (c^2*x*(1 - a^2*x^2)^(3/2))/4 + (c^2*(1 - a
^2*x^2)^(5/2))/(5*a) + (3*c^2*ArcSin[a*x])/(8*a)
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = c^2\int(1-ax)\left(1-a^2x^2\right)^{3/2}\,dx$$
$$= \frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a} + c^2\int\left(1-a^2x^2\right)^{3/2}\,dx$$
$$= \frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2} + \frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a} + \frac{1}{4}\left(3c^2\right)\int\sqrt{1-a^2x^2}\,dx$$
$$= \frac{3}{8}c^2x\sqrt{1-a^2x^2} + \frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2} + \frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a} + \frac{1}{8}\left(3c^2\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{3}{8}c^2x\sqrt{1-a^2x^2} + \frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2} + \frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a} + \frac{3c^2\sin^{-1}(ax)}{8a}$$

Mathematica [A] time = 0.0948982, size = 75, normalized size = 0.9

$$\frac{c^2\left(\sqrt{1-a^2x^2}\left(8a^4x^4-10a^3x^3-16a^2x^2+25ax+8\right)-30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{40a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^2/E^ArcTanh[a*x],x]
```

```
[Out] (c^2*(Sqrt[1 - a^2*x^2]*(8 + 25*a*x - 16*a^2*x^2 - 10*a^3*x^3 + 8*a^4*x^4)
- 30*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(40*a)
```

Maple [A] time = 0.037, size = 109, normalized size = 1.3

$$-\frac{ac^2x^2}{5}\left(-a^2x^2+1\right)^{\frac{3}{2}} + \frac{c^2}{5\,a}\left(-a^2x^2+1\right)^{\frac{3}{2}} + \frac{xc^2}{4}\left(-a^2x^2+1\right)^{\frac{3}{2}} + \frac{3\,xc^2}{8}\sqrt{-a^2x^2+1} + \frac{3\,c^2}{8}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/5*c^2*a*x^2*(-a^2*x^2+1)^(3/2)+1/5*c^2*(-a^2*x^2+1)^(3/2)/a+1/4*c^2*x*(-
a^2*x^2+1)^(3/2)+3/8*c^2*x*(-a^2*x^2+1)^(1/2)+3/8*c^2/(a^2)^(1/2)*arctan((a
^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.45793, size = 122, normalized size = 1.47

$$-\frac{1}{5}\left(-a^2x^2+1\right)^{\frac{3}{2}}ac^2x^2 + \frac{1}{4}\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2x + \frac{3}{8}\sqrt{-a^2x^2+1}c^2x + \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}{5\,a} + \frac{3\,c^2\arcsin\left(ax\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/5*(-a^2*x^2 + 1)^(3/2)*a*c^2*x^2 + 1/4*(-a^2*x^2 + 1)^(3/2)*c^2*x + 3/8*
sqrt(-a^2*x^2 + 1)*c^2*x + 1/5*(-a^2*x^2 + 1)^(3/2)*c^2/a + 3/8*c^2*arcsin(
```

```
a*x)/a
```

Fricas [A] time = 2.40067, size = 201, normalized size = 2.42

$$-\frac{30\,c^{2}\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)-\left(8\,a^{4}c^{2}x^{4}-10\,a^{3}c^{2}x^{3}-16\,a^{2}c^{2}x^{2}+25\,ac^{2}x+8\,c^{2}\right)\sqrt{-a^{2}x^{2}+1}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/40*(30*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (8*a^4*c^2*x^4 - 10*
a^3*c^2*x^3 - 16*a^2*c^2*x^2 + 25*a*c^2*x + 8*c^2)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [C] time = 6.20297, size = 337, normalized size = 4.06

$$a^{3}c^{2}\left(\begin{cases}\frac{x^{4}\sqrt{-a^{2}x^{2}+1}}{5}-\frac{x^{2}\sqrt{-a^{2}x^{2}+1}}{15a^{2}}-\frac{2\sqrt{-a^{2}x^{2}+1}}{15a^{4}} & \text{for}\ a\neq 0\\ \frac{x^{4}}{4} & \text{otherwise}\end{cases}\right)-a^{2}c^{2}\left(\begin{cases}\frac{ia^{2}x^{5}}{4\sqrt{a^{2}x^{2}-1}}-\frac{3ix^{3}}{8\sqrt{a^{2}x^{2}-1}}+\frac{ix}{8a^{2}\sqrt{a^{2}x^{2}-1}}-\frac{i\operatorname{acosh}(ax)}{8a^{3}} & \\ -\frac{a^{2}x^{5}}{4\sqrt{-a^{2}x^{2}+1}}+\frac{3x^{3}}{8\sqrt{-a^{2}x^{2}+1}}-\frac{x}{8a^{2}\sqrt{-a^{2}x^{2}+1}}+\frac{\operatorname{asin}(ax)}{8a^{3}} & \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] a**3*c**2*Piecewise((x**4*sqrt(-a**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 + 1
)/(15*a**2) - 2*sqrt(-a**2*x**2 + 1)/(15*a**4), Ne(a, 0)), (x**4/4, True))
- a**2*c**2*Piecewise((I*a**2*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8*sq
rt(a**2*x**2 - 1)) + I*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*a**
3), Abs(a**2*x**2) > 1), (-a**2*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(8*s
qrt(-a**2*x**2 + 1)) - x/(8*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a**3)
, True)) - a*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2
)/(3*a**2), True)) + c**2*Piecewise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)) -
I*x/(2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (x*s
qrt(-a**2*x**2 + 1)/2 + asin(a*x)/(2*a), True))
```

Giac [A] time = 1.21018, size = 105, normalized size = 1.27

$$\frac{3\,c^{2}\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{8\,|a|}+\frac{1}{40}\,\sqrt{-a^{2}x^{2}+1}\left(\left(25\,c^{2}-2\left(8\,ac^{2}-\left(4\,a^{3}c^{2}x-5\,a^{2}c^{2}\right)x\right)x\right)x+\frac{8\,c^{2}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*c^2*arcsin(a*x)*sgn(a)/abs(a) + 1/40*sqrt(-a^2*x^2 + 1)*((25*c^2 - 2*(8
*a*c^2 - (4*a^3*c^2*x - 5*a^2*c^2)*x)*x)*x + 8*c^2/a)
```

3.1195 $\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)dx$

Optimal. Leaf size=55

$$\frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}$$

```
[Out] (c*x*Sqrt[1 - a^2*x^2])/2 + (c*(1 - a^2*x^2)^(3/2))/(3*a) + (c*ArcSin[a*x])
/(2*a)
```

Rubi [A] time = 0.0299767, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6139, 641, 195, 216}

$$\frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/E^ArcTanh[a*x],x]
```

```
[Out] (c*x*Sqrt[1 - a^2*x^2])/2 + (c*(1 - a^2*x^2)^(3/2))/(3*a) + (c*ArcSin[a*x])
/(2*a)
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] && !Int
egerQ[p - n/2]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)dx &= c\int (1-ax)\sqrt{1-a^2x^2}\,dx \\
&= \frac{c\left(1-a^2x^2\right)^{3/2}}{3a} + c\int \sqrt{1-a^2x^2}\,dx \\
&= \frac{1}{2}cx\sqrt{1-a^2x^2} + \frac{c\left(1-a^2x^2\right)^{3/2}}{3a} + \frac{1}{2}c\int \frac{1}{\sqrt{1-a^2x^2}}\,dx \\
&= \frac{1}{2}cx\sqrt{1-a^2x^2} + \frac{c\left(1-a^2x^2\right)^{3/2}}{3a} + \frac{c\sin^{-1}(ax)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.0807683, size = 57, normalized size = 1.04

$$
\frac{c\left(\left(-2a^2x^2+3ax+2\right)\sqrt{1-a^2x^2}-6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{6a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)/E^ArcTanh[a*x],x]
```

```
[Out] (c*((2 + 3*a*x - 2*a^2*x^2)*Sqrt[1 - a^2*x^2] - 6*ArcSin[Sqrt[1 - a*x]/Sqrt
[2]]))/(6*a)
```

Maple [A] time = 0.03, size = 64, normalized size = 1.2

$$
\frac{c}{3a}\left(-a^2x^2+1\right)^{\frac{3}{2}}+\frac{cx}{2}\sqrt{-a^2x^2+1}+\frac{c}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/3*c*(-a^2*x^2+1)^(3/2)/a+1/2*c*x*(-a^2*x^2+1)^(1/2)+1/2*c/(a^2)^(1/2)*arc
tan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.45104, size = 61, normalized size = 1.11

$$
\frac{1}{2}\sqrt{-a^2x^2+1}cx+\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}c}{3a}+\frac{c\arcsin(ax)}{2a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*c*x + 1/3*(-a^2*x^2 + 1)^(3/2)*c/a + 1/2*c*arcsin(a*
x)/a
```

Fricas [A] time = 2.16619, size = 140, normalized size = 2.55

$$-\frac{6\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^2cx^2-3\,acx-2\,c\right)\sqrt{-a^2x^2+1}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(6*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^2*c*x^2 - 3*a*c*x -
 2*c)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [C] time = 3.65976, size = 109, normalized size = 1.98

$$-ac\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3a^2} & \text{otherwise}\end{cases}\right)+c\left(\begin{cases}\frac{ia^2x^3}{2\sqrt{a^2x^2-1}}-\frac{ix}{2\sqrt{a^2x^2-1}}-\frac{i\operatorname{acosh}(ax)}{2a} & \text{for}\ \left|a^2x^2\right|>1\\ \frac{x\sqrt{-a^2x^2+1}}{2}+\frac{\operatorname{asin}(ax)}{2a} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -a*c*Piecewise((x**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2)/(3*a**2), T
rue)) + c*Piecewise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)) - I*x/(2*sqrt(a**2
*x**2 - 1)) - I*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (x*sqrt(-a**2*x**2 +
 1)/2 + asin(a*x)/(2*a), True))
```

Giac [A] time = 1.26892, size = 62, normalized size = 1.13

$$\frac{c\arcsin(ax)\,\mathrm{sgn}(a)}{2\,|a|}-\frac{1}{6}\,\sqrt{-a^2x^2+1}\left((2\,acx-3\,c)x-\frac{2\,c}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*c*arcsin(a*x)*sgn(a)/abs(a) - 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c*x - 3*c)*x
 - 2*c/a)
```

3.1196 $\int \frac{e^{-\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=16

$$-\frac{e^{-\tanh^{-1}(ax)}}{ac}$$

```
[Out] -(1/(a*c*E^ArcTanh[a*x]))
```

Rubi [A] time = 0.0311933, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6137}

$$-\frac{e^{-\tanh^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)),x]

[Out] -(1/(a*c*E^ArcTanh[a*x]))
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{e^{-\tanh^{-1}(ax)}}{ac}$$

Mathematica [A] time = 0.0101139, size = 27, normalized size = 1.69

$$-\frac{\sqrt{1-ax}}{ac\sqrt{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)),x]

[Out] -(Sqrt[1 - a*x]/(a*c*Sqrt[1 + a*x]))
```

Maple [A] time = 0.03, size = 28, normalized size = 1.8

$$-\frac{1}{(ax+1)\,ac}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c),x)
```

```
[Out] -1/a/c/(a*x+1)*(-a^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-a^2x^2+1}}{\left(a^2cx^2-c\right)(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c),x, algorithm="maxima"
)
```

```
[Out] -integrate(sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)*(a*x + 1)), x)
```

Fricas [A] time = 2.0768, size = 66, normalized size = 4.12

$$-\frac{ax+\sqrt{-a^2x^2+1}+1}{a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c),x, algorithm="fricas"
)
```

```
[Out] -(a*x + sqrt(-a^2*x^2 + 1) + 1)/(a^2*c*x + a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c),x)
```

```
[Out] Integral(1/(a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c
```

Giac [A] time = 1.1947, size = 50, normalized size = 3.12

$$\frac{2}{c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

3.1197 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=53

$$\frac{2x}{3c^2\sqrt{1-a^2x^2}}-\frac{1-ax}{3ac^2\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] -(1 - a*x)/(3*a*c^2*(1 - a^2*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0431805, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6139, 639, 191}

$$\frac{2x}{3c^2\sqrt{1-a^2x^2}}-\frac{1-ax}{3ac^2\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^2),x]
```

```
[Out] -(1 - a*x)/(3*a*c^2*(1 - a^2*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[1 - a^2*x^2])
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{1-ax}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}\\
&= -\frac{1-ax}{3ac^2\left(1-a^2x^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^2}\\
&= -\frac{1-ax}{3ac^2\left(1-a^2x^2\right)^{3/2}}+\frac{2x}{3c^2\sqrt{1-a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0173273, size = 43, normalized size = 0.81

$$\frac{2a^2x^2+2ax-1}{3ac^2\sqrt{1-ax}(ax+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^2),x]
```

```
[Out] (-1 + 2*a*x + 2*a^2*x^2)/(3*a*c^2*Sqrt[1 - a*x]*(1 + a*x)^(3/2))
```

Maple [A] time = 0.03, size = 42, normalized size = 0.8

$$\frac{2\,a^2x^2+2\,ax-1}{(3\,ax+3)\,ac^2}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/3/(-a^2*x^2+1)^(1/2)*(2*a^2*x^2+2*a*x-1)/(a*x+1)/a/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{\left(a^2cx^2-c\right)^2(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)^2*(a*x + 1)), x)
```

Fricas [A] time = 2.33526, size = 174, normalized size = 3.28

$$-\frac{a^3x^3+a^2x^2-ax+\left(2\,a^2x^2+2\,ax-1\right)\sqrt{-a^2x^2+1}-1}{3\left(a^4c^2x^3+a^3c^2x^2-a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="frica
s")
```

```
[Out] -1/3*(a^3*x^3 + a^2*x^2 - a*x + (2*a^2*x^2 + 2*a*x - 1)*sqrt(-a^2*x^2 + 1)
- 1)/(a^4*c^2*x^3 + a^3*c^2*x^2 - a^2*c^2*x - a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{-a^3x^3\sqrt{-a^2x^2+1}-a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(1/(-a**3*x**3*sqrt(-a**2*x**2 + 1) - a**2*x**2*sqrt(-a**2*x**2 + 1
) + a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{\left(a^2cx^2-c\right)^2(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)^2*(a*x + 1)), x)
```

3.1198 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=75

$$\frac{8x}{15c^3\sqrt{1-a^2x^2}}+\frac{4x}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{1-ax}{5ac^3\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] -(1 - a*x)/(5*a*c^3*(1 - a^2*x^2)^(5/2)) + (4*x)/(15*c^3*(1 - a^2*x^2)^(3/2
)) + (8*x)/(15*c^3*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0499653, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6139, 639, 192, 191}

$$\frac{8x}{15c^3\sqrt{1-a^2x^2}}+\frac{4x}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{1-ax}{5ac^3\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^3),x]
```

```
[Out] -(1 - a*x)/(5*a*c^3*(1 - a^2*x^2)^(5/2)) + (4*x)/(15*c^3*(1 - a^2*x^2)^(3/2
)) + (8*x)/(15*c^3*Sqrt[1 - a^2*x^2])
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{\int \frac{1-ax}{(1-a^2x^2)^{7/2}}\,dx}{c^3}$$

$$= -\frac{1-ax}{5ac^3(1-a^2x^2)^{5/2}} + \frac{4\int \frac{1}{(1-a^2x^2)^{5/2}}\,dx}{5c^3}$$

$$= -\frac{1-ax}{5ac^3(1-a^2x^2)^{5/2}} + \frac{4x}{15c^3(1-a^2x^2)^{3/2}} + \frac{8\int \frac{1}{(1-a^2x^2)^{3/2}}\,dx}{15c^3}$$

$$= -\frac{1-ax}{5ac^3(1-a^2x^2)^{5/2}} + \frac{4x}{15c^3(1-a^2x^2)^{3/2}} + \frac{8x}{15c^3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0276046, size = 59, normalized size = 0.79

$$-\frac{8a^4x^4+8a^3x^3-12a^2x^2-12ax+3}{15ac^3(1-ax)^{3/2}(ax+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^3),x]
```

```
[Out] -(3 - 12*a*x - 12*a^2*x^2 + 8*a^3*x^3 + 8*a^4*x^4)/(15*a*c^3*(1 - a*x)^(3/2
)*(1 + a*x)^(5/2))
```

Maple [A] time = 0.03, size = 58, normalized size = 0.8

$$-\frac{8\,x^4a^4+8\,x^3a^3-12\,a^2x^2-12\,ax+3}{(15\,ax+15)\,c^3a}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/15/(-a^2*x^2+1)^(3/2)*(8*a^4*x^4+8*a^3*x^3-12*a^2*x^2-12*a*x+3)/(a*x+1)/
c^3/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-a^2x^2+1}}{(a^2cx^2-c)^3(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)^3*(a*x + 1)), x)
```

Fricas [B] time = 2.48133, size = 294, normalized size = 3.92

$$-\frac{3\,a^5x^5+3\,a^4x^4-6\,a^3x^3-6\,a^2x^2+3\,ax+\left(8\,a^4x^4+8\,a^3x^3-12\,a^2x^2-12\,ax+3\right)\sqrt{-a^2x^2+1}+3}{15\left(a^6c^3x^5+a^5c^3x^4-2\,a^4c^3x^3-2\,a^3c^3x^2+a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="frica
s")
```

```
[Out] -1/15*(3*a^5*x^5 + 3*a^4*x^4 - 6*a^3*x^3 - 6*a^2*x^2 + 3*a*x + (8*a^4*x^4 +
 8*a^3*x^3 - 12*a^2*x^2 - 12*a*x + 3)*sqrt(-a^2*x^2 + 1) + 3)/(a^6*c^3*x^5
+ a^5*c^3*x^4 - 2*a^4*c^3*x^3 - 2*a^3*c^3*x^2 + a^2*c^3*x + a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^5x^5\sqrt{-a^2x^2+1}+a^4x^4\sqrt{-a^2x^2+1}-2a^3x^3\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(1/(a**5*x**5*sqrt(-a**2*x**2 + 1) + a**4*x**4*sqrt(-a**2*x**2 + 1)
 - 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + a*
x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-a^2x^2+1}}{\left(a^2cx^2-c\right)^3(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac"
)
```

```
[Out] integrate(-sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)^3*(a*x + 1)), x)
```

3.1199 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=97

$$\frac{16x}{35c^4\sqrt{1-a^2x^2}}+\frac{8x}{35c^4\left(1-a^2x^2\right)^{3/2}}+\frac{6x}{35c^4\left(1-a^2x^2\right)^{5/2}}-\frac{1-ax}{7ac^4\left(1-a^2x^2\right)^{7/2}}$$

```
[Out] -(1 - a*x)/(7*a*c^4*(1 - a^2*x^2)^(7/2)) + (6*x)/(35*c^4*(1 - a^2*x^2)^(5/2
)) + (8*x)/(35*c^4*(1 - a^2*x^2)^(3/2)) + (16*x)/(35*c^4*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0566229, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6139, 639, 192, 191}

$$\frac{16x}{35c^4\sqrt{1-a^2x^2}}+\frac{8x}{35c^4\left(1-a^2x^2\right)^{3/2}}+\frac{6x}{35c^4\left(1-a^2x^2\right)^{5/2}}-\frac{1-ax}{7ac^4\left(1-a^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^4),x]
```

```
[Out] -(1 - a*x)/(7*a*c^4*(1 - a^2*x^2)^(7/2)) + (6*x)/(35*c^4*(1 - a^2*x^2)^(5/2
)) + (8*x)/(35*c^4*(1 - a^2*x^2)^(3/2)) + (16*x)/(35*c^4*Sqrt[1 - a^2*x^2])
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \frac{\int \frac{1-ax}{\left(1-a^2x^2\right)^{9/2}}\,dx}{c^4}$$

$$= -\frac{1-ax}{7ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{6\int \frac{1}{\left(1-a^2x^2\right)^{7/2}}\,dx}{7c^4}$$

$$= -\frac{1-ax}{7ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{6x}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{24\int \frac{1}{\left(1-a^2x^2\right)^{5/2}}\,dx}{35c^4}$$

$$= -\frac{1-ax}{7ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{6x}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{8x}{35c^4\left(1-a^2x^2\right)^{3/2}} + \frac{16\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{35c^4}$$

$$= -\frac{1-ax}{7ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{6x}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{8x}{35c^4\left(1-a^2x^2\right)^{3/2}} + \frac{16x}{35c^4\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0344725, size = 75, normalized size = 0.77

$$\frac{16a^6x^6+16a^5x^5-40a^4x^4-40a^3x^3+30a^2x^2+30ax-5}{35ac^4(1-ax)^{5/2}(ax+1)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^4),x]
```

```
[Out] (-5 + 30*a*x + 30*a^2*x^2 - 40*a^3*x^3 - 40*a^4*x^4 + 16*a^5*x^5 + 16*a^6*x
^6)/(35*a*c^4*(1 - a*x)^(5/2)*(1 + a*x)^(7/2))
```

Maple [A] time = 0.03, size = 74, normalized size = 0.8

$$\frac{16\,x^6a^6+16\,x^5a^5-40\,x^4a^4-40\,x^3a^3+30\,a^2x^2+30\,ax-5}{(35\,ax+35)\,c^4a}\left(-a^2x^2+1\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] 1/35/(-a^2*x^2+1)^(5/2)*(16*a^6*x^6+16*a^5*x^5-40*a^4*x^4-40*a^3*x^3+30*a^2
*x^2+30*a*x-5)/(a*x+1)/c^4/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{\left(a^2cx^2-c\right)^4(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)^4*(a*x + 1)), x)
```

Fricas [B] time = 2.72434, size = 413, normalized size = 4.26

$$-\frac{5\,a^7x^7+5\,a^6x^6-15\,a^5x^5-15\,a^4x^4+15\,a^3x^3+15\,a^2x^2-5\,ax+\left(16\,a^6x^6+16\,a^5x^5-40\,a^4x^4-40\,a^3x^3+30\,a^2x^2+3}{35\left(a^8c^4x^7+a^7c^4x^6-3\,a^6c^4x^5-3\,a^5c^4x^4+3\,a^4c^4x^3+3\,a^3c^4x^2-a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="frica
s")
```

```
[Out] -1/35*(5*a^7*x^7 + 5*a^6*x^6 - 15*a^5*x^5 - 15*a^4*x^4 + 15*a^3*x^3 + 15*a^
2*x^2 - 5*a*x + (16*a^6*x^6 + 16*a^5*x^5 - 40*a^4*x^4 - 40*a^3*x^3 + 30*a^2
*x^2 + 30*a*x - 5)*sqrt(-a^2*x^2 + 1) - 5)/(a^8*c^4*x^7 + a^7*c^4*x^6 - 3*a
^6*c^4*x^5 - 3*a^5*c^4*x^4 + 3*a^4*c^4*x^3 + 3*a^3*c^4*x^2 - a^2*c^4*x - a*
c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{-a^7x^7\sqrt{-a^2x^2+1}-a^6x^6\sqrt{-a^2x^2+1}+3a^5x^5\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Integral(1/(-a**7*x**7*sqrt(-a**2*x**2 + 1) - a**6*x**6*sqrt(-a**2*x**2 + 1
) + 3*a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 + 1) - 3
*a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + a*x*sq
rt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{\left(a^2cx^2-c\right)^4(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)^4*(a*x + 1)), x)
```

3.1200 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-a^2cx^2)^5} dx$

Optimal. Leaf size=119

$$\frac{128x}{315c^5\sqrt{1-a^2x^2}} + \frac{64x}{315c^5\left(1-a^2x^2\right)^{3/2}} + \frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} - \frac{1-ax}{9ac^5\left(1-a^2x^2\right)^{9/2}}$$

```
[Out] -(1 - a*x)/(9*a*c^5*(1 - a^2*x^2)^(9/2)) + (8*x)/(63*c^5*(1 - a^2*x^2)^(7/2
)) + (16*x)/(105*c^5*(1 - a^2*x^2)^(5/2)) + (64*x)/(315*c^5*(1 - a^2*x^2)^(
3/2)) + (128*x)/(315*c^5*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0659426, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6139, 639, 192, 191}

$$\frac{128x}{315c^5\sqrt{1-a^2x^2}} + \frac{64x}{315c^5\left(1-a^2x^2\right)^{3/2}} + \frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} - \frac{1-ax}{9ac^5\left(1-a^2x^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^5),x]
```

```
[Out] -(1 - a*x)/(9*a*c^5*(1 - a^2*x^2)^(9/2)) + (8*x)/(63*c^5*(1 - a^2*x^2)^(7/2
)) + (16*x)/(105*c^5*(1 - a^2*x^2)^(5/2)) + (64*x)/(315*c^5*(1 - a^2*x^2)^(
3/2)) + (128*x)/(315*c^5*Sqrt[1 - a^2*x^2])
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^5}\,dx = \frac{\int \frac{1-ax}{\left(1-a^2x^2\right)^{11/2}}dx}{c^5}$$

$$= -\frac{1-ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8\int \frac{1}{\left(1-a^2x^2\right)^{9/2}}dx}{9c^5}$$

$$= -\frac{1-ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} + \frac{16\int \frac{1}{\left(1-a^2x^2\right)^{7/2}}dx}{21c^5}$$

$$= -\frac{1-ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} + \frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}} + \frac{64\int \frac{1}{\left(1-a^2x^2\right)^{5/2}}dx}{105c^5}$$

$$= -\frac{1-ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} + \frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}} + \frac{64x}{315c^5\left(1-a^2x^2\right)^{3/2}} + \frac{128\int \frac{}{(1-}}{31}$$

$$= -\frac{1-ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} + \frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}} + \frac{64x}{315c^5\left(1-a^2x^2\right)^{3/2}} + \frac{128}{315c^5\sqrt{1}}$$

Mathematica [A] time = 0.0395975, size = 91, normalized size = 0.76

$$-\frac{128a^8x^8+128a^7x^7-448a^6x^6-448a^5x^5+560a^4x^4+560a^3x^3-280a^2x^2-280ax+35}{315ac^5(1-ax)^{7/2}(ax+1)^{9/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^5),x]
```

```
[Out] -(35 - 280*a*x - 280*a^2*x^2 + 560*a^3*x^3 + 560*a^4*x^4 - 448*a^5*x^5 - 44
8*a^6*x^6 + 128*a^7*x^7 + 128*a^8*x^8)/(315*a*c^5*(1 - a*x)^(7/2)*(1 + a*x)
^(9/2))
```

Maple [A] time = 0.032, size = 90, normalized size = 0.8

$$-\frac{128\,a^8x^8+128\,a^7x^7-448\,x^6a^6-448\,x^5a^5+560\,x^4a^4+560\,x^3a^3-280\,a^2x^2-280\,ax+35}{(315\,ax+315)\,c^5a}\left(-a^2x^2+1\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^5,x)
```

```
[Out] -1/315/(-a^2*x^2+1)^(7/2)*(128*a^8*x^8+128*a^7*x^7-448*a^6*x^6-448*a^5*x^5+
560*a^4*x^4+560*a^3*x^3-280*a^2*x^2-280*a*x+35)/(a*x+1)/c^5/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-a^2x^2+1}}{\left(a^2cx^2-c\right)^5(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^5,x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)^5*(a*x + 1)), x)
```

Fricas [B] time = 3.50518, size = 555, normalized size = 4.66

$$-\frac{35\,a^9x^9+35\,a^8x^8-140\,a^7x^7-140\,a^6x^6+210\,a^5x^5+210\,a^4x^4-140\,a^3x^3-140\,a^2x^2+35\,ax+\left(128\,a^8x^8+128\,a^7x^7-\right.}{315\left(a^{10}c^5x^9+a^9c^5x^8-4\,a^8c^5x^7-4\,a^7c^5x^6+6\,a^6c^5x^5+6\,a^5c^5x^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^5,x, algorithm="frica
s")
```

```
[Out] -1/315*(35*a^9*x^9 + 35*a^8*x^8 - 140*a^7*x^7 - 140*a^6*x^6 + 210*a^5*x^5 +
 210*a^4*x^4 - 140*a^3*x^3 - 140*a^2*x^2 + 35*a*x + (128*a^8*x^8 + 128*a^7*
x^7 - 448*a^6*x^6 - 448*a^5*x^5 + 560*a^4*x^4 + 560*a^3*x^3 - 280*a^2*x^2 -
 280*a*x + 35)*sqrt(-a^2*x^2 + 1) + 35)/(a^10*c^5*x^9 + a^9*c^5*x^8 - 4*a^8
*c^5*x^7 - 4*a^7*c^5*x^6 + 6*a^6*c^5*x^5 + 6*a^5*c^5*x^4 - 4*a^4*c^5*x^3 -
4*a^3*c^5*x^2 + a^2*c^5*x + a*c^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^9x^9\sqrt{-a^2x^2+1}+a^8x^8\sqrt{-a^2x^2+1}-4a^7x^7\sqrt{-a^2x^2+1}-4a^6x^6\sqrt{-a^2x^2+1}+6a^5x^5\sqrt{-a^2x^2+1}+6a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}-4a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}+\sqrt{}}}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**5,x)
```

```
[Out] Integral(1/(a**9*x**9*sqrt(-a**2*x**2 + 1) + a**8*x**8*sqrt(-a**2*x**2 + 1)
 - 4*a**7*x**7*sqrt(-a**2*x**2 + 1) - 4*a**6*x**6*sqrt(-a**2*x**2 + 1) + 6*
a**5*x**5*sqrt(-a**2*x**2 + 1) + 6*a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*
x**3*sqrt(-a**2*x**2 + 1) - 4*a**2*x**2*sqrt(-a**2*x**2 + 1) + a*x*sqrt(-a*
*2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-a^2x^2+1}}{\left(a^2cx^2-c\right)^5(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^5,x, algorithm="giac"
)
```

```
[Out] integrate(-sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)^5*(a*x + 1)), x)
```

3.1201 $\int e^{-\tanh^{-1}(ax)} x^m \sqrt{c - a^2 c x^2}\, dx$

Optimal. Leaf size=83

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}-\frac{ax^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}$$

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) - (a*x^(2 + m)*
Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.177692, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 43}

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}-\frac{ax^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^ArcTanh[a*x],x]
```

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) - (a*x^(2 + m)*
Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-\tanh^{-1}(ax)}x^m\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int x^m(1-ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(x^m-ax^{1+m}\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}} - \frac{ax^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0447579, size = 50, normalized size = 0.6

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}\left(\frac{1}{m+1}-\frac{ax}{m+2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^ArcTanh[a*x],x]
```

```
[Out] (x^(1 + m)*((1 + m)^(-1) - (a*x)/(2 + m))*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2
*x^2]
```

Maple [A] time = 0.03, size = 68, normalized size = 0.8

$$\frac{x^{1+m}\,(amx+ax-m-2)}{(2+m)\,(1+m)\,(ax-1)\,(ax+1)}\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] x^(1+m)*(a*m*x+a*x-m-2)*(-a^2*c*x^2+c)^(1/2)*(-a^2*x^2+1)^(1/2)/(2+m)/(1+m)
/(a*x-1)/(a*x+1)
```

Maxima [A] time = 1.04055, size = 85, normalized size = 1.02

$$-\frac{\left(a\sqrt{c}(m+1)x^2-\sqrt{c}(m+2)x\right)(ax+1)(ax-1)x^m}{\left(m^2+3\,m+2\right)a^2x^2-m^2-3\,m-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm=
"maxima")
```

```
[Out] -(a*sqrt(c)*(m + 1)*x^2 - sqrt(c)*(m + 2)*x)*(a*x + 1)*(a*x - 1)*x^m/((m^2
+ 3*m + 2)*a^2*x^2 - m^2 - 3*m - 2)
```

Fricas [A] time = 2.65412, size = 166, normalized size = 2.

$$\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\left((am+a)x^2-(m+2)x\right)x^m}{\left(a^2m^2+3\,a^2m+2\,a^2\right)x^2-m^2-3\,m-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm=
"fricas")
```

```
[Out] sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((a*m + a)*x^2 - (m + 2)*x)*x^m/((a
^2*m^2 + 3*a^2*m + 2*a^2)*x^2 - m^2 - 3*m - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^m\sqrt{-(ax-1)(ax+1)}\sqrt{-c(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/(a*x
+ 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^m/(a*x + 1), x)
```

3.1202 $\int e^{-\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=74

$$\frac{x^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}-\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}$$

```
[Out] (x^3*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - a^2*x^2]) - (a*x^4*Sqrt[c - a^2*c*x^2
])/(4*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.188724, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 43}

$$\frac{x^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}-\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^ArcTanh[a*x],x]
```

```
[Out] (x^3*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - a^2*x^2]) - (a*x^4*Sqrt[c - a^2*c*x^2
])/(4*Sqrt[1 - a^2*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-\tanh^{-1}(ax)}x^2\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int x^2(1-ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(x^2-ax^3\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{x^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}} - \frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0242038, size = 42, normalized size = 0.57

$$-\frac{x^3(3ax-4)\sqrt{c-a^2cx^2}}{12\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^ArcTanh[a*x],x]
```

```
[Out] -(x^3*(-4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.027, size = 51, normalized size = 0.7

$$\frac{x^3(3ax-4)}{(12ax-12)(ax+1)}\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/12*x^3*(3*a*x-4)*(-a^2*c*x^2+c)^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(a*x+1)
```

Maxima [A] time = 1.0029, size = 55, normalized size = 0.74

$$-\frac{\left(3a\sqrt{c}x^4-4\sqrt{c}x^3\right)(ax+1)(ax-1)}{12\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm=
"maxima")
```

```
[Out] -1/12*(3*a*sqrt(c)*x^4 - 4*sqrt(c)*x^3)*(a*x + 1)*(a*x - 1)/(a^2*x^2 - 1)
```

Fricas [A] time = 2.29637, size = 105, normalized size = 1.42

$$\frac{\sqrt{-a^2cx^2+c}\left(3ax^4-4x^3\right)\sqrt{-a^2x^2+1}}{12\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm=
"fricas")
```

```
[Out] 1/12*sqrt(-a^2*c*x^2 + c)*(3*a*x^4 - 4*x^3)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sqrt{-(ax-1)(ax+1)}\sqrt{-c(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/(a*x
+ 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^2}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^2/(a*x + 1), x)
```

3.1203 $\int e^{-\tanh^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=74

$$\frac{x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}$$

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/(2*Sqrt[1 - a^2*x^2]) - (a*x^3*Sqrt[c - a^2*c*x^2
])/(3*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.130159, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 43}

$$\frac{x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^ArcTanh[a*x],x]
```

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/(2*Sqrt[1 - a^2*x^2]) - (a*x^3*Sqrt[c - a^2*c*x^2
])/(3*Sqrt[1 - a^2*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int x(1-ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(x-ax^2\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}} - \frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0218962, size = 42, normalized size = 0.57

$$-\frac{x^2(2ax-3)\sqrt{c-a^2cx^2}}{6\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^ArcTanh[a*x],x]
```

```
[Out] -(x^2*(-3 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(6*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.028, size = 51, normalized size = 0.7

$$\frac{x^2\,(2\,ax-3)}{(6\,ax-6)\,(ax+1)}\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/6*x^2*(2*a*x-3)*(-a^2*c*x^2+c)^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(a*x+1)
```

Maxima [A] time = 1.00856, size = 55, normalized size = 0.74

$$-\frac{\left(2\,a\sqrt{c}x^3-3\,\sqrt{c}x^2\right)(ax+1)(ax-1)}{6\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="m
axima")
```

```
[Out] -1/6*(2*a*sqrt(c)*x^3 - 3*sqrt(c)*x^2)*(a*x + 1)*(a*x - 1)/(a^2*x^2 - 1)
```

Fricas [A] time = 2.39566, size = 104, normalized size = 1.41

$$\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\left(2\,ax^3-3\,x^2\right)}{6\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/6*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(2*a*x^3 - 3*x^2)/(a^2*x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-(ax-1)(ax+1)}\sqrt{-c(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/(a*x + 1
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x/(a*x + 1), x)
```

3.1204 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=69

$$\frac{x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}$$

```
[Out] (x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] - (a*x^2*Sqrt[c - a^2*c*x^2])/(2*
Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0713259, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6143, 6140}

$$\frac{x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^ArcTanh[a*x],x]
```

```
[Out] (x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] - (a*x^2*Sqrt[c - a^2*c*x^2])/(2*
Sqrt[1 - a^2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{c-a^2cx^2}\int (1-ax)\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0155949, size = 40, normalized size = 0.58

$$\frac{\left(x-\frac{ax^2}{2}\right)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^ArcTanh[a*x],x]
```

```
[Out] ((x - (a*x^2)/2)*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.027, size = 48, normalized size = 0.7

$$\frac{x(ax-2)}{(2ax-2)(ax+1)}\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*x*(a*x-2)*(-a^2*c*x^2+c)^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(a*x+1)
```

Maxima [A] time = 1.00756, size = 65, normalized size = 0.94

$$-\frac{\left(a^2\sqrt{c}x^2-2a\sqrt{c}x+2\sqrt{c}\right)(ax+1)(ax-1)}{2\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/2*(a^2*sqrt(c)*x^2 - 2*a*sqrt(c)*x + 2*sqrt(c))*(a*x + 1)*(a*x - 1)/(a^3
*x^2 - a)
```

Fricas [A] time = 2.42799, size = 99, normalized size = 1.43

$$\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\left(ax^2-2x\right)}{2\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x^2 - 2*x)/(a^2*x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(ax-1)(ax+1)}\sqrt{-c(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/(a*x + 1),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

3.1205 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=66

$$\frac{\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

```
[Out] -((a*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2]) + (Sqrt[c - a^2*c*x^2]*Log[x
])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.176813, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 43}

$$\frac{\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^ArcTanh[a*x]*x),x]
```

```
[Out] -((a*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2]) + (Sqrt[c - a^2*c*x^2]*Log[x
])/Sqrt[1 - a^2*x^2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{1-ax}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-a+\frac{1}{x}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}} + \frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.019214, size = 37, normalized size = 0.56

$$\frac{\sqrt{c-a^2cx^2}(\log(x)-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^ArcTanh[a*x]*x),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-(a*x) + Log[x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.085, size = 47, normalized size = 0.7

$$\frac{ax-\ln(x)}{a^2x^2-1}\sqrt{-c\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] (a*x-ln(x))*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/((a*x + 1)*x), x)
```

Fricas [B] time = 2.58263, size = 555, normalized size = 8.41

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\left(x^4-1\right)\sqrt{c}-c}{a^2x^4-x^2}\right)+2\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}(ax-a)}{2\left(a^2x^2-1\right)}, \frac{\left(a^2x^2-1\right)\sqrt{-c}\operatorname{arctan}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="f
ricas")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 - sqrt(-a^2*
c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x^4 - 1)*sqrt(c) - c)/(a^2*x^4 - x^2)) + 2*s
qrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x - a))/(a^2*x^2 - 1), ((a^2*x^2
- 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x^2 + 1)*sqrt
(-c)/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) + sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^
2 + 1)*(a*x - a))/(a^2*x^2 - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\sqrt{-c(ax-1)(ax+1)}}{x(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/(x*(a*x +
1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/((a*x + 1)*x), x)
```

3.1206 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{a\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

```
[Out] -(Sqrt[c - a^2*c*x^2]/(x*Sqrt[1 - a^2*x^2])) - (a*Sqrt[c - a^2*c*x^2]*Log[x
])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.178884, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 43}

$$-\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{a\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(x*Sqrt[1 - a^2*x^2])) - (a*Sqrt[c - a^2*c*x^2]*Log[x
])/Sqrt[1 - a^2*x^2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{1-ax}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}-\frac{a}{x}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}} - \frac{a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0230615, size = 41, normalized size = 0.59

$$\frac{\sqrt{c-a^2cx^2}\left(-a\log(x)-\frac{1}{x}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-x^(-1) - a*Log[x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.088, size = 49, normalized size = 0.7

$$\frac{a\ln(x)x+1}{x\left(a^2x^2-1\right)}\sqrt{-c\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] (-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(a*ln(x)*x+1)/(a^2*x^2-1)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/((a*x + 1)*x^2), x)
```

Fricas [B] time = 2.46878, size = 556, normalized size = 8.06

$$\left[\frac{\left(a^3x^3-ax\right)\sqrt{c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\left(x^4-1\right)\sqrt{c}-c}{a^2x^4-x^2}\right)-2\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}(x-1)}{2\left(a^2x^3-x\right)},-\frac{\left(a^3x^3-ax\right)\sqrt{-c}\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm=
"fricas")
```

```
[Out] [1/2*((a^3*x^3 - a*x)*sqrt(c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 + sqrt(-a^
2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x^4 - 1)*sqrt(c) - c)/(a^2*x^4 - x^2)) - 2
*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x - 1))/(a^2*x^3 - x), -((a^3*x^3
 - a*x)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x^2 + 1)*s
qrt(-c)/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) + sqrt(-a^2*c*x^2 + c)*sqrt(-a^2
*x^2 + 1)*(x - 1))/(a^2*x^3 - x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(ax-1)(ax+1)}\sqrt{-c(ax-1)(ax+1)}}{x^2(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/(x**2*(a*x
 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/((a*x + 1)*x^2), x)
```

3.1207 $\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=91

$$\frac{c(1-ax)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}-\frac{2c(1-ax)^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}$$

```
[Out] (-2*c*(1 - a*x)^3*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) + (c*(1 - a*
x)^4*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0882952, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$\frac{c(1-ax)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}-\frac{2c(1-ax)^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^ArcTanh[a*x],x]
```

```
[Out] (-2*c*(1 - a*x)^3*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) + (c*(1 - a*
x)^4*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}dx=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(1-ax)^2(1+ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int\left(2(1-ax)^2-(1-ax)^3\right)dx}{\sqrt{1-a^2x^2}}$$
$$=-\frac{2c(1-ax)^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}+\frac{c(1-ax)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.030942, size = 57, normalized size = 0.63

$$\frac{cx\left(3a^3x^3-4a^2x^2-6ax+12\right)\sqrt{c-a^2cx^2}}{12\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^ArcTanh[a*x],x]
```

```
[Out] (c*x*Sqrt[c - a^2*c*x^2]*(12 - 6*a*x - 4*a^2*x^2 + 3*a^3*x^3))/(12*Sqrt[1 -
 a^2*x^2])
```

Maple [A] time = 0.029, size = 65, normalized size = 0.7

$$\frac{x\left(3\,x^3a^3-4\,a^2x^2-6\,ax+12\right)}{12\,(ax+1)^2\,(ax-1)^2}\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/12*x*(3*a^3*x^3-4*a^2*x^2-6*a*x+12)*(-a^2*c*x^2+c)^(3/2)*(-a^2*x^2+1)^(1/
2)/(a*x+1)^2/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

Fricas [A] time = 2.20506, size = 149, normalized size = 1.64

$$-\frac{\left(3\,a^3cx^4-4\,a^2cx^3-6\,acx^2+12\,cx\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{12\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/12*(3*a^3*c*x^4 - 4*a^2*c*x^3 - 6*a*c*x^2 + 12*c*x)*sqrt(-a^2*c*x^2 + c)
*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{3}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)/(a*x +
1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

3.1208 $\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=140

$$-\frac{c^2(1-ax)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}+\frac{4c^2(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}-\frac{c^2(1-ax)^4\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}$$

```
[Out] -((c^2*(1 - a*x)^4*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2])) + (4*c^2*(1
- a*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) - (c^2*(1 - a*x)^6*Sq
rt[c - a^2*c*x^2])/(6*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0974888, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$-\frac{c^2(1-ax)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}+\frac{4c^2(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}-\frac{c^2(1-ax)^4\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^ArcTanh[a*x],x]
```

```
[Out] -((c^2*(1 - a*x)^4*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2])) + (4*c^2*(1
- a*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) - (c^2*(1 - a*x)^6*Sq
rt[c - a^2*c*x^2])/(6*a*Sqrt[1 - a^2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)} \left(c-a^2cx^2\right)^{5/2} dx = \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int e^{-\tanh^{-1}(ax)} \left(1-a^2x^2\right)^{5/2} dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int (1-ax)^3(1+ax)^2 dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int \left(4(1-ax)^3-4(1-ax)^4+(1-ax)^5\right) dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{c^2(1-ax)^4\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}} + \frac{4c^2(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}} - \frac{c^2(1-ax)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0421286, size = 60, normalized size = 0.43

$$-\frac{c^2(ax-1)^4\left(5a^2x^2+14ax+11\right)\sqrt{c-a^2cx^2}}{30a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^ArcTanh[a*x],x]
```

```
[Out] -(c^2*(-1 + a*x)^4*(11 + 14*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(30*a*Sqr
t[1 - a^2*x^2])
```

Maple [A] time = 0.029, size = 81, normalized size = 0.6

$$\frac{x\left(5x^5a^5-6x^4a^4-15x^3a^3+20a^2x^2+15ax-30\right)}{30\,(ax+1)^3\,(ax-1)^3}\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/30*x*(5*a^5*x^5-6*a^4*x^4-15*a^3*x^3+20*a^2*x^2+15*a*x-30)*(-a^2*c*x^2+c)
^(5/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^3/(a*x-1)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

Fricas [A] time = 2.34548, size = 205, normalized size = 1.46

$$\frac{\left(5\,a^5c^2x^6-6\,a^4c^2x^5-15\,a^3c^2x^4+20\,a^2c^2x^3+15\,ac^2x^2-30\,c^2x\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{30\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/30*(5*a^5*c^2*x^6 - 6*a^4*c^2*x^5 - 15*a^3*c^2*x^4 + 20*a^2*c^2*x^3 + 15*
a*c^2*x^2 - 30*c^2*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{5}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(5/2)/(a*x +
1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

3.1209 $\int e^{-\tanh^{-1}(ax)} \left(c - a^2 c x^2\right)^{7/2} dx$

Optimal. Leaf size=187

$$\frac{c^3(1-ax)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}} - \frac{6c^3(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}} + \frac{2c^3(1-ax)^6\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}} - \frac{8c^3(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}$$

```
[Out] (-8*c^3*(1 - a*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) + (2*c^3*(
1 - a*x)^6*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) - (6*c^3*(1 - a*x)^7*
Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) + (c^3*(1 - a*x)^8*Sqrt[c - a^
2*c*x^2])/(8*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.116011, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$\frac{c^3(1-ax)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}} - \frac{6c^3(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}} + \frac{2c^3(1-ax)^6\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}} - \frac{8c^3(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(7/2)/E^ArcTanh[a*x],x]
```

```
[Out] (-8*c^3*(1 - a*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) + (2*c^3*(
1 - a*x)^6*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) - (6*c^3*(1 - a*x)^7*
Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) + (c^3*(1 - a*x)^8*Sqrt[c - a^
2*c*x^2])/(8*a*Sqrt[1 - a^2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}dx=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{7/2}dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int(1-ax)^4(1+ax)^3dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int\left(8(1-ax)^4-12(1-ax)^5+6(1-ax)^6-(1-ax)^7\right)dx}{\sqrt{1-a^2x^2}}$$
$$=-\frac{8c^3(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{2c^3(1-ax)^6\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{6c^3(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{c^3(1-}{8}$$

Mathematica [A] time = 0.0481089, size = 68, normalized size = 0.36

$$\frac{c^3(ax-1)^5\left(35a^3x^3+135a^2x^2+185ax+93\right)\sqrt{c-a^2cx^2}}{280a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(7/2)/E^ArcTanh[a*x],x]
```

```
[Out] (c^3*(-1 + a*x)^5*Sqrt[c - a^2*c*x^2]*(93 + 185*a*x + 135*a^2*x^2 + 35*a^3*
x^3))/(280*a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.03, size = 97, normalized size = 0.5

$$\frac{x\left(35\,a^7x^7-40\,x^6a^6-140\,x^5a^5+168\,x^4a^4+210\,x^3a^3-280\,a^2x^2-140\,ax+280\right)}{280\,(ax+1)^4\,(ax-1)^4}\left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/280*x*(35*a^7*x^7-40*a^6*x^6-140*a^5*x^5+168*a^4*x^4+210*a^3*x^3-280*a^2*
x^2-140*a*x+280)*(-a^2*c*x^2+c)^(7/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^4/(a*x-1)^
4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

Fricas [A] time = 2.27553, size = 265, normalized size = 1.42

$$-\frac{\left(35\,a^7c^3x^8-40\,a^6c^3x^7-140\,a^5c^3x^6+168\,a^4c^3x^5+210\,a^3c^3x^4-280\,a^2c^3x^3-140\,ac^3x^2+280\,c^3x\right)\sqrt{-a^2cx^2+c}\sqrt{-a}}{280\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/280*(35*a^7*c^3*x^8 - 40*a^6*c^3*x^7 - 140*a^5*c^3*x^6 + 168*a^4*c^3*x^5
 + 210*a^3*c^3*x^4 - 280*a^2*c^3*x^3 - 140*a*c^3*x^2 + 280*c^3*x)*sqrt(-a^2
*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(7/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

3.1210 $\int e^{-\tanh^{-1}(ax)} \left(c-a^2cx^2\right)^{9/2} dx$

Optimal. Leaf size=234

$$-\frac{c^4(1-ax)^{10}\sqrt{c-a^2cx^2}}{10a\sqrt{1-a^2x^2}}+\frac{8c^4(1-ax)^9\sqrt{c-a^2cx^2}}{9a\sqrt{1-a^2x^2}}-\frac{3c^4(1-ax)^8\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}+\frac{32c^4(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{8c^4(1-ax)^6}{3a\sqrt{1-}}$$

```
[Out] (-8*c^4*(1 - a*x)^6*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) + (32*c^4*
(1 - a*x)^7*Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) - (3*c^4*(1 - a*x)
^8*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) + (8*c^4*(1 - a*x)^9*Sqrt[c -
 a^2*c*x^2])/(9*a*Sqrt[1 - a^2*x^2]) - (c^4*(1 - a*x)^10*Sqrt[c - a^2*c*x^2
])/(10*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.119275, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$-\frac{c^4(1-ax)^{10}\sqrt{c-a^2cx^2}}{10a\sqrt{1-a^2x^2}}+\frac{8c^4(1-ax)^9\sqrt{c-a^2cx^2}}{9a\sqrt{1-a^2x^2}}-\frac{3c^4(1-ax)^8\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}+\frac{32c^4(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{8c^4(1-ax)^6}{3a\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(9/2)/E^ArcTanh[a*x],x]
```


```
[Out] (-8*c^4*(1 - a*x)^6*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) + (32*c^4*
(1 - a*x)^7*Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) - (3*c^4*(1 - a*x)
^8*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) + (8*c^4*(1 - a*x)^9*Sqrt[c -
 a^2*c*x^2])/(9*a*Sqrt[1 - a^2*x^2]) - (c^4*(1 - a*x)^10*Sqrt[c - a^2*c*x^2
])/(10*a*Sqrt[1 - a^2*x^2])
```


Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}dx = \frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{9/2}dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int (1-ax)^5(1+ax)^4\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int \left(16(1-ax)^5-32(1-ax)^6+24(1-ax)^7-8(1-ax)^8+(1-ax)^9\right)}{\sqrt{1-a^2x^2}}$$
$$= -\frac{8c^4(1-ax)^6\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}+\frac{32c^4(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{3c^4(1-ax)^8\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}+8$$

Mathematica [A] time = 0.0595504, size = 76, normalized size = 0.32

$$-\frac{c^4(ax-1)^6\left(63a^4x^4+308a^3x^3+588a^2x^2+528ax+193\right)\sqrt{c-a^2cx^2}}{630a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(9/2)/E^ArcTanh[a*x],x]
```

```
[Out] -(c^4*(-1 + a*x)^6*Sqrt[c - a^2*c*x^2]*(193 + 528*a*x + 588*a^2*x^2 + 308*a
^3*x^3 + 63*a^4*x^4))/(630*a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.03, size = 113, normalized size = 0.5

$$\frac{x\left(63a^9x^9-70x^8a^8-315a^7x^7+360x^6a^6+630x^5a^5-756x^4a^4-630x^3a^3+840a^2x^2+315ax-630\right)}{630\,(ax+1)^5\,(ax-1)^5}\left(-a^2cx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/630*x*(63*a^9*x^9-70*a^8*x^8-315*a^7*x^7+360*a^6*x^6+630*a^5*x^5-756*a^4*
x^4-630*a^3*x^3+840*a^2*x^2+315*a*x-630)*(-a^2*c*x^2+c)^(9/2)*(-a^2*x^2+1)^
(1/2)/(a*x+1)^5/(a*x-1)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

Fricas [A] time = 2.44713, size = 313, normalized size = 1.34

$$\frac{\left(63\,a^9c^4x^{10} - 70\,a^8c^4x^9 - 315\,a^7c^4x^8 + 360\,a^6c^4x^7 + 630\,a^5c^4x^6 - 756\,a^4c^4x^5 - 630\,a^3c^4x^4 + 840\,a^2c^4x^3 + 315\,ac^4x^2 - 63\right.}{630\left(a^2x^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/630*(63*a^9*c^4*x^10 - 70*a^8*c^4*x^9 - 315*a^7*c^4*x^8 + 360*a^6*c^4*x^7
 + 630*a^5*c^4*x^6 - 756*a^4*c^4*x^5 - 630*a^3*c^4*x^4 + 840*a^2*c^4*x^3 +
315*a*c^4*x^2 - 630*c^4*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2
 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(9/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2 + c\right)^{\frac{9}{2}}\sqrt{-a^2x^2 + 1}}{ax + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

3.1211 $\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{1-a^2x^2}\log(ax+1)}{a\sqrt{c-a^2cx^2}}$$

```
[Out] (Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(a*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.0746729, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 31}

$$\frac{\sqrt{1-a^2x^2}\log(ax+1)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(a*Sqrt[c - a^2*c*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{1}{1+ax}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\log(1+ax)}{a\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0195006, size = 39, normalized size = 1.

$$\frac{\sqrt{1-a^2x^2}\log(ax+1)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(a*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.081, size = 40, normalized size = 1.

$$\frac{\ln(ax+1)}{ac}\sqrt{-c\left(a^2x^2-1\right)}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/(-a^2*x^2+1)^(1/2)/c/a*(-c*(a^2*x^2-1))^(1/2)*ln(a*x+1)
```

Maxima [A] time = 0.988283, size = 18, normalized size = 0.46

$$\frac{\log(ax+1)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] log(a*x + 1)/(a*sqrt(c))
```

Fricas [B] time = 2.6056, size = 493, normalized size = 12.64

$$\left[\frac{\log\left(\frac{a^6cx^6+4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2-4\,acx-\left(a^4x^4+4\,a^3x^3+6\,a^2x^2+4\,ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-2\,c}{a^4x^4+2\,a^3x^3-2\,ax-1}\right)}{2\,a\sqrt{c}}, \frac{\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\left(a^2x^2+2\,ax+2\right)\sqrt{-a^2x^2}}{a^4cx^4+2\,a^3cx^3-a^2cx^2-2\,ac}\right)}{ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/2*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 - 4*a*c*x - (
a^4*x^4 + 4*a^3*x^3 + 6*a^2*x^2 + 4*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2
 + 1)*sqrt(c) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1))/(a*sqrt(c)), sqrt(-
c)*arctan(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqr
```

```
t(-c)/(a^4*c*x^4 + 2*a^3*c*x^3 - a^2*c*x^2 - 2*a*c*x))/(a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{\sqrt{-a^2cx^2+c}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/(sqrt(-a^2*c*x^2 + c)*(a*x + 1)), x)
```

3.1212 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2ac\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2ac(ax+1)\sqrt{c-a^2cx^2}}$$

```
[Out] -Sqrt[1 - a^2*x^2]/(2*a*c*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^
2]*ArcTanh[a*x])/(2*a*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.0991248, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6143, 6140, 44, 207}

$$\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2ac\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2ac(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*a*c*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^
2]*ArcTanh[a*x])/(2*a*c*Sqrt[c - a^2*c*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)(1+ax)^2}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{2(1+ax)^2}-\frac{1}{2\left(-1+a^2x^2\right)}\right)dx}{c\sqrt{c-a^2cx^2}}$$
$$= -\frac{\sqrt{1-a^2x^2}}{2ac(1+ax)\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int \frac{1}{-1+a^2x^2}\,dx}{2c\sqrt{c-a^2cx^2}}$$
$$= -\frac{\sqrt{1-a^2x^2}}{2ac(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2ac\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0397485, size = 59, normalized size = 0.66

$$\frac{\sqrt{1-a^2x^2}\left(\frac{\tanh^{-1}(ax)}{2a}-\frac{1}{2a(ax+1)}\right)}{c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-1/(2*a*(1 + a*x)) + ArcTanh[a*x]/(2*a)))/(c*Sqrt[c - a
^2*c*x^2])
```

Maple [A] time = 0.093, size = 88, normalized size = 1.

$$-\frac{ax\ln(ax+1)-\ln(ax-1)xa+\ln(ax+1)-\ln(ax-1)-2}{\left(4a^2x^2-4\right)c^2a(ax+1)}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*x*ln(a*x+1)-ln(a*x-1)*x*a
+ln(a*x+1)-ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a/(a*x+1)
```

Maxima [A] time = 1.01435, size = 68, normalized size = 0.76

$$-\frac{\sqrt{c}}{2\left(a^2c^2x+ac^2\right)}+\frac{\log(ax+1)}{4ac^{\frac{3}{2}}}-\frac{\log(ax-1)}{4ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] -1/2*sqrt(c)/(a^2*c^2*x + a*c^2) + 1/4*log(a*x + 1)/(a*c^(3/2)) - 1/4*log(a
*x - 1)/(a*c^(3/2))
```

Fricas [A] time = 2.7514, size = 707, normalized size = 7.86

$$\left[-\frac{4\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}ax-\left(a^3x^3+a^2x^2-ax-1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5\,a^4cx^4-5\,a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-c}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}\right)}{8\left(a^4c^2x^3+a^3c^2x^2-a^2c^2x-ac^2\right)}, -2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] [-1/8*(4*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x - (a^3*x^3 + a^2*x^2 -
 a*x - 1)*sqrt(c)*log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*x^3
+ a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^
4*x^4 + 3*a^2*x^2 - 1)))/(a^4*c^2*x^3 + a^3*c^2*x^2 - a^2*c^2*x - a*c^2), -
1/4*(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x - (a^3*x^3 + a^2*x^2 - a
*x - 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqrt(-c
)*x/(a^4*c*x^4 - c)))/(a^4*c^2*x^3 + a^3*c^2*x^2 - a^2*c^2*x - a*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{(-c(ax-1)(ax+1))^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x +
 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)), x)
```

3.1213 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=183

$$\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{4ac^2(ax+1)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(ax+1)^2\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8ac^2\sqrt{c-a^2cx^2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(8*a*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^
2]/(8*a*c^2*(1 + a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(4*a*c^2*(
1 + a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*a*c^2
*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.115071, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6143, 6140, 44, 207}

$$\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{4ac^2(ax+1)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(ax+1)^2\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8ac^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(8*a*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^
2]/(8*a*c^2*(1 + a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(4*a*c^2*(
1 + a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*a*c^2
*Sqrt[c - a^2*c*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{-\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^2(1+ax)^3}\,dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{8(-1+ax)^2}+\frac{1}{4(1+ax)^3}+\frac{1}{4(1+ax)^2}-\frac{3}{8\left(-1+a^2x^2\right)}\right)dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(1+ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{4ac^2(1+ax)\sqrt{c-a^2cx^2}}-\frac{\left(3\sqrt{1-a^2x^2}\right)\int \frac{}{}}{8c^2\sqrt{c-a^2}}\\
&= \frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(1+ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{4ac^2(1+ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\tanh}{8ac^2\sqrt{c-a^2c}}
\end{aligned}
$$

Mathematica [A] time = 0.0648622, size = 83, normalized size = 0.45

$$
\frac{\sqrt{1-a^2x^2}\left(-3a^2x^2-3ax+3(ax-1)(ax+1)^2\tanh^{-1}(ax)+2\right)}{8a(ax-1)(acx+c)^2\sqrt{c-a^2cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2 - 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)*(1 + a*x)^2*ArcTan
h[a*x]))/(8*a*(-1 + a*x)*(c + a*c*x)^2*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.094, size = 166, normalized size = 0.9

$$
-\frac{3a^3x^3\ln(ax+1)-3\ln(ax-1)x^3a^3+3\ln(ax+1)a^2x^2-3\ln(ax-1)a^2x^2-6a^2x^2-3ax\ln(ax+1)+3\ln(ax-1)}{\left(16a^2x^2-16\right)c^3a\left(ax+1\right)^2\left(ax-1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*x^3*ln(a*x+1)-3*ln(a
*x-1)*x^3*a^3+3*ln(a*x+1)*a^2*x^2-3*ln(a*x-1)*a^2*x^2-6*a^2*x^2-3*a*x*ln(a*
x+1)+3*ln(a*x-1)*x*a-6*a*x-3*ln(a*x+1)+3*ln(a*x-1)+4)/(a^2*x^2-1)/c^3/a/(a*
x+1)^2/(a*x-1)
```

Maxima [A] time = 1.0127, size = 112, normalized size = 0.61

$$
-\frac{3a^2x^2+3ax-2}{8\left(a^4c^{\frac{5}{2}}x^3+a^3c^{\frac{5}{2}}x^2-a^2c^{\frac{5}{2}}x-ac^{\frac{5}{2}}\right)}+\frac{3\log(ax+1)}{16ac^{\frac{5}{2}}}-\frac{3\log(ax-1)}{16ac^{\frac{5}{2}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="m
axima")
```

```
[Out] -1/8*(3*a^2*x^2 + 3*a*x - 2)/(a^4*c^(5/2)*x^3 + a^3*c^(5/2)*x^2 - a^2*c^(5/
2)*x - a*c^(5/2)) + 3/16*log(a*x + 1)/(a*c^(5/2)) - 3/16*log(a*x - 1)/(a*c^
(5/2))
```

Fricas [A] time = 2.6323, size = 934, normalized size = 5.1

$$\left[\frac{3\left(a^5x^5+a^4x^4-2\,a^3x^3-2\,a^2x^2+ax+1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5\,a^4cx^4-5\,a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-c}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}\right)-4\left(2\,a^3x^3-a\right.}{32\left(a^6c^3x^5+a^5c^3x^4-2\,a^4c^3x^3-2\,a^3c^3x^2+a^2c^3x+ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="f
ricas")
```

```
[Out] [1/32*(3*(a^5*x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*sqrt(c)*log(
-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*x^3 + a*x)*sqrt(-a^2*c*x^2
 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)
) - 4*(2*a^3*x^3 - a^2*x^2 - 5*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)
)/(a^6*c^3*x^5 + a^5*c^3*x^4 - 2*a^4*c^3*x^3 - 2*a^3*c^3*x^2 + a^2*c^3*x +
a*c^3), 1/16*(3*(a^5*x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*sqrt(
-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*c*x^
4 - c)) - 2*(2*a^3*x^3 - a^2*x^2 - 5*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^
2 + 1))/(a^6*c^3*x^5 + a^5*c^3*x^4 - 2*a^4*c^3*x^3 - 2*a^3*c^3*x^2 + a^2*c^
3*x + a*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{(-c(ax-1)(ax+1))^{\frac{5}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x +
 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)), x)
```

3.1214 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=276

$$\frac{\sqrt{1-a^2x^2}}{8ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}}{16ac^3(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}}{32ac^3(ax+1)^2\sqrt{c-a^2cx^2}}-\frac{\quad}{24ac^3}$$

```
[Out] Sqrt[1 - a^2*x^2]/(32*a*c^3*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2
*x^2]/(8*a*c^3*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(24*a*c^3
*(1 + a*x)^3*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2])/(32*a*c^3*(1 + a*
x)^2*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2])/(16*a*c^3*(1 + a*x)*Sqrt[
c - a^2*c*x^2]) + (5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(16*a*c^3*Sqrt[c - a^2
*c*x^2])
```

Rubi [A] time = 0.138208, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6143, 6140, 44, 207}

$$\frac{\sqrt{1-a^2x^2}}{8ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}}{16ac^3(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}}{32ac^3(ax+1)^2\sqrt{c-a^2cx^2}}-\frac{\quad}{24ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^(7/2)),x]
```


```
[Out] Sqrt[1 - a^2*x^2]/(32*a*c^3*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2
*x^2]/(8*a*c^3*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(24*a*c^3
*(1 + a*x)^3*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2])/(32*a*c^3*(1 + a*
x)^2*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2])/(16*a*c^3*(1 + a*x)*Sqrt[
c - a^2*c*x^2]) + (5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(16*a*c^3*Sqrt[c - a^2
*c*x^2])
```


Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^3(1+ax)^4}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{16(-1+ax)^3}+\frac{1}{8(-1+ax)^2}+\frac{1}{8(1+ax)^4}+\frac{3}{16(1+ax)^3}+\frac{3}{16(1+ax)^2}-\frac{5}{16\left(-1+a^2x^2\right)}\right)dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{24ac^3(1+ax)^3\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-}}{32ac^3(1+ax)}$$

$$= \frac{\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{24ac^3(1+ax)^3\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-}}{32ac^3(1+ax)}$$

Mathematica [A] time = 0.0868012, size = 101, normalized size = 0.37

$$\frac{\sqrt{1-a^2x^2}\left(-15a^4x^4-15a^3x^3+25a^2x^2+25ax+15(ax-1)^2(ax+1)^3\tanh^{-1}(ax)-8\right)}{48a(ax-1)^2(acx+c)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-8 + 25*a*x + 25*a^2*x^2 - 15*a^3*x^3 - 15*a^4*x^4 + 15
*(-1 + a*x)^2*(1 + a*x)^3*ArcTanh[a*x]))/(48*a*(-1 + a*x)^2*(c + a*c*x)^3*S
qrt[c - a^2*c*x^2])
```

Maple [A] time = 0.095, size = 238, normalized size = 0.9

$$-\frac{15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5+15\ln(ax+1)a^4x^4-15\ln(ax-1)a^4x^4-30x^4a^4-30a^3x^3\ln(ax+1)+30}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/96*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x+1)*x^5*a^5-15*ln
(a*x-1)*x^5*a^5+15*ln(a*x+1)*a^4*x^4-15*ln(a*x-1)*a^4*x^4-30*x^4*a^4-30*a^3
*x^3*ln(a*x+1)+30*ln(a*x-1)*x^3*a^3-30*x^3*a^3-30*ln(a*x+1)*a^2*x^2+30*ln(a
*x-1)*a^2*x^2+50*a^2*x^2+15*a*x*ln(a*x+1)-15*ln(a*x-1)*x*a+50*a*x+15*ln(a*x
+1)-15*ln(a*x-1)-16)/(a^2*x^2-1)/c^4/a/(a*x+1)^3/(a*x-1)^2
```

Maxima [A] time = 1.0279, size = 182, normalized size = 0.66

$$-\frac{15\,a^4\sqrt{c}x^4+15\,a^3\sqrt{c}x^3-25\,a^2\sqrt{c}x^2-25\,a\sqrt{c}x+8\,\sqrt{c}}{48\,\left(a^6c^4x^5+a^5c^4x^4-2\,a^4c^4x^3-2\,a^3c^4x^2+a^2c^4x+ac^4\right)}+\frac{5\,\log\left(ax+1\right)}{32\,ac^{\frac{7}{2}}}-\frac{5\,\log\left(ax-1\right)}{32\,ac^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="m
axima")
```

```
[Out] -1/48*(15*a^4*sqrt(c)*x^4 + 15*a^3*sqrt(c)*x^3 - 25*a^2*sqrt(c)*x^2 - 25*a*
sqrt(c)*x + 8*sqrt(c))/(a^6*c^4*x^5 + a^5*c^4*x^4 - 2*a^4*c^4*x^3 - 2*a^3*c
^4*x^2 + a^2*c^4*x + a*c^4) + 5/32*log(a*x + 1)/(a*c^(7/2)) - 5/32*log(a*x
- 1)/(a*c^(7/2))
```

Fricas [A] time = 2.64875, size = 1165, normalized size = 4.22

$$\left[\frac{15\,\left(a^7x^7+a^6x^6-3\,a^5x^5-3\,a^4x^4+3\,a^3x^3+3\,a^2x^2-ax-1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5\,a^4cx^4-5\,a^2cx^2-4\,\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}\right.}{192\,\left(a^8c^4x^7+a^7c^4x^6-3\,a^6c^4x^5-3\,a^5c^4x^4+3\,a^4c^4x^3+3\,a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="f
ricas")
```

```
[Out] [1/192*(15*(a^7*x^7 + a^6*x^6 - 3*a^5*x^5 - 3*a^4*x^4 + 3*a^3*x^3 + 3*a^2*x
^2 - a*x - 1)*sqrt(c)*log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*
x^3 + a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 -
3*a^4*x^4 + 3*a^2*x^2 - 1)) - 4*(8*a^5*x^5 - 7*a^4*x^4 - 31*a^3*x^3 + 9*a^2
*x^2 + 33*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^8*c^4*x^7 + a^7*
c^4*x^6 - 3*a^6*c^4*x^5 - 3*a^5*c^4*x^4 + 3*a^4*c^4*x^3 + 3*a^3*c^4*x^2 - a
^2*c^4*x - a*c^4), 1/96*(15*(a^7*x^7 + a^6*x^6 - 3*a^5*x^5 - 3*a^4*x^4 + 3*
a^3*x^3 + 3*a^2*x^2 - a*x - 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(
-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*c*x^4 - c)) - 2*(8*a^5*x^5 - 7*a^4*x^4 - 31
*a^3*x^3 + 9*a^2*x^2 + 33*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^
8*c^4*x^7 + a^7*c^4*x^6 - 3*a^6*c^4*x^5 - 3*a^5*c^4*x^4 + 3*a^4*c^4*x^3 + 3
*a^3*c^4*x^2 - a^2*c^4*x - a*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x +
 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((-a^2*c*x^2 + c)^(7/2)*(a*x + 1)), x)
```

3.1215 $\int e^{-\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=137

$$\frac{x^{m+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1}{2}-p,\frac{m+3}{2},a^2x^2\right)}{m+1}-\frac{ax^{m+2}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hype}}{m}$$

```
[Out] (x^(1 + m)*(c - a^2*c*x^2)^p*Hypergeometric2F1[(1 + m)/2, 1/2 - p, (3 + m)/
2, a^2*x^2])/((1 + m)*(1 - a^2*x^2)^p) - (a*x^(2 + m)*(c - a^2*c*x^2)^p*Hyp
ergeometric2F1[(2 + m)/2, 1/2 - p, (4 + m)/2, a^2*x^2])/((2 + m)*(1 - a^2*x
^2)^p)
```

Rubi [A] time = 0.163696, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6153, 6149, 808, 364}

$$\frac{x^{m+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p {}_2F_1\left(\frac{m+1}{2},\frac{1}{2}-p;\frac{m+3}{2};a^2x^2\right)}{m+1}-\frac{ax^{m+2}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p {}_2F_1\left(\frac{m+2}{2},\frac{1}{2}-p;\frac{m+4}{2};a^2\right.}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(c - a^2*c*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] (x^(1 + m)*(c - a^2*c*x^2)^p*Hypergeometric2F1[(1 + m)/2, 1/2 - p, (3 + m)/
2, a^2*x^2])/((1 + m)*(1 - a^2*x^2)^p) - (a*x^(2 + m)*(c - a^2*c*x^2)^p*Hyp
ergeometric2F1[(2 + m)/2, 1/2 - p, (4 + m)/2, a^2*x^2])/((2 + m)*(1 - a^2*x
^2)^p)
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{-\tanh^{-1}(ax)}x^m\left(1-a^2x^2\right)^p\,dx\\ &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^m(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\ &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^m\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx-\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int\\ &= \frac{x^{1+m}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1+m}{2},\frac{1}{2}-p;\frac{3+m}{2};a^2x^2\right)}{1+m}-\frac{ax^{2+m}\left(1-a^2x^2\right)^{-p}\left(c-\right.}{}\end{aligned}$$

Mathematica [A] time = 0.0474386, size = 115, normalized size = 0.84

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1}{2}-p,\frac{m+1}{2}+1,a^2x^2\right)}{m+1}-\frac{ax^{m+2}\text{Hypergeometric2F1}\left(\frac{m+2}{2},\frac{1}{2}\right.}{m+2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(c - a^2*c*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] ((c - a^2*c*x^2)^p*((x^(1 + m)*Hypergeometric2F1[(1 + m)/2, 1/2 - p, 1 + (1
 + m)/2, a^2*x^2]))/(1 + m) - (a*x^(2 + m)*Hypergeometric2F1[(2 + m)/2, 1/2
- p, 1 + (2 + m)/2, a^2*x^2])/(2 + m)))/(1 - a^2*x^2)^p
```

Maple [F] time = 0.44, size = 0, normalized size = 0.

$$\int\frac{x^m\left(-a^2cx^2+c\right)^p}{ax+1}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^px^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^m/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^px^m}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^m/(a*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a**2*c*x**2+c)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^px^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^m/(a*x + 1), x)
```

3.1216 $\int e^{-\tanh^{-1}(ax)}x^3\left(1-a^2x^2\right)^p\,dx$

Optimal. Leaf size=85

$$-\frac{1}{5}ax^5\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-p,\frac{7}{2},a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^4(2p+1)}+\frac{\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^4(2p+3)}$$

```
[Out] -((1 - a^2*x^2)^(1/2 + p)/(a^4*(1 + 2*p))) + (1 - a^2*x^2)^(3/2 + p)/(a^4*(
3 + 2*p)) - (a*x^5*Hypergeometric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/5
```

Rubi [A] time = 0.11718, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6149, 764, 266, 43, 364}

$$-\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^4(2p+1)}+\frac{\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^4(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 - a^2*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] -((1 - a^2*x^2)^(1/2 + p)/(a^4*(1 + 2*p))) + (1 - a^2*x^2)^(3/2 + p)/(a^4*(
3 + 2*p)) - (a*x^5*Hypergeometric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/5
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}x^3\left(1-a^2x^2\right)^p\,dx &= \int x^3(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\ &= -\left(a\int x^4\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\right)+\int x^3\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\ &= -\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{2}\text{Subst}\left(\int x\left(1-a^2x\right)^{-\frac{1}{2}+p}\,dx,x,x^2\right)\\ &= -\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{2}\text{Subst}\left(\int\left(\frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{a^2}-\frac{\left(1-a^2x\right)^{\frac{1}{2}+p}}{a^2}\right)dx,x\right.\\ &= -\frac{\left(1-a^2x^2\right)^{\frac{1}{2}+p}}{a^4(1+2p)}+\frac{\left(1-a^2x^2\right)^{\frac{3}{2}+p}}{a^4(3+2p)}-\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0723435, size = 77, normalized size = 0.91

$$-\frac{1}{5}ax^5\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-p,\frac{7}{2},a^2x^2\right)-\frac{\left(a^2(2p+1)x^2+2\right)\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^4\left(4p^2+8p+3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 - a^2*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] -(((1 - a^2*x^2)^(1/2 + p)*(2 + a^2*(1 + 2*p)*x^2))/(a^4*(3 + 8*p + 4*p^2))
) - (a*x^5*Hypergeometric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/5
```

Maple [F] time = 0.431, size = 0, normalized size = 0.

$$\int \frac{x^3\left(-a^2x^2+1\right)^p}{ax+1}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^3*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{p+\frac{1}{2}}x^3}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-a^2*x^2 + 1)^(p + 1/2)*x^3/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x^3}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x^3/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-(ax-1)(ax+1)}\,(-(ax-1)(ax+1))^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*x**2+1)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-(a*x - 1)*(a*x + 1))*(-(a*x - 1)*(a*x + 1))**p/(a*x + 1
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x^3}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x^3/(a*x + 1), x)
```

3.1217 $\int e^{-\tanh^{-1}(ax)} x^2 \left(1-a^2x^2\right)^p \, dx$

Optimal. Leaf size=84

$$\frac{1}{3}x^3\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-p,\frac{5}{2},a^2x^2\right)+\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^3(2p+1)}-\frac{\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^3(2p+3)}$$

```
[Out] (1 - a^2*x^2)^(1/2 + p)/(a^3*(1 + 2*p)) - (1 - a^2*x^2)^(3/2 + p)/(a^3*(3 +
 2*p)) + (x^3*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/3
```

Rubi [A] time = 0.120778, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6149, 764, 364, 266, 43}

$$\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^3(2p+1)}-\frac{\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^3(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - a^2*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] (1 - a^2*x^2)^(1/2 + p)/(a^3*(1 + 2*p)) - (1 - a^2*x^2)^(3/2 + p)/(a^3*(3 +
 2*p)) + (x^3*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/3
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}x^2\left(1-a^2x^2\right)^p dx &= \int x^2(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p} dx\\ &= -\left(a\int x^3\left(1-a^2x^2\right)^{-\frac{1}{2}+p} dx\right)+\int x^2\left(1-a^2x^2\right)^{-\frac{1}{2}+p} dx\\ &= \frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{1}{2}a\,\text{Subst}\left(\int x\left(1-a^2x\right)^{-\frac{1}{2}+p} dx,x,x^2\right)\\ &= \frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{1}{2}a\,\text{Subst}\left(\int\left(\frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{a^2}-\frac{\left(1-a^2x\right)^{\frac{1}{2}+p}}{a^2}\right)dx,x,x^2\right)\\ &= \frac{\left(1-a^2x^2\right)^{\frac{1}{2}+p}}{a^3(1+2p)}-\frac{\left(1-a^2x^2\right)^{\frac{3}{2}+p}}{a^3(3+2p)}+\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0555171, size = 75, normalized size = 0.89

$$\frac{1}{3}x^3\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-p,\frac{5}{2},a^2x^2\right)+\frac{\left(a^2(2p+1)x^2+2\right)\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^3\left(4p^2+8p+3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - a^2*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] ((1 - a^2*x^2)^(1/2 + p)*(2 + a^2*(1 + 2*p)*x^2))/(a^3*(3 + 8*p + 4*p^2)) +
 (x^3*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/3
```

Maple [F] time = 0.424, size = 0, normalized size = 0.

$$\int \frac{x^2\left(-a^2x^2+1\right)^p}{ax+1}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^2*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{p+\frac{1}{2}}x^2}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-a^2*x^2 + 1)^(p + 1/2)*x^2/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x^2}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x^2/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-(ax-1)(ax+1)}\,(-(ax-1)(ax+1))^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))*(-(a*x - 1)*(a*x + 1))**p/(a*x + 1
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x^2}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x^2/(a*x + 1), x)
```

3.1218 $\int e^{-\tanh^{-1}(ax)}x\left(1-a^2x^2\right)^p\,dx$

Optimal. Leaf size=58

$$-\frac{1}{3}ax^3\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-p,\frac{5}{2},a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^2(2p+1)}$$

```
[Out] -((1 - a^2*x^2)^(1/2 + p)/(a^2*(1 + 2*p))) - (a*x^3*Hypergeometric2F1[3/2,
1/2 - p, 5/2, a^2*x^2])/3
```

Rubi [A] time = 0.0648003, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6149, 764, 261, 364}

$$-\frac{1}{3}ax^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - a^2*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] -((1 - a^2*x^2)^(1/2 + p)/(a^2*(1 + 2*p))) - (a*x^3*Hypergeometric2F1[3/2,
1/2 - p, 5/2, a^2*x^2])/3
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}x\left(1-a^2x^2\right)^p\,dx=\int x(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx$$
$$=-\left(a\int x^2\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\right)+\int x\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx$$
$$=-\frac{\left(1-a^2x^2\right)^{\frac{1}{2}+p}}{a^2(1+2p)}-\frac{1}{3}ax^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)$$

Mathematica [A] time = 0.0247705, size = 60, normalized size = 1.03

$$-\frac{1}{3}ax^3\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-p,\frac{5}{2},a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{2a^2\left(p+\frac{1}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 - a^2*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] -(1 - a^2*x^2)^(1/2 + p)/(2*a^2*(1/2 + p)) - (a*x^3*Hypergeometric2F1[3/2,
1/2 - p, 5/2, a^2*x^2])/3
```

Maple [F] time = 0.429, size = 0, normalized size = 0.

$$\int\frac{x\left(-a^2x^2+1\right)^p}{ax+1}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2x^2+1\right)^{p+\frac{1}{2}}x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((-a^2*x^2 + 1)^(p + 1/2)*x/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^px}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-(ax-1)(ax+1)}\,(-(ax-1)(ax+1))^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a*x - 1)*(a*x + 1))*(-(a*x - 1)*(a*x + 1))**p/(a*x + 1),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x/(a*x + 1), x)
```

3.1219 $\int e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx$

Optimal. Leaf size=59

$$-\frac{2^{p+\frac{1}{2}}(1-ax)^{p+\frac{3}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2}-p,p+\frac{3}{2},p+\frac{5}{2},\frac{1}{2}(1-ax)\right)}{a(2p+3)}$$

```
[Out] -((2^(1/2 + p)*(1 - a*x)^(3/2 + p)*Hypergeometric2F1[1/2 - p, 3/2 + p, 5/2
+ p, (1 - a*x)/2])/(a*(3 + 2*p)))
```

Rubi [A] time = 0.0412697, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6140, 69}

$$-\frac{2^{p+\frac{1}{2}}(1-ax)^{p+\frac{3}{2}}\,{}_2F_1\left(\frac{1}{2}-p,p+\frac{3}{2};p+\frac{5}{2};\frac{1}{2}(1-ax)\right)}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^p/E^ArcTanh[a*x],x]
```

```
[Out] -((2^(1/2 + p)*(1 - a*x)^(3/2 + p)*Hypergeometric2F1[1/2 - p, 3/2 + p, 5/2
+ p, (1 - a*x)/2])/(a*(3 + 2*p)))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx &= \int (1-ax)^{\frac{1}{2}+p}(1+ax)^{-\frac{1}{2}+p}\,dx\\ &= -\frac{2^{\frac{1}{2}+p}(1-ax)^{\frac{3}{2}+p}\,{}_2F_1\left(\frac{1}{2}-p,\frac{3}{2}+p;\frac{5}{2}+p;\frac{1}{2}(1-ax)\right)}{a(3+2p)}\end{aligned}$$

Mathematica [A] time = 0.0230162, size = 56, normalized size = 0.95

$$\frac{(ax-1)(2-2ax)^{p+\frac{1}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2}-p,p+\frac{3}{2},p+\frac{5}{2},\frac{1}{2}(1-ax)\right)}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^p/E^ArcTanh[a*x],x]
```

```
[Out] ((2 - 2*a*x)^(1/2 + p)*(-1 + a*x)*Hypergeometric2F1[1/2 - p, 3/2 + p, 5/2 +
 p, (1 - a*x)/2])/(a*(3 + 2*p))
```

Maple [F] time = 0.422, size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^p}{ax+1}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{p+\frac{1}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(p + 1/2)/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,(-(ax-1)(ax+1))^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-(a*x - 1)*(a*x + 1))**p/(a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/(a*x + 1), x)
```

3.1220 $\int \frac{e^{-\tanh^{-1}(ax)}(1-a^2x^2)^p}{x}\,dx$

Optimal. Leaf size=73

$$-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}\text{Hypergeometric2F1}\left(1,p+\frac{1}{2},p+\frac{3}{2},1-a^2x^2\right)}{2p+1}-ax\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-p,\frac{3}{2},a^2x^2\right)$$

[Out] -(a*x*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2]) - ((1 - a^2*x^2)^(1/2 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)

Rubi [A] time = 0.0949322, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6149, 764, 266, 65, 245}

$$-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^p/(E^ArcTanh[a*x]*x),x]
```

```
[Out] -(a*x*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2]) - ((1 - a^2*x^2)^(1/2
+ p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x}\,dx = \int \frac{(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx$$
$$= -\left(a\int \left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\right) + \int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx$$
$$= -ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{x}\,dx, x, x^2\right)$$
$$= -ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right) - \frac{\left(1-a^2x^2\right)^{\frac{1}{2}+p}\,{}_2F_1\left(1,\frac{1}{2}+p;\frac{3}{2}+p;1-a^2x^2\right)}{1+2p}$$

Mathematica [A] time = 0.0273997, size = 75, normalized size = 1.03

$$-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}\,\text{Hypergeometric2F1}\left(1,p+\frac{1}{2},p+\frac{3}{2},1-a^2x^2\right)}{2\left(p+\frac{1}{2}\right)} - ax\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-p,\frac{3}{2},a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^p/(E^ArcTanh[a*x]*x),x]
```

```
[Out] -(a*x*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2]) - ((1 - a^2*x^2)^(1/2
+ p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(2*(1/2 + p))
```

Maple [F] time = 0.431, size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^p}{(ax+1)\,x}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] int((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{p+\frac{1}{2}}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate((-a^2*x^2 + 1)^(p + 1/2)/((a*x + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{ax^2+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/(a*x^2 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,(-(ax-1)(ax+1))^p}{x(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**p/(a*x+1)*(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-(a*x - 1)*(a*x + 1))**p/(x*(a*x + 1))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/((a*x + 1)*x), x)
```

3.1221 $\int \frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^2}\,dx$

Optimal. Leaf size=74

$$\frac{a\left(1-a^2x^2\right)^{p+\frac{1}{2}}\text{Hypergeometric2F1}\left(1,p+\frac{1}{2},p+\frac{3}{2},1-a^2x^2\right)}{2p+1}-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-p,\frac{1}{2},a^2x^2\right)}{x}$$

```
[Out] -(Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2]/x) + (a*(1 - a^2*x^2)^(1/2
 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rubi [A] time = 0.105648, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6149, 764, 364, 266, 65}

$$\frac{a\left(1-a^2x^2\right)^{p+\frac{1}{2}}{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^p/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2]/x) + (a*(1 - a^2*x^2)^(1/2
 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
```

```
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^2}\,dx &= \int \frac{(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx\\
&= -\left(a\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx\right)+\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx\\
&= -\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}-\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{x}\,dx,x,x^2\right)\\
&= -\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}+\frac{a\left(1-a^2x^2\right)^{\frac{1}{2}+p}{}_2F_1\left(1,\frac{1}{2}+p;\frac{3}{2}+p;1-a^2x^2\right)}{1+2p}
\end{aligned}$$

Mathematica [A] time = 0.0264227, size = 74, normalized size = 1.

$$\frac{a\left(1-a^2x^2\right)^{p+\frac{1}{2}}\text{Hypergeometric2F1}\left(1,p+\frac{1}{2},p+\frac{3}{2},1-a^2x^2\right)}{2p+1}-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-p,\frac{1}{2},a^2x^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^p/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2]/x) + (a*(1 - a^2*x^2)^(1/2
 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Maple [F] time = 0.424, size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^p}{(ax+1)\,x^2}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] int((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{p+\frac{1}{2}}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate((-a^2*x^2 + 1)^(p + 1/2)/((a*x + 1)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{ax^3+x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/(a*x^3 + x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,(-(ax-1)(ax+1))^p}{x^2(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**p/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-(a*x - 1)*(a*x + 1))**p/(x**2*(a*x +
1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/((a*x + 1)*x^2), x)
```

3.1222 $\int e^{-\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=134

$$-\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-p,\frac{7}{2},a^2x^2\right)+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^4(2p+3)}-\frac{\sqrt{1-a^2x^2}\left(c-\right.}{a^4(2p+1}$$

```
[Out] -((Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^4*(1 + 2*p))) + ((1 - a^2*x^2)^(
3/2)*(c - a^2*c*x^2)^p)/(a^4*(3 + 2*p)) - (a*x^5*(c - a^2*c*x^2)^p*Hypergeo
metric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/(5*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.188389, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6153, 6149, 764, 266, 43, 364}

$$-\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^4(2p+3)}-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^4(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c - a^2*c*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^4*(1 + 2*p))) + ((1 - a^2*x^2)^(
3/2)*(c - a^2*c*x^2)^p)/(a^4*(3 + 2*p)) - (a*x^5*(c - a^2*c*x^2)^p*Hypergeo
metric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/(5*(1 - a^2*x^2)^p)
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^p\,dx = \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{-\tanh^{-1}(ax)}x^3\left(1-a^2x^2\right)^p\,dx$$
$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^3(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx$$
$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^3\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx - \left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)$$
$$= -\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2c\right.\right.$$
$$= -\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2c\right.\right.$$
$$= -\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^4(1+2p)}+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^4(3+2p)}-\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2c\right.$$

Mathematica [A] time = 0.0601477, size = 119, normalized size = 0.89

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{1}{2}\left(\frac{2\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^4(2p+3)}-\frac{2\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^4(2p+1)}\right)-\frac{1}{5}ax^5\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-p,\frac{7}{2},a^2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c - a^2*c*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] ((c - a^2*c*x^2)^p*(((-2*(1 - a^2*x^2)^(1/2 + p))/(a^4*(1 + 2*p)) + (2*(1 -
 a^2*x^2)^(3/2 + p))/(a^4*(3 + 2*p)))/2 - (a*x^5*Hypergeometric2F1[5/2, 1/2
 - p, 7/2, a^2*x^2])/5))/(1 - a^2*x^2)^p
```

Maple [F] time = 0.421, size = 0, normalized size = 0.

$$\int \frac{x^3\left(-a^2cx^2+c\right)^p}{ax+1}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^3*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x^3}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^3/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x^3}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^3/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*c*x**2+c)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p/(a*x +
 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x^3}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^3/(a*x + 1), x)
```

3.1223 $\int e^{-\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=133

$$\frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-p,\frac{5}{2},a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^3(2p+3)}+\frac{\sqrt{1-a^2x^2}\left(c-\right.}{a^3(2p+}$$

```
[Out] (Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^3*(1 + 2*p)) - ((1 - a^2*x^2)^(3/2
)*(c - a^2*c*x^2)^p)/(a^3*(3 + 2*p)) + (x^3*(c - a^2*c*x^2)^p*Hypergeometri
c2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/(3*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.186643, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6153, 6149, 764, 364, 266, 43}

$$\frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^3(2p+3)}+\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c - a^2*c*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^3*(1 + 2*p)) - ((1 - a^2*x^2)^(3/2
)*(c - a^2*c*x^2)^p)/(a^3*(3 + 2*p)) + (x^3*(c - a^2*c*x^2)^p*Hypergeometri
c2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/(3*(1 - a^2*x^2)^p)
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{-\tanh^{-1}(ax)}x^2\left(1-a^2x^2\right)^p\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^2(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^2\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx-\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int\\
&= \frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{1}{2}\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right.\\
&= \frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{1}{2}\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right.\\
&= \frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^3(1+2p)}-\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^3(3+2p)}+\frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2
\end{aligned}
$$

Mathematica [A] time = 0.0802161, size = 102, normalized size = 0.77

$$
\frac{1}{3}\left(c-a^2cx^2\right)^p\left(x^3\left(1-a^2x^2\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-p,\frac{5}{2},a^2x^2\right)+\frac{3\sqrt{1-a^2x^2}\left(a^2(2p+1)x^2+2\right)}{a^3\left(4p^2+8p+3\right)}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c - a^2*c*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] ((c - a^2*c*x^2)^p*((3*Sqrt[1 - a^2*x^2]*(2 + a^2*(1 + 2*p)*x^2))/(a^3*(3 +
 8*p + 4*p^2)) + (x^3*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/(1 - a
^2*x^2)^p))/3
```

Maple [F] time = 0.418, size = 0, normalized size = 0.

$$
\int \frac{x^2\left(-a^2cx^2+c\right)^p}{ax+1}\sqrt{-a^2x^2+1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^2*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x^2}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^2/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x^2}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^2/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-(ax-1)(ax+1)}\,(-c\,(ax-1)(ax+1))^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p/(a*x +
 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x^2}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^2/(a*x + 1), x)
```

3.1224 $\int e^{-\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=96

$$-\frac{1}{3}ax^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-p,\frac{5}{2},a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^2(2p+1)}$$

```
[Out] -((Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^2*(1 + 2*p))) - (a*x^3*(c - a^2*
c*x^2)^p*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/(3*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.112623, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6153, 6149, 764, 261, 364}

$$-\frac{1}{3}ax^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c - a^2*c*x^2)^p)/E^ArcTanh[a*x],x]
```


```
[Out] -((Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^2*(1 + 2*p))) - (a*x^3*(c - a^2*
c*x^2)^p*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/(3*(1 - a^2*x^2)^p)
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{-\tanh^{-1}(ax)}x\left(1-a^2x^2\right)^p\,dx\\ &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\ &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx-\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int\\ &= -\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^2(1+2p)}-\frac{1}{3}ax^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0295533, size = 88, normalized size = 0.92

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(-\frac{1}{3}ax^3\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-p,\frac{5}{2},a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{2a^2\left(p+\frac{1}{2}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c - a^2*c*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] ((c - a^2*c*x^2)^p*(-(1 - a^2*x^2)^(1/2 + p)/(2*a^2*(1/2 + p)) - (a*x^3*Hyp
ergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/3))/(1 - a^2*x^2)^p
```

Maple [F] time = 0.43, size = 0, normalized size = 0.

$$\int\frac{x\left(-a^2cx^2+c\right)^p}{ax+1}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^px}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p/(a*x + 1)
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x/(a*x + 1), x)
```

3.1225 $\int e^{-\tanh^{-1}(ax)} \left(c-a^2cx^2\right)^p \, dx$

Optimal. Leaf size=86

$$-\frac{2^{p+\frac{1}{2}}(1-ax)^{p+\frac{3}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2}-p,p+\frac{3}{2},p+\frac{5}{2},\frac{1}{2}(1-ax)\right)}{a(2p+3)}$$

```
[Out] -((2^(1/2 + p)*(1 - a*x)^(3/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2
- p, 3/2 + p, 5/2 + p, (1 - a*x)/2])/(a*(3 + 2*p)*(1 - a^2*x^2)^p))
```

Rubi [A] time = 0.0664606, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6143, 6140, 69}

$$-\frac{2^{p+\frac{1}{2}}(1-ax)^{p+\frac{3}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p {}_2F_1\left(\frac{1}{2}-p,p+\frac{3}{2};p+\frac{5}{2};\frac{1}{2}(1-ax)\right)}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^ArcTanh[a*x],x]
```


```
[Out] -((2^(1/2 + p)*(1 - a*x)^(3/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2
- p, 3/2 + p, 5/2 + p, (1 - a*x)/2])/(a*(3 + 2*p)*(1 - a^2*x^2)^p))
```


Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p \, dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p \, dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int (1-ax)^{\frac{1}{2}+p}(1+ax)^{-\frac{1}{2}+p} \, dx\\
&= -\frac{2^{\frac{1}{2}+p}(1-ax)^{\frac{3}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p {}_2F_1\left(\frac{1}{2}-p,\frac{3}{2}+p;\frac{5}{2}+p;\frac{1}{2}(1-ax)\right)}{a(3+2p)}
\end{aligned}$$

Mathematica [A] time = 0.0307036, size = 83, normalized size = 0.97

$$\frac{(ax-1)(2-2ax)^{p+\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2}-p,p+\frac{3}{2},p+\frac{5}{2},\frac{1}{2}(1-ax)\right)}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^ArcTanh[a*x],x]
```

```
[Out] ((2 - 2*a*x)^(1/2 + p)*(-1 + a*x)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2 -
 p, 3/2 + p, 5/2 + p, (1 - a*x)/2])/(a*(3 + 2*p)*(1 - a^2*x^2)^p)
```

Maple [F] time = 0.422, size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^p}{ax+1}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p/(a*x + 1),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

3.1226 $\int \frac{e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x}\,dx$

Optimal. Leaf size=111

$$-ax\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-p,\frac{3}{2},a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(\right.}{2p+1}$$

```
[Out] -((a*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2])/(1
- a^2*x^2)^p) - (Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p*Hypergeometric2F1[1, 1
/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rubi [A] time = 0.15838, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6153, 6149, 764, 266, 65, 245}

$$-ax\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \,_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p \,_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/(E^ArcTanh[a*x]*x),x]
```

```
[Out] -((a*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2])/(1
- a^2*x^2)^p) - (Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p*Hypergeometric2F1[1, 1
/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x}\,dx = \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x}\,dx$$

$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx$$

$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx - \left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int (1$$

$$= -ax\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)$$

$$= -ax\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1\right.}{1+2p}$$

Mathematica [A] time = 0.0318164, size = 103, normalized size = 0.93

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}\text{Hypergeometric2F1}\left(1,p+\frac{1}{2},p+\frac{3}{2},1-a^2x^2\right)}{2\left(p+\frac{1}{2}\right)}-ax\text{Hypergeometric2F}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^p/(E^ArcTanh[a*x]*x),x]
```

```
[Out] ((c - a^2*c*x^2)^p*(-(a*x*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2]) -
((1 - a^2*x^2)^(1/2 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2
])/(2*(1/2 + p))))/(1 - a^2*x^2)^p
```

Maple [F] time = 0.42, size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^p}{(ax+1)\,x}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] int((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/((a*x + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{ax^2+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x^2 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^p}{x(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p/(a*x+1)*(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p/(x*(a*x + 1
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/((a*x + 1)*x), x)
```

3.1227 $\int \frac{e^{-\tanh^{-1}(ax)}(c-a^2cx^2)^p}{x^2}\,dx$

Optimal. Leaf size=112

$$\frac{a\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(1,p+\frac{1}{2},p+\frac{3}{2},1-a^2x^2\right)}{2p+1}-\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometr}}{x}$$

```
[Out] -(((c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2])/(x*(1
- a^2*x^2)^p)) + (a*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p*Hypergeometric2F1[1
, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rubi [A] time = 0.174016, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6153, 6149, 764, 364, 266, 65}

$$\frac{a\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p {}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p {}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] -(((c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2])/(x*(1
- a^2*x^2)^p)) + (a*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p*Hypergeometric2F1[1
, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x^2}\,dx = \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^2}\,dx$$
$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx$$
$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx - \left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1-$$
$$= -\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x} - \frac{1}{2}\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\text{Su}$$
$$= -\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x} + \frac{a\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,\frac{1}{2}\right.}{1+2p}$$

Mathematica [A] time = 0.0353348, size = 102, normalized size = 0.91

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{a\left(1-a^2x^2\right)^{p+\frac{1}{2}}\text{Hypergeometric2F1}\left(1,p+\frac{1}{2},p+\frac{3}{2},1-a^2x^2\right)}{2p+1} - \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2}\right.}{x}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^p/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] ((c - a^2*c*x^2)^p*(-(Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2]/x) + (
a*(1 - a^2*x^2)^(1/2 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^
2])/(1 + 2*p)))/(1 - a^2*x^2)^p
```

Maple [F] time = 0.419, size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^p}{(ax+1)\,x^2}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] int((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/((a*x + 1)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{ax^3+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x^3 + x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^p}{x^2(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p/(x**2*(a*x
+ 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/((a*x + 1)*x^2), x)
```

3.1228 $\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=73

$$\frac{c^4(1-ax)^9}{9a}-\frac{3c^4(1-ax)^8}{4a}+\frac{12c^4(1-ax)^7}{7a}-\frac{4c^4(1-ax)^6}{3a}$$

```
[Out] (-4*c^4*(1 - a*x)^6)/(3*a) + (12*c^4*(1 - a*x)^7)/(7*a) - (3*c^4*(1 - a*x)^
8)/(4*a) + (c^4*(1 - a*x)^9)/(9*a)
```

Rubi [A] time = 0.0553365, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$\frac{c^4(1-ax)^9}{9a}-\frac{3c^4(1-ax)^8}{4a}+\frac{12c^4(1-ax)^7}{7a}-\frac{4c^4(1-ax)^6}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^4/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-4*c^4*(1 - a*x)^6)/(3*a) + (12*c^4*(1 - a*x)^7)/(7*a) - (3*c^4*(1 - a*x)^
8)/(4*a) + (c^4*(1 - a*x)^9)/(9*a)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx &= c^4\int(1-ax)^5(1+ax)^3\,dx\\ &= c^4\int\left(8(1-ax)^5-12(1-ax)^6+6(1-ax)^7-(1-ax)^8\right)dx\\ &= -\frac{4c^4(1-ax)^6}{3a}+\frac{12c^4(1-ax)^7}{7a}-\frac{3c^4(1-ax)^8}{4a}+\frac{c^4(1-ax)^9}{9a}\end{aligned}$$

Mathematica [A] time = 0.026989, size = 39, normalized size = 0.53

$$-\frac{c^4(ax-1)^6\left(28a^3x^3+105a^2x^2+138ax+65\right)}{252a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^4/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c^4*(-1 + a*x)^6*(65 + 138*a*x + 105*a^2*x^2 + 28*a^3*x^3))/(252*a)
```

Maple [A] time = 0.024, size = 61, normalized size = 0.8

$$c^{4}\left(-\frac{x^{9}a^{8}}{9}+\frac{a^{7}x^{8}}{4}+\frac{2\,x^{7}a^{6}}{7}-x^{6}a^{5}+\frac{3\,x^{4}a^{3}}{2}-\frac{2\,x^{3}a^{2}}{3}-ax^{2}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^4/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] c^4*(-1/9*x^9*a^8+1/4*a^7*x^8+2/7*x^7*a^6-x^6*a^5+3/2*x^4*a^3-2/3*x^3*a^2-a
*x^2+x)
```

Maxima [A] time = 0.963072, size = 109, normalized size = 1.49

$$-\frac{1}{9}\,a^{8}c^{4}x^{9}+\frac{1}{4}\,a^{7}c^{4}x^{8}+\frac{2}{7}\,a^{6}c^{4}x^{7}-a^{5}c^{4}x^{6}+\frac{3}{2}\,a^{3}c^{4}x^{4}-\frac{2}{3}\,a^{2}c^{4}x^{3}-ac^{4}x^{2}+c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 + 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^
3*c^4*x^4 - 2/3*a^2*c^4*x^3 - a*c^4*x^2 + c^4*x
```

Fricas [A] time = 2.07214, size = 167, normalized size = 2.29

$$-\frac{1}{9}\,a^{8}c^{4}x^{9}+\frac{1}{4}\,a^{7}c^{4}x^{8}+\frac{2}{7}\,a^{6}c^{4}x^{7}-a^{5}c^{4}x^{6}+\frac{3}{2}\,a^{3}c^{4}x^{4}-\frac{2}{3}\,a^{2}c^{4}x^{3}-ac^{4}x^{2}+c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 + 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^
3*c^4*x^4 - 2/3*a^2*c^4*x^3 - a*c^4*x^2 + c^4*x
```

Sympy [A] time = 0.112576, size = 87, normalized size = 1.19

$$-\frac{a^{8}c^{4}x^{9}}{9}+\frac{a^{7}c^{4}x^{8}}{4}+\frac{2a^{6}c^{4}x^{7}}{7}-a^{5}c^{4}x^{6}+\frac{3a^{3}c^{4}x^{4}}{2}-\frac{2a^{2}c^{4}x^{3}}{3}-ac^{4}x^{2}+c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**4/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -a**8*c**4*x**9/9 + a**7*c**4*x**8/4 + 2*a**6*c**4*x**7/7 - a**5*c**4*x**6
+ 3*a**3*c**4*x**4/2 - 2*a**2*c**4*x**3/3 - a*c**4*x**2 + c**4*x
```

Giac [A] time = 1.13551, size = 105, normalized size = 1.44

$$-\frac{\left(28\,c^{4}-\frac{315\,c^{4}}{ax+1}+\frac{1440\,c^{4}}{(ax+1)^{2}}-\frac{3360\,c^{4}}{(ax+1)^{3}}+\frac{4032\,c^{4}}{(ax+1)^{4}}-\frac{2016\,c^{4}}{(ax+1)^{5}}\right)(ax+1)^{9}}{252\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/252*(28*c^4 - 315*c^4/(a*x + 1) + 1440*c^4/(a*x + 1)^2 - 3360*c^4/(a*x +
 1)^3 + 4032*c^4/(a*x + 1)^4 - 2016*c^4/(a*x + 1)^5)*(a*x + 1)^9/a
```

3.1229 $\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=55

$$-\frac{c^3(1-ax)^7}{7a}+\frac{2c^3(1-ax)^6}{3a}-\frac{4c^3(1-ax)^5}{5a}$$

```
[Out] (-4*c^3*(1 - a*x)^5)/(5*a) + (2*c^3*(1 - a*x)^6)/(3*a) - (c^3*(1 - a*x)^7)/
(7*a)
```

Rubi [A] time = 0.0467867, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$-\frac{c^3(1-ax)^7}{7a}+\frac{2c^3(1-ax)^6}{3a}-\frac{4c^3(1-ax)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-4*c^3*(1 - a*x)^5)/(5*a) + (2*c^3*(1 - a*x)^6)/(3*a) - (c^3*(1 - a*x)^7)/
(7*a)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx&=c^3\int(1-ax)^4(1+ax)^2\,dx\\&=c^3\int\left(4(1-ax)^4-4(1-ax)^5+(1-ax)^6\right)dx\\&=-\frac{4c^3(1-ax)^5}{5a}+\frac{2c^3(1-ax)^6}{3a}-\frac{c^3(1-ax)^7}{7a}\end{aligned}$$

Mathematica [A] time = 0.0227724, size = 31, normalized size = 0.56

$$\frac{c^3(ax-1)^5\left(15a^2x^2+40ax+29\right)}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (c^3*(-1 + a*x)^5*(29 + 40*a*x + 15*a^2*x^2))/(105*a)
```

Maple [A] time = 0.026, size = 52, normalized size = 1.

$$c^{3}\left(\frac{x^{7}a^{6}}{7}-\frac{x^{6}a^{5}}{3}-\frac{x^{5}a^{4}}{5}+x^{4}a^{3}-\frac{x^{3}a^{2}}{3}-ax^{2}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] c^3*(1/7*x^7*a^6-1/3*x^6*a^5-1/5*x^5*a^4+x^4*a^3-1/3*x^3*a^2-a*x^2+x)
```

Maxima [A] time = 0.975515, size = 93, normalized size = 1.69

$$\frac{1}{7}a^{6}c^{3}x^{7}-\frac{1}{3}a^{5}c^{3}x^{6}-\frac{1}{5}a^{4}c^{3}x^{5}+a^{3}c^{3}x^{4}-\frac{1}{3}a^{2}c^{3}x^{3}-ac^{3}x^{2}+c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 - 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 - 1/3*a^2
*c^3*x^3 - a*c^3*x^2 + c^3*x
```

Fricas [A] time = 2.20074, size = 142, normalized size = 2.58

$$\frac{1}{7}a^{6}c^{3}x^{7}-\frac{1}{3}a^{5}c^{3}x^{6}-\frac{1}{5}a^{4}c^{3}x^{5}+a^{3}c^{3}x^{4}-\frac{1}{3}a^{2}c^{3}x^{3}-ac^{3}x^{2}+c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 - 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 - 1/3*a^2
*c^3*x^3 - a*c^3*x^2 + c^3*x
```

Sympy [A] time = 0.105803, size = 70, normalized size = 1.27

$$\frac{a^{6}c^{3}x^{7}}{7}-\frac{a^{5}c^{3}x^{6}}{3}-\frac{a^{4}c^{3}x^{5}}{5}+a^{3}c^{3}x^{4}-\frac{a^{2}c^{3}x^{3}}{3}-ac^{3}x^{2}+c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] a**6*c**3*x**7/7 - a**5*c**3*x**6/3 - a**4*c**3*x**5/5 + a**3*c**3*x**4 - a
**2*c**3*x**3/3 - a*c**3*x**2 + c**3*x
```

Giac [A] time = 1.15144, size = 89, normalized size = 1.62

$$\frac{\left(15\,c^3-\frac{140\,c^3}{ax+1}+\frac{504\,c^3}{(ax+1)^2}-\frac{840\,c^3}{(ax+1)^3}+\frac{560\,c^3}{(ax+1)^4}\right)(ax+1)^7}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/105*(15*c^3 - 140*c^3/(a*x + 1) + 504*c^3/(a*x + 1)^2 - 840*c^3/(a*x + 1)
^3 + 560*c^3/(a*x + 1)^4)*(a*x + 1)^7/a
```

3.1230 $\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=37

$$\frac{c^2(1-ax)^5}{5a}-\frac{c^2(1-ax)^4}{2a}$$

```
[Out] -(c^2*(1 - a*x)^4)/(2*a) + (c^2*(1 - a*x)^5)/(5*a)
```

Rubi [A] time = 0.0367321, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$\frac{c^2(1-ax)^5}{5a}-\frac{c^2(1-ax)^4}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c^2*(1 - a*x)^4)/(2*a) + (c^2*(1 - a*x)^5)/(5*a)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx&=c^2\int(1-ax)^3(1+ax)\,dx\\&=c^2\int\left(2(1-ax)^3-(1-ax)^4\right)\,dx\\&=-\frac{c^2(1-ax)^4}{2a}+\frac{c^2(1-ax)^5}{5a}\end{aligned}$$

Mathematica [A] time = 0.0160266, size = 32, normalized size = 0.86

$$c^2\left(-\frac{1}{5}a^4x^5+\frac{a^3x^4}{2}-ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^2/E^(2*ArcTanh[a*x]),x]
```

```
[Out] c^2*(x - a*x^2 + (a^3*x^4)/2 - (a^4*x^5)/5)
```

Maple [A] time = 0.027, size = 29, normalized size = 0.8

$$c^2\left(-\frac{x^5a^4}{5}+\frac{x^4a^3}{2}-ax^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] c^2*(-1/5*x^5*a^4+1/2*x^4*a^3-a*x^2+x)
```

Maxima [A] time = 0.967953, size = 50, normalized size = 1.35

$$-\frac{1}{5}a^4c^2x^5+\frac{1}{2}a^3c^2x^4-ac^2x^2+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 + c^2*x
```

Fricas [A] time = 2.19162, size = 76, normalized size = 2.05

$$-\frac{1}{5}a^4c^2x^5+\frac{1}{2}a^3c^2x^4-ac^2x^2+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 + c^2*x
```

Sympy [A] time = 0.097322, size = 36, normalized size = 0.97

$$-\frac{a^4c^2x^5}{5}+\frac{a^3c^2x^4}{2}-ac^2x^2+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -a**4*c**2*x**5/5 + a**3*c**2*x**4/2 - a*c**2*x**2 + c**2*x
```

Giac [A] time = 1.1257, size = 73, normalized size = 1.97

$$-\frac{\left(2\,c^2-\frac{15\,c^2}{ax+1}+\frac{40\,c^2}{(ax+1)^2}-\frac{40\,c^2}{(ax+1)^3}\right)(ax+1)^5}{10\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/10*(2*c^2 - 15*c^2/(a*x + 1) + 40*c^2/(a*x + 1)^2 - 40*c^2/(a*x + 1)^3)*
(a*x + 1)^5/a
```

3.1231 $\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=16

$$-\frac{c(1-ax)^3}{3a}$$

[Out] -(c*(1 - a*x)^3)/(3*a)

Rubi [A] time = 0.0189674, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6140, 32}

$$-\frac{c(1-ax)^3}{3a}$$

Antiderivative was successfully verified.

[In] Int[(c - a^2*c*x^2)/E^(2*ArcTanh[a*x]),x]

[Out] -(c*(1 - a*x)^3)/(3*a)

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx &= c\int(1-ax)^2\,dx\\ &= -\frac{c(1-ax)^3}{3a}\end{aligned}$$

Mathematica [A] time = 0.0103685, size = 20, normalized size = 1.25

$$c\left(\frac{a^2x^3}{3}-ax^2+x\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c - a^2*c*x^2)/E^(2*ArcTanh[a*x]),x]

[Out] c*(x - a*x^2 + (a^2*x^3)/3)

Maple [A] time = 0.026, size = 14, normalized size = 0.9

$$\frac{c\left(ax-1\right)^{3}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/3*c*(a*x-1)^3/a
```

Maxima [A] time = 0.950734, size = 27, normalized size = 1.69

$$\frac{1}{3}\,a^{2}cx^{3}-acx^{2}+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c*x^3 - a*c*x^2 + c*x
```

Fricas [A] time = 2.13448, size = 42, normalized size = 2.62

$$\frac{1}{3}\,a^{2}cx^{3}-acx^{2}+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*a^2*c*x^3 - a*c*x^2 + c*x
```

Sympy [A] time = 0.150038, size = 19, normalized size = 1.19

$$\frac{a^{2}cx^{3}}{3}-acx^{2}+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] a**2*c*x**3/3 - a*c*x**2 + c*x
```

Giac [B] time = 1.1363, size = 46, normalized size = 2.88

$$\frac{\left(ax+1\right)^{3}\left(c-\frac{6\,c}{ax+1}+\frac{12\,c}{\left(ax+1\right)^{2}}\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*(a*x + 1)^3*(c - 6*c/(a*x + 1) + 12*c/(a*x + 1)^2)/a
```

3.1232 $\int \frac{e^{-2\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{ac(ax+1)}$$

[Out] -(1/(a*c*(1 + a*x)))

Rubi [A] time = 0.0325568, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 32}

$$-\frac{1}{ac(ax+1)}$$

Antiderivative was successfully verified.

[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)),x]

[Out] -(1/(a*c*(1 + a*x)))

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx &= \frac{\int \frac{1}{(1+ax)^2}\,dx}{c}\\ &= -\frac{1}{ac(1+ax)}\end{aligned}$$

Mathematica [C] time = 0.0164715, size = 18, normalized size = 1.2

$$-\frac{e^{-2\tanh^{-1}(ax)}}{2ac}$$

Antiderivative was successfully verified.

[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)),x]

[Out] -1/(2*a*c*E^(2*ArcTanh[a*x]))

Maple [A] time = 0.024, size = 16, normalized size = 1.1

$$-\frac{1}{ac\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c),x)
```

```
[Out] -1/a/c/(a*x+1)
```

Maxima [A] time = 0.958754, size = 19, normalized size = 1.27

$$-\frac{1}{a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/(a^2*c*x + a*c)
```

Fricas [A] time = 2.06551, size = 27, normalized size = 1.8

$$-\frac{1}{a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/(a^2*c*x + a*c)
```

Sympy [A] time = 0.340862, size = 12, normalized size = 0.8

$$-\frac{1}{a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c),x)
```

```
[Out] -1/(a**2*c*x + a*c)
```

Giac [A] time = 1.14865, size = 20, normalized size = 1.33

$$-\frac{1}{(ax+1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -1/((a*x + 1)*a*c)
```

3.1233 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{4ac^2(ax+1)}-\frac{1}{4ac^2(ax+1)^2}+\frac{\tanh^{-1}(ax)}{4ac^2}$$

```
[Out] -1/(4*a*c^2*(1 + a*x)^2) - 1/(4*a*c^2*(1 + a*x)) + ArcTanh[a*x]/(4*a*c^2)
```

Rubi [A] time = 0.0501079, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6140, 44, 207}

$$-\frac{1}{4ac^2(ax+1)}-\frac{1}{4ac^2(ax+1)^2}+\frac{\tanh^{-1}(ax)}{4ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] -1/(4*a*c^2*(1 + a*x)^2) - 1/(4*a*c^2*(1 + a*x)) + ArcTanh[a*x]/(4*a*c^2)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{1}{(1-ax)(1+ax)^3}\,dx}{c^2}\\
&= \frac{\int \left(\frac{1}{2(1+ax)^3}+\frac{1}{4(1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^2}\\
&= -\frac{1}{4ac^2(1+ax)^2}-\frac{1}{4ac^2(1+ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^2}\\
&= -\frac{1}{4ac^2(1+ax)^2}-\frac{1}{4ac^2(1+ax)}+\frac{\tanh^{-1}(ax)}{4ac^2}
\end{aligned}$$

Mathematica [A] time = 0.0250277, size = 33, normalized size = 0.67

$$\frac{-ax + (ax + 1)^2 \tanh^{-1}(ax) - 2}{4a(acx + c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] (-2 - a*x + (1 + a*x)^2*ArcTanh[a*x])/(4*a*(c + a*c*x)^2)
```

Maple [A] time = 0.034, size = 60, normalized size = 1.2

$$-\frac{1}{4\,ac^2\,(ax+1)^2} - \frac{1}{4\,ac^2\,(ax+1)} + \frac{\ln(ax+1)}{8\,ac^2} - \frac{\ln(ax-1)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/4/a/c^2/(a*x+1)^2-1/4/a/c^2/(a*x+1)+1/8*ln(a*x+1)/a/c^2-1/8/c^2/a*ln(a*x
-1)
```

Maxima [A] time = 0.961268, size = 85, normalized size = 1.73

$$-\frac{ax+2}{4\left(a^3c^2x^2 + 2\,a^2c^2x + ac^2\right)} + \frac{\log(ax+1)}{8\,ac^2} - \frac{\log(ax-1)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a*x + 2)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2) + 1/8*log(a*x + 1)/(a*c^
2) - 1/8*log(a*x - 1)/(a*c^2)
```

Fricas [A] time = 2.0748, size = 173, normalized size = 3.53

$$-\frac{2\,ax - \left(a^2x^2 + 2\,ax + 1\right)\log(ax+1) + \left(a^2x^2 + 2\,ax + 1\right)\log(ax-1) + 4}{8\left(a^3c^2x^2 + 2\,a^2c^2x + ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a*x - (a^2*x^2 + 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 + 2*a*x + 1)*lo
g(a*x - 1) + 4)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)
```

Sympy [A] time = 0.521674, size = 56, normalized size = 1.14

$$-\frac{ax+2}{4a^3c^2x^2+8a^2c^2x+4ac^2}-\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -(a*x + 2)/(4*a**3*c**2*x**2 + 8*a**2*c**2*x + 4*a*c**2) - (log(x - 1/a)/8
- log(x + 1/a)/8)/(a*c**2)
```

Giac [A] time = 1.16565, size = 74, normalized size = 1.51

$$-\frac{\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{8\,ac^2}-\frac{\frac{ac^2}{ax+1}+\frac{ac^2}{(ax+1)^2}}{4\,a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(-2/(a*x + 1) + 1))/(a*c^2) - 1/4*(a*c^2/(a*x + 1) + a*c^2/(a*x
 + 1)^2)/(a^2*c^4)
```

3.1234 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=84

$$\frac{1}{16ac^3(1-ax)}-\frac{3}{16ac^3(ax+1)}-\frac{1}{8ac^3(ax+1)^2}-\frac{1}{12ac^3(ax+1)^3}+\frac{\tanh^{-1}(ax)}{4ac^3}$$

```
[Out] 1/(16*a*c^3*(1 - a*x)) - 1/(12*a*c^3*(1 + a*x)^3) - 1/(8*a*c^3*(1 + a*x)^2)
 - 3/(16*a*c^3*(1 + a*x)) + ArcTanh[a*x]/(4*a*c^3)
```

Rubi [A] time = 0.0677399, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6140, 44, 207}

$$\frac{1}{16ac^3(1-ax)}-\frac{3}{16ac^3(ax+1)}-\frac{1}{8ac^3(ax+1)^2}-\frac{1}{12ac^3(ax+1)^3}+\frac{\tanh^{-1}(ax)}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] 1/(16*a*c^3*(1 - a*x)) - 1/(12*a*c^3*(1 + a*x)^3) - 1/(8*a*c^3*(1 + a*x)^2)
 - 3/(16*a*c^3*(1 + a*x)) + ArcTanh[a*x]/(4*a*c^3)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{1}{(1-ax)^2(1+ax)^4}\,dx}{c^3}$$
$$= \frac{\int \left(\frac{1}{16(-1+ax)^2}+\frac{1}{4(1+ax)^4}+\frac{1}{4(1+ax)^3}+\frac{3}{16(1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^3}$$
$$= \frac{1}{16ac^3(1-ax)}-\frac{1}{12ac^3(1+ax)^3}-\frac{1}{8ac^3(1+ax)^2}-\frac{3}{16ac^3(1+ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^3}$$
$$= \frac{1}{16ac^3(1-ax)}-\frac{1}{12ac^3(1+ax)^3}-\frac{1}{8ac^3(1+ax)^2}-\frac{3}{16ac^3(1+ax)}+\frac{\tanh^{-1}(ax)}{4ac^3}$$

Mathematica [A] time = 0.0415584, size = 61, normalized size = 0.73

$$-\frac{3a^3x^3+6a^2x^2+ax-3(ax-1)(ax+1)^3\tanh^{-1}(ax)-4}{12a(ax-1)(acx+c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] -(-4 + a*x + 6*a^2*x^2 + 3*a^3*x^3 - 3*(-1 + a*x)*(1 + a*x)^3*ArcTanh[a*x])
/(12*a*(-1 + a*x)*(c + a*c*x)^3)
```

Maple [A] time = 0.036, size = 90, normalized size = 1.1

$$-\frac{1}{12\,ac^3\,(ax+1)^3}-\frac{1}{8\,ac^3\,(ax+1)^2}-\frac{3}{16\,ac^3\,(ax+1)}+\frac{\ln(ax+1)}{8\,ac^3}-\frac{1}{16\,ac^3\,(ax-1)}-\frac{\ln(ax-1)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/12/a/c^3/(a*x+1)^3-1/8/a/c^3/(a*x+1)^2-3/16/a/c^3/(a*x+1)+1/8*ln(a*x+1)/
a/c^3-1/16/c^3/a/(a*x-1)-1/8/c^3/a*ln(a*x-1)
```

Maxima [A] time = 0.967795, size = 123, normalized size = 1.46

$$-\frac{3\,a^3x^3+6\,a^2x^2+ax-4}{12\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}+\frac{\log(ax+1)}{8\,ac^3}-\frac{\log(ax-1)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^3*x^3 + 6*a^2*x^2 + a*x - 4)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^
2*c^3*x - a*c^3) + 1/8*log(a*x + 1)/(a*c^3) - 1/8*log(a*x - 1)/(a*c^3)
```

Fricas [A] time = 2.34261, size = 267, normalized size = 3.18

$$-\frac{6\,a^{3}x^{3}+12\,a^{2}x^{2}+2\,ax-3\left(a^{4}x^{4}+2\,a^{3}x^{3}-2\,ax-1\right)\log\left(ax+1\right)+3\left(a^{4}x^{4}+2\,a^{3}x^{3}-2\,ax-1\right)\log\left(ax-1\right)-8}{24\left(a^{5}c^{3}x^{4}+2\,a^{4}c^{3}x^{3}-2\,a^{2}c^{3}x-ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^3*x^3 + 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)
*log(a*x + 1) + 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x - 1) - 8)/(a^5*
c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

Sympy [A] time = 0.689245, size = 83, normalized size = 0.99

$$-\frac{3a^{3}x^{3}+6a^{2}x^{2}+ax-4}{12a^{5}c^{3}x^{4}+24a^{4}c^{3}x^{3}-24a^{2}c^{3}x-12ac^{3}}+\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(3*a**3*x**3 + 6*a**2*x**2 + a*x - 4)/(12*a**5*c**3*x**4 + 24*a**4*c**3*x*
*3 - 24*a**2*c**3*x - 12*a*c**3) + (-log(x - 1/a)/8 + log(x + 1/a)/8)/(a*c*
*3)
```

Giac [A] time = 1.14445, size = 131, normalized size = 1.56

$$-\frac{\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{8\,ac^{3}}+\frac{1}{32\,ac^{3}\left(\frac{2}{ax+1}-1\right)}-\frac{\frac{9\,a^{5}c^{6}}{ax+1}+\frac{6\,a^{5}c^{6}}{(ax+1)^{2}}+\frac{4\,a^{5}c^{6}}{(ax+1)^{3}}}{48\,a^{6}c^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(-2/(a*x + 1) + 1))/(a*c^3) + 1/32/(a*c^3*(2/(a*x + 1) - 1)) -
1/48*(9*a^5*c^6/(a*x + 1) + 6*a^5*c^6/(a*x + 1)^2 + 4*a^5*c^6/(a*x + 1)^3)/
(a^6*c^9)
```

3.1235 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=119

$$\frac{5}{64ac^4(1-ax)}-\frac{5}{32ac^4(ax+1)}+\frac{1}{64ac^4(1-ax)^2}-\frac{3}{32ac^4(ax+1)^2}-\frac{1}{16ac^4(ax+1)^3}-\frac{1}{32ac^4(ax+1)^4}+\frac{15\tanh^{-1}(a}{64ac^4}$$

```
[Out] 1/(64*a*c^4*(1 - a*x)^2) + 5/(64*a*c^4*(1 - a*x)) - 1/(32*a*c^4*(1 + a*x)^4
) - 1/(16*a*c^4*(1 + a*x)^3) - 3/(32*a*c^4*(1 + a*x)^2) - 5/(32*a*c^4*(1 +
a*x)) + (15*ArcTanh[a*x])/(64*a*c^4)
```

Rubi [A] time = 0.0868269, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6140, 44, 207}

$$\frac{5}{64ac^4(1-ax)}-\frac{5}{32ac^4(ax+1)}+\frac{1}{64ac^4(1-ax)^2}-\frac{3}{32ac^4(ax+1)^2}-\frac{1}{16ac^4(ax+1)^3}-\frac{1}{32ac^4(ax+1)^4}+\frac{15\tanh^{-1}(a}{64ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] 1/(64*a*c^4*(1 - a*x)^2) + 5/(64*a*c^4*(1 - a*x)) - 1/(32*a*c^4*(1 + a*x)^4
) - 1/(16*a*c^4*(1 + a*x)^3) - 3/(32*a*c^4*(1 + a*x)^2) - 5/(32*a*c^4*(1 +
a*x)) + (15*ArcTanh[a*x])/(64*a*c^4)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \frac{\int \frac{1}{(1-ax)^3(1+ax)^5}\,dx}{c^4}$$

$$= \frac{\int \left(-\frac{1}{32(-1+ax)^3}+\frac{5}{64(-1+ax)^2}+\frac{1}{8(1+ax)^5}+\frac{3}{16(1+ax)^4}+\frac{3}{16(1+ax)^3}+\frac{5}{32(1+ax)^2}-\frac{15}{64\left(-1+a^2x^2\right)}\right)dx}{c^4}$$

$$= \frac{1}{64ac^4(1-ax)^2}+\frac{5}{64ac^4(1-ax)}-\frac{1}{32ac^4(1+ax)^4}-\frac{1}{16ac^4(1+ax)^3}-\frac{3}{32ac^4(1+ax)^2}-\frac{5}{32ac^4(1+a}$$

$$= \frac{1}{64ac^4(1-ax)^2}+\frac{5}{64ac^4(1-ax)}-\frac{1}{32ac^4(1+ax)^4}-\frac{1}{16ac^4(1+ax)^3}-\frac{3}{32ac^4(1+ax)^2}-\frac{5}{32ac^4(1+a}$$

Mathematica [A] time = 0.061964, size = 80, normalized size = 0.67

$$\frac{-15a^5x^5-30a^4x^4+10a^3x^3+50a^2x^2+17ax+15(ax-1)^2(ax+1)^4\tanh^{-1}(ax)-16}{64a(ax-1)^2(acx+c)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] (-16 + 17*a*x + 50*a^2*x^2 + 10*a^3*x^3 - 30*a^4*x^4 - 15*a^5*x^5 + 15*(-1
+ a*x)^2*(1 + a*x)^4*ArcTanh[a*x])/(64*a*(-1 + a*x)^2*(c + a*c*x)^4)
```

Maple [A] time = 0.036, size = 120, normalized size = 1.

$$-\frac{1}{32\,ac^4\,(ax+1)^4}-\frac{1}{16\,ac^4\,(ax+1)^3}-\frac{3}{32\,ac^4\,(ax+1)^2}-\frac{5}{32\,ac^4\,(ax+1)}+\frac{15\,\ln(ax+1)}{128\,ac^4}+\frac{1}{64\,ac^4\,(ax-1)^2}-\frac{5}{64\,ac^4\,(ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/32/a/c^4/(a*x+1)^4-1/16/a/c^4/(a*x+1)^3-3/32/a/c^4/(a*x+1)^2-5/32/a/c^4/
(a*x+1)+15/128*ln(a*x+1)/a/c^4+1/64/c^4/a/(a*x-1)^2-5/64/c^4/a/(a*x-1)-15/1
28/c^4/a*ln(a*x-1)
```

Maxima [A] time = 1.00513, size = 189, normalized size = 1.59

$$-\frac{15\,a^5x^5+30\,a^4x^4-10\,a^3x^3-50\,a^2x^2-17\,ax+16}{64\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4\right)}+\frac{15\,\log(ax+1)}{128\,ac^4}-\frac{15\,\log(ax-1)}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] -1/64*(15*a^5*x^5 + 30*a^4*x^4 - 10*a^3*x^3 - 50*a^2*x^2 - 17*a*x + 16)/(a^
7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a
^2*c^4*x + a*c^4) + 15/128*log(a*x + 1)/(a*c^4) - 15/128*log(a*x - 1)/(a*c^
4)
```

Fricas [B] time = 2.35021, size = 458, normalized size = 3.85

$$-\frac{30\,a^5x^5+60\,a^4x^4-20\,a^3x^3-100\,a^2x^2-34\,ax-15\left(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1\right)\log\left(ax+1\right)+}{128\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/128*(30*a^5*x^5 + 60*a^4*x^4 - 20*a^3*x^3 - 100*a^2*x^2 - 34*a*x - 15*(a
^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x + 1
) + 15*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*lo
g(a*x - 1) + 32)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3
 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [A] time = 1.04576, size = 143, normalized size = 1.2

$$-\frac{15a^5x^5+30a^4x^4-10a^3x^3-50a^2x^2-17ax+16}{64a^7c^4x^6+128a^6c^4x^5-64a^5c^4x^4-256a^4c^4x^3-64a^3c^4x^2+128a^2c^4x+64ac^4}-\frac{\frac{15\log\left(x-\frac{1}{a}\right)}{128}-\frac{15\log\left(x+\frac{1}{a}\right)}{128}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] -(15*a**5*x**5 + 30*a**4*x**4 - 10*a**3*x**3 - 50*a**2*x**2 - 17*a*x + 16)/
(64*a**7*c**4*x**6 + 128*a**6*c**4*x**5 - 64*a**5*c**4*x**4 - 256*a**4*c**4
*x**3 - 64*a**3*c**4*x**2 + 128*a**2*c**4*x + 64*a*c**4) - (15*log(x - 1/a)
/128 - 15*log(x + 1/a)/128)/(a*c**4)
```

Giac [A] time = 1.16423, size = 165, normalized size = 1.39

$$-\frac{15\,\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{128\,ac^4}+\frac{\frac{24}{ax+1}-11}{256\,ac^4\left(\frac{2}{ax+1}-1\right)^2}-\frac{\frac{5\,a^{11}c^{12}}{ax+1}+\frac{3\,a^{11}c^{12}}{(ax+1)^2}+\frac{2\,a^{11}c^{12}}{(ax+1)^3}+\frac{a^{11}c^{12}}{(ax+1)^4}}{32\,a^{12}c^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -15/128*log(abs(-2/(a*x + 1) + 1))/(a*c^4) + 1/256*(24/(a*x + 1) - 11)/(a*c
^4*(2/(a*x + 1) - 1)^2) - 1/32*(5*a^11*c^12/(a*x + 1) + 3*a^11*c^12/(a*x +
1)^2 + 2*a^11*c^12/(a*x + 1)^3 + a^11*c^12/(a*x + 1)^4)/(a^12*c^16)
```

3.1236 $\int e^{-2\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=137

$$-\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

```
[Out] (-3*x^2*Sqrt[c - a^2*c*x^2])/(5*a^2) + (x^3*Sqrt[c - a^2*c*x^2])/(2*a) - (x
^4*Sqrt[c - a^2*c*x^2])/5 - (3*(8 - 5*a*x)*Sqrt[c - a^2*c*x^2])/(20*a^4) -
(3*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(4*a^4)
```

Rubi [A] time = 0.327147, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6152, 1809, 833, 780, 217, 203}

$$-\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a^2*c*x^2])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-3*x^2*Sqrt[c - a^2*c*x^2])/(5*a^2) + (x^3*Sqrt[c - a^2*c*x^2])/(2*a) - (x
^4*Sqrt[c - a^2*c*x^2])/5 - (3*(8 - 5*a*x)*Sqrt[c - a^2*c*x^2])/(20*a^4) -
(3*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(4*a^4)
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx &= c\int \frac{x^3(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx \\
&= -\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{\int \frac{x^3\left(-9a^2c+10a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{5a^2} \\
&= \frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{\int \frac{x^2\left(-30a^3c^2+36a^4c^2x\right)}{\sqrt{c-a^2cx^2}}\,dx}{20a^4c} \\
&= -\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{\int \frac{x\left(-72a^4c^3+90a^5c^3x\right)}{\sqrt{c-a^2cx^2}}\,dx}{60a^6c^2} \\
&= -\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}-\frac{(3c)\ \ldots}{\ldots} \\
&= -\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}-\frac{(3c)\ \ldots}{\ldots} \\
&= -\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}-\frac{3\sqrt{c}\ \ldots}{\ldots}
\end{aligned}
$$

Mathematica [A] time = 0.126838, size = 96, normalized size = 0.7

$$
\frac{\left(-4a^4x^4+10a^3x^3-12a^2x^2+15ax-24\right)\sqrt{c-a^2cx^2}+15\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{20a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a^2*c*x^2])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-24 + 15*a*x - 12*a^2*x^2 + 10*a^3*x^3 - 4*a^4*x^4) +
 15*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(20
*a^4)
```

Maple [A] time = 0.042, size = 202, normalized size = 1.5

$$\frac{x^2}{5a^2c}\left(-a^2cx^2+c\right)^{\frac{3}{2}}+\frac{4}{5ca^4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{x}{2a^3c}\left(-a^2cx^2+c\right)^{\frac{3}{2}}+\frac{5x}{4a^3}\sqrt{-a^2cx^2+c}+\frac{5c}{4a^3}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/5*x^2*(-a^2*c*x^2+c)^(3/2)/a^2/c+4/5/c/a^4*(-a^2*c*x^2+c)^(3/2)-1/2/a^3*x
*(-a^2*c*x^2+c)^(3/2)/c+5/4/a^3*x*(-a^2*c*x^2+c)^(1/2)+5/4/a^3*c/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/a^4*(-c*a^2*(x+1/a)^2+2*
a*c*(x+1/a))^(1/2)-2/a^3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x+
1/a)^2+2*a*c*(x+1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.48803, size = 433, normalized size = 3.16

$$\left[-\frac{2\left(4a^4x^4-10a^3x^3+12a^2x^2-15ax+24\right)\sqrt{-a^2cx^2+c}-15\sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{40a^4},-\frac{\left(4a^4x^4-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fri
cas")
```

```
[Out] [-1/40*(2*(4*a^4*x^4 - 10*a^3*x^3 + 12*a^2*x^2 - 15*a*x + 24)*sqrt(-a^2*c*x
^2 + c) - 15*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
 - c))/a^4, -1/20*((4*a^4*x^4 - 10*a^3*x^3 + 12*a^2*x^2 - 15*a*x + 24)*sqrt
(-a^2*c*x^2 + c) - 15*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*
c*x^2 - c)))/a^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x^3\sqrt{-a^2cx^2+c}}{ax+1}\,dx-\int \frac{ax^4\sqrt{-a^2cx^2+c}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x**3*sqrt(-a**2*c*x**2 + c)/(a*x + 1), x) - Integral(a*x**4*sqrt
(-a**2*c*x**2 + c)/(a*x + 1), x)
```

Giac [B] time = 1.31113, size = 371, normalized size = 2.71

$$\frac{\left(480\,a^{6}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-c+\frac{2\,c}{ax+1}}}{\sqrt{c}}\right)\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}\left(a\right)-\frac{\left(65\,a^{6}\left(c-\frac{2\,c}{ax+1}\right)^{4}c^{2}\sqrt{-c+\frac{2\,c}{ax+1}}\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}(a)-90\,a^{6}\left(c-\frac{2\,c}{ax+1}\right)^{3}c^{3}\sqrt{-c+\frac{2\,c}{ax+1}}\operatorname{sgn}\left(\frac{1}{ax+1}\right)\cdots}{\cdots}\right.}{320\,a^{11}c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="gia
c")
```

```
[Out] 1/320*(480*a^6*c^(3/2)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1/(a*x
+ 1))*sgn(a) - (65*a^6*(c - 2*c/(a*x + 1))^4*c^2*sqrt(-c + 2*c/(a*x + 1))*s
gn(1/(a*x + 1))*sgn(a) - 90*a^6*(c - 2*c/(a*x + 1))^3*c^3*sqrt(-c + 2*c/(a*
x + 1))*sgn(1/(a*x + 1))*sgn(a) + 144*a^6*(c - 2*c/(a*x + 1))^2*c^4*sqrt(-c
 + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) + 15*a^6*c^6*sqrt(-c + 2*c/(a*x +
 1))*sgn(1/(a*x + 1))*sgn(a) + 70*a^6*c^5*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/
(a*x + 1))*sgn(a))*(a*x + 1)^5/c^5)*abs(a)/(a^11*c)
```

3.1237 $\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=112

$$-\frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}+\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

```
[Out] (2*x^2*Sqrt[c - a^2*c*x^2])/(3*a) - (x^3*Sqrt[c - a^2*c*x^2])/4 + ((32 - 21
*a*x)*Sqrt[c - a^2*c*x^2])/(24*a^3) + (7*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(8*a^3)
```

Rubi [A] time = 0.28425, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6152, 1809, 833, 780, 217, 203}

$$-\frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}+\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (2*x^2*Sqrt[c - a^2*c*x^2])/(3*a) - (x^3*Sqrt[c - a^2*c*x^2])/4 + ((32 - 21
*a*x)*Sqrt[c - a^2*c*x^2])/(24*a^3) + (7*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(8*a^3)
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx &= c\int \frac{x^2(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{\int \frac{x^2\left(-7a^2c+8a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{4a^2}\\
&= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} - \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{\int \frac{x\left(-16a^3c^2+21a^4c^2x\right)}{\sqrt{c-a^2cx^2}}\,dx}{12a^4c}\\
&= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} - \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3} + \frac{(7c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{8a^2}\\
&= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} - \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3} + \frac{(7c)\operatorname{Subst}\left(\int \frac{1}{1+a^2cx^2}\right.}{8a^2}\\
&= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} - \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3} + \frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}
\end{aligned}$$

Mathematica [A] time = 0.0949311, size = 88, normalized size = 0.79

$$\frac{\left(-6a^3x^3+16a^2x^2-21ax+32\right)\sqrt{c-a^2cx^2}-21\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(32 - 21*a*x + 16*a^2*x^2 - 6*a^3*x^3) - 21*Sqrt[c]*Ar
cTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(24*a^3)
```

Maple [A] time = 0.039, size = 178, normalized size = 1.6

$$\frac{x}{4a^2c}\left(-a^2cx^2+c\right)^{\frac{3}{2}} - \frac{9x}{8a^2}\sqrt{-a^2cx^2+c} - \frac{9c}{8a^2}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}} - \frac{2}{3a^3c}\left(-a^2cx^2+c\right)^{\frac{3}{2}} + 2\,\frac{\sqrt{-ca^2}\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/4*x*(-a^2*c*x^2+c)^(3/2)/a^2/c-9/8/a^2*x*(-a^2*c*x^2+c)^(1/2)-9/8/a^2*c/(
a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/3/a^3*(-a^2*c*x
^2+c)^(3/2)/c+2/a^3*(-c*a^2*(x+1/a)^2+2*a*c*(x+1/a))^(1/2)+2/a^2*c/(a^2*c)^
(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x+1/a)^2+2*a*c*(x+1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.36431, size = 398, normalized size = 3.55

$$\left[-\frac{2\left(6\,a^3x^3-16\,a^2x^2+21\,ax-32\right)\sqrt{-a^2cx^2+c}-21\,\sqrt{-c}\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{48\,a^3},-\frac{\left(6\,a^3x^3-16\,a^2x^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fri
cas")
```

```
[Out] [-1/48*(2*(6*a^3*x^3 - 16*a^2*x^2 + 21*a*x - 32)*sqrt(-a^2*c*x^2 + c) - 21*
sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^3, -
1/24*((6*a^3*x^3 - 16*a^2*x^2 + 21*a*x - 32)*sqrt(-a^2*c*x^2 + c) + 21*sqrt
(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x^2\sqrt{-a^2cx^2+c}}{ax+1}\,dx-\int \frac{ax^3\sqrt{-a^2cx^2+c}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x**2*sqrt(-a**2*c*x**2 + c)/(a*x + 1), x) - Integral(a*x**3*sqrt
(-a**2*c*x**2 + c)/(a*x + 1), x)
```

Giac [B] time = 1.26439, size = 305, normalized size = 2.72

$$-\frac{\left(336\,a^{5}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-c+\frac{2\,c}{ax+1}}}{\sqrt{c}}\right)\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}\left(a\right)+\frac{\left(75\,a^{5}\left(c-\frac{2\,c}{ax+1}\right)^{3}c^{2}\sqrt{-c+\frac{2\,c}{ax+1}}\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}(a)-83\,a^{5}\left(c-\frac{2\,c}{ax+1}\right)^{2}c^{3}\sqrt{-c+\frac{2\,c}{ax+1}}\operatorname{sgn}\left(\frac{1}{ax+1}\right.\right.}{}\right.}{192\,a^{9}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="gia
c")
```

```
[Out] -1/192*(336*a^5*c^(3/2)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1/(a*x
 + 1))*sgn(a) + (75*a^5*(c - 2*c/(a*x + 1))^3*c^2*sqrt(-c + 2*c/(a*x + 1))*
sgn(1/(a*x + 1))*sgn(a) - 83*a^5*(c - 2*c/(a*x + 1))^2*c^3*sqrt(-c + 2*c/(a
*x + 1))*sgn(1/(a*x + 1))*sgn(a) - 21*a^5*c^5*sqrt(-c + 2*c/(a*x + 1))*sgn(
1/(a*x + 1))*sgn(a) - 77*a^5*c^4*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/(a*x + 1)
)*sgn(a))*(a*x + 1)^4/c^4)*abs(a)/(a^9*c)
```

3.1238 $\int e^{-2\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=85

$$-\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

```
[Out] -(x^2*Sqrt[c - a^2*c*x^2])/3 - ((5 - 3*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2) -
(Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/a^2
```

Rubi [A] time = 0.183082, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6152, 1809, 780, 217, 203}

$$-\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(x^2*Sqrt[c - a^2*c*x^2])/3 - ((5 - 3*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2) -
(Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/a^2
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= c\int \frac{x(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{\int \frac{x\left(-5a^2c+6a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{3a^2}\\
&= -\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{c\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a}\\
&= -\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{c\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)}{a}\\
&= -\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.0813597, size = 80, normalized size = 0.94

$$\frac{3\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)-\left(a^2x^2-3ax+5\right)\sqrt{c-a^2cx^2}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-((5 - 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2]) + 3*Sqrt[c]*ArcTan[(a*x*Sqrt[
c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^2)
```

Maple [B] time = 0.036, size = 154, normalized size = 1.8

$$\frac{1}{3\,a^2c}\left(-a^2cx^2+c\right)^{\frac{3}{2}}+\frac{x}{a}\sqrt{-a^2cx^2+c}+\frac{c}{a}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}-2\frac{\sqrt{-ca^2\left(x+a^{-1}\right)^2+2\,ac\left(x+a^{-1}\right)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/3*(-a^2*c*x^2+c)^(3/2)/a^2/c+x/a*(-a^2*c*x^2+c)^(1/2)+1/a*c/(a^2*c)^(1/2)
*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/a^2*(-c*a^2*(x+1/a)^2+2*a*c
*(x+1/a))^(1/2)-2/a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x+1/a)^
2+2*a*c*(x+1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.41889, size = 347, normalized size = 4.08

$$\left[-\frac{2\sqrt{-a^2cx^2+c}\left(a^2x^2-3\,ax+5\right)-3\sqrt{-c}\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}x-c\right)}{6\,a^2},-\frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-3\,ax+5\right)-3}{3\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="frica
s")
```

```
[Out] [-1/6*(2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 3*a*x + 5) - 3*sqrt(-c)*log(2*a^2*
c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^2, -1/3*(sqrt(-a^2*c*x^
2 + c)*(a^2*x^2 - 3*a*x + 5) - 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt
(c)*x/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x\sqrt{-a^2cx^2+c}}{ax+1}\,dx-\int \frac{ax^2\sqrt{-a^2cx^2+c}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x*sqrt(-a**2*c*x**2 + c)/(a*x + 1), x) - Integral(a*x**2*sqrt(-a
**2*c*x**2 + c)/(a*x + 1), x)
```

Giac [B] time = 1.30079, size = 242, normalized size = 2.85

$$\frac{\left(24\,a^4c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-c+\frac{2\,c}{ax+1}}}{\sqrt{c}}\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}\left(a\right)-\frac{\left(9\,a^4\left(c-\frac{2\,c}{ax+1}\right)^2c^2\sqrt{-c+\frac{2\,c}{ax+1}}\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)+3\,a^4c^4\sqrt{-c+\frac{2\,c}{ax+1}}\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)+8\,a^4c^3\left(-c+\right.\right.}{c^3}\right.}{12\,a^7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac"
)
```

```
[Out] 1/12*(24*a^4*c^(3/2)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1/(a*x +
1))*sgn(a) - (9*a^4*(c - 2*c/(a*x + 1))^2*c^2*sqrt(-c + 2*c/(a*x + 1))*sgn(
```

```
1/(a*x + 1))*sgn(a) + 3*a^4*c^4*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*s
gn(a) + 8*a^4*c^3*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/(a*x + 1))*sgn(a))*(a*x
+ 1)^3/c^3)*abs(a)/(a^7*c)
```

3.1239 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=87

$$\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(2*a) + ((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a) + (3*
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rubi [A] time = 0.0753709, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6142, 671, 641, 217, 203}

$$\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(2*a) + ((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a) + (3*
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = c\int \frac{(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{(1-ax)\sqrt{c-a^2cx^2}}{2a} + \frac{1}{2}(3c)\int \frac{1-ax}{\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{3\sqrt{c-a^2cx^2}}{2a} + \frac{(1-ax)\sqrt{c-a^2cx^2}}{2a} + \frac{1}{2}(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{3\sqrt{c-a^2cx^2}}{2a} + \frac{(1-ax)\sqrt{c-a^2cx^2}}{2a} + \frac{1}{2}(3c)\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)$$
$$= \frac{3\sqrt{c-a^2cx^2}}{2a} + \frac{(1-ax)\sqrt{c-a^2cx^2}}{2a} + \frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

Mathematica [A] time = 0.0506014, size = 99, normalized size = 1.14

$$\frac{\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(a^2x^2-5ax+4\right)-6\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2) - 6*Sqrt[1 - a*x]
*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.034, size = 126, normalized size = 1.5

$$-\frac{x}{2}\sqrt{-a^2cx^2+c}-\frac{c}{2}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}+2\frac{\sqrt{-ca^2\left(x+a^{-1}\right)^2+2\,ac\left(x+a^{-1}\right)}}{a}+2\frac{c}{\sqrt{a^2c}}\arctan\left(\frac{}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/2*x*(-a^2*c*x^2+c)^(1/2)-1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^
2*c*x^2+c)^(1/2))+2/a*(-c*a^2*(x+1/a)^2+2*a*c*(x+1/a))^(1/2)+2*c/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x+1/a)^2+2*a*c*(x+1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.43303, size = 309, normalized size = 3.55

$$\left[-\frac{2\sqrt{-a^2cx^2+c}(ax-4)-3\sqrt{-c}\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{4a}, -\frac{\sqrt{-a^2cx^2+c}(ax-4)+3\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] [-1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x - 4) - 3*sqrt(-c)*log(2*a^2*c*x^2 + 2*sq
rt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, -1/2*(sqrt(-a^2*c*x^2 + c)*(a*x -
4) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{-a^2cx^2+c}}{ax+1}\,dx - \int \frac{ax\sqrt{-a^2cx^2+c}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-sqrt(-a**2*c*x**2 + c)/(a*x + 1), x) - Integral(a*x*sqrt(-a**2*c
*x**2 + c)/(a*x + 1), x)
```

Giac [A] time = 1.20922, size = 177, normalized size = 2.03

$$-\frac{\left(12a^3c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-c+\frac{2c}{ax+1}}}{\sqrt{c}}\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)-\frac{\left(3a^3c^3\sqrt{-c+\frac{2c}{ax+1}}\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)+5a^3c^2\left(-c+\frac{2c}{ax+1}\right)^{\frac{3}{2}}\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)\right)(ax+1)^2}{c^2}\right)|a|}{4a^5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*(12*a^3*c^(3/2)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1/(a*x +
1))*sgn(a) - (3*a^3*c^3*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) +
5*a^3*c^2*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/(a*x + 1))*sgn(a))*(a*x + 1)^2/c
^2)*abs(a)/(a^5*c)
```

3.1240 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=78

$$-\sqrt{c-a^2cx^2}-2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -Sqrt[c - a^2*c*x^2] - 2*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]
- Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.244762, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6152, 1809, 844, 217, 203, 266, 63, 208}

$$-\sqrt{c-a^2cx^2}-2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] -Sqrt[c - a^2*c*x^2] - 2*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]
- Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= c\int \frac{(1-ax)^2}{x\sqrt{c-a^2cx^2}}\,dx\\
&= -\sqrt{c-a^2cx^2} - \frac{\int \frac{-a^2c+2a^3cx}{x\sqrt{c-a^2cx^2}}\,dx}{a^2}\\
&= -\sqrt{c-a^2cx^2} + c\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx - (2ac)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\sqrt{c-a^2cx^2} + \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) - (2ac)\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{}{\sqrt{c}}\right.\\
&= -\sqrt{c-a^2cx^2} - 2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)}{a^2}\\
&= -\sqrt{c-a^2cx^2} - 2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - \sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0800474, size = 99, normalized size = 1.27

$$
-\sqrt{c-a^2cx^2} - \sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) + 2\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right) + \sqrt{c}\log(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] -Sqrt[c - a^2*c*x^2] + 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*
(-1 + a^2*x^2))] + Sqrt[c]*Log[x] - Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*
x^2]]
```

Maple [A] time = 0.04, size = 120, normalized size = 1.5

$$\sqrt{-a^2cx^2+c}-\sqrt{c}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right)-2\sqrt{-ca^2\left(x+a^{-1}\right)^2+2ac\left(x+a^{-1}\right)}-2\frac{ac}{\sqrt{a^2c}}\arctan\left(\frac{\quad}{\sqrt{-ca^2\left(x\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x)
```

```
[Out] (-a^2*c*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-2*(
-c*a^2*(x+1/a)^2+2*a*c*(x+1/a))^(1/2)-2*a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1
/2)*x/(-c*a^2*(x+1/a)^2+2*a*c*(x+1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)}{(ax+1)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)/((a*x + 1)^2*x), x)
```

Fricas [A] time = 2.40191, size = 443, normalized size = 5.68

$$\left[2\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+\frac{1}{2}\sqrt{c}\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)-\sqrt{-a^2cx^2+c},-\sqrt{-c}\arctan\left(\frac{\sqrt{-a}}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="frica
s")
```

```
[Out] [2*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2*s
qrt(c)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) - sqrt(
-a^2*c*x^2 + c), -sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2
- c)) + sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c)
 - sqrt(-a^2*c*x^2 + c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int-\frac{\sqrt{-a^2cx^2+c}}{ax^2+x}\,dx-\int\frac{ax\sqrt{-a^2cx^2+c}}{ax^2+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x,x)
```

```
[Out] -Integral(-sqrt(-a**2*c*x**2 + c)/(a*x**2 + x), x) - Integral(a*x*sqrt(-a**
2*c*x**2 + c)/(a*x**2 + x), x)
```

Giac [A] time = 1.23677, size = 167, normalized size = 2.14

$$-\left(\frac{(ax+1)\sqrt{-c+\frac{2c}{ax+1}}\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}(a)}{a^2}-\frac{2c\arctan\left(\frac{\sqrt{-c+\frac{2c}{ax+1}}}{\sqrt{-c}}\right)\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}(a)}{a^2\sqrt{-c}}-\frac{4\sqrt{c}\arctan\left(\frac{\sqrt{-c+\frac{2c}{ax+1}}}{\sqrt{c}}\right)\operatorname{sgn}\left(\frac{1}{ax+1}\right.}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="giac"
)
```

```
[Out] -((a*x + 1)*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a)/a^2 - 2*c*arct
an(sqrt(-c + 2*c/(a*x + 1))/sqrt(-c))*sgn(1/(a*x + 1))*sgn(a)/(a^2*sqrt(-c)
) - 4*sqrt(c)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1/(a*x + 1))*sgn
(a)/a^2)*a*abs(a)
```

3.1241 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=82

$$-\frac{\sqrt{c-a^2cx^2}}{x}+a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -(Sqrt[c - a^2*c*x^2]/x) + a*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^
2]] + 2*a*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.248316, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6152, 1807, 844, 217, 203, 266, 63, 208}

$$-\frac{\sqrt{c-a^2cx^2}}{x}+a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/x) + a*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^
2]] + 2*a*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= c\int \frac{(1-ax)^2}{x^2\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{x}-\int \frac{2ac-a^2cx}{x\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{x}-(2ac)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx+\left(a^2c\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{x}-(ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)+\left(a^2c\right)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\right.\\
&= -\frac{\sqrt{c-a^2cx^2}}{x}+a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)}{a}\\
&= -\frac{\sqrt{c-a^2cx^2}}{x}+a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0887855, size = 106, normalized size = 1.29

$$
-\frac{\sqrt{c-a^2cx^2}}{x}+2a\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)-a\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)-2a\sqrt{c}\log(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/x) - a*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt
[c]*(-1 + a^2*x^2))] - 2*a*Sqrt[c]*Log[x] + 2*a*Sqrt[c]*Log[c + Sqrt[c]*Sqr
t[c - a^2*c*x^2]]
```

Maple [B] time = 0.04, size = 203, normalized size = 2.5

$$-\frac{1}{cx}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-a^2x\sqrt{-a^2cx^2+c}-a^2c\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}+2\ln\left(\frac{2c+2\sqrt{c}\sqrt{-a^2cx^2+c}}{x}\right)\sqrt{c}a\cdot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x)
```

```
[Out] -1/c/x*(-a^2*c*x^2+c)^(3/2)-a^2*x*(-a^2*c*x^2+c)^(1/2)-a^2*c/(a^2*c)^(1/2)*
arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2*ln((2*c+2*c^(1/2)*(-a^2*c*x^
2+c)^(1/2))/x)*c^(1/2)*a-2*(-a^2*c*x^2+c)^(1/2)*a+2*a*(-c*a^2*(x+1/a)^2+2*a
*c*(x+1/a))^(1/2)+2*a^2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x+1
/a)^2+2*a*c*(x+1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)}{(ax+1)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)/((a*x + 1)^2*x^2), x)
```

Fricas [A] time = 2.54189, size = 478, normalized size = 5.83

$$\left[-\frac{a\sqrt{c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)-a\sqrt{c}x\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+\sqrt{-a^2cx^2+c}}{x},\frac{4a\sqrt{-c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="fri
cas")
```

```
[Out] [-(a*sqrt(c)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - a
*sqrt(c)*x*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + s
qrt(-a^2*c*x^2 + c))/x, 1/2*(4*a*sqrt(-c)*x*arctan(sqrt(-a^2*c*x^2 + c)*sqr
t(-c)/(a^2*c*x^2 - c)) + a*sqrt(-c)*x*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 +
 c)*a*sqrt(-c)*x - c) - 2*sqrt(-a^2*c*x^2 + c))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{-a^2cx^2+c}}{ax^3+x^2}\,dx-\int \frac{ax\sqrt{-a^2cx^2+c}}{ax^3+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**2,x)
```

```
[Out] -Integral(-sqrt(-a**2*c*x**2 + c)/(a*x**3 + x**2), x) - Integral(a*x*sqrt(-
a**2*c*x**2 + c)/(a*x**3 + x**2), x)
```

Giac [B] time = 1.19943, size = 227, normalized size = 2.77

$$-\left(\frac{4\,c\arctan\left(\frac{\sqrt{-c+\frac{2\,c}{ax+1}}}{\sqrt{-c}}\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}\left(a\right)}{\sqrt{-c}}+2\,\sqrt{c}\arctan\left(\frac{\sqrt{-c+\frac{2\,c}{ax+1}}}{\sqrt{c}}\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}\left(a\right)-\frac{\left(\pi c+2\,\sqrt{-c}\sqrt{c}\arctan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="gia
c")
```

```
[Out] -(4*c*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(-c))*sgn(1/(a*x + 1))*sgn(a)/sqr
t(-c) + 2*sqrt(c)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1/(a*x + 1))
*sgn(a) - (pi*c + 2*sqrt(-c)*sqrt(c)*arctan(sqrt(-c)/sqrt(c)) - c)*sgn(1/(a
*x + 1))*sgn(a)/sqrt(-c) + c*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(
a)/(c - c/(a*x + 1)))*abs(a)
```

3.1242 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=78

$$\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -Sqrt[c - a^2*c*x^2]/(2*x^2) + (2*a*Sqrt[c - a^2*c*x^2])/x - (3*a^2*Sqrt[c]
*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rubi [A] time = 0.237261, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6152, 1807, 807, 266, 63, 208}

$$\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(2*x^2) + (2*a*Sqrt[c - a^2*c*x^2])/x - (3*a^2*Sqrt[c]
*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx &= c\int \frac{(1-ax)^2}{x^3\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{1}{2}\int \frac{4ac-3a^2cx}{x^2\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{1}{2}\left(3a^2c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{1}{4}\left(3a^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)\\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.126432, size = 76, normalized size = 0.97

$$\frac{1}{2}\left(\frac{(4ax-1)\sqrt{c-a^2cx^2}}{x^2}-3a^2\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+3a^2\sqrt{c}\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] (((-1 + 4*a*x)*Sqrt[c - a^2*c*x^2])/x^2 + 3*a^2*Sqrt[c]*Log[x] - 3*a^2*Sqrt
[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/2
```

Maple [B] time = 0.042, size = 231, normalized size = 3.

$$2\,\frac{a\left(-a^2cx^2+c\right)^{3/2}}{cx}+2\,a^3x\sqrt{-a^2cx^2+c}+2\,\frac{a^3c}{\sqrt{a^2c}}\arctan\left(\frac{\sqrt{a^2c}x}{\sqrt{-a^2cx^2+c}}\right)-\frac{3\,a^2}{2}\sqrt{c}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right)+\frac{3\,a}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x)
```

```
[Out] 2*a/c/x*(-a^2*c*x^2+c)^(3/2)+2*a^3*x*(-a^2*c*x^2+c)^(1/2)+2*a^3*c/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-3/2*c^(1/2)*ln((2*c+2*c^(
```

```
1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a^2+3/2*(-a^2*c*x^2+c)^(1/2)*a^2-2*a^2*(-c*a^
2*(x+1/a)^2+2*a*c*(x+1/a))^(1/2)-2*a^3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)
*x/(-c*a^2*(x+1/a)^2+2*a*c*(x+1/a))^(1/2))-1/2/c/x^2*(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)}{(ax+1)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)/((a*x + 1)^2*x^3), x)
```

Fricas [A] time = 2.35702, size = 339, normalized size = 4.35

$$\left[\frac{3\,a^2\sqrt{c}x^2\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\,\sqrt{-a^2cx^2+c}(4\,ax-1)}{4\,x^2},-\frac{3\,a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)-\sqrt{-a^2cx^2+c(}}{2\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="fri
cas")
```

```
[Out] [1/4*(3*a^2*sqrt(c)*x^2*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(4*a*x - 1))/x^2, -1/2*(3*a^2*sqrt(-c)*x
^2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt(-a^2*c*x^2
+ c)*(4*a*x - 1))/x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{-a^2cx^2+c}}{ax^4+x^3}\,dx-\int \frac{ax\sqrt{-a^2cx^2+c}}{ax^4+x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**3,x)
```

```
[Out] -Integral(-sqrt(-a**2*c*x**2 + c)/(a*x**4 + x**3), x) - Integral(a*x*sqrt(-
a**2*c*x**2 + c)/(a*x**4 + x**3), x)
```

Giac [B] time = 1.17133, size = 205, normalized size = 2.63

$$\frac{1}{4}\left(\frac{12\,ac\arctan\left(\frac{\sqrt{-c+\frac{2\,c}{ax+1}}}{\sqrt{-c}}\right)\text{sgn}\left(\frac{1}{ax+1}\right)\text{sgn}\left(a\right)}{\sqrt{-c}}-\frac{(3\,\pi ac-8\,ac)\text{sgn}\left(\frac{1}{ax+1}\right)\text{sgn}\left(a\right)}{\sqrt{-c}}+\frac{3\,ac^2\sqrt{-c+\frac{2\,c}{ax+1}}\text{sgn}\left(\frac{1}{ax+1}\right)\text{sgn}\,(}{(c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="gia
c")
```

```
[Out] 1/4*(12*a*c*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(-c))*sgn(1/(a*x + 1))*sgn(
a)/sqrt(-c) - (3*pi*a*c - 8*a*c)*sgn(1/(a*x + 1))*sgn(a)/sqrt(-c) + (3*a*c^
2*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) - 5*a*c*(-c + 2*c/(a*x +
 1))^(3/2)*sgn(1/(a*x + 1))*sgn(a))/(c - c/(a*x + 1))^2)*abs(a)
```

3.1243 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=99

$$-\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -Sqrt[c - a^2*c*x^2]/(3*x^3) + (a*Sqrt[c - a^2*c*x^2])/x^2 - (5*a^2*Sqrt[c
- a^2*c*x^2])/(3*x) + a^3*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.274834, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6152, 1807, 835, 807, 266, 63, 208}

$$-\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x^4),x]
```


```
[Out] -Sqrt[c - a^2*c*x^2]/(3*x^3) + (a*Sqrt[c - a^2*c*x^2])/x^2 - (5*a^2*Sqrt[c
- a^2*c*x^2])/(3*x) + a^3*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```


Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
```

```
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx &= c\int \frac{(1-ax)^2}{x^4\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{1}{3}\int \frac{6ac-5a^2cx}{x^3\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{\int \frac{10a^2c^2-6a^3c^2x}{x^2\sqrt{c-a^2cx^2}}\,dx}{6c}\\
&= -\frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\left(a^3c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\frac{1}{2}\left(a^3c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+a\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)\\
&= -\frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.127972, size = 82, normalized size = 0.83

$$\frac{\left(-5a^2x^2+3ax-1\right)\sqrt{c-a^2cx^2}}{3x^3}+a^3\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+a^3\left(-\sqrt{c}\right)\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] ((-1 + 3*a*x - 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*x^3) - a^3*Sqrt[c]*Log[x]
 + a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

Maple [B] time = 0.047, size = 253, normalized size = 2.6

$$-2\frac{a^2\left(-a^2cx^2+c\right)^{3/2}}{cx}-2a^4x\sqrt{-a^2cx^2+c}-2\frac{a^4c}{\sqrt{a^2c}}\arctan\left(\frac{\sqrt{a^2c}x}{\sqrt{-a^2cx^2+c}}\right)+\sqrt{c}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right)a^3-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x)
```

```
[Out] -2*a^2/c/x*(-a^2*c*x^2+c)^(3/2)-2*a^4*x*(-a^2*c*x^2+c)^(1/2)-2*a^4*c/(a^2*c
)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+c^(1/2)*ln((2*c+2*c^(1
/2)*(-a^2*c*x^2+c)^(1/2))/x)*a^3-(-a^2*c*x^2+c)^(1/2)*a^3+2*a^3*(-c*a^2*(x+
1/a)^2+2*a*c*(x+1/a))^(1/2)+2*a^4*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-
c*a^2*(x+1/a)^2+2*a*c*(x+1/a))^(1/2))+a/c/x^2*(-a^2*c*x^2+c)^(3/2)-1/3/c/x^
3*(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)}{(ax+1)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)/((a*x + 1)^2*x^4), x)
```

Fricas [A] time = 2.4601, size = 370, normalized size = 3.74

$$\left[\frac{3a^3\sqrt{c}x^3\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)-2\sqrt{-a^2cx^2+c}\left(5a^2x^2-3ax+1\right)}{6x^3},\frac{3a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)-\sqrt{-a^2}}{3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="fri
cas")
```

```
[Out] [1/6*(3*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) - 2*sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 - 3*a*x + 1))/x^3, 1/3*(3*a^3
*sqrt(-c)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt(
-a^2*c*x^2 + c)*(5*a^2*x^2 - 3*a*x + 1))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int-\frac{\sqrt{-a^2cx^2+c}}{ax^5+x^4}\,dx-\int\frac{ax\sqrt{-a^2cx^2+c}}{ax^5+x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**4,x)
```

```
[Out] -Integral(-sqrt(-a**2*c*x**2 + c)/(a*x**5 + x**4), x) - Integral(a*x*sqrt(-
a**2*c*x**2 + c)/(a*x**5 + x**4), x)
```

Giac [B] time = 1.21167, size = 284, normalized size = 2.87

$$-\frac{1}{12}\left(\frac{24\,a^2c\arctan\left(\frac{\sqrt{-c+\frac{2c}{ax+1}}}{\sqrt{-c}}\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}\left(a\right)}{\sqrt{-c}}-\frac{2\left(3\,\pi a^2c-10\,a^2c\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}\left(a\right)}{\sqrt{-c}}+\frac{9\,a^2\left(c-\frac{2c}{ax+1}\right)^2c\sqrt{-c+\frac{2c}{ax+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="gia
c")
```

```
[Out] -1/12*(24*a^2*c*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(-c))*sgn(1/(a*x + 1))*
sgn(a)/sqrt(-c) - 2*(3*pi*a^2*c - 10*a^2*c)*sgn(1/(a*x + 1))*sgn(a)/sqrt(-c
) + (9*a^2*(c - 2*c/(a*x + 1))^2*c*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1)
)*sgn(a) + 3*a^2*c^3*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) - 8*a
^2*c^2*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/(a*x + 1))*sgn(a))/(c - c/(a*x + 1)
)^3)*abs(a)
```

3.1244 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=130

$$\frac{4a^3\sqrt{c-a^2cx^2}}{3x}-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -Sqrt[c - a^2*c*x^2]/(4*x^4) + (2*a*Sqrt[c - a^2*c*x^2])/(3*x^3) - (7*a^2*S
qrt[c - a^2*c*x^2])/(8*x^2) + (4*a^3*Sqrt[c - a^2*c*x^2])/(3*x) - (7*a^4*Sq
rt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```

Rubi [A] time = 0.308009, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6152, 1807, 835, 807, 266, 63, 208}

$$\frac{4a^3\sqrt{c-a^2cx^2}}{3x}-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x^5),x]
```


```
[Out] -Sqrt[c - a^2*c*x^2]/(4*x^4) + (2*a*Sqrt[c - a^2*c*x^2])/(3*x^3) - (7*a^2*S
qrt[c - a^2*c*x^2])/(8*x^2) + (4*a^3*Sqrt[c - a^2*c*x^2])/(3*x) - (7*a^4*Sq
rt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```


Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx &= c\int \frac{(1-ax)^2}{x^5\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{1}{4}\int \frac{8ac-7a^2cx}{x^4\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{\int \frac{21a^2c^2-16a^3c^2x}{x^3\sqrt{c-a^2cx^2}}\,dx}{12c}\\
&= -\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{\int \frac{32a^3c^3-21a^4c^3x}{x^2\sqrt{c-a^2cx^2}}\,dx}{24c^2}\\
&= -\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{1}{8}\left(7a^4c\right)\int \frac{1}{x\sqrt{c-a^2}}\\
&= -\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{1}{16}\left(7a^4c\right)\text{Subst}\left(\int\right.\\
&= -\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{4a^3\sqrt{c-a^2cx^2}}{3x}-\frac{1}{8}\left(7a^2\right)\text{Subst}\left(\int\right.\\
&= -\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{4a^3\sqrt{c-a^2cx^2}}{3x}-\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-}}{}\right.
\end{aligned}$$

Mathematica [A] time = 0.134225, size = 95, normalized size = 0.73

$$\frac{\left(32a^3x^3-21a^2x^2+16ax-6\right)\sqrt{c-a^2cx^2}}{24x^4}-\frac{7}{8}a^4\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+\frac{7}{8}a^4\sqrt{c}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-6 + 16*a*x - 21*a^2*x^2 + 32*a^3*x^3))/(24*x^4) + (7
*a^4*Sqrt[c]*Log[x])/8 - (7*a^4*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]
])/8
```

Maple [B] time = 0.052, size = 279, normalized size = 2.2

$$-\frac{1}{4cx^4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{9a^2}{8cx^2}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{7a^4}{8}\sqrt{c}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right)+\frac{7a^4}{8}\sqrt{-a^2cx^2+c}+2\frac{a^3\left(-a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x)
```

```
[Out] -1/4/c/x^4*(-a^2*c*x^2+c)^(3/2)-9/8*a^2/c/x^2*(-a^2*c*x^2+c)^(3/2)-7/8*a^4*
c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)+7/8*a^4*(-a^2*c*x^2+c)^(
1/2)+2*a^3/c/x*(-a^2*c*x^2+c)^(3/2)+2*a^5*x*(-a^2*c*x^2+c)^(1/2)+2*a^5*c/(a
^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2*a^4*(-c*a^2*(x+1
/a)^2+2*a*c*(x+1/a))^(1/2)-2*a^5*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c
*a^2*(x+1/a)^2+2*a*c*(x+1/a))^(1/2))+2/3*a/c/x^3*(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)}{(ax+1)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)/((a*x + 1)^2*x^5), x)
```

Fricas [A] time = 2.4791, size = 417, normalized size = 3.21

$$\left[\frac{21a^4\sqrt{c}x^4\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+2\left(32a^3x^3-21a^2x^2+16ax-6\right)\sqrt{-a^2cx^2+c}}{48x^4},-\frac{21a^4\sqrt{-c}x^4\arctan\left(\frac{\sqrt{-a^2c}}{a^2c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="fri
cas")
```

```
[Out] [1/48*(21*a^4*sqrt(c)*x^4*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) + 2*(32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqrt(-a^2*c*x^2 + c)
)/x^4, -1/24*(21*a^4*sqrt(-c)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2
*c*x^2 - c)) - (32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqrt(-a^2*c*x^2 + c))
/x^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{-a^2cx^2+c}}{ax^6+x^5}\,dx - \int \frac{ax\sqrt{-a^2cx^2+c}}{ax^6+x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**5,x)
```

```
[Out] -Integral(-sqrt(-a**2*c*x**2 + c)/(a*x**6 + x**5), x) - Integral(a*x*sqrt(-
a**2*c*x**2 + c)/(a*x**6 + x**5), x)
```

Giac [B] time = 1.17048, size = 348, normalized size = 2.68

$$\frac{1}{192}\left(\frac{336\,a^3c\arctan\left(\frac{\sqrt{-c+\frac{2c}{ax+1}}}{\sqrt{-c}}\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}\left(a\right)}{\sqrt{-c}} - \frac{4\left(21\,\pi a^3c-64\,a^3c\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}\left(a\right)}{\sqrt{-c}} + \frac{75\,a^3\left(c-\frac{2c}{ax+1}\right)^3c\sqrt{-c+}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="gia
c")
```

```
[Out] 1/192*(336*a^3*c*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(-c))*sgn(1/(a*x + 1))
*sgn(a)/sqrt(-c) - 4*(21*pi*a^3*c - 64*a^3*c)*sgn(1/(a*x + 1))*sgn(a)/sqrt(
-c) + (75*a^3*(c - 2*c/(a*x + 1))^3*c*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x +
 1))*sgn(a) + 83*a^3*(c - 2*c/(a*x + 1))^2*c^2*sqrt(-c + 2*c/(a*x + 1))*sgn
(1/(a*x + 1))*sgn(a) + 21*a^3*c^4*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))
*sgn(a) - 77*a^3*c^3*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/(a*x + 1))*sgn(a))/(c
 - c/(a*x + 1))^4)*abs(a)
```

3.1245 $\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=108

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}+\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

```
[Out] (5*c*x*Sqrt[c - a^2*c*x^2])/8 + (5*(c - a^2*c*x^2)^(3/2))/(12*a) + ((1 - a*
x)*(c - a^2*c*x^2)^(3/2))/(4*a) + (5*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
a^2*c*x^2]])/(8*a)
```

Rubi [A] time = 0.0865184, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6142, 671, 641, 195, 217, 203}

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}+\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (5*c*x*Sqrt[c - a^2*c*x^2])/8 + (5*(c - a^2*c*x^2)^(3/2))/(12*a) + ((1 - a*
x)*(c - a^2*c*x^2)^(3/2))/(4*a) + (5*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
a^2*c*x^2]])/(8*a)
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx &= c\int (1-ax)^2\sqrt{c-a^2cx^2}\,dx\\ &= \frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{1}{4}(5c)\int (1-ax)\sqrt{c-a^2cx^2}\,dx\\ &= \frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{1}{4}(5c)\int \sqrt{c-a^2cx^2}\,dx\\ &= \frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{1}{8}\left(5c^2\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,d\\ &= \frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{1}{8}\left(5c^2\right)\text{Subst}\left(\int \frac{1}{1+a}\right.\\ &= \frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}\end{aligned}$$

Mathematica [A] time = 0.0970476, size = 117, normalized size = 1.08

$$-\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(6a^4x^4-22a^3x^3+25a^2x^2+7ax-16\right)+30\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(-16 + 7*a*x + 25*a^2*x^2 - 22*a^3*x
^3 + 6*a^4*x^4) + 30*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(24*a*Sq
rt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.035, size = 174, normalized size = 1.6

$$-\frac{x}{4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{3\,cx}{8}\sqrt{-a^2cx^2+c}-\frac{3\,c^2}{8}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}+\frac{2}{3\,a}\left(-ca^2\left(x+a^{-1}\right)^2+2\,ac\left(x+a^{-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/4*x*(-a^2*c*x^2+c)^(3/2)-3/8*c*x*(-a^2*c*x^2+c)^(1/2)-3/8*c^2/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/3/a*(-c*a^2*(x+1/a)^2+2*
```

```
a*c*(x+1/a))^(3/2)+c*(-c*a^2*(x+1/a)^2+2*a*c*(x+1/a))^(1/2)*x+c^2/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x+1/a)^2+2*a*c*(x+1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.45324, size = 413, normalized size = 3.82

$$\left[\frac{15\sqrt{-c}c\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+2\left(6a^3cx^3-16a^2cx^2+9acx+16c\right)\sqrt{-a^2cx^2+c}}{48a}, -\frac{15c^{\frac{3}{2}}\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] [1/48*(15*sqrt(-c)*c*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c) + 2*(6*a^3*c*x^3 - 16*a^2*c*x^2 + 9*a*c*x + 16*c)*sqrt(-a^2*c*x^2 + c)
)/a, -1/24*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 -
 c)) - (6*a^3*c*x^3 - 16*a^2*c*x^2 + 9*a*c*x + 16*c)*sqrt(-a^2*c*x^2 + c))/
a]
```

Sympy [C] time = 8.39055, size = 340, normalized size = 3.15

$$a^2c\left(\begin{cases}\frac{ia^2\sqrt{c}x^5}{4\sqrt{a^2x^2-1}}-\frac{3i\sqrt{c}x^3}{8\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}x}{8a^2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\operatorname{acosh}(ax)}{8a^3} & \text{for } |a^2x^2|>1\\ -\frac{a^2\sqrt{c}x^5}{4\sqrt{-a^2x^2+1}}+\frac{3\sqrt{c}x^3}{8\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\operatorname{asin}(ax)}{8a^3} & \text{otherwise}\end{cases}\right)-2ac\left(\begin{cases}0 & \text{for } c=0\\ \frac{\sqrt{c}x^2}{2} & \text{for } a^2=0\\ -\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3a^2c} & \text{otherwise}\end{cases}\right)+c\left(\begin{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] a**2*c*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*sqrt(c)
*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x**2 - 1)) -
I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**5/(4*
sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1)) - sqrt(c)*x
/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), True)) - 2*a*c
*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 +
c)**(3/2)/(3*a**2*c), True)) + c*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**
2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(
```

```
2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin
(a*x)/(2*a), True))
```

Giac [B] time = 1.25445, size = 302, normalized size = 2.8

$$-\frac{\left(240\,a^{5}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-c+\frac{2\,c}{ax+1}}}{\sqrt{c}}\right)\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}\left(a\right)-\frac{\left(15\,a^{5}\left(c-\frac{2\,c}{ax+1}\right)^{3}c^{2}\sqrt{-c+\frac{2\,c}{ax+1}}\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}(a)+73\,a^{5}\left(c-\frac{2\,c}{ax+1}\right)^{2}c^{3}\sqrt{-c+\frac{2\,c}{ax+1}}\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sg}\cdots\right.}{c^{4}\cdots}\right.}{192\,a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/192*(240*a^5*c^(3/2)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1/(a*x
 + 1))*sgn(a) - (15*a^5*(c - 2*c/(a*x + 1))^3*c^2*sqrt(-c + 2*c/(a*x + 1))*
sgn(1/(a*x + 1))*sgn(a) + 73*a^5*(c - 2*c/(a*x + 1))^2*c^3*sqrt(-c + 2*c/(a
*x + 1))*sgn(1/(a*x + 1))*sgn(a) + 15*a^5*c^5*sqrt(-c + 2*c/(a*x + 1))*sgn(
1/(a*x + 1))*sgn(a) + 55*a^5*c^4*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/(a*x + 1)
)*sgn(a))*(a*x + 1)^4/c^4)*abs(a)/a^7
```

3.1246 $\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=131

$$\frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

```
[Out] (7*c^2*x*Sqrt[c - a^2*c*x^2])/16 + (7*c*x*(c - a^2*c*x^2)^(3/2))/24 + (7*(c
 - a^2*c*x^2)^(5/2))/(30*a) + ((1 - a*x)*(c - a^2*c*x^2)^(5/2))/(6*a) + (7*
c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a)
```

Rubi [A] time = 0.0968, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6142, 671, 641, 195, 217, 203}

$$\frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (7*c^2*x*Sqrt[c - a^2*c*x^2])/16 + (7*c*x*(c - a^2*c*x^2)^(3/2))/24 + (7*(c
 - a^2*c*x^2)^(5/2))/(30*a) + ((1 - a*x)*(c - a^2*c*x^2)^(5/2))/(6*a) + (7*
c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a)
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx = c\int (1-ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$= \frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a} + \frac{1}{6}(7c)\int (1-ax)\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$= \frac{7\left(c-a^2cx^2\right)^{5/2}}{30a} + \frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a} + \frac{1}{6}(7c)\int \left(c-a^2cx^2\right)^{3/2}\,dx$$
$$= \frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2} + \frac{7\left(c-a^2cx^2\right)^{5/2}}{30a} + \frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a} + \frac{1}{8}\left(7c^2\right)\int \sqrt{c-a^2c}$$
$$= \frac{7}{16}c^2x\sqrt{c-a^2cx^2} + \frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2} + \frac{7\left(c-a^2cx^2\right)^{5/2}}{30a} + \frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a} +$$
$$= \frac{7}{16}c^2x\sqrt{c-a^2cx^2} + \frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2} + \frac{7\left(c-a^2cx^2\right)^{5/2}}{30a} + \frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a} +$$
$$= \frac{7}{16}c^2x\sqrt{c-a^2cx^2} + \frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2} + \frac{7\left(c-a^2cx^2\right)^{5/2}}{30a} + \frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a} +$$

Mathematica [A] time = 0.139402, size = 135, normalized size = 1.03

$$\frac{c^2\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(40a^6x^6-136a^5x^5+86a^4x^4+202a^3x^3-327a^2x^2+39ax+96\right)-210\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{240a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(96 + 39*a*x - 327*a^2*x^2 + 202*a^
3*x^3 + 86*a^4*x^4 - 136*a^5*x^5 + 40*a^6*x^6) - 210*Sqrt[1 - a*x]*ArcSin[S
qrt[1 - a*x]/Sqrt[2]]))/(240*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.036, size = 226, normalized size = 1.7

$$-\frac{x}{6}\left(-a^2cx^2+c\right)^{\frac{5}{2}} - \frac{5\,cx}{24}\left(-a^2cx^2+c\right)^{\frac{3}{2}} - \frac{5\,xc^2}{16}\sqrt{-a^2cx^2+c} - \frac{5\,c^3}{16}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}} + \frac{2}{5\,a}\left(-ca^2\left(x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/6*x*(-a^2*c*x^2+c)^(5/2)-5/24*c*x*(-a^2*c*x^2+c)^(3/2)-5/16*c^2*x*(-a^2*
c*x^2+c)^(1/2)-5/16*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)
^(1/2))+2/5/a*(-c*a^2*(x+1/a)^2+2*a*c*(x+1/a))^(5/2)+1/2*c*(-c*a^2*(x+1/a)^
2+2*a*c*(x+1/a))^(3/2)*x+3/4*c^2*(-c*a^2*(x+1/a)^2+2*a*c*(x+1/a))^(1/2)*x+3
/4*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x+1/a)^2+2*a*c*(x+1/a)
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.67876, size = 545, normalized size = 4.16

$$\left[\frac{105\,\sqrt{-c}c^2\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\left(40\,a^5c^2x^5-96\,a^4c^2x^4-10\,a^3c^2x^3+192\,a^2c^2x^2-135\,ac^2x-\right.}{480\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] [1/480*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c
)*x - c) - 2*(40*a^5*c^2*x^5 - 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 192*a^2*c^
2*x^2 - 135*a*c^2*x - 96*c^2)*sqrt(-a^2*c*x^2 + c))/a, -1/240*(105*c^(5/2)*
arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (40*a^5*c^2*x^5
- 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 192*a^2*c^2*x^2 - 135*a*c^2*x - 96*c^2)
*sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [C] time = 13.758, size = 478, normalized size = 3.65

$$-a^4c^2\left(\begin{cases}\frac{ia^2\sqrt{c}x^7}{6\sqrt{a^2x^2-1}}-\frac{5i\sqrt{c}x^5}{24\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}x^3}{48a^2\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}x}{16a^4\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,\mathrm{acosh}\,(ax)}{16a^5} & \text{for } \left|a^2x^2\right|>1\\ -\frac{a^2\sqrt{c}x^7}{6\sqrt{-a^2x^2+1}}+\frac{5\sqrt{c}x^5}{24\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}x^3}{48a^2\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}x}{16a^4\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\,\mathrm{asin}\,(ax)}{16a^5} & \text{otherwise}\end{cases}\right)+2a^3c^2\left(\begin{cases}\frac{x^4\sqrt{-a^2cx^2+c}}{5}-\\ \frac{\sqrt{c}x^4}{4}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -a**4*c**2*Piecewise((I*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqr
t(c)*x**5/(24*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2
 - 1)) + I*sqrt(c)*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(
```

```
16*a**5), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1))
 + 5*sqrt(c)*x**5/(24*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a
**2*x**2 + 1)) - sqrt(c)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*
x)/(16*a**5), True)) + 2*a**3*c**2*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5
 - x**2*sqrt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**
4), Ne(a, 0)), (sqrt(c)*x**4/4, True)) - 2*a*c**2*Piecewise((0, Eq(c, 0)),
(sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True
)) + c**2*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c
)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) >
1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin(a*x)/(2*a), True))
```

Giac [B] time = 1.38837, size = 432, normalized size = 3.3

$$-\frac{\left(6720\,a^{7}c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-c+\frac{2\,c}{ax+1}}}{\sqrt{c}}\right)\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}\left(a\right)-\frac{\left(105\,a^{7}\left(c-\frac{2\,c}{ax+1}\right)^{5}c^{3}\sqrt{-c+\frac{2\,c}{ax+1}}\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}(a)-595\,a^{7}\left(c-\frac{2\,c}{ax+1}\right)^{4}c^{4}\sqrt{-c+\frac{2\,c}{ax+1}}\operatorname{sgn}\left(\frac{1}{ax+1}\right.\cdots\right.}{\cdots}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/7680*(6720*a^7*c^(5/2)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1/(a
*x + 1))*sgn(a) - (105*a^7*(c - 2*c/(a*x + 1))^5*c^3*sqrt(-c + 2*c/(a*x + 1
))*sgn(1/(a*x + 1))*sgn(a) - 595*a^7*(c - 2*c/(a*x + 1))^4*c^4*sqrt(-c + 2*
c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) - 1686*a^7*(c - 2*c/(a*x + 1))^3*c^5*s
qrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) + 1386*a^7*(c - 2*c/(a*x +
1))^2*c^6*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) + 105*a^7*c^8*sq
rt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) + 595*a^7*c^7*(-c + 2*c/(a*x
 + 1))^(3/2)*sgn(1/(a*x + 1))*sgn(a))*(a*x + 1)^6/c^6)*abs(a)/a^9
```

3.1247 $\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=154

$$\frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{45c^{7/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{128a}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{(1-ax)\left(c-a^2cx^2\right)^{7/2}}{8a}+$$

```
[Out] (45*c^3*x*Sqrt[c - a^2*c*x^2])/128 + (15*c^2*x*(c - a^2*c*x^2)^(3/2))/64 +
(3*c*x*(c - a^2*c*x^2)^(5/2))/16 + (9*(c - a^2*c*x^2)^(7/2))/(56*a) + ((1 -
 a*x)*(c - a^2*c*x^2)^(7/2))/(8*a) + (45*c^(7/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(128*a)
```

Rubi [A] time = 0.116381, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6142, 671, 641, 195, 217, 203}

$$\frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{45c^{7/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{128a}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{(1-ax)\left(c-a^2cx^2\right)^{7/2}}{8a}+$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(7/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (45*c^3*x*Sqrt[c - a^2*c*x^2])/128 + (15*c^2*x*(c - a^2*c*x^2)^(3/2))/64 +
(3*c*x*(c - a^2*c*x^2)^(5/2))/16 + (9*(c - a^2*c*x^2)^(7/2))/(56*a) + ((1 -
 a*x)*(c - a^2*c*x^2)^(7/2))/(8*a) + (45*c^(7/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(128*a)
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = c\int(1-ax)^2\left(c-a^2cx^2\right)^{5/2}\,dx$$
$$= \frac{(1-ax)\left(c-a^2cx^2\right)^{7/2}}{8a}+\frac{1}{8}(9c)\int(1-ax)\left(c-a^2cx^2\right)^{5/2}\,dx$$
$$= \frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1-ax)\left(c-a^2cx^2\right)^{7/2}}{8a}+\frac{1}{8}(9c)\int\left(c-a^2cx^2\right)^{5/2}\,dx$$
$$= \frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1-ax)\left(c-a^2cx^2\right)^{7/2}}{8a}+\frac{1}{16}\left(15c^2\right)\int\left(c-a^2\right.$$
$$= \frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1-ax)\left(c-a^2cx^2\right)^{7/2}}{8a}$$
$$= \frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+$$
$$= \frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+$$
$$= \frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+$$

Mathematica [A] time = 0.129328, size = 151, normalized size = 0.98

$$-\frac{c^3\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(112a^8x^8-368a^7x^7+88a^6x^6+936a^5x^5-978a^4x^4-558a^3x^3+1349a^2x^2-325ax-256\right)+630\right.}{896a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(7/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c^3*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(-256 - 325*a*x + 1349*a^2*x^2 - 5
58*a^3*x^3 - 978*a^4*x^4 + 936*a^5*x^5 + 88*a^6*x^6 - 368*a^7*x^7 + 112*a^8
*x^8) + 630*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(896*a*Sqrt[1 - a
*x]*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.037, size = 276, normalized size = 1.8

$$-\frac{x}{8}\left(-a^2cx^2+c\right)^{\frac{7}{2}}-\frac{7\,cx}{48}\left(-a^2cx^2+c\right)^{\frac{5}{2}}-\frac{35\,xc^2}{192}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{35\,c^3x}{128}\sqrt{-a^2cx^2+c}-\frac{35\,c^4}{128}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/8*x*(-a^2*c*x^2+c)^(7/2)-7/48*c*x*(-a^2*c*x^2+c)^(5/2)-35/192*c^2*x*(-a^
2*c*x^2+c)^(3/2)-35/128*c^3*x*(-a^2*c*x^2+c)^(1/2)-35/128*c^4/(a^2*c)^(1/2)
*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/7/a*(-c*a^2*(x+1/a)^2+2*a*c
*(x+1/a))^(7/2)+1/3*c*(-c*a^2*(x+1/a)^2+2*a*c*(x+1/a))^(5/2)*x+5/12*c^2*(-c
*a^2*(x+1/a)^2+2*a*c*(x+1/a))^(3/2)*x+5/8*c^3*(-c*a^2*(x+1/a)^2+2*a*c*(x+1/
a))^(1/2)*x+5/8*c^4/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x+1/a)^2+
2*a*c*(x+1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.50984, size = 655, normalized size = 4.25

$$\left[\frac{315\sqrt{-c}c^3\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+2\left(112a^7c^3x^7-256a^6c^3x^6-168a^5c^3x^5+768a^4c^3x^4-210a^3c\right.}{1792a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] [1/1792*(315*sqrt(-c)*c^3*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) + 2*(112*a^7*c^3*x^7 - 256*a^6*c^3*x^6 - 168*a^5*c^3*x^5 + 768*a^
4*c^3*x^4 - 210*a^3*c^3*x^3 - 768*a^2*c^3*x^2 + 581*a*c^3*x + 256*c^3)*sqrt
(-a^2*c*x^2 + c))/a, -1/896*(315*c^(7/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt
(c)*x/(a^2*c*x^2 - c)) - (112*a^7*c^3*x^7 - 256*a^6*c^3*x^6 - 168*a^5*c^3*x
^5 + 768*a^4*c^3*x^4 - 210*a^3*c^3*x^3 - 768*a^2*c^3*x^2 + 581*a*c^3*x + 25
6*c^3)*sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(7/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42158, size = 562, normalized size = 3.65

$$-\frac{\left(80640\,a^9c^{\frac{7}{2}}\arctan\left(\frac{\sqrt{-c+\frac{2c}{ax+1}}}{\sqrt{c}}\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}\left(a\right)-\frac{\left(315\,a^9\left(c-\frac{2c}{ax+1}\right)^7c^4\sqrt{-c+\frac{2c}{ax+1}}\,\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)-2415\,a^9\left(c-\frac{2c}{ax+1}\right)^6c^5\sqrt{-c+\frac{2c}{ax+1}}\,\mathrm{sgn}\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/114688*(80640*a^9*c^(7/2)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1
/(a*x + 1))*sgn(a) - (315*a^9*(c - 2*c/(a*x + 1))^7*c^4*sqrt(-c + 2*c/(a*x
+ 1))*sgn(1/(a*x + 1))*sgn(a) - 2415*a^9*(c - 2*c/(a*x + 1))^6*c^5*sqrt(-c
+ 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) + 8043*a^9*(c - 2*c/(a*x + 1))^5*c
^6*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) + 17609*a^9*(c - 2*c/(a
*x + 1))^4*c^7*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) - 15159*a^9
*(c - 2*c/(a*x + 1))^3*c^8*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a)
 + 8043*a^9*(c - 2*c/(a*x + 1))^2*c^9*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x +
 1))*sgn(a) + 315*a^9*c^11*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a)
 + 2415*a^9*c^10*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/(a*x + 1))*sgn(a))*(a*x +
 1)^8/c^8)*abs(a)/a^11
```

3.1248 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}-\frac{\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] (-2*(1 - a*x))/(a*Sqrt[c - a^2*c*x^2]) - ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*
c*x^2]]/(a*Sqrt[c])
```

Rubi [A] time = 0.0669253, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6142, 653, 217, 203}

$$-\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}-\frac{\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (-2*(1 - a*x))/(a*Sqrt[c - a^2*c*x^2]) - ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*
c*x^2]]/(a*Sqrt[c])
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$
$$= -\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}} - \int \frac{1}{\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}} - \text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)$$
$$= -\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}} - \frac{\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

Mathematica [A] time = 0.0639273, size = 100, normalized size = 1.64

$$\frac{2\sqrt{1-a^2x^2}\left(\sqrt{ax+1}(ax-1)+\sqrt{1-ax}(ax+1)\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-ax}(ax+1)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2]*((-1 + a*x)*Sqrt[1 + a*x] + Sqrt[1 - a*x]*(1 + a*x)*Ar
cSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*Sqrt[1 - a*x]*(1 + a*x)*Sqrt[c - a^2*c*x^2
])
```

Maple [A] time = 0.033, size = 74, normalized size = 1.2

$$-\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}} - 2\frac{\sqrt{-ca^2\left(x+a^{-1}\right)^2+2\,ac\left(x+a^{-1}\right)}}{a^2c\left(x+a^{-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/a^2/c/(x+1/
a)*(-c*a^2*(x+1/a)^2+2*a*c*(x+1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.51008, size = 338, normalized size = 5.54

$$\left[-\frac{(ax+1)\sqrt{-c}\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+4\,\sqrt{-a^2cx^2+c}}{2\left(a^2cx+ac\right)},\ \frac{(ax+1)\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)-2\,\sqrt{-a\right.}{a^2cx+ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/2*((a*x + 1)*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) + 4*sqrt(-a^2*c*x^2 + c))/(a^2*c*x + a*c), ((a*x + 1)*sqrt(c)*arc
tan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*c*x^2 +
 c))/(a^2*c*x + a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{ax\sqrt{-a^2cx^2+c}+\sqrt{-a^2cx^2+c}}\,dx-\int -\frac{1}{ax\sqrt{-a^2cx^2+c}+\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x*sqrt(-a**2*c*x**2 + c) + sqrt(-a**2*c*x**2 + c)), x) - I
ntegral(-1/(a*x*sqrt(-a**2*c*x**2 + c) + sqrt(-a**2*c*x**2 + c)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a^2x^2-1}{\sqrt{-a^2cx^2+c}(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(-(a^2*x^2 - 1)/(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^2), x)
```

3.1249 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{x}{3c\sqrt{c-a^2cx^2}}-\frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] (-2*(1 - a*x))/(3*a*(c - a^2*c*x^2)^(3/2)) + x/(3*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.0642532, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6142, 653, 191}

$$\frac{x}{3c\sqrt{c-a^2cx^2}}-\frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (-2*(1 - a*x))/(3*a*(c - a^2*c*x^2)^(3/2)) + x/(3*c*Sqrt[c - a^2*c*x^2])
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx\\ &= -\frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}+\frac{1}{3}\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\ &= -\frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}+\frac{x}{3c\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0362213, size = 63, normalized size = 1.21

$$-\frac{\sqrt{1-ax}(ax+2)\sqrt{1-a^2x^2}}{3ac(ax+1)^{3/2}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -(Sqrt[1 - a*x]*(2 + a*x)*Sqrt[1 - a^2*x^2])/(3*a*c*(1 + a*x)^(3/2)*Sqrt[c
- a^2*c*x^2])
```

Maple [A] time = 0.029, size = 31, normalized size = 0.6

$$-\frac{(ax-1)^2(ax+2)}{3a}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/3*(a*x-1)^2*(a*x+2)/(-a^2*c*x^2+c)^(3/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.46851, size = 99, normalized size = 1.9

$$-\frac{\sqrt{-a^2cx^2+c}(ax+2)}{3\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/3*sqrt(-a^2*c*x^2 + c)*(a*x + 2)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{ax}{-a^3cx^3\sqrt{-a^2cx^2+c}-a^2cx^2\sqrt{-a^2cx^2+c}+acx\sqrt{-a^2cx^2+c}+c\sqrt{-a^2cx^2+c}}\,dx-\int-\frac{}{-a^3cx^3\sqrt{-a^2cx^2+c}-a^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(a*x/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) - a**2*c*x**2*sqrt(-a**2
*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) + c*sqrt(-a**2*c*x**2 + c)), x)
 - Integral(-1/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) - a**2*c*x**2*sqrt(-a**
2*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) + c*sqrt(-a**2*c*x**2 + c)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a^2x^2-1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(-(a^2*x^2 - 1)/((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^2), x)
```

3.1250 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=75

$$\frac{2x}{5c^2\sqrt{c-a^2cx^2}}+\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}-\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] (-2*(1 - a*x))/(5*a*(c - a^2*c*x^2)^(5/2)) + x/(5*c*(c - a^2*c*x^2)^(3/2))
+ (2*x)/(5*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.0705397, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6142, 653, 192, 191}

$$\frac{2x}{5c^2\sqrt{c-a^2cx^2}}+\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}-\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (-2*(1 - a*x))/(5*a*(c - a^2*c*x^2)^(5/2)) + x/(5*c*(c - a^2*c*x^2)^(3/2))
+ (2*x)/(5*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] && !IntegerQ[p] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{7/2}}\,dx$$
$$= -\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}} + \frac{3}{5}\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx$$
$$= -\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}} + \frac{x}{5c\left(c-a^2cx^2\right)^{3/2}} + \frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{5c}$$
$$= -\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}} + \frac{x}{5c\left(c-a^2cx^2\right)^{3/2}} + \frac{2x}{5c^2\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0456087, size = 79, normalized size = 1.05

$$\frac{\sqrt{1-a^2x^2}\left(2a^3x^3+4a^2x^2+ax-2\right)}{5ac^2\sqrt{1-ax}(ax+1)^{5/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2 + a*x + 4*a^2*x^2 + 2*a^3*x^3))/(5*a*c^2*Sqrt[1 - a*
x]*(1 + a*x)^(5/2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.03, size = 47, normalized size = 0.6

$$\frac{(ax-1)^2\left(2x^3a^3+4a^2x^2+ax-2\right)}{5a}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/5*(a*x-1)^2*(2*a^3*x^3+4*a^2*x^2+a*x-2)/(-a^2*c*x^2+c)^(5/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.91879, size = 153, normalized size = 2.04

$$-\frac{\left(2 a^{3} x^{3}+4 a^{2} x^{2}+a x-2\right) \sqrt{-a^{2} c x^{2}+c}}{5\left(a^{5} c^{3} x^{4}+2 a^{4} c^{3} x^{3}-2 a^{2} c^{3} x-a c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/5*(2*a^3*x^3 + 4*a^2*x^2 + a*x - 2)*sqrt(-a^2*c*x^2 + c)/(a^5*c^3*x^4 +
2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{a^5c^2x^5\sqrt{-a^2cx^2+c}+a^4c^2x^4\sqrt{-a^2cx^2+c}-2a^3c^2x^3\sqrt{-a^2cx^2+c}-2a^2c^2x^2\sqrt{-a^2cx^2+c}+ac^2x\sqrt{-a^2cx^2+c}+c^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] -Integral(a*x/(a**5*c**2*x**5*sqrt(-a**2*c*x**2 + c) + a**4*c**2*x**4*sqrt(
-a**2*c*x**2 + c) - 2*a**3*c**2*x**3*sqrt(-a**2*c*x**2 + c) - 2*a**2*c**2*x
**2*sqrt(-a**2*c*x**2 + c) + a*c**2*x*sqrt(-a**2*c*x**2 + c) + c**2*sqrt(-a
**2*c*x**2 + c)), x) - Integral(-1/(a**5*c**2*x**5*sqrt(-a**2*c*x**2 + c) +
 a**4*c**2*x**4*sqrt(-a**2*c*x**2 + c) - 2*a**3*c**2*x**3*sqrt(-a**2*c*x**2
 + c) - 2*a**2*c**2*x**2*sqrt(-a**2*c*x**2 + c) + a*c**2*x*sqrt(-a**2*c*x**
2 + c) + c**2*sqrt(-a**2*c*x**2 + c)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a^{2} x^{2}-1}{\left(-a^{2} c x^{2}+c\right)^{\frac{5}{2}}(a x+1)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(-(a^2*x^2 - 1)/((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^2), x)
```

3.1251 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=98

$$\frac{8x}{21c^3\sqrt{c-a^2cx^2}}+\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

```
[Out] (-2*(1 - a*x))/(7*a*(c - a^2*c*x^2)^(7/2)) + x/(7*c*(c - a^2*c*x^2)^(5/2))
+ (4*x)/(21*c^2*(c - a^2*c*x^2)^(3/2)) + (8*x)/(21*c^3*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.08145, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6142, 653, 192, 191}

$$\frac{8x}{21c^3\sqrt{c-a^2cx^2}}+\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (-2*(1 - a*x))/(7*a*(c - a^2*c*x^2)^(7/2)) + x/(7*c*(c - a^2*c*x^2)^(5/2))
+ (4*x)/(21*c^2*(c - a^2*c*x^2)^(3/2)) + (8*x)/(21*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{9/2}}\,dx$$
$$= -\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}} + \frac{5}{7}\int \frac{1}{\left(c-a^2cx^2\right)^{7/2}}\,dx$$
$$= -\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}} + \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} + \frac{4\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{7c}$$
$$= -\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}} + \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} + \frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}} + \frac{8\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{21c^2}$$
$$= -\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}} + \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} + \frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}} + \frac{8x}{21c^3\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0544708, size = 96, normalized size = 0.98

$$-\frac{\sqrt{1-a^2x^2}\left(8a^5x^5+16a^4x^4-4a^3x^3-24a^2x^2-9ax+6\right)}{21ac^3(1-ax)^{3/2}(ax+1)^{7/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]*(6 - 9*a*x - 24*a^2*x^2 - 4*a^3*x^3 + 16*a^4*x^4 + 8*a^
5*x^5))/(21*a*c^3*(1 - a*x)^(3/2)*(1 + a*x)^(7/2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.03, size = 64, normalized size = 0.7

$$-\frac{(ax-1)^2\left(8x^5a^5+16x^4a^4-4x^3a^3-24a^2x^2-9ax+6\right)}{21a}\left(-a^2cx^2+c\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/21*(a*x-1)^2*(8*a^5*x^5+16*a^4*x^4-4*a^3*x^3-24*a^2*x^2-9*a*x+6)/(-a^2*c
*x^2+c)^(7/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.60181, size = 251, normalized size = 2.56

$$-\frac{\left(8\,a^{5}x^{5}+16\,a^{4}x^{4}-4\,a^{3}x^{3}-24\,a^{2}x^{2}-9\,ax+6\right)\sqrt{-a^{2}cx^{2}+c}}{21\left(a^{7}c^{4}x^{6}+2\,a^{6}c^{4}x^{5}-a^{5}c^{4}x^{4}-4\,a^{4}c^{4}x^{3}-a^{3}c^{4}x^{2}+2\,a^{2}c^{4}x+ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="frica
s")
```

```
[Out] -1/21*(8*a^5*x^5 + 16*a^4*x^4 - 4*a^3*x^3 - 24*a^2*x^2 - 9*a*x + 6)*sqrt(-a
^2*c*x^2 + c)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 -
a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax}{-a^{7}c^{3}x^{7}\sqrt{-a^{2}cx^{2}+c}-a^{6}c^{3}x^{6}\sqrt{-a^{2}cx^{2}+c}+3a^{5}c^{3}x^{5}\sqrt{-a^{2}cx^{2}+c}+3a^{4}c^{3}x^{4}\sqrt{-a^{2}cx^{2}+c}-3a^{3}c^{3}x^{3}\sqrt{-a^{2}cx^{2}+c}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] -Integral(a*x/(-a**7*c**3*x**7*sqrt(-a**2*c*x**2 + c) - a**6*c**3*x**6*sqrt
(-a**2*c*x**2 + c) + 3*a**5*c**3*x**5*sqrt(-a**2*c*x**2 + c) + 3*a**4*c**3*
x**4*sqrt(-a**2*c*x**2 + c) - 3*a**3*c**3*x**3*sqrt(-a**2*c*x**2 + c) - 3*a
**2*c**3*x**2*sqrt(-a**2*c*x**2 + c) + a*c**3*x*sqrt(-a**2*c*x**2 + c) + c*
*3*sqrt(-a**2*c*x**2 + c)), x) - Integral(-1/(-a**7*c**3*x**7*sqrt(-a**2*c*
x**2 + c) - a**6*c**3*x**6*sqrt(-a**2*c*x**2 + c) + 3*a**5*c**3*x**5*sqrt(-
a**2*c*x**2 + c) + 3*a**4*c**3*x**4*sqrt(-a**2*c*x**2 + c) - 3*a**3*c**3*x*
*3*sqrt(-a**2*c*x**2 + c) - 3*a**2*c**3*x**2*sqrt(-a**2*c*x**2 + c) + a*c**
3*x*sqrt(-a**2*c*x**2 + c) + c**3*sqrt(-a**2*c*x**2 + c)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a^{2}x^{2}-1}{\left(-a^{2}cx^{2}+c\right)^{\frac{7}{2}}(ax+1)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate(-(a^2*x^2 - 1)/((-a^2*c*x^2 + c)^(7/2)*(a*x + 1)^2), x)
```

3.1252 $\int e^{-2\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=172

$$\frac{c(2m+3)\sqrt{1-a^2x^2}x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}-\frac{2ac\sqrt{1-a^2x^2}x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\ldots\right)}{(m+2)\sqrt{c-a^2cx^2}}$$

```
[Out] -((x^(1 + m)*Sqrt[c - a^2*c*x^2])/(2 + m)) + (c*(3 + 2*m)*x^(1 + m)*Sqrt[1
- a^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*
(2 + m)*Sqrt[c - a^2*c*x^2]) - (2*a*c*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeom
etric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.27619, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6152, 1809, 808, 365, 364}

$$\frac{c(2m+3)\sqrt{1-a^2x^2}x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}-\frac{2ac\sqrt{1-a^2x^2}x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -((x^(1 + m)*Sqrt[c - a^2*c*x^2])/(2 + m)) + (c*(3 + 2*m)*x^(1 + m)*Sqrt[1
- a^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*
(2 + m)*Sqrt[c - a^2*c*x^2]) - (2*a*c*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeom
etric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= c\int\frac{x^m(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\\ &= -\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-\frac{\int\frac{x^m\left(-a^2c(3+2m)+2a^3c(2+m)x\right)}{\sqrt{c-a^2cx^2}}\,dx}{a^2(2+m)}\\ &= -\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-(2ac)\int\frac{x^{1+m}}{\sqrt{c-a^2cx^2}}\,dx+\frac{(c(3+2m))\int\frac{x^m}{\sqrt{c-a^2cx^2}}\,dx}{2+m}\\ &= -\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-\frac{\left(2ac\sqrt{1-a^2x^2}\right)\int\frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}+\frac{\left(c(3+2m)\sqrt{1-a^2x^2}\right)\int\frac{x^m}{\sqrt{1-a^2x^2}}\,d}{(2+m)\sqrt{c-a^2cx^2}}\\ &= -\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}+\frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}}-\frac{2acx^{2+m}\sqrt{1-}}{(2}\end{aligned}$$

Mathematica [C] time = 0.167008, size = 111, normalized size = 0.65

$$\frac{x^{m+1}\left(\frac{2\sqrt{c-acx}F_1\left(m+1;\frac{1}{2},-\frac{1}{2};m+2;-ax,ax\right)}{\sqrt{1-ax}}-\frac{\sqrt{c-a^2cx^2}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{\sqrt{1-a^2x^2}}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (x^(1 + m)*((2*Sqrt[c - a*c*x]*AppellF1[1 + m, 1/2, -1/2, 2 + m, -(a*x), a*
x])/Sqrt[1 - a*x] - (Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2,
 (3 + m)/2, a^2*x^2])/Sqrt[1 - a^2*x^2]))/(1 + m)
```

Maple [F] time = 0.312, size = 0, normalized size = 0.

$$\int\frac{x^m\left(-a^2x^2+1\right)}{(ax+1)^2}\sqrt{-a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] int(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)x^m}{\left(ax+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)*x^m/(a*x + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2cx^2+c}(ax-1)x^m}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x^m\sqrt{-a^2cx^2+c}}{ax+1}\,dx - \int \frac{axx^m\sqrt{-a^2cx^2+c}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x**m*sqrt(-a**2*c*x**2 + c)/(a*x + 1), x) - Integral(a*x*x**m*sq
rt(-a**2*c*x**2 + c)/(a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)x^m}{\left(ax+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="gia
c")
```

```
[Out] integrate(-sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)*x^m/(a*x + 1)^2, x)
```

3.1253 $\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=54

$$\frac{2^{p+1}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-p-1,p,p+1,\frac{1}{2}(ax+1)\right)}{ap}$$

```
[Out] (2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 + a*x)/
2])/(a*p*(1 - a*x)^p)
```

Rubi [A] time = 0.0585352, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6142, 678, 69}

$$\frac{2^{p+1}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-1,p;p+1;\frac{1}{2}(ax+1)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 + a*x)/
2])/(a*p*(1 - a*x)^p)
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=c\int(1-ax)^2\left(c-a^2cx^2\right)^{-1+p}\,dx$$
$$=\left(c(1-ax)^{-p}(c+acx)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1-ax)^{1+p}(c+acx)^{-1+p}\,dx$$
$$=\frac{2^{1+p}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-p,p;1+p;\frac{1}{2}(1+ax)\right)}{ap}$$

Mathematica [A] time = 0.0235417, size = 74, normalized size = 1.37

$$-\frac{2^{p-1}(1-ax)^{p+2}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(1-p,p+2,p+3,\frac{1}{2}(1-ax)\right)}{a(p+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -((2^(-1 + p)*(1 - a*x)^(2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1 - p,
2 + p, 3 + p, (1 - a*x)/2])/(a*(2 + p)*(1 - a^2*x^2)^p))
```

Maple [F] time = 0.327, size = 0, normalized size = 0.

$$\int\frac{\left(-a^2cx^2+c\right)^p\left(-a^2x^2+1\right)}{\left(ax+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] int((-a^2*c*x^2+c)^p/(a*x+1)^2*(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(a^2x^2-1\right)\left(-a^2cx^2+c\right)^p}{\left(ax+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ax-1)\left(-a^2cx^2+c\right)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-(a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

Sympy [C] time = 11.127, size = 651, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -a*Piecewise((0**p*x/a + 0**p*log(1/(a**2*x**2))/(2*a**2) - 0**p*log(-1 + 1
/(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 - c**p*x**2*gamma(p)*gamm
a(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma
(-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(
-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), 1/(a**2*x**2))/(2*a*gamma(1/
2 - p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a + 0**p*log(1/(a**2*x
**2))/(2*a**2) - 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atanh(1/(a*x))
/a**2 - c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x
**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2
*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,
), 1/(a**2*x**2))/(2*a*gamma(1/2 - p)*gamma(p + 1)), True)) + Piecewise((0*
*p*log(a**2*x**2 - 1)/(2*a) + 0**p*acoth(a*x)/a + a*c**p*x**2*gamma(p)*gamm
a(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma
(-p)*gamma(p + 1)) + a**(2*p)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/
2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3
/2 - p)*gamma(p + 1)), Abs(a**2*x**2) > 1), (0**p*log(-a**2*x**2 + 1)/(2*a)
 + 0**p*atanh(a*x)/a + a*c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p
), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) + a**(2*
p)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 -
p), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3/2 - p)*gamma(p + 1)), True
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(a^2x^2-1\right)\left(-a^2cx^2+c\right)^p}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1)^2, x)
```

3.1254 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=167

$$\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{7/2}}{9a}+\frac{11c^4(1-ax)\left(1-a^2x^2\right)^{7/2}}{72a}+\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/}$$

```
[Out] (55*c^4*x*Sqrt[1 - a^2*x^2])/128 + (55*c^4*x*(1 - a^2*x^2)^(3/2))/192 + (11
*c^4*x*(1 - a^2*x^2)^(5/2))/48 + (11*c^4*(1 - a^2*x^2)^(7/2))/(56*a) + (11*
c^4*(1 - a*x)*(1 - a^2*x^2)^(7/2))/(72*a) + (c^4*(1 - a*x)^2*(1 - a^2*x^2)^
(7/2))/(9*a) + (55*c^4*ArcSin[a*x])/(128*a)
```

Rubi [A] time = 0.0943462, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6139, 671, 641, 195, 216}

$$\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{7/2}}{9a}+\frac{11c^4(1-ax)\left(1-a^2x^2\right)^{7/2}}{72a}+\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^4/E^(3*ArcTanh[a*x]),x]
```


```
[Out] (55*c^4*x*Sqrt[1 - a^2*x^2])/128 + (55*c^4*x*(1 - a^2*x^2)^(3/2))/192 + (11
*c^4*x*(1 - a^2*x^2)^(5/2))/48 + (11*c^4*(1 - a^2*x^2)^(7/2))/(56*a) + (11*
c^4*(1 - a*x)*(1 - a^2*x^2)^(7/2))/(72*a) + (c^4*(1 - a*x)^2*(1 - a^2*x^2)^
(7/2))/(9*a) + (55*c^4*ArcSin[a*x])/(128*a)
```


Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = c^4\int (1-ax)^3\left(1-a^2x^2\right)^{5/2}\,dx$$
$$= \frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{7/2}}{9a}+\frac{1}{9}\left(11c^4\right)\int (1-ax)^2\left(1-a^2x^2\right)^{5/2}\,dx$$
$$= \frac{11c^4(1-ax)\left(1-a^2x^2\right)^{7/2}}{72a}+\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{7/2}}{9a}+\frac{1}{8}\left(11c^4\right)\int (1-ax)\left(1-a^2x^2\right)^{5/}$$
$$= \frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}+\frac{11c^4(1-ax)\left(1-a^2x^2\right)^{7/2}}{72a}+\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{7/2}}{9a}+\frac{1}{8}\left(11c^4\right)\int$$
$$= \frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}+\frac{11c^4(1-ax)\left(1-a^2x^2\right)^{7/2}}{72a}+\frac{c^4(1-ax)^2\left(1-\right.}{9a}$$
$$= \frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}+\frac{11c^4(1-ax)\left(1-a^2x\right.}{72a}$$
$$= \frac{55}{128}c^4x\sqrt{1-a^2x^2}+\frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}+$$
$$= \frac{55}{128}c^4x\sqrt{1-a^2x^2}+\frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}+$$

Mathematica [A] time = 0.147153, size = 107, normalized size = 0.64

$$-\frac{c^4\left(\sqrt{1-a^2x^2}\left(896a^8x^8-3024a^7x^7+1024a^6x^6+7224a^5x^5-8448a^4x^4-3066a^3x^3+10240a^2x^2-4599ax-3712\right)+6\right.}{8064a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^4/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(c^4*(Sqrt[1 - a^2*x^2]*(-3712 - 4599*a*x + 10240*a^2*x^2 - 3066*a^3*x^3 -
 8448*a^4*x^4 + 7224*a^5*x^5 + 1024*a^6*x^6 - 3024*a^7*x^7 + 896*a^8*x^8) +
 6930*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(8064*a)
```

Maple [A] time = 0.05, size = 173, normalized size = 1.

$$-\frac{c^4a^3x^4}{9}\left(-a^2x^2+1\right)^{\frac{5}{2}}-\frac{22\,c^4ax^2}{63}\left(-a^2x^2+1\right)^{\frac{5}{2}}+\frac{29\,c^4}{63\,a}\left(-a^2x^2+1\right)^{\frac{5}{2}}+\frac{3\,a^2c^4x^3}{8}\left(-a^2x^2+1\right)^{\frac{5}{2}}-\frac{7\,c^4x}{48}\left(-a^2x^2+1\right)^{\frac{5}{2}}+\frac{55}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^4/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/9*c^4*a^3*x^4*(-a^2*x^2+1)^(5/2)-22/63*c^4*a*x^2*(-a^2*x^2+1)^(5/2)+29/6
3*c^4*(-a^2*x^2+1)^(5/2)/a+3/8*c^4*a^2*x^3*(-a^2*x^2+1)^(5/2)-7/48*c^4*x*(-
```

```
a^2*x^2+1)^(5/2)+55/192*c^4*x*(-a^2*x^2+1)^(3/2)+55/128*c^4*x*(-a^2*x^2+1)^
(1/2)+55/128*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.4669, size = 208, normalized size = 1.25

$$-\frac{1}{9}\left(-a^2x^2+1\right)^{\frac{5}{2}}a^3c^4x^4+\frac{3}{8}\left(-a^2x^2+1\right)^{\frac{5}{2}}a^2c^4x^3-\frac{22}{63}\left(-a^2x^2+1\right)^{\frac{5}{2}}ac^4x^2-\frac{7}{48}\left(-a^2x^2+1\right)^{\frac{5}{2}}c^4x+\frac{55}{192}\left(-a^2x^2+1\right)^{\frac{3}{2}}c^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/9*(-a^2*x^2 + 1)^(5/2)*a^3*c^4*x^4 + 3/8*(-a^2*x^2 + 1)^(5/2)*a^2*c^4*x^
3 - 22/63*(-a^2*x^2 + 1)^(5/2)*a*c^4*x^2 - 7/48*(-a^2*x^2 + 1)^(5/2)*c^4*x
+ 55/192*(-a^2*x^2 + 1)^(3/2)*c^4*x + 29/63*(-a^2*x^2 + 1)^(5/2)*c^4/a + 55
/128*sqrt(-a^2*x^2 + 1)*c^4*x + 55/128*c^4*arcsin(a*x)/a
```

Fricas [A] time = 2.65421, size = 325, normalized size = 1.95

$$-\frac{6930\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(896\,a^8c^4x^8-3024\,a^7c^4x^7+1024\,a^6c^4x^6+7224\,a^5c^4x^5-8448\,a^4c^4x^4-3066\,a^3c^4x^3\right.}{8064\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/8064*(6930*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (896*a^8*c^4*x^8
 - 3024*a^7*c^4*x^7 + 1024*a^6*c^4*x^6 + 7224*a^5*c^4*x^5 - 8448*a^4*c^4*x^
4 - 3066*a^3*c^4*x^3 + 10240*a^2*c^4*x^2 - 4599*a*c^4*x - 3712*c^4)*sqrt(-a
^2*x^2 + 1))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**4/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17329, size = 170, normalized size = 1.02

$$\frac{55\,c^4\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{128\,|a|}+\frac{1}{8064}\sqrt{-a^2x^2+1}\left(\frac{3712\,c^4}{a}+\left(4599\,c^4-2\left(5120\,ac^4-\left(1533\,a^2c^4+4\left(1056\,a^3c^4-\left(903\,a^4c\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac"
)
```

```
[Out] 55/128*c^4*arcsin(a*x)*sgn(a)/abs(a) + 1/8064*sqrt(-a^2*x^2 + 1)*(3712*c^4/
a + (4599*c^4 - 2*(5120*a*c^4 - (1533*a^2*c^4 + 4*(1056*a^3*c^4 - (903*a^4*
c^4 + 2*(64*a^5*c^4 + 7*(8*a^7*c^4*x - 27*a^6*c^4)*x)*x)*x)*x)*x)*x)*x)
```

3.1255 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=145

$$\frac{c^3(1-ax)^2\left(1-a^2x^2\right)^{5/2}}{7a}+\frac{3c^3(1-ax)\left(1-a^2x^2\right)^{5/2}}{14a}+\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a}+\frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{9}{16}c^3x\sqrt{1-a^2x^2}+\frac{9c^3}{}$$

```
[Out] (9*c^3*x*Sqrt[1 - a^2*x^2])/16 + (3*c^3*x*(1 - a^2*x^2)^(3/2))/8 + (3*c^3*(
1 - a^2*x^2)^(5/2))/(10*a) + (3*c^3*(1 - a*x)*(1 - a^2*x^2)^(5/2))/(14*a) +
 (c^3*(1 - a*x)^2*(1 - a^2*x^2)^(5/2))/(7*a) + (9*c^3*ArcSin[a*x])/(16*a)
```

Rubi [A] time = 0.0866331, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6139, 671, 641, 195, 216}

$$\frac{c^3(1-ax)^2\left(1-a^2x^2\right)^{5/2}}{7a}+\frac{3c^3(1-ax)\left(1-a^2x^2\right)^{5/2}}{14a}+\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a}+\frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{9}{16}c^3x\sqrt{1-a^2x^2}+\frac{9c^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (9*c^3*x*Sqrt[1 - a^2*x^2])/16 + (3*c^3*x*(1 - a^2*x^2)^(3/2))/8 + (3*c^3*(
1 - a^2*x^2)^(5/2))/(10*a) + (3*c^3*(1 - a*x)*(1 - a^2*x^2)^(5/2))/(14*a) +
 (c^3*(1 - a*x)^2*(1 - a^2*x^2)^(5/2))/(7*a) + (9*c^3*ArcSin[a*x])/(16*a)
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3 dx = c^3\int (1-ax)^3\left(1-a^2x^2\right)^{3/2} dx$$
$$= \frac{c^3(1-ax)^2\left(1-a^2x^2\right)^{5/2}}{7a} + \frac{1}{7}\left(9c^3\right)\int (1-ax)^2\left(1-a^2x^2\right)^{3/2} dx$$
$$= \frac{3c^3(1-ax)\left(1-a^2x^2\right)^{5/2}}{14a} + \frac{c^3(1-ax)^2\left(1-a^2x^2\right)^{5/2}}{7a} + \frac{1}{2}\left(3c^3\right)\int (1-ax)\left(1-a^2x^2\right)^{3/2}$$
$$= \frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a} + \frac{3c^3(1-ax)\left(1-a^2x^2\right)^{5/2}}{14a} + \frac{c^3(1-ax)^2\left(1-a^2x^2\right)^{5/2}}{7a} + \frac{1}{2}\left(3c^3\right)\int (1$$
$$= \frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2} + \frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a} + \frac{3c^3(1-ax)\left(1-a^2x^2\right)^{5/2}}{14a} + \frac{c^3(1-ax)^2\left(1-a^2\right.}{7a}$$
$$= \frac{9}{16}c^3x\sqrt{1-a^2x^2} + \frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2} + \frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a} + \frac{3c^3(1-ax)\left(1-a^2x^2\right)^{5/2}}{14a} +$$
$$= \frac{9}{16}c^3x\sqrt{1-a^2x^2} + \frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2} + \frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a} + \frac{3c^3(1-ax)\left(1-a^2x^2\right)^{5/2}}{14a} +$$

Mathematica [A] time = 0.119178, size = 91, normalized size = 0.63

$$\frac{c^3\left(\sqrt{1-a^2x^2}\left(80a^6x^6-280a^5x^5+208a^4x^4+350a^3x^3-656a^2x^2+245ax+368\right)-630\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{560a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^3*(Sqrt[1 - a^2*x^2]*(368 + 245*a*x - 656*a^2*x^2 + 350*a^3*x^3 + 208*a^
4*x^4 - 280*a^5*x^5 + 80*a^6*x^6) - 630*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(56
0*a)
```

Maple [A] time = 0.038, size = 127, normalized size = 0.9

$$\frac{c^3ax^2}{7}\left(-a^2x^2+1\right)^{\frac{5}{2}} + \frac{23\,c^3}{35\,a}\left(-a^2x^2+1\right)^{\frac{5}{2}} - \frac{c^3x}{2}\left(-a^2x^2+1\right)^{\frac{5}{2}} + \frac{3\,c^3x}{8}\left(-a^2x^2+1\right)^{\frac{3}{2}} + \frac{9\,c^3x}{16}\sqrt{-a^2x^2+1} + \frac{9\,c^3}{16}\arctan\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/7*c^3*a*x^2*(-a^2*x^2+1)^(5/2)+23/35*c^3*(-a^2*x^2+1)^(5/2)/a-1/2*c^3*x*(
-a^2*x^2+1)^(5/2)+3/8*c^3*x*(-a^2*x^2+1)^(3/2)+9/16*c^3*x*(-a^2*x^2+1)^(1/2
)+9/16*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.45898, size = 146, normalized size = 1.01

$$\frac{1}{7}\left(-a^2x^2+1\right)^{\frac{5}{2}}ac^3x^2-\frac{1}{2}\left(-a^2x^2+1\right)^{\frac{5}{2}}c^3x+\frac{3}{8}\left(-a^2x^2+1\right)^{\frac{3}{2}}c^3x+\frac{23\left(-a^2x^2+1\right)^{\frac{5}{2}}c^3}{35\,a}+\frac{9}{16}\sqrt{-a^2x^2+1}c^3x+\frac{9\,c^3\,\mathrm{arcs}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxim
a")
```

```
[Out] 1/7*(-a^2*x^2 + 1)^(5/2)*a*c^3*x^2 - 1/2*(-a^2*x^2 + 1)^(5/2)*c^3*x + 3/8*(
-a^2*x^2 + 1)^(3/2)*c^3*x + 23/35*(-a^2*x^2 + 1)^(5/2)*c^3/a + 9/16*sqrt(-a
^2*x^2 + 1)*c^3*x + 9/16*c^3*arcsin(a*x)/a
```

Fricas [A] time = 2.6148, size = 261, normalized size = 1.8

$$-\frac{630\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(80\,a^6c^3x^6-280\,a^5c^3x^5+208\,a^4c^3x^4+350\,a^3c^3x^3-656\,a^2c^3x^2+245\,ac^3x+368\,c^3\right)\sqrt{}}{560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/560*(630*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (80*a^6*c^3*x^6 -
280*a^5*c^3*x^5 + 208*a^4*c^3*x^4 + 350*a^3*c^3*x^3 - 656*a^2*c^3*x^2 + 245
*a*c^3*x + 368*c^3)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [C] time = 34.4053, size = 632, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] a**5*c**3*Piecewise((x**6*sqrt(-a**2*x**2 + 1)/7 - x**4*sqrt(-a**2*x**2 + 1
)/(35*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)/(105*a**4) - 8*sqrt(-a**2*x**2 +
1)/(105*a**6), Ne(a, 0)), (x**6/6, True)) - 3*a**4*c**3*Piecewise((I*a**2*x
**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*x**5/(24*sqrt(a**2*x**2 - 1)) - I*x**3/(4
8*a**2*sqrt(a**2*x**2 - 1)) + I*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*acosh(a
*x)/(16*a**5), Abs(a**2*x**2) > 1), (-a**2*x**7/(6*sqrt(-a**2*x**2 + 1)) +
5*x**5/(24*sqrt(-a**2*x**2 + 1)) + x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - x/
(16*a**4*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(16*a**5), True)) + 2*a**3*c**3*
Piecewise((x**4*sqrt(-a**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 + 1)/(15*a**2
) - 2*sqrt(-a**2*x**2 + 1)/(15*a**4), Ne(a, 0)), (x**4/4, True)) + 2*a**2*c
**3*Piecewise((I*a**2*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8*sqrt(a**2*
x**2 - 1)) + I*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*a**3), Abs(
a**2*x**2) > 1), (-a**2*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(8*sqrt(-a**
2*x**2 + 1)) - x/(8*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a**3), True))
 - 3*a*c**3*Piecewise((x**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2)/(3*a
**2), True)) + c**3*Piecewise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)) - I*x/(2
```

```
*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (x*sqrt(-a
**2*x**2 + 1)/2 + asin(a*x)/(2*a), True))
```

Giac [A] time = 1.15974, size = 138, normalized size = 0.95

$$\frac{9\,c^3\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{16\,|a|}+\frac{1}{560}\,\sqrt{-a^2x^2+1}\left(\frac{368\,c^3}{a}+\left(245\,c^3-2\left(328\,ac^3-\left(175\,a^2c^3+4\left(26\,a^3c^3+5\left(2\,a^5c^3x-7\,a^4c^3\right)x\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac"
)
```

```
[Out] 9/16*c^3*arcsin(a*x)*sgn(a)/abs(a) + 1/560*sqrt(-a^2*x^2 + 1)*(368*c^3/a +
(245*c^3 - 2*(328*a*c^3 - (175*a^2*c^3 + 4*(26*a^3*c^3 + 5*(2*a^5*c^3*x - 7
*a^4*c^3)*x)*x)*x)*x)*x)
```

3.1256 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=123

$$\frac{c^2(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{7c^2(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7}{8}c^2x\sqrt{1-a^2x^2}+\frac{7c^2\sin^{-1}(ax)}{8a}$$

```
[Out] (7*c^2*x*Sqrt[1 - a^2*x^2])/8 + (7*c^2*(1 - a^2*x^2)^(3/2))/(12*a) + (7*c^2
*(1 - a*x)*(1 - a^2*x^2)^(3/2))/(20*a) + (c^2*(1 - a*x)^2*(1 - a^2*x^2)^(3/
2))/(5*a) + (7*c^2*ArcSin[a*x])/(8*a)
```

Rubi [A] time = 0.0745975, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6139, 671, 641, 195, 216}

$$\frac{c^2(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{7c^2(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7}{8}c^2x\sqrt{1-a^2x^2}+\frac{7c^2\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (7*c^2*x*Sqrt[1 - a^2*x^2])/8 + (7*c^2*(1 - a^2*x^2)^(3/2))/(12*a) + (7*c^2
*(1 - a*x)*(1 - a^2*x^2)^(3/2))/(20*a) + (c^2*(1 - a*x)^2*(1 - a^2*x^2)^(3/
2))/(5*a) + (7*c^2*ArcSin[a*x])/(8*a)
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = c^2\int(1-ax)^3\sqrt{1-a^2x^2}\,dx$$
$$=\frac{c^2(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{5}\left(7c^2\right)\int(1-ax)^2\sqrt{1-a^2x^2}\,dx$$
$$=\frac{7c^2(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{c^2(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{4}\left(7c^2\right)\int(1-ax)\sqrt{1-a^2x^2}\,dx$$
$$=\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7c^2(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{c^2(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{4}\left(7c^2\right)\int\sqrt{\ }$$
$$=\frac{7}{8}c^2x\sqrt{1-a^2x^2}+\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7c^2(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{c^2(1-ax)^2\left(1-a^2x^2\right)}{5a}$$
$$=\frac{7}{8}c^2x\sqrt{1-a^2x^2}+\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7c^2(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{c^2(1-ax)^2\left(1-a^2x^2\right)}{5a}$$

Mathematica [A] time = 0.0998186, size = 75, normalized size = 0.61

$$-\frac{c^2\left(\sqrt{1-a^2x^2}\left(24a^4x^4-90a^3x^3+112a^2x^2-15ax-136\right)+210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(c^2*(Sqrt[1 - a^2*x^2]*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3 + 24*a^4
*x^4) + 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(120*a)
```

Maple [A] time = 0.034, size = 189, normalized size = 1.5

$$-\frac{c^2}{5\,a}\left(-a^2x^2+1\right)^{\frac{5}{2}}-\frac{3\,xc^2}{4}\left(-a^2x^2+1\right)^{\frac{3}{2}}-\frac{9\,xc^2}{8}\sqrt{-a^2x^2+1}-\frac{9\,c^2}{8}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{4\,c^2}{3\,a}\left(-a^2\left(x+a^{-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/5*c^2*(-a^2*x^2+1)^(5/2)/a-3/4*c^2*x*(-a^2*x^2+1)^(3/2)-9/8*c^2*x*(-a^2*
x^2+1)^(1/2)-9/8*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+4
/3*c^2/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+2*c^2*(-a^2*(x+1/a)^2+2*a*(x+1/
a))^(1/2)*x+2*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1
/a))^(1/2))
```

Maxima [C] time = 1.45503, size = 198, normalized size = 1.61

$$-\frac{3}{4}\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2x-\frac{\left(-a^2x^2+1\right)^{\frac{5}{2}}c^2}{5\,a}+2\,\sqrt{a^2x^2+4\,ax+3}c^2x-\frac{9}{8}\,\sqrt{-a^2x^2+1}c^2x+\frac{4\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}{3\,a}-\frac{2i\,c^2\arcsin}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -3/4*(-a^2*x^2 + 1)^(3/2)*c^2*x - 1/5*(-a^2*x^2 + 1)^(5/2)*c^2/a + 2*sqrt(a
^2*x^2 + 4*a*x + 3)*c^2*x - 9/8*sqrt(-a^2*x^2 + 1)*c^2*x + 4/3*(-a^2*x^2 +
1)^(3/2)*c^2/a - 2*I*c^2*arcsin(a*x + 2)/a - 9/8*c^2*arcsin(a*x)/a + 4*sqrt
(a^2*x^2 + 4*a*x + 3)*c^2/a
```

Fricas [A] time = 2.57265, size = 209, normalized size = 1.7

$$-\frac{210\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(24\,a^4c^2x^4-90\,a^3c^2x^3+112\,a^2c^2x^2-15\,ac^2x-136\,c^2\right)\sqrt{-a^2x^2+1}}{120\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/120*(210*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (24*a^4*c^2*x^4 -
90*a^3*c^2*x^3 + 112*a^2*c^2*x^2 - 15*a*c^2*x - 136*c^2)*sqrt(-a^2*x^2 + 1)
)/a
```

Sympy [C] time = 12.5228, size = 340, normalized size = 2.76

$$-a^3c^2\left(\begin{cases}\frac{x^4\sqrt{-a^2x^2+1}}{5}-\frac{x^2\sqrt{-a^2x^2+1}}{15a^2}-\frac{2\sqrt{-a^2x^2+1}}{15a^4} & \text{for } a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)+3a^2c^2\left(\begin{cases}\frac{ia^2x^5}{4\sqrt{a^2x^2-1}}-\frac{3ix^3}{8\sqrt{a^2x^2-1}}+\frac{ix}{8a^2\sqrt{a^2x^2-1}}-\frac{i\,\mathrm{acosh}\,(ax)}{8a^3}\\ -\frac{a^2x^5}{4\sqrt{-a^2x^2+1}}+\frac{3x^3}{8\sqrt{-a^2x^2+1}}-\frac{x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\mathrm{asin}\,(ax)}{8a^3}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -a**3*c**2*Piecewise((x**4*sqrt(-a**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 +
1)/(15*a**2) - 2*sqrt(-a**2*x**2 + 1)/(15*a**4), Ne(a, 0)), (x**4/4, True))
 + 3*a**2*c**2*Piecewise((I*a**2*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8
*sqrt(a**2*x**2 - 1)) + I*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*
a**3), Abs(a**2*x**2) > 1), (-a**2*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(
8*sqrt(-a**2*x**2 + 1)) - x/(8*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a*
*3), True)) - 3*a*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)*
*(3/2)/(3*a**2), True)) + c**2*Piecewise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1
)) - I*x/(2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1),
 (x*sqrt(-a**2*x**2 + 1)/2 + asin(a*x)/(2*a), True))
```

Giac [A] time = 1.17667, size = 105, normalized size = 0.85

$$\frac{7\,c^2\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{8\,|a|}+\frac{1}{120}\,\sqrt{-a^2x^2+1}\left(\left(15\,c^2-2\left(56\,ac^2+3\left(4\,a^3c^2x-15\,a^2c^2\right)x\right)x\right)x+\frac{136\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac"
)
```

```
[Out] 7/8*c^2*arcsin(a*x)*sgn(a)/abs(a) + 1/120*sqrt(-a^2*x^2 + 1)*((15*c^2 - 2*(
56*a*c^2 + 3*(4*a^3*c^2*x - 15*a^2*c^2)*x)*x)*x + 136*c^2/a)
```

3.1257 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)dx$

Optimal. Leaf size=93

$$\frac{c\sqrt{1-a^2x^2}(1-ax)^2}{3a}+\frac{5c\sqrt{1-a^2x^2}(1-ax)}{6a}+\frac{5c\sqrt{1-a^2x^2}}{2a}+\frac{5c\sin^{-1}(ax)}{2a}$$

```
[Out] (5*c*Sqrt[1 - a^2*x^2])/(2*a) + (5*c*(1 - a*x)*Sqrt[1 - a^2*x^2])/(6*a) + (
c*(1 - a*x)^2*Sqrt[1 - a^2*x^2])/(3*a) + (5*c*ArcSin[a*x])/(2*a)
```

Rubi [A] time = 0.0567689, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6139, 671, 641, 216}

$$\frac{c\sqrt{1-a^2x^2}(1-ax)^2}{3a}+\frac{5c\sqrt{1-a^2x^2}(1-ax)}{6a}+\frac{5c\sqrt{1-a^2x^2}}{2a}+\frac{5c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (5*c*Sqrt[1 - a^2*x^2])/(2*a) + (5*c*(1 - a*x)*Sqrt[1 - a^2*x^2])/(6*a) + (
c*(1 - a*x)^2*Sqrt[1 - a^2*x^2])/(3*a) + (5*c*ArcSin[a*x])/(2*a)
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)dx = c\int \frac{(1-ax)^3}{\sqrt{1-a^2x^2}}dx$$
$$= \frac{c(1-ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{1}{3}(5c)\int \frac{(1-ax)^2}{\sqrt{1-a^2x^2}}dx$$
$$= \frac{5c(1-ax)\sqrt{1-a^2x^2}}{6a}+\frac{c(1-ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{1}{2}(5c)\int \frac{1-ax}{\sqrt{1-a^2x^2}}dx$$
$$= \frac{5c\sqrt{1-a^2x^2}}{2a}+\frac{5c(1-ax)\sqrt{1-a^2x^2}}{6a}+\frac{c(1-ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{1}{2}(5c)\int \frac{1}{\sqrt{1-a^2x^2}}dx$$
$$= \frac{5c\sqrt{1-a^2x^2}}{2a}+\frac{5c(1-ax)\sqrt{1-a^2x^2}}{6a}+\frac{c(1-ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{5c\sin^{-1}(ax)}{2a}$$

Mathematica [A] time = 0.0856578, size = 70, normalized size = 0.75

$$\frac{c\left(\frac{\sqrt{ax+1}(-2a^3x^3+11a^2x^2-31ax+22)}{\sqrt{1-ax}}-30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c*((Sqrt[1 + a*x]*(22 - 31*a*x + 11*a^2*x^2 - 2*a^3*x^3))/Sqrt[1 - a*x] -
30*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(6*a)
```

Maple [A] time = 0.04, size = 133, normalized size = 1.4

$$2\,\frac{c\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right)^{5/2}}{a^3\left(x+a^{-1}\right)^2}+\frac{5\,c}{3\,a}\left(-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)\right)^{\frac{3}{2}}+\frac{5\,cx}{2}\sqrt{-a^2\left(x+a^{-1}\right)^2+2\,a\left(x+a^{-1}\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 2*c/a^3/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)+5/3*c/a*(-a^2*(x+1/a)^
2+2*a*(x+1/a))^(3/2)+5/2*c*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x+5/2*c/(a^2)
^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

Maxima [C] time = 1.46424, size = 165, normalized size = 1.77

$$-\frac{1}{2}\sqrt{a^2x^2+4\,ax+3}cx+\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}c}{a^2x+a}-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}c}{3\,a}+\frac{i\,c\arcsin\left(ax+2\right)}{2\,a}+\frac{3\,c\arcsin\left(ax\right)}{a}-\frac{\sqrt{a^2x^2+4\,ax+3}c}{a}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*sqrt(a^2*x^2 + 4*a*x + 3)*c*x + (-a^2*x^2 + 1)^(3/2)*c/(a^2*x + a) - 1
/3*(-a^2*x^2 + 1)^(3/2)*c/a + 1/2*I*c*arcsin(a*x + 2)/a + 3*c*arcsin(a*x)/a
 - sqrt(a^2*x^2 + 4*a*x + 3)*c/a + 3*sqrt(-a^2*x^2 + 1)*c/a
```

Fricas [A] time = 2.71979, size = 143, normalized size = 1.54

$$-\frac{30\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(2\,a^2cx^2-9\,acx+22\,c\right)\sqrt{-a^2x^2+1}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/6*(30*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (2*a^2*c*x^2 - 9*a*c*x
+ 22*c)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx+\int -\frac{2a^2x^2\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx+\int \frac{a^4x^4\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] c*(Integral(sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1), x)
+ Integral(-2*a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a
*x + 1), x) + Integral(a**4*x**4*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x
**2 + 3*a*x + 1), x))
```

Giac [A] time = 1.16713, size = 62, normalized size = 0.67

$$\frac{5\,c\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{2\,|a|}+\frac{1}{6}\,\sqrt{-a^2x^2+1}\left(\left(2\,acx-9\,c\right)x+\frac{22\,c}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] 5/2*c*arcsin(a*x)*sgn(a)/abs(a) + 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c*x - 9*c)*x
 + 22*c/a)
```

3.1258 $\int \frac{e^{-3\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=18

$$-\frac{e^{-3\tanh^{-1}(ax)}}{3ac}$$

```
[Out] -1/(3*a*c*E^(3*ArcTanh[a*x]))
```

Rubi [A] time = 0.0332538, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6137}

$$-\frac{e^{-3\tanh^{-1}(ax)}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] -1/(3*a*c*E^(3*ArcTanh[a*x]))
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{e^{-3\tanh^{-1}(ax)}}{3ac}$$

Mathematica [A] time = 0.0147526, size = 29, normalized size = 1.61

$$-\frac{(1-ax)^{3/2}}{3ac(ax+1)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] -(1 - a*x)^(3/2)/(3*a*c*(1 + a*x)^(3/2))
```

Maple [A] time = 0.029, size = 28, normalized size = 1.6

$$-\frac{1}{3\,(ax+1)^3\,ca}\left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c),x)
```

```
[Out] -1/3/a/c/(a*x+1)^3*(-a^2*x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(a^2cx^2-c\right)(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -integrate((-a^2*x^2 + 1)^(3/2)/((a^2*c*x^2 - c)*(a*x + 1)^3), x)
```

Fricas [B] time = 2.69853, size = 120, normalized size = 6.67

$$-\frac{a^2x^2 + 2\,ax - \sqrt{-a^2x^2+1}(ax-1) + 1}{3\left(a^3cx^2 + 2\,a^2cx + ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] -1/3*(a^2*x^2 + 2*a*x - sqrt(-a^2*x^2 + 1)*(a*x - 1) + 1)/(a^3*c*x^2 + 2*a^
2*c*x + a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c),x)
```

```
[Out] Integral(sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1), x)/c
```

Giac [B] time = 1.1546, size = 89, normalized size = 4.94

$$\frac{2\left(\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+1\right)}{3\,c\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] 2/3*(3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^4*x^2) + 1)/(c*((sqrt(-a^2*x^2
+ 1)*abs(a) + a)/(a^2*x) + 1)^3*abs(a))
```

3.1259 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=94

$$-\frac{2\sqrt{1-a^2x^2}}{15ac^2(ax+1)}-\frac{2\sqrt{1-a^2x^2}}{15ac^2(ax+1)^2}-\frac{\sqrt{1-a^2x^2}}{5ac^2(ax+1)^3}$$

```
[Out] -Sqrt[1 - a^2*x^2]/(5*a*c^2*(1 + a*x)^3) - (2*Sqrt[1 - a^2*x^2])/(15*a*c^2*
(1 + a*x)^2) - (2*Sqrt[1 - a^2*x^2])/(15*a*c^2*(1 + a*x))
```

Rubi [A] time = 0.0797878, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6139, 655, 659, 651}

$$-\frac{2\sqrt{1-a^2x^2}}{15ac^2(ax+1)}-\frac{2\sqrt{1-a^2x^2}}{15ac^2(ax+1)^2}-\frac{\sqrt{1-a^2x^2}}{5ac^2(ax+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(5*a*c^2*(1 + a*x)^3) - (2*Sqrt[1 - a^2*x^2])/(15*a*c^2*
(1 + a*x)^2) - (2*Sqrt[1 - a^2*x^2])/(15*a*c^2*(1 + a*x))
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] && !Int
egerQ[p - n/2]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}dx = \frac{\int \frac{(1-ax)^3}{\left(1-a^2x^2\right)^{7/2}}dx}{c^2}$$
$$= \frac{\int \frac{1}{(1+ax)^3\sqrt{1-a^2x^2}}dx}{c^2}$$
$$= -\frac{\sqrt{1-a^2x^2}}{5ac^2(1+ax)^3} + \frac{2\int \frac{1}{(1+ax)^2\sqrt{1-a^2x^2}}dx}{5c^2}$$
$$= -\frac{\sqrt{1-a^2x^2}}{5ac^2(1+ax)^3} - \frac{2\sqrt{1-a^2x^2}}{15ac^2(1+ax)^2} + \frac{2\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}dx}{15c^2}$$
$$= -\frac{\sqrt{1-a^2x^2}}{5ac^2(1+ax)^3} - \frac{2\sqrt{1-a^2x^2}}{15ac^2(1+ax)^2} - \frac{2\sqrt{1-a^2x^2}}{15ac^2(1+ax)}$$

Mathematica [A] time = 0.0185846, size = 43, normalized size = 0.46

$$-\frac{\sqrt{1-ax}\left(2a^2x^2+6ax+7\right)}{15ac^2(ax+1)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] -(Sqrt[1 - a*x]*(7 + 6*a*x + 2*a^2*x^2))/(15*a*c^2*(1 + a*x)^(5/2))
```

Maple [A] time = 0.029, size = 42, normalized size = 0.5

$$-\frac{2\,a^2x^2+6\,ax+7}{15\,(ax+1)^3\,c^2a}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/15*(-a^2*x^2+1)^(1/2)*(2*a^2*x^2+6*a*x+7)/(a*x+1)^3/c^2/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(a^2cx^2-c\right)^2(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a^2*c*x^2 - c)^2*(a*x + 1)^3), x)
```

Fricas [A] time = 3.11633, size = 192, normalized size = 2.04

$$-\frac{7\,a^3x^3+21\,a^2x^2+21\,ax+\left(2\,a^2x^2+6\,ax+7\right)\sqrt{-a^2x^2+1}+7}{15\left(a^4c^2x^3+3\,a^3c^2x^2+3\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] -1/15*(7*a^3*x^3 + 21*a^2*x^2 + 21*a*x + (2*a^2*x^2 + 6*a*x + 7)*sqrt(-a^2*
x^2 + 1) + 7)/(a^4*c^2*x^3 + 3*a^3*c^2*x^2 + 3*a^2*c^2*x + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{a^3x^3\sqrt{-a^2x^2+1}+3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(1/(a**3*x**3*sqrt(-a**2*x**2 + 1) + 3*a**2*x**2*sqrt(-a**2*x**2 +
1) + 3*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**2
```

Giac [A] time = 1.1756, size = 196, normalized size = 2.09

$$\frac{2\left(\frac{20\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x}+\frac{40\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2}+\frac{30\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3}+\frac{15\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4}+7\right)}{15\,c^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] 2/15*(20*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 40*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)^2/(a^4*x^2) + 30*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) + 1
5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 7)/(c^2*((sqrt(-a^2*x^2 + 1
)*abs(a) + a)/(a^2*x) + 1)^5*abs(a))
```

3.1260 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=116

$$\frac{8x}{35c^3\sqrt{1-a^2x^2}}-\frac{4}{35ac^3(ax+1)\sqrt{1-a^2x^2}}-\frac{4}{35ac^3(ax+1)^2\sqrt{1-a^2x^2}}-\frac{1}{7ac^3(ax+1)^3\sqrt{1-a^2x^2}}$$

```
[Out] (8*x)/(35*c^3*Sqrt[1 - a^2*x^2]) - 1/(7*a*c^3*(1 + a*x)^3*Sqrt[1 - a^2*x^2]
) - 4/(35*a*c^3*(1 + a*x)^2*Sqrt[1 - a^2*x^2]) - 4/(35*a*c^3*(1 + a*x)*Sqrt
[1 - a^2*x^2])
```

Rubi [A] time = 0.084844, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6139, 655, 659, 191}

$$\frac{8x}{35c^3\sqrt{1-a^2x^2}}-\frac{4}{35ac^3(ax+1)\sqrt{1-a^2x^2}}-\frac{4}{35ac^3(ax+1)^2\sqrt{1-a^2x^2}}-\frac{1}{7ac^3(ax+1)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] (8*x)/(35*c^3*Sqrt[1 - a^2*x^2]) - 1/(7*a*c^3*(1 + a*x)^3*Sqrt[1 - a^2*x^2]
) - 4/(35*a*c^3*(1 + a*x)^2*Sqrt[1 - a^2*x^2]) - 4/(35*a*c^3*(1 + a*x)*Sqrt
[1 - a^2*x^2])
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{(1-ax)^3}{\left(1-a^2x^2\right)^{9/2}}\,dx}{c^3}$$

$$= \frac{\int \frac{1}{(1+ax)^3\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$

$$= -\frac{1}{7ac^3(1+ax)^3\sqrt{1-a^2x^2}} + \frac{4\int \frac{1}{(1+ax)^2\left(1-a^2x^2\right)^{3/2}}\,dx}{7c^3}$$

$$= -\frac{1}{7ac^3(1+ax)^3\sqrt{1-a^2x^2}} - \frac{4}{35ac^3(1+ax)^2\sqrt{1-a^2x^2}} + \frac{12\int \frac{1}{(1+ax)\left(1-a^2x^2\right)^{3/2}}\,dx}{35c^3}$$

$$= -\frac{1}{7ac^3(1+ax)^3\sqrt{1-a^2x^2}} - \frac{4}{35ac^3(1+ax)^2\sqrt{1-a^2x^2}} - \frac{4}{35ac^3(1+ax)\sqrt{1-a^2x^2}} + \frac{8\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}}{35c^3}$$

$$= \frac{8x}{35c^3\sqrt{1-a^2x^2}} - \frac{1}{7ac^3(1+ax)^3\sqrt{1-a^2x^2}} - \frac{4}{35ac^3(1+ax)^2\sqrt{1-a^2x^2}} - \frac{4}{35ac^3(1+ax)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0231819, size = 59, normalized size = 0.51

$$\frac{8a^4x^4+24a^3x^3+20a^2x^2-4ax-13}{35ac^3\sqrt{1-ax}(ax+1)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] (-13 - 4*a*x + 20*a^2*x^2 + 24*a^3*x^3 + 8*a^4*x^4)/(35*a*c^3*Sqrt[1 - a*x]
*(1 + a*x)^(7/2))
```

Maple [A] time = 0.03, size = 58, normalized size = 0.5

$$\frac{8\,x^4a^4+24\,x^3a^3+20\,a^2x^2-4\,ax-13}{35\,(ax+1)^3\,c^3a}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/35/(-a^2*x^2+1)^(1/2)*(8*a^4*x^4+24*a^3*x^3+20*a^2*x^2-4*a*x-13)/(a*x+1)^
3/c^3/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(a^2cx^2-c\right)^3(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*x^2 + 1)^(3/2)/((a^2*c*x^2 - c)^3*(a*x + 1)^3), x)
```

Fricas [A] time = 2.72811, size = 309, normalized size = 2.66

$$-\frac{13\,a^{5}x^{5}+39\,a^{4}x^{4}+26\,a^{3}x^{3}-26\,a^{2}x^{2}-39\,ax+\left(8\,a^{4}x^{4}+24\,a^{3}x^{3}+20\,a^{2}x^{2}-4\,ax-13\right)\sqrt{-a^{2}x^{2}+1}-13}{35\left(a^{6}c^{3}x^{5}+3\,a^{5}c^{3}x^{4}+2\,a^{4}c^{3}x^{3}-2\,a^{3}c^{3}x^{2}-3\,a^{2}c^{3}x-ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] -1/35*(13*a^5*x^5 + 39*a^4*x^4 + 26*a^3*x^3 - 26*a^2*x^2 - 39*a*x + (8*a^4*
x^4 + 24*a^3*x^3 + 20*a^2*x^2 - 4*a*x - 13)*sqrt(-a^2*x^2 + 1) - 13)/(a^6*c
^3*x^5 + 3*a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^3*c^3*x^2 - 3*a^2*c^3*x - a*c^
3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{-a^{5}x^{5}\sqrt{-a^{2}x^{2}+1}-3a^{4}x^{4}\sqrt{-a^{2}x^{2}+1}-2a^{3}x^{3}\sqrt{-a^{2}x^{2}+1}+2a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}+3ax\sqrt{-a^{2}x^{2}+1}+\sqrt{-a^{2}x^{2}+1}}\, dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(1/(-a**5*x**5*sqrt(-a**2*x**2 + 1) - 3*a**4*x**4*sqrt(-a**2*x**2 +
 1) - 2*a**3*x**3*sqrt(-a**2*x**2 + 1) + 2*a**2*x**2*sqrt(-a**2*x**2 + 1) +
 3*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(a^{2}cx^{2}-c\right)^{3}(ax+1)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] integrate(-(-a^2*x^2 + 1)^(3/2)/((a^2*c*x^2 - c)^3*(a*x + 1)^3), x)
```

3.1261 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=138

$$\frac{16x}{63c^4\sqrt{1-a^2x^2}}+\frac{8x}{63c^4\left(1-a^2x^2\right)^{3/2}}-\frac{2}{21ac^4(ax+1)\left(1-a^2x^2\right)^{3/2}}-\frac{2}{21ac^4(ax+1)^2\left(1-a^2x^2\right)^{3/2}}-\frac{1}{9ac^4(ax+1)^3\left(1-\right.}$$

```
[Out] (8*x)/(63*c^4*(1 - a^2*x^2)^(3/2)) - 1/(9*a*c^4*(1 + a*x)^3*(1 - a^2*x^2)^(
3/2)) - 2/(21*a*c^4*(1 + a*x)^2*(1 - a^2*x^2)^(3/2)) - 2/(21*a*c^4*(1 + a*x
)*(1 - a^2*x^2)^(3/2)) + (16*x)/(63*c^4*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0930441, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6139, 655, 659, 192, 191}

$$\frac{16x}{63c^4\sqrt{1-a^2x^2}}+\frac{8x}{63c^4\left(1-a^2x^2\right)^{3/2}}-\frac{2}{21ac^4(ax+1)\left(1-a^2x^2\right)^{3/2}}-\frac{2}{21ac^4(ax+1)^2\left(1-a^2x^2\right)^{3/2}}-\frac{1}{9ac^4(ax+1)^3\left(1-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] (8*x)/(63*c^4*(1 - a^2*x^2)^(3/2)) - 1/(9*a*c^4*(1 + a*x)^3*(1 - a^2*x^2)^(
3/2)) - 2/(21*a*c^4*(1 + a*x)^2*(1 - a^2*x^2)^(3/2)) - 2/(21*a*c^4*(1 + a*x
)*(1 - a^2*x^2)^(3/2)) + (16*x)/(63*c^4*Sqrt[1 - a^2*x^2])
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \frac{\int \frac{(1-ax)^3}{\left(1-a^2x^2\right)^{11/2}}\,dx}{c^4}$$

$$= \frac{\int \frac{1}{(1+ax)^3\left(1-a^2x^2\right)^{5/2}}\,dx}{c^4}$$

$$= -\frac{1}{9ac^4(1+ax)^3\left(1-a^2x^2\right)^{3/2}} + \frac{2\int \frac{1}{(1+ax)^2\left(1-a^2x^2\right)^{5/2}}\,dx}{3c^4}$$

$$= -\frac{1}{9ac^4(1+ax)^3\left(1-a^2x^2\right)^{3/2}} - \frac{2}{21ac^4(1+ax)^2\left(1-a^2x^2\right)^{3/2}} + \frac{10\int \frac{1}{(1+ax)\left(1-a^2x^2\right)^{5/2}}\,dx}{21c^4}$$

$$= -\frac{1}{9ac^4(1+ax)^3\left(1-a^2x^2\right)^{3/2}} - \frac{2}{21ac^4(1+ax)^2\left(1-a^2x^2\right)^{3/2}} - \frac{2}{21ac^4(1+ax)\left(1-a^2x^2\right)^{3/2}} + \frac{8\int \frac{}{(1-}}{2}$$

$$= \frac{8x}{63c^4\left(1-a^2x^2\right)^{3/2}} - \frac{1}{9ac^4(1+ax)^3\left(1-a^2x^2\right)^{3/2}} - \frac{2}{21ac^4(1+ax)^2\left(1-a^2x^2\right)^{3/2}} - \frac{2}{21ac^4(1+ax)\left(1-\right.}$$

$$= \frac{8x}{63c^4\left(1-a^2x^2\right)^{3/2}} - \frac{1}{9ac^4(1+ax)^3\left(1-a^2x^2\right)^{3/2}} - \frac{2}{21ac^4(1+ax)^2\left(1-a^2x^2\right)^{3/2}} - \frac{2}{21ac^4(1+ax)\left(1-\right.}$$

Mathematica [A] time = 0.0323553, size = 75, normalized size = 0.54

$$-\frac{16a^6x^6+48a^5x^5+24a^4x^4-56a^3x^3-66a^2x^2-6ax+19}{63ac^4(1-ax)^{3/2}(ax+1)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] -(19 - 6*a*x - 66*a^2*x^2 - 56*a^3*x^3 + 24*a^4*x^4 + 48*a^5*x^5 + 16*a^6*x
^6)/(63*a*c^4*(1 - a*x)^(3/2)*(1 + a*x)^(9/2))
```

Maple [A] time = 0.03, size = 74, normalized size = 0.5

$$-\frac{16\,x^6a^6+48\,x^5a^5+24\,x^4a^4-56\,x^3a^3-66\,a^2x^2-6\,ax+19}{63\,(ax+1)^3\,c^4a}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/63/(-a^2*x^2+1)^(3/2)*(16*a^6*x^6+48*a^5*x^5+24*a^4*x^4-56*a^3*x^3-66*a^
2*x^2-6*a*x+19)/(a*x+1)^3/c^4/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(a^2cx^2-c\right)^4(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a^2*c*x^2 - c)^4*(a*x + 1)^3), x)
```

Fricas [A] time = 3.07722, size = 419, normalized size = 3.04

$$-\frac{19\,a^7x^7+57\,a^6x^6+19\,a^5x^5-95\,a^4x^4-95\,a^3x^3+19\,a^2x^2+57\,ax+\left(16\,a^6x^6+48\,a^5x^5+24\,a^4x^4-56\,a^3x^3-66\,a^2x}{63\left(a^8c^4x^7+3\,a^7c^4x^6+a^6c^4x^5-5\,a^5c^4x^4-5\,a^4c^4x^3+a^3c^4x^2+3\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="fri
cas")
```

```
[Out] -1/63*(19*a^7*x^7 + 57*a^6*x^6 + 19*a^5*x^5 - 95*a^4*x^4 - 95*a^3*x^3 + 19*
a^2*x^2 + 57*a*x + (16*a^6*x^6 + 48*a^5*x^5 + 24*a^4*x^4 - 56*a^3*x^3 - 66*
a^2*x^2 - 6*a*x + 19)*sqrt(-a^2*x^2 + 1) + 19)/(a^8*c^4*x^7 + 3*a^7*c^4*x^6
 + a^6*c^4*x^5 - 5*a^5*c^4*x^4 - 5*a^4*c^4*x^3 + a^3*c^4*x^2 + 3*a^2*c^4*x
+ a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(a^2cx^2-c\right)^4(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="gia
c")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a^2*c*x^2 - c)^4*(a*x + 1)^3), x)
```

3.1262 $\int e^{-3\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=225

$$\frac{ax^5\sqrt{c-a^2cx^2}}{5\sqrt{1-a^2x^2}}-\frac{3x^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}+\frac{4x^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{2x^2\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}+\frac{4x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^4\sqrt{1-a^2x^2}}$$

```
[Out] (4*x*Sqrt[c - a^2*c*x^2])/(a^3*Sqrt[1 - a^2*x^2]) - (2*x^2*Sqrt[c - a^2*c*x
^2])/(a^2*Sqrt[1 - a^2*x^2]) + (4*x^3*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^
2*x^2]) - (3*x^4*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - a^2*x^2]) + (a*x^5*Sqrt[c
 - a^2*c*x^2])/(5*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])
/(a^4*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.224854, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$\frac{ax^5\sqrt{c-a^2cx^2}}{5\sqrt{1-a^2x^2}}-\frac{3x^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}+\frac{4x^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{2x^2\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}+\frac{4x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a^2*c*x^2])/E^(3*ArcTanh[a*x]),x]
```


```
[Out] (4*x*Sqrt[c - a^2*c*x^2])/(a^3*Sqrt[1 - a^2*x^2]) - (2*x^2*Sqrt[c - a^2*c*x
^2])/(a^2*Sqrt[1 - a^2*x^2]) + (4*x^3*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^
2*x^2]) - (3*x^4*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - a^2*x^2]) + (a*x^5*Sqrt[c
 - a^2*c*x^2])/(5*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])
/(a^4*Sqrt[1 - a^2*x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-3\tanh^{-1}(ax)}x^3\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x^3(1-ax)^2}{1+ax}dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a^3}-\frac{4x}{a^2}+\frac{4x^2}{a}-3x^3+ax^4-\frac{4}{a^3(1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{4x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-a^2x^2}}-\frac{2x^2\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}+\frac{4x^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{3x^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}+\frac{ax^5\sqrt{c-a^2cx^2}}{5\sqrt{1-a^2x^2}}-$$

Mathematica [A] time = 0.0573029, size = 81, normalized size = 0.36

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{2x^2}{a^2}+\frac{4x}{a^3}-\frac{4\log(ax+1)}{a^4}+\frac{ax^5}{5}+\frac{4x^3}{3a}-\frac{3x^4}{4}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a^2*c*x^2])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4*x)/a^3 - (2*x^2)/a^2 + (4*x^3)/(3*a) - (3*x^4)/4 +
 (a*x^5)/5 - (4*Log[1 + a*x])/a^4))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.086, size = 88, normalized size = 0.4

$$\frac{-12x^5a^5+45x^4a^4-80x^3a^3+120a^2x^2-240ax+240\ln(ax+1)}{(60a^2x^2-60)a^4}\sqrt{-c(a^2x^2-1)}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/60*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(-12*x^5*a^5+45*x^4*a^4-80*x
^3*a^3+120*a^2*x^2-240*a*x+240*ln(a*x+1))/(a^2*x^2-1)/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)*x^3/(a*x + 1)^3, x)
```

Fricas [A] time = 3.56111, size = 869, normalized size = 3.86

$$\left[\frac{120\left(a^{2}x^{2}-1\right)\sqrt{c}\log\left(\frac{a^{6}cx^{6}+4a^{5}cx^{5}+5a^{4}cx^{4}-4a^{2}cx^{2}-4acx+\left(a^{4}x^{4}+4a^{3}x^{3}+6a^{2}x^{2}+4ax\right)\sqrt{-a^{2}cx^{2}+c}\sqrt{-a^{2}x^{2}+1}\sqrt{c}-2c}{a^{4}x^{4}+2a^{3}x^{3}-2ax-1}\right)-\left(12a^{5}x^{5}-45a^{4}x\right.}{60\left(a^{6}x^{2}-a^{4}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="fricas")
```

```
[Out] [1/60*(120*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4
 - 4*a^2*c*x^2 - 4*a*c*x + (a^4*x^4 + 4*a^3*x^3 + 6*a^2*x^2 + 4*a*x)*sqrt(-
a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a
*x - 1)) - (12*a^5*x^5 - 45*a^4*x^4 + 80*a^3*x^3 - 120*a^2*x^2 + 240*a*x)*s
qrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^6*x^2 - a^4), -1/60*(240*(a^2*x^
2 - 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 2*a*x + 2)*sqrt(-a^2
*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 + 2*a^3*c*x^3 - a^2*c*x^2 - 2*a*c*x)) + (12*a
^5*x^5 - 45*a^4*x^4 + 80*a^3*x^3 - 120*a^2*x^2 + 240*a*x)*sqrt(-a^2*c*x^2 +
 c)*sqrt(-a^2*x^2 + 1))/(a^6*x^2 - a^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(a
*x + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^{2}cx^{2}+c}\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}x^{3}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)*x^3/(a*x + 1)^3, x)
```

3.1263 $\int e^{-3\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=184

$$\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}-\frac{x^3\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{2x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^3\sqrt{1-a^2x^2}}$$

```
[Out] (-4*x*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - a^2*x^2]) + (2*x^2*Sqrt[c - a^2*c*
x^2])/(a*Sqrt[1 - a^2*x^2]) - (x^3*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] +
 (a*x^4*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - a^2*x^2]) + (4*Sqrt[c - a^2*c*x^2]
*Log[1 + a*x])/(a^3*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.218766, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}-\frac{x^3\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{2x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-4*x*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - a^2*x^2]) + (2*x^2*Sqrt[c - a^2*c*
x^2])/(a*Sqrt[1 - a^2*x^2]) - (x^3*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] +
 (a*x^4*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - a^2*x^2]) + (4*Sqrt[c - a^2*c*x^2]
*Log[1 + a*x])/(a^3*Sqrt[1 - a^2*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-3\tanh^{-1}(ax)}x^2\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x^2(1-ax)^2}{1+ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4}{a^2}+\frac{4x}{a}-3x^2+ax^3+\frac{4}{a^2(1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}+\frac{2x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{x^3\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}+\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-}}$$

Mathematica [A] time = 0.0466033, size = 69, normalized size = 0.38

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{4x}{a^2}+\frac{4\log(ax+1)}{a^3}+\frac{ax^4}{4}+\frac{2x^2}{a}-x^3\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((-4*x)/a^2 + (2*x^2)/a - x^3 + (a*x^4)/4 + (4*Log[1 +
 a*x])/a^3))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.085, size = 79, normalized size = 0.4

$$-\frac{x^4a^4-4x^3a^3+8a^2x^2-16ax+16\ln(ax+1)}{(4a^2x^2-4)a^3}\sqrt{-c(a^2x^2-1)}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/4*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(x^4*a^4-4*x^3*a^3+8*a^2*x^2
-16*a*x+16*ln(a*x+1))/(a^2*x^2-1)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\,(-a^2x^2+1)^{\frac{3}{2}}x^2}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)*x^2/(a*x + 1)^3, x)
```

Fricas [A] time = 2.95694, size = 807, normalized size = 4.39

$$\left[\frac{8\left(a^2x^2-1\right)\sqrt{c}\log\left(\frac{a^6cx^6+4a^5cx^5+5a^4cx^4-4a^2cx^2-4acx-\left(a^4x^4+4a^3x^3+6a^2x^2+4ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-2c}{a^4x^4+2a^3x^3-2ax-1}\right)-\left(a^4x^4-4a^3x^3+8a^2x^2\right.}{4\left(a^5x^2-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="fricas")
```

```
[Out] [1/4*(8*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 -
4*a^2*c*x^2 - 4*a*c*x - (a^4*x^4 + 4*a^3*x^3 + 6*a^2*x^2 + 4*a*x)*sqrt(-a^2
*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a*x
- 1)) - (a^4*x^4 - 4*a^3*x^3 + 8*a^2*x^2 - 16*a*x)*sqrt(-a^2*c*x^2 + c)*sqr
t(-a^2*x^2 + 1))/(a^5*x^2 - a^3), 1/4*(16*(a^2*x^2 - 1)*sqrt(-c)*arctan(sqr
t(-a^2*c*x^2 + c)*(a^2*x^2 + 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*
x^4 + 2*a^3*c*x^3 - a^2*c*x^2 - 2*a*c*x)) - (a^4*x^4 - 4*a^3*x^3 + 8*a^2*x^
2 - 16*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^5*x^2 - a^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{-c(ax-1)(ax+1)}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(a
*x + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)*x^2/(a*x + 1)^3, x)
```

3.1264 $\int e^{-3\tanh^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=149

$$\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}+\frac{4x\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^2\sqrt{1-a^2x^2}}$$

```
[Out] (4*x*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) - (3*x^2*Sqrt[c - a^2*c*x^2
])/(2*Sqrt[1 - a^2*x^2]) + (a*x^3*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - a^2*x^2]
) - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^2*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.148544, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 77}

$$\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}+\frac{4x\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (4*x*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) - (3*x^2*Sqrt[c - a^2*c*x^2
])/(2*Sqrt[1 - a^2*x^2]) + (a*x^3*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - a^2*x^2]
) - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^2*Sqrt[1 - a^2*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-3\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x(1-ax)^2}{1+ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a}-3x+ax^2-\frac{4}{a(1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{4x\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}+\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0481811, size = 63, normalized size = 0.42

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(ax+1)}{a^2}+\frac{ax^3}{3}+\frac{4x}{a}-\frac{3x^2}{2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4*x)/a - (3*x^2)/2 + (a*x^3)/3 - (4*Log[1 + a*x])/a^
2))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.085, size = 72, normalized size = 0.5

$$\frac{-2\,x^3a^3+9\,a^2x^2-24\,ax+24\,\ln(ax+1)}{\left(6\,a^2x^2-6\right)a^2}\sqrt{-c\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/6*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(-2*x^3*a^3+9*a^2*x^2-24*a*x+
24*ln(a*x+1))/(a^2*x^2-1)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}x}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)*x/(a*x + 1)^3, x)
```

Fricas [A] time = 2.98392, size = 783, normalized size = 5.26

$$\left[\frac{12\left(a^2x^2-1\right)\sqrt{c}\log\left(\frac{a^6cx^6+4a^5cx^5+5a^4cx^4-4a^2cx^2-4acx+\left(a^4x^4+4a^3x^3+6a^2x^2+4ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-2c}{a^4x^4+2a^3x^3-2ax-1}\right)-\left(2a^3x^3-9a^2x^2+\right.}{6\left(a^4x^2-a^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(12*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 -
 4*a^2*c*x^2 - 4*a*c*x + (a^4*x^4 + 4*a^3*x^3 + 6*a^2*x^2 + 4*a*x)*sqrt(-a^
2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a*x
 - 1)) - (2*a^3*x^3 - 9*a^2*x^2 + 24*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^
2 + 1))/(a^4*x^2 - a^2), -1/6*(24*(a^2*x^2 - 1)*sqrt(-c)*arctan(sqrt(-a^2*c
*x^2 + c)*(a^2*x^2 + 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 + 2*
a^3*c*x^3 - a^2*c*x^2 - 2*a*c*x)) + (2*a^3*x^3 - 9*a^2*x^2 + 24*a*x)*sqrt(-
a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^4*x^2 - a^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*(-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(a*x
+ 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}x}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)*x/(a*x + 1)^3, x)
```

3.1265 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=110

$$\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{3x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a\sqrt{1-a^2x^2}}$$

```
[Out] (-3*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] + (a*x^2*Sqrt[c - a^2*c*x^2])/
(2*Sqrt[1 - a^2*x^2]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a*Sqrt[1 - a^
2*x^2])
```

Rubi [A] time = 0.0833562, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{3x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-3*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] + (a*x^2*Sqrt[c - a^2*c*x^2])/
(2*Sqrt[1 - a^2*x^2]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a*Sqrt[1 - a^
2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{1+ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-3+ax+\frac{4}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{3x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0259275, size = 53, normalized size = 0.48

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{ax^2}{2}+\frac{4\log(ax+1)}{a}-3x\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-3*x + (a*x^2)/2 + (4*Log[1 + a*x])/a))/Sqrt[1 - a^2*
x^2]
```

Maple [A] time = 0.084, size = 63, normalized size = 0.6

$$-\frac{a^2x^2-6\,ax+8\,\ln(ax+1)}{(2\,a^2x^2-2)\,a}\sqrt{-c\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(a^2*x^2-6*a*x+8*ln(a*x+1))/
(a^2*x^2-1)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

Fricas [A] time = 2.89631, size = 733, normalized size = 6.66

$$\left[\frac{4\left(a^2x^2-1\right)\sqrt{c}\log\left(\frac{a^6cx^6+4a^5cx^5+5a^4cx^4-4a^2cx^2-4acx-\left(a^4x^4+4a^3x^3+6a^2x^2+4ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-2c}{a^4x^4+2a^3x^3-2ax-1}\right)-\sqrt{-a^2cx^2+c}\left(a^2x^2-6a\right.}{2\left(a^3x^2-a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="f
ricas")
```

```
[Out] [1/2*(4*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 -
4*a^2*c*x^2 - 4*a*c*x - (a^4*x^4 + 4*a^3*x^3 + 6*a^2*x^2 + 4*a*x)*sqrt(-a^2
*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a*x
- 1)) - sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 6*a*x)*sqrt(-a^2*x^2 + 1))/(a^3*x^2
 - a), 1/2*(8*(a^2*x^2 - 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 +
 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 + 2*a^3*c*x^3 - a^2*c*x^
2 - 2*a*c*x)) - sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 6*a*x)*sqrt(-a^2*x^2 + 1))/
(a^3*x^2 - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(a*x +
1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

3.1266 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=102

$$\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] (a*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] + (Sqrt[c - a^2*c*x^2]*Log[x])/
Sqrt[1 - a^2*x^2] - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.196604, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 72}

$$\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] (a*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] + (Sqrt[c - a^2*c*x^2]*Log[x])/
Sqrt[1 - a^2*x^2] - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/Sqrt[1 - a^2*x^2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}} + \frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}} - \frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0253727, size = 44, normalized size = 0.43

$$\frac{\sqrt{c-a^2cx^2}(ax-4\log(ax+1)+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x + Log[x] - 4*Log[1 + a*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.088, size = 56, normalized size = 0.6

$$\frac{-ax-\ln(x)+4\ln(ax+1)}{a^2x^2-1}\sqrt{-c\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x)
```

```
[Out] (-a*x-ln(x)+4*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/(a^2*x^2
-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}(ax-1)}{a^2x^3+2\,ax^2+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x - 1)/(a^2*x^3 + 2*a*
x^2 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(-(ax-1)(ax+1))^{\frac{3}{2}}\sqrt{-c(ax-1)(ax+1)}}{x(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(x*(a*x
 + 1)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x), x)
```

3.1267 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{3a\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{4a\sqrt{c-a^2cx^2}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] -(Sqrt[c - a^2*c*x^2]/(x*Sqrt[1 - a^2*x^2])) - (3*a*Sqrt[c - a^2*c*x^2]*Log
[x])/Sqrt[1 - a^2*x^2] + (4*a*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/Sqrt[1 - a^
2*x^2]
```

Rubi [A] time = 0.202444, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{3a\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{4a\sqrt{c-a^2cx^2}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(x*Sqrt[1 - a^2*x^2])) - (3*a*Sqrt[c - a^2*c*x^2]*Log
[x])/Sqrt[1 - a^2*x^2] + (4*a*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/Sqrt[1 - a^
2*x^2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x^2(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}} - \frac{3a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}} + \frac{4a\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0316537, size = 50, normalized size = 0.47

$$\frac{\sqrt{c-a^2cx^2}\left(-3a\log(x)+4a\log(ax+1)-\frac{1}{x}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-x^(-1) - 3*a*Log[x] + 4*a*Log[1 + a*x]))/Sqrt[1 - a^
2*x^2]
```

Maple [A] time = 0.095, size = 60, normalized size = 0.6

$$\frac{3\,a\ln(x)\,x-4\,ax\ln(ax+1)+1}{x\left(a^2x^2-1\right)}\sqrt{-c\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x)
```

```
[Out] (-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(3*a*ln(x)*x-4*a*x*ln(a*x+1)+1)/(
a^2*x^2-1)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}(ax-1)}{a^2x^4+2\,ax^3+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorith
m="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x - 1)/(a^2*x^4 + 2*a*
x^3 + x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(-(ax-1)(ax+1))^{\frac{3}{2}}\sqrt{-c(ax-1)(ax+1)}}{x^2(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(x**2*(
a*x + 1)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^2), x)
```

3.1268 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=148

$$\frac{3a\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}+\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-a^2cx^2}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] -Sqrt[c - a^2*c*x^2]/(2*x^2*Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - a^2*c*x^2])/
(x*Sqrt[1 - a^2*x^2]) + (4*a^2*Sqrt[c - a^2*c*x^2]*Log[x])/Sqrt[1 - a^2*x^2
] - (4*a^2*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.20577, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$\frac{3a\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}+\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-a^2cx^2}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x^3),x]
```


```
[Out] -Sqrt[c - a^2*c*x^2]/(2*x^2*Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - a^2*c*x^2])/
(x*Sqrt[1 - a^2*x^2]) + (4*a^2*Sqrt[c - a^2*c*x^2]*Log[x])/Sqrt[1 - a^2*x^2
] - (4*a^2*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/Sqrt[1 - a^2*x^2]
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^3}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x^3(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^3}-\frac{3a}{x^2}+\frac{4a^2}{x}-\frac{4a^3}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}+\frac{4a^2\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0419929, size = 62, normalized size = 0.42

$$\frac{\sqrt{c-a^2cx^2}\left(4a^2\log(x)-4a^2\log(ax+1)+\frac{3a}{x}-\frac{1}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/(2*x^2) + (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1 + a
*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.094, size = 73, normalized size = 0.5

$$-\frac{8\,a^2\ln(x)\,x^2-8\,\ln(ax+1)\,a^2x^2+6\,ax-1}{\left(2\,a^2x^2-2\right)x^2}\sqrt{-c\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x)
```

```
[Out] -1/2*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(8*a^2*ln(x)*x^2-8*ln(a*x+1)
*a^2*x^2+6*a*x-1)/(a^2*x^2-1)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^3), x)
```

Fricas [A] time = 3.23728, size = 959, normalized size = 6.48

$$\left[\frac{4\left(a^4x^4-a^2x^2\right)\sqrt{c}\log\left(\frac{4a^5cx^5+\left(2a^6+4a^5+6a^4+4a^3+a^2\right)cx^6+\left(4a^4-4a^3-6a^2-4a-1\right)cx^4-5a^2cx^2-4acx+\Big(4a^3x^3-\left(4a^3+6a^2+4a+1\right)x^4+6a^2x^2+4}{a^4x^6+2a^3x^5-2ax^3-x^2}\right.}{2\left(a^2x^4-x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorith
m="fricas")
```

```
[Out] [1/2*(4*(a^4*x^4 - a^2*x^2)*sqrt(c)*log((4*a^5*c*x^5 + (2*a^6 + 4*a^5 + 6*a
^4 + 4*a^3 + a^2)*c*x^6 + (4*a^4 - 4*a^3 - 6*a^2 - 4*a - 1)*c*x^4 - 5*a^2*c
*x^2 - 4*a*c*x + (4*a^3*x^3 - (4*a^3 + 6*a^2 + 4*a + 1)*x^4 + 6*a^2*x^2 + 4
*a*x + 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^4*x^6 + 2
*a^3*x^5 - 2*a*x^3 - x^2)) + sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((6*a
- 1)*x^2 - 6*a*x + 1))/(a^2*x^4 - x^2), 1/2*(8*(a^4*x^4 - a^2*x^2)*sqrt(-c)
*arctan(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((2*a^2 + 2*a + 1)*x^2 + 2
*a*x + 1)*sqrt(-c)/(2*a^3*c*x^3 - (2*a^3 + a^2)*c*x^4 + (a^2 + 2*a + 1)*c*x
^2 - 2*a*c*x - c)) + sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((6*a - 1)*x^2
 - 6*a*x + 1))/(a^2*x^4 - x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{x^3\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(x**3*(
a*x + 1)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^3), x)
```

3.1269 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=187

$$-\frac{4a^2\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{3x^3\sqrt{1-a^2x^2}}-\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{4a^3\sqrt{c-a^2cx^2}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] -Sqrt[c - a^2*c*x^2]/(3*x^3*Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - a^2*c*x^2])/
(2*x^2*Sqrt[1 - a^2*x^2]) - (4*a^2*Sqrt[c - a^2*c*x^2])/(x*Sqrt[1 - a^2*x^2
]) - (4*a^3*Sqrt[c - a^2*c*x^2]*Log[x])/Sqrt[1 - a^2*x^2] + (4*a^3*Sqrt[c -
 a^2*c*x^2]*Log[1 + a*x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.207641, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{4a^2\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{3x^3\sqrt{1-a^2x^2}}-\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{4a^3\sqrt{c-a^2cx^2}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(3*x^3*Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - a^2*c*x^2])/
(2*x^2*Sqrt[1 - a^2*x^2]) - (4*a^2*Sqrt[c - a^2*c*x^2])/(x*Sqrt[1 - a^2*x^2
]) - (4*a^3*Sqrt[c - a^2*c*x^2]*Log[x])/Sqrt[1 - a^2*x^2] + (4*a^3*Sqrt[c -
 a^2*c*x^2]*Log[1 + a*x])/Sqrt[1 - a^2*x^2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^4}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x^4(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^4}-\frac{3a}{x^3}+\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{3x^3\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{4a^3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}}+\frac{4a^3\sqrt{c-}}{}$$

Mathematica [A] time = 0.0426764, size = 72, normalized size = 0.39

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{4a^2}{x}-4a^3\log(x)+4a^3\log(ax+1)+\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/(3*x^3) + (3*a)/(2*x^2) - (4*a^2)/x - 4*a^3*Log[x]
 + 4*a^3*Log[1 + a*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.094, size = 81, normalized size = 0.4

$$\frac{24\,a^3\ln(x)\,x^3-24\,a^3x^3\ln(ax+1)+24\,a^2x^2-9\,ax+2}{\left(6\,a^2x^2-6\right)x^3}\sqrt{-c\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x)
```

```
[Out] 1/6*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(24*a^3*ln(x)*x^3-24*a^3*x^3*
ln(a*x+1)+24*a^2*x^2-9*a*x+2)/(a^2*x^2-1)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^4), x)
```

Fricas [A] time = 3.21139, size = 1022, normalized size = 5.47

$$\left[\frac{12\left(a^{5}x^{5}-a^{3}x^{3}\right)\sqrt{c}\log\left(\frac{4\,a^{5}cx^{5}+\left(2\,a^{6}+4\,a^{5}+6\,a^{4}+4\,a^{3}+a^{2}\right)cx^{6}+\left(4\,a^{4}-4\,a^{3}-6\,a^{2}-4\,a-1\right)cx^{4}-5\,a^{2}cx^{2}-4\,acx-\left(4\,a^{3}x^{3}-\left(4\,a^{3}+6\,a^{2}+4\,a+1\right)x^{4}+6\,a^{2}x^{2}+4\,ax\cdots}{a^{4}x^{6}+2\,a^{3}x^{5}-2\,ax^{3}-x^{2}}\cdots}{6\left(a^{2}x^{5}-x^{3}\right)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorith
m="fricas")
```

```
[Out] [1/6*(12*(a^5*x^5 - a^3*x^3)*sqrt(c)*log((4*a^5*c*x^5 + (2*a^6 + 4*a^5 + 6*
a^4 + 4*a^3 + a^2)*c*x^6 + (4*a^4 - 4*a^3 - 6*a^2 - 4*a - 1)*c*x^4 - 5*a^2*
c*x^2 - 4*a*c*x - (4*a^3*x^3 - (4*a^3 + 6*a^2 + 4*a + 1)*x^4 + 6*a^2*x^2 +
4*a*x + 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^4*x^6 +
2*a^3*x^5 - 2*a*x^3 - x^2)) + sqrt(-a^2*c*x^2 + c)*(24*a^2*x^2 - (24*a^2 -
9*a + 2)*x^3 - 9*a*x + 2)*sqrt(-a^2*x^2 + 1))/(a^2*x^5 - x^3), -1/6*(24*(a^
5*x^5 - a^3*x^3)*sqrt(-c)*arctan(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(
(2*a^2 + 2*a + 1)*x^2 + 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^3 - (2*a^3 + a^2)*c*
x^4 + (a^2 + 2*a + 1)*c*x^2 - 2*a*c*x - c)) - sqrt(-a^2*c*x^2 + c)*(24*a^2*
x^2 - (24*a^2 - 9*a + 2)*x^3 - 9*a*x + 2)*sqrt(-a^2*x^2 + 1))/(a^2*x^5 - x^
3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{x^{4}\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(x**4*(
a*x + 1)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^{2}cx^{2}+c}\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^{3}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^4), x)
```

3.1270 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=221

$$\frac{4a^3\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{4x^4\sqrt{1-a^2x^2}}+\frac{4a^4\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^4\sqrt{c-a^2cx^2}\log(ax+1}{\sqrt{1-a^2x^2}}$$

```
[Out] -Sqrt[c - a^2*c*x^2]/(4*x^4*Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - a^2*c*x^2])/(x
^3*Sqrt[1 - a^2*x^2]) - (2*a^2*Sqrt[c - a^2*c*x^2])/(x^2*Sqrt[1 - a^2*x^2])
 + (4*a^3*Sqrt[c - a^2*c*x^2])/(x*Sqrt[1 - a^2*x^2]) + (4*a^4*Sqrt[c - a^2*
c*x^2]*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])
/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.211821, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$\frac{4a^3\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{4x^4\sqrt{1-a^2x^2}}+\frac{4a^4\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^4\sqrt{c-a^2cx^2}\log(ax+1}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x^5),x]
```


```
[Out] -Sqrt[c - a^2*c*x^2]/(4*x^4*Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - a^2*c*x^2])/(x
^3*Sqrt[1 - a^2*x^2]) - (2*a^2*Sqrt[c - a^2*c*x^2])/(x^2*Sqrt[1 - a^2*x^2])
 + (4*a^3*Sqrt[c - a^2*c*x^2])/(x*Sqrt[1 - a^2*x^2]) + (4*a^4*Sqrt[c - a^2*
c*x^2]*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])
/Sqrt[1 - a^2*x^2]
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^5}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x^5(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^5}-\frac{3a}{x^4}+\frac{4a^2}{x^3}-\frac{4a^3}{x^2}+\frac{4a^4}{x}-\frac{4a^5}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-a^2x^2}}+\frac{4a^3\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}+\frac{4a^4\sqrt{c-a^2cx^2}\log(}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0494624, size = 77, normalized size = 0.35

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{2a^2}{x^2}+\frac{4a^3}{x}+4a^4\log(x)-4a^4\log(ax+1)+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/(4*x^4) + a/x^3 - (2*a^2)/x^2 + (4*a^3)/x + 4*a^4*
Log[x] - 4*a^4*Log[1 + a*x]))/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.089, size = 89, normalized size = 0.4

$$-\frac{16\,a^4\ln(x)\,x^4-16\,\ln(ax+1)\,a^4x^4+16\,x^3a^3-8\,a^2x^2+4\,ax-1}{\left(4\,a^2x^2-4\right)x^4}\sqrt{-c\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x)
```

```
[Out] -1/4*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(16*a^4*ln(x)*x^4-16*ln(a*x+
1)*a^4*x^4+16*x^3*a^3-8*a^2*x^2+4*a*x-1)/(a^2*x^2-1)/x^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^5), x)
```

Fricas [A] time = 3.18486, size = 1073, normalized size = 4.86

$$\left[\frac{8\left(a^6x^6 - a^4x^4\right)\sqrt{c}\log\left(\frac{4a^5cx^5+(2a^6+4a^5+6a^4+4a^3+a^2)cx^6+(4a^4-4a^3-6a^2-4a-1)cx^4-5a^2cx^2-4acx+(4a^3x^3-(4a^3+6a^2+4a+1)x^4+6a^2x^2+4}{a^4x^6+2a^3x^5-2ax^3-x^2}\right.}{4\left(a^2x^6-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorith
m="fricas")
```

```
[Out] [1/4*(8*(a^6*x^6 - a^4*x^4)*sqrt(c)*log((4*a^5*c*x^5 + (2*a^6 + 4*a^5 + 6*a
^4 + 4*a^3 + a^2)*c*x^6 + (4*a^4 - 4*a^3 - 6*a^2 - 4*a - 1)*c*x^4 - 5*a^2*c
*x^2 - 4*a*c*x + (4*a^3*x^3 - (4*a^3 + 6*a^2 + 4*a + 1)*x^4 + 6*a^2*x^2 + 4
*a*x + 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^4*x^6 + 2
*a^3*x^5 - 2*a*x^3 - x^2)) - (16*a^3*x^3 - (16*a^3 - 8*a^2 + 4*a - 1)*x^4 -
 8*a^2*x^2 + 4*a*x - 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^2*x^6 -
 x^4), 1/4*(16*(a^6*x^6 - a^4*x^4)*sqrt(-c)*arctan(-sqrt(-a^2*c*x^2 + c)*sq
rt(-a^2*x^2 + 1)*((2*a^2 + 2*a + 1)*x^2 + 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^3
- (2*a^3 + a^2)*c*x^4 + (a^2 + 2*a + 1)*c*x^2 - 2*a*c*x - c)) - (16*a^3*x^3
 - (16*a^3 - 8*a^2 + 4*a - 1)*x^4 - 8*a^2*x^2 + 4*a*x - 1)*sqrt(-a^2*c*x^2
+ c)*sqrt(-a^2*x^2 + 1))/(a^2*x^6 - x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}\sqrt{-c(ax-1)(ax+1)}}{x^5(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**5,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(x**5*(
a*x + 1)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^5), x)
```

3.1271 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

Optimal. Leaf size=189

$$\frac{c^4(1-ax)^{10}\sqrt{c-a^2cx^2}}{10a\sqrt{1-a^2x^2}}-\frac{2c^4(1-ax)^9\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}+\frac{3c^4(1-ax)^8\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}}-\frac{8c^4(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}$$

```
[Out] (-8*c^4*(1 - a*x)^7*Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) + (3*c^4*(
1 - a*x)^8*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - a^2*x^2]) - (2*c^4*(1 - a*x)^
9*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) + (c^4*(1 - a*x)^10*Sqrt[c -
 a^2*c*x^2])/(10*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.109391, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$\frac{c^4(1-ax)^{10}\sqrt{c-a^2cx^2}}{10a\sqrt{1-a^2x^2}}-\frac{2c^4(1-ax)^9\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}+\frac{3c^4(1-ax)^8\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}}-\frac{8c^4(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(9/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-8*c^4*(1 - a*x)^7*Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) + (3*c^4*(
1 - a*x)^8*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - a^2*x^2]) - (2*c^4*(1 - a*x)^
9*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) + (c^4*(1 - a*x)^10*Sqrt[c -
 a^2*c*x^2])/(10*a*Sqrt[1 - a^2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}dx = \frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{9/2}dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int (1-ax)^6(1+ax)^3\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int \left(8(1-ax)^6-12(1-ax)^7+6(1-ax)^8-(1-ax)^9\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{8c^4(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{3c^4(1-ax)^8\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}}-\frac{2c^4(1-ax)^9\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}+\frac{c^4}{}$$

Mathematica [A] time = 0.0538006, size = 68, normalized size = 0.36

$$\frac{c^4(ax-1)^7\left(21a^3x^3+77a^2x^2+98ax+44\right)\sqrt{c-a^2cx^2}}{210a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(9/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^4*(-1 + a*x)^7*Sqrt[c - a^2*c*x^2]*(44 + 98*a*x + 77*a^2*x^2 + 21*a^3*x^
3))/(210*a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.03, size = 97, normalized size = 0.5

$$\frac{x\left(21\,a^9x^9-70\,x^8a^8+240\,x^6a^6-210\,x^5a^5-252\,x^4a^4+420\,x^3a^3-315\,ax+210\right)}{210\,(ax+1)^6\,(ax-1)^6}\left(-a^2cx^2+c\right)^{\frac{9}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/210*x*(21*a^9*x^9-70*a^8*x^8+240*a^6*x^6-210*a^5*x^5-252*a^4*x^4+420*a^3*
x^3-315*a*x+210)*(-a^2*c*x^2+c)^(9/2)*(-a^2*x^2+1)^(3/2)/(a*x+1)^6/(a*x-1)^
6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

Fricas [A] time = 2.88936, size = 266, normalized size = 1.41

$$-\frac{\left(21\,a^9c^4x^{10}-70\,a^8c^4x^9+240\,a^6c^4x^7-210\,a^5c^4x^6-252\,a^4c^4x^5+420\,a^3c^4x^4-315\,ac^4x^2+210\,c^4x\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x}}{210\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="f
ricas")
```

```
[Out] -1/210*(21*a^9*c^4*x^10 - 70*a^8*c^4*x^9 + 240*a^6*c^4*x^7 - 210*a^5*c^4*x^
6 - 252*a^4*c^4*x^5 + 420*a^3*c^4*x^4 - 315*a*c^4*x^2 + 210*c^4*x)*sqrt(-a^
2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(9/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

3.1272 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=142

$$-\frac{c^3(1-ax)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{4c^3(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{2c^3(1-ax)^6\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}$$

```
[Out] (-2*c^3*(1 - a*x)^6*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) + (4*c^3*(
1 - a*x)^7*Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) - (c^3*(1 - a*x)^8*
Sqrt[c - a^2*c*x^2])/(8*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.103736, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$-\frac{c^3(1-ax)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{4c^3(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{2c^3(1-ax)^6\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(7/2)/E^(3*ArcTanh[a*x]),x]
```


```
[Out] (-2*c^3*(1 - a*x)^6*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) + (4*c^3*(
1 - a*x)^7*Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) - (c^3*(1 - a*x)^8*
Sqrt[c - a^2*c*x^2])/(8*a*Sqrt[1 - a^2*x^2])
```


Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}dx = \frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{7/2}dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int(1-ax)^5(1+ax)^2dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int\left(4(1-ax)^5-4(1-ax)^6+(1-ax)^7\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{2c^3(1-ax)^6\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}+\frac{4c^3(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{c^3(1-ax)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0494028, size = 60, normalized size = 0.42

$$-\frac{c^3(ax-1)^6\left(21a^2x^2+54ax+37\right)\sqrt{c-a^2cx^2}}{168a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(7/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(c^3*(-1 + a*x)^6*(37 + 54*a*x + 21*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(168*a*S
qrt[1 - a^2*x^2])
```

Maple [A] time = 0.03, size = 97, normalized size = 0.7

$$\frac{x\left(21\,a^7x^7-72\,x^6a^6+28\,x^5a^5+168\,x^4a^4-210\,x^3a^3-56\,a^2x^2+252\,ax-168\right)}{168\,(ax+1)^5\,(ax-1)^5}\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/168*x*(21*a^7*x^7-72*a^6*x^6+28*a^5*x^5+168*a^4*x^4-210*a^3*x^3-56*a^2*x^
2+252*a*x-168)*(-a^2*c*x^2+c)^(7/2)*(-a^2*x^2+1)^(3/2)/(a*x+1)^5/(a*x-1)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

Fricas [A] time = 3.16564, size = 261, normalized size = 1.84

$$\frac{\left(21\,a^7c^3x^8-72\,a^6c^3x^7+28\,a^5c^3x^6+168\,a^4c^3x^5-210\,a^3c^3x^4-56\,a^2c^3x^3+252\,ac^3x^2-168\,c^3x\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2}}{168\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/168*(21*a^7*c^3*x^8 - 72*a^6*c^3*x^7 + 28*a^5*c^3*x^6 + 168*a^4*c^3*x^5 -
 210*a^3*c^3*x^4 - 56*a^2*c^3*x^3 + 252*a*c^3*x^2 - 168*c^3*x)*sqrt(-a^2*c*
x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(7/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

3.1273 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=95

$$\frac{c^2(1-ax)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}-\frac{2c^2(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}$$

```
[Out] (-2*c^2*(1 - a*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) + (c^2*(1
- a*x)^6*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0888439, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$\frac{c^2(1-ax)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}-\frac{2c^2(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-2*c^2*(1 - a*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) + (c^2*(1
- a*x)^6*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - a^2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}dx = \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int(1-ax)^4(1+ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int\left(2(1-ax)^4-(1-ax)^5\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{2c^2(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{c^2(1-ax)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0392596, size = 52, normalized size = 0.55

$$\frac{c^2(ax-1)^5(5ax+7)\sqrt{c-a^2cx^2}}{30a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^2*(-1 + a*x)^5*(7 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(30*a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.028, size = 81, normalized size = 0.9

$$\frac{x\left(5x^5a^5-18x^4a^4+15x^3a^3+20a^2x^2-45ax+30\right)}{30(ax+1)^4(ax-1)^4}\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/30*x*(5*a^5*x^5-18*a^4*x^4+15*a^3*x^3+20*a^2*x^2-45*a*x+30)*(-a^2*c*x^2+c
)^(5/2)*(-a^2*x^2+1)^(3/2)/(a*x+1)^4/(a*x-1)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

Fricas [A] time = 3.05641, size = 208, normalized size = 2.19

$$-\frac{\left(5\,a^{5}c^{2}x^{6}-18\,a^{4}c^{2}x^{5}+15\,a^{3}c^{2}x^{4}+20\,a^{2}c^{2}x^{3}-45\,ac^{2}x^{2}+30\,c^{2}x\right)\sqrt{-a^{2}cx^{2}+c}\sqrt{-a^{2}x^{2}+1}}{30\,\left(a^{2}x^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="f
ricas")
```

```
[Out] -1/30*(5*a^5*c^2*x^6 - 18*a^4*c^2*x^5 + 15*a^3*c^2*x^4 + 20*a^2*c^2*x^3 - 4
5*a*c^2*x^2 + 30*c^2*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 -
1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

3.1274 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=45

$$-\frac{c(1-ax)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

```
[Out] -(c*(1 - a*x)^4*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0801597, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 32}

$$-\frac{c(1-ax)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(c*(1 - a*x)^4*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx &= \frac{\left(c\sqrt{c-a^2cx^2}\right)\int e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\left(c\sqrt{c-a^2cx^2}\right)\int (1-ax)^3\,dx}{\sqrt{1-a^2x^2}}\\ &= -\frac{c(1-ax)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0305275, size = 58, normalized size = 1.29

$$\frac{c\left(-\frac{1}{4}a^3x^4+a^2x^3-\frac{3ax^2}{2}+x\right)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(x - (3*a*x^2)/2 + a^2*x^3 - (a^3*x^4)/4))/Sqrt[1 -
a^2*x^2]
```

Maple [A] time = 0.026, size = 64, normalized size = 1.4

$$\frac{x\left(x^3a^3-4a^2x^2+6ax-4\right)}{4(ax-1)^3(ax+1)^3}\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/4*x*(a^3*x^3-4*a^2*x^2+6*a*x-4)*(-a^2*c*x^2+c)^(3/2)*(-a^2*x^2+1)^(3/2)/(
a*x-1)^3/(a*x+1)^3
```

Maxima [A] time = 1.04349, size = 95, normalized size = 2.11

$$-\frac{\left(a^4c^{\frac{3}{2}}x^4-4a^3c^{\frac{3}{2}}x^3+6a^2c^{\frac{3}{2}}x^2-4ac^{\frac{3}{2}}x+4c^{\frac{3}{2}}\right)(ax+1)(ax-1)}{4\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="m
axima")
```

```
[Out] -1/4*(a^4*c^(3/2)*x^4 - 4*a^3*c^(3/2)*x^3 + 6*a^2*c^(3/2)*x^2 - 4*a*c^(3/2)
*x + 4*c^(3/2))*(a*x + 1)*(a*x - 1)/(a^3*x^2 - a)
```

Fricas [A] time = 3.08914, size = 142, normalized size = 3.16

$$\frac{\left(a^3cx^4-4a^2cx^3+6acx^2-4cx\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{4\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/4*(a^3*c*x^4 - 4*a^2*c*x^3 + 6*a*c*x^2 - 4*c*x)*sqrt(-a^2*c*x^2 + c)*sqrt
(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}(-c(ax-1)(ax+1))^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(a*x - 1)*(a*x + 1))**(3/2)/(a*x
 + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

3.1275 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=82

$$-\frac{2\sqrt{1-a^2x^2}}{a(ax+1)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(ax+1)}{a\sqrt{c-a^2cx^2}}$$

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^
2]*Log[1 + a*x])/(a*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.0875578, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$-\frac{2\sqrt{1-a^2x^2}}{a(ax+1)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(ax+1)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^
2]*Log[1 + a*x])/(a*Sqrt[c - a^2*c*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1-ax}{(1+ax)^2}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{-1-ax}+\frac{2}{(1+ax)^2}\right)dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{2\sqrt{1-a^2x^2}}{a(1+ax)\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\log(1+ax)}{a\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0332752, size = 54, normalized size = 0.66

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{2}{a(ax+1)}-\frac{\log(ax+1)}{a}\right)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2/(a*(1 + a*x)) - Log[1 + a*x]/a))/Sqrt[c - a^2*c*x^2]
```

Maple [A] time = 0.087, size = 69, normalized size = 0.8

$$\frac{ax\ln(ax+1)+\ln(ax+1)+2}{c\left(a^2x^2-1\right)a\,(ax+1)}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*x*ln(a*x+1)+ln(a*x+1)+2)/(a^2*
x^2-1)/c/a/(a*x+1)
```

Maxima [A] time = 0.993197, size = 45, normalized size = 0.55

$$-\frac{\log(ax+1)}{a\sqrt{c}} - \frac{2}{a^2\sqrt{c}x + a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] -log(a*x + 1)/(a*sqrt(c)) - 2/(a^2*sqrt(c)*x + a*sqrt(c))
```

Fricas [B] time = 3.02338, size = 806, normalized size = 9.83

$$\left[-\frac{4\,\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}ax-\left(a^3x^3+a^2x^2-ax-1\right)\sqrt{c}\log\left(\frac{a^6cx^6+4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2-4\,acx+\left(a^4x^4+4\,a^3x^3+6\,a^2x^2+4\,ax\right)\sqrt{-a^2cx^2+c}}{a^4x^4+2\,a^3x^3-2\,ax-1}\right.}{2\left(a^4cx^3+a^3cx^2-a^2cx-ac\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [-1/2*(4*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x - (a^3*x^3 + a^2*x^2 -
 a*x - 1)*sqrt(c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2
- 4*a*c*x + (a^4*x^4 + 4*a^3*x^3 + 6*a^2*x^2 + 4*a*x)*sqrt(-a^2*c*x^2 + c)*
sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)))/(a^4*
c*x^3 + a^3*c*x^2 - a^2*c*x - a*c), -(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2
+ 1)*a*x + (a^3*x^3 + a^2*x^2 - a*x - 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 +
c)*(a^2*x^2 + 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 + 2*a^3*c*x
^3 - a^2*c*x^2 - 2*a*c*x)))/(a^4*c*x^3 + a^3*c*x^2 - a^2*c*x - a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x +
 1)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\sqrt{-a^2cx^2+c}(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3), x)
```

3.1276 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{\sqrt{1-a^2x^2}}{2ac(ax+1)^2\sqrt{c-a^2cx^2}}$$

```
[Out] -Sqrt[1 - a^2*x^2]/(2*a*c*(1 + a*x)^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.0812225, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 32}

$$-\frac{\sqrt{1-a^2x^2}}{2ac(ax+1)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*a*c*(1 + a*x)^2*Sqrt[c - a^2*c*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1+ax)^3}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= -\frac{\sqrt{1-a^2x^2}}{2ac(1+ax)^2\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0434178, size = 53, normalized size = 1.15

$$\frac{\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{2ac^2(ax-1)(ax+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*Sqrt[c - a^2*c*x^2])/(2*a*c^2*(-1 + a*x)*(1 + a*x)^3)
```

Maple [A] time = 0.027, size = 38, normalized size = 0.8

$$-\frac{1}{2\left(ax+1\right)^2a}\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2/(a*x+1)^2/a*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2)
```

Maxima [A] time = 0.996838, size = 39, normalized size = 0.85

$$-\frac{1}{2\left(a^3c^{\frac{3}{2}}x^2+2\,a^2c^{\frac{3}{2}}x+ac^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -1/2/(a^3*c^(3/2)*x^2 + 2*a^2*c^(3/2)*x + a*c^(3/2))
```

Fricas [A] time = 2.45908, size = 146, normalized size = 3.17

$$-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\left(ax^2+2\,x\right)}{2\left(a^4c^2x^4+2\,a^3c^2x^3-2\,ac^2x-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -1/2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x^2 + 2*x)/(a^4*c^2*x^4 + 2
*a^3*c^2*x^3 - 2*a*c^2*x - c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}}{(-c(ax-1)(ax+1))^{\frac{3}{2}}(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*
x + 1)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^3), x)
```

3.1277 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=182

$$-\frac{\sqrt{1-a^2x^2}}{8ac^2(ax+1)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(ax+1)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{6ac^2(ax+1)^3\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8ac^2\sqrt{c-a^2cx^2}}$$

```
[Out] -Sqrt[1 - a^2*x^2]/(6*a*c^2*(1 + a*x)^3*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2
*x^2]/(8*a*c^2*(1 + a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a*c^
2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*a*c^
2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.110792, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6143, 6140, 44, 207}

$$-\frac{\sqrt{1-a^2x^2}}{8ac^2(ax+1)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(ax+1)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{6ac^2(ax+1)^3\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8ac^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(6*a*c^2*(1 + a*x)^3*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2
*x^2]/(8*a*c^2*(1 + a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a*c^
2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*a*c^
2*Sqrt[c - a^2*c*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)(1+ax)^4}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{2(1+ax)^4}+\frac{1}{4(1+ax)^3}+\frac{1}{8(1+ax)^2}-\frac{1}{8\left(-1+a^2x^2\right)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= -\frac{\sqrt{1-a^2x^2}}{6ac^2(1+ax)^3\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(1+ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(1+ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8c^2\sqrt{c}}$$
$$= -\frac{\sqrt{1-a^2x^2}}{6ac^2(1+ax)^3\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(1+ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(1+ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2\sqrt{c}}$$

Mathematica [A] time = 0.0610418, size = 73, normalized size = 0.4

$$\frac{\sqrt{1-a^2x^2}\left(-3a^2x^2-9ax+3(ax+1)^3\tanh^{-1}(ax)-10\right)}{24ac^2(ax+1)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-10 - 9*a*x - 3*a^2*x^2 + 3*(1 + a*x)^3*ArcTanh[a*x]))/
(24*a*c^2*(1 + a*x)^3*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.092, size = 159, normalized size = 0.9

$$-\frac{3a^3x^3\ln(ax+1)-3\ln(ax-1)x^3a^3+9\ln(ax+1)a^2x^2-9\ln(ax-1)a^2x^2-6a^2x^2+9ax\ln(ax+1)-9\ln(ax-}{\left(48a^2x^2-48\right)c^3a(ax+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/48*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*x^3*ln(a*x+1)-3*ln(a
*x-1)*x^3*a^3+9*ln(a*x+1)*a^2*x^2-9*ln(a*x-1)*a^2*x^2-6*a^2*x^2+9*a*x*ln(a*
x+1)-9*ln(a*x-1)*x*a-18*a*x+3*ln(a*x+1)-3*ln(a*x-1)-20)/(a^2*x^2-1)/c^3/a/(
a*x+1)^3
```

Maxima [A] time = 1.00669, size = 126, normalized size = 0.69

$$-\frac{3a^2\sqrt{c}x^2+9a\sqrt{c}x+10\sqrt{c}}{24\left(a^4c^3x^3+3a^3c^3x^2+3a^2c^3x+ac^3\right)}+\frac{\log(ax+1)}{16ac^{\frac{5}{2}}}-\frac{\log(ax-1)}{16ac^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] -1/24*(3*a^2*sqrt(c)*x^2 + 9*a*sqrt(c)*x + 10*sqrt(c))/(a^4*c^3*x^3 + 3*a^3
*c^3*x^2 + 3*a^2*c^3*x + a*c^3) + 1/16*log(a*x + 1)/(a*c^(5/2)) - 1/16*log(
a*x - 1)/(a*c^(5/2))
```

Fricas [A] time = 2.92127, size = 969, normalized size = 5.32

$$\left[\frac{3\left(a^5x^5+3\,a^4x^4+2\,a^3x^3-2\,a^2x^2-3\,ax-1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5\,a^4cx^4-5\,a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-c}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}\right)-4\left(10\,a^3x^3+\right.}{96\left(a^6c^3x^5+3\,a^5c^3x^4+2\,a^4c^3x^3-2\,a^3c^3x^2-3\,a^2c^3x-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] [1/96*(3*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 - 3*a*x - 1)*sqrt(c)*
log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*x^3 + a*x)*sqrt(-a^2*c
*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
- 1)) - 4*(10*a^3*x^3 + 27*a^2*x^2 + 21*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2
*x^2 + 1))/(a^6*c^3*x^5 + 3*a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^3*c^3*x^2 - 3
*a^2*c^3*x - a*c^3), 1/48*(3*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 -
 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqr
t(-c)*x/(a^4*c*x^4 - c)) - 2*(10*a^3*x^3 + 27*a^2*x^2 + 21*a*x)*sqrt(-a^2*c
*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^6*c^3*x^5 + 3*a^5*c^3*x^4 + 2*a^4*c^3*x^3
- 2*a^3*c^3*x^2 - 3*a^2*c^3*x - a*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*
x + 1)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^3), x)
```

3.1278 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=275

$$\frac{\sqrt{1-a^2x^2}}{32ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^3(ax+1)\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}}{32ac^3(ax+1)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{12ac^3(ax+1)^3\sqrt{c-a^2cx^2}}-\frac{}{16ac^3(ax}$$

```
[Out] Sqrt[1 - a^2*x^2]/(32*a*c^3*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x
^2]/(16*a*c^3*(1 + a*x)^4*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(12*a*c^
3*(1 + a*x)^3*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2])/(32*a*c^3*(1 + a
*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a*c^3*(1 + a*x)*Sqrt[c -
a^2*c*x^2]) + (5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(32*a*c^3*Sqrt[c - a^2*c*x
^2])
```

Rubi [A] time = 0.140019, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6143, 6140, 44, 207}

$$\frac{\sqrt{1-a^2x^2}}{32ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^3(ax+1)\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}}{32ac^3(ax+1)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{12ac^3(ax+1)^3\sqrt{c-a^2cx^2}}-\frac{}{16ac^3(ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(32*a*c^3*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x
^2]/(16*a*c^3*(1 + a*x)^4*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(12*a*c^
3*(1 + a*x)^3*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2])/(32*a*c^3*(1 + a
*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a*c^3*(1 + a*x)*Sqrt[c -
a^2*c*x^2]) + (5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(32*a*c^3*Sqrt[c - a^2*c*x
^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^2(1+ax)^5}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{32(-1+ax)^2}+\frac{1}{4(1+ax)^5}+\frac{1}{4(1+ax)^4}+\frac{3}{16(1+ax)^3}+\frac{1}{8(1+ax)^2}-\frac{5}{32\left(-1+a^2x^2\right)}\right)dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}}{32ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{16ac^3(1+ax)^4\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{12ac^3(1+ax)^3\sqrt{c-a^2cx^2}}-\frac{3\sqrt{}}{32ac^3(1+}$$

$$= \frac{\sqrt{1-a^2x^2}}{32ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{16ac^3(1+ax)^4\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{12ac^3(1+ax)^3\sqrt{c-a^2cx^2}}-\frac{3\sqrt{}}{32ac^3(1+}$$

Mathematica [A] time = 0.0849209, size = 101, normalized size = 0.37

$$\frac{\sqrt{1-a^2x^2}\left(-15a^4x^4-45a^3x^3-35a^2x^2+15ax+15(ax-1)(ax+1)^4\tanh^{-1}(ax)+32\right)}{96ac^3(ax-1)(ax+1)^4\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(32 + 15*a*x - 35*a^2*x^2 - 45*a^3*x^3 - 15*a^4*x^4 + 15
*(-1 + a*x)*(1 + a*x)^4*ArcTanh[a*x]))/(96*a*c^3*(-1 + a*x)*(1 + a*x)^4*Sqr
t[c - a^2*c*x^2])
```

Maple [A] time = 0.098, size = 238, normalized size = 0.9

$$-\frac{15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5+45\ln(ax+1)a^4x^4-45\ln(ax-1)a^4x^4-30x^4a^4+30a^3x^3\ln(ax+1)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/192*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x+1)*x^5*a^5-15*l
n(a*x-1)*x^5*a^5+45*ln(a*x+1)*a^4*x^4-45*ln(a*x-1)*a^4*x^4-30*x^4*a^4+30*a^
3*x^3*ln(a*x+1)-30*ln(a*x-1)*x^3*a^3-90*x^3*a^3-30*ln(a*x+1)*a^2*x^2+30*ln(
a*x-1)*a^2*x^2-70*a^2*x^2-45*a*x*ln(a*x+1)+45*ln(a*x-1)*x*a+30*a*x-15*ln(a*
x+1)+15*ln(a*x-1)+64)/(a^2*x^2-1)/c^4/a/(a*x+1)^4/(a*x-1)
```

Maxima [A] time = 1.03142, size = 165, normalized size = 0.6

$$-\frac{15\,a^4x^4+45\,a^3x^3+35\,a^2x^2-15\,ax-32}{96\left(a^6c^{\frac{7}{2}}x^5+3\,a^5c^{\frac{7}{2}}x^4+2\,a^4c^{\frac{7}{2}}x^3-2\,a^3c^{\frac{7}{2}}x^2-3\,a^2c^{\frac{7}{2}}x-ac^{\frac{7}{2}}\right)}+\frac{5\,\log\left(ax+1\right)}{64\,ac^{\frac{7}{2}}}-\frac{5\,\log\left(ax-1\right)}{64\,ac^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm=
"maxima")
```

```
[Out] -1/96*(15*a^4*x^4 + 45*a^3*x^3 + 35*a^2*x^2 - 15*a*x - 32)/(a^6*c^(7/2)*x^5
 + 3*a^5*c^(7/2)*x^4 + 2*a^4*c^(7/2)*x^3 - 2*a^3*c^(7/2)*x^2 - 3*a^2*c^(7/2
)*x - a*c^(7/2)) + 5/64*log(a*x + 1)/(a*c^(7/2)) - 5/64*log(a*x - 1)/(a*c^(
7/2))
```

Fricas [A] time = 2.94204, size = 1175, normalized size = 4.27

$$\left[\frac{15\left(a^7x^7+3\,a^6x^6+a^5x^5-5\,a^4x^4-5\,a^3x^3+a^2x^2+3\,ax+1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5\,a^4cx^4-5\,a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-c}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}\right.}{384\left(a^8c^4x^7+3\,a^7c^4x^6+a^6c^4x^5-5\,a^5c^4x^4-5\,a^4c^4x^3+a^3c^4x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm=
"fricas")
```

```
[Out] [1/384*(15*(a^7*x^7 + 3*a^6*x^6 + a^5*x^5 - 5*a^4*x^4 - 5*a^3*x^3 + a^2*x^2
 + 3*a*x + 1)*sqrt(c)*log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*
x^3 + a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 -
3*a^4*x^4 + 3*a^2*x^2 - 1)) - 4*(32*a^5*x^5 + 81*a^4*x^4 + 19*a^3*x^3 - 99*
a^2*x^2 - 81*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^8*c^4*x^7 + 3
*a^7*c^4*x^6 + a^6*c^4*x^5 - 5*a^5*c^4*x^4 - 5*a^4*c^4*x^3 + a^3*c^4*x^2 +
3*a^2*c^4*x + a*c^4), 1/192*(15*(a^7*x^7 + 3*a^6*x^6 + a^5*x^5 - 5*a^4*x^4
- 5*a^3*x^3 + a^2*x^2 + 3*a*x + 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*s
qrt(-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*c*x^4 - c)) - 2*(32*a^5*x^5 + 81*a^4*x^
4 + 19*a^3*x^3 - 99*a^2*x^2 - 81*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 +
1))/(a^8*c^4*x^7 + 3*a^7*c^4*x^6 + a^6*c^4*x^5 - 5*a^5*c^4*x^4 - 5*a^4*c^4*
x^3 + a^3*c^4*x^2 + 3*a^2*c^4*x + a*c^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm=
"giac")

[Out] integrate((-a^2*x^2 + 1)^(3/2)/((-a^2*c*x^2 + c)^(7/2)*(a*x + 1)^3), x)
```

3.1279 $\int e^{-3\tanh^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=136

$$\frac{4x^{m+1}\sqrt{c-a^2cx^2}\text{Hypergeometric2F1}(1,m+1,m+2,-ax)}{(m+1)\sqrt{1-a^2x^2}}-\frac{3x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}+\frac{ax^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}$$

```
[Out] (-3*x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) + (a*x^(2 +
m)*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2]) + (4*x^(1 + m)*Sqrt[c -
 a^2*c*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/((1 + m)*Sqrt[1 - a
^2*x^2])
```

Rubi [A] time = 0.201518, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6153, 6150, 88, 64}

$$\frac{4x^{m+1}\sqrt{c-a^2cx^2}\,{}_2F_1(1,m+1;m+2;-ax)}{(m+1)\sqrt{1-a^2x^2}}-\frac{3x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}+\frac{ax^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^(3*ArcTanh[a*x]),x]
```


```
[Out] (-3*x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) + (a*x^(2 +
m)*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2]) + (4*x^(1 + m)*Sqrt[c -
 a^2*c*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/((1 + m)*Sqrt[1 - a
^2*x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
```

```
&&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\tanh^{-1}(ax)}x^m\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{x^m(1-ax)^2}{1+ax}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-3x^m+ax^{1+m}+\frac{4x^m}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}\\
&= -\frac{3x^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}}+\frac{ax^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}}+\frac{\left(4\sqrt{c-a^2cx^2}\right)\int \frac{x^m}{1+ax}\,dx}{\sqrt{1-a^2x^2}}\\
&= -\frac{3x^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}}+\frac{ax^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}}+\frac{4x^{1+m}\sqrt{c-a^2cx^2}\,{}_2F_1(1,1+m;2+m;-}{(1+m)\sqrt{1-a^2x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0500204, size = 74, normalized size = 0.54

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}(4(m+2)\text{Hypergeometric2F1}(1,m+1,m+2,-ax)+m(ax-3)+ax-6)}{(m+1)(m+2)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2]*(-6 + a*x + m*(-3 + a*x) + 4*(2 + m)*Hyperge
ometric2F1[1, 1 + m, 2 + m, -(a*x)]))/((1 + m)*(2 + m)*Sqrt[1 - a^2*x^2])
```

Maple [F] time = 0.435, size = 0, normalized size = 0.

$$\int \frac{x^m}{(ax+1)^3}\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}x^m}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)*x^m/(a*x + 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}(ax-1)x^m}{a^2x^2+2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^m/(a^2*x^2 +
2*a*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}x^m}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)*x^m/(a*x + 1)^3, x)
```

3.1280 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=86

$$-\frac{2^{p-\frac{1}{2}}(1-ax)^{p+\frac{5}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(\frac{3}{2}-p,p+\frac{5}{2},p+\frac{7}{2},\frac{1}{2}(1-ax)\right)}{a(2p+5)}$$

```
[Out] -((2^(-1/2 + p)*(1 - a*x)^(5/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[3/2
 - p, 5/2 + p, 7/2 + p, (1 - a*x)/2])/(a*(5 + 2*p)*(1 - a^2*x^2)^p))
```

Rubi [A] time = 0.0867313, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6143, 6140, 69}

$$-\frac{2^{p-\frac{1}{2}}(1-ax)^{p+\frac{5}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2}-p,p+\frac{5}{2};p+\frac{7}{2};\frac{1}{2}(1-ax)\right)}{a(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((2^(-1/2 + p)*(1 - a*x)^(5/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[3/2
 - p, 5/2 + p, 7/2 + p, (1 - a*x)/2])/(a*(5 + 2*p)*(1 - a^2*x^2)^p))
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx&=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx\\&=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1-ax)^{\frac{3}{2}+p}(1+ax)^{-\frac{3}{2}+p}\,dx\\&=-\frac{2^{-\frac{1}{2}+p}(1-ax)^{\frac{5}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2}-p,\frac{5}{2}+p;\frac{7}{2}+p;\frac{1}{2}(1-ax)\right)}{a(5+2p)}\end{aligned}$$

Mathematica [A] time = 0.0288671, size = 86, normalized size = 1.

$$-\frac{2^{p-\frac{3}{2}}(1-ax)^{p+\frac{5}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(\frac{3}{2}-p,p+\frac{5}{2},p+\frac{7}{2},\frac{1}{2}(1-ax)\right)}{a\left(p+\frac{5}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((2^(-3/2 + p)*(1 - a*x)^(5/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[3/2
 - p, 5/2 + p, 7/2 + p, (1 - a*x)/2])/(a*(5/2 + p)*(1 - a^2*x^2)^p))
```

Maple [F] time = 0.431, size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^p}{(ax+1)^3}\left(-a^2x^2+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(-a^2cx^2+c\right)^p}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(-a^2*c*x^2 + c)^p/(a*x + 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}(ax-1)\left(-a^2cx^2+c\right)^p}{a^2x^2+2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(a*x - 1)*(-a^2*c*x^2 + c)^p/(a^2*x^2 + 2*a*x
 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}(-c(ax-1)(ax+1))^p}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(a*x - 1)*(a*x + 1))**p/(a*x + 1
)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-a^2x^2+1)^{\frac{3}{2}}(-a^2cx^2+c)^p}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(-a^2*c*x^2 + c)^p/(a*x + 1)^3, x)
```

3.1281 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=359

$$-\frac{(ax+1)^{11/4}(1-ax)^{13/4}}{6a}-\frac{11(ax+1)^{7/4}(1-ax)^{13/4}}{60a}-\frac{77(ax+1)^{3/4}(1-ax)^{13/4}}{480a}+\frac{77(ax+1)^{3/4}(1-ax)^{9/4}}{960a}+\frac{231(ax+1)^{3/4}}{128}$$

```
[Out] (231*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(512*a) + (231*(1 - a*x)^(5/4)*(1 + a
*x)^(3/4))/(1280*a) + (77*(1 - a*x)^(9/4)*(1 + a*x)^(3/4))/(960*a) - (77*(1
 - a*x)^(13/4)*(1 + a*x)^(3/4))/(480*a) - (11*(1 - a*x)^(13/4)*(1 + a*x)^(7
/4))/(60*a) - ((1 - a*x)^(13/4)*(1 + a*x)^(11/4))/(6*a) + (231*ArcTan[1 - (
Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(512*Sqrt[2]*a) - (231*ArcTan[1
+ (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(512*Sqrt[2]*a) + (231*Log[1
+ Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])
/(1024*Sqrt[2]*a) - (231*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1
- a*x)^(1/4))/(1 + a*x)^(1/4)])/(1024*Sqrt[2]*a)
```

Rubi [A] time = 0.306347, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6140, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(ax+1)^{11/4}(1-ax)^{13/4}}{6a}-\frac{11(ax+1)^{7/4}(1-ax)^{13/4}}{60a}-\frac{77(ax+1)^{3/4}(1-ax)^{13/4}}{480a}+\frac{77(ax+1)^{3/4}(1-ax)^{9/4}}{960a}+\frac{231(ax+1)^{3/4}}{128}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*(1 - a^2*x^2)^(5/2),x]
```


```
[Out] (231*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(512*a) + (231*(1 - a*x)^(5/4)*(1 + a
*x)^(3/4))/(1280*a) + (77*(1 - a*x)^(9/4)*(1 + a*x)^(3/4))/(960*a) - (77*(1
 - a*x)^(13/4)*(1 + a*x)^(3/4))/(480*a) - (11*(1 - a*x)^(13/4)*(1 + a*x)^(7
/4))/(60*a) - ((1 - a*x)^(13/4)*(1 + a*x)^(11/4))/(6*a) + (231*ArcTan[1 - (
Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(512*Sqrt[2]*a) - (231*ArcTan[1
+ (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(512*Sqrt[2]*a) + (231*Log[1
+ Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])
/(1024*Sqrt[2]*a) - (231*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1
- a*x)^(1/4))/(1 + a*x)^(1/4)])/(1024*Sqrt[2]*a)
```


Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}dx=\int (1-ax)^{9/4}(1+ax)^{11/4}\,dx$$

$$=-\frac{(1-ax)^{13/4}(1+ax)^{11/4}}{6a}+\frac{11}{12}\int (1-ax)^{9/4}(1+ax)^{7/4}\,dx$$

$$=-\frac{11(1-ax)^{13/4}(1+ax)^{7/4}}{60a}-\frac{(1-ax)^{13/4}(1+ax)^{11/4}}{6a}+\frac{77}{120}\int (1-ax)^{9/4}(1+ax)^{3/4}\,dx$$

$$=-\frac{77(1-ax)^{13/4}(1+ax)^{3/4}}{480a}-\frac{11(1-ax)^{13/4}(1+ax)^{7/4}}{60a}-\frac{(1-ax)^{13/4}(1+ax)^{11/4}}{6a}+\frac{77}{320}\int$$

$$=\frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a}-\frac{77(1-ax)^{13/4}(1+ax)^{3/4}}{480a}-\frac{11(1-ax)^{13/4}(1+ax)^{7/4}}{60a}-\frac{(1-a}{}$$

$$=\frac{231(1-ax)^{5/4}(1+ax)^{3/4}}{1280a}+\frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a}-\frac{77(1-ax)^{13/4}(1+ax)^{3/4}}{480a}-\frac{11(1-}{}$$

$$=\frac{231\sqrt[4]{1-ax}(1+ax)^{3/4}}{512a}+\frac{231(1-ax)^{5/4}(1+ax)^{3/4}}{1280a}+\frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a}-\frac{77(1-}{}$$

$$=\frac{231\sqrt[4]{1-ax}(1+ax)^{3/4}}{512a}+\frac{231(1-ax)^{5/4}(1+ax)^{3/4}}{1280a}+\frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a}-\frac{77(1-}{}$$

$$=\frac{231\sqrt[4]{1-ax}(1+ax)^{3/4}}{512a}+\frac{231(1-ax)^{5/4}(1+ax)^{3/4}}{1280a}+\frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a}-\frac{77(1-}{}$$

$$=\frac{231\sqrt[4]{1-ax}(1+ax)^{3/4}}{512a}+\frac{231(1-ax)^{5/4}(1+ax)^{3/4}}{1280a}+\frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a}-\frac{77(1-}{}$$

$$=\frac{231\sqrt[4]{1-ax}(1+ax)^{3/4}}{512a}+\frac{231(1-ax)^{5/4}(1+ax)^{3/4}}{1280a}+\frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a}-\frac{77(1-}{}$$

$$=\frac{231\sqrt[4]{1-ax}(1+ax)^{3/4}}{512a}+\frac{231(1-ax)^{5/4}(1+ax)^{3/4}}{1280a}+\frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a}-\frac{77(1-}{}$$

$$=\frac{231\sqrt[4]{1-ax}(1+ax)^{3/4}}{512a}+\frac{231(1-ax)^{5/4}(1+ax)^{3/4}}{1280a}+\frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a}-\frac{77(1-}{}$$

Mathematica [C] time = 0.0185667, size = 42, normalized size = 0.12

$$-\frac{16\,2^{3/4}(1-ax)^{13/4}\text{Hypergeometric2F1}\left(-\frac{11}{4},\frac{13}{4},\frac{17}{4},\frac{1}{2}(1-ax)\right)}{13a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)*(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (-16*2^(3/4)*(1 - a*x)^(13/4)*Hypergeometric2F1[-11/4, 13/4, 17/4, (1 - a*x
)/2])/(13*a)
```

Maple [F] time = 0.201, size = 0, normalized size = 0.

$$\int \sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2x^2+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(5/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2x^2+1\right)^{\frac{5}{2}}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(5/2),x, algorith
m="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(5/2)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

Fricas [B] time = 2.96471, size = 1405, normalized size = 3.91

$$-\frac{13860\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\sqrt{2}a\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-1\right)+13860}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(5/2),x, algorith
m="fricas")
```

```
[Out] -1/30720*(13860*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt((sqrt(2)*(a^
4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^
2)*sqrt(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a
*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) - 1) + 13860*sqrt(2)*a*
(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^
2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a
^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1))*(a^(-4))^(1/4) + 1) + 3465*sqrt(2)*a*(a^(-4))^(1/4)*log((sqrt(2
)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x
 - a^2)*sqrt(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 3465*sqrt(2)*a*(a^(
-4))^(1/4)*log(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*
(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)
) - 4*(1280*a^5*x^5 + 128*a^4*x^4 - 4144*a^3*x^3 - 520*a^2*x^2 + 5174*a*x +
 1547)*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*(-a**2*x**2+1)**(5/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2x^2+1\right)^{\frac{5}{2}} \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(5/2),x, algorith
m="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(5/2)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

3.1282 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=307

$$-\frac{(ax+1)^{7/4}(1-ax)^{9/4}}{4a}-\frac{7(ax+1)^{3/4}(1-ax)^{9/4}}{24a}+\frac{7(ax+1)^{3/4}(1-ax)^{5/4}}{32a}+\frac{35(ax+1)^{3/4}\sqrt[4]{1-ax}}{64a}+\frac{35\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\right.}{128\sqrt{2}}$$

```
[Out] (35*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(64*a) + (7*(1 - a*x)^(5/4)*(1 + a*x)^
(3/4))/(32*a) - (7*(1 - a*x)^(9/4)*(1 + a*x)^(3/4))/(24*a) - ((1 - a*x)^(9/
4)*(1 + a*x)^(7/4))/(4*a) + (35*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a
*x)^(1/4)])/(64*Sqrt[2]*a) - (35*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 +
a*x)^(1/4)])/(64*Sqrt[2]*a) + (35*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sq
rt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a) - (35*Log[1 + Sqrt
[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*
Sqrt[2]*a)
```

Rubi [A] time = 0.23989, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6140, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(ax+1)^{7/4}(1-ax)^{9/4}}{4a}-\frac{7(ax+1)^{3/4}(1-ax)^{9/4}}{24a}+\frac{7(ax+1)^{3/4}(1-ax)^{5/4}}{32a}+\frac{35(ax+1)^{3/4}\sqrt[4]{1-ax}}{64a}+\frac{35\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\right.}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*(1 - a^2*x^2)^(3/2),x]
```


```
[Out] (35*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(64*a) + (7*(1 - a*x)^(5/4)*(1 + a*x)^
(3/4))/(32*a) - (7*(1 - a*x)^(9/4)*(1 + a*x)^(3/4))/(24*a) - ((1 - a*x)^(9/
4)*(1 + a*x)^(7/4))/(4*a) + (35*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a
*x)^(1/4)])/(64*Sqrt[2]*a) - (35*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 +
a*x)^(1/4)])/(64*Sqrt[2]*a) + (35*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sq
rt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a) - (35*Log[1 + Sqrt
[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*
Sqrt[2]*a)
```


Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}\,dx &= \int (1-ax)^{5/4}(1+ax)^{7/4}\,dx \\
&= -\frac{(1-ax)^{9/4}(1+ax)^{7/4}}{4a}+\frac{7}{8}\int (1-ax)^{5/4}(1+ax)^{3/4}\,dx \\
&= -\frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a}-\frac{(1-ax)^{9/4}(1+ax)^{7/4}}{4a}+\frac{7}{16}\int \frac{(1-ax)^{5/4}}{\sqrt[4]{1+ax}}\,dx \\
&= \frac{7(1-ax)^{5/4}(1+ax)^{3/4}}{32a}-\frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a}-\frac{(1-ax)^{9/4}(1+ax)^{7/4}}{4a}+\frac{35}{64}\int \frac{\sqrt[4]{1}}{\sqrt[4]{1}} \\
&= \frac{35\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a}+\frac{7(1-ax)^{5/4}(1+ax)^{3/4}}{32a}-\frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a}-(1-ax)^{9/} \\
&= \frac{35\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a}+\frac{7(1-ax)^{5/4}(1+ax)^{3/4}}{32a}-\frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a}-(1-ax)^{9/} \\
&= \frac{35\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a}+\frac{7(1-ax)^{5/4}(1+ax)^{3/4}}{32a}-\frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a}-(1-ax)^{9/} \\
&= \frac{35\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a}+\frac{7(1-ax)^{5/4}(1+ax)^{3/4}}{32a}-\frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a}-(1-ax)^{9/} \\
&= \frac{35\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a}+\frac{7(1-ax)^{5/4}(1+ax)^{3/4}}{32a}-\frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a}-(1-ax)^{9/} \\
&= \frac{35\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a}+\frac{7(1-ax)^{5/4}(1+ax)^{3/4}}{32a}-\frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a}-(1-ax)^{9/} \\
&= \frac{35\sqrt[4]{1-ax}(1+ax)^{3/4}}{64a}+\frac{7(1-ax)^{5/4}(1+ax)^{3/4}}{32a}-\frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a}-(1-ax)^{9/}
\end{aligned}
$$

Mathematica [C] time = 0.0143421, size = 42, normalized size = 0.14

$$
-\frac{8\,2^{3/4}(1-ax)^{9/4}\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}(1-ax)\right)}{9a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)*(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-8*2^(3/4)*(1 - a*x)^(9/4)*Hypergeometric2F1[-7/4, 9/4, 13/4, (1 - a*x)/2]
)/(9*a)
```

Maple [F] time = 0.201, size = 0, normalized size = 0.

$$
\int \sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2x^2+1\right)^{\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

Fricas [B] time = 2.95697, size = 1346, normalized size = 4.38

$$-\frac{420\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\sqrt{2}a\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-1\right)+420\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(3/2),x, algorith
m="fricas")
```

```
[Out] -1/768*(420*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt((sqrt(2)*(a^4*x
 - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*s
qrt(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqr
t(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) - 1) + 420*sqrt(2)*a*(a^(-4
))^(1/4)*arctan(sqrt(2)*a*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2
 + 1)/(a*x - 1))*(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2
 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
 - 1))*(a^(-4))^(1/4) + 1) + 105*sqrt(2)*a*(a^(-4))^(1/4)*log((sqrt(2)*(a^4*
x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)
*sqrt(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 105*sqrt(2)*a*(a^(-4))^(1/
4)*log(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))
^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 4*(4
8*a^3*x^3 + 8*a^2*x^2 - 118*a*x - 43)*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^
2 + 1)/(a*x - 1)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(3/2),x, algorith
m="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

3.1283 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx$

Optimal. Leaf size=255

$$-\frac{(ax+1)^{3/4}(1-ax)^{5/4}}{2a}+\frac{3(ax+1)^{3/4}\sqrt[4]{1-ax}}{4a}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a}+\frac{3\tan^{-1}\left(\right.}{4}$$

```
[Out] (3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*a) - ((1 - a*x)^(5/4)*(1 + a*x)^(3/4
))/(2*a) + (3*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqr
t[2]*a) - (3*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt
[2]*a) + (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))
/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a) - (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] +
 (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a)
```

Rubi [A] time = 0.187567, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6140, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(ax+1)^{3/4}(1-ax)^{5/4}}{2a}+\frac{3(ax+1)^{3/4}\sqrt[4]{1-ax}}{4a}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a}+\frac{3\tan^{-1}\left(\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*Sqrt[1 - a^2*x^2],x]
```

```
[Out] (3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*a) - ((1 - a*x)^(5/4)*(1 + a*x)^(3/4
))/(2*a) + (3*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqr
t[2]*a) - (3*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt
[2]*a) + (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))
/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a) - (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] +
 (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx = \int \sqrt[4]{1-ax}(1+ax)^{3/4}\,dx$$
$$= -\frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a} + \frac{3}{4}\int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx$$
$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a} + \frac{3}{8}\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$
$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)}{2a}$$
$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a}$$
$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a} - \frac{3\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4a} - \frac{3\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8a} - \frac{3\,\text{Subs}}{}$$
$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a} + \frac{3\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8\sqrt{2}a} - \frac{3\log\left(1+\right.}{}$$
$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}}{4a} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a} + \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}a} - \frac{3\tan^{-1}\left(1+\right.}{4\sqrt{2}a}$$

Mathematica [C] time = 0.0108107, size = 42, normalized size = 0.16

$$-\frac{4\,2^{3/4}(1-ax)^{5/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-ax)\right)}{5a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)*Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-4*2^(3/4)*(1 - a*x)^(5/4)*Hypergeometric2F1[-3/4, 5/4, 9/4, (1 - a*x)/2])
/(5*a)
```

Maple [F] time = 0.198, size = 0, normalized size = 0.

$$\int \sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

Fricas [B] time = 2.8568, size = 1299, normalized size = 5.09

$$-\frac{12\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\sqrt{2}a\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-1\right)+12\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] -1/16*(12*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt((sqrt(2)*(a^4*x -
a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*sqr
t(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(
-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) - 1) + 12*sqrt(2)*a*(a^(-4))^
(1/4)*arctan(sqrt(2)*a*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1
)/(a*x - 1))*(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 +
1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1
))*(a^(-4))^(1/4) + 1) + 3*sqrt(2)*a*(a^(-4))^(1/4)*log((sqrt(2)*(a^4*x - a
^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*sqrt
(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 3*sqrt(2)*a*(a^(-4))^(1/4)*log(
-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4)
- (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 4*sqrt(-a^2
*x^2 + 1)*(2*a*x + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\sqrt{-(ax-1)(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))*sqrt(-(a*x - 1)*(a*x + 1)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

3.1284 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=193

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{a}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{a}$$

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a - (Sqrt[2
]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a + Log[1 + Sqrt[1
 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(Sqrt[2]
*a) - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 +
a*x)^(1/4)]/(Sqrt[2]*a)
```

Rubi [A] time = 0.142258, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {6140, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{a}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a - (Sqrt[2
]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a + Log[1 + Sqrt[1
 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(Sqrt[2]
*a) - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 +
a*x)^(1/4)]/(Sqrt[2]*a)
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx = \int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$
$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$
$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{2\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,\right.}{\sqrt{2}a}$$
$$= \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} - \frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right.}{a}$$
$$= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} - \frac{\log\left(1+\frac{\sqrt{1-}}{\sqrt{1+}}\right.}{}$$

Mathematica [C] time = 0.0107849, size = 40, normalized size = 0.21

$$-\frac{2\ 2^{3/4}\sqrt[4]{1-ax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-ax)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-2*2^(3/4)*(1 - a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (1 - a*x)/2])/
a
```

Maple [F] time = 0.21, size = 0, normalized size = 0.

$$\int \sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/sqrt(-a^2*x^2 + 1), x)
```

Fricas [B] time = 2.77645, size = 1183, normalized size = 6.13

$$-2\sqrt{2}\frac{1}{a^4}^{\frac{1}{4}}\arctan\left(\sqrt{2}a\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^4}^{\frac{3}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^4}^{\frac{1}{4}}-\sqrt{2}a\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^4}^{\frac{1}{4}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] -2*sqrt(2)*(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt((sqrt(2)*(a^4*x - a^3)*sqrt
(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*sqrt(a^(-4))
 - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-sqrt(-a^
2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) - 1) - 2*sqrt(2)*(a^(-4))^(1/4)*arctan
(sqrt(2)*a*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))
*(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1
))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^
(1/4) + 1) - 1/2*sqrt(2)*(a^(-4))^(1/4)*log((sqrt(2)*(a^4*x - a^3)*sqrt(-sq
rt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*sqrt(a^(-4)) - s
qrt(-a^2*x^2 + 1))/(a*x - 1)) + 1/2*sqrt(2)*(a^(-4))^(1/4)*log(-(sqrt(2)*(a
^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) - (a^3*x - a
^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/sqrt(-(a*x - 1)*(a*x + 1)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/sqrt(-a^2*x^2 + 1), x)
```

3.1285 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{2(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{3a\sqrt{1-a^2x^2}}$$

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 2*a*x))/(3*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0399511, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {6135}

$$-\frac{2(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 2*a*x))/(3*a*Sqrt[1 - a^2*x^2])
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx = -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-2ax)}{3a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0128866, size = 32, normalized size = 0.86

$$\frac{2(2ax-1)}{3a(1-ax)^{3/4}\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (2*(-1 + 2*a*x))/(3*a*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))
```

Maple [A] time = 0.031, size = 54, normalized size = 1.5

$$-\frac{(2\,ax-2)\,(ax+1)\,(2\,ax-1)}{3\,a}\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -2/3*(a*x-1)*(a*x+1)*(2*a*x-1)*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/a/(-a^2*x
^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 2.45104, size = 117, normalized size = 3.16

$$-\frac{2\sqrt{-a^2x^2+1}(2\,ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{3\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(3/2),x, algorith
m="fricas")
```

```
[Out] -2/3*sqrt(-a^2*x^2 + 1)*(2*a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))/(a^
3*x^2 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/(-(a*x - 1)*(a*x + 1))**(3/2)
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(3/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*x^2 + 1)^(3/2), x)
```

3.1286 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{2(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{35a\left(1-a^2x^2\right)^{3/2}}-\frac{16(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{35a\sqrt{1-a^2x^2}}$$

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 6*a*x))/(35*a*(1 - a^2*x^2)^(3/2)) - (16*E^(Arc
Tanh[a*x]/2)*(1 - 2*a*x))/(35*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0793326, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6136, 6135}

$$-\frac{2(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{35a\left(1-a^2x^2\right)^{3/2}}-\frac{16(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{35a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 6*a*x))/(35*a*(1 - a^2*x^2)^(3/2)) - (16*E^(Arc
Tanh[a*x]/2)*(1 - 2*a*x))/(35*a*Sqrt[1 - a^2*x^2])
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx &= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{35a\left(1-a^2x^2\right)^{3/2}}+\frac{24}{35}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx\\ &= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{35a\left(1-a^2x^2\right)^{3/2}}-\frac{16e^{\frac{1}{2}\tanh^{-1}(ax)}(1-2ax)}{35a\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0198521, size = 48, normalized size = 0.64

$$-\frac{2\left(16a^3x^3-8a^2x^2-22ax+9\right)}{35a(1-ax)^{7/4}(ax+1)^{5/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (-2*(9 - 22*a*x - 8*a^2*x^2 + 16*a^3*x^3))/(35*a*(1 - a*x)^(7/4)*(1 + a*x)^
(5/4))
```

Maple [A] time = 0.033, size = 70, normalized size = 0.9

$$\frac{(2\,ax-2)\,(ax+1)\left(16\,x^3a^3-8\,a^2x^2-22\,ax+9\right)}{35\,a}\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2x^2+1\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(5/2),x)
```

```
[Out] 2/35*(a*x-1)*(a*x+1)*(16*a^3*x^3-8*a^2*x^2-22*a*x+9)*((a*x+1)/(-a^2*x^2+1)^
(1/2))^(1/2)/a/(-a^2*x^2+1)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(5/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*x^2 + 1)^(5/2), x)
```

Fricas [A] time = 2.59857, size = 170, normalized size = 2.27

$$-\frac{2\left(16\,a^3x^3-8\,a^2x^2-22\,ax+9\right)\sqrt{-a^2x^2+1}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{35\left(a^5x^4-2\,a^3x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(5/2),x, algorith
m="fricas")
```

```
[Out] -2/35*(16*a^3*x^3 - 8*a^2*x^2 - 22*a*x + 9)*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-
a^2*x^2 + 1)/(a*x - 1))/(a^5*x^4 - 2*a^3*x^2 + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*x**2+1)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(5/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*x^2 + 1)^(5/2), x)
```

3.1287 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{2(1-10ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{99a\left(1-a^2x^2\right)^{5/2}}-\frac{256(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{693a\sqrt{1-a^2x^2}}-\frac{32(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{693a\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 10*a*x))/(99*a*(1 - a^2*x^2)^(5/2)) - (32*E^(Ar
cTanh[a*x]/2)*(1 - 6*a*x))/(693*a*(1 - a^2*x^2)^(3/2)) - (256*E^(ArcTanh[a*
x]/2)*(1 - 2*a*x))/(693*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.117185, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6136, 6135}

$$-\frac{2(1-10ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{99a\left(1-a^2x^2\right)^{5/2}}-\frac{256(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{693a\sqrt{1-a^2x^2}}-\frac{32(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{693a\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(1 - a^2*x^2)^(7/2),x]
```


```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 10*a*x))/(99*a*(1 - a^2*x^2)^(5/2)) - (32*E^(Ar
cTanh[a*x]/2)*(1 - 6*a*x))/(693*a*(1 - a^2*x^2)^(3/2)) - (256*E^(ArcTanh[a*
x]/2)*(1 - 2*a*x))/(693*a*Sqrt[1 - a^2*x^2])
```


Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{7/2}}\,dx = -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{99a\left(1-a^2x^2\right)^{5/2}} + \frac{80}{99}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{99a\left(1-a^2x^2\right)^{5/2}} - \frac{32e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{693a\left(1-a^2x^2\right)^{3/2}} + \frac{128}{231}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{99a\left(1-a^2x^2\right)^{5/2}} - \frac{32e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{693a\left(1-a^2x^2\right)^{3/2}} - \frac{256e^{\frac{1}{2}\tanh^{-1}(ax)}(1-2ax)}{693a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0303329, size = 64, normalized size = 0.57

$$\frac{2\left(256a^5x^5-128a^4x^4-608a^3x^3+272a^2x^2+422ax-151\right)}{693a(1-ax)^{11/4}(ax+1)^{9/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] (2*(-151 + 422*a*x + 272*a^2*x^2 - 608*a^3*x^3 - 128*a^4*x^4 + 256*a^5*x^5)
)/(693*a*(1 - a*x)^(11/4)*(1 + a*x)^(9/4))
```

Maple [A] time = 0.03, size = 86, normalized size = 0.8

$$-\frac{(2\,ax-2)\,(ax+1)\left(256\,x^5a^5-128\,x^4a^4-608\,x^3a^3+272\,a^2x^2+422\,ax-151\right)}{693\,a}\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2x^2+1\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(7/2),x)
```

```
[Out] -2/693*(a*x-1)*(a*x+1)*(256*a^5*x^5-128*a^4*x^4-608*a^3*x^3+272*a^2*x^2+422
*a*x-151)*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/a/(-a^2*x^2+1)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2x^2+1\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(7/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*x^2 + 1)^(7/2), x)
```

Fricas [A] time = 2.56681, size = 234, normalized size = 2.09

$$-\frac{2\left(256\,a^{5}x^{5}-128\,a^{4}x^{4}-608\,a^{3}x^{3}+272\,a^{2}x^{2}+422\,ax-151\right)\sqrt{-a^{2}x^{2}+1}\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}}{693\left(a^{7}x^{6}-3\,a^{5}x^{4}+3\,a^{3}x^{2}-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(7/2),x, algorith
m="fricas")
```

```
[Out] -2/693*(256*a^5*x^5 - 128*a^4*x^4 - 608*a^3*x^3 + 272*a^2*x^2 + 422*a*x - 1
51)*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))/(a^7*x^6 - 3*a^5
*x^4 + 3*a^3*x^2 - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*x**2+1)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}}{\left(-a^{2}x^{2}+1\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(7/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*x^2 + 1)^(7/2), x)
```

3.1288 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{(1-a^2x^2)^{9/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{2(1-14ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{195a(1-a^2x^2)^{7/2}}-\frac{2048(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435a\sqrt{1-a^2x^2}}-\frac{256(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435a(1-a^2x^2)^{3/2}}-\frac{112(1-10ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435a(1-a^2x^2)^{5/2}}$$

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 14*a*x))/(195*a*(1 - a^2*x^2)^(7/2)) - (112*E^(
ArcTanh[a*x]/2)*(1 - 10*a*x))/(6435*a*(1 - a^2*x^2)^(5/2)) - (256*E^(ArcTan
h[a*x]/2)*(1 - 6*a*x))/(6435*a*(1 - a^2*x^2)^(3/2)) - (2048*E^(ArcTanh[a*x]
/2)*(1 - 2*a*x))/(6435*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.168022, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6136, 6135}

$$-\frac{2(1-14ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{195a(1-a^2x^2)^{7/2}}-\frac{2048(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435a\sqrt{1-a^2x^2}}-\frac{256(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435a(1-a^2x^2)^{3/2}}-\frac{112(1-10ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435a(1-a^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 14*a*x))/(195*a*(1 - a^2*x^2)^(7/2)) - (112*E^(
ArcTanh[a*x]/2)*(1 - 10*a*x))/(6435*a*(1 - a^2*x^2)^(5/2)) - (256*E^(ArcTan
h[a*x]/2)*(1 - 6*a*x))/(6435*a*(1 - a^2*x^2)^(3/2)) - (2048*E^(ArcTanh[a*x]
/2)*(1 - 2*a*x))/(6435*a*Sqrt[1 - a^2*x^2])
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{9/2}}\,dx = -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-14ax)}{195a\left(1-a^2x^2\right)^{7/2}} + \frac{56}{65}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-14ax)}{195a\left(1-a^2x^2\right)^{7/2}} - \frac{112e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{6435a\left(1-a^2x^2\right)^{5/2}} + \frac{896\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx}{1287}$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-14ax)}{195a\left(1-a^2x^2\right)^{7/2}} - \frac{112e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{6435a\left(1-a^2x^2\right)^{5/2}} - \frac{256e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{6435a\left(1-a^2x^2\right)^{3/2}} + \frac{1024\int \frac{e^{\frac{1}{2}\tanh^{-1}}}{\left(1-a^2x^2\right)}}{2145}$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-14ax)}{195a\left(1-a^2x^2\right)^{7/2}} - \frac{112e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{6435a\left(1-a^2x^2\right)^{5/2}} - \frac{256e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{6435a\left(1-a^2x^2\right)^{3/2}} - \frac{2048e^{\frac{1}{2}\tanh^{-1}(ax}}{6435a\sqrt{1-}}$$

Mathematica [A] time = 0.0303226, size = 80, normalized size = 0.54

$$-\frac{2\left(2048a^7x^7-1024a^6x^6-6912a^5x^5+3200a^4x^4+8240a^3x^3-3384a^2x^2-3838ax+1241\right)}{6435a(1-ax)^{15/4}(ax+1)^{13/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] (-2*(1241 - 3838*a*x - 3384*a^2*x^2 + 8240*a^3*x^3 + 3200*a^4*x^4 - 6912*a^
5*x^5 - 1024*a^6*x^6 + 2048*a^7*x^7))/(6435*a*(1 - a*x)^(15/4)*(1 + a*x)^(1
3/4))
```

Maple [A] time = 0.031, size = 102, normalized size = 0.7

$$\frac{(2\,ax-2)\,(ax+1)\left(2048\,a^7x^7-1024\,x^6a^6-6912\,x^5a^5+3200\,x^4a^4+8240\,x^3a^3-3384\,a^2x^2-3838\,ax+1241\right)}{6435\,a}\sqrt{(ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(9/2),x)
```

```
[Out] 2/6435*(a*x-1)*(a*x+1)*(2048*a^7*x^7-1024*a^6*x^6-6912*a^5*x^5+3200*a^4*x^4
+8240*a^3*x^3-3384*a^2*x^2-3838*a*x+1241)*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2
)/a/(-a^2*x^2+1)^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2x^2+1\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(9/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*x^2 + 1)^(9/2), x)
```

Fricas [A] time = 2.53286, size = 300, normalized size = 2.01

$$-\frac{2\left(2048\,a^7x^7-1024\,a^6x^6-6912\,a^5x^5+3200\,a^4x^4+8240\,a^3x^3-3384\,a^2x^2-3838\,ax+1241\right)\sqrt{-a^2x^2+1}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{6435\left(a^9x^8-4\,a^7x^6+6\,a^5x^4-4\,a^3x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(9/2),x, algorith
m="fricas")
```

```
[Out] -2/6435*(2048*a^7*x^7 - 1024*a^6*x^6 - 6912*a^5*x^5 + 3200*a^4*x^4 + 8240*a
^3*x^3 - 3384*a^2*x^2 - 3838*a*x + 1241)*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2
*x^2 + 1)/(a*x - 1))/(a^9*x^8 - 4*a^7*x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*x**2+1)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2x^2+1\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(9/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*x^2 + 1)^(9/2), x)
```

3.1289 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=679

$$-\frac{c^2(ax+1)^{11/4}(1-ax)^{13/4}\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}-\frac{11c^2(ax+1)^{7/4}(1-ax)^{13/4}\sqrt{c-a^2cx^2}}{60a\sqrt{1-a^2x^2}}-\frac{77c^2(ax+1)^{3/4}(1-ax)^{13/4}\sqrt{c-a^2cx^2}}{480a\sqrt{1-a^2x^2}}+$$

```
[Out] (231*c^2*(1 - a*x)^(1/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(512*a*Sqrt[1
 - a^2*x^2]) + (231*c^2*(1 - a*x)^(5/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2]
)/(1280*a*Sqrt[1 - a^2*x^2]) + (77*c^2*(1 - a*x)^(9/4)*(1 + a*x)^(3/4)*Sqrt
[c - a^2*c*x^2])/(960*a*Sqrt[1 - a^2*x^2]) - (77*c^2*(1 - a*x)^(13/4)*(1 +
a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(480*a*Sqrt[1 - a^2*x^2]) - (11*c^2*(1 - a*
x)^(13/4)*(1 + a*x)^(7/4)*Sqrt[c - a^2*c*x^2])/(60*a*Sqrt[1 - a^2*x^2]) - (
c^2*(1 - a*x)^(13/4)*(1 + a*x)^(11/4)*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - a^
2*x^2]) + (231*c^2*Sqrt[c - a^2*c*x^2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))
/(1 + a*x)^(1/4)])/(512*Sqrt[2]*a*Sqrt[1 - a^2*x^2]) - (231*c^2*Sqrt[c - a^
2*c*x^2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(512*Sqrt[2
]*a*Sqrt[1 - a^2*x^2]) + (231*c^2*Sqrt[c - a^2*c*x^2]*Log[1 + Sqrt[1 - a*x]
/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(1024*Sqrt[2]*
a*Sqrt[1 - a^2*x^2]) - (231*c^2*Sqrt[c - a^2*c*x^2]*Log[1 + Sqrt[1 - a*x]/S
qrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(1024*Sqrt[2]*a*
Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.392367, antiderivative size = 679, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {6143, 6140, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{c^2(ax+1)^{11/4}(1-ax)^{13/4}\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}-\frac{11c^2(ax+1)^{7/4}(1-ax)^{13/4}\sqrt{c-a^2cx^2}}{60a\sqrt{1-a^2x^2}}-\frac{77c^2(ax+1)^{3/4}(1-ax)^{13/4}\sqrt{c-a^2cx^2}}{480a\sqrt{1-a^2x^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*(c - a^2*c*x^2)^(5/2),x]
```


```
[Out] (231*c^2*(1 - a*x)^(1/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(512*a*Sqrt[1
 - a^2*x^2]) + (231*c^2*(1 - a*x)^(5/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2]
)/(1280*a*Sqrt[1 - a^2*x^2]) + (77*c^2*(1 - a*x)^(9/4)*(1 + a*x)^(3/4)*Sqrt
[c - a^2*c*x^2])/(960*a*Sqrt[1 - a^2*x^2]) - (77*c^2*(1 - a*x)^(13/4)*(1 +
a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(480*a*Sqrt[1 - a^2*x^2]) - (11*c^2*(1 - a*
x)^(13/4)*(1 + a*x)^(7/4)*Sqrt[c - a^2*c*x^2])/(60*a*Sqrt[1 - a^2*x^2]) - (
c^2*(1 - a*x)^(13/4)*(1 + a*x)^(11/4)*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - a^
2*x^2]) + (231*c^2*Sqrt[c - a^2*c*x^2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))
/(1 + a*x)^(1/4)])/(512*Sqrt[2]*a*Sqrt[1 - a^2*x^2]) - (231*c^2*Sqrt[c - a^
2*c*x^2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(512*Sqrt[2
]*a*Sqrt[1 - a^2*x^2]) + (231*c^2*Sqrt[c - a^2*c*x^2]*Log[1 + Sqrt[1 - a*x]
/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(1024*Sqrt[2]*
a*Sqrt[1 - a^2*x^2]) - (231*c^2*Sqrt[c - a^2*c*x^2]*Log[1 + Sqrt[1 - a*x]/S
qrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(1024*Sqrt[2]*a*
Sqrt[1 - a^2*x^2])
```


Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
```

```
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}dx = \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int(1-ax)^{9/4}(1+ax)^{11/4}dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{c^2(1-ax)^{13/4}(1+ax)^{11/4}\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}} + \frac{\left(11c^2\sqrt{c-a^2cx^2}\right)\int(1-ax)^{9/4}(1+ax)^{7/4}dx}{12\sqrt{1-a^2x^2}}$$

$$= -\frac{11c^2(1-ax)^{13/4}(1+ax)^{7/4}\sqrt{c-a^2cx^2}}{60a\sqrt{1-a^2x^2}} - \frac{c^2(1-ax)^{13/4}(1+ax)^{11/4}\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}} + \frac{(7}{}$$

$$= -\frac{77c^2(1-ax)^{13/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{480a\sqrt{1-a^2x^2}} - \frac{11c^2(1-ax)^{13/4}(1+ax)^{7/4}\sqrt{c-a^2cx^2}}{60a\sqrt{1-a^2x^2}} - \frac{c}{}$$

$$= \frac{77c^2(1-ax)^{9/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{960a\sqrt{1-a^2x^2}} - \frac{77c^2(1-ax)^{13/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{480a\sqrt{1-a^2x^2}} - \frac{11c}{}$$

$$= \frac{231c^2(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{1280a\sqrt{1-a^2x^2}} + \frac{77c^2(1-ax)^{9/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{960a\sqrt{1-a^2x^2}} - \frac{77}{}$$

$$= \frac{231c^2\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{512a\sqrt{1-a^2x^2}} + \frac{231c^2(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{1280a\sqrt{1-a^2x^2}} + \frac{77c}{}$$

$$= \frac{231c^2\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{512a\sqrt{1-a^2x^2}} + \frac{231c^2(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{1280a\sqrt{1-a^2x^2}} + \frac{77c}{}$$

$$= \frac{231c^2\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{512a\sqrt{1-a^2x^2}} + \frac{231c^2(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{1280a\sqrt{1-a^2x^2}} + \frac{77c}{}$$

$$= \frac{231c^2\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{512a\sqrt{1-a^2x^2}} + \frac{231c^2(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{1280a\sqrt{1-a^2x^2}} + \frac{77c}{}$$

$$= \frac{231c^2\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{512a\sqrt{1-a^2x^2}} + \frac{231c^2(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{1280a\sqrt{1-a^2x^2}} + \frac{77c}{}$$

$$= \frac{231c^2\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{512a\sqrt{1-a^2x^2}} + \frac{231c^2(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{1280a\sqrt{1-a^2x^2}} + \frac{77c}{}$$

$$= \frac{231c^2\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{512a\sqrt{1-a^2x^2}} + \frac{231c^2(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{1280a\sqrt{1-a^2x^2}} + \frac{77c}{}$$

Mathematica [C] time = 0.0362318, size = 74, normalized size = 0.11

$$-\frac{16\ 2^{3/4}c^2(1-ax)^{13/4}\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},\frac{13}{4},\frac{17}{4},\frac{1}{2}(1-ax)\right)}{13a\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-16*2^(3/4)*c^2*(1 - a*x)^(13/4)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[-11
/4, 13/4, 17/4, (1 - a*x)/2])/(13*a*Sqrt[1 - a^2*x^2])
```

Maple [F] time = 0.198, size = 0, normalized size = 0.

$$\int \sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2cx^2+c\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algori
thm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algori
thm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

3.1290 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=547

$$-\frac{c(ax+1)^{7/4}(1-ax)^{9/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}-\frac{7c(ax+1)^{3/4}(1-ax)^{9/4}\sqrt{c-a^2cx^2}}{24a\sqrt{1-a^2x^2}}+\frac{7c(ax+1)^{3/4}(1-ax)^{5/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}+\frac{35c(ax+$$

```
[Out] (35*c*(1 - a*x)^(1/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(64*a*Sqrt[1 - a
^2*x^2]) + (7*c*(1 - a*x)^(5/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(32*a*
Sqrt[1 - a^2*x^2]) - (7*c*(1 - a*x)^(9/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^
2])/(24*a*Sqrt[1 - a^2*x^2]) - (c*(1 - a*x)^(9/4)*(1 + a*x)^(7/4)*Sqrt[c -
a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x^2]) + (35*c*Sqrt[c - a^2*c*x^2]*ArcTan[1 -
(Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]*a*Sqrt[1 - a^2*x^2]
) - (35*c*Sqrt[c - a^2*c*x^2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x
)^(1/4)])/(64*Sqrt[2]*a*Sqrt[1 - a^2*x^2]) + (35*c*Sqrt[c - a^2*c*x^2]*Log[
1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)
])/(128*Sqrt[2]*a*Sqrt[1 - a^2*x^2]) - (35*c*Sqrt[c - a^2*c*x^2]*Log[1 + Sq
rt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(12
8*Sqrt[2]*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.315305, antiderivative size = 547, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {6143, 6140, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{c(ax+1)^{7/4}(1-ax)^{9/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}-\frac{7c(ax+1)^{3/4}(1-ax)^{9/4}\sqrt{c-a^2cx^2}}{24a\sqrt{1-a^2x^2}}+\frac{7c(ax+1)^{3/4}(1-ax)^{5/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}+\frac{35c(ax+$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*(c - a^2*c*x^2)^(3/2),x]
```


```
[Out] (35*c*(1 - a*x)^(1/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(64*a*Sqrt[1 - a
^2*x^2]) + (7*c*(1 - a*x)^(5/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(32*a*
Sqrt[1 - a^2*x^2]) - (7*c*(1 - a*x)^(9/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^
2])/(24*a*Sqrt[1 - a^2*x^2]) - (c*(1 - a*x)^(9/4)*(1 + a*x)^(7/4)*Sqrt[c -
a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x^2]) + (35*c*Sqrt[c - a^2*c*x^2]*ArcTan[1 -
(Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]*a*Sqrt[1 - a^2*x^2]
) - (35*c*Sqrt[c - a^2*c*x^2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x
)^(1/4)])/(64*Sqrt[2]*a*Sqrt[1 - a^2*x^2]) + (35*c*Sqrt[c - a^2*c*x^2]*Log[
1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)
])/(128*Sqrt[2]*a*Sqrt[1 - a^2*x^2]) - (35*c*Sqrt[c - a^2*c*x^2]*Log[1 + Sq
rt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(12
8*Sqrt[2]*a*Sqrt[1 - a^2*x^2])
```


Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
```

```
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}dx=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}dx}{\sqrt{1-a^2x^2}}$$

$$=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(1-ax)^{5/4}(1+ax)^{7/4}dx}{\sqrt{1-a^2x^2}}$$

$$=-\frac{c(1-ax)^{9/4}(1+ax)^{7/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}+\frac{\left(7c\sqrt{c-a^2cx^2}\right)\int(1-ax)^{5/4}(1+ax)^{3/4}dx}{8\sqrt{1-a^2x^2}}$$

$$=-\frac{7c(1-ax)^{9/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{24a\sqrt{1-a^2x^2}}-\frac{c(1-ax)^{9/4}(1+ax)^{7/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}+\frac{\left(7c\sqrt{c-a^2c\right.}{16\sqrt{}}$$

$$=\frac{7c(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}-\frac{7c(1-ax)^{9/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{24a\sqrt{1-a^2x^2}}-\frac{c(1-ax)^{9/4}(}{4a}$$

$$=\frac{35c\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{64a\sqrt{1-a^2x^2}}+\frac{7c(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}-\frac{7c(1-ax)^{9/4}}{24}$$

$$=\frac{35c\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{64a\sqrt{1-a^2x^2}}+\frac{7c(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}-\frac{7c(1-ax)^{9/4}}{24}$$

$$=\frac{35c\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{64a\sqrt{1-a^2x^2}}+\frac{7c(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}-\frac{7c(1-ax)^{9/4}}{24}$$

$$=\frac{35c\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{64a\sqrt{1-a^2x^2}}+\frac{7c(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}-\frac{7c(1-ax)^{9/4}}{24}$$

$$=\frac{35c\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{64a\sqrt{1-a^2x^2}}+\frac{7c(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}-\frac{7c(1-ax)^{9/4}}{24}$$

$$=\frac{35c\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{64a\sqrt{1-a^2x^2}}+\frac{7c(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}-\frac{7c(1-ax)^{9/4}}{24}$$

$$=\frac{35c\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{64a\sqrt{1-a^2x^2}}+\frac{7c(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}-\frac{7c(1-ax)^{9/4}}{24}$$

Mathematica [C] time = 0.0299938, size = 72, normalized size = 0.13

$$-\frac{8\ 2^{3/4}c(1-ax)^{9/4}\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}(1-ax)\right)}{9a\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(ArcTanh[a*x]/2)*(c - a^2*c*x^2)^(3/2),x]

[Out] (-8*2^(3/4)*c*(1 - a*x)^(9/4)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[-7/4, 9/4, 13/4, (1 - a*x)/2])/(9*a*Sqrt[1 - a^2*x^2])

Maple [F] time = 0.202, size = 0, normalized size = 0.

$$\int \sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2cx^2+c\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(3/2),x)

[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")

[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*(-a**2*c*x**2+c)**(3/2),x)

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algori
thm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

3.1291 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=429

$$-\frac{(ax+1)^{3/4}(1-ax)^{5/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}}+\frac{3(ax+1)^{3/4}\sqrt[4]{1-ax}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}+\frac{3\sqrt{c-a^2cx^2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a\sqrt{1-a^2x^2}}-\frac{3\sqrt{c}}{}$$

```
[Out] (3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x
^2]) - ((1 - a*x)^(5/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 -
a^2*x^2]) + (3*Sqrt[c - a^2*c*x^2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1
+ a*x)^(1/4)])/(4*Sqrt[2]*a*Sqrt[1 - a^2*x^2]) - (3*Sqrt[c - a^2*c*x^2]*Arc
Tan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt[2]*a*Sqrt[1 - a
^2*x^2]) + (3*Sqrt[c - a^2*c*x^2]*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sq
rt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a*Sqrt[1 - a^2*x^2]) -
(3*Sqrt[c - a^2*c*x^2]*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 -
a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.257195, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {6143, 6140, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(ax+1)^{3/4}(1-ax)^{5/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}}+\frac{3(ax+1)^{3/4}\sqrt[4]{1-ax}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}+\frac{3\sqrt{c-a^2cx^2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a\sqrt{1-a^2x^2}}-\frac{3\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*Sqrt[c - a^2*c*x^2],x]
```



```
[Out] (3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x
^2]) - ((1 - a*x)^(5/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 -
a^2*x^2]) + (3*Sqrt[c - a^2*c*x^2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1
+ a*x)^(1/4)])/(4*Sqrt[2]*a*Sqrt[1 - a^2*x^2]) - (3*Sqrt[c - a^2*c*x^2]*Arc
Tan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt[2]*a*Sqrt[1 - a
^2*x^2]) + (3*Sqrt[c - a^2*c*x^2]*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sq
rt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a*Sqrt[1 - a^2*x^2]) -
(3*Sqrt[c - a^2*c*x^2]*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 -
a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a*Sqrt[1 - a^2*x^2])
```


Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 240

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/n]

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{\frac{1}{2}\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \sqrt[4]{1-ax}(1+ax)^{3/4}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{c-a^2cx^2}\right)\int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx}{4\sqrt{1-a^2x^2}}$$
$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{c-a^2cx^2}\right)\int}{8\sqrt{1-}}$$
$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} - \frac{\left(3\sqrt{c-a^2cx^2}\right)\mathrm{Su}}{2}$$
$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} - \frac{\left(3\sqrt{c-a^2cx^2}\right)\mathrm{Su}}{2a}$$
$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} - \frac{\left(3\sqrt{c-a^2cx^2}\right)\mathrm{Su}}{4a}$$
$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} - \frac{\left(3\sqrt{c-a^2cx^2}\right)\mathrm{Su}}{}$$
$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} + \frac{3\sqrt{c-a^2cx^2}\log\Big(}{8\sqrt{2}a}$$
$$= \frac{3\sqrt[4]{1-ax}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} + \frac{3\sqrt{c-a^2cx^2}\tan^{-}}{4\sqrt{2}a\sqrt{1}}$$

Mathematica [C] time = 0.0235308, size = 71, normalized size = 0.17

$$-\frac{4\ 2^{3/4}(1-ax)^{5/4}\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-ax)\right)}{5a\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-4*2^(3/4)*(1 - a*x)^(5/4)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[-3/4, 5/4
, 9/4, (1 - a*x)/2])/(5*a*Sqrt[1 - a^2*x^2])
```

Maple [F] time = 0.203, size = 0, normalized size = 0.

$$\int \sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\sqrt{-a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

3.1292 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=309

$$\frac{\sqrt{1-a^2x^2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a\sqrt{c-a^2cx^2}}+\frac{\sqrt{2}\sqrt{1-a^2x^2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{a\sqrt{c-a^2cx^2}}-\ldots$$

```
[Out] (Sqrt[2]*Sqrt[1 - a^2*x^2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(
1/4)])/(a*Sqrt[c - a^2*c*x^2]) - (Sqrt[2]*Sqrt[1 - a^2*x^2]*ArcTan[1 + (Sqr
t[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(a*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 -
 a^2*x^2]*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(
1 + a*x)^(1/4)])/(Sqrt[2]*a*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*Log[1
 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]
)/(Sqrt[2]*a*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.204799, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6143, 6140, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt{1-a^2x^2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a\sqrt{c-a^2cx^2}}+\frac{\sqrt{2}\sqrt{1-a^2x^2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{a\sqrt{c-a^2cx^2}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[2]*Sqrt[1 - a^2*x^2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(
1/4)])/(a*Sqrt[c - a^2*c*x^2]) - (Sqrt[2]*Sqrt[1 - a^2*x^2]*ArcTan[1 + (Sqr
t[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(a*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 -
 a^2*x^2]*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(
1 + a*x)^(1/4)])/(Sqrt[2]*a*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*Log[1
 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]
)/(Sqrt[2]*a*Sqrt[c - a^2*c*x^2])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}} \\
&= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx}{\sqrt{c-a^2cx^2}} \\
&= -\frac{\left(4\sqrt{1-a^2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a\sqrt{c-a^2cx^2}} \\
&= -\frac{\left(4\sqrt{1-a^2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a\sqrt{c-a^2cx^2}} \\
&= -\frac{\left(2\sqrt{1-a^2x^2}\right)\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a\sqrt{c-a^2cx^2}} - \frac{\left(2\sqrt{1-a^2x^2}\right)\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a\sqrt{c-a^2cx^2}} \\
&= -\frac{\sqrt{1-a^2x^2}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-a}\cdots}{\cdots} \\
&= \frac{\sqrt{1-a^2x^2}\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a\sqrt{c-a^2cx^2}} - \frac{\left(\sqrt{2}\sqrt{1-a^2x^2}\right)\mathrm{Sub}\cdots}{\cdots} \\
&= \frac{\sqrt{2}\sqrt{1-a^2x^2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a\sqrt{c-a^2cx^2}} - \frac{\sqrt{2}\sqrt{1-a^2x^2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-a^2x^2}\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\cdots\right.}{\sqrt{2}a\sqrt{c-a^2cx\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.0219701, size = 69, normalized size = 0.22

$$
-\frac{2\,2^{3/4}\sqrt[4]{1-ax}\sqrt{1-a^2x^2}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-ax)\right)}{a\sqrt{c-a^2cx^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-2*2^(3/4)*(1 - a*x)^(1/4)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1/4, 1/4, 5
/4, (1 - a*x)/2])/(a*Sqrt[c - a^2*c*x^2])
```

Maple [F] time = 0.202, size = 0, normalized size = 0.

$$
\int \sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\frac{1}{\sqrt{-a^2cx^2+c}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/sqrt(-c*(a*x - 1)*(a*x + 1)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/sqrt(-a^2*c*x^2 + c), x)
```

3.1293 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{2(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{3ac\sqrt{c-a^2cx^2}}$$

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 2*a*x))/(3*a*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.0452908, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6135}

$$-\frac{2(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{3ac\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 2*a*x))/(3*a*c*Sqrt[c - a^2*c*x^2])
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-2ax)}{3ac\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.024708, size = 64, normalized size = 1.56

$$\frac{2(2ax-1)\sqrt{1-a^2x^2}}{3ac(1-ax)^{3/4}\sqrt[4]{ax+1}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*(-1 + 2*a*x)*Sqrt[1 - a^2*x^2])/(3*a*c*(1 - a*x)^(3/4)*(1 + a*x)^(1/4)*S
qrt[c - a^2*c*x^2])
```

Maple [A] time = 0.028, size = 55, normalized size = 1.3

$$-\frac{(2ax-2)(ax+1)(2ax-1)}{3a}\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -2/3*(a*x-1)*(a*x+1)*(2*a*x-1)*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/a/(-a^2*c
*x^2+c)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(3/2), x)
```

3.1294 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{16(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{35ac^2\sqrt{c-a^2cx^2}}-\frac{2(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{35ac\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 6*a*x))/(35*a*c*(c - a^2*c*x^2)^(3/2)) - (16*E^
(ArcTanh[a*x]/2)*(1 - 2*a*x))/(35*a*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.093848, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6136, 6135}

$$-\frac{16(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{35ac^2\sqrt{c-a^2cx^2}}-\frac{2(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{35ac\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 6*a*x))/(35*a*c*(c - a^2*c*x^2)^(3/2)) - (16*E^
(ArcTanh[a*x]/2)*(1 - 2*a*x))/(35*a*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{35ac\left(c-a^2cx^2\right)^{3/2}}+\frac{24\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{35c}\\ &= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{35ac\left(c-a^2cx^2\right)^{3/2}}-\frac{16e^{\frac{1}{2}\tanh^{-1}(ax)}(1-2ax)}{35ac^2\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0366685, size = 80, normalized size = 0.96

$$-\frac{2\sqrt{1-a^2x^2}\left(16a^3x^3-8a^2x^2-22ax+9\right)}{35ac^2(1-ax)^{7/4}(ax+1)^{5/4}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2]*(9 - 22*a*x - 8*a^2*x^2 + 16*a^3*x^3))/(35*a*c^2*(1 -
 a*x)^(7/4)*(1 + a*x)^(5/4)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.03, size = 71, normalized size = 0.9

$$\frac{(2\,ax-2)\,(ax+1)\left(16\,x^3a^3-8\,a^2x^2-22\,ax+9\right)}{35\,a}\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 2/35*(a*x-1)*(a*x+1)*(16*a^3*x^3-8*a^2*x^2-22*a*x+9)*((a*x+1)/(-a^2*x^2+1)^
(1/2))^(1/2)/a/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/2), x)
```

3.1295 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{256(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{693ac^3\sqrt{c-a^2cx^2}}-\frac{32(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{693ac^2\left(c-a^2cx^2\right)^{3/2}}-\frac{2(1-10ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{99ac\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 10*a*x))/(99*a*c*(c - a^2*c*x^2)^(5/2)) - (32*E
^(ArcTanh[a*x]/2)*(1 - 6*a*x))/(693*a*c^2*(c - a^2*c*x^2)^(3/2)) - (256*E^(
ArcTanh[a*x]/2)*(1 - 2*a*x))/(693*a*c^3*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.143283, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6136, 6135}

$$-\frac{256(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{693ac^3\sqrt{c-a^2cx^2}}-\frac{32(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{693ac^2\left(c-a^2cx^2\right)^{3/2}}-\frac{2(1-10ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{99ac\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 10*a*x))/(99*a*c*(c - a^2*c*x^2)^(5/2)) - (32*E
^(ArcTanh[a*x]/2)*(1 - 6*a*x))/(693*a*c^2*(c - a^2*c*x^2)^(3/2)) - (256*E^(
ArcTanh[a*x]/2)*(1 - 2*a*x))/(693*a*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{99ac\left(c-a^2cx^2\right)^{5/2}} + \frac{80\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}dx}{99c}$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{99ac\left(c-a^2cx^2\right)^{5/2}} - \frac{32e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{693ac^2\left(c-a^2cx^2\right)^{3/2}} + \frac{128\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}dx}{231c^2}$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{99ac\left(c-a^2cx^2\right)^{5/2}} - \frac{32e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{693ac^2\left(c-a^2cx^2\right)^{3/2}} - \frac{256e^{\frac{1}{2}\tanh^{-1}(ax)}(1-2ax)}{693ac^3\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0473499, size = 96, normalized size = 0.77

$$\frac{2\sqrt{1-a^2x^2}\left(256a^5x^5-128a^4x^4-608a^3x^3+272a^2x^2+422ax-151\right)}{693ac^3(1-ax)^{11/4}(ax+1)^{9/4}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2]*(-151 + 422*a*x + 272*a^2*x^2 - 608*a^3*x^3 - 128*a^4*
x^4 + 256*a^5*x^5))/(693*a*c^3*(1 - a*x)^(11/4)*(1 + a*x)^(9/4)*Sqrt[c - a^
2*c*x^2])
```

Maple [A] time = 0.03, size = 87, normalized size = 0.7

$$-\frac{(2ax-2)(ax+1)\left(256x^5a^5-128x^4a^4-608x^3a^3+272a^2x^2+422ax-151\right)}{693a}\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2cx^2+c\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -2/693*(a*x-1)*(a*x+1)*(256*a^5*x^5-128*a^4*x^4-608*a^3*x^3+272*a^2*x^2+422
*a*x-151)*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/a/(-a^2*c*x^2+c)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(7/2), x)
```

3.1296 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{2048(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435ac^4\sqrt{c-a^2cx^2}}-\frac{256(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435ac^3\left(c-a^2cx^2\right)^{3/2}}-\frac{112(1-10ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435ac^2\left(c-a^2cx^2\right)^{5/2}}-\frac{2(1-14ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{195ac\left(c-a^2cx^2\right)^{7/2}}$$

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 14*a*x))/(195*a*c*(c - a^2*c*x^2)^(7/2)) - (112
*E^(ArcTanh[a*x]/2)*(1 - 10*a*x))/(6435*a*c^2*(c - a^2*c*x^2)^(5/2)) - (256
*E^(ArcTanh[a*x]/2)*(1 - 6*a*x))/(6435*a*c^3*(c - a^2*c*x^2)^(3/2)) - (2048
*E^(ArcTanh[a*x]/2)*(1 - 2*a*x))/(6435*a*c^4*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.197033, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6136, 6135}

$$-\frac{2048(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435ac^4\sqrt{c-a^2cx^2}}-\frac{256(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435ac^3\left(c-a^2cx^2\right)^{3/2}}-\frac{112(1-10ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435ac^2\left(c-a^2cx^2\right)^{5/2}}-\frac{2(1-14ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{195ac\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(9/2),x]
```


```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 14*a*x))/(195*a*c*(c - a^2*c*x^2)^(7/2)) - (112
*E^(ArcTanh[a*x]/2)*(1 - 10*a*x))/(6435*a*c^2*(c - a^2*c*x^2)^(5/2)) - (256
*E^(ArcTanh[a*x]/2)*(1 - 6*a*x))/(6435*a*c^3*(c - a^2*c*x^2)^(3/2)) - (2048
*E^(ArcTanh[a*x]/2)*(1 - 2*a*x))/(6435*a*c^4*Sqrt[c - a^2*c*x^2])
```


Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-14ax)}{195ac\,(c-a^2cx^2)^{7/2}} + \frac{56\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx}{65c}$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-14ax)}{195ac\,(c-a^2cx^2)^{7/2}} - \frac{112e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{6435ac^2\,(c-a^2cx^2)^{5/2}} + \frac{896\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx}{1287c^2}$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-14ax)}{195ac\,(c-a^2cx^2)^{7/2}} - \frac{112e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{6435ac^2\,(c-a^2cx^2)^{5/2}} - \frac{256e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{6435ac^3\,(c-a^2cx^2)^{3/2}} + \frac{1024\int \frac{e^{\frac{1}{2}\tanh^{-}}}{(c-a^2cx^2}}{2145c^3}$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-14ax)}{195ac\,(c-a^2cx^2)^{7/2}} - \frac{112e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{6435ac^2\,(c-a^2cx^2)^{5/2}} - \frac{256e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{6435ac^3\,(c-a^2cx^2)^{3/2}} - \frac{2048e^{\frac{1}{2}\tanh^{-1}(a}}{6435ac^4\sqrt{c}}$$

Mathematica [A] time = 0.0527619, size = 112, normalized size = 0.68

$$-\frac{2\sqrt{1-a^2x^2}\left(2048a^7x^7-1024a^6x^6-6912a^5x^5+3200a^4x^4+8240a^3x^3-3384a^2x^2-3838ax+1241\right)}{6435ac^4(1-ax)^{15/4}(ax+1)^{13/4}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2]*(1241 - 3838*a*x - 3384*a^2*x^2 + 8240*a^3*x^3 + 3200
*a^4*x^4 - 6912*a^5*x^5 - 1024*a^6*x^6 + 2048*a^7*x^7))/(6435*a*c^4*(1 - a*
x)^(15/4)*(1 + a*x)^(13/4)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.031, size = 103, normalized size = 0.6

$$\frac{(2\,ax-2)\,(ax+1)\left(2048\,a^7x^7-1024\,x^6a^6-6912\,x^5a^5+3200\,x^4a^4+8240\,x^3a^3-3384\,a^2x^2-3838\,ax+1241\right)}{6435\,a}\sqrt{(ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/2),x)
```

```
[Out] 2/6435*(a*x-1)*(a*x+1)*(2048*a^7*x^7-1024*a^6*x^6-6912*a^5*x^5+3200*a^4*x^4
+8240*a^3*x^3-3384*a^2*x^2-3838*a*x+1241)*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2
)/a/(-a^2*c*x^2+c)^(9/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(9/2), x)
```

3.1297 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/4}}\,dx$

Optimal. Leaf size=201

$$-\frac{2\sqrt[4]{1-a^2x^2}(1-ax)^{3/2}}{3a^4c\sqrt[4]{c-a^2cx^2}}+\frac{2\sqrt[4]{1-a^2x^2}\sqrt{1-ax}}{a^4c\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}}{a^4c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}a^4c\sqrt[4]{c-a^2cx^2}}$$

```
[Out] (1 - a^2*x^2)^(1/4)/(a^4*c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) + (2*Sqrt[1
 - a*x]*(1 - a^2*x^2)^(1/4))/(a^4*c*(c - a^2*c*x^2)^(1/4)) - (2*(1 - a*x)^(
3/2)*(1 - a^2*x^2)^(1/4))/(3*a^4*c*(c - a^2*c*x^2)^(1/4)) + ((1 - a^2*x^2)^
(1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*a^4*c*(c - a^2*c*x^2)^(1/4))
```

Rubi [A] time = 0.299329, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {6153, 6150, 87, 43, 783, 78, 63, 207}

$$-\frac{2\sqrt[4]{1-a^2x^2}(1-ax)^{3/2}}{3a^4c\sqrt[4]{c-a^2cx^2}}+\frac{2\sqrt[4]{1-a^2x^2}\sqrt{1-ax}}{a^4c\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}}{a^4c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}a^4c\sqrt[4]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(ArcTanh[a*x]/2)*x^3)/(c - a^2*c*x^2)^(5/4),x]
```



```
[Out] (1 - a^2*x^2)^(1/4)/(a^4*c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) + (2*Sqrt[1
 - a*x]*(1 - a^2*x^2)^(1/4))/(a^4*c*(c - a^2*c*x^2)^(1/4)) - (2*(1 - a*x)^(
3/2)*(1 - a^2*x^2)^(1/4))/(3*a^4*c*(c - a^2*c*x^2)^(1/4)) + ((1 - a^2*x^2)^
(1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*a^4*c*(c - a^2*c*x^2)^(1/4))
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 783

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{5/4}}dx = \frac{\sqrt[4]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{5/4}}dx}{c\sqrt[4]{c-a^2cx^2}}$$

$$= \frac{\sqrt[4]{1-a^2x^2}\int \frac{x^3}{(1-ax)^{3/2}(1+ax)}dx}{c\sqrt[4]{c-a^2cx^2}}$$

$$= \frac{\sqrt[4]{1-a^2x^2}\int \left(-\frac{x}{a^2\sqrt{1-ax}}-\frac{x}{a^2\sqrt{1-ax}\left(-1+a^2x^2\right)}\right)dx}{c\sqrt[4]{c-a^2cx^2}}$$

$$= -\frac{\sqrt[4]{1-a^2x^2}\int \frac{x}{\sqrt{1-ax}}dx}{a^2c\sqrt[4]{c-a^2cx^2}} - \frac{\sqrt[4]{1-a^2x^2}\int \frac{x}{\sqrt{1-ax}\left(-1+a^2x^2\right)}dx}{a^2c\sqrt[4]{c-a^2cx^2}}$$

$$= -\frac{\sqrt[4]{1-a^2x^2}\int \frac{x}{(-1-ax)(1-ax)^{3/2}}dx}{a^2c\sqrt[4]{c-a^2cx^2}} - \frac{\sqrt[4]{1-a^2x^2}\int \left(\frac{1}{a\sqrt{1-ax}}-\frac{\sqrt{1-ax}}{a}\right)dx}{a^2c\sqrt[4]{c-a^2cx^2}}$$

$$= \frac{\sqrt[4]{1-a^2x^2}}{a^4c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}} + \frac{2\sqrt{1-ax}\sqrt[4]{1-a^2x^2}}{a^4c\sqrt[4]{c-a^2cx^2}} - \frac{2(1-ax)^{3/2}\sqrt[4]{1-a^2x^2}}{3a^4c\sqrt[4]{c-a^2cx^2}} + \frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{(-1-ax)\sqrt{1-ax}}}{2a^3c\sqrt[4]{c-a^2cx^2}}$$

$$= \frac{\sqrt[4]{1-a^2x^2}}{a^4c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}} + \frac{2\sqrt{1-ax}\sqrt[4]{1-a^2x^2}}{a^4c\sqrt[4]{c-a^2cx^2}} - \frac{2(1-ax)^{3/2}\sqrt[4]{1-a^2x^2}}{3a^4c\sqrt[4]{c-a^2cx^2}} - \frac{\sqrt[4]{1-a^2x^2}\,\text{Subst}\left(\int \frac{1}{-2+}\right.}{a^4c\sqrt[4]{c-a^2}}$$

$$= \frac{\sqrt[4]{1-a^2x^2}}{a^4c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}} + \frac{2\sqrt{1-ax}\sqrt[4]{1-a^2x^2}}{a^4c\sqrt[4]{c-a^2cx^2}} - \frac{2(1-ax)^{3/2}\sqrt[4]{1-a^2x^2}}{3a^4c\sqrt[4]{c-a^2cx^2}} + \frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right.}{\sqrt{2}a^4c\sqrt[4]{c-a^2cx^2}}$$

Mathematica [C] time = 0.0644657, size = 84, normalized size = 0.42

$$-\frac{\sqrt[4]{1-a^2x^2}\left(3\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-ax)\right)+2\left(a^2x^2+ax-5\right)\right)}{3a^4c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(ArcTanh[a*x]/2)*x^3)/(c - a^2*c*x^2)^(5/4),x]
```

```
[Out] -((1 - a^2*x^2)^(1/4)*(2*(-5 + a*x + a^2*x^2) + 3*Hypergeometric2F1[-1/2, 1
, 1/2, (1 - a*x)/2]))/(3*a^4*c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4))
```

Maple [F] time = 0.217, size = 0, normalized size = 0.

$$\int x^3\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2cx^2+c\right)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/4),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="maxima")
```

```
[Out] integrate(x^3*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x**3/(-a**2*c*x**2+c)**(5/
4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="giac")
```

```
[Out] integrate(x^3*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/4), x)
```

3.1298 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/4}}\,dx$

Optimal. Leaf size=153

$$\frac{2\sqrt{1-ax}\sqrt[4]{1-a^2x^2}}{a^3c\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}}{a^3c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}a^3c\sqrt[4]{c-a^2cx^2}}$$

```
[Out] (1 - a^2*x^2)^(1/4)/(a^3*c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) + (2*Sqrt[1
 - a*x]*(1 - a^2*x^2)^(1/4))/(a^3*c*(c - a^2*c*x^2)^(1/4)) - ((1 - a^2*x^2)
^(1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*a^3*c*(c - a^2*c*x^2)^(1/4)
)
```

Rubi [A] time = 0.259916, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {6153, 6150, 87, 627, 51, 63, 207}

$$\frac{2\sqrt{1-ax}\sqrt[4]{1-a^2x^2}}{a^3c\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}}{a^3c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}a^3c\sqrt[4]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(ArcTanh[a*x]/2)*x^2)/(c - a^2*c*x^2)^(5/4),x]
```


```
[Out] (1 - a^2*x^2)^(1/4)/(a^3*c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) + (2*Sqrt[1
 - a*x]*(1 - a^2*x^2)^(1/4))/(a^3*c*(c - a^2*c*x^2)^(1/4)) - ((1 - a^2*x^2)
^(1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*a^3*c*(c - a^2*c*x^2)^(1/4)
)
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
```

```
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{5/4}}\,dx &= \frac{\sqrt[4]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{5/4}}\,dx}{c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}\int \frac{x^2}{(1-ax)^{3/2}(1+ax)}\,dx}{c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}\int \left(-\frac{1}{a^2\sqrt{1-ax}}-\frac{1}{a^2\sqrt{1-ax}\left(-1+a^2x^2\right)}\right)dx}{c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{2\sqrt{1-ax}\sqrt[4]{1-a^2x^2}}{a^3c\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{\sqrt{1-ax}\left(-1+a^2x^2\right)}\,dx}{a^2c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{2\sqrt{1-ax}\sqrt[4]{1-a^2x^2}}{a^3c\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{(-1-ax)(1-ax)^{3/2}}\,dx}{a^2c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{a^3c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{2\sqrt{1-ax}\sqrt[4]{1-a^2x^2}}{a^3c\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{(-1-ax)\sqrt{1-ax}}\,dx}{2a^2c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{a^3c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{2\sqrt{1-ax}\sqrt[4]{1-a^2x^2}}{a^3c\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\,\mathrm{Subst}\left(\int \frac{1}{-2+x^2}\,dx,x,\sqrt{1-ax}\right)}{a^3c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{a^3c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{2\sqrt{1-ax}\sqrt[4]{1-a^2x^2}}{a^3c\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}a^3c\sqrt[4]{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.0431518, size = 70, normalized size = 0.46

$$\frac{\sqrt[4]{1-a^2x^2}\left(\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-ax)\right)-2ax+2\right)}{a^3c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(ArcTanh[a*x]/2)*x^2)/(c - a^2*c*x^2)^(5/4),x]
```

```
[Out] ((1 - a^2*x^2)^(1/4)*(2 - 2*a*x + Hypergeometric2F1[-1/2, 1, 1/2, (1 - a*x)
/2]))/(a^3*c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4))
```

Maple [F] time = 0.217, size = 0, normalized size = 0.

$$\int x^2\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2cx^2+c\right)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/4),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="maxima")
```

```
[Out] integrate(x^2*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x**2/(-a**2*c*x**2+c)**(5/
4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="giac")
```

```
[Out] integrate(x^2*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/4), x)
```

3.1299 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt[4]{1-a^2x^2}}{a^2c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}a^2c\sqrt[4]{c-a^2cx^2}}$$

```
[Out] (1 - a^2*x^2)^(1/4)/(a^2*c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) + ((1 - a^2
*x^2)^(1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*a^2*c*(c - a^2*c*x^2)^
(1/4))
```

Rubi [A] time = 0.162256, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6153, 6150, 78, 63, 206}

$$\frac{\sqrt[4]{1-a^2x^2}}{a^2c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}a^2c\sqrt[4]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(ArcTanh[a*x]/2)*x)/(c - a^2*c*x^2)^(5/4),x]
```

```
[Out] (1 - a^2*x^2)^(1/4)/(a^2*c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) + ((1 - a^2
*x^2)^(1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*a^2*c*(c - a^2*c*x^2)^
(1/4))
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{5/4}}\,dx &= \frac{\sqrt[4]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x}{\left(1-a^2x^2\right)^{5/4}}\,dx}{c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}\int \frac{x}{(1-ax)^{3/2}(1+ax)}\,dx}{c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{a^2c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx}{2ac\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{a^2c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1-ax}\right)}{a^2c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{a^2c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}a^2c\sqrt[4]{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0531292, size = 74, normalized size = 0.7

$$\frac{\sqrt[4]{1-a^2x^2}\left(\frac{1}{a^2\sqrt{1-ax}}+\frac{\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}a^2}\right)}{c\sqrt[4]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(ArcTanh[a*x]/2)*x)/(c - a^2*c*x^2)^(5/4),x]
```

```
[Out] ((1 - a^2*x^2)^(1/4)*(1/(a^2*Sqrt[1 - a*x]) + ArcTanh[Sqrt[1 - a*x]/Sqrt[2]
]/(Sqrt[2]*a^2)))/(c*(c - a^2*c*x^2)^(1/4))
```

Maple [F] time = 0.214, size = 0, normalized size = 0.

$$\int x\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2cx^2+c\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(5/4),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(5/4),x, algo
rithm="maxima")
```

```
[Out] integrate(x*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(5/4),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x/(-a**2*c*x**2+c)**(5/4),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(5/4),x, algo
rithm="giac")
```

```
[Out] integrate(x*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/4), x)
```

3.1300 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/4}}\,dx$

Optimal. Leaf size=106

$$\frac{\sqrt[4]{1-a^2x^2}}{ac\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}ac\sqrt[4]{c-a^2cx^2}}$$

```
[Out] (1 - a^2*x^2)^(1/4)/(a*c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) - ((1 - a^2*x
^2)^(1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*a*c*(c - a^2*c*x^2)^(1/4
))
```

Rubi [A] time = 0.0978393, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6143, 6140, 51, 63, 206}

$$\frac{\sqrt[4]{1-a^2x^2}}{ac\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}ac\sqrt[4]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(5/4),x]
```


```
[Out] (1 - a^2*x^2)^(1/4)/(a*c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) - ((1 - a^2*x
^2)^(1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*a*c*(c - a^2*c*x^2)^(1/4
))
```


Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/4}}\,dx &= \frac{\sqrt[4]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{(1-a^2x^2)^{5/4}}\,dx}{c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{(1-ax)^{3/2}(1+ax)}\,dx}{c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{ac\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx}{2c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{ac\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1-ax}\right)}{ac\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{ac\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}ac\sqrt[4]{c-a^2cx^2}}\end{aligned}$$

Mathematica [C] time = 0.0166965, size = 64, normalized size = 0.6

$$\frac{\sqrt[4]{1-a^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-ax)\right)}{ac\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(5/4),x]
```

```
[Out] ((1 - a^2*x^2)^(1/4)*Hypergeometric2F1[-1/2, 1, 1/2, (1 - a*x)/2])/(a*c*Sqr
t[1 - a*x]*(c - a^2*c*x^2)^(1/4))
```

Maple [F] time = 0.213, size = 0, normalized size = 0.

$$\int \sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2cx^2+c\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/4),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/4),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/4),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*c*x**2+c)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/4),x, algori
thm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/4), x)
```

3.1301 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=144

$$\frac{\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{2\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-ax}\right)}{c\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}c\sqrt[4]{c-a^2cx^2}}$$

```
[Out] (1 - a^2*x^2)^(1/4)/(c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) - (2*(1 - a^2*x
^2)^(1/4)*ArcTanh[Sqrt[1 - a*x]])/(c*(c - a^2*c*x^2)^(1/4)) + ((1 - a^2*x^2
)^(1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*c*(c - a^2*c*x^2)^(1/4))
```

Rubi [A] time = 0.235062, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {6153, 6150, 85, 156, 63, 208, 206}

$$\frac{\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{2\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-ax}\right)}{c\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}c\sqrt[4]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(x*(c - a^2*c*x^2)^(5/4)),x]
```


```
[Out] (1 - a^2*x^2)^(1/4)/(c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) - (2*(1 - a^2*x
^2)^(1/4)*ArcTanh[Sqrt[1 - a*x]])/(c*(c - a^2*c*x^2)^(1/4)) + ((1 - a^2*x^2
)^(1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*c*(c - a^2*c*x^2)^(1/4))
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/4}}\,dx &= \frac{\sqrt[4]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{5/4}}\,dx}{c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{x(1-ax)^{3/2}(1+ax)}\,dx}{c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\int \frac{2a+a^2x}{x\sqrt{1-ax}(1+ax)}\,dx}{2ac\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{x\sqrt{1-ax}}\,dx}{c\sqrt[4]{c-a^2cx^2}}-\frac{\left(a\sqrt[4]{1-a^2x^2}\right)\int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx}{2c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1-ax}\right)}{c\sqrt[4]{c-a^2cx^2}}-\frac{\left(2\sqrt[4]{1-a^2x^2}\right)\text{Subst}\left(\int \frac{}{\frac{1}{a}}\right.}{ac\sqrt[4]{c-a^2c}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{2\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-ax}\right)}{c\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}c\sqrt[4]{c-a^2cx^2}}
\end{aligned}$$

Mathematica [C] time = 0.0316578, size = 79, normalized size = 0.55

$$-\frac{\sqrt[4]{1-a^2x^2}\left(\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-ax)\right)-2\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-ax\right)\right)}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(x*(c - a^2*c*x^2)^(5/4)),x]
```

```
[Out] -(((1 - a^2*x^2)^(1/4)*(Hypergeometric2F1[-1/2, 1, 1/2, (1 - a*x)/2] - 2*Hy
pergeometric2F1[-1/2, 1, 1/2, 1 - a*x]))/(c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(
```

```
1/4)))
```

Maple [F] time = 0.211, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2cx^2+c\right)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(5/4),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(5/4),x, algo
rithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/((-a^2*c*x^2 + c)^(5/4)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(5/4),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x/(-a**2*c*x**2+c)**(5/4),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(5/4),x, algo
rithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/((-a^2*c*x^2 + c)^(5/4)*x), x)
```

3.1302 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^{5/4}}\,dx$

Optimal. Leaf size=196

$$\frac{2a\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}}{cx\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{a\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-ax}\right)}{c\sqrt[4]{c-a^2cx^2}}-\frac{a\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}c\sqrt[4]{c-a^2cx^2}}$$

```
[Out] (2*a*(1 - a^2*x^2)^(1/4))/(c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) - (1 - a^
2*x^2)^(1/4)/(c*x*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) - (a*(1 - a^2*x^2)^(
1/4)*ArcTanh[Sqrt[1 - a*x]])/(c*(c - a^2*c*x^2)^(1/4)) - (a*(1 - a^2*x^2)^(
1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*c*(c - a^2*c*x^2)^(1/4))
```

Rubi [A] time = 0.249028, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {6153, 6150, 103, 152, 156, 63, 208, 206}

$$\frac{2a\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}}{cx\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{a\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-ax}\right)}{c\sqrt[4]{c-a^2cx^2}}-\frac{a\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}c\sqrt[4]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(x^2*(c - a^2*c*x^2)^(5/4)),x]
```


```
[Out] (2*a*(1 - a^2*x^2)^(1/4))/(c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) - (1 - a^
2*x^2)^(1/4)/(c*x*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) - (a*(1 - a^2*x^2)^(
1/4)*ArcTanh[Sqrt[1 - a*x]])/(c*(c - a^2*c*x^2)^(1/4)) - (a*(1 - a^2*x^2)^(
1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*c*(c - a^2*c*x^2)^(1/4))
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/4}}\,dx = \frac{\sqrt[4]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^2\left(1-a^2x^2\right)^{5/4}}\,dx}{c\sqrt[4]{c-a^2cx^2}}$$

$$= \frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{x^2(1-ax)^{3/2}(1+ax)}\,dx}{c\sqrt[4]{c-a^2cx^2}}$$

$$= -\frac{\sqrt[4]{1-a^2x^2}}{cx\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}} - \frac{\sqrt[4]{1-a^2x^2}\int \frac{-\frac{a}{2}-\frac{3a^2x}{2}}{x(1-ax)^{3/2}(1+ax)}\,dx}{c\sqrt[4]{c-a^2cx^2}}$$

$$= \frac{2a\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}} - \frac{\sqrt[4]{1-a^2x^2}}{cx\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}} + \frac{\sqrt[4]{1-a^2x^2}\int \frac{\frac{a^2}{2}+a^3x}{x\sqrt{1-ax}(1+ax)}\,dx}{ac\sqrt[4]{c-a^2cx^2}}$$

$$= \frac{2a\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}} - \frac{\sqrt[4]{1-a^2x^2}}{cx\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}} + \frac{\left(a\sqrt[4]{1-a^2x^2}\right)\int \frac{1}{x\sqrt{1-ax}}\,dx}{2c\sqrt[4]{c-a^2cx^2}} + \frac{\left(a^2\sqrt[4]{1-a^2x^2}\right)\int -$$

$$= \frac{2a\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}} - \frac{\sqrt[4]{1-a^2x^2}}{cx\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}} - \frac{\sqrt[4]{1-a^2x^2}\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-ax}\right)}{c\sqrt[4]{c-a^2cx^2}} - (a$$

$$= \frac{2a\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}} - \frac{\sqrt[4]{1-a^2x^2}}{cx\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}} - \frac{a\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-ax}\right)}{c\sqrt[4]{c-a^2cx^2}} - \frac{a\sqrt[4]{1-a^2x^2}\tan}{\sqrt{2}c\sqrt[4]{c-}}$$

Mathematica [C] time = 0.0350511, size = 86, normalized size = 0.44

$$\frac{\sqrt[4]{1-a^2x^2}\left(ax\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-ax)\right)+ax\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-ax\right)-1\right)}{cx\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(x^2*(c - a^2*c*x^2)^(5/4)),x]
```

```
[Out] ((1 - a^2*x^2)^(1/4)*(-1 + a*x*Hypergeometric2F1[-1/2, 1, 1/2, (1 - a*x)/2]
 + a*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 - a*x]))/(c*x*Sqrt[1 - a*x]*(c - a
^2*c*x^2)^(1/4))
```

Maple [F] time = 0.223, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2cx^2+c\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/4),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/((-a^2*c*x^2 + c)^(5/4)*x^2),
x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**2/(-a**2*c*x**2+c)**(5/
4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/((-a^2*c*x^2 + c)^(5/4)*x^2),
x)
```

3.1303 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^{9/8}}\,dx$

Optimal. Leaf size=200

$$\frac{64\sqrt[8]{2}(1-ax)^{5/8}\sqrt[8]{1-a^2x^2}\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{7}{8},\frac{13}{8},\frac{1}{2}(1-ax)\right)}{105a^4c\sqrt[8]{c-a^2cx^2}}-\frac{4x^2\sqrt[8]{ax+1}\sqrt[8]{1-a^2x^2}}{7a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}+\frac{8(6-ax)\sqrt[8]{ax+1}\sqrt[8]{\cdots}}{21a^4c(1-ax)^{3/8}\sqrt[8]{c\cdots}}$$

```
[Out] (-4*x^2*(1 + a*x)^(1/8)*(1 - a^2*x^2)^(1/8))/(7*a^2*c*(1 - a*x)^(3/8)*(c -
a^2*c*x^2)^(1/8)) + (8*(6 - a*x)*(1 + a*x)^(1/8)*(1 - a^2*x^2)^(1/8))/(21*a
^4*c*(1 - a*x)^(3/8)*(c - a^2*c*x^2)^(1/8)) + (64*2^(1/8)*(1 - a*x)^(5/8)*(
1 - a^2*x^2)^(1/8)*Hypergeometric2F1[5/8, 7/8, 13/8, (1 - a*x)/2])/(105*a^4
*c*(c - a^2*c*x^2)^(1/8))
```

Rubi [A] time = 0.25395, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {6153, 6150, 100, 146, 69}

$$\frac{64\sqrt[8]{2}(1-ax)^{5/8}\sqrt[8]{1-a^2x^2}\,{}_2F_1\left(\frac{5}{8},\frac{7}{8};\frac{13}{8};\frac{1}{2}(1-ax)\right)}{105a^4c\sqrt[8]{c-a^2cx^2}}-\frac{4x^2\sqrt[8]{ax+1}\sqrt[8]{1-a^2x^2}}{7a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}+\frac{8(6-ax)\sqrt[8]{ax+1}\sqrt[8]{1-a^2x^2}}{21a^4c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(ArcTanh[a*x]/2)*x^3)/(c - a^2*c*x^2)^(9/8),x]
```

```
[Out] (-4*x^2*(1 + a*x)^(1/8)*(1 - a^2*x^2)^(1/8))/(7*a^2*c*(1 - a*x)^(3/8)*(c -
a^2*c*x^2)^(1/8)) + (8*(6 - a*x)*(1 + a*x)^(1/8)*(1 - a^2*x^2)^(1/8))/(21*a
^4*c*(1 - a*x)^(3/8)*(c - a^2*c*x^2)^(1/8)) + (64*2^(1/8)*(1 - a*x)^(5/8)*(
1 - a^2*x^2)^(1/8)*Hypergeometric2F1[5/8, 7/8, 13/8, (1 - a*x)/2])/(105*a^4
*c*(c - a^2*c*x^2)^(1/8))
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{9/8}}\,dx &= \frac{\sqrt[8]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{9/8}}\,dx}{c\sqrt[8]{c-a^2cx^2}}\\
&= \frac{\sqrt[8]{1-a^2x^2}\int \frac{x^3}{(1-ax)^{11/8}(1+ax)^{7/8}}\,dx}{c\sqrt[8]{c-a^2cx^2}}\\
&= -\frac{4x^2\sqrt[8]{1+ax}\sqrt[8]{1-a^2x^2}}{7a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}-\frac{\left(4\sqrt[8]{1-a^2x^2}\right)\int \frac{x\left(-2-\frac{ax}{2}\right)}{(1-ax)^{11/8}(1+ax)^{7/8}}\,dx}{7a^2c\sqrt[8]{c-a^2cx^2}}\\
&= -\frac{4x^2\sqrt[8]{1+ax}\sqrt[8]{1-a^2x^2}}{7a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}+\frac{8(6-ax)\sqrt[8]{1+ax}\sqrt[8]{1-a^2x^2}}{21a^4c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}-\frac{\left(16\sqrt[8]{1-a^2x^2}\right)\int \frac{1}{(1-ax)^{3/8}(1+ax)^{7/8}}\,d}{21a^3c\sqrt[8]{c-a^2cx^2}}\\
&= -\frac{4x^2\sqrt[8]{1+ax}\sqrt[8]{1-a^2x^2}}{7a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}+\frac{8(6-ax)\sqrt[8]{1+ax}\sqrt[8]{1-a^2x^2}}{21a^4c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}+\frac{64\sqrt[8]{2}(1-ax)^{5/8}\sqrt[8]{1-a^2x^2}\,{}_2F_1\left(\frac{5}{8}\right.}{105a^4c\sqrt[8]{c-a^2cx}}
\end{aligned}$$

Mathematica [A] time = 0.0694418, size = 107, normalized size = 0.54

$$-\frac{4\sqrt[8]{1-a^2x^2}\left(16\sqrt[8]{2}(ax-1)\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{7}{8},\frac{13}{8},\frac{1}{2}(1-ax)\right)+5\sqrt[8]{ax+1}\left(3a^2x^2+2ax-12\right)\right)}{105a^4c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(ArcTanh[a*x]/2)*x^3)/(c - a^2*c*x^2)^(9/8),x]
```

```
[Out] (-4*(1 - a^2*x^2)^(1/8)*(5*(1 + a*x)^(1/8)*(-12 + 2*a*x + 3*a^2*x^2) + 16*2
^(1/8)*(-1 + a*x)*Hypergeometric2F1[5/8, 7/8, 13/8, (1 - a*x)/2]))/(105*a^4
*c*(1 - a*x)^(3/8)*(c - a^2*c*x^2)^(1/8))
```

Maple [F] time = 0.215, size = 0, normalized size = 0.

$$\int x^{3} \sqrt{(ax+1) \frac{1}{\sqrt{-a^{2} x^{2}+1}}}\, \left(-a^{2} c \,x^{2}+c \right)^{-\frac{9}{8}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(9/8),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(9/8),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3} \sqrt{\frac{ax+1}{\sqrt{-a^{2} x^{2}+1}}}}{\left(-a^{2} c x^{2}+c\right)^{\frac{9}{8}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(9/8),x, al
gorithm="maxima")
```

```
[Out] integrate(x^3*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(9/8), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(9/8),x, al
gorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x**3/(-a**2*c*x**2+c)**(9/
8),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{9}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(9/8),x, al
gorithm="giac")
```

```
[Out] integrate(x^3*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(9/8), x)
```

3.1304 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{9/8}}\,dx$

Optimal. Leaf size=41

$$\frac{4(2-ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{3a^3c\sqrt[8]{c-a^2cx^2}}$$

```
[Out] (4*E^(ArcTanh[a*x]/2)*(2 - a*x))/(3*a^3*c*(c - a^2*c*x^2)^(1/8))
```

Rubi [A] time = 0.119413, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {6146}

$$\frac{4(2-ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{3a^3c\sqrt[8]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(ArcTanh[a*x]/2)*x^2)/(c - a^2*c*x^2)^(9/8),x]

[Out] (4*E^(ArcTanh[a*x]/2)*(2 - a*x))/(3*a^3*c*(c - a^2*c*x^2)^(1/8))
```

Rule 6146

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[((1 - a*n*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*d*n*(n^
2 - 1)), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && EqQ[n^2 + 2*(
p + 1), 0] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{9/8}}\,dx = \frac{4e^{\frac{1}{2}\tanh^{-1}(ax)}(2-ax)}{3a^3c\sqrt[8]{c-a^2cx^2}}$$

Mathematica [A] time = 0.0329122, size = 63, normalized size = 1.54

$$-\frac{4(ax-2)\sqrt[8]{ax+1}\sqrt[8]{1-a^2x^2}}{3a^3c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(ArcTanh[a*x]/2)*x^2)/(c - a^2*c*x^2)^(9/8),x]

[Out] (-4*(-2 + a*x)*(1 + a*x)^(1/8)*(1 - a^2*x^2)^(1/8))/(3*a^3*c*(1 - a*x)^(3/8
)*(c - a^2*c*x^2)^(1/8))
```

Maple [A] time = 0.028, size = 54, normalized size = 1.3

$$\frac{(4\,ax-4)\,(ax+1)\,(ax-2)}{3\,a^{3}}\sqrt{(ax+1)\frac{1}{\sqrt{-a^{2}x^{2}+1}}}\left(-a^{2}cx^{2}+c\right)^{-\frac{9}{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(9/8),x)
```

```
[Out] 4/3*(a*x-1)*(a*x+1)*(a*x-2)*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/a^3/(-a^2*c*
x^2+c)^(9/8)
```

Maxima [A] time = 2.53288, size = 38, normalized size = 0.93

$$-\frac{4\,(ax+1)^{\frac{1}{8}}(ax-2)}{3\,(-ax+1)^{\frac{3}{8}}a^{3}c^{\frac{9}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(9/8),x, al
gorithm="maxima")
```

```
[Out] -4/3*(a*x + 1)^(1/8)*(a*x - 2)/((-a*x + 1)^(3/8)*a^3*c^(9/8))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(9/8),x, al
gorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x**2/(-a**2*c*x**2+c)**(9/
8),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{9}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(9/8),x, al
gorithm="giac")
```

```
[Out] integrate(x^2*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(9/8), x)
```

3.1305 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{9/8}}\,dx$

Optimal. Leaf size=133

$$\frac{8\sqrt[8]{2}(1-ax)^{5/8}\sqrt[8]{1-a^2x^2}\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{7}{8},\frac{13}{8},\frac{1}{2}(1-ax)\right)}{15a^2c\sqrt[8]{c-a^2cx^2}}+\frac{4\sqrt[8]{ax+1}\sqrt[8]{1-a^2x^2}}{3a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

```
[Out] (4*(1 + a*x)^(1/8)*(1 - a^2*x^2)^(1/8))/(3*a^2*c*(1 - a*x)^(3/8)*(c - a^2*c
*x^2)^(1/8)) + (8*2^(1/8)*(1 - a*x)^(5/8)*(1 - a^2*x^2)^(1/8)*Hypergeometri
c2F1[5/8, 7/8, 13/8, (1 - a*x)/2])/(15*a^2*c*(c - a^2*c*x^2)^(1/8))
```

Rubi [A] time = 0.168761, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6153, 6150, 78, 69}

$$\frac{8\sqrt[8]{2}(1-ax)^{5/8}\sqrt[8]{1-a^2x^2}\,{}_2F_1\left(\frac{5}{8},\frac{7}{8};\frac{13}{8};\frac{1}{2}(1-ax)\right)}{15a^2c\sqrt[8]{c-a^2cx^2}}+\frac{4\sqrt[8]{ax+1}\sqrt[8]{1-a^2x^2}}{3a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(ArcTanh[a*x]/2)*x)/(c - a^2*c*x^2)^(9/8),x]
```

```
[Out] (4*(1 + a*x)^(1/8)*(1 - a^2*x^2)^(1/8))/(3*a^2*c*(1 - a*x)^(3/8)*(c - a^2*c
*x^2)^(1/8)) + (8*2^(1/8)*(1 - a*x)^(5/8)*(1 - a^2*x^2)^(1/8)*Hypergeometri
c2F1[5/8, 7/8, 13/8, (1 - a*x)/2])/(15*a^2*c*(c - a^2*c*x^2)^(1/8))
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{9/8}}\,dx &= \frac{\sqrt[8]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x}{\left(1-a^2x^2\right)^{9/8}}\,dx}{c\sqrt[8]{c-a^2cx^2}}\\
&= \frac{\sqrt[8]{1-a^2x^2}\int \frac{x}{(1-ax)^{11/8}(1+ax)^{7/8}}\,dx}{c\sqrt[8]{c-a^2cx^2}}\\
&= \frac{4\sqrt[8]{1+ax}\sqrt[8]{1-a^2x^2}}{3a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}-\frac{\left(2\sqrt[8]{1-a^2x^2}\right)\int \frac{1}{(1-ax)^{3/8}(1+ax)^{7/8}}\,dx}{3ac\sqrt[8]{c-a^2cx^2}}\\
&= \frac{4\sqrt[8]{1+ax}\sqrt[8]{1-a^2x^2}}{3a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}+\frac{8\sqrt[8]{2}(1-ax)^{5/8}\sqrt[8]{1-a^2x^2}\,{}_2F_1\left(\frac{5}{8},\frac{7}{8};\frac{13}{8};\frac{1}{2}(1-ax)\right)}{15a^2c\sqrt[8]{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0387695, size = 93, normalized size = 0.7

$$-\frac{4\sqrt[8]{1-a^2x^2}\left(2\sqrt[8]{2}(ax-1)\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{7}{8},\frac{13}{8},\frac{1}{2}(1-ax)\right)-5\sqrt[8]{ax+1}\right)}{15a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(ArcTanh[a*x]/2)*x)/(c - a^2*c*x^2)^(9/8),x]
```

```
[Out] (-4*(1 - a^2*x^2)^(1/8)*(-5*(1 + a*x)^(1/8) + 2*2^(1/8)*(-1 + a*x)*Hypergeo
metric2F1[5/8, 7/8, 13/8, (1 - a*x)/2]))/(15*a^2*c*(1 - a*x)^(3/8)*(c - a^2
*c*x^2)^(1/8))
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int x\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2cx^2+c\right)^{-\frac{9}{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(9/8),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(9/8),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{9}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(9/8),x, algo
rithm="maxima")
```

```
[Out] integrate(x*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(9/8), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(9/8),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x/(-a**2*c*x**2+c)**(9/8),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{9}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(9/8),x, algo
rithm="giac")
```

```
[Out] integrate(x*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(9/8), x)
```

3.1306 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/8}}\,dx$

Optimal. Leaf size=74

$$\frac{4\sqrt[8]{2}\sqrt[8]{1-a^2x^2}\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{7}{8},\frac{5}{8},\frac{1}{2}(1-ax)\right)}{3ac(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

```
[Out] (4*2^(1/8)*(1 - a^2*x^2)^(1/8)*Hypergeometric2F1[-3/8, 7/8, 5/8, (1 - a*x)/
2])/(3*a*c*(1 - a*x)^(3/8)*(c - a^2*c*x^2)^(1/8))
```

Rubi [A] time = 0.0903437, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6143, 6140, 69}

$$\frac{4\sqrt[8]{2}\sqrt[8]{1-a^2x^2}\,{}_2F_1\left(-\frac{3}{8},\frac{7}{8};\frac{5}{8};\frac{1}{2}(1-ax)\right)}{3ac(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(9/8),x]
```

```
[Out] (4*2^(1/8)*(1 - a^2*x^2)^(1/8)*Hypergeometric2F1[-3/8, 7/8, 5/8, (1 - a*x)/
2])/(3*a*c*(1 - a*x)^(3/8)*(c - a^2*c*x^2)^(1/8))
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/8}}\,dx = \frac{\sqrt[8]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{9/8}}\,dx}{c\sqrt[8]{c-a^2cx^2}}$$
$$= \frac{\sqrt[8]{1-a^2x^2}\int \frac{1}{(1-ax)^{11/8}(1+ax)^{7/8}}\,dx}{c\sqrt[8]{c-a^2cx^2}}$$
$$= \frac{4\sqrt[8]{2}\sqrt[8]{1-a^2x^2}\,{}_2F_1\left(-\frac{3}{8},\frac{7}{8};\frac{5}{8};\frac{1}{2}(1-ax)\right)}{3ac(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

Mathematica [A] time = 0.0256603, size = 69, normalized size = 0.93

$$\frac{4\sqrt[8]{2-2a^2x^2}\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{7}{8},\frac{5}{8},\frac{1}{2}(1-ax)\right)}{3ac(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(9/8),x]
```

```
[Out] (4*(2 - 2*a^2*x^2)^(1/8)*Hypergeometric2F1[-3/8, 7/8, 5/8, (1 - a*x)/2])/(3
*a*c*(1 - a*x)^(3/8)*(c - a^2*c*x^2)^(1/8))
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int \sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2cx^2+c\right)^{-\frac{9}{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/8),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/8),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{9}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/8),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(9/8), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/8),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*c*x**2+c)**(9/8),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{9}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/8),x, algori
thm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(9/8), x)
```

3.1307 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{9/8}}\,dx$

Optimal. Leaf size=73

$$-\frac{2\ 2^{5/8}\sqrt[8]{ax+1}\sqrt[8]{1-a^2x^2}F_1\left(\frac{1}{8};\frac{11}{8},1;\frac{9}{8};\frac{1}{2}(ax+1),ax+1\right)}{c\sqrt[8]{c-a^2cx^2}}$$

```
[Out] (-2*2^(5/8)*(1 + a*x)^(1/8)*(1 - a^2*x^2)^(1/8)*AppellF1[1/8, 11/8, 1, 9/8,
 (1 + a*x)/2, 1 + a*x])/(c*(c - a^2*c*x^2)^(1/8))
```

Rubi [A] time = 0.22463, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {6153, 6150, 136}

$$-\frac{2\ 2^{5/8}\sqrt[8]{ax+1}\sqrt[8]{1-a^2x^2}F_1\left(\frac{1}{8};\frac{11}{8},1;\frac{9}{8};\frac{1}{2}(ax+1),ax+1\right)}{c\sqrt[8]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(x*(c - a^2*c*x^2)^(9/8)),x]
```

```
[Out] (-2*2^(5/8)*(1 + a*x)^(1/8)*(1 - a^2*x^2)^(1/8)*AppellF1[1/8, 11/8, 1, 9/8,
 (1 + a*x)/2, 1 + a*x])/(c*(c - a^2*c*x^2)^(1/8))
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{9/8}}\,dx = \frac{\sqrt[8]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{9/8}}\,dx}{c\sqrt[8]{c-a^2cx^2}}$$
$$= \frac{\sqrt[8]{1-a^2x^2}\int \frac{1}{x(1-ax)^{11/8}(1+ax)^{7/8}}\,dx}{c\sqrt[8]{c-a^2cx^2}}$$
$$= -\frac{2\,2^{5/8}\sqrt[8]{1+ax}\sqrt[8]{1-a^2x^2}F_1\left(\frac{1}{8};\frac{11}{8},1;\frac{9}{8};\frac{1}{2}(1+ax),1+ax\right)}{c\sqrt[8]{c-a^2cx^2}}$$

Mathematica [F] time = 0.564326, size = 0, normalized size = 0.

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{9/8}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(x*(c - a^2*c*x^2)^(9/8)),x]
```

```
[Out] Integrate[E^(ArcTanh[a*x]/2)/(x*(c - a^2*c*x^2)^(9/8)), x]
```

Maple [F] time = 0.217, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}}\left(-a^2cx^2+c\right)^{-\frac{9}{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(9/8),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(9/8),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{9}{8}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(9/8),x, algo
rithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/((-a^2*c*x^2 + c)^(9/8)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(9/8),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x/(-a**2*c*x**2+c)**(9/8),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{9}{8}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(9/8),x, algo
rithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/((-a^2*c*x^2 + c)^(9/8)*x), x)
```

3.1308 $\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=68

$$-\frac{c2^{\frac{n}{2}+2}(1-ax)^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(-\frac{n}{2}-1,2-\frac{n}{2},3-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a(4-n)}$$

```
[Out] -((2^(2 + n/2)*c*(1 - a*x)^(2 - n/2)*Hypergeometric2F1[-1 - n/2, 2 - n/2, 3
 - n/2, (1 - a*x)/2])/(a*(4 - n)))
```

Rubi [A] time = 0.0441174, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6140, 69}

$$-\frac{c2^{\frac{n}{2}+2}(1-ax)^{2-\frac{n}{2}}\,{}_2F_1\left(-\frac{n}{2}-1,2-\frac{n}{2};3-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(4-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -((2^(2 + n/2)*c*(1 - a*x)^(2 - n/2)*Hypergeometric2F1[-1 - n/2, 2 - n/2, 3
 - n/2, (1 - a*x)/2])/(a*(4 - n)))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx &= c\int(1-ax)^{1-\frac{n}{2}}(1+ax)^{1+\frac{n}{2}}\,dx\\ &= -\frac{2^{2+\frac{n}{2}}c(1-ax)^{2-\frac{n}{2}}\,{}_2F_1\left(-1-\frac{n}{2},2-\frac{n}{2};3-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(4-n)}\end{aligned}$$

Mathematica [A] time = 0.0190884, size = 65, normalized size = 0.96

$$\frac{c2^{\frac{n}{2}+2}(1-ax)^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(-\frac{n}{2}-1,2-\frac{n}{2},3-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a(n-4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] (2^(2 + n/2)*c*(1 - a*x)^(2 - n/2)*Hypergeometric2F1[-1 - n/2, 2 - n/2, 3 -
 n/2, (1 - a*x)/2])/(a*(-4 + n))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \mathrm{e}^{n\,\mathrm{Artanh}(ax)} \left(-a^2 c x^2 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(a^2 c x^2 - c\right) \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^2 c x^2 - c\right) \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int a^2 x^2 e^{n\,\mathrm{atanh}\,(ax)}\,dx + \int -e^{n\,\mathrm{atanh}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a**2*c*x**2+c),x)
```

```
[Out] -c*(Integral(a**2*x**2*exp(n*atanh(a*x)), x) + Integral(-exp(n*atanh(a*x)),
 x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(a^2cx^2 - c\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.1309 $\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=70

$$-\frac{c^2 2^{\frac{n}{2}+3}(1-ax)^{3-\frac{n}{2}}\text{Hypergeometric2F1}\left(-\frac{n}{2}-2,3-\frac{n}{2},4-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a(6-n)}$$

```
[Out] -((2^(3 + n/2)*c^2*(1 - a*x)^(3 - n/2)*Hypergeometric2F1[-2 - n/2, 3 - n/2,
 4 - n/2, (1 - a*x)/2])/(a*(6 - n)))
```

Rubi [A] time = 0.0536608, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 69}

$$-\frac{c^2 2^{\frac{n}{2}+3}(1-ax)^{3-\frac{n}{2}}\,{}_2F_1\left(-\frac{n}{2}-2,3-\frac{n}{2};4-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(6-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] -((2^(3 + n/2)*c^2*(1 - a*x)^(3 - n/2)*Hypergeometric2F1[-2 - n/2, 3 - n/2,
 4 - n/2, (1 - a*x)/2])/(a*(6 - n)))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx &= c^2\int(1-ax)^{2-\frac{n}{2}}(1+ax)^{2+\frac{n}{2}}\,dx\\ &= -\frac{2^{3+\frac{n}{2}}c^2(1-ax)^{3-\frac{n}{2}}\,{}_2F_1\left(-2-\frac{n}{2},3-\frac{n}{2};4-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(6-n)}\end{aligned}$$

Mathematica [A] time = 0.0205565, size = 67, normalized size = 0.96

$$\frac{c^2 2^{\frac{n}{2}+3}(1-ax)^{3-\frac{n}{2}}\text{Hypergeometric2F1}\left(-\frac{n}{2}-2,3-\frac{n}{2},4-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a(n-6)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (2^(3 + n/2)*c^2*(1 - a*x)^(3 - n/2)*Hypergeometric2F1[-2 - n/2, 3 - n/2, 4
 - n/2, (1 - a*x)/2])/(a*(-6 + n))
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{Artanh}(ax)} \left(-a^2 c x^2 + c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 c x^2 - c\right)^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^4 c^2 x^4 - 2\,a^2 c^2 x^2 + c^2\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*((a*x + 1)/(a*x - 1))^(1/2*n),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2 \left(\int -2a^2x^2 e^{n\,\mathrm{atanh}\,(ax)}\,dx + \int a^4x^4 e^{n\,\mathrm{atanh}\,(ax)}\,dx + \int e^{n\,\mathrm{atanh}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a**2*c*x**2+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2*exp(n*atanh(a*x)), x) + Integral(a**4*x**4*exp(
n*atanh(a*x)), x) + Integral(exp(n*atanh(a*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2-c\right)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.1310 $\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=70

$$-\frac{c^3 2^{\frac{n}{2}+4}(1-ax)^{4-\frac{n}{2}}\text{Hypergeometric2F1}\left(-\frac{n}{2}-3,4-\frac{n}{2},5-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a(8-n)}$$

```
[Out] -((2^(4 + n/2)*c^3*(1 - a*x)^(4 - n/2)*Hypergeometric2F1[-3 - n/2, 4 - n/2,
 5 - n/2, (1 - a*x)/2])/(a*(8 - n)))
```

Rubi [A] time = 0.0563535, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 69}

$$-\frac{c^3 2^{\frac{n}{2}+4}(1-ax)^{4-\frac{n}{2}}\,{}_2F_1\left(-\frac{n}{2}-3,4-\frac{n}{2};5-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(8-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -((2^(4 + n/2)*c^3*(1 - a*x)^(4 - n/2)*Hypergeometric2F1[-3 - n/2, 4 - n/2,
 5 - n/2, (1 - a*x)/2])/(a*(8 - n)))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx &= c^3\int(1-ax)^{3-\frac{n}{2}}(1+ax)^{3+\frac{n}{2}}\,dx\\ &= -\frac{2^{4+\frac{n}{2}}c^3(1-ax)^{4-\frac{n}{2}}\,{}_2F_1\left(-3-\frac{n}{2},4-\frac{n}{2};5-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(8-n)}\end{aligned}$$

Mathematica [A] time = 0.0253843, size = 67, normalized size = 0.96

$$\frac{c^3 2^{\frac{n}{2}+4}(1-ax)^{4-\frac{n}{2}}\text{Hypergeometric2F1}\left(-\frac{n}{2}-3,4-\frac{n}{2},5-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a(n-8)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (2^(4 + n/2)*c^3*(1 - a*x)^(4 - n/2)*Hypergeometric2F1[-3 - n/2, 4 - n/2, 5
 - n/2, (1 - a*x)/2])/(a*(-8 + n))
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int e^{n\mathrm{Artanh}(ax)}\left(-a^{2}c x^{2}+c\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^3,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(a^{2}c x^{2}-c\right)^{3}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate((a^2*c*x^2 - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^{6}c^{3}x^{6}-3\,a^{4}c^{3}x^{4}+3\,a^{2}c^{3}x^{2}-c^{3}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*((a*x + 1)/(a
*x - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^{3}\left(\int 3a^{2}x^{2}e^{n\operatorname{atanh}(ax)}\,dx+\int -3a^{4}x^{4}e^{n\operatorname{atanh}(ax)}\,dx+\int a^{6}x^{6}e^{n\operatorname{atanh}(ax)}\,dx+\int -e^{n\operatorname{atanh}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2*exp(n*atanh(a*x)), x) + Integral(-3*a**4*x**4*e
xp(n*atanh(a*x)), x) + Integral(a**6*x**6*exp(n*atanh(a*x)), x) + Integral(
-exp(n*atanh(a*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(a^2cx^2-c\right)^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.1311 $\int \frac{e^{n\tanh^{-1}(ax)}x^4}{c-a^2cx^2}\,dx$

Optimal. Leaf size=209

$$\frac{2^{\frac{n}{2}-1}n\left(n^2+8\right)(1-ax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{3a^5c(2-n)}+\frac{(ax+1)^{n/2}\left(-a\left(n^2+6\right)nx+n^3+\right.}{6a^5cn}$$

```
[Out] -(n*x^2*(1 + a*x)^(n/2))/(6*a^3*c*(1 - a*x)^(n/2)) - (x^3*(1 + a*x)^(n/2))/
(3*a^2*c*(1 - a*x)^(n/2)) + ((1 + a*x)^(n/2)*(6 + 8*n + n^2 + n^3 - a*n*(6
+ n^2)*x))/(6*a^5*c*n*(1 - a*x)^(n/2)) + (2^(-1 + n/2)*n*(8 + n^2)*(1 - a*x
)^(1 - n/2)*Hypergeometric2F1[1 - n/2, 1 - n/2, 2 - n/2, (1 - a*x)/2])/(3*a
^5*c*(2 - n))
```

Rubi [A] time = 0.257294, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6150, 100, 153, 143, 69}

$$\frac{2^{\frac{n}{2}-1}n\left(n^2+8\right)(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},1-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{3a^5c(2-n)}+\frac{(ax+1)^{n/2}\left(-a\left(n^2+6\right)nx+n^3+n^2+8n+6\right)(1-}{6a^5cn}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2),x]
```



```
[Out] -(n*x^2*(1 + a*x)^(n/2))/(6*a^3*c*(1 - a*x)^(n/2)) - (x^3*(1 + a*x)^(n/2))/
(3*a^2*c*(1 - a*x)^(n/2)) + ((1 + a*x)^(n/2)*(6 + 8*n + n^2 + n^3 - a*n*(6
+ n^2)*x))/(6*a^5*c*n*(1 - a*x)^(n/2)) + (2^(-1 + n/2)*n*(8 + n^2)*(1 - a*x
)^(1 - n/2)*Hypergeometric2F1[1 - n/2, 1 - n/2, 2 - n/2, (1 - a*x)/2])/(3*a
^5*c*(2 - n))
```


Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && !(SumSimplerQ[n, 1] && !SumSimplerQ[m, 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^4}{c-a^2cx^2}\,dx = \frac{\int x^4(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}\,dx}{c}$$
$$= -\frac{x^3(1-ax)^{-n/2}(1+ax)^{n/2}}{3a^2c} - \frac{\int x^2(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}(-3-anx)\,dx}{3a^2c}$$
$$= -\frac{nx^2(1-ax)^{-n/2}(1+ax)^{n/2}}{6a^3c} - \frac{x^3(1-ax)^{-n/2}(1+ax)^{n/2}}{3a^2c} + \frac{\int x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}\left(2an+a^2\left(\right.\right.}{6a^4c}$$
$$= -\frac{nx^2(1-ax)^{-n/2}(1+ax)^{n/2}}{6a^3c} - \frac{x^3(1-ax)^{-n/2}(1+ax)^{n/2}}{3a^2c} + \frac{(1-ax)^{-n/2}(1+ax)^{n/2}\left(6+8n+n^2+n^{3}\right.}{6a^5cn}$$
$$= -\frac{nx^2(1-ax)^{-n/2}(1+ax)^{n/2}}{6a^3c} - \frac{x^3(1-ax)^{-n/2}(1+ax)^{n/2}}{3a^2c} + \frac{(1-ax)^{-n/2}(1+ax)^{n/2}\left(6+8n+n^2+n^{3}\right.}{6a^5cn}$$

Mathematica [A] time = 0.202656, size = 172, normalized size = 0.82

$$\frac{(1-ax)^{-n/2}\left(2^{\frac{n}{2}+1}n\left(n^2-2n+2\right)(ax-1)\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right)-(n-2)\left((ax+1)^{n/2}\left(2n\left(a\right.\right.\right.\right.}{6a^5c(n-2)n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2),x]
```

```
[Out] (2^(1 + n/2)*n*(2 - 2*n + n^2)*(-1 + a*x)*Hypergeometric2F1[1 - n/2, -n/2,
2 - n/2, (1 - a*x)/2] - (-2 + n)*((1 + a*x)^(n/2)*(-6 + (n + a*n*x)^2 + 2*n
*(3 + 2*a*x + a^3*x^3)) - 2^(2 + n/2)*n*(3 + n)*Hypergeometric2F1[-n/2, -n/
2, 1 - n/2, (1 - a*x)/2]))/(6*a^5*c*(-2 + n)*n*(1 - a*x)^(n/2))
```

Maple [F] time = 0.203, size = 0, normalized size = 0.

$$\int \frac{e^{n\text{Artanh}(ax)}x^4}{-a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2 - c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -integrate(x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^4 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2 - c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^4 e^{n \operatorname{atanh}(ax)}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**4/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(x**4*exp(n*atanh(a*x))/(a**2*x**2 - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^4 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2 - c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c),x, algorithm="giac")

[Out] integrate(-x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

3.1312 $\int \frac{e^{n \tanh^{-1}(ax)} x^3}{c-a^2cx^2} \, dx$

Optimal. Leaf size=160

$$\frac{2^{\frac{n}{2}-1}\left(n^2+2\right)(1-ax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a^4c(2-n)}+\frac{(ax+1)^{n/2}\left(-an^2x+n^2+n+2\right)(1}{2a^4cn}$$

```
[Out] -(x^2*(1 + a*x)^(n/2))/(2*a^2*c*(1 - a*x)^(n/2)) + ((1 + a*x)^(n/2)*(2 + n
+ n^2 - a*n^2*x))/(2*a^4*c*n*(1 - a*x)^(n/2)) + (2^(-1 + n/2)*(2 + n^2)*(1
- a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, 1 - n/2, 2 - n/2, (1 - a*x)/2])
/(a^4*c*(2 - n))
```

Rubi [A] time = 0.205022, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6150, 100, 143, 69}

$$\frac{2^{\frac{n}{2}-1}\left(n^2+2\right)(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},1-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^4c(2-n)}+\frac{(ax+1)^{n/2}\left(-an^2x+n^2+n+2\right)(1-ax)^{-n/2}}{2a^4cn}-\frac{x^2(ax}{}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2),x]
```

```
[Out] -(x^2*(1 + a*x)^(n/2))/(2*a^2*c*(1 - a*x)^(n/2)) + ((1 + a*x)^(n/2)*(2 + n
+ n^2 - a*n^2*x))/(2*a^4*c*n*(1 - a*x)^(n/2)) + (2^(-1 + n/2)*(2 + n^2)*(1
- a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, 1 - n/2, 2 - n/2, (1 - a*x)/2])
/(a^4*c*(2 - n))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^3}{c-a^2cx^2}\,dx = \frac{\int x^3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}\,dx}{c}$$
$$= -\frac{x^2(1-ax)^{-n/2}(1+ax)^{n/2}}{2a^2c} - \frac{\int x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}(-2-anx)\,dx}{2a^2c}$$
$$= -\frac{x^2(1-ax)^{-n/2}(1+ax)^{n/2}}{2a^2c} + \frac{(1-ax)^{-n/2}(1+ax)^{n/2}\left(2+n+n^2-an^2x\right)}{2a^4cn} - \frac{\left(2+n^2\right)\int(1-ax)^{-n/2}(\cdots}{2a^3c}$$
$$= -\frac{x^2(1-ax)^{-n/2}(1+ax)^{n/2}}{2a^2c} + \frac{(1-ax)^{-n/2}(1+ax)^{n/2}\left(2+n+n^2-an^2x\right)}{2a^4cn} + \frac{2^{-1+\frac{n}{2}}\left(2+n^2\right)(1-ax)\cdots}{\cdots}$$

Mathematica [A] time = 0.0612192, size = 120, normalized size = 0.75

$$\frac{(1-ax)^{-n/2}\left(2^{n/2}n\left(n^2+2\right)(ax-1)\text{Hypergeometric2F1}\left(1-\frac{n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right)-(n-2)(ax+1)^{n/2}\left(n\left(a^2x^2-1\cdots\right.\right.\right.}{2a^4c(n-2)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2),x]
```

```
[Out] (-((-2 + n)*(1 + a*x)^(n/2)*(-2 + n^2*(-1 + a*x) + n*(-1 + a^2*x^2))) + 2^(
n/2)*n*(2 + n^2)*(-1 + a*x)*Hypergeometric2F1[1 - n/2, 1 - n/2, 2 - n/2, (1
 - a*x)/2])/(2*a^4*c*(-2 + n)*n*(1 - a*x)^(n/2))
```

Maple [F] time = 0.188, size = 0, normalized size = 0.

$$\int \frac{e^{n\,\mathrm{Artanh}(ax)}x^3}{-a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c),x, algorithm="maxima")

[Out] -integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c),x, algorithm="fricas")

[Out] integral(-x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^3e^{n\operatorname{atanh}(ax)}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**3/(-a**2*c*x**2+c),x)

[Out] -Integral(x**3*exp(n*atanh(a*x))/(a**2*x**2 - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c),x, algorithm="giac")

[Out] integrate(-x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

3.1313 $\int \frac{e^{n \tanh^{-1}(ax)} x^2}{c-a^2cx^2} dx$

Optimal. Leaf size=86

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a^3c(2-n)}+\frac{e^{n\tanh^{-1}(ax)}}{a^3cn}$$

```
[Out] E^(n*ArcTanh[a*x])/(a^3*c*n) + (2^(1 + n/2)*(1 - a*x)^(1 - n/2)*Hypergeomet
ric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2])/(a^3*c*(2 - n))
```

Rubi [A] time = 0.135161, antiderivative size = 127, normalized size of antiderivative = 1.48, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6150, 90, 79, 69}

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^3c}-\frac{x(ax+1)^{n/2}(1-ax)^{-n/2}}{a^2c}+\frac{(1-n)(ax+1)^{n/2}(1-ax)^{-n/2}}{a^3cn}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2),x]
```

```
[Out] ((1 - n)*(1 + a*x)^(n/2))/(a^3*c*n*(1 - a*x)^(n/2)) - (x*(1 + a*x)^(n/2))/(
a^2*c*(1 - a*x)^(n/2)) + (2^(1 + n/2)*Hypergeometric2F1[-n/2, -n/2, 1 - n/2
, (1 - a*x)/2])/(a^3*c*(1 - a*x)^(n/2))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{n\tanh^{-1}(ax)}x^2}{c-a^2cx^2}\,dx &= \frac{\int x^2(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}\,dx}{c}\\
&= -\frac{x(1-ax)^{-n/2}(1+ax)^{n/2}}{a^2c}-\frac{\int (1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}(-1-anx)\,dx}{a^2c}\\
&= \frac{(1-n)(1-ax)^{-n/2}(1+ax)^{n/2}}{a^3cn}-\frac{x(1-ax)^{-n/2}(1+ax)^{n/2}}{a^2c}+\frac{n\int (1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}\,dx}{a^2c}\\
&= \frac{(1-n)(1-ax)^{-n/2}(1+ax)^{n/2}}{a^3cn}-\frac{x(1-ax)^{-n/2}(1+ax)^{n/2}}{a^2c}+\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\right.}{a^3c}
\end{aligned}
$$

Mathematica [A] time = 0.0631519, size = 82, normalized size = 0.95

$$
\frac{(1-ax)^{-n/2}\left(2^{\frac{n}{2}+1}n\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}(1-ax)\right)-(ax+1)^{n/2}(anx+n-1)\right)}{a^3cn}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2),x]
```

```
[Out] (-((1 + a*x)^(n/2)*(-1 + n + a*n*x)) + 2^(1 + n/2)*n*Hypergeometric2F1[-n/2
, -n/2, 1 - n/2, (1 - a*x)/2])/(a^3*c*n*(1 - a*x)^(n/2))
```

Maple [F] time = 0.185, size = 0, normalized size = 0.

$$
\int \frac{e^{n\operatorname{Artanh}(ax)}x^2}{-a^2cx^2+c}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
-\int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^2e^{n\,\text{atanh}\,(ax)}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**2/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(x**2*exp(n*atanh(a*x))/(a**2*x**2 - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

3.1314 $\int \frac{e^{n\tanh^{-1}(ax)}x}{c-a^2cx^2}\,dx$

Optimal. Leaf size=94

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a^2cn}-\frac{(1-ax)^{-n/2}(ax+1)^{n/2}}{a^2cn}$$

```
[Out] -((1 + a*x)^(n/2)/(a^2*c*n*(1 - a*x)^(n/2))) + (2^(1 + n/2)*Hypergeometric2
F1[-n/2, -n/2, 1 - n/2, (1 - a*x)/2])/(a^2*c*n*(1 - a*x)^(n/2))
```

Rubi [A] time = 0.0828106, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6150, 79, 69}

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^2cn}-\frac{(1-ax)^{-n/2}(ax+1)^{n/2}}{a^2cn}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x)/(c - a^2*c*x^2),x]
```

```
[Out] -((1 + a*x)^(n/2)/(a^2*c*n*(1 - a*x)^(n/2))) + (2^(1 + n/2)*Hypergeometric2
F1[-n/2, -n/2, 1 - n/2, (1 - a*x)/2])/(a^2*c*n*(1 - a*x)^(n/2))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x}{c-a^2cx^2}\,dx = \frac{\int x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}\,dx}{c}$$
$$= -\frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{a^2cn} + \frac{\int (1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}\,dx}{ac}$$
$$= -\frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{a^2cn} + \frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^2cn}$$

Mathematica [A] time = 0.055578, size = 74, normalized size = 0.79

$$\frac{(1-ax)^{-n/2}\left(2^{\frac{n}{2}+1}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}(1-ax)\right)-(ax+1)^{n/2}\right)}{a^2cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x)/(c - a^2*c*x^2),x]
```

```
[Out] (-(1 + a*x)^(n/2) + 2^(1 + n/2)*Hypergeometric2F1[-n/2, -n/2, 1 - n/2, (1 -
 a*x)/2])/(a^2*c*n*(1 - a*x)^(n/2))
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int \frac{e^{n\text{Artanh}(ax)}x}{-a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{xe^{n\operatorname{atanh}(ax)}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(x*exp(n*atanh(a*x))/(a**2*x**2 - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

3.1315 $\int \frac{e^{n\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{e^{n\tanh^{-1}(ax)}}{acn}$$

```
[Out] E^(n*ArcTanh[a*x])/(a*c*n)
```

Rubi [A] time = 0.0343823, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6137}

$$\frac{e^{n\tanh^{-1}(ax)}}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] E^(n*ArcTanh[a*x])/(a*c*n)
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{n\tanh^{-1}(ax)}}{acn}$$

Mathematica [A] time = 0.006812, size = 33, normalized size = 1.83

$$\frac{(1-ax)^{-n/2}(ax+1)^{n/2}}{acn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] (1 + a*x)^(n/2)/(a*c*n*(1 - a*x)^(n/2))
```

Maple [A] time = 0.029, size = 18, normalized size = 1.

$$\frac{e^{n\operatorname{Artanh}(ax)}}{can}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c),x)
```

```
[Out] exp(n*arctanh(a*x))/a/c/n
```

Maxima [A] time = 0.964448, size = 41, normalized size = 2.28

$$\frac{e^{\left(\frac{1}{2}\,n\log(ax+1)-\frac{1}{2}\,n\log(ax-1)\right)}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] e^(1/2*n*log(a*x + 1) - 1/2*n*log(a*x - 1))/(a*c*n)
```

Fricas [A] time = 2.19292, size = 53, normalized size = 2.94

$$\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] ((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*n)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a**2*c*x**2+c),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

3.1316 $\int \frac{e^{n\tanh^{-1}(ax)}}{x(c-a^2cx^2)}\,dx$

Optimal. Leaf size=90

$$\frac{(1-ax)^{-n/2}(ax+1)^{n/2}}{cn}-\frac{2(1-ax)^{-n/2}(ax+1)^{n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n+2}{2},\frac{ax+1}{1-ax}\right)}{cn}$$

```
[Out] (1 + a*x)^(n/2)/(c*n*(1 - a*x)^(n/2)) - (2*(1 + a*x)^(n/2)*Hypergeometric2F
1[1, n/2, (2 + n)/2, (1 + a*x)/(1 - a*x)])/(c*n*(1 - a*x)^(n/2))
```

Rubi [A] time = 0.105493, antiderivative size = 100, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6150, 96, 131}

$$\frac{(1-ax)^{-n/2}(ax+1)^{n/2}}{cn}-\frac{2(1-ax)^{1-\frac{n}{2}}(ax+1)^{\frac{n-2}{2}}\,{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{c(2-n)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x*(c - a^2*c*x^2)),x]
```

```
[Out] (1 + a*x)^(n/2)/(c*n*(1 - a*x)^(n/2)) - (2*(1 - a*x)^(1 - n/2)*(1 + a*x)^((
-2 + n)/2)*Hypergeometric2F1[1, 1 - n/2, 2 - n/2, (1 - a*x)/(1 + a*x)])/(c*
(2 - n))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)}dx = \frac{\int \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}}{x}dx}{c}$$
$$= \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{cn} + \frac{\int \frac{(1-ax)^{-n/2}(1+ax)^{-1+\frac{n}{2}}}{x}dx}{c}$$
$$= \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{cn} - \frac{2(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{1+ax}\right)}{c(2-n)}$$

Mathematica [A] time = 0.0352695, size = 85, normalized size = 0.94

$$\frac{(1-ax)^{-n/2}(ax+1)^{\frac{n}{2}-1}\left((n-2)(ax+1)-2n(ax-1)\text{Hypergeometric2F1}\left(1,1-\frac{n}{2},2-\frac{n}{2},\frac{1-ax}{ax+1}\right)\right)}{c(n-2)n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x*(c - a^2*c*x^2)),x]
```

```
[Out] ((1 + a*x)^(-1 + n/2)*((-2 + n)*(1 + a*x) - 2*n*(-1 + a*x)*Hypergeometric2F
1[1, 1 - n/2, 2 - n/2, (1 - a*x)/(1 + a*x)]))/(c*(-2 + n)*n*(1 - a*x)^(n/2)
)
```

Maple [F] time = 0.19, size = 0, normalized size = 0.

$$\int \frac{e^{n\text{Artanh}(ax)}}{x\left(-a^2cx^2+c\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((a^2*c*x^2 - c)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^3-cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^3 - c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{e^{n\,\text{atanh}\,(ax)}}{a^2x^3-x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(exp(n*atanh(a*x))/(a**2*x**3 - x), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(a^2cx^2-c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/((a^2*c*x^2 - c)*x), x)
```

3.1317 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)}\,dx$

Optimal. Leaf size=123

$$-\frac{2a(ax+1)^{n/2}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n+2}{2},\frac{ax+1}{1-ax}\right)}{c}+\frac{a(n+1)(ax+1)^{n/2}(1-ax)^{-n/2}}{cn}-\frac{(ax+1)^{n/2}(1-a}{cx}$$

```
[Out] (a*(1 + n)*(1 + a*x)^(n/2))/(c*n*(1 - a*x)^(n/2)) - (1 + a*x)^(n/2)/(c*x*(1
 - a*x)^(n/2)) - (2*a*(1 + a*x)^(n/2)*Hypergeometric2F1[1, n/2, (2 + n)/2,
(1 + a*x)/(1 - a*x)])/(c*(1 - a*x)^(n/2))
```

Rubi [A] time = 0.143568, antiderivative size = 137, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6150, 129, 155, 12, 131}

$$-\frac{2an(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{c(2-n)}+\frac{a(n+1)(ax+1)^{n/2}(1-ax)^{-n/2}}{cn}-\frac{(ax+1)^{n/2}(1-ax)^{-n/2}}{cx}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)),x]
```

```
[Out] (a*(1 + n)*(1 + a*x)^(n/2))/(c*n*(1 - a*x)^(n/2)) - (1 + a*x)^(n/2)/(c*x*(1
 - a*x)^(n/2)) - (2*a*n*(1 - a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2)*Hypergeo
metric2F1[1, 1 - n/2, 2 - n/2, (1 - a*x)/(1 + a*x)])/(c*(2 - n))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
```

```
1]) && !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)}\,dx = \frac{\int \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}}{x^2}\,dx}{c}$$

$$= -\frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{cx} - \frac{\int \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}(-an-a^2x)}{x}\,dx}{c}$$

$$= \frac{a(1+n)(1-ax)^{-n/2}(1+ax)^{n/2}}{cn} - \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{cx} + \frac{\int \frac{a^2n^2(1-ax)^{-n/2}(1+ax)^{-1+\frac{n}{2}}}{x}\,dx}{acn}$$

$$= \frac{a(1+n)(1-ax)^{-n/2}(1+ax)^{n/2}}{cn} - \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{cx} + \frac{(an)\int \frac{(1-ax)^{-n/2}(1+ax)^{-1+\frac{n}{2}}}{x}\,dx}{c}$$

$$= \frac{a(1+n)(1-ax)^{-n/2}(1+ax)^{n/2}}{cn} - \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{cx} - \frac{2an(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(1,1\right.}{c(2-n)}$$

Mathematica [A] time = 0.044348, size = 103, normalized size = 0.84

$$\frac{(1-ax)^{-n/2}(ax+1)^{\frac{n}{2}-1}\left((n-2)(ax+1)(n(ax-1)+ax)-2an^2x(ax-1)\text{Hypergeometric2F1}\left(1,1-\frac{n}{2},2-\frac{n}{2},\frac{1-ax}{ax+1}\right)\right)}{c(n-2)nx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)),x]
```

```
[Out] ((1 + a*x)^(-1 + n/2)*((-2 + n)*(1 + a*x)*(a*x + n*(-1 + a*x)) - 2*a*n^2*x*
(-1 + a*x)*Hypergeometric2F1[1, 1 - n/2, 2 - n/2, (1 - a*x)/(1 + a*x)]))/(c
*(-2 + n)*n*x*(1 - a*x)^(n/2))
```

Maple [F] time = 0.185, size = 0, normalized size = 0.

$$\int \frac{e^{n\text{Artanh}(ax)}}{x^2(-a^2cx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((a^2*c*x^2 - c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^4-cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^4 - c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{e^{n \operatorname{atanh}(ax)}}{a^2x^4-x^2}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**2/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(exp(n*atanh(a*x))/(a**2*x**4 - x**2), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/((a^2*c*x^2 - c)*x^2), x)
```

3.1318 $\int \frac{e^{n\tanh^{-1}(ax)}x^4}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=376

$$-\frac{2^{n/2}n(1-ax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a^5c^2(2-n)}-\frac{(n+3)\left(2-n^2\right)(ax+1)^{n/2}(1-ax)^{-\frac{n}{2}-1}}{a^5c^2\left(4-n^2\right)}-\frac{(n$$

```
[Out] ((1 - n)*(3 + n)*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^5*c^2*(2 -
 n)) + ((3 + n)*x*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^4*c^2) -
(x^3*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^2*c^2) + ((1 - a*x)^(1
 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^5*c^2*(2 - n)) - (1 + a*x)^((-2 + n)/2)/
(a^5*c^2*(1 - a*x)^(n/2)) - ((3 + n)*(2 - n^2)*(1 - a*x)^(-1 - n/2)*(1 + a*
x)^(n/2))/(a^5*c^2*(4 - n^2)) - ((3 + n)*(2 - n^2)*(1 + a*x)^(n/2))/(a^5*c^
2*n*(4 - n^2)*(1 - a*x)^(n/2)) - (2^(n/2)*n*(1 - a*x)^(1 - n/2)*Hypergeomet
ric2F1[(2 - n)/2, 1 - n/2, 2 - n/2, (1 - a*x)/2])/(a^5*c^2*(2 - n))
```

Rubi [A] time = 0.38817, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {6150, 100, 159, 89, 79, 69, 90, 45, 37}

$$-\frac{2^{n/2}n(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(\frac{2-n}{2},1-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^5c^2(2-n)}-\frac{(n+3)\left(2-n^2\right)(ax+1)^{n/2}(1-ax)^{-\frac{n}{2}-1}}{a^5c^2\left(4-n^2\right)}-\frac{(n+3)\left(2-n^2\right)(ax+}{a^5c^2n\left(4\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2)^2,x]
```


```
[Out] ((1 - n)*(3 + n)*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^5*c^2*(2 -
 n)) + ((3 + n)*x*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^4*c^2) -
(x^3*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^2*c^2) + ((1 - a*x)^(1
 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^5*c^2*(2 - n)) - (1 + a*x)^((-2 + n)/2)/
(a^5*c^2*(1 - a*x)^(n/2)) - ((3 + n)*(2 - n^2)*(1 - a*x)^(-1 - n/2)*(1 + a*
x)^(n/2))/(a^5*c^2*(4 - n^2)) - ((3 + n)*(2 - n^2)*(1 + a*x)^(n/2))/(a^5*c^
2*n*(4 - n^2)*(1 - a*x)^(n/2)) - (2^(n/2)*n*(1 - a*x)^(1 - n/2)*Hypergeomet
ric2F1[(2 - n)/2, 1 - n/2, 2 - n/2, (1 - a*x)/2])/(a^5*c^2*(2 - n))
```


Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

1]

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^2}\,dx = \frac{\int x^4(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\,dx}{c^2}$$

$$= -\frac{x^3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^2c^2} - \frac{\int x^2(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}(-3-anx)\,dx}{a^2c^2}$$

$$= -\frac{x^3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^2c^2} - \frac{n\int x^2(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\,dx}{a^2c^2} + \frac{(3+n)\int x^2(1-ax)^{-2-}}{a^2c^2}$$

$$= \frac{(3+n)x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2} - \frac{x^3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^2c^2} - \frac{(1-ax)^{-n/2}(1+ax)^{\frac{1}{2}(-2}}{a^5c^2}$$

$$= \frac{(1-n)(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^5c^2(2-n)} + \frac{(3+n)x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2} - \frac{x^3(1-ax)^{-}}{}$$

$$= \frac{(1-n)(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^5c^2(2-n)} + \frac{(3+n)x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2} - \frac{x^3(1-ax)^{-}}{}$$

$$= \frac{(1-n)(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^5c^2(2-n)} + \frac{(3+n)x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2} - \frac{x^3(1-ax)^{-}}{}$$

Mathematica [A] time = 0.156389, size = 178, normalized size = 0.47

$$-\frac{(1-ax)^{-\frac{n}{2}-1}\left((ax+1)^{n/2}\left(n^2\left(1-2a^2x^2\right)+n\left(-4a^3x^3+4a^2x^2+6ax-4\right)+6a^2x^2+n^3(ax-1)^2(ax+1)-6\right)-2^{n/2}n\right.}{a^5c^2(n-2)n(n+2)(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(((1 - a*x)^(-1 - n/2)*((1 + a*x)^(n/2)*(-6 + 6*a^2*x^2 + n^3*(-1 + a*x)^2
*(1 + a*x) + n^2*(1 - 2*a^2*x^2) + n*(-4 + 6*a*x + 4*a^2*x^2 - 4*a^3*x^3))
- 2^(n/2)*n^2*(2 + n)*(-1 + a*x)^2*(1 + a*x)*Hypergeometric2F1[1 - n/2, 1 -
 n/2, 2 - n/2, (1 - a*x)/2]))/(a^5*c^2*(-2 + n)*n*(2 + n)*(1 + a*x)))
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int \frac{e^{n\mathrm{Artanh}(ax)}x^4}{\left(-a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2 - c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^4 - 2\,a^2c^2x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^4 e^{n \operatorname{atanh}(ax)}}{a^4x^4 - 2a^2x^2 + 1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**4/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**4*exp(n*atanh(a*x))/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2 - c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

3.1319 $\int \frac{e^{n\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=311

$$\frac{2^{\frac{n}{2}+2}(1-ax)^{-\frac{n}{2}-1}\text{Hypergeometric2F1}\left(-\frac{n}{2}-1,-\frac{n}{2}-1,-\frac{n}{2},\frac{1}{2}(1-ax)\right)}{a^4c^2(n+2)}+\frac{2(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}}{a^4c^2n(4-n^2)}-\frac{(ax+1)^{\frac{n-2}{2}}(1-}{a^4c^2(n+2}$$

```
[Out] -(((1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^4*c^2*(2 + n))) + (2*(1
- a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^4*c^2*n*(4 - n^2)) - (2*(1 + a*
x)^((-2 + n)/2))/(a^4*c^2*n*(2 + n)*(1 - a*x)^(n/2)) + (3*(1 - a*x)^(-1 - n
/2)*(1 + a*x)^(n/2))/(a^4*c^2*(2 + n)) + (3*(1 + a*x)^(n/2))/(a^4*c^2*n*(2
+ n)*(1 - a*x)^(n/2)) - (3*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((2 + n)/2))/(a^4
*c^2*(2 + n)) + (2^(2 + n/2)*(1 - a*x)^(-1 - n/2)*Hypergeometric2F1[-1 - n/
2, -1 - n/2, -n/2, (1 - a*x)/2])/(a^4*c^2*(2 + n))
```

Rubi [A] time = 0.252191, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6150, 128, 45, 37, 69}

$$\frac{2^{\frac{n}{2}+2}(1-ax)^{-\frac{n}{2}-1}\,{}_2F_1\left(-\frac{n}{2}-1,-\frac{n}{2}-1;-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^4c^2(n+2)}+\frac{2(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}}{a^4c^2n(4-n^2)}-\frac{(ax+1)^{\frac{n-2}{2}}(1-ax)^{-\frac{n}{2}-1}}{a^4c^2(n+2)}+\frac{3(ax+1}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^2,x]
```


```
[Out] -(((1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^4*c^2*(2 + n))) + (2*(1
- a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^4*c^2*n*(4 - n^2)) - (2*(1 + a*
x)^((-2 + n)/2))/(a^4*c^2*n*(2 + n)*(1 - a*x)^(n/2)) + (3*(1 - a*x)^(-1 - n
/2)*(1 + a*x)^(n/2))/(a^4*c^2*(2 + n)) + (3*(1 + a*x)^(n/2))/(a^4*c^2*n*(2
+ n)*(1 - a*x)^(n/2)) - (3*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((2 + n)/2))/(a^4
*c^2*(2 + n)) + (2^(2 + n/2)*(1 - a*x)^(-1 - n/2)*Hypergeometric2F1[-1 - n/
2, -1 - n/2, -n/2, (1 - a*x)/2])/(a^4*c^2*(2 + n))
```


Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 128

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (IGtQ[m, 0] || (ILtQ[
m, 0] && ILtQ[n, 0]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
```

```
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^2}\,dx = \frac{\int x^3(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\,dx}{c^2}$$

$$= \frac{\int \left(-\frac{(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}}{a^3}+\frac{3(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}}{a^3}+\frac{(1-ax)^{-2-\frac{n}{2}}(1+ax)^{1+\frac{n}{2}}}{a^3}-\frac{3(1-ax)^{-2-\frac{n}{2}}(1+ax)^{n/2}}{a^3}\right)dx}{c^2}$$

$$= -\frac{\int (1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\,dx}{a^3c^2}+\frac{\int (1-ax)^{-2-\frac{n}{2}}(1+ax)^{1+\frac{n}{2}}\,dx}{a^3c^2}+\frac{3\int (1-ax)^{-2-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}\,dx}{a^3c^2}$$

$$= -\frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2(2+n)}+\frac{3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}}{a^4c^2(2+n)}-\frac{3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{a^4c^2(2+n)}+\frac{2^{2+\frac{n}{2}}(1-a}{}$$

$$= -\frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2(2+n)}-\frac{2(1-ax)^{-n/2}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2n(2+n)}+\frac{3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}}{a^4c^2(2+n)}+\frac{3(1-ax}{a^4}$$

$$= -\frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2(2+n)}+\frac{2(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2n\left(4-n^2\right)}-\frac{2(1-ax)^{-n/2}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2n(2+n)}+\frac{3(1-}{}$$

Mathematica [A] time = 5.19257, size = 122, normalized size = 0.39

$$-\frac{e^{n\tanh^{-1}(ax)}\left(-2\left(n^2-4\right)\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n}{2}+1,-e^{2\tanh^{-1}(ax)}\right)+2(n-2)ne^{2\tanh^{-1}(ax)}\text{Hypergeometric2F1}}{2a^4c^2n\left(n^2-4\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(E^(n*ArcTanh[a*x])*(2*E^(2*ArcTanh[a*x])*(-2 + n)*n*Hypergeometric2F1[1,
1 + n/2, 2 + n/2, -E^(2*ArcTanh[a*x])] - 2*(-4 + n^2)*Hypergeometric2F1[1,
n/2, 1 + n/2, -E^(2*ArcTanh[a*x])] + n*(2*Cosh[2*ArcTanh[a*x]] - n*Sinh[2*A
rcTanh[a*x]])))/(2*a^4*c^2*n*(-4 + n^2))
```

Maple [F] time = 0.192, size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{Artanh}(ax)} x^3}{\left(-a^2 c x^2 + c\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2 c x^2 - c\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4 c^2 x^4 - 2\, a^2 c^2 x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3 e^{n \operatorname{atanh}(ax)}}{a^4 x^4 - 2a^2 x^2 + 1} dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**3/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**3*exp(n*atanh(a*x))/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2 - c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

3.1320 $\int \frac{e^{n\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=79

$$-\frac{(n-2ax)e^{n\tanh^{-1}(ax)}}{a^3c^2(4-n^2)(1-a^2x^2)}-\frac{(2-n^2)e^{n\tanh^{-1}(ax)}}{a^3c^2n(4-n^2)}$$

```
[Out] -((E^(n*ArcTanh[a*x])*(2 - n^2))/(a^3*c^2*n*(4 - n^2))) - (E^(n*ArcTanh[a*x
])*(n - 2*a*x))/(a^3*c^2*(4 - n^2)*(1 - a^2*x^2))
```

Rubi [A] time = 0.127441, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6147, 6137}

$$-\frac{(n-2ax)e^{n\tanh^{-1}(ax)}}{a^3c^2(4-n^2)(1-a^2x^2)}-\frac{(2-n^2)e^{n\tanh^{-1}(ax)}}{a^3c^2n(4-n^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -((E^(n*ArcTanh[a*x])*(2 - n^2))/(a^3*c^2*n*(4 - n^2))) - (E^(n*ArcTanh[a*x
])*(n - 2*a*x))/(a^3*c^2*(4 - n^2)*(1 - a^2*x^2))
```

Rule 6147

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> -Simp[((n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a
*d*(n^2 - 4*(p + 1)^2)), x] + Dist[(n^2 + 2*(p + 1))/(d*(n^2 - 4*(p + 1)^2)
), Int[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}
, x] && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p
+ 1)^2, 0] && IntegerQ[2*p]
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^2}\,dx &= -\frac{e^{n\tanh^{-1}(ax)}(n-2ax)}{a^3c^2(4-n^2)(1-a^2x^2)}-\frac{(2-n^2)\int \frac{e^{n\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx}{a^2c(4-n^2)}\\ &= -\frac{e^{n\tanh^{-1}(ax)}(2-n^2)}{a^3c^2n(4-n^2)}-\frac{e^{n\tanh^{-1}(ax)}(n-2ax)}{a^3c^2(4-n^2)(1-a^2x^2)}\end{aligned}$$

Mathematica [A] time = 0.052017, size = 65, normalized size = 0.82

$$-\frac{(1-ax)^{-\frac{n}{2}-1}(ax+1)^{\frac{n}{2}-1}\left(-a^2(n^2-2)x^2+2anx-2\right)}{a^3c^2n(n^2-4)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(((1 - a*x)^(-1 - n/2)*(1 + a*x)^(-1 + n/2)*(-2 + 2*a*n*x - a^2*(-2 + n^2)
*x^2))/(a^3*c^2*n*(-4 + n^2)))
```

Maple [A] time = 0.028, size = 62, normalized size = 0.8

$$-\frac{e^{n\text{Artanh}(ax)}\left(a^{2}n^{2}x^{2}-2\,a^{2}x^{2}-2\,nax+2\right)}{\left(a^{2}x^{2}-1\right)c^{2}a^{3}n\left(n^{2}-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -exp(n*arctanh(a*x))*(a^2*n^2*x^2-2*a^2*x^2-2*a*n*x+2)/(a^2*x^2-1)/c^2/a^3/
n/(n^2-4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^{2}cx^{2}-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

Fricas [A] time = 2.17578, size = 178, normalized size = 2.25

$$-\frac{\left(2\,anx-\left(a^{2}n^{2}-2\,a^{2}\right)x^{2}-2\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^{3}c^{2}n^{3}-4\,a^{3}c^{2}n-\left(a^{5}c^{2}n^{3}-4\,a^{5}c^{2}n\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*n*x - (a^2*n^2 - 2*a^2)*x^2 - 2)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^3*c
^2*n^3 - 4*a^3*c^2*n - (a^5*c^2*n^3 - 4*a^5*c^2*n)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

3.1321 $\int \frac{e^{n\tanh^{-1}(ax)}x}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=69

$$\frac{(2-anx)e^{n\tanh^{-1}(ax)}}{a^2c^2(4-n^2)(1-a^2x^2)}-\frac{e^{n\tanh^{-1}(ax)}}{a^2c^2(4-n^2)}$$

```
[Out] -(E^(n*ArcTanh[a*x])/(a^2*c^2*(4 - n^2))) + (E^(n*ArcTanh[a*x])*(2 - a*n*x)
)/(a^2*c^2*(4 - n^2)*(1 - a^2*x^2))
```

Rubi [A] time = 0.149231, antiderivative size = 102, normalized size of antiderivative = 1.48, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6145, 6136, 6137}

$$\frac{n(n-2ax)e^{n\tanh^{-1}(ax)}}{2a^2c^2(4-n^2)(1-a^2x^2)}-\frac{e^{n\tanh^{-1}(ax)}}{a^2c^2(4-n^2)}+\frac{e^{n\tanh^{-1}(ax)}}{2a^2c^2(1-a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(E^(n*ArcTanh[a*x])/(a^2*c^2*(4 - n^2))) + E^(n*ArcTanh[a*x])/(2*a^2*c^2*(
1 - a^2*x^2)) + (E^(n*ArcTanh[a*x])*n*(n - 2*a*x))/(2*a^2*c^2*(4 - n^2)*(1
- a^2*x^2))
```

Rule 6145

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[((c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(2*d*(p + 1)), x] - Dist[
(a*c*n)/(2*d*(p + 1)), Int[(c + d*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && I
ntegerQ[2*p]
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^2}\,dx = \frac{e^{n\tanh^{-1}(ax)}}{2a^2c^2\left(1-a^2x^2\right)} - \frac{n\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{2a}$$
$$= \frac{e^{n\tanh^{-1}(ax)}}{2a^2c^2\left(1-a^2x^2\right)} + \frac{e^{n\tanh^{-1}(ax)}n(n-2ax)}{2a^2c^2\left(4-n^2\right)\left(1-a^2x^2\right)} - \frac{n\int \frac{e^{n\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx}{ac\left(4-n^2\right)}$$
$$= -\frac{e^{n\tanh^{-1}(ax)}}{a^2c^2\left(4-n^2\right)} + \frac{e^{n\tanh^{-1}(ax)}}{2a^2c^2\left(1-a^2x^2\right)} + \frac{e^{n\tanh^{-1}(ax)}n(n-2ax)}{2a^2c^2\left(4-n^2\right)\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.0340597, size = 56, normalized size = 0.81

$$-\frac{(1-ax)^{-\frac{n}{2}-1}(ax+1)^{\frac{n}{2}-1}\left(a^2x^2-anx+1\right)}{a^2c^2\left(n^2-4\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(((1 - a*x)^(-1 - n/2)*(1 + a*x)^(-1 + n/2)*(1 - a*n*x + a^2*x^2))/(a^2*c^
2*(-4 + n^2)))
```

Maple [A] time = 0.03, size = 47, normalized size = 0.7

$$\frac{e^{n\mathrm{Artanh}(ax)}\left(a^2x^2-nax+1\right)}{\left(a^2x^2-1\right)c^2a^2\left(n^2-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^2,x)
```

```
[Out] exp(n*arctanh(a*x))*(a^2*x^2-a*n*x+1)/(a^2*x^2-1)/c^2/a^2/(n^2-4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

Fricas [A] time = 2.2321, size = 151, normalized size = 2.19

$$-\frac{\left(a^{2} x^{2}-a n x+1\right)\left(\frac{a x+1}{a x-1}\right)^{\frac{1}{2} n}}{a^{2} c^{2} n^{2}-4 a^{2} c^{2}-\left(a^{4} c^{2} n^{2}-4 a^{4} c^{2}\right) x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*x^2 - a*n*x + 1)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*n^2 - 4*a^2*c
^2 - (a^4*c^2*n^2 - 4*a^4*c^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{a x+1}{a x-1}\right)^{\frac{1}{2} n}}{\left(a^{2} c x^{2}-c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

3.1322 $\int \frac{e^{n\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=72

$$\frac{2e^{n\tanh^{-1}(ax)}}{ac^2n\left(4-n^2\right)}-\frac{(n-2ax)e^{n\tanh^{-1}(ax)}}{ac^2\left(4-n^2\right)\left(1-a^2x^2\right)}$$

```
[Out] (2*E^(n*ArcTanh[a*x]))/(a*c^2*n*(4 - n^2)) - (E^(n*ArcTanh[a*x])*(n - 2*a*x
))/(a*c^2*(4 - n^2)*(1 - a^2*x^2))
```

Rubi [A] time = 0.074394, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6136, 6137}

$$\frac{2e^{n\tanh^{-1}(ax)}}{ac^2n\left(4-n^2\right)}-\frac{(n-2ax)e^{n\tanh^{-1}(ax)}}{ac^2\left(4-n^2\right)\left(1-a^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] (2*E^(n*ArcTanh[a*x]))/(a*c^2*n*(4 - n^2)) - (E^(n*ArcTanh[a*x])*(n - 2*a*x
))/(a*c^2*(4 - n^2)*(1 - a^2*x^2))
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx &= -\frac{e^{n\tanh^{-1}(ax)}(n-2ax)}{ac^2\left(4-n^2\right)\left(1-a^2x^2\right)}+\frac{2\int \frac{e^{n\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx}{c\left(4-n^2\right)}\\ &= \frac{2e^{n\tanh^{-1}(ax)}}{ac^2n\left(4-n^2\right)}-\frac{e^{n\tanh^{-1}(ax)}(n-2ax)}{ac^2\left(4-n^2\right)\left(1-a^2x^2\right)}\end{aligned}$$

Mathematica [A] time = 0.0291504, size = 68, normalized size = 0.94

$$-\frac{(1-ax)^{-\frac{n}{2}-1}(ax+1)^{\frac{n}{2}-1}\left(-2a^2x^2+2anx-n^2+2\right)}{ac^2(n-2)n(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(((1 - a*x)^(-1 - n/2)*(1 + a*x)^(-1 + n/2)*(2 - n^2 + 2*a*n*x - 2*a^2*x^2
))/(a*c^2*(-2 + n)*n*(2 + n)))
```

Maple [A] time = 0.029, size = 55, normalized size = 0.8

$$-\frac{e^{n\,\mathrm{Artanh}(ax)}\left(2\,a^{2}x^{2}-2\,nax+n^{2}-2\right)}{\left(a^{2}x^{2}-1\right)c^{2}an\left(n^{2}-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -exp(n*arctanh(a*x))*(2*a^2*x^2-2*a*n*x+n^2-2)/(a^2*x^2-1)/c^2/a/n/(n^2-4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(a^{2}cx^{2}-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

Fricas [A] time = 2.34176, size = 163, normalized size = 2.26

$$\frac{\left(2\,a^{2}x^{2}-2\,anx+n^{2}-2\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^{2}n^{3}-4\,ac^{2}n-\left(a^{3}c^{2}n^{3}-4\,a^{3}c^{2}n\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] (2*a^2*x^2 - 2*a*n*x + n^2 - 2)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n^3 -
4*a*c^2*n - (a^3*c^2*n^3 - 4*a^3*c^2*n)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

3.1323 $\int \frac{e^{n\tanh^{-1}(ax)}}{x(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=190

$$-\frac{2(ax+1)^{n/2}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n+2}{2},\frac{ax+1}{1-ax}\right)}{c^2n}-\frac{\left(-n^2-n+4\right)(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}}{c^2n\left(4-n^2\right)}+\frac{(ax+1)^{\frac{n-2}{2}}(1}{c^2(n+}$$

```
[Out] ((1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(c^2*(2 + n)) - ((4 - n - n^2
)*(1 - a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2))/(c^2*n*(4 - n^2)) + ((4 + n)*
(1 + a*x)^((-2 + n)/2))/(c^2*n*(2 + n)*(1 - a*x)^(n/2)) - (2*(1 + a*x)^(n/2
)*Hypergeometric2F1[1, n/2, (2 + n)/2, (1 + a*x)/(1 - a*x)])/(c^2*n*(1 - a*
x)^(n/2))
```

Rubi [A] time = 0.202266, antiderivative size = 200, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6150, 129, 155, 12, 131}

$$-\frac{2(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{c^2(2-n)}-\frac{\left(-n^2-n+4\right)(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}}{c^2n\left(4-n^2\right)}+\frac{(ax+1)^{\frac{n-2}{2}}(1-ax)^{-\frac{n}{2}-1}}{c^2(n+2)}+$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^2),x]
```

```
[Out] ((1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(c^2*(2 + n)) - ((4 - n - n^2
)*(1 - a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2))/(c^2*n*(4 - n^2)) + ((4 + n)*
(1 + a*x)^((-2 + n)/2))/(c^2*n*(2 + n)*(1 - a*x)^(n/2)) - (2*(1 - a*x)^(1 -
 n/2)*(1 + a*x)^((-2 + n)/2)*Hypergeometric2F1[1, 1 - n/2, 2 - n/2, (1 - a*
x)/(1 + a*x)])/(c^2*(2 - n))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}}{x}\,dx}{c^2}$$

$$= \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{\int \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\left(-a(2+n)-2a^2x\right)}{x}\,dx}{ac^2(2+n)}$$

$$= \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} + \frac{(4+n)(1-ax)^{-n/2}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2n(2+n)} + \frac{\int \frac{(1-ax)^{-n/2}(1+ax)^{-2+\frac{n}{2}}\left(a^2n(2+n\right.}{x}}{a^2c^2n(2+n)}$$

$$= \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{\left(4-n-n^2\right)(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2n\left(4-n^2\right)} + \frac{(4+n)(1-ax)^{-n/2}(1-}{c^2n(2+n)}$$

$$= \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{\left(4-n-n^2\right)(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2n\left(4-n^2\right)} + \frac{(4+n)(1-ax)^{-n/2}(1-}{c^2n(2+n)}$$

$$= \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{\left(4-n-n^2\right)(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2n\left(4-n^2\right)} + \frac{(4+n)(1-ax)^{-n/2}(1-}{c^2n(2+n)}$$

Mathematica [A] time = 0.069729, size = 121, normalized size = 0.64

$$-\frac{(1-ax)^{-\frac{n}{2}-1}(ax+1)^{\frac{n}{2}-1}\left(-2(n+2)n(ax-1)^2\text{Hypergeometric2F1}\left(1,1-\frac{n}{2},2-\frac{n}{2},\frac{1-ax}{ax+1}\right)+n^2\left(a^2x^2-ax-1\right)+a^2\right.}{c^2n\left(n^2-4\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^2),x]
```

```
[Out] -(((1 - a*x)^(-1 - n/2)*(1 + a*x)^(-1 + n/2)*(4 + n - 4*a^2*x^2 + a^2*n*x^2
 + n^2*(-1 - a*x + a^2*x^2) - 2*n*(2 + n)*(-1 + a*x)^2*Hypergeometric2F1[1,
 1 - n/2, 2 - n/2, (1 - a*x)/(1 + a*x)]))/(c^2*n*(-4 + n^2)))
```

Maple [F] time = 0.183, size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{Artanh}(ax)}}{x\left(-a^{2} c\,x^{2}+c\right)^{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(a^{2} c x^{2}-c\right)^{2} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((a^2*c*x^2 - c)^2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^{4} c^{2} x^{5}-2\,a^{2} c^{2} x^{3}+c^{2} x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^5 - 2*a^2*c^2*x^3 + c^2*x
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^{n \operatorname{atanh}(ax)}}{a^{4}x^{5}-2a^{2}x^{3}+x}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(exp(n*atanh(a*x))/(a**4*x**5 - 2*a**2*x**3 + x), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((a^2*c*x^2 - c)^2*x), x)
```

3.1324 $\int \frac{e^{n \tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=239

$$-\frac{2a(ax+1)^{n/2}(1-ax)^{-n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n+2}{2},\frac{ax+1}{1-ax}\right)}{c^2}-\frac{a\left(-n^3-n^2+4n+6\right)(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}}{c^2n\left(4-n^2\right)}+\frac{a\left(n^2\right.}{}$$

```
[Out] (a*(3 + n)*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(c^2*(2 + n)) - ((1
 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(c^2*x) - (a*(6 + 4*n - n^2 - n^
3)*(1 - a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2))/(c^2*n*(4 - n^2)) + (a*(6 +
4*n + n^2)*(1 + a*x)^((-2 + n)/2))/(c^2*n*(2 + n)*(1 - a*x)^(n/2)) - (2*a*(
1 + a*x)^(n/2)*Hypergeometric2F1[1, n/2, (2 + n)/2, (1 + a*x)/(1 - a*x)])/(
c^2*(1 - a*x)^(n/2))
```

Rubi [A] time = 0.251409, antiderivative size = 253, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6150, 129, 155, 12, 131}

$$-\frac{2an(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{c^2(2-n)}-\frac{a\left(-n^3-n^2+4n+6\right)(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}}{c^2n\left(4-n^2\right)}+\frac{a\left(n^2+4n+6\right)}{c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^2),x]
```

```
[Out] (a*(3 + n)*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(c^2*(2 + n)) - ((1
 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(c^2*x) - (a*(6 + 4*n - n^2 - n^
3)*(1 - a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2))/(c^2*n*(4 - n^2)) + (a*(6 +
4*n + n^2)*(1 + a*x)^((-2 + n)/2))/(c^2*n*(2 + n)*(1 - a*x)^(n/2)) - (2*a*n
*(1 - a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2)*Hypergeometric2F1[1, 1 - n/2, 2
 - n/2, (1 - a*x)/(1 + a*x)])/(c^2*(2 - n))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}}{x^2}\,dx}{c^2}$$

$$= -\frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2x} - \frac{\int \frac{(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\left(-an-3a^2x\right)}{x}\,dx}{c^2}$$

$$= \frac{a(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2x} + \frac{\int \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\left(a^2n\right.}{x}}{ac^2(2+}$$

$$= \frac{a(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2x} + \frac{a\left(6+4n+n^2\right)(1-ax)}{c^2n(2+}$$

$$= \frac{a(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2x} - \frac{a\left(6+4n-n^2-n^3\right)(1-}{c^2n\left(4\right.}$$

$$= \frac{a(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2x} - \frac{a\left(6+4n-n^2-n^3\right)(1-}{c^2n\left(4\right.}$$

$$= \frac{a(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2x} - \frac{a\left(6+4n-n^2-n^3\right)(1-}{c^2n\left(4\right.}$$

Mathematica [A] time = 0.0881545, size = 163, normalized size = 0.68

$$-\frac{(1-ax)^{-\frac{n}{2}-1}(ax+1)^{\frac{n}{2}-1}\left(-2a(n+2)n^2x(ax-1)^2\text{Hypergeometric2F1}\left(1,1-\frac{n}{2},2-\frac{n}{2},\frac{1-ax}{ax+1}\right)+an^2x\left(a^2x^2-2\right)+n\right.}{c^2(n-2)n(n+2)x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^2),x]
```

```
[Out] -(((1 - a*x)^(-1 - n/2)*(1 + a*x)^(-1 + n/2)*(6*a*x - 6*a^3*x^3 + n^3*(-1 +
 a*x)^2*(1 + a*x) + a*n^2*x*(-2 + a^2*x^2) + n*(-4 + 4*a*x + 6*a^2*x^2 - 4*
a^3*x^3) - 2*a*n^2*(2 + n)*x*(-1 + a*x)^2*Hypergeometric2F1[1, 1 - n/2, 2 -
 n/2, (1 - a*x)/(1 + a*x)]))/(c^2*(-2 + n)*n*(2 + n)*x))
```

Maple [F] time = 0.187, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{n \operatorname{Artanh}(ax)}}{x^2\left(-a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((a^2*c*x^2 - c)^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^6-2\,a^2c^2x^4+c^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^6 - 2*a^2*c^2*x^4 + c^2*x
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^{n\operatorname{atanh}(ax)}}{a^4x^6-2a^2x^4+x^2}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(exp(n*atanh(a*x))/(a**4*x**6 - 2*a**2*x**4 + x**2), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((a^2*c*x^2 - c)^2*x^2), x)
```

3.1325 $\int \frac{e^{n\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=127

$$-\frac{(n-4ax)e^{n\tanh^{-1}(ax)}}{ac^3(16-n^2)(1-a^2x^2)^2}-\frac{12(n-2ax)e^{n\tanh^{-1}(ax)}}{ac^3(4-n^2)(16-n^2)(1-a^2x^2)}+\frac{24e^{n\tanh^{-1}(ax)}}{ac^3n(n^4-20n^2+64)}$$

```
[Out] (24*E^(n*ArcTanh[a*x]))/(a*c^3*n*(64 - 20*n^2 + n^4)) - (E^(n*ArcTanh[a*x])
*(n - 4*a*x))/(a*c^3*(16 - n^2)*(1 - a^2*x^2)^2) - (12*E^(n*ArcTanh[a*x])*(
n - 2*a*x))/(a*c^3*(4 - n^2)*(16 - n^2)*(1 - a^2*x^2))
```

Rubi [A] time = 0.134426, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6136, 6137}

$$-\frac{(n-4ax)e^{n\tanh^{-1}(ax)}}{ac^3(16-n^2)(1-a^2x^2)^2}-\frac{12(n-2ax)e^{n\tanh^{-1}(ax)}}{ac^3(4-n^2)(16-n^2)(1-a^2x^2)}+\frac{24e^{n\tanh^{-1}(ax)}}{ac^3n(n^4-20n^2+64)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (24*E^(n*ArcTanh[a*x]))/(a*c^3*n*(64 - 20*n^2 + n^4)) - (E^(n*ArcTanh[a*x])
*(n - 4*a*x))/(a*c^3*(16 - n^2)*(1 - a^2*x^2)^2) - (12*E^(n*ArcTanh[a*x])*(
n - 2*a*x))/(a*c^3*(4 - n^2)*(16 - n^2)*(1 - a^2*x^2))
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{e^{n\tanh^{-1}(ax)}(n-4ax)}{ac^3(16-n^2)(1-a^2x^2)^2} + \frac{12\int \frac{e^{n\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{c(16-n^2)}$$
$$= -\frac{e^{n\tanh^{-1}(ax)}(n-4ax)}{ac^3(16-n^2)(1-a^2x^2)^2} - \frac{12e^{n\tanh^{-1}(ax)}(n-2ax)}{ac^3(4-n^2)(16-n^2)(1-a^2x^2)} + \frac{24\int \frac{e^{n\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx}{c^2(64-20n^2+n^4)}$$
$$= \frac{24e^{n\tanh^{-1}(ax)}}{ac^3n(64-20n^2+n^4)} - \frac{e^{n\tanh^{-1}(ax)}(n-4ax)}{ac^3(16-n^2)(1-a^2x^2)^2} - \frac{12e^{n\tanh^{-1}(ax)}(n-2ax)}{ac^3(4-n^2)(16-n^2)(1-a^2x^2)}$$

Mathematica [A] time = 0.075697, size = 111, normalized size = 0.87

$$\frac{(1-ax)^{-\frac{n}{2}-2}(ax+1)^{\frac{n}{2}-2}\left(4n^2(3a^2x^2-4)-8anx(3a^2x^2-5)+24(a^2x^2-1)^2-4an^3x+n^4\right)}{ac^3(n-4)(n-2)n(n+2)(n+4)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] ((1 - a*x)^(-2 - n/2)*(1 + a*x)^(-2 + n/2)*(n^4 - 4*a*n^3*x + 24*(-1 + a^2*
x^2)^2 - 8*a*n*x*(-5 + 3*a^2*x^2) + 4*n^2*(-4 + 3*a^2*x^2)))/(a*c^3*(-4 + n
)*(-2 + n)*n*(2 + n)*(4 + n))
```

Maple [A] time = 0.032, size = 101, normalized size = 0.8

$$\frac{(24x^4a^4-24x^3a^3n+12a^2n^2x^2-4an^3x-48a^2x^2+n^4+40nax-16n^2+24)\,e^{n\mathrm{Artanh}(ax)}}{(a^2x^2-1)^2c^3a(n^2-16)(n^2-4)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^3,x)
```

```
[Out] (24*a^4*x^4-24*a^3*n*x^3+12*a^2*n^2*x^2-4*a*n^3*x-48*a^2*x^2+n^4+40*a*n*x-1
6*n^2+24)*exp(n*arctanh(a*x))/(a^2*x^2-1)^2/c^3/a/(n^2-16)/(n^2-4)/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(a^2cx^2-c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^3, x)
```

Fricas [A] time = 2.55521, size = 371, normalized size = 2.92

$$\frac{\left(24\,a^4x^4-24\,a^3nx^3+n^4+12\left(a^2n^2-4\,a^2\right)x^2-16\,n^2-4\left(an^3-10\,an\right)x+24\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^3n^5-20\,ac^3n^3+64\,ac^3n+\left(a^5c^3n^5-20\,a^5c^3n^3+64\,a^5c^3n\right)x^4-2\left(a^3c^3n^5-20\,a^3c^3n^3+64\,a^3c^3n\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] (24*a^4*x^4 - 24*a^3*n*x^3 + n^4 + 12*(a^2*n^2 - 4*a^2)*x^2 - 16*n^2 - 4*(a
*n^3 - 10*a*n)*x + 24)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^5 - 20*a*c^3*
n^3 + 64*a*c^3*n + (a^5*c^3*n^5 - 20*a^5*c^3*n^3 + 64*a^5*c^3*n)*x^4 - 2*(a
^3*c^3*n^5 - 20*a^3*c^3*n^3 + 64*a^3*c^3*n)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(a^2cx^2-c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^3, x)
```

3.1326 $\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=197

$$-\frac{(n-6ax)e^{n\tanh^{-1}(ax)}}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}-\frac{360(n-2ax)e^{n\tanh^{-1}(ax)}}{ac^4\left(4-n^2\right)\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)}-\frac{30(n-4ax)e^{n\tanh^{-1}(ax)}}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)^2}+\frac{}{ac^4n}$$

```
[Out] (720*E^(n*ArcTanh[a*x]))/(a*c^4*n*(36 - n^2)*(64 - 20*n^2 + n^4)) - (E^(n*A
rcTanh[a*x])*(n - 6*a*x))/(a*c^4*(36 - n^2)*(1 - a^2*x^2)^3) - (30*E^(n*Arc
Tanh[a*x])*(n - 4*a*x))/(a*c^4*(16 - n^2)*(36 - n^2)*(1 - a^2*x^2)^2) - (36
0*E^(n*ArcTanh[a*x])*(n - 2*a*x))/(a*c^4*(4 - n^2)*(16 - n^2)*(36 - n^2)*(1
 - a^2*x^2))
```

Rubi [A] time = 0.185916, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6136, 6137}

$$-\frac{(n-6ax)e^{n\tanh^{-1}(ax)}}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}-\frac{360(n-2ax)e^{n\tanh^{-1}(ax)}}{ac^4\left(4-n^2\right)\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)}-\frac{30(n-4ax)e^{n\tanh^{-1}(ax)}}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)^2}+\frac{}{ac^4n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^4,x]
```


```
[Out] (720*E^(n*ArcTanh[a*x]))/(a*c^4*n*(36 - n^2)*(64 - 20*n^2 + n^4)) - (E^(n*A
rcTanh[a*x])*(n - 6*a*x))/(a*c^4*(36 - n^2)*(1 - a^2*x^2)^3) - (30*E^(n*Arc
Tanh[a*x])*(n - 4*a*x))/(a*c^4*(16 - n^2)*(36 - n^2)*(1 - a^2*x^2)^2) - (36
0*E^(n*ArcTanh[a*x])*(n - 2*a*x))/(a*c^4*(4 - n^2)*(16 - n^2)*(36 - n^2)*(1
 - a^2*x^2))
```


Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = -\frac{e^{n\tanh^{-1}(ax)}(n-6ax)}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3} + \frac{30\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx}{c\left(36-n^2\right)}$$

$$= -\frac{e^{n\tanh^{-1}(ax)}(n-6ax)}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3} - \frac{30e^{n\tanh^{-1}(ax)}(n-4ax)}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)^2} + \frac{360\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{c^2\left(576-52n^2+n^4\right)}$$

$$= -\frac{e^{n\tanh^{-1}(ax)}(n-6ax)}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3} - \frac{30e^{n\tanh^{-1}(ax)}(n-4ax)}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)^2} - \frac{360e^{n\tanh^{-1}(ax)}(n-2ax}{ac^4\left(4-n^2\right)\left(576-52n^2+n^4\right)(}$$

$$= \frac{720e^{n\tanh^{-1}(ax)}}{ac^4n\left(4-n^2\right)\left(576-52n^2+n^4\right)} - \frac{e^{n\tanh^{-1}(ax)}(n-6ax)}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3} - \frac{30e^{n\tanh^{-1}(ax)}(n-4ax)}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)^2} -$$

Mathematica [A] time = 0.100065, size = 170, normalized size = 0.86

$$-\frac{(1-ax)^{-\frac{n}{2}-3}(ax+1)^{\frac{n}{2}-3}\left(n^4\left(50-30a^2x^2\right)+120an^3x\left(a^2x^2-2\right)-8n^2\left(45a^4x^4-105a^2x^2+68\right)+48anx\left(15a^4x^4-40a\right.\right.}{ac^4(n-6)(n-4)(n-2)n(n+2)(n+4)(n+6)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -(((1 - a*x)^(-3 - n/2)*(1 + a*x)^(-3 + n/2)*(-n^6 + 6*a*n^5*x + n^4*(50 -
30*a^2*x^2) + 120*a*n^3*x*(-2 + a^2*x^2) - 720*(-1 + a^2*x^2)^3 + 48*a*n*x*
(33 - 40*a^2*x^2 + 15*a^4*x^4) - 8*n^2*(68 - 105*a^2*x^2 + 45*a^4*x^4)))/(a
*c^4*(-6 + n)*(-4 + n)*(-2 + n)*n*(2 + n)*(4 + n)*(6 + n)))
```

Maple [A] time = 0.033, size = 167, normalized size = 0.9

$$-\frac{\left(720\,x^6a^6-720\,a^5x^5n+360\,a^4n^2x^4-120\,a^3n^3x^3-2160\,x^4a^4+30\,a^2n^4x^2+1920\,x^3a^3n-6\,an^5x-840\,a^2n^2x^2+n^6+\right.}{\left(a^2x^2-1\right)^3c^4an\left(n^6-56\,n^4+784\,n^2-2304\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -(720*a^6*x^6-720*a^5*n*x^5+360*a^4*n^2*x^4-120*a^3*n^3*x^3-2160*a^4*x^4+30
*a^2*n^4*x^2+1920*a^3*n*x^3-6*a*n^5*x-840*a^2*n^2*x^2+n^6+240*a*n^3*x+2160*
a^2*x^2-50*n^4-1584*a*n*x+544*n^2-720)*exp(n*arctanh(a*x))/(a^2*x^2-1)^3/c^
4/a/n/(n^6-56*n^4+784*n^2-2304)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^4, x)
```

Fricas [A] time = 2.59453, size = 682, normalized size = 3.46

$$\frac{\left(720\,a^6x^6 - 720\,a^5nx^5 + n^6 + 360\left(a^4n^2 - 6\,a^4\right)x^4 - 50\,n^4 - 120\left(a^3n^3 - 16\,a^3n\right)x^3 + 30\left(a^2n^4 - 28\,a^2\right.\right.}{ac^4n^7 - 56\,ac^4n^5 + 784\,ac^4n^3 - \left(a^7c^4n^7 - 56\,a^7c^4n^5 + 784\,a^7c^4n^3 - 2304\,a^7c^4n\right)x^6 - 2304\,ac^4n + 3\left(a^5c^4n^7 - 56\,a^5c^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] (720*a^6*x^6 - 720*a^5*n*x^5 + n^6 + 360*(a^4*n^2 - 6*a^4)*x^4 - 50*n^4 - 1
20*(a^3*n^3 - 16*a^3*n)*x^3 + 30*(a^2*n^4 - 28*a^2*n^2 + 72*a^2)*x^2 + 544*
n^2 - 6*(a*n^5 - 40*a*n^3 + 264*a*n)*x - 720)*((a*x + 1)/(a*x - 1))^(1/2*n)
/(a*c^4*n^7 - 56*a*c^4*n^5 + 784*a*c^4*n^3 - (a^7*c^4*n^7 - 56*a^7*c^4*n^5
+ 784*a^7*c^4*n^3 - 2304*a^7*c^4*n)*x^6 - 2304*a*c^4*n + 3*(a^5*c^4*n^7 - 5
6*a^5*c^4*n^5 + 784*a^5*c^4*n^3 - 2304*a^5*c^4*n)*x^4 - 3*(a^3*c^4*n^7 - 56
*a^3*c^4*n^5 + 784*a^3*c^4*n^3 - 2304*a^3*c^4*n)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(a^2cx^2 - c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^4, x)
```

3.1327 $\int e^{n\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=256

$$-\frac{2^{\frac{n-1}{2}}n\left(n^2+11\right)\sqrt{c-a^2cx^2}(1-ax)^{\frac{3-n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-1),\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-ax)\right)}{15a^4(3-n)\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}(ax+1)^{\frac{n+3}{2}}\left(\right.}{60a^4\sqrt{}}$$

```
[Out] -(x^2*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((3 + n)/2)*Sqrt[c - a^2*c*x^2])/(5*a
^2*Sqrt[1 - a^2*x^2]) - ((1 - a*x)^((3 - n)/2)*(1 + a*x)^((3 + n)/2)*(8 + n
^2 + 3*a*n*x)*Sqrt[c - a^2*c*x^2])/(60*a^4*Sqrt[1 - a^2*x^2]) - (2^((-1 + n
)/2)*n*(11 + n^2)*(1 - a*x)^((3 - n)/2)*Sqrt[c - a^2*c*x^2]*Hypergeometric2
F1[(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(15*a^4*(3 - n)*Sqrt[1 -
 a^2*x^2])
```

Rubi [A] time = 0.33956, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6153, 6150, 100, 147, 69}

$$-\frac{2^{\frac{n-1}{2}}n\left(n^2+11\right)\sqrt{c-a^2cx^2}(1-ax)^{\frac{3-n}{2}}{}_2F_1\left(\frac{1}{2}(-n-1),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{15a^4(3-n)\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}(ax+1)^{\frac{n+3}{2}}\left(3anx+n^2+8\right)(1-}{60a^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(x^2*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((3 + n)/2)*Sqrt[c - a^2*c*x^2])/(5*a
^2*Sqrt[1 - a^2*x^2]) - ((1 - a*x)^((3 - n)/2)*(1 + a*x)^((3 + n)/2)*(8 + n
^2 + 3*a*n*x)*Sqrt[c - a^2*c*x^2])/(60*a^4*Sqrt[1 - a^2*x^2]) - (2^((-1 + n
)/2)*n*(11 + n^2)*(1 - a*x)^((3 - n)/2)*Sqrt[c - a^2*c*x^2]*Hypergeometric2
F1[(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(15*a^4*(3 - n)*Sqrt[1 -
 a^2*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{n\tanh^{-1}(ax)}x^3\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int x^3(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{x^2(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{5a^2\sqrt{1-a^2x^2}} - \frac{\sqrt{c-a^2cx^2}\int x(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}(-2-$$
$$= -\frac{x^2(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{5a^2\sqrt{1-a^2x^2}} - \frac{(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\left(8+n^2+3anx\right)\sqrt{c-}}{60a^4\sqrt{1-a^2x^2}}$$
$$= -\frac{x^2(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{5a^2\sqrt{1-a^2x^2}} - \frac{(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\left(8+n^2+3anx\right)\sqrt{c-}}{60a^4\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.506088, size = 236, normalized size = 0.92

$$\frac{\sqrt{c-a^2cx^2}(1-ax)^{\frac{3}{2}-\frac{n}{2}}\left(-2^{\frac{n+7}{2}}n\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-5),\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-ax)\right)+2^{\frac{n+7}{2}}(n-1)\text{Hypergeometri}\right.}{5a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((1 - a*x)^(3/2 - n/2)*Sqrt[c - a^2*c*x^2]*(a^2*(-3 + n)*x^2*(1 + a*x)^((3
+ n)/2) - 2^((7 + n)/2)*n*Hypergeometric2F1[(-5 - n)/2, (3 - n)/2, (5 - n)/
2, (1 - a*x)/2] + 2^((7 + n)/2)*(-1 + n)*Hypergeometric2F1[(-3 - n)/2, (3 -
 n)/2, (5 - n)/2, (1 - a*x)/2] - 2^((3 + n)/2)*(-2 + n)*Hypergeometric2F1[(
-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2]))/(5*a^4*(3 - n)*Sqrt[1 - a^2
*x^2])
```

Maple [F] time = 0.203, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{Artanh}(ax)} x^3 \sqrt{-a^2 c x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^3*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^3*(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2 c x^2 + c}\, x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-a^2 c x^2 + c}\, x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**3*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2 + c}x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.1328 $\int e^{n\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=244

$$-\frac{2^{\frac{n-1}{2}}\left(n^2+3\right)\sqrt{c-a^2cx^2}(1-ax)^{\frac{3-n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-1),\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-ax)\right)}{3a^3(3-n)\sqrt{1-a^2x^2}}-\frac{n\sqrt{c-a^2cx^2}(ax+1)^{\frac{n+3}{2}}(1}{12a^3\sqrt{1-a^2x^2}}$$

```
[Out] -(n*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((3 + n)/2)*Sqrt[c - a^2*c*x^2])/(12*a^
3*Sqrt[1 - a^2*x^2]) - (x*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((3 + n)/2)*Sqrt[
c - a^2*c*x^2])/(4*a^2*Sqrt[1 - a^2*x^2]) - (2^((-1 + n)/2)*(3 + n^2)*(1 -
a*x)^((3 - n)/2)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[(-1 - n)/2, (3 - n)/
2, (5 - n)/2, (1 - a*x)/2])/(3*a^3*(3 - n)*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.312724, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6153, 6150, 90, 80, 69}

$$-\frac{2^{\frac{n-1}{2}}\left(n^2+3\right)\sqrt{c-a^2cx^2}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{3a^3(3-n)\sqrt{1-a^2x^2}}-\frac{n\sqrt{c-a^2cx^2}(ax+1)^{\frac{n+3}{2}}(1-ax)^{\frac{3-n}{2}}}{12a^3\sqrt{1-a^2x^2}}-\frac{x\sqrt{c-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(n*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((3 + n)/2)*Sqrt[c - a^2*c*x^2])/(12*a^
3*Sqrt[1 - a^2*x^2]) - (x*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((3 + n)/2)*Sqrt[
c - a^2*c*x^2])/(4*a^2*Sqrt[1 - a^2*x^2]) - (2^((-1 + n)/2)*(3 + n^2)*(1 -
a*x)^((3 - n)/2)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[(-1 - n)/2, (3 - n)/
2, (5 - n)/2, (1 - a*x)/2])/(3*a^3*(3 - n)*Sqrt[1 - a^2*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{n\tanh^{-1}(ax)}x^2\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int x^2(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{x(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{4a^2\sqrt{1-a^2x^2}} - \frac{\sqrt{c-a^2cx^2}\int(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}(-1-an}{4a^2\sqrt{1-a^2x^2}}$$
$$= -\frac{n(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{12a^3\sqrt{1-a^2x^2}} - \frac{x(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{4a^2\sqrt{1-a^2x^2}} + \frac{((3+n}{}$$
$$= -\frac{n(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{12a^3\sqrt{1-a^2x^2}} - \frac{x(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{4a^2\sqrt{1-a^2x^2}} - \frac{2^{\frac{1}{2}(-1+n}}{}$$

Mathematica [A] time = 0.191262, size = 135, normalized size = 0.55

$$\frac{\sqrt{c-a^2cx^2}(1-ax)^{\frac{3}{2}-\frac{n}{2}}\left(2^{\frac{n+3}{2}}\left(n^2+3\right)\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-1),\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-ax)\right)-(n-3)(ax+1)^{\frac{n+3}{2}}(3ax}{12a^3(n-3)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((1 - a*x)^(3/2 - n/2)*Sqrt[c - a^2*c*x^2]*(-((-3 + n)*(1 + a*x)^((3 + n)/2
)*(n + 3*a*x)) + 2^((3 + n)/2)*(3 + n^2)*Hypergeometric2F1[(-1 - n)/2, (3 -
 n)/2, (5 - n)/2, (1 - a*x)/2]))/(12*a^3*(-3 + n)*Sqrt[1 - a^2*x^2])
```

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}x^2\sqrt{-a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+cx^2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{-a^2cx^2+cx^2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{-c\,(ax-1)\,(ax+1)}e^{n\,\operatorname{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+cx^2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.1329 $\int e^{n\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=173

$$-\frac{2^{\frac{n+3}{2}}n\sqrt{c-a^2cx^2}(1-ax)^{\frac{3-n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-1),\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-ax)\right)}{3a^2(3-n)\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}(ax+1)^{\frac{n+3}{2}}(1-ax)^{\frac{3-n}{2}}}{3a^2\sqrt{1-a^2x^2}}$$

```
[Out] -((1 - a*x)^((3 - n)/2)*(1 + a*x)^((3 + n)/2)*Sqrt[c - a^2*c*x^2])/(3*a^2*S
qrt[1 - a^2*x^2]) - (2^((3 + n)/2)*n*(1 - a*x)^((3 - n)/2)*Sqrt[c - a^2*c*x
^2]*Hypergeometric2F1[(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(3*a^
2*(3 - n)*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.187153, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6153, 6150, 80, 69}

$$-\frac{2^{\frac{n+3}{2}}n\sqrt{c-a^2cx^2}(1-ax)^{\frac{3-n}{2}}{}_2F_1\left(\frac{1}{2}(-n-1),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{3a^2(3-n)\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}(ax+1)^{\frac{n+3}{2}}(1-ax)^{\frac{3-n}{2}}}{3a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((1 - a*x)^((3 - n)/2)*(1 + a*x)^((3 + n)/2)*Sqrt[c - a^2*c*x^2])/(3*a^2*S
qrt[1 - a^2*x^2]) - (2^((3 + n)/2)*n*(1 - a*x)^((3 - n)/2)*Sqrt[c - a^2*c*x
^2]*Hypergeometric2F1[(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(3*a^
2*(3 - n)*Sqrt[1 - a^2*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
a*d))]])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int e^{n\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{n\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{c-a^2cx^2}\int x(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{1-a^2x^2}}\\
&= -\frac{(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-a^2x^2}} + \frac{\left(n\sqrt{c-a^2cx^2}\right)\int(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}\,dx}{3a\sqrt{1-a^2x^2}}\\
&= -\frac{(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-a^2x^2}} - \frac{2^{\frac{3+n}{2}}n(1-ax)^{\frac{3-n}{2}}\sqrt{c-a^2cx^2}\,{}_2F_1\left(\frac{1}{2}(-1-n),\frac{3-n}{2};\frac{5-}{2}\right.}{3a^2(3-n)\sqrt{1-a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.12782, size = 125, normalized size = 0.72

$$\frac{\sqrt{c-a^2cx^2}(1-ax)^{\frac{3}{2}-\frac{n}{2}}\left(2^{\frac{n+3}{2}}n\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-1),\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-ax)\right)-(n-3)(ax+1)^{\frac{n+3}{2}}\right)}{3a^2(n-3)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((1 - a*x)^(3/2 - n/2)*Sqrt[c - a^2*c*x^2]*(-((-3 + n)*(1 + a*x)^((3 + n)/2
)) + 2^((3 + n)/2)*n*Hypergeometric2F1[(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1
 - a*x)/2]))/(3*a^2*(-3 + n)*Sqrt[1 - a^2*x^2])
```

Maple [F] time = 0.2, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}x\sqrt{-a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a^2cx^2+cx}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-a^2cx^2+cx}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{-c\,(ax-1)\,(ax+1)}e^{n\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a^2cx^2+cx}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.1330 $\int e^{n\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=104

$$-\frac{2^{\frac{n+3}{2}}\sqrt{c-a^2cx^2}(1-ax)^{\frac{3-n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-1),\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-ax)\right)}{a(3-n)\sqrt{1-a^2x^2}}$$

```
[Out] -((2^((3 + n)/2)*(1 - a*x)^((3 - n)/2)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F
1[(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(a*(3 - n)*Sqrt[1 - a^2*x
^2]))
```

Rubi [A] time = 0.096049, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 69}

$$-\frac{2^{\frac{n+3}{2}}\sqrt{c-a^2cx^2}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{a(3-n)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((2^((3 + n)/2)*(1 - a*x)^((3 - n)/2)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F
1[(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(a*(3 - n)*Sqrt[1 - a^2*x
^2]))
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{n\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int (1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{2^{\frac{3+n}{2}}(1-ax)^{\frac{3-n}{2}}\sqrt{c-a^2cx^2}\,{}_2F_1\left(\frac{1}{2}(-1-n),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{a(3-n)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0494407, size = 101, normalized size = 0.97

$$\frac{2^{\frac{n+3}{2}}\sqrt{c-a^2cx^2}(1-ax)^{\frac{3}{2}-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-1),\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-ax)\right)}{a(n-3)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2^((3 + n)/2)*(1 - a*x)^(3/2 - n/2)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[
(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(a*(-3 + n)*Sqrt[1 - a^2*x^
2])
```

Maple [F] time = 0.205, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}\sqrt{-a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c\,(ax-1)\,(ax+1)}e^{n\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.1331 $\int \frac{e^{n \tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=269

$$\frac{2^{\frac{n+1}{2}} n\sqrt{c-a^2cx^2}(1-ax)^{\frac{3-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-ax)\right)}{\left(n^2-4n+3\right)\sqrt{1-a^2x^2}}+\frac{2\sqrt{c-a^2cx^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\text{Hy}}{(1-n)\sqrt{}}$$

```
[Out] -(((1 - a*x)^((3 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[c - a^2*c*x^2])/((1 -
n)*Sqrt[1 - a^2*x^2])) + (2*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sq
rt[c - a^2*c*x^2]*Hypergeometric2F1[1, (-1 + n)/2, (1 + n)/2, (1 + a*x)/(1
- a*x)])/((1 - n)*Sqrt[1 - a^2*x^2]) + (2^((1 + n)/2)*n*(1 - a*x)^((3 - n)/
2)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[(1 - n)/2, (3 - n)/2, (5 - n)/2, (
1 - a*x)/2])/((3 - 4*n + n^2)*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.293015, antiderivative size = 299, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6153, 6150, 105, 69, 131}

$$\frac{2\sqrt{c-a^2cx^2}(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1}{2}(-n-1)}\,{}_2F_1\left(1,\frac{1}{2}(-n-1);\frac{1-n}{2};\frac{1-ax}{ax+1}\right)}{(n+1)\sqrt{1-a^2x^2}}-\frac{2^{\frac{n+3}{2}}\sqrt{c-a^2cx^2}(1-ax)^{\frac{1}{2}(-n-1)}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{1}{2}\right.}{(n+1)\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(E^(n*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] (2*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[c - a^2*c*x^2]*Hyperge
ometric2F1[1, (-1 - n)/2, (1 - n)/2, (1 - a*x)/(1 + a*x)])/((1 + n)*Sqrt[1
- a^2*x^2]) - (2^((3 + n)/2)*(1 - a*x)^((-1 - n)/2)*Sqrt[c - a^2*c*x^2]*Hyp
ergeometric2F1[(-1 - n)/2, (-1 - n)/2, (1 - n)/2, (1 - a*x)/2])/((1 + n)*Sq
rt[1 - a^2*x^2]) + (2^((3 + n)/2)*(1 - a*x)^((1 - n)/2)*Sqrt[c - a^2*c*x^2]
*Hypergeometric2F1[(-1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - a*x)/2])/((1 - n)
*Sqrt[1 - a^2*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
```

```
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{n\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}}{x}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}}{x}\,dx}{\sqrt{1-a^2x^2}} - \frac{\left(a\sqrt{c-a^2cx^2}\right)\int (1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{2^{\frac{3+n}{2}}(1-ax)^{\frac{1-n}{2}}\sqrt{c-a^2cx^2}\,{}_2F_1\left(\frac{1}{2}(-1-n),\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{(1-n)\sqrt{1-a^2x^2}} + \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(}{x}}{\sqrt{1-a^2x^2}}$$

$$= \frac{2(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1+n}{2}}\sqrt{c-a^2cx^2}\,{}_2F_1\left(1,\frac{1}{2}(-1-n);\frac{1-n}{2};\frac{1-ax}{1+ax}\right)}{(1+n)\sqrt{1-a^2x^2}} - \frac{2^{\frac{3+n}{2}}(1-ax)^{\frac{1}{2}(-1-n)}}{}$$

Mathematica [A] time = 0.156866, size = 207, normalized size = 0.77

$$\frac{2\sqrt{c-a^2cx^2}(1-ax)^{\frac{1}{2}(-n-1)}\left((n-1)(ax+1)^{\frac{n+1}{2}}\text{Hypergeometric2F1}\left(1,-\frac{n}{2}-\frac{1}{2},\frac{1}{2}-\frac{n}{2},\frac{1-ax}{ax+1}\right)+2^{\frac{n+1}{2}}\left((n+1)(ax-1)\text{Hyp}\right.\right.}{\left(n^2-1\right)\sqrt{1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] (2*(1 - a*x)^((-1 - n)/2)*Sqrt[c - a^2*c*x^2]*((-1 + n)*(1 + a*x)^((1 + n)/
2)*Hypergeometric2F1[1, -1/2 - n/2, 1/2 - n/2, (1 - a*x)/(1 + a*x)] + 2^((1
 + n)/2)*(-((-1 + n)*Hypergeometric2F1[-1/2 - n/2, -1/2 - n/2, 1/2 - n/2, 1
/2 - (a*x)/2]) + (1 + n)*(-1 + a*x)*Hypergeometric2F1[-1/2 - n/2, 1/2 - n/2
, 3/2 - n/2, 1/2 - (a*x)/2])))/((-1 + n^2)*Sqrt[1 - a^2*x^2])
```

Maple [F] time = 0.201, size = 0, normalized size = 0.

$$\int \frac{e^{n\,\mathrm{Artanh}(ax)}}{x}\sqrt{-a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}e^{n\,\mathrm{atanh}\,(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*atanh(a*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

3.1332 $\int \frac{e^{n\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=268

$$-\frac{2an\sqrt{c-a^2cx^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{1-ax}{ax+1}\right)}{(1-n)\sqrt{1-a^2x^2}}+\frac{a2^{\frac{n+1}{2}}\sqrt{c-a^2cx^2}(1-ax)^{\frac{1-n}{2}}\text{Hyper}}{(1-r}$$

```
[Out] -(((1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[c - a^2*c*x^2])/(x*Sqrt
[1 - a^2*x^2])) - (2*a*n*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[
c - a^2*c*x^2]*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (1 - a*x)/(1 + a*
x)])/((1 - n)*Sqrt[1 - a^2*x^2]) + (2^((1 + n)/2)*a*(1 - a*x)^((1 - n)/2)*S
qrt[c - a^2*c*x^2]*Hypergeometric2F1[(1 - n)/2, (1 - n)/2, (3 - n)/2, (1 -
a*x)/2])/((1 - n)*Sqrt[1 - a^2*x^2])
```

Rubi [C] time = 0.233537, antiderivative size = 97, normalized size of antiderivative = 0.36, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 136}

$$\frac{a2^{\frac{3}{2}-\frac{n}{2}}\sqrt{c-a^2cx^2}(ax+1)^{\frac{n+3}{2}}F_1\left(\frac{n+3}{2};\frac{n-1}{2},2;\frac{n+5}{2};\frac{1}{2}(ax+1),ax+1\right)}{(n+3)\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(E^(n*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] (2^(3/2 - n/2)*a*(1 + a*x)^((3 + n)/2)*Sqrt[c - a^2*c*x^2]*AppellF1[(3 + n)
/2, (-1 + n)/2, 2, (5 + n)/2, (1 + a*x)/2, 1 + a*x])/((3 + n)*Sqrt[1 - a^2*
x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{n\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{2^{\frac{3}{2}-\frac{n}{2}}a(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}F_1\left(\frac{3+n}{2};\frac{1}{2}(-1+n),2;\frac{5+n}{2};\frac{1}{2}(1+ax),1+ax\right)}{(3+n)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.423031, size = 138, normalized size = 0.51

$$-\frac{c\sqrt{1-a^2x^2}e^{n\tanh^{-1}(ax)}\left(2axe^{\tanh^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-e^{2\tanh^{-1}(ax)}\right)+2anxe^{\tanh^{-1}(ax)}\text{Hypergeomet}\right.}{(n+1)x\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] -((c*E^(n*ArcTanh[a*x])*Sqrt[1 - a^2*x^2]*((1 + n)*Sqrt[1 - a^2*x^2] + 2*a*
E^ArcTanh[a*x]*x*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -E^(2*ArcTanh[a
*x])] + 2*a*E^ArcTanh[a*x]*n*x*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E
^(2*ArcTanh[a*x])]))/((1 + n)*x*Sqrt[c - a^2*c*x^2]))
```

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int \frac{e^{n\mathrm{Artanh}(ax)}}{x^2}\sqrt{-a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c(ax-1)(ax+1)}e^{n\operatorname{atanh}(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*atanh(a*x))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

3.1333 $\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=105

$$-\frac{c2^{\frac{n+5}{2}}\sqrt{c-a^2cx^2}(1-ax)^{\frac{5-n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-3),\frac{5-n}{2},\frac{7-n}{2},\frac{1}{2}(1-ax)\right)}{a(5-n)\sqrt{1-a^2x^2}}$$

```
[Out] -((2^((5 + n)/2)*c*(1 - a*x)^((5 - n)/2)*Sqrt[c - a^2*c*x^2]*Hypergeometric
2F1[(-3 - n)/2, (5 - n)/2, (7 - n)/2, (1 - a*x)/2])/(a*(5 - n)*Sqrt[1 - a^2
*x^2]))
```

Rubi [A] time = 0.112971, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 69}

$$-\frac{c2^{\frac{n+5}{2}}\sqrt{c-a^2cx^2}(1-ax)^{\frac{5-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-3),\frac{5-n}{2};\frac{7-n}{2};\frac{1}{2}(1-ax)\right)}{a(5-n)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((2^((5 + n)/2)*c*(1 - a*x)^((5 - n)/2)*Sqrt[c - a^2*c*x^2]*Hypergeometric
2F1[(-3 - n)/2, (5 - n)/2, (7 - n)/2, (1 - a*x)/2])/(a*(5 - n)*Sqrt[1 - a^2
*x^2]))
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}dx=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int e^{n\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(1-ax)^{\frac{3}{2}-\frac{n}{2}}(1+ax)^{\frac{3}{2}+\frac{n}{2}}dx}{\sqrt{1-a^2x^2}}$$
$$=-\frac{2^{\frac{5+n}{2}}c(1-ax)^{\frac{5-n}{2}}\sqrt{c-a^2cx^2}\,{}_2F_1\left(\frac{1}{2}(-3-n),\frac{5-n}{2};\frac{7-n}{2};\frac{1}{2}(1-ax)\right)}{a(5-n)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0741913, size = 102, normalized size = 0.97

$$\frac{c2^{\frac{n+5}{2}}\sqrt{c-a^2cx^2}(1-ax)^{\frac{5}{2}-\frac{n}{2}}\text{Hypergeometric2F1}\left(-\frac{n}{2}-\frac{3}{2},\frac{5}{2}-\frac{n}{2},\frac{7}{2}-\frac{n}{2},\frac{1}{2}-\frac{ax}{2}\right)}{a(n-5)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (2^((5 + n)/2)*c*(1 - a*x)^(5/2 - n/2)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F
1[-3/2 - n/2, 5/2 - n/2, 7/2 - n/2, 1/2 - (a*x)/2])/(a*(-5 + n)*Sqrt[1 - a^
2*x^2])
```

Maple [F] time = 0.21, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}\left(-a^2cx^2+c\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2cx^2-c\right)\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.1334 $\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=275

$$-\frac{2^{\frac{n-1}{2}}n\left(n^2+5\right)\sqrt{1-a^2x^2}(1-ax)^{\frac{3-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-ax)\right)}{3a^4(1-n)(3-n)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}(ax+1)^{\frac{n+1}{2}}\left(a(1-\right.}{6a^4(1-n)}$$

```
[Out] -(x^2*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(3*a^2
*Sqrt[c - a^2*c*x^2]) - ((1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*(4 + n
 + n^2 + a*(1 - n)*n*x)*Sqrt[1 - a^2*x^2])/(6*a^4*(1 - n)*Sqrt[c - a^2*c*x^
2]) - (2^((-1 + n)/2)*n*(5 + n^2)*(1 - a*x)^((3 - n)/2)*Sqrt[1 - a^2*x^2]*H
ypergeometric2F1[(1 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(3*a^4*(1 -
 n)*(3 - n)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.380919, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6153, 6150, 100, 146, 69}

$$-\frac{2^{\frac{n-1}{2}}n\left(n^2+5\right)\sqrt{1-a^2x^2}(1-ax)^{\frac{3-n}{2}}{}_2F_1\left(\frac{1-n}{2},\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{3a^4(1-n)(3-n)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}(ax+1)^{\frac{n+1}{2}}\left(a(1-n)nx+n^2+n+4\right)}{6a^4(1-n)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^3)/Sqrt[c - a^2*c*x^2],x]
```


```
[Out] -(x^2*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(3*a^2
*Sqrt[c - a^2*c*x^2]) - ((1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*(4 + n
 + n^2 + a*(1 - n)*n*x)*Sqrt[1 - a^2*x^2])/(6*a^4*(1 - n)*Sqrt[c - a^2*c*x^
2]) - (2^((-1 + n)/2)*n*(5 + n^2)*(1 - a*x)^((3 - n)/2)*Sqrt[1 - a^2*x^2]*H
ypergeometric2F1[(1 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(3*a^4*(1 -
 n)*(3 - n)*Sqrt[c - a^2*c*x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
```

```
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int x^3(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{c-a^2cx^2}}\\
&= -\frac{x^2(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{3a^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\int x(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}(-2-anx)\,dx}{3a^2\sqrt{c-a^2cx^2}}\\
&= -\frac{x^2(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{3a^2\sqrt{c-a^2cx^2}}-\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\left(4+n+n^2+a(1-n)nx\right)\sqrt{1-a^2x^2}}{6a^4(1-n)\sqrt{c-a^2cx^2}}\\
&= -\frac{x^2(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{3a^2\sqrt{c-a^2cx^2}}-\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\left(4+n+n^2+a(1-n)nx\right)\sqrt{1-a^2x^2}}{6a^4(1-n)\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.261604, size = 187, normalized size = 0.68

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}-\frac{n}{2}}\left(2^{\frac{n}{2}+1}n\left(n^2+5\right)(ax-1)\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},\frac{5}{2}-\frac{n}{2},\frac{1}{2}-\frac{ax}{2}\right)-\sqrt{2}(n-3)(ax+1)^{\frac{n+}{2}}\right.}{6\sqrt{2}a^4(n-3)(n-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^3)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((1 - a*x)^(1/2 - n/2)*Sqrt[1 - a^2*x^2]*(-(Sqrt[2]*(-3 + n)*(1 + a*x)^((1
+ n)/2)*(n^2*(-1 + a*x) - 2*(2 + a^2*x^2) + n*(-1 - a*x + 2*a^2*x^2))) + 2^
(1 + n/2)*n*(5 + n^2)*(-1 + a*x)*Hypergeometric2F1[1/2 - n/2, 3/2 - n/2, 5/
2 - n/2, 1/2 - (a*x)/2]))/(6*Sqrt[2]*a^4*(-3 + n)*(-1 + n)*Sqrt[c - a^2*c*x
```

```
^2])
```

Maple [F] time = 0.209, size = 0, normalized size = 0.

$$\int e^{n \operatorname{Artanh}(ax)} x^{3} \frac{1}{\sqrt{-a^{2} c x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\, n}}{\sqrt{-a^{2} c x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^{2} c x^{2}+c}\, x^{3} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\, n}}{a^{2} c x^{2}-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2
 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3} e^{n \operatorname{atanh}{\left(ax \right)}}}{\sqrt{-c \left(ax-1\right) \left(ax+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**3/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*exp(n*atanh(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

3.1335 $\int \frac{e^{n\tanh^{-1}(ax)}x^2}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=253

$$-\frac{2^{\frac{n+1}{2}}\left(n^2+1\right)\sqrt{1-a^2x^2}(1-ax)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-1),\frac{1-n}{2},\frac{3-n}{2},\frac{1}{2}(1-ax)\right)}{a^3\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}(ax+1)^{\frac{n+1}{2}}(1}{2a^2\sqrt{c-a^2cx^2}}$$

```
[Out] ((1 - n)*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(2*
a^3*(1 + n)*Sqrt[c - a^2*c*x^2]) - (x*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 +
 n)/2)*Sqrt[1 - a^2*x^2])/(2*a^2*Sqrt[c - a^2*c*x^2]) - (2^((1 + n)/2)*(1 +
 n^2)*(1 - a*x)^((1 - n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[(-1 - n)/2,
 (1 - n)/2, (3 - n)/2, (1 - a*x)/2])/(a^3*(1 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.329219, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6153, 6150, 90, 79, 69}

$$-\frac{2^{\frac{n+1}{2}}\left(n^2+1\right)\sqrt{1-a^2x^2}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a^3\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{2a^2\sqrt{c-a^2cx^2}}+\frac{(1-n}{}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((1 - n)*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(2*
a^3*(1 + n)*Sqrt[c - a^2*c*x^2]) - (x*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 +
 n)/2)*Sqrt[1 - a^2*x^2])/(2*a^2*Sqrt[c - a^2*c*x^2]) - (2^((1 + n)/2)*(1 +
 n^2)*(1 - a*x)^((1 - n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[(-1 - n)/2,
 (1 - n)/2, (3 - n)/2, (1 - a*x)/2])/(a^3*(1 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^2}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}x^2}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int x^2(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= -\frac{x(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2a^2\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int (1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}(-1-anx)\,dx}{2a^2\sqrt{c-a^2cx^2}}$$

$$= \frac{(1-n)(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2a^3(1+n)\sqrt{c-a^2cx^2}} - \frac{x(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2a^2\sqrt{c-a^2cx^2}} + \frac{\left((1+n^2)\sqrt{1-a^2x\right.}{2a^2(}$$

$$= \frac{(1-n)(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2a^3(1+n)\sqrt{c-a^2cx^2}} - \frac{x(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2a^2\sqrt{c-a^2cx^2}} - \frac{2^{\frac{1+n}{2}}(1+n^2)(1-a}{}$$

Mathematica [A] time = 0.164513, size = 141, normalized size = 0.56

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}-\frac{n}{2}}\left(2^{\frac{n+3}{2}}\left(n^2+1\right)\text{Hypergeometric2F1}\left(-\frac{n}{2}-\frac{1}{2},\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},\frac{1}{2}-\frac{ax}{2}\right)-(n-1)(ax+1)^{\frac{n+1}{2}}(anx+ax\right.}{2a^3\left(n^2-1\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((1 - a*x)^(1/2 - n/2)*Sqrt[1 - a^2*x^2]*(-((-1 + n)*(1 + a*x)^((1 + n)/2)*
(-1 + n + a*x + a*n*x)) + 2^((3 + n)/2)*(1 + n^2)*Hypergeometric2F1[-1/2 -
n/2, 1/2 - n/2, 3/2 - n/2, 1/2 - (a*x)/2]))/(2*a^3*(-1 + n^2)*Sqrt[c - a^2*
c*x^2])
```

Maple [F] time = 0.213, size = 0, normalized size = 0.

$$\int e^{n\mathrm{Artanh}(ax)}x^2\frac{1}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2
 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 e^{n \operatorname{atanh}(ax)}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**2/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*exp(n*atanh(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

3.1336 $\int \frac{e^{n\tanh^{-1}(ax)}x}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=176

$$-\frac{2^{\frac{n+3}{2}}n\sqrt{1-a^2x^2}(1-ax)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-1),\frac{1-n}{2},\frac{3-n}{2},\frac{1}{2}(1-ax)\right)}{a^2\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{a^2(n+1)\sqrt{c-a^2cx^2}}$$

```
[Out] -(((1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(a^2*(1 +
 n)*Sqrt[c - a^2*c*x^2])) - (2^((3 + n)/2)*n*(1 - a*x)^((1 - n)/2)*Sqrt[1 -
 a^2*x^2]*Hypergeometric2F1[(-1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - a*x)/2])
/(a^2*(1 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.201562, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6153, 6150, 79, 69}

$$-\frac{2^{\frac{n+3}{2}}n\sqrt{1-a^2x^2}(1-ax)^{\frac{1-n}{2}}{}_2F_1\left(\frac{1}{2}(-n-1),\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a^2\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{a^2(n+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(((1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(a^2*(1 +
 n)*Sqrt[c - a^2*c*x^2])) - (2^((3 + n)/2)*n*(1 - a*x)^((1 - n)/2)*Sqrt[1 -
 a^2*x^2]*Hypergeometric2F1[(-1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - a*x)/2])
/(a^2*(1 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\tanh^{-1}(ax)}x}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int x(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{c-a^2cx^2}}\\
&= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{a^2(1+n)\sqrt{c-a^2cx^2}} + \frac{\left(n\sqrt{1-a^2x^2}\right)\int (1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1+n}{2}}\,dx}{a(1+n)\sqrt{c-a^2cx^2}}\\
&= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{a^2(1+n)\sqrt{c-a^2cx^2}} - \frac{2^{\frac{3+n}{2}}n(1-ax)^{\frac{1-n}{2}}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1}{2}(-1-n),\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a^2\left(1-n^2\right)\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0927741, size = 124, normalized size = 0.7

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}-\frac{n}{2}}\left(2^{\frac{n+3}{2}}n\text{Hypergeometric2F1}\left(-\frac{n}{2}-\frac{1}{2},\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},\frac{1}{2}-\frac{ax}{2}\right)-(n-1)(ax+1)^{\frac{n+1}{2}}\right)}{a^2\left(n^2-1\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((1 - a*x)^(1/2 - n/2)*Sqrt[1 - a^2*x^2]*(-((-1 + n)*(1 + a*x)^((1 + n)/2))
 + 2^((3 + n)/2)*n*Hypergeometric2F1[-1/2 - n/2, 1/2 - n/2, 3/2 - n/2, 1/2
- (a*x)/2]))/(a^2*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

Maple [F] time = 0.211, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}x\frac{1}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 -
 c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^{n\,\text{atanh}\,(ax)}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*exp(n*atanh(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

3.1337 $\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=104

$$-\frac{2^{\frac{n+1}{2}}\sqrt{1-a^2x^2}(1-ax)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{1}{2}(1-ax)\right)}{a(1-n)\sqrt{c-a^2cx^2}}$$

```
[Out] -((2^((1 + n)/2)*(1 - a*x)^((1 - n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[
(1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - a*x)/2])/(a*(1 - n)*Sqrt[c - a^2*c*x^
2]))
```

Rubi [A] time = 0.105508, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 69}

$$-\frac{2^{\frac{n+1}{2}}\sqrt{1-a^2x^2}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a(1-n)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((2^((1 + n)/2)*(1 - a*x)^((1 - n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[
(1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - a*x)/2])/(a*(1 - n)*Sqrt[c - a^2*c*x^
2]))
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int (1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{2^{\frac{1+n}{2}}(1-ax)^{\frac{1-n}{2}}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a(1-n)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0352407, size = 101, normalized size = 0.97

$$\frac{2^{\frac{n+1}{2}}\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{n}{2},\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},\frac{1}{2}-\frac{ax}{2}\right)}{a(n-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2^((1 + n)/2)*(1 - a*x)^(1/2 - n/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1/
2 - n/2, 1/2 - n/2, 3/2 - n/2, 1/2 - (a*x)/2])/(a*(-1 + n)*Sqrt[c - a^2*c*x
^2])
```

Maple [F] time = 0.203, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}\frac{1}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{atanh}(ax)}}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

3.1338 $\int \frac{e^{n\tanh^{-1}(ax)}}{x\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=101

$$-\frac{2\sqrt{1-a^2x^2}(1-ax)^{\frac{1-n}{2}}(ax+1)^{\frac{n-1}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{1-ax}{ax+1}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

```
[Out] (-2*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2]*Hypergeo
metric2F1[1, (1 - n)/2, (3 - n)/2, (1 - a*x)/(1 + a*x)])/((1 - n)*Sqrt[c -
a^2*c*x^2])
```

Rubi [A] time = 0.230984, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 131}

$$-\frac{2\sqrt{1-a^2x^2}(1-ax)^{\frac{1-n}{2}}(ax+1)^{\frac{n-1}{2}}\,_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{1-ax}{ax+1}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(x*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (-2*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2]*Hypergeo
metric2F1[1, (1 - n)/2, (3 - n)/2, (1 - a*x)/(1 + a*x)])/((1 - n)*Sqrt[c -
a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}}{x}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= -\frac{2(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{1-ax}{1+ax}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0482765, size = 99, normalized size = 0.98

$$\frac{2\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}-\frac{n}{2}}(ax+1)^{\frac{n-1}{2}}\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},\frac{1-ax}{ax+1}\right)}{(n-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*(1 - a*x)^(1/2 - n/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2]*Hypergeom
etric2F1[1, 1/2 - n/2, 3/2 - n/2, (1 - a*x)/(1 + a*x)])/((-1 + n)*Sqrt[c -
a^2*c*x^2])
```

Maple [F] time = 0.205, size = 0, normalized size = 0.

$$\int \frac{e^{n\text{Artanh}(ax)}}{x}\frac{1}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(sqrt(-a^2*c*x^2 + c)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^3-cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^3 - c
*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{atanh}(ax)}}{x\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/(x*sqrt(-c*(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(sqrt(-a^2*c*x^2 + c)*x), x)
```

3.1339 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=167

$$-\frac{2an\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{1-ax}{ax+1}\right)}{(1-n)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{x\sqrt{c-a^2cx^2}}$$

```
[Out] -(((1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(x*Sqrt[c
 - a^2*c*x^2])) - (2*a*n*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[
1 - a^2*x^2]*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (1 - a*x)/(1 + a*x)
])/((1 - n)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.249247, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6153, 6150, 96, 131}

$$-\frac{2an\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{1-ax}{ax+1}\right)}{(1-n)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{x\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(x^2*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] -(((1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(x*Sqrt[c
 - a^2*c*x^2])) - (2*a*n*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[
1 - a^2*x^2]*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (1 - a*x)/(1 + a*x)
])/((1 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}}{x^2}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}} + \frac{\left(an\sqrt{1-a^2x^2}\right)\int \frac{(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}}{x}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}} - \frac{2an(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{1-}{1-}\right.}{(1-n)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0677058, size = 117, normalized size = 0.7

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}-\frac{n}{2}}(ax+1)^{\frac{n-1}{2}}\left(2anx\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},\frac{1-ax}{ax+1}\right)-(n-1)(ax+1)\right)}{(n-1)x\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x^2*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] ((1 - a*x)^(1/2 - n/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2]*(-((-1 + n)
*(1 + a*x)) + 2*a*n*x*Hypergeometric2F1[1, 1/2 - n/2, 3/2 - n/2, (1 - a*x)/
(1 + a*x)]))/((-1 + n)*x*Sqrt[c - a^2*c*x^2])
```

Maple [F] time = 0.202, size = 0, normalized size = 0.

$$\int \frac{e^{n\text{Artanh}(ax)}}{x^2}\frac{1}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(sqrt(-a^2*c*x^2 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^4-cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^4 - c
*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n\,\text{atanh}\,(ax)}}{x^2\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**2/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(sqrt(-a^2*c*x^2 + c)*x^2), x)
```

3.1340 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^3\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=242

$$-\frac{a^2(n^2+1)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{1-ax}{ax+1}\right)}{(1-n)\sqrt{c-a^2cx^2}}-\frac{an\sqrt{1-a^2x^2}(ax+1)^{\frac{n+1}{2}}(1-}{2x\sqrt{c-a^2cx^2}}$$

```
[Out] -((1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(2*x^2*Sqr
t[c - a^2*c*x^2]) - (a*n*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1
 - a^2*x^2])/(2*x*Sqrt[c - a^2*c*x^2]) - (a^2*(1 + n^2)*(1 - a*x)^((1 - n)/
2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (1 - n)/2,
 (3 - n)/2, (1 - a*x)/(1 + a*x)])/((1 - n)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.273594, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6153, 6150, 129, 151, 12, 131}

$$-\frac{a^2(n^2+1)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{1-ax}{ax+1}\right)}{(1-n)\sqrt{c-a^2cx^2}}-\frac{an\sqrt{1-a^2x^2}(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{2x\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(x^3*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] -((1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(2*x^2*Sqr
t[c - a^2*c*x^2]) - (a*n*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1
 - a^2*x^2])/(2*x*Sqrt[c - a^2*c*x^2]) - (a^2*(1 + n^2)*(1 - a*x)^((1 - n)/
2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (1 - n)/2,
 (3 - n)/2, (1 - a*x)/(1 + a*x)])/((1 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
```

```
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^3\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x^3\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}}{x^3}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}\left(-an-a^2x\right)}{x^2}\,dx}{2\sqrt{c-a^2cx^2}}$$

$$= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}} - \frac{an(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2x\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-a^2x^2}\int \frac{a^2\left(1+n^2\right)(1-}{}}{2\sqrt{c-a}}$$

$$= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}} - \frac{an(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2x\sqrt{c-a^2cx^2}} + \frac{\left(a^2\left(1+n^2\right)\sqrt{1-a^2x^2}\right.}{2\sqrt{c}}$$

$$= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}} - \frac{an(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2x\sqrt{c-a^2cx^2}} - \frac{a^2\left(1+n^2\right)(1-ax)^{\frac{1-n}{2}}(}{}$$

Mathematica [A] time = 0.0842499, size = 134, normalized size = 0.55

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}-\frac{n}{2}}(ax+1)^{\frac{n-1}{2}}\left(2a^2\left(n^2+1\right)x^2\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},\frac{1-ax}{ax+1}\right)-(n-1)(ax+1)(anx+1)\right.}{2(n-1)x^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x^3*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] ((1 - a*x)^(1/2 - n/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2]*(-((-1 + n)
*(1 + a*x)*(1 + a*n*x)) + 2*a^2*(1 + n^2)*x^2*Hypergeometric2F1[1, 1/2 - n/
2, 3/2 - n/2, (1 - a*x)/(1 + a*x)]))/(2*(-1 + n)*x^2*Sqrt[c - a^2*c*x^2])
```

Maple [F] time = 0.208, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{n\operatorname{Artanh}(ax)}}{x^3}\frac{1}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(sqrt(-a^2*c*x^2 + c)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^5-cx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^5 - c
*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n\operatorname{atanh}(ax)}}{x^3\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**3/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(sqrt(-a^2*c*x^2 + c)*x^3), x)
```

3.1341 $\int \frac{e^{n\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{2^{\frac{n-1}{2}}n\sqrt{1-a^2x^2}(1-ax)^{\frac{3-n}{2}}\text{Hypergeometric2F1}\left(\frac{3-n}{2},\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-ax)\right)}{a^4c(3-n)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}\left(-a(2n+3)nx+\right.}{a^4c\left(1-n^2\right)\sqrt{c}}$$

```
[Out] -((x^2*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(a^
2*c*Sqrt[c - a^2*c*x^2])) + ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*
(2 + 2*n + n^2 - a*n*(3 + 2*n)*x)*Sqrt[1 - a^2*x^2])/(a^4*c*(1 - n^2)*Sqrt[
c - a^2*c*x^2]) - (2^((-1 + n)/2)*n*(1 - a*x)^((3 - n)/2)*Sqrt[1 - a^2*x^2]
*Hypergeometric2F1[(3 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(a^4*c*(3
 - n)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.363132, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6153, 6150, 100, 145, 69}

$$-\frac{2^{\frac{n-1}{2}}n\sqrt{1-a^2x^2}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{3-n}{2},\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{a^4c(3-n)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}\left(-a(2n+3)nx+n^2+2n+2\right)(1-}{a^4c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((x^2*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(a^
2*c*Sqrt[c - a^2*c*x^2])) + ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*
(2 + 2*n + n^2 - a*n*(3 + 2*n)*x)*Sqrt[1 - a^2*x^2])/(a^4*c*(1 - n^2)*Sqrt[
c - a^2*c*x^2]) - (2^((-1 + n)/2)*n*(1 - a*x)^((3 - n)/2)*Sqrt[1 - a^2*x^2]
*Hypergeometric2F1[(3 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(a^4*c*(3
 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
```

```
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int x^3(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= -\frac{x^2(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{a^2c\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int x(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}(-2-anx)\,d}{a^2c\sqrt{c-a^2cx^2}}$$

$$= -\frac{x^2(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{a^2c\sqrt{c-a^2cx^2}} + \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\left(2+2n+n^2-an(3-\right.}{a^4c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

$$= -\frac{x^2(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{a^2c\sqrt{c-a^2cx^2}} + \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\left(2+2n+n^2-an(3-\right.}{a^4c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.2145, size = 186, normalized size = 0.69

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{-\frac{n}{2}-\frac{1}{2}}\left(\frac{a^2 2^{\frac{n+3}{2}}n(ax-1)^2\text{Hypergeometric2F1}\left(\frac{3}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2};\frac{5}{2}-\frac{n}{2};\frac{1}{2}-\frac{ax}{2}\right)}{n-3}+\frac{4a^2\left(n^2(2ax-1)+n(3ax-2)-2\right)(ax+1)^{\frac{n-1}{2}}}{n^2-1}-4a^4x^2(ax+\right.}{4a^6c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - a*x)^(-1/2 - n/2)*Sqrt[1 - a^2*x^2]*(-4*a^4*x^2*(1 + a*x)^((-1 + n)/2
) + (4*a^2*(1 + a*x)^((-1 + n)/2)*(-2 + n^2*(-1 + 2*a*x) + n*(-2 + 3*a*x)))
/(-1 + n^2) + (2^((3 + n)/2)*a^2*n*(-1 + a*x)^2*Hypergeometric2F1[3/2 - n/2
, 3/2 - n/2, 5/2 - n/2, 1/2 - (a*x)/2])/(-3 + n)))/(4*a^6*c*Sqrt[c - a^2*c*
x^2])
```

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int e^{n\operatorname{Artanh}(ax)} x^3 \left(-a^2 c x^2 + c\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2 c x^2 + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^2 c x^2 + c}\, x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4 c^2 x^4 - 2\, a^2 c^2 x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^
4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.1342 $\int \frac{e^{n\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=153

$$\frac{2^{\frac{n+1}{2}}\sqrt{1-a^2x^2}(1-ax)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{1}{2}(1-ax)\right)}{a^3c(1-n)\sqrt{c-a^2cx^2}}-\frac{(n-ax)e^{n\tanh^{-1}(ax)}}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

```
[Out] -((E^(n*ArcTanh[a*x])*(n - a*x))/(a^3*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])) + (
2^((1 + n)/2)*(1 - a*x)^((1 - n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[(1
- n)/2, (1 - n)/2, (3 - n)/2, (1 - a*x)/2])/(a^3*c*(1 - n)*Sqrt[c - a^2*c*x
^2])
```

Rubi [A] time = 0.246333, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6147, 6143, 6140, 69}

$$\frac{2^{\frac{n+1}{2}}\sqrt{1-a^2x^2}(1-ax)^{\frac{1-n}{2}}{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a^3c(1-n)\sqrt{c-a^2cx^2}}-\frac{(n-ax)e^{n\tanh^{-1}(ax)}}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((E^(n*ArcTanh[a*x])*(n - a*x))/(a^3*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])) + (
2^((1 + n)/2)*(1 - a*x)^((1 - n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[(1
- n)/2, (1 - n)/2, (3 - n)/2, (1 - a*x)/2])/(a^3*c*(1 - n)*Sqrt[c - a^2*c*x
^2])
```

Rule 6147

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> -Simp[((n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a
*d*(n^2 - 4*(p + 1)^2)), x] + Dist[(n^2 + 2*(p + 1))/(d*(n^2 - 4*(p + 1)^2)
), Int[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}
, x] && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p
+ 1)^2, 0] && IntegerQ[2*p]
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= -\frac{e^{n\tanh^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx}{a^2c}\\
&= -\frac{e^{n\tanh^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{a^2c\sqrt{c-a^2cx^2}}\\
&= -\frac{e^{n\tanh^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\int (1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}\,dx}{a^2c\sqrt{c-a^2cx^2}}\\
&= -\frac{e^{n\tanh^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}+\frac{2^{\frac{1+n}{2}}(1-ax)^{\frac{1-n}{2}}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a^3c(1-n)\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.217666, size = 155, normalized size = 1.01

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{-\frac{n}{2}-\frac{1}{2}}\left(2^{\frac{n+1}{2}}(n+1)(ax-1)\sqrt{ax+1}\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{n}{2},\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},\frac{1}{2}-\frac{ax}{2}\right)+(n-ax)(ax+1\right.}{a^3c(n-1)(n+1)\sqrt{ax+1}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - a*x)^(-1/2 - n/2)*Sqrt[1 - a^2*x^2]*((n - a*x)*(1 + a*x)^(n/2) + 2^((
1 + n)/2)*(1 + n)*(-1 + a*x)*Sqrt[1 + a*x]*Hypergeometric2F1[1/2 - n/2, 1/2
 - n/2, 3/2 - n/2, 1/2 - (a*x)/2]))/(a^3*c*(-1 + n)*(1 + n)*Sqrt[1 + a*x]*S
qrt[c - a^2*c*x^2])
```

Maple [F] time = 0.203, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}x^2\left(-a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^4-2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^
4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2e^{n\,\text{atanh}\,(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*exp(n*atanh(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.1343 $\int \frac{e^{n\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{(1-anx)e^{n\tanh^{-1}(ax)}}{a^2c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

```
[Out] (E^(n*ArcTanh[a*x])*(1 - a*n*x))/(a^2*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.0932925, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {6144}

$$\frac{(1-anx)e^{n\tanh^{-1}(ax)}}{a^2c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^(n*ArcTanh[a*x])*(1 - a*n*x))/(a^2*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6144

```
Int[(E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[((1 - a*n*x)*E^(n*ArcTanh[a*x]))/(d*(n^2 - 1)*Sqrt[c + d*x^2]),
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{3/2}}\,dx=\frac{e^{n\tanh^{-1}(ax)}(1-anx)}{a^2c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0576016, size = 81, normalized size = 1.76

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{-\frac{n}{2}-\frac{1}{2}}(ax+1)^{\frac{n-1}{2}}(anx-1)}{a^2c(n-1)(n+1)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - a*x)^(-1/2 - n/2)*(1 + a*x)^((-1 + n)/2)*(-1 + a*n*x)*Sqrt[1 - a^2*x^
2])/(a^2*c*(-1 + n)*(1 + n)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.029, size = 49, normalized size = 1.1

$$-\frac{(nax-1)(ax-1)(ax+1)\,\mathrm{e}^{n\,\mathrm{Artanh}(ax)}}{a^2(n^2-1)}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -(a*x-1)*(a*x+1)*(a*n*x-1)*exp(n*arctanh(a*x))/a^2/(n^2-1)/(-a^2*c*x^2+c)^(
3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A] time = 2.25363, size = 159, normalized size = 3.46

$$\frac{\sqrt{-a^2cx^2+c}(anx-1)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2c^2n^2-a^2c^2-\left(a^4c^2n^2-a^4c^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(-a^2*c*x^2 + c)*(a*n*x - 1)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*n^2
 - a^2*c^2 - (a^4*c^2*n^2 - a^4*c^2)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^{n\,\mathrm{atanh}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x*exp(n*atanh(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.1344 $\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{(n-ax)e^{n\tanh^{-1}(ax)}}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

```
[Out] -((E^(n*ArcTanh[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))
```

Rubi [A] time = 0.0526932, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6135}

$$-\frac{(n-ax)e^{n\tanh^{-1}(ax)}}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((E^(n*ArcTanh[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{e^{n\tanh^{-1}(ax)}(n-ax)}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0533181, size = 81, normalized size = 1.76

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}(-n-1)}(n-ax)(ax+1)^{\frac{n-1}{2}}}{ac(n-1)(n+1)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - a*x)^((-1 - n)/2)*(n - a*x)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])
/(a*c*(-1 + n)*(1 + n)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.027, size = 49, normalized size = 1.1

$$\frac{(ax-n)\,(ax-1)\,(ax+1)\,\mathrm{e}^{n\mathrm{Artanh}(ax)}}{\left(n^{2}-1\right)a}\left(-a^{2}cx^{2}+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(a*x-n)*exp(n*arctanh(a*x))/(n^2-1)/a/(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A] time = 2.22744, size = 153, normalized size = 3.33

$$-\frac{\sqrt{-a^{2}cx^{2}+c}(ax-n)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^{2}n^{2}-ac^{2}-\left(a^{3}c^{2}n^{2}-a^{3}c^{2}\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*(a*x - n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n^2 -
a*c^2 - (a^3*c^2*n^2 - a^3*c^2)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n\,\mathrm{atanh}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.1345 $\int \frac{e^{n\tanh^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=243

$$\frac{2\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-1}{2},\frac{n+1}{2},\frac{ax+1}{1-ax}\right)}{c(1-n)\sqrt{c-a^2cx^2}}-\frac{(n+2)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}}{c\left(1-n^2\right)\sqrt{c-a^2cx^2}}+$$

```
[Out] ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(c*(1 + n
)*Sqrt[c - a^2*c*x^2]) - ((2 + n)*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)
/2)*Sqrt[1 - a^2*x^2])/(c*(1 - n^2)*Sqrt[c - a^2*c*x^2]) + (2*(1 - a*x)^((1
 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (-1
+ n)/2, (1 + n)/2, (1 + a*x)/(1 - a*x)])/(c*(1 - n)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.324249, antiderivative size = 247, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6153, 6150, 129, 155, 12, 131}

$$-\frac{2\sqrt{1-a^2x^2}(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(1,\frac{3-n}{2};\frac{5-n}{2};\frac{1-ax}{ax+1}\right)}{c(3-n)\sqrt{c-a^2cx^2}}-\frac{(n+2)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}}{c\left(1-n^2\right)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}(ax+1)}{c(n+1)\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(c*(1 + n
)*Sqrt[c - a^2*c*x^2]) - ((2 + n)*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)
/2)*Sqrt[1 - a^2*x^2])/(c*(1 - n^2)*Sqrt[c - a^2*c*x^2]) - (2*(1 - a*x)^((3
 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (3 -
 n)/2, (5 - n)/2, (1 - a*x)/(1 + a*x)])/(c*(3 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
```

```
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}}{x}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}\left(-a(1+n)-a^2x\right)}{x}\,dx}{ac(1+n)\sqrt{c-a^2cx^2}}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{(2+n)(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c\left(1-n^2\right)\sqrt{c-a^2cx^2}} + \frac{\sqrt{1 \cdots}}{\cdots}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{(2+n)(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c\left(1-n^2\right)\sqrt{c-a^2cx^2}} + \frac{\sqrt{1 \cdots}}{\cdots}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{(2+n)(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c\left(1-n^2\right)\sqrt{c-a^2cx^2}} - \frac{2(1 \cdots}{\cdots}$$

Mathematica [A] time = 0.144052, size = 149, normalized size = 0.61

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}(-n-1)}(ax+1)^{\frac{n-3}{2}}\left(2\left(n^2-1\right)(ax-1)^2\text{Hypergeometric2F1}\left(1,\frac{3}{2}-\frac{n}{2},\frac{5}{2}-\frac{n}{2},\frac{1-ax}{ax+1}\right)-(n-3)(ax+1 \cdots\right.}{c(n-3)(n-1)(n+1)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(-((-3 + n
)*(1 + a*x)*(-1 + 2*a*x + n*(-2 + a*x))) + 2*(-1 + n^2)*(-1 + a*x)^2*Hyperg
eometric2F1[1, 3/2 - n/2, 5/2 - n/2, (1 - a*x)/(1 + a*x)]))/(c*(-3 + n)*(-1
 + n)*(1 + n)*Sqrt[c - a^2*c*x^2])
```

Maple [F] time = 0.209, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{n\operatorname{Artanh}(ax)}}{x}\left(-a^{2}c x^{2}+c\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^{2}cx^{2}+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^{4}c^{2}x^{5}-2\,a^{2}c^{2}x^{3}+c^{2}x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^5 -
2*a^2*c^2*x^3 + c^2*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x), x)
```

3.1346 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=321

$$\frac{2an\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-1}{2},\frac{n+1}{2},\frac{ax+1}{1-ax}\right)}{c(1-n)\sqrt{c-a^2cx^2}}-\frac{a\left(n^2+2n+2\right)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-}{c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

```
[Out] (a*(2 + n)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])
/(c*(1 + n)*Sqrt[c - a^2*c*x^2]) - ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 +
 n)/2)*Sqrt[1 - a^2*x^2])/(c*x*Sqrt[c - a^2*c*x^2]) - (a*(2 + 2*n + n^2)*(1
 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(c*(1 - n^2)*
Sqrt[c - a^2*c*x^2]) + (2*a*n*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)/2)*
Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (-1 + n)/2, (1 + n)/2, (1 + a*x)/(1
- a*x)])/(c*(1 - n)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.351142, antiderivative size = 325, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6153, 6150, 129, 155, 12, 131}

$$-\frac{2an\sqrt{1-a^2x^2}(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(1,\frac{3-n}{2};\frac{5-n}{2};\frac{1-ax}{ax+1}\right)}{c(3-n)\sqrt{c-a^2cx^2}}-\frac{a\left(n^2+2n+2\right)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}}{c\left(1-n^2\right)\sqrt{c-a^2cx^2}}+\frac{a(n+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a*(2 + n)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])
/(c*(1 + n)*Sqrt[c - a^2*c*x^2]) - ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 +
 n)/2)*Sqrt[1 - a^2*x^2])/(c*x*Sqrt[c - a^2*c*x^2]) - (a*(2 + 2*n + n^2)*(1
 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(c*(1 - n^2)*
Sqrt[c - a^2*c*x^2]) - (2*a*n*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((-3 + n)/2)*
Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (3 - n)/2, (5 - n)/2, (1 - a*x)/(1 +
 a*x)])/(c*(3 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\left(1-a^2x^2\right)^{3/2}}dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}}{x^2}dx}{c\sqrt{c-a^2cx^2}}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}\left(-an-2a^2x\right)}{x}dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{a(2+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}} + \cdots$$

$$= \frac{a(2+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}} - \frac{a}{\cdots}$$

$$= \frac{a(2+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}} - \frac{a}{\cdots}$$

$$= \frac{a(2+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}} - \frac{a}{\cdots}$$

Mathematica [A] time = 0.167206, size = 173, normalized size = 0.54

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}(-n-1)}(ax+1)^{\frac{n-3}{2}}\left(2an\left(n^2-1\right)x(ax-1)^2\text{Hypergeometric2F1}\left(1,\frac{3}{2}-\frac{n}{2},\frac{5}{2}-\frac{n}{2},\frac{1-ax}{ax+1}\right)-(n-3)(ax+\cdots\right.}{c(n-3)(n-1)(n+1)x\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(-((-3 + n
)*(1 + a*x)*(-1 + 2*a^2*x^2 + n^2*(-1 + a*x)^2 + a*n*x*(-3 + 2*a*x))) + 2*a
*n*(-1 + n^2)*x*(-1 + a*x)^2*Hypergeometric2F1[1, 3/2 - n/2, 5/2 - n/2, (1
- a*x)/(1 + a*x)]))/(c*(-3 + n)*(-1 + n)*(1 + n)*x*Sqrt[c - a^2*c*x^2])
```

Maple [F] time = 0.21, size = 0, normalized size = 0.

$$\int \frac{e^{n\text{Artanh}(ax)}}{x^2}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^6-2\,a^2c^2x^4+c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^6 -
2*a^2*c^2*x^4 + c^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x^2), x)
```

3.1347 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=417

$$\frac{a^2(n^2+3)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-1}{2},\frac{n+1}{2},\frac{ax+1}{1-ax}\right)}{c(1-n)\sqrt{c-a^2cx^2}}+\frac{a^2(n^2+2n+3)\sqrt{1-a^2x^2}(ax+$$

```
[Out] (a^2*(3 + 2*n + n^2)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 -
 a^2*x^2])/(2*c*(1 + n)*Sqrt[c - a^2*c*x^2]) - ((1 - a*x)^((-1 - n)/2)*(1 +
 a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c*x^2*Sqrt[c - a^2*c*x^2]) - (a*n*
(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c*x*Sqr
t[c - a^2*c*x^2]) - (a^2*(6 + 5*n + 2*n^2 + n^3)*(1 - a*x)^((1 - n)/2)*(1 +
 a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]) +
 (a^2*(3 + n^2)*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x
^2]*Hypergeometric2F1[1, (-1 + n)/2, (1 + n)/2, (1 + a*x)/(1 - a*x)])/(c*(1
 - n)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.420699, antiderivative size = 422, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6153, 6150, 129, 151, 155, 12, 131}

$$-\frac{a^2(n^2+3)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(1,\frac{3-n}{2};\frac{5-n}{2};\frac{1-ax}{ax+1}\right)}{c(3-n)\sqrt{c-a^2cx^2}}+\frac{a^2(n^2+2n+3)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1}{2}(-n}}{2c(n+1)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^2*(3 + 2*n + n^2)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 -
 a^2*x^2])/(2*c*(1 + n)*Sqrt[c - a^2*c*x^2]) - ((1 - a*x)^((-1 - n)/2)*(1 +
 a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c*x^2*Sqrt[c - a^2*c*x^2]) - (a*n*
(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c*x*Sqr
t[c - a^2*c*x^2]) - (a^2*(6 + 5*n + 2*n^2 + n^3)*(1 - a*x)^((1 - n)/2)*(1 +
 a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]) -
 (a^2*(3 + n^2)*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x
^2]*Hypergeometric2F1[1, (3 - n)/2, (5 - n)/2, (1 - a*x)/(1 + a*x)])/(c*(3
- n)*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}}{x^3}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}\left(-an-3a^2x\right)}{x^2}\,dx}{2c\sqrt{c-a^2cx^2}}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}} - \frac{an(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2cx\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-}}{}$$

$$= \frac{a^2\left(3+2n+n^2\right)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a}}{2cx^2\sqrt{c-a^2cx^2}}$$

$$= \frac{a^2\left(3+2n+n^2\right)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a}}{2cx^2\sqrt{c-a^2cx^2}}$$

$$= \frac{a^2\left(3+2n+n^2\right)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a}}{2cx^2\sqrt{c-a^2cx^2}}$$

$$= \frac{a^2\left(3+2n+n^2\right)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a}}{2cx^2\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.215807, size = 219, normalized size = 0.53

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}(-n-1)}(ax+1)^{\frac{n-3}{2}}\left(2a^2\left(n^4+2n^2-3\right)x^2(ax-1)^2\text{Hypergeometric2F1}\left(1,\frac{3}{2}-\frac{n}{2},\frac{5}{2}-\frac{n}{2},\frac{1-ax}{ax+1}\right)-(n-3\right.}{2c(n-3)(n-1)(n+1)x^2\sqrt{c-}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(-((-3 + n
)*(1 + a*x)*(-1 - 3*a^2*x^2 + 6*a^3*x^3 + a*n^3*x*(-1 + a*x)^2 + n^2*(-1 +
a*x)^2*(1 + 2*a*x) + a*n*x*(-1 - 6*a*x + 5*a^2*x^2))) + 2*a^2*(-3 + 2*n^2 +
 n^4)*x^2*(-1 + a*x)^2*Hypergeometric2F1[1, 3/2 - n/2, 5/2 - n/2, (1 - a*x)
/(1 + a*x)]))/(2*c*(-3 + n)*(-1 + n)*(1 + n)*x^2*Sqrt[c - a^2*c*x^2])
```

Maple [F] time = 0.201, size = 0, normalized size = 0.

$$\int \frac{e^{n\text{Artanh}(ax)}}{x^3}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^7-2\,a^2c^2x^5+c^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^7 -
2*a^2*c^2*x^5 + c^2*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x^3), x)
```

3.1348 $\int \frac{e^{n\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=407

$$-\frac{3(2-n)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{1}{2}(-n-1)}}{a^4c^2\left(9-n^2\right)\sqrt{c-a^2cx^2}}+\frac{3\left(-n^2+2n+1\right)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1}{2}(-n-1)}}{a^4c^2(3-n)(n+1)(n+3)\sqrt{c-a^2cx^2}}-\frac{3\left(-n^2+2n\right.}{a^4c^2}$$

```
[Out] (x^3*(1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(a*c^
2*(3 + n)*Sqrt[c - a^2*c*x^2]) - (3*(2 - n)*(1 - a*x)^((-1 - n)/2)*(1 + a*x
)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(a^4*c^2*(9 - n^2)*Sqrt[c - a^2*c*x^2]) -
 (3*x*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(a^3
*c^2*(3 + n)*Sqrt[c - a^2*c*x^2]) + (3*(1 + 2*n - n^2)*(1 - a*x)^((-1 - n)/
2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(a^4*c^2*(3 - n)*(1 + n)*(3 +
n)*Sqrt[c - a^2*c*x^2]) - (3*(1 + 2*n - n^2)*(1 - a*x)^((1 - n)/2)*(1 + a*x
)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(a^4*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*
c*x^2])
```

Rubi [A] time = 0.486462, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6153, 6150, 94, 90, 79, 45, 37}

$$-\frac{3(2-n)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{1}{2}(-n-1)}}{a^4c^2\left(9-n^2\right)\sqrt{c-a^2cx^2}}+\frac{3\left(-n^2+2n+1\right)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1}{2}(-n-1)}}{a^4c^2(3-n)(n+1)(n+3)\sqrt{c-a^2cx^2}}-\frac{3\left(-n^2+2n\right.}{a^4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^(5/2),x]
```


```
[Out] (x^3*(1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(a*c^
2*(3 + n)*Sqrt[c - a^2*c*x^2]) - (3*(2 - n)*(1 - a*x)^((-1 - n)/2)*(1 + a*x
)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(a^4*c^2*(9 - n^2)*Sqrt[c - a^2*c*x^2]) -
 (3*x*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(a^3
*c^2*(3 + n)*Sqrt[c - a^2*c*x^2]) + (3*(1 + 2*n - n^2)*(1 - a*x)^((-1 - n)/
2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(a^4*c^2*(3 - n)*(1 + n)*(3 +
n)*Sqrt[c - a^2*c*x^2]) - (3*(1 + 2*n - n^2)*(1 - a*x)^((1 - n)/2)*(1 + a*x
)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(a^4*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*
c*x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int x^3(1-ax)^{-\frac{5}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{x^3(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{ac^2(3+n)\sqrt{c-a^2cx^2}} - \frac{\left(3\sqrt{1-a^2x^2}\right)\int x^2(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}\,dx}{ac^2(3+n)\sqrt{c-a^2cx^2}}$$
$$= \frac{x^3(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{ac^2(3+n)\sqrt{c-a^2cx^2}} - \frac{3x(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{a^3c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(3\sqrt{}}{}$$
$$= \frac{x^3(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{ac^2(3+n)\sqrt{c-a^2cx^2}} - \frac{3(2-n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{a^4c^2\left(9-n^2\right)\sqrt{c-a^2cx^2}} -$$
$$= \frac{x^3(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{ac^2(3+n)\sqrt{c-a^2cx^2}} - \frac{3(2-n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{a^4c^2\left(9-n^2\right)\sqrt{c-a^2cx^2}} -$$
$$= \frac{x^3(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{ac^2(3+n)\sqrt{c-a^2cx^2}} - \frac{3(2-n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{a^4c^2\left(9-n^2\right)\sqrt{c-a^2cx^2}} -$$

Mathematica [A] time = 0.197095, size = 112, normalized size = 0.28

$$-\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}(-n-3)}(ax+1)^{\frac{n-3}{2}}\left(-a^3n\left(n^2-7\right)x^3+3a^2\left(n^2-3\right)x^2-6anx+6\right)}{a^4c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(6 - 6*a
*n*x + 3*a^2*(-3 + n^2)*x^2 - a^3*n*(-7 + n^2)*x^3))/(a^4*c^2*(9 - 10*n^2 +
 n^4)*Sqrt[c - a^2*c*x^2]))
```

Maple [A] time = 0.028, size = 93, normalized size = 0.2

$$-\frac{\left(a^3n^3x^3-7\,nx^3a^3-3\,a^2n^2x^2+9\,a^2x^2+6\,nax-6\right)(ax-1)(ax+1)\,{\rm e}^{n{\rm Artanh}(ax)}}{a^4\left(n^4-10\,n^2+9\right)}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -(a*x-1)*(a*x+1)*(a^3*n^3*x^3-7*a^3*n*x^3-3*a^2*n^2*x^2+9*a^2*x^2+6*a*n*x-6
)*exp(n*arctanh(a*x))/a^4/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 2.41749, size = 352, normalized size = 0.86

$$\frac{\sqrt{-a^2cx^2+c}\left(\left(a^3n^3-7\,a^3n\right)x^3+6\,anx-3\left(a^2n^2-3\,a^2\right)x^2-6\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^3n^4-10\,a^4c^3n^2+9\,a^4c^3+\left(a^8c^3n^4-10\,a^8c^3n^2+9\,a^8c^3\right)x^4-2\left(a^6c^3n^4-10\,a^6c^3n^2+9\,a^6c^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] sqrt(-a^2*c*x^2 + c)*((a^3*n^3 - 7*a^3*n)*x^3 + 6*a*n*x - 3*(a^2*n^2 - 3*a^
2)*x^2 - 6)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^3*n^4 - 10*a^4*c^3*n^2 + 9
*a^4*c^3 + (a^8*c^3*n^4 - 10*a^8*c^3*n^2 + 9*a^8*c^3)*x^4 - 2*(a^6*c^3*n^4
- 10*a^6*c^3*n^2 + 9*a^6*c^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**3/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.1349 $\int \frac{e^{n\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=102

$$\frac{(3-n^2)(n-ax)e^{n\tanh^{-1}(ax)}}{a^3c^2(n^4-10n^2+9)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\tanh^{-1}(ax)}}{a^3c(9-n^2)(c-a^2cx^2)^{3/2}}$$

```
[Out] -((E^(n*ArcTanh[a*x])*(n - 3*a*x))/(a^3*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)))
 + (E^(n*ArcTanh[a*x])*(3 - n^2)*(n - a*x))/(a^3*c^2*(9 - 10*n^2 + n^4)*Sqr
t[c - a^2*c*x^2])
```

Rubi [A] time = 0.199066, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6147, 6135}

$$\frac{(3-n^2)(n-ax)e^{n\tanh^{-1}(ax)}}{a^3c^2(n^4-10n^2+9)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\tanh^{-1}(ax)}}{a^3c(9-n^2)(c-a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(5/2),x]
```


```
[Out] -((E^(n*ArcTanh[a*x])*(n - 3*a*x))/(a^3*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)))
 + (E^(n*ArcTanh[a*x])*(3 - n^2)*(n - a*x))/(a^3*c^2*(9 - 10*n^2 + n^4)*Sqr
t[c - a^2*c*x^2])
```


Rule 6147

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> -Simp[((n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a
*d*(n^2 - 4*(p + 1)^2)), x] + Dist[(n^2 + 2*(p + 1))/(d*(n^2 - 4*(p + 1)^2)
), Int[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}
, x] && EqQ[a^2*c + d, 0] && LtQ[p, -1] && !IntegerQ[n] && NeQ[n^2 - 4*(p
+ 1)^2, 0] && IntegerQ[2*p]
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx &= -\frac{e^{n\tanh^{-1}(ax)}(n-3ax)}{a^3c(9-n^2)(c-a^2cx^2)^{3/2}}-\frac{(3-n^2)\int \frac{e^{n\tanh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx}{a^2c(9-n^2)}\\ &= -\frac{e^{n\tanh^{-1}(ax)}(n-3ax)}{a^3c(9-n^2)(c-a^2cx^2)^{3/2}}+\frac{e^{n\tanh^{-1}(ax)}(3-n^2)(n-ax)}{a^3c^2(9-10n^2+n^4)\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.183813, size = 125, normalized size = 1.23

$$-\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}(-n-3)}(ax+1)^{\frac{n-3}{2}}\left(-a^2n^3x^2+an^2x\left(a^2x^2+2\right)+n\left(3a^2x^2-2\right)-3a^3x^3\right)}{a^3c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(-(a^2*n
^3*x^2) - 3*a^3*x^3 + a*n^2*x*(2 + a^2*x^2) + n*(-2 + 3*a^2*x^2)))/(a^3*c^2
*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2]))
```

Maple [A] time = 0.03, size = 96, normalized size = 0.9

$$\frac{\left(a^3n^2x^3-a^2n^3x^2-3x^3a^3+3nx^2a^2+2n^2xa-2n\right)(ax-1)(ax+1)\,e^{n\operatorname{Artanh}(ax)}}{\left(n^4-10n^2+9\right)a^3}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(a^3*n^2*x^3-a^2*n^3*x^2-3*a^3*x^3+3*a^2*n*x^2+2*a*n^2*x-2*
n)*exp(n*arctanh(a*x))/(n^4-10*n^2+9)/a^3/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 2.24438, size = 356, normalized size = 3.49

$$-\frac{\sqrt{-a^2cx^2+c}\left(2\,an^2x+\left(a^3n^2-3\,a^3\right)x^3-\left(a^2n^3-3\,a^2n\right)x^2-2\,n\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3+\left(a^7c^3n^4-10\,a^7c^3n^2+9\,a^7c^3\right)x^4-2\left(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*(2*a*n^2*x + (a^3*n^2 - 3*a^3)*x^3 - (a^2*n^3 - 3*a^2
*n)*x^2 - 2*n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^3*c^3*n^4 - 10*a^3*c^3*n^2
+ 9*a^3*c^3 + (a^7*c^3*n^4 - 10*a^7*c^3*n^2 + 9*a^7*c^3)*x^4 - 2*(a^5*c^3*n
^4 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2 + c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.1350 $\int \frac{e^{n\tanh^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=133

$$\frac{2n(n-ax)e^{n\tanh^{-1}(ax)}}{a^2c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}+\frac{n(n-3ax)e^{n\tanh^{-1}(ax)}}{3a^2c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}+\frac{e^{n\tanh^{-1}(ax)}}{3a^2c\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] E^(n*ArcTanh[a*x])/(3*a^2*c*(c - a^2*c*x^2)^(3/2)) + (E^(n*ArcTanh[a*x])*n*
(n - 3*a*x))/(3*a^2*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)) + (2*E^(n*ArcTanh[a*
x])*n*(n - a*x))/(a^2*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.213821, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6145, 6136, 6135}

$$\frac{2n(n-ax)e^{n\tanh^{-1}(ax)}}{a^2c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}+\frac{n(n-3ax)e^{n\tanh^{-1}(ax)}}{3a^2c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}+\frac{e^{n\tanh^{-1}(ax)}}{3a^2c\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] E^(n*ArcTanh[a*x])/(3*a^2*c*(c - a^2*c*x^2)^(3/2)) + (E^(n*ArcTanh[a*x])*n*
(n - 3*a*x))/(3*a^2*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)) + (2*E^(n*ArcTanh[a*
x])*n*(n - a*x))/(a^2*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Rule 6145

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[((c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(2*d*(p + 1)), x] - Dist[
(a*c*n)/(2*d*(p + 1)), Int[(c + d*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && I
ntegerQ[2*p]
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \frac{e^{n\tanh^{-1}(ax)}}{3a^2c\,(c-a^2cx^2)^{3/2}} - \frac{n\int \frac{e^{n\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx}{3a}$$

$$= \frac{e^{n\tanh^{-1}(ax)}}{3a^2c\,(c-a^2cx^2)^{3/2}} + \frac{e^{n\tanh^{-1}(ax)}n(n-3ax)}{3a^2c\,(9-n^2)\,(c-a^2cx^2)^{3/2}} - \frac{(2n)\int \frac{e^{n\tanh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx}{ac\,(9-n^2)}$$

$$= \frac{e^{n\tanh^{-1}(ax)}}{3a^2c\,(c-a^2cx^2)^{3/2}} + \frac{e^{n\tanh^{-1}(ax)}n(n-3ax)}{3a^2c\,(9-n^2)\,(c-a^2cx^2)^{3/2}} + \frac{2e^{n\tanh^{-1}(ax)}n(n-ax)}{a^2c^2\,(9-10n^2+n^4)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.165181, size = 114, normalized size = 0.86

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}(-n-3)}(ax+1)^{\frac{n-3}{2}}\left(-n^2\left(2a^2x^2+1\right)+anx\left(2a^2x^2-3\right)+an^3x+3\right)}{a^2c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(3 + a*n^3
*x + a*n*x*(-3 + 2*a^2*x^2) - n^2*(1 + 2*a^2*x^2)))/(a^2*c^2*(9 - 10*n^2 +
n^4)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.03, size = 86, normalized size = 0.7

$$-\frac{\left(2\,a^3nx^3-2\,n^2x^2a^2+an^3x-3\,nax-n^2+3\right)(ax-1)(ax+1)\,e^{n\,\mathrm{Artanh}(ax)}}{a^2\left(n^4-10\,n^2+9\right)}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -(a*x-1)*(a*x+1)*(2*a^3*n*x^3-2*a^2*n^2*x^2+a*n^3*x-3*a*n*x-n^2+3)*exp(n*ar
ctanh(a*x))/a^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 2.10473, size = 344, normalized size = 2.59

$$\frac{\left(2\,a^3nx^3-2\,a^2n^2x^2-n^2+\left(an^3-3\,an\right)x+3\right)\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^2c^3n^4-10\,a^2c^3n^2+9\,a^2c^3+\left(a^6c^3n^4-10\,a^6c^3n^2+9\,a^6c^3\right)x^4-2\left(a^4c^3n^4-10\,a^4c^3n^2+9\,a^4c^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] (2*a^3*n*x^3 - 2*a^2*n^2*x^2 - n^2 + (a*n^3 - 3*a*n)*x + 3)*sqrt(-a^2*c*x^2
 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^3*n^4 - 10*a^2*c^3*n^2 + 9*a^2*c
^3 + (a^6*c^3*n^4 - 10*a^6*c^3*n^2 + 9*a^6*c^3)*x^4 - 2*(a^4*c^3*n^4 - 10*a
^4*c^3*n^2 + 9*a^4*c^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.1351 $\int \frac{e^{n\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{6(n-ax)e^{n\tanh^{-1}(ax)}}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\tanh^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}$$

```
[Out] -((E^(n*ArcTanh[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2))) -
 (6*E^(n*ArcTanh[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2*c
*x^2])
```

Rubi [A] time = 0.106349, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6136, 6135}

$$-\frac{6(n-ax)e^{n\tanh^{-1}(ax)}}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\tanh^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((E^(n*ArcTanh[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2))) -
 (6*E^(n*ArcTanh[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2*c
*x^2])
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx &= -\frac{e^{n\tanh^{-1}(ax)}(n-3ax)}{ac(9-n^2)(c-a^2cx^2)^{3/2}}+\frac{6\int \frac{e^{n\tanh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx}{c(9-n^2)}\\ &= -\frac{e^{n\tanh^{-1}(ax)}(n-3ax)}{ac(9-n^2)(c-a^2cx^2)^{3/2}}-\frac{6e^{n\tanh^{-1}(ax)}(n-ax)}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.142717, size = 123, normalized size = 1.21

$$-\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}(-n-3)}(ax+1)^{\frac{n-3}{2}}\left(n\left(7-6a^2x^2\right)+6a^3x^3+3an^2x-9ax-n^3\right)}{ac^2(n-3)(n-1)(n+1)(n+3)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(-n^3 -
9*a*x + 3*a*n^2*x + 6*a^3*x^3 + n*(7 - 6*a^2*x^2)))/(a*c^2*(-3 + n)*(-1 + n
)*(1 + n)*(3 + n)*Sqrt[c - a^2*c*x^2]))
```

Maple [A] time = 0.03, size = 84, normalized size = 0.8

$$\frac{\left(6x^3a^3-6na^2x^2+3an^2x-n^3-9ax+7n\right)(ax-1)(ax+1)e^{n\mathrm{Artanh}(ax)}}{a\left(n^4-10n^2+9\right)}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(6*a^3*x^3-6*a^2*n*x^2+3*a*n^2*x-n^3-9*a*x+7*n)*exp(n*arcta
nh(a*x))/a/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 2.13011, size = 335, normalized size = 3.28

$$-\frac{\left(6a^3x^3-6a^2nx^2-n^3+3\left(an^2-3a\right)x+7n\right)\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^3n^4-10ac^3n^2+\left(a^5c^3n^4-10a^5c^3n^2+9a^5c^3\right)x^4+9ac^3-2\left(a^3c^3n^4-10a^3c^3n^2+9a^3c^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -(6*a^3*x^3 - 6*a^2*n*x^2 - n^3 + 3*(a*n^2 - 3*a)*x + 7*n)*sqrt(-a^2*c*x^2
+ c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^4 - 10*a*c^3*n^2 + (a^5*c^3*n^4
```

```
 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^4 + 9*a*c^3 - 2*(a^3*c^3*n^4 - 10*a^3*c^3*
n^2 + 9*a^3*c^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.1352 $\int \frac{e^{n\tanh^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=417

$$\frac{2\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-1}{2},\frac{n+1}{2},\frac{ax+1}{1-ax}\right)}{c^2(1-n)\sqrt{c-a^2cx^2}}-\frac{(n^2+6n+15)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-3}{2}}(1-ax}{c^2(n+3)(1-n^2)\sqrt{c-a^2cx^2}}$$

```
[Out] ((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(3 +
 n)*Sqrt[c - a^2*c*x^2]) + ((6 + n)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 +
 n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(1 + n)*(3 + n)*Sqrt[c - a^2*c*x^2]) - ((15
+ 6*n + n^2)*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]
)/(c^2*(3 + n)*(1 - n^2)*Sqrt[c - a^2*c*x^2]) + ((18 + 7*n - 2*n^2 - n^3)*(
1 - a*x)^((3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(9 - 10
*n^2 + n^4)*Sqrt[c - a^2*c*x^2]) + (2*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-1
+ n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (-1 + n)/2, (1 + n)/2, (1 +
a*x)/(1 - a*x)])/(c^2*(1 - n)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.457503, antiderivative size = 421, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6153, 6150, 129, 155, 12, 131}

$$-\frac{2\sqrt{1-a^2x^2}(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(1,\frac{3-n}{2};\frac{5-n}{2};\frac{1-ax}{ax+1}\right)}{c^2(3-n)\sqrt{c-a^2cx^2}}-\frac{(n^2+6n+15)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{1-n}{2}}}{c^2(n+3)(1-n^2)\sqrt{c-a^2cx^2}}+\frac{(-n^3-2n}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] ((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(3 +
 n)*Sqrt[c - a^2*c*x^2]) + ((6 + n)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 +
 n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(1 + n)*(3 + n)*Sqrt[c - a^2*c*x^2]) - ((15
+ 6*n + n^2)*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]
)/(c^2*(3 + n)*(1 - n^2)*Sqrt[c - a^2*c*x^2]) + ((18 + 7*n - 2*n^2 - n^3)*(
1 - a*x)^((3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(9 - 10
*n^2 + n^4)*Sqrt[c - a^2*c*x^2]) - (2*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((-3
+ n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (3 - n)/2, (5 - n)/2, (1 - a
*x)/(1 + a*x)])/(c^2*(3 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{5/2}}dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{5}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}}{x}dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}\left(-a(3+n)-3a^2x\right)}{x}dx}{ac^2(3+n)\sqrt{c-a^2cx^2}}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} + \frac{(6+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(1+n)(3+n)\sqrt{c-a^2cx^2}} + \frac{\sqrt{1}}{}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} + \frac{(6+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(1+n)(3+n)\sqrt{c-a^2cx^2}} - \frac{(15}{}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} + \frac{(6+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(1+n)(3+n)\sqrt{c-a^2cx^2}} - \frac{(15}{}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} + \frac{(6+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(1+n)(3+n)\sqrt{c-a^2cx^2}} - \frac{(15}{}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} + \frac{(6+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(1+n)(3+n)\sqrt{c-a^2cx^2}} - \frac{(15}{}$$

Mathematica [A] time = 0.242677, size = 222, normalized size = 0.53

$$-\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}(-n-3)}(ax+1)^{\frac{n-3}{2}}\left(2\left(n^3+3n^2-n-3\right)(ax-1)^3\text{Hypergeometric2F1}\left(1,\frac{3}{2}-\frac{n}{2},\frac{5}{2}-\frac{n}{2},\frac{1-ax}{ax+1}\right)+n^3(\cdots\right.}{c^2\left(n^4-10n^2+9\right.\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(a*n^2*x
*(2 + 3*a*x - 2*a^2*x^2) - n^3*(2 - 2*a^2*x^2 + a^3*x^3) + 3*(2 - 6*a*x - 3
*a^2*x^2 + 6*a^3*x^3) + n*(18 - 6*a*x - 18*a^2*x^2 + 7*a^3*x^3) + 2*(-3 - n
 + 3*n^2 + n^3)*(-1 + a*x)^3*Hypergeometric2F1[1, 3/2 - n/2, 5/2 - n/2, (1
- a*x)/(1 + a*x)]))/(c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2]))
```

Maple [F] time = 0.211, size = 0, normalized size = 0.

$$\int \frac{e^{n\text{Artanh}(ax)}}{x}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^6c^3x^7-3\,a^4c^3x^5+3\,a^2c^3x^3-c^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^6*c^3*x^7 -
 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 - c^3*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x), x)
```

3.1353 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=507

$$\frac{2an\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-1}{2},\frac{n+1}{2},\frac{ax+1}{1-ax}\right)}{c^2(1-n)\sqrt{c-a^2cx^2}}+\frac{a\left(n^2+6n+12\right)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-3}{2}}(1}{c^2(n+1)(n+3)\sqrt{c-a^2cx^2}}$$

```
[Out] (a*(4 + n)*(1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])
/(c^2*(3 + n)*Sqrt[c - a^2*c*x^2]) - ((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3
 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*x*Sqrt[c - a^2*c*x^2]) + (a*(12 + 6*n + n^
2)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(1
 + n)*(3 + n)*Sqrt[c - a^2*c*x^2]) - (a*(24 + 15*n + 6*n^2 + n^3)*(1 - a*x)
^((1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(3 + n)*(1 - n^
2)*Sqrt[c - a^2*c*x^2]) + (a*(24 + 18*n + 7*n^2 - 2*n^3 - n^4)*(1 - a*x)^((
3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(9 - 10*n^2 + n^4)
*Sqrt[c - a^2*c*x^2]) + (2*a*n*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)/2)
*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (-1 + n)/2, (1 + n)/2, (1 + a*x)/(1
 - a*x)])/(c^2*(1 - n)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.535474, antiderivative size = 511, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6153, 6150, 129, 155, 12, 131}

$$-\frac{2an\sqrt{1-a^2x^2}(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(1,\frac{3-n}{2};\frac{5-n}{2};\frac{1-ax}{ax+1}\right)}{c^2(3-n)\sqrt{c-a^2cx^2}}+\frac{a\left(n^2+6n+12\right)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{1}{2}(-n-1)}}{c^2(n+1)(n+3)\sqrt{c-a^2cx^2}}-\frac{a}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a*(4 + n)*(1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])
/(c^2*(3 + n)*Sqrt[c - a^2*c*x^2]) - ((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3
 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*x*Sqrt[c - a^2*c*x^2]) + (a*(12 + 6*n + n^
2)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(1
 + n)*(3 + n)*Sqrt[c - a^2*c*x^2]) - (a*(24 + 15*n + 6*n^2 + n^3)*(1 - a*x)
^((1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(3 + n)*(1 - n^
2)*Sqrt[c - a^2*c*x^2]) + (a*(24 + 18*n + 7*n^2 - 2*n^3 - n^4)*(1 - a*x)^((
3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(9 - 10*n^2 + n^4)
*Sqrt[c - a^2*c*x^2]) - (2*a*n*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((-3 + n)/2)
*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (3 - n)/2, (5 - n)/2, (1 - a*x)/(1
+ a*x)])/(c^2*(3 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{5}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}}{x^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{5}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}\left(-an-4a^2x\right)}{x}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{a(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}} + \cdots$$

$$= \frac{a(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}} + \frac{a}{\cdots}$$

$$= \frac{a(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}} + \frac{a}{\cdots}$$

$$= \frac{a(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}} + \frac{a}{\cdots}$$

$$= \frac{a(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}} + \frac{a}{\cdots}$$

$$= \frac{a(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}} + \frac{a}{\cdots}$$

Mathematica [A] time = 0.303994, size = 268, normalized size = 0.53

$$-\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}(-n-3)}(ax+1)^{\frac{n-3}{2}}\left(2a\left(n^3+3n^2-n-3\right)nx(ax-1)^3\text{Hypergeometric2F1}\left(1,\frac{3}{2}-\frac{n}{2},\frac{5}{2}-\frac{n}{2},\frac{1-ax}{ax+1}\right)+\cdots\right)}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(9 - 36*
a^2*x^2 + 24*a^4*x^4 - n^4*(-1 + a*x)^3*(1 + a*x) + a*n^3*x*(-4 + 2*a*x + 3
*a^2*x^2 - 2*a^3*x^3) + a*n*x*(34 - 18*a*x - 33*a^2*x^2 + 18*a^3*x^3) + n^2
*(-10 + 18*a*x + 6*a^2*x^2 - 18*a^3*x^3 + 7*a^4*x^4) + 2*a*n*(-3 - n + 3*n^
2 + n^3)*x*(-1 + a*x)^3*Hypergeometric2F1[1, 3/2 - n/2, 5/2 - n/2, (1 - a*x
)/(1 + a*x)]))/(c^2*(-3 + n)*(-1 + n)*(1 + n)*(3 + n)*x*Sqrt[c - a^2*c*x^2]
))
```

Maple [F] time = 0.211, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{n\mathrm{Artanh}(ax)}}{x^2}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^6c^3x^8-3\,a^4c^3x^6+3\,a^2c^3x^4-c^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^6*c^3*x^8 -
 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 - c^3*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x^2), x)
```

3.1354 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=623

$$\frac{a^2(n^2+5)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-1}{2},\frac{n+1}{2},\frac{ax+1}{1-ax}\right)}{c^2(1-n)\sqrt{c-a^2cx^2}}+\frac{a^2(n^2+4n+5)\sqrt{1-a^2x^2}(a}{2c^2(n+3)\sqrt{c}}$$

```
[Out] (a^2*(5 + 4*n + n^2)*(1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 -
 a^2*x^2])/(2*c^2*(3 + n)*Sqrt[c - a^2*c*x^2]) - ((1 - a*x)^((-3 - n)/2)*(1
 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c^2*x^2*Sqrt[c - a^2*c*x^2]) - (
a*n*(1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c^2
*x*Sqrt[c - a^2*c*x^2]) + (a^2*(30 + 17*n + 6*n^2 + n^3)*(1 - a*x)^((-1 - n
)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c^2*(1 + n)*(3 + n)*Sqrt[
c - a^2*c*x^2]) - (a^2*(75 + 54*n + 20*n^2 + 6*n^3 + n^4)*(1 - a*x)^((1 - n
)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c^2*(3 + n)*(1 - n^2)*Sqr
t[c - a^2*c*x^2]) + (a^2*(90 + 59*n + 8*n^2 + 2*n^3 - 2*n^4 - n^5)*(1 - a*x
)^((3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c^2*(9 - 10*n^2
+ n^4)*Sqrt[c - a^2*c*x^2]) + (a^2*(5 + n^2)*(1 - a*x)^((1 - n)/2)*(1 + a*x
)^((-1 + n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (-1 + n)/2, (1 + n)/2
, (1 + a*x)/(1 - a*x)])/(c^2*(1 - n)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.646728, antiderivative size = 628, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6153, 6150, 129, 151, 155, 12, 131}

$$-\frac{a^2(n^2+5)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(1,\frac{3-n}{2};\frac{5-n}{2};\frac{1-ax}{ax+1}\right)}{c^2(3-n)\sqrt{c-a^2cx^2}}+\frac{a^2(n^2+4n+5)\sqrt{1-a^2x^2}(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{1}{2}}}{2c^2(n+3)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^(5/2)),x]
```


```
[Out] (a^2*(5 + 4*n + n^2)*(1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 -
 a^2*x^2])/(2*c^2*(3 + n)*Sqrt[c - a^2*c*x^2]) - ((1 - a*x)^((-3 - n)/2)*(1
 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c^2*x^2*Sqrt[c - a^2*c*x^2]) - (
a*n*(1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c^2
*x*Sqrt[c - a^2*c*x^2]) + (a^2*(30 + 17*n + 6*n^2 + n^3)*(1 - a*x)^((-1 - n
)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c^2*(1 + n)*(3 + n)*Sqrt[
c - a^2*c*x^2]) - (a^2*(75 + 54*n + 20*n^2 + 6*n^3 + n^4)*(1 - a*x)^((1 - n
)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c^2*(3 + n)*(1 - n^2)*Sqr
t[c - a^2*c*x^2]) + (a^2*(90 + 59*n + 8*n^2 + 2*n^3 - 2*n^4 - n^5)*(1 - a*x
)^((3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c^2*(9 - 10*n^2
+ n^4)*Sqrt[c - a^2*c*x^2])
```

```
 - (a^2*(5 + n^2)*(1 - a*x)^((3 - n)/2)*(1 + a*x
)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (3 - n)/2, (5 - n)/2,
 (1 - a*x)/(1 + a*x)])/(c^2*(3 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
```

tegerQ[n/2]

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x^3\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{5}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}}{x^3}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2x^2\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{5}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}\left(-an-5a^2x\right)}{x^2}\,dx}{2c^2\sqrt{c-a^2cx^2}}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2x^2\sqrt{c-a^2cx^2}} - \frac{an(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2x\sqrt{c-a^2cx^2}} + \frac{\sqrt{1}}{}$$

$$= \frac{a^2\left(5+4n+n^2\right)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1}}{2c^2x^2\sqrt{c-a^2cx^2}}$$

$$= \frac{a^2\left(5+4n+n^2\right)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1}}{2c^2x^2\sqrt{c-a^2cx^2}}$$

$$= \frac{a^2\left(5+4n+n^2\right)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1}}{2c^2x^2\sqrt{c-a^2cx^2}}$$

$$= \frac{a^2\left(5+4n+n^2\right)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1}}{2c^2x^2\sqrt{c-a^2cx^2}}$$

$$= \frac{a^2\left(5+4n+n^2\right)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1}}{2c^2x^2\sqrt{c-a^2cx^2}}$$

$$= \frac{a^2\left(5+4n+n^2\right)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1}}{2c^2x^2\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.658955, size = 267, normalized size = 0.43

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}(-n-3)}(ax+1)^{\frac{n-3}{2}}\left(-\frac{a^2(1-ax)\left(-(ax-1)^2\left(2\left(n^6-5n^4-41n^2+45\right)\text{Hypergeometric2F1}\left(1,\frac{3}{2}-\frac{n}{2},\frac{5}{2}-\frac{n}{2},\frac{1-ax}{ax+1}\right)-n^6+n^5+8n^4+2n^3+3\right.\right.}{(n-3)^2(n-1)(n+1)(}\right.}{2c^2\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] ((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*((a^2*(5 +
 4*n + n^2))/(3 + n) - x^(-2) - (a*n)/x - (a^2*(1 - a*x)*(-((-3 + n)^2*(-1
+ n)*(30 + 17*n + 6*n^2 + n^3)) + (-3 + n)^2*(75 + 54*n + 20*n^2 + 6*n^3 +
n^4)*(-1 + a*x) - (-1 + a*x)^2*(-270 - 87*n + 35*n^2 + 2*n^3 + 8*n^4 + n^5
- n^6 + 2*(45 - 41*n^2 - 5*n^4 + n^6)*Hypergeometric2F1[1, 3/2 - n/2, 5/2 -
 n/2, (1 - a*x)/(1 + a*x)])))/((-3 + n)^2*(-1 + n)*(1 + n)*(3 + n))))/(2*c^
2*Sqrt[c - a^2*c*x^2])
```

Maple [F] time = 0.204, size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{Artanh}(ax)}}{x^{3}} \left(-a^{2} c \,x^{2}+c \right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^{2}cx^{2}+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^{6}c^{3}x^{9}-3\,a^{4}c^{3}x^{7}+3\,a^{2}c^{3}x^{5}-c^{3}x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^6*c^3*x^9 -
 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 - c^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**3/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x^3), x)
```

3.1355 $\int \frac{e^{n\tanh^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{120(n-ax)e^{n\tanh^{-1}(ax)}}{ac^3(1-n^2)(9-n^2)(25-n^2)\sqrt{c-a^2cx^2}}-\frac{20(n-3ax)e^{n\tanh^{-1}(ax)}}{ac^2(9-n^2)(25-n^2)(c-a^2cx^2)^{3/2}}-\frac{(n-5ax)e^{n\tanh^{-1}(ax)}}{ac(25-n^2)(c-a^2cx^2)^{5/2}}$$

```
[Out] -((E^(n*ArcTanh[a*x])*(n - 5*a*x))/(a*c*(25 - n^2)*(c - a^2*c*x^2)^(5/2)))
- (20*E^(n*ArcTanh[a*x])*(n - 3*a*x))/(a*c^2*(9 - n^2)*(25 - n^2)*(c - a^2*
c*x^2)^(3/2)) - (120*E^(n*ArcTanh[a*x])*(n - a*x))/(a*c^3*(1 - n^2)*(9 - n^
2)*(25 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.186363, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6136, 6135}

$$-\frac{120(n-ax)e^{n\tanh^{-1}(ax)}}{ac^3(1-n^2)(9-n^2)(25-n^2)\sqrt{c-a^2cx^2}}-\frac{20(n-3ax)e^{n\tanh^{-1}(ax)}}{ac^2(9-n^2)(25-n^2)(c-a^2cx^2)^{3/2}}-\frac{(n-5ax)e^{n\tanh^{-1}(ax)}}{ac(25-n^2)(c-a^2cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^(7/2),x]
```


```
[Out] -((E^(n*ArcTanh[a*x])*(n - 5*a*x))/(a*c*(25 - n^2)*(c - a^2*c*x^2)^(5/2)))
- (20*E^(n*ArcTanh[a*x])*(n - 3*a*x))/(a*c^2*(9 - n^2)*(25 - n^2)*(c - a^2*
c*x^2)^(3/2)) - (120*E^(n*ArcTanh[a*x])*(n - a*x))/(a*c^3*(1 - n^2)*(9 - n^
2)*(25 - n^2)*Sqrt[c - a^2*c*x^2])
```


Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{e^{n\tanh^{-1}(ax)}(n-5ax)}{ac\left(25-n^2\right)\left(c-a^2cx^2\right)^{5/2}} + \frac{20\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{c\left(25-n^2\right)}$$

$$= -\frac{e^{n\tanh^{-1}(ax)}(n-5ax)}{ac\left(25-n^2\right)\left(c-a^2cx^2\right)^{5/2}} - \frac{20e^{n\tanh^{-1}(ax)}(n-3ax)}{ac^2\left(9-n^2\right)\left(25-n^2\right)\left(c-a^2cx^2\right)^{3/2}} + \frac{120\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{c^2\left(9-n^2\right)\left(25-n^2\right)}$$

$$= -\frac{e^{n\tanh^{-1}(ax)}(n-5ax)}{ac\left(25-n^2\right)\left(c-a^2cx^2\right)^{5/2}} - \frac{20e^{n\tanh^{-1}(ax)}(n-3ax)}{ac^2\left(9-n^2\right)\left(25-n^2\right)\left(c-a^2cx^2\right)^{3/2}} - \frac{120e^{n\tanh^{-1}(ax)}(}{ac^3\left(1-n^2\right)\left(9-n^2\right)(25}$$

Mathematica [A] time = 0.221295, size = 182, normalized size = 1.1

$$-\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}(-n-5)}(ax+1)^{\frac{n-5}{2}}\left(n^3\left(30-20a^2x^2\right)+10an^2x\left(6a^2x^2-11\right)+n\left(-120a^4x^4+260a^2x^2-149\right)+15a\right.}{ac^3(n-5)(n-3)(n-1)(n+1)(n+3)(n+5)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -(((1 - a*x)^((-5 - n)/2)*(1 + a*x)^((-5 + n)/2)*Sqrt[1 - a^2*x^2]*(-n^5 +
5*a*n^4*x + n^3*(30 - 20*a^2*x^2) + 10*a*n^2*x*(-11 + 6*a^2*x^2) + n*(-149
+ 260*a^2*x^2 - 120*a^4*x^4) + 15*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)))/(a*c^
3*(-5 + n)*(-3 + n)*(-1 + n)*(1 + n)*(3 + n)*(5 + n)*Sqrt[c - a^2*c*x^2]))
```

Maple [A] time = 0.03, size = 140, normalized size = 0.8

$$\frac{\left(120\,x^5a^5-120\,na^4x^4+60\,a^3n^2x^3-20\,a^2n^3x^2-300\,x^3a^3+5\,an^4x+260\,na^2x^2-n^5-110\,an^2x+30\,n^3+225\,ax-1\right.}{a\left(n^6-35\,n^4+259\,n^2-225\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(120*a^5*x^5-120*a^4*n*x^4+60*a^3*n^2*x^3-20*a^2*n^3*x^2-30
0*a^3*x^3+5*a*n^4*x+260*a^2*n*x^2-n^5-110*a*n^2*x+30*n^3+225*a*x-149*n)*exp
(n*arctanh(a*x))/a/(n^6-35*n^4+259*n^2-225)/(-a^2*c*x^2+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(7/2), x)
```

Fricas [A] time = 2.2288, size = 622, normalized size = 3.75

$$-\frac{\left(120\,a^{5}x^{5}-120\,a^{4}nx^{4}-n^{5}+60\left(a^{3}n^{2}-5\,a^{3}\right)x^{3}+30\,n^{3}-20\left(a^{2}n^{3}-13\,a^{2}n\right)x^{2}+5\left(an^{4}-\right.\right.}{ac^{4}n^{6}-35\,ac^{4}n^{4}+259\,ac^{4}n^{2}-\left(a^{7}c^{4}n^{6}-35\,a^{7}c^{4}n^{4}+259\,a^{7}c^{4}n^{2}-225\,a^{7}c^{4}\right)x^{6}-225\,ac^{4}+3\left(a^{5}c^{4}n^{6}-35\,a^{5}c^{4}n^{4}+25\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -(120*a^5*x^5 - 120*a^4*n*x^4 - n^5 + 60*(a^3*n^2 - 5*a^3)*x^3 + 30*n^3 - 2
0*(a^2*n^3 - 13*a^2*n)*x^2 + 5*(a*n^4 - 22*a*n^2 + 45*a)*x - 149*n)*sqrt(-a
^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^4*n^6 - 35*a*c^4*n^4 + 259
*a*c^4*n^2 - (a^7*c^4*n^6 - 35*a^7*c^4*n^4 + 259*a^7*c^4*n^2 - 225*a^7*c^4)
*x^6 - 225*a*c^4 + 3*(a^5*c^4*n^6 - 35*a^5*c^4*n^4 + 259*a^5*c^4*n^2 - 225*
a^5*c^4)*x^4 - 3*(a^3*c^4*n^6 - 35*a^3*c^4*n^4 + 259*a^3*c^4*n^2 - 225*a^3*
c^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^{2}cx^{2}+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(7/2), x)
```

3.1356 $\int e^{n\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=42

$$\frac{c^2x^{m+1}F_1\left(m+1;\frac{n-4}{2},-\frac{n}{2}-2;m+2;ax,-ax\right)}{m+1}$$

```
[Out] (c^2*x^(1 + m)*AppellF1[1 + m, (-4 + n)/2, -2 - n/2, 2 + m, a*x, -(a*x)])/(
1 + m)
```

Rubi [A] time = 0.0911683, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 133}

$$\frac{c^2x^{m+1}F_1\left(m+1;\frac{n-4}{2},-\frac{n}{2}-2;m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*x^(1 + m)*AppellF1[1 + m, (-4 + n)/2, -2 - n/2, 2 + m, a*x, -(a*x)])/(
1 + m)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^2\,dx&=c^2\int x^m(1-ax)^{2-\frac{n}{2}}(1+ax)^{2+\frac{n}{2}}\,dx\\&=\frac{c^2x^{1+m}F_1\left(1+m;\frac{1}{2}(-4+n),-2-\frac{n}{2};2+m;ax,-ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.289885, size = 0, normalized size = 0.

$$\int e^{n\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^2,x]

[Out] Integrate[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^2, x]
```

Maple [F] time = 0.162, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{Artanh}(ax)} x^m \left(-a^2 c x^2 + c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^2,x)

[Out] int(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 c x^2 - c\right)^2 x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^2,x, algorithm="maxima")

[Out] integrate((a^2*c*x^2 - c)^2*x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^4 c^2 x^4 - 2\,a^2 c^2 x^2 + c^2\right) x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^2,x, algorithm="fricas")

[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*x^m*((a*x + 1)/(a*x - 1))^(1/2
*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**m*(-a**2*c*x**2+c)**2,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 - c\right)^2 x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 - c)^2*x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.1357 $\int e^{n\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=40

$$\frac{cx^{m+1}F_1\left(m+1;\frac{n-2}{2},-\frac{n}{2}-1;m+2;ax,-ax\right)}{m+1}$$

```
[Out] (c*x^(1 + m)*AppellF1[1 + m, (-2 + n)/2, -1 - n/2, 2 + m, a*x, -(a*x)])/(1
+ m)
```

Rubi [A] time = 0.0738821, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 133}

$$\frac{cx^{m+1}F_1\left(m+1;\frac{n-2}{2},-\frac{n}{2}-1;m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^(1 + m)*AppellF1[1 + m, (-2 + n)/2, -1 - n/2, 2 + m, a*x, -(a*x)])/(1
+ m)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)\,dx &= c\int x^m(1-ax)^{1-\frac{n}{2}}(1+ax)^{1+\frac{n}{2}}\,dx\\ &= \frac{cx^{1+m}F_1\left(1+m;\frac{1}{2}(-2+n),-1-\frac{n}{2};2+m;ax,-ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.262303, size = 0, normalized size = 0.

$$\int e^{n\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2),x]
```

```
[Out] Integrate[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2), x]
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{Artanh}(ax)} x^m \left(-a^2 c x^2 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(a^2 c x^2 - c\right) x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*c*x^2 - c)*x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^2 c x^2 - c\right) x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)*x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**m*(-a**2*c*x**2+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(a^2cx^2 - c\right)x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)*x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.1358 $\int \frac{e^{n \tanh^{-1}(ax)} x^m}{c-a^2cx^2} \, dx$

Optimal. Leaf size=42

$$\frac{x^{m+1}F_1\left(m+1;\frac{n+2}{2},1-\frac{n}{2};m+2;ax,-ax\right)}{c(m+1)}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, (2 + n)/2, 1 - n/2, 2 + m, a*x, -(a*x)])/(c*(1 + m))
```

Rubi [A] time = 0.102289, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{n+2}{2},1-\frac{n}{2};m+2;ax,-ax\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, (2 + n)/2, 1 - n/2, 2 + m, a*x, -(a*x)])/(c*(1 + m))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n \tanh^{-1}(ax)} x^m}{c-a^2cx^2} \, dx &= \frac{\int x^m(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}} \, dx}{c}\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{2+n}{2},1-\frac{n}{2};2+m;ax,-ax\right)}{c(1+m)}\end{aligned}$$

Mathematica [B] time = 0.195915, size = 106, normalized size = 2.52

$$\frac{x^m\left(e^{-2\tanh^{-1}(ax)}-1\right)^m\left(e^{-2\tanh^{-1}(ax)}+1\right)^m\left(-e^{-4\tanh^{-1}(ax)}\left(e^{2\tanh^{-1}(ax)}-1\right)^2\right)^{-m}e^{n\tanh^{-1}(ax)}F_1\left(-\frac{n}{2};m,-m;1-\frac{n}{2};-e^{-2\dots}\right.}{acn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2),x]
```

```
[Out] (E^(n*ArcTanh[a*x])*(-1 + E^(-2*ArcTanh[a*x]))^m*(1 + E^(-2*ArcTanh[a*x]))^
m*x^m*AppellF1[-n/2, m, -m, 1 - n/2, -E^(-2*ArcTanh[a*x]), E^(-2*ArcTanh[a*
x])])/(a*c*(-((-1 + E^(2*ArcTanh[a*x]))^2/E^(4*ArcTanh[a*x])))^m*n)
```

Maple [F] time = 0.203, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{n \operatorname{Artanh}(ax)} x^{m}}{-a^{2} c \,x^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^{m} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^{2} c x^{2} - c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -integrate(x^m*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{x^{m} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^{2} c x^{2} - c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-x^m*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^{m} e^{n \operatorname{atanh}{\left(ax \right)}}}{a^{2} x^{2} - 1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**m/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(x**m*exp(n*atanh(a*x))/(a**2*x**2 - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2 - c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-x^m*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

3.1359 $\int \frac{e^{n\tanh^{-1}(ax)}x^m}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=42

$$\frac{x^{m+1}F_1\left(m+1;\frac{n+4}{2},2-\frac{n}{2};m+2;ax,-ax\right)}{c^2(m+1)}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, (4 + n)/2, 2 - n/2, 2 + m, a*x, -(a*x)])/(c^2*(1
 + m))
```

Rubi [A] time = 0.098707, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{n+4}{2},2-\frac{n}{2};m+2;ax,-ax\right)}{c^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, (4 + n)/2, 2 - n/2, 2 + m, a*x, -(a*x)])/(c^2*(1
 + m))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int x^m(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\,dx}{c^2}\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{4+n}{2},2-\frac{n}{2};2+m;ax,-ax\right)}{c^2(1+m)}\end{aligned}$$

Mathematica [F] time = 0.44451, size = 0, normalized size = 0.

$$\int \frac{e^{n\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^2,x]
```

```
[Out] Integrate[(E^(n*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^2, x]
```

Maple [F] time = 0.21, size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{Artanh}(ax)} x^{m}}{\left(-a^{2} c \,x^{2}+c \right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(a^{2} c x^{2} - c\right)^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{x^{m} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^{4} c^{2} x^{4} - 2\,a^{2} c^{2} x^{2} + c^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c
^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**m/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

3.1360 $\int e^{n\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=70

$$\frac{x^{m+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^pF_1\left(m+1;\frac{1}{2}(n-2p),-\frac{n}{2}-p;m+2;ax,-ax\right)}{m+1}$$

```
[Out] (x^(1 + m)*(c - a^2*c*x^2)^p*AppellF1[1 + m, (n - 2*p)/2, -n/2 - p, 2 + m,
a*x, -(a*x)])/((1 + m)*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.161412, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 133}

$$\frac{x^{m+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^pF_1\left(m+1;\frac{1}{2}(n-2p),-\frac{n}{2}-p;m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^p,x]
```

```
[Out] (x^(1 + m)*(c - a^2*c*x^2)^p*AppellF1[1 + m, (n - 2*p)/2, -n/2 - p, 2 + m,
a*x, -(a*x)])/((1 + m)*(1 - a^2*x^2)^p)
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^p\,dx&=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{n\tanh^{-1}(ax)}x^m\left(1-a^2x^2\right)^p\,dx\\&=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^m(1-ax)^{-\frac{n}{2}+p}(1+ax)^{\frac{n}{2}+p}\,dx\\&=\frac{x^{1+m}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^pF_1\left(1+m;\frac{1}{2}(n-2p),-\frac{n}{2}-p;2+m;ax,-ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.478661, size = 0, normalized size = 0.

$$\int e^{n\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^p,x]
```

```
[Out] Integrate[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^p, x]
```

Maple [F] time = 0.307, size = 0, normalized size = 0.

$$\int e^{n\mathrm{Artanh}(ax)}x^m\left(-a^2cx^2+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a^2cx^2+c\right)^p x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-a^2cx^2+c\right)^p x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((-a^2*c*x^2 + c)^p*x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**m*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^p x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.1361 $\int e^{n\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=177

$$-\frac{n2^{\frac{n}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p(1-ax)^{-\frac{n}{2}+p+1}\text{Hypergeometric2F1}\left(-\frac{n}{2}-p,-\frac{n}{2}+p+1,-\frac{n}{2}+p+2,\frac{1}{2}(1-ax)\right)}{a^2(p+1)(-n+2p+2)}-\frac{(1}{}$$

```
[Out] -((1 - a*x)^(1 - n/2 + p)*(1 + a*x)^(1 + n/2 + p)*(c - a^2*c*x^2)^p)/(2*a^2
*(1 + p)*(1 - a^2*x^2)^p) - (2^(n/2 + p)*n*(1 - a*x)^(1 - n/2 + p)*(c - a^2
*c*x^2)^p*Hypergeometric2F1[-n/2 - p, 1 - n/2 + p, 2 - n/2 + p, (1 - a*x)/2
])/(a^2*(1 + p)*(2 - n + 2*p)*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.183648, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6153, 6150, 80, 69}

$$-\frac{n2^{\frac{n}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p(1-ax)^{-\frac{n}{2}+p+1}{}_2F_1\left(-\frac{n}{2}-p,-\frac{n}{2}+p+1;-\frac{n}{2}+p+2;\frac{1}{2}(1-ax)\right)}{a^2(p+1)(-n+2p+2)}-\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x*(c - a^2*c*x^2)^p,x]
```


```
[Out] -((1 - a*x)^(1 - n/2 + p)*(1 + a*x)^(1 + n/2 + p)*(c - a^2*c*x^2)^p)/(2*a^2
*(1 + p)*(1 - a^2*x^2)^p) - (2^(n/2 + p)*n*(1 - a*x)^(1 - n/2 + p)*(c - a^2
*c*x^2)^p*Hypergeometric2F1[-n/2 - p, 1 - n/2 + p, 2 - n/2 + p, (1 - a*x)/2
])/(a^2*(1 + p)*(2 - n + 2*p)*(1 - a^2*x^2)^p)
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{n\tanh^{-1}(ax)}x\left(1-a^2x^2\right)^p\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x(1-ax)^{-\frac{n}{2}+p}(1+ax)^{\frac{n}{2}+p}\,dx$$
$$=-\frac{(1-ax)^{1-\frac{n}{2}+p}(1+ax)^{1+\frac{n}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{2a^2(1+p)}+\frac{\left(n\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right.\right.}{2a}$$
$$=-\frac{(1-ax)^{1-\frac{n}{2}+p}(1+ax)^{1+\frac{n}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{2a^2(1+p)}-\frac{2^{\frac{n}{2}+p}n(1-ax)^{1-\frac{n}{2}+p}\left(1-a\right.}{}$$

Mathematica [A] time = 0.0984656, size = 136, normalized size = 0.77

$$-\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p(1-ax)^{-\frac{n}{2}+p+1}\left(n2^{\frac{n}{2}+p+1}\text{Hypergeometric2F1}\left(-\frac{n}{2}-p,-\frac{n}{2}+p+1,-\frac{n}{2}+p+2,\frac{1}{2}(1-ax)\right)\right.}{2a^2(p+1)(-n+2p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((1 - a*x)^(1 - n/2 + p)*(c - a^2*c*x^2)^p*(-((n - 2*(1 + p))*(1 + a*x)^(1
 + n/2 + p)) + 2^(1 + n/2 + p)*n*Hypergeometric2F1[-n/2 - p, 1 - n/2 + p, 2
 - n/2 + p, (1 - a*x)/2]))/(2*a^2*(1 + p)*(2 - n + 2*p)*(1 - a^2*x^2)^p)
```

Maple [F] time = 0.27, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{Artanh}(ax)}x\left(-a^2cx^2+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a^2cx^2+c\right)^p x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-a^2cx^2+c\right)^p x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((-a^2*c*x^2 + c)^p*x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(-c\left(ax-1\right)\left(ax+1\right)\right)^p e^{n\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral(x*(-c*(a*x - 1)*(a*x + 1))**p*exp(n*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^p x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.1362 $\int e^{n \tanh^{-1}(ax)} \left(c - a^2cx^2\right)^p \, dx$

Optimal. Leaf size=103

$$-\frac{2^{\frac{n}{2}+p+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p(1-ax)^{-\frac{n}{2}+p+1}\text{Hypergeometric2F1}\left(-\frac{n}{2}-p,-\frac{n}{2}+p+1,-\frac{n}{2}+p+2,\frac{1}{2}(1-ax)\right)}{a(-n+2p+2)}$$

```
[Out] -((2^(1 + n/2 + p)*(1 - a*x)^(1 - n/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric
2F1[-n/2 - p, 1 - n/2 + p, 2 - n/2 + p, (1 - a*x)/2])/(a*(2 - n + 2*p)*(1 -
 a^2*x^2)^p))
```

Rubi [A] time = 0.085118, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6143, 6140, 69}

$$-\frac{2^{\frac{n}{2}+p+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p(1-ax)^{-\frac{n}{2}+p+1}{}_2F_1\left(-\frac{n}{2}-p,-\frac{n}{2}+p+1;-\frac{n}{2}+p+2;\frac{1}{2}(1-ax)\right)}{a(-n+2p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((2^(1 + n/2 + p)*(1 - a*x)^(1 - n/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric
2F1[-n/2 - p, 1 - n/2 + p, 2 - n/2 + p, (1 - a*x)/2])/(a*(2 - n + 2*p)*(1 -
 a^2*x^2)^p))
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{n\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1-ax)^{-\frac{n}{2}+p}(1+ax)^{\frac{n}{2}+p}\,dx$$
$$=-\frac{2^{1+\frac{n}{2}+p}(1-ax)^{1-\frac{n}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{n}{2}-p,1-\frac{n}{2}+p;2-\frac{n}{2}+p;\frac{1}{2}(1-\right.}{a(2-n+2p)}$$

Mathematica [A] time = 0.0250776, size = 102, normalized size = 0.99

$$-\frac{2^{\frac{n}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p(1-ax)^{-\frac{n}{2}+p+1}\text{Hypergeometric2F1}\left(-\frac{n}{2}-p,-\frac{n}{2}+p+1,-\frac{n}{2}+p+2,\frac{1}{2}(1-ax)\right)}{a\left(-\frac{n}{2}+p+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((2^(n/2 + p)*(1 - a*x)^(1 - n/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[
-n/2 - p, 1 - n/2 + p, 2 - n/2 + p, (1 - a*x)/2])/(a*(1 - n/2 + p)*(1 - a^2
*x^2)^p))
```

Maple [F] time = 0.24, size = 0, normalized size = 0.

$$\int e^{n\text{Artanh}(ax)}\left(-a^2cx^2+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a^2cx^2+c\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-a^2cx^2+c\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c\left(ax-1\right)\left(ax+1\right)\right)^p e^{n\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*exp(n*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^p \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.1363 $\int e^{2(1+p)\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{-p}\,dx$

Optimal. Leaf size=41

$$\frac{(1-ax)^{1-2p}}{a(1-2p)}+\frac{(1-ax)^{-2p}}{ap}$$

```
[Out] (1 - a*x)^(1 - 2*p)/(a*(1 - 2*p)) + 1/(a*p*(1 - a*x)^(2*p))
```

Rubi [A] time = 0.0478388, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6140, 43}

$$\frac{(1-ax)^{1-2p}}{a(1-2p)}+\frac{(1-ax)^{-2p}}{ap}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(1 + p)*ArcTanh[a*x])/(1 - a^2*x^2)^p,x]
```

```
[Out] (1 - a*x)^(1 - 2*p)/(a*(1 - 2*p)) + 1/(a*p*(1 - a*x)^(2*p))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2(1+p)\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{-p}\,dx &= \int (1-ax)^{-1-2p}(1+ax)\,dx\\ &= \int \left(2(1-ax)^{-1-2p}-(1-ax)^{-2p}\right)dx\\ &= \frac{(1-ax)^{1-2p}}{a(1-2p)}+\frac{(1-ax)^{-2p}}{ap}\end{aligned}$$

Mathematica [A] time = 0.0187566, size = 31, normalized size = 0.76

$$\frac{(1-ax)^{-2p}(apx+p-1)}{ap(2p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(1 + p)*ArcTanh[a*x])/(1 - a^2*x^2)^p,x]
```

```
[Out] (-1 + p + a*p*x)/(a*p*(-1 + 2*p)*(1 - a*x)^(2*p))
```

Maple [A] time = 0.026, size = 59, normalized size = 1.4

$$-\frac{\left(apx+p-1\right)(ax-1)\,\mathrm{e}^{2\,(1+p)\operatorname{Artanh}(ax)}}{(ax+1)\,ap\left(2\,p-1\right)\left(-a^2x^2+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*(1+p)*arctanh(a*x))/((-a^2*x^2+1)^p),x)
```

```
[Out] -(a*x-1)*(a*p*x+p-1)*exp(2*(1+p)*arctanh(a*x))/a/p/(2*p-1)/(a*x+1)/((-a^2*x
^2+1)^p)
```

Maxima [A] time = 1.00252, size = 54, normalized size = 1.32

$$-\frac{apx+p-1}{\left(2\,(-1)^p\,p^2-(-1)^p\,p\right)(ax-1)^{2\,p}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*(1+p)*arctanh(a*x))/((-a^2*x^2+1)^p),x, algorithm="maxima")
```

```
[Out] -(a*p*x + p - 1)/((2*(-1)^p*p^2 - (-1)^p*p)*(a*x - 1)^(2*p)*a)
```

Fricas [A] time = 2.14788, size = 158, normalized size = 3.85

$$-\frac{\left(a^2px^2-ax-p+1\right)\left(\frac{ax+1}{ax-1}\right)^{p+1}}{\left(2\,ap^2-ap+\left(2\,a^2p^2-a^2p\right)x\right)\left(-a^2x^2+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*(1+p)*arctanh(a*x))/((-a^2*x^2+1)^p),x, algorithm="fricas")
```

```
[Out] -(a^2*p*x^2 - a*x - p + 1)*((a*x + 1)/(a*x - 1))^(p + 1)/((2*a*p^2 - a*p +
(2*a^2*p^2 - a^2*p)*x)*(-a^2*x^2 + 1)^p)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*(1+p)*atanh(a*x))/((-a**2*x**2+1)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{p+1}}{\left(-a^2x^2+1\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*(1+p)*arctanh(a*x))/((-a^2*x^2+1)^p),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(p + 1)/(-a^2*x^2 + 1)^p, x)
```

3.1364 $\int e^{2(1+p)\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{-p}\,dx$

Optimal. Leaf size=95

$$\frac{\left(1-a^2x^2\right)^p(1-ax)^{1-2p}\left(c-a^2cx^2\right)^{-p}}{a(1-2p)}+\frac{\left(1-a^2x^2\right)^p(1-ax)^{-2p}\left(c-a^2cx^2\right)^{-p}}{ap}$$

```
[Out] ((1 - a*x)^(1 - 2*p)*(1 - a^2*x^2)^p)/(a*(1 - 2*p)*(c - a^2*c*x^2)^p) + (1
- a^2*x^2)^p/(a*p*(1 - a*x)^(2*p)*(c - a^2*c*x^2)^p)
```

Rubi [A] time = 0.0903334, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6143, 6140, 43}

$$\frac{\left(1-a^2x^2\right)^p(1-ax)^{1-2p}\left(c-a^2cx^2\right)^{-p}}{a(1-2p)}+\frac{\left(1-a^2x^2\right)^p(1-ax)^{-2p}\left(c-a^2cx^2\right)^{-p}}{ap}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(1 + p)*ArcTanh[a*x])/(c - a^2*c*x^2)^p,x]
```

```
[Out] ((1 - a*x)^(1 - 2*p)*(1 - a^2*x^2)^p)/(a*(1 - 2*p)*(c - a^2*c*x^2)^p) + (1
- a^2*x^2)^p/(a*p*(1 - a*x)^(2*p)*(c - a^2*c*x^2)^p)
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2(1+p)\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{-p}\,dx &= \left(\left(1-a^2x^2\right)^p\left(c-a^2cx^2\right)^{-p}\right)\int e^{2(1+p)\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{-p}\,dx\\
&= \left(\left(1-a^2x^2\right)^p\left(c-a^2cx^2\right)^{-p}\right)\int (1-ax)^{-1-2p}(1+ax)\,dx\\
&= \left(\left(1-a^2x^2\right)^p\left(c-a^2cx^2\right)^{-p}\right)\int \left(2(1-ax)^{-1-2p}-(1-ax)^{-2p}\right)dx\\
&= \frac{(1-ax)^{1-2p}\left(1-a^2x^2\right)^p\left(c-a^2cx^2\right)^{-p}}{a(1-2p)}+\frac{(1-ax)^{-2p}\left(1-a^2x^2\right)^p\left(c-a^2cx^2\right)^{-p}}{ap}
\end{aligned}$$

Mathematica [A] time = 0.0209287, size = 58, normalized size = 0.61

$$\frac{(1-ax)^{-2p}(apx+p-1)\left(1-a^2x^2\right)^p\left(c-a^2cx^2\right)^{-p}}{ap(2p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(1 + p)*ArcTanh[a*x])/(c - a^2*c*x^2)^p,x]
```

```
[Out] ((-1 + p + a*p*x)*(1 - a^2*x^2)^p)/(a*p*(-1 + 2*p)*(1 - a*x)^(2*p)*(c - a^2
*c*x^2)^p)
```

Maple [A] time = 0.029, size = 60, normalized size = 0.6

$$-\frac{\left(pxa+p-1\right)(ax-1)\,e^{2\,(1+p)\mathrm{Artanh}(ax)}}{(ax+1)\,ap\left(2\,p-1\right)\left(-a^2cx^2+c\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*(1+p)*arctanh(a*x))/((-a^2*c*x^2+c)^p),x)
```

```
[Out] -(a*x-1)*(a*p*x+p-1)*exp(2*(1+p)*arctanh(a*x))/a/p/(2*p-1)/(a*x+1)/((-a^2*c
*x^2+c)^p)
```

Maxima [A] time = 0.995822, size = 55, normalized size = 0.58

$$-\frac{apx+p-1}{\left(2\,p^2-p\right)(ax-1)^{2\,p}a\,(-c)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*(1+p)*arctanh(a*x))/((-a^2*c*x^2+c)^p),x, algorithm="maxima
")
```

```
[Out] -(a*p*x + p - 1)/((2*p^2 - p)*(a*x - 1)^(2*p)*a*(-c)^p)
```

Fricas [A] time = 2.20877, size = 161, normalized size = 1.69

$$-\frac{\left(a^2px^2-ax-p+1\right)\left(\frac{ax+1}{ax-1}\right)^{p+1}}{\left(2\,ap^2-ap+\left(2\,a^2p^2-a^2p\right)x\right)\left(-a^2cx^2+c\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*(1+p)*arctanh(a*x))/((-a^2*c*x^2+c)^p),x, algorithm="fricas
")
```

```
[Out] -(a^2*p*x^2 - a*x - p + 1)*((a*x + 1)/(a*x - 1))^(p + 1)/((2*a*p^2 - a*p +
(2*a^2*p^2 - a^2*p)*x)*(-a^2*c*x^2 + c)^p)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*(1+p)*atanh(a*x))/((-a**2*c*x**2+c)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{p+1}}{\left(-a^2cx^2+c\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*(1+p)*arctanh(a*x))/((-a^2*c*x^2+c)^p),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(p + 1)/(-a^2*c*x^2 + c)^p, x)
```

3.1365 $\int e^{2p\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=49

$$\frac{(ax+1)^{2p+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{a(2p+1)}$$

```
[Out] ((1 + a*x)^(1 + 2*p)*(c - a^2*c*x^2)^p)/(a*(1 + 2*p)*(1 - a^2*x^2)^p)
```

Rubi [A] time = 0.0728544, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6143, 6140, 32}

$$\frac{(ax+1)^{2p+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*p*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((1 + a*x)^(1 + 2*p)*(c - a^2*c*x^2)^p)/(a*(1 + 2*p)*(1 - a^2*x^2)^p)
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{2p\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx&=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{2p\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx\\&=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1+ax)^{2p}\,dx\\&=\frac{(1+ax)^{1+2p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{a(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.0248821, size = 36, normalized size = 0.73

$$\frac{(ax+1)\left(c-a^2cx^2\right)^p e^{2p\tanh^{-1}(ax)}}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*p*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (E^(2*p*ArcTanh[a*x])*(1 + a*x)*(c - a^2*c*x^2)^p)/(a + 2*a*p)
```

Maple [A] time = 0.024, size = 38, normalized size = 0.8

$$\frac{(ax+1)\,\mathrm{e}^{2p\mathrm{Artanh}(ax)}\left(-a^2cx^2+c\right)^p}{\left(1+2p\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arctanh(a*x))*(-a^2*c*x^2+c)^p,x)
```

```
[Out] (a*x+1)/(1+2*p)/a*exp(2*p*arctanh(a*x))*(-a^2*c*x^2+c)^p
```

Maxima [A] time = 0.982954, size = 46, normalized size = 0.94

$$\frac{\left(a\left(-c\right)^p x+\left(-c\right)^p\right)(ax+1)^{2p}}{a\left(2p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] (a*(-c)^p*x + (-c)^p)*(a*x + 1)^(2*p)/(a*(2*p + 1))
```

Fricas [A] time = 2.14824, size = 89, normalized size = 1.82

$$\frac{(ax+1)\left(-a^2cx^2+c\right)^p\left(\frac{ax+1}{ax-1}\right)^p}{2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] (a*x + 1)*(-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^p/(2*a*p + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*atanh(a*x))*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^p \left(\frac{ax+1}{ax-1}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^p, x)
```

3.1366 $\int e^{-2p\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=51

$$-\frac{(1-ax)^{2p+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{a(2p+1)}$$

```
[Out] -(((1 - a*x)^(1 + 2*p)*(c - a^2*c*x^2)^p)/(a*(1 + 2*p)*(1 - a^2*x^2)^p))
```

Rubi [A] time = 0.0705453, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6143, 6140, 32}

$$-\frac{(1-ax)^{2p+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^(2*p*ArcTanh[a*x]),x]
```

```
[Out] -(((1 - a*x)^(1 + 2*p)*(c - a^2*c*x^2)^p)/(a*(1 + 2*p)*(1 - a^2*x^2)^p))
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{-2p\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{-2p\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx\\ &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int (1-ax)^{2p}\,dx\\ &= -\frac{(1-ax)^{1+2p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{a(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.0277983, size = 36, normalized size = 0.71

$$\frac{(ax-1)\left(c-a^2cx^2\right)^p e^{-2p\tanh^{-1}(ax)}}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(2*p*ArcTanh[a*x]),x]
```

```
[Out] ((-1 + a*x)*(c - a^2*c*x^2)^p)/(E^(2*p*ArcTanh[a*x])*(a + 2*a*p))
```

Maple [A] time = 0.026, size = 40, normalized size = 0.8

$$\frac{(ax-1)\left(-a^2cx^2+c\right)^p}{a\left(1+2p\right)\mathrm{e}^{2p\,\mathrm{Artanh}(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/exp(2*p*arctanh(a*x)),x)
```

```
[Out] (a*x-1)/a/(1+2*p)*(-a^2*c*x^2+c)^p/exp(2*p*arctanh(a*x))
```

Maxima [A] time = 0.9907, size = 49, normalized size = 0.96

$$\frac{\left(a\,(-c)^p\,x-(-c)^p\right)(ax-1)^{2\,p}}{a\left(2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arctanh(a*x)),x, algorithm="maxima")
```

```
[Out] (a*(-c)^p*x - (-c)^p)*(a*x - 1)^(2*p)/(a*(2*p + 1))
```

Fricas [A] time = 2.13192, size = 92, normalized size = 1.8

$$\frac{(ax-1)\left(-a^2cx^2+c\right)^p}{\left(2\,ap+a\right)\left(\frac{ax+1}{ax-1}\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arctanh(a*x)),x, algorithm="fricas")
```

```
[Out] (a*x - 1)*(-a^2*c*x^2 + c)^p/((2*a*p + a)*((a*x + 1)/(a*x - 1))^p)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p/exp(2*p*atanh(a*x)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arctanh(a*x)),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p/((a*x + 1)/(a*x - 1))^p, x)
```

3.1367 $\int e^{n \tanh^{-1}(ax)} x^2 \left(c - a^2 c x^2\right)^{-1-\frac{n^2}{2}} dx$

Optimal. Leaf size=53

$$\frac{(1 - anx)\left(c - a^2 c x^2\right)^{-\frac{n^2}{2}} e^{n \tanh^{-1}(ax)}}{a^3 c n \left(1 - n^2\right)}$$

```
[Out] (E^(n*ArcTanh[a*x])*(1 - a*n*x))/(a^3*c*n*(1 - n^2)*(c - a^2*c*x^2)^(n^2/2)
)
```

Rubi [A] time = 0.109377, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {6146}

$$\frac{(1 - anx)\left(c - a^2 c x^2\right)^{-\frac{n^2}{2}} e^{n \tanh^{-1}(ax)}}{a^3 c n \left(1 - n^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^(-1 - n^2/2),x]
```

```
[Out] (E^(n*ArcTanh[a*x])*(1 - a*n*x))/(a^3*c*n*(1 - n^2)*(c - a^2*c*x^2)^(n^2/2)
)
```

Rule 6146

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[((1 - a*n*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*d*n*(n^
2 - 1)), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && EqQ[n^2 + 2*(
p + 1), 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{n \tanh^{-1}(ax)} x^2 \left(c - a^2 c x^2\right)^{-1-\frac{n^2}{2}} dx = \frac{e^{n \tanh^{-1}(ax)} (1 - anx)\left(c - a^2 c x^2\right)^{-\frac{n^2}{2}}}{a^3 c n \left(1 - n^2\right)}$$

Mathematica [A] time = 0.0735886, size = 92, normalized size = 1.74

$$\frac{(1 - ax)^{-\frac{1}{2} n(n+1)} (ax + 1)^{-\frac{1}{2}(n-1)n} (anx - 1)\left(1 - a^2 x^2\right)^{\frac{n^2}{2}} \left(c - a^2 c x^2\right)^{-\frac{n^2}{2}}}{a^3 c (n - 1) n (n + 1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^(-1 - n^2/2),x]
```

```
[Out] ((-1 + a*n*x)*(1 - a^2*x^2)^(n^2/2))/(a^3*c*(-1 + n)*n*(1 + n)*(1 - a*x)^((
n*(1 + n))/2)*(1 + a*x)^(((-1 + n)*n)/2)*(c - a^2*c*x^2)^(n^2/2))
```

Maple [A] time = 0.03, size = 58, normalized size = 1.1

$$-\frac{(nax-1)(ax-1)(ax+1)\,\mathrm{e}^{n\operatorname{Artanh}(ax)}}{a^3n\left(n^2-1\right)}\left(-a^2cx^2+c\right)^{-1-\frac{n^2}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(-1-1/2*n^2),x)
```

```
[Out] -(a*x-1)*(a*x+1)*(a*n*x-1)*exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(-1-1/2*n^2)/
a^3/n/(n^2-1)
```

Maxima [A] time = 1.02798, size = 101, normalized size = 1.91

$$\frac{(anx-1)e^{\left(-\frac{1}{2}\,n^2\log(ax+1)-\frac{1}{2}\,n^2\log(ax-1)+\frac{1}{2}\,n\log(ax+1)-\frac{1}{2}\,n\log(ax-1)\right)}}{\left(n^3-n\right)a^3\left(-c\right)^{\frac{1}{2}\,n^2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm=
"maxima")
```

```
[Out] (a*n*x - 1)*e^(-1/2*n^2*log(a*x + 1) - 1/2*n^2*log(a*x - 1) + 1/2*n*log(a*x
 + 1) - 1/2*n*log(a*x - 1))/((n^3 - n)*a^3*(-c)^(1/2*n^2)*c)
```

Fricas [A] time = 2.18517, size = 157, normalized size = 2.96

$$-\frac{\left(a^3nx^3-a^2x^2-anx+1\right)\left(-a^2cx^2+c\right)^{-\frac{1}{2}\,n^2-1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^3n^3-a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm=
"fricas")
```

```
[Out] -(a^3*n*x^3 - a^2*x^2 - a*n*x + 1)*(-a^2*c*x^2 + c)^(-1/2*n^2 - 1)*((a*x +
1)/(a*x - 1))^(1/2*n)/(a^3*n^3 - a^3*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**2*(-a**2*c*x**2+c)**(-1-1/2*n**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{-\frac{1}{2}n^2-1}x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm=
"giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(-1/2*n^2 - 1)*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)
, x)
```

3.1368 $\int \frac{e^{6\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{19}}\,dx$

Optimal. Leaf size=31

$$-\frac{1-6ax}{210a^3c^{19}(1-ax)^{21}(ax+1)^{15}}$$

```
[Out] -(1 - 6*a*x)/(210*a^3*c^19*(1 - a*x)^21*(1 + a*x)^15)
```

Rubi [A] time = 0.0914841, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 81}

$$-\frac{1-6ax}{210a^3c^{19}(1-ax)^{21}(ax+1)^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(6*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^19,x]
```

```
[Out] -(1 - 6*a*x)/(210*a^3*c^19*(1 - a*x)^21*(1 + a*x)^15)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{6\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{19}}\,dx &= \frac{\int \frac{x^2}{(1-ax)^{22}(1+ax)^{16}}\,dx}{c^{19}}\\ &= -\frac{1-6ax}{210a^3c^{19}(1-ax)^{21}(1+ax)^{15}}\end{aligned}$$

Mathematica [A] time = 0.974625, size = 30, normalized size = 0.97

$$\frac{1-6ax}{210a^3c^{19}(ax-1)^{21}(ax+1)^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(6*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^19,x]
```

```
[Out] (1 - 6*a*x)/(210*a^3*c^19*(-1 + a*x)^21*(1 + a*x)^15)
```

Maple [B] time = 0.059, size = 426, normalized size = 13.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^6/(-a^2*x^2+1)^3*x^2/(-a^2*c*x^2+c)^19,x)
```

```
[Out] 1/c^19*(-13/16777216/a^3/(a*x+1)^13+969/7340032/a^3/(a*x-1)^14-969/8388608/
a^3/(a*x-1)^13+3553/50331648/a^3/(a*x-1)^12-7429/83886080/a^3/(a*x-1)^10+37
145/201326592/a^3/(a*x-1)^9-37145/134217728/a^3/(a*x-1)^8+334305/939524096/
a^3/(a*x-1)^7-111435/268435456/a^3/(a*x-1)^6+1938969/4294967296/a^3/(a*x-1)
^5-3991995/8589934592/a^3/(a*x-1)^4+1964315/4294967296/a^3/(a*x-1)^3-930465
/2147483648/a^3/(a*x-1)^2+3411705/8589934592/a^3/(a*x-1)-1/65536/a^3/(a*x-1
)^19-3411705/8589934592/a^3/(a*x+1)-1/62914560/a^3/(a*x+1)^15-9/58720256/a^
3/(a*x+1)^14-275/100663296/a^3/(a*x+1)^12-253/33554432/a^3/(a*x+1)^11-5819/
335544320/a^3/(a*x+1)^10-13915/402653184/a^3/(a*x+1)^9-16445/268435456/a^3/
(a*x+1)^8-740025/7516192768/a^3/(a*x+1)^7-312455/2147483648/a^3/(a*x+1)^6-1
/1376256/a^3/(a*x-1)^21+3/655360/a^3/(a*x-1)^20+7/196608/a^3/(a*x-1)^18-17/
262144/a^3/(a*x-1)^17+51/524288/a^3/(a*x-1)^16-323/2621440/a^3/(a*x-1)^15-8
58429/4294967296/a^3/(a*x+1)^5-2211105/8589934592/a^3/(a*x+1)^4-1344005/429
4967296/a^3/(a*x+1)^3-1550775/4294967296/a^3/(a*x+1)^2)
```

Maxima [B] time = 2.00376, size = 512, normalized size = 16.52

$$-\frac{}{210\left(a^{39}c^{19}x^{36} - 6\,a^{38}c^{19}x^{35} + 70\,a^{36}c^{19}x^{33} - 105\,a^{35}c^{19}x^{32} - 336\,a^{34}c^{19}x^{31} + 896\,a^{33}c^{19}x^{30} + 720\,a^{32}c^{19}x^{29} - 3900\,a^{31}c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^6/(-a^2*x^2+1)^3*x^2/(-a^2*c*x^2+c)^19,x, algorithm="maxi
ma")
```

```
[Out] -1/210*(6*a*x - 1)/(a^39*c^19*x^36 - 6*a^38*c^19*x^35 + 70*a^36*c^19*x^33 -
 105*a^35*c^19*x^32 - 336*a^34*c^19*x^31 + 896*a^33*c^19*x^30 + 720*a^32*c^
19*x^29 - 3900*a^31*c^19*x^28 + 280*a^30*c^19*x^27 + 10752*a^29*c^19*x^26 -
 6552*a^28*c^19*x^25 - 20020*a^27*c^19*x^24 + 21840*a^26*c^19*x^23 + 24960*
a^25*c^19*x^22 - 43472*a^24*c^19*x^21 - 18018*a^23*c^19*x^20 + 60060*a^22*c
^19*x^19 - 60060*a^20*c^19*x^17 + 18018*a^19*c^19*x^16 + 43472*a^18*c^19*x^
15 - 24960*a^17*c^19*x^14 - 21840*a^16*c^19*x^13 + 20020*a^15*c^19*x^12 + 6
552*a^14*c^19*x^11 - 10752*a^13*c^19*x^10 - 280*a^12*c^19*x^9 + 3900*a^11*c
^19*x^8 - 720*a^10*c^19*x^7 - 896*a^9*c^19*x^6 + 336*a^8*c^19*x^5 + 105*a^7
*c^19*x^4 - 70*a^6*c^19*x^3 + 6*a^4*c^19*x - a^3*c^19)
```

Fricas [B] time = 5.52788, size = 987, normalized size = 31.84

$$-\frac{}{210\left(a^{39}c^{19}x^{36} - 6\,a^{38}c^{19}x^{35} + 70\,a^{36}c^{19}x^{33} - 105\,a^{35}c^{19}x^{32} - 336\,a^{34}c^{19}x^{31} + 896\,a^{33}c^{19}x^{30} + 720\,a^{32}c^{19}x^{29} - 3900\,a^{31}c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^6/(-a^2*x^2+1)^3*x^2/(-a^2*c*x^2+c)^19,x, algorithm="fric
as")
```

```
[Out] -1/210*(6*a*x - 1)/(a^39*c^19*x^36 - 6*a^38*c^19*x^35 + 70*a^36*c^19*x^33 -
 105*a^35*c^19*x^32 - 336*a^34*c^19*x^31 + 896*a^33*c^19*x^30 + 720*a^32*c^
19*x^29 - 3900*a^31*c^19*x^28 + 280*a^30*c^19*x^27 + 10752*a^29*c^19*x^26 -
 6552*a^28*c^19*x^25 - 20020*a^27*c^19*x^24 + 21840*a^26*c^19*x^23 + 24960*
a^25*c^19*x^22 - 43472*a^24*c^19*x^21 - 18018*a^23*c^19*x^20 + 60060*a^22*c
^19*x^19 - 60060*a^20*c^19*x^17 + 18018*a^19*c^19*x^16 + 43472*a^18*c^19*x^
15 - 24960*a^17*c^19*x^14 - 21840*a^16*c^19*x^13 + 20020*a^15*c^19*x^12 + 6
552*a^14*c^19*x^11 - 10752*a^13*c^19*x^10 - 280*a^12*c^19*x^9 + 3900*a^11*c
^19*x^8 - 720*a^10*c^19*x^7 - 896*a^9*c^19*x^6 + 336*a^8*c^19*x^5 + 105*a^7
*c^19*x^4 - 70*a^6*c^19*x^3 + 6*a^4*c^19*x - a^3*c^19)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**6/(-a**2*x**2+1)**3*x**2/(-a**2*c*x**2+c)**19,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22541, size = 404, normalized size = 13.03

$$-\frac{358229025\,a^{14}x^{14} + 5340869100\,a^{13}x^{13} + 37114698075\,a^{12}x^{12} + 159416118225\,a^{11}x^{11} + 473088806190\,a^{10}x^{10} + 10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^6/(-a^2*x^2+1)^3*x^2/(-a^2*c*x^2+c)^19,x, algorithm="giac
")
```

```
[Out] -1/901943132160*(358229025*a^14*x^14 + 5340869100*a^13*x^13 + 37114698075*a
^12*x^12 + 159416118225*a^11*x^11 + 473088806190*a^10*x^10 + 1026819468675*
a^9*x^9 + 1682288472150*a^8*x^8 + 2115551402250*a^7*x^7 + 2054435046125*a^6
*x^6 + 1535397250002*a^5*x^5 + 870854759775*a^4*x^4 + 364307533205*a^3*x^3
+ 106553746740*a^2*x^2 + 19571887695*a*x + 1710785408)/((a*x + 1)^15*a^3*c^
19) + 1/901943132160*(358229025*a^20*x^20 - 7555375800*a^19*x^19 + 75901131
600*a^18*x^18 - 483051354975*a^17*x^17 + 2184946607340*a^16*x^16 - 74692054
50840*a^15*x^15 + 20031221295000*a^14*x^14 - 43177004037300*a^13*x^13 + 760
13078916950*a^12*x^12 - 110448380006328*a^11*x^11 + 133277726128008*a^10*x^
10 - 133908931763530*a^9*x^9 + 111933156213900*a^8*x^8 - 77492989590120*a^7
*x^7 + 44041557267624*a^6*x^6 - 20244576347604*a^5*x^5 + 7349182966545*a^4*
x^4 - 2026362494800*a^3*x^3 + 396520754280*a^2*x^2 - 48177926223*a*x + 2584
181888)/((a*x - 1)^21*a^3*c^19)
```

3.1369 $\int \frac{e^{4\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^9}\,dx$

Optimal. Leaf size=31

$$-\frac{1-4ax}{60a^3c^9(1-ax)^{10}(ax+1)^6}$$

```
[Out] -(1 - 4*a*x)/(60*a^3*c^9*(1 - a*x)^10*(1 + a*x)^6)
```

Rubi [A] time = 0.0890639, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 81}

$$-\frac{1-4ax}{60a^3c^9(1-ax)^{10}(ax+1)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(4*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^9,x]
```

```
[Out] -(1 - 4*a*x)/(60*a^3*c^9*(1 - a*x)^10*(1 + a*x)^6)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^9}\,dx &= \frac{\int \frac{x^2}{(1-ax)^{11}(1+ax)^7}\,dx}{c^9}\\ &= -\frac{1-4ax}{60a^3c^9(1-ax)^{10}(1+ax)^6}\end{aligned}$$

Mathematica [A] time = 0.175405, size = 30, normalized size = 0.97

$$\frac{4ax-1}{60a^3c^9(ax-1)^{10}(ax+1)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(4*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^9,x]
```

```
[Out] (-1 + 4*a*x)/(60*a^3*c^9*(-1 + a*x)^10*(1 + a*x)^6)
```

Maple [B] time = 0.04, size = 186, normalized size = 6.

$$\frac{1}{c^9}\left(-\frac{1}{12288\,a^3\,(ax+1)^6}-\frac{7}{20480\,a^3\,(ax+1)^5}-\frac{11}{8192\,a^3\,(ax+1)^3}-\frac{121}{65536\,a^3\,(ax+1)^2}-\frac{143}{65536\,a^3\,(ax+1)}-\frac{}{16384\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*x^2/(-a^2*c*x^2+c)^9,x)
```

```
[Out] 1/c^9*(-1/12288/a^3/(a*x+1)^6-7/20480/a^3/(a*x+1)^5-11/8192/a^3/(a*x+1)^3-1
21/65536/a^3/(a*x+1)^2-143/65536/a^3/(a*x+1)-13/16384/a^3/(a*x+1)^4+1/1280/
a^3/(a*x-1)^10-1/768/a^3/(a*x-1)^9-7/6144/a^3/(a*x-1)^6+21/10240/a^3/(a*x-1
)^5-21/8192/a^3/(a*x-1)^4+11/4096/a^3/(a*x-1)^3-165/65536/a^3/(a*x-1)^2+143
/65536/a^3/(a*x-1)+1/1024/a^3/(a*x-1)^8)
```

Maxima [B] time = 1.11362, size = 228, normalized size = 7.35

$$\frac{4\,ax-1}{60\left(a^{19}c^9x^{16}-4\,a^{18}c^9x^{15}+20\,a^{16}c^9x^{13}-20\,a^{15}c^9x^{12}-36\,a^{14}c^9x^{11}+64\,a^{13}c^9x^{10}+20\,a^{12}c^9x^9-90\,a^{11}c^9x^8+20\,a^{10}c^9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^2/(-a^2*c*x^2+c)^9,x, algorithm="maxim
a")
```

```
[Out] 1/60*(4*a*x - 1)/(a^19*c^9*x^16 - 4*a^18*c^9*x^15 + 20*a^16*c^9*x^13 - 20*a
^15*c^9*x^12 - 36*a^14*c^9*x^11 + 64*a^13*c^9*x^10 + 20*a^12*c^9*x^9 - 90*a
^11*c^9*x^8 + 20*a^10*c^9*x^7 + 64*a^9*c^9*x^6 - 36*a^8*c^9*x^5 - 20*a^7*c^
9*x^4 + 20*a^6*c^9*x^3 - 4*a^4*c^9*x + a^3*c^9)
```

Fricas [B] time = 2.14771, size = 370, normalized size = 11.94

$$\frac{4\,ax-1}{60\left(a^{19}c^9x^{16}-4\,a^{18}c^9x^{15}+20\,a^{16}c^9x^{13}-20\,a^{15}c^9x^{12}-36\,a^{14}c^9x^{11}+64\,a^{13}c^9x^{10}+20\,a^{12}c^9x^9-90\,a^{11}c^9x^8+20\,a^{10}c^9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^2/(-a^2*c*x^2+c)^9,x, algorithm="frica
s")
```

```
[Out] 1/60*(4*a*x - 1)/(a^19*c^9*x^16 - 4*a^18*c^9*x^15 + 20*a^16*c^9*x^13 - 20*a
^15*c^9*x^12 - 36*a^14*c^9*x^11 + 64*a^13*c^9*x^10 + 20*a^12*c^9*x^9 - 90*a
^11*c^9*x^8 + 20*a^10*c^9*x^7 + 64*a^9*c^9*x^6 - 36*a^8*c^9*x^5 - 20*a^7*c^
9*x^4 + 20*a^6*c^9*x^3 - 4*a^4*c^9*x + a^3*c^9)
```

Sympy [B] time = 132.855, size = 178, normalized size = 5.74

$$\frac{4ax-1}{60a^{19}c^9x^{16}-240a^{18}c^9x^{15}+1200a^{16}c^9x^{13}-1200a^{15}c^9x^{12}-2160a^{14}c^9x^{11}+3840a^{13}c^9x^{10}+1200a^{12}c^9x^9-5400a^{11}c^9x^8+\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*x**2/(-a**2*c*x**2+c)**9,x)
```

```
[Out] (4*a*x - 1)/(60*a**19*c**9*x**16 - 240*a**18*c**9*x**15 + 1200*a**16*c**9*x
**13 - 1200*a**15*c**9*x**12 - 2160*a**14*c**9*x**11 + 3840*a**13*c**9*x**1
0 + 1200*a**12*c**9*x**9 - 5400*a**11*c**9*x**8 + 1200*a**10*c**9*x**7 + 38
40*a**9*c**9*x**6 - 2160*a**8*c**9*x**5 - 1200*a**7*c**9*x**4 + 1200*a**6*c
**9*x**3 - 240*a**4*c**9*x + 60*a**3*c**9)
```

Giac [B] time = 1.13442, size = 188, normalized size = 6.06

$$-\frac{2145\,a^5x^5+12540\,a^4x^4+30030\,a^3x^3+37080\,a^2x^2+23841\,ax+6476}{983040\,(ax+1)^6a^3c^9}+\frac{2145\,a^9x^9-21780\,a^8x^8+99660\,a^7x^7-2704\dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^2/(-a^2*c*x^2+c)^9,x, algorithm="giac"
)
```

```
[Out] -1/983040*(2145*a^5*x^5 + 12540*a^4*x^4 + 30030*a^3*x^3 + 37080*a^2*x^2 + 2
3841*a*x + 6476)/((a*x + 1)^6*a^3*c^9) + 1/983040*(2145*a^9*x^9 - 21780*a^8
*x^8 + 99660*a^7*x^7 - 270480*a^6*x^6 + 481446*a^5*x^5 - 584920*a^4*x^4 + 4
86220*a^3*x^3 - 265680*a^2*x^2 + 84065*a*x - 9908)/((a*x - 1)^10*a^3*c^9)
```

3.1370 $\int \frac{e^{2\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=31

$$-\frac{1-2ax}{6a^3c^3(1-ax)^3(ax+1)}$$

```
[Out] -(1 - 2*a*x)/(6*a^3*c^3*(1 - a*x)^3*(1 + a*x))
```

Rubi [A] time = 0.0879717, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 81}

$$-\frac{1-2ax}{6a^3c^3(1-ax)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^3,x]
```

```
[Out] -(1 - 2*a*x)/(6*a^3*c^3*(1 - a*x)^3*(1 + a*x))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^3}\,dx &= \frac{\int \frac{x^2}{(1-ax)^4(1+ax)^2}\,dx}{c^3}\\
&= -\frac{1-2ax}{6a^3c^3(1-ax)^3(1+ax)}
\end{aligned}$$

Mathematica [A] time = 0.0306313, size = 30, normalized size = 0.97

$$\frac{1-2ax}{6a^3c^3(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (1 - 2*a*x)/(6*a^3*c^3*(-1 + a*x)^3*(1 + a*x))
```

Maple [A] time = 0.034, size = 54, normalized size = 1.7

$$\frac{1}{c^3}\left(-\frac{1}{16\,a^3\,(ax+1)}-\frac{1}{12\,a^3\,(ax-1)^3}-\frac{1}{8\,a^3\,(ax-1)^2}+\frac{1}{16\,a^3\,(ax-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/c^3*(-1/16/a^3/(a*x+1)-1/12/a^3/(a*x-1)^3-1/8/a^3/(a*x-1)^2+1/16/a^3/(a*x
-1))
```

Maxima [A] time = 0.9849, size = 66, normalized size = 2.13

$$-\frac{2\,ax-1}{6\,\left(a^7c^3x^4-2\,a^6c^3x^3+2\,a^4c^3x-a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -1/6*(2*a*x - 1)/(a^7*c^3*x^4 - 2*a^6*c^3*x^3 + 2*a^4*c^3*x - a^3*c^3)
```

Fricas [A] time = 1.90582, size = 97, normalized size = 3.13

$$-\frac{2\,ax-1}{6\,\left(a^7c^3x^4-2\,a^6c^3x^3+2\,a^4c^3x-a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/6*(2*a*x - 1)/(a^7*c^3*x^4 - 2*a^6*c^3*x^3 + 2*a^4*c^3*x - a^3*c^3)
```

Sympy [A] time = 0.567505, size = 49, normalized size = 1.58

$$-\frac{2ax-1}{6a^7c^3x^4-12a^6c^3x^3+12a^4c^3x-6a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(2*a*x - 1)/(6*a**7*c**3*x**4 - 12*a**6*c**3*x**3 + 12*a**4*c**3*x - 6*a**
3*c**3)
```

Giac [A] time = 1.1264, size = 61, normalized size = 1.97

$$-\frac{1}{16\,(ax+1)a^3c^3}+\frac{3\,a^2x^2-12\,ax+5}{48\,(ax-1)^3a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/16/((a*x + 1)*a^3*c^3) + 1/48*(3*a^2*x^2 - 12*a*x + 5)/((a*x - 1)^3*a^3*
c^3)
```

3.1371 $\int \frac{e^{-2\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=31

$$\frac{2ax+1}{6a^3c^3(1-ax)(ax+1)^3}$$

```
[Out] (1 + 2*a*x)/(6*a^3*c^3*(1 - a*x)*(1 + a*x)^3)
```

Rubi [A] time = 0.0893645, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 81}

$$\frac{2ax+1}{6a^3c^3(1-ax)(ax+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] (1 + 2*a*x)/(6*a^3*c^3*(1 - a*x)*(1 + a*x)^3)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^3}\,dx &= \frac{\int \frac{x^2}{(1-ax)^2(1+ax)^4}\,dx}{c^3}\\ &= \frac{1+2ax}{6a^3c^3(1-ax)(1+ax)^3}\end{aligned}$$

Mathematica [A] time = 0.0304598, size = 30, normalized size = 0.97

$$-\frac{2ax+1}{6a^3c^3(ax-1)(ax+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] -(1 + 2*a*x)/(6*a^3*c^3*(-1 + a*x)*(1 + a*x)^3)
```

Maple [A] time = 0.033, size = 54, normalized size = 1.7

$$\frac{1}{c^3}\left(-\frac{1}{12\,a^3\,(ax+1)^3}+\frac{1}{8\,a^3\,(ax+1)^2}+\frac{1}{16\,a^3\,(ax+1)}-\frac{1}{16\,a^3\,(ax-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/c^3*(-1/12/a^3/(a*x+1)^3+1/8/a^3/(a*x+1)^2+1/16/a^3/(a*x+1)-1/16/a^3/(a*x
-1))
```

Maxima [A] time = 0.959972, size = 66, normalized size = 2.13

$$-\frac{2\,ax+1}{6\left(a^7c^3x^4+2\,a^6c^3x^3-2\,a^4c^3x-a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -1/6*(2*a*x + 1)/(a^7*c^3*x^4 + 2*a^6*c^3*x^3 - 2*a^4*c^3*x - a^3*c^3)
```

Fricas [A] time = 1.90108, size = 97, normalized size = 3.13

$$-\frac{2\,ax+1}{6\left(a^7c^3x^4+2\,a^6c^3x^3-2\,a^4c^3x-a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/6*(2*a*x + 1)/(a^7*c^3*x^4 + 2*a^6*c^3*x^3 - 2*a^4*c^3*x - a^3*c^3)
```

Sympy [A] time = 0.580487, size = 49, normalized size = 1.58

$$-\frac{2ax+1}{6a^7c^3x^4+12a^6c^3x^3-12a^4c^3x-6a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(2*a*x + 1)/(6*a**7*c**3*x**4 + 12*a**6*c**3*x**3 - 12*a**4*c**3*x - 6*a**
3*c**3)
```

Giac [B] time = 1.136, size = 103, normalized size = 3.32

$$\frac{1}{32\,a^3c^3\left(\frac{2}{ax+1}-1\right)}+\frac{\frac{3\,a^3c^6}{ax+1}+\frac{6\,a^3c^6}{(ax+1)^2}-\frac{4\,a^3c^6}{(ax+1)^3}}{48\,a^6c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/32/(a^3*c^3*(2/(a*x + 1) - 1)) + 1/48*(3*a^3*c^6/(a*x + 1) + 6*a^3*c^6/(a
*x + 1)^2 - 4*a^3*c^6/(a*x + 1)^3)/(a^6*c^9)
```

3.1372 $\int \frac{e^{-4\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^9}\,dx$

Optimal. Leaf size=31

$$\frac{4ax+1}{60a^3c^9(1-ax)^6(ax+1)^{10}}$$

```
[Out] (1 + 4*a*x)/(60*a^3*c^9*(1 - a*x)^6*(1 + a*x)^10)
```

Rubi [A] time = 0.0868716, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6150, 81}

$$\frac{4ax+1}{60a^3c^9(1-ax)^6(ax+1)^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^9),x]
```

```
[Out] (1 + 4*a*x)/(60*a^3*c^9*(1 - a*x)^6*(1 + a*x)^10)
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-4\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^9}\,dx &= \frac{\int \frac{x^2}{(1-ax)^7(1+ax)^{11}}\,dx}{c^9}\\ &= \frac{1+4ax}{60a^3c^9(1-ax)^6(1+ax)^{10}}\end{aligned}$$

Mathematica [A] time = 0.169063, size = 30, normalized size = 0.97

$$\frac{4ax+1}{60a^3c^9(ax-1)^6(ax+1)^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^9),x]
```

```
[Out] (1 + 4*a*x)/(60*a^3*c^9*(-1 + a*x)^6*(1 + a*x)^10)
```

Maple [B] time = 0.04, size = 186, normalized size = 6.

$$\frac{1}{c^9}\left(-\frac{1}{1280\,a^3\,(ax+1)^{10}}-\frac{1}{768\,a^3\,(ax+1)^9}+\frac{7}{6144\,a^3\,(ax+1)^6}+\frac{21}{10240\,a^3\,(ax+1)^5}+\frac{21}{8192\,a^3\,(ax+1)^4}+\frac{11}{4096\,a^3\,(ax+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)^4*(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^9,x)
```

```
[Out] 1/c^9*(-1/1280/a^3/(a*x+1)^10-1/768/a^3/(a*x+1)^9+7/6144/a^3/(a*x+1)^6+21/1
0240/a^3/(a*x+1)^5+21/8192/a^3/(a*x+1)^4+11/4096/a^3/(a*x+1)^3+165/65536/a^
3/(a*x+1)^2+143/65536/a^3/(a*x+1)-1/1024/a^3/(a*x+1)^8+1/12288/a^3/(a*x-1)^
6-7/20480/a^3/(a*x-1)^5-11/8192/a^3/(a*x-1)^3+121/65536/a^3/(a*x-1)^2-143/6
5536/a^3/(a*x-1)+13/16384/a^3/(a*x-1)^4)
```

Maxima [B] time = 1.10271, size = 228, normalized size = 7.35

$$\frac{4\,ax+1}{60\left(a^{19}c^9x^{16}+4\,a^{18}c^9x^{15}-20\,a^{16}c^9x^{13}-20\,a^{15}c^9x^{12}+36\,a^{14}c^9x^{11}+64\,a^{13}c^9x^{10}-20\,a^{12}c^9x^9-90\,a^{11}c^9x^8-20\,a^{10}c^9x^7-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^4*(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^9,x, algorithm="maxim
a")
```

```
[Out] 1/60*(4*a*x + 1)/(a^19*c^9*x^16 + 4*a^18*c^9*x^15 - 20*a^16*c^9*x^13 - 20*a
^15*c^9*x^12 + 36*a^14*c^9*x^11 + 64*a^13*c^9*x^10 - 20*a^12*c^9*x^9 - 90*a
^11*c^9*x^8 - 20*a^10*c^9*x^7 + 64*a^9*c^9*x^6 + 36*a^8*c^9*x^5 - 20*a^7*c^
9*x^4 - 20*a^6*c^9*x^3 + 4*a^4*c^9*x + a^3*c^9)
```

Fricas [B] time = 2.08409, size = 370, normalized size = 11.94

$$\frac{4\,ax+1}{60\left(a^{19}c^9x^{16}+4\,a^{18}c^9x^{15}-20\,a^{16}c^9x^{13}-20\,a^{15}c^9x^{12}+36\,a^{14}c^9x^{11}+64\,a^{13}c^9x^{10}-20\,a^{12}c^9x^9-90\,a^{11}c^9x^8-20\,a^{10}c^9x^7-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^4*(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^9,x, algorithm="frica
s")
```

```
[Out] 1/60*(4*a*x + 1)/(a^19*c^9*x^16 + 4*a^18*c^9*x^15 - 20*a^16*c^9*x^13 - 20*a
^15*c^9*x^12 + 36*a^14*c^9*x^11 + 64*a^13*c^9*x^10 - 20*a^12*c^9*x^9 - 90*a
^11*c^9*x^8 - 20*a^10*c^9*x^7 + 64*a^9*c^9*x^6 + 36*a^8*c^9*x^5 - 20*a^7*c^
9*x^4 - 20*a^6*c^9*x^3 + 4*a^4*c^9*x + a^3*c^9)
```

Sympy [B] time = 100.454, size = 178, normalized size = 5.74

$$\frac{4ax+1}{60a^{19}c^9x^{16}+240a^{18}c^9x^{15}-1200a^{16}c^9x^{13}-1200a^{15}c^9x^{12}+2160a^{14}c^9x^{11}+3840a^{13}c^9x^{10}-1200a^{12}c^9x^9-5400a^{11}c^9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)**4*(-a**2*x**2+1)**2/(-a**2*c*x**2+c)**9,x)
```

```
[Out] (4*a*x + 1)/(60*a**19*c**9*x**16 + 240*a**18*c**9*x**15 - 1200*a**16*c**9*x
**13 - 1200*a**15*c**9*x**12 + 2160*a**14*c**9*x**11 + 3840*a**13*c**9*x**1
0 - 1200*a**12*c**9*x**9 - 5400*a**11*c**9*x**8 - 1200*a**10*c**9*x**7 + 38
40*a**9*c**9*x**6 + 2160*a**8*c**9*x**5 - 1200*a**7*c**9*x**4 - 1200*a**6*c
**9*x**3 + 240*a**4*c**9*x + 60*a**3*c**9)
```

Giac [B] time = 1.14097, size = 188, normalized size = 6.06

$$-\frac{2145\,a^5x^5-12540\,a^4x^4+30030\,a^3x^3-37080\,a^2x^2+23841\,ax-6476}{983040\,(ax-1)^6a^3c^9}+\frac{2145\,a^9x^9+21780\,a^8x^8+99660\,a^7x^7+27$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^4*(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^9,x, algorithm="giac"
)
```

```
[Out] -1/983040*(2145*a^5*x^5 - 12540*a^4*x^4 + 30030*a^3*x^3 - 37080*a^2*x^2 + 2
3841*a*x - 6476)/((a*x - 1)^6*a^3*c^9) + 1/983040*(2145*a^9*x^9 + 21780*a^8
*x^8 + 99660*a^7*x^7 + 270480*a^6*x^6 + 481446*a^5*x^5 + 584920*a^4*x^4 + 4
86220*a^3*x^3 + 265680*a^2*x^2 + 84065*a*x + 9908)/((a*x + 1)^10*a^3*c^9)
```

3.1373 $\int \frac{e^{5\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{27/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{(1-5ax)\sqrt{1-a^2x^2}}{120a^3c^{13}(1-ax)^{15}(ax+1)^{10}\sqrt{c-a^2cx^2}}$$

```
[Out] -((1 - 5*a*x)*Sqrt[1 - a^2*x^2])/(120*a^3*c^13*(1 - a*x)^15*(1 + a*x)^10*Sq
rt[c - a^2*c*x^2])
```

Rubi [A] time = 0.230833, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 81}

$$-\frac{(1-5ax)\sqrt{1-a^2x^2}}{120a^3c^{13}(1-ax)^{15}(ax+1)^{10}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(5*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(27/2),x]
```

```
[Out] -((1 - 5*a*x)*Sqrt[1 - a^2*x^2])/(120*a^3*c^13*(1 - a*x)^15*(1 + a*x)^10*Sq
rt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\int \frac{e^{5\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{27/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{5\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{27/2}}\,dx}{c^{13}\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{(1-ax)^{16}(1+ax)^{11}}\,dx}{c^{13}\sqrt{c-a^2cx^2}}$$
$$= -\frac{(1-5ax)\sqrt{1-a^2x^2}}{120a^3c^{13}(1-ax)^{15}(1+ax)^{10}\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.447027, size = 59, normalized size = 0.98

$$\frac{(1-5ax)\sqrt{1-a^2x^2}}{120a^3c^{13}(ax-1)^{15}(ax+1)^{10}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(5*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(27/2),x]
```

```
[Out] ((1 - 5*a*x)*Sqrt[1 - a^2*x^2])/(120*a^3*c^13*(-1 + a*x)^15*(1 + a*x)^10*Sq
rt[c - a^2*c*x^2])
```

Maple [A] time = 0.04, size = 49, normalized size = 0.8

$$-\frac{(ax-1)(ax+1)^6(5\,ax-1)}{120\,a^3}\left(-a^2x^2+1\right)^{-\frac{5}{2}}\left(-a^2cx^2+c\right)^{-\frac{27}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^5/(-a^2*x^2+1)^(5/2)*x^2/(-a^2*c*x^2+c)^(27/2),x)
```

```
[Out] -1/120*(a*x-1)*(a*x+1)^6*(5*a*x-1)/a^3/(-a^2*x^2+1)^(5/2)/(-a^2*c*x^2+c)^(2
7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^5x^2}{\left(-a^2cx^2+c\right)^{\frac{27}{2}}\left(-a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^5/(-a^2*x^2+1)^(5/2)*x^2/(-a^2*c*x^2+c)^(27/2),x, algorit
hm="maxima")
```

```
[Out] integrate((a*x + 1)^5*x^2/((-a^2*c*x^2 + c)^(27/2)*(-a^2*x^2 + 1)^(5/2)), x
)
```

Fricas [B] time = 2.90849, size = 1227, normalized size = 20.45

$$-\frac{\left(a^{22}x^{25} - 5\,a^{21}x^{24} + 40\,a^{19}x^{22} - 50\,a^{18}x^{21} - 126\,a^{17}x^{20} + 2\right.}{120\left(a^{27}c^{14}x^{27} - 5\,a^{26}c^{14}x^{26} - a^{25}c^{14}x^{25} + 45\,a^{24}c^{14}x^{24} - 50\,a^{23}c^{14}x^{23} - 166\,a^{22}c^{14}x^{22} + 330\,a^{21}c^{14}x^{21} + 286\,a^{20}c^{14}x^{20} - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^5/(-a^2*x^2+1)^(5/2)*x^2/(-a^2*c*x^2+c)^(27/2),x, algorit
hm="fricas")
```

```
[Out] -1/120*(a^22*x^25 - 5*a^21*x^24 + 40*a^19*x^22 - 50*a^18*x^21 - 126*a^17*x^
20 + 280*a^16*x^19 + 160*a^15*x^18 - 765*a^14*x^17 + 105*a^13*x^16 + 1248*a
^12*x^15 - 720*a^11*x^14 - 1260*a^10*x^13 + 1260*a^9*x^12 + 720*a^8*x^11 -
1248*a^7*x^10 - 105*a^6*x^9 + 765*a^5*x^8 - 160*a^4*x^7 - 280*a^3*x^6 + 126
*a^2*x^5 + 50*a*x^4 - 40*x^3)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^27
*c^14*x^27 - 5*a^26*c^14*x^26 - a^25*c^14*x^25 + 45*a^24*c^14*x^24 - 50*a^2
3*c^14*x^23 - 166*a^22*c^14*x^22 + 330*a^21*c^14*x^21 + 286*a^20*c^14*x^20
- 1045*a^19*c^14*x^19 - 55*a^18*c^14*x^18 + 2013*a^17*c^14*x^17 - 825*a^16*
c^14*x^16 - 2508*a^15*c^14*x^15 + 1980*a^14*c^14*x^14 + 1980*a^13*c^14*x^13
 - 2508*a^12*c^14*x^12 - 825*a^11*c^14*x^11 + 2013*a^10*c^14*x^10 - 55*a^9*
c^14*x^9 - 1045*a^8*c^14*x^8 + 286*a^7*c^14*x^7 + 330*a^6*c^14*x^6 - 166*a^
5*c^14*x^5 - 50*a^4*c^14*x^4 + 45*a^3*c^14*x^3 - a^2*c^14*x^2 - 5*a*c^14*x
+ c^14)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**5/(-a**2*x**2+1)**(5/2)*x**2/(-a**2*c*x**2+c)**(27/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^5 x^2}{\left(-a^2cx^2+c\right)^{\frac{27}{2}}\left(-a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^5/(-a^2*x^2+1)^(5/2)*x^2/(-a^2*c*x^2+c)^(27/2),x, algorit
hm="giac")
```

```
[Out] integrate((a*x + 1)^5*x^2/((-a^2*c*x^2 + c)^(27/2)*(-a^2*x^2 + 1)^(5/2)), x
)
```

3.1374 $\int \frac{e^{3\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{11/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{(1-3ax)\sqrt{1-a^2x^2}}{24a^3c^5(1-ax)^6(ax+1)^3\sqrt{c-a^2cx^2}}$$

```
[Out] -((1 - 3*a*x)*Sqrt[1 - a^2*x^2])/(24*a^3*c^5*(1 - a*x)^6*(1 + a*x)^3*Sqrt[c
 - a^2*c*x^2])
```

Rubi [A] time = 0.23957, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 81}

$$-\frac{(1-3ax)\sqrt{1-a^2x^2}}{24a^3c^5(1-ax)^6(ax+1)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(11/2),x]
```

```
[Out] -((1 - 3*a*x)*Sqrt[1 - a^2*x^2])/(24*a^3*c^5*(1 - a*x)^6*(1 + a*x)^3*Sqrt[c
 - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{11/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{3\tanh^{-1}(ax)}x^2}{(1-a^2x^2)^{11/2}}\,dx}{c^5\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{(1-ax)^7(1+ax)^4}\,dx}{c^5\sqrt{c-a^2cx^2}}$$
$$= -\frac{(1-3ax)\sqrt{1-a^2x^2}}{24a^3c^5(1-ax)^6(1+ax)^3\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.094181, size = 59, normalized size = 0.98

$$\frac{(3ax-1)\sqrt{1-a^2x^2}}{24a^3c^5(ax-1)^6(ax+1)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(11/2),x]
```

```
[Out] ((-1 + 3*a*x)*Sqrt[1 - a^2*x^2])/(24*a^3*c^5*(-1 + a*x)^6*(1 + a*x)^3*Sqrt[
c - a^2*c*x^2])
```

Maple [A] time = 0.031, size = 49, normalized size = 0.8

$$-\frac{(ax-1)(ax+1)^4(3ax-1)}{24a^3}\left(-a^2x^2+1\right)^{-\frac{3}{2}}\left(-a^2cx^2+c\right)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2/(-a^2*c*x^2+c)^(11/2),x)
```

```
[Out] -1/24*(a*x-1)*(a*x+1)^4*(3*a*x-1)/a^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(11
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3x^2}{\left(-a^2cx^2+c\right)^{\frac{11}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2/(-a^2*c*x^2+c)^(11/2),x, algorit
hm="maxima")
```

```
[Out] integrate((a*x + 1)^3*x^2/((-a^2*c*x^2 + c)^(11/2)*(-a^2*x^2 + 1)^(3/2)), x
)
```

Fricas [B] time = 2.23565, size = 393, normalized size = 6.55

$$-\frac{\left(a^{6}x^{9}-3\,a^{5}x^{8}+8\,a^{3}x^{6}-6\,a^{2}x^{5}-6\,ax^{4}+8\,x^{3}\right)\sqrt{-a^{2}cx^{2}+c}\sqrt{-a^{2}x^{2}+1}}{24\,\left(a^{11}c^{6}x^{11}-3\,a^{10}c^{6}x^{10}-a^{9}c^{6}x^{9}+11\,a^{8}c^{6}x^{8}-6\,a^{7}c^{6}x^{7}-14\,a^{6}c^{6}x^{6}+14\,a^{5}c^{6}x^{5}+6\,a^{4}c^{6}x^{4}-11\,a^{3}c^{6}x^{3}+a^{2}c^{6}x^{2}+3\,ac^{6}x-c^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2/(-a^2*c*x^2+c)^(11/2),x, algorit
hm="fricas")
```

```
[Out] -1/24*(a^6*x^9 - 3*a^5*x^8 + 8*a^3*x^6 - 6*a^2*x^5 - 6*a*x^4 + 8*x^3)*sqrt(
-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^11*c^6*x^11 - 3*a^10*c^6*x^10 - a^9*c
^6*x^9 + 11*a^8*c^6*x^8 - 6*a^7*c^6*x^7 - 14*a^6*c^6*x^6 + 14*a^5*c^6*x^5 +
 6*a^4*c^6*x^4 - 11*a^3*c^6*x^3 + a^2*c^6*x^2 + 3*a*c^6*x - c^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2/(-a**2*c*x**2+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^{3}x^{2}}{\left(-a^{2}cx^{2}+c\right)^{\frac{11}{2}}\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2/(-a^2*c*x^2+c)^(11/2),x, algorit
hm="giac")
```

```
[Out] integrate((a*x + 1)^3*x^2/((-a^2*c*x^2 + c)^(11/2)*(-a^2*x^2 + 1)^(3/2)), x
)
```

3.1375 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{1-a^2x^2}}{2a^3c(1-ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(ax+1)}{4a^3c\sqrt{c-a^2cx^2}}$$

```
[Out] Sqrt[1 - a^2*x^2]/(2*a^3*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2
*x^2]*Log[1 - a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*Log[
1 + a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.240193, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6153, 6150, 88}

$$\frac{\sqrt{1-a^2x^2}}{2a^3c(1-ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(ax+1)}{4a^3c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a^3*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2
*x^2]*Log[1 - a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*Log[
1 + a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{2a^2(-1+ax)^2}+\frac{3}{4a^2(-1+ax)}+\frac{1}{4a^2(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}}{2a^3c(1-ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(1+ax)}{4a^3c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0472776, size = 76, normalized size = 0.55

$$\frac{\sqrt{1-a^2x^2}\left(\frac{1}{2a^3(1-ax)}+\frac{3\log(1-ax)}{4a^3}+\frac{\log(ax+1)}{4a^3}\right)}{c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1/(2*a^3*(1 - a*x)) + (3*Log[1 - a*x])/(4*a^3) + Log[1
+ a*x]/(4*a^3)))/(c*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.09, size = 90, normalized size = 0.7

$$-\frac{ax\ln(ax+1)+3\ln(ax-1)xa-\ln(ax+1)-3\ln(ax-1)-2}{\left(4a^2x^2-4\right)c^2a^3(ax-1)}\sqrt{-a^2x^2+1}\sqrt{-c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*x*ln(a*x+1)+3*ln(a*x-1)*x
*a-ln(a*x+1)-3*ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a^3/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^2}{a^5c^2x^5-a^4c^2x^4-2\,a^3c^2x^3+2\,a^2c^2x^2+ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^2/(a^5*c^2*x^5 - a^4*c^
2*x^4 - 2*a^3*c^2*x^3 + 2*a^2*c^2*x^2 + a*c^2*x - c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\,(ax+1)}{\sqrt{-\,(ax-1)\,(ax+1)}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)x^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

3.1376 $\int \frac{e^{-\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{1-a^2x^2}}{2a^3c(ax+1)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\log(ax+1)}{4a^3c\sqrt{c-a^2cx^2}}$$

```
[Out] -Sqrt[1 - a^2*x^2]/(2*a^3*c*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*
x^2]*Log[1 - a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2]*Log
[1 + a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.257435, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{\sqrt{1-a^2x^2}}{2a^3c(ax+1)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\log(ax+1)}{4a^3c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```


```
[Out] -Sqrt[1 - a^2*x^2]/(2*a^3*c*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*
x^2]*Log[1 - a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2]*Log
[1 + a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2])
```


Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-\tanh^{-1}(ax)}x^2}{(1-a^2x^2)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{(1-ax)(1+ax)^2}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{4a^2(-1+ax)}+\frac{1}{2a^2(1+ax)^2}-\frac{3}{4a^2(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$
$$= -\frac{\sqrt{1-a^2x^2}}{2a^3c(1+ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\log(1+ax)}{4a^3c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0497776, size = 72, normalized size = 0.52

$$-\frac{\sqrt{1-a^2x^2}((ax+1)\log(1-ax)+3(ax+1)\log(ax+1)+2)}{4a^3(acx+c)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]*(2 + (1 + a*x)*Log[1 - a*x] + 3*(1 + a*x)*Log[1 + a*x])
)/(4*a^3*(c + a*c*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.094, size = 88, normalized size = 0.6

$$\frac{3\,ax\ln(ax+1)+\ln(ax-1)\,xa+3\,\ln(ax+1)+\ln(ax-1)+2}{(4\,a^2x^2-4)\,c^2a^3\,(ax+1)}\sqrt{-a^2x^2+1}\sqrt{-c\,(a^2x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(3*a*x*ln(a*x+1)+ln(a*x-1)*x*
a+3*ln(a*x+1)+ln(a*x-1)+2)/(a^2*x^2-1)/c^2/a^3/(a*x+1)
```

Maxima [A] time = 0.989786, size = 70, normalized size = 0.51

$$-\frac{\sqrt{c}}{2\,(a^4c^2x+a^3c^2)}-\frac{3\,\log(ax+1)}{4\,a^3c^{\frac{3}{2}}}-\frac{\log(ax-1)}{4\,a^3c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -1/2*sqrt(c)/(a^4*c^2*x + a^3*c^2) - 3/4*log(a*x + 1)/(a^3*c^(3/2)) - 1/4*l
og(a*x - 1)/(a^3*c^(3/2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}x^2}{a^5c^2x^5+a^4c^2x^4-2\,a^3c^2x^3-2\,a^2c^2x^2+ac^2x+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^2/(a^5*c^2*x^5 + a^4*c^2
*x^4 - 2*a^3*c^2*x^3 - 2*a^2*c^2*x^2 + a*c^2*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\sqrt{-(ax-1)(ax+1)}}{(-c(ax-1)(ax+1))^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(
a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}x^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2/((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)), x)
```

3.1377 $\int \frac{e^{-3\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{11/2}}\,dx$

Optimal. Leaf size=60

$$\frac{(3ax+1)\sqrt{1-a^2x^2}}{24a^3c^5(1-ax)^3(ax+1)^6\sqrt{c-a^2cx^2}}$$

```
[Out] ((1 + 3*a*x)*Sqrt[1 - a^2*x^2])/(24*a^3*c^5*(1 - a*x)^3*(1 + a*x)^6*Sqrt[c
- a^2*c*x^2])
```

Rubi [A] time = 0.236635, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 81}

$$\frac{(3ax+1)\sqrt{1-a^2x^2}}{24a^3c^5(1-ax)^3(ax+1)^6\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(11/2)),x]
```

```
[Out] ((1 + 3*a*x)*Sqrt[1 - a^2*x^2])/(24*a^3*c^5*(1 - a*x)^3*(1 + a*x)^6*Sqrt[c
- a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{11/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-3\tanh^{-1}(ax)}x^2}{(1-a^2x^2)^{11/2}}\,dx}{c^5\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{(1-ax)^4(1+ax)^7}\,dx}{c^5\sqrt{c-a^2cx^2}}$$
$$= \frac{(1+3ax)\sqrt{1-a^2x^2}}{24a^3c^5(1-ax)^3(1+ax)^6\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0915533, size = 59, normalized size = 0.98

$$-\frac{(3ax+1)\sqrt{1-a^2x^2}}{24a^3c^5(ax-1)^3(ax+1)^6\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(11/2)),x]
```

```
[Out] -((1 + 3*a*x)*Sqrt[1 - a^2*x^2])/(24*a^3*c^5*(-1 + a*x)^3*(1 + a*x)^6*Sqrt[
c - a^2*c*x^2])
```

Maple [A] time = 0.032, size = 49, normalized size = 0.8

$$-\frac{(3\,ax+1)\,(ax-1)}{24\,a^3\,(ax+1)^2}\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(-a^2cx^2+c\right)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(11/2),x)
```

```
[Out] -1/24*(a*x-1)*(3*a*x+1)*(-a^2*x^2+1)^(3/2)/(a*x+1)^2/a^3/(-a^2*c*x^2+c)^(11
/2)
```

Maxima [A] time = 1.1023, size = 126, normalized size = 2.1

$$-\frac{3\,ax+1}{24\left(a^{12}c^{\frac{11}{2}}x^9+3\,a^{11}c^{\frac{11}{2}}x^8-8\,a^9c^{\frac{11}{2}}x^6-6\,a^8c^{\frac{11}{2}}x^5+6\,a^7c^{\frac{11}{2}}x^4+8\,a^6c^{\frac{11}{2}}x^3-3\,a^4c^{\frac{11}{2}}x-a^3c^{\frac{11}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(11/2),x, algorit
hm="maxima")
```

```
[Out] -1/24*(3*a*x + 1)/(a^12*c^(11/2)*x^9 + 3*a^11*c^(11/2)*x^8 - 8*a^9*c^(11/2)
*x^6 - 6*a^8*c^(11/2)*x^5 + 6*a^7*c^(11/2)*x^4 + 8*a^6*c^(11/2)*x^3 - 3*a^4
*c^(11/2)*x - a^3*c^(11/2))
```

Fricas [B] time = 2.17869, size = 392, normalized size = 6.53

$$\frac{\left(a^{6}x^{9}+3\,a^{5}x^{8}-8\,a^{3}x^{6}-6\,a^{2}x^{5}+6\,ax^{4}+8\,x^{3}\right)\sqrt{-a^{2}cx^{2}+c}\sqrt{-a^{2}x^{2}+1}}{24\left(a^{11}c^{6}x^{11}+3\,a^{10}c^{6}x^{10}-a^{9}c^{6}x^{9}-11\,a^{8}c^{6}x^{8}-6\,a^{7}c^{6}x^{7}+14\,a^{6}c^{6}x^{6}+14\,a^{5}c^{6}x^{5}-6\,a^{4}c^{6}x^{4}-11\,a^{3}c^{6}x^{3}-a^{2}c^{6}x^{2}+3\,ac^{6}x\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(11/2),x, algorit
hm="fricas")
```

```
[Out] 1/24*(a^6*x^9 + 3*a^5*x^8 - 8*a^3*x^6 - 6*a^2*x^5 + 6*a*x^4 + 8*x^3)*sqrt(-
a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^11*c^6*x^11 + 3*a^10*c^6*x^10 - a^9*c^
6*x^9 - 11*a^8*c^6*x^8 - 6*a^7*c^6*x^7 + 14*a^6*c^6*x^6 + 14*a^5*c^6*x^5 -
6*a^4*c^6*x^4 - 11*a^3*c^6*x^3 - a^2*c^6*x^2 + 3*a*c^6*x + c^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}x^{2}}{\left(-a^{2}cx^{2}+c\right)^{\frac{11}{2}}(ax+1)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(11/2),x, algorit
hm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^2/((-a^2*c*x^2 + c)^(11/2)*(a*x + 1)^3), x
)
```

3.1378 $\int \frac{e^{-5\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{27/2}}\,dx$

Optimal. Leaf size=60

$$\frac{(5ax+1)\sqrt{1-a^2x^2}}{120a^3c^{13}(1-ax)^{10}(ax+1)^{15}\sqrt{c-a^2cx^2}}$$

```
[Out] ((1 + 5*a*x)*Sqrt[1 - a^2*x^2])/(120*a^3*c^13*(1 - a*x)^10*(1 + a*x)^15*Sqr
t[c - a^2*c*x^2])
```

Rubi [A] time = 0.237762, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 81}

$$\frac{(5ax+1)\sqrt{1-a^2x^2}}{120a^3c^{13}(1-ax)^{10}(ax+1)^{15}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^(5*ArcTanh[a*x])*(c - a^2*c*x^2)^(27/2)),x]
```

```
[Out] ((1 + 5*a*x)*Sqrt[1 - a^2*x^2])/(120*a^3*c^13*(1 - a*x)^10*(1 + a*x)^15*Sqr
t[c - a^2*c*x^2])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\int \frac{e^{-5\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{27/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-5\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{27/2}}\,dx}{c^{13}\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{(1-ax)^{11}(1+ax)^{16}}\,dx}{c^{13}\sqrt{c-a^2cx^2}}$$
$$= \frac{(1+5ax)\sqrt{1-a^2x^2}}{120a^3c^{13}(1-ax)^{10}(1+ax)^{15}\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.466298, size = 59, normalized size = 0.98

$$\frac{(5ax+1)\sqrt{1-a^2x^2}}{120a^3c^{13}(ax-1)^{10}(ax+1)^{15}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^(5*ArcTanh[a*x])*(c - a^2*c*x^2)^(27/2)),x]
```

```
[Out] ((1 + 5*a*x)*Sqrt[1 - a^2*x^2])/(120*a^3*c^13*(-1 + a*x)^10*(1 + a*x)^15*Sq
rt[c - a^2*c*x^2])
```

Maple [A] time = 0.037, size = 49, normalized size = 0.8

$$-\frac{(5\,ax+1)\,(ax-1)}{120\,a^3\,(ax+1)^4}\left(-a^2x^2+1\right)^{\frac{5}{2}}\left(-a^2cx^2+c\right)^{-\frac{27}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)^5*(-a^2*x^2+1)^(5/2)/(-a^2*c*x^2+c)^(27/2),x)
```

```
[Out] -1/120*(a*x-1)*(5*a*x+1)*(-a^2*x^2+1)^(5/2)/(a*x+1)^4/a^3/(-a^2*c*x^2+c)^(2
7/2)
```

Maxima [B] time = 3.73154, size = 369, normalized size = 6.15

$$\frac{}{120\left(a^{28}c^{14}x^{25}+5\,a^{27}c^{14}x^{24}-40\,a^{25}c^{14}x^{22}-50\,a^{24}c^{14}x^{21}+126\,a^{23}c^{14}x^{20}+280\,a^{22}c^{14}x^{19}-160\,a^{21}c^{14}x^{18}-765\,a^{20}c^{14}x^{1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^5*(-a^2*x^2+1)^(5/2)/(-a^2*c*x^2+c)^(27/2),x, algorit
hm="maxima")
```

```
[Out] 1/120*(5*a*sqrt(c)*x + sqrt(c))/(a^28*c^14*x^25 + 5*a^27*c^14*x^24 - 40*a^2
5*c^14*x^22 - 50*a^24*c^14*x^21 + 126*a^23*c^14*x^20 + 280*a^22*c^14*x^19 -
 160*a^21*c^14*x^18 - 765*a^20*c^14*x^17 - 105*a^19*c^14*x^16 + 1248*a^18*c
^14*x^15 + 720*a^17*c^14*x^14 - 1260*a^16*c^14*x^13 - 1260*a^15*c^14*x^12 +
 720*a^14*c^14*x^11 + 1248*a^13*c^14*x^10 - 105*a^12*c^14*x^9 - 765*a^11*c^
14*x^8 - 160*a^10*c^14*x^7 + 280*a^9*c^14*x^6 + 126*a^8*c^14*x^5 - 50*a^7*c
```

```
^14*x^4 - 40*a^6*c^14*x^3 + 5*a^4*c^14*x + a^3*c^14)
```

Fricas [B] time = 3.0069, size = 1226, normalized size = 20.43

$$\frac{\left(a^{22}x^{25} + 5\,a^{21}x^{24} - 40\,a^{19}x^{22} - 50\,a^{18}x^{21} + 126\,a^{17}x^{20} + \right.}{120\left(a^{27}c^{14}x^{27} + 5\,a^{26}c^{14}x^{26} - a^{25}c^{14}x^{25} - 45\,a^{24}c^{14}x^{24} - 50\,a^{23}c^{14}x^{23} + 166\,a^{22}c^{14}x^{22} + 330\,a^{21}c^{14}x^{21} - 286\,a^{20}c^{14}x^{20}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^5*(-a^2*x^2+1)^(5/2)/(-a^2*c*x^2+c)^(27/2),x, algorit
hm="fricas")
```

```
[Out] 1/120*(a^22*x^25 + 5*a^21*x^24 - 40*a^19*x^22 - 50*a^18*x^21 + 126*a^17*x^2
0 + 280*a^16*x^19 - 160*a^15*x^18 - 765*a^14*x^17 - 105*a^13*x^16 + 1248*a^
12*x^15 + 720*a^11*x^14 - 1260*a^10*x^13 - 1260*a^9*x^12 + 720*a^8*x^11 + 1
248*a^7*x^10 - 105*a^6*x^9 - 765*a^5*x^8 - 160*a^4*x^7 + 280*a^3*x^6 + 126*
a^2*x^5 - 50*a*x^4 - 40*x^3)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^27*
c^14*x^27 + 5*a^26*c^14*x^26 - a^25*c^14*x^25 - 45*a^24*c^14*x^24 - 50*a^23
*c^14*x^23 + 166*a^22*c^14*x^22 + 330*a^21*c^14*x^21 - 286*a^20*c^14*x^20 -
 1045*a^19*c^14*x^19 + 55*a^18*c^14*x^18 + 2013*a^17*c^14*x^17 + 825*a^16*c
^14*x^16 - 2508*a^15*c^14*x^15 - 1980*a^14*c^14*x^14 + 1980*a^13*c^14*x^13
+ 2508*a^12*c^14*x^12 - 825*a^11*c^14*x^11 - 2013*a^10*c^14*x^10 - 55*a^9*c
^14*x^9 + 1045*a^8*c^14*x^8 + 286*a^7*c^14*x^7 - 330*a^6*c^14*x^6 - 166*a^5
*c^14*x^5 + 50*a^4*c^14*x^4 + 45*a^3*c^14*x^3 + a^2*c^14*x^2 - 5*a*c^14*x -
 c^14)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)**5*(-a**2*x**2+1)**(5/2)/(-a**2*c*x**2+c)**(27/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 29.0573, size = 664, normalized size = 11.07

$$-\frac{1}{2013265920}\left(\frac{2451570\,\sqrt{c}\left(\frac{2}{ax+1}-1\right)^{9} + 1514205\,\sqrt{c}\left(\frac{2}{ax+1}-1\right)^{8} + 769120\,\sqrt{c}\left(\frac{2}{ax+1}-1\right)^{7} + 318780\,\sqrt{c}\left(\frac{2}{ax+1}-1\right)^{6} + }{a^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^5*(-a^2*x^2+1)^(5/2)/(-a^2*c*x^2+c)^(27/2),x, algorit
hm="giac")
```

```
[Out] -1/2013265920*((2451570*sqrt(c)*(2/(a*x + 1) - 1)^9 + 1514205*sqrt(c)*(2/(a
*x + 1) - 1)^8 + 769120*sqrt(c)*(2/(a*x + 1) - 1)^7 + 318780*sqrt(c)*(2/(a*
x + 1) - 1)^6 + 106260*sqrt(c)*(2/(a*x + 1) - 1)^5 + 27830*sqrt(c)*(2/(a*x
```

```
+ 1) - 1)^4 + 5520*sqrt(c)*(2/(a*x + 1) - 1)^3 + 780*sqrt(c)*(2/(a*x + 1) -
 1)^2 + 70*sqrt(c)*(2/(a*x + 1) - 1) + 3*sqrt(c))/(a^4*c^14*(2/(a*x + 1) -
1)^10) - (2*a^56*c^(393/2)*(2/(a*x + 1) - 1)^15 + 45*a^56*c^(393/2)*(2/(a*x
 + 1) - 1)^14 + 480*a^56*c^(393/2)*(2/(a*x + 1) - 1)^13 + 3220*a^56*c^(393/
2)*(2/(a*x + 1) - 1)^12 + 15180*a^56*c^(393/2)*(2/(a*x + 1) - 1)^11 + 53130
*a^56*c^(393/2)*(2/(a*x + 1) - 1)^10 + 141680*a^56*c^(393/2)*(2/(a*x + 1) -
 1)^9 + 288420*a^56*c^(393/2)*(2/(a*x + 1) - 1)^8 + 432630*a^56*c^(393/2)*(
2/(a*x + 1) - 1)^7 + 408595*a^56*c^(393/2)*(2/(a*x + 1) - 1)^6 - 891480*a^5
6*c^(393/2)*(2/(a*x + 1) - 1)^4 - 2080120*a^56*c^(393/2)*(2/(a*x + 1) - 1)^
3 - 3120180*a^56*c^(393/2)*(2/(a*x + 1) - 1)^2 - 3565920*a^56*c^(393/2)*(2/
(a*x + 1) - 1))/(a^60*c^210))*abs(a)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
             fi;
             #both result and optimal complex
             if leaf_count_result<=2*leaf_count_optimal then
                return "A";
             else
                return "B";
             end if
          else #result contains complex but optimal is not
             if debug then
                print("result contains complex but optimal is not");
             fi;
             return "C";
          end if
       else # result do not contain complex
            # this assumes optimal do not as well
            if debug then
                print("result do not contain complex, this assumes optimal do
    not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
               if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
               fi;
               return "A";
            else
               if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
               fi;
               return "B";
            end if
       end if
  else #ExpnType(result) > ExpnType(optimal)
       if debug then
          print("ExpnType(result) > ExpnType(optimal)");
       fi;
       return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
   (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                    #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```